Universality of temperature distribution in granular gas mixtures with a steep particle size distribution

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Abstract –Distribution of granular temperatures in granular gas mixtures is investigated analytically and numerically. We analyze space uniform systems in a homogeneous cooling state (HCS) and under a uniform heating with a mass-dependent heating rate $\Gamma_k \sim m_k^{\gamma}$. We demonstrate that for steep size distributions of particles the granular temperatures obey a *universal* power-law distribution, $T_k \sim m_k^{\alpha}$, where the exponent α does not depend on a particular form of the size distribution, the number of species and inelasticity of the grains. Moreover, α is a universal constant for a HCS and depends piecewise linearly on γ for heated gases. The predictions of our scaling theory agree well with the numerical results.

Introduction. – Granular materials, such as gravel, sand or different types of powders, are ubiquitous in nature and widely used in industry [1]. Rarified granular systems, where the volume of a solid phase is small as compared to the total volume, are termed as granular gases [2]. In the Earth conditions these may be obtained by placing granular matter into a container with vibrating [3] or rotating [4] walls, applying electrostatic [5] or magnetic [6] forces etc. Extraterrestrial granular gases are also common. Many astrophysical objects, like protoplanetary discs, Planetary Rings and interstellar dust clouds contain granular gases as one of the important component [7].

Granular particles collide inelastically, loosing their kinetic energy during the collisions, which transforms into heat (the energy of the internal degrees of freedom). If no energy is supplied to a system from an external source, the particles gradually slow down. Such state of granular gases is called a homogeneous cooling state (HCS) [2]; it can be realized in a microgravity environment, e.g. [8]. In the HCS the average kinetic energy of particles decreases and after some time a space homogeneous distribution looses its stability, e.g. [2]. Contrary, driven gases may permanently remain homogeneous. In the present study only space uniform systems are addressed.

Real granular materials are not comprised of identical particles but rather represent a polydisperse mixture of grains of different mass and size. Particular forms of particle size distributions may be very different, depending on the nature of granular systems. Power-law size distribution is, however, rather common. It is observed in Planetary Rings [7] and may be used to describe industrial materials, e.g. [9].

A statistical description of a space homogeneous gas is based on the velocity distribution functions $f_k(\mathbf{v}_k, t)$, which gives the number of particles of sort k with velocity $\mathbf{v}_k$ in a unit volume at time t. Correspondingly,

$$n_k = \int f_k\left(\mathbf{v}_k, t\right) d\mathbf{v}_k \qquad (1)$$

are number densities of particles of the sort k, which keep constant in a homogeneous gas. An important characteristic of a granular system is a mean kinetic energy of the gas particles of sort k, e.g. [11, 12],

$$\frac{3}{2} n_k T_k(t) = \int d\mathbf{v}_k f_k\left(\mathbf{v}_k, t\right) \frac{m_k v_k^2}{2}\,, \qquad (2)$$

where m_k is the corresponding mass of the grains. T_k are also termed "partial granular temperatures". In a sharp contrast with molecular gases, the energy equipartition does not hold in a mixture of granular gases, that is $T_k \neq T_l$ for $k \neq l$. This has been shown in experiments [3] and explained theoretically [2, 11, 13, 14]. Physically, this follows from the nature of a dissipative collision between particles of different mass: The larger the mass, the smaller the part of a particle energy lost in a collision.

Quantitatively, the dissipation of energy in such collisions is described by a restitution coefficient:

$$\varepsilon_{ik} = \left| \frac{(\mathbf{v}'_{ik} \cdot \mathbf{e})}{(\mathbf{v}_{ik} \cdot \mathbf{e})} \right| . \qquad (3)$$

Here $\mathbf{v}'_{ik} = \mathbf{v}'_k - \mathbf{v}'_i$ and $\mathbf{v}_{ik} = \mathbf{v}_k - \mathbf{v}_i$ are the relative velocities of two particles of a sort k and i after and before a collision, and $\mathbf{e}$ is a unit vector connecting particles' centers at the collision instant. Note, that generally ε_{ik} depends not only on the particular species, involved in an impact, but also on the relative velocities of the particles, e.g. [2, 15, 16]. In what follows, however, we will use the simplifying approximation, $\varepsilon_{ik} = \text{const}$.

The behavior of polydisperse granular gases is rather well studied, not only for a simple binary distribution [3, 10, 11, 13], but also for more complex distributions, including a continuous one. Namely, polydisperse mixtures with uniform [12] and power-law [17] size distributions, as well as mixtures of particles of equal mass, which differ by restitution coefficients [18] have been investigated theoretically [12, 18] and numerically [12, 17, 18]. In Refs. [19, 20] theory of inhomogeneous polydisperse granular gases has been developed.

Although elaborated theory of granular mixtures allows to compute partial temperatures T_k, it may be done only numerically; a simple analytical expression which allows to describe distribution of temperatures is still lacking. While it is known [11, 12] that for a constant restitution coefficient the law of equal cooling rates $\dot{T}_k/T_k = \dot{T}_l/T_l$ in a HCS[1] holds true for all k, l, there is no general relation between temperature distribution and particle size distribution. It would be rather desirable to have such simple relation, at least for particular cases.

In the present study we show that if the particle size distribution $n_k(k)$ in a granular mixture of N different species is steep enough, the temperature distribution $T_k \propto m_k^{\alpha}$ is *universal*, that is, the scaling exponent α depends neither on inelasticity of particles nor on a particular form of the distribution $n_k(m_k)$; it is also (asymptotically) independent on the total number of species for $N \gg 1$. Our conclusion holds true for a force-free gas in a HCS as well as for a gas under a uniform heating, with a power-law dependence of the heating rate on a mass.

Model. – We consider a system with discrete particles' masses: $m_k = m_1 k$, where $k = 1, 2, \ldots N$, so that N is the total number of different species. In the context of Planetary Rings, where particles are aggregates built up of smaller grains, m_1 may be interpreted as a mass of the smallest particles, hence k characterizes the size of the particles. All particles have the same density. For simplicity we assume that the grains are spherical with the diameters of the species $\sigma_k = \sigma_1 k^{1/3}$ and smooth (i.e. the rotational motion is neglected). We also assume that the number of large particles in the system is significantly smaller than that of small particles, that is, we consider a *steep* particle size distribution. While a quantitative definition of a steep distribution is given below, we wish to emphasize now, that system's kinetics is mainly determined in this case by small and intermediate particles. For space homogeneous granular mixtures addressed here the number densities of species, $n_k(m_k)$, are stationary, $\dot{n}_k = 0$, and the partial velocity distribution functions $f_k(\mathbf{v}_k, t)$ evolve subjected to the Boltzmann equation:

$$\frac{\partial}{\partial t} f_k\left(\mathbf{v}_k, t\right) = I_k^{\text{res}} + I_k^{\text{heat}} . \qquad (4)$$

The first term in the right-hand side of Eq. (4) is the collision integral for restitutive collisions in a multi-component granular gas [2, 19, 20]:

$$I_k^{\text{res}} = \sum_{j=1}^{N} \sigma_{kj}^2 g_2(\sigma_{kj}) \int d\mathbf{v}_j \int d\mathbf{e}\, \Theta(-\mathbf{v}_{kj}\cdot\mathbf{e})\, |\mathbf{v}_{kj}\cdot\mathbf{e}| \times \left[\frac{1}{\varepsilon_{kj}^2} f_k(\mathbf{v}_k{}'', t) f_j(\mathbf{v}_j{}'', t) - f_k(\mathbf{v}_k, t) f_j(\mathbf{v}_j, t) \right] , \qquad (5)$$

where $\sigma_{kj} = \left(\sigma_k + \sigma_j\right)/2$ and $g_2(\sigma_{kj})$ is a contact value of the pair correlation function for species j and k, which takes into account the effects of excluded volume. In what follows we employ the approximation $g_2(\sigma_{kj}) = 1$, valid for dilute systems. The summation in Eq. (5) is performed over all species and the rest of the notations has its usual meaning, e.g. [2]: $\mathbf{v}_k''$ and $\mathbf{v}_j''$ denote pre-collision velocities in the so-called inverse collision, resulting in the post-collision velocities $\mathbf{v}_k$ and $\mathbf{v}_j$. The Heaviside step-function $\Theta(-\mathbf{v}_{kj}\cdot\mathbf{e})$ selects the approaching particles, etc.

The second term I_k^{heat} describes the heating of the system. It quantifies the energy injection into a granular gas to compensate its losses in dissipative collisions; it is zero for a gas in a HCS. Here we consider a uniform heating – the case when the grains suffer small random uncorrelated kicks throughout the volume [22]. To mimic the external driving forces a few types of thermostat have been proposed [23]. For a thermostat with a Gaussian white noise, the heating term has the Fokker-Planck form [24]:

$$I_k^{\text{heat}} = \frac{1}{2}\frac{\Gamma_k}{m_k}\frac{\partial^2}{\partial \mathbf{v}_\mathbf{k}^2} f_k(\mathbf{v}_\mathbf{k}, t). \qquad (6)$$

Here the constant Γ_k characterizes the strength of the driving force. It may vary for different species, depending on the type of driving [12, 25]. If all species are supplied with the same energy, $\Gamma_k = \Gamma_1 = \text{const}$. In the case of the force controlled driving, $\Gamma_k \propto 1/m_k$, while in the case of the velocity controlled driving, $\Gamma_k \propto m_k$ [12]. In our study we analyze a more general case of a power-law dependence of Γ_k on a particle mass, namely, $\Gamma_k \sim m_k^{\gamma}$ or $\Gamma_k = \Gamma_1 k^{\gamma}$. However, we will not consider the case of driving, based on a local velocity of the granular gas [25].

[1] For a velocity-dependent restitution coefficient [2, 15] the temperature ratio is not a constant but demonstrates a complicated non-monotonous evolution, giving rise to super and sub-diffusion in granular Brownian motion [21].

We multiply the Boltzmann Eqs. (4) for $k = 1, \dots N$ with $m_k v_k^2/2$ and integrate over $\mathbf{v_k}$. Using the Maxwellian distribution with the granular temperatures $T_k(t)$, as a good approximation [12, 20] for the velocity distribution functions $f_k(\mathbf{v}_k, t)$, we obtain,

$$\frac{d}{dt} T_k = -T_k \sum_{i=1}^{N} \xi_{ki} + \Gamma_k \qquad k = 1, \dots N\,, \tag{7}$$

where the cooling rates ξ_{ik} read,

$$\begin{aligned} \xi_{ki}(t) &= \frac{8}{3}\sqrt{2\pi} n_i \sigma_{ik}^2 g_2(\sigma_{ik}) \frac{\mu_{ik}}{m_k} \left(\frac{T_k}{m_k} + \frac{T_i}{m_i} \right)^{1/2} \\ &\times (1+\varepsilon_{ik}) \left[1 - \frac{1}{2}(1+\varepsilon_{ik}) \frac{\mu_{ik}}{T_k} \left(\frac{T_k}{m_k} + \frac{T_i}{m_i} \right) \right], \end{aligned} \tag{8}$$

with the reduced mass $\mu_{ik} = m_i m_k/(m_i + m_k)$. The above expressions for ξ_{ik} coincides with the ones reported in Refs. [12, 20].

Scaling theory . –

Homogeneous cooling state. In a homogeneous cooling state no external energy is supplied to a system. In this case the heating is lacking, $\Gamma_k = 0$, and the evolution equations for the granular temperatures read:

$$\frac{1}{T_k} \frac{dT_k}{dt} = -\sum_{i=1}^{N} \xi_{ki}, \qquad k = 1, \dots N\,. \tag{9}$$

The energy of grains decrease due to dissipative collisions, their motion slows down until all particles come to a rest. From a general analysis of the Boltzmann equation it may be shown that after a short relaxation time the solution attains a *normal form*, when a distribution function depends on space and time only through hydrodynamic fields. In this regime the cooling rates $T_k^{-1} dT_k/dt$ for all species with $k = 1, \dots N$ become equal [11, 12], that is,

$$\sum_{i=1}^{N} \xi_{ki} = G(t) \qquad \text{for all} \qquad k. \tag{10}$$

Here $G(t)$ depends on the relevant characteristics of the system – concentrations, masses, etc. From Eqs.(9)-(10) follows that in the above regime the dependence of $T_k(t)$ on k and t factorizes – it may be generally written as $T_k(t) = T_1(t)F(k)$ with $F(1) = 1$. This also implies the steady-state ratios of granular temperatures T_k/T_l $(k, l = 1, \dots, N)$ for a granular gas mixture in a HCS.

Let the number of species in a granular mixture be large, $N \gg 1$, so that the scaling analysis may be applied. Analysing the form of ξ_{ki} in Eq. (8) we notice that a reasonable assumption for $F(k)$ to fulfil the condition (10) would be $F(k) \sim m_k^{\alpha}$, or, since $m_k = m_1 k$,

$$T_k = T_1 k^{\alpha}\,, \tag{11}$$

and search for the exponent α. We focus on the sum $\sum_{i=1}^{N} \xi_{ki}$ and analyze the consequences of the condition (10). Approximating for $N \gg 1$ the summation in Eq. (10) by integration, we obtain,

$$\begin{aligned} \sum_{i=1}^{N} \xi_{ki} &= a \int_1^N n_i \left(i^{1/3} + k^{1/3} \right)^2 \frac{i}{k+i} \left(i^{\alpha-1} + k^{\alpha-1} \right)^{1/2} \\ &\times \left[1 - \frac{1}{2}(1+\varepsilon) \frac{i}{k+i} \left(1 + \left(\frac{i}{k} \right)^{\alpha-1} \right) \right] di \end{aligned} \tag{12}$$

with

$$a = \frac{2}{3}\sqrt{2\pi}\sigma_1^2 \left(\frac{T_1}{m_1} \right)^{\frac{1}{2}} (1+\varepsilon)\,, \tag{13}$$

where we assume that $\varepsilon_{ik} = \varepsilon = \text{const}$. Since the condition (10) holds for any k, we choose $k \gg 1$. Then, if the particle size-distribution $n_i = n_i(i)$ is steep enough the main contribution to the integral in Eq. (12) comes from $i \ll k$. Expanding the integrand in Eq. (12) with respect to $(i/k) \ll 1$ and keeping only the leading terms in the expansion we arrive at

$$\sum_{i=1}^{N} \xi_{ki} = \begin{cases} a k^{\frac{\alpha}{2} - \frac{5}{6}} \int_1^N i\, n_i\, di & \text{if} \quad \alpha \geq 1 \\ a k^{-\frac{1}{3}} \int_1^N i^{\frac{\alpha+1}{2}} n_i\, di & \text{if} \quad 0 < \alpha < 1\,. \end{cases} \tag{14}$$

Here we exclude $\alpha < 0$, since it may yield for $i \ll k$ a negative sign for the factor in the square brackets of the integrand in Eq. (12). The result of the integration, however, should be positive, as it gives the cooling rate. For steep distributions n_i one can approximate N in the upper limits of the integrals in Eq. (14) by the infinity,

$$\int_1^N i^p\, n_i\, di \simeq \int_1^{\infty} i^p\, n_i\, di = \text{const}\,, \tag{15}$$

where $p = 1$ for $\alpha > 1$ and $p = (\alpha+1)/2$ for $0 < \alpha < 1$ (see Eq. (14)), so that the sum in (10) does not (asymptotically, for $N \gg 1$) depend on N. Since it neither depends on k [see Eq. (10)], it follows from Eq. (14) that $\alpha/2 - 5/6 = 0$. That is, we conclude that in a HCS *all* granular gas mixtures with a steep distribution of species size have the same universal power-law distribution of temperatures (11) with the exponent

$$\alpha = \frac{5}{3} \approx 1.67\,. \tag{16}$$

Heated granular mixtures. If energy is injected into a granular gas to compensate its losses in dissipative collisions, the system rapidly settles into a nonequilibrium steady-state [23, 24]. In the case of a heated granular mixtures all granular temperatures T_k attain, after a short relaxation time, constant values, so that $dT_k/dt = 0$. Then Eqs. (7) read,

$$T_k \sum_{i=1}^{N} \xi_{ki} = \Gamma_1 k^{\gamma}\,. \tag{17}$$

We again assume that the size distribution n_i is steep enough, so that in the scaling domain $k \gg 1$ one can use Eqs. (14)-(15) for the sum $\sum_{i=1}^{N} \xi_{ki}$. Substituting into the above equation $T_k = T_1 k^\alpha$, along with Eqs. (14)-(15), we obtain, taking into account that the exponents of k in the both sides of the equation must be equal:

$$\alpha = \begin{cases} \frac{5}{9} + \frac{2}{3}\gamma & \text{if} \quad \gamma \geq \frac{2}{3} \\ \gamma + \frac{1}{3} & \text{if} \quad -\frac{1}{3} \leq \gamma \leq \frac{2}{3}. \end{cases} \tag{18}$$

Steep size distributions. The most important for practice are the power-law size distributions $n_k = n_1 k^{-\theta}$. These are very common in nature and industry, as obtained e.g. in fragmentation processes. The condition of a steep distribution (15) reads for a power-law distribution,

$$\begin{aligned} &\theta > 2 && \text{if} \quad \alpha > 1 \\ &\theta > \frac{\alpha}{2} + \frac{3}{2} && \text{if} \quad 0 < \alpha < 1 \,. \end{aligned} \tag{19}$$

In the case of the exponential distribution, $n_k = n_1 \exp\left(-bk^\beta\right)$, the condition of steepness takes in the form: $bN^\beta >> 1$.

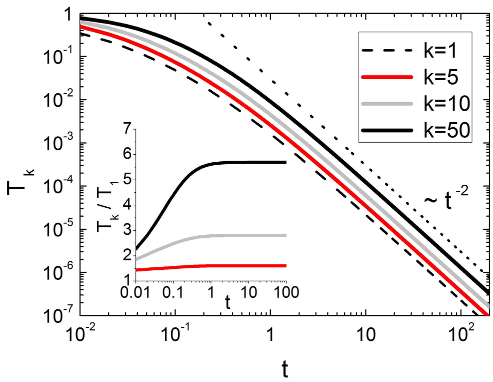

Fig. 1: The time-dependence of granular temperatures $T_k(t)$ and their ratios T_k/T_1 (inset) in a granular mixture with a power-law size distribution, $n_k \sim k^{-3}$, in a HCS state. The granular temperatures of all species decrease with the same cooling rate according to Haff's law $T_k \sim 1/t^2$ (shown by the dotted line), while the ratios T_k/T_1 rapidly evolve to the steady state values (see the inset). The restitution coefficient $\varepsilon = 0.5$.

Note that strictly speaking, the above scaling theory is valid only for $\varepsilon_{ik} = \varepsilon =$ const. The realistic dependence of ε on mass and size of particles is however rather weak (see e.g. [2] (a)) so that the deviations of $(1 + \varepsilon_{ik})$ from some average value are small. Hence, the scaling theory is expected to remain reasonably accurate.

Numerical analysis versus theory. – In order to check the predictions of the scaling theory, we solve numerically the system of differential equations (7) for granular mixtures in a homogeneous cooling state, as well as in a state under a uniform heating. In the HCS state the granular temperatures of all species rapidly relax to a state with an equal cooling rate for all components. All granular temperatures decay under these conditions according

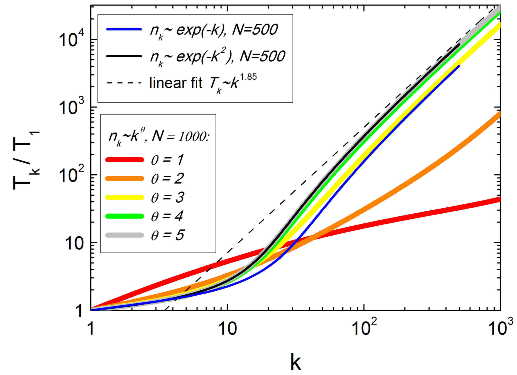

Fig. 2: The dependence of T_k/T_1 on $k = m_k/m_1$ in a granular mixture in a homogeneous cooling state. The masses of particles are distributed according to the power law $n_k \sim k^{-\theta}$, exponential distribution $n_k \sim \exp(-k)$ and Gaussian distribution $n_k \sim \exp\left(-k^2\right)$. It may be fitted for $k \gg 1$ by the power law $T_k/T_1 = k^\alpha$, where $\alpha = 1.85$ is the same for exponential, Gaussian and power-law distributions with $\theta > 2$. Lines from bottom to top correspond to $n_k \sim k^{-1}$, $n_k \sim k^{-2}$, $n_k \sim \exp(-k)$, $n_k \sim k^{-3}$, $n_k \sim k^{-4}$, $n_k \sim k^{-5}$, $n_k \sim \exp(-k^2)$.

to the Haff's law $T_k \sim t^{-2}$ [2], while the ratios T_k/T_1 remain fixed, Fig. 1. The dependence of T_k/T_1 on k in the log-log scale can be fitted for $k \gg 1$ with a straight line, Fig. 2; this confirms our scaling hypothesis, $T_k \sim m_k^\alpha$. The fitting of the numerical solution yields the slope $\alpha = 1.85$. This slope, $\alpha = 1.85$, is the same for exponential, Gaussian and power-law distributions with $\theta > 2$. Note however, that the universality of the temperature distribution is violated for $\theta = 2$ and $\theta = 1$ – these size-distributions do not fulfill the condition of steepness (19). The discrepancy between the numerical and scaling result of $\alpha = 1.67$ may be attributed to the systematic error due to replacement of the finite N by the infinite one and the summation by integration. In the case of a uniform heating, the granular temperatures of all species rapidly evolve to their steady-state values where the ratios T_k/T_1 are kept constant. As in the case of a HCS the log-log plot of T_k/T_1 on the dimensionless mass $k = m_k/m_1$ demonstrates a linear dependence for the scaling domain, $k \gg 1$, Fig. 3. As it may be seen from Fig. 3, the slope of the temperature distribution does not depend on the number of species in the system N and the particular form of the size distribution, as the scaling analysis predicts.

We additionally check independence of the temperature distribution on the particle size distribution for the case of power-law distribution $n_k \sim k^{-\theta}$. In particular, we analyze the threshold value of θ, above which the scaling theory is valid. In Fig. 4 (left panel) we show the numerical results for $\gamma = 1$. As it may be seen from the figure, the exponent α is indeed independent on θ, provided it is larger than the threshold $\theta_0 > 2$, which is close to 2, as it follows from the scaling theory. In the numerical solution

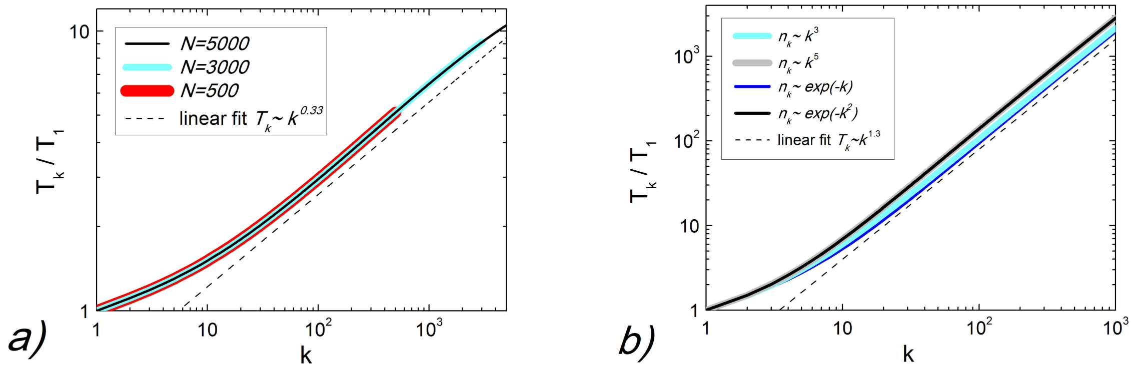

Fig. 3: The dependence of the ratios of granular temperatures T_k/T_1 on the dimensionless particles' mass $k = m_k/m_1$ for a heated granular mixture. The left panel, $\gamma = 0$ (equal energy supply for all species), $n_k \sim k^{-\theta}$ ($\theta = 5 > 2$), illustrates independence of the temperature distribution on the number of species in the system N. The right panel, $\gamma = 1$ (the velocity controlled driving), shows independence of the temperature distribution on the particular form of the size distribution, provided it is steep enough. Lines from the bottom to top correspond to $n_k \sim \exp(-k)$, $n_k \sim k^{-3}$, $n_k \sim k^{-5}$ and $n_k \sim \exp(-k^2)$. The scaling predictions for the temperature distribution $T_k \sim m_k^{\alpha}$ with α from Eq. (18) are depicted as the dashed lines, with $\alpha = 1/3$ for $\gamma = 0$ (left panel) and $\alpha = 11/9$ for $\gamma = 1$ (right panel).

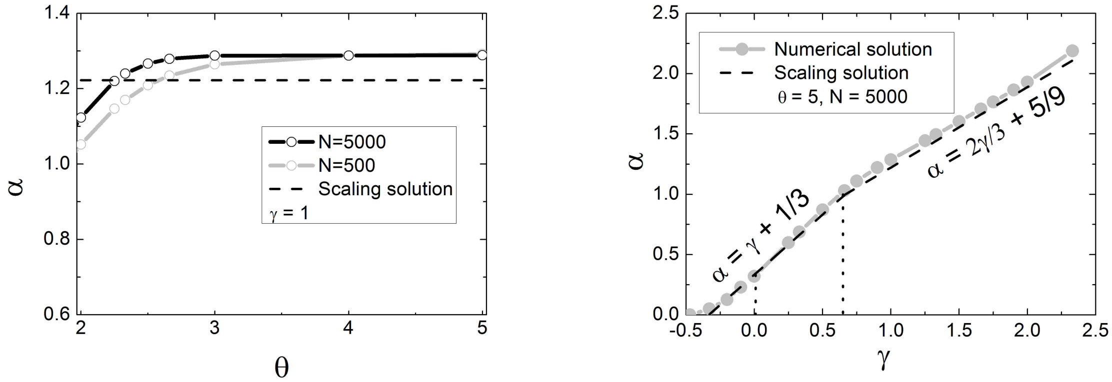

Fig. 4: The dependence of the exponent α of the granular temperature distribution, $T_k \sim m_k^{\alpha}$ on the exponent θ of the particle size distribution, $n_k \sim m_k^{-\theta}$ (left panel) and on the exponent γ of the mass dependence of the heating rate, $\Gamma \sim m_k^{\gamma}$ (right panel). Thin dashed black lines show the scaling solution.

we observe that the threshold θ_0 depends on N, tending to the predicted value of $\theta_0 = 2$, as N tends to infinity. The numerically found plateau value of $\alpha = 1.28$ differs from the scaling prediction $\alpha = 11/9 = 1.22$, Eq. (18), by less than 5%. Note that for the case of heated gas the replacement in Eq. (8) of the finite N by the infinite one and of the summation by integration leads to smaller error as compared to the case of the HCS, since in the latter case evolution of granular temperatures are described by differential equations (9), while in the former one - by an algebraic equations (17)[2].

Finally, we study numerically the dependence of α on the power exponent γ of the heating rate $\Gamma_k \sim m_k^{\gamma}$. As it follows from Fig. 4 (right panel), the scaling theory agrees very well with the numerical data, which also demonstrate two different slopes on the dependence of α of γ: $\alpha = 1/3 + \gamma$ for $\gamma < 2/3$ and $\alpha = 5/9 + 2/3\gamma$ for $\gamma \geq 2/3$.

It is interesting to analyze distribution of granular temperatures $T_k \sim k^{\alpha}$ for some particular values of γ. For the *equal energy* supply for all species, $\gamma = 0$, the violation of energy equipartition nevertheless takes place, with $\alpha = 1/3$. This happens because smaller particles loose in collisions larger part of their energy than bigger ones. To compensate this "bias" in the collision losses of small particles, more energy is to be supplied for the smaller grains than to the larger ones, that is, the exponent γ should be negative. Indeed, for $\gamma = -1/3$ the energy equipartition, $T_k = \text{const.}$ is achieved, see Fig. 4 (right panel). For another important case of $\gamma = 2/3$, the exponent α is unity,

[2]As it follows from Eqs. (9) and (10), $T_k \sim \exp(-\int^t G(\tau)d\tau)$, which implies exponentially strong dependence of the distribution T_k on systematic errors in a HCS.

$\alpha = 1$, and the distribution of the characteristic velocities, $\bar{v}_k = (2T_k/m_k)^{1/2} = \text{const.}$, is flat. Noteworthy, the distribution of characteristic velocities for different particle sizes in Planetary Rings is seemingly also rather flat [26].

Conclusion. – We have studied by means of a scaling approach and numerically granular gas mixtures with steep size distributions of components, when the number density of small particles significantly exceeds that of large particles. We explored space uniform systems, both in a homogeneous cooling state (HCS) and under a uniform heating; the latter has been modeled by a white-noise thermostat. We analyzed mass-dependent heating rates Γ_k, which depended on species masses as a power-law, $\Gamma_k \sim m_k^{\gamma}$. We have shown that for *all* steep size distributions, the distribution of granular temperatures obeys a *universal* power-law distribution, $T_k \sim m_k^{\alpha}$, where the exponent α does not depend on inelasticity, number of species and a particular form of the size distribution. For a HCS we have found numerically the universal exponent $\alpha = 1.85$, which is close to $\alpha = 5/3 \simeq 1.67$, predicted by the scaling theory. For heated system we have revealed a piecewise linear dependence of α on γ, namely, $\alpha = 5/9 + 2/3\gamma$ for $\gamma \geq 2/3$ and $\alpha = 1/3 + \gamma$ for $2/3 \geq \gamma \geq -1/3$; the predictions of the scaling theory have been confirmed numerically. The results of our study may be important for rapid granular flows of highly polydisperse particles and in studying dynamics and structure of Planetary Rings.

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