Project Description:

Project Name

submitted by
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Specific academic field (specialization): Theoretical Physics/Space Science

Supervisor and first expert reviewer for recommendation letters: Prof. Dr. Frank Spahn

Further expert reviewers (name, academic field; these do not have to be identical with the second reviewer of the PhD thesis): Prof. Dr. Jürgen Schmidt

Working environment: The planned studies will be carried place in the *theoretical planetology* group of Prof. Dr. Frank Spahn of the *Dpt. of theoretical physics*, chair Prof. Dr. Ralf Metzler, where room, working place/space and computer environment will be provided (see signed affirmation attached).

Abstract

Our work concerning cosmic granular gases (planetary rings) has been greatly inspired by the stunning results of the unique *Cassini*-mission to the giant-planet Saturn[1]. The occultation- (UVIS) and the imaging (ISS) experiments have spotted quite unexpected *fine-scale* structures during the *grande finale* phase of that gorgeous mission – clustering structures which occur on the scale of the ring-aggregate sizes as an expression of the non-equilibrium character of the huge granular gas disks.

We plan to explain these observations with generalized kinetic Boltzmann-type equations addressing aggregation, fragmentation and restitution (Spahn et al., 2004) as collisional results. The latter are quantified by visco-elastic adhering collision dynamics of granular icy aggregates (Brilliantov et al., 2007).

In a first step, the evolution of a homogeneous, unperturbed ring composed of aggregates with different masses (k index) is analyzed with the zeroth'- and second order mean field-equations, i.e. the particle number density $n_k(\vec{r}, t)$ and corresponding granular temperatures $T_k(\vec{r}, t)$. For a given size(mass) distribution (DF) $n_k \propto k^{-\alpha}$, we have found mass (k) dependent temperatures T_k as a stationary expression for the non-equilibrium character of the sheared granular gas a planetary ring is made of.

In order to investigate cluster formation under external excitations, the *stability* of the steady state solutions will be analyzed against gravitational (resonant) perturbations, i.e. we will aim at responses of the size distribution n_k vs granular temperatures T_k under external driving.

Furthermore, the generalized kinetic model allows to study the momentum and energy transports between different mass classes of aggregates relevant for cluster processes and the establishment of vertical stratification. The different mobilities corresponding to ring aggregates of different masses may cause size(mass) segregation processes in a perturbed multi-disperse ring. This may lead to a separation of regions containing larger aggregates from those composed of rather small grains – effects which are potentially suitable to describe small scale structural phenomena discovered by *Cassini*.

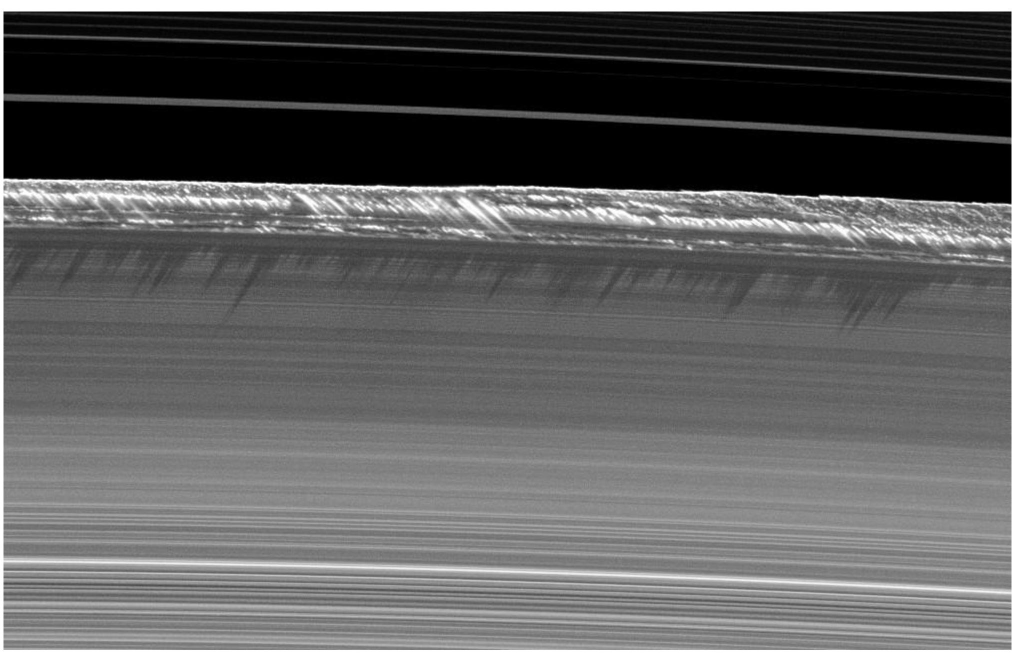


Figure 1: This Cassini-image taken in August 2009 (Saturn's equinox) shows the outer edge (bright fuzzy horizontal region in the upper half of the picture) of Saturn's dense B ring. The direction from the bottom to the top points radially away from Saturn. This edge – confined by the 2:1 mean motion resonance of the moon Mimas, the strongest inner resonance acting in the main rings of Saturn – is dominated by inclined, sheared bright features. They cast long shadows onto the ring plane indicating a stunning vertical extent of a few kilometers of these remarkable features (Image credit: NASA-JPL).

[1] Frank Spahn is a member (CoI) of the *Cassini*-CDA (cosmic dust analyzer) *Science* team since 1995.

A. Summary of the research topic

Introduction & State of "the Art"

Saturns rings do not only fascinate the observer by their miraculous beauty, but they are also natural "laboratories" or even proxies for the dynamics of all the cosmic disks of quite different size and distance from us. These larger "brothers" of planetary rings – e.g. pre-planetary gas-dust disks as the "nurseries" of planets, the huge galactic disks or spirals – have many physical processes in common with dense planetary rings as for instance: differential rotation or quite a low ratio between vertical and lateral extent. The advantage of those cosmic disks in our cosmic neighborhood is that we can inspect these objects in situ with space-vehicles.

During its 13 years-journey through the Saturnian system, the *Cassini*-spacecraft has detected a wealth of new structures and phenomena in Saturn's dense, icy granular rings. For instance, there have been a confirmation of "viscous overstabilities" (Thomson et al., 2007) or "propeller" structures caused by "skyscraper"-sized tiny ring-moons (dubbed as moonlets; Tiscareno et al., 2006) – both formerly theoretically predicted in our group (Schmidt et al., 2001; Spahn and Sremčević, 2000).
Density- and bending waves – driven by the gravity action of the numerous Kronian satellites or structural inhomogeneities inside Saturn and first spotted by the *Voyager*-cameras – have now been caught with the camera-system (Imaging Sub-System - ISS) of the *Cassini*-spacecraft at an unprecedented spatial resolution (French et al., 2016; Tiscareno and Harries, 2018). These spiral waves are dominated by inertia forces – acting in a differentially rotating Keplerian ring – balanced by the self-gravity of the ring-matter providing a possibility to estimate the local mass density of the ring. Apart from those already expected ring-phenomena, new features have

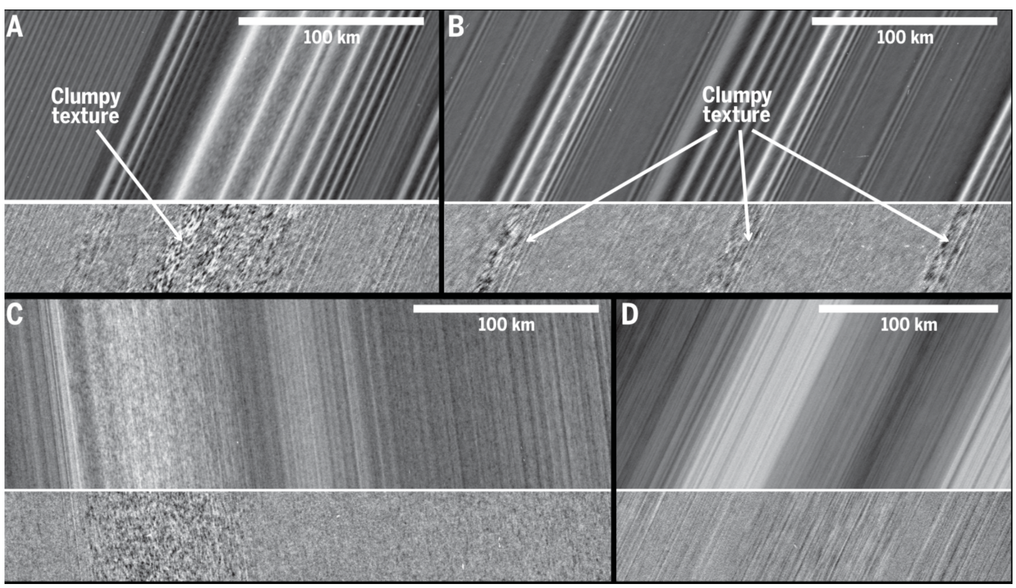


Figure 2: In resonantly driven density waves in Saturn's rings (upper panels A & B) a clustering texture mainly appears in the wave troughs, characterized by low granular temperatures usually corresponding to larger mean sizes (Goldhirsch and Zanetti, 1993; Esposito et al., 2012) (Image from Tiscareno et al. (2019)).

been caught by *Cassini's* cameras (ISS-data) as presented in Figure 1. Here the B ring edge – confined by the strongest resonance in the rings: the 2:1 inner Mimas Lindblad resonance – is shown with its tilted *fine-structures* in the grazing Sun-light during the sunset at the rings (equinox in August 2009). Under these illumination conditions all outstanding vertical features cast shadows whose lengths provide a measure for the vertical extent of these structures. Those visible in Figure 1 suggest a 3.5 kilometre altitude of vertical ring-matter excursions (Spitale and Porco, 2010). Having a closer look at this image, it seems that the shadows are closely tied to the inclined bright streaky features/clusters suggesting their quite large vertical reach.

Further examples of unexpected material clustering are shown in Figure 2 which occur in regions of resonantly driven waves in Saturns rings.

All these observations indicate that larger clusters preferably evolve in "colder", unperturbed regions where the collision frequency and -intensity is reduced. Esposito et al. (2012) have described these facts with a two-dimensional, scale-based kinetic model quantifying evolution of two state variables: the *mean effective mass* $\approx \langle k \rangle$ along with the *mean velocity dispersion* $\approx \langle T_k \rangle$ (mean granular temperature), which covers astonishingly well the *Cassini*-observations[2].

Based on a generalized kinetic theory (Spahn et al., 2004; Spahn et al., 2014) we have calculated the size-spectrum n_k under near-equilibrium conditions (Brilliantov et al., 2015), i.e. assuming energy equipartition of all mass-classes $T_k \to T$. The result is shown in Figure 3 attesting an astonishingly well agreement with spacecraft measurements.

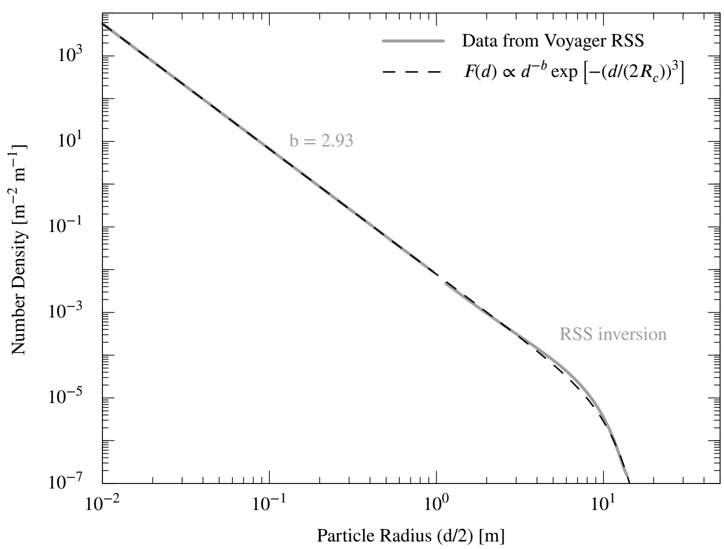


Figure 3: The particle size distribution obtained by Brilliantov et al. (2015) compared to the Voyager radio-science (RSS) data. A quite good agreement between theory and observation has been found.

However, even these obvious successes of the above models do not hide the fact that neither a two-dimensional reduction (Esposito et al., 2012) nor the assumption energy-equipartition (Spahn et al., 2004; Brilliantov et al., 2015) are sufficient to explain the non-equilibrium character of the fine-scale structures observed in Saturn's rings. Consequently, we have started to investigate the coupled dynamics of the granular temperatures $\{T_k\}$ with the mass-frequencies $\{n_k\}$ of an undisturbed dense ring[3]. A splitting up of an universal temperature into a mass-dependent spectrum $T \to \{T_k\}$ has been found as an expression of the non-equilibrium character of the undisturbed multi-disperse sheared granular gas!

In the planned project, we will analyze the stability of that steady state under external perturbations in order to understand the cluster formation, and thus, to complete the goals of my Ph.D.-thesis.

Scientific Objectives

Unlike molecular gases, a granular gas is always subject to collisional dissipation of inner energy, hence temperature decay over time (Haff et al., 1983; Brilliantov and Pöschel, 2004). Even more interesting phenomena can be observed when granular gas consists of species with different masses. In this case, due to non-equal partitioning of energy, each species attain unique granular temperatures (Garzó et al., 2007; Osinsky et al., 2020).

The model we consider is a granular gas with polydisperse size-distribution of constituents in a sheared environment. We start our kinetic description of such a medium with an introduction of a one-particle DFs $f_k(\vec{r}, \vec{v}, t)$

[2] The mass-index k labels the number of constituents forming a ring-aggregate of mass $m = km_0$

[3] The curly brackets denote the $\mathcal{N}$-tuples of the mass-classes.

for each species k. Time evolution of such functions obey Boltzmann-type kinetic equations (Haff et al., 1983; Brilliantov and Pöschel, 2004):

$$\frac{\partial f_k}{\partial t} + \vec{v} \cdot \frac{\partial f_k}{\partial \vec{r}} - \frac{1}{m_k} \frac{\partial U}{\partial \vec{r}} \cdot \frac{\partial f_k}{\partial \vec{v}} = \sum_i n_i I_c(f_k, f_i) \; , \tag{1}$$

where $U(r)$ is the gravitational potential of the planet and $I_c(f_k, f_i)$ is the collision integral. Using these DFs we can introduce macroscopic fields as corresponding velocity moments of DFs:

$$\begin{aligned} \rho_k(\vec{r}, t) &= m_k \int f_k(\vec{r}, \vec{v}, t)\, d\vec{v} \; , \\ \rho_k(\vec{r}, t)\vec{u}(\vec{r}, t) &= \int \vec{v} f_k(\vec{r}, \vec{v}, t)\, d\vec{v} \; , \\ T_k(\vec{r}, t) &= \frac{2}{3} \int \frac{m_k c^2}{2} f_k(\vec{r}, \vec{v}, t)\, d\vec{v} \; , \end{aligned} \tag{2}$$

where $\rho_k = m_k n_k$ is the mass density, $\vec{u}$ is the mean velocity and $\vec{c} = \vec{v} - \vec{u}$ is the peculiar velocity. Multiplying Eq. (1) by necessary power of velocity and integrating over the whole velocity space, we obtain corresponding mean field balance equations. In a generic form, for any given macroscopic field $F_k(\vec{r}, t)$, these balance equations are in the next form:

$$\left\langle \frac{dF_k}{dt} \right\rangle_{convective} = \left\langle \frac{dF_k}{dt} \right\rangle_{collisional} \; , \tag{3}$$

meaning that convective changes of the field are balanced by the collisional changes. Angular brackets denote an average over the velocity space. In our first step, we consider only restitutive collisions, meaning that size-distribution function does not change. In this case, due to mass and momentum conservation, the zeroth and first order moment equations have no collisional changes. The second moment equations, i.e. temperature evolution equations, give us much more interesting information, since they have non-zero collisional changes. In a generalized form, these equations can be written as:

$$\frac{dT_k}{dt} \propto H_k - A_k T_k + \sum_i B_{ki}(T_k - T_i) \; , \tag{4}$$

where $A_k > 0$ is the parameter describing dissipation due to collisions, which depends on size distribution function, collision frequency and restitution. The parameter $B_{ki} = B_{ik} > 0$ describes the inter-species heat flow. This is the main reason why species tend to have different temperatures. H_k is a certain external heating function. If one introduces an external energy pump into the system, the temperatures of species don't reach zero, but attain certain stationary and still unique values, balancing the outer heating (Bodrova et al., 2014).

One caveat of these heating models is that they are all artificial. Our goal is to investigate the model with a more realistic heating term, by including an appropriate potential $U(r)$ in Eq. (1). Such a heating is known to be in the next form:

$$H_k \propto \nu_k \Omega^2 \; , \tag{5}$$

where Ω is the mean orbital frequency around the considered location of the system, and ν_k is the shear viscosity term. In the case of planetary rings, the viscosity term is split into *local* and *non-local* parts $\nu = \nu_l + \nu_{nl}$ (Seiß and Spahn, 2011; Spahn and Schmidt, 2006; Stewart et al., 1984). However, these terms are given only for the mono-disperse case. In the case of a gas with different species, we need to know the viscosity terms for each species. This is the first goal of our project, to obtain the kinetic transport coefficients for each species from the microscopic level of description.

In order to test the theoretical results, we have developed a molecular dynamics (MD) code, simulating granular particles of different sizes in a Hill's box. *Here some* $\ln T$ *over* $\ln t$ *simulation results graph Fig. 4*

These temperature separation effects are most interesting to investigate if we allow the size-distribution to change due to collisions, e.g. including coagulation and fragmentation processes. This is done by introduction of specific coagulation and fragmentation kernels into the collision integrals. The main difference here is that

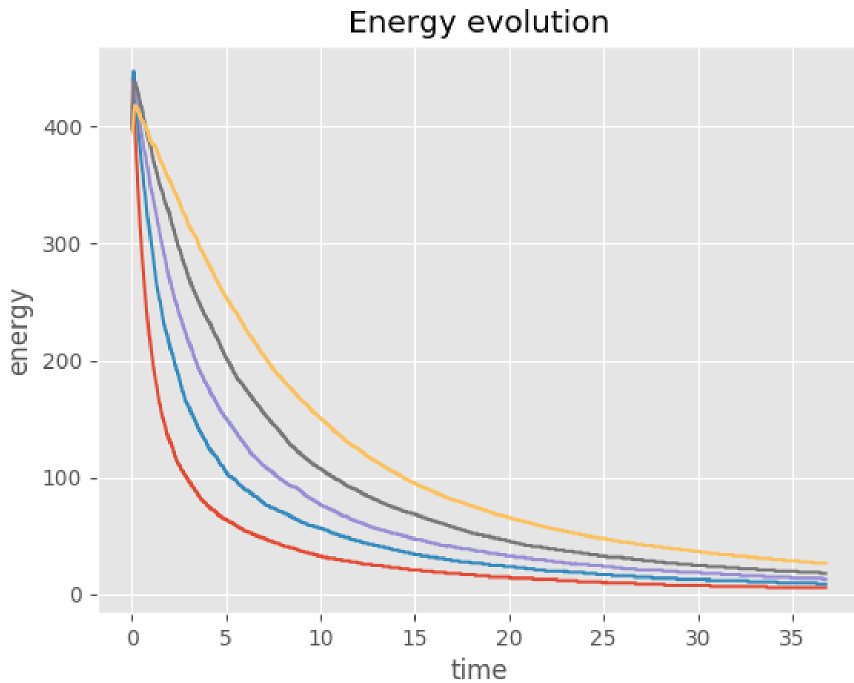


Figure 4: This is only an example plot. Should be replaced by my new simulation results

together with the temperature evolution equations, we have to consider the size-distribution evolution equations, since their collisional part (Eq. 3) does not vanish anymore. In a generic form we have to analyze a system of equations of the next form:

$$\begin{aligned} \left\langle \frac{dn_k}{Dt} \right\rangle_{convective} &= \left\langle \frac{dn_k}{dt} \right\rangle_{gain} - \left\langle \frac{dn_k}{dt} \right\rangle_{loss} , \\ \left\langle \frac{dT_k}{Dt} \right\rangle_{convective} &= \left\langle \frac{dT_k}{dt} \right\rangle_{gain} - \left\langle \frac{dT_k}{dt} \right\rangle_{loss} , \end{aligned} \tag{6}$$

where we specifically separated the collisional gain and loss terms. The gain term in n_k is the contribution of coagulation processes, and loss term in n_k is the contribution of fragmentation processes. Purely restitutive collisions do not contribute into the change of size-DF. In the second set of equations, the natural dissipation of granular gases obviously contribute to the loss term in T_k. However, this time we need to consider change of temperatures due to change in concentration of particles. Mainly, the change of concentration affects the collision frequencies of constituents, which in tern leads to faster/slower temperature decay. On the other hand, the separation of temperatures affects the mobilities of particles, which in tern lead to larger/smaller aggregates. In our previous work (Baibolatov and Spahn, 2012), we have estimated the most probable size of an aggregate for given temperatures of a granular gas, showing that different temperatures lead to different sizes of aggregates.

As a next step, we are planning to analyze the system Eq. (6) under influence of expternal periodic perturbations from moon resonances. This effect can be described as a contribution to the gain term of T_k of a cyclic heating profile.

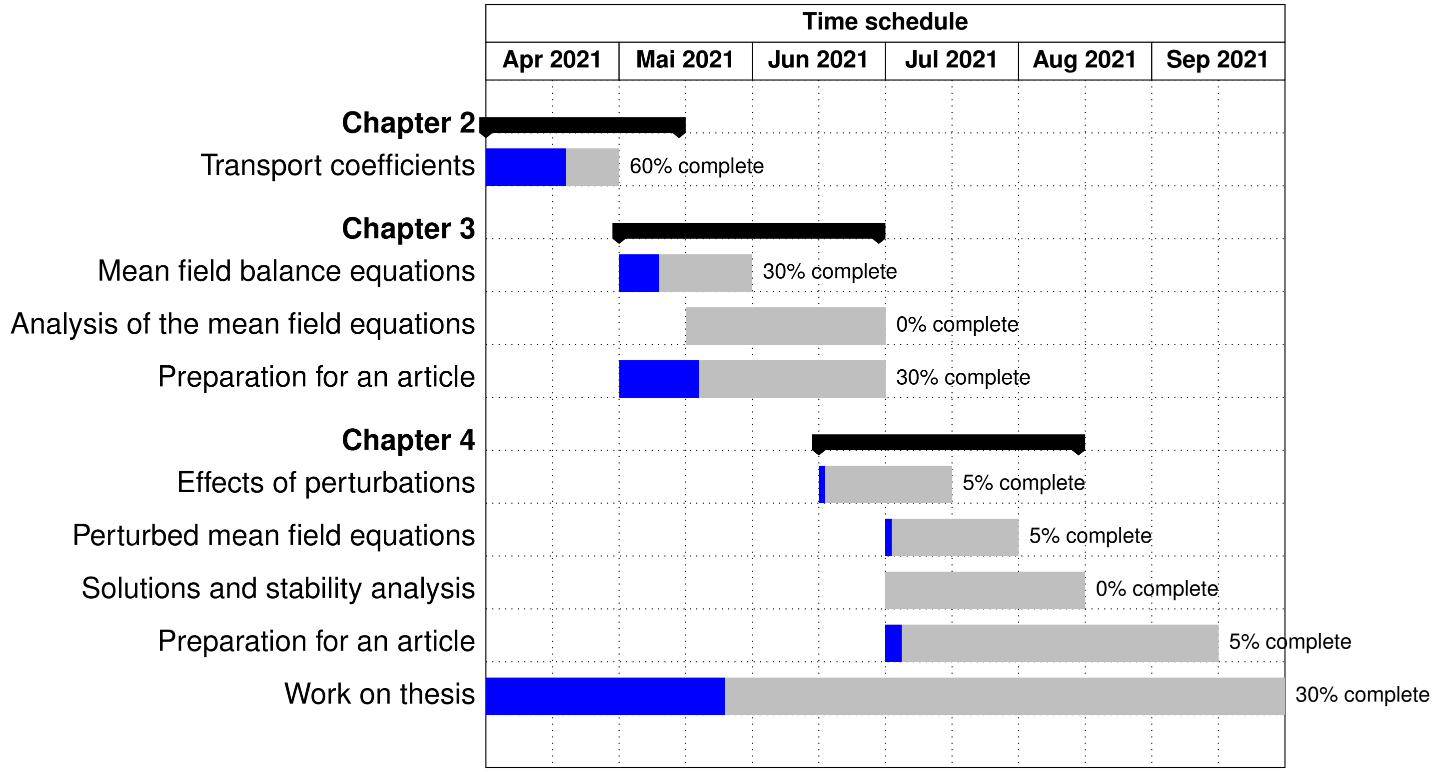
A little more meat about external kicks and stability analysis

B.Table of contents and outline of the thesis with an overview of accomplished (sub-) chapter

Table of contents of the PhD thesis (subject to minor changes):

The second chapter of the thesis, is almost complete. The third chapter is complete up to the point where mean field balance equations are derived and further analyzed. Chapter four is planned to be accomplished during the period of the project.

C. Working program and intended completion date



In the chart above, the planned work on thesis and articles is shown. Obvioulsly, the completion percentages of each task are only rough estimates, however they seem to be realistic. As shown in the chart, the planned completion time is at September 2021, however I kindly ask the commitee to take into account the global COVID-19 pandemic events, and consider the shift of the project by few months. The main reason is that it might be problematic to get my residence permit in Germany during the time-period shown in the timeline of the project.

Our work is purely theoretical, and we do not plan to perform any experiments. The main *validity check experiments* we do, are numerical simulations using molecular dynamics methods.

Something about our planned approach to the problems

- Working-Program, time schedule (max. 4 pages)Detailed information about your planned approach, especially a thorough explanation of the meth-odology, that you will apply in the completion phase of your PhD. A time schedule (e. g. in graphic or tabular form) for the period of funding should clearly demonstrate the steps in your research project that are planned, have already started or are completed. If your PhD includes experiments, please indicate the experiments that have already been conducted and that will still be conducted. The quality of the research approach and the characterization of the accomplished and planned steps are of utmost importance.

- Please specify your intended completion date.

D. Literature

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