(b) Find the Bayesian Nash equilibrium of this game. Also, make assumptions concerning a_H, a_L, θ, and c such that all equilibrium quantities are positive. (12 points)

from the calculation, finally we get BNE like this,

$$\boxed{q_1^*(a_H) = \frac{(3-\theta)\cdot a_H - (1-\theta)\cdot a_L - 2c}{6}, \quad q_1^*(a_L) = \frac{(\theta+2)\cdot a_L - \theta a_H - 2c}{6} \qquad q_2^* = \frac{\theta\cdot a_H + (1-\theta)a_L - c}{3}}$$

then we could know that $a_H > a_L$ and by assumption of Cournot Model,

we know $a_H > a_L > c$.

And we could know $q_1^*(a_H)$ and q_2^* always positive whatever c is.

Let's suppose c is a_L.

$$q_2^* = \frac{\theta \times a_H + (1-\theta)a_L - a_L}{3} = \frac{\theta(a_H - a_L)}{3} > 0 \text{ because } a_H - a_L > 0$$

$$q_1^*(a_H) = \frac{(3-\theta)\cdot a_H - (1-\theta)\cdot a_L - 2a_L}{3} = \frac{(3-\theta)a_H - (3-\theta)a_L}{3} = \frac{(3-\theta)(a_H-a_L)}{3} > 0 \quad \text{because } \begin{array}{l} 3-\theta > 0 \\ a_H - a_L > 0 \end{array}$$

but, $q_1^*(a_L)$ ~~is~~ could be negative ~~d~~ by "C" value. So, we could make assumption.

$$q_1^*(a_L) = \frac{(\theta+2)a_L - \theta a_H - 2c}{6} > 0 \Rightarrow \theta a_L + 2a_L - \theta a_H - 2c > 0 \quad 2(a_L - c) - \theta(a_H - a_L) > 0$$

$$\boxed{\therefore \theta < \frac{2(a_L - c)}{a_H - a_L}}$$

we need this assumption.