Game Theory

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1. T
2. T
3. T
4. F
5. F ⇐ discount 생각해라! infinite 생각

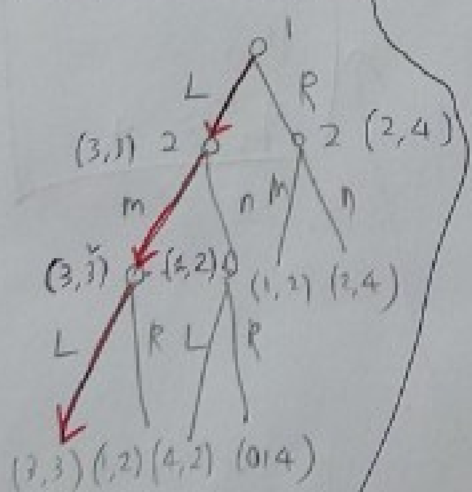
6. payoff matrix is like this

(a)

1 \ 2	mm	mn	nm	nn
LL	3, 3	3, 3	4, 2	4, 2
LR	1, 2	1, 2	0, 4	0, 4
RL	1, 2	2, 4	1, 2	2, 4
RR	1, 2	2, 4	1, 2	2, 4

Nash equilibria is (LL, mm), (LL, mn)

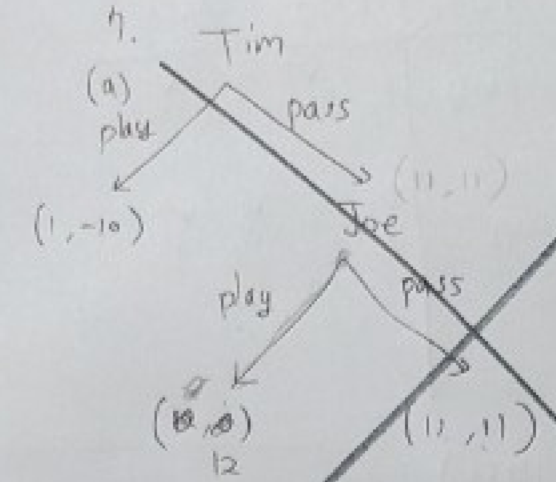
(b) Tree is like



SPNE = $\boxed{(LL, mn)}$

7.

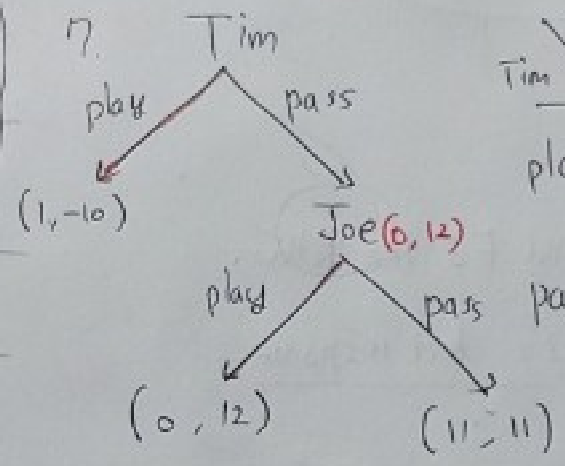
(a)



Tim \ Joe	play	pass
play	1, -10	1, -10
pass	12, 0	12, 0

~~(b) Nash equilibria is (pass, play) (pass, pass)~~

7.

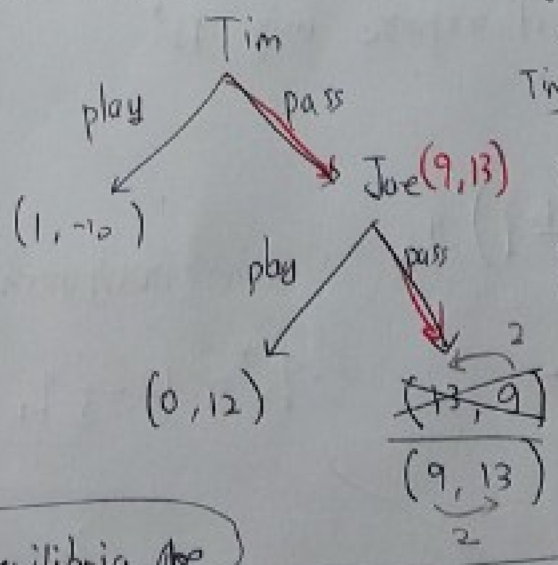


Tim \ Joe	play	pass
play	1, -10	1, -10
pass	0, 12	11, 11

(b) Nash equilibrium is (play, play)

SPNE is also (play, play)

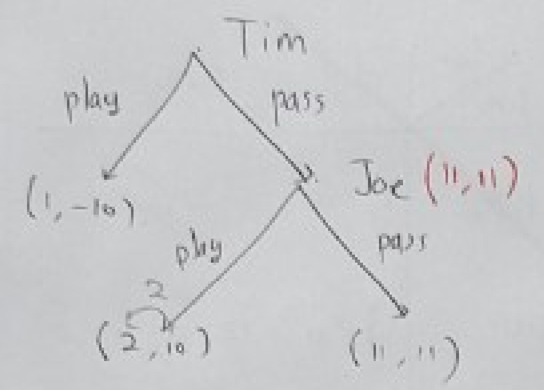
(c) consider the condition game change like this



Tim \ Joe	play	pass
play	1, -10	1, -10
pass	0, 12	9, 13

Nash equilibria are (play, play) (pass, pass) and $\boxed{\text{SPNE is (pass, pass)}}$ So, Equilibrium Change.

(d) game change like this



Tim \ Joe	play	pass
play	1, -10	1, -10
pass	2, 10	11, 11

Nash equilibrium is (pass, pass)

SPNE also (pass, pass)

So NE changes.

8. $Q = 24 - P$

$Q = q_1 + q_2 + q_3$

F_1, F_2 simultaneously and F3 then follow.

So, first we could get F3's best response.

$\pi_3 : (24 - q_1 - q_2 - q_3) \cdot q_3 \Rightarrow 24 q_3 - q_1 q_3 - q_2 q_3 - q_3^2$

$\Rightarrow 24 - q_1 - q_2 - 2q_3 = 0$

$\therefore q_3^* = \dfrac{24 - q_1 - q_2}{2}$

then get F_1, F_2's best response using q_3^*

$\pi_1 \Rightarrow$

$\left(24 - q_1 - q_2 - \left\{\dfrac{24 - q_1 - q_2}{2}\right\}\right) \cdot q_1$

$12 q_1 - \frac{1}{2} q_1^2 - \frac{1}{2} q_1 \cdot q_2$

$12 - q_1 - \frac{1}{2} q_2 = 0$

$q_1^* = 12 - \frac{1}{2} q_2$

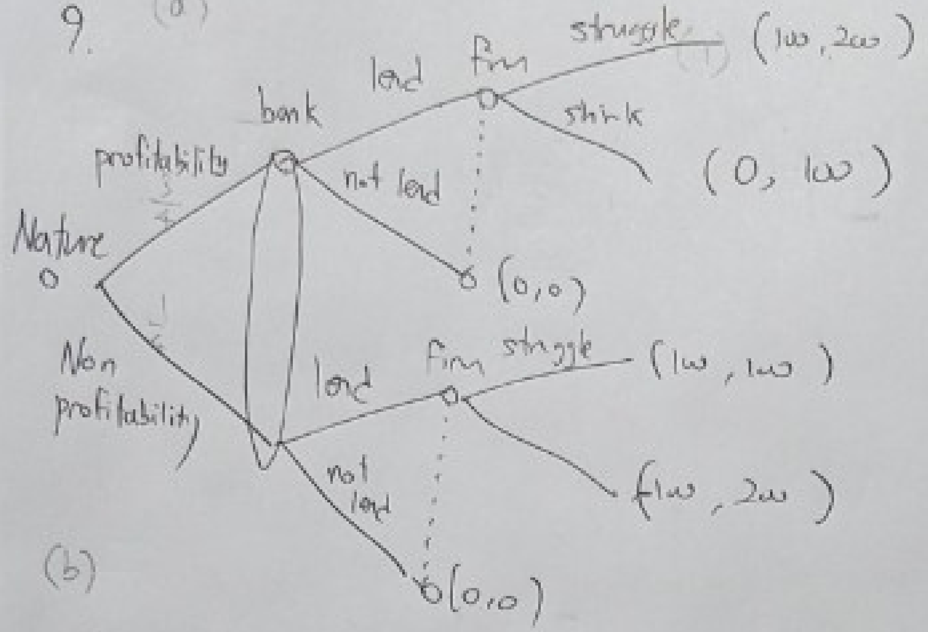
Same mechanism

$q_2^* = 12 - \frac{1}{2} q_1$

So, solve this equation,

$$\boxed{\begin{array}{l} q_1^* = q_2^* = 8. \\ \text{then} \\ q_3^* = 4 \end{array}}$$

9. (a)



(b)

First let's see bank case.

Bank's belief like this to firm's type.

$P(t_P | t_b) = \frac{3}{4}$, $P(t_{np} | t_b) = \frac{1}{4}$ and his type is only t_b.

then let's see.

i) Picks 'L',

$$E(U_b(L, S_F^*) | t_b) = U_b(L, S_F^*(t_P) | t_b) \times P(t_P | t_b) + U_b(L, S_F^*(t_{np}) | t_b) \times P(t_{np} | t_b)$$
$$= 100 \times \frac{3}{4} + (-100) \times \frac{1}{4} \Rightarrow \frac{200}{4} \Rightarrow \boxed{50}$$

ii) Picks 'NL',

$$E(U_b(NL, S_F^*) | t_b) = U_b(NL, S_F^*(t_P) | t_b) \times P(t_P | t_b) + U_b(NL, S_F^*(t_{np}) | t_b) \times P(t_{np} | t_b)$$
$$= 0 \times \frac{3}{4} + 0 \times \frac{1}{4} \Rightarrow \boxed{0}$$

So, bank's best response is 'L'

How about Firms,

Firm's belief is like this, $P(t_b | t_P) = P(t_b | t_{np}) = 1$

And he has two type t_P, t_{np}.

i) type t_P,

① pick T

$$E(U_F(T, S_b^*) | t_P) = U_F(T, S^*(t_b) | t_P) \times 1 = 200$$ ✓

② pick S

$$E(U_F(S, S_b^*) | t_{np}) = U_F(T, S^*(t_b) | t_{np}) \times 1 = 100$$

ii) type t_{np}

① pick T

$$E(U_F(T, S_b^*) | t_{np}) = U_F(T, S_b^*(t_b) | t_{np}) \times 1 = 100$$

$$E(U_F(S, S_b^*) | t_{np}) = U_F(S, S_b^*(t_b) | t_{np}) \times 1 = 200.$$

So, BNE is like this,

$$\boxed{\begin{aligned} S_b^*(t_b) &= L \text{ (Lend)} \\ S_f^*(t_p) &= T \text{ (struggle)} \\ S_f^*(t_{np}) &= S \text{ (shirk)} \end{aligned}}$$

so, if profitable project is chosen, bank's BNE is L, and firm's BNE is T. So, we can verify that note like this flow.

10.

(a) $V_C = 6 + 6 \cdot \delta + \cdots = \frac{6}{1-\delta}$

$V_D = 4 + 4 \cdot \delta + \cdots = \frac{4}{1-\delta}$

if history is C, payoff matrix is like this,

1 \ 2	C		D	
C	$\underline{6+\delta \cdot V_C}$	$6+\delta \cdot V_C$	$\underline{2+\delta \cdot V_D}$	$8+\delta \cdot V_D$
D	$\underline{8+\delta \cdot V_D}$	$2+\delta \cdot V_D$	$\underline{4+\delta \cdot V_D}$	$4+\delta \cdot V_D$

~~if~~ to pass single deviaton test with grim trigger strategy,

$\boxed{6+\delta \cdot V_C \geq 8+\delta \cdot V_D}$ ~~and $2+\delta \cdot V_D$~~

$6+\delta \cdot \frac{6}{1-\delta} \geq 8+\delta \cdot \frac{4}{1-\delta}$

$\frac{2\delta}{1-\delta} \geq 2$

$2 \cdot \delta \geq 2 - 2\delta$

$4\delta \geq 2$

$\delta \geq \frac{1}{2}$

$\boxed{\therefore \frac{1}{2} \leq \delta < 1}$

we need this condition

(b) history, D payoff is like this,

1 \ 2	C	D
C	$6+\delta \cdot V_D, 6+\delta \cdot V_D$	$2+\delta V_D$, $8+\delta V_D$
D	$8+\delta V_D, 2+\delta \cdot V_D$	$4+\delta V_D$, $4+\delta V_D$

in this case, already pick (D,D) without any condition in δ.

So, this could be pass single deviation test.

(c) So, Finally this grim & trigger strategy ~~is~~

could be stay in (C,C) when $\frac{1}{2} \leq \delta < 1$.

And this could be pass single deviation test because ~~they~~

both players do not have incentive to chose D

because C is more beneficial.

So, these strategies could pass single deviation test.