Boltzmann

(1)

$$\partial_t f_\alpha + \vec{v}_\alpha \cdot \nabla_\alpha f_\alpha + \vec{a}_\alpha \cdot \frac{\partial f_\alpha}{\partial \vec{v}_\alpha} = I \dots \qquad (*)$$

derive $\vec{u}_\alpha = \frac{1}{n_\alpha} \int d^3\vec{v}_\alpha \, \vec{v}_\alpha \, f(\vec{v}_\alpha)$

by assuming Maxwellian $\underline{f_\alpha \sim \exp\left\{-\frac{m(\vec{v}_\alpha - \vec{u}_\alpha)^2}{2T_\alpha}\right\}}$

with mass dependent temp. $\underline{T_\alpha = \frac{1}{n_\alpha} \int d^3\vec{v}_\alpha \frac{m_\alpha}{2} \vec{c}_\alpha^{\,2} f_\alpha}$

$$\underline{\vec{c}_\alpha = \vec{v}_\alpha - \vec{u}_\alpha}$$

for a given size distribution

$\underline{n(s_\alpha) \simeq n_\alpha \sim s_\alpha^{-\gamma}}$ (power law)

(1) Hill's equations $\Rightarrow$ see my Celest.-Mech. Skrip[t]
at my home page!

Hill's equations (without perturbation)

$$\boxed{\begin{aligned} \ddot{x} - 2\Omega_0 \dot{y} - 3\Omega_0^2 x &= 0 \quad (1) \\ \ddot{y} + 2\Omega_0 \dot{x} &= 0 \; (2) \; ; \quad \ddot{z} + z\Omega_0^2 = 0 \; (3) \end{aligned}}$$

For a stationary mean field $\vec{u} = (\dot{x}, \dot{y}, \dot{z})^T \iff \ddot{x} = \ddot{y} = 0$

$\Rightarrow$ Eq. (1) $\Rightarrow \boxed{-2v_y - 3\Omega_0 x = 0}$

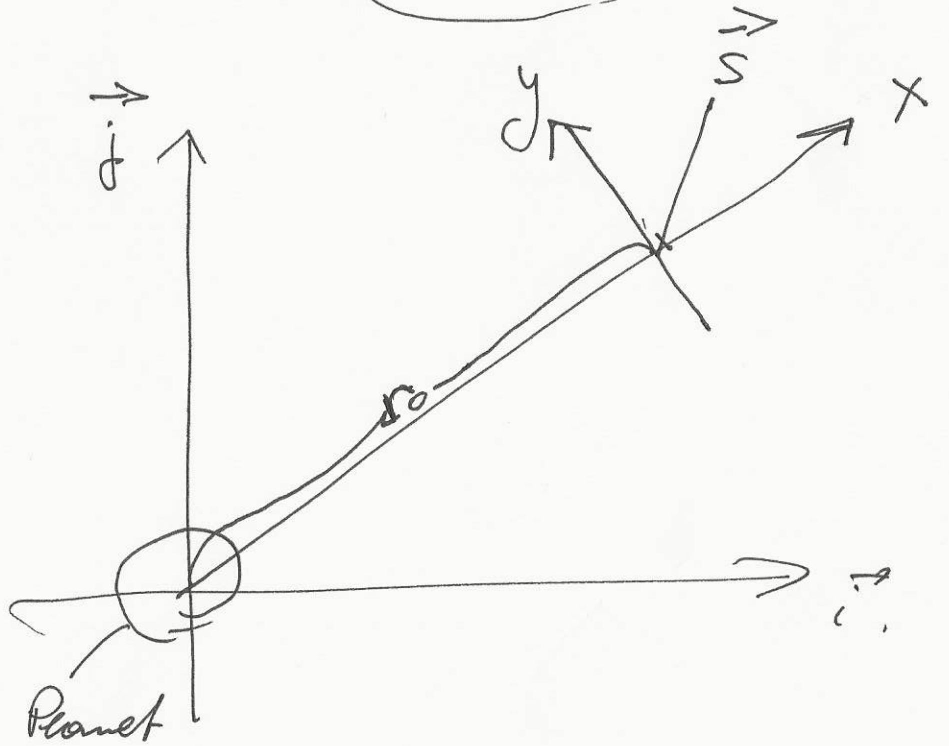
$v_y = \dot{y} = -\frac{3}{2}\Omega_0 x \; ; \quad v_x = \dot{x} = 0$

$\hat{=}$ corresponds to circular orbits

$\Rightarrow$ Eq. (*) + Maxwellian we have $\boxed{\vec{u}_d = -\frac{3}{2}\Omega_0 x \, e_y}$

($\Rightarrow$ plug in in Maxwellian for f_d)

$\vec{S} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$



Multiplying Eqs. (1), (2) & (3) by $\dot{x}, \dot{y}, \dot{z}$ and then

(3)

adding them we obtain:

$$\ddot{x}\dot{x} + \ddot{y}\dot{y} + \ddot{z}\dot{z} = \underbrace{\cancel{2\Omega_0 \dot{x}\dot{y}} - \cancel{2\Omega_0 \dot{x}\dot{y}}}_{\text{Coriolis does not exert work}} + 3\Omega_0^2 \dot{x}x - \dot{z}z\Omega_0^2$$

$$\Rightarrow \frac{d}{dt}\left(\frac{v^2}{2}\right) = \frac{3}{2}\Omega_0 \frac{d}{dt}\left(\frac{x^2}{2}\right) - \Omega_0^2 \frac{d}{dt}\left(\frac{z^2}{2}\right)$$

This allows for formulating a potential ϕ_H Hills

$$\phi_H = -\frac{3}{2}\Omega_0^2 x^2 + \frac{(\Omega_0 z)^2}{2} \quad \text{with:}$$

$$3\Omega_0^2 x = -\frac{\partial \phi_H}{\partial x}, \quad -\Omega_0^2 z = -\frac{\partial \phi_H}{\partial z} \Rightarrow \text{giving our}$$

desired acceleration for particle α:

$$\boxed{\vec{\omega}_\alpha = 3\Omega_0^2 x_\alpha \vec{e}_x - \Omega_0^2 z_\alpha \vec{e}_z}$$

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Fazit / Result

$$\vec{U}_\alpha = -\frac{3}{2}\Omega_0 x\, \vec{e}_y$$

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$$\vec{\omega}_\alpha = 3\Omega_0^2 x_\alpha \vec{e}_x - \Omega_0^2 z_\alpha \vec{e}_z$$

Bgl.

$$\boxed{\partial_t f_\alpha + \vec{v}_\alpha \cdot \nabla_\alpha f_\alpha + \vec{\omega}_\alpha \cdot \frac{\partial f_\alpha}{\partial \vec{v}_\alpha} = \ldots I}$$

$$\boxed{\frac{\partial}{\partial \vec{s}_\alpha} = \left(\vec{e}_x \frac{\partial}{\partial x_\alpha} + \vec{e}_y \frac{\partial}{\partial y_\alpha} + \vec{e}_z \frac{\partial}{\partial z_\alpha}\right)}$$

and $\vec{c}_\alpha = \vec{v}_\alpha - \vec{U}_\alpha$

In the Integral $I \Rightarrow$ we will use:

$$f_\alpha = \left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{3/2} \exp\left\{-\frac{m(\vec{v}_\alpha - \vec{c}_\alpha)^2}{2T_\alpha}\right\}$$