Minimal HW

Problem 1:

a. $q = 2, r = 5$

b. $q = -11, r = -1$

c. $q = 34, r = 7$

d. $q = 77, r = 0$

e. $q = 0, r = 0$

f. $q = 0, r = 3$

g. $q = -1, r = 2$

h. $q = 4, r = 0$

Problem 2:

Let k we want to find the smallest absolute value that is congruent to $a \bmod m$ where $m > 0$.

$k \equiv a \bmod m$ where the set of integers for k would be:

$k = \{ -m/2, ..., -1, 0, 1, ..., m/2 \}$ depending on m (even or odd).

Problem 3:

a. $ac \cong bc \pmod{m}$ with $m \geq 2$ but $a \ncong b \pmod{m}$ if $a = 0, b = 2, c = 2, m = 4$ then $0 \cong 4 \pmod{4}$ then $0 \ncong 2 \pmod{4}$.

b. $a \cong b \pmod{m}$ and $c \cong d \pmod{m}$ but $a^c \ncong b^d \pmod{m}$ if $a = 2, b = 1, c = 2, d = 3, m = 4$ then $2 \cong 1 \pmod{4}$ and $2 \cong 3 \pmod{4}$ but $2^2 \ncong 1^3 \pmod{4}$

Problem 4:

$a \cong b \pmod{m}$ then $a^k \cong b^k \pmod{m}$.

We know that $a \cdot a \cong b \cdot b \pmod{m}$ based on the theorem. Then $a^3 \cong b^3 \cong \pmod{m}$. Based on the induction k can be any integer that proves the statement.

Problem 5:

Based on the commutative law the table consist of values $a \leq b$:

0 + 0 = 0	1 + 1 = 2	2 + 3 = 0
0 + 1 = 1	1 + 2 = 3	2 + 4 = 1
0 + 2 = 2	1 + 3 = 4	3 + 3 = 1
0 + 3 = 3	1 + 4 = 0	3 + 4 = 2
0 + 4 = 4	2 + 2 = 4	4 + 4 = 3

0 ∗ 0 = 0	1 ∗ 1 = 1	2 ∗ 3 = 1
0 ∗ 1 = 0	1 ∗ 2 = 2	2 ∗ 4 = 3
0 ∗ 2 = 0	1 ∗ 3 = 3	3 ∗ 3 = 4
0 ∗ 3 = 0	1 ∗ 4 = 4	3 ∗ 4 = 2
0 ∗ 4 = 0	2 ∗ 2 = 4	4 ∗ 4 = 1

Problem 6:

$f(a) = a \div d$ and $g(a) = a \bmod d$ where d is fixed.

For $d = 1 \rightarrow f(a) = a$ and $g(a) = 0$ then $f(a)$ is one-to-one and onto, although $g(a)$ is none of them.

For $d > 1 \rightarrow f(kd) = k$ for any integer k then $f(a)$ is not one-to-one but onto. Furthermore $g(kd) = kd \bmod d$ is not onto because $kd \bmod d$ ranges from 0 to $d - 1$ inclusively. In addition to that $g(a)$ is not one-to-one as well since $g(0) = g(d) = d$.

Problem 7:

a. $231 = 11100111_2$

b. $4532 = 1000110110100_2$

c. $97644 = 10111110101101100_2$

Problem 8:

a. $11111_2 = 2^4 + 2^3 + 2^2 + 2 + 1 = 31$

b. $1000000001_2 = 2^9 + 2^0 = 513$

c. $101010101_2 = 2^8 + 2^6 + 2^4 + 2^2 + 2^0 = 341$

d. $110100100010000_2 = 2^{14} + 2^{13} + 2^{11} + 2^8 + 2^4$

Problem 9:

a. $1000111_2 + 1110111_2 = 10111110_2$

b. $11101111_2 + 10111101_2 = 110101100$

c. $1010101010 + 111110000 = 10010011010$

d. $1000000001 + 1111111111 = 11000000000$

Problem 10:

1. $88 = 2^3 \cdot 11$
2. $126 = 2 \cdot 3^2 \cdot 7$
3. $729 = 3^6$
4. $1001 = 7 \cdot 11 \cdot 13$
5. $1111 = 11 \cdot 101$
6. $90909 = 3^2 \cdot 7 \cdot 11 \cdot 13 \cdot 101$

Problem 11:

The number of 0 at the end of $100!$ can be determined by how many times we divide this number by 10.

10 can be factorized as $2 \cdot 5$.

Let's find out the $5s$ since they are much less than 2s.

Since $100! = 100 \cdot 99 \cdot ... \cdot 50 \cdot ... \cdot 20 \cdot ... \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$100/5 + 100/5^2 + 100/5^3 + ... = 20 + 4 + 0 + ... + 0 = 24.$

Problem 12:

Euclidean algorithm is $a = bq + r$ then $gcd(a, b) = gcd(b, r) \rightarrow$
$gcd(a, b) = gcd(b, a \bmod b)$

a. $gcd(12, 18) = gcd(12, 6) = 6$

b. $gcd(111, 201) = gcd(111, 90) = gcd(90, 21) = gcd(21, 6) = gcd(6, 3) = gcd(3, 0) = 3$

c. $gcd(1001, 1331) = gcd(1001, 330) = gcd(330, 11) = gcd(11, 0) = 11$

d. $gcd(12345, 54321) = gcd(12345, 4941) = gcd(4941, 2463) = gcd(2463, 15) = gcd(15, 3) = gcd(3, 0) = 3$

e. $gcd(1000, 5040) = gcd(1000, 40) = gcd(40, 0) = 40$

f. $gcd(9888, 6060) = gcd(6060, 3828) = gcd(3828, 2232) = gcd(2232, 1596) = gcd(1596, 636) = gcd(636, 324) = gcd(324, 312) = gcd(312, 12) = (12, 0) = 12$