Size-polydisperse dust in molecular gas: Energy equipartition versus non-equipartition

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We investigate numerically and analytically size-polydisperse granular mixtures immersed into a molecular gas. We show that the equipartition of granular temperatures of particles of different sizes is established, however, the granular temperatures significantly differs from the temperature of the molecular gas. This result is surprising since generally, the energy equipartition is strongly violated in driven granular mixtures. Qualitatively, the obtained results do not depend on the collision model, being valid for a constant restitution coefficient ε, as well as for the ε for viscoelastic particles. Our findings may be important for astrophysical applications, such as protoplanetary disks, interstellar dust clouds and comets.

I. INTRODUCTION

What is common between very different objects, such as interstellar dust, protoplanetary discs [1–3], comets [4] and dust devils on Mars [5], Earth [6] and possibly other planets? All these systems are comprised of size and mass polydisperse dust particles immersed in molecular gas. Interstellar molecular clouds possess high density regions, so called clumps. The density of matter there is large enough to trigger gravitational collapse, which eventually leads to the formation of stars. In the present study we assume that the density of matter is below this threshold. Moreover, we assume that the granular gas is rarified, so that the surrounding molecular gas may be treated as a thermostat which is not affected by the granular gas.

The dust particles addressed here are macroscopic, but small enough grains, so that the gravitational interactions between the grains may be neglected. Hence, we have a granular mixture, driven by the molecular gas. Usually, in granular mixtures the energy equipartition between particles of different size is violated. This has been predicted theoretically [7–11] and confirmed in experiments [12, 13] and computer simulations [11, 14, 15]. The same is true for such natural systems as Saturn rings, which are essentially, granular gas mixtures of particles with a size, ranging from 10^{-3}m to 1m [16, 17]. The size poldispersity of the rings particles stems from the permanent aggregation and fragmentation of the constituents, which keeps the steady state size distribution [17, 18]. The energy equipartition for the rings particles does not hold as shown theoretically [19, 20] and in computer experiments [21].

Let a granular mixture contain N different sorts of particles, of mass m_k and diameter σ_k with $k = 1, \dots N$. We assume that the particles are uniform spheres with the mass density of the material ρ, then $m_k = \pi\sigma_k^3\rho/6$. Without the loss of generality we assume that all masses are multiples of some minimal mass m_1, that is, $m_k = km_1$. Let the number density of particles of mass m_k be equal n_k. We consider a space uniform system, so that the particles of mass m_k may be characterized by the velocity distribution function $f(\mathbf{v}_k, t)$. It quantifies the number of such particles with velocity $\mathbf{v}_k$ at time t in a unit volume. The number density may be expressed in terms of the distribution function as $n_k = \int d\mathbf{v}_k f(\mathbf{v}_k, t)$. The average energy of particles of mass m_k is characterized by the corresponding granular temperature, T_k, defined as [7, 10],

$$\frac{3}{2} n_k T_k = \int d\mathbf{v}_k f(\mathbf{v}_k) \frac{m_k v_k^2}{2}. \tag{1}$$

If all inter-particle collisions were elastic, the energy equipartition between all sorts of the particles would hold. The violation of equipartition stems from the dissipative nature of inter-particle collisions, which are quantified by the restitution coefficient ε [7],

$$\varepsilon = \left| \frac{(\mathbf{v}_{ki}' \cdot \mathbf{e})}{(\mathbf{v}_{ki} \cdot \mathbf{e})} \right|, \tag{2}$$

where $\mathbf{v}_{ki}' = \mathbf{v}_k' - \mathbf{v}_i'$ and $\mathbf{v}_{ki} = \mathbf{v}_k - \mathbf{v}_i$ are the relative velocities of two particles after and before a collision, correspondingly, and $\mathbf{e}$ is a unit vector connecting their centers at the collision instant. We do not consider very soft particles where the definition of the restitution coefficient is more subtle [22]. The post-collision velocities are related to the pre-collision velocities $\mathbf{v}_k$ and $\mathbf{v}_i$ as follows [7]:

$$\mathbf{v}_{k/i}' = \mathbf{v}_{k/i} \pm \frac{m_{\text{eff}}}{m_{k/i}} (1+\varepsilon) (\mathbf{v}_{ki} \cdot \mathbf{e}) \mathbf{e}. \tag{3}$$

Here $m_{\text{eff}} = m_k m_i / (m_k + m_i)$ is the effective mass of the colliding particles. To date, most studies of granular gases have been focused on the case of a constant restitution coefficient [23–31]. This assumption contradicts, however, experimental observations [32–34], along with basic mechanical laws [35, 36], which indicate that ε does depend on the impact velocity [34, 35, 37–39]. This dependence may be obtained by solving the equations of motion for colliding particles with the explicit account for the dissipative forces acting between the grains. The simplest first-principle model of inelastic collisions assumes viscoelastic properties of particles' material, which results in viscoelastic inter-particle force [37] and finally in the restitution coefficient [35, 39, 40]:

$$\varepsilon_{ki} = 1 + \sum_{j=1}^{20} h_j \left(A\kappa_{ki}^{2/5} \right)^{j/2} |(\mathbf{v}_{ki} \cdot \mathbf{e})|^{j/10}, \tag{4}$$

Here h_k are numerical coefficients [40]. The elastic constant

$$\kappa_{ki} = \frac{\kappa}{\sqrt{2}} \frac{k+i}{k^{5/6} i^{5/6} \sqrt{k^{1/3} + i^{1/3}}} \tag{5}$$

where

$$\kappa = \left(\frac{3}{2}\right)^{3/2} \frac{Y}{1-\nu^2} \left(\frac{6}{\pi \rho m_1^2}\right)^{\frac{1}{3}} \tag{6}$$

is a function of the Young's modulus Y and Poisson ratio ν; the constant A quantifies the viscous properties of the particles' material [41, 42]:

$$A = \frac{1}{Y} \frac{(1+\nu)}{(1-\nu)} \left(\frac{4}{3} \eta_1 \left(1 - \nu + \nu^2\right) + \eta_2 \left(1 - 2\nu\right)^2 \right) \tag{7}$$

where η_1 and η_2 are the viscosity coefficients.

Recently we have shown that the distribution of granular temperatures in polydisperse mixtures of granular particles follows the power law $T_k = T_1 k^{\alpha}$, if the size distribution in a mixture is steep enough [43]. The exponent α is universal for all steep size distributions for force-free granular mixtures. For driven granular mixtures α strongly depends on the agitation mode, in particular on the dependence of the driving force on the particle size. In the current study we investigate the distribution of temperatures in a mixture of granular particles emersed in a molecular gas. We assume that the particles are small enough, similar to the dust particles in sand devils, tornado on Earth, interstellar dust, comets and protoplanetary disks. In this case the presence of a molecular gas becomes important. Moreover we assume that the action of the molecular gas on the granular mixture keeps it in a steady state. We analyze both models of the restitution coefficient – the simplified model of a constant ε, as well as the realistic, first-principle model of visco-elastic particles. In either cases we obtain qualitatively same and somewhat unexpected result: The energy equipartition for the different granular species, along with the strong violation of the equipartition between the granular mixture and molecular gas. This looks surprising, since generally, a strong violation of the energy equipartition in driven granular mixture is expected [43]. Our theoretical predictions have been checked by numerical simulations. Namely, we performed the Direct Simulation Monte Carlo (DSMC) and confirmed the analytical findings. Interestingly, our conclusion supports a conjecture of the energy equipartition in a granular mixture, immersed in a molecular gas, proposed in Ref. [44]. The rest of the study is organized as follows. In the next Section II we specify the model and derive the granular temperatures for all species, which is done for both models of the restitution coefficient. In Section III we discuss the details of the numerical simulations and compare the numerical and analytical results. Finally, in Section IV we summarize our findings.

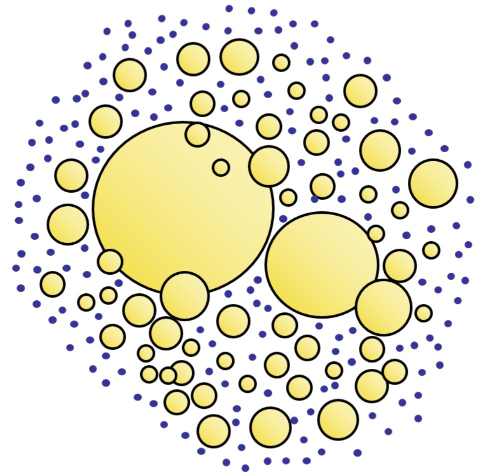

FIG. 1. The granular particles of different masses m_k (depicted with yellow circles) are immersed into a molecular gas composed of molecules of mass $m_g \ll m_k$ (shown as small blue circles).

II. TEMPERATURE DISTRIBUTION IN A GRANULAR MIXTURE

We consider a granular mixture, comprised on N species of mass $m_k = k\, m_1$ immersed in a molecular gas with temperature T_g and molecular mass m_g (see Fig. 1). Although being small, the dust particles are still much heavier than the gas molecules, that is, $m_g \ll m_1$. The collisions between the granular particles and gas molecules are elastic. Since the velocity distribution functions are close to Maxwellian distributions [7], we assume for simplicity that $f_k(\mathbf{v}_k, t)$ are Maxwellian:

$$f_k(\mathbf{v}) = \left(\frac{m_k}{2\pi T_k}\right)^{3/2} \exp\left(-\frac{m_k v_k^2}{2T_k}\right). \tag{8}$$

These functions evolve according to the Boltzmann equation [7],

$$\frac{\partial}{\partial t} f_k(\mathbf{v}_k, t) = I_k^{\mathrm{coll}} + I_k^{\mathrm{m.g.}}. \tag{9}$$

In Eq. (9) I_k^{coll} is the Boltzmann collision integral [7]:

$$I_k^{\mathrm{coll}} = \sum_{i=1}^{N} \sigma_{ki}^2 \int d\mathbf{v}_i \int d\mathbf{e}\, \Theta(-\mathbf{v}_{ki}\cdot\mathbf{e})\, |\mathbf{v}_{ki}\cdot\mathbf{e}\,| \left[\chi f_k(\mathbf{v}_k{''}, t) f_i(\mathbf{v}_i{''}, t) - f_k(\mathbf{v}_k, t) f_i(\mathbf{v}_i, t)\right], \tag{10}$$

where $\sigma_{ki} = (\sigma_k + \sigma_i)/2$, with $\sigma_k = \left(6 m_k/(\pi\rho)\right)^{1/3}$. The summation is performed over all species in the system. $\mathbf{v}_k''$ and $\mathbf{v}_i''$ are pre-collision velocities in the so-called inverse collision, resulting in the post-collision velocities $\mathbf{v}_k$ and $\mathbf{v}_i$. The Heaviside step-function $\Theta(-\mathbf{v}_{ki}\cdot\mathbf{e})$ selects the approaching particles and the factor χ equals the product of the Jacobian of the transformation $(\mathbf{v}_k'', \mathbf{v}_i'') \to (\mathbf{v}_k, \mathbf{v}_i)$ and the ratio of the lengths of the collision cylinders of the inverse and the direct collisions [7]. In the case of a constant restitution coefficient $\chi = 1/\varepsilon^2$. For viscoelastic particles it has a more complicated form [7]; in what follows we do not need its explicit expression.

The second term $I_k^{\rm m.g.}$ describes the driving of the system due to collisions with the surrounding molecular gas. It quantifies the energy injection into the granular mixture to compensate its losses in dissipative collisions. Since the mass ratio of the gas and grain particles m_g/m_k is very small, the collision integral may be written using the Kramers-Moyal expansion [7]:

$$I_k^{\rm m.g.} = \frac{\partial}{\partial \mathbf{v_k}} \left(\gamma_k \mathbf{v}_k + \bar{\gamma}_k \frac{\partial}{\partial \mathbf{v}_k} \right) f_k(\mathbf{v_k}, t). \qquad (11)$$

We investigate the evolution of granular temperatures, defined by Eq. (1). Multiplying the Boltzmann equation (9) by $m_k v_k^2/2$ for $k = 1...N$ and performing integration over $\mathbf{v}_k$, we get the following system of equations for evolution of the granular temperatures T_k of species of different masses m_k:

$$\begin{cases} \frac{d}{dt} T_1 = -T_1 \sum_{i=1}^N \xi_{1i} + 2\gamma_k \left(T_g - T_1\right) \\ \dots \\ \frac{d}{dt} T_k = -T_k \sum_{i=1}^N \xi_{ki} + 2\gamma_k \left(T_g - T_k\right) \\ \dots \\ \frac{d}{dt} T_N = -T_N \sum_{i=1}^N \xi_{Ni} + 2\gamma_k \left(T_g - T_N\right) . \end{cases} \qquad (12)$$

Here $\gamma_k = \gamma_{0k}\sqrt{T_g}$, $\gamma_{0k} = \frac{4}{3} n_g \sigma_k^2 \sqrt{2\pi m_g}/m_k$ and $\bar{\gamma}_k = \gamma_k T_g/m_k$ [7, 44]. The cooling rates ξ_{ki} describe the decrease of temperature of the species of mass m_k due to collisions with the species of mass m_i. For the case of a constant restitution coefficient these quantities read [43]:

$$\xi_{ki}(t) = \frac{8}{3}\sqrt{2\pi} n_i \sigma_{ki}^2 \left(\frac{T_k m_i + T_i m_k}{m_i m_k} \right)^{1/2} (1+\varepsilon) \left(\frac{m_i}{m_i + m_k} \right) \left[1 - \frac{1}{2}(1+\varepsilon) \frac{T_i m_k + T_k m_i}{T_k (m_i + m_k)} \right] . \qquad (13)$$

We assume that the restitution coefficient ε is the same for the collisions of particles of all sizes. In the case of viscoelastic particles the cooling rates have the form:

$$\xi_{ki}(t) = \frac{16}{3}\sqrt{2\pi} n_i \sigma_{ki}^2 \left(\frac{T_k m_i + T_i m_k}{m_i m_k} \right)^{1/2} \left(\frac{m_i}{m_i + m_k} \right) \times \left[1 - \frac{T_k m_i + T_i m_k}{T_k (m_i + m_k)} + \sum_{n=2}^{20} B_n \left(h_n - \frac{1}{2} \frac{T_k m_i + T_i m_k}{T_k (m_i + m_k)} A_n \right) \right] \qquad (14)$$

where $A_n = 4h_n + \sum_{j+k=n} h_j h_k$ are pure numbers and

$$B_n(t) = \left(A \kappa_{ki}^{2/5} \right)^{\frac{n}{2}} \left(\frac{2T_k}{m_k} + \frac{2T_i}{m_i} \right)^{\frac{n}{20}} \left(\frac{(20+n)\, n}{800} \right) \Gamma\left(\frac{n}{20}\right) \qquad (15)$$

with $\Gamma(x)$ being the Gamma-function. Driven granular systems rapidly settle into a non-equilibrium steady state and all granular temperatures attain, after a short time, some constant values, so that $dT_k/dt = 0$. The system of equations (12) turns then into a set of algebraic equations,

$$T_k \sum_{i=1}^N \xi_{ki} = 2\gamma_k \left(T_g - T_k\right) . \qquad (16)$$

Let us assume that the size distribution of the dust particles is steep enough and the number density of the granular mixture scales according to the power law, $n_k = n_1 k^{-\theta}$, with $\theta > 2$. Let the distribution of granular temperatures also scale according to the power-law: $T_k = T_1 k^{\alpha}$. Then the following approximate relation holds [43]:

$$\sum_{i=1}^N \xi_{ki} \simeq \begin{cases} k^{\frac{\alpha}{2} - \frac{5}{6}} \int_1^N i\, n_i \, di & \text{if } \alpha \geq 1 \\ k^{-\frac{1}{3}} \int_1^N i^{\frac{\alpha+1}{2}} n_i \, di & \text{if } 0 < \alpha < 1 \, . \end{cases} \qquad (17)$$

Substituting Eq. (17) into Eq. (16) and taking into account that $\gamma_k = \gamma_1 k^{-1/3}$, we conclude that the system (17) is compatible only for $\alpha = 0$. This immediately implies the equality of the granular temperatures, $T_k = T_1$, that is, the energy equipartition, and justification of the conjecture of Ref. [44].

We present the sum over the cooling rates in the form

$$\sum_{i=1}^N \xi_{ki} = \sqrt{T_1} \xi_{0k}, \qquad (18)$$

where for a constant restitution coefficient:

$$\xi_{0k} = \sum_{i=1}^N \frac{8}{3}\sqrt{2\pi} n_i \sigma_{ki}^2 (1+\varepsilon) \sqrt{\frac{m_i}{m_k (m_i + m_k)}} \times \left(1 - \frac{1}{2}(1+\varepsilon) \right) \qquad (19)$$

and for viscoelastic particles:

$$\xi_{0k} = \sum_{i=1}^N \frac{16}{3}\sqrt{2\pi} n_i \sigma_{ki}^2 \sqrt{\frac{m_i}{m_k (m_i + m_k)}} \times \sum_{n=2}^{20} B_n \left(\frac{1}{2} \sum_{j+k=n} h_j h_k - h_n \right) \qquad (20)$$

where the coefficients $B_n(t)$ given by Eq. (15) now take the form

$$B_n(t) = \left(A \kappa_{ki}^{2/5} \right)^{\frac{n}{2}} (2T_1)^{\frac{n}{20}} \left(\frac{m_i + m_k}{m_i m_k} \right)^{\frac{n}{20}} \times \left(\frac{(20+n)\, n}{800} \right) \Gamma\left(\frac{n}{20}\right) .$$

The granular temperature of the smallest particles (monomers) T_1 can be found from the equation:

$$T_1 \sqrt{T_1} + b\sqrt{T_g} T_1 - b\sqrt{T_g} T_g = 0, \qquad (21)$$

where we introduce the notation, $b = 2\gamma_{01}/\xi_{01}$. The quantity b is a function of T_1 for granular particles, colliding with velocity-dependent restitution coefficient, $b = b(T_1)$, so that Eq. ((21)) is a transcendental equation. However for the case of a constant restitution coefficient b is constant. Introducing $x = \sqrt{T_1/T_g}$, we recast Eq. (21) in a cubic equation:

$$x^3 + bx^2 - b = 0, \tag{22}$$

with the solution in the form:

$$x = \frac{1}{3}\left(-b + zb^2 + \frac{1}{z}\right), \tag{23}$$

where $z = 2^{1/3}\left(27b - 2b^3 + 3\sqrt{3}\sqrt{27b^2 - 4b^4}\right)^{-1/3}$. This yields the explicit expression for temperatures in a granular mixture in terms of the molecular gas temperature T_g:

$$T_k = T_1 = \frac{T_g}{9}\left(-b + zb^2 + \frac{1}{z}\right)^2. \tag{24}$$

It may be shown that $T_k = T_1$ is always smaller than T_g, which also follows from the physical nature of these quantities.

For viscoelastic granular mixture $T_k = T_1$ may be found from the numerical solution of Eq. (21), where the dependence of b on T_1 is to be taken into account. In both cases of constant ε, as well as for ε for viscoelastic particles, the energy equipartition for all granular species is observed. At the same time the granular temperature significantly differs from the temperature of the molecular gas, $T_k > T_g$. Physically this implies a steady energy flux from the molecular gas to the granular mixture, which permanently lose energy in dissipative collisions.

III. COMPUTER SIMULATION AND SIMULATION RESULTS

To check the prediction of our theory, we perform Direct Simulation Monte Carlo (DSMC), modified for the application to multi-species systems. The detailed description of the DSMC may be found elsewhere, see e.g. [45]. Here we briefly sketch some detail of the simulation method, with the focus on the implementation of the thermostat, see also [46, 47].

The simulation of collisions between granular particles has been performed according to the following scheme:

1. Choose the sizes of colliding particles i and j.

2. Choose the particles with speeds $\mathbf{v}_i$ and $\mathbf{v}_j$ with the probability, proportional to $|(\mathbf{v}_i - \mathbf{v}_j)\cdot\mathbf{e}|$, where $\mathbf{e}$ is the collision direction (a random unit vector).

3. Update the speeds of the colliding particles according to the collision rules.

The action of the molecular gas is is described by the term $I_k^{\text{m.g.}}$ of the Boltzmann equation (9). Obviously, this term plays a role of thermostat. In the lack of collisions between dust particles, the equation for the distribution function reads,

$$\frac{\partial f(\mathbf{v_k}, \mathbf{t})}{\partial t} = I_k^{\text{m.g.}}. \tag{25}$$

This equation, with $I_k^{\text{m.g.}}$ given by Eq. (11) is a Fokker-Planck equation, which corresponds to the Langevin equation (see e.g. [7]):

$$\frac{d\mathbf{v}_k}{dt} = -\gamma_k \mathbf{v}_k + \mathbf{F}_k^{st}, \tag{26}$$

$$\left\langle \mathbf{F}_k^{st}\right\rangle = 0, \quad \left\langle \mathbf{F}_k^{st}(t)\mathbf{F}_k^{st}(t')\right\rangle = \frac{2}{3}\hat{\mathrm{I}}\bar{\gamma}_k\delta(t-t'), \tag{27}$$

where $\hat{\mathrm{I}}$ is the unit matrix and a direct (diadic) product is implied in the second part of Eq. (27). The solution of the stochastic Langevin equation may be written as

$$\mathbf{v}_k(t+\Delta t) = \mathbf{v}_k(t)e^{-\gamma_k\Delta t} + \xi\mathbf{e}\cdot\sqrt{3\frac{T_g}{m_k}\left(1 - e^{-2\gamma_k\Delta t}\right)},$$
$$\xi \sim \mathcal{N}(0,1), \tag{28}$$

where $\mathcal{N}(0,1)$ denotes the normal distribution with zero mean and unit dispersion.

Numerically the thermostat is implemented by changing the speed of each particle independently, according to Eq. (28). The chosen time interval Δt corresponds to the time of $N_p h$ collisions, where N_p is the total number of particles. $h = 0.1$ is a parameter, which should be sufficiently small, to guarantee that each particle experiences several times the action of the thermostat between collisions with other granular particles. We used $N_1 = 3\times 10^8$ particles of minimal mass m_1 (monomers), so that there were $N_k = \lfloor N_1/k^3\rfloor = \lfloor 3\cdot 10^8/k^3\rfloor$ particles of mass m_k (k-mers).

The distribution of granular temperatures T_k, obtained by DSMC for different values of temperature of a molecular gas T_g and restitution coefficient ε is given in Fig. 2. Fig. 2a corresponds to a constant restitution coefficient and in Fig. 2b the distribution of granular temperatures in a mixture of viscoelastic particles is shown for steep size distribution $n_k \sim k^{-\theta}$ with $\theta > 2$. The granular temperatures rapidly tend to a steady-state values, where the equipartition in a granular mixture is practically established: $T_k \approx T_1$ for any k. The temperatures of the granular particles T_k differ, however, from the temperature of the molecular gas, $T_k < T_g$ as it is predicted by the theory and expected from simple "physical" arguments of the heat flux from the molecular gas to the granular mixture. The smaller the value of the restitution coefficient, the larger the difference between temperatures of the gas and the mixture. For $\varepsilon = 1$ both temperatures become equal and full equipartition is established: $T_k = T_g$. Strictly speaking, the "true" equipartition should follow for $k \gg 1$, as Eq. (17) is valid in this case. In practice

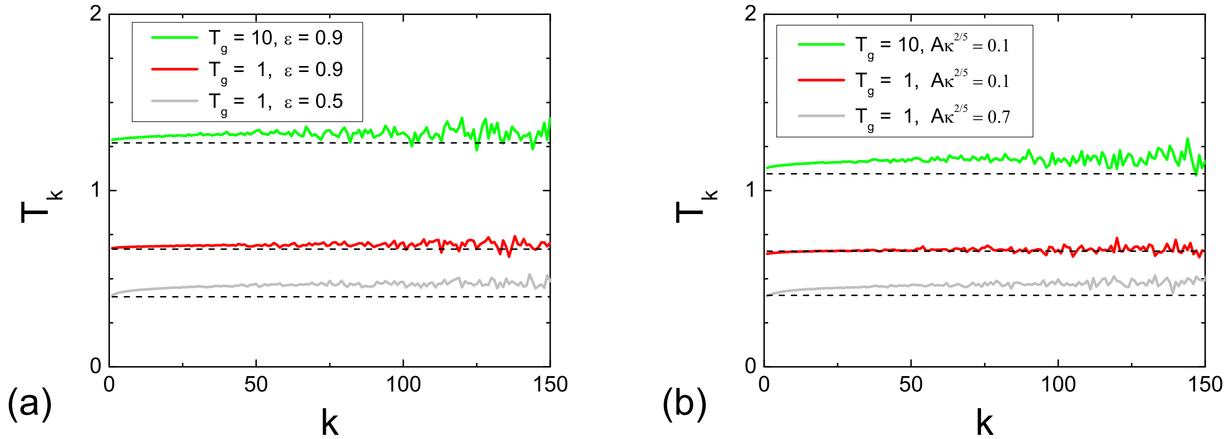

FIG. 2. Granular temperatures T_k of granular particles of mass m_k immersed into a molecular gas of temperature T_g. The granular particles collide with (a) constant restitution coefficient and (b) viscoelastic restitution coefficient. The parameter values: $\sigma_1 = 1$, $m_1 = 1$ $\gamma_{01} = T_g^{-3/2}/2$, $n_k = n_1 k^{-\theta}$ with $n_1 = 1$ and $\theta = 3$. Colored solid lines show simulation results, dashed black lines - analytical results for T_1 (Eq. (24)) for $\varepsilon = \text{const}$ and numerical solution of Eq. (21) for viscoelastic particles. One can clearly see that the temperature distribution in granular mixture for the both cases is close to the equipartition, $T_k \simeq T_1$. There is no energy equipartition between the granular mixture and molecular gas, $T_k < T_g$, since a permanent energy flux from the gas to the mixture supports the steady state.

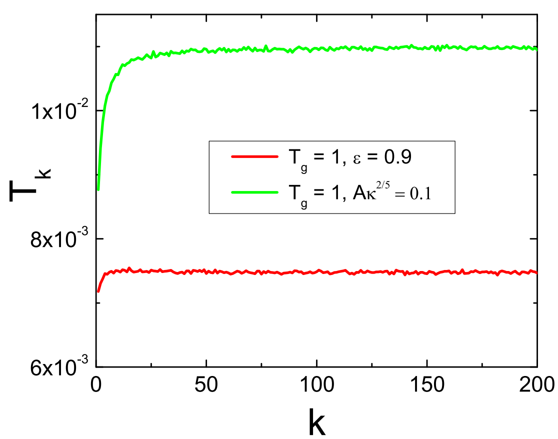

FIG. 3. The temperature distribution for the granular mixture with flat size distribution, $n_k = n_1\Theta(k_{\max} - k)$, immersed into a molecular gas of temperature T_g.

it is observed already for $k > 10$. For other size distributions, e.g. for a flat distribution, $n_k = n_1\Theta(k_{\max} - k)$, the DSMC also shows constant distribution of temperature, $T_k \approx T_1$ (see Fig. 3).

IV. CONCLUSION

We investigate numerically and theoretically a size-polydisperse granular gas mixture immersed into a molecular gas. We assume that the molecular gas with temperature T_g is not affected by the granular gas and plays a role of thermostat. The mixture is comprised of N different species of masses m_k ($k = 1, \ldots N$). We consider two models of dissipative collisions – a simplified model of a constant restitution coefficient, $\varepsilon = \text{const.}$ and a realistic model of viscoelastic particles, where the restitution coefficient depends on the relative velocities of colliding particles and their masses and sizes. For the both models we observe qualitatively similar behavior:the granular mixture rapidly relaxes to a steady state where granular temperatures of all species become equal, $T_k = T_1$ for all $k = 1, \ldots N$, that is, the energy equipartition is observed. At the same time the granular temperatures are not equal to the temperature of the molecular gas, $T_k < T_g$. This may be explained by the permanent energy flux from the molecular gas to the granular mixture in the steady state which compensates the energy losses in dissipative collisions of the grains. The observed energy equipartition in granular mixtures seems somewhat surprising, since generally, the equipartition does not hold in driven granular granular gases with different particle sizes.

The results of our study may be important to understand the properties of molecular gas-dust mixtures – the systems, where small dust particles are immersed in a surrounding molecular gas, like protoplanetary discs, interstellar dust clouds, sand storms, tornados, etc.

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[1] D. Apai, D. S. Lauretta, *Protoplanetary Dust. Astrophysical and Cosmochemical Perspectives.* (Cambridge University Press, 2010).
[2] J. Blum, Advances in Physics, **55**, 881 (2006).
[3] J. Blum, G. Wurm, Annu. Rev. Astron. Astrophys, **46**, 21 (2008).
[4] A.-C. Levasseur-Regourd, J. Agarwal, H. Cottin, C. Engrand, G. Flynn, M. Fulle, T. Gombosi, Y. Langevin, J. Lasue, Th. Mannel, S. Merouane, O. Poch, N. Thomas, A. Westphal, Space Sci Rev. **214**, 64 (2018).
[5] P. Thomas, P. J. Gierasch, Science **230**, 175 (1985).
[6] N. O. Renno, M. L. Burkett, and M. P. Larkin, J. Atmosph. Sci., **55**, 3244 (1998).
[7] N. V. Brilliantov and T. Pöschel, *Kinetic theory of Granular Gases* (Oxford: Oxford University Press, 2004).
[8] J. J. Brey, M. J. Ruiz-Montero, R. Garcia-Rojo and J. W. Dufty, Phys. Rev. E **60**, 7174-7181 (1999).
[9] J. W. Dufty, and J. J. Brey, New J. of Phys. **7**, 20 (2005).
[10] V. Garzo and J. W. Dufty, Phys. Rev. E **60**, 5706 (1999).
[11] S. R. Dahl, C. M. Hrenya, V. Garzo and J. W. Dufty, Phys. Rev. E **66**, 041301 (2002).
[12] R. D. Wildman and D. J. Parker, Phys. Rev. Lett **88**, 064301 (2002).
[13] K. Feitosa and N. Menon, Phys. Rev. Lett **88**, 198301 (2002).
[14] A. Bodrova, A. K. Dubey, S. Puri and N. Brilliantov, Phys. Rev. Lett., **109** 178001 (2012).
[15] N. Brilliantov, T. Poeschel, and A. Formella, Nature Communications, **9** 797 (2018).
[16] J. Schmidt, K. Ohtsuki, N. Rappaport, H. Salo, and F. Spahn, *Dynamics of Saturn's Dense Rings.* In: Dougherty, M. K., Esposito, L. W., Krimigis, S. M. (Eds.) *Saturn from Cassini-Huygens* (Springer) **413** (2009).
[17] N. Brilliantov, P. L. Krapivsky, A. Bodrova, F. Spahn, H. Hayakawa, V. Stadnichuk and J. Schmidt, Proc. Natl. Acad. Sci. USA **112**, 9536 (2015).
[18] F. Spahn, N. Albers, M. Sremčević, C. Thornton, Europhys. Lett., **67**, 545 (2004).
[19] K. Ohtsuki, Icarus **137**, 152 (1999).
[20] K. Ohtsuki, Icarus **183**, 384 (2006).
[21] H. Salo, Icarus **96**, 85 (1992).
[22] K. Saitoh, A. Bodrova, H. Hayakawa and N.V. Brilliantov, Phys. Rev. Lett., **105** 238001 (2010).
[23] P. Haff, J. Fluid Mech. **134**, 401 (1983).
[24] I. Goldhirsch and G. Zanetti, Phys. Rev. Lett. **70**, 1619 (1993).
[25] S. Das, and S. Puri, Europhys. Lett. **61**, 749 (2003).
[26] S.K. Das and S. Puri, Phys. Rev. E **68**, 011302 (2003).
[27] H. Nakanishi, Phys. Rev. E **67**, 010301(R) (2003).
[28] T.P.C. van Noije, M.H. Ernst, R. Brito and J.A.G. Orza, Phys. Rev. Lett. **79**, 411 (1997).
[29] T.P.C. van Noije, M.H. Ernst, and R. Brito, *Phys. Rev. E* **57**, R4891 (1998).
[30] S.R. Ahmad and S. Puri, Europhys. Lett. **75**, 56 (2006).
[31] S.R. Ahmad and S. Puri, Phys. Rev. E **75**, 031302 (2007).
[32] W. Goldsmit, *The Theory and Physical Behavior of Colliding Solids* (Oxford, Oxford University Press, 2004)
[33] F. Bridges, A. Hatzes and D. Lin, Nature **309** 333 (1984).
[34] G. Kuwabara and K. J. Kono, Appl. Phys. Part 1 **26**, 1230 (1987).
[35] R. Ramirez, T. Pöschel, N.V. Brilliantov, and T. Schwager, Phys. Rev. E **60**, 4465 (1999).
[36] T. Tanaka, T. Ishida and Y. Tsuji, Jap. Soc. Mech. Eng. **57**, 456 (1991).
[37] N.V. Brilliantov, F. Spahn, J.M. Hertzsch, and T. Pöschel, Phys. Rev. E **53**, 5382 (1996).
[38] W.A.M. Morgado and I. Oppenheim, Phys. Rev. E **55**, 1940 (1997).
[39] T. Schwager, and T. Pöschel, Phys. Rev. E **57**, 650 (1998).
[40] T. Schwager and T. Pöschel, Phys. Rev. E **78**, 051304 (2008).
[41] N.V. Brilliantov, A.V. Pimenova and D.S. Goldobin, Europhys. Lett., **109**, 14005 (2015).
[42] D. S. Goldobin, E. A. Susloparov, A. V. Pimenova, N. V. Brilliantov, Eur. Phys. J. E **38**, 55 (2015).
[43] A. Bodrova, D. Levchenko, N.V. Brilliantov, Europhys. Lett. **106**, 14001 (2014).
[44] N. Brilliantov and F. Spahn, Math. Comp. Simul., **72**, 93 (2006).
[45] T. Poeschel and T. Schwager, Computational Granular Dynamics: Models and Algorithms (Springer, Berlin, 2005).
[46] J. M. Montanero and A. Santos, Gran. Mat. **2**, 53 (2000).
[47] D. R. M. Williams and F. C. MacKintosh, Phys. Rev. E **54**, R9 (1996).