Minimal HW

Problem 1: Well-known Logarithmic Formulas

$log_2(\frac{8\sqrt{2}}{16}) + log_2(32) - 2log_2(4)$

$log_2(\frac{1\sqrt{2}}{2}) + 5 - 4 \rightarrow log_2(2^{-1}2^{1/2}) + 1$

Answer: $-1/2 + 1 = 1/2$

Problem 2: Well-Known Logarithmic Formulas

$log_3(x-1) + log_3(x+1) = 2$ solve for x

$log_3(x-1)(x+1) = 2 \rightarrow log_3(x^2-1) = 2$

$log_3(x^2-1) = 2 \rightarrow x^2 - 1 = 3^2$

Answer: $x^2 = 9 + 1 \rightarrow x = \sqrt{10}$

Problem 3: Compound Interest Exercises

$A = P(1 + \frac{r}{n})^{nt}$

$20000 \geq 10000(1 + \frac{0.06}{4})^{4t}$

$ln(2) \geq ln(1 + \frac{0.06}{4})^{4t} \rightarrow ln(2) \geq 4t \times ln(1 + \frac{0.06}{4})$

Answer: $0.693 \geq 4t \times 0.0149 \rightarrow t \leq 11.6$ **years**

Problem 4: Radioactive Decay Exercises

$N(t) = N_0 e^{-kt}$

$N(t_{1/2}) = \frac{N_0}{2}$

$\frac{N_0}{2} = N_0 e^{-kt_{1/2}}$

$\frac{1}{2} = e^{-k5} \rightarrow ln(2^{-1}) = ln(e^{-k5})$

Answer: $-ln(2) = -5k \rightarrow k = \frac{ln(2)}{5} \approx 0.138$ per year.

Problem 5: Radioactive Decay Exercises

$N(3) = 70 \rightarrow 70 = 100e^{-3k}$

$0.7 = e^{-3k} \rightarrow ln(0.7) = -3k$

$-0.35 = -3k \rightarrow k \approx 0,12$

Find $N(20) - t?$

$20 = 100 \times e^{-0.12 \times t}$

$ln(0.2) = -0.12 \times t \rightarrow -1.6 = -0.12 \times t$

Answer: $t = 13.41$ hours.

Problem 6: Geometric

Find $\hat{u} = \frac{\overrightarrow{AB}}{|AB|}$ for $A(1,2,3)$ to point $B(4,6,9)$

$\overrightarrow{AB} = \langle 4-1|6-2|9-3\rangle$

$|AB| = \sqrt{(9-3)^2 + (6-2)^2 + (4-1)^2} = \sqrt{36+16+9} = \sqrt{61}$

Answer: $\hat{u} = \langle \frac{3}{\sqrt{61}} | \frac{4}{\sqrt{61}} | \frac{6}{\sqrt{61}} \rangle$

Problem 7: Matrix Form

Answers:

$$\hat{u} = 7\hat{i} - 2\hat{j} + 4\hat{k} \rightarrow \hat{v} = \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix}$$

$$|\hat{u}| = \sqrt{7^2 + (-2)^2 + 4^2} = \sqrt{49+4+16} = \sqrt{69}$$

Problem 8: Adding and Scaling Vectors

$3\overrightarrow{a} - 2\overrightarrow{b} \rightarrow \langle 3\times 2|3\times -1|3\times 3\rangle - \langle 2\times -1|2\times 4|2\times 2\rangle$

Answer: $\langle 8|-11|5\rangle$

Problem 9: Dot product

$cos\theta = \frac{\overrightarrow{p}\cdot\overrightarrow{q}}{|\overrightarrow{p}|\times|\overrightarrow{q}|}$

Answer: $cos\theta = \frac{4-10+18}{\sqrt{1+2^2+3^2}\times\sqrt{4^2+(-5)^2+6^2}} = \frac{12}{\sqrt{14}\sqrt{77}}$

Problem 10: Dot product Application

$cos\theta = 0$ if vectors are orthogonal where $\overrightarrow{p}\cdot\overrightarrow{q}$ is 0 as well.

$cos\theta = \frac{-16-4-64}{\sqrt{4+1+16}\sqrt{64+16+256}} = \frac{-84}{\sqrt{21}\sqrt{336}}$

Answer: the following vectors are not orthogonal.

Problem 11: Adding and Subtracting Matrices

$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix}$

$2A - 3B \rightarrow \begin{bmatrix} 2 \times 2 - 3 \times 4 & -1 \times 2 - 3 \times 5 \\ 2 \times 0 - 3 \times -2 & 3 \times 2 - 1 \times 3 \end{bmatrix}$

Answer: $\begin{bmatrix} -8 & -17 \\ 6 & 3 \end{bmatrix}$

Problem 12: Multiplying Matrices

$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$E = CD \rightarrow \begin{bmatrix} 1 \times 5 + 2 \times 7 & 1 \times 6 + 2 \times 8 \\ 3 \times 5 + 4 \times 7 & 3 \times 6 + 4 \times 8 \end{bmatrix}$

Answer: $E = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

Problem 13: Row Operations

$\begin{cases} x + y + z = 6 \\ 2x - y + 3z = 14 \\ -3x + 2y - 2z = -10 \end{cases}$ use Gaussian elimination to solve the system

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 2 & -1 & 3 & | & 14 \\ -3 & 2 & -2 & | & -10 \end{bmatrix}$$

eliminate x using first row where $r2 = r2 - 2r1$ and $r3 = r3 + 3r1$

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 0 & -3 & 1 & | & 2 \\ 0 & 5 & 1 & | & 8 \end{bmatrix}$$

eliminate y using second row where $r2 = r2 + 3r1$

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 0 & -3 & 1 & | & 20 \\ 0 & 0 & 8/3 & | & 8 \end{bmatrix}$$

Answers:

solve for $z \rightarrow 8/3z = 8 \rightarrow z = 3$

solve for $y \rightarrow -3y + 3 = 20 \rightarrow y = -17/3$

solve for $x \rightarrow x - 17/3 + 3 = 6 \rightarrow x = 26/3$

Problem 14: Reduced Row Echelon Form

$$B = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

First eliminate 1 where $r1 = r1 + r3$

$$B = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Eliminate 3 where $r2 = r2 - 3r3$

$$B = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Eliminate 2 where $r1 = r1 - 2r2$

Answer: $B = \begin{bmatrix} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

Problem 15: Matrix Inverse and RREF Relationship

$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ find A^{-1}

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ where there is a need to transform A into identity matrix

$r1 = r1/2 \rightarrow A = \begin{bmatrix} 1 & 1/2 \\ 5 & 3 \end{bmatrix}$ and $I = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}$

$r2 = r2 - 5r1 \rightarrow A = \begin{bmatrix} 1 & 1/2 \\ 0 & 1/2 \end{bmatrix}$ and $I = \begin{bmatrix} 1/2 & 0 \\ -5/2 & 1 \end{bmatrix}$

$r2 = r2/(1/2) \rightarrow \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1/2 & 0 \\ -5 & 2 \end{bmatrix}$

$r1 = r1 - (1/2)r2 \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1/2 & 0 \\ -5 & 2 \end{bmatrix}$

Answer: $\begin{bmatrix} 1/2 & 0 \\ -5 & 2 \end{bmatrix}$