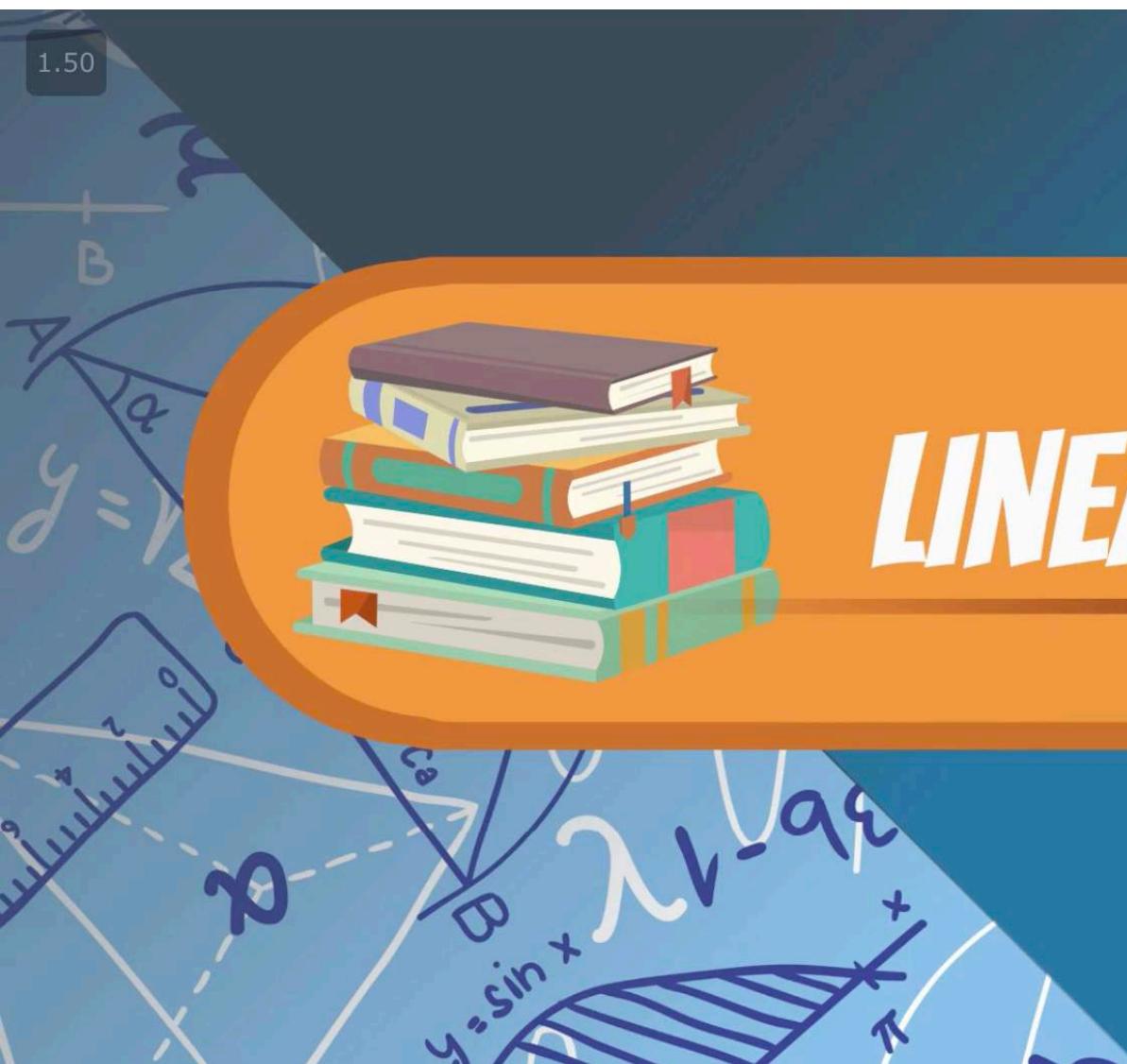


1.50



LINEAR ALGEBRA



A matrix is a collection of numbers
ordered in rows and columns.
Here is one.

element

$$\begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}$$

A is a 2-by-3 matrix

$$\underline{\mathbf{A}} = \begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}$$

2 rows

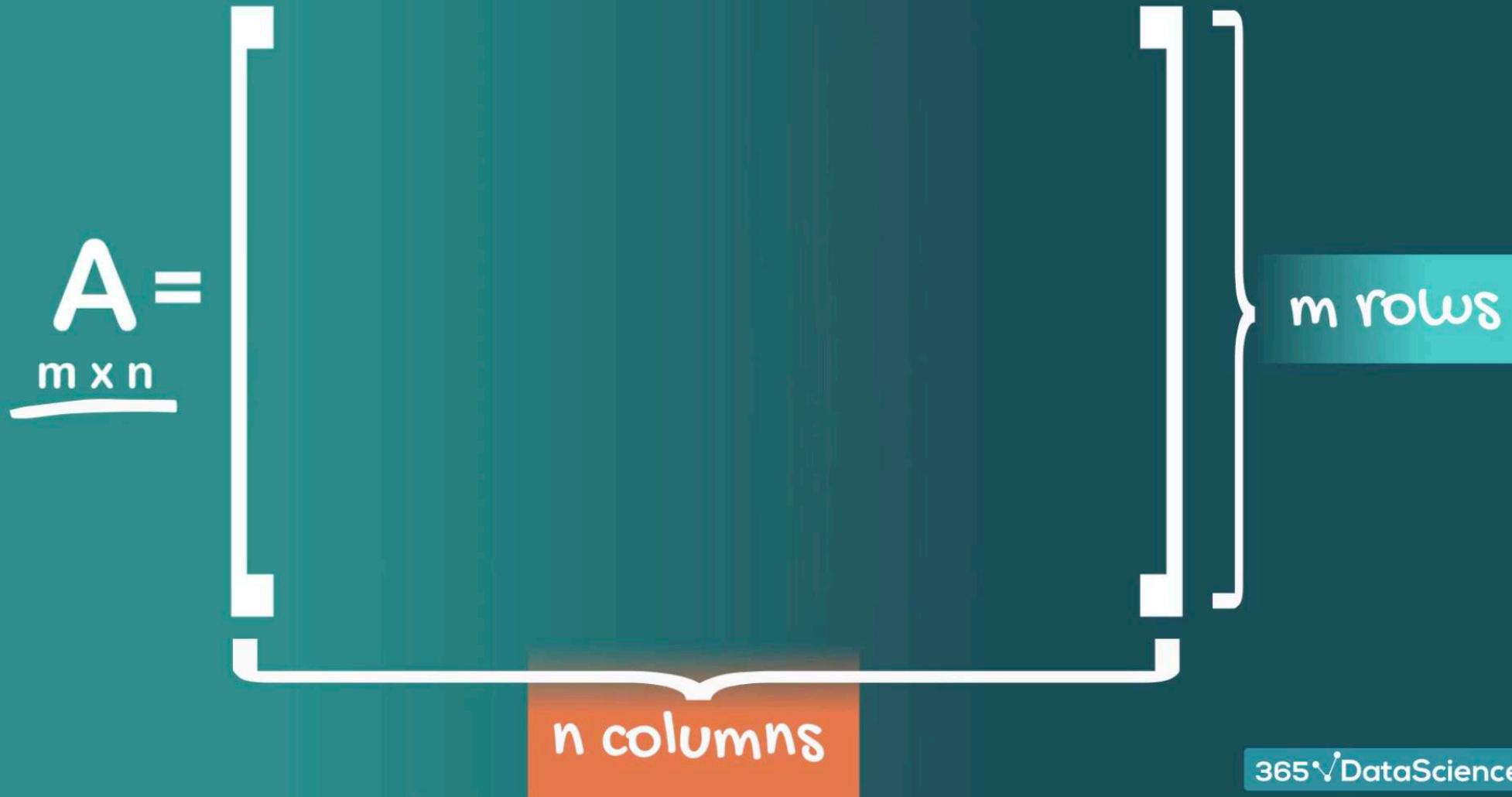
3 columns

A matrix can only contain numbers, symbols, or expressions

$$A = \begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}$$

$$B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$C = \begin{bmatrix} a-x & b+5 \\ d+e & e \end{bmatrix}$$



$A =$
 $m \times n$



a_{ij}
 j -th
column

i -th row

$$A = [\begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & a_{ij} & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{array}]$$

m x n

i-th row

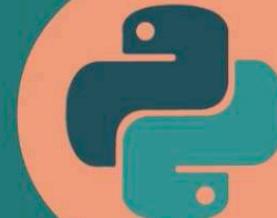
j-th column

MATHEMATICS**A =****m x n**

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & a_{ij} & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

PROGRAMMING

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & \dots & a_{0(n-1)} \\ a_{10} & a_{11} & a_{12} & \dots & a_{1(n-1)} \\ a_{20} & a_{21} & a_{22} & \dots & a_{2(n-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{(m-1)0} & a_{(m-1)1} & a_{(m-1)2} & \dots & a_{(m-1)(n-1)} \end{bmatrix}$$



MATRICES

$$A = m \times n \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & a_{ij} & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

rows

columns

2D

MATRICES = SCALARS

$$\left[\begin{matrix} a_{11} \end{matrix} \right]$$

1 column

1 row

SCALARS HAVE 0 DIMENSIONS

[15]; [1]; [2]; [-5]; []

All numbers we know from algebra are referred to as scalars in linear algebra

VECTORS

[15]

Scalar

$$\begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

Vector
 3×1 matrix

$$\begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}$$

Matrix

VECTORS

$$\mathbf{U} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

3 rows

1 column

VECTORS

A vector is practically the
simplest linear algebraic object

$$\begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}$$

vect. vect. vect.
1 2 3

TYPES OF VECTORS

column

$$\begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

row

$$[3 4 5 8]$$

LENGTH

Df: The number of elements in a vector

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

Length = m
 $m \times 1$

Length = m
 $1 \times m$

$$\left[x_1 \ x_2 \ \dots \ x_m \right]$$

SUMMARY

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & a_{ij} & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}_{m \times n}$$

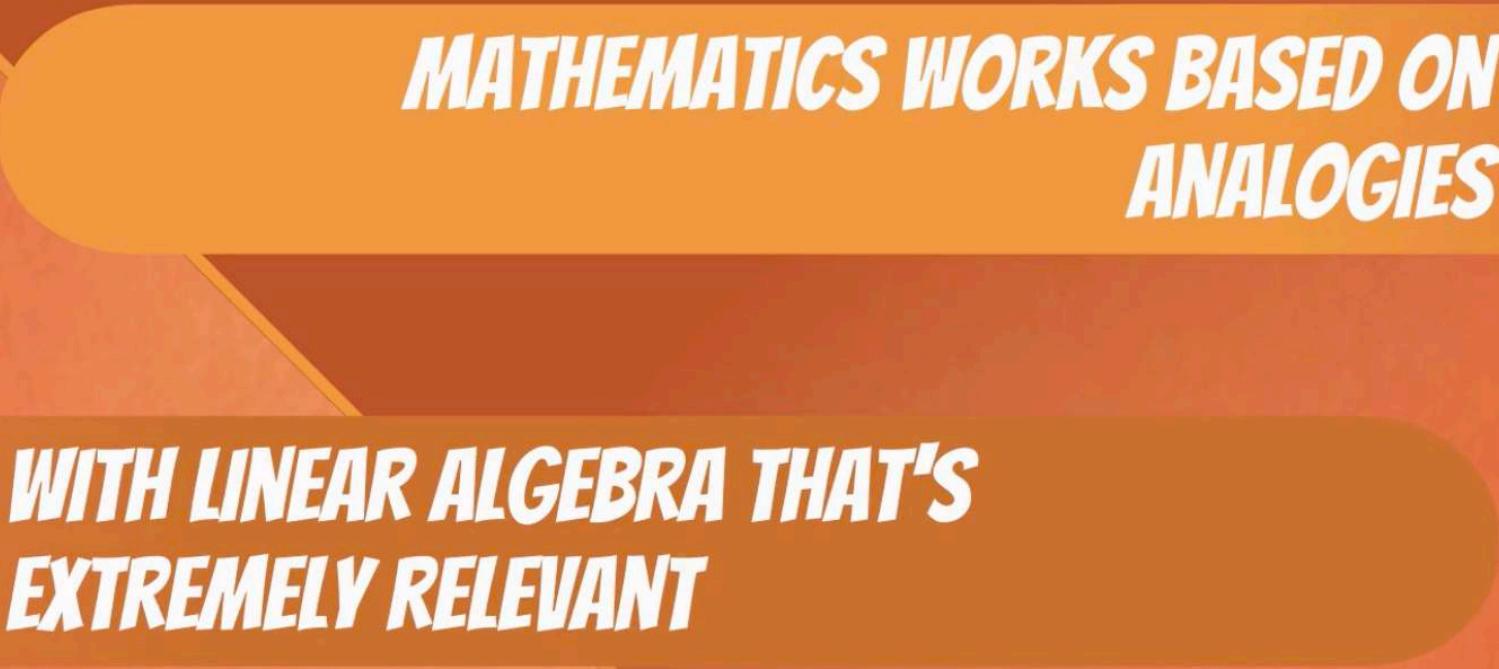
2D Matrix
 $m \times n$

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}_{m \times 1}$$

1D Vector
 $m \times 1$

$$\begin{bmatrix} x \end{bmatrix}_{1 \times 1}$$

0D Scalar
 1×1



**MATHEMATICS WORKS BASED ON
ANALOGIES**

**WITH LINEAR ALGEBRA THAT'S
EXTREMELY RELEVANT**

LINEAR ALGEBRA AND GEOMETRY

[X]

OD

Scalar
 1×1



OD

Point

LINEAR ALGEBRA AND GEOMETRY

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

1D

Vector
 $m \times 1$

1D

Line



LINEAR ALGEBRA AND GEOMETRY

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

2D

Vectors
 $m \times 1$ $m \times 1$

2D

Line
(Plane)



LINEAR ALGEBRA AND GEOMETRY

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_m & y_m \end{bmatrix}$$

2D

Matrix
 $m \times 2$

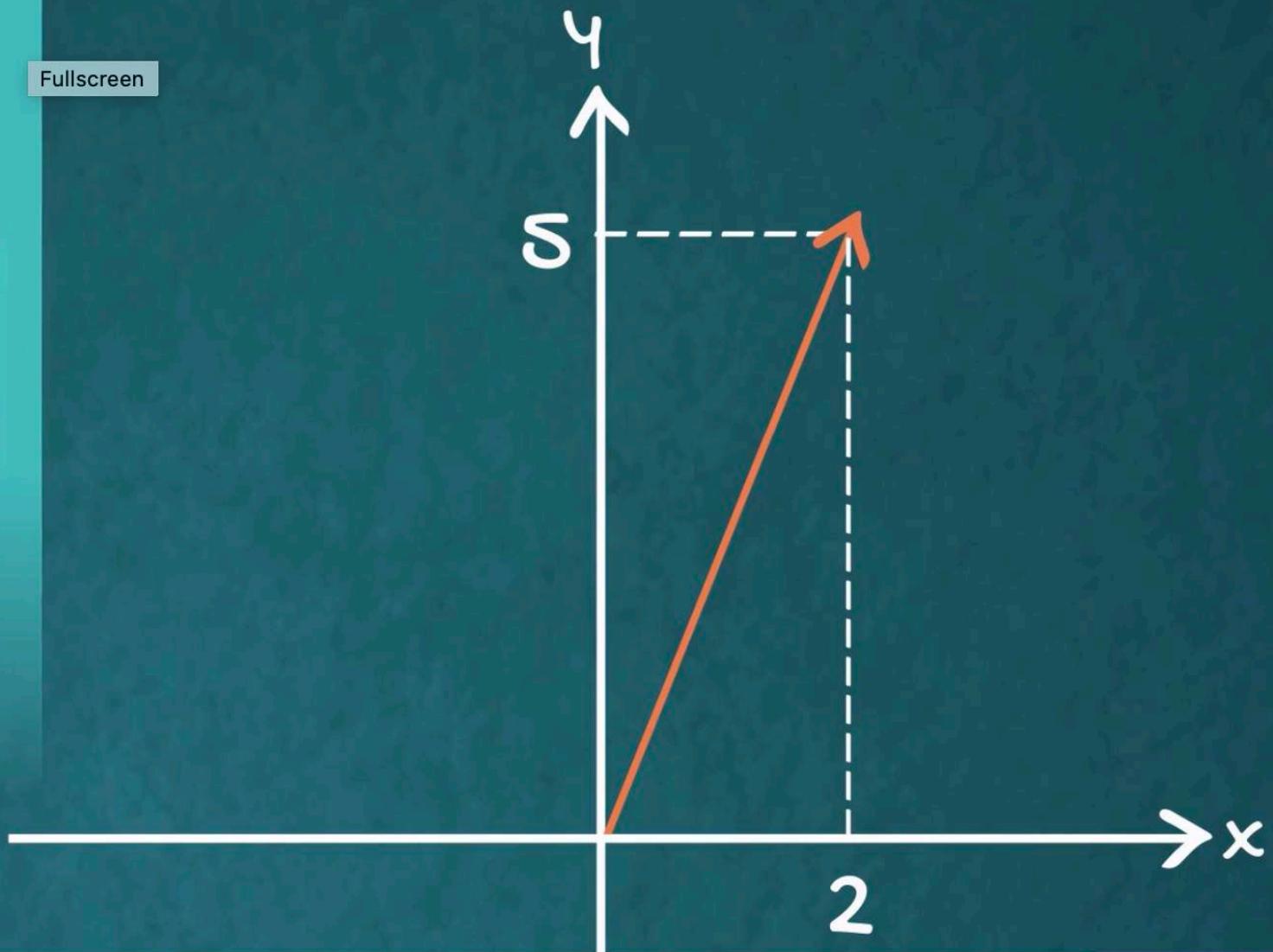
2D

Line
(Plane)



$\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

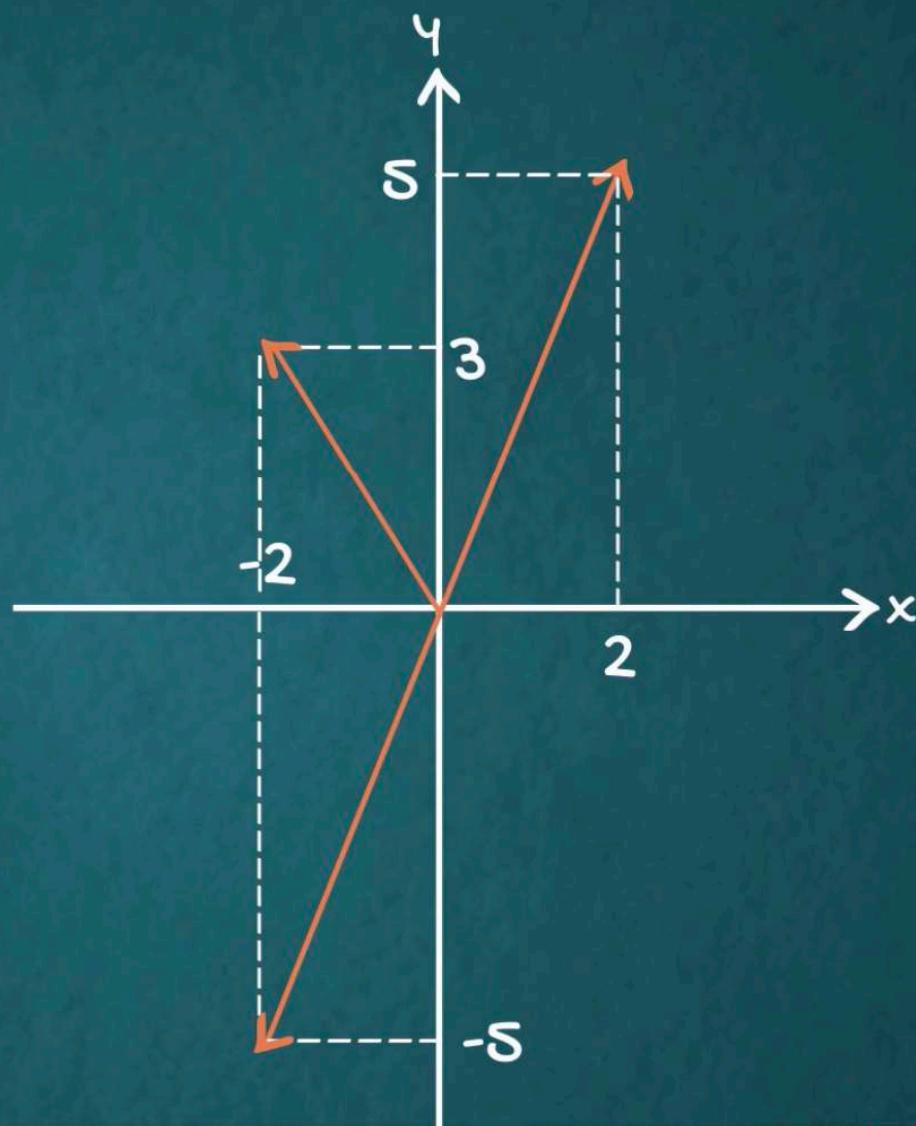
Fullscreen



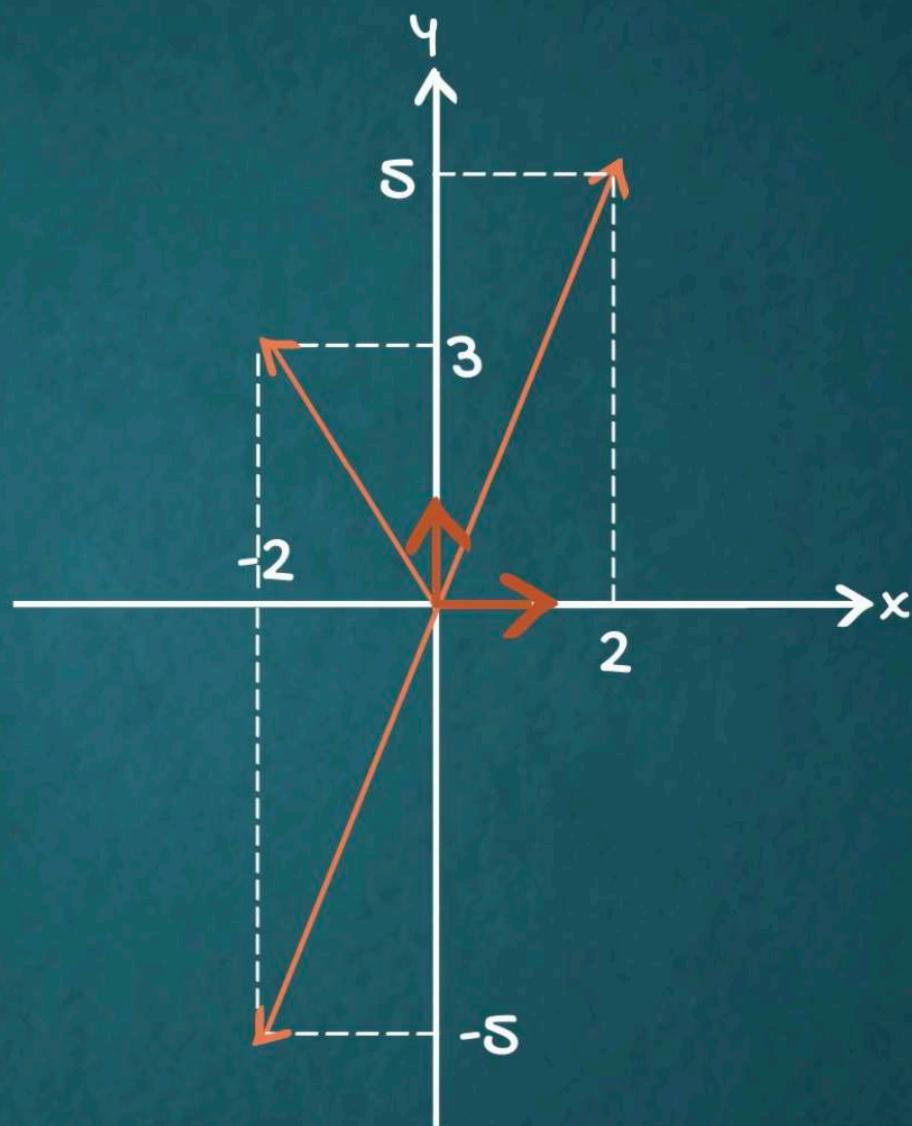
$$\begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 3 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

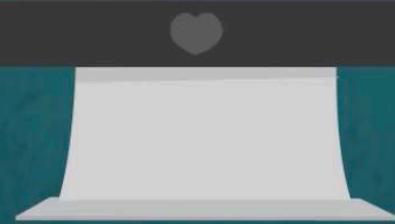


3D VECTOR PLOTTER

BY ACADEMO.ORG



In this way, you will get a better grasp of the geometrical concept



Import the relevant libraries

```
In [ ]: import numpy as np
```

The simplest most flexible way to work with matrices in Python is by using arrays

Declaring scalars, vectors, and matrices

Scalars

```
In [2]: s = 5
```

```
In [3]: s
```

```
Out[3]: 5
```

Vectors

By default, 'v' is a row vector

```
In [4]: v = np.array([5,-2,4])
```

```
In [5]: v
```

```
Out[5]: array([ 5, -2,  4])
```

$$\begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

Matrices

```
In [6]: m = np.array([[5,12,6],[-3,0,14]])
```

```
In [7]: m
```

```
Out[7]: array([[ 5, 12,  6],  
[-3,  0, 14]])
```

$$\begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}$$

Data types

```
In [8]: type(s)
```

```
Out[8]: int
```

```
In [9]: type(v)
```

```
Out[9]: numpy.ndarray
```

ndarray = n-dimensional array

```
In [10]: type(m)
```

```
Out[10]: numpy.ndarray
```

```
In [11]: s_array = np.array(5)  
s_array
```

```
Out[11]: array(5)
```

```
In [12]: type(s_array)
```

```
Out[12]: numpy.ndarray
```

Data shapes

```
In [13]: m.shape
```

shape returns the shape (dimensions) of a variable

```
Out[13]: (2, 3)
```

```
In [14]: v.shape
```

```
Out[14]: (3,)
```

In the memory of the computer, this object has 3 (ordered) elements

Creating a column vector

```
In [17]: v.reshape(1,3)
```

```
Out[17]: array([[ 5, -2,  4]])
```

reshape gives an array a new shape, without changing its data

```
In [18]: v.reshape(3,1)
```

```
Out[18]: array([[ 5],  
                 [-2],  
                 [ 4]])
```

```
In [15]: s.shape
```

```
-----  
AttributeError                                Traceback (most recent call last)  
<ipython-input-15-1f0fd037afc4> in <module>()  
----> 1 s.shape
```

```
AttributeError: 'int' object has no attribute 'shape'
```

```
In [16]: s_array.shape
```

0-dimensional objects don't have shape

```
Out[16]: ()
```

1.00



TENSORS



WHAT IS A TENSOR

[15]



A vector of lenght 1

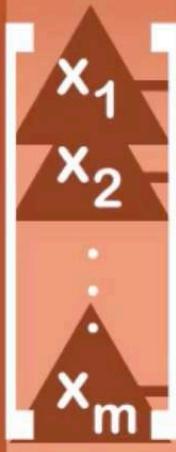


A 1×1 matrix

Scalar

WHAT IS A TENSOR

[15]



- ▲ each element is a scalar
- ▲ $m \times 1$ or $1 \times m$ matrix

Scalar Vector
 1×1

WHAT IS A TENSOR

[15]

$$\mathbf{A} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}_{m \times n}$$

$\mathbf{A} =$

a_{11}	a_{12}	a_{13}	\dots	a_{1n}
a_{21}	a_{22}	a_{23}	\dots	a_{2n}
a_{31}	a_{32}	a_{33}	\dots	a_{3n}
\vdots	\vdots	\vdots	\ddots	\vdots
a_{m1}	a_{m2}	a_{m3}	\dots	a_{mn}

➤ a collection of vectors

➤ a collection of scalars

Scalar
 1×1

Vector
 $m \times 1$

Matrix
 $m \times n$

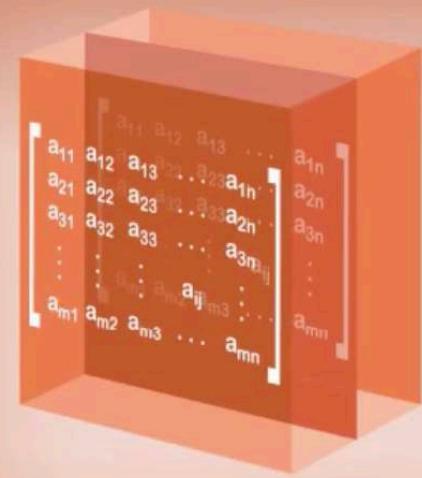
TENSORS ARE SIMPLY A GENERALIZATION OF THE CONCEPTS WE HAVE SEEN SO FAR

[15]

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & a_{ij} & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

\mathbf{x}_1
 \mathbf{x}_2
 \vdots
 \vdots
 \mathbf{x}_m

$m \times n$



RANK 0

Tensor
 1×1

RANK 1

Tensor
 $m \times 1$

RANK 2

Tensor
 $m \times n$

RANK 3

Tensor
 $k \times m \times n$

Import the relevant libraries

```
In [1]: import numpy as np
```

Creating a tensor

```
In [2]: m1 = np.array([[5,12,6],[-3,0,14]])
m1
```

```
Out[2]: array([[ 5, 12,  6],
[-3,  0, 14]])
```

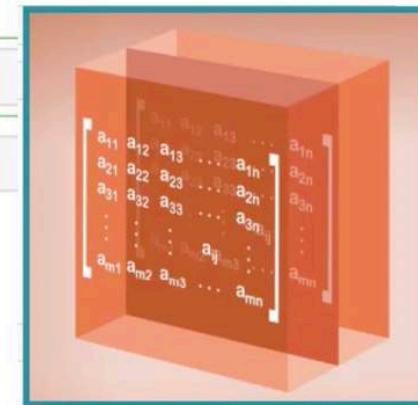
```
In [3]: m2 = np.array([[9,8,7],[1,3,-5]])
m2
```

```
Out[3]: array([[ 9,  8,  7],
[ 1,  3, -5]])
```

```
In [ ]: t = np.array([m1,m2])
```

```
In [5]: t
```

```
Out[5]: array([[[ 5, 12,  6],
[-3,  0, 14]],
[[ 9,  8,  7],
[ 1,  3, -5]]])
```



Checking its shape

```
In [6]: t.shape
```

```
Out[6]: (2, 2, 3)
```

Manually creating a tensor

```
In [7]: t_manual = np.array([[[ 5, 12,  6], [-3,  0, 14]], [[ 9,  8,  7], [ 1,  3, -5]]])
```

ADDITION AND SUBTRACTION



ADDITION

Very easy

1 condition:

The two matrices must
have the same
dimensions

$$\begin{matrix} \mathbf{M}_1 \\ 2 \times 3 \end{matrix} + \begin{matrix} \mathbf{M}_2 \\ 2 \times 3 \end{matrix} = \begin{matrix} \mathbf{X} \\ 2 \times 3 \end{matrix}$$

$$\begin{bmatrix}
 5 & 12 & 6 \\
 -3 & 0 & 14
 \end{bmatrix} +
 \begin{bmatrix}
 9 & 8 & 7 \\
 1 & 3 & -5
 \end{bmatrix} =
 \begin{bmatrix}
 5 + 9 & 12 + 8 & 6 + 7 \\
 -3 + 1 & 0 + 3 & 14 - 5
 \end{bmatrix} \\
 =
 \begin{bmatrix}
 14 & 20 & 13 \\
 -2 & 3 & 9
 \end{bmatrix}$$

result:

Import the relevant libraries

```
In [1]: import numpy as np
```

Addition

```
In [2]: m1 = np.array([[5,12,6],[-3,0,14]])  
m1
```

```
Out[2]: array([[ 5, 12,  6],  
               [-3,  0, 14]])
```

```
In [3]: m2 = np.array([[9,8,7],[1,3,-5]])  
m2
```

```
Out[3]: array([[ 9,  8,  7],  
               [ 1,  3, -5]])
```

```
In [4]: m1 + m2
```

```
Out[4]: array([[14, 20, 13],  
               [-2,  3,  9]])
```

$$\begin{matrix} \mathbf{M}_1 \\ 2 \times 3 \end{matrix} + \begin{matrix} \mathbf{M}_2 \\ 2 \times 3 \end{matrix} = \begin{matrix} \mathbf{X} \\ 2 \times 3 \end{matrix}$$

$$\begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix} + \begin{bmatrix} 9 & 8 & 7 \\ 1 & 3 & -5 \end{bmatrix} = \begin{bmatrix} 5+9 & 12+8 & 6+7 \\ -3+1 & 0+3 & 14-5 \end{bmatrix}$$

result:

$$= \begin{bmatrix} 14 & 20 & 13 \\ -2 & 3 & 9 \end{bmatrix}$$

```
[ -340.21,    1.06],  
[  30.41,  424.99]])
```

```
In [11]: m5 - m6
```

```
Out[11]: array([[-108.8 , -453.02],  
[ 137.07, 1198.96],  
[ -26.19, -190.89]]))
```

Adding vectors together

```
In [12]: v1 = np.array([1,2,3,4,5])  
v2 = np.array([5,4,3,2,1])
```

```
In [13]: v1 + v2
```

```
Out[13]: array([6, 6, 6, 6, 6])
```

```
In [14]: v1 - v2
```

```
Out[14]: array([-4, -2,  0,  2,  4])
```

SUBTRACTION OF VECTORS

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} + \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 6 \\ 6 \\ 6 \end{bmatrix}$$
$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} - \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 0 \\ 2 \\ 4 \end{bmatrix}$$

length=5 length=5 length=5

This result has a meaning in terms of arrays, but not in terms of linear algebra

Exceptions (addition with a scalar)

Adding scalars to matrices

```
In [11]: m1
```

```
Out[11]: array([[ 5, 12,  6],  
                 [-3,  0, 14]])
```

```
In [12]: m1 + 1
```

```
Out[12]: array([[ 6, 13,  7],  
                 [-2,  1, 15]])
```

```
In [13]: v1
```

```
Out[13]: array([1, 2, 3, 4, 5])
```

```
In [14]: v1 + 1
```

```
Out[14]: array([2, 3, 4, 5, 6])
```

```
In [15]: m1 + np.array([1])
```

```
Out[15]: array([[ 6, 13,  7],  
                 [-2,  1, 15]])
```

```
In [16]: v1 + np.array([1])
```

```
Out[16]: array([2, 3, 4, 5, 6])
```

TRANSPOSE OF A MATRIX

TRANSPOSING VECTORS

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}^\top = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

x x^\top

TRANSPOSING VECTORS

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

x^T

$(x^T)^T = x$

TRANSPOSING VECTORS

▲ The values are not changing or transforming;
only their position is

▲ Transposing the same vector (object) twice
yields the initial vector (object)

▲ A 3×1 matrix transposed is a 1×3 matrix

TRANSPOSING MATRICES

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}^T = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}_{n \times m}$$

TRANSPOSING MATRICES

$$\underline{\mathbf{A} = \begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}}$$

$$\underline{\mathbf{A}^T = \begin{bmatrix} 5 & -3 \\ 12 & 0 \\ 6 & 14 \end{bmatrix}}$$

TRANSPOSING MATRICES

$$A = \begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}_{2 \times 3}$$

$$A^T = \begin{bmatrix} 5 & -3 \\ 12 & 0 \\ 6 & 14 \end{bmatrix}_{3 \times 2}$$

TRANSPOSING MATRICES

$$\mathbf{A} = \begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}$$

2x3

$$\mathbf{A}^T = \begin{bmatrix} 5 & -3 \\ 12 & 0 \\ 6 & 14 \end{bmatrix}$$

3x2

$$\mathbf{A} = \begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}$$

2x3

$$\mathbf{A}^T = \begin{bmatrix} 5 & -3 \\ 12 & 0 \\ 6 & 14 \end{bmatrix}$$

3x2

Import the relevant libraries

```
In [1]: import numpy as np
```

Transposing matrices

```
In [6]: C = np.array([[4,-5],[8,12],[-2,-3],[19,0]])  
C
```

```
Out[6]: array([[ 4, -5],  
               [ 8, 12],  
               [-2, -3],  
               [19,  0]])
```

```
In [7]: C.T
```

```
Out[7]: array([[ 4,  8, -2, 19],  
               [-5, 12, -3,  0]])
```

array.T returns the transpose of an array (matrix)

Transposing vectors

```
In [10]: x = np.array([1,2,3])  
x
```

```
Out[10]: array([1, 2, 3])
```

```
In [11]: x.T
```

```
Out[11]: array([1, 2, 3])
```

```
In [12]: x.shape
```

```
Out[12]: (3,)
```

In Python, 1D arrays don't really get transposed (in the memory of the computer)

```
In [13]: x_reshaped = x.reshape(1,3)  
x_reshaped
```

```
Out[13]: array([[1, 2, 3]])
```

```
In [14]: x_reshaped.T
```

```
Out[14]: array([[1],  
                [2],  
                [3]])
```

DOT PRODUCT



SCALAR MULTIPLICATION

$$[6] \cdot [5] = [30]$$

$$[10] \cdot [-2] = [-20]$$

VECTOR MULTIPLICATION

$$\begin{bmatrix} 2 \\ 8 \\ -4 \end{bmatrix} \times \begin{bmatrix} 1 \\ -7 \\ 3 \end{bmatrix}$$

Condition: They must have the same length

DOT PRODUCT

$$\begin{bmatrix} 2 \\ 8 \\ -4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -7 \\ 3 \end{bmatrix} = [-66]$$

the 'sum of the products of the corresponding elements'

Vector

Vector

Scalar

Import relevant libraries

```
In [1]: import numpy as np
```

Dot product

np.dot() returns the dot product of two objects



```
In [2]: x = np.array([2,8,-4])  
y = np.array([1,-7,3])
```

```
In [3]: np.dot(x,y)
```

Out[3]: -66

```
In [4]: u = np.array([0,2,5,8])  
v = np.array([20,3,4,-1])
```

```
In [5]: np.dot(u,v)
```

$$0x(20) + (2x3) + (5x4) + 8x(-1)$$

Out[5]: 18

```
In [ ]:
```

```
In [ ]:
```

Scalar * Scalar

```
In [6]: np.dot(5,6)
```

```
Out[6]: 30
```

```
In [7]: np.dot(10,-2)
```

```
Out[7]: -20
```

Scalar * Vector

We get a vector with the same lenght!

```
In [8]: x
```

```
Out[8]: array([ 2,  8, -4])
```

```
In [9]: 5*x
```

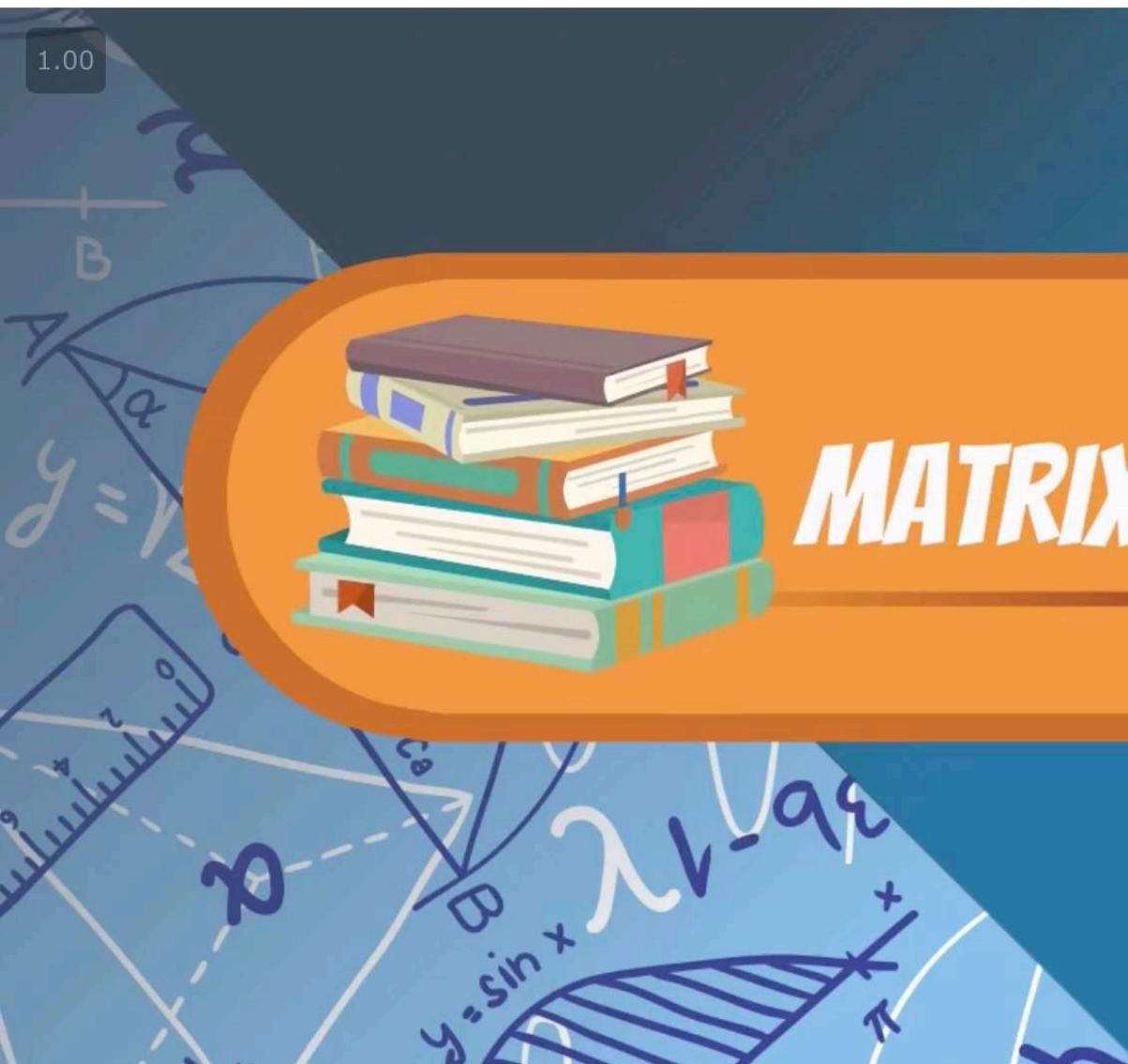
```
Out[9]: array([ 10,  40, -20])
```

The initial vector had been SCALED 5 times (thus the name)

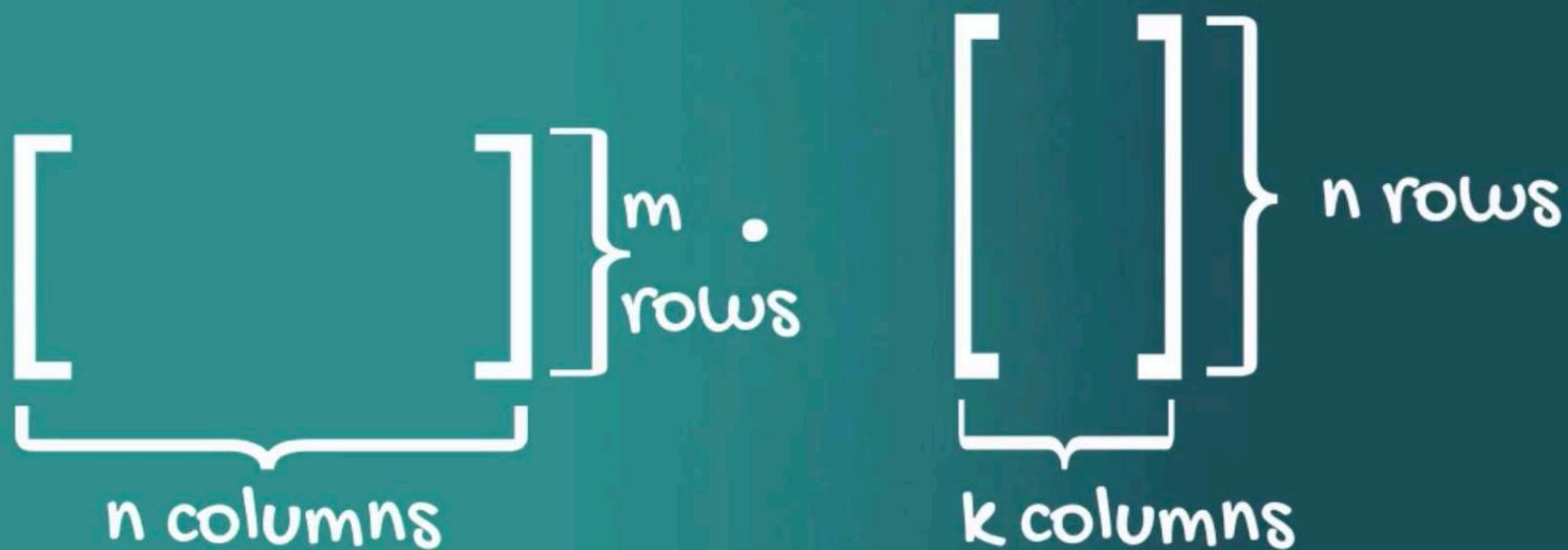
1.00



MATRIX MULTIPLICATION



DOT PRODUCT



Condition: We can only multiply an $m \times n$ with an $n \times k$ matrix

DOT PRODUCT

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

2×3

.

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \end{bmatrix}$$

$3 \times k$

Condition: We can only multiply an $m \times n$ with an $n \times k$ matrix

DOT PRODUCT

$$[\underset{m \times n}{\square}] \cdot [\underset{n \times k}{\square}] = [\underset{m \times k}{\square}]$$

$$A_{2 \times 3} \cdot B_{3 \times 6} = C_{2 \times 6}$$

$$A_{3 \times 4} \cdot B_{4 \times 2} = C_{3 \times 2}$$

$$A_{100 \times 300} \cdot B_{300 \times 3} = C_{100 \times 3}$$

DOT PRODUCT

$$\begin{bmatrix} 2 & 8 & -4 \end{bmatrix}_{1 \times 3} \cdot \begin{bmatrix} 1 \\ -7 \\ 3 \end{bmatrix}_{3 \times 1} = [-66]$$

Scalar

Compatibility: they need to have matching forms

DOT PRODUCT

$$\begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}_{2 \times 3} \cdot \begin{bmatrix} 2 & -1 \\ 8 & 0 \\ 3 & 0 \end{bmatrix}_{3 \times 2} = \begin{bmatrix} \quad & \quad \end{bmatrix}_{2 \times 2}$$

- 1) Matrices are nothing more than a collection of vectors
- 2) When we have a dot product, we always multiply a row vector times a column vector

DOT PRODUCT

$$\begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}_{2 \times 3} \cdot \begin{bmatrix} 2 & -1 \\ 8 & 0 \\ 3 & 0 \end{bmatrix}_{3 \times 2} = \begin{bmatrix} 124 \\ \dots \end{bmatrix}_{2 \times 2}$$

$$5 \times 2 + 12 \times 8 + 6 \times 3$$

DOT PRODUCT

$$\begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}_{2 \times 3} \cdot \begin{bmatrix} 2 & -1 \\ 8 & 0 \\ 3 & 0 \end{bmatrix}_{3 \times 2} = \begin{bmatrix} 124 & -5 \end{bmatrix}_{2 \times 2}$$

$$(5 \times -1) + (12 \times 0) + (6 \times 0)$$

DOT PRODUCT

$$\begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}_{2 \times 3} \cdot \begin{bmatrix} 2 & -1 \\ 8 & 0 \\ 3 & 0 \end{bmatrix}_{3 \times 2} = \begin{bmatrix} 124 & -5 \\ 36 & \end{bmatrix}_{2 \times 2}$$

$$(-3 \times 2) + (0 \times 8) + (14 \times 3)$$

DOT PRODUCT

$$\begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}_{2 \times 3} \cdot \begin{bmatrix} 2 & -1 \\ 8 & 0 \\ 3 & 0 \end{bmatrix}_{3 \times 2} = \begin{bmatrix} 124 & -5 \\ 36 & 3 \end{bmatrix}_{2 \times 2}$$

Row vectors determine the row, column vectors → the column compatibility: they need to have matching forms

DOT PRODUCT

$$\begin{bmatrix} -12 & 5 & -5 & 1 & 6 \\ 6 & -2 & 0 & 0 & -3 \\ 10 & 2 & 0 & 8 & 0 \\ 9 & -4 & 8 & 3 & -6 \end{bmatrix}$$

4×5

$$\begin{bmatrix} 6 & -1 \\ 8 & -4 \\ 2 & -2 \\ 7 & 4 \\ -6 & -9 \end{bmatrix}$$

5×2

$\cdot =$

$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}$$

4×2

DOT PRODUCT

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \cdot \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \text{X}$$

3 × 4 2 × 3

NOT COMPATIBLE

Scalar * Matrix

```
In [9]: A = np.array([[5,12,6],[-3,0,14]])  
A
```

```
Out[9]: array([[ 5, 12,  6],  
[-3,  0, 14]])
```

```
In [ ]: 3*A
```

365 DataS

```
Out[10]: array([[15, 36, 18],  
[-9,  0, 42]])
```

The initial matrix had been SCALED 3 times

Matrix * Matrix

Example 1

```
In [11]: B = np.array([[2,-1],[8,0],[3,0]])  
B
```

```
Out[11]: array([[ 2, -1],  
[ 8,  0],  
[ 3,  0]])
```

```
In [12]: np.dot(A,B)
```

```
Out[12]: array([[124, -5],  
[ 36,  3]])
```

DOT PRODUCT

$$\begin{bmatrix} 5 & 12 & 6 \\ -3 & 0 & 14 \end{bmatrix}_{2 \times 3} \cdot \begin{bmatrix} 2 & -1 \\ 8 & 0 \\ 3 & 0 \end{bmatrix}_{3 \times 2} = \begin{bmatrix} 124 & -5 \\ 36 & 3 \end{bmatrix}_{2 \times 2}$$

WHY IS LINEAR ALGEBRA ACTUALLY USEFUL?

Applications in data science

Vectorized code

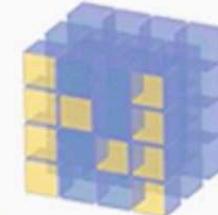
Image recognition

Dimensionality reduction

VECTORIZED CODE

Whenever we are using linear algebra
to compute many values simultaneously

much
faster



NUMPY

VECTORIZED CODE

Whenever we are using linear algebra
to compute many values simultaneously

10,190 + 223 *



MANUALLY

$$10,190 + 223 * 693 = 164729$$

$$10,190 + 223 * 656 = 156478$$

$$10,190 + 223 * 1060 = 246570$$

$$10,190 + 223 * 487 = 118791$$

$$10,190 + 223 * 1275 = 294515$$

WITH A LOOP

```
for i in range(100):  
    print(10190 + 223 * size[i])
```

1.90

$$\begin{bmatrix} 1 & 693 \\ 1 & 656 \\ 1 & 1060 \\ 1 & 487 \\ 1 & 1275 \end{bmatrix} \cdot \begin{bmatrix} 10190 \\ 223 \end{bmatrix} = \begin{bmatrix} 1 * 10,190 + 223 * 693 \\ 1 * 10,190 + 223 * 656 \\ 1 * 10,190 + 223 * 1060 \\ 1 * 10,190 + 223 * 487 \\ 1 * 10,190 + 223 * 1275 \end{bmatrix}$$

5×2 2×1 5×1

$$\begin{bmatrix} 1 & 693 \\ 1 & 656 \\ 1 & 1060 \\ 1 & 487 \\ 1 & 1275 \end{bmatrix} \cdot \begin{bmatrix} 10190 \\ 223 \end{bmatrix} = \begin{bmatrix} 164729 \\ 156478 \\ 246570 \\ 118791 \\ 294515 \end{bmatrix}$$

5×2 2×1 5×1

inputs

weights
(coefficients)

outputs

$$10190 + 223 *$$

 inputs
 10000×2

$\cdot \begin{bmatrix} 10190 \\ 223 \end{bmatrix} =$
 2×1

 outputs
 10000×1

inputs

weights
(coefficients)

outputs

IMAGE RECOGNITION

Deep learning
CNNs (convolutional neural networks)

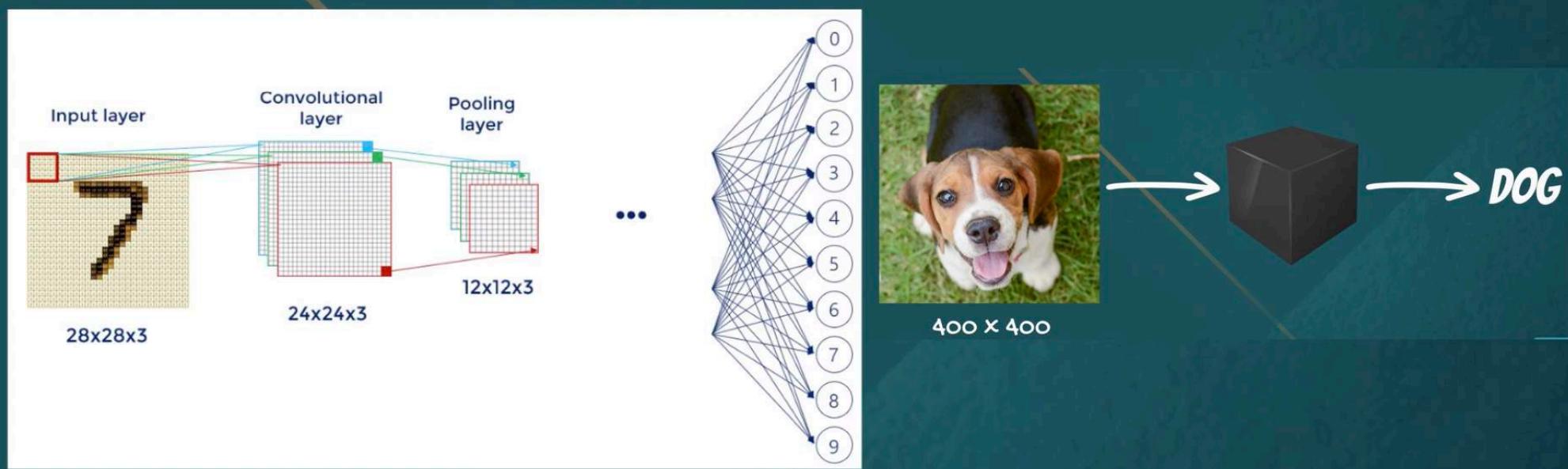
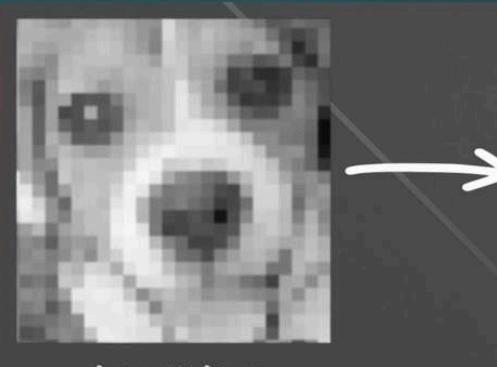
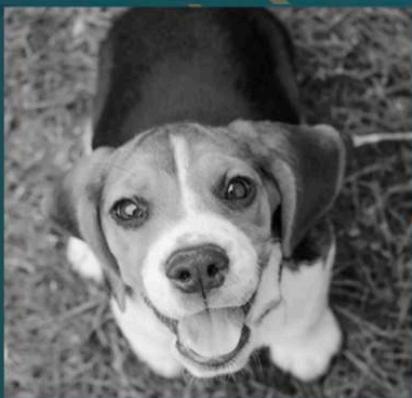


IMAGE RECOGNITION

- Zoom in until we see pixels
- Zoom out to reach the photo



010101010101
101010101010
010101010101
101010101010
010101010101
101010101010

height = 400

400 x 400
width = 400

- 256 shades of gray

- 0 = totally white
- 255 = totally black

- shows the intensity of grey

IMAGE RECOGNITION



400 x 400

How to represent colour

R G B

› RGB scale

255	0	0	›	
0	255	0	›	
0	0	255	›	
138	106	214	›	

IMAGE RECOGNITION



B
G
R

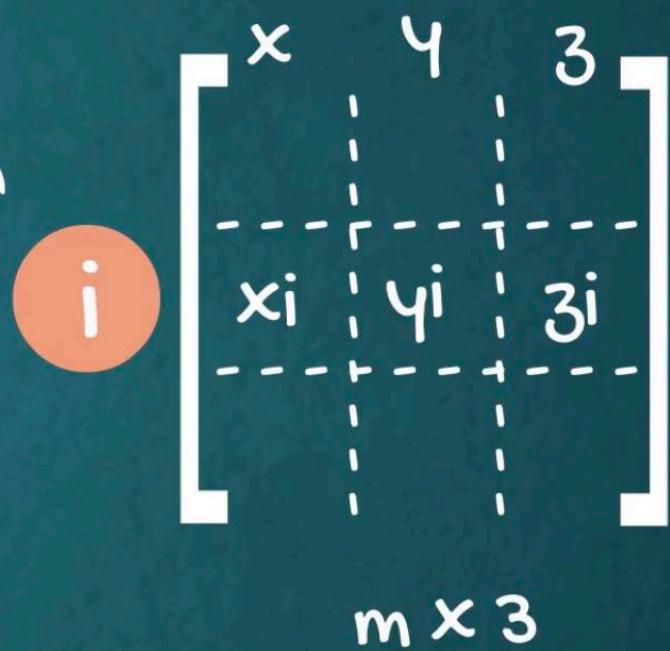
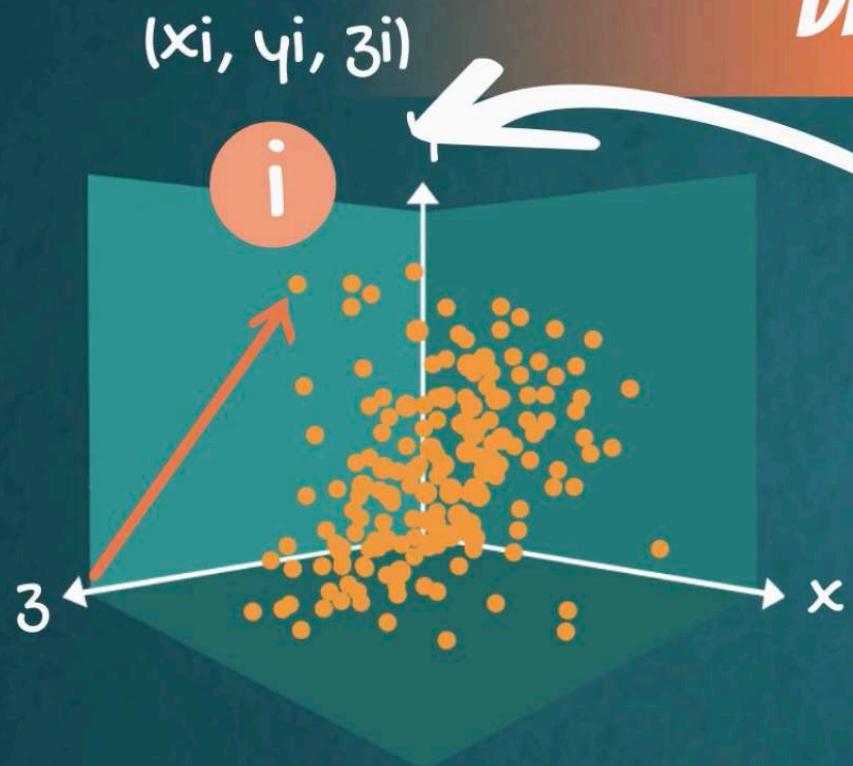
how to represent colour

› RGB scale

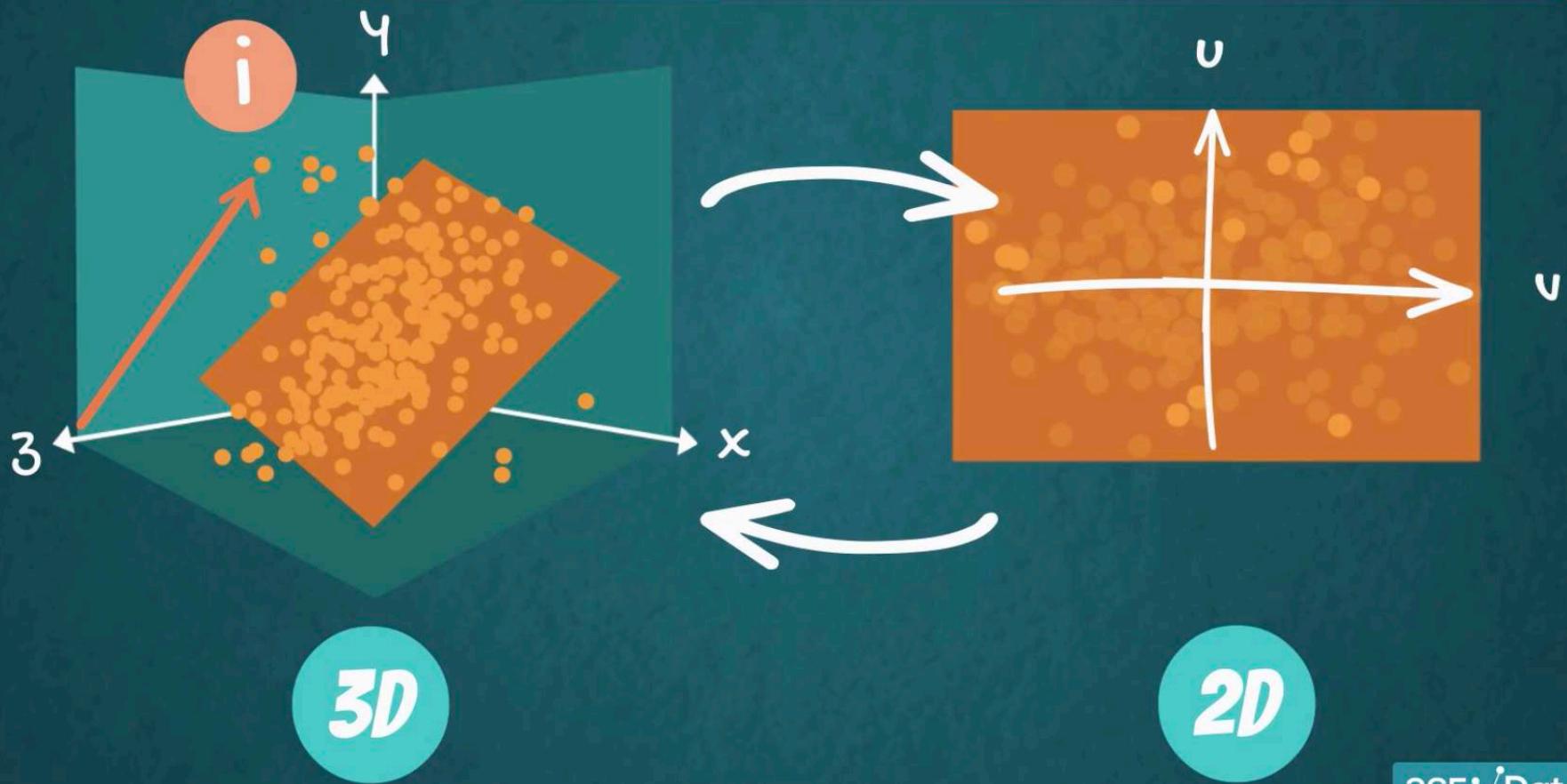
› Tensors

1.90

DIMENSIONALITY REDUCTION



DIMENSIONALITY REDUCTION



DIMENSIONALITY REDUCTION

INITIAL

i

$$\begin{bmatrix} x & y & z \\ x_i & y_i & z_i \end{bmatrix}$$

$m \times 3$

3D

TRANSFORMED

$$\begin{bmatrix} u & v \\ u_i & v_i \end{bmatrix}$$

$m \times 2$

2D

DIMENSIONALITY REDUCTION

PLEASE RATE FROM 1 TO 5

1) I feel comfortable around people

1 2 3 4 5

2) I easily make friends

1 2 3 4 5

3) I like going out

1 2 3 4 5

:

so)

THROUGH LINEAR ALGEBRA

LEVEL OF EXTROVERSION