Probability Theory

Conditional Probability

- If A and B are two dependent events then the probability of occurrence of A given that B has already occurred (is denoted by P(A/B)) is given by

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}; P(A \cap B) = P(B) x P\left(\frac{A}{B}\right)$$

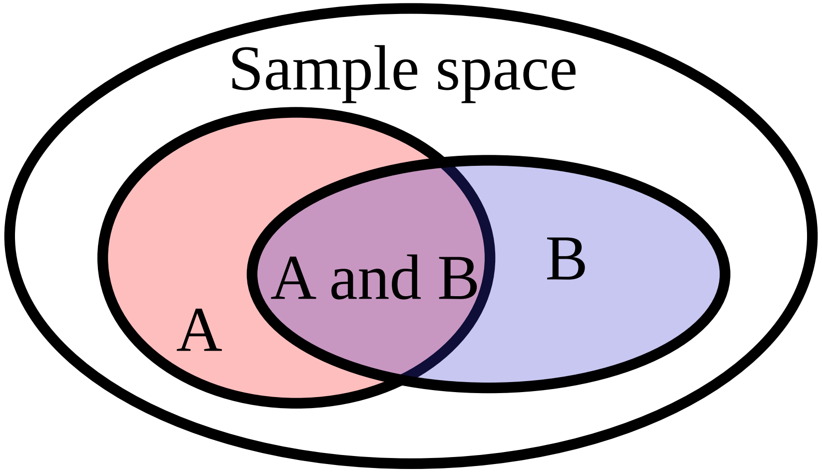
- Proof: $P(A \cap B) = \frac{P(A \cap B)}{P(S)} = \frac{P(A \cap B)}{P(A)} x \left(\frac{P(A)}{P(S)}\right)$

$$= \left(\frac{P(A)}{P(S)}\right) x \frac{P(A \cap B)}{P(A)} \qquad \because \frac{P(A \cap B)}{P(A)} = P\left(\frac{B}{A}\right)$$

$$= P(A) x P\left(\frac{B}{A}\right) \text{..................equation (i)}$$

Conditional Probability

- The probability of an event A based on the occurrence of another event B is termed conditional Probability. It is denoted as **P(A|B)** and represents the probability of A when event B has already happened.
- $P(A \mid B) = P(A \cap B)/ P(B)$



Joint Probability:

The probability of two more events occurring together and at the same time is measured it is termed as Joint Probability. Joint probability for two events A and B is denoted as, $P(A \cap B)$.

Conditional Probability

- **$P(A \mid B) = P(A \cap B) / P(B)$**
- If the two events are independent:
  - $P(A \cap B) = P(A) * P(B)$
  - $P(A/B) = P(A)$

Bayes Theorem

- Bayes theorem, also known as the Bayes Rule, is used to determine the conditional probability of event A when event B has already happened.
- “The conditional probability of an event A, given the occurrence of another event B, is equal to the product of the event of B, given A and the probability of A divided by the probability of event B.” i.e.

$$P(A|B) = P(B|A)P(A) / P(B) \quad \text{given} \quad P(B) \neq 0$$

- where,
  - P(A) and P(B) are the probabilities of events A and B
  - P(A|B) is the probability of event A when event B happens
  - P(B|A) is the probability of event B when event A happens

Bayes Theorem

$$P(A|B) = P(B|A)P(A) / P(B)$$

Proof:

By conditional probability we know

$$P(A \mid B)P(B) = P(A \text{ and } B)$$

$$P(B \mid A)P(A) = P(A \text{ and } B)$$

Thus, $P(A \mid B)P(B) = P(B \mid A)P(A)$

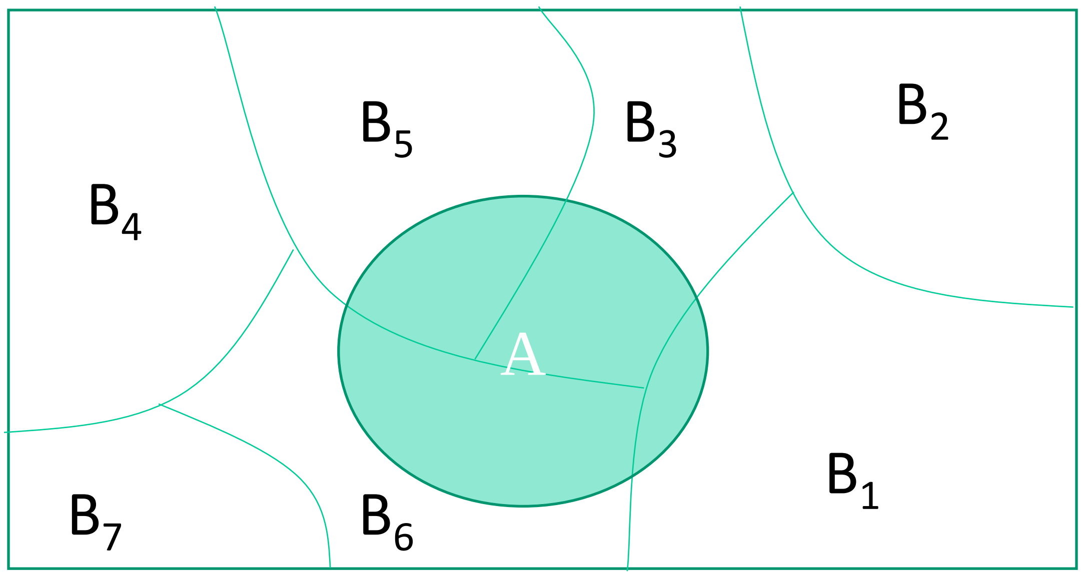
$$P(A \mid B) = P(B \mid A)P(A) / P(B)$$

Theorem of Total Probability

- Let E1, E2,…………..En are mutually exclusive and exhaustive events associated with a random experiment and lets E be an event that occurs with some Ei.
- Then,

$$P(E) = {}^{n}\sum_{i=1} P(E/E_i) \,.\, P(E_i)$$

Theorem of Total Probability



$$p(A) = \sum P(B_i) P(A \mid B_i)$$

Questions

Q1. Two dice are thrown. The events A, B, C, D, E, F

A = getting even number on first die.

B= getting an odd number on the first die.

C = getting a sum of the number on dice ≤ 5

D = getting a sum of the number on dice > 5 but less than 10.

Show that:

1. A, B are a mutually exclusive event and Exhaustive Event.
2. A, C are not mutually exclusive.
3. C, D are a mutually exclusive event but not Exhaustive Event.
4. $A' \cap B'$ are a mutually exclusive and exhaustive event.

Questions

Q2. A bag contains 5 green and 7 red balls. Two balls are drawn. Find the probability that one is green and the other is red.

Q3. Find the probability of drawing a heart on each of two consecutive draws from well shuffled-packs of cards if the card is not replaced after the draw.

Q4. “X+Y=6 or X+Y=7” – given this (and only this), what is the probability of Y=5?

Q5. There are three urns containing 3 white and 2 black balls; 2 white and 3 black balls; 1 black and 4 white balls respectively. There is an equal probability of each urn being chosen. One ball is equal probability chosen at random. what is the probability that a white ball is drawn?