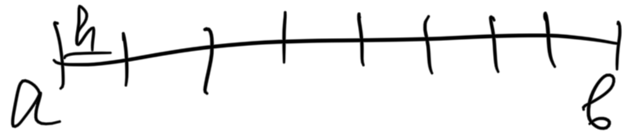
$$\int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$\int_a^b x(\tau)\, y(t-\tau)\, d\tau$$



$$(x*y)(t) = \sum_k{}' x(\tau_k)\, y(t-\tau_k)\cdot h$$

$$\tau_k = a + k\cdot h$$

$t \to$ by discrete value on uniform grid with the same step h

$$t_l = a' + l\cdot h$$

$$\sum_k{}' h\, \underline{x(a+k\cdot h)}\, \underline{y(a'+(l-k)h - a)}$$

$$x_k \cdot y_{l-k}$$

$$A_{ij} = y_{i-j}$$

y_0	y_{-1}	y_{-2}	y_{-3}
y_1	y_0	y_{-1}	y_{-2}
y_2	y_1	y	$-y_{-1}$
y_3	y_2	y_1	0

$2n-1$ param.

$$C = \frac{1}{n} F^* \Lambda F$$

Λ - diagonal

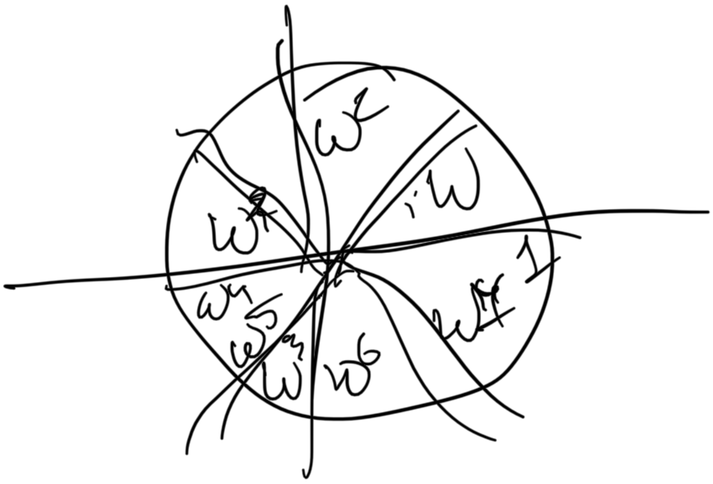
$$F_{kl} = \omega^{kl}$$

$i^2 = -1$

$$w = e^{+\frac{2\pi i}{n}}$$

$$\omega^n = e^{\frac{2\pi i}{n} n} = e^{2\pi i}$$

$$= (-1)^2 = 1$$



$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$\delta(1)$ $\delta(-1)$

$$n = 2^{d} \cdot 3^{d} \cdot 5^{d} \cdots$$

$\boxed{511}$

$$\begin{pmatrix} F_{n/2} & 0 \\ 0 & F_{n/2} \end{pmatrix} \begin{pmatrix} I & I \\ W_{n/2} & -W_{n/2} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\left(0 \quad W_{n/2} \right)$$

$$= \begin{pmatrix} x + y \\ W_{n/2}x + W_{n/2}y \end{pmatrix}$$

Diag $O(n)$

$O(n)$

$$= \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} F_{n/2}x' \\ F_{n/2}y' \end{pmatrix}$$

$$\begin{pmatrix} F_{n/2} & 0 \\ 0 & F_{n/2} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$F_n \to c \cdot n + 2F_{n/2}$$

$$\begin{bmatrix} t_0 & t_{-1} & & \\ t_1 & \ddots & \ddots & \\ & \ddots & \ddots & t_{-1} \\ & & t_1 & t_0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|cc} t_0 & t_{-1} & t_{-2} & t_{-3} & t_{-4} \\ t_1 & t_0 & t_{-1} & t_{-2} & t_{-3} \\ t_2 & t_1 & t_0 & t_{-1} & t_{-2} \\ \hline t_3 & t_2 & t_1 & t_0 & t_{-1} \end{array}\right]$$

$|t_1 \ t \ t_2 | t_4 \ t_0|$ $\neq 0$

$$C = \begin{bmatrix} T & * \\ * & * \end{bmatrix} \begin{bmatrix} q \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} Tq \\ * \end{bmatrix}$$

$y = F \cdot x \quad x \in \mathbb{R}^n \to y \in \mathbb{R}^n$

2D signal

$n \times m$ matrix

or (nm) vector

output is nm vecto

2D FFT is then

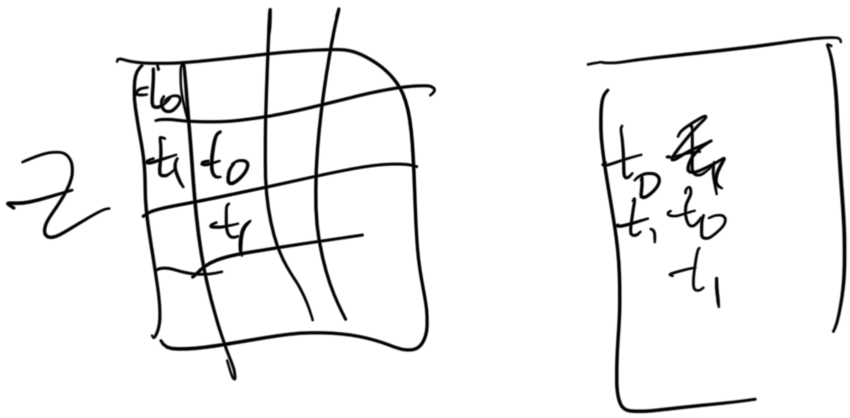
given by $nm \times nm$

0 0 matrix

$$F_2 = F_n \otimes F_m$$

и

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \dots \\ \vdots & & \\ \vdots & & \end{bmatrix}$$



e - scaled circulant

$$\begin{pmatrix} t_0 & & & & e\,t_1 \\ t_{-1} & t_0 & & & \\ & t_{-1} & t_0 & & \\ & & & t_0 & \\ & & & & t_0 \end{pmatrix}$$

$e = 1 \to$ circulant

$e = -1$ anti-circula

· Hankel $A_{ij} = a_{i+j}$

$$\begin{bmatrix} & & & a_4 \\ & & a_4 & a_7 \\ & a_4 & a_7 & \\ a_4 & a_7 & & \end{bmatrix}$$

Vandermonde

$$A_{ij} = (x_j)^k$$

Cauchy

$$A_{ij} = \frac{1}{x_i - y_j}$$

$O(n^2)$

$O(n)$

$$x_i A_{ij} - A_{ij} y_j = 1$$

c

$$D(x)\cdot A - A\,D(y) = R_1$$

$$\begin{pmatrix} 1 & & * \\ & \ddots & \\ & & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} \lambda_1 & & * \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$$

$$T^2 = \begin{pmatrix} \lambda_1^2 & & \\ & \ddots & \\ & & \lambda_n^2 \end{pmatrix}$$

$$f(T) = \begin{pmatrix} f(\lambda_1) & & \\ & \ddots & \\ & & f(\lambda_n) \end{pmatrix}$$

$$F(T)\,T - T\,F(T) = 0$$

$$\begin{bmatrix} f_{11} & f_{12} \\ & f_{22} \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} \\ 0 & t_{22} \end{bmatrix}$$

$$= \begin{bmatrix} t_{11} & t_{12} \\ 0 & t_{22} \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} \\ 0 & f_{22} \end{bmatrix}$$

$$= \left[\begin{array}{c|c} f_{11}t_{11} & f_{11}t_{12} + f_{12}t_{22} \\ \hline 0 & f_{22}t_{2} \end{array}\right]$$

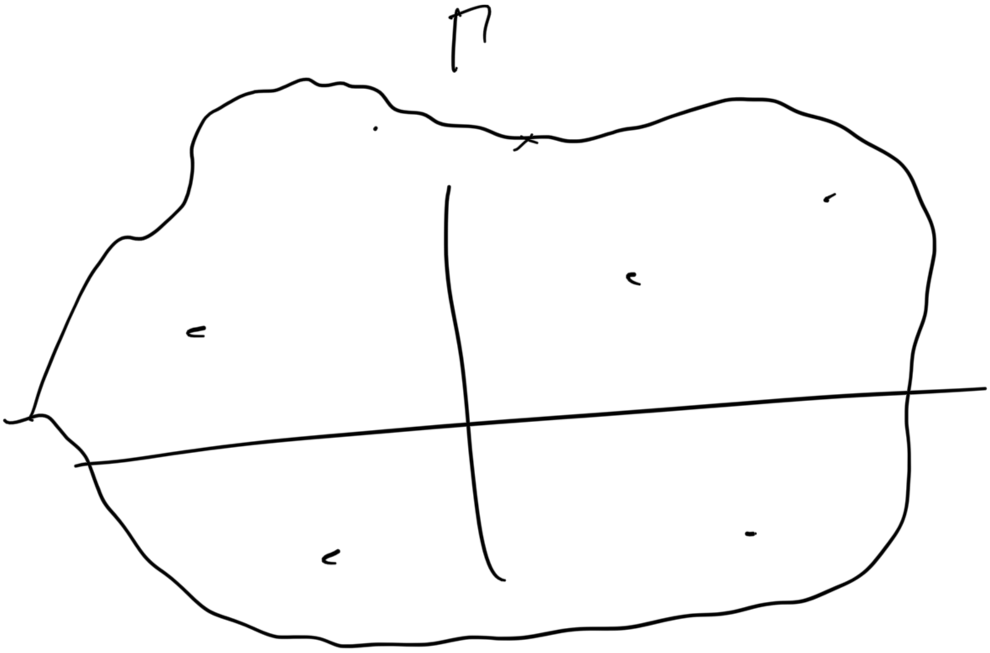
$$= \left[\begin{array}{c|c} f_{11}t_{11} & t_{11}f_{12} + t_{12}f_{22} \\ \hline 0 & f_{22}t_{22} \end{array}\right]$$

$$f_{12}(t_{11} - t_{22}) = t_{12}\cancel{f_{11}}$$

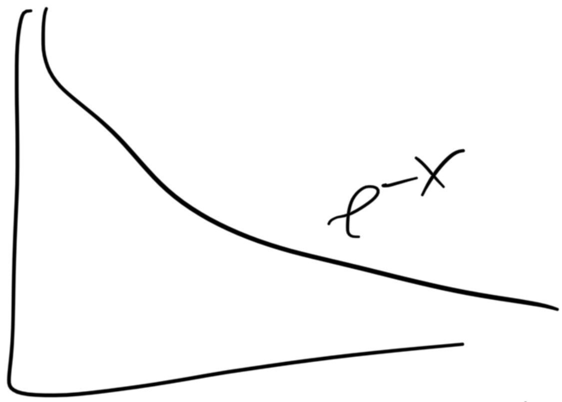
$$f_{11}t_{12} - t_{12}f_{22}$$

$$f_{12} = \frac{f_{11}t_{12} - t_{12}f_{22}}{t_{11} - t_{22}}$$

$$f(A) = \oint \frac{f(z)(zI - A)^{-1} dz}{}$$

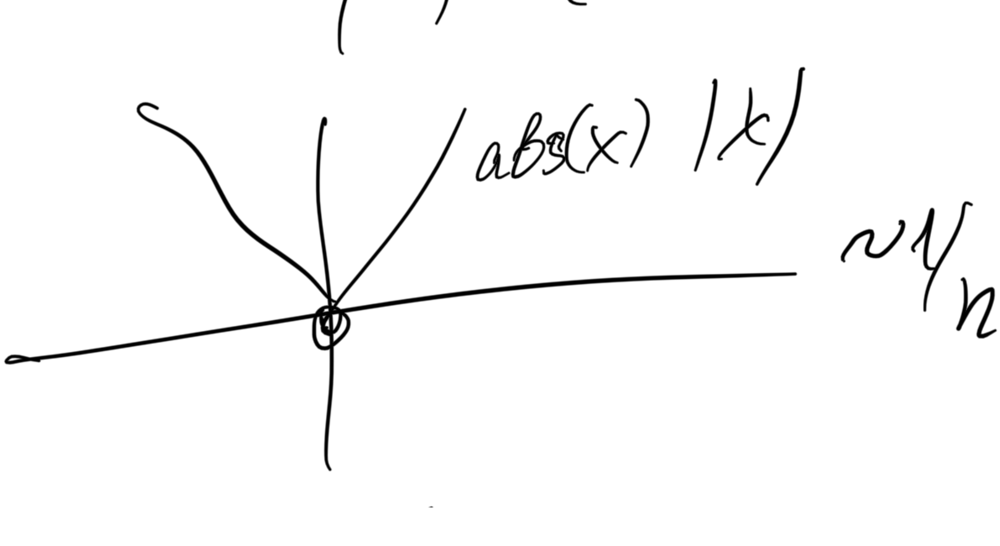


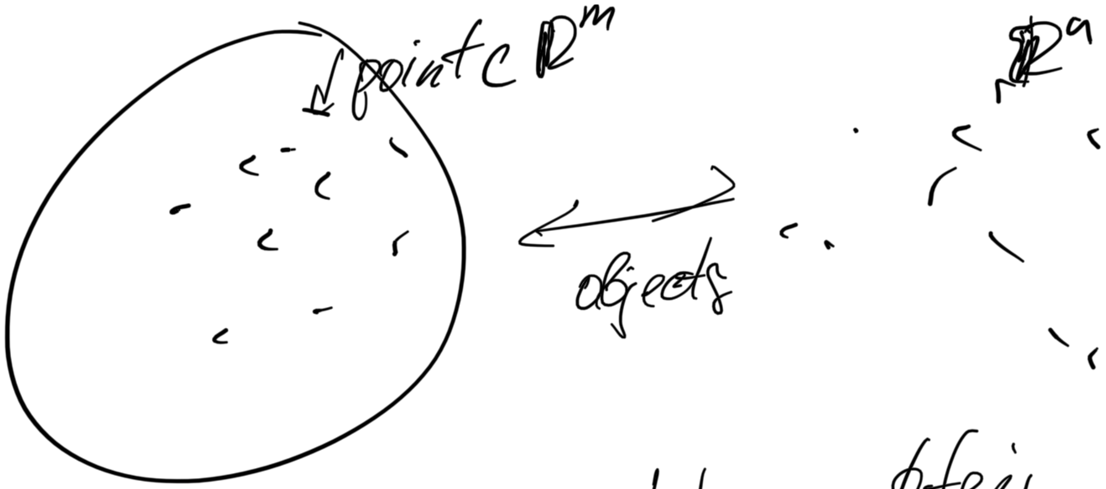
$p \approx f$



$$\min_p \| e^{-x} - p_n(x) \| = \varphi(n)$$

$$\varphi(n) \sim e^{-cn}$$



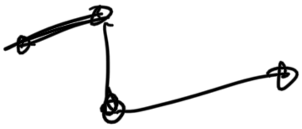


Gromov-Wasserstein distance

$$GW(M,N) \geq \sup_t e^{-2(t+t^{-1})} \left| hkt(M) - hkt(N) \right|$$

Point-cloud → graph

Nearest



$G \to$ Laplacian of the graph

$$\mathcal{L}_M \qquad \mathcal{L}_N$$

$$\cancel{hkt} \; e^{-\mathcal{L}_M t}$$

$$\mathrm{Tr}_M\left(e^{-\mathcal{L}_M t}\right) = hkt(t)$$

$z \sim N(0,1)$ $\quad A$

$$y = C \cdot z$$

$$E\, yy^T = CC^T = A$$

$$C = C^T = A^{1/2} \ (?)$$