

ADVENTURES IN RATSUB

Adam Ponting

Nov 2019



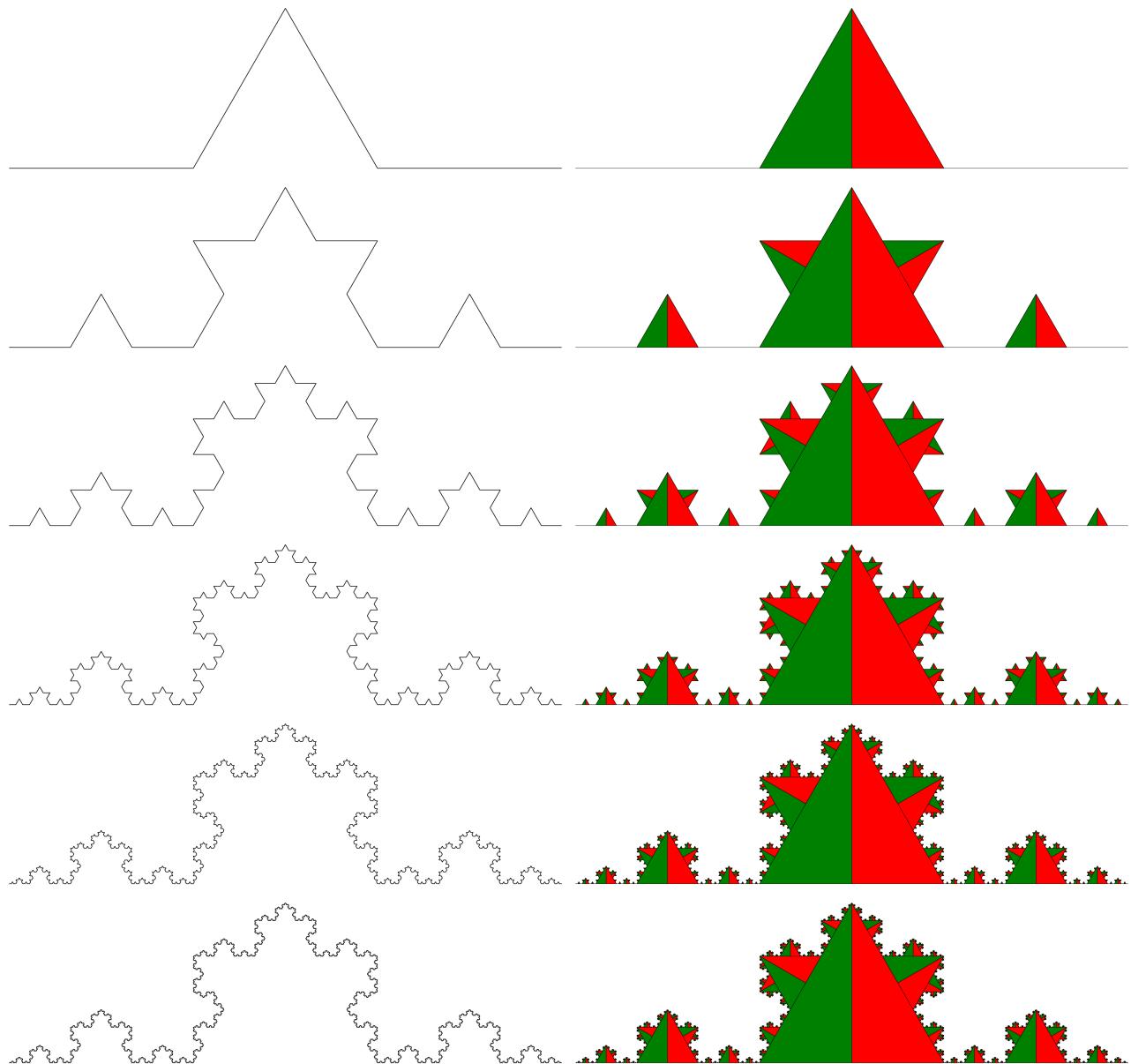
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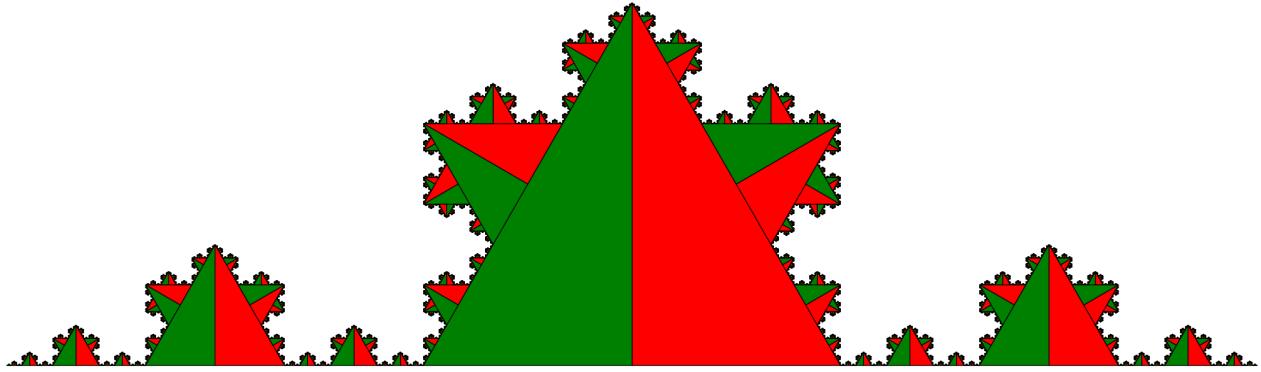
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1. Line fractals

Koch curve

(Listing 19).

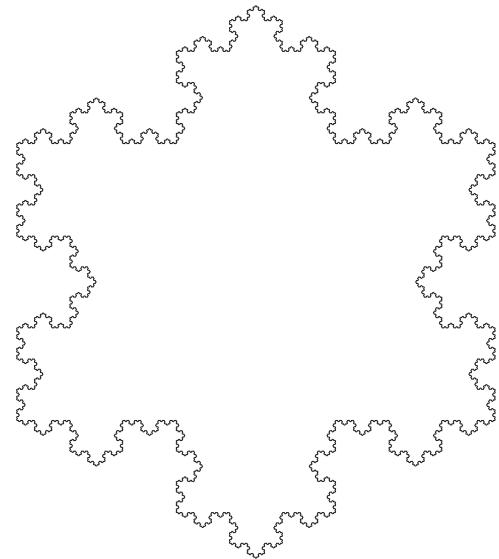




```

only edges
margin 1 26 1 1
pgon 3 50 30
def tri 3
  line :21
  line :10
  line :02
def line 2
  p2..4 p0 2..4/6 p1
  p5 p0 1 p3 left 0.57735
  line :02
  line :25
  line :54
  line :41

```



```

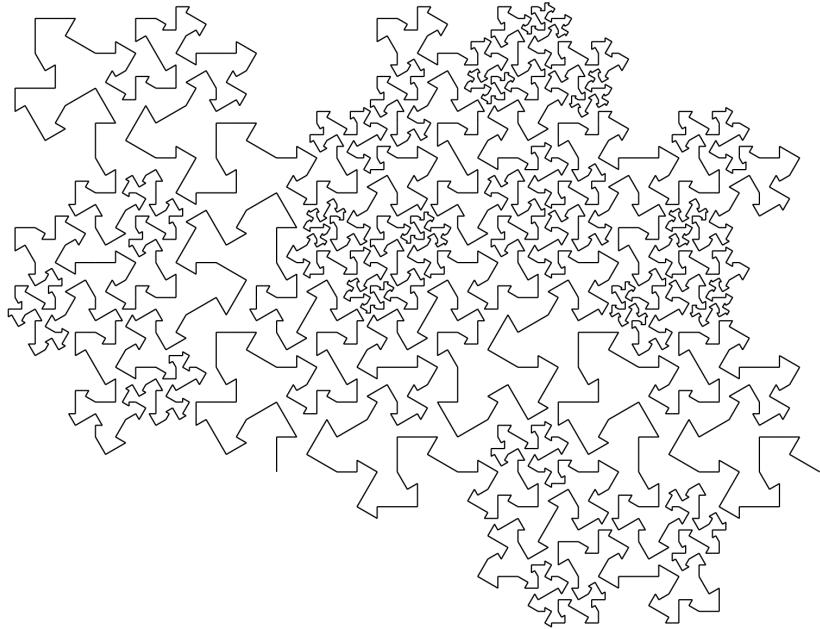
no edges
margin .5 13 .5 .5
pgon 3 25 30
def tri 3
  black
  p3..5 p0.. .5 p1..0 right .1428
  tri :345 + 1 1 0
  draw :012 + 0 1 0
  line :21 + 0 1 0
  line :10 + 0 1 0
  line :02 + 0 1 0
def line 2
  p2..4 p0 2..4/6 p1
  p5 p0 1 p3 left 0.57735
  tri :254 + 0 0 1
  draw :254 =
  line :02 + -2 -1 3
  line :25 + 1 -3 2
  line :54 + 2 -3 1
  line :41 + 3 -1 -2

```



Ventrella fractals

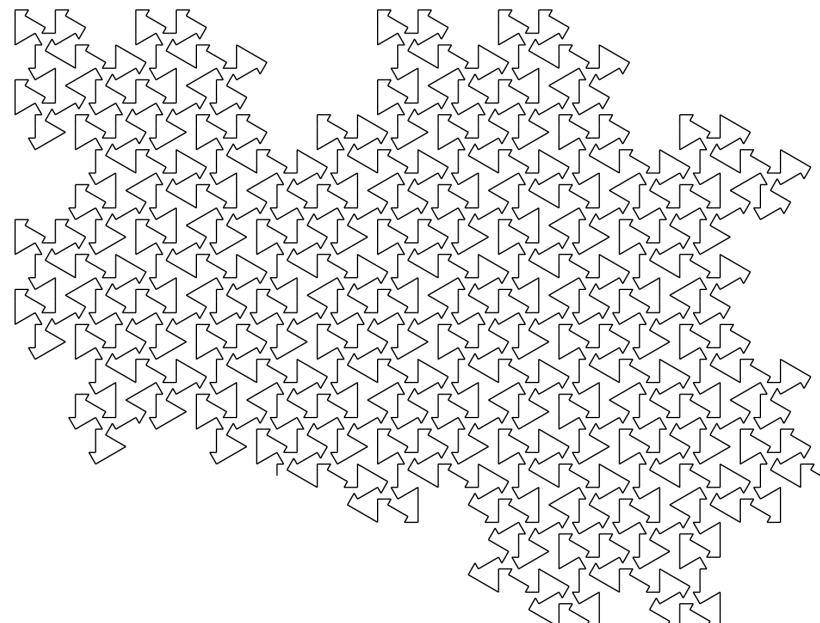
```
only edges
margin 35 50 15 20
p1 50 0
def line 2
  p2 p0 .5 p1 left .288675
  p3 p1 2 p2
  p4 p0 1 p3 right 1.1547
  p5 p3 .5 p4
  line :30 flip
  line :53
  line :54
  line :42
  line :21 flip
```



Level 5

```
only edges
margin 40 50 25 25
p1 50 0
def line 2
  p2 p0 .5 p1 left .288675
  p3 p1 2 p2
  p4 p0 1 p3 right 1.1547
  p5 p3 .5 p4
  line2 :30
  line :53 wait
  line :54 wait
  line :42 wait
  line2 :21
def line2 2
  p2 p0 .5 p1 right .288675
  p3 p1 2 p2
  p4 p0 1 p3 left 1.1547
  p5 p3 .5 p4
  line :30
  line2 :53 wait
  line2 :54 wait
  line2 :42 wait
  line :21
```

Level 7



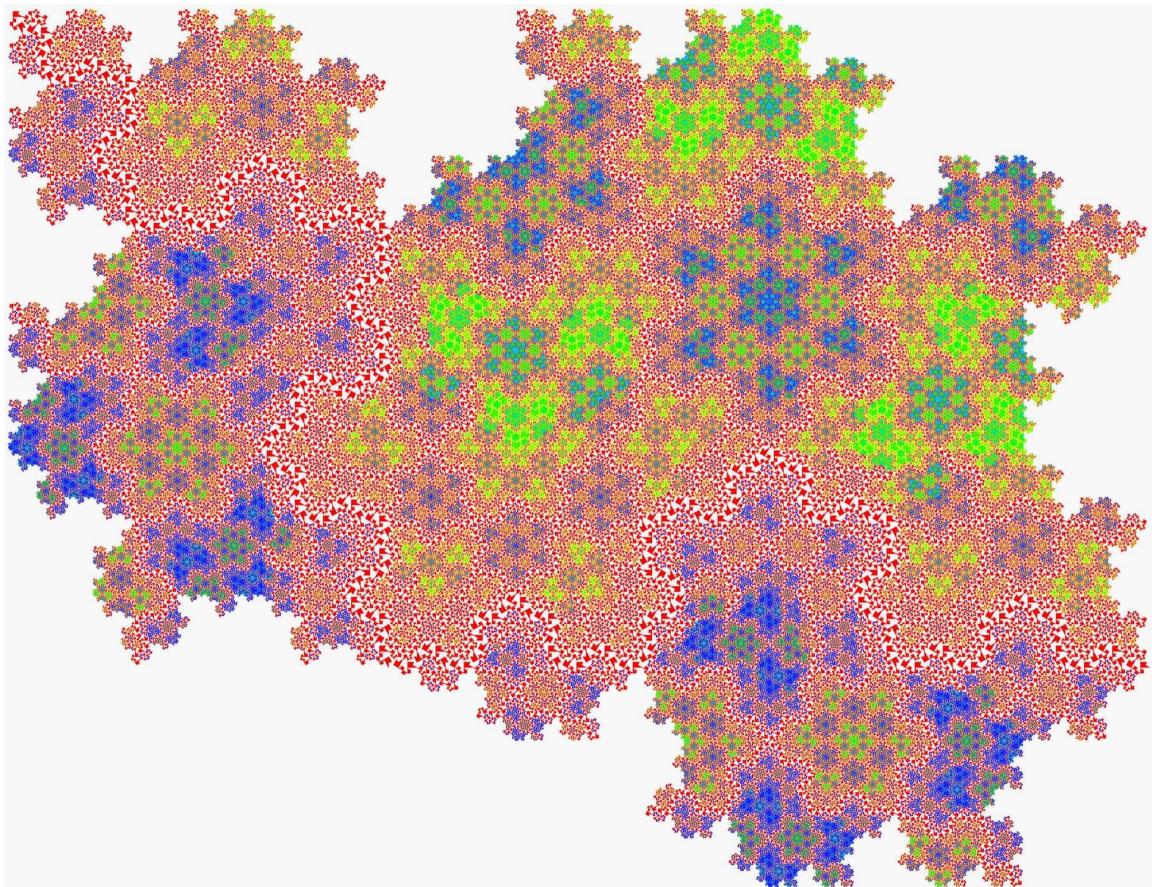
The same program, but with wait shapes for the 3 smaller segments:

```

no edges
margin 27 46 3 17
p1 50 0
def line 2
    grey
    p2 p0 .5 p1 left .288675
    p3 p1 2 p2
    p4 p0 1 p3 right 1.1547
    p5 p3 .5 p4
    hex :035421 + 1 1 -1
def line2 2
    p2 p0 .5 p1 right .288675
    p3 p1 2 p2
    p4 p0 1 p3 left 1.1547
    p5 p3 .5 p4
    hex2 :035421 - 1 1 -1
def hex 6
    line2 :10 + 1 -1 -1
    line :21 - 1 -1 -1
    line :23 - 1 -1 -1
    line :34 - 1 -1 -1
    line2 :45 + 1 -1 -1
def hex2 6
    line :10 + 1 -1 -1
    line2 :21 - 1 -1 -1
    line2 :23 - 1 -1 -1
    line2 :34 - 1 -1 -1
    line :45 + 1 -1 -1

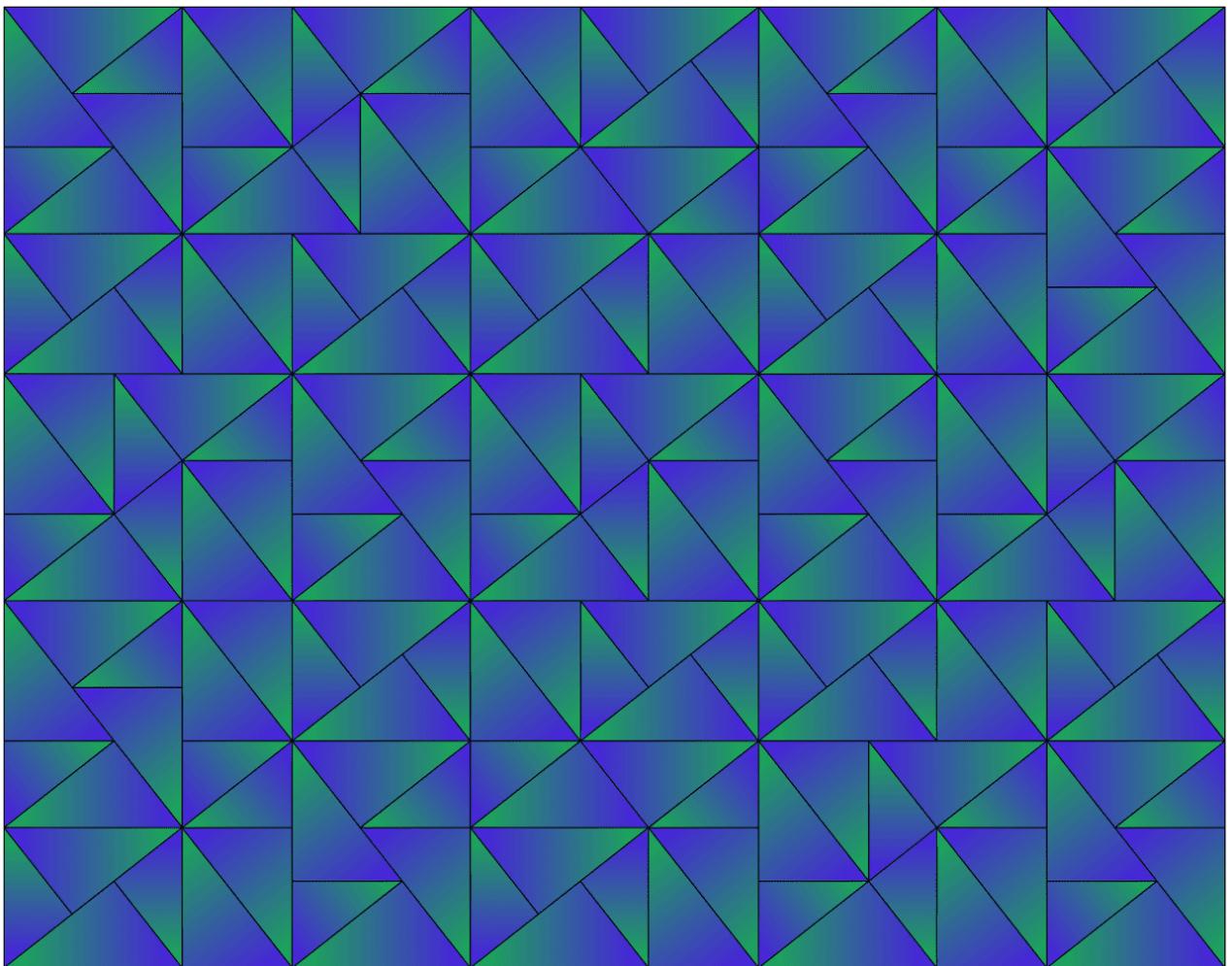
```

Below: Level 17 of the same curve, program adapted for colouring (image corrected). This program would be half as long by using `flip`, but the 2 shape calls to `hex` use different colouring.



2. Triangles

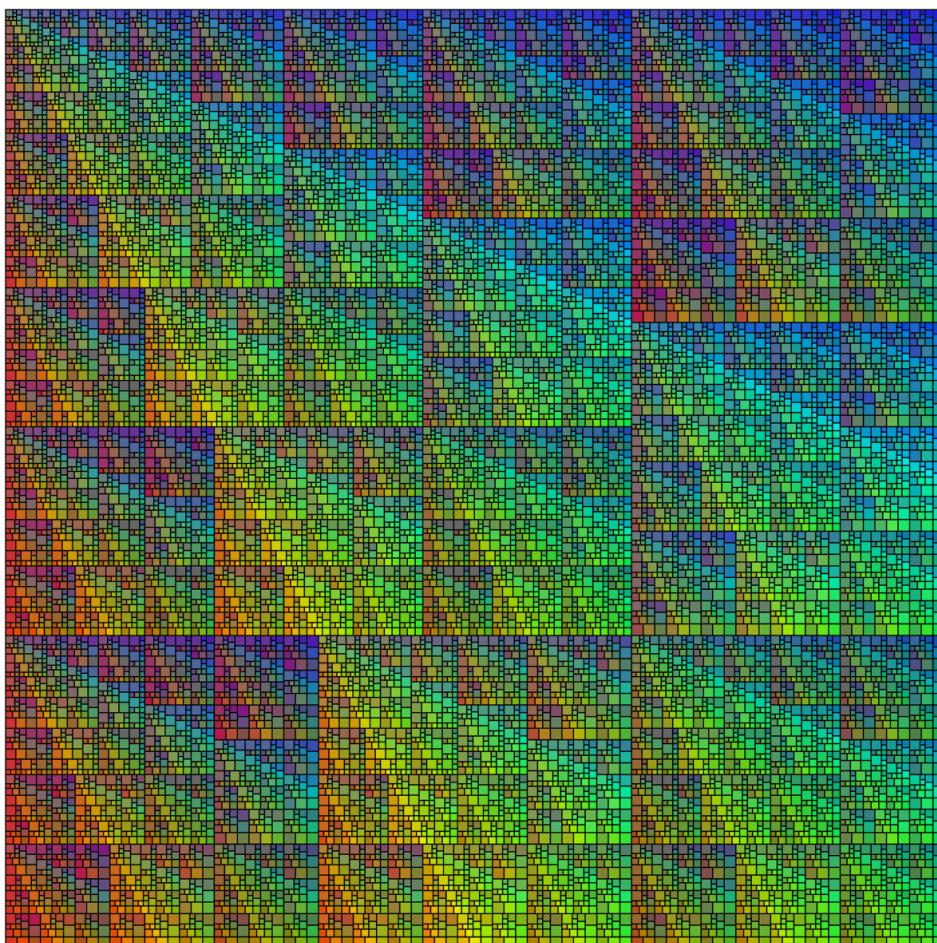
```
pbox 127.2 100
def rect 4
  tri :320
  tri :102
def tri 3
  p1 0.3 0.1 0.9 p2 0.1 0.7 0.3
  p3 p1 0.382 p2
  tri :310 wait
  tri :302
```



Hmm this is counted as one shape, not rectangles+triangles? I guess because the rectangle isn't repeatedly subdivided.

3. Squares

```
def sq 4
grey
p4 p0 1/3 p1
p5..7 p1.. 2/3 p2..0
p8..9 p2.. 1/3 p3..0
p10..11 p4 1..2/3 p6
p12 p5 1/3 p9
sq p4,1,5,11
sq p0,4,10,7 + 1 -1 -1 wait 2
sq p7,10,11,9 + 1 1 -2 wait 2
sq p9,11,6,3 + -1 1 -1 wait 2
sq p11,12,8,6 + -2 1 1 wait 2
sq p12,5,2,8 + -1 -1 1 wait 2
```



```
def sq 4
p4 p1 1/2 p2
p5 p0 1/2 p3
sq :1450 white
sq :3542 black
```

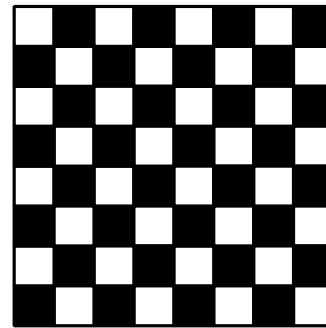
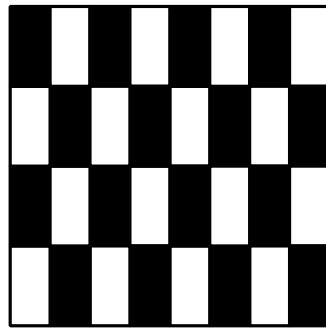
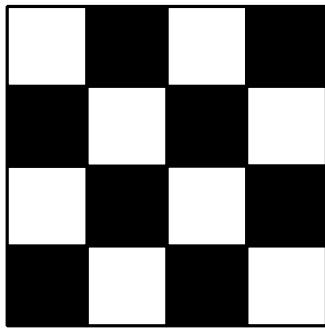
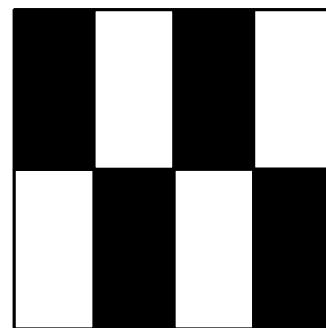
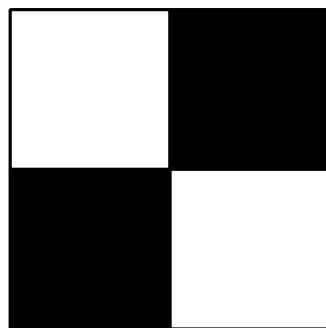
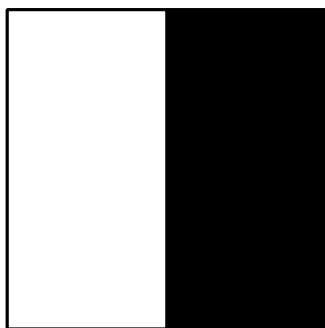
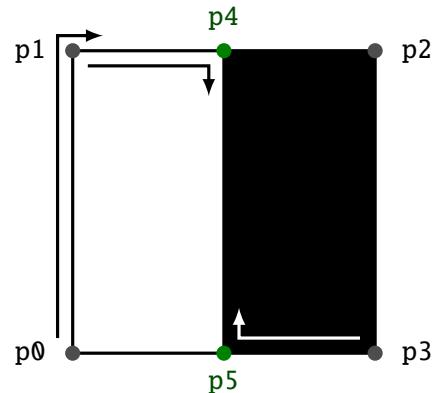
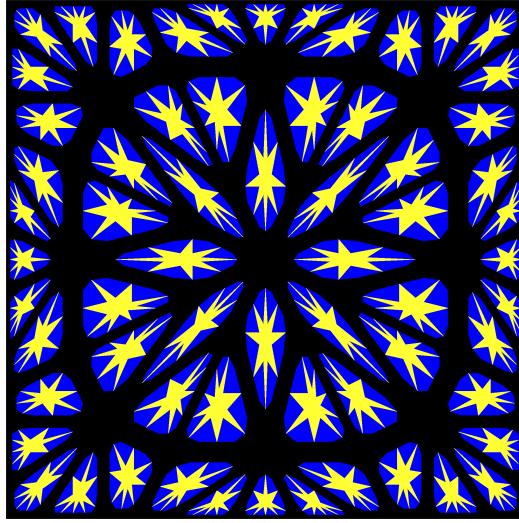
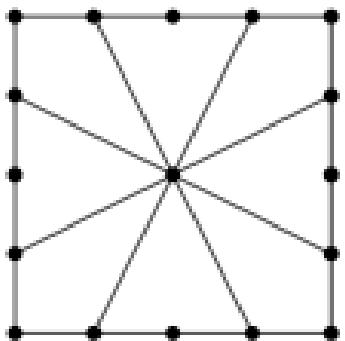


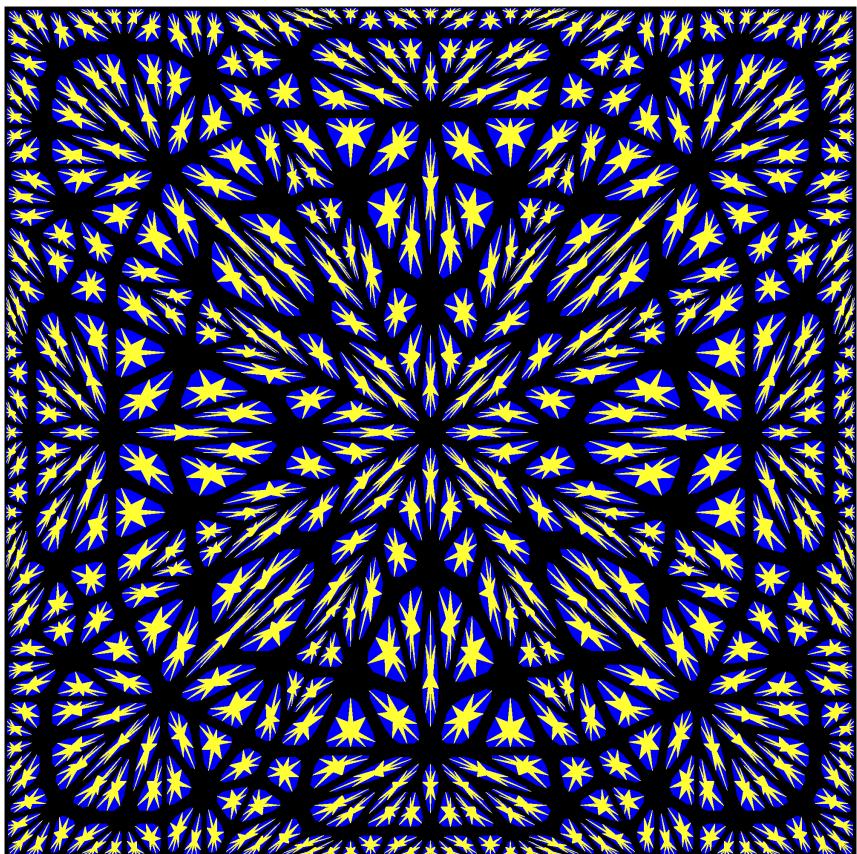
Figure 3.1: Levels 1-6

4. Quadrilaterals

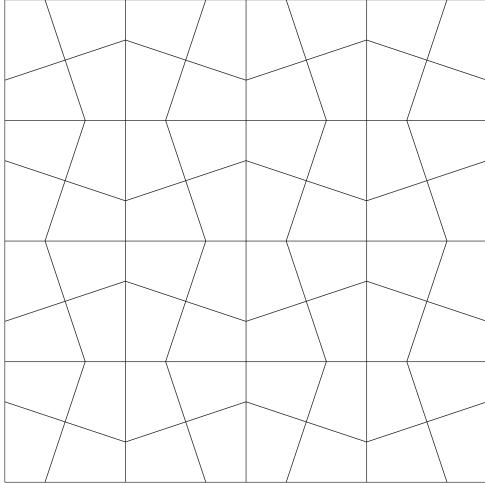


Level 4 and 5.

```
no edges
def sq 4
black
p4..7 p0.. 1/4 p1..0
p8..11 p0.. 2/4 p1..0
p12..15 p0.. 3/4 p1..0
p16 c
sq p0,4,16,15
sq p1,5,16,12
sq p2,6,16,13 1 1 .2
sq p3,7,16,14
sq p8,12,16,4
sq p9,13,16,5 blue
sq p10,14,16,6 blue
sq p11,15,16,7
```

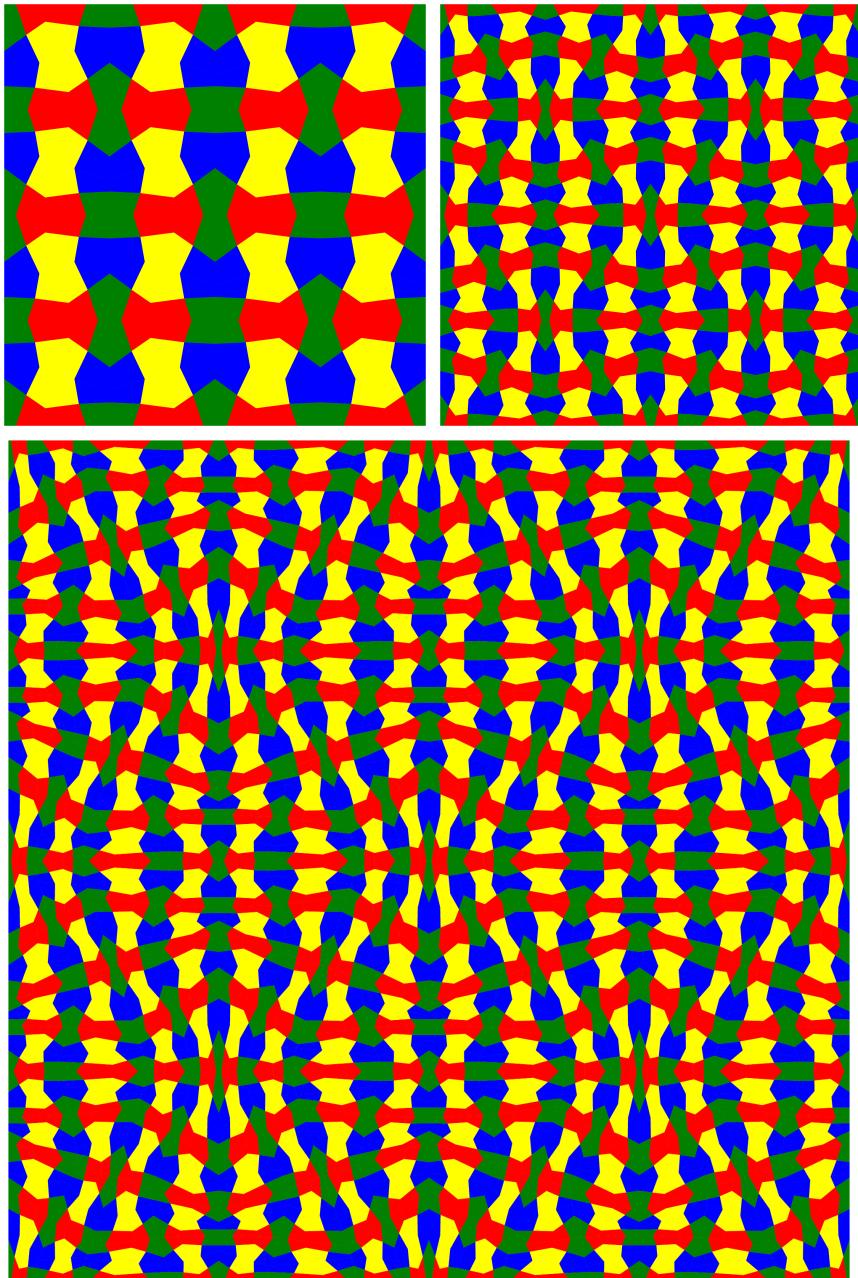


```
only edges
def sq0 4
    p4..7 p0.. 1/2 p1..0
    sq0 :415c
    sq0 :625c
    sq0 :7c40
    sq0 :7c63
=1
    sq :415c
    sq :625c
    sq :7c40
    sq :7c63
def sq 4
    p4..7 p0.. 1/3 p1..0
    sq :415c
    sq :407c
    sq :637c
    sq :625c
```



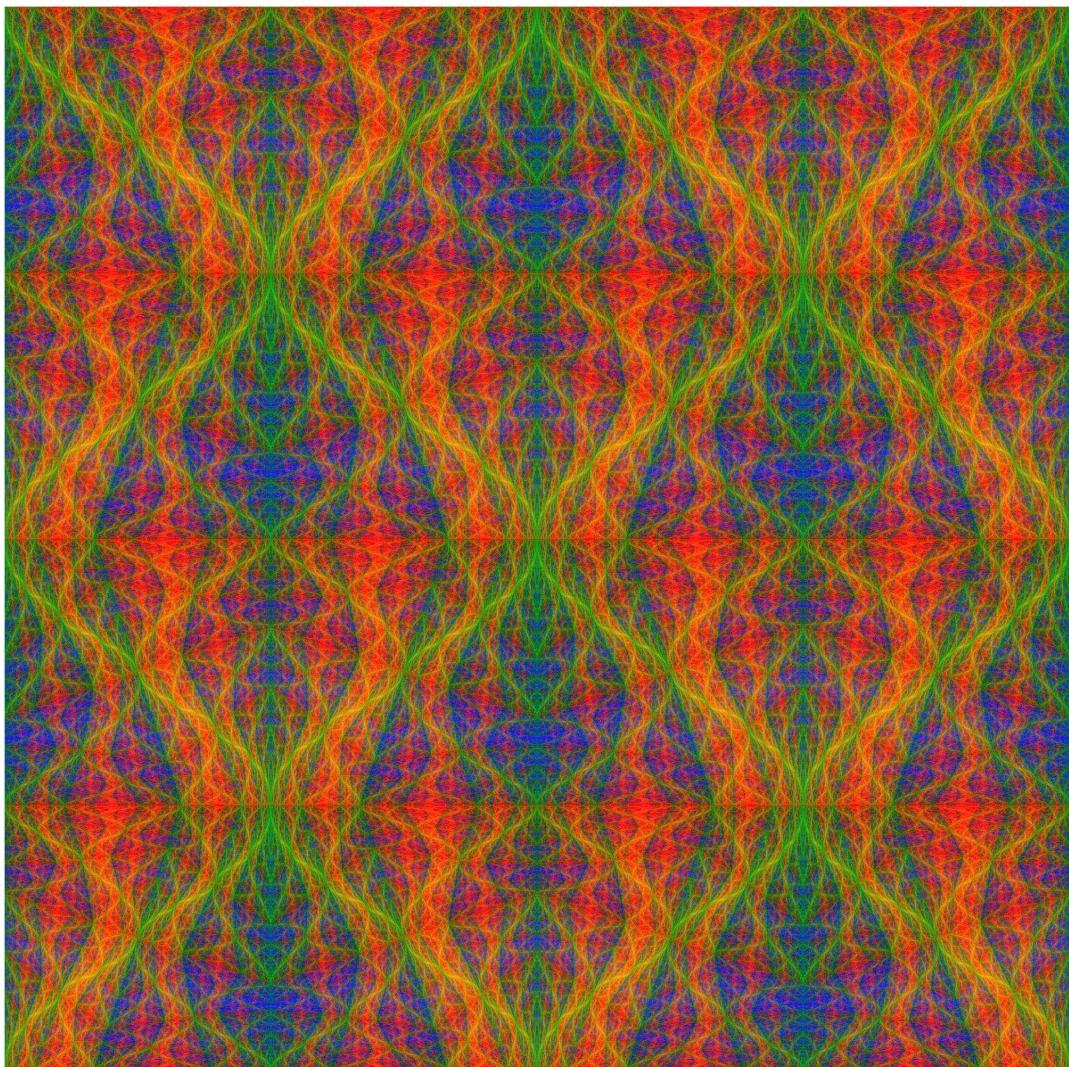
With primary colours:

```
only edges
def sq0 4
  p4..7 p0.. 1/2 p1..0
  sq0 :415c
  sq0 :625c
  sq0 :7c40
  sq0 :7c63
=1
  sq :415c
  sq :625c
  sq :7c40
  sq :7c63
def sq 4
  p4..7 p0.. 1/3 p1..0
  sq :415c green
  sq :407c blue
  sq :637c yellow
  sq :625c red
```



With colour-add:

```
only edges
def sq0 4
  .5 .3 .5
  p4..7 p0.. 1/2 p1..0
  sq0 :415c
  sq0 :625c
  sq0 :7c40
  sq0 :7c63
=1
  sq :415c
  sq :625c
  sq :7c40
  sq :7c63
def sq 4
  p4..7 p0.. 1/3 p1..0
  sq :415c + -1 .5 -1
  sq :407c + -1 -.5 1
  sq :637c + 1 .5 -1
  sq :625c + 1 -.5 -1
```



5. Two different polygons

Squares + triangles

(Listing 19).

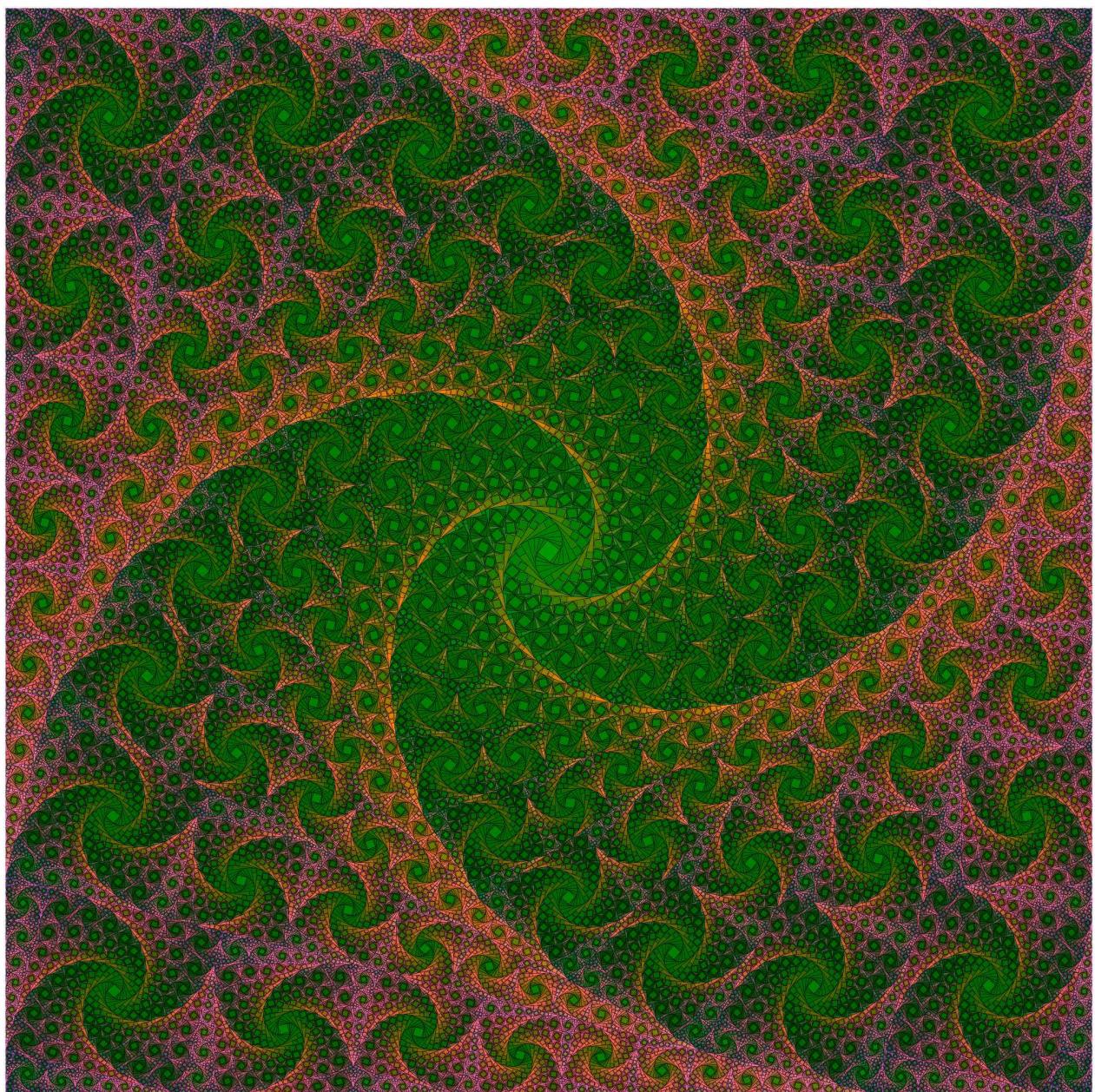


Figure 5.1: compdiss2a.sdv, level 22

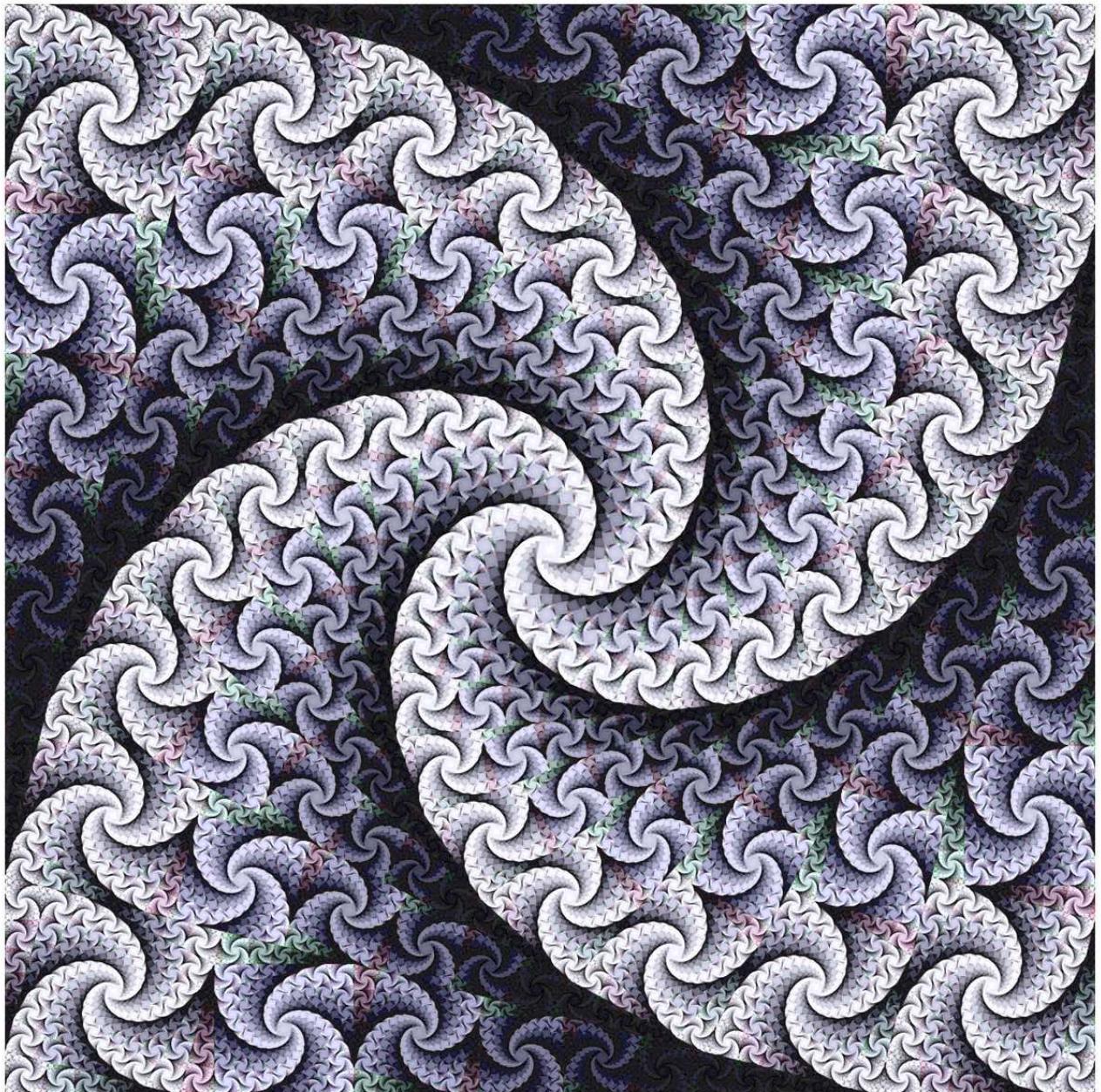


Figure 5.2: compdiss2c.sdv, level 25

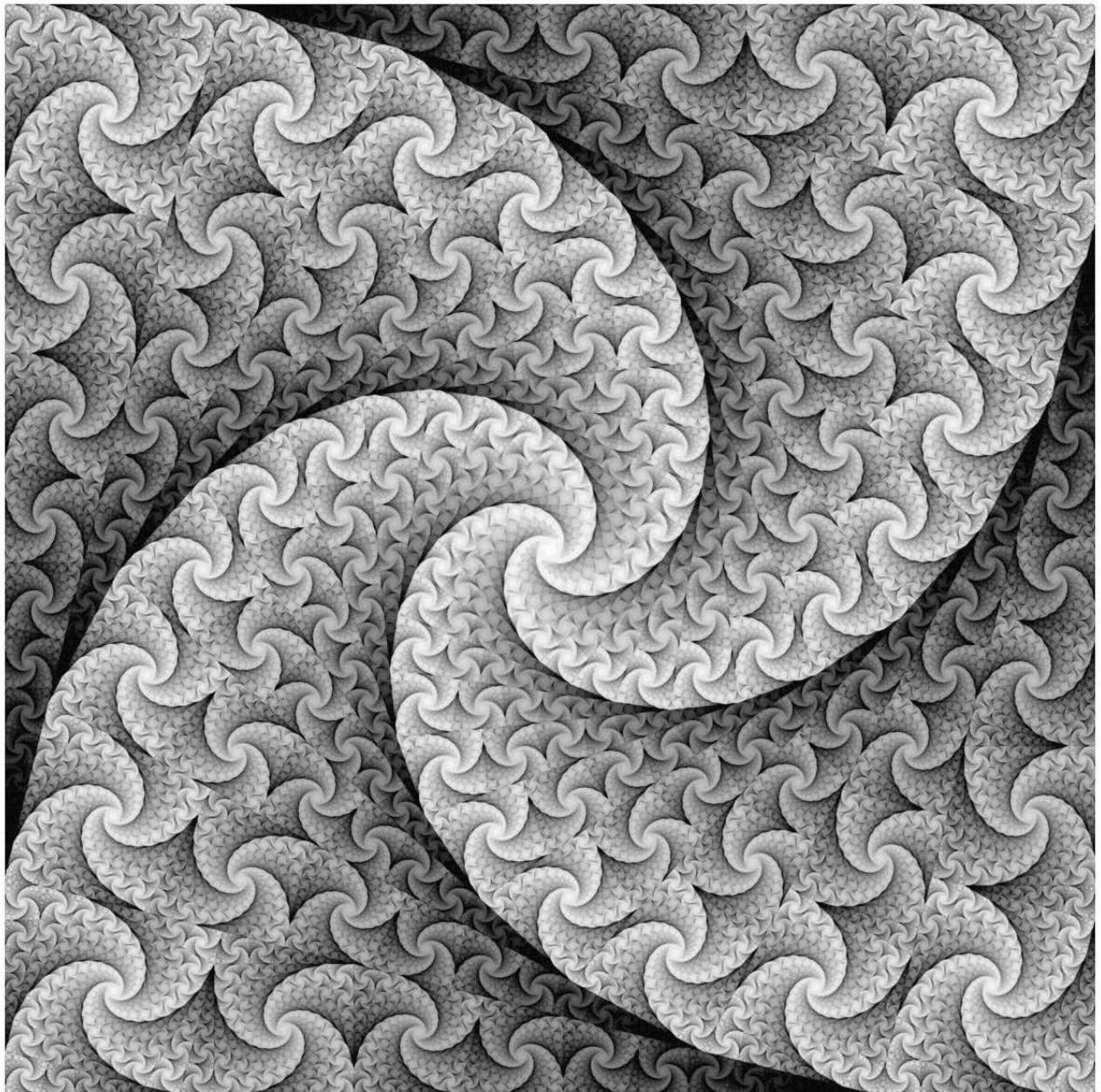


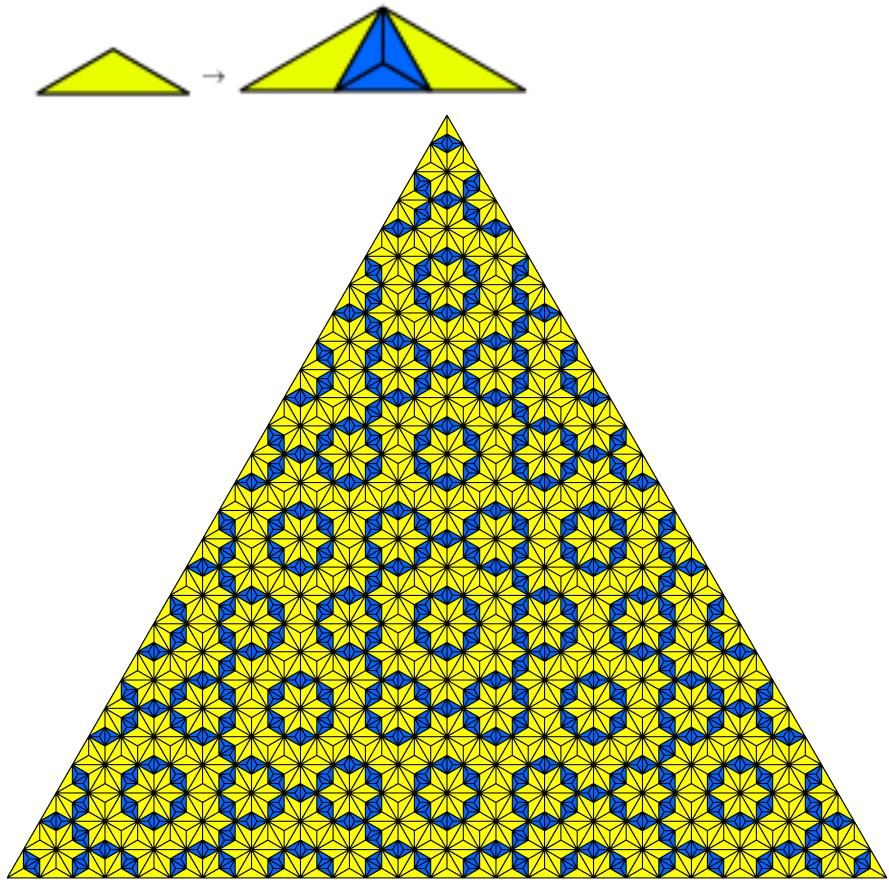
Figure 5.3: compdiss2e.sdv, level 25

6. Substitution tilings

Danzer's T2000



```
pgon 3 70 90
def init 3
  blue :0c1
  blue :2c0
  blue :1c2
def blue 3
  0 .4 1
  y :012 yellow
def y 3
  yellow
  p3..4 p0 1..2/3 p2
  y :130 yellow
  y :241 yellow
  blue :1c3 0 .4 1
  blue :3c4 0 .4 1
  blue :4c1 0 .4 1
```

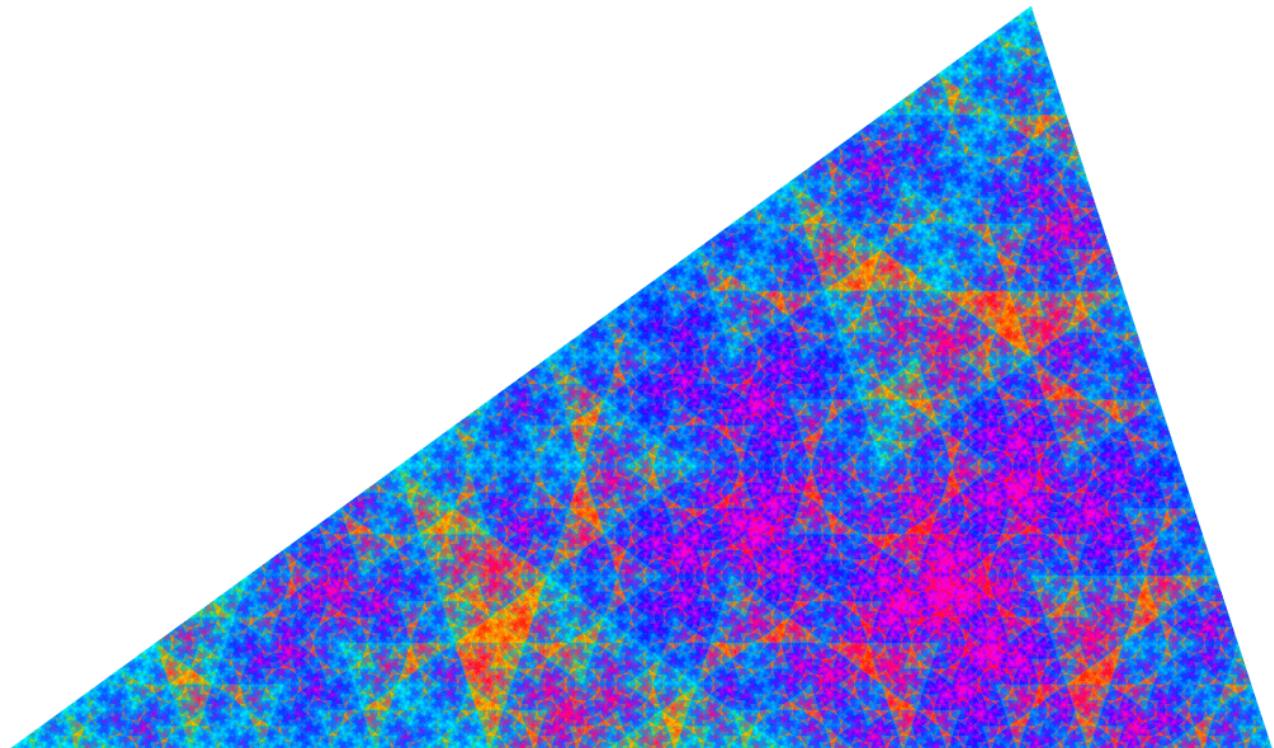


7. Aperiodic tilings

Penrose tiling

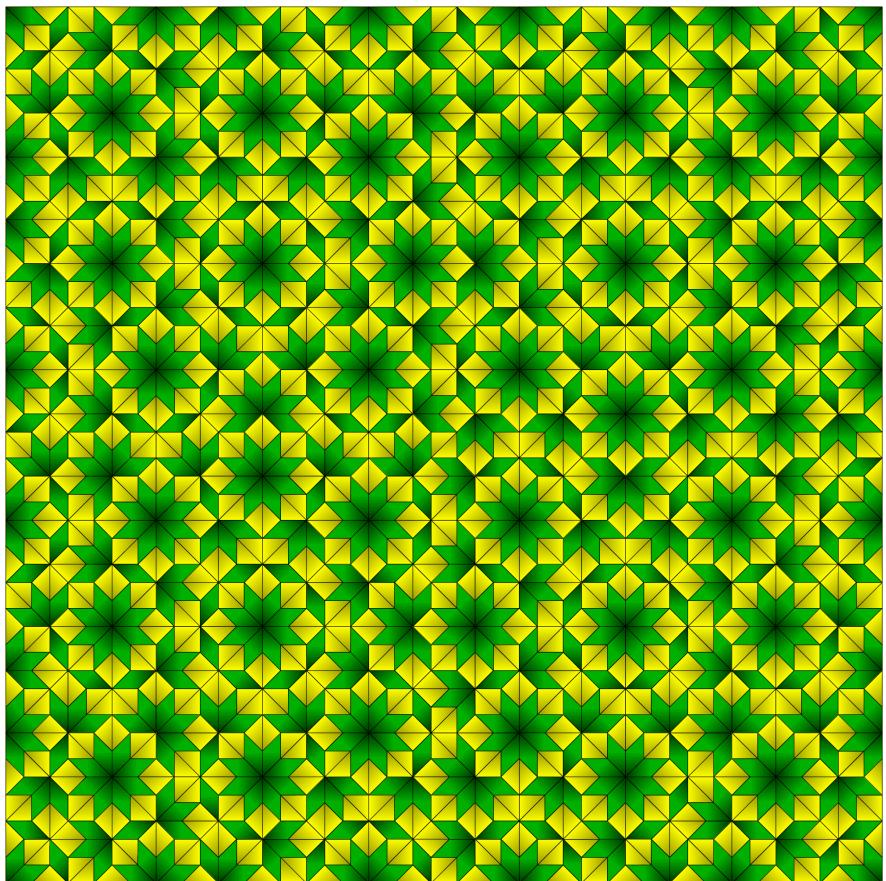
```
noedges
grid 36
p1 0 100
p2 100 0
def halfkite 3
  p3 p0 .381966 p1
  p4 p2 .381966 p0
  halfdart :034 + -2 0 2
  halfkite :134 + -1 0 1
  halfkite :124 + 1 -1 0
def halfdart 3
  p3 p2 .381966 p0
  halfdart :231 + 2 0 -2
  halfkite :031 + 0 1 -1
```

Level 12.

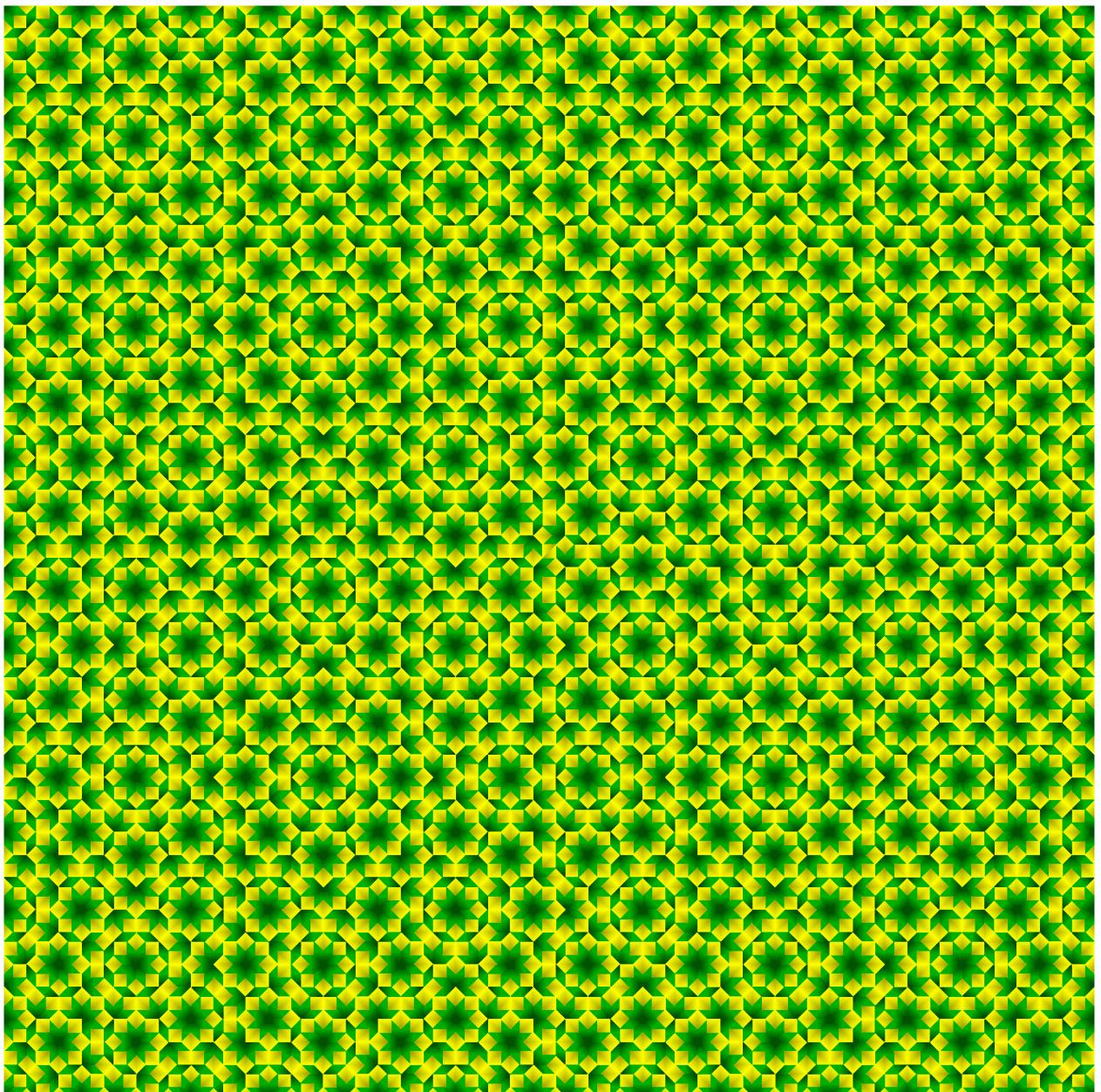


Amman-Beenker tiling

```
def sq 4
  tri :012
  tri :230
def tri 3
  p1 1 1 0 p0 .6 .6 0
  p3..4 p0.. 0.5858 p1..
  p5 p0 0.7071 p2
  p6 p2 " p0
  p7 p2 " p3
  tri :360
  tri :576
  tri :471
  loz :1367
  loz :2574
def loz 4
  p0 0 .2 0 p1 0 .7 0
  p4 p1 .5858 p0
  p5 p1 " p2
  p6 p3 " p2
  p7 p3 " p0
  p8 p2 " p0
  p9 p0 " p2
  loz :0487
  loz :3819
  loz :2596
  tri :783
  tri :481
  tri :591
  tri :693
```



Level 5

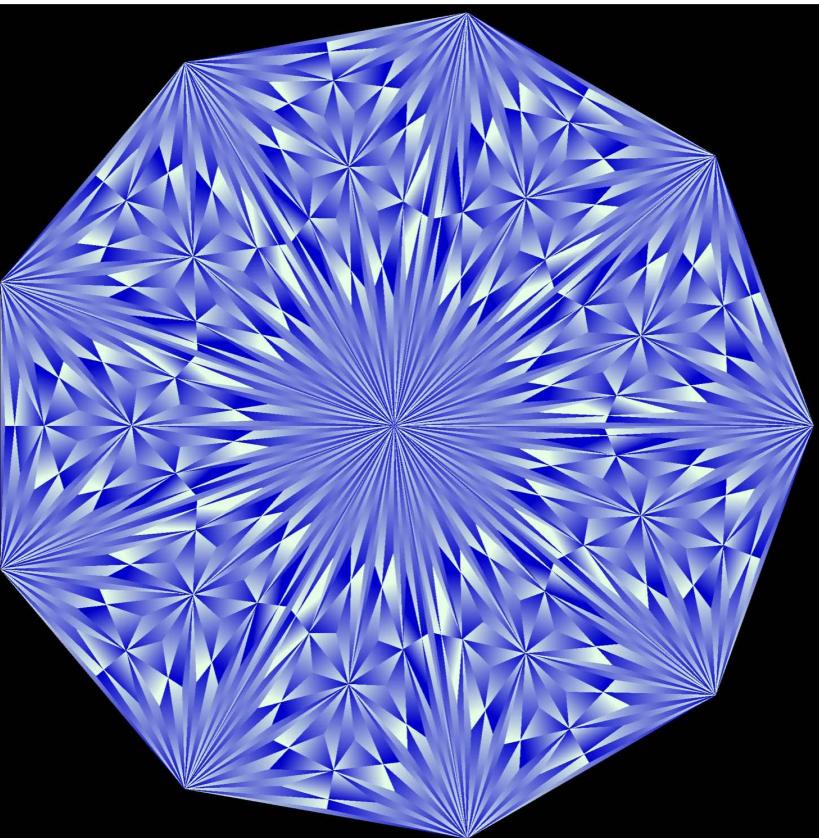


Amman-Beenker level 6, no edges.

8. Gradient shading

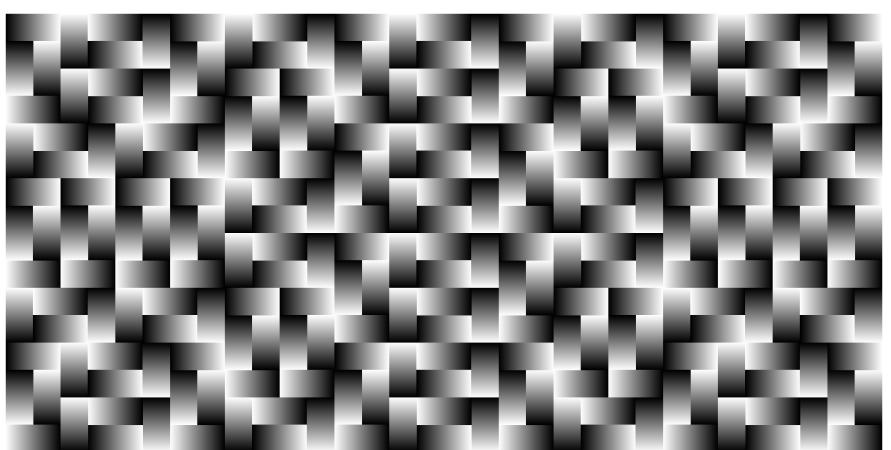
```
background black
no edges
pgon 9 50
def non 9
  tri :01c
  tri :12c
  tri :23c
  tri :34c
  tri :45c
  tri :56c
  tri :67c
  tri :78c
  tri :80c
def tri 3
  p0 .9 1 .9 p1 0 0 .8
  tri :20c
  tri :12c
  tri :01c
```

Level 5



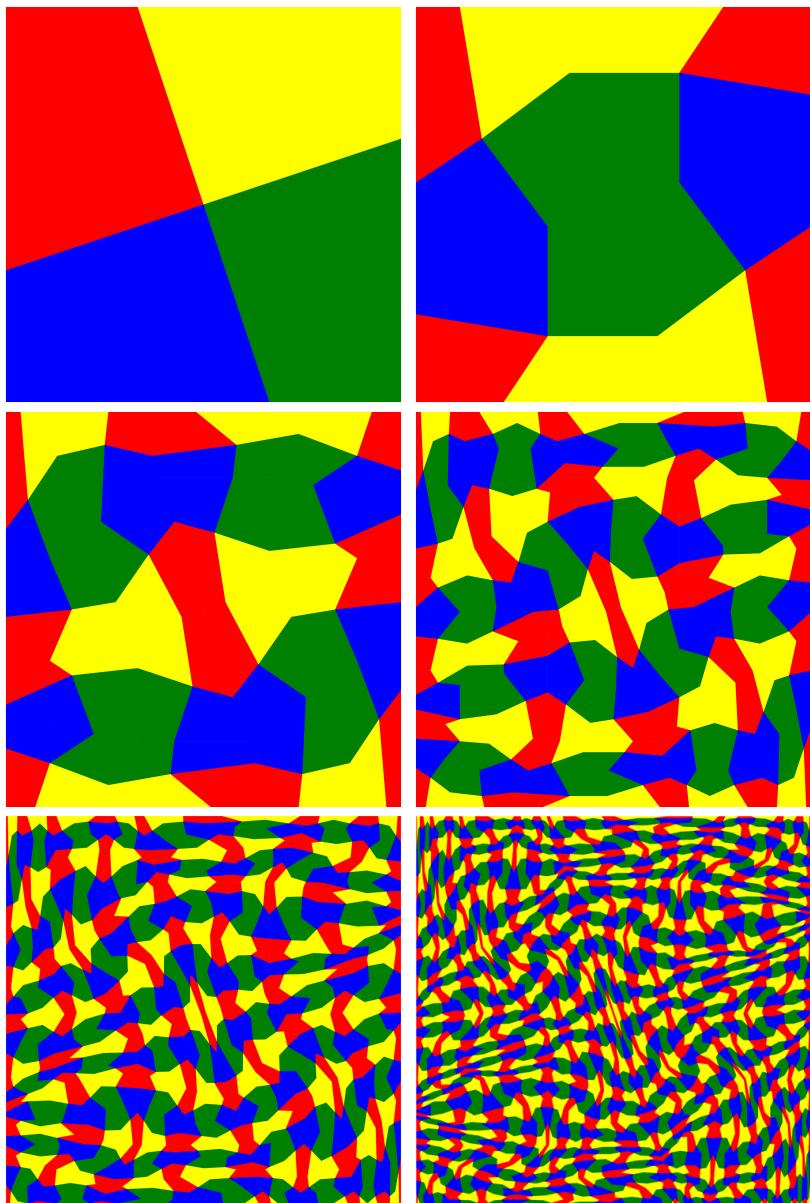
```
noedges
pbox 100 50
def rect 4
  p0 black p3 white
  p4..5 p1 1,3/4 p2
  p6..7 p3 1,3/4 p0
  p8..9 p4.. 1/2 p7..6
rect :7014
rect :5984
rect :7896
rect :5236
```

Level 4



```
no edges
def sq 4
p4..7 p0.. 1/3 p1..0
sq :407c blue
sq :415c red
sq :625c yellow
sq :637c green
```

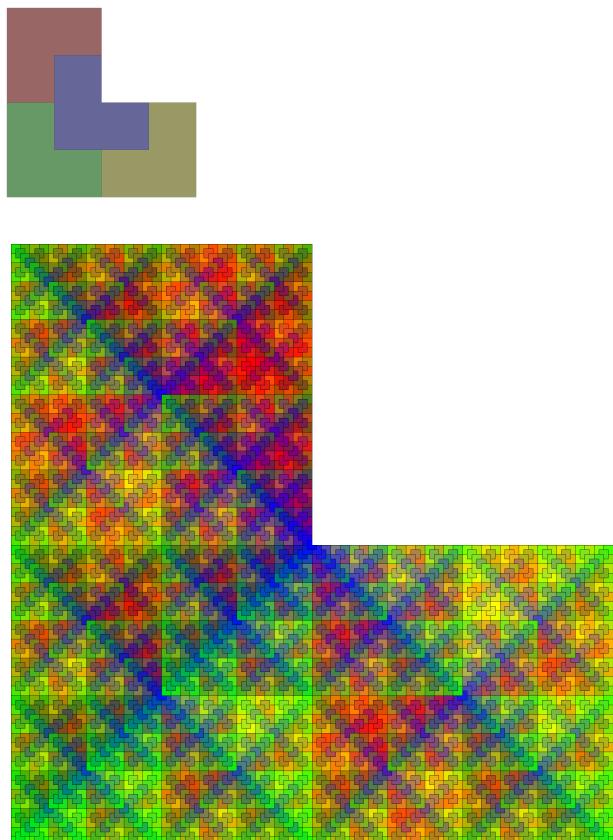
Levels 1 to 6



9. Reptiles

Trionimo

```
width .02
p1 0 100
p2 50 100
p3 50 50
p4 100 50
p5 100 0
def t 6
  p6..8 p0.. 1/2 p3
  p9..10 p4.. 1/2 p3
  p11 p0 1/2 p5
  p12 p0 1/2 p1
  p13..14 p11.. 1/2 p3
  t p1,2,8,7,14,12 + 1 -1 -1
  t p0,12,14,6,13,11 + -1 1 -1
  t p6,7,8,3,9,10 + -1 -1 1
  t p5,11,13,10,9,4 + 1 1 -1
```

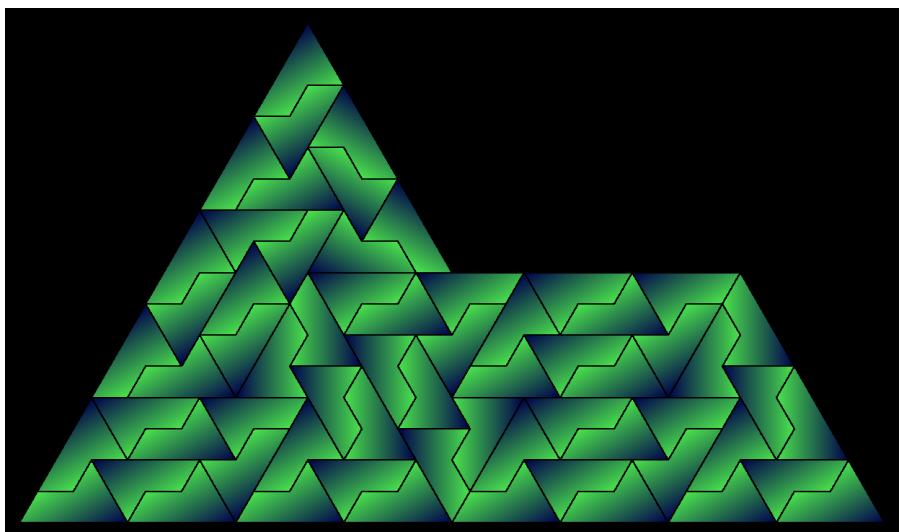
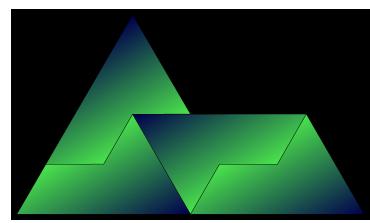


Levels 1 and 6

Sphinx

```
background black
width .05
p1 20 34.4641
p2 30 17.3205
p3 50 "
p4 60 0
def s 5
p0 0 0 .3 p1 .3 .9 .3
p5 p0 1/4 p1
p6 p0 1/2 p2
p7 p0 " p1
p8 p0 " p4
p9 p4 " p7
p10 p3 " p4
p11 p2 " p7
p12 p9 " p10
s p1,2,11,6,5
s p8,11,6,5,0
s p4,3,12,9,8
s p11,8,9,12,3
```

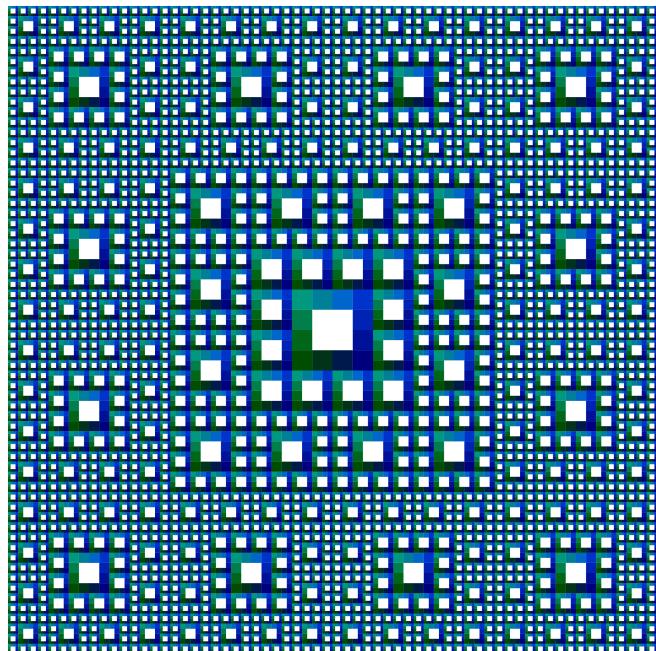
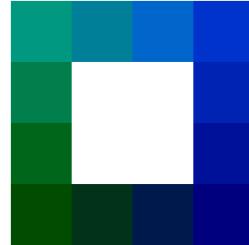
Level 1 and 5



Kenyon carpet

Levels 1 and 4

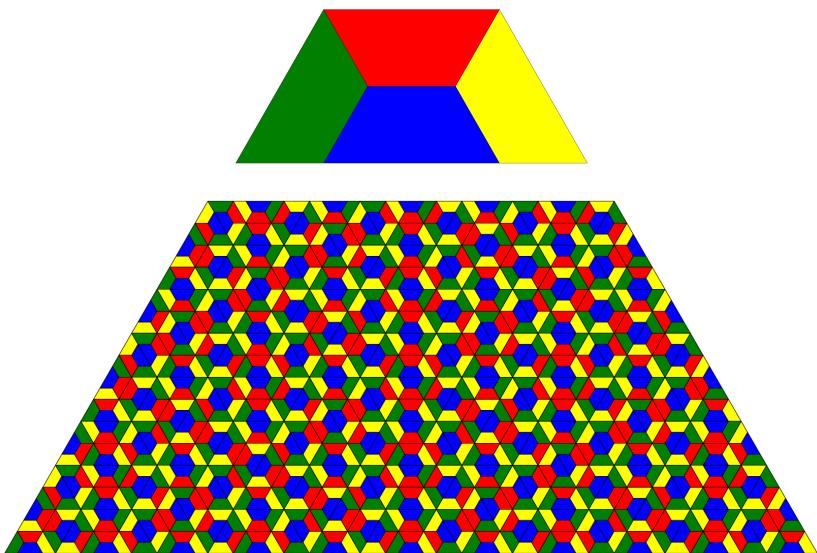
```
no edges
def sq 4
  p4..6 p0 1..3/4 p1
  p7..9 p1 1..3/4 p2
  p10..12 p2 1..3/4 p3
  p13..15 p3 1..3/4 p0
  p16..18 p6 1..3/4 p10
  p19 p5 1/4 p11
  p20 p5 3/4 p11
  p21..23 p4 1..3/4 p12
  sq p6,1,7,16 0 .6 .5
  sq p16,7,8,17 0 .5 .6
  sq p17,8,9,18 0 .4 .8
  sq p18,9,2,10 0 .2 .8
  sq p5,6,16,19 0 .5 .3
  sq p20,18,10,11 0 .14 .7
  sq p4,5,19,21 0 .4 .1
  sq p23,20,11,12 0 .07 .6
  sq p0,4,21,15 0 .3 0
  sq p15,21,22,14 0 .2 .1
  sq p14,22,23,13 0 .1 .3
  sq p13,23,12,3 0 0 .5
  sq p21,16,18,23 white
```



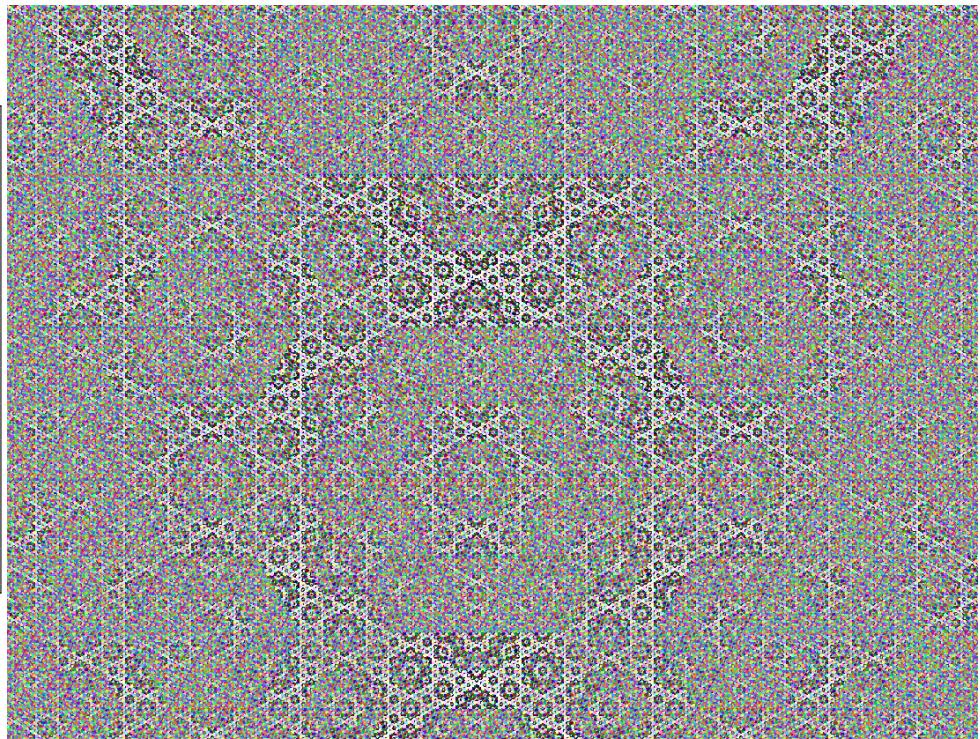
Half-hex

Levels 1 and 5

```
width 0.02
grid 60
p1 0 20
p2 20 20
p3 40 0
def hh 4
  p4..6 p0 1..3/4 p3
  p7..8 p1.. .5 p5
  hh :1740 green
  hh :2871 red
  hh :3682 yellow
  hh :4786 blue
```



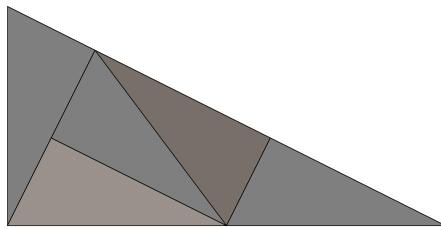
```
no edges
grid 60
p1 0 20
p2 20 20
p3 40 0
def hh 4
  .8 .5 .1
  p4..6 p0 1..3/4 p3
  p7..8 p1.. .5 p5
  hh :1740 rot2
  hh :2871 + 1
  hh :3682 rot
  hh :4786 !
```



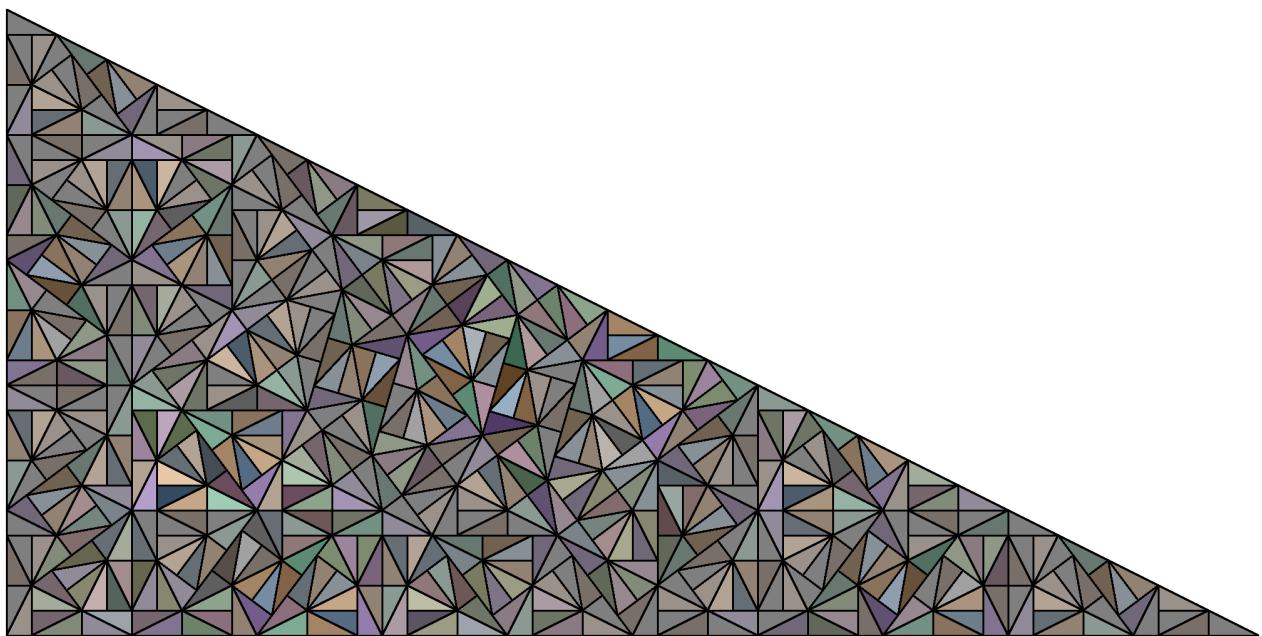
Detail of level 10

Pinwheel

```
p1 0 50
p2 100 0
def tri 3
  p3..4 p1 1,3/5 p2
  p5 p0 1/2 p3
  p6 p0 1/2 p2
  tri :310 rot2
  tri :506 + 1 .7 .4
  tri :536 !
  tri :463 - .3 .6 .9
  tri :462 rot
```



Levels 1 and 4

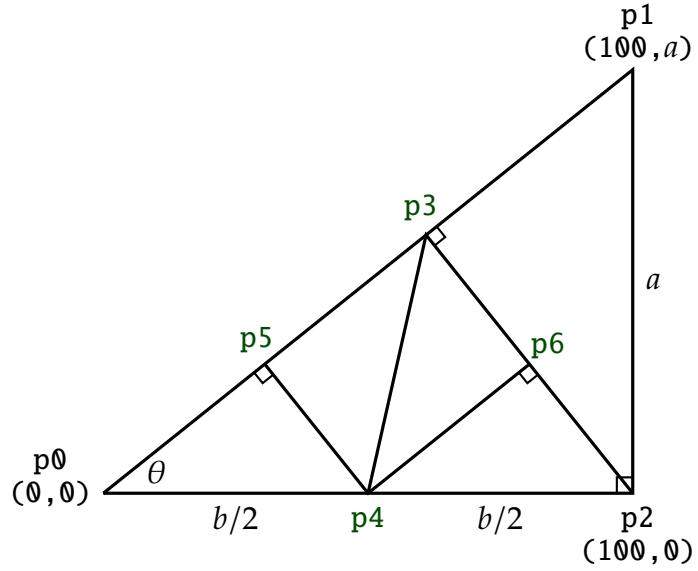


10. Irreptiles

These are shapes that can only be made from *differently-sized* smaller copies of themselves.

Sadun's generalized pinwheel

```
only edges
p1 100 80
p2 100 0
def tri 3
  p3 p0 .609756 p1
  p4 p0 1/2 p2
  p5 p0 1/2 p3
  p6 p3 1/2 p2
  tri :045 wait
  tri :345 wait
  tri :436 wait
  tri :426 wait
  tri :213
```



Only the larger triangles are subdivided each time—this is achieved in Ratsub by having the small triangles call a wait shape.

To place p_3 , we need to know what fraction of the way p_3 is from p_0 to p_1 . This is given by

$$\frac{10000}{a^2 + 10000}.$$

Trying $a = 80$ gives $0.609756\dots$ (Figure 10.1).

But we need to make sure θ/π is irrational for an infinite number of orientations, so try $\theta = 15e^\circ = 40.7742274\dots^\circ$. This gives $a = 86.2392187\dots$.

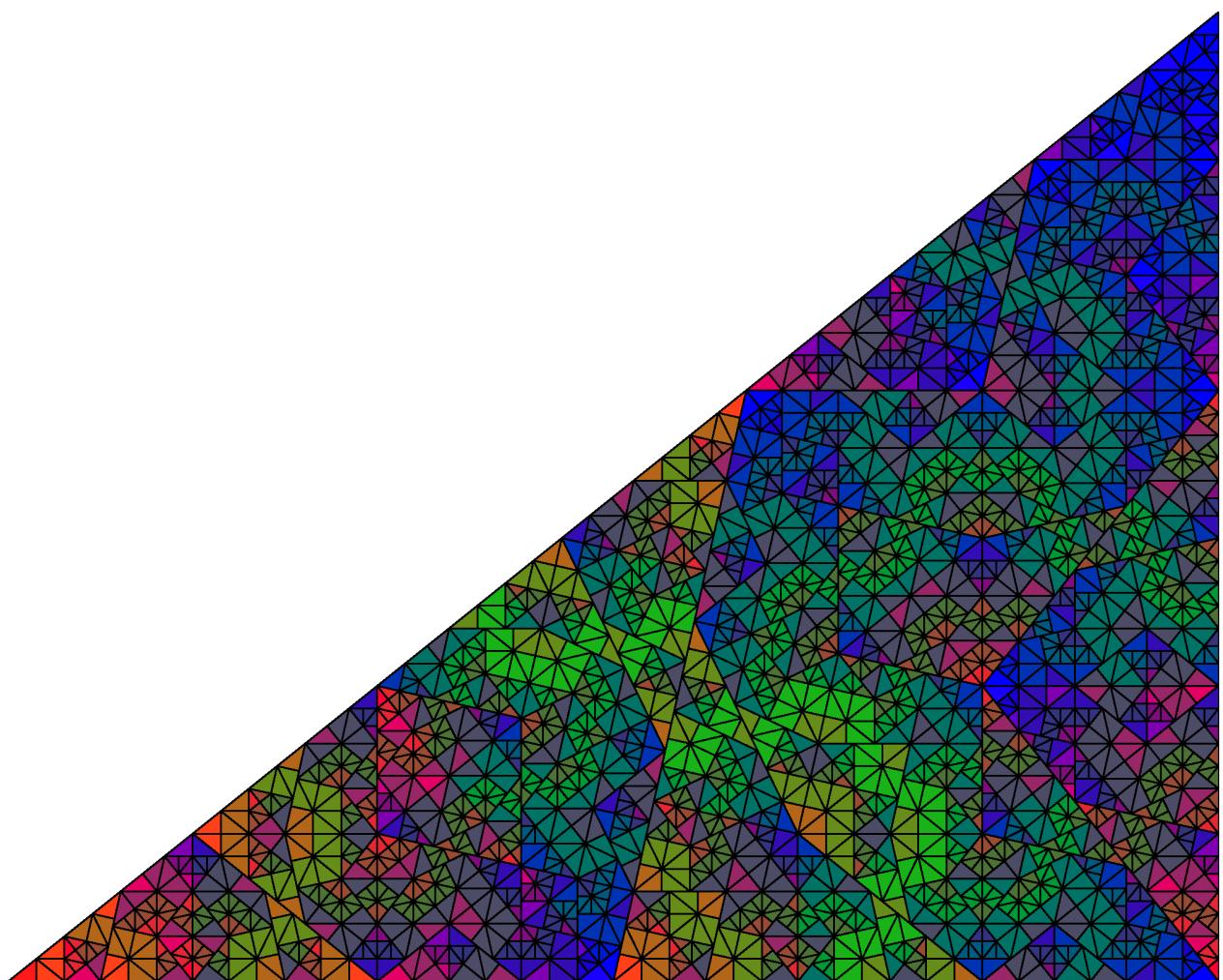
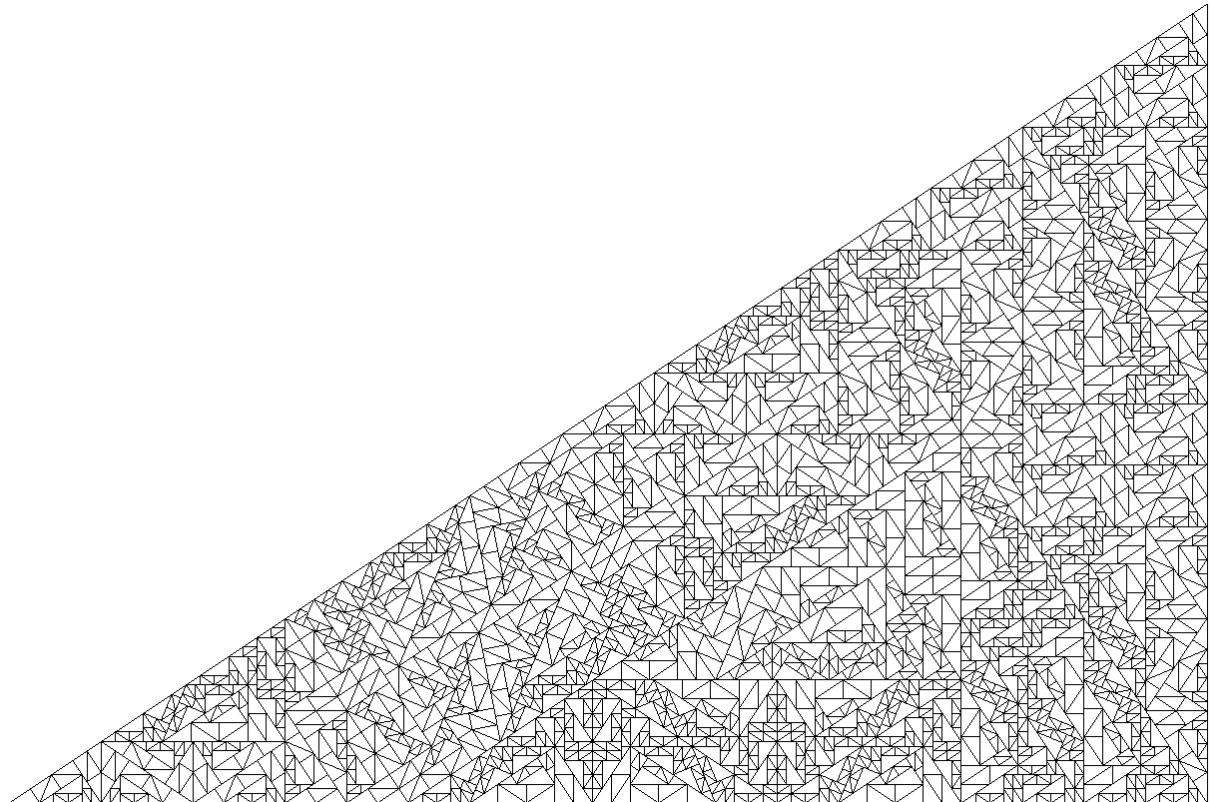
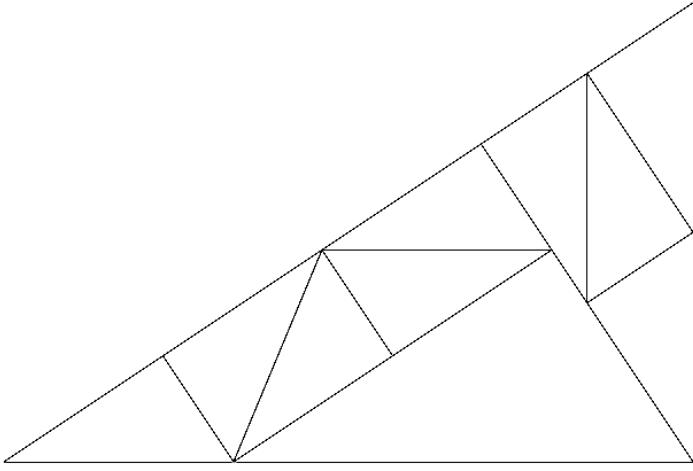


Figure 10.1: Level 8 Sadun pinwheel with $a/b = 0.8$, $\theta \sim 38.659808^\circ$

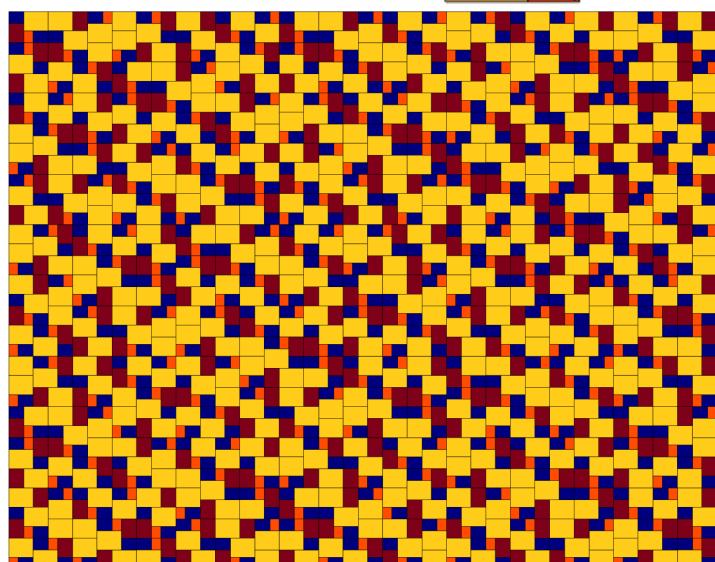
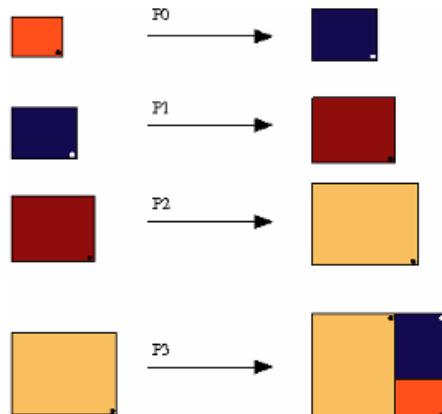
Altered (2,3) pinwheel

```
only edges
width .01
p0 30 0
p1 0 0
p2 30 20
def tri 3
  p3..4 p0 1..2/3 p1
  p5..8 p1 3,6,9,11/13 p2
  p9 p0 1/2 p2
  p10 p0 1/2 p7
  p11..12 p0 1,2/3 p7
  p13 p3 1/2 p6
  tri p12,4,0 !
  tri p5,1,4 wait
  tri p13,4,6 wait
  tri p5,6,4 wait
  tri p7,6,12 wait
  tri p13,12,6 wait
  tri p10,0,9 wait
  tri p7,10,8 wait
  tri p9,8,10 wait
  tri p8,9,2 wait
```



Rectangulo dorado

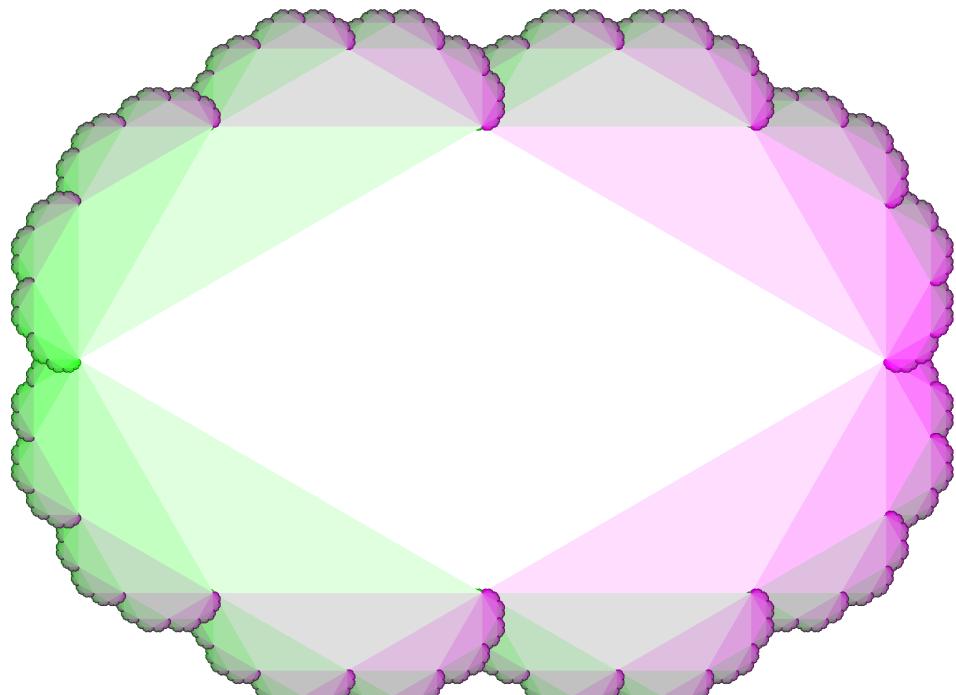
```
width .01
pbox 33.3019 26.18034
def rec3 4
  1 .8 .1
  p4 p1 .618034 p2
  p5 p2 " p3
  p6 p0 " p3
  p7 p4 " p6
  rec3 :6014
  rec1 :5742
  rec0 :6753
def rec0 4
  orange red
  rec1 :0123
def rec1 4
  navy
  rec2 :0123
def rec2 4
  .5 0 .1
  rec3 :0123
```



Level 14

11. Fractals

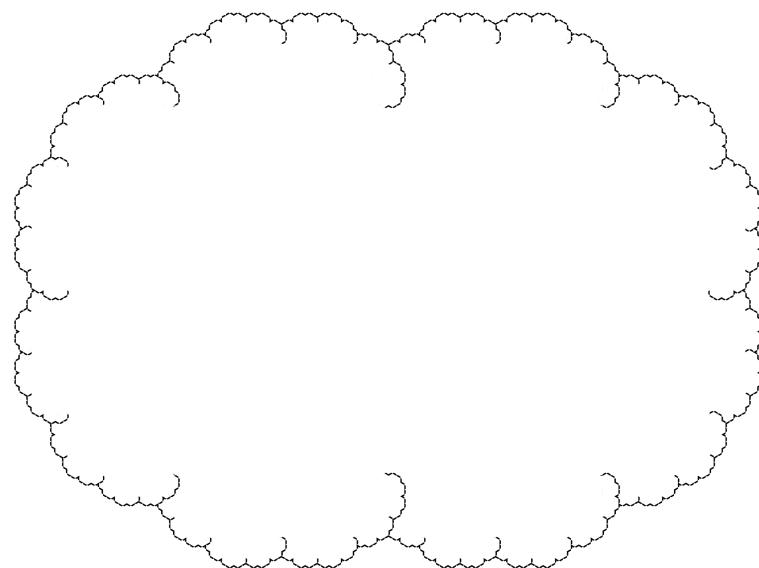
```
margin 20
noedges
p1 50 28.86751
p2 100 0
p3 50 -28.86751
def quad 4
white
tri :012
tri :032
def tri 3
  p3..4 p0 1..2/3 p2
  p5..6 p0.. .5 p1..
  p7..8 p3.. 2 p5..
  tri :071 - 1 0 1
  tri :182 - 0 1 0
draw :012
```



Level 15

```
margin 20
noedges
p1 50 28.867513
p2 100 0
p3 50 -28.867513
def quad 4
  tri p0,1,2
  tri p0,3,2
def tri 3
  black
  p3..4 p0 1..2/3 p2
  p5..6 p0.. .5 p1..
  p7..8 p3.. 2 p5..
  tri :071
  tri :182
  tri1 :012 black
def tri1 3
  tri1 :012 white
```

To get just the fractal boundary, no edges and change the rands to black and white. The black in tri1 p0,1,2 black works like a wait shape, making the boundary thicker.



Level 11

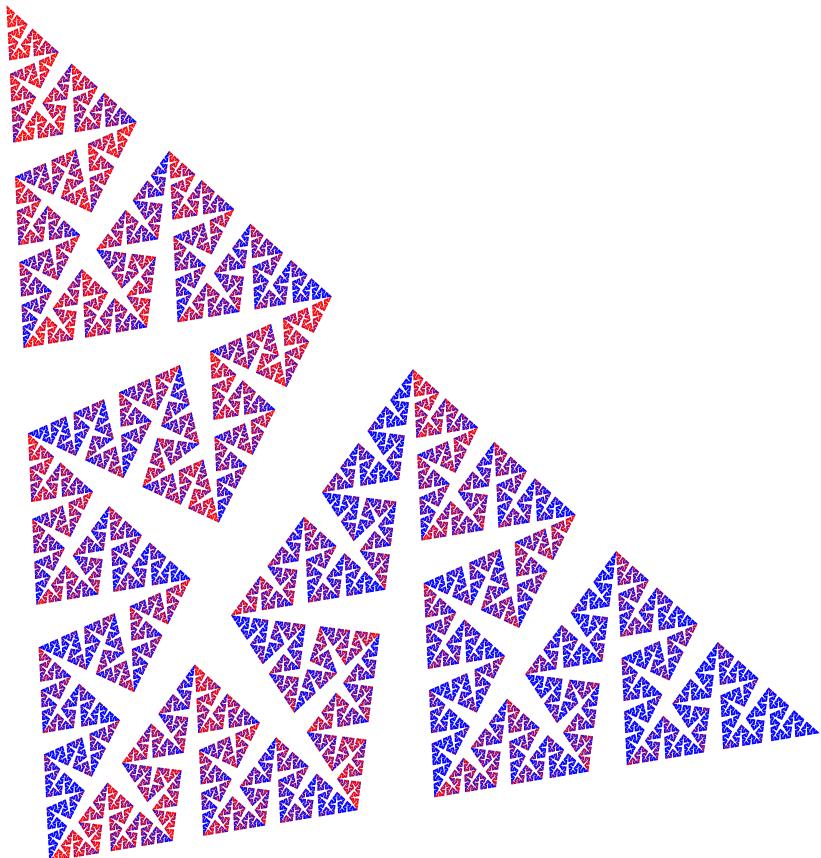
This may look a lot better with the bottom half flipped, so it has rotational symmetry!

Plane-filling curves

Pólya's curve

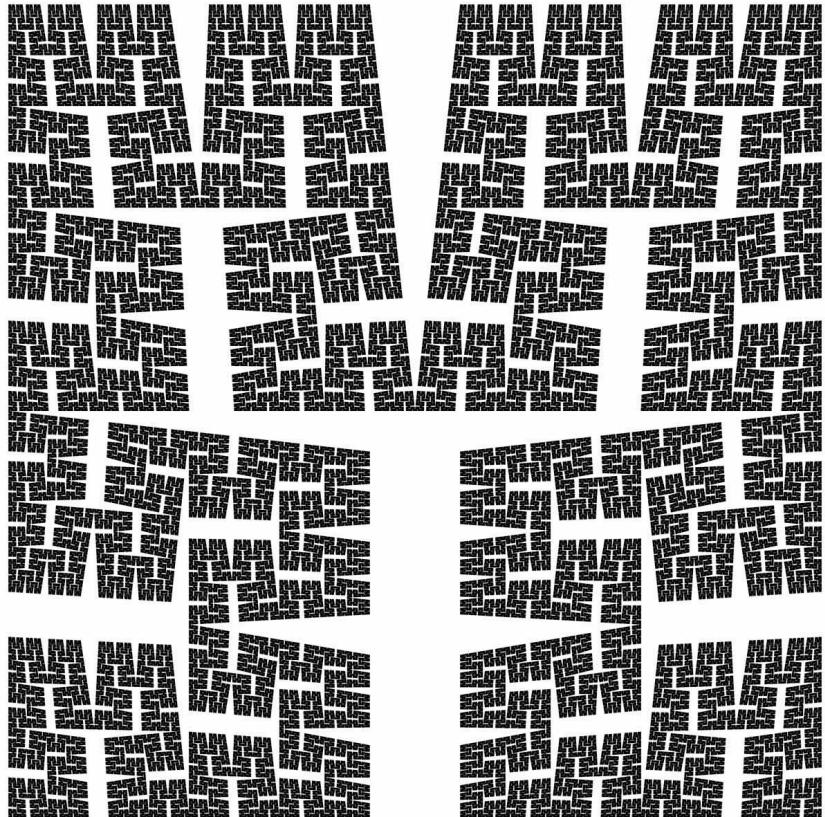
```
no edges
p1 -5 100
p2 90 15
def tri 3
  p3..4 p1 4..5/10 p2
  tri :310 - -1 1 1
  tri :402 - 1 1 -1
```

Level 15



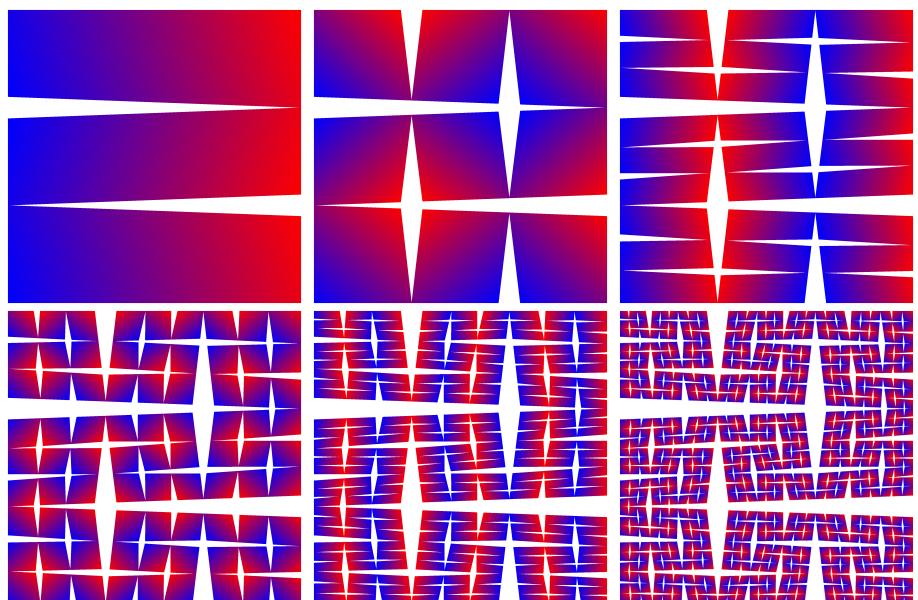
Hilbert curve

```
noedges
def sq 4
  p4 p0 .5 p1
  p5 p2 .5 p3
  p6 c
  p7..8 p1 4..5/9 p2
  p9..10 p0 4..5/9 p3
  p11..12 p9.. .45 p7..
  sq p4,11,9,0
  sq p4,1,7,6
  sq p6,8,2,5
  sq p3,10,12,5
```



Peano curve

```
noedges
def sq 4
  p0 blue p2 red
  p4 p0 1/3 p1
  p5 p2 1/3 p3
  p6..7 p4 4..5/9 p1
  p8..9 p5 4..5/9 p3
  sq :7521
  sq :6584
  sq :0394
```



12. IFS

Random IFS

```
width .005
strand 7
pbox -10 -10 10 10
def sq 4
    p4..7 p0.. crand p1..0
    p8..11 p0.. crand p1..0
    p12..15 p0.. crand p1..0
    sq p4..7 - -1 1 1
    sq p8..11 - 1 1 -1
    sq p12..15 - 1 -1 1
```

See Figure 12.1 and Figure 12.2.

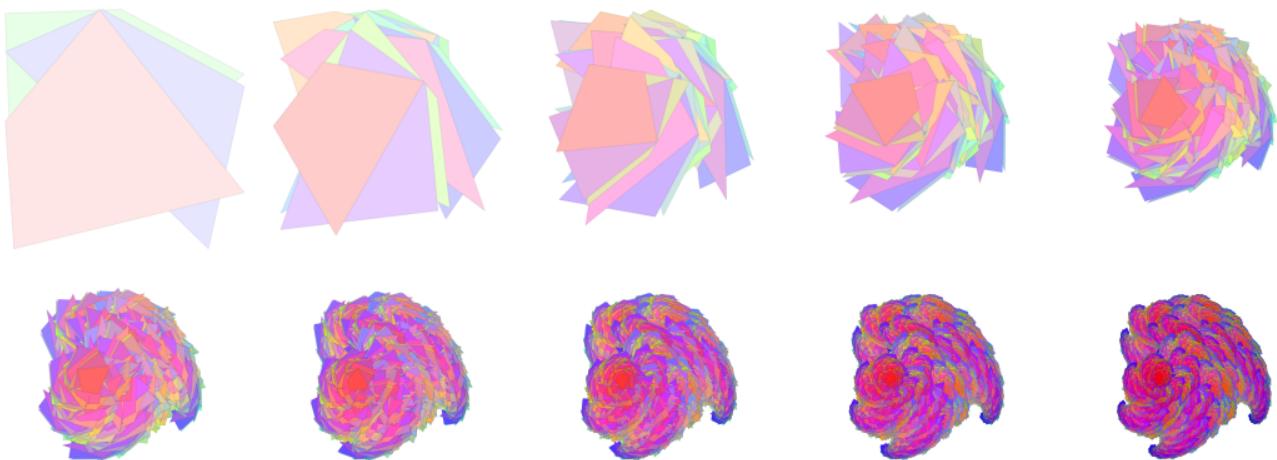


Figure 12.1: Random IFS, levels 1 to 10.

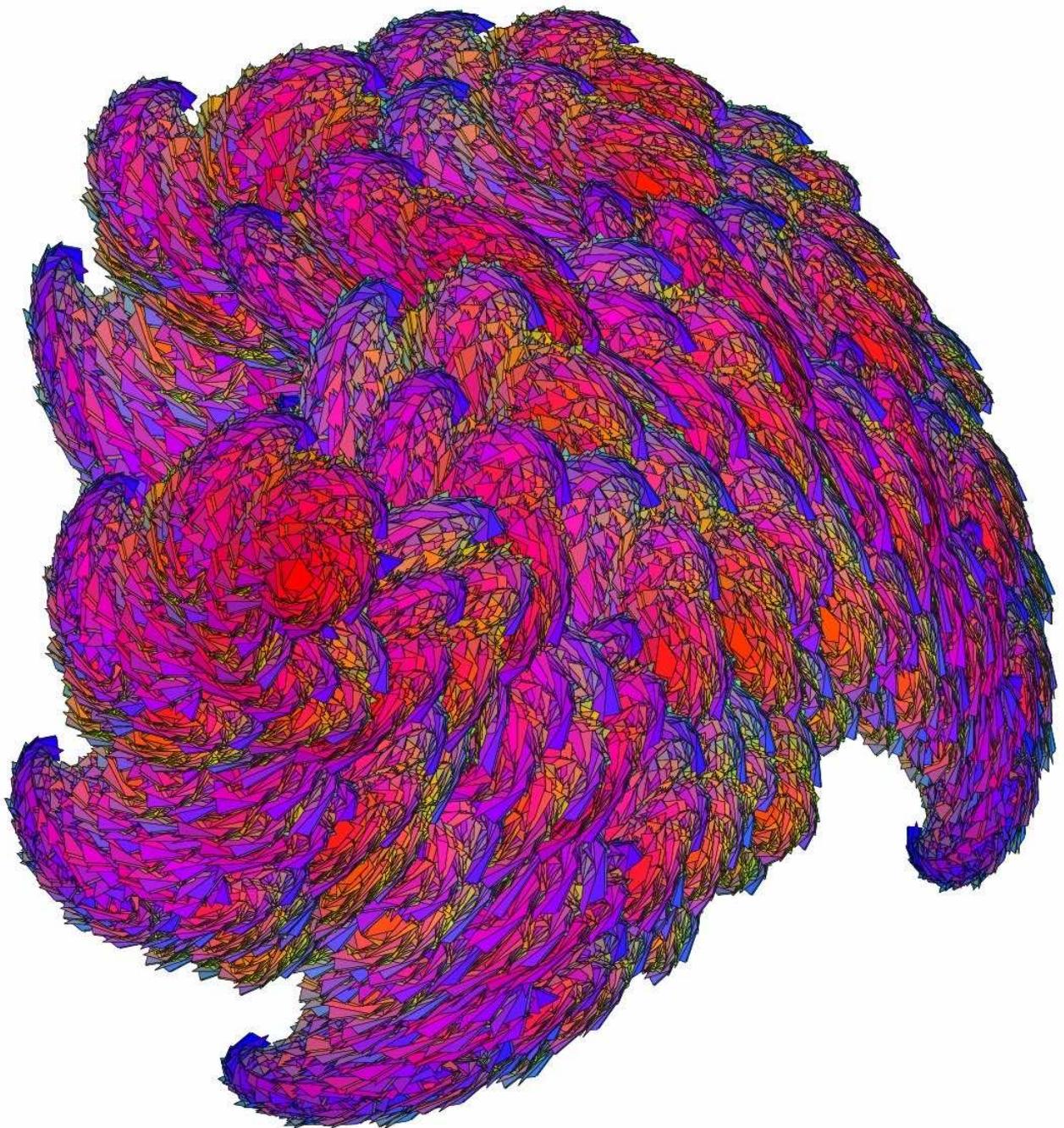
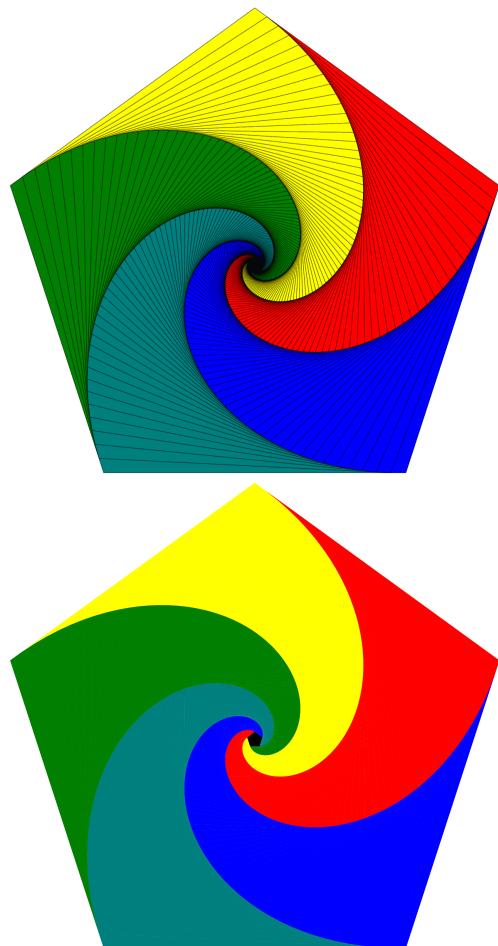


Figure 12.2: Random IFS, level 11

13. Curves

```
width 0.05
pgon 5 50 18
def pent 5
  p5..9 p0.. .95 p1..0
  p10..11 p9 1..2/3 p5
  p12..13 p5 1..2/3 p6
  p14..15 p6 1..2/3 p7
  p16..17 p7 1..2/3 p8
  p18..19 p8 1..2/3 p9
draw p0,9,10,11,5 red
draw p1,5,12,13,6 yellow
draw p2,6,14,15,7 green
draw p3,7,16,17,8 teal
draw p4,8,18,19,9 blue
pent :95678
```

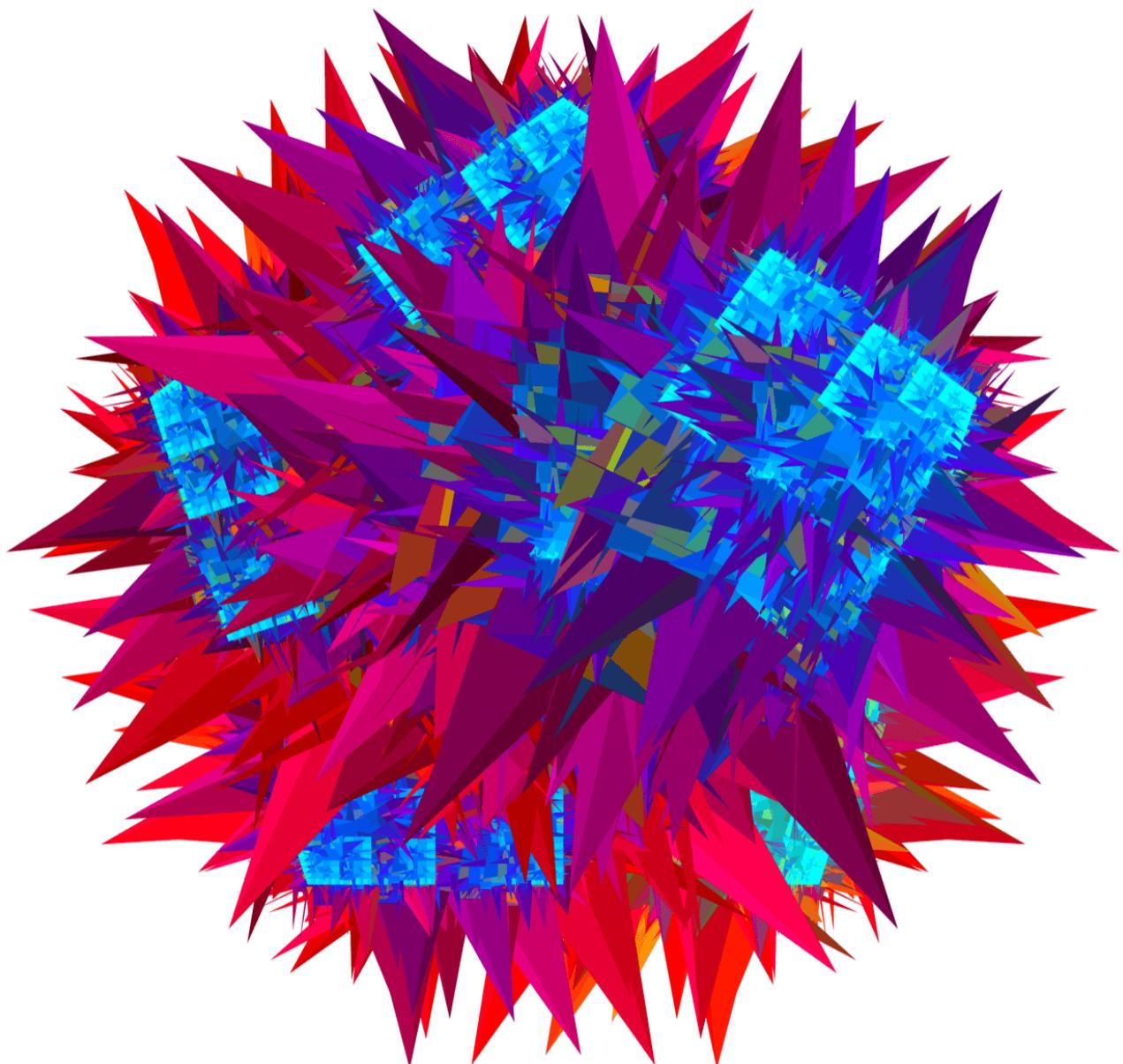


Level 102, (a) width 0.05 (b) no edges.

14. Turning

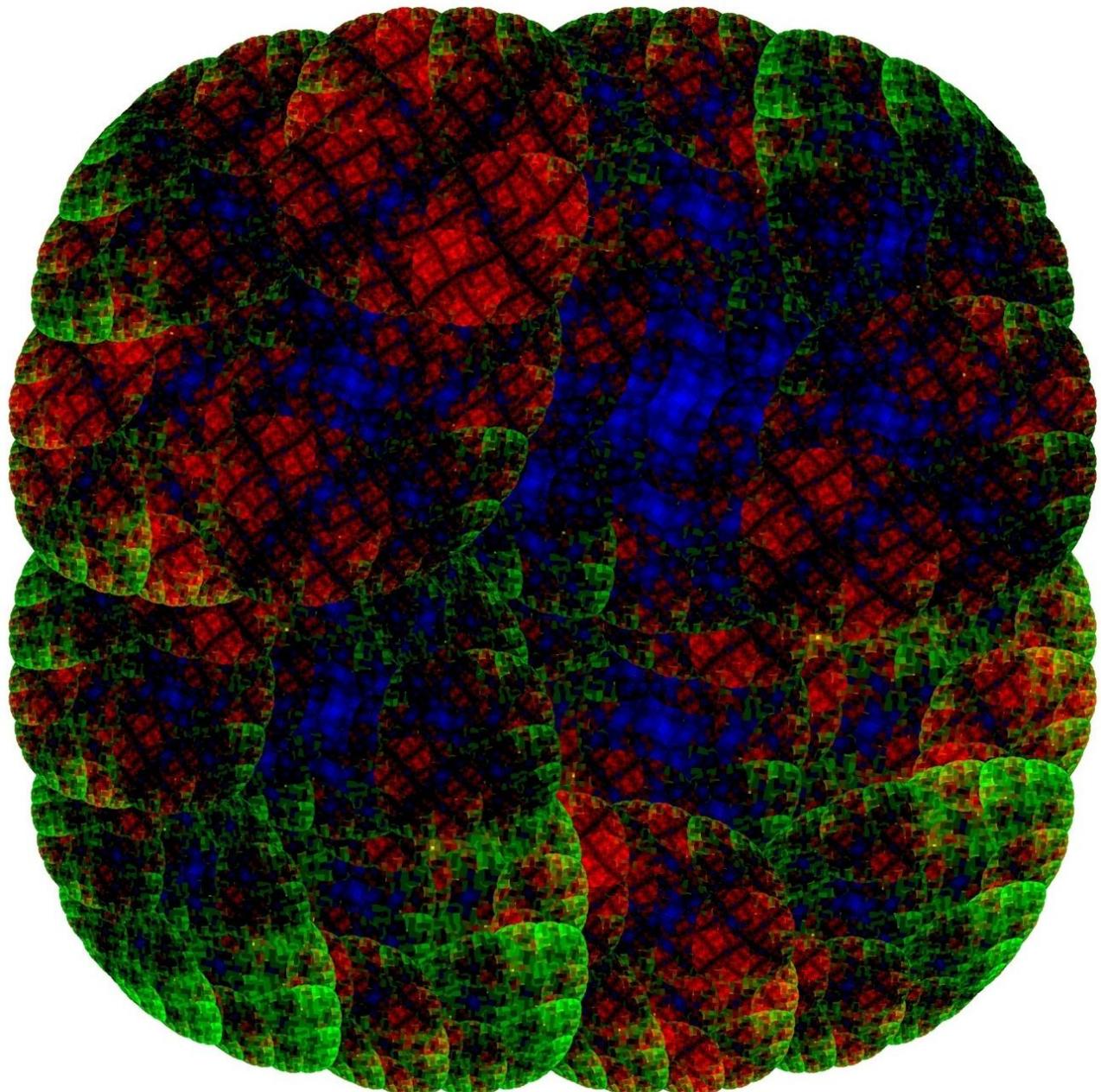
```
noedges
margin 30
pgon 5 50 18
def pent 5
  p5..9 p0.. 1/2 p1..0
  p10..14 p0.. 1 p5.. left 1.5
  pent p0,5,10,14,9 + -2 1 2
  pent p1,6,11,10,5 + -1 0 1
  pent p2,7,12,11,6 + 0 -1 -1
  pent p3,8,13,12,7 + 1 -2 0
  pent p4,9,14,13,8 + 2 2 -2
```

Level 7



```
margin 25
no edges
def sq 4
  p4..7 p0.. .5 p1..0 left .1
  p8 p5 .5 p7 right .1
  sq :1584 - 0 2 2
  sq :0487 - 2 0 2
  sq :2685 - 2 2 0
  sq :3786 - 0 0 2
```

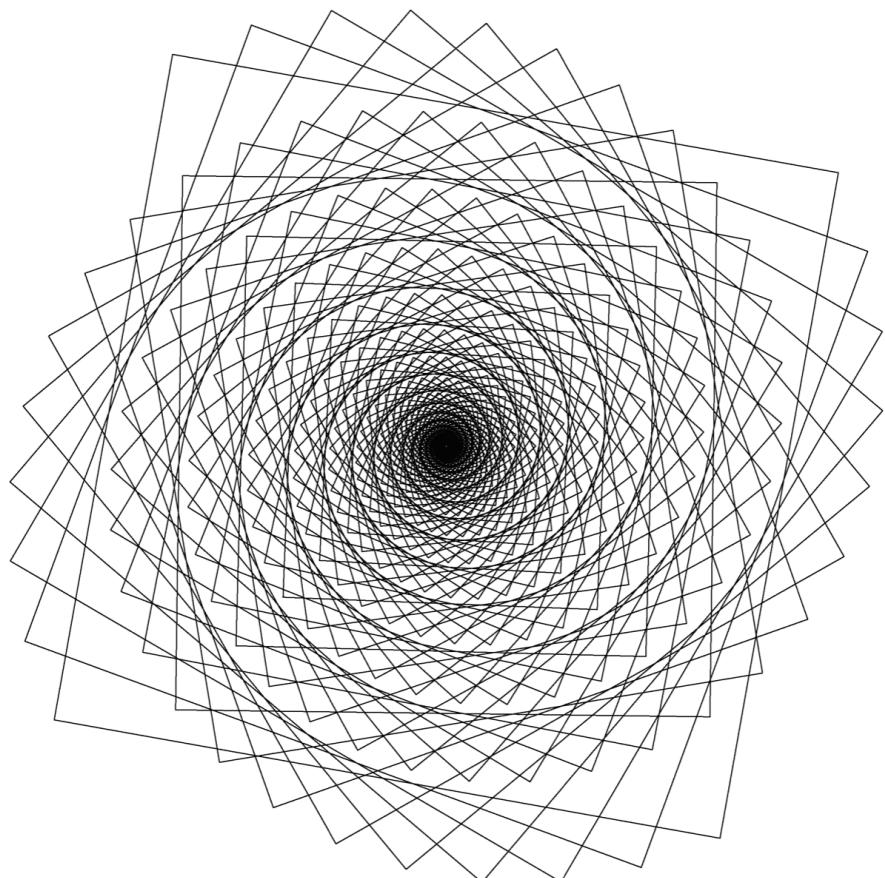
Level 9



15. Rotating shapes

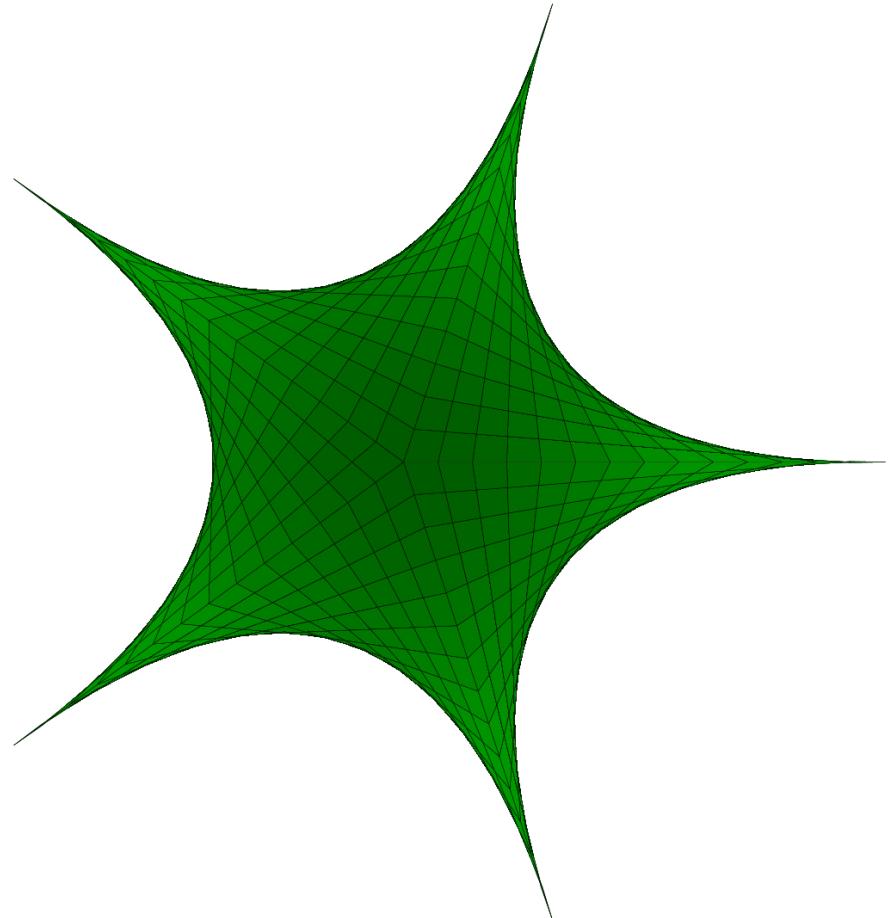
```
margin 25
only edges
def sq 4
  p4..7 p0... .1 p1..0
  p8..11 p4.. 1.07 p5..4
  draw p11,8,9,10
  sq p11,8,9,10
```

levels 200



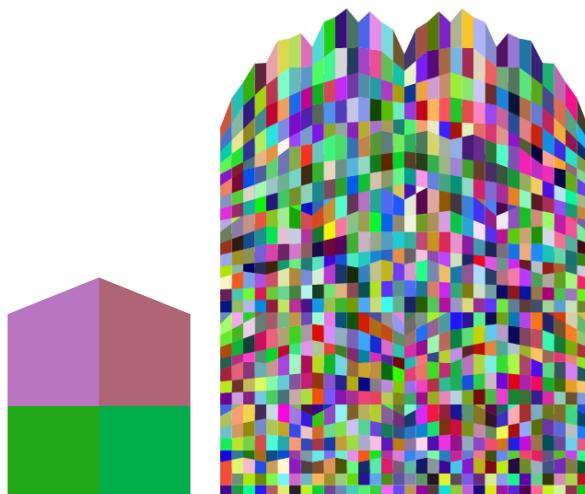
16. Going outside the interval

```
width 0.01
margin 10
pgon 5 20
def pent 5
  p5 c
  p6..10 p0.. .9 p5
  p11..15 p6.. 2.5 p5
quad p5,7,14,6
quad p5,8,15,7
quad p5,9,11,8
quad p5,10,12,9
quad p5,6,13,10
def quad 4
  ip6 0 .1 0
  ip0..5 0 .6 0
  p4 p0 2 p1
  p5 p3 2 p2
  p6 p1 2 p2
  p7 p0 2 p3
  keep :0123
  quad :1452
  quad :3267
def keep 4
  ip6 0 .1 0
  ip0..5 0 .6 0
```



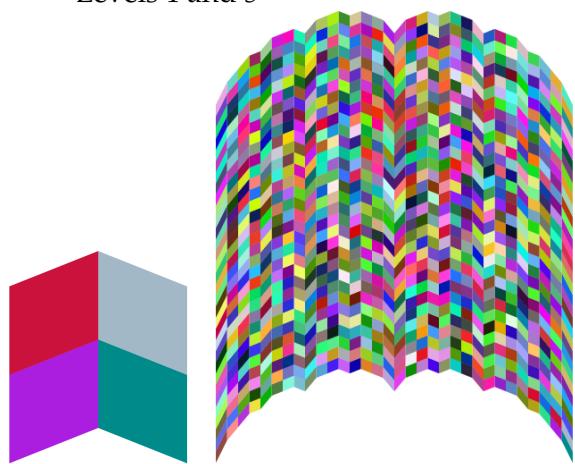
Level 14

```
no edges
margin 10
pbox 20 20
def sq 4
  rand
  p4..7 p0.. .5 p1..0
  p8 p5 .5 p7
  p9 p7 1.2 p5
  sq p4,1,9,8
  sq p8,9,2,6
  sq p0,4,8,7
  sq p7,8,6,3
```



```
no edges
margin 10
pbox 20 20
def sq 4
rand
p4..7 p0.. .5 p1..0
p8 p5 .5 p7
p9 p7 1.2 p5
p10 p5 .8 p7
p11 p10 .5 p9
sq p4,1,9,11
sq p11,9,2,6
sq p0,4,11,10
sq p10,11,6,3
```

Levels 1 and 5

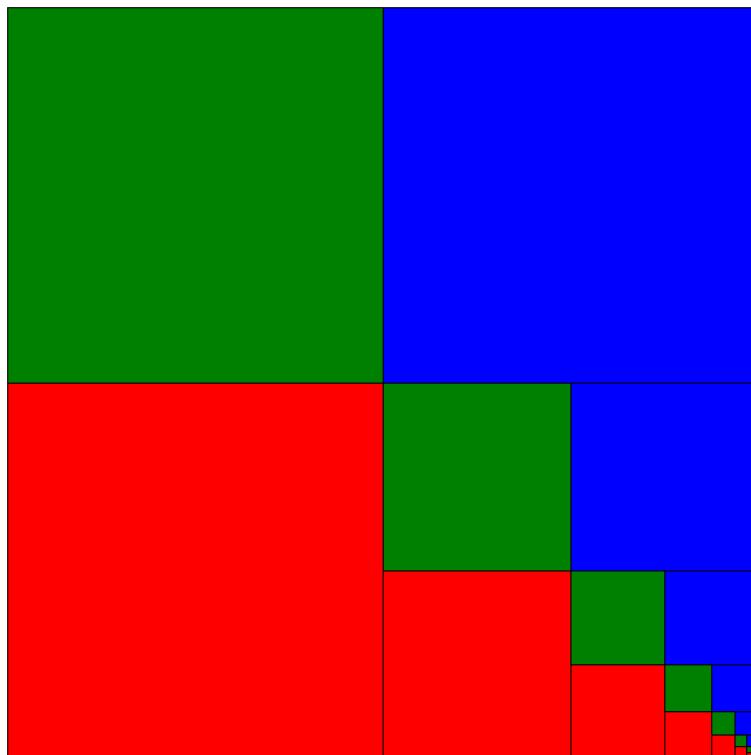


17. Proofs without words

$$\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \cdots = \frac{1}{3}$$

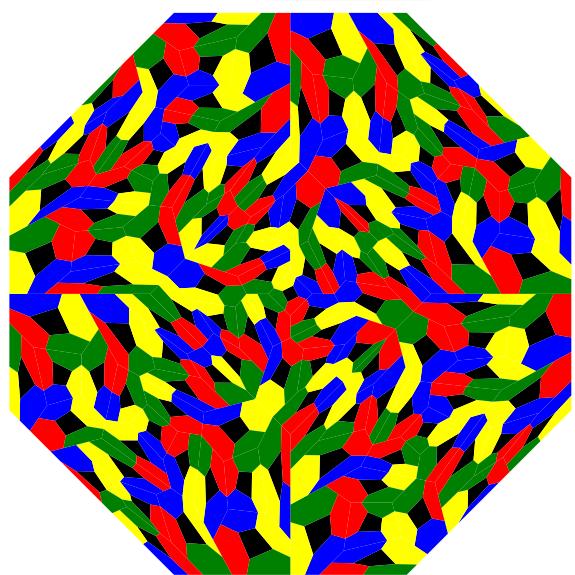
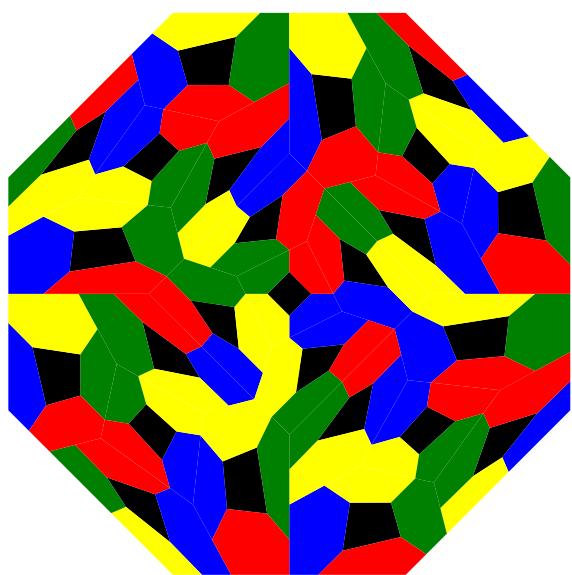
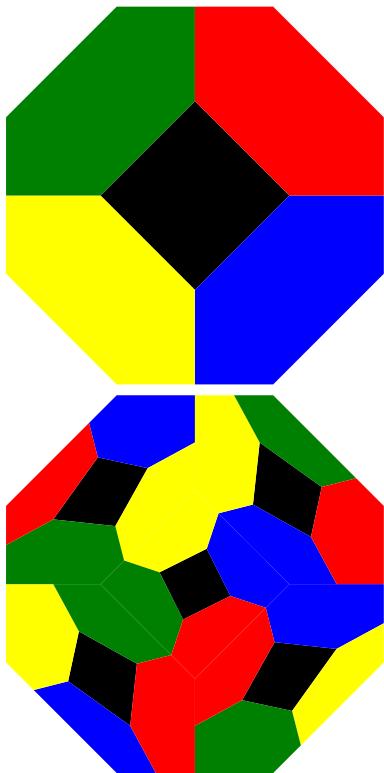
```
def sq 4
p4..7 p0.. .5 p1..0
draw :415c green
draw :c526 blue
draw :04c7 red
sq :7c63
```

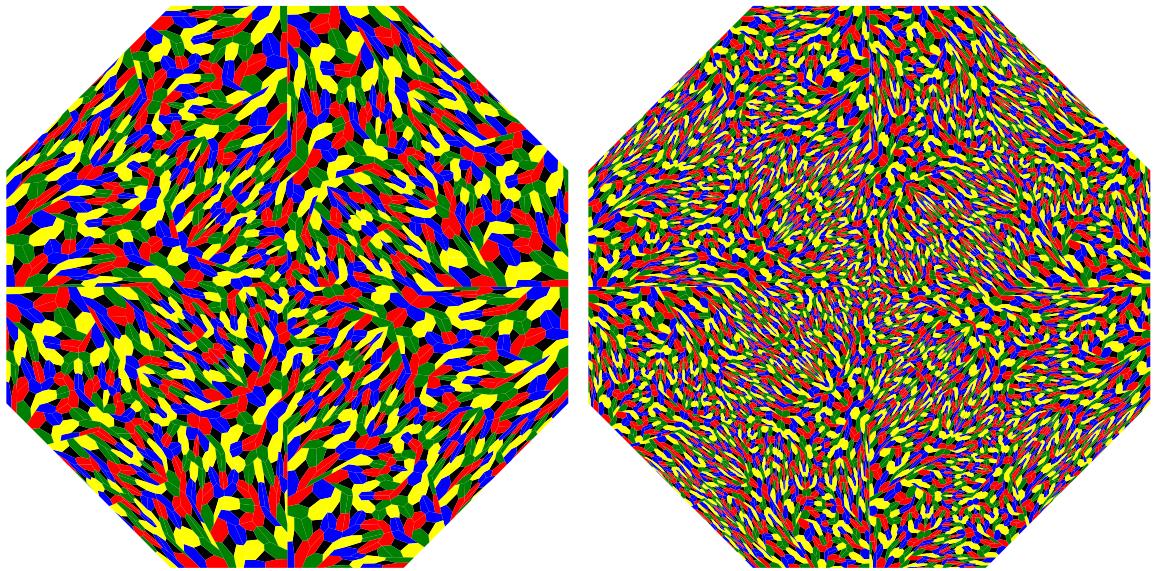
Level 15



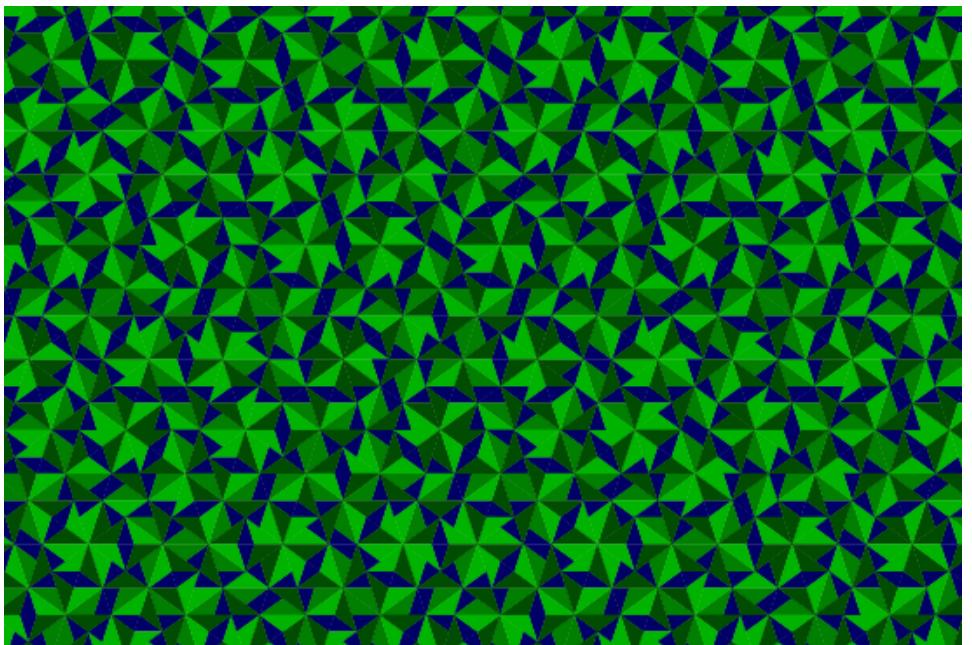
18. Others

```
no edges
pgon 8 50 22.5
def oct 8
  p8..15 p0.. .5 p1..0
  p16 p15 1/4 p11
  p17 p15 3/4 p11
  p18 p9 1/4 p13
  p19 p9 3/4 p13
  p20 p16 1/2 p18
  p21 p18 1/2 p17
  p22 p17 1/2 p19
  p23 p19 1/2 p16
  oct p15,0,8,1,9,18,20,16 red
  oct p10,3,11,17,21,18,9,2 green
  oct p13,19,22,17,11,4,12,5 yellow
  oct p23,19,13,6,14,7,15,16 blue
  oct p23,19,22,17,21,18,20,16 black
```





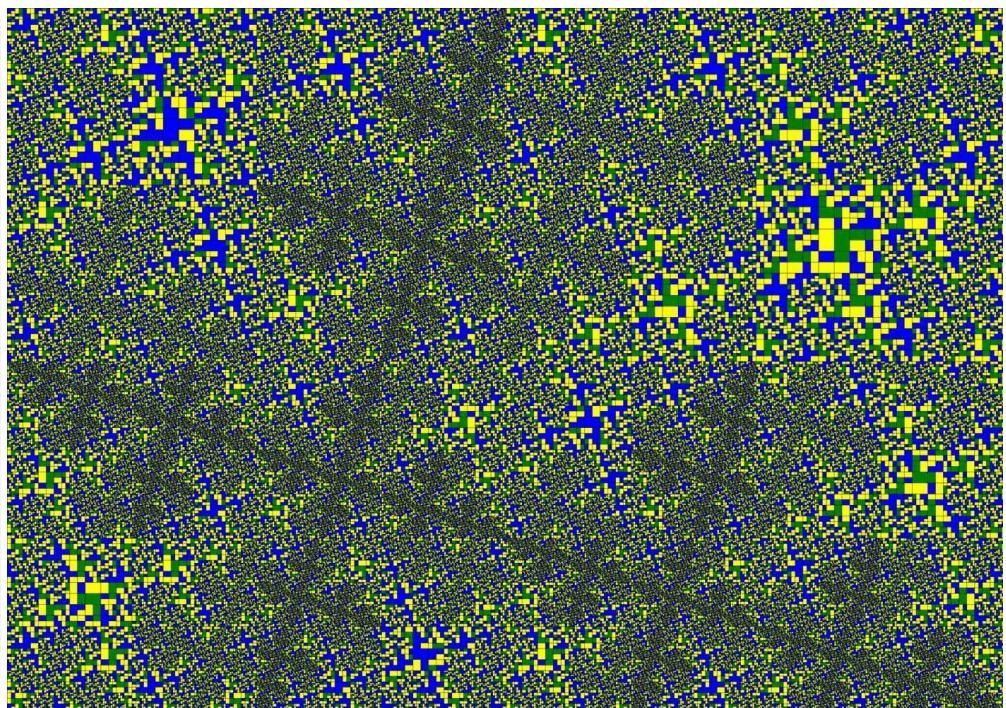
```
no edges
pgon 5 50 18
def pent 5
  p5 p0 0.618034 p3
  tri1 :012
  tri1 :250
  tri1 :432
  tri2 :405
def tri1 3
  p3 p2 0.618034 p0
  p4 p0 " p2
  tri1 :130 0 .3 0
  tri1 :241 0 .7 0
  tri2 :314
def tri2 3
  0 0 .4
  p3 p1 0.618034 p0
  tri1 :231 0 .5 0
  tri2 :320
```



Detail of level 11

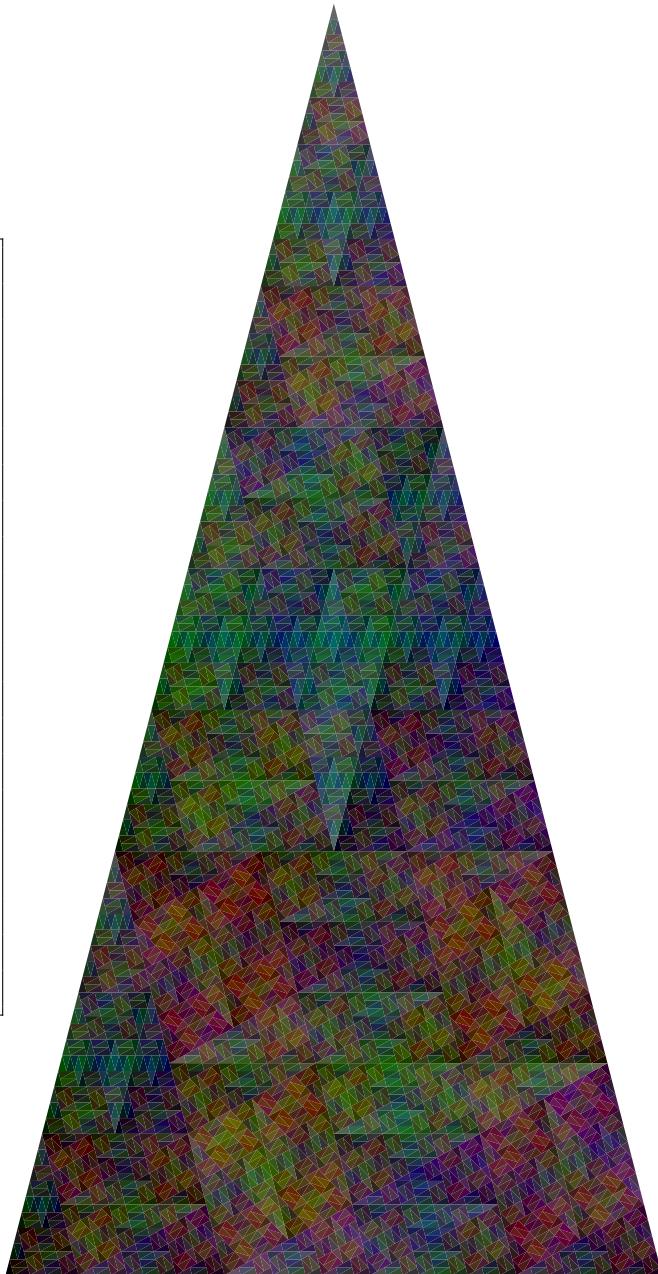
```
width 0.02
pbox 141.421 100
def r1 4
yellow
p4 p1 1/2 p2
p5 p0 " p3
r1 :5014
r2 :3542
def r2 4
blue
p4 p1 1/2 p2
p5 p0 " p3
r1 :1450
r3 :4235
def r3 4
green
r2 :0123
```

level 20



Danzer's Unknown

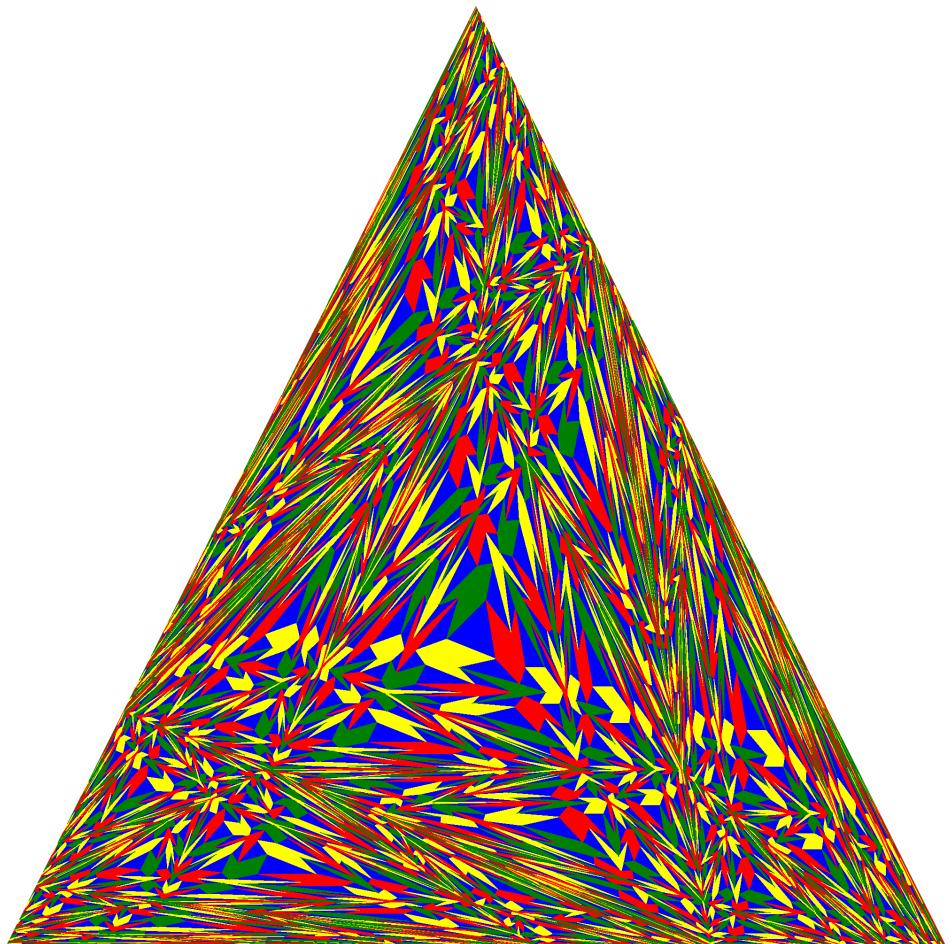
```
no edges
p1 50 193.649
p2 100 0
def tri 3
  black
  p3..4 p0 1..2/3 p1
  p5..6 p1 1..2/3 p2
  p7 p0 1/3 p2
  p8 p3 1/2 p6
  p9 p3 1/2 p7
  p10 p6 1/2 p2
  tri p4,1,5 + 1 1 1
  tri p3,4,8 + 0 1 0
  tri p5,8,4 + 0 1 1
  tri p8,5,6 + 0 0 1
  tri p0,3,7
  tri p3,6,9 + 1 0 0
  tri p9,10,7 + 1 1 0
  tri p2,7,10 + 1 0 1
  tri p10,9,6 + 1 .5
  0
```



As related by Dirk Frettlöh - this and pinwheel(2,3) both came from DF's show-qc25.pdf

```
no edges
p1 50 100
p2 100 0
def tri 3
  p3 p1 1/4 p2
  p4 p0 1/4 p1
  p5 p2 1/4 p0
  tri :134 green
  tri :532 red
  tri :504 yellow
  tri :534 blue
```

Level 7



19. Program listings

Title page, *Pythagorean tree* pyth.sdv

```
no edges
margin 1 1 1 -11
background black
pbox 350 240
def first 4
  p4 p0 180/350 p3 left .02857
  p5 p0 180/350 p3 left .17143
  p6 p0 230/350 p3 left .02857
  p7 p0 230/350 p3 left .17143
  p8 p0 .13 p1
  p9 p3 .21 p2
  sq :4576
  grass :0893
  sky :8129
def sq 4
  p1 red p0 white
  p4 p1 16/25 p2 left .48
  p5 p1 1 p4 left 1
  p6 p4 1 p1 right 1
  p7 p2 1 p4 right 1
  p8 p4 1 p2 left 1
  sqd :0123
  trid :142
  sq :1654
  sqw :4782
def trid 3
  p0 .4 .7 .1 p2 .2 .1 .1
def sqd 4
  p1 .4 .7 .1 p2 .2 .1 .1
def sqw 4
  p1 red p0 white
  sq :0123
def sky 4
  p0 orange p1 purple
def grass 4
  p1 dark green p0 black
```

Lines

kochline.sdv

```
only edges
margin 1 31 1 1
p1 100 0
def line 2
  p2..3 p0 1..2/3 p1
  p4 c
  p5 p0 1 p4 left 0.57735
  line :02
  line :25
  line :53
  line :31
```

kochline-col.sdv

```
width .05
margin 1 31 1 1
p1 100 0
def line 2
  p2..3 p0 1..2/3 p1
  p4 c
  p5 p0 1 p4 left 0.57735
  line :02
  line :25
  line :53
  line :31
  draw :254 green
  draw :354 red
```

Polygons

dodecnew.sdv

```
width .2
def sq 4
  red
  p4..6 p0 2..4/6 p1
  p7..9 p1 2..4/6 p2
  p10..12 p2 2..4/6 p3
  p13..15 p3 2..4/6 p0
  p16 p5 .3 p11
  p17 p5 .7 p11
  p18 p8 .3 p14
  p19 p8 .7 p14
  pent p16,6,1,7,18
  tri p9,7,18
  pent p9,2,10,17,18
  tri p10,17,12
```

```

pent p3,13,19,17,12
tri p15,13,19
pent p15,19,16,4,0
tri p16,4,6
sq p16,18,17,19 blue
def tri 3
orange
p3 c
p4..5 p0 1..2/3 p1
p6..7 p1 1..2/3 p2
p8..9 p2 1..2/3 p0
p10..12 p0.. .7 p3
sq p0,4,10,9
sq p5,4,10,11 dark orange
sq p5,1,6,11
sq p12,7,6,11 dark orange
sq p12,7,2,8
sq p9,10,12,8 dark orange
tri p10,11,12 yellow
def pent 5
red
p5 c
p6..10 p0.. 1/3 p1..0
p11..15 p0.. 2/3 p1..0
p16..20 p0.. 1.35 p5
pent p6,19,18,15,0
tri p6,19,11
pent p1,7,20,19,11
tri p20,7,12
pent p2,8,16,20,12
tri p13,8,16
pent p13,3,9,17,16
tri p9,14,17
pent p14,17,18,10,4
tri p10,18,15
pent p16,17,18,19,20 blue

```

```

halfdart :231 red
halfkite :031 orange

```

square + triangles

(a) compdiss2a Figure 5.1

```

width .01
def sq 4
.6 .6 .6
p4..7 p0.. .2 p1..0
sq :4567 - 1 0 1
tri4 :047 - 0 1 0
tri4 :154 - 0 1 0
tri4 :265 - 0 1 0
tri4 :376 - 0 1 0
def tri 3
p3 p0 .2 p1
p4 p0 .2 p2
p5 p1 .25 p2
p6 p1 .2 p5
tri4 :613 + 1 0 1
tri :542 + 1 0 0
tri4 :034 + 0 0 1
sq1 :3654 - 1 0 1
def tri4 3
tri3 :012
def tri3 3
tri2 :012
def tri2 3
tri1 :012
def tri1 3
tri :012
def sq1 4
sq :0123

```

Penrose tiling

halfkite.sdv

```

width .01
p1 80.9017 58.7785
p2 100 0
def halfkite 3
p3 p0 .381966 p1
p4 p2 .381966 p0
halfdart :034 blue
halfkite :134 dark green
halfkite :124 0 .6 .6
def halfdart 3
p3 p2 .381966 p0

```

(b) compdiss2c Figure 5.2

```

no edges
def sq 4
white
p4..7 p0.. .2 p1..0
sq :4567
tri4 :047 - .5 .5 0
tri4 :154 - .5 .5 0
tri4 :265 - .5 .5 0
tri4 :376 - .5 .5 0
def tri 3
p3 p0 .2 p1
p4 p0 .2 p2

```

```

p5 p1 .25 p2
p6 p1 .2 p5
tri4 :613 + 1 0 0
tri :542 - 2 2 2
tri4 :034 + 0 1 0
sq1 :3654 + 1 1 1
def tri4 3
  tri3 :012
def tri3 3
  tri2 :012
def tri2 3
  tri1 :012
def tri1 3
  tri :012
def sq1 4
  sq :0123

```

NB use wait shapes!
(c) compdiss2e Figure 5.3

```

no edges
def sq 4
  white
  p4..7 p0.. .2 p1..0
  sq :4567
  tri4 :047
  tri4 :154
  tri4 :265
  tri4 :376
def tri 3
  p3 p0 .2 p1
  p4 p0 .2 p2
  p5 p1 .25 p2
  p6 p1 .2 p5
  tri4 :613
  tri :542 - 1 1 1
  tri4 :034
  sq1 :3654
def tri4 3
  tri3 :012 - .5 .5 .5
def tri3 3
  tri2 :012
def tri2 3
  tri1 :012
def tri1 3
  tri :012
def sq1 4
  sq :0123

```

Notes and sources

- p4. See Ventrella's *Brain Filling Curves*, and Mandelbrot's *FGN*.
- p7. Idea for this from a paper of Barnsley & Vince.
- p10. Cannon, Floyd & Parry, *Finite Subdivision Rules*, Figure 19a.
- p11. This is G & S's Fig. 3.4.2 (p137)
- p17. See Natalie Frank, *Introduction to hierarchical tiling dynamical systems*, p19.
- p28. L. Sadun, *Some Generalizations of the Pinwheel Tiling*, Discrete Comput Geom 20:79–110 (1998)
- p32. See Robert Fathauer, *Self-similar Tilings Based on Prototiles Constructed from Segments of Regular Polygons*
- p32. See James Cannon, *Two-Dimensional Spaces*, Vol. 2, 2017, p46-7