Knowledge Representation Techniques

Resolution Method

- Resolution method is an inference rule which is used in Propositional Logic for proving the satisfiability of a sentence.
- In resolution method, we use Proof by Refutation/contradiction technique to prove the given statement.
- The process followed for resolution method in propositional logic is:
  - Convert the given proposition into normal form.
    - Remove bi-conditions and implications using logical equivalence.
    - $P \rightarrow Q \equiv (\neg P \vee Q)$ and $P \Leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$
    - Negation(¬) appears only in literals, therefore we move it inwards
  - Apply and proof the given goal using negation rule.

Normal forms

Sentences in the propositional logic can be transformed into one of the normal forms. This can simplify the inferences.

Normal forms used:

Conjunctive normal form (CNF) Product of Sums (AND of ORs)

- conjunction of clauses (clauses include disjunctions of literals)

$$(A \vee B) \wedge (\neg A \vee \neg C \vee D)$$

Disjunctive normal form (DNF) Sum of Products (OR of ANDs)

- Disjunction of terms (terms include conjunction of literals)

$$(A \wedge \neg B) \vee (\neg A \wedge C) \vee (C \wedge \neg D)$$

Conversion to a CNF

Assume: $\neg(A \Rightarrow B) \vee (C \Rightarrow A)$

1. Eliminate $\Rightarrow, \Leftrightarrow$

$$\neg(\neg A \vee B) \vee (\neg C \vee A)$$

2. Reduce the scope of signs through DeMorgan Laws and double negation

$$(A \wedge \neg B) \vee (\neg C \vee A)$$

3. Convert to CNF using the associative and distributive laws

$$(A \vee \neg C \vee A) \wedge (\neg B \vee \neg C \vee A)$$

and

$$(A \vee \neg C) \wedge (\neg B \vee \neg C \vee A)$$

Inference problem and satisfiability

Inference problem:

- we want to show that the sentence α is entailed by KB

Satisfiability:

- The sentence is satisfiable if there is some assignment (interpretation) under which the sentence evaluates to true

Connection:

$KB \models \alpha$ if and only if
$(KB \wedge \neg \alpha)$ is **unsatisfiable**

- **proof by contradiction**
  - **Disproving:** $KB, \neg \alpha$
  - **Proves the entailment** $KB \models \alpha$

Resolution algorithm

Algorithm:

- **Convert KB to the CNF form;**
- **Apply iteratively the resolution rule** starting from $KB, \neg \alpha$ (in CNF form)
- **Stop when**:
  - Contradiction (empty clause) is reached:
    - $A, \neg A \rightarrow \emptyset$
    - proves entailment.
  - No more new sentences can be derived
    - disproves it.

Example. Resolution.

KB: $(P \wedge Q) \wedge (P \Rightarrow R) \wedge [(Q \wedge R) \Rightarrow S]$ **Theorem:** S

Step 1. convert KB to CNF:

- $P \wedge Q \longrightarrow P \wedge Q$
- $P \Rightarrow R \longrightarrow (\neg P \vee R)$
- $(Q \wedge R) \Rightarrow S \longrightarrow (\neg Q \vee \neg R \vee S)$

KB: $P \quad Q \quad (\neg P \vee R) \quad (\neg Q \vee \neg R \vee S)$

Step 2. Negate the theorem to prove it via refutation

$S \longrightarrow \neg S$

Step 3. Run resolution on the set of clauses

$$P \quad Q \quad (\neg P \vee R) \quad (\neg Q \vee \neg R \vee S) \quad \neg S$$

Example. Resolution.

KB: $(P \wedge Q) \wedge (P \Rightarrow R) \wedge [(Q \wedge R) \Rightarrow S]$ **Theorem:** S

$$P \quad Q \quad (\neg P \vee R) \quad (\neg Q \vee \neg R \vee S) \quad \neg S$$

$$R$$

Example. Resolution.

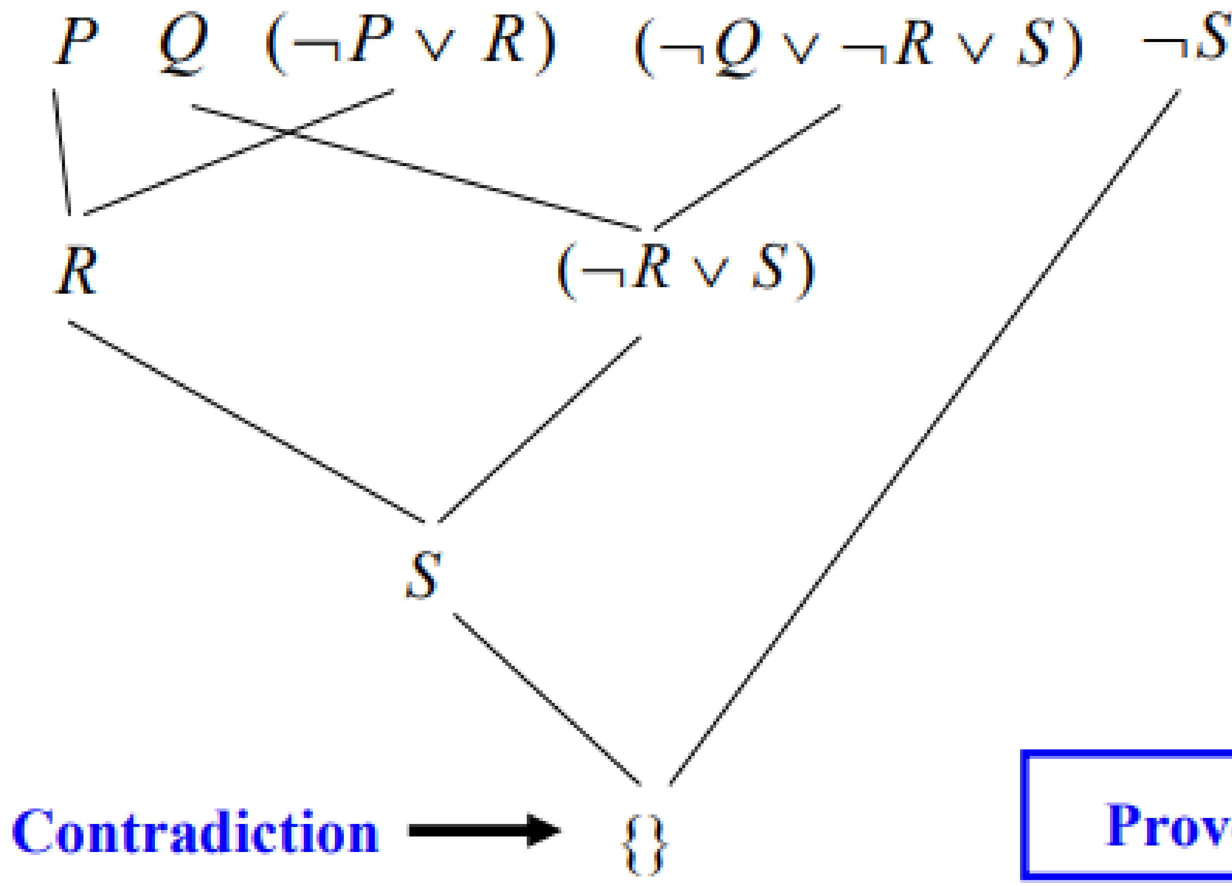
KB: $(P \wedge Q) \wedge (P \Rightarrow R) \wedge [(Q \wedge R) \Rightarrow S]$ **Theorem:** S

$$P \quad Q \quad (\neg P \vee R) \quad (\neg Q \vee \neg R \vee S) \quad \neg S$$

$$R \qquad\qquad (\neg R \vee S)$$

Example. Resolution.

KB: $(P \wedge Q) \wedge (P \Rightarrow R) \wedge [(Q \wedge R) \Rightarrow S]$ **Theorem:** S



Proved: $\boxed{S}$

Example: Resolution

Consider the following Knowledge Base:

1. The humidity is high or the sky is cloudy.
2. If the sky is cloudy, then it will rain.
3. If the humidity is high, then it is hot.
4. It is not hot.

Goal: It will rain.

Use propositional logic and apply resolution method to prove that the goal is derivable from the given knowledge base.

Example: Resolution

Solution: Construct propositions of the given sentences:

P: Humidity is high.

Q: Sky is cloudy.

R: It will rain.

S: It is hot.

KB:

1. The humidity is high or the sky is cloudy.	$P \vee Q$
2. If the sky is cloudy, then it will rain.	$Q \rightarrow R$
3. If the humidity is high, then it is hot.	$P \rightarrow S$
4. It is not hot.	$\neg S$

Example: Resolution

KB:

1. The humidity is high or the sky is cloudy. $P \vee Q$
2. If the sky is cloudy, then it will rain. $Q \rightarrow R$
3. If the humidity is high, then it is hot. $P \rightarrow S$
4. It is not hot. $\neg S$

Applying resolution method:

In (2), $Q \rightarrow R$ will be converted as $(\neg Q \vee R)$

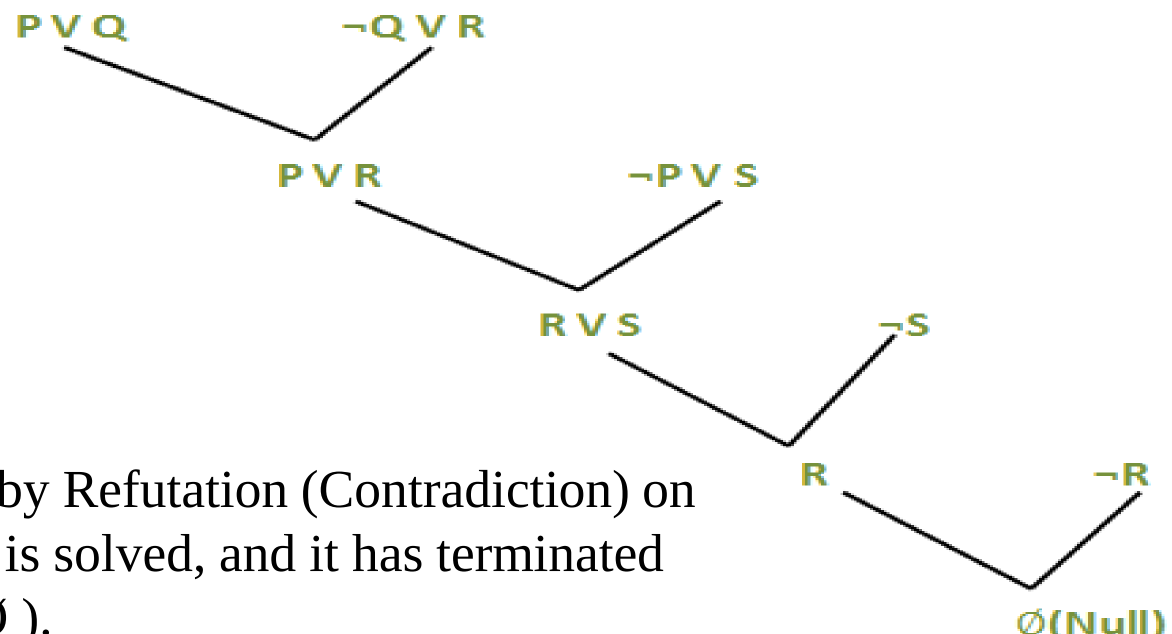
In (3), $P \rightarrow S$ will be converted as $(\neg P \vee S)$

Negation of Goal ($\neg$R): It will not rain.

Example: Resolution

KB:

1. The humidity is high or the sky is cloudy. P ∨ Q
2. If the sky is cloudy, then it will rain. (¬Q V R)
3. If the humidity is high, then it is hot. (¬P V S)
4. It is not hot. ¬ S



After applying Proof by Refutation (Contradiction) on the goal, the problem is solved, and it has terminated with a Null clause (Ø).
Hence, the goal is achieved. Thus, It is not raining.

Predicate Logic

Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

New Additions in Proposition (First Order Logic)

Variables, Constants, Predicate Symbols and

New Connectors: ∃ (there exists), ∀(for all)

- **Constant symbols,** which represent individuals in the world
  - Mary
  - 3
  - Green
- **Function symbols,** which map individuals to individuals
  - father-of(Mary) = John
  - color-of(Sky) = Blue
- **Predicate symbols,** which map individuals to truth values
  - greater(5,3)
  - green(Grass)
  - color(Grass, Green)

Formulating Predicate Logic Statements

New Additions in Proposition (First Order Logic)
Variables, Constants, Predicate Symbols and
New Connectors: ∃ (there exists), ∀(for all)

Wherever Mary goes, so does the Lamb. Mary goes to School. So the Lamb goes to School.

Predicate: goes(x,y) to represent x goes to y

New Connectors: ∃ (there exists), ∀(for all)

F1: ∀x(goes(Mary, x) → goes(Lamb, x))

F2: goes(Mary, School)

G: goes(Lamb, School)

To prove: (F1 ∧ F2) → G) is always true

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

<u>Predicates</u>: contractor(x), dependable(x), engineer(x)

F1: ∀x(contractor(x) → ~dependable(x))

[Alternative: ~∃x (contractor(x) ∧ dependable(x))]

F2: ∃x(engineer(x) ∧ contractor(x))

G: ∃x(engineer(x) ∧ ~dependable(x))

To prove: (F1 ∧ F2) → G) is always true

Formulating Predicate Logic Statements

Every gardener likes the sun.

$\forall x\ gardener(x) \rightarrow likes(x,Sun)$

You can fool some of the people all of the time.

$\exists x\ \forall t\ \ person(x) \wedge time(t) \rightarrow can\text{-}fool(x,t)$

You can fool all of the people some of the time.

$\forall x\ \exists t\ (person(x) \rightarrow time(t) \wedge can\text{-}fool(x,t))$

$\forall x\ (person(x) \rightarrow \exists t\ (time(t) \wedge can\text{-}fool(x,t)))$

Equivalent

All purple mushrooms are poisonous.

$\forall x\ (mushroom(x) \wedge purple(x)) \rightarrow poisonous(x)$

No purple mushroom is poisonous.

$\neg\exists x\ purple(x) \wedge mushroom(x) \wedge poisonous(x)$

$\forall x\ \ (mushroom(x) \wedge purple(x)) \rightarrow \neg poisonous(x)$

Equivalent

There are exactly two purple mushrooms.

$\exists x\ \exists y\ mushroom(x) \wedge purple(x) \wedge mushroom(y) \wedge purple(y)\ \hat{}\ \neg(x=y) \wedge \forall z\ (mushroom(z) \wedge purple(z)) \rightarrow ((x=z) \vee (y=z))$

Clinton is not tall.

$\neg tall(Clinton)$

X is above Y iff X is on directly on top of Y or there is a pile of one or more other objects directly on top of one another starting with X and ending with Y.

$\forall x\ \forall y\ above(x,y) \leftrightarrow (on(x,y) \vee\ \exists z\ (on(x,z) \wedge above(z,y)))$

Assignment 2

Q5. Suppose we know that:

“if Arjun is thin, then Mohit is not bearded, or Julia is not tall” and “if Julia is tall then Devika is graceful”

and “if Devika is graceful and Mohit is bearded then Arjun is thin” and “Mohit is bearded”.

Can we deduce that “Julia is not tall” ?

Q6. All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.