Sheared Granular Gas Dynamics

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I. NON-UNIFORM GRANULAR GAS SYSTEM

In this work we consider a rotating disk of granular gases with non-uniform size distribution of constituents under external gravitational shear. Granular gases are well known for its intrinsic dissipative nature and hence decay of velocity dispersion or *granular temperature* if the system has nonzero initial energy. Let us start our work with notations and definitions of necessary parameters. The total number of constituents in the system is N. The number of species in our system is s and the total number of constituents of species α is N_α, where $\alpha \in [1, s]$. Hence, we can write

$$\chi_\alpha = \frac{N_\alpha}{N} \ , \ \ \sum_{\alpha=1}^{s} \chi_\alpha = 1 \ , \tag{1}$$

where χ_α is the concentration of constituents of species α. This means that there are N_α identical constituents with masses m_α. Obviously $m_\alpha \neq m_\beta$ if $\alpha \neq \beta$. Since a system of certain species consists of large number of constituents N_α, it is considered as a statistical system and we analyze it through certain macroscopic parameters. Let us say that P_α is one of the macroscopic parameters of species α, then for the whole system we can write

$$P = \sum_{\alpha=1}^{s} \chi_\alpha \cdot P_\alpha \ . \tag{2}$$

This is the mean value of macroparameter for the whole system across all species.

In order to define the macroparameter P_α itself, we need to introduce the one-particle distribution function in the phase space of dynamic variables. The only dynamic variables of a single particle are coordinate $\mathbf{x}$ and velocity $\mathbf{v}$. Now, the one-particle distribution function, or simply distribution function, for species α is written as $F_\alpha(\mathbf{x}, \mathbf{v})$. This function should have the next property

$$\int F_\alpha(\mathbf{x}, \mathbf{v}) d\mathbf{x} d\mathbf{v} = n_\alpha = \frac{N_\alpha}{V} \ , \tag{3}$$

where the integration is performed over all phase space and n_α is the number density of species α, V is the total volume of the system. Note that n_α here is the average number density across the whole system volume V. We can write separately coordinate and velocity distribution functions as

$$\begin{aligned} g_\alpha(\mathbf{x}) &= \int F_\alpha(\mathbf{x}, \mathbf{v}) d\mathbf{v} \ , \\ f_\alpha(\mathbf{v}) &= \int F_\alpha(\mathbf{x}, \mathbf{v}) d\mathbf{x} \ , \end{aligned} \tag{4}$$

and obviously

$$\int g_\alpha(\mathbf{x})d\mathbf{x} = \int f_\alpha(\mathbf{v})d\mathbf{v} = n_\alpha \ . \tag{5}$$

Further, we will mostly use the velocity distribution function, since we know that all informative macroscopic parameters describing statistical systems are certain moments of dynamic functions of velocity $p_\alpha = p_\alpha(\mathbf{v})$. Hence corresponding macroparameter P_α is obtained from

$$n_\alpha P_\alpha = \int p_\alpha(\mathbf{v}) f_\alpha(\mathbf{v})d\mathbf{v} \ . \tag{6}$$

First three velocity moments corresponding to three physical values of mass, momentum and energy are written as

$$\begin{aligned} \rho_\alpha = m_\alpha n_\alpha &= \int m_\alpha f_\alpha(\mathbf{v})d\mathbf{v} \ , \\ \rho_\alpha \mathbf{u}_\alpha &= \int m_\alpha \mathbf{v} f_\alpha(\mathbf{v})d\mathbf{v} \ , \\ \frac{D}{2} n_\alpha T_\alpha &= \int \frac{m_\alpha c^2}{2} f_\alpha(\mathbf{v})d\mathbf{v} \ , \end{aligned} \tag{7}$$

where $\mathbf{c} = \mathbf{v} - \mathbf{u}_\alpha$, $D = 2$ for two dimensional and $D = 3$ for three dimensional systems. These moments are mass density ρ_α, momentum density $\rho_\alpha \mathbf{u}_\alpha$ and granular temperature T_α correspondingly.

Let us now define the macroparameters for the whole system as

$$\begin{aligned} \rho &= \sum_{\alpha=1}^{s} \chi_\alpha \cdot \rho_\alpha \ , \\ \rho \mathbf{u} &= \sum_{\alpha=1}^{s} \chi_\alpha \cdot \rho_\alpha \mathbf{u}_\alpha \ , \\ nT &= \sum_{\alpha=1}^{s} \chi_\alpha \cdot n_\alpha T_\alpha \ , \end{aligned} \tag{8}$$

where

$$n = \sum_{\alpha=1}^{s} n_\alpha = \frac{N}{V} \ . \tag{9}$$

If we make the assumption that our system is very large and highly non-uniform, i.e. we consider the limits $N \to \infty$ and $s \to \infty$, hence

$$\chi_\alpha \to \chi(\alpha) \ , \tag{10}$$

our number density becomes the size distribution function, and (2) turns into integration over distribution function

$$P = \int_{1}^{\infty} P(\alpha)\chi(\alpha)d\alpha = \int P(\alpha)d\chi(\alpha) , \tag{11}$$

and $P_\alpha \to P(\alpha)$ becomes the function of α.

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