Chapter 3

Multiple Linear Regression

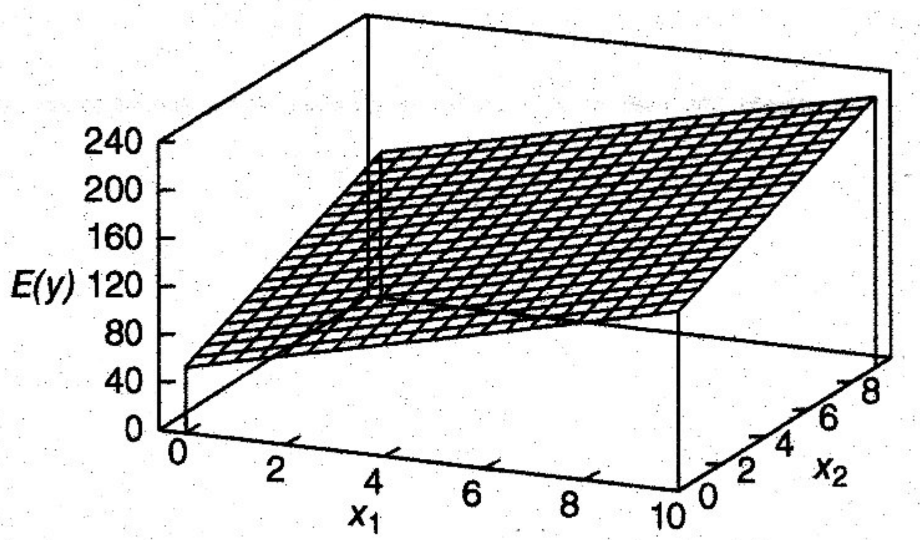
Chapter 3a

2 regressor variable?

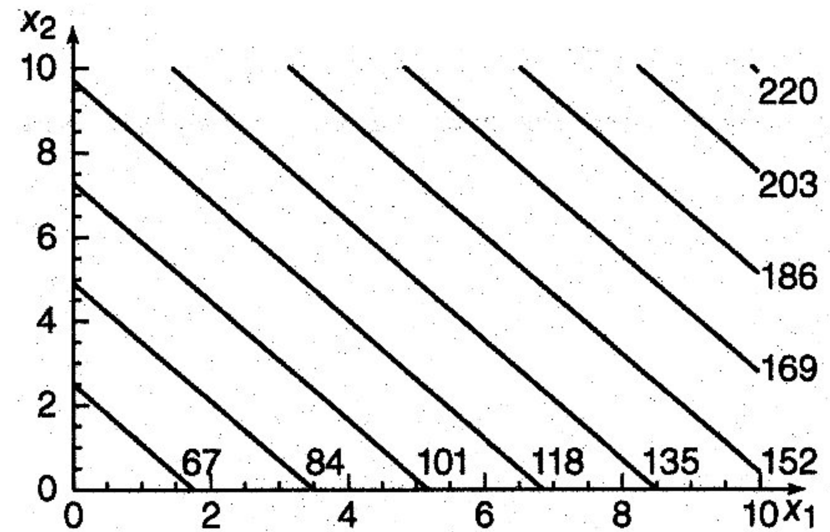
Multiple linear regression models

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

$$E(y) = 50 + 10x_1 + 7x_2$$



regression plane

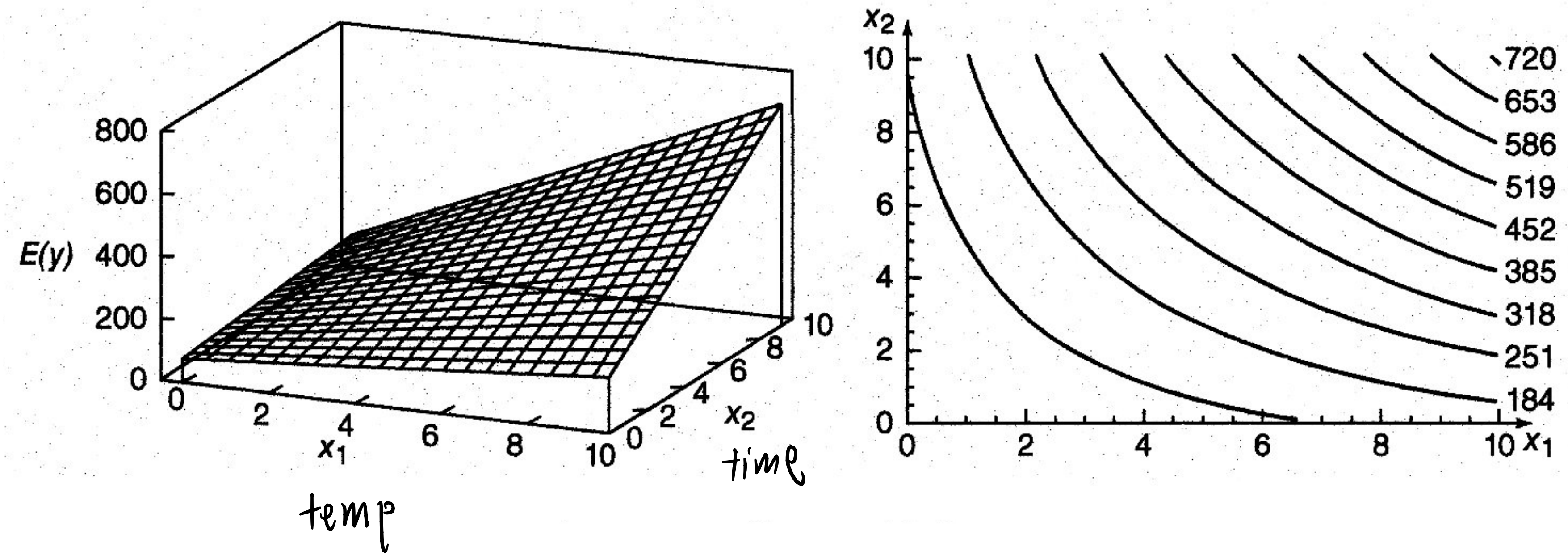


contour plot

interaction term

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} \underline{x_1 x_2} + \varepsilon$$

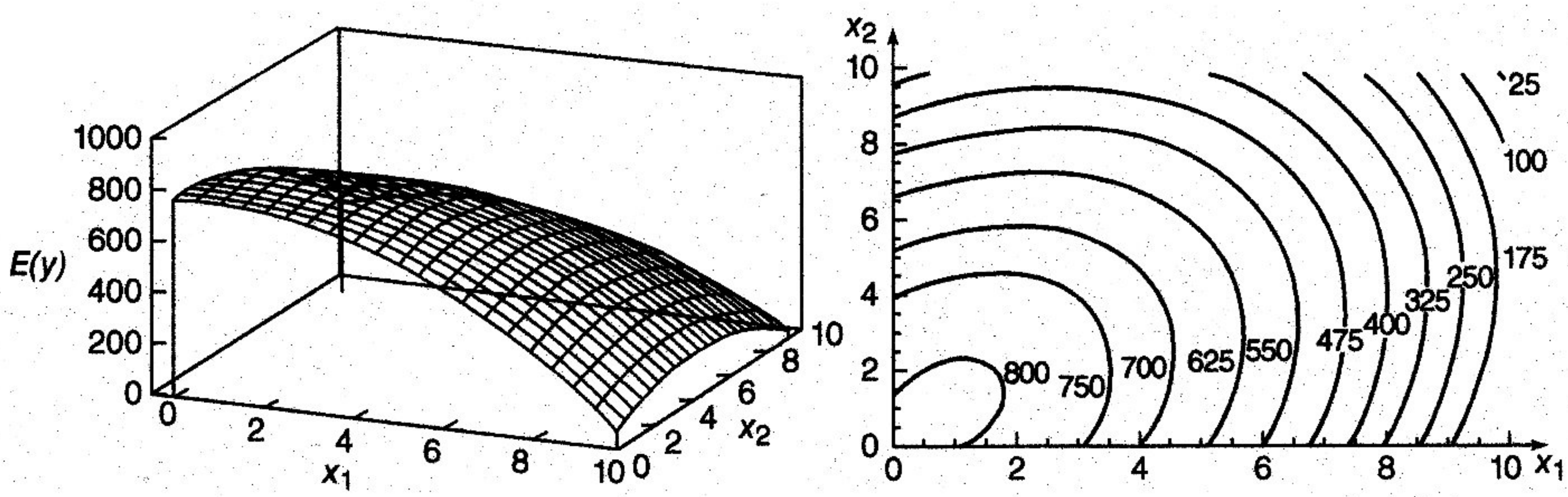
$$E(y) = 50 + 10x_1 + 7x_2 + 5x_1 x_2$$



quadratic component

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2 + \varepsilon$$

$$E(y) = 800 + 10x_1 + 7x_2 - 8.5x_1^2 - 5x_2^2 + 4x_1 x_2$$



Multiple linear regression model

The multiple linear regression model for a response variable y and regressor variables $x_1, x_2, ..., x_k$ can be written as

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_k x_{ik} + \epsilon_i, \;\; i = 1, 2, ..., n, \\ &= \beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + \epsilon_i, \;\; i = 1, 2, ..., n, \end{aligned}$$

K regressor variables

where $E(\epsilon_i) = 0$, $Var(\epsilon_i) = \sigma^2, \;\; i = 1, 2, ..., n$ and ϵ_i's are independent. Note that the number of regressor variables is k and the number of regression coefficients is $p = k + 1$.

The coefficients $\beta_j, j = 1, 2, ..., k$ are called partial regression coefficients. The parameter β_j represents the expected change in the response per unit change in x_j when all the remaining regressor variables are held constant.

Data for Multiple Linear Regression

Observation, i	Response, y	Regressors x_1	x_2	$\cdots$	x_k
1	y_1	x_{11}	x_{12}	$\cdots$	x_{1k}
2	y_2	x_{21}	x_{22}	$\cdots$	x_{2k}
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$
n	y_n	x_{n1}	x_{n2}	$\cdots$	x_{nk}

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik} + \varepsilon_i \quad i = 1, 2, \ldots, n$$

$$\varepsilon_i = y_i - (\beta_0 + \beta_1 x_{i1} + \ldots \beta_k x_{ik})$$

least square function (in terms of k+1 variable) $= \sum \varepsilon_i^2$

$$\underbrace{\frac{\partial S}{\partial \beta_0} = 0, \quad \frac{\partial S}{\partial \beta_1} = 0 \quad \ldots \quad \frac{\partial S}{\partial \beta_k} = 0}_{k+1 \text{ equations}}$$

The delivery time data

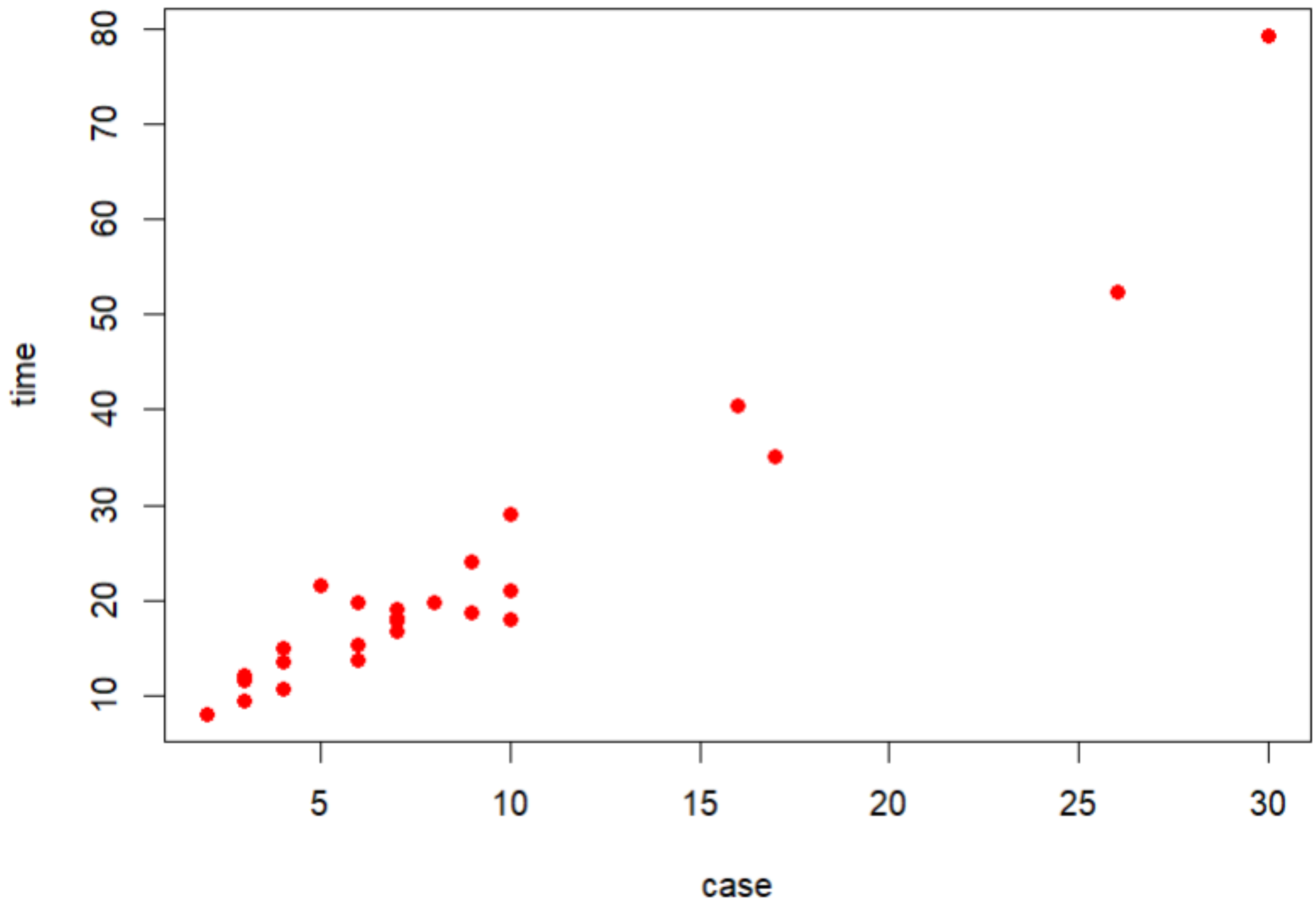
The response y is the amount of time in minutes required by a route driver to stock the vending machines with beverage products and do minor maintenance of the machines in an outlet. The two regressor variables are (i) number of cases of products stocked (x_1), and (ii) the distance walked in feet (x_2). The data set is displayed below.

response variable

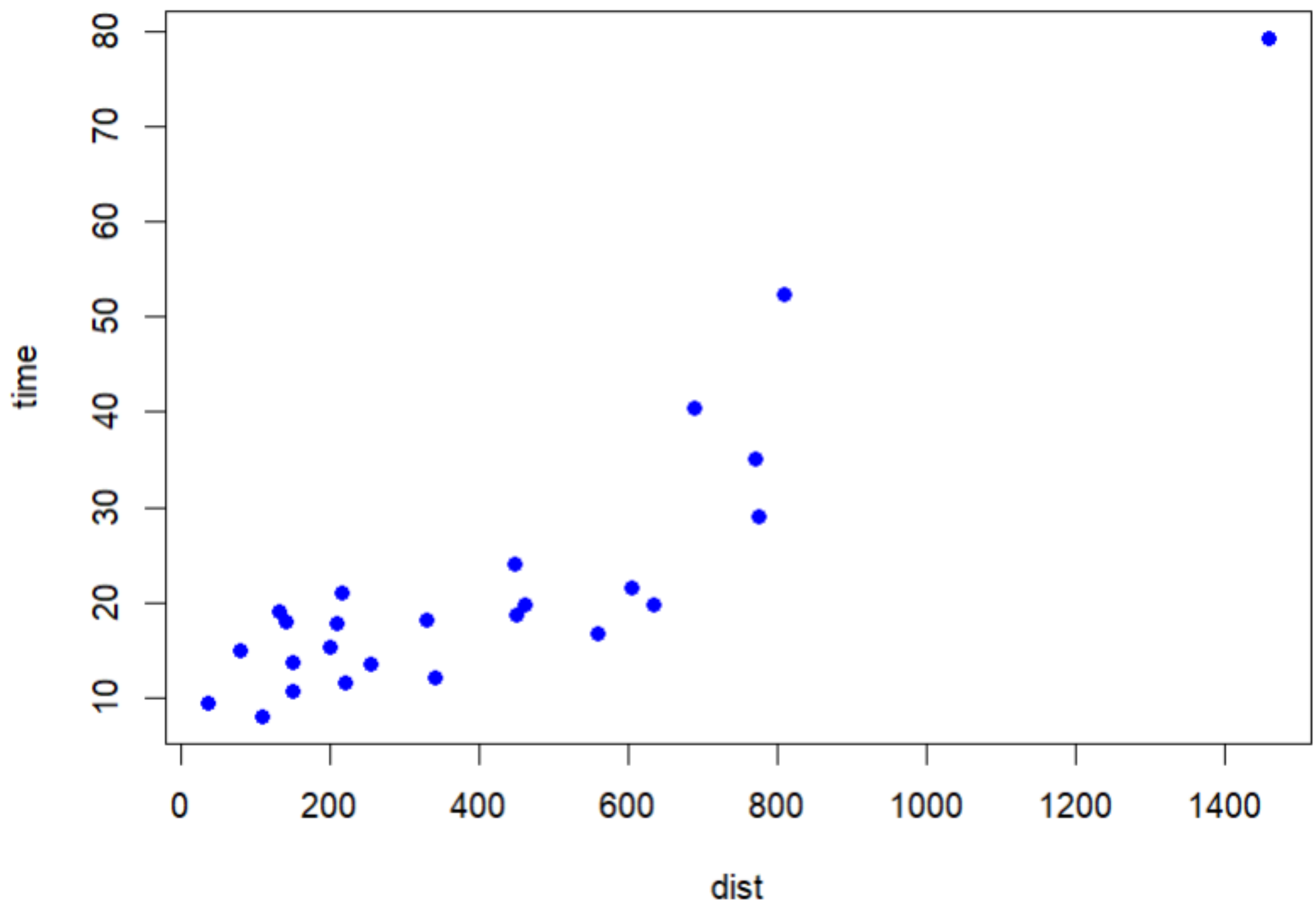
regressor variable

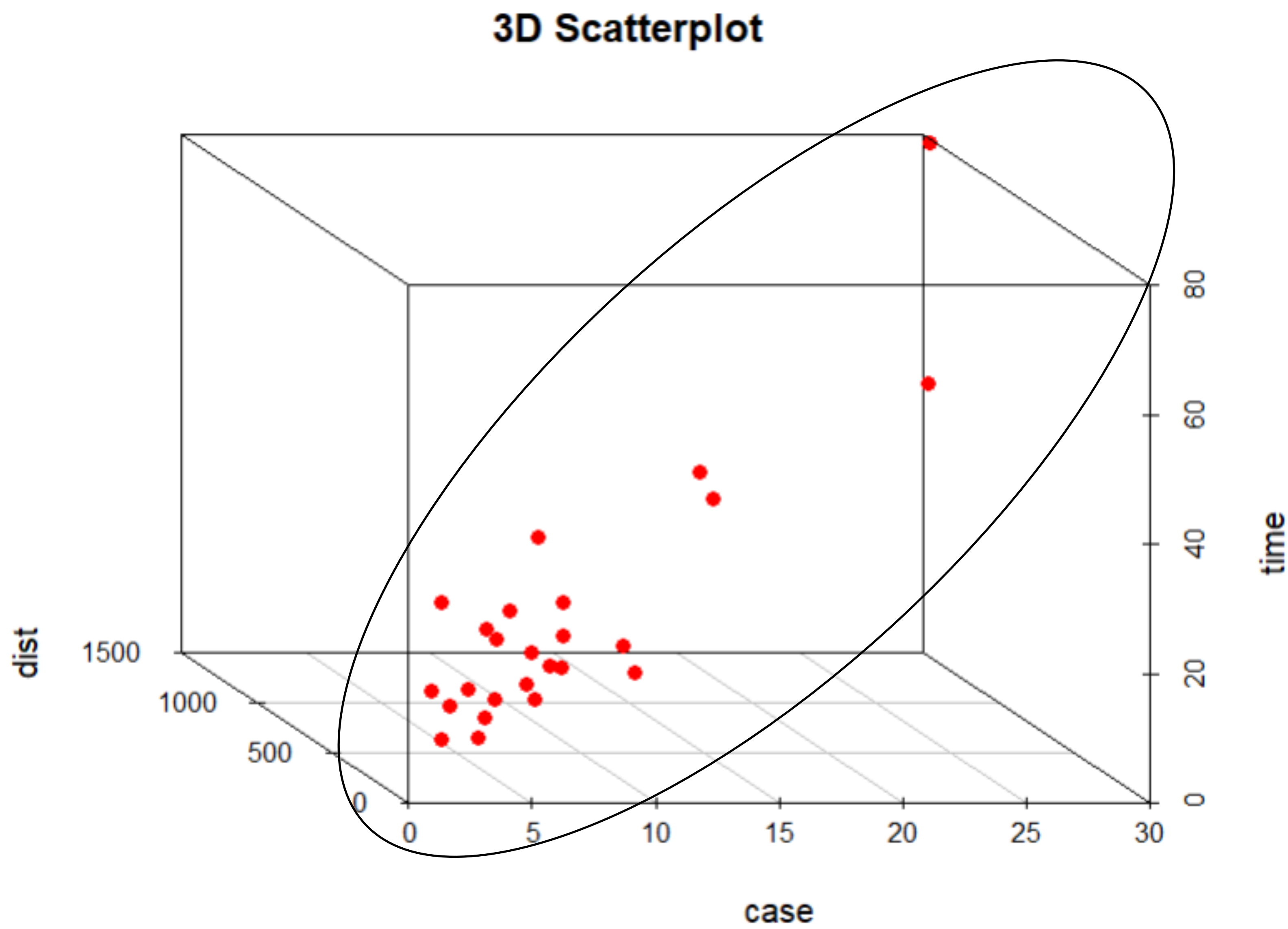
	time	case	dist
1	16.68	7	560
2	11.50	3	220
3	12.03	3	340
4	14.88	4	80
5	13.75	6	150
6	18.11	7	330
7	8.00	2	110
8	17.83	7	210
9	79.24	30	1460
10	21.50	5	605
11	40.33	16	688
12	21.00	10	215
13	13.50	4	255
14	19.75	6	462
15	24.00	9	448
16	29.00	10	776
17	15.35	6	200
18	19.00	7	132
19	9.50	3	36
20	35.10	17	770
21	17.90	10	140
22	52.32	26	810
23	18.75	9	450
24	19.83	8	635
25	10.75	4	150

```
#install.packages("scatterplot3d") # install package
library(scatterplot3d)
library(MASS)
rm(list = ls())
dat <- read.csv("D:\\nus_teaching\\st3131\\data\\Delivery_Time.csv",
                header = T, sep=",")
dat
obs <- dat[,1]
time <- dat[,2]
case <- dat[,3]
dist <- dat[,4]

# 2d scatter plots
plot(case,time,pch=16,col="red")
plot(dist,time,pch=16,col="blue")

# 3d scatter plot
scatterplot3d(case,dist,time,main="3D Scatterplot",
              pch=16,color="red",angle=145)
```



time
case
10
20
30
40
50
60
70
80
5
10
15
20
25
30


time
80
70
60
50
40
30
20
10
0
200
400
600
800
1000
1200
1400
dist


3D Scatterplot
dist
1500
1000
500
0
0
5
10
15
20
25
30
case
0
20
40
60
80
time

Least-squares function and least-squares regression

The least-squares function is the error sum of squares

$$S = \sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij} \right)^2 .$$

Least-squares regression means fitting a model that minimizes the error sum of squares.

S

$$\frac{\partial S}{\partial \beta_0} = \sum_{i=1}^{n} 2 \left\{ y_i - \beta_0 - \sum_{j=1}^{k} \beta_x x_{ij} \right\}^1 (-1)$$

Least-squares normal equations and estimates of regression coefficients

The least-squares estimates $\hat{\beta}_0, \hat{\beta}_1, ..., \hat{\beta}_k$ of $\beta_0, \beta_1, ..., \beta_k$ are obtained by minimizing the error sum of squares with respect to $\beta_0, \beta_1, ..., \beta_k$. This is done by (i) Differentiate S with respect to $\beta_0, \beta_1, ..., \beta_k$ to obtain $\frac{\partial S}{\partial \beta_0}, \frac{\partial S}{\partial \beta_1}, ..., \frac{\partial S}{\partial \beta_k}$, and (ii) Set $\frac{\partial S}{\partial \beta_0}, \frac{\partial S}{\partial \beta_1}, ..., \frac{\partial S}{\partial \beta_k}$ equal to zero to find $\hat{\beta}_0, \hat{\beta}_1, ..., \hat{\beta}_k$.

(i) Differentiate S with respect to $\beta_0, \beta_1, ..., \beta_k$,

$$\frac{\partial S}{\partial \beta_0} = \sum_{i=1}^{n} 2\left(y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij}\right)(-1),$$

$$\frac{\partial S}{\partial \beta_j} = \sum_{i=1}^{n} 2\left(y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij}\right)(-x_{ij}),\ j = 1, 2, ..., k.$$

(ii) Set $\frac{\partial S}{\partial \beta_0}, \frac{\partial S}{\partial \beta_1}, ..., \frac{\partial S}{\partial \beta_k}$ equal to zero:

$$0 = -2\sum_{i=1}^{n}\left(y_i - \hat{\beta}_0 - \sum_{j=1}^{k} \hat{\beta}_j x_{ij}\right),$$

$$0 = -2\left[\sum_{i=1}^{n} x_{ij} y_i - \hat{\beta}_0 \sum_{i=1}^{n} x_{ij} - \sum_{i=1}^{n} x_{ij}\left(\sum_{j=1}^{k} \hat{\beta}_j x_{ij}\right)\right],\ j = 1, 2, ..., k.$$

use hat as it is now an estimate

The $k+1$ equations can be simplified to obtain

$$n\hat{\beta}_0 + \hat{\beta}_1 \sum_{i=1}^{n} x_{i1} + \hat{\beta}_2 \sum_{i=1}^{n} x_{i2} + ... + \hat{\beta}_k \sum_{i=1}^{n} x_{ik} = \sum_{i=1}^{n} y_i$$

$$\hat{\beta}_0 \sum_{i=1}^{n} x_{i1} + \hat{\beta}_1 \sum_{i=1}^{n} x_{i1}^2 + \hat{\beta}_2 \sum_{i=1}^{n} x_{i1}x_{i2} + ... + \hat{\beta}_k \sum_{i=1}^{n} x_{i1}x_{ik} = \sum_{i=1}^{n} x_{i1}y_i$$

$$\hat{\beta}_0 \sum_{i=1}^{n} x_{i2} + \hat{\beta}_1 \sum_{i=1}^{n} x_{i2}x_{i1} + \hat{\beta}_2 \sum_{i=1}^{n} x_{i2}^2 + ... + \hat{\beta}_k \sum_{i=1}^{n} x_{i2}x_{ik} = \sum_{i=1}^{n} x_{i2}y_i$$

$$\vdots$$

$$\hat{\beta}_0 \sum_{i=1}^{n} x_{ik} + \hat{\beta}_1 \sum_{i=1}^{n} x_{ik}x_{i1} + \hat{\beta}_2 \sum_{i=1}^{n} x_{ik}x_{i2} + ... + \hat{\beta}_k \sum_{i=1}^{n} x_{ik}^2 = \sum_{i=1}^{n} x_{ik}y_i$$

The $k+1$ equations can then solved to find $\hat{\beta}_0, \hat{\beta}_1, ..., \hat{\beta}_k$.

Delivary time data
Fitting multiple linear regression model

```r
# fit multiple linear regression model
fitted.model <- lm(time ~ case + dist)
summary(fitted.model)
```

19:4 (Top Level)

Console | Background Jobs

R 3.4.1

```
> # fit multiple linear regression model
> fitted.model <- lm(time ~ case + dist)
> summary(fitted.model)

Call:
lm(formula = time ~ case + dist)

Residuals:
    Min      1Q  Median      3Q     Max 
-5.7880 -0.6629  0.4364  1.1566  7.4197 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2.341231   1.096730   2.135 0.044170 *  
case        1.615907   0.170735   9.464 3.25e-09 ***
dist        0.014385   0.003613   3.981 0.000631 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.259 on 22 degrees of freedom
Multiple R-squared:  0.9596,	Adjusted R-squared:  0.9559 
F-statistic: 261.2 on 2 and 22 DF,  p-value: 4.687e-16
```

Least-squares estimation - matrix approach

1. Multiple linear regression model in terms of the n observations:

$$y_1 = \beta_0 + \beta_1 x_{11} + \beta_2 x_{12} + ... + \beta_k x_{1k} + \epsilon_1$$

$$y_2 = \beta_0 + \beta_1 x_{21} + \beta_2 x_{22} + ... + \beta_k x_{2k} + \epsilon_2$$

$$.$$

$$.$$

$$y_n = \beta_0 + \beta_1 x_{n1} + \beta_2 x_{n2} + ... + \beta_k x_{nk} + \epsilon_n$$

vector
y

2. Multiple linear regression model in matrix notation:

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

regressor variable

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_n \end{bmatrix}_{n\times 1} \qquad \boldsymbol{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \ldots & x_{1k} \\ 1 & x_{21} & x_{22} & \ldots & x_{2k} \\ . & . & . & & . \\ . & . & . & & . \\ . & . & . & & . \\ 1 & x_{n1} & x_{n2} & \ldots & x_{nk} \end{bmatrix}_{n\times(k+1)}$$

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ . \\ . \\ \beta_k \end{bmatrix}_{(k+1)\times 1} \qquad \boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ . \\ . \\ . \\ \epsilon_n \end{bmatrix}_{n\times 1}$$

$$(\varepsilon_1 \ \varepsilon_2 \ \ldots \ \varepsilon_n)\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

3. Least-squares function

$y = X\beta + \varepsilon$

$\Rightarrow \varepsilon = y - X\beta$

$$
\begin{aligned}
S &= \sum_{i=1}^{n} \epsilon_i^2 \\
&= \boldsymbol{\epsilon}'\boldsymbol{\epsilon} \\
&= (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})'(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}) \\
&= (\boldsymbol{y}' - \boldsymbol{\beta}'\boldsymbol{X}')(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}) \\
&= \boldsymbol{y}'\boldsymbol{y} - \boldsymbol{y}'\boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{\beta}'\boldsymbol{X}'\boldsymbol{y} + \boldsymbol{\beta}'\boldsymbol{X}'\boldsymbol{X}\boldsymbol{\beta} \\
&= \boldsymbol{y}'\boldsymbol{y} - 2\boldsymbol{y}'\boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\beta}'\boldsymbol{X}'\boldsymbol{X}\boldsymbol{\beta} \quad \because \boldsymbol{y}'\boldsymbol{X}\boldsymbol{\beta} = \boldsymbol{\beta}'\boldsymbol{X}'\boldsymbol{y}
\end{aligned}
$$

4. Least-squares normal equations

scalar quantity

$$\frac{\partial S}{\partial \boldsymbol{\beta}} = -2\boldsymbol{X}'\boldsymbol{y} + 2\boldsymbol{X}'\boldsymbol{X}\boldsymbol{\beta} \qquad \because \frac{\partial(\boldsymbol{a}'\boldsymbol{\beta})}{\partial \boldsymbol{\beta}} = \boldsymbol{a}, \quad \frac{\partial(\boldsymbol{\beta}'\boldsymbol{A}\boldsymbol{\beta})}{\partial \boldsymbol{\beta}} = 2\boldsymbol{A}\boldsymbol{\beta}$$

vector

$$\boldsymbol{0} = -2\boldsymbol{X}'\boldsymbol{y} + 2\boldsymbol{X}'\boldsymbol{X}\hat{\boldsymbol{\beta}}$$

$$\boxed{\boldsymbol{X}'\boldsymbol{X}\hat{\boldsymbol{\beta}} = \boldsymbol{X}'\boldsymbol{y}}$$

same

$$\hat{\beta} = (X'X)^{-1}X'y$$

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i2} & \dots & \sum_{i=1}^{n} x_{ik} \\ \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i1}^2 & \sum_{i=1}^{n} x_{i1}x_{i2} & \dots & \sum_{i=1}^{n} x_{i1}x_{ik} \\ \sum_{i=1}^{n} x_{i2} & \sum_{i=1}^{n} x_{i2}x_{i1} & \sum_{i=1}^{n} x_{i2}^2 & \dots & \sum_{i=1}^{n} x_{i2}x_{ik} \\ . & . & . & & . \\ . & . & . & & . \\ . & . & . & & . \\ \sum_{i=1}^{n} x_{ik} & \sum_{i=1}^{n} x_{ik}x_{i1} & \sum_{i=1}^{n} x_{ik}x_{i2} & \dots & \sum_{i=1}^{n} x_{ik}^2 \end{bmatrix} \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \\ . \\ . \\ . \\ \hat{\beta}_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{i1}y_i \\ \sum_{i=1}^{n} x_{i2}y_i \\ . \\ . \\ . \\ \sum_{i=1}^{n} x_{ik}y_i \end{bmatrix}$$

5. Least-squares estimates of $\hat{\boldsymbol{\beta}}$

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$$

6. Predicted response $\hat{\boldsymbol{y}}$ and residual $\boldsymbol{e}$

$y = X\beta + \varepsilon$

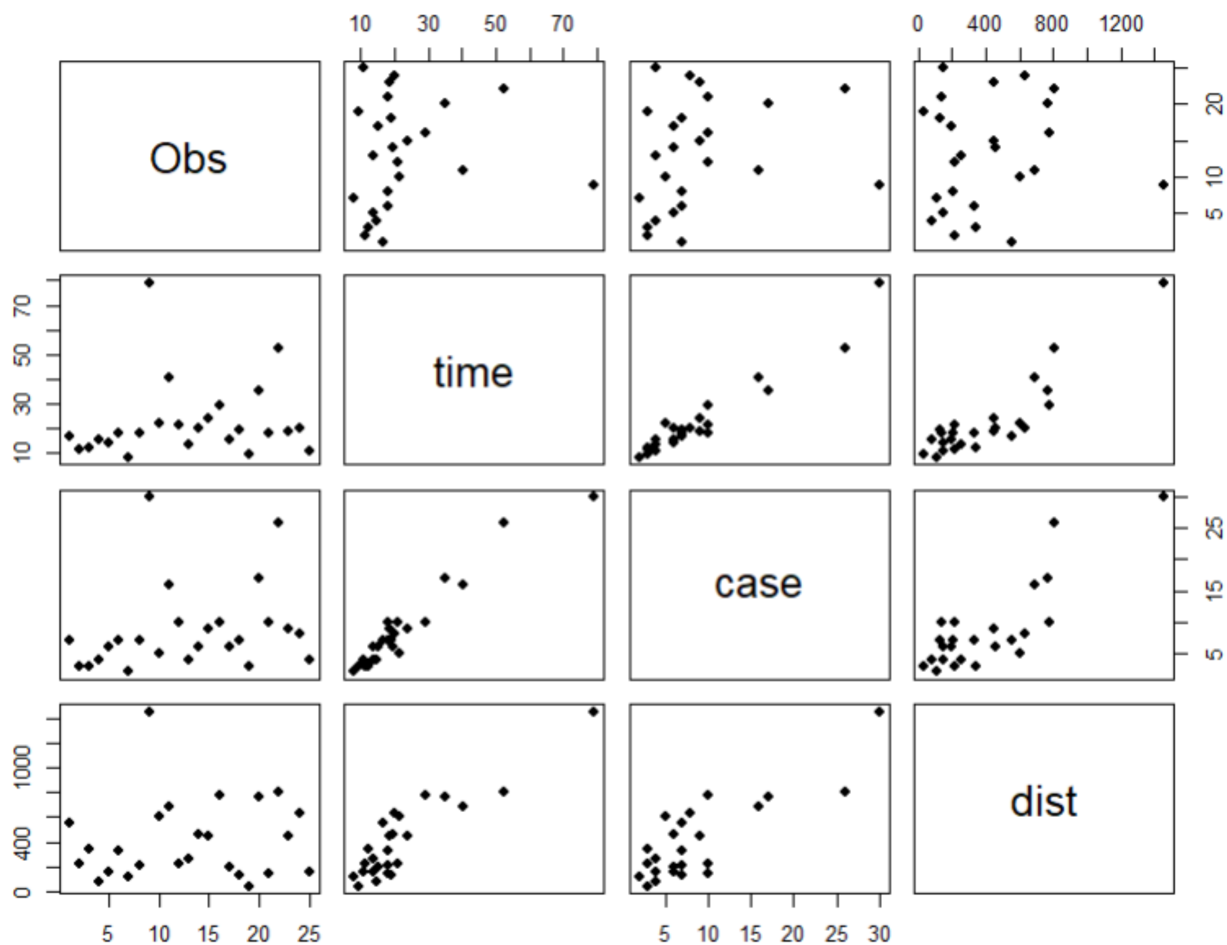
$$\begin{aligned}\hat{\boldsymbol{y}} &= \boldsymbol{X}\hat{\boldsymbol{\beta}} \\ &= \boxed{\boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'}\boldsymbol{y} \\ &= \boldsymbol{H}\boldsymbol{y} \quad \text{where } \boldsymbol{H} \text{ is called the hat matrix.}\end{aligned}$$

(observed - predicted)

residual
$$\begin{aligned}\boldsymbol{e} &= \boldsymbol{y} - \hat{\boldsymbol{y}} \\ &= \boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}} \\ &= \boldsymbol{y} - \boldsymbol{H}\boldsymbol{y} \\ &= (\boldsymbol{I}_n - \boldsymbol{H})\boldsymbol{y}\end{aligned}$$

Delivery time data

```r
# scatter matrix plot
pairs(dat, pch=16)
```



Delivery time data

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

$$\hat{y} = 2.34123 + 1.61591 x_1 + 0.01438 x_2$$

$$\mathbf{X} = \begin{bmatrix} 1 & 7 & 560 \\ 1 & 3 & 220 \\ 1 & 3 & 340 \\ 1 & 4 & 80 \\ 1 & 6 & 150 \\ 1 & 7 & 330 \\ 1 & 2 & 110 \\ 1 & 7 & 210 \\ 1 & 30 & 1460 \\ 1 & 5 & 605 \\ 1 & 16 & 688 \\ 1 & 10 & 215 \\ 1 & 4 & 255 \\ 1 & 6 & 462 \\ 1 & 9 & 448 \\ 1 & 10 & 776 \\ 1 & 6 & 200 \\ 1 & 7 & 132 \\ 1 & 3 & 36 \\ 1 & 17 & 770 \\ 1 & 10 & 140 \\ 1 & 26 & 810 \\ 1 & 9 & 450 \\ 1 & 8 & 635 \\ 1 & 4 & 150 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 16.68 \\ 11.50 \\ 12.03 \\ 14.88 \\ 13.75 \\ 18.11 \\ 8.00 \\ 17.83 \\ 79.24 \\ 21.50 \\ 40.33 \\ 21.00 \\ 13.50 \\ 19.75 \\ 24.00 \\ 29.00 \\ 15.35 \\ 19.00 \\ 9.50 \\ 35.10 \\ 17.90 \\ 52.32 \\ 18.75 \\ 19.83 \\ 10.75 \end{bmatrix}$$

$$\mathbf{X}'\mathbf{X} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 7 & 3 & \cdots & 4 \\ 560 & 220 & \cdots & 150 \end{bmatrix}}_{3\times n} \underbrace{\begin{bmatrix} 1 & 7 & 560 \\ 1 & 3 & 220 \\ \vdots & \vdots & \vdots \\ 1 & 4 & 150 \end{bmatrix}}_{n\times 3} = \underbrace{\begin{bmatrix} 25 & 219 & 10{,}232 \\ 219 & 3{,}055 & 133{,}899 \\ 10{,}232 & 133{,}899 & 6{,}725{,}688 \end{bmatrix}}_{3\times 3}$$

$$\mathbf{X}'\mathbf{y} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 7 & 3 & \cdots & 4 \\ 560 & 220 & \cdots & 150 \end{bmatrix}}_{3\times n} \underbrace{\begin{bmatrix} 16.68 \\ 11.50 \\ \vdots \\ 10.75 \end{bmatrix}}_{n\times 1} = \underbrace{\begin{bmatrix} 559.60 \\ 7{,}375.44 \\ 337{,}072.00 \end{bmatrix}}_{3\times 1}$$

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

$$\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} = \begin{bmatrix} 25 & 219 & 10{,}232 \\ 219 & 3{,}055 & 133{,}899 \\ 10{,}232 & 133{,}899 & 6{,}725{,}688 \end{bmatrix}^{-1} \begin{bmatrix} 559.60 \\ 7{,}375.44 \\ 337{,}072.00 \end{bmatrix}$$

$$= \begin{bmatrix} 0.11321518 & -0.00444859 & -0.00008367 \\ -0.00444859 & 0.00274378 & -0.00004786 \\ -0.00008367 & -0.00004786 & 0.00000123 \end{bmatrix} \begin{bmatrix} 559.60 \\ 7{,}375.44 \\ 337{,}072.00 \end{bmatrix}$$

$$= \begin{bmatrix} 2.34123115 \\ 1.61590712 \\ 0.01438483 \end{bmatrix}$$

```r
# matrices and finding betas
one <- rep(1,length(time))
X <- array(c(one,case,dist), dim=c(length(time),3))
y <- array(time, dim=c(length(time),1))
XPX <- t(X) %*% X
```

matrix multiplication

```r
XPX
XPy <- t(X) %*% y
XPy
betahat <- solve(XPX, XPy)
betahat
```

33:1 (Top Level)

Console **Background Jobs**

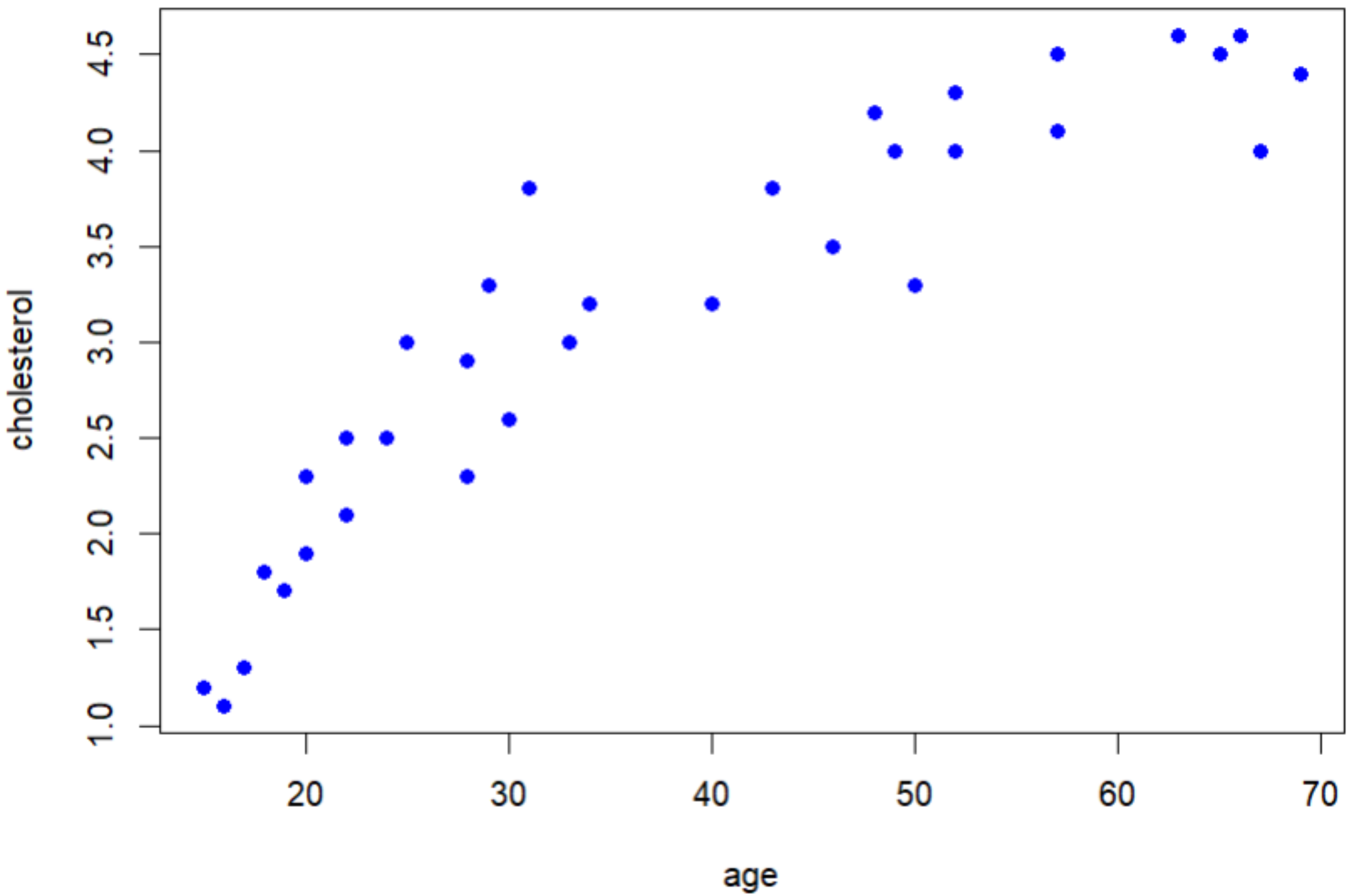
R 3.4.1 ·

```
> y <- array(time, dim=c(length(time),1))
> XPX <- t(X) %*% X
> XPX
      [,1]   [,2]    [,3]
[1,]    25    219   10232
[2,]   219   3055  133899
[3,] 10232 133899 6725688
> XPy <- t(X) %*% y
> XPy
          [,1]
[1,]    559.60
[2,]   7375.44
[3,] 337071.69
> betahat <- solve(XPX, XPy)
> betahat
           [,1]
[1,] 2.34123115
[2,] 1.61590721
[3,] 0.01438483
```

Cholesterol age data

	age	cholesterol_level
1	15	1.2
2	17	1.3
3	16	1.1
4	19	1.7
5	18	1.8
6	20	2.3
7	20	1.9
8	22	2.1
9	22	2.5
10	24	2.5
11	25	3.0
12	28	2.3
13	28	2.9
14	29	3.3
15	30	2.6
16	33	3.0
17	34	3.2

	age	cholesterol_level
18	31	3.8
19	40	3.2
20	43	3.8
21	46	3.5
22	48	4.2
23	49	4.0
24	50	3.3
25	52	4.0
26	52	4.3
27	57	4.1
28	57	4.5
29	63	4.6
30	65	4.5
31	66	4.6
32	67	4.0
33	69	4.4



cholesterol
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
20
30
40
50
60
70
age

Cholesterol age data

Straight line model

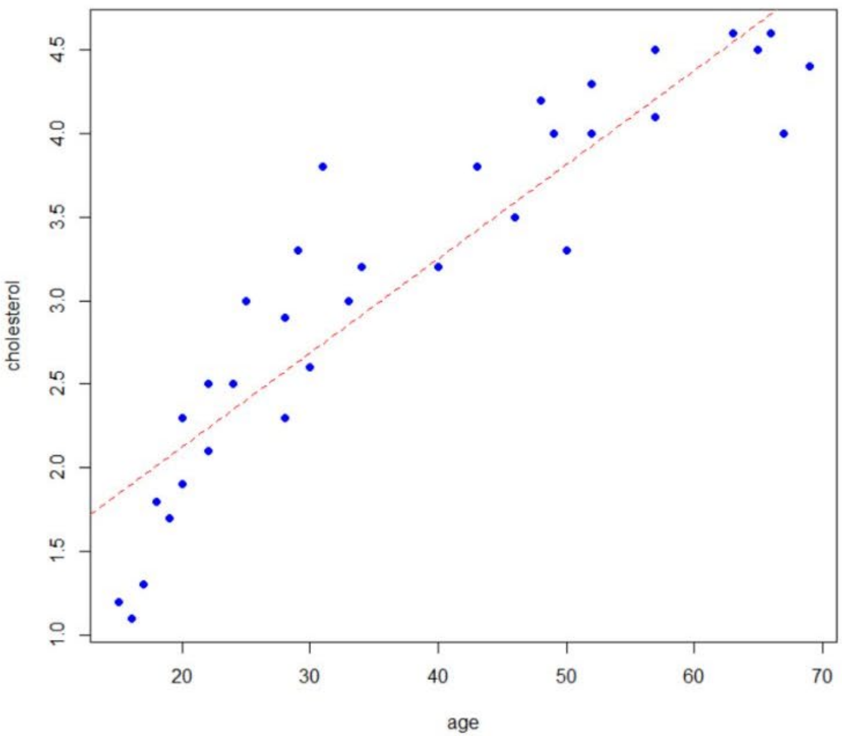
$$cholesterol = \beta_0 + \beta_1 \; age + \epsilon$$

Quadratic line model

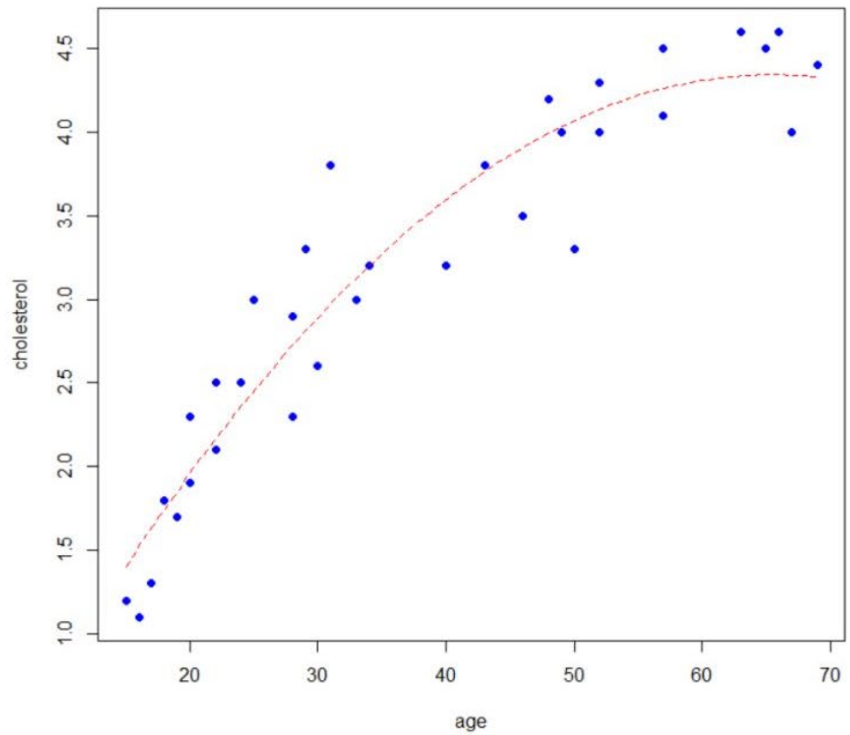
$$cholesterol = \beta_0 + \beta_1 \; age + \beta_2 \; age^2 + \epsilon$$

Cholesterol age data

```r
#install.packages("np")
library(MASS)
library(np)
rm(list=ls())

dat <- read.csv("D:\\nus_teaching\\st3131\\data\\cholesterol_age.csv",
                header = T, sep=",")
age <- dat[,1]
cholesterol <- dat[,2]

#Fit straight line model
mod1 <- lm(cholesterol~age)
COEF <- coef(mod1)
names(COEF) <- NULL
beta0 <- COEF[1]
beta1 <- COEF[2]
plot(age,cholesterol,pch=16,col="blue")
abline(beta0,beta1,lty=1,col="red")
summary(mod1)

#Fit quadratic model
mod2 <-lm(cholesterol~age + I(age^2))
mod2
COEF2 <- coef(mod2)
names(COEF2) <- NULL
COEF2
beta2.0 <- COEF2[1]
beta2.1 <- COEF2[2]
beta2.2 <- COEF2[3]
beta2.0
beta2.1
beta2.2
summary(mod2)
plot(age,cholesterol,pch=16,col="blue")
curve(beta2.0+x*beta2.1+x*x*beta2.2, add=TRUE, lty=1,col="red")
```

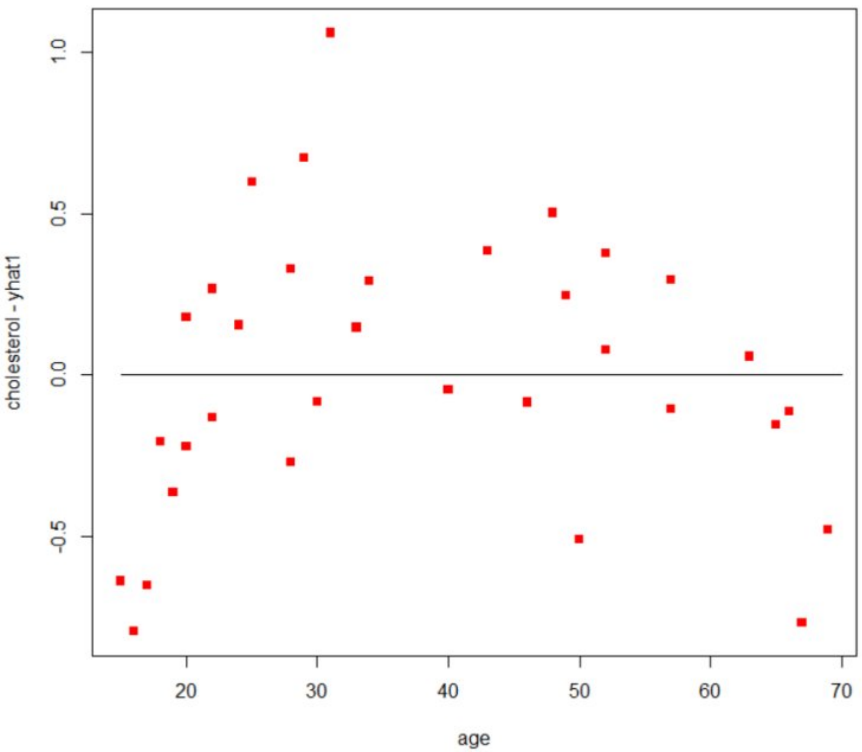


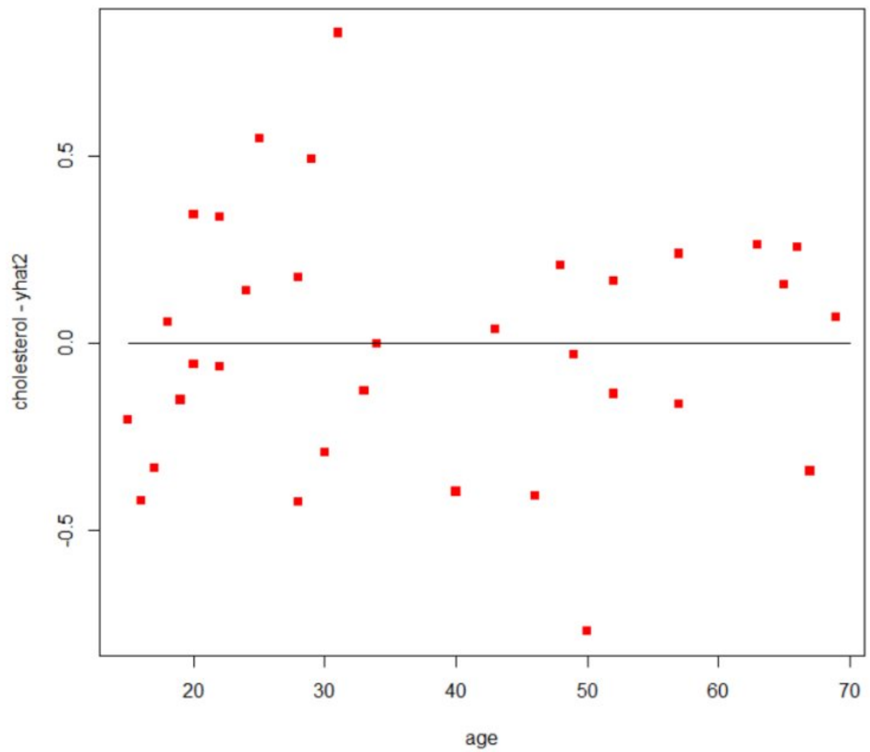
Multiple R-squared: 0.839



Multiple R-squared: 0.9041

Straight line model **Residual plots** quadratic model





The End