Minimal HW

Problem 1:

a. $P(\{a, b, \{a, b\}\})$: 3 distinct elements and 2^3 in total.

b. $P(\{\emptyset, a, \{a\}, \{\{a\}\}\})$: 4 distinct elements and 2^4 in total.

c. $P(P(\emptyset))$: 1 distinct element and 2^1 in total.

Problem 2:

A^n when A has m elements and $n > 0$.

There are m^n elements in total.

Problem 3:

a. $P(x) : x^2 < 3$, $\{x \in Z \mid x^2 < 3\} = \{-1, 0, 1\}$

b. $Q(x) : x^2 > x$, $\{x \in Z \mid x^2 > x\} = Z - \{0, 1\}$

c. $R(x) : 2x + 1 = 0$, $\{x \in Z \mid 2x + 1 = 0\} = \emptyset$ since $x = -\frac{1}{2}$ is not in the domain of Z (integers).

Problem 4:

First, find the number of distinct elements in the finite set.

Second, find the number of Power set by the formula 2^n where n is the number of distinct elements.

This can be done recursively by eliminating one element at a time. For instance, there is a finite set: $\{a, b\}$. We reduce 0 elements in the set and keep increasing until the total number of distinct elements. Finally, we get:

recursive step 1: $\{a, b\}$ - since we removed 0 elements.

recursive step 2: $\{a\}$ - since we removed 1 element which is b.

recursive step 3:$\{b\}$ - since we removed 1 element which is a.

recursive step 4: $\emptyset$ - since we removed 2 elements which is a and b.

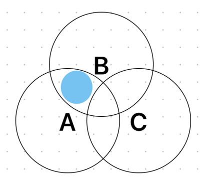
Problem 5:

$$\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$$
$$\overline{(A \cap B) \cap C} = \overline{(A \cap B)} \cup \overline{C} = \overline{A} \cup \overline{B} \cup \overline{C}$$

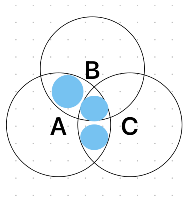
A	B	C	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{A \cap B \cap C}$	$\overline{A} \cup$
0	0	0	1	1	1	0	0
0	0	1	1	1	0	1	1
0	1	0	1	0	1	1	1
0	1	1	1	0	0	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	0	1	1
1	1	0	0	0	1	1	1
1	1	1	0	0	0	1	1

Problem 6:

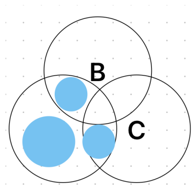
a.



b.



c.



Problem 7:

a. $A \cup B = A$ then $B \subset A$.

b. $A \cap B = A$ then $A \subset B$.

c. $A - B = A$ then $A \cap B$ is $\emptyset$.

d. $A \cap B = B \cap A$ this is always true.

e. $A - B = B - A$ then $A = B$.

Problem 8:

$A \oplus C = B \oplus C$ then $A = B$.

$A \cup C - (A \cap C) = B \cup C - (B \cap C)$.

If $x \in A$ then it implies that $x \in B$ as well.

If $x \in B$ then by definition $x \notin A \oplus C$. That justifies that set A is equal to set B.

Problem 9:

a. $\bigcup_1^\infty A_i = \{-\infty, ..., -1, 0, 1, ..., \infty\} \equiv Z$ and $\bigcap_1^\infty A_i$ is $\{-1, 0, 1\}$.

b. $\bigcup_1^\infty A_i = \{-1, 1, -2, 2, ..., -n, n\} - \{0\} \equiv Z - \{0\}$ and $\bigcap_1^\infty A_i = \emptyset$

c. $\bigcup_1^\infty A_i = \{-\infty, ..., \infty\} \equiv R$ and $\bigcap_1^\infty A_i = \{-1, ..., 1\} \equiv R - (-1, 1)$

d. $\bigcup_1^\infty A_i = \{1, ..., \infty\} \equiv R - (-\infty, 1)$ and $\bigcap_1^\infty A_i = \emptyset$