Minimal HW

Problem 1: Sigma Notation

$S = 5 + 9 + 13 + ... + 89$

$a_1 = 5$ and difference $d = 4$

$a_n = a_1 + d(n-1)$

Let's fit a_1 and d values into equation:

$89 = 5 + 4n - 4$

$88 = 4n \rightarrow n = 22$

Answer: $S = \sum_{k=1}^{22} 5 + 4(k-1)$

Problem 2: Shifting Index in Sigma Notation

$\sum_{k=3}^{15}(2k+1)$

start at $k = 1$ then $\sum_{k=3-2}^{15-2}(2(k+2)+1)$

Answer: $S = \sum_{k=1}^{13} 2k + 5$

Problem 3: Recursive Notation

$a_1 = 12$ and $a_n = a_{n-1} + d$.

if $a_{10} = 57$ then $d = \frac{57-12}{10-1} = 5$

Answer: $a_{25} = 12 + 5(25-1) = 132$

Problem 4: Applied Problem

if we check the first and last multiples of 7 then we can easily find d and the sum.

The first multiple above 100 is 105 where 7×15 since $100 ... 104$ are not multiples of 7.

The last multiple below 1000 is 994 where 7×142 since $1000, 999, 998, 997, 996, 995$ are not multiples of 7.

$n = \frac{a_n - a_1}{d} + 1 \to \frac{994-105}{7} + 1 = 127 + 1 = 128$

Answers: $S = \frac{a_1 + a_n}{2} \times n = \frac{105+994}{2} \times 128 = 1099 \times 64 = 70336$

Problem 5: Sigma notation

$2650 = \sum_{k=1}^{n}(3k+2)$

$S = \frac{a_1+a_n}{2} \times n$ and $a_1 = 3 \times 1 + 2 = 5$, $a_n = 3n+2$

$S = \frac{5+3n+2}{2} \times n \to 5300 = n \times (3n+7)$

$3n^2 + 7n - 5300 = 0 \Rightarrow D = 49 + 4 \times 3 \times 5300 = 63649$

$n = \frac{-7 \pm \sqrt{63649}}{6} \approx 41$

Check if n satisfies the $S = 2650$

$S = \frac{5+3\times41+2}{2} \times 41 \to 41 \times 65 = 2665$ slight larger than S

Check if $n = 40$ satisfies the $S = 2650$

$S = \frac{5+3\times40+2}{2} \times 40 \to 20 \times 127 = 2540$ less than S

Answer: there is no n such that $S = 2650$

Problem 6: Mean Formula

$a_{10} = a_1 + 9d$. Let's find a_1 and d

$a_5 = a_1 + 4d$ and $a_{15} = a_1 + 14d$

$a_{15} - a_5 = 10d \to 60 - 20 = 10d \to d = 4$

Answer: $a_{10} - a_5 = 5d \to a_{10} = 20 + 20 = 40$

Problem 7: Applied Problem

$a_{20} = a_1 + 19d \to a_{20} = 5 + 19 \times 0.5 = 29/2$

Answer: $S = \frac{a_{20}+a_1}{2} \times n \to S = \frac{29/2+5}{2} \times 20 = 10 \times (14.5+5) = 195$

Problem 8: Sum of the First N terms

$a_1 = 11$ and $d = 3$

$1000 = \frac{11+a_n}{2} \times n \to 2000/n = 11 + a_n \to 2000/n = 11 + a1 + 3 \times (n-1)$

$2000/n = 22 + 3n - 3 \to 2000/n = 19 + 3n$

$3n^2 + 19n - 2000 > 0$

Answer: $n = \frac{-19 \pm \sqrt{19^2 - 4 \times 3 \times -2000}}{2 \times 3} \approx \frac{-19+156}{6} \approx 23$

Problem 9: Shifting Index in Sigma Notation

$\sum_{k=3}^{12} 4(\frac{1}{2})^k$ find S starting from $k=0$

$\sum_{k-3}^{12-3} 4(\frac{1}{2})^{k+3} \to \sum_{k=0}^{9} 4(\frac{1}{2})^k(\frac{1}{8})$

Answer:$\sum_{k=0}^{9}(\frac{1}{2})^k(\frac{1}{2})$

Problem 10: N-th Term Formula of geometric sequence

$a_2 = a_1 r$ and $a_5 = a_1 r^4$

$\frac{48}{-6} = r^3 \to r = -2$

Answer: $\frac{a_{10}}{a_2} = r^8 \to a_{10} = -6 * (-2)^8 = -1536$

Problem 11: Finding Common Ratio

Answer: $\frac{a_7}{a_4} = r^3 \to \frac{1458}{54} = r^3 \to r = 3$

Problem 12: Sum of the First N terms

$S_n = a_1 \frac{1-r^n}{1-r}$

Answer: $S_{15} = 8 \times \frac{1-(3/4)^{15}}{1-3/4} \approx 8 \times 4 = 32$

Problem 13: Definitions

$P(x) = x^5 - 4x^3 + x^2 - 7$

Answer: degree is 5 and there are 4 terms which are 1,-4, 1,-7

Problem 14: Adding Polynomials

Simplify $(2x^4 - 3x^3 + x - 5) + (x^3 - 2x^2 + 4x + 7)$

Answer: $2x^4 - 2x^3 + 5x + 2$

Problem 15: Multiplying Polynomials

$(x^2 - x + 2)(x^2 + x + 1)$

$(x(x-1)+2)(x(x+1)+1) \to (x^2 - x) \times (x^2 + x) + x(x-1) + 2x(x+1) + 2$

Answer: $x^4 - x^2 + x^2 - x + 2x^2 + 2x + 2 = x^4 + 2x^2 + x + 2$

Problem 16: GCD and LCM of Monomials

GCD for $24x^3y^2z^5$ and $36x^5y^3z^2$

$2, 2, 2, 3, x^3, y^2, z^2, z^3$ and $2, 2, 3, 3, x^2, x^3, y, y^2, z^2$

$24, 48, 72$ and $36, 72$

Answers: GCD is $12x^3y^2z^2$ and LCM is $72x^5y^3z^5$

Problem 17: Factoring Quadratics

Factor $x^4 - 13x^2 + 36$

If $u = x^2$ then $u^2 - 13u + 36 \to (u-4)(u-9)$

Answer: $(x^2 - 2^2)(x^2 - 3^2) = (x-2)(x+2)(x-3)(x+3)$

Problem 18: Special Binomial Products

Expand $(2x + 3y)^5$

$C_0^5(2x)^5 + C_1^5(2x)^4(3y) + C_2^5(2x)^(3y)^2 + C_3^5(2x^2)(3y)^3 +$
$C_4^5(2x)(3y)^4 + C_5^5(3y)^5$

Answer: $32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$

Problem 19: Polynomial division

$6x^3 + 11x^2 - 31x + 15$ divide by $3x - 2$

divide by $2x^2$ then $6x^3 + 11x^2 - (6x^3 - 4x^2) = 15x^2 - 31x + 1$

divide by $5x$ then $15x^2 - 31x - (15x^2 - 10x) = -21x + 15$

divide by -7 then $-21x + 15 - (-21x + 14) = 1$

Answer: $2x^2 + 5x - 7$ remainder 1.

Problem 20: The remainder theorem

List all possible rational zeros of $f(x) = 2x^4 - 5x^3 + x^2 - 4$

$a_0 = -4$ where $p = \pm 1, \pm 2, \pm 4$

$a_n = 2$ where $q = \pm 1, \pm 2$

Answer: $\frac{p}{q} = \{\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{4}{1}, \pm\frac{4}{2}\} \to \{\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4\}$