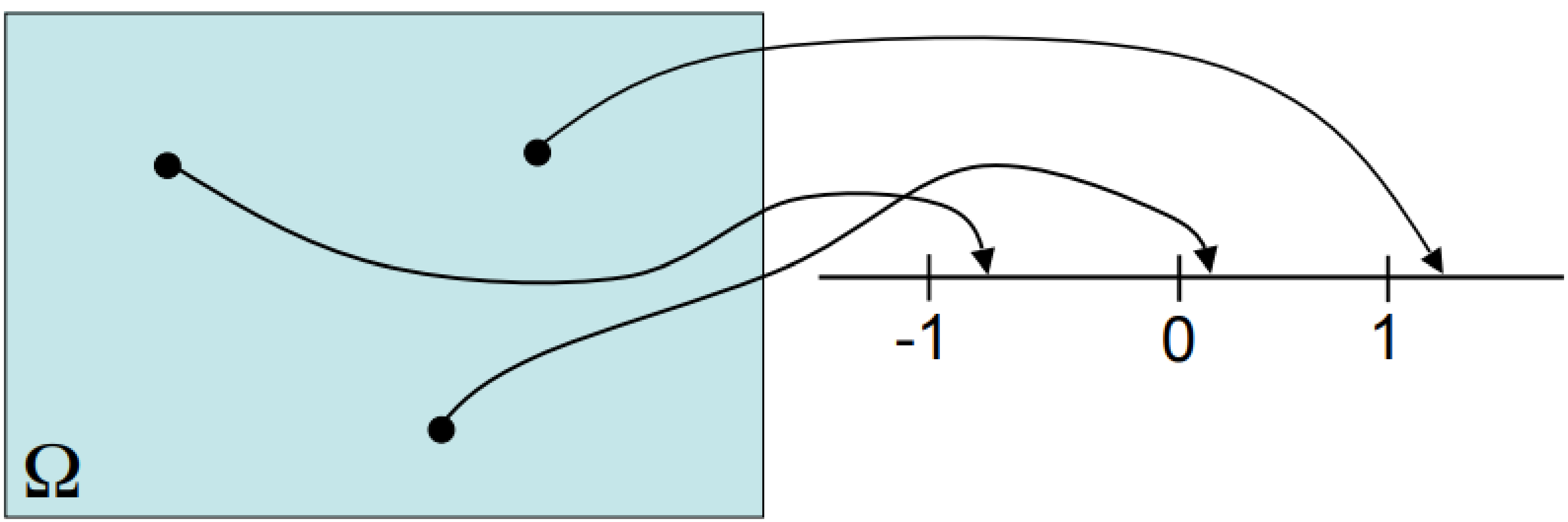
Random Variable

Random Variable

- A random variable is a numerical quantity that is generated by a random experiment.
- A RV is any rule (i.e., function) that associates a number with each outcome in the sample space.



Random Variable

- A RV is any rule (i.e., function) that associates a number with each outcome in the sample space.

Example 1 : Machine Breakdowns

- Sample space : $S = \{electrical, mechanical, misuse\}$
- Each of these failures may be associated with a repair cost
- State space : $\{50, 200, 350\}$
- Cost is a random variable : 50, 200, and 350

Random Variable

- We will denote random variables by capital letters, such as X or Z, and the actual values that they can take by lowercase letters, such as x and z.

Experiment	**Number X**	**Possible Values of X**
Roll two fair dice	Sum of the number of dots on the top faces	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
Flip a fair coin repeatedly	Number of tosses until the coin lands heads	1, 2, 3,4, …
Measure the voltage at an electrical outlet	Voltage measured	$118 \le x \le 122$
Operate a light bulb until it burns out	Time until the bulb burns out	$0 \le x < \infty$

Random Variable

- A random variable is called **discrete** if its possible values form a finite or countable set.
- A random variable is called **continuous** if its possible values contain a whole interval of numbers.
- For instance, a random variable representing the number of automobiles sold at a particular dealership on one day would be discrete, while a random variable representing the weight of a person in kilograms (or pounds) would be continuous.

Probability Distribution

- The probability distribution for a random variable describes how the probabilities are distributed over the values of the random variable.
- The probabilities in the probability distribution of a random variable X must satisfy the following two conditions:
  - Each probability P(x) must be between 0 to 1:

    $$\mathbf{0 \leq P(x) \leq 1}.$$

  - The sum of all the possible probabilities is 1:

    $$\mathbf{\sum P(x) = 1}.$$

Probability Mass Function

- For a **discrete random variable**, *x*, the probability distribution is defined by a **probability mass function**, denoted by *f(x)*.
- This function provides the probability for each value of the random variable.

Probability Mass Function (p.m.f.)

- A set of probability value p_i assigned to each of the values taken by the discrete random variable x_i
- $0 \le p_i \le 1$ and $\sum_i p_i = 1$
- Probability : $P(X = x_i) = p_i$

Probability Mass Function

Q1. A fair coin is tossed twice. Let X be the number of heads that are observed.

- Construct the probability distribution of X.
- Find the probability that at least one head is observed.

Solution:

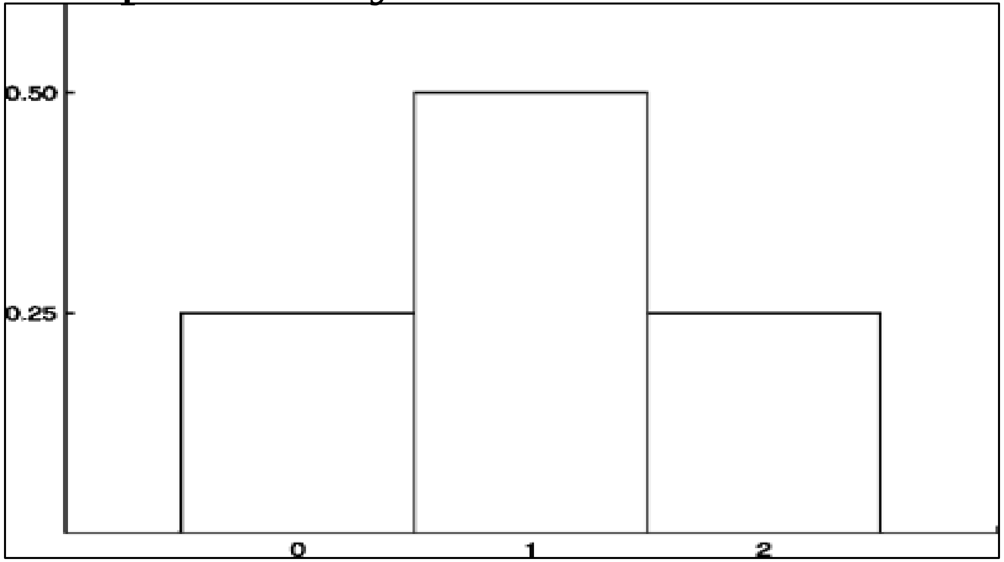
- The possible values that X can take are 0, 1, and 2.
- Each of these numbers corresponds to an event in the sample space S={hh,ht,th,tt} of equally likely outcomes for this experiment:
  - $X=0$ to{tt}, $X=1$ to{ht,th}, and $X=2$ to{hh}.

Probability Mass Function

- S={hh,ht,th,tt}   RV: X=0 to{tt}, X=1 to{ht,th}, and X=2 to{hh}
- The probability of each of these events, hence of the corresponding value of X, is given by

x	0	1	2
P(x)	0.25	0.50	0.25

- This table is the probability distribution of X.



Probability Mass Function

- S={hh,ht,th,tt}   RV: X=0 to{tt}, X=1 to{ht,th}, and X=2 to{hh}
- "At least one head" is the event $X \geq 1$, which is the union of the mutually exclusive events $X=1$ and $X=2$.
- Thus, $P(X \geq 1) = P(1)+P(2)$

  $=0.50+0.25 =0.75$

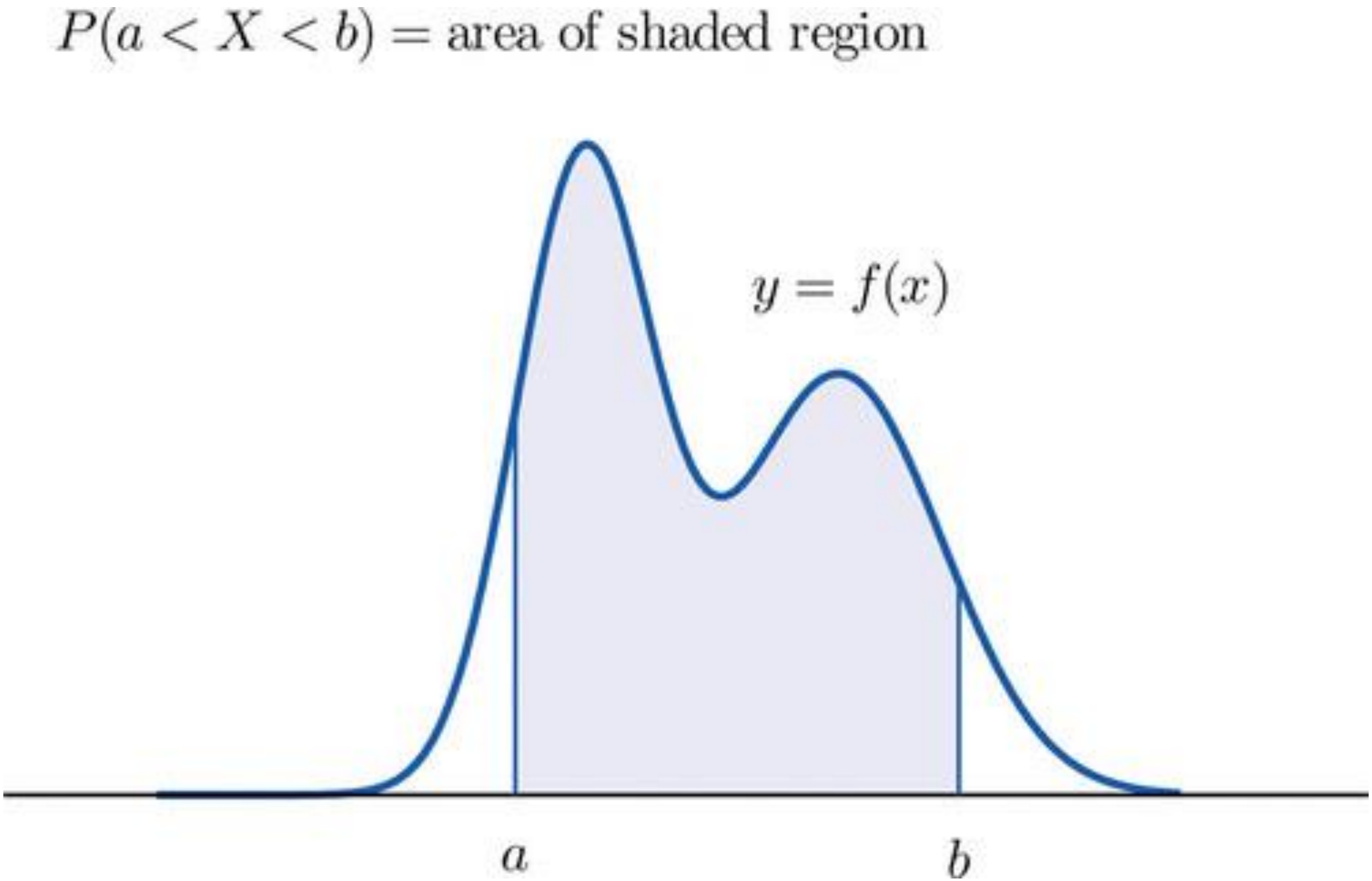
Q2: A pair of fair dice is rolled. Let X denote the sum of the number of dots on the top faces.

  – Construct probability distribution of X for a pair of fair dice.
  – Find $P(X \geq 9)$ Ans: 10/36
  – Find the probability that X takes an even value.   Ans: 0.5

Probability Density Function

- The probability distribution of a continuous random variable X is an assignment of probabilities to intervals of decimal numbers using a function f(x), called a **Probability Density Function**.
- The probability that X assumes a value in interval [a,b] is equal to the area of the region that is bounded above by the graph of the equation y=f(x), bounded below by the x-axis, and bounded on the left and right by the vertical lines through a and b.
- Density Function f(x) must satisfy the following two conditions:
  - For all numbers x, $f(x) \geq 0$, so that the graph of y=f(x) never drops below the x-axis.
  - The area of the region under the graph of y=f(x) and above the x-axis is 1.

Probability Density Function



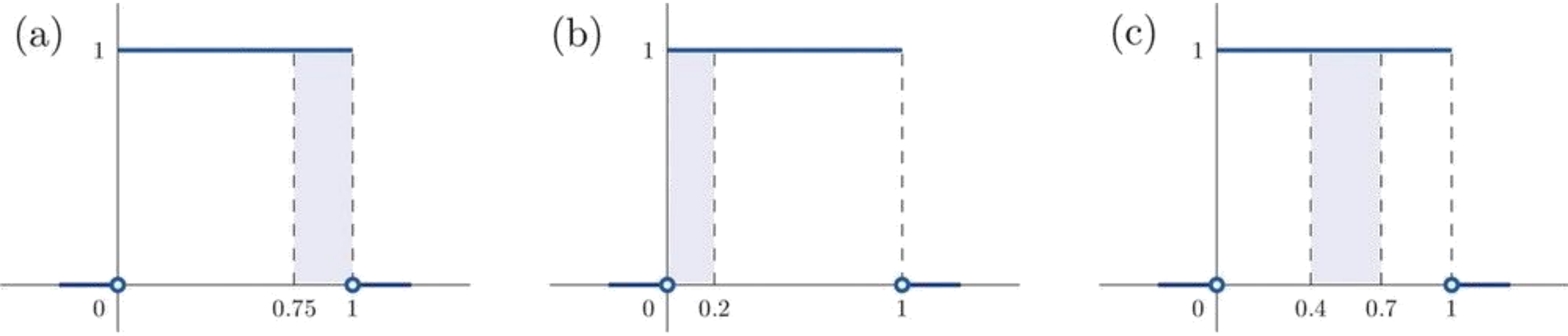
Probability Density Function

Q3. A random variable X has the uniform distribution on the interval $[0,1]$: the density function is $f(x)=1$ if x is between 0 and 1 and $f(x)=0$ for all other values of x (a uniform distribution).

- Find $P(X>0.75)$, the probability that X assumes a value greater than 0.75
- Find $P(X\leq 0.2)$, the probability that X assumes a value less than or equal to 0.2
- Find $P(0.4<X<0.7)$, the probability that X assumes a value between 0.4 and 0.7

Probability Density Function

- $P(X>0.75)$ is the area of the rectangle of height 1 and base length $1-0.75=0.25$, hence is base×height=$(0.25)\cdot(1) = 0.25$
- $P(X\leq 0.2)$ is the area of the rectangle of height 1 and base length $0.2-0=0.2$, hence is base×height=$(0.2)\cdot(1)=0.2$
- $P(0.4<X<0.7)$ is the area of the rectangle of height 1 and length $0.7-0.4=0.3$, hence is base×height=$(0.3)\cdot(1)=0.3$



Probability Density Function

Q4. A man arrives at a bus stop at a random time (that is, with no regard for the scheduled service) to catch the next bus. Buses run every 30 minutes without fail, hence the next bus will come any time during the next 30 minutes with evenly distributed probability (a uniform distribution). Find the probability that a bus will come within the next 10 minutes.