Homework 2

Name:

ID(학번):

Part 1. Answer **True** or **False** at following sentences. (25 points, 5 points/question)

1.   A player may have some information that is not known by some other players. Such informational asymmetry can be modeled by Nature's moves. (   )

2.   With the use of grim-trigger strategies, almost any repeated-game payoff can be achieved in equilibrium with impatient players by Folk theorem. (   )

3.   A subgame-perfect Nash equilibrium is always a Nash equilibrium.   (   )

4.   In a game with infinitely many nodes, backward induction always results in a Nash equilibrium. (   )

5.   In a sequential game, a player's ability to commit is always good. (   )

Part 2. Solve the following problems. (75 points)

6.   Consider a three-period bargaining model. Player 1 and 2 are bargaining over one dollar. A detailed description of the timing of the three-period bargaining game is as follows.

(1a) At the beginning of the first period, player 1 proposes to take a share s_1 of the dollar, leaving $1 - s_1$ for player 2.

(1b) Player 2 either accepts the offer (in which case the game ends and the payoffs s_1 to player 1 and $1 - s_1$ to player 2 are immediately received) or rejects the offer (in which case play continues to the second period).

(2a) At the beginning of the second period, player 2 proposes that player 1 takes a share s_2 of the dollar, leaving $1 - s_2$ for player 2. (Note the convention that s_t always goes to player 1, regardless of who made the offer.)

(2b) Player 1 either accepts the offer (in which case the game ends and the payoffs s_2 to player 1 and $1 - s_2$ to player 2 are immediately received) or rejects the offer (in which case play continues to the third period).

(3) At the beginning of the third period, player 1 receives a share s of the dollar, leaving $1 - s$ for player 2, where $0 < s < 1$. (Notice that the third-period settlement $(s, 1 - s)$ is given exogenously.)

Assume that each player will accept an offer if indifferent between accepting and rejecting. The discount factor is δ, where $0 < \delta < 1$. Then, find a Nash equilibrium through using backward induction. (10 points)

7. Everyone gives lip service to the fact that saving is a good idea. Unfortunately, few people actually do it. Part of the reason for the reluctance to save is that individuals recognize that society won't let them starve, so there is a good chance they will be bailed out later on. To formulate this in a game between the generations, let's consider two strategies for the older generation: save or squander. The younger generation likewise has two strategies: support their elders or save for their own retirement. A possible game matrix is shown below.

Old \ Young	Support	Refrain
Save	2, -1	1, 0
Squander	3, -1	-2, -2

(a) Find pure strategy Nash equilibria. (4 points)

(b) The above game setting ignores the time structure: one of the (few) advantages of being old is that you get to move first. Draw the game tree for the extensive game. (4 points)

(c) Then draw the payoff matrix of the extensive game of (b). Find Nash equilibria from the payoff matrix. (8 points)
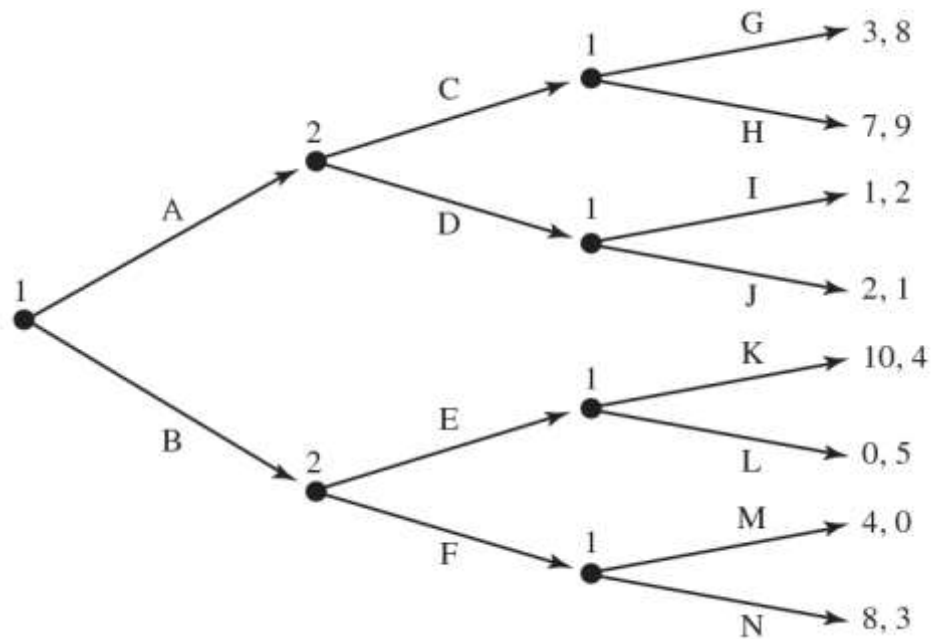
(d) Lastly, find the subgame-perfect Nash equilibrium. (6 points)

8.   Suppose that two motorcycle manufacturers, Honda and Suzuki, are considering offering 10-year full coverage warranties for their new motorcycles. Although the warranties are expensive to offer, it could be disastrous for one firm if it does not offer a warranty while its competitor does. Let's assume the payoffs for the firms are as follows:

Honda \ Suzuki	Offer Warranty	Don't Offer
Offer Warranty	20, 20	120, 10
Don't Offer	10, 120	50, 50

(a) If the game is played once, what is the Nash equilibrium? (3 points)

(b) Suppose the game is repeated three times. Will the outcome of (a) change from your answer in (a)? Explain. (6 points)

(c) Now, suppose the game is infinitely repeated and Suzuki and Honda formed an agreement to "not offer" warranties to their customers. Each firm plans the use of a grim-trigger strategy to encourage compliance with the agreement. At what level of δ (discount factor) would Honda be indifferent about keeping the agreement vs. cheating on it? Explain. (8 points)

9. Consider the extensive form game. By using backward induction, find an equilibrium. (6 points)

G → 3, 8
H → 7, 9
I → 1, 2
J → 2, 1
K → 10, 4
L → 0, 5
M → 4, 0
N → 8, 3

(Nodes: 1, A, 2, B, C, D, E, F, with player labels 1 and 2)

10. Consider a Cournot duopoly operating in a market with inverse demand $P(Q) = a - Q$, where $Q = q_1 + q_2$ is the aggregate quantity on the market. Both firms have total costs $c_i(q_i) = c \cdot q_i$, but demand is uncertain: it is high ($a = a_H$) with probability θ and low ($a = a_L$) with probability $1 - \theta$. Furthermore, information is asymmetric: firm 1 knows whether demand is high or low, but firm 2 does not. All of this is common knowledge. The two firms simultaneously choose quantities.

(a) What are the best responses for the two firms, respectively? (8 points)

(b) Find the Bayesian Nash equilibrium of this game. Also, make assumptions concerning a_H, a_L, θ, and c such that all equilibrium quantities are positive. (12 points)