

Minimal HW

Problem 1:

Given $n = 2^m$

$$O(n) = n \times \log(n) \times (1 + 2 + \dots + n) = n^2 \log(n) = n^2$$

$$\text{Substitute } n \text{ with } 2^m: O(n) = (2^m)^2 \log(2^m) = m4^m = 4^m$$

Problem 2:

$f(n) = \theta(f(n/2))$ is always true or false.

Counterexample: let $f(n) = 2^n$ then $f(n/2) = 2^{n/2} = \sqrt{2^n}$ which is false.

Problem 3:

$$T(n) = aT(n/b) + \theta(n^k \log^p n)$$

- $T(n) = 5T(n/3) + n \log(n) \rightarrow a = 5, b = 3$ where $a > b$ then $T(n) = \theta(n^{\log_3 5})$
- $T(n) = 3T(n/3) + n/\log(n) \rightarrow a = 3, b = 3$ where $a = b$ then $n^{\log_3 3} = n$. $p = -1$ then $T(n) = \theta(n \log \log(n))$
- $T(n) = 8T(n/2) + n^3 \sqrt{n} \rightarrow n^{\log_2 8} + n^{3.5}$ then $T(n) = \theta(n^{3.5})$
- $T(n) = 2T(n/2 - 2) + n/2 \rightarrow n^{\log_2 2} + 1/2n$ then $T(n) = \theta(n)$
- $T(n) = 2T(n/2) + n/\log(n) \rightarrow n^{\log_2 2} + n \log^{-1} n$ then $T(n) = \theta(n \log \log n)$
- $T(n) = T(n/2) + T(n/4) + T(n/8) + n \rightarrow n^{\log_2 1} + n^{\log_4 1} + n^{\log_8 1} + n$ then $T(n) = \theta(3 + n) = \theta(n)$
- $T(n) = T(n-1) + 1/n \rightarrow 1/n$ limits to 0 then $T(n) = T(n-1) = \theta(\log n)$
- $T(n) = T(n-1) + \log n \rightarrow T(n) = \theta(n \log n)$
- $T(n) = T(n-2) + 1/\log n \rightarrow T(n) = \theta(n/\log n)$
- $T(n) = \sqrt{n}T(\sqrt{n}) + n \rightarrow T(n) = \theta(n \log \log n)$