Minimal HW

1. **Answer:** There are two cases where Lydia and Marty have two phone numbers, implying that the relation between names and phone numbers is **not a function**. This relation contradicts the statement that each function **input value** leads to a **single output value**.
2. **Answers:**
    a. For $y = x^2 + 1$, let's check if any of the values of x yields exactly one solution for y:
        i. if $x = 1$ then $y = 2$
        ii. if $x = 2$ then $y = 5$
        iii. if $x = 3$ then $y = 10$
        iv. and so on ...
        v. hence it is a function since for any x the equation yields exactly one solution for y.
    b. For $y^2 = x + 1 \to y = \sqrt{x+1}$ let's check if any of the values of x yields exactly one solution for y:
        i. if $x = 1$ then $y = \sqrt{2}$
        ii. if $x = 2$ then $y = \sqrt{3}$
        iii. if $x = 3$ then $y = \sqrt{4}$ ⇒ ± 2 where y yields two solutions so it's not a function.
        iv. if $x = 8$ then $y = \sqrt{9}$ ⇒ ± 3 where y yields two solutions as well hence it's not a function.
3. **Answers:**
    a. $f(n) = 3n$ is **not surjective** since there is no integer n such that $3n = 5$. So 5 is missing from the range of the codomain $3\mathbb{Z}$ which is multiples of 3. Whereas the function **is injective** since if $3x_1 = 3x_2$, dividing both sides by 3 yields $x_1 = x_2$ where all distinct elements of the codomain are mapped to at most one element in the domain.

b. given $g : \{1, 2, 3\} \to \{a, b, c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$ It is **not surjective** since there is no x such that g(x)=b. So, the b value is missing from the range of the codomain. In addition, the function is **not injective** as well because g yields the same result a a for the inputs 2 and 3.

c. given $g : \{1, 2, 3\} \to \{1, 2, 3\}$ defined as $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ **Is a surjective** function since all values in the codomain are in the range of domain. Furthermore, the function is **injective** since all input values are distinct and mapped to all distinct output values once. Hence the function is **bijective**.

4. **Answer:**

   a. Is $g = f^{-1}$, let's solve this for $g(f(x))$:

      i. $g(f(x)) = \frac{1}{\frac{1}{x+2}} - 2$ ⇒ $x + 2 - 2 = x$ so $g = f^{-1}$

5. **Answer:**

   a. $y = 2 + \sqrt{x - 4} \to x = 2 + \sqrt{y - 4}$

   b. solve for y

      i. $x - 2 = \sqrt{y - 4}$

      ii. $x^2 - 4x + 4 = y - 4$

      iii. $f^1(x) = x^2 - 4x + 8$

6. **Answer:**

   a. $C = \frac{5}{9}(F - 32)$

   b. $9C = 5F - 160$

   c. $F = \frac{9C+160}{5} \to \frac{9}{5}C + 32$

7. **Answer:**

   a. $g(x) = 2\sqrt{x - 4}$

   b. domain: $x - 4 \geq 0 \to x \geq 4$ ⇒ $D : [4, \infty+)$

   c. The range of the function:

      i. $x = 4, g(4) = 0$

ii. $x = 5, g(5) = 2$

iii. $x = 6, g(6) = 2\sqrt{2}$

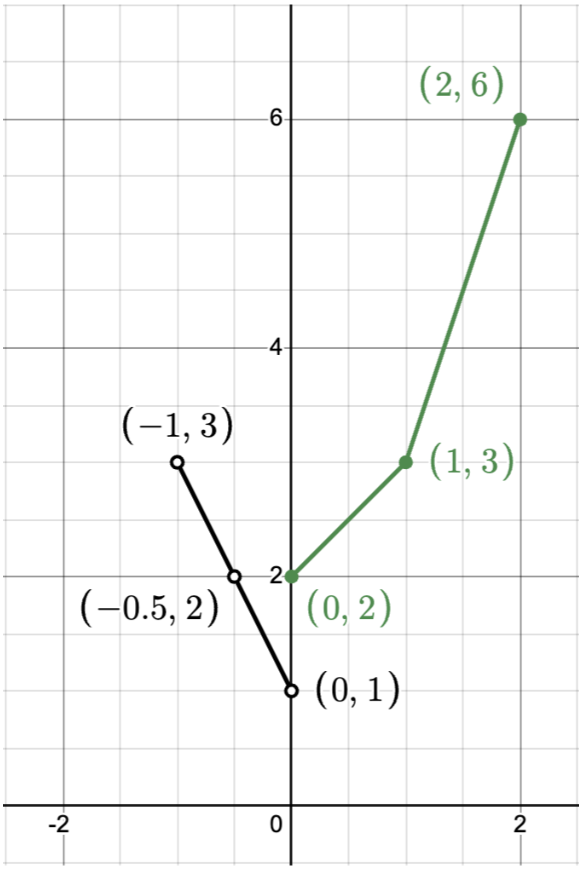
iv. $R : [0, \infty)$

8. **Answer:**
   a. $h(x) = -2x^2 + 4x - 9$
   b. The x value can be any real number hence $D : (-\infty, \infty)$.
   c. Since the function is quadratic let's graph the parabola to identify the range of the parabola we need to find the vertex and either the function is up or down.
9. x - vertex = $\frac{-b}{2a} = \frac{-4}{2 \cdot -2} = 1$
10. y - vertex = $-2 \cdot 1^2 + 4 \cdot 1 - 9 = -7$
11. so the vertex is $(1, -7)$ and quadratic term is **negative**, the parabola **opens down** where the $R : (-\infty, -7]$.
12. **Answer:**
    a. $f(x) = \frac{x-4}{x^2-2x-15}$
    b. The x value can be any real number in the numerator part.
    c. Let's check the x value in the denominator part:
       i. $(x-5)(x+3) = 0$
       ii. $x = 5$ and $x = -3$
    d. $R : (-\infty, -3) \cup (-3, 5) \cup (5, \infty)$
13. **Answer:**
    a. For the given value of x:

$$f(x) = \begin{cases} -2x + 1 & -1 \leq x < 0 \\ x^2 + 2 & 0 \leq x \leq 2 \end{cases}$$

       a. $f(-1) = 3$, $f(-1/2) = 2$, $f(0) = 1$ not in the domain.
       b. $f(0) = 2$, $f(1) = 3$, $f(2) = 6$.

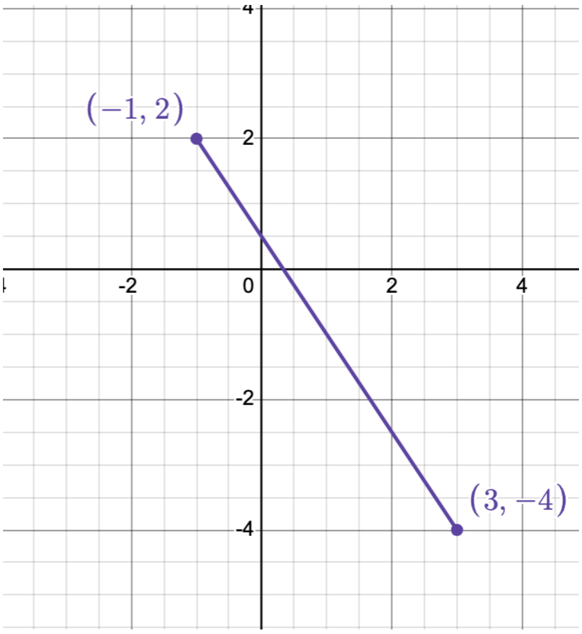
    b. Graph:



14. **Answer:**

    a. find $slope = \frac{y_2 - y_1}{x_2 - x_1}$ for the points $(-1, 2)$ and $(3, -4)$.

    b. $slope = \frac{-4-2}{3+1} = -\frac{3}{2} = -1.5$

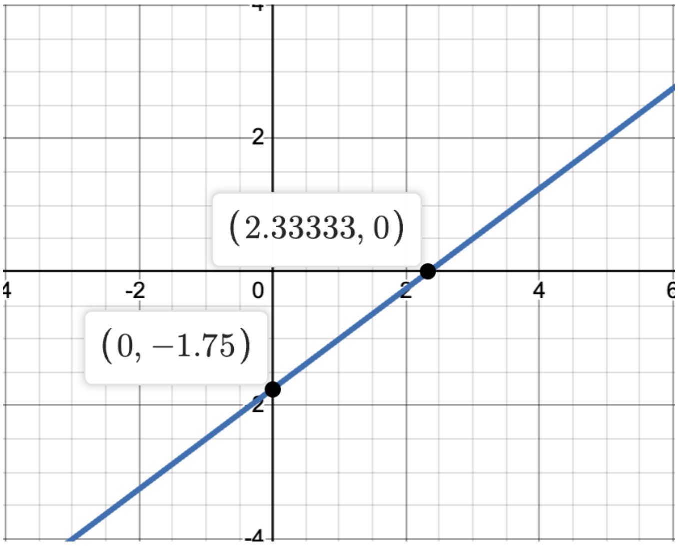


15. **Answer**

a. $y = mx + b$ where $m = \frac{3}{4}$ going through the point $(1, -1)$

b. $b = (-1) - \frac{3}{4} \cdot 1 = -\frac{7}{4}$

c. $y = \frac{3}{4}x - \frac{7}{4}$



16. **Answer**

a. $f(x) = (x-1)^2$

b. The average rate of change in the interval $[-1, 2]$ will be:

$\frac{f(2)-f(-1)}{2-(-1)} = \frac{1-4}{3} = -1$

x	y
-1	4
0	1
1	0
2	1

17. **Answer**

a. $f(x) = x^2 - \frac{1}{x}$

b. The average rate of change on the interval $[2, 4]$ is $\frac{f(4)-f(2)}{4-2} = \frac{\frac{63}{4} - \frac{7}{2}}{2} = \frac{\frac{49}{4}}{2} = \frac{49}{8}$

x	y
2	4 - 1/2=7/2
4	16-1/4=63/4

18. **Answer**
    a. $f(t) = t^2 - t$ and $h(x) = 3x + 2$, evaluate $(f(h(1))$
    b. $h(1) = 5, f(h(1)) = 5^2 - 5 = 20$
19. **Answer**
    a. Find the domain $(f \cdot g)(x)$ where $f(x) = \frac{5}{x-1}$ and $g(x) = \frac{4}{3x-2}$
    b. $f(g(x)) \to \frac{5}{\frac{4}{3x-2}-1} = \frac{15x-10}{-3x+6}$
    c. Let's check the denominator part:
        i. $-3(x-2) = 0 \to x = 2$
        ii. $x \neq 2$ or $x \neq \frac{2}{3}$
        iii. $D : (-\infty, \frac{2}{3}) \cup (\frac{2}{3}, 2) \cup (2, \infty)$
20. **Answer**
    a. $f(x) = x - 1$ and $g(x) = x^2 - 1$
    b. $g(x) - f(x) \to x^2 - x = x(x-1)$
    c. $\frac{g(x)}{f(x)} \to \frac{x^2-1}{x-1} = x + 1$
    d. $f(x)$ and $g(x)$ are not same
21. **Answer**
    a. $y = a\sqrt{x-h} + k$ where the $h(x) = \sqrt{x-1} + 2$
22. **Answer**
    a. $f(x) = \frac{1}{x}$ shifts the function one unit right $\to \frac{1}{x-1}$ and one unit up $\to \frac{1}{x-1} + 1$
23. **Answer**
    a. $f(x) = x^3 + 2x$
    b. $f(-1) = -1 - 2 = -3$ is odd
24. **Answer**
    a. $f(s) = s^4 + 3s^2 + 7$
    b. $f(-1) = 1 + 3 + 7 = 11$ is even
25. **Answer**

a. $m = \frac{6}{3} = 2, b = 7 - 16 = -9$

b. $y = 2x - 9$

26. **Answer**

a. $m = \frac{1-(-2)}{8-3} = \frac{3}{5}$

b. increasing function

27. **Answer**

a. Absolute maxima $\{\, f(-2) = 16, f(2) = 16 \,\}$

b. Absolute minima $\{\, f(3) = -10 \,\}$

28. **Answer**

a. Local maxima $f(1) = 2$

b. Local minima $f(-1) = -2$

29. **Answer**

a. $\begin{cases} f(x) = 2x + 3 \\ j(x) = 2x - 6 \end{cases} \to 2x + 3 = 2x - 6 \to 0 \neq -9$ ⇒ $f(x)$ and $j(x)$ are parallel

b. $\begin{cases} g(x) = \frac{1}{2}x - 4 \\ h(x) = -2x + 2 \end{cases} \to \frac{5}{2}x = 6 \to x = \frac{12}{5}$ ⇒ $f(x)$ and $h(x)$ are perpendicular

30. **Answer**

a. Let's solve for x : $\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases} \to \begin{cases} 4x + 2y = 14 \\ x - 2y = 6 \end{cases} \to 5x = 20 \to x = 4$

b. Then solve for y : $y = 7 - 8 = -1$

31. **Answer**

a. let's solve for x : $\begin{cases} 4x + 2y = 4 \\ 6x - y = 8 \end{cases} \to \begin{cases} 4x + 2y = 4 \\ 12x - 2y = 16 \end{cases} \to 16x = 20 \to x = \frac{5}{4}$

b. Solve for y : $y = \frac{15}{2} - 8 = -\frac{1}{2}$

32. **Answer**
    a. $f(x) = 2x^2 - 6x + 7$
    b. $h = -\frac{b}{2a}, -\frac{-6}{4} = \frac{3}{2}$
    c. $k = f(h) \to f(\frac{3}{2}) = 2\frac{3}{2}^2 - 6\frac{3}{2} + 7 = \frac{5}{2}$
    d. standard form of parabola functions is $2(x - \frac{3}{2})^2 + \frac{5}{2}$.
33. **Answer**
    a. $f(x) = -5x^2 + 9x - 1$
    b. x can be any real value so the domain of the function is $(-\infty, \infty)$.
    c. a is negative, so the parabola opens down and has a maximum.
    d. $h = -\frac{9}{2 \cdot -5} = \frac{9}{10}$
    e. $k = f(h) \to f(\frac{9}{10}) = -5(\frac{9}{10})^2 + 9(\frac{9}{10}) - 1 = -\frac{81}{20} + \frac{81}{10} - 1$
    f. $k = \frac{81}{20} - 1 = \frac{61}{20}$
    g. Range of the function is $(-\infty, \frac{61}{20}]$.
34. **Answer**
    a. $f(x) = 3x^2 + 5x - 2$
    b. Find the y-intercept $f(0) = -2 \to (0, -2)$
    c. Find the x-intercept $(3x_1 - 1)(x_2 + 2) = 0 \to (\frac{1}{3}, 0), (-2, 0)$
    d. $3x - 1 = 0 \to 3x = 1 \to x_1 = \frac{1}{3}$
    e. $x + 2 = 0 \to x_2 = -2$
35. **Answer**
    a. $-1 \leq 2x - 5 < 7$
        i. $4 \leq 2x < 12 \to 2 \leq x < 6$
        ii. The interval is $[2, 6)$.
    b. $x^2 + 7x + 10 < 0 \to (x + 5)(x + 2) < 0$
        i. $x + 5$ is positive and $x + 2$ is negative
            1. $x + 5 > 0 \to x > -5$
            2. $x + 2 < 0 \to x < -2$

ii. $x + 5$ is negative and $x + 2$ is positive

1. $x + 5 < 0 \to x < -5$
2. $x + 2 > 0 \to x > -2$
3. no solution since x can't conform both inequalities.

iii. $(-5, -2)$

c. $-6 < x - 2 < 4$

i. $-4 < x < 6$

ii. The interval is $(-4, 6)$.

36. **Answer**

a. $10 - (2y + 1) \leq -4(3y + 2) - 3 \to -2y + 9 \leq -12y - 11$

b. $10y \leq -20 \to y \leq -2$

c. The interval is $\{\, y \mid y \leq -2 \,\}, (-\infty, -2]$

37. **Answer**

a. $x(x + 3)^2(x - 4) < 0$

i. $x < 0$ and $x - 4 < 0$ is always positive if x is less than 0.

ii. $x - 4 < 0 \to x < 4$.

iii. $(x + 3)^2 < 0$ the domain is always positive.

iv. The range is $(0, 4)$.

38. **Answer**

a. $2x^4 > 3x^3 + 9x^2 \to x^2(2x^2 - 3x - 9) > 0$

i. $x^2 > 0$ is always positive with any value of x

ii. $(2x + 3)(x - 3)$ > 0

1. if $2x + 3 > 0$ and $x - 3 < 0$ then $x > -\frac{3}{2}$ and $x < 3$
2. if $2x + 3 < 0$ and $x - 3 > 0$ then $x < -\frac{3}{2}$ and $x > 3$

iii. The range is $(-\infty, -\frac{3}{2}) \cup (3, \infty)$

39. **Answer**

a. $f(x) = -\frac{1}{2}|4x-5|+3$ determine x - values for which $f(x)$ values are negative.

   i. $-\frac{1}{2}|4x-5|+3<0 \rightarrow |4x-5|>6$

   ii. $4x-5>6 \rightarrow x>\frac{11}{4}$

   iii. $4x-5<-6 \rightarrow x<-\frac{1}{4}$

   iv. $R:(-\infty,-\frac{1}{4})\cup(\frac{11}{4},\infty)$

40. **Answer**

   a. $f(x)=13-2|4x-7|\leq 3 \rightarrow |4x-7|\geq 5$

      i. $4x-7\geq 5 \rightarrow x\geq 3$

      ii. $4x-7\leq -5 \rightarrow x\leq \frac{1}{2}$

      iii. $R:(-\infty,\frac{1}{2}]\cup[3,\infty)$