Game Theory
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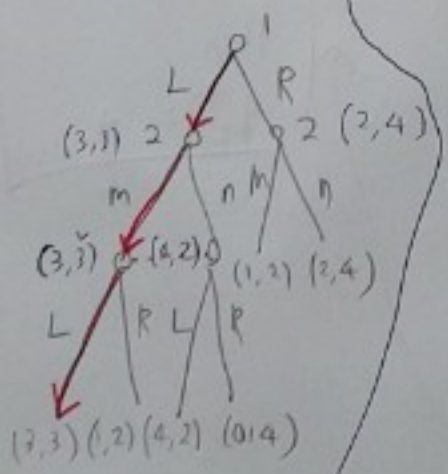
1. T
2. T
3. T
4. T
5. F ← discount 없으면 infinite 됨.
6. payoff matrix is like this

(a)

1 \ 2	mm	mn	nm	nn
LL	3,3	3,3	4,2	4,2
LR	1,2	1,2	0,4	0,4
RL	1,2	2,4	1,2	2,4
RR	1,2	2,4	1,2	2,4

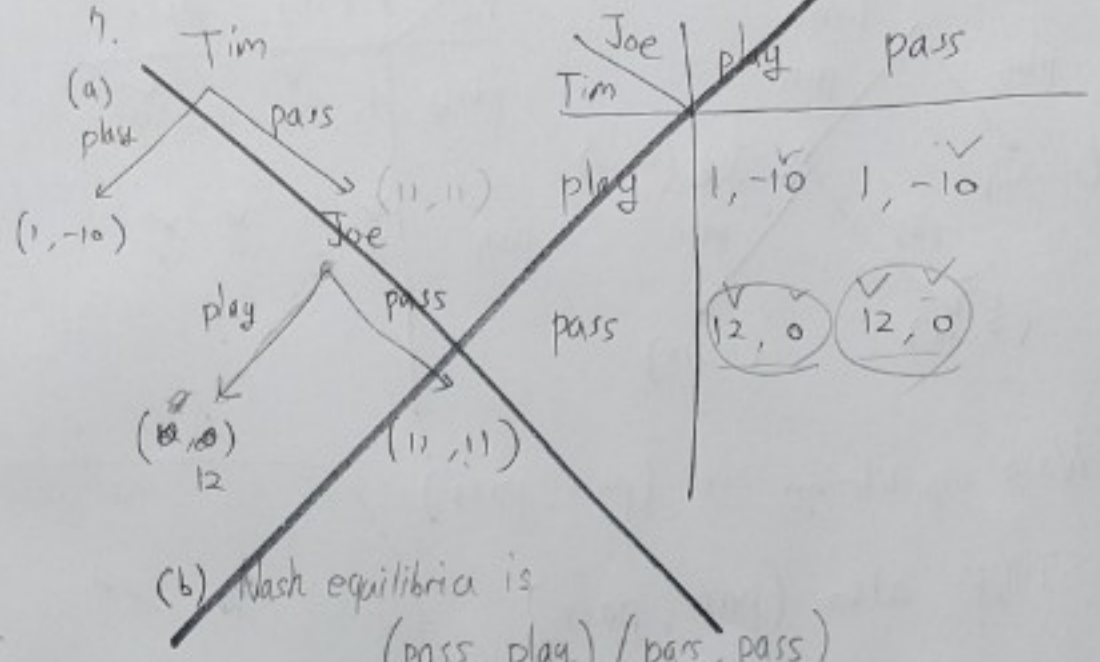
Nash equilibria is
(LL, mm), (LL, mn)

(b) Tree is like

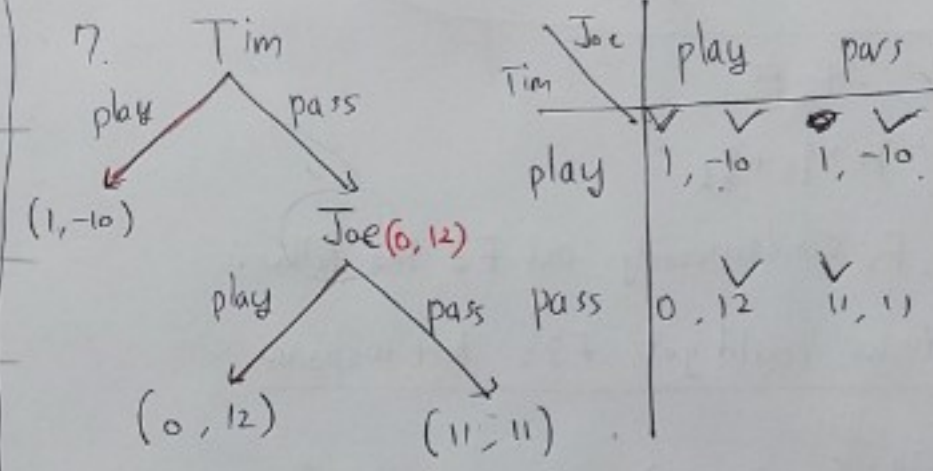


SPNE = (LL, mn)

7.

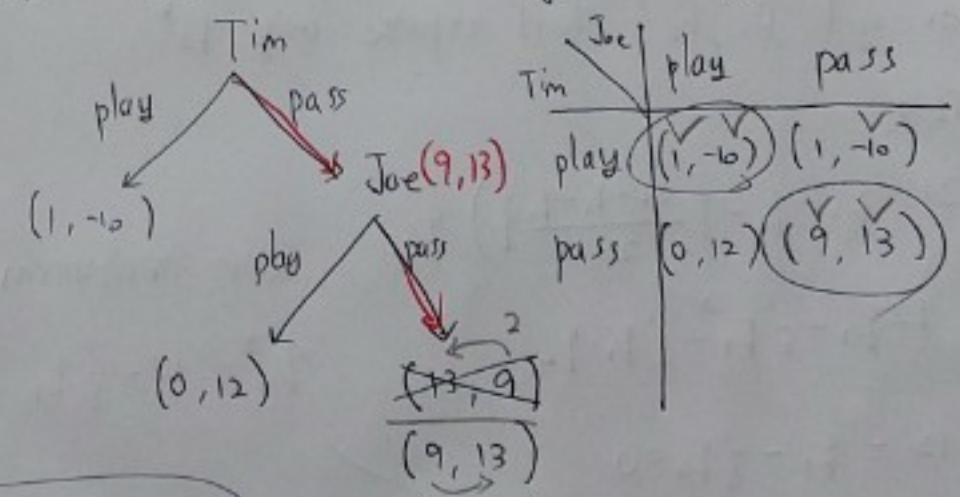


7.



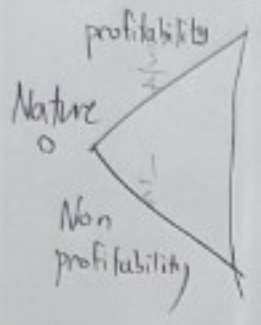
(b) Nash equilibrium is
(play, play)
SPNE is also (play, play)

(c) consider the condition game change like this



Nash equilibria are
(play, play) and (pass, pass)
So, SPNE is (pass, pass)
Equilibrium Change.

9. (a)



(b)

First let's

Bank's be

$P(t_p|t$

then let's

i) Picks

$E(U_b(L,S$

ii) Picks

$E(U_b(NL,S_1$

How abou

Firm's beli

And he

i) type

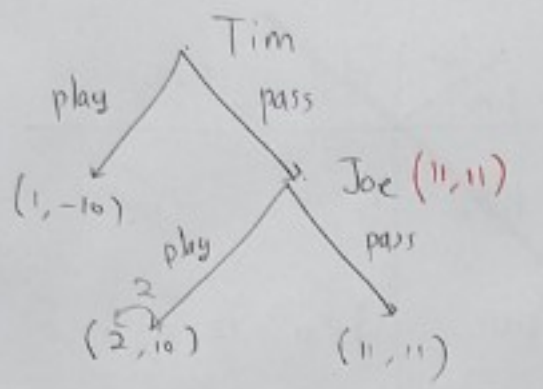
① pick

$E(U_F(T,S$

② pick

$E(U_F(S,S_b$

(d) game change like this



Tim \ Joe	play	pass
play	1, -10 ✓	1, -10 ✓
pass	2, 10 ✓	11, 11 ✓ ✓

Nash equilibrium is (pass, pass)

SPNE also (pass, pass)

So NE changes.

8. $Q = 24 - P$

$Q = q_1 + q_2 + q_3$

F_1, F_2 simultaneously and F3 then follow.

So, first we could get F3's best response.

$\pi_3 : (24 - q_1 - q_2 - q_3) \cdot q_3 \Rightarrow 24q_3 - q_1q_3 - q_2q_3 - q_3^2$

$\Rightarrow 24 - q_1 - q_2 - 2q_3 = 0$

$\therefore q_3^* = \frac{24 - q_1 - q_2}{2}$

then get F_1, F_2's best response using q_3^*

$\pi_1 \Rightarrow$

$\left(24 - q_1 - q_2 - \left[\frac{24 - q_1 - q_2}{2}\right]\right) \cdot q_1$

$12q_1 - \frac{1}{2}q_1^2 - \frac{1}{2}q_1 \cdot q_2$

$12 - q_1 - \frac{1}{2}q_2 = 0$

$q_1^* = 12 - \frac{1}{2}q_2$

Same mechanism

$q_2^* = 12 - \frac{1}{2}q_1$

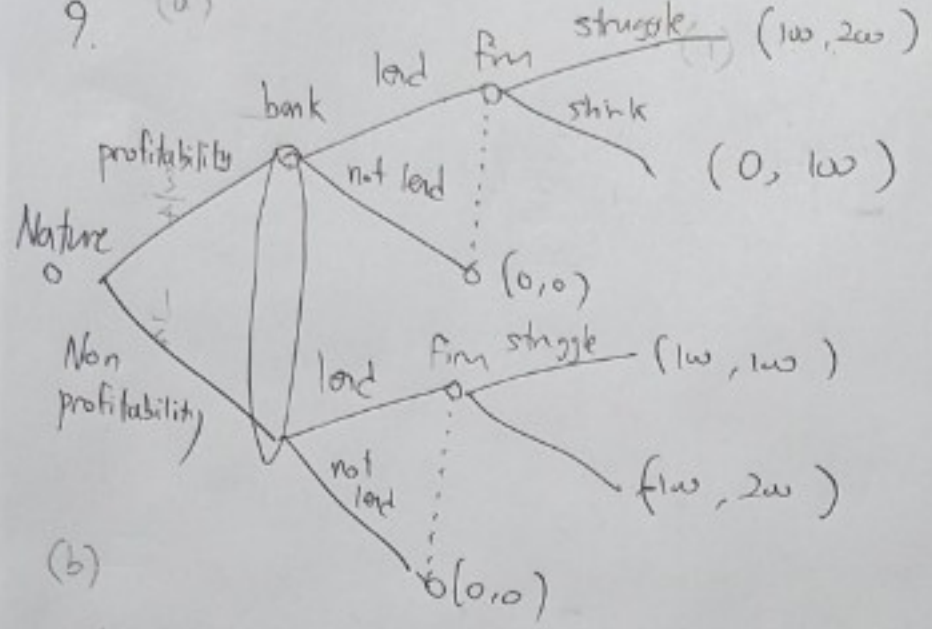
So, solve this equation,

$q_1^* = q_2^* = 8.$

then

$q_3^* = 4$

9. (a)



(b)

First Let's see bank case.

Bank's belief like this to firm's type.

$P(t_p \mid t_b) = \frac{3}{4}$, $P(t_{np} \mid t_b) = \frac{1}{4}$ and his type is only t_b

then let's see.

i) Picks 'L',

$$E(U_b(L, S_F^*) \mid t_b) = U_b(L, S_F^*(t_p) \mid t_b) \times P(t_p \mid t_b) + U_b(L, S_F^*(t_{np}) \mid t_b) \times p(t_{np} \mid t_b)$$

$$= 100 \times \frac{3}{4} + (-100) \times \frac{1}{4} \Rightarrow \frac{200}{4} \Rightarrow \boxed{50}$$

ii) Picks 'NL',

$$E(U_b(NL, S_F^*) \mid t_b) = U_b(NL, S_F^*(t_p) \mid t_b) \times p(t_p \mid t_b) + U_b(NL, S_F^*(t_{np}) \mid t_b) \times p(t_{np} \mid t_b)$$

$$= 0 \times \frac{3}{4} + 0 \times \frac{1}{4} \Rightarrow \boxed{0}$$

So, bank's best response is 'L'

How about Firms,

Firm's belief is like this, $P(t_b \mid t_p) = P(t_b \mid t_{np}) = 1$

And he has two type t_p, t_{np}.

i) type t_p,

① pick T

$$E(U_F(T, S_b^*) \mid t_p) = U_F(T, S^*(t_b) \mid t_p) \times 1 = 200$$ ✓

② pick S

$$E(U_F(S, S_b^*) \mid t_{np}) = U_F(T, S^*(t_b) \mid t_{np}) \times 1 = 100$$

ii) type t_{np}

★ ① pick T

$$E(U_F(T, S_b^*) \mid t_{np}) = U_F(T, S_b^*(t_b) \mid t_{np}) \times 1 = 100$$

$$E(U_F(S, S_b^*) \mid t_{np}) = U_F(S, S_b^*(t_b) \mid t_{np}) \times 1 = \overbrace{200}.$$

So, BNE is like this,

$S_b^*(t_b) = L$ (Lend)

$S_f^*(t_p) = T$ (struggle)

$S_f^*(t_{np}) = S$ (shirk)

so, if profitable project is chosen, bank's BNE is L, and firm's BNE is T. So, we can verify that note like this flow.

10.

(a) $V_C = 6 + 6\cdot\delta + \cdots = \frac{6}{1-\delta}$

$V_D = 4 + 4\cdot\delta + \cdots = \frac{4}{1-\delta}$

if history is C, payoff matrix is like this,

1 \ 2	C	D
C	$\underline{6+\delta\cdot V_C}$ $6+\delta\cdot V_C$	$\underline{2+\delta\cdot V_D}$ $8+\delta\cdot V_D$
D	$\underline{8+\delta\cdot V_D}$ $2+\delta\cdot V_D$	$\underline{4+\delta\cdot V_D}$ $4+\delta\cdot V_D$

~~If~~ to pass single deviation test with grim trigger strategy,

$6+\delta\cdot V_C \geq 8+\delta\cdot V_D$ ~~and $2+\delta\cdot V_D$~~

$6+\delta\cdot\frac{6}{1-\delta} \geq 8+\delta\cdot\frac{4}{1-\delta}$

$\frac{2\delta}{1-\delta} \geq 2$

$2\cdot\delta \geq 2-2\delta$

$4\delta \geq 2$

$\delta \geq \frac{1}{2}$

$\therefore \frac{1}{2} \leq \delta < 1$

we need this condition

(b) history, D payoff is like this,

1 \ 2	C	D
C	$6+\delta V_D$, $6+\delta V_D$	$2+\delta V_D$, $8+\delta V_D$ ✓
D	$8+\delta V_D$ ✓, $2+\delta V_D$	$4+\delta V_D$ ✓, $4+\delta V_D$ ✓

in this case, already pick (D,D) without any condition in δ.

So, this could be pass single deviation test.

(c) So, Finally this grim & trigger strategy ~~is~~
could be stay in (C,C) when $\frac{1}{2} \leq \delta < 1$.

And this could be pass single deviation test because ~~they~~
both players do not have incentive to chose D
because C is more beneficial.

So, these strategies could pass sigle deviation test.