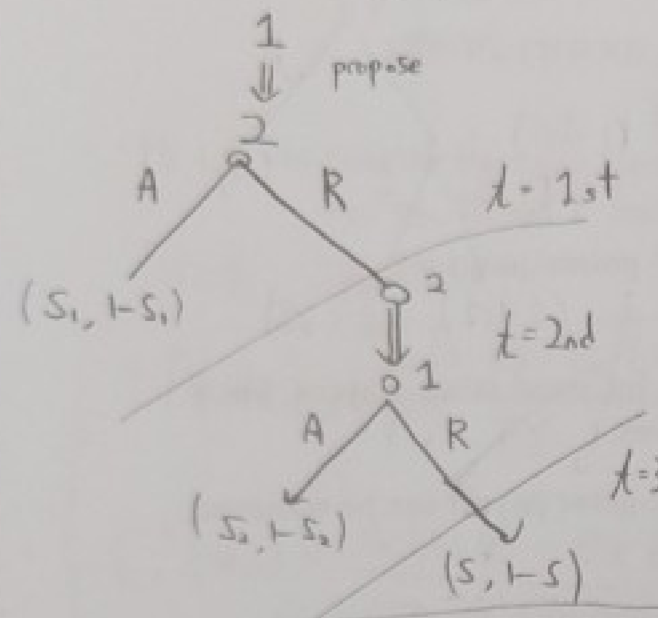
본 교재의 순차협상부분을 참고해보라.

discount factor를 어떻게 적용시킬까.

Assume that each player will accept an offer if indifferent between accepting and rejecting. The discount factor is δ, where $0 < \delta < 1$. Then, find a Nash equilibrium through using backward induction. (10 points)

Game is like this graph



by backward induction, let's see 2nd period.

player 1 compare his payoff S_2 and S and we know S's present value in 2nd is $\delta \times S$, and player 1's best response in 2nd period is like this,

player 1 action $\begin{cases} A, & S_2 \geq \delta \cdot S \text{ (if both are same, we chose A)} \\ R, & S_2 < \delta \cdot S \end{cases}$

And then, for player 2, she knows that when player 2 chose A, she can maximize her payoff by suggesting $S_2 = \delta \times S$.

Then, let's check it is better than payoff that she would got when player 1 reject. And that payoff is $1-S$, and it's present value is $\delta \times (1-S)$.

$1-\delta S$ (player 1 accept in $S_2 = \delta \times S$) $\geq \delta(1-S)$, so player 1's suggestion $S_2 = \delta \times S$ is best response.

Then, Let's see 1st period. player 2 got $1-S_1$ when chose A, otherwise $1-\delta S$. And player 2's best response in 1st period is like this, present value

player 2 $\begin{cases} A, & (1-S_1) \geq \delta(1-\delta S) \rightarrow S_1 \leq 1-\delta(1-\delta S) \\ R, & (1-S_1) < \delta(1-\delta S) \rightarrow S_1 > 1-\delta(1-\delta S) \end{cases}$

then player want to make player 2 accept his propose and maximize his payoff, $S_1 = 1-\delta(1-\delta S)$ may be satisfied. Then he compare this results with player 2's reject. then he get $\delta \cdot S$.

$1-\delta(1-\delta S) > \delta S$, so he suppose $\boxed{S_1 = 1-\delta(1-\delta S)}$

So, nash equilibrium is just like this,

player 1	player 2
1st : $S_1 = 1-\delta(1-\delta S)$	1st $\begin{cases} A, & S_1 \leq 1-\delta(1-\delta S) \\ R, & S_1 > 1-\delta(1-\delta S) \end{cases}$
2nd $\begin{cases} A, & S_2 \geq \delta \cdot S \\ R, & S_2 < \delta \cdot S \end{cases}$	2nd $S_2 = \delta S$

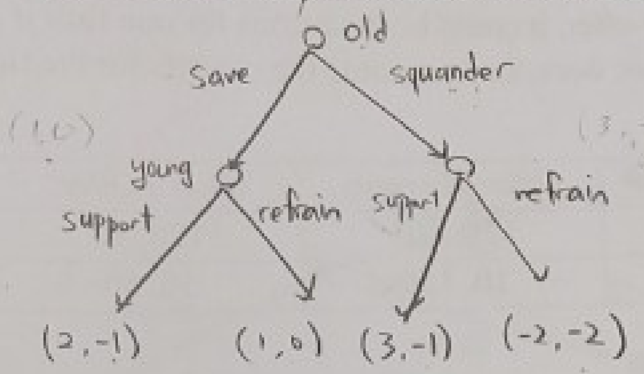
7. Everyone gives lip service to the fact that saving is a good idea. Unfortunately, few people actually do it. Part of the reason for the reluctance to save is that individuals recognize that society won't let them starve, so there is a good chance they will be bailed out later on. To formulate this in a game between the generations, let's consider two strategies for the older generation: save or squander. The younger generation likewise has two strategies: support their elders or save for their own retirement. A possible game matrix is shown below.

Old \ Young	*Support*	*Refrain*
Save	2, -1	1, 0
Squander	3, -1	-2, -2

(a) Find pure strategy Nash equilibria. (4 points)

(squander, support), (save, refrain)

(b) The above game setting ignores the time structure: one of the (few) advantages of being old is that you get to move first. Draw the game tree for the extensive game. (4 points)



(c) Then draw the payoff matrix of the extensive game of (b). Find Nash equilibria from the payoff matrix. (8 points) Let's suppose S → support , R → refrain

old \ young	SS	SR	RS	RR
save	2, -1	2, -1	1, 0	1, 0
Squander	3, -1	-2, -2	3, -1	-2, -2

∴ NE

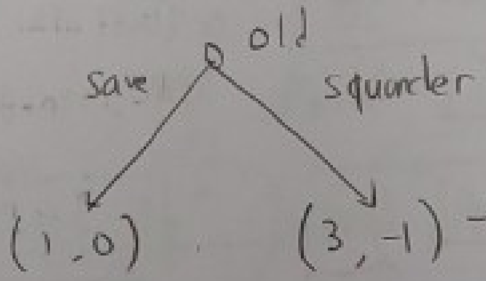
⇒ (save , RR)

(squander , SS)

(squander , RS)

(d) Lastly, find the subgame-perfect Nash equilibrium. (6 points)

After consider young's payoff, game reformed just like this,



young would choose refrain ← (1,0)

(3,-1) → young would choose support

So, then old chose squander

$\boxed{\text{(squander, RS)}}$

8. Suppose that two motorcycle manufacturers, Honda and Suzuki, are considering offering 10-year full coverage warranties for their new motorcycles. Although the warranties are expensive to offer, it could be disastrous for one firm if it does not offer a warranty while its competitor does. Let's assume the payoffs for the firms are as follows:

Honda \ Suzuki	*Offer Warranty*	*Don't Offer*
Offer Warranty	20, 20	120, 10
Don't Offer	10, 120	50, 50

(a) If the game is played once, what is the Nash equilibrium? (3 points)

(offer Warranty, offer warranty)

(b) Suppose the game is repeated three times. Will the outcome of (a) change from your answer in (a)? Explain. (6 points)

In this finite repeated game, each game's nash eq. is independent from other stage. So Nash equilibrium isn't change.

문제를 맞게 이해한건~

(c) Now, suppose the game is infinitely repeated and Suzuki and Honda formed an agreement to "not offer" warranties to their customers. Each firm plans the use of a grim-trigger strategy to encourage compliance with the agreement. At what level of δ (discount factor) would Honda be indifferent about keeping the agreement vs. cheating on it? Explain. (8 points)

to use augmented game, let's get present value from agreement and cheating

$V_a = 50 + 50\delta + \dots = \frac{50}{1-\delta}$

$V_c = 20 + 20\delta + \dots = \frac{20}{1-\delta}$

and then, payoff matrix is like when history is agreement

	Offer	Don't offer
Offer	$20+\delta V_c$, $20+\delta V_c$	$120+\delta V_c$, $10+\delta V_c$
Don't	$10+\delta V_c$, $120+\delta V_c$	$50+\delta V_a$, $50+\delta V_a$

in history aggrement to (Don't offer, Don't offer) be SPNE, $120 + \delta \cdot \frac{20}{1-\delta} \leq 50 + \delta \frac{50}{1-\delta}$

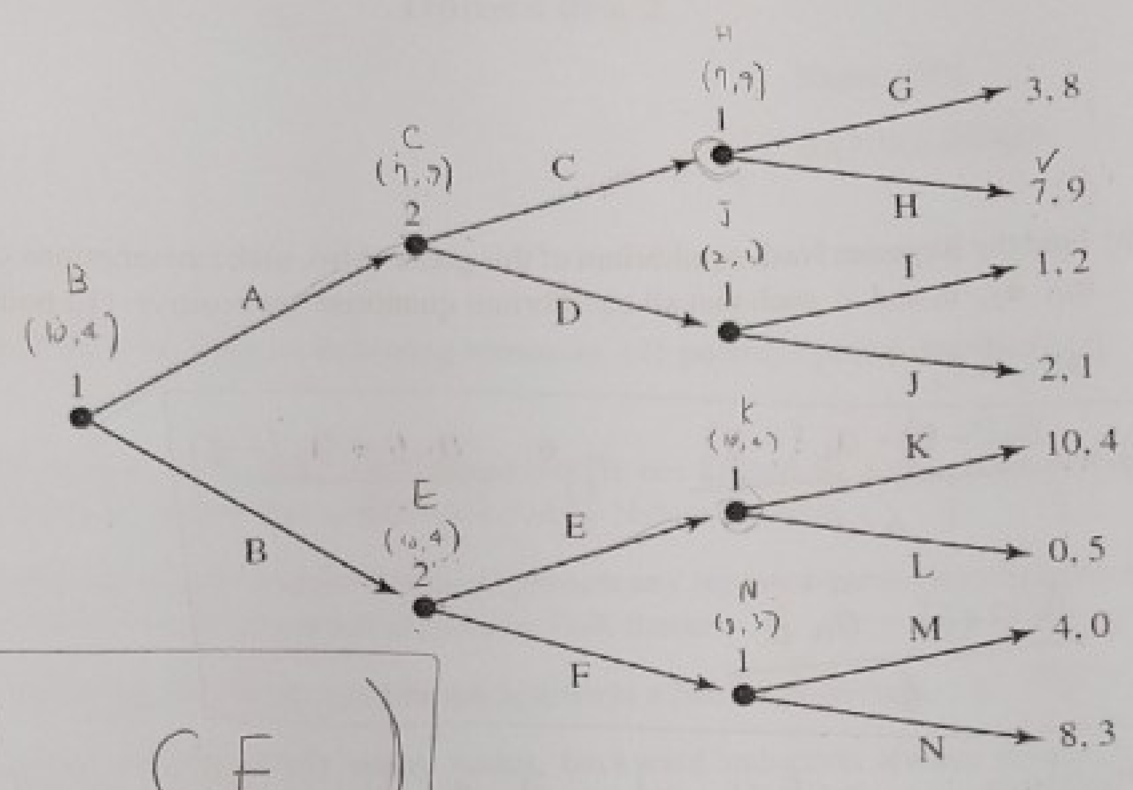
$\boxed{\therefore \frac{7}{10} \leq \delta < 1}$ //

when history is cheating →

	offer	Don't
offer	$\boxed{20+\delta V_c, 20+\delta V_c}$	$120+\delta V_c$, $10+\delta V_c$
Don't	$10+\delta V_c$, $120+\delta V_c$	$50+\delta V_c$, $50+\delta V_c$

absolutely offer, offer in history cheating

9. Consider the extensive form game. By using backward induction, find an equilibrium. (6 points)



(BKN , CE)

프린트 여백이 부족해서 풀이 과정은 따로 첨부하였습니다

10. Consider a Cournot duopoly operating in a market with inverse demand $P(Q) = a - Q$, where $Q = q_1 + q_2$ is the aggregate quantity on the market. Both firms have total costs $c_i(q_i) = c \cdot q_i$, but demand is uncertain: it is high ($a = a_H$) with probability θ and low ($a = a_L$) with probability $1 - \theta$. Furthermore, information is asymmetric: firm 1 knows whether demand is high or low, but firm 2 does not. All of this is common knowledge. The two firms simultaneously choose quantities.

(a) What are the best responses for the two firms, respectively? (8 points)

First, Firm 1 knows demand type. So we can divide by two cases

1) a_H.

$\pi_1 : (a_H - q_1(a_H) - q_2^*) \cdot c q_1$

$\therefore q_1^*(a_H) = \dfrac{a_H - q_2^*}{2}$

2) a_L

$\pi_1 : (a_L - q_1(a_L) - q_2^*) \cdot c \cdot q_1$ by FOC

$\therefore q_1^*(a_L) = \dfrac{a_L - q_2^*}{2}$ ⇒ this is best response.

at Firm 2,

$\pi_2 : (a_H - q_1^*(a_H) - q_2) \times c \times q_2 \times \theta + (a_L - q_1^*(a_L) - q_2) \times c \times q_2 \times (1-\theta)$ by FOC

$\therefore q_2^* = \dfrac{a_H\theta + a_L(1-\theta)}{3}$

⇒ So, $q_1^*(a_H) = \dfrac{a_H(3-\theta) - a_L(1-\theta)}{6}$ $\quad q_2^* = \dfrac{a_H\theta + a_L(1-\theta)}{3}$

$q_1^*(a_L) = \dfrac{a_L(2+\theta) - a_H\theta}{6}$

(b) Find the Bayesian Nash equilibrium of this game. Also, make assumptions concerning a_H, a_L, θ, and c such that all equilibrium quantities are positive. (12 points)

BNE like this,

$$\boxed{\begin{aligned} q_1^*(a_H) &= \frac{a_H(3-\theta) - a_L(1-\theta)}{6} \qquad q_2^* = \frac{a_H\theta + a_L(1-\theta)}{3} \\ q_1^*(a_L) &= \frac{a_L(2+\theta) - a_H\theta}{6} \end{aligned}}$$

in this equation, $0<\theta<1$ and a_H, a_L also >0 because of Cournot model's assumption.

So, all these equilibrium quantities are positive.

$\leq 50 + \delta \cdot \frac{50}{1-\delta}$

$P(Q) = a - Q$ [high, low]

q_1 q_2 ?

$\leq 50 + \frac{50\delta}{1-\delta}$

$c = q_1 c$

$q_1^*(a_H)$ $q_1^*(a_L)$

구하는 과정.

$\frac{50\delta}{\delta}$

$\delta \geq \frac{7}{10}$

1] π_1

1) $a = a_H$

$(P - Q) \cdot c$

$\Rightarrow (a_H - q_1 - q_2^*) \cdot c \cdot q_1$

$\Rightarrow a_H c q_1 - c q_1^2 - c q_2^* q_1$

$\Rightarrow a_H c - 2c q_1 - c q_2^* = 0$

$\cancel{2c q_1 = \frac{c(a_H - q_2^*)}{2c}}$

$\cancel{2c} q_1 = \frac{a_H c - c q_2^*}{2c}$

$q_1 = \frac{a_H - q_2^*}{2}$

$q_1 = \frac{a_L - q_2^*}{2}$

$q_2(1-\theta)$

$c\theta q_2^2$

$q_2(1-\theta) - c(1-\theta) \cdot q_2^2$

θq_2

$c(1-\theta) - 2c(1-\theta) \cdot q_2 = 0.$

F1

$$q_2^* = \frac{a_H\theta + a_L(1-\theta)}{3}$$

$\cancel{q_1}$

$$(a_H - q_1(a_H) - q_2^*)\cdot c\, q_1$$

$$q_1^*(a_H) = \frac{a_H - q_2^*}{2} \qquad q_1^*(a_L) = \frac{a_L - q_2^*}{2}$$

$(\cancel{a_H - q_1^* - q_2}$ q_2^* 구하는 과정.

$$q_1^*(a_H) = \frac{1}{2}\left\{a_H - \frac{(a_H\theta + a_L - a_L\theta)}{3}\right\}$$

$$= \frac{1}{2}\left\{\frac{\cancel{3a_H} - \cancel{a_H}\theta - a_L + a_L\theta}{3}\right\}$$

$$= \frac{1}{2}\left\{\frac{a_H(3-\theta) - a_L(1-\theta)}{3}\right\}$$

$$\theta\times(a_H - q_1^*(a_H) - q_2)\times c\, q_2 + (a_L - q_1^*(a_L) - q_2)\times c\, q_2 \times (1-\theta)$$

$$q_1^*(a_L) = \frac{1}{2}\left\{\frac{\overset{2a_L}{3a_L - a_L} - a_H\theta + a_L\theta}{3}\right\}$$

$$= \frac{a_L(2+\theta) - a_H\theta}{6}$$

$$\theta a_H c\, q_2 - q_1^*$$

$$\theta a_H c - \theta c\, q_1^*(a_H) - 2c\theta q_2 + a_L(1-\theta)\cdot c - q_1^*(a_L)\cdot c\cdot(1-\theta) - 2c(1-\theta)\cdot q_2 = 0$$

$$\theta a_H \cancel{c} - \theta \cancel{c}\, q_1^*(a_H) + (1-\theta)\cdot a_L\cdot \cancel{c} - (1-\theta)\cdot \cancel{c}\cdot q_1^*(a_L) = 2c(1-\theta)\cdot q_2 + 2c\theta q_2$$

$$= (2c\theta + 2c(1-\theta))\, q_2$$

$$\frac{\theta a_H - q_1^*(a_H) + (1-\theta)\cdot a_L - (1-\theta)\cdot q_1^*(a_L)}{2} = \cancel{2c}\, q_2^*$$

$$\frac{\theta\{a_H - q_1^*(a_H)\} + (1-\theta)\{a_L - q_1^*(a_L)\}}{2} = q_2^*$$

$$\theta\left(a_H - \frac{(a_H - q_2^*)}{2}\right) + (1-\theta)\left(a_L - \frac{(a_L - q_2^*)}{2}\right) = 2q_2^*$$

$$a_H\theta - \frac{\theta(a_H - q_2^*)}{2} + (1-\theta)a_L - \frac{(a_L - q_2^*)(1-\theta)}{2} = 2q_2^*$$

$$2a_H\theta - \theta(a_H - q_2^*) + 2(1-\theta)a_L - (a_L - q_2^*)(1-\theta) = 4q_2^*$$

$$2a_H\theta - a_H\theta + \cancel{\theta q_2^*} + 2(1-\theta)a_L - (1-\theta)a_L + \cancel{q_2^*(1-\theta)} = \cancel{4q_2^*}$$

$$\{4 - (1-\theta) - \theta\}\, q_2^*$$

$$= \frac{\cancel{2a_H\theta} - \cancel{a_H\theta} + \cancel{2a_L} - \cancel{2a_L\theta} - \cancel{a_L} + \cancel{\theta a_L}}{}$$

$$= \frac{a_H\theta + a_L - a_L\theta}{}$$

$$4 - 1 + \theta - \theta$$

$$\frac{q_2^*}{\cancel{\theta}} = \frac{a_H\theta + a_L(1-\theta)}{3}$$

Homework 2

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Part 1. Answer **True** or **False** at following sentences. (25 points, 5 points/question)

1. A player may have some information that is not known by some other players. Such informational asymmetry can be modeled by Nature's moves. (T)
2. With the use of grim-trigger strategies, almost any repeated-game payoff can be achieved in equilibrium with impatient players by Folk theorem. (F)
3. A subgame-perfect Nash equilibrium is always a Nash equilibrium. (T)
4. In a game with infinitely many nodes, backward induction always results in a Nash equilibrium. (F)
5. In a sequential game, a player's ability to commit is always good. (F)

Part 2. Solve the following problems. (75 points)

6. Consider a three-period bargaining model. Player 1 and 2 are bargaining over one dollar. A detailed description of the timing of the three-period bargaining game is as follows.

   (1a) At the beginning of the first period, player 1 proposes to take a share s_1 of the dollar, leaving $1 - s_1$ for player 2.

   (1b) Player 2 either accepts the offer (in which case the game ends and the payoffs s_1 to player 1 and $1 - s_1$ to player 2 are immediately received) or rejects the offer (in which case play continues to the second period).

   (2a) At the beginning of the second period, player 2 proposes that player 1 takes a share s_2 of the dollar, leaving $1 - s_2$ for player 2. (Note the convention that s_t always goes to player 1, regardless of who made the offer.)

   (2b) Player 1 either accepts the offer (in which case the game ends and the payoffs s_2 to player 1 and $1 - s_2$ to player 2 are immediately received) or rejects the offer (in which case play continues to the third period).

   (3) At the beginning of the third period, player 1 receives a share s of the dollar, leaving $1 - s$ for player 2, where $0 < s < 1$. (Notice that the third-period settlement $(s, 1 - s)$ is given exogenously.)