Project Description:

Project Name

submitted by
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Specific academic field (specialization): Theoretical Physics/Space Science

Supervisor and first expert reviewer for recommendation letters: Prof. Dr. Frank Spahn

Further expert reviewers (name, academic field; these do not have to be identical with the second reviewer of the PhD thesis): Prof. Dr. Jürgen Schmidt

Working environment: The planned studies will be carried place in the *theoretical planetology* group of Prof. Dr. Frank Spahn of the *Dpt. of theoretical physics*, chair Prof. Dr. Ralf Metzler, where room, working place/space and computer environment will be provided (see signed affirmation attached).

Abstract

Our work concerning cosmic granular gases (planetary rings) has been greatly inspired by the stunning results of the unique *Cassini*-mission to the giant-planet Saturn[1]. The occultation- (UVIS) and the imaging (ISS) experiments have spotted quite unexpected *fine-scale* structures during the *grande finale* phase of that gorgeous mission – clustering structures which occur on the scale of the ring-aggregate sizes as an expression of the non-equilibrium character of the huge granular gas disks.

We plan to explain these observations with generalized kinetic Boltzmann-type equations addressing aggregation, fragmentation and restitution (Spahn et al., 2004) as collisional results. The latter are quantified by visco-elastic adhering collision dynamics of granular icy aggregates (Brilliantov et al., 2007).

In a first step, the evolution of a homogeneous, unperturbed ring composed of aggregates with different masses (k index) is analyzed with the zeroth'- and second order mean field-equations, i.e. the particle number density $n_k(\vec{r}, t)$ and corresponding granular temperatures $T_k(\vec{r}, t)$. For a given size(mass) distribution (DF) $n_k \propto k^{-\alpha}$, we have found mass (k) dependent temperatures T_k as a stationary expression for the non-equilibrium character of the sheared granular gas a planetary ring is made of.

In order to investigate cluster formation under external excitations, the *stability* of the steady state solutions will be analyzed against gravitational (resonant) perturbations, i.e. we will aim at responses of the size distribution n_k vs granular temperatures T_k under external driving.

Furthermore, the generalized kinetic model allows to study the momentum and energy transports between different mass classes of aggregates relevant for cluster processes and the establishment of vertical stratification. The different mobilities corresponding to ring aggregates of different masses may cause size(mass) segregation processes in a perturbed multi-disperse ring. This may lead to a separation of regions containing larger aggregates from those composed of rather small grains – effects which are potentially suitable to describe small scale structural phenomena discovered by *Cassini*.

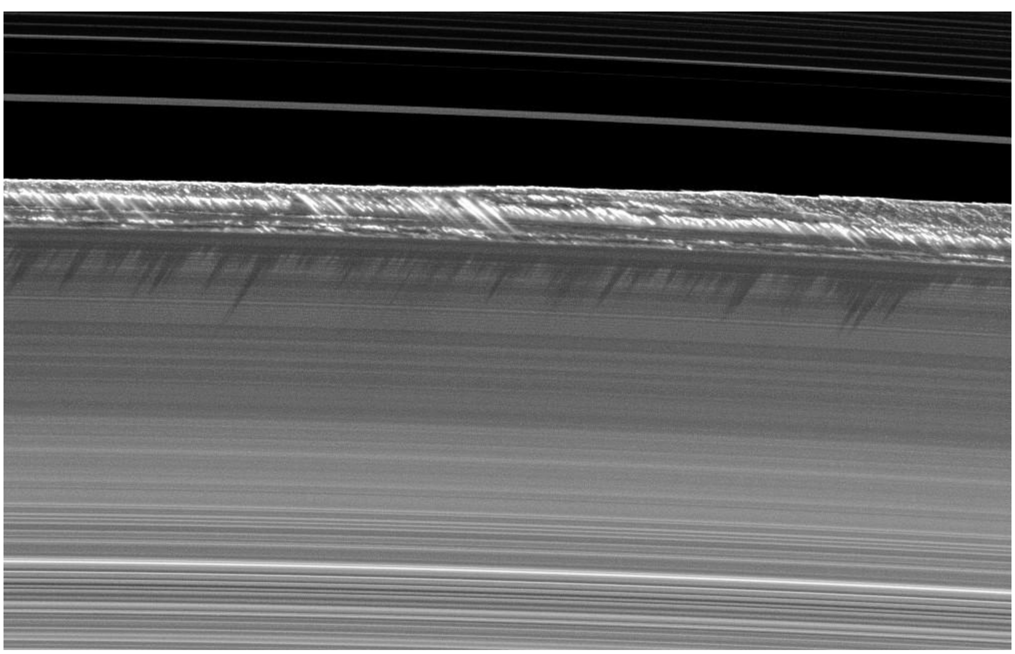


Figure 1: This Cassini-image taken in August 2009 (Saturn's equinox) shows the outer edge (bright fuzzy horizontal region in the upper half of the picture) of Saturn's dense B ring. The direction from the bottom to the top points radially away from Saturn. This edge – confined by the 2:1 mean motion resonance of the moon Mimas, the strongest inner resonance acting in the main rings of Saturn – is dominated by inclined, sheared bright features. They cast long shadows onto the ring plane indicating a stunning vertical extent of a few kilometers of these remarkable features (Image credit: NASA-JPL).

[1] Frank Spahn is a member (CoI) of the *Cassini*-CDA (cosmic dust analyzer) *Science* team since 1995.

A. Summary of the research topic

Introduction & State of "the Art"

Saturns rings do not only fascinate the observer by their miraculous beauty, but they are also natural "laboratories" or even proxies for the dynamics of all the cosmic disks of quite different size and distance from us. These larger "brothers" of planetary rings – e.g. pre-planetary gas-dust disks as the "nurseries" of planets, the huge galactic disks or spirals – have many physical processes in common with dense planetary rings as for instance: differential rotation or quite a low ratio between vertical and lateral extent. The advantage of those cosmic disks in our cosmic neighborhood is that we can inspect these objects in situ with space-vehicles.

During its 13 years-journey through the Saturnian system, the *Cassini*-spacecraft has detected a wealth of new structures and phenomena in Saturn's dense, icy granular rings. For instance, there have been a confirmation of "viscous overstabilities" (Thomson et al., 2007) or "propeller" structures caused by "skyscraper"-sized tiny ring-moons (dubbed as moonlets; Tiscareno et al., 2006) – both formerly theoretically predicted in our group (Schmidt et al., 2001; Spahn and Sremčević, 2000).
Density- and bending waves – driven by the gravity action of the numerous Kronian satellites or structural inhomogeneities inside Saturn and first spotted by the *Voyager*-cameras – have now been caught with the camera-system (Imaging Sub-System - ISS) of the *Cassini*-spacecraft at an unprecedented spatial resolution (French et al., 2016; Tiscareno and Harries, 2018). These spiral waves are dominated by inertia forces – acting in a differentially rotating Keplerian ring – balanced by the self-gravity of the ring-matter providing a possibility to estimate the local mass density of the ring. Apart from those already expected ring-phenomena, new features have

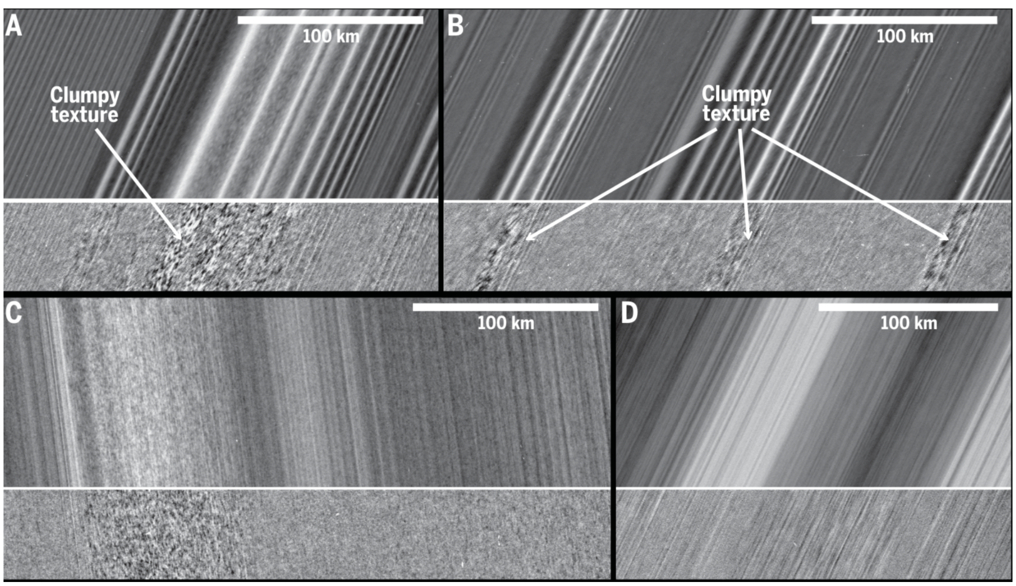


Figure 2: In resonantly driven density waves in Saturn's rings (upper panels A & B) a clustering texture mainly appears in the wave troughs, characterized by low granular temperatures usually corresponding to larger mean sizes (Goldhirsch and Zanetti, 1993; Esposito et al., 2012) (Image from Tiscareno et al. (2019)).

been caught by *Cassini's* cameras (ISS-data) as presented in Figure 1. Here the B ring edge – confined by the strongest resonance in the rings: the 2:1 inner Mimas Lindblad resonance – is shown with its tilted *fine-structures* in the grazing Sun-light during the sunset at the rings (equinox in August 2009). Under these illumination conditions all outstanding vertical features cast shadows whose lengths provide a measure for the vertical extent of these structures. Those visible in Figure 1 suggest a 3.5 kilometre altitude of vertical ring-matter excursions (Spitale and Porco, 2010). Having a closer look at this image, it seems that the shadows are closely tied to the inclined bright streaky features/clusters suggesting their quite large vertical reach.

Further examples of unexpected material clustering are shown in Figure 2 which occur in regions of resonantly driven waves in Saturns rings.

All these observations indicate that larger clusters preferably evolve in “colder”, unperturbed regions where the collision frequency and -intensity is reduced. Esposito et al. (2012) have described these facts with a two-dimensional, scale-based kinetic model quantifying evolution of two state variables: the *mean effective mass* $\approx \langle k \rangle$ along with the *mean velocity dispersion* $\approx \langle T_k \rangle$ (mean granular temperature), which covers astonishingly well the *Cassini*-observations[2].

Based on a generalized kinetic theory (Spahn et al., 2004; Spahn et al., 2014) we have calculated the size-spectrum n_k under near-equilibrium conditions (Brilliantov et al., 2015), i.e. assuming energy equipartition of all mass-classes $T_k \to T$. The result is shown in Figure 3 attesting an astonishingly well agreement with spacecraft measurements.

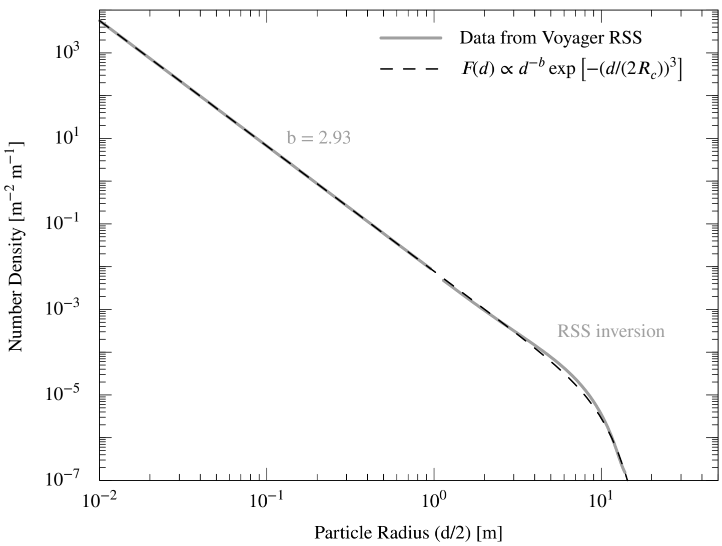


Figure 3: The particle size distribution obtained by Brilliantov et al. (2015) compared to the Voyager radio-science (RSS) data. A quite good agreement between theory and observation has been found.

However, even these obvious successes of the above models do not hide the fact that neither a two-dimensional reduction (Esposito et al., 2012) nor the assumption energy-equipartition (Spahn et al., 2004; Brilliantov et al., 2015) are sufficient to explain the non-equilibrium character of the fine-scale structures observed in Saturn’s rings. Consequently, we have started to investigate the coupled dynamics of the granular temperatures $\{T_k\}$ with the mass-frequencies $\{n_k\}$ of an undisturbed dense ring[3]. A splitting up of an universal temperature into a mass-dependent spectrum $T \to \{T_k\}$ has been found as an expression of the non-equilibrium character of the undisturbed multi-disperse sheared granular gas!

In the planned project, we will analyze the stability of that steady state under external perturbations in order to understand the cluster formation, and thus, to complete the goals of my Ph.D.-thesis.

Scientific Objectives

Unlike molecular gases, a granular gas is always subject to collisional dissipation of inner energy, hence temperature decay over time (Haff et al., 1983; Brilliantov and Pöschel, 2004). Even more interesting phenomena can be observed when granular gas consists of species with different masses. In this case, due to non-equal partitioning of energy, each species attain unique granular temperatures (Garzó et al., 2007; Osinsky et al., 2020).

Our approach to explain the observed *fine structures* in Saturn’s dense rings is divided into *two steps*:

[2]The mass-index k labels the number of constituents forming a ring-aggregate of mass $m = km_0$

[3]The curly brackets denote the $\mathcal{N}$-tuples of the mass-classes.

1. derivation of the stationary field evolution equations: particle number- resp. size distribution $\dot{n}_k$ and granular temperatures $\dot{T}_k$, of a spatially homogeneous Keplerian planetary ring (i.e. $\vec{u} \propto r^{-1/2}\vec{e}_\varphi$);
2. perturbation- and stability analyses of the unperturbed mean fields n_k and T_k against resonant gravitational perturbations exerted by satellites.

Concerning 1: The model we consider is a granular gas with polydisperse size-distribution of constituents in a sheared environment. We start our kinetic description of such a medium with an introduction of a one-particle DFs $f_k(\vec{r}, \vec{v}, t)$ for each species k. Time evolution of such functions obey Boltzmann-type kinetic equations (Haff et al., 1983; Brilliantov and Pöschel, 2004; Spahn et al., 2004):

$$\frac{\partial f_k}{\partial t} + \vec{v} \cdot \frac{\partial f_k}{\partial \vec{r}} - \frac{1}{m_k}\frac{\partial U}{\partial \vec{r}} \cdot \frac{\partial f_k}{\partial \vec{v}} = \sum_i n_i I_c(f_k, f_i) \ , \tag{1}$$

where $U(r)$ is the gravitational potential of the planet and $I_c(f_k, f_i)$ is the collision integral. With the aid of the DFs f_k we can define macroscopic fields as corresponding velocity moments of DFs:

$$\begin{aligned} \rho_k(\vec{r}, t) &= m_k \int f_k(\vec{r}, \vec{v}, t)\, d\vec{v}_k \ , \\ \rho_k(\vec{r}, t)\vec{u}(\vec{r}, t) &= m_k \int \vec{v} f_k(\vec{r}, \vec{v}, t)\, d\vec{v}_k \ , \\ T_k(\vec{r}, t) &= \frac{2}{3}\int \frac{m_k c^2}{2} f_k(\vec{r}, \vec{v}, t)\, d\vec{v}_k \ , \end{aligned} \tag{2}$$

where $\rho_k = m_k n_k$ is the mass density, $\vec{u}$ is the mean velocity and $\vec{c} = \vec{v} - \vec{u}$ is the peculiar velocity. Performing these first three moments of Eq. (1), we obtain corresponding mean field balance equations of mass (particle number), momentum and thermal energy. In a generic form, for any given macroscopic field $\Psi_k(\vec{r}, t)$ (here $m, m\vec{v}$, and $m\vec{c}^2/2$), these balance equations are of the form:

$$\left\langle \frac{D\Psi_k}{Dt} \right\rangle_{convective} = \left\langle \frac{d\Psi_k}{dt} \right\rangle_{collisional} \ , \tag{3}$$

meaning that convective changes of the mean fields $\Psi(\vec{r}, t)$ are balanced by the collisional changes. Angular brackets denote an average over the velocity space.

In our first step, only restitutive collisions occurring in a homogeneous Keplerian disk of a given size-DF n_k are considered. The question is, how do the granular temperatures T_k evolve as functions of the mass k of the aggregates as an expression of the non-equilibrium character of the granular ring. While the collisional changes of the zeroth- and first order will vanish for restitutive collisions, the second moments suffer non-zero collisional changes.

In a generalized form, these equations can be written as:

$$\frac{dT_k}{dt} \propto H_k - A_k T_k + \sum_i B_{ki}(T_k - T_i) \ , \tag{4}$$

where $A_k > 0$ is the parameter describing dissipation due to collisions – dubbed as *granular cooling*– which depends on size distribution function, collision frequency and restitution. The latter describes the dissipative character of the collisions between the ring-aggregates. The parameter $B_{ki} = B_{ik} > 0$ quantifies the inter-species heat flow causing different temperatures for different aggregate masses.
The heating function H_k guarantees an establishment of stationary granular temperatures (Bodrova et al., 2014). The "heat-source" in a planetary ring provided by the central potential $U(r) \propto -1/r$ caused a steady shear $\propto \partial\vec{u}/\partial\vec{r}$ and the related viscous friction. For a Keplerian disk the heating takes the form:

$$H_k \propto \nu_k \Omega^2 \ , \tag{5}$$

where Ω is the mean orbital frequency around the considered location of the system, and ν_k is the shear viscosity term.

In the case of planetary rings, the viscosity consists of *local* and *non-local* parts $\nu = \nu_l + \nu_{nl}$ (Spahn and Schmidt, 2006; Schmidt et al., 2009; Seiß and Spahn, 2011), dependent whether one considers finite size effect in the collisions or not. In the case of a gas with different species, we need to know the viscosity terms for each species, where especially the non-local contributions ν_{nl} matter in dense rings. With these transports the heat-source H_k can be quantified, and with it, allowing the granular temperatures to establish stationary levels depending on the mass k – i.e. the deviation from the energy equipartition increases with decreasing mass k.

This has constituted the first goal of our project to derive kinetic transport coefficients for each species from the microscopic (collisional) level of description and to characterize the establishment of a temperature-vector $\{T_k\}$.

In order to test our first theoretical results, we have developed a molecular dynamics (MD) code, simulating granular particles of different sizes in a Hill's box. *Here some* $\ln T$ *over* $\ln t$ *simulation results graph Fig. 4*

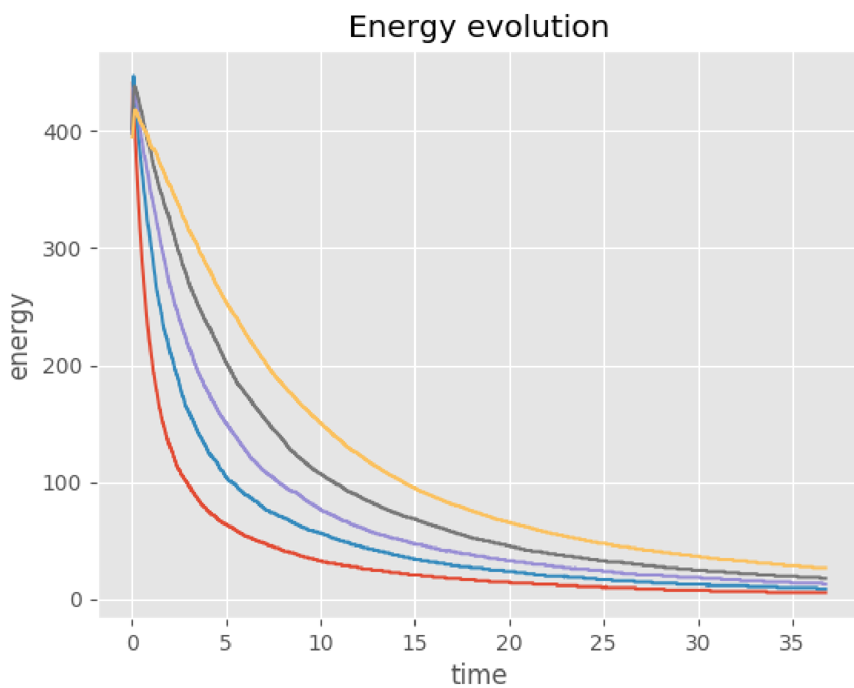


Figure 4: This is only an example plot. Should be replaced by my new simulation results

In our *step 1.* studies – unperturbed, homogeneous dense planetary rings – the momentum balance is well fulfilled for a Keplerian (circular) velocity field $\vec{u} \approx \vec{e}_\varphi \sqrt{\mu/r}$ with the gravity parameter of the planet $\mu = GM_p$ (G - gravity constant, M_p - mass of the planet). The precision is of the order of the dissipative migration towards the planet (Spahn and Sremčević, 2000) $u_r \approx -3\nu/(2r) \approx 10^{-10}$m s^{-1} (for $\nu \approx 10^{-2}$m^{2}s^{-1}, and $r \approx 10^8 m$), which has to be related to the orbital speed $|\vec{u}| \approx 10$km s^{-1} $= 10^4$m s^{-1}. That means, only the balances of mass (zeroth-order n_k) and thermal energy (granular temperatures T_k, second-order) matter for the simple *homogeneous* case 1.

The temperature splitting becomes important when allowing the size-distribution n_k to react on variations of the temperature vector $\{T_k\}$ via aggregating and fragmenting collisions. To this aim specific coagulation- and fragmentation kernels (domains) have to be considered in the collision integrals (Spahn et al., 2004).
Performing the zeroth order mean of the corresponding kinetic equation we arrive at the coupled system of

equations:

$$
\begin{aligned}
\left\langle \frac{Dn_k}{Dt} \right\rangle_{convective} &= \left\langle \frac{dn_k}{dt} \right\rangle_{gain} - \left\langle \frac{dn_k}{dt} \right\rangle_{loss} \,, \\
\left\langle \frac{DT_k}{Dt} \right\rangle_{convective} &= \left\langle \frac{dT_k}{dt} \right\rangle_{gain} - \left\langle \frac{dT_k}{dt} \right\rangle_{loss} \,,
\end{aligned} \tag{6}
$$

where we distinguish between collisional *gain* and *loss* terms, either reducing or rising the number of grains in the mass class k, respectively – both originated by the aggregation and fragmentation.
In contrast, restitutive collisions do not contribute to changes of size-DF but they thermalize the velocity part of the DF! In other words, the collisions between ring-aggregates create *loss* in thermal motion owing to *granular cooling* and the viscous shear *gain* balances the losses.
However, the coupled balances (6) quantify simultaneously the evolution of both state value-vectors $\{T_k\}$ and $\{n_k\}$. Mainly, the change of concentrations n_k affects the collision frequencies of constituents, which in turn leads to faster/slower temperature relaxation. On the other hand, the mass-dependency of temperatures affects the mobilities of the ring-aggregates, which may have an effect on their growth or dissolution. In a former work, Baibolatov and Spahn (2012) has estimated the most probable size of an aggregate for given temperatures of a granular gas, showing that different temperatures lead to different sizes of aggregates – supporting also the above findings.

Concerning 2: we are planning to analyze the system Eq. (6) under influence of external periodic gravitational perturbations driven at resonances of satellites. For weak and moderate perturbations, this will be modeled by periodically driving the evolution for $\{T_k\}$ in Eqs. (6) – where the mass-spectrum $\{n_k\}$ will have to respond to those excitations. Esposito et al. (2012) dubbed such a behavior as *"predator-pray"* system, where the average temperature $\langle T \rangle \propto c^2$ (velocity dispersion: c^2) takes the role of the *"predator"* and the mean particle mass $\langle k \rangle = m_{eff}$ reacts as the *"pray"*. An averaging over certain mass-classes (or even the whole spectrum) provide a substantial test for that 2D-low dimensional dynamical model (Esposito et al., 2012). However, our model is far more than this, we can indeed describe interactions between different mass-classes, as for instance segregation and related regions of virtually different mean sizes (-masses), effects which can be checked with analyses of *Cassini*-data.

For stronger perturbations, the momentum-balance $\varrho_k \vec{u}_k$ has to be taken into account because the deviations from the circular Kepler-$\vec{u}$ field become decisive changing the shear $\nabla \vec{u}_k$. A general streamline-approach of strongly disturbed dense rings has been proposed by Borderies et al. (1983) – where even shear stress reversals and with it momentum flux reversals can occur as shown in our own work (Grätz et al., 2019). However, many of these fine-scale structures are observed near resonances of moderate strength so that we will concentrate on analyses of Eqs. (6) in this PhD-thesis.

B.Table of contents and outline of the thesis with an overview of accomplished (sub-) chapter

Table of contents of the PhD thesis (subject to minor changes):

1. **Introcution**

   State of the Art description of the problem

2. **Breakage of energy equipartition in planetary rings**

   2.1 Hill equations

   2.2 Contact mechanics of binary collisions

   2.3 Enskog-Boltzmann kinetic equations

   2.4 Local and nonlocal split of collision integrals

   2.5 Hydrodynamic approximation and mean field equations

   2.6 Transport coefficients

   2.7 Maxwellian distribution assumption and temperature evolution equations

   2.8 Numerical simulations

3. **Aggregation and fragmentation processes**

   3.1 Adhesive contact forces

   3.2 Equilibrium model of aggregate formation

   3.3 Aggregation and fragmentaion kernels of collision integrals

   3.4 Mean field balance equations

   3.5 Analysis of the mean field equations

   3.6 Numerical simulations

4. **Periodic moon perturbations**

   4.1 2:1 resonances with Mimas

   4.2 Effects of perturbations on temperatures

   4.3 Perturbed mean field equations

   4.4 Solutions of mean field equations and stability analysis

   4.5 Numerical simulations

5. **Conclusions**

   Final thoughts and outline of the work

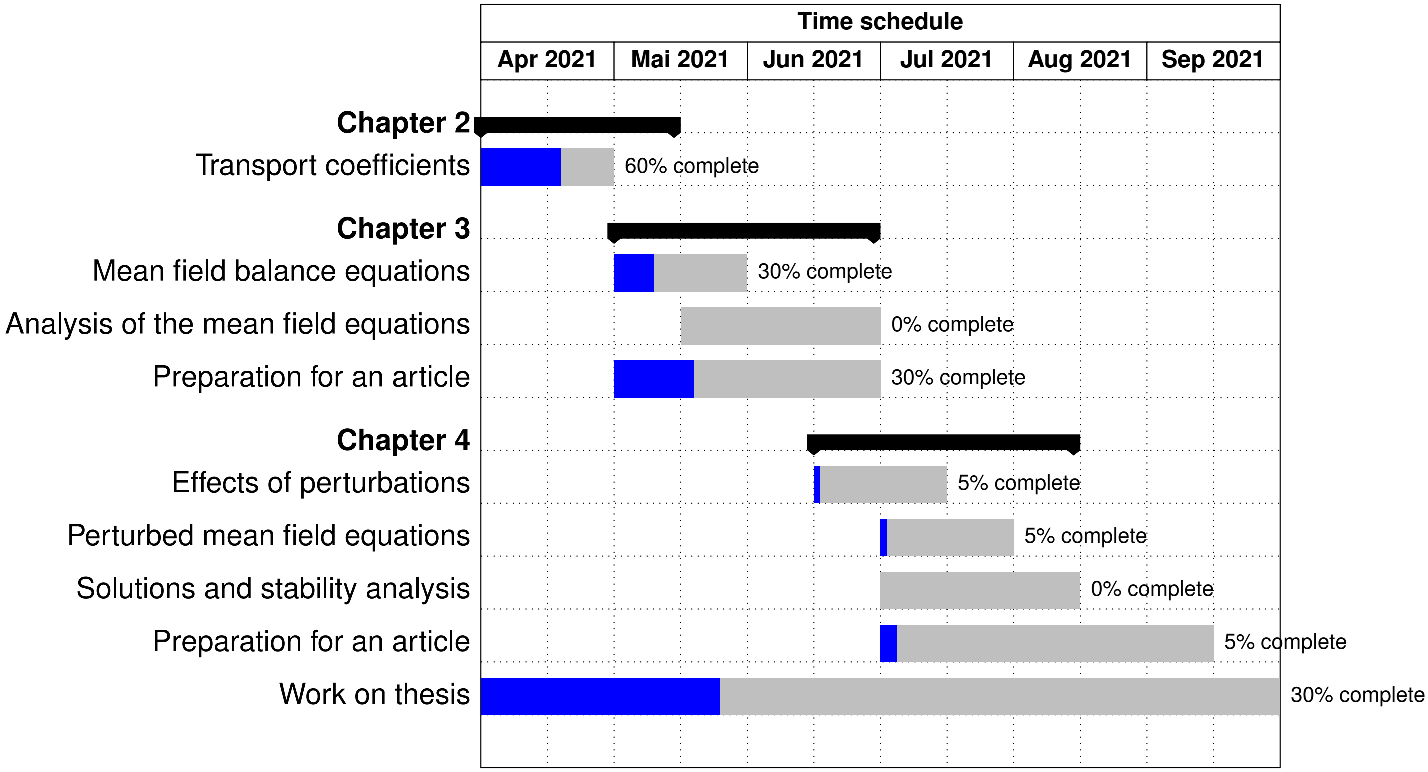
6. **Appendix**

   A.1 Overview of the numerial simulation methods

   A.2 and further will be devoted to tedious calculations

The second chapter of the thesis, is almost complete. The third chapter is complete up to the point where mean field balance equations are derived and further analyzed. Chapter four is planned to be accomplished during the period of the project.

C. Working program and intended completion date



In the chart above, the planned work on thesis and articles is shown. Obvioulsly, the completion percentages of each task are only rough estimates, however they seem to be realistic. As shown in the chart, the planned completion time is at September 2021, however I kindly ask the commitee to take into account the global COVID-19 pandemic events, and consider the shift of the project by few months. The main reason is that it might be problematic to get my residence permit in Germany during the time-period shown in the timeline of the project.

Our work is purely theoretical, and we do not plan to perform any experiments. The main *validity check experiments* we do, are numerical simulations using molecular dynamics methods.

Something about our planned approach to the problems

- Working-Program, time schedule (max. 4 pages)Detailed information about your planned approach, especially a thorough explanation of the meth-odology, that you will apply in the completion phase of your PhD. A time schedule (e. g. in graphic or tabular form) for the period of funding should clearly demonstrate the steps in your research project that are planned, have already started or are completed. If your PhD includes experiments, please indicate the experiments that have already been conducted and that will still be conducted. The quality of the research approach and the characterization of the accomplished and planned steps are of utmost importance.

- Please specify your intended completion date.

References

Baibolatov, Y. and Spahn, F. (2012). The role of adhesion for ensembles of mesoscopic particles. *Gran. Matt.*, 14:197–202.

Bodrova, A., Levchenko, D., and Brilliantov, N. (2014). Universality of temperature distribution in granular gas mixtures with a steep particle size distribution. *Europhys. Lett.*, 106:14001.

Borderies, N., Goldreich, P., and Tremain, S. (1983). Perturbed particle disks. *Icarus*, 55:124–132.

Brilliantov, N. V., Albers, N., Spahn, F., and Pöschel, T. (2007). Collision dynamics of granular particles with adhesion. *Phys. Rev. E*, 76(5):051302.

Brilliantov, N. V., Krapivsky, P. L., Bodrova, A., Spahn, F., Hayakawa, H., Stadnichuk, V., and Schmidt, J. (2015). Size distribution of particles in Saturn s rings from aggregation and fragmentation. *PNAS*, 112(31):9536–9541.

Brilliantov, N. V. and Pöschel, T. (2004). *Kinetic theory of granular gases*. Oxford Univ. Press.

Esposito, L. W., Albers, N., Meinke, B. K., Sremčević, M., Madhusudhanan, P., Colwell, J. E., and Jerousek, R. G. (2012). A predator-prey model for moon-triggered clumping in Saturn's rings. *Icarus*, 217:103–114.

French, R. G., et al., and et al. (2016). Decipheringthe embedded wave in Saturn's Maxwell ringlet. *Icarus*, 279:62–77.

Garzó, V., Dufty, J. W., and Hrenya, C. M. (2007). Enskog theory for polydisperse granular mixtures. I. Navier-Stokes order transport. *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, 76(3).

Goldhirsch, I. and Zanetti, G. (1993). Clustering instability in dissipative gases. *Phys. Rev. Lett.*, 70:1619–1622.

Grätz, F., Seiß, M., Schmidt, J., Colwell, J., and Spahn, F. (2019). Sharp gap edges in dense planetary rings: An axisymmetric diffusion model. *The Astrophysical Journal*, 872(2):153.

Haff, P. K., Eviatar, A., and Siscoe, G. (1983). Ring and Plasma: The enigmae of Enceladus. *Icarus*, 56:426–438.

Osinsky, A., Bodrova, A. S., and Brilliantov, N. V. (2020). Size-polydisperse dust in molecular gas : Energy equipartition versus nonequipartition. *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, 022903:37–39.

Schmidt, J., Brilliantov, N., Spahn, F., and Kempf, S. (2009). Formation of Enceladus' Dust Plume. *submitted to Icarus*.

Schmidt, J., Salo, H., Spahn, F., and Petzschmann, O. (2001). Viscous overstability in Saturn's b ring: II hydrodynamic theory and comparison to simulations. *Icarus*, 153:316–331.

Seiß, M. and Spahn, F. (2011). Hydrodynamics of saturn's dense rings. *Math. Model. Nat. Phenom.*, 4:191–218.

Spahn, F., Albers, N., Sremčević, M., and Thornton, C. (2004). Kinetic description of coagulation and fragmentation in dilute granular particle ensembles. *Europhys. Lett.*, 67:545–551.

Spahn, F. and Schmidt, J. (2006). Hydrodynamic description of planetary rings. *GAMM-Mitteilungen*, 29:118–143.

Spahn, F. and Sremčević, M. (2000). Density patterns induced by small moonlets in Saturn's rings? *Astron. & Astrophys.*, 358:368–372.

Spahn, F., Vieira-Neto, E., Guimarães, A. H. F., Gorban, A. N., and Brilliantov, N. V. (2014). A statistical model of aggregate fragmentation. *New J. Phys.*, 14(1-11).

Spitale, J. N. and Porco, C. C. (2010). Detection of Free Unstable Modes and Massive Bodies in Saturn's Outer B Ring. *Astron. J.*, 140:1747–1757.

Thomson, F. S., Marouf, E. A., Tyler, G. L., French, R. G., and Rappoport, N. J. (2007). Periodic microstructure in Saturn's rings A and B. *Geophys. Res. Lett.*, 34:24203–+.

Tiscareno, M. S., Burns, J. A., Hedman, M. M., Porco, C. C., Weiss, J. W., Murray, C. D., and Dones, L. (2006). Observation of "propellers" indicates 100-metre diameter moonlets reside in saturn's a-ring. *Nature*, 440:648–650.

Tiscareno, M. S., et al., and et al. (2019). Close-range remote sensing of Saturn's rings during Cassini's ring-grazing orbits and Grand Finale. *Science*, 364:1054–1065.

Tiscareno, M. S. and Harries, B. E. (2018). Mapping spiral waves and other radial features in Saturn's rings. *Icarus*, 312:157–171.