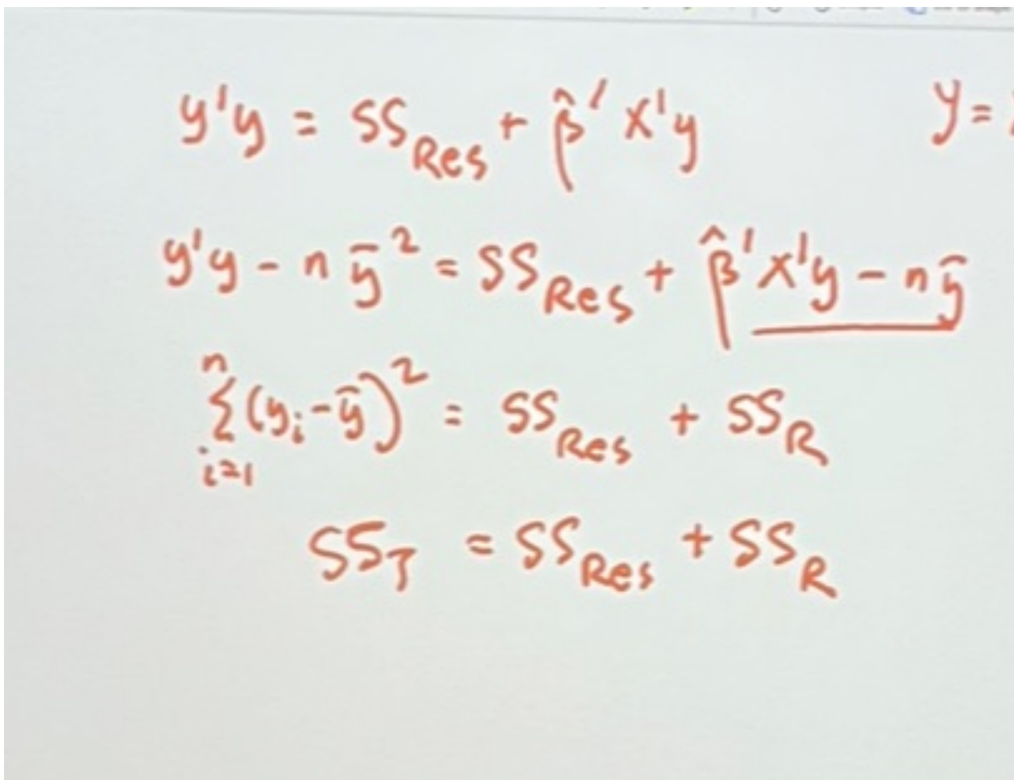
Chapter 3

Multiple Linear Regression



Chapter 3e

$SS_R = SS_R(\beta) - SS_R(\beta_0)$

$= SS_R(\beta_0, \beta_1 \ldots \beta_k) - SS_R(\beta_0)$

$= SS_R(\beta_1, \beta_2 \ldots \beta_k \mid \beta_0)$

$H_0: \beta_1 = \beta_2 = \ldots \beta_k = 0$

H_1: At least 1 of $\beta_1, \beta_2 \ldots \beta_k$ are non-zero

Extra-sum-of-squares method (pages 3-7)

- **Examples A – pr2103 data (pages 8-10)**
- **Examples B – pr2103 data (pages 11-13)**
- **Examples C – pr2103 data (pages 14-16)**
- **Difference between Example A and Example C (page 17)**
- **More examples (pages 18-26)**

Extra sum of squares method

1. Consider the multiple linear regression model with k regressor variables

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k + \epsilon, \quad p = k + 1$$

$$y = (\beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_{k-r} x_{k-r}) + (\beta_{k-r+1} x_{k-r+1} + ... + \beta_k x_k) + \epsilon$$

In matrix notation:

(Full model)

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon} = (\boldsymbol{X_1}, \boldsymbol{X_2}) \begin{bmatrix} \boldsymbol{\beta_1} \\ \boldsymbol{\beta_2} \end{bmatrix} + \boldsymbol{\epsilon} = \boldsymbol{X_1\beta_1} + \boldsymbol{X_2\beta_2} + \boldsymbol{\epsilon}$$

$\boldsymbol{\beta_1}$ contains the first $p - r$ regression coefficients.

$y = X_1 \beta_1 + \varepsilon$
(reduced model)

$\boldsymbol{\beta_2}$ contains the last r regression coefficients.

$H_0: \beta_2 = \underset{\sim}{0}$ (zero vector)

4. The F statistic for testing the hypotheses is given as

$$F = \frac{SS_R(\boldsymbol{\beta}_2|\boldsymbol{\beta}_1)/r}{\boxed{SS_{Res}(\underbrace{FM}_{\text{full model (k regressor variables)}})/(n-p)}}. \to MS_{Res}$$

Reject H_0 if $F > F_{\alpha,r,n-p}$

5. The sum of squares $SS_R(\boldsymbol{\beta}_2|\boldsymbol{\beta}_1)$ is known as the **extra** sum of squares due to $\boldsymbol{\beta_2}$ because it measures the increase in the regression sum of squares that results from adding $\boldsymbol{\beta_2}$ to a model that already contains $\boldsymbol{\beta_1}$.

6. $SS_R(\boldsymbol{\beta}_2|\boldsymbol{\beta}_1)$ can be expressed in terms of residual sum of squares of the two models as

$$SS_R(\boldsymbol{\beta}_2|\boldsymbol{\beta}_1) = SS_{Res}(RM) - SS_{Res}(FM)$$

$\hat{\beta}' X' y$

$n\bar{y}^2$

$$\begin{aligned} SS_R(\boldsymbol{\beta}_2|\boldsymbol{\beta}_1) &= SS_R(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2) - SS_R(\boldsymbol{\beta}_1) \\ &= [SS_R(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2) - SS_R(\beta_0)] - [SS_R(\boldsymbol{\beta}_1) - SS_R(\beta_0)] \\ &= [SS_T - SS_{Res}(FM)] - [SS_T - SS_{Res}(RM)] \\ &= SS_{Res}(RM) - SS_{Res}(FM) \end{aligned}$$

7. The degrees of freedom (DF) of $SS_R(\boldsymbol{\beta}_2|\boldsymbol{\beta}_1)$ can be expressed in terms of degrees of freedom of the two models as

DF of $SS_R(\boldsymbol{\beta}_2|\boldsymbol{\beta}_1)$ = DF of $SS_{Res}(RM)$ - DF of $SS_{Res}(FM)$

DF of $SS_{Res}(RM)$ - DF of $SS_{Res}(FM)$

$= [n-(p-r)]-[n-p]$

$= r$

= DF of $SS_R(\boldsymbol{\beta}_2|\boldsymbol{\beta}_1)$

$H_0: \beta_2 = \underset{\sim}{0}$ (0 vector)

r parameters

8. R can be used to compare the full and reduced models to obtain extra sum of squares $SS_R(\boldsymbol{\beta}_2|\boldsymbol{\beta}_1)$, degrees of freedom and p-value for testing the hypotheses.

For example, suppose the full and reduced models are given as

Full model (FM): $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \epsilon$

Reduced model (RM): $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon$

We want to test

$H_0 : \beta_4 = \beta_5 = 0$

H_1 : At least one of the two β's is not zero

big ⇒ big variation in y

By comparing the full and reduced models, the following R code will produce extra sum of squares $SS_R(\beta_4, \beta_5|\beta_0, \beta_1, \beta_2, \beta_3)$, degrees of freedom and p-value for testing the hypotheses,

with introduction of new regression variables

compare 2 models

```
anova(lm(y ~ x1+x2+x3), lm(y ~ x1+x2+x3+x4+x5))
```

RM FM

Example A - pr2103 data

Suppose we consider the following multiple linear regression model

$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \epsilon$

We want to test

$H_0 : \beta_4 = \beta_5 = 0$

H_1 : At least one of the β's is not zero

they are not too useful in predicting response

The p-value is 0.9224 which is large, so we do not reject $H_0 : \beta_4 = \beta_5 = 0$.

rm fm

```
> anova(lm(y~x1+x2+x3),lm(y~x1+x2+x3+x4+x5))
Analysis of Variance Table

Model 1: y ~ x1 + x2 + x3
Model 2: y ~ x1 + x2 + x3 + x4 + x5
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     79 8914.3
2     77 8895.7  2    18.675 0.0808 0.9224
```

rm: Model 1; fm: Model 2

RSS: SSres; minus

Sum of Sq: $SS_R(\beta_4, \beta_5 \mid \beta_0, \beta_1, \beta_2, \beta_3)$

Pr(>F) (P-value)

F: $\dfrac{18.675/2}{MS_{Res}(FM)}$

```r
#ch3_pr2103.R
library(MASS)
rm(list = ls())
dat <- read.csv("D:\\nus_teaching\\st3131\\data\\pr2103.csv",
                 header = T, sep=",")
dat
names(dat)

# to test H0: beta1=beta2=beta3=0
anova(lm(y~x1+x2+x3))
anova(lm(y~1))
anova(lm(y~1),lm(y~x1+x2+x3))

# to test H0: beta4=beta5=0
anova(lm(y~x1+x2+x3+x4+x5))
anova(lm(y~x1+x2+x3))
anova(lm(y~x1+x2+x3),lm(y~x1+x2+x3+x4+x5))

# to test H0: beta1=beta2=beta3=0
anova(lm(y~x1+x2+x3+x4+x5))
anova(lm(y~x4+x5))
anova(lm(y~x4+x5),lm(y~x1+x2+x3+x4+x5))
```

```
> # to test H0: beta4=beta5=0
> anova(lm(y~x1+x2+x3+x4+x5))
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)
x1         1 8413.4  8413.4 72.8254 9.510e-13 ***
x2         1 1009.6  1009.6  8.7388  0.004133 **
x3         1 2327.8  2327.8 20.1491 2.474e-05 ***
x4         1    1.5     1.5  0.0129  0.909910
x5         1   17.2    17.2  0.1488  0.700788
Residuals 77 8895.7   115.5
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

FM

```
> anova(lm(y~x1+x2+x3))
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)
x1         1 8413.4  8413.4  74.560 5.028e-13 ***
x2         1 1009.6  1009.6   8.947  0.003706 **
x3         1 2327.8  2327.8  20.629 1.973e-05 ***
Residuals 79 8914.3   112.8
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

RM

```
> anova(lm(y~x1+x2+x3),lm(y~x1+x2+x3+x4+x5))
Analysis of Variance Table
```

RM
```
Model 1: y ~ x1 + x2 + x3
```
FM
```
Model 2: y ~ x1 + x2 + x3 + x4 + x5
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     79 8914.3
2     77 8895.7  2    18.675 0.0808 0.9224
```

8914.3 - 8895.7 = 18.6

comparing both models

Example B - pr2103 data

Suppose we consider the following multiple linear regression model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \epsilon$$

We want to test

$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

H_1 : At least one of the β's is not zero

The p-value is 8.8×10^{-11} which is very small, so we reject $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$.

rm

$y \sim x_4 + x_5 + x_1 + x_2 + x_3$

```
> anova(lm(y~x4+x5),lm(y~x1+x2+x3+x4+x5))
Analysis of Variance Table

Model 1: y ~ x4 + x5
Model 2: y ~ x1 + x2 + x3 + x4 + x5
  Res.Df     RSS Df Sum of Sq      F    Pr(>F)
1     80 16918.4
2     77  8895.7  3    8022.7 23.148 8.808e-11 ***
```

```r
#ch3_pr2103.R
library(MASS)
rm(list = ls())
dat <- read.csv("D:\\nus_teaching\\st3131\\data\\pr2103.csv",
                    header = T, sep=",")
dat
names(dat)

# to test H0: beta1=beta2=beta3=0
anova(lm(y~x1+x2+x3))
anova(lm(y~1))
anova(lm(y~1),lm(y~x1+x2+x3))
```

$= y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$

```r
# to test H0: beta4=beta5=0
anova(lm(y~x1+x2+x3+x4+x5))
anova(lm(y~x1+x2+x3))
anova(lm(y~x1+x2+x3),lm(y~x1+x2+x3+x4+x5))
```

```r
# to test H0: beta1=beta2=beta3=0
anova(lm(y~x1+x2+x3+x4+x5))
```

FM

```r
anova(lm(y~x4+x5))
```

RM

```r
anova(lm(y~x4+x5),lm(y~x1+x2+x3+x4+x5))
```

```
> # to test H0: beta1=beta2=beta3=0
> anova(lm(y~x1+x2+x3+x4+x5))
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)    
x1         1 8413.4  8413.4 72.8254 9.510e-13 ***
x2         1 1009.6  1009.6  8.7388  0.004133 ** 
x3         1 2327.8  2327.8 20.1491 2.474e-05 ***
x4         1    1.5     1.5  0.0129  0.909910    
x5         1   17.2    17.2  0.1488  0.700788    
Residuals 77 8895.7   115.5                      
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> anova(lm(y~x4+x5))
Analysis of Variance Table

Response: y
          Df  Sum Sq Mean Sq F value    Pr(>F)    
x4         1  3649.8  3649.8 17.2584 8.113e-05 ***
x5         1    96.9    96.9  0.4581    0.5005    
Residuals 80 16918.4   211.5                      
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> anova(lm(y~x4+x5),lm(y~x1+x2+x3+x4+x5))
Analysis of Variance Table

Model 1: y ~ x4 + x5
Model 2: y ~ x1 + x2 + x3 + x4 + x5
  Res.Df     RSS Df Sum of Sq      F    Pr(>F)    
1     80 16918.4                                  
2     77  8895.7  3    8022.7 23.148 8.808e-11 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

$H_0: \beta_1=\beta_2=\beta_3=0$

Example C - pr2103 data

Suppose we consider the following multiple linear regression model

$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon$

We want to test

$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

H_1 : At least one of the β's is not zero

The p-value is 2.056×10^{-14} which is very small, so we reject $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$.

```
> anova(lm(y~1),lm(y~x1+x2+x3))
Analysis of Variance Table

Model 1: y ~ 1
Model 2: y ~ x1 + x2 + x3
  Res.Df     RSS Df Sum of Sq      F    Pr(>F)
1     82 20665.1
2     79  8914.3  3     11751 34.712 2.056e-14 ***
```

```r
#ch3_pr2103.R
library(MASS)
rm(list = ls())
dat <- read.csv("D:\\nus_teaching\\st3131\\data\\pr2103.csv",
                header = T, sep=",")
dat
names(dat)

# to test H0: beta1=beta2=beta3=0
anova(lm(y~x1+x2+x3))
anova(lm(y~1))
anova(lm(y~1),lm(y~x1+x2+x3))

# to test H0: beta4=beta5=0
anova(lm(y~x1+x2+x3+x4+x5))
anova(lm(y~x1+x2+x3))
anova(lm(y~x1+x2+x3),lm(y~x1+x2+x3+x4+x5))

# to test H0: beta1=beta2=beta3=0
anova(lm(y~x1+x2+x3+x4+x5))
anova(lm(y~x4+x5))
anova(lm(y~x4+x5),lm(y~x1+x2+x3+x4+x5))
```

```
> # to test H0: beta1=beta2=beta3=0
> anova(lm(y~x1+x2+x3))
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)    
x1         1 8413.4  8413.4  74.560 5.028e-13 ***
x2         1 1009.6  1009.6   8.947  0.003706 ** 
x3         1 2327.8  2327.8  20.629 1.973e-05 ***
Residuals 79 8914.3   112.8                      
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> anova(lm(y~1))
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals 82  20665  252.01               
> anova(lm(y~1),lm(y~x1+x2+x3))
Analysis of Variance Table

Model 1: y ~ 1
Model 2: y ~ x1 + x2 + x3
  Res.Df     RSS Df Sum of Sq      F    Pr(>F)    
1     82 20665.1                                  
2     79  8914.3  3     11751 34.712 2.056e-14 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

extra sum of sq due to $\beta_1, \beta_2, \beta_3$

In Example B, we use $SS_R(\beta_1, \beta_2, \beta_3|\beta_0, \beta_4, \beta_5)$ to test $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ given that β_4 and β_5 are already included in the model.

In Example C, we use $SS_R(\beta_1, \beta_2, \beta_3|\beta_0)$ to test $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$, without considering β_4 and β_5.

anova(lm(y ~ x_1))

$SS_T = SS_{Res} + SS_R$

$y = \beta_0 + \beta_1 x_1 + \varepsilon$

Analysis of Variance

Source of Variation	Sum of Squares	DF	Mean Square
x_1 $SS_R(\beta_0, \beta_1) - SS_R(\beta_0)$	$SS_R(\beta_0 \vert \beta_1) = SS_R$	1	$MS_R(\beta_0 \vert \beta_1)$
Residual	$SS_{Res}(\beta_0, \beta_1) = SS_{Res}$	$n-2$	$MS_{Res}(\beta_0, \beta_1)$
Total	SST	$n-1$	

$$F = MS_R(\beta_0 | \beta_1)/MS_{Res}(\beta_0, \beta_1)$$

To test $H_0 : \beta_1 = 0$, reject H_0 if $F > F_{\alpha,1,n-2}$

Note: $SS_{Res}(\beta_0, \beta_1) = SS_{Res}$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$$

Analysis of Variance

Source of Variation	Sum of Squares	DF	Mean Square
x_1, x_2, x_3	$SS_R(\beta_1, \beta_2, \beta_3 \mid \beta_0)$	3	$MS_R(\beta_1, \beta_2, \beta_3 \mid \beta_0)$
Residual	$SS_{Res}(\beta_0, \beta_1, \beta_2, \beta_3)$	$n-4$	$MS_{Res}(\beta_0, \beta_1, \beta_2, \beta_3)$
Total	SST	$n-1$	

$$F = MS_R(\beta_1, \beta_2, \beta_3 \mid \beta_0) \, / MS_{Res}(\beta_0, \beta_1, \beta_2, \beta_3)$$

To test $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$,

reject H_0 if $F > F_{\alpha,3,n-4}$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$$

Analysis of Variance

Source of Variation	Sum of Squares	DF	Mean Square
x_1	$SS_R(\beta_1\|\beta_0)$ $H_0: \beta_1 = 0$	1	$MS_R(\beta_1\|\beta_0)$
$x_2, x_3\|x_1$	$SS_R(\beta_2, \beta_3\|\beta_0, \beta_1)$	2	$MS_R(\beta_2, \beta_3\|\beta_0, \beta_1)$
Residual	$SS_{Res}(\beta_0, \beta_1, \beta_2, \beta_3)$	$n-4$	$MS_{Res}(\beta_0, \beta_1, \beta_2, \beta_3)$
Total	SST	$n-1$	

$$F = MS_R(\beta_2, \beta_3|\beta_0, \beta_1)/MS_{Res}(\beta_0, \beta_1, \beta_2, \beta_3)$$

To test $H_0: \beta_2 = \beta_3 = 0$,

reject H_0 if $F > F_{\alpha,2,n-4}$

Note: $SS_R(\beta_2, \beta_3|\beta_0, \beta_1) = SS_R(\beta_1, \beta_2, \beta_3|\beta_0) - SS_R(\beta_1|\beta_0)$

```
> anova(lm(y ~ x1+x2))
Analysis of Variance Table
```

Test $H_0 : \beta_1 = 0$

Test $H_0 : y = \beta_0 + \epsilon$ versus $H_A : y = \beta_0 + \beta_1 x_1 + \epsilon$

$SS_R(\beta_1|\beta_0)$

```
Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)
x1         1 866.50  866.50 87.6186 5.748e-10
x2         1   5.60    5.60  0.5659    0.4584
Residuals 27 267.01    9.89
```

$SS_R(\beta_2|\beta_0, \beta_1)$

Test $H_0 : \beta_2 = 0$

Test $H_0 : y = \beta_0 + \beta_1 x_1 + \epsilon$ versus $H_A : y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$

An alternative way of getting $SS_R(\beta_2|\beta_0, \beta_1)$:

```
> anova(lm(y ~ x1),lm(y ~ x1+x2))
Analysis of Variance Table

Model 1: y ~ x1
Model 2: y ~ x1 + x2
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     28 272.61
2     27 267.01  1    5.5959 0.5659 0.4584
```

$$SS_R(\beta_2|\beta_0, \beta_1) = SS_{Res}(RM) - SS_{Res}(FM)$$

Reduced Model (RM):

$$y = \beta_0 + \beta_1 x_1 + \epsilon$$

```
> anova(lm(y ~ x1))
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)
x1         1 866.50  866.50  88.999 3.429e-10
Residuals 28 272.61    9.74
```

Full Model (FM):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

```
> anova(lm(y ~ x1+x2))
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)
x1         1 866.50  866.50 87.6186 5.748e-10
x2         1   5.60    5.60  0.5659    0.4584
Residuals 27 267.01    9.89
```

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

Test $H_0 : \beta_1 = \beta_2 = 0$ versus $H_A :$ At least one of the β's is not zero.

```
> anova(lm(y ~ 1),lm(y ~ x1+x2))
Analysis of Variance Table

Model 1: y ~ 1
Model 2: y ~ x1 + x2
  Res.Df     RSS Df Sum of Sq      F    Pr(>F)
1     29 1139.11
2     27  267.01  2    872.09 44.092 3.123e-09
```

$$SS_R(\beta_1, \beta_2 | \beta_0)$$

Full Model (FM):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon$$

Reduced Model (RM):

$$y = \beta_0 + \beta_1 x_1 + \epsilon$$

Test $H_0 : \beta_2 = \beta_3 = 0$ versus $H_A :$ At least one of the β's is not zero.

```
> anova(lm(y ~ x1),lm(y ~ x1+x2+x3))
Analysis of Variance Table

Model 1: y ~ x1
Model 2: y ~ x1 + x2 + x3
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     28 272.61
2     26 263.31  2    9.2957 0.4589  0.637
```

$SS_R(\beta_2, \beta_3 | \beta_0, \beta_1)$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 + \beta_8 x_8 + \epsilon$$

Test $H_0 : \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0$ versus $H_A :$ At least one of the β's is not zero.

Reduced Model (RM):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon$$

Full Model (FM):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 + \beta_8 x_8 + \epsilon$$

```
> anova(lm(y ~ x1+x2+x3), lm(y ~ x1+x2+x3+x4+x5+x6+x7+x8))
Analysis of Variance Table

Model 1: y ~ x1 + x2 + x3
Model 2: y ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     26 263.31
2     21 202.87  5     60.44 1.2513 0.3212
```

$$SS_R(\beta_4, \beta_5, \beta_6, \beta_7, \beta_8 | \beta_0, \beta_1, \beta_2, \beta_3)$$

$SS_R(\beta_1|\beta_0)$

$SS_R(\beta_2|\beta_0,\beta_1)$

$SS_R(\beta_3|\beta_0,\beta_1,\beta_2)$

not the same

```
> anova(lm(y ~ x1+x2+x3))
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)
x1         1 866.50  866.50 85.5590 1.048e-09
x2         1   5.60    5.60  0.5525    0.4639
x3         1   3.70    3.70  0.3653    0.5508
Residuals 26 263.31   10.13
```

Partial or marginal tests

```
> summary(lm(y ~ x1+x2+x3))

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 32.620594   2.050397  15.909 6.45e-15
x1          -0.077808   0.036767  -2.116   0.0441
x2           0.007284   0.053111   0.137   0.8920
x3           0.039820   0.065881   0.604   0.5508
```

$F=t^2$

$SS_R(\beta_1|\beta_0,\beta_2,\beta_3)$

$SS_R(\beta_2|\beta_0,\beta_1,\beta_3)$

$SS_R(\beta_3|\beta_0,\beta_1,\beta_2)$

$H_0: \beta_3 = 0$

The End