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Particle Properties and the Large-Scale Structure of Planetary Rings: Rebound Characteristics and Viscosity

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In a swarm of particles on Keplerian orbits, like a planetary ring, systematic flow is circular with velocity decreasing with distance from the planet. Viscosity ordinarily transports momentum from a faster to a slower moving region, i.e., outward. But any individual at apocenter of its orbit moves slower, and at pericenter faster, than the mean flow, suggesting a reversed momentum transport or negative viscosity. Resolution of this seeming paradox illuminates the importance of physical properties of particles. A conventional model has uniform, highly elastic, perfectly slippery, spherical particles, with a particular dependence of elasticity on impact velocity. Even slight deviation from that ideal affects viscosity and leads to inconsistencies with observed ring structure. If particles are less elastic they may clump into large temporary agglomerations. The size of the larger bodies in the rings probably determines random velocities, viscosity, and much about ring structure.

I. INTRODUCTION

A substantial body of dynamical theory has been developed in recent years to explain the large-scale structure of planetary rings revealed by Voyager spacecraft and by improved ground-based observations. The key issues addressed by this work have included angular momentum transport, spreading of material outward from and in toward the planet, vertical thickness, instabilities that may allow clumping into narrow ringlets, formation of gaps, and structure of density waves and bending waves. Interrelated parameters that are essential to all of this theory are the characteristic random relative velocity among ring particles, the coefficient of restitution for collisions, and the viscosity of the fluid-like system.

A considerable body of dynamical theory is based largely on the highly idealized physical model of particle collisions by Goldreich and Tremaine (1978). All particles are assumed to be identical in size, shape, and properties. Each is spherical and highly elastic (large coefficient of restitution). Very little energy is lost in collisions. Moreover, it is implicit that the surfaces must be smooth and slippery, ensuring that there is no exchange of energy between particles' rotation and their relative translational motion. The slippery surfaces also mean that in a collision the component of relative velocity tangential to the contact surface is preserved. Thus, even if the coefficient of restitution were small, very little energy would be lost in the case of a glancing blow.

These idealized physical properties seem very different from the sorts of materials generally found in nature. In this paper I show how alternative, and possibly more realistic, properties might change the large-scale dynamical picture. For example, conventional dynamical theory depends strongly on both implications of the slipperiness of particles: (i) the neglect of particle spin and (ii) the strong side-scattering from glancing blows. Perhaps ring particles are perfectly round, smooth, elastic, and slippery; collisions may have

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beaten them into that shape. But that picture is so idealized and speculative that we need to remain aware of the degree to which dynamical theory depends on it.

Whatever one's prejudices may be concerning the physical properties of a single particle, the assumption that all particles are the same size is surely incorrect. The Voyager radio experiment (Marouf *et al.* 1983) showed that, at least in regions where the radio beam could penetrate Saturn's rings, particle sizes cover a wide range. As Greenberg *et al.* (1977) predicted, the distribution is so broad that no single characteristic size can be meaningfully defined. The range is at least from centimeter-sized particles, which are sufficiently numerous that they provide most of the reflective cross section of the rings, to bodies several meters across, which contain most of the mass of the system (Fig. 1).

Thus the small particles define the visible large-scale structure of the system, while the large particles must play a major role in dynamical processes. For example, wave structure depends on self-gravity of the rings and thus on the presence of the large particles with all their mass. The small bodies are also important to dynamics, because with their great total surface area their collisions are most common and yield most of the energy dissipation of the system.

The distinction in the particle size distribution for Saturn's rings between small bodies with most of the area and large ones with most of the mass also helps clarify a seeming conflict between optical and dynamical estimates of ring thickness. The "opposition effect" of extra brightness at low phase angles (Franklin and Cook 1965) suggested a shadowing model with a layer many particles thick (e.g., Bobrov 1970), while collisions might be expected to damp random relative velocities to the extent that the system would rapidly flatten to a monolayer (Jeffreys 1947). Now it seems likely that the large bodies do nearly form a monolayer due to collisional damping, while the small ones, which govern optical scattering, are many particles thick in a disk not much more vertically extended than the large particles (e.g., Cuzzi *et al.* 1984).

The size distribution, combined with the idea that real particles might be less than ideally elastic, led to the suggestion that the larger ring particles are "dynamic, ephemeral bodies" (DEBs), temporary aggregations of centimeter-sized building blocks (Greenberg *et al.* 1983, Weidenschilling *et al.* 1984). In that model, inelastic collisions among the small objects result in accretion of the DEBs, each of which soon disrupts under tidal influence. If the lifetime of each DEB were greater than a couple of weeks, Saturn's rings would become much fainter as the numbers of small bodies and the corresponding optical thickness diminished.

The possibility that ring particles may

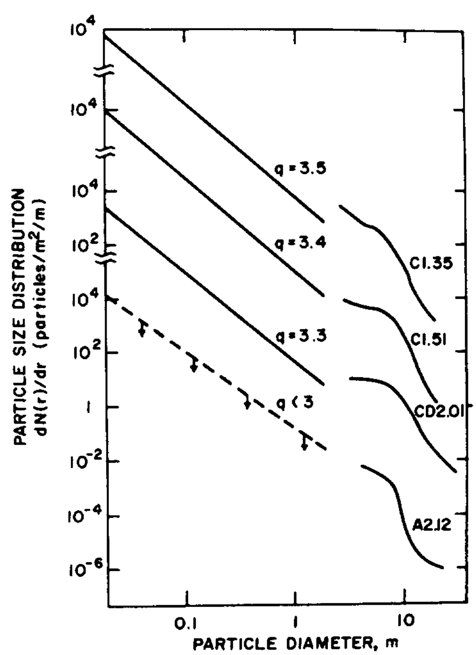

FIG. 1. Size distribution of ring particles measured by the Voyager radio science experiment (Marouf *et al.* 1983) at four locations in Saturn's rings: two in the C Ring, one in the inner A Ring, and one in the Cassini Division.

differ to some extent (although perhaps not so radically as in the DEB model) from the conventional, idealized model requires that we have some understanding of the sensitivity of dynamical theory to actual particle properties. A key collective property of the system is its viscosity, which characterizes transport of momentum. Physical interpretation of this process shows it to be very sensitive to particle properties and indicates how an alternative set of particle properties (the DEB model is one example) could govern observed ring structure.

II. WHAT IS VISCOSITY?

Viscosity is the property of a fluid that determines the shear stress on a surface within the fluid, for a given gradient in the velocity parallel to the surface. Such a shear stress can transport momentum and is therefore critical to the evolution of planetary rings. In rings the systematic flow of the swarm of ring particles is circular around the planet and follows Kepler's third law: The velocity decreases with distance from the planet. Thus there is the possibility of a shear stress across any planetocentric circle (more precisely a cylinder) within a ring (Fig. 2).

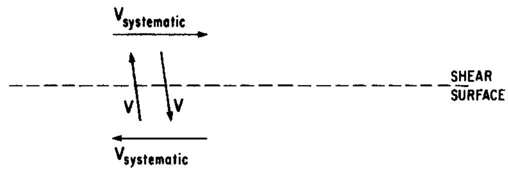

FIG. 2. Velocity shear in a planetary ring across a cylindrical surface centered on the planet. Systematic velocity decreases with distance from the planet according to Kepler's third law. As viewed here in a reference frame rotating at the Keplerian angular velocity at the shear surface, the relative systematic velocity goes in opposite directions on either side of the surface. Momentum is transported thanks to the random or "thermal" velocity V which allows particles to cross the shear surface and deposit momentum via collisions.

Superimposed over the systematic flow of the shearing disk is the random velocity of particles. From the celestial mechanical point of view, the random motion represents a particle's orbital eccentricity e (inplane component of the random velocity) and inclination i (out-of-plane component). If all e's and i's were zero, each particle would move on a circular path matching the Keplerian flow of the disk as a whole. With nonzero e's and i's, the systematic flow of the disk is unchanged, but the random motion gives it a finite dynamical "temperature." The random motion allows an exchange of particles across a shear surface and thus makes momentum transport possible, as each particle delivers some momentum to its new home via collision.

The finite size of ring particles can also play a role in the transport of angular momentum. Even if there were no random motion, only circular orbits, collisions and momentum transport could occur as particles move by one another to Keplerian shear. Here I assume, following Goldreich and Tremaine (1978), that particle size is negligible compared with the effect of random motion, but I return to this issue later in this paper.

The shear stress across the circle in Fig. 2 is given by:

$$\begin{aligned}\text{Shear stress} &= \text{momentum transfer/area/time}\\ &= (\text{momentum transport/collision})\ (\text{No. of particles crossing/area/time})\\ &= (m\ \Delta V_{\text{systematic}})\ (\text{No. of particles/volume})\ V, \end{aligned} \tag{1}$$

where m is the mass of each particle, V is the mean random velocity in the system, and $\Delta V_{\text{systematic}}$ is the difference in the local systematic velocity between the sites of successive collisions of a typical particle. The latter is roughly the mean free path times the velocity gradient, or $(V/\omega_c) \times (r\ d\Omega/dr)$, where ω_c is collision frequency, r is distance from the planet, and Ω is the angular velocity at r. The number of particles per volume can be expressed as ρ/m, where ρ is the mass density of the ring.

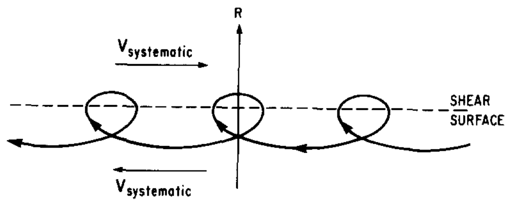

FIG. 3. Motion of a particle with random velocity in a case of rare collisions (small τ). The reference frame here is fixed to a point on a chosen shear surface, which is moving on a circular orbit at Keplerian angular velocity; the reference is also rotating at that same angular velocity. In this frame the particle, with a slightly smaller semimajor axis, follows the looped path shown. Radial transport distance is limited to the scale of the epicycles of Keplerian motion. Epicyclic velocity is such that collisions would seem to enhance, rather than reduce, the gradient in the systematic flow.

With these substitutions we obtain

$$\text{Stress} = (V^2/\omega_c)\rho r d\Omega/dr. \qquad (2)$$

By definition the quantity in parentheses is kinematic viscosity, ν.

The collision frequency for this special case in which all particles are the same size can be expressed approximately in terms of the optical thickness τ of the ring. A typical particle moves up and down through the thickness of the ring once per orbit. In fact, the physical thickness of the ring is defined by the characteristic vertical random motion in the system. In this vertical motion the probability of an impact is roughly the same as that for a beam of light trying to pass through the ring. Therefore

$$\omega_c = \tau\Omega. \qquad (3)$$

With this expression for ω_c, viscosity becomes

$$\nu = (V^2/\Omega)(1/\tau). \qquad (4)$$

We can compare this expression with the now-standard formula derived by Goldreich and Tremaine (1978), assuming a steady-state value for V and the idealized spherical, slippery particles:

$$\nu \sim (V^2/\Omega)(\tau/(\tau^2 + 1)). \qquad (5)$$

Our expression agrees perfectly for $\tau \gg 1$, but not for $\tau < 1$. The heuristic derivation above fails in the latter case, because it depends on the radial separation of successive collisions being comparable to the mean free path. If collisions are rare, a particle's radial motion will reverse direction twice in each orbit around the planet as it executes Keplerian elliptical motion. In that case, particles do not move as far radially between collisions as assumed in our derivation, so momentum transport and viscosity are much smaller than that given by (4).

Considered in terms of epicyclic motion in a reference frame rotating with the angular velocity of the ring, the limit on radial travel becomes even more graphically apparent (Fig. 3). For this case we must modify the above derivation by replacing the mean free path with the characteristic radial distance traveled on an epicycle, which is $\sim er = V/r\Omega$. Also, because momentum is not delivered by every particle on every crossing of the shear surface, we must include a correction factor ω_c/Ω, the fraction of orbits on which momentum is delivered. With these corrections, the viscosity expression becomes

$$\nu = (V^2/\Omega)\,\tau, \qquad (6)$$

in agreement with the standard formula (Eq. (5)) for small τ.

However, more complete consideration of this epicyclic viewpoint leads to a change of sign in the above formula and to some very important physical questions. In Fig. 3 we see that when a particle from inside the shear surface moves outside, it tends to be moving slower than the mean flow in the region it is visiting, even though it comes from a region where the mean flow is faster. In fact if the particle has moved radially a distance $\sim er$ between collisions as assumed in the derivation of Eq. (6), it is likely to be near apocenter of its Keplerian orbit, where its velocity relative to the local mean is $\sim -\Delta V_{\text{systematic}}$. If correct, this argument would mean that momentum transport is in the opposite direction, but the same magnitude, as estimated above. The sign for the viscosity formula for small τ (Eq. (6)) would be reversed, the standard formula (Eq. (5)) would be wrong, and momentum would be transported from the region of slow flow to the region of fast flow.

What is the correct sign of viscosity? The answer is critical to ring dynamics, affecting matters as fundamental as whether rings tend to spread radially or contract into narrow ringlets. With positive viscosity, angular momentum is taken from the inside of the ring and added to the outside, so the ring gets ever wider. If viscosity were negative, the opposite would occur.

One way to find the answer is by energy and angular momentum considerations (next section), which show that contraction is possible only under special or temporary conditions. In general, on a global scale spreading must occur. Apparently the heuristic discussion above is not adequate to incorporate all the essential physics of the momentum transport process. Global constraints are discussed in the next section. Then I give a more complete microscopic physical description which shows the crucial roles of the statistics of collision events and of the collisional scattering law.

III. ENERGY AND ANGULAR MOMENTUM CONSIDERATIONS

As a ring system loses energy due to particle collisions, conservation of angular momentum requires that mean random motion (orbital eccentricities of individual particles) must decrease and/or the ring must widen radially. A single particle's energy is given by

$$E = -GMm/2a, \tag{7}$$

where a is its orbital semimajor axis and M is the mass of the primary. A general loss of energy throughout the ring will tend to reduce semimajor axes. Now a particle's angular momentum is given by

$$H = m[GMa(1 - e^2)]^{1/2}. \tag{8}$$

Equation (8) shows how damping e can balance the decrease in semimajor axes. However, in a real ring e is probably damped to some equilibrium value very quickly (within a few orbits unless $\tau \ll 1$). In that case the net H for the system can be preserved by having some particles move outward, even while many move inward with the net decrease in E; i.e., the ring gets wider.

Integrating Eqs. (7) and (8) over the width w of a ring gives the relation between energy loss and spreading rates, assuming that eccentricities have already reached a constant mean value:

$$dE/dt \sim -(M_{\text{ring}}\Omega^2)d(w^2)/dt. \tag{9}$$

Because every particle has a collision with frequency $\omega_c = \tau\Omega$, the rate of energy dissipation in the ring is

$$dE/dt \sim -fM_{\text{ring}}V^2\tau\Omega, \tag{10}$$

where f is the fraction of impact kinetic energy lost in each collision. From Eqs. (9) and (10) we obtain

$$d(w^2)/dt \sim fV^2\tau/\Omega. \tag{11}$$

The rate of widening of the ring can also be compared with the viscous transport of angular momentum, yielding

$$d(w^2)/dt \sim \nu. \tag{12}$$

Comparing Eqs. (11) and (12) yields

$$\nu \sim fV^2\tau/\Omega. \tag{13}$$

Equation (13) is consistent with Goldreich and Tremaine's standard expression for viscosity (Eq. (5) above). When $\tau < 1$, they found that the steady-state condition requires $f \sim 1/2$, in which case Eq. (13) agrees with Eq. (5). In the case of $\tau \gg 1$, they found that a steady state requires f near zero, so Eq. (13) is consistent with Eq. (5) in that regime as well.

These global energy and angular momentum considerations still admit the possibility of fine-scale contraction into narrow ringlets; they do not rule out a negative effective viscosity on a local scale. Also angular momentum can be conserved during energy loss if particles' orbits become more circular, i.e., as the system cools down, so that spreading would be temporarily unnecessary. But in the long run on the whole any ring must spread. The spreading is consistent with the positive viscosity found by Goldreich and Tremaine for all values of τ. It demonstrates the inadequacy (at least for small τ) of the heuristic derivation in Section II, which was based on a microscopic picture with consideration of particle collisions. Yet, as discussed in the Introduction, the microscopic view is needed if we are to develop an appreciation of the connection between visible structure of rings and the physics of the collision processes.

IV. A MORE COMPREHENSIVE MICROSCOPIC VIEW

The problem with the microscopic view was not an intrinsic failure of that general approach, but rather that the version described in Section II was incomplete. It did not properly account for the statistics of where in particles' epicyclic motion sequential pairs of collisions took place and what the average momentum transfer was at each collision.

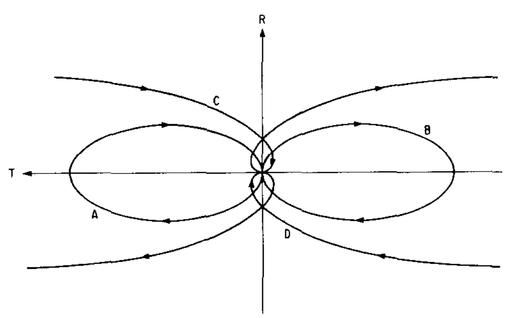

FIG. 4. Four of the many orbits with a given eccentricity that can pass through a typical point in a ring. The trajectories are shown in a rotating coordinate system centered on the point. The R axis points radially outward from the planet, and the T axis points in the direction tangential to the systematic circular flow. In this reference frame, trajectories with the same semimajor axes (A and B) as the point under consideration appear as closed epicycles, while others are open loops as in Fig. 3.

A way to consider angular momentum transport from the microscopic point of view and to incorporate the critical statistics is to evaluate the local velocity distribution in the neighborhood of a typical point in the ring, as did Goldreich and Tremaine. To develop an intuitive understanding of the processes involved, first suppose that the ring consists of a swarm of bodies all having exactly the same orbital eccentricity and uniformly distributed in semimajor axes and azimuthally about the planet. For simplicity I neglect out-of-plane motion (inclinations) in this discussion.

The velocity distribution relative to a given point revolving about the planet at the local Keplerian mean motion is obtained by considering the relative velocity of each orbit that can pass through that point (Fig. 4). These velocities are expressed relative to a reference frame rotating with the orbital angular velocity of the point under consideration. The locus in velocity space is an ellipse elongated in the radial direction (V_R) by a factor of 2 relative to the direction (V_T) tangent to the mean circular flow of the swarm (Fig. 5), assuming standard epicyclic motion to first order in orbital eccentricity. It can also be readily shown that the probability distribution along the velocity ellipse is not uniform; however, the distribution is such that if there is a narrow range of eccentricities de in the system, there is a uniform probability distribution over the narrow area in V_R, V_T space between the ellipses for e and $e + de$.

If there is a wide range of eccentricities represented in the swarm, the probability distribution in V_R, V_T space can be represented by a set of contours consisting of constant-e ellipses (Fig. 5). For a Gaussian distribution of V_T, there must also be a Gaussian distribution of V_R with twice the variance.

Now the viscous transport results of Goldreich and Tremaine can be explained in the following way (R. Narayan 1986, private communication). Contrary to the Keplerian orbital behavior described

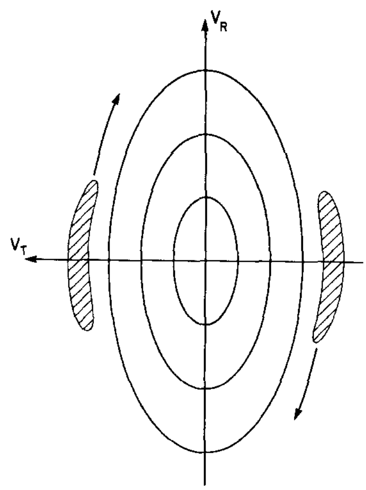

FIG. 5. Velocity distribution for particles in the neighborhood of a typical point in a ring. The locus for all particles with a single orbital eccentricity follows one of the ellipses shown. Collisions tend to circularize this distribution (assuming a side-scattering component to the scattering law), populating the grey areas. Keplerian motion tends to spread (arrows) this distribution along the elongated ellipses. This interplay tilts the distribution so as to give momentum transport.

above, collisions among particles tend to equalize the mean random motion in the radial and tangential directions. Collisions tend to drive the "temperature" toward equality in all directions. The probability distribution in Fig. 5 becomes more circular as particles are removed from extreme positions on the V_R axis and effectively added to the right and left extremes on the V_T axis in Fig. 5. Orbits like A and B in Fig. 4 tend to be replaced by orbits like C (with positive V_T) and D (with negative V_T). Because the collision frequency per particle is $\sim\tau\Omega$, in any time interval dt a fraction of particles $\sim\tau\Omega dt$ becomes roughly isotropically distributed in velocity space, with variance $\sim V$ as long as little energy is lost in each impact.

If we consider the population of particles set on such new orbits in a given time increment dt, we expect them to continue on these orbits over at least one epicycle if τ is small and collisions are rare. As they orbit, they will tend to become redistributed around the elongated ellipses in velocity space. Particles placed on the + and − V_T axes by collisions tend to be moved respectively into the $(+V_T, +V_R)$ and $(-V_T, -V_R)$ quadrants by Keplerian motion (see Figs. 4 and 5) and then around the Keplerian velocity ellipsoid. Keplerian motion destroys any circular symmetry that collisions tend to create. Although collisions are infrequent, the number of these particles decays exponentially (time constant $\sim 1/(\tau\Omega)$) as they are picked off by collisions: More will be removed in the first quarter of an orbit than in the second, in the third than in the fourth, etc. For such exponential decay, at any given time in a steady state, the fraction of particles moved 0° to 90° or 180° to 270° since their last collision, minus the fraction moved 90° to 180° or 270° to 360°, is $\sim\tau$.

Thus the continual play-off between the tendency of collisions to circularize the velocity distribution and the tendency of Keplerian motion to spread the particles back onto the elliptical velocity distribution tilts the distribution in velocity space. Particles moving outward $(+V_R)$ generally have a greater than average tangential velocity $(+V_T)$; those moving inward have smaller than average tangential velocity. Now the viscosity can be expressed (as an extension of Eq. (1)) as $\nu \sim \langle V_R V_T\rangle/\Omega$. Most of the distribution is symmetrical so as not to contribute to this average cross-product, except for the excess fraction τ which is elongated and oriented roughly 45° from the axes (Fig. 5). For this fraction of the population, $\langle V_R V_T\rangle \sim V^2$, so averaged over all particles $\nu \sim V^2\tau/\Omega$. This result is in perfect agreement with the results of Goldreich and Tremaine (Eq. (5)) for low τ.

According to these results, viscosity approaches zero in the extreme of very low τ, because collisions are so infrequent that Keplerian motion is able to keep particles well distributed around the ellipses in velocity space, eliminating the tilted component of the distribution needed for mo-

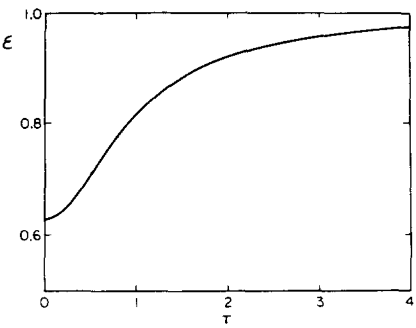

FIG. 6. Equilibrium values of coefficient of restitution ε as a function of optical thickness τ for ideal, spherical, slippery particles (Goldreich and Tremaine 1978). For lower ε the random velocity damps down. For ideal material this would raise ε, until the system reaches equilibrium. Similarly, if ε is too large, the system also evolves toward equilibrium.

mentum transport. In the extreme of very large τ, collisions are so frequent that the velocity distribution is circularized and Keplerian motion can have little effect; again symmetry prevents momentum transport.

This explanation of the viscous transport process has several important implications. The continual tendency of collisions to circularize the velocity distribution also tends to increase orbital eccentricities; particles are moved to higher constant-e lines in velocity space. On the other hand, energy loss in collisions tends to damp random relative motion. Thus steady-state random motion requires that the rate of damping as governed by the coefficient of restitution ε must exactly balance the pumping that accompanies viscous transport. For the ideal particles the critical value of ε is 0.63 for $\tau \ll 1$ and ranges up to 1 for $\tau \gg 1$ (Fig. 6). In other words, the ideal particles, already assumed to be smooth, round, and slippery, would also have to be very elastic.

Goldreich and Tremaine expected not only that smooth ice spheres would have such large values of ε, but also that for that material ε might increase with V (Fig. 7). They suggested that ε would evolve toward the value required for a steady state, be-

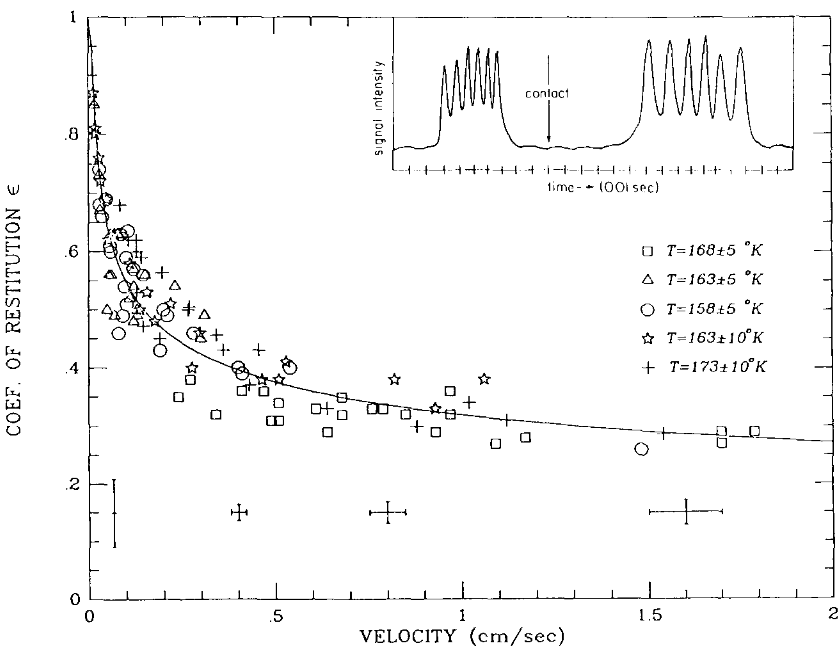

FIG. 7. Experimentally determined relation between ε and V for ice spheres (Bridges *et al.* 1984), which determines equilibrium random velocity in Saturn's rings according to Goldreich and Tremaine's (1978) theory.

cause if ε were too small (or too large), V would be damped (or pumped) until ε reached the steady-state value. Laboratory experiments with cold ice spheres (Bridges *et al.* 1984) have confirmed those predictions about the elasticity and its dependence on velocity. According to Goldreich and Tremaine's theory, for any given value of τ the V vs ε relation for the material determines the equilibrium value of V and thus of the viscosity.

This result has been invoked to explain contraction of material into many narrow ringlets (Lin and Bodenheimer 1981, Ward 1981). The ringlet instability comes from the smaller steady-state V in very dense (large τ) regions, which yields a weaker viscous stress and thus a transport of material into the denser regions. The system conserves angular momentum, despite the contraction, by lowering V (equivalently, e).

Whether such instabilities are the correct explanation of the narrow ringlet structure in much of Saturn's rings remains problematical. Even if the idealized particle-property assumptions on which it is based were valid, it is not clear that the distribution of optical thickness observed in the rings is consistent with that predicted by that instability model. Moreover, the instability model does not work in the small τ regime, because the direct dependence of viscosity on τ (see Eq. (6)) means viscous stress increases with τ and contraction cannot occur.

There is also a potentially important inconsistency in the theory that predicts equilibrium values for V, ε, and ν. For any τ, the equilibrium value of ε (Fig. 6) was computed (Goldreich and Tremaine 1978) on the assumption of a gaussian distribution of random velocities. In such a system collisions occur over a wide range of velocities, but ε was taken to be constant for all collisions in Goldreich and Tremaine's equilibrium solution. Then, once the equilibrium value of ε was established, the V vs ε relation was invoked to explain why the system should always evolve toward equilibrium. In effect, Goldreich and Tremaine took the coefficient of restitution to be dependent on the rms random velocity of the system rather than on the actual velocity of individual collisions.

In order to make the theory self-consistent, the V vs ε relation should be incorporated into the equilibrium solution. Other corrections to the dynamics could have significant implications. For example, in the discussion in this section, as well as in the analysis by Goldreich and Tremaine, collisions were assumed to occur among particles all at the same place in space. In fact, due to the finite size of particles, collisions involve particles in slightly different places where the velocity distribution is not identical (Araki and Tremaine 1986). When such nonlocal collisions and velocity-dependent ε are taken into account, the velocity distribution may well differ from a Gaussian.

These and other improvements to the theory of ring dynamics are being carried out by several researchers (e.g., Araki 1987, Brophy and Esposito 1987). The physical approach to interpretation of the dynamics as introduced in this paper (especially in this section) will be useful in interpreting the new results. An immediate benefit of this physical approach is the ability to consider (in Section V) the critical role that particles' physical properties can play in viscous transport.

V. PARTICLE PROPERTIES REQUIRED FOR VISCOUS TRANSPORT

The heuristic explanation for viscosity in the low τ regime given in the previous section depends on a tendency of collisions to circularize the distribution of particles in velocity space. However, such circularization requires a specific property of ring particle collisions: They must be fairly strongly side-scattering; i.e., a substantial fraction of impacts must result in rebound velocity with a strong component perpendicular to the relative velocity before impact. This requirement becomes apparent after the following considerations.

The relative velocity between two particles is the vector connecting the points in V space (Fig. 5). Suppose an impact is completely inelastic. Then after the impact the relative velocity is zero, and the two particles occupy the same point in V space, midway between their original velocities. If the impact is instead partially elastic, then the final velocities will each be part-way back from the midpoint to the original positions on the V plane. If the impact is strongly side-scattering, then their final velocities lie well off the line connecting the original velocities. For example, consider an impact between a particle initially on the relatively well-populated $+V_R$ axis with another on the equally well-populated $-V_R$ axis. With no sidescattering, the final velocities will both lie on the V_R axis. The only way such a collision could contribute to the viscosity story, by helping them occupy the $+V_T$ and $-V_T$ axes, would be with side-scattering.

With no side-scattering, it can be proven that collisions would on average preserve the elongated elliptical distribution in velocity space, because the statistical redistribution is identical to that in a foreshortened isotropic Gaussian distribution in velocity space. The rms velocity decreases, but the shape of the distribution is preserved. Thus side-scattering is required in order for the viscosity mechanism to work.

The slippery sphere model (Goldreich and Tremaine 1978) is fairly strongly side-scattering (Fig. 8b). Head-on collisions result in reduction of relative velocity V_{rel} to εV_{rel}, with no side-scattering of course. But for more glancing blows, very little energy is lost and those events are both more common and strongly side-scattering. For a given impact velocity V_{rel}, final velocities are distributed with uniform probability over a sphere of radius $(1 + \varepsilon)\, V_{rel}/2$. Even with $\varepsilon = 0$, a substantial fraction of collisions yields side-scattering at $\sim V_{rel}/2$.

If the particles were not so strongly side-

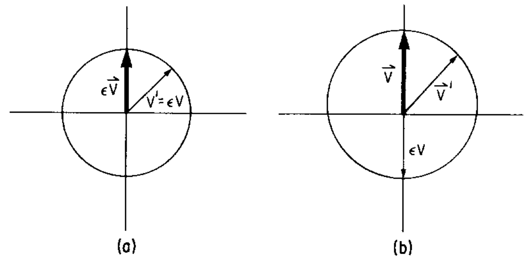

FIG. 8. (a) An isotropic scattering law, and (b) the scattering law for slippery, elastic spheres (Goldreich and Tremaine 1978). In both cases, the post-impact relative velocity (V') distribution is uniform over a sphere in velocity space. In (a) the radius of the sphere is εV and in (b) the radius is $V(1 + \varepsilon)/2$.

scattering, the viscosity would be reduced. Moreover, the tendency to increase random velocities that accompanies the circularization of the velocity distribution must also decrease. Therefore for equilibrium of random velocity, the coefficient of restitution would need to be even larger than that required by Goldreich and Tremaine.

Suppose the scattering law were perfectly isotropic (Fig. 8a): final relative velocities distributed uniformly over a sphere in velocity space, centered on the origin, with radius εV_{rel}. If $\varepsilon < 1$, this law is less strongly side-scattering than slippery spheres, except in the special case of perfect elasticity. Numerical simulations of ring evolution with isotropic scattering (Spaute and Greenberg 1987) show that the value of ε necessary for equilibrium is 0.83 for $\tau \ll 1$, indeed greater than the slippery-sphere value $\varepsilon = 0.63$.

Another relatively weakly side-scattering model can be constructed by abandoning the slipperiness requirement, while otherwise embracing the uniform-elastic-sphere assumptions. This change reduces side-scattering in a couple of ways. First, because the tangential component of relative velocity at the impact point is not conserved, glancing blows lose more energy than in the slippery model. Second, a partitioning of random motion into particles' rotation is important because collisions between spinning particles tend to be more uniformly scattering. Without slipperiness, a steady-state solution (Araki 1987) accordingly requires very large ε, ranging from 0.9 at $\tau \ll 1$ to nearly 1 for $\tau \gg 1$. Material in the rings would have to be remarkably elastic.

The experimental results for ice (Fig. 7) show that such elasticity is possible, but only if random velocities are ~0.01 cm/sec. Such a low V would solve one dynamical problem. The viscosity of the rings would be reduced by a factor of 100 compared to its value given the widely accepted random velocity $V \sim 0.1$ cm/sec, so the problem of the embarrassingly short time scale for spreading of the A Ring (Esposito 1986) is eliminated. However, studies of the observed damping of density waves and bending waves are consistent with the higher values of V and the larger viscosities to which they correspond. This discussion shows that simply abandoning the assumption of extreme slipperiness of particles has significant effects on the dynamical model.

VI. OTHER KINDS OF PARTICLES

The ideal-sphere model seems to be self-consistent only if the particles are perfectly slippery, and even then it contradicts the observed size distribution in Saturn's rings. But there is a wide range of alternative particle models that could and should be explored theoretically and tested against observations. One advantage of the slippery sphere model was relative ease of incorporation into a dynamical theory. The messier behavior of real particles may not be so simple.

On the other hand, one kind of particle that seems intuitively plausible actually implies what may be the simplest dynamics possible consistent with observations of rings. Suppose that particles are very inelastic, as might be the case if they are not very smooth or if they have a thin coating of fragmented debris on their surfaces. (Levitation of such "regolith" material from the surfaces has been invoked as a source of fine material in the temporary spokes in Saturn's rings.) In effect each collision would simply reduce both particles' velocities to their average value.

With nearly zero ε the particles become gravitationally bound to one another. They may accrete into large agglomerations (the DEBs described in the Introduction) so it becomes necessary to treat the dynamics of a system of bodies with a range of sizes. But that degree of complication is necessary in any case because we know from observations that particles really do have a range of sizes.

Very inelastic collisions do not tend to circularize the velocity distribution, so the viscosity mechanism for small τ would become inoperative. The predominant effect of collisions would be to reduce random velocities. The system would evolve toward a quiescent steady state with purely circular orbits. With collision frequency assumed to be governed by the random motions among particles, as in the discussion above, there would be no further collisions.

However, there are effects that prevent random motion from damping to zero, even when there is no stirring due to collisional side-scattering. One effect is gravitational scattering among particles, which converts systematic Keplerian shear into random motion. Velocities tend to be on the order of the escape velocities from the dominant scatterers (Safronov 1972). Gravitational scattering would give random velocities of only $\sim 10^{-3}$ cm/sec if centimeter-sized bodies dominated the system, but the presence of 5-m-diameter bodies dominating the system in terms of mass means that random velocities can be pumped to ~0.1 cm/sec, just the value needed to explain observed wave structure (Cuzzi *et al.* 1979).

Another effect that sets a lower limit on the damping of random motion is the physical size of particles. Even if all particles were on perfectly circular orbits, collisions would occur as they shear by one another within a distance smaller than their cross-section. Such collisions would induce random motion comparable to the size of the bodies, that is to say the subsequent radial and vertical excursion would be comparable to the body size (Brahic 1977, Araki and Tremaine 1986). For centimeter particles, the random motion induced that way would be negligible, again $\sim 10^{-3}$ cm/sec. However, several-meter-size bodies would be large enough to induce the required ~ 0.1 cm/sec.

In the DEB model, mutual collisions among the larger agglomerations might cause the bodies to fall apart rather than to divert onto new orbits, because the larger bodies are assumed to be already near the point of tidal disruption. The DEBs are prone to fall apart in the planet's tidal potential once they get large because, as agglomerations, they are effectively weaker and less dense than the small bodies which they comprise. Moreover, as mentioned in the Introduction, if these agglomerations did not break up into their constituent centimeter-sized building-blocks after only a short lifetime, the small bodies would be rapidly depleted.

The very act of falling apart may be a means by which the large DEBs launch the centimeter-size building blocks onto orbits with random motion of ~ 0.1 cm/sec. While still part of a DEB, a building block is forced to move with the DEB's mean motion but is typically held away from the center of the DEB at a distance comparable to the DEB's size. As the DEB comes apart due to tidal disruption the building block finds itself on an independent, now eccentric, orbit. The building block's next collision will begin its accumulation into another DEB. In this model, each centimeter-size particle is likely to go through this cycle every couple of weeks, so random motions would be governed by separation from DEBs.

The random velocity among ring particles depends directly on the size of the largest particles in the swarm, whether random motion is governed by gravitational stirring (Cuzzi *et al.* 1979), by jostling (Brahic 1977, Araki and Tremaine 1986), or by tidal breakup of DEBs. Particle sizes measured by Voyager are just right to give the random velocity and viscosity required to explain wave structure. This interpretation contrasts with that of Goldreich and Tremaine in which the V vs ε relation for perfect ice spheres determined the steady-state random motion, independent of particle size.

The viscous instability mechanism (discussed in Section IV) for making narrow ringlets no longer seems viable, because it depended on Goldreich and Tremaine's mechanism for maintaining random velocity. Besides, it was never very successful at explaining the observed nature of Saturn's ringlet structure (Borderies *et al.* 1984).

However, the dependence of viscosity on particles' physical properties suggests considering other viscous instability mechanisms. For example, suppose DEBs were more readily broken apart in regions with higher optical thickness, so they could not get as large as elsewhere. Then random motion would be cooler where τ is larger, precisely the condition required for viscous instability. Whether such speculation can stand up to quantitative scrutiny remains to be seen.

Another ringlet instability mechanism, also independent of the Goldreich–Tremaine velocity model, follows from the description of momentum transport in Section IV. If there were a radial density gradient, then the Keplerian 2/1 ellipses representing the velocity distribution (e.g., Fig 5) would be offset in the positive or negative V_T directions (depending on whether the radial density gradient were positive or negative, respectively). Then, as the collision products are smeared onto ellipses in velocity space, the resulting distribution represents not only angular momentum transport, but also mass transport, which can be toward the direction of greatest density (see also Stewart 1987).

VII. CONCLUSION

While some of these ideas about alternative models of particles' physical properties are rather qualitative and speculative, they do indicate the extent to which dynamical analyses may be affected by such changes. Theories that explain the large-scale structure of planetary rings are strongly dependent on assumptions about the physical properties of individual particles.

By considering the relative rates of collisional and Keplerian processes, it has been possible to reproduce very simply and to elucidate many of the important theoretical results based on the highly simplified (round, smooth, slippery, uniform) particle model. This approach may be useful in examining more realistic particle models, such as a model with inelastic collisions that might lead to growth of DEBs, which have the potential for producing an internally consistent dynamical theory.

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