Game Theory (2020-1)

Homework 1

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Part 1. Answer **True** or **False** at following sentences. (25 points, 5 points/question)

1. A contract is said to be automatically enforced if the players have individual incentives to abide by the terms of the contract. (F)

2. If a strategy profile s^* is a Nash equilibrium, then s^* is rationalizable for every player i. (T)

3. Incomplete information in strategic settings means that some player is uncertain about another player's preference. (T)

4. A set of mixed strategies includes a set of pure strategies. (T)

5. A strategy s_i is a best response to some belief if and only if s_i is not dominated. (F) = weakly dominate

$A \geq B$ (A is dominate) $\neq$

$A \geq B$ (A is dominate)

$A \leq B$ (A is dominated)

$A > B$ (A is not dominated)

Part 2. Solve the following problems. (75 points)

6. Compute the set of rationalizable strategies in the following game. (10 points)

1\2	~~a~~	B	c	~~d~~
w	0, 1	0, 3	3, 1	0, 0
~~x~~	~~3, 2~~	~~0, 0~~	~~2, 1~~	~~1, 4~~
~~y~~	~~2, 2~~	~~3, 4~~	~~1, 1~~	~~9, 2~~
z	0, 3	5, 5	1, 8	0, 2

① ③ ② ④

Step 1] There are no dominant strategies for each other among player 1 and player 2 strategies so use [IESDS] to get rationaliziable strategies.

Step 2] player 2 cannot rationalize strategy "a" so remove strategy a and game resetting

Step 3] And then in resetting game, player 1 cannot rationalize strategy "x", remove x.

Step 4] And then, player 2 cannot rationalize strategy "d", remove d.

Step 5] And then, player 1 cannot rationalize strategy "y", remove y.

So then game just like this,

1\2	B	c
W	0, 3	3, 1
Z	5, 5	1, 8

→ there is no any strictly dominated strategy.

So (W, B), (W, C), (Z, B), (Z, C) are rationalizable strategies

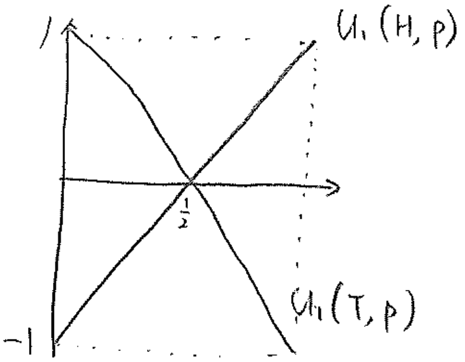
(d) Show that the mixed-strategy Nash equilibrium is also a maxmin strategy. (10 points)

player choose H in prob p and chuse T in prob $1-p$

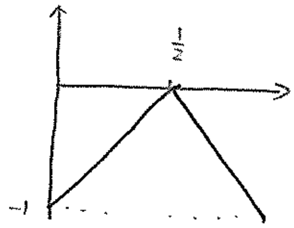
When player 2 chuse H, player 1's payoff $U_1(H, p) = p + (-1)\times(1-p) = 2p-1$

When player 2 chuse T, player 1's payoff $U_1(T, p) = -p + (1-p) = -2p+1$

And we can draw graph like this



But player 2 want to maximize player 1's payoff since this is strictly competitive game, $p < \frac{1}{2}$, player 2 chuse H, $p > \frac{1}{2}$ player 2 chuse T. So graph apply this therom is like this



The maximizing strategy among minimizing payoff stragies is $p = \frac{1}{2}$ and same mechanism can apply to player 2. So, this is same as Nash eq.

8. Consider the duopoly with linear demand function $P = 4 - Q$, where P is the price and $Q = q_1 + q_2$ is the total supply. Firm 1 and 2 simultaneously produce q_1 and q_2, and they sell at price P. Both Firm 1 and 2 have an identical marginal cost, 2. The two Firms are rational and all of above is common knowledge.

(a) Find a Nash equilibrium in this game. (4 points)

$\pi_1 = [4 - (q_1 + q_2)] \cdot q_1 - 2q_1 = 4q_1 - q_1^2 - q_1 q_2 - 2q_1$

$\pi_1' = 4 - 2q_1 - q_2 - 2 = 0 \quad 2q_1 = 4 - q_2 - 2 \quad \boxed{q_1^* = 1 - \frac{1}{2} q_2^*}$

$\pi_2 \Rightarrow [4 - (q_1 + q_2)] \cdot q_2 - 2q_2 = 4q_2 - q_1 q_2 - q_2^2 - 2q_2$

$\pi_2' \Rightarrow 4 - q_1 - 2q_2 - 2 = 0 \quad \boxed{q_2^* = 1 - \frac{1}{2} q_1^*}$

$2q_1^* = 2 - q_2^*$

$2q_2^* = 2 - q_1^*$

$2q_2^* = 4 - 4q_1^*$

$2 = 3q_1^* \quad 2 = 3q_2^*$

$\boxed{\therefore q_1^* = q_2^* = \frac{2}{3}}$

(b) Consider an extension to the n-firm case with demand function $P = 4 - Q$, where P is the price and $Q = \sum_{i=1}^{n} q_i$ is the total supply. Every Firm i has an identical marginal cost, 2. Find a Nash equilibrium in the extension of the previous game. (8 points)

firm i, other firms $-i$

The Nash equilibrium is same as like provious question. like,

$$q_i^* = 1 - \frac{1}{2} q_{-i}^*$$

and in equilibrium, every firm's quantity is same, So we could write like this.

$$q = 1 - \frac{(n-1)}{2} q \quad [n-1 \text{ is other company's number}]$$

So, we get $q^* = \frac{2}{n+1}$. And this is Nash Equilibrium

10. Briefly describe what John Nash did in the game theory. (10 points)

In previous theory, game theories have solved the decision-making problem of one strategy dominating the other strategies (dominant strategy) and eliminating strategies that do not (IESDS) and this problem are very simple and rough. In fact, issues of interest are often beyond this net. But John Nash has provided a good indicator of decision making in numerous disciplines since then, by finding the optimal response to the opponent's strategy and find the Nash equilibrium, a set of strategies that has no incentive to switch to different strategies