

# Minimal HW

## Problem 1: Well-known Logarithmic Formulas

$$\log_2\left(\frac{8\sqrt{2}}{16}\right) + \log_2(32) - 2\log_2(4)$$

$$\log_2\left(\frac{1\sqrt{2}}{2}\right) + 5 - 4 \rightarrow \log_2(2^{-1}2^{1/2}) + 1$$

$$\text{Answer: } -1/2 + 1 = 1/2$$

## Problem 2: Well-Known Logarithmic Formulas

$$\log_3(x - 1) + \log_3(x + 1) = 2 \text{ solve for } x$$

$$\log_3(x - 1)(x + 1) = 2 \rightarrow \log_3(x^2 - 1) = 2$$

$$\log_3(x^2 - 1) = 2 \rightarrow x^2 - 1 = 3^2$$

$$\text{Answer: } x^2 = 9 + 1 \rightarrow x = \sqrt{10}$$

## Problem 3: Compound Interest Exercises

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$20000 \geq 10000\left(1 + \frac{0.06}{4}\right)^{4t}$$

$$\ln(2) \geq \ln\left(1 + \frac{0.06}{4}\right)^{4t} \rightarrow \ln(2) \geq 4t \times \ln\left(1 + \frac{0.06}{4}\right)$$

$$\text{Answer: } 0.693 \geq 4t \times 0.0149 \rightarrow t \leq 11.6 \text{ years}$$

## Problem 4: Radioactive Decay Exercises

$$N(t) = N_0 e^{-kt}$$

$$N(t_{1/2}) = \frac{N_0}{2}$$

$$\frac{N_0}{2} = N_0 e^{-kt_{1/2}}$$

$$\frac{1}{2} = e^{-k5} \rightarrow \ln(2^{-1}) = \ln(e^{-k5})$$

$$\text{Answer: } -\ln(2) = -5k \rightarrow k = \frac{\ln(2)}{5} \approx 0.138 \text{ per year.}$$

## Problem 5: Radioactive Decay Exercises

$$N(3) = 70 \rightarrow 70 = 100e^{-3k}$$

$$0.7 = e^{-3k} \rightarrow \ln(0.7) = -3k$$

$$-0.35 = -3k \rightarrow k \approx 0.12$$

Find  $N(20) - t$ ?

$$20 = 100 \times e^{-0.12 \times t}$$

$$\ln(0.2) = -0.12 \times t \rightarrow -1.6 = -0.12 \times t$$

**Answer:**  $t = 13.41$  hours.

### **Problem 6: Geometric**

Find  $\hat{u} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$  for  $A(1, 2, 3)$  to point  $B(4, 6, 9)$

$$\overrightarrow{AB} = \langle 4 - 1 | 6 - 2 | 9 - 3 \rangle$$

$$|\overrightarrow{AB}| = \sqrt{(9 - 3)^2 + (6 - 2)^2 + (4 - 1)^2} = \sqrt{36 + 16 + 9} = \sqrt{61}$$

$$\text{Answer: } \hat{u} = \left\langle \frac{3}{\sqrt{61}} \mid \frac{4}{\sqrt{61}} \mid \frac{6}{\sqrt{61}} \right\rangle$$

### **Problem 7: Matrix Form**

$$\hat{u} = 7\hat{i} - 2\hat{j} + 4\hat{k} \rightarrow \hat{v} = \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix}$$

$$|\hat{u}| = \sqrt{7^2 + (-2)^2 + 4^2} = \sqrt{49 + 4 + 16} = \sqrt{69}$$

### **Problem 8: Adding and Scaling Vectors**

$$3\vec{a} - 2\vec{b} \rightarrow \langle 3 \times 2 | 3 \times -1 | 3 \times 3 \rangle - \langle 2 \times -1 | 2 \times 4 | 2 \times 2 \rangle$$

$$\text{Answer: } \langle 8 | -11 | 5 \rangle$$

### **Problem 9: Dot product**

$$\cos\theta = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}| \times |\vec{q}|}$$

$$\text{Answer: } \cos\theta = \frac{4 - 10 + 18}{\sqrt{1 + 2^2 + 3^2} \times \sqrt{4^2 + (-5)^2 + 6^2}} = \frac{12}{\sqrt{14}\sqrt{77}}$$

### **Problem 10: Dot product Application**

$\cos\theta = 0$  if vectors are orthogonal where  $\vec{p} \cdot \vec{q}$  is 0 as well.

$$\cos\theta = \frac{-16 - 4 - 64}{\sqrt{4 + 1 + 16}\sqrt{64 + 16 + 256}} = \frac{-84}{\sqrt{21}\sqrt{336}}$$

Answers: the following vectors are not orthogonal.