Minimal HW

Problem 1:

$shirts = \{ S_1, S_2, S_3 \}$ and $pants = \{ P_1, P_2, P_3, P_4 \}$

Cartesian product $shirts \times pants = \{ (S_1, P_1), ..., (S_3, P_4) \} = 12$ in total.

Problem 2:

7 characters for license plate: $10 \times 25 \times 35^5$ combinations.

Problem 3:

a. $25 \times 24 \times 23 \times 22$

b. 25^4

Problem 4:

5 letters in the alphabet. Each word has no more than 3 letters.

With 3 letters: $5 \cdot 5 \cdot 5 = 125$

With 2 letters: $5 \cdot 5 = 25$

With 1 letter: 5

In total: $125 + 25 + 5 = 155$ words.

Problem 5:

4 sons and 3 daughters.

At least 2 sons should seat next to each other.

There are many ways of how at least 2 sons can sit next to each other.

But let's think of how many ways we have when there is no such cases:

$BGBGBGB$ where sons are not sitting next to each other.

So there are $4! \cdot 3! = 24 \cdot 6 = 144$ combinations.

Since we are looking for those sons who sit next to each other:

In total we have $7! - 144$ combinations.

Problem 6:

There are 1-5 integers means 5 choices for each digit and 7 digits. In total there are 5^7 sequences.

No two adjacent digits means we need to exclude the cases such as $11234, 111211$ or any case with duplicated digits.

First integer might be chosen 5 times and next integer 4 times. The third integer may include the first integer again so we can chose it 4 times as well and so on. So in total we have:

$5 \cdot 4^4 = 5 \cdot 256 = 20480$ combinations.

Problem 7:

We can choose Ali and Brenda: $20 \cdot 19 \cdot 18$ ways.

In case if Ali is officer: 3 ways.

There are 2 cases left for Brenda and other 18 ways for member.

In total we have $20 \cdot 19 \cdot 18 - 18 \cdot 3 \cdot 2$.

Problem 8:

4 choose $2 = \frac{4!}{2!2!} = 6$.

Problem 9:

There are $n(n-1)/2$ pairs of vertices of edges. We need to exclude n pairs since they do not exist.

Problem 10:

$C_6^{48} = \frac{48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 \cdot 43}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 47 \cdot 46 \cdot 43 \cdot 22 \cdot 9 \cdot 4$

Problem 11:

There are 12 players. 5 players in a team. C_5^{12} total number of combinations.

Two of them refuse to play together. C_3^{10} number of combinations with those two players in a team.

In total we have: $C_5^{12} - C_3^{10}$.

Problem 12:

$(x + 2y^2)^6$ and y^8 in it.

$(2y^2)^4 = 16y^8$

From the total of 6 terms we need choose 4 of $2y^2$s.

There are C_4^6 combinations in total.