Game Theory (2020-1)

Homework 1

Name:

ID(학번):

Part 1. Answer **True** or **False** at following sentences. (25 points, 5 points/question)

1. A contract is said to be automatically enforced if the players have individual incentives to abide by the terms of the contract. ( )
2. If a strategy profile s^* is a Nash equilibrium, then s^* is rationalizable for every player i. ( )
3. Incomplete information in strategic settings means that some player is uncertain about another player's preference. ( )
4. A set of mixed strategies includes a set of pure strategies. ( )
5. A strategy s_i is a best response to some belief if and only if s_i is not dominated. ( )

Part 2. Solve the following problems. (75 points)

6. Compute the set of rationalizable strategies in the following game. (10 points)

1\2	*a*	*B*	*c*	*d*
w	0, 1	0, 3	3, 1	0, 0
x	3, 2	0, 0	2, 1	1, 4
y	2, 2	3, 4	1, 1	9, 2
z	0, 3	5, 5	1, 8	0, 2

7. Consider a following payoff matrix of matching pennies.

1\2	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

(a) Show the above game is strictly competitive. (4 points)

(b) Suppose that there is a mixed-strategy Nash equilibrium, $\sigma^* = (p, q)$, where the probability of Player 1's strategy H is denoted by p, and the probability of Player 2's strategy H is denoted by q. Find each player's best response function expressed by p and q. (6 points)

(c) Find a mixed-strategy Nash equilibrium. (5 points)

(d) Show that the mixed-strategy Nash equilibrium is also a maxmin strategy. (10 points)

8. Consider the duopoly with linear demand function $P = 4 - Q$, where P is the price and $Q = q_1 + q_2$ is the total supply. Firm 1 and 2 simultaneously produce q_1 and q_2, and they sell at price P. Both Firm 1 and 2 have an identical marginal cost, 2. The two Firms are rational and all of above is common knowledge.

(a) Find a Nash equilibrium in this game. (4 points)

(b) Consider an extension to the n-firm case with demand function $P = 4 - Q$, where P is the price and $Q = \sum_{i=1}^{n} q_i$ is the total supply. Every Firm i has an identical marginal cost, 2. Find a Nash equilibrium in the extension of the previous game. (8 points)

(c) Consider the result of (b) where the Nash equilibrium is q^* and $Q^* = \sum_{i=1}^{n} q_i^*$. When the number of firms (n) goes to infinity, compute the total supply Q^*. Discuss your finding briefly, considering an implication of a perfectly competitive market. (6 points)

9. Assume that two players are rational.

(a) Find a pure strategy Nash equilibrium in the following game. (4 points)

1\2	*L*	*M*	*R*
S	10, 9	8, 7	8, 5
T	9, 7	9, 6	7, 5
U	8, 8	7, 8	9, 6

(b) Find a Nash equilibrium in the following game. (8 points)

1\2	*L*	*M*	*R*
S	1,0	0,1	5,0
T	0,2	2,1	1,0

10. Briefly describe what John Nash did in the game theory. (10 points)