

▼ Code

▼ 0. Import library

```
1  # Import libraries
2
3  # math library
4  import numpy as np
5  import torch
6
7  # visualization library
8  %matplotlib inline
9  from IPython.display import set_matplotlib_formats
10 set_matplotlib_formats('png2x','pdf')
11 import matplotlib.pyplot as plt
```

▼ 1. Load and plot the dataset

```
1  import numpy as np
2  import matplotlib.pyplot as plt
3  # data path
4  test_path = '/content/drive/My Drive/ML_Assignment/data/testing.txt'
5  train_path = '/content/drive/My Drive/ML_Assignment/data/training.txt'
6
7  # import data with numpy
8  data_train = np.loadtxt(train_path, delimiter=',')
9  data_test = np.loadtxt(test_path, delimiter=',')
10
11 # number of training data
12 number_data_train = data_train.shape[0]
13 number_data_test = data_test.shape[0]
14
15 # training data
16 x1_train = data_train[:,0] # feature 1
17 x2_train = data_train[:,1] # feature 2
18 idx_class0_train = (data_train[:,2]==0) # index of class0
19 idx_class1_train = (data_train[:,2]==1) # index of class1
20
21 # testing data
22 x1_test = data_test[:,0] # feature 1
23 x2_test = data_test[:,1] # feature 2
24 idx_class0_test = (data_test[:,2]==0) # index of class0
25 idx_class1_test = (data_test[:,2]==1) # index of clas
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29 # gradient function definition
30 def grad_loss(y_pred,y,X):
31     n = len(y)
32     grad = torch.sum((y_pred - y)* X[:,i].unsqueeze(1))/n
33     return grad
34
35
36 # gradient descent function definition
37 def grad_desc(X_train, X_test, y_train, y_test, tau, max_iter, lamb):
38     L_iters_train = np.zeros([max_iter]) # record the loss values
39     L_iters_test = np.zeros([max_iter])
40     w = np.array([[0.01] for j in range(100)]) # initialization
41     w = torch.tensor(w)
42     print('===== Running Start =====')
43     for i in range(max_iter): # loop over the iterations
44         train_pred = g_func(X_train, w) # linear predication function
45         test_pred = g_func(X_test, w)
46         for n in range(0, 100, 1) :
47             w[n] = (1 - tau*lamb) * w[n] - tau*torch.sum((train_pred - y_train)* X_train[:,n].unsqueeze(1))/len(y_train)
48             J_train = loss_logreg(train_pred,y_train,w,lamb)
49             J_test = loss_logreg(test_pred,y_test,w,lamb)
50             L_iters_train[i] = J_train # save the current loss value
51             L_iters_test[i] = J_test
52             if i % 10000 == 0:
53                 print('Epoch: {:6d}, train_cost: {:.10f}, test_cost: {:.10f}'.format(i,J_train,J_test))
54
55     return w, L_iters_train, L_iters_test

```

[Change Numpy to Tensor]

```

1  train_x1 = torch.DoubleTensor(x1_train)
2  train_x2 = torch.DoubleTensor(x2_train)
3  train_label = torch.DoubleTensor(data_train[:,2])
4
5  test_x1 = torch.DoubleTensor(x1_test)
6  test_x2 = torch.DoubleTensor(x2_test)
7  test_label = torch.DoubleTensor(data_test[:,2])
8
9  train_x1 = train_x1.unsqueeze(1)
10 train_x2 = train_x2.unsqueeze(1)
11 train_label = train_label.unsqueeze(1)
12
13 test_x1 = test_x1.unsqueeze(1)
14 test_x2 = test_x2.unsqueeze(1)
15 test_label = test_label.unsqueeze(1)
16
17 # construct the data matrix X
18 X_train = function(train_x1, train_x2)
19 X_test = function(test_x1, test_x2)

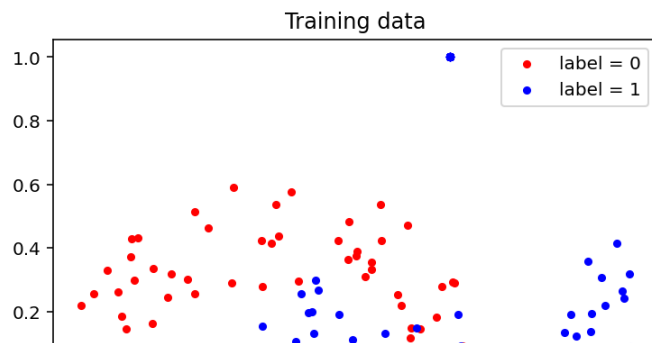
```

▼ 3. define a prediction function and run a gradient descent algorithm

```

1  # run gradient descent algorithm
2  tau = 1e-7; max_iter = 100000;
3  w_0, train_L_0, test_L_0 = grad_desc(X_train, X_test, train_label, test_label, tau, max_iter, 0.00001)
4  w_1, train_L_1, test_L_1 = grad_desc(X_train, X_test, train_label, test_label, tau, max_iter, 0.0001)
5  w_2, train_L_2, test_L_2 = grad_desc(X_train, X_test, train_label, test_label, tau, max_iter, 0.001)
6  w_3, train_L_3, test_L_3 = grad_desc(X_train, X_test, train_label, test_label, tau, max_iter, 0.01)
7  w_4, train_L_4, test_L_4 = grad_desc(X_train, X_test, train_label, test_label, tau, max_iter, 0.1)

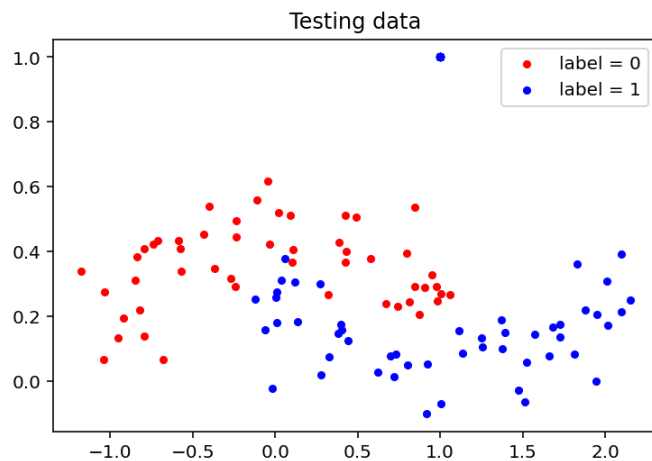
```



```

1 fig_2 = plt.figure()
2 plt.scatter(x1_test[idx_class0_test], x2_test[idx_class0_test], s=50, c='r', marker='.', label='label = 0')
3 plt.scatter(x1_test[idx_class1_test], x2_test[idx_class1_test], s=50, c='b', marker='.', label='label = 1')
4 plt.title('Testing data')
5 plt.legend()
6 plt.show()
7 fig_2.savefig('Testing data.png')

```



▼ 2. Define a logistic regression loss function and its gradient

```

1 # make X
2 def function(x, y):
3     func_list = []
4     for i in range (0, 10, 1):
5         for j in range (0, 10, 1):
6             func_list.append((x**i)*(y**j))
7     func_list = torch.stack(func_list).squeeze(-1)
8     return func_list.T
9
10 # g(x, y, w)
11 def g_func(X, w):
12     pred = sigmoid(torch.matmul(X, w))
13     return pred
14
15 # sigmoid function
16 def sigmoid(z):
17     sigmoid_f = 1.0 / (1.0 + torch.exp(-z))
18     return sigmoid_f
19
20 # loss function definition
21 def loss_logreg(y_pred, y, w, lamb):
22     n = len(y)
23     cross_entropy = torch.sum(-y * torch.log(y_pred) - (1 - y) * torch.log(1 - y_pred)) / n
24     r = torch.sum(w**2) * lamb * 0.5
25     loss = cross_entropy + r
26     return loss
27

```

```

===== Running Start =====
Epoch:    0, train_cost:  0.440711, test_cost:  0.442080
Epoch:  10000, train_cost:  0.438475, test_cost:  0.439784
Epoch:  20000, train_cost:  0.436311, test_cost:  0.437562
Epoch:  30000, train_cost:  0.434214, test_cost:  0.435408
Epoch:  40000, train_cost:  0.432178, test_cost:  0.433316
Epoch:  50000, train_cost:  0.430200, test_cost:  0.431284
Epoch:  60000, train_cost:  0.428276, test_cost:  0.429307
Epoch:  70000, train_cost:  0.426403, test_cost:  0.427383
Epoch:  80000, train_cost:  0.424578, test_cost:  0.425507
Epoch:  90000, train_cost:  0.422799, test_cost:  0.423679

```

```

===== Running Start =====
Epoch:    0, train_cost:  0.440711, test_cost:  0.442080
Epoch:  10000, train_cost:  0.438476, test_cost:  0.439785
Epoch:  20000, train_cost:  0.436312, test_cost:  0.437563
Epoch:  30000, train_cost:  0.434215, test_cost:  0.435408
Epoch:  40000, train_cost:  0.432179, test_cost:  0.433317
Epoch:  50000, train_cost:  0.430201, test_cost:  0.431285
Epoch:  60000, train_cost:  0.428277, test_cost:  0.429308
Epoch:  70000, train_cost:  0.426404, test_cost:  0.427383
Epoch:  80000, train_cost:  0.424579, test_cost:  0.425508
Epoch:  90000, train_cost:  0.422800, test_cost:  0.423680

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===== Running Start =====
Epoch:    0, train_cost:  0.440716, test_cost:  0.442085
Epoch:  10000, train_cost:  0.438480, test_cost:  0.439790
Epoch:  20000, train_cost:  0.436317, test_cost:  0.437568
Epoch:  30000, train_cost:  0.434220, test_cost:  0.435414
Epoch:  40000, train_cost:  0.432184, test_cost:  0.433323
Epoch:  50000, train_cost:  0.430207, test_cost:  0.431290
Epoch:  60000, train_cost:  0.428283, test_cost:  0.429314
Epoch:  70000, train_cost:  0.426410, test_cost:  0.427389
Epoch:  80000, train_cost:  0.424586, test_cost:  0.425515
Epoch:  90000, train_cost:  0.422807, test_cost:  0.423687

```

```

===== Running Start =====
Epoch:    0, train_cost:  0.440761, test_cost:  0.442130
Epoch:  10000, train_cost:  0.438528, test_cost:  0.439837
Epoch:  20000, train_cost:  0.436367, test_cost:  0.437618
Epoch:  30000, train_cost:  0.434272, test_cost:  0.435466
Epoch:  40000, train_cost:  0.432240, test_cost:  0.433378
Epoch:  50000, train_cost:  0.430264, test_cost:  0.431348
Epoch:  60000, train_cost:  0.428343, test_cost:  0.429374
Epoch:  70000, train_cost:  0.426473, test_cost:  0.427452
Epoch:  80000, train_cost:  0.424650, test_cost:  0.425579
Epoch:  90000, train_cost:  0.422874, test_cost:  0.423754

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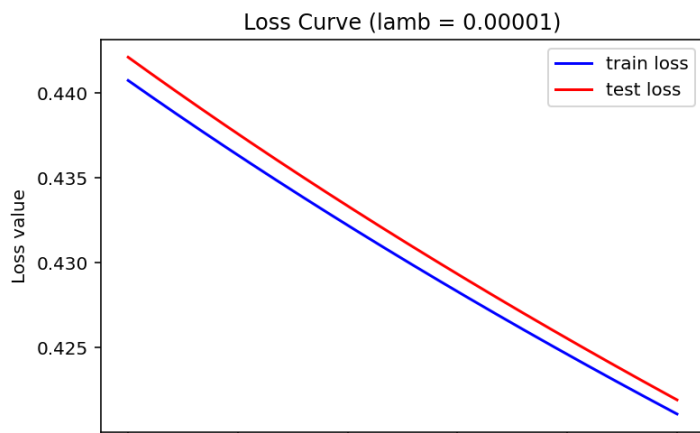
===== Running Start =====
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Epoch:  40000, train_cost:  0.432790, test_cost:  0.433929
Epoch:  50000, train_cost:  0.430839, test_cost:  0.431924
Epoch:  60000, train_cost:  0.428942, test_cost:  0.429974
Epoch:  70000, train_cost:  0.427096, test_cost:  0.428077
Epoch:  80000, train_cost:  0.425298, test_cost:  0.426228

```

```

1  # plot
2  fig_3 = plt.figure()
3  plt.plot(np.array(range(max_iter)), train_L_0, c = 'b', label='train loss')
4  plt.plot(np.array(range(max_iter)), test_L_0, c = 'r', label='test loss')
5  plt.xlabel('Iterations')
6  plt.ylabel('Loss value')
7  plt.title('Loss Curve (lamb = 0.00001)')
8  plt.legend()
9  plt.show()
10 fig_3.savefig('Loss(lamb = 0.00001).png')

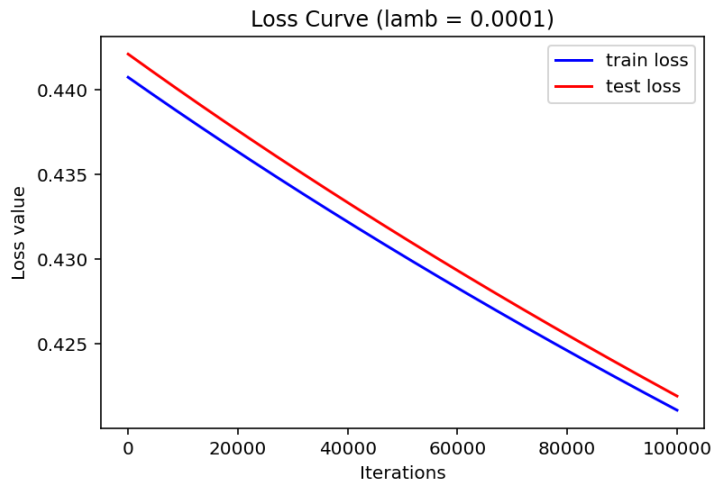
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```

1 # plot
2 fig_4 = plt.figure()
3 plt.plot(np.array(range(max_iter)), train_L_1, c = 'b', label='train loss')
4 plt.plot(np.array(range(max_iter)), test_L_1, c = 'r', label='test loss')
5 plt.xlabel('Iterations')
6 plt.ylabel('Loss value')
7 plt.title('Loss Curve (lamb = 0.0001)')
8 plt.legend()
9 plt.show()
10 fig_4.savefig('Loss(lamb = 0.0001).png')

```



```

1 # plot
2 fig_5 = plt.figure()
3 plt.plot(np.array(range(max_iter)), train_L_2, c = 'b', label='train loss')
4 plt.plot(np.array(range(max_iter)), test_L_2, c = 'r', label='test loss')
5 plt.xlabel('Iterations')
6 plt.ylabel('Loss value')
7 plt.title('Loss Curve (lamb = 0.001)')
8 plt.legend()
9 plt.show()
10 fig_5.savefig('Loss(lamb = 0.001).png')

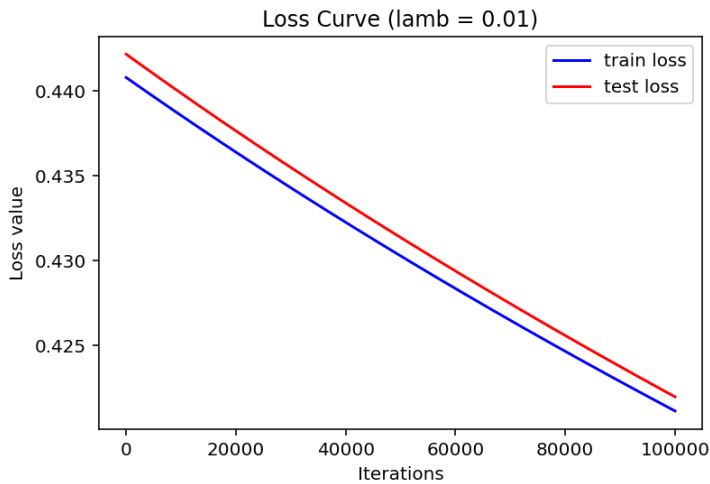
```

Loss Curve (lamb = 0.001)

```

1 # plot
2 fig_6 = plt.figure()
3 plt.plot(np.array(range(max_iter)), train_L_3, c = 'b', label='train loss')
4 plt.plot(np.array(range(max_iter)), test_L_3, c = 'r', label='test loss')
5 plt.xlabel('Iterations')
6 plt.ylabel('Loss value')
7 plt.title('Loss Curve (lamb = 0.01)')
8 plt.legend()
9 plt.show()
10 fig_6.savefig('Loss(lamb = 0.01).png')

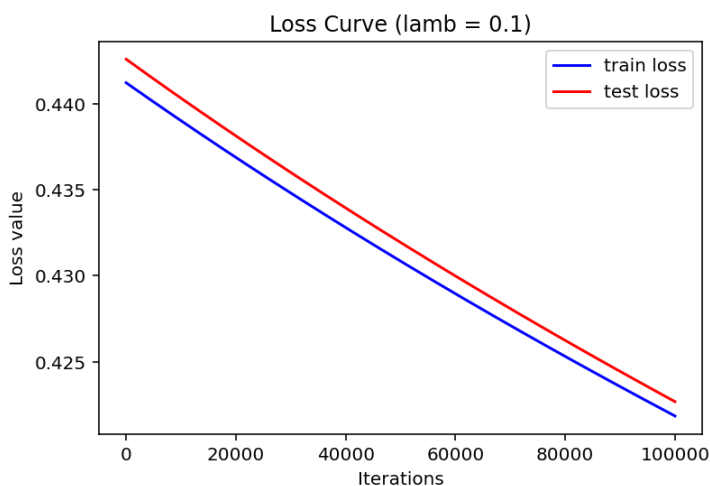
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```

1 # plot
2 fig_7 = plt.figure()
3 plt.plot(np.array(range(max_iter)), train_L_4, c = 'b', label='train loss')
4 plt.plot(np.array(range(max_iter)), test_L_4, c = 'r', label='test loss')
5 plt.xlabel('Iterations')
6 plt.ylabel('Loss value')
7 plt.title('Loss Curve (lamb = 0.1)')
8 plt.legend()
9 plt.show()
10 fig_7.savefig('Loss(lamb = 0.1).png')

```



▼ 4. Plot the decision boundary with probability map

```

1 # compute values p(x) for multiple data points x
2 x1_min, x1_max = x1_test.min(), x1_test.max() # min and max of grade 1
3 x2_min, x2_max = x2_test.min(), x2_test.max() # min and max of grade 2
4 xx1, xx2 = np.meshgrid(np.linspace(x1_min, x1_max), np.linspace(x2_min, x2_max)) # create meshgrid
5 xx1 = torch.DoubleTensor(xx1)

```

```

6  xx2 = torch.DoubleTensor(xx2)
7  X2 = function(xx1.reshape(-1), xx2.reshape(-1))

1  def pic_train_commons(ax, xx1, xx2, p):
2      ax.legend(loc = 'upper right')
3      ax.set_title('train')
4      ax.contourf(xx1,xx2,p,100,vmin=0,vmax=1,cmap='coolwarm', alpha=0.6)
5      ax.contour(xx1, xx2, p, [0.5], linewidths=2, colors='k')
6      ax.scatter(x1_train[idx_class0_train], x2_train[idx_class0_train], s=50, c='r', marker='.', label='label = 0')
7      ax.scatter(x1_train[idx_class1_train], x2_train[idx_class1_train], s=50, c='b', marker='.', label='label = 1')
8
9  def pic_test_commons(ax, xx1, xx2, p):
10     ax.legend(loc = 'upper right')
11     ax.set_title('test')
12     ax.contourf(xx1,xx2,p,100,vmin=0,vmax=1,cmap='coolwarm', alpha=0.6)
13     ax.contour(xx1, xx2, p, [0.5], linewidths=2, colors='k')
14     ax.scatter(x1_test[idx_class0_train], x2_test[idx_class0_train], s=50, c='r', marker='.', label='label = 0')
15     ax.scatter(x1_test[idx_class1_train], x2_test[idx_class1_train], s=50, c='b', marker='.', label='label = 1')

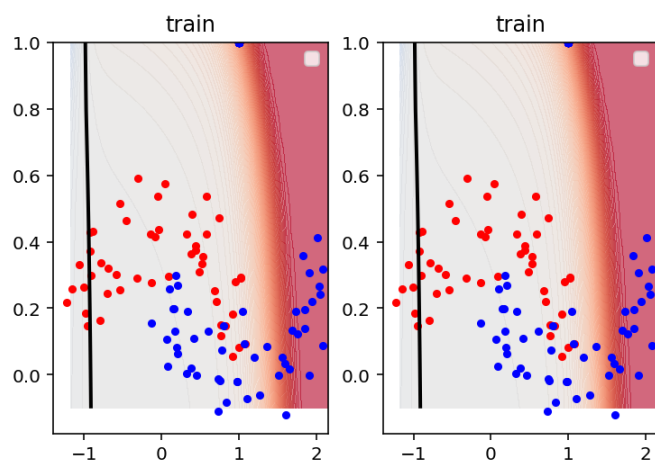
```

```

1  p = g_func(X2,w_0)
2  p = p.reshape(50,50)
3  p = p.numpy()
4
5  # plot
6  fig_8 = plt.figure()
7  fig_8, ax = plt.subplots(1,2)
8  plt.title('lambda = 0.00001')
9  pic_train_commons(ax[0], xx1, xx2, p)
10 pic_train_commons(ax[1], xx1, xx2, p)
11 plt.show()
12 fig_8.savefig('Probability Map (Lambda = 0.00001).png')

```

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<Figure size 432x288 with 0 Axes>

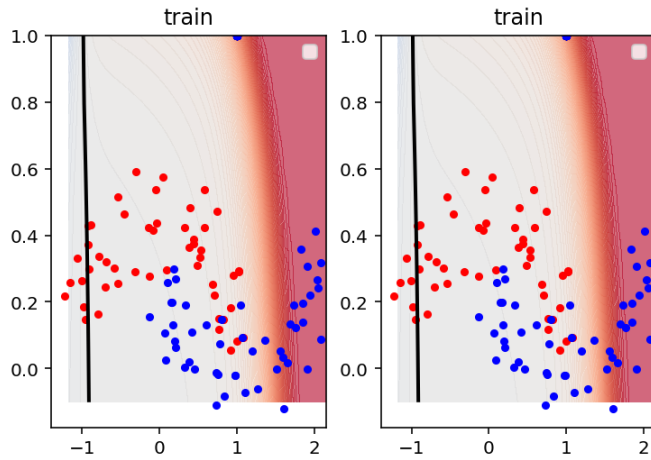


```

1  p = g_func(X2,w_1)
2  p = p.reshape(50,50)
3  p = p.numpy()
4
5  # plot
6  fig_9 = plt.figure()
7  fig_9, ax = plt.subplots(1,2)
8  plt.title('lambda = 0.0001')
9  pic_train_commons(ax[0], xx1, xx2, p)
10 pic_train_commons(ax[1], xx1, xx2, p)
11 plt.show()
12 fig_9.savefig('Probability Map (Lambda = 0.0001).png')

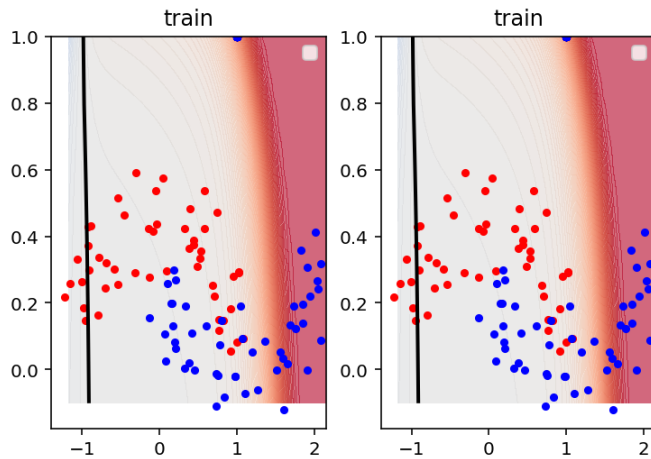
```

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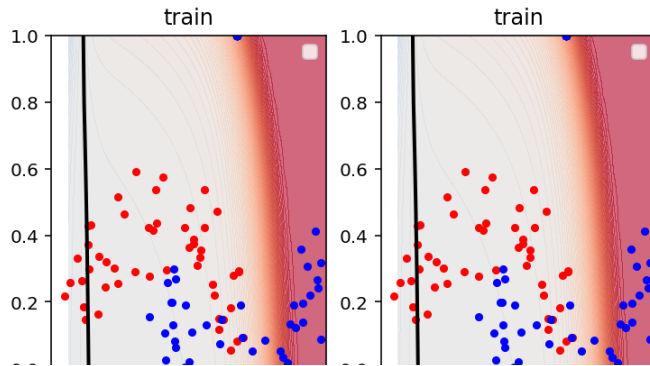
```
1 p = g_func(X2,w_2)
2 p = p.reshape(50,50)
3 p = p.numpy()
4
5 # plot
6 fig_10 = plt.figure()
7 fig_10, ax = plt.subplots(1,2)
8 plt.title('lambda = 0.001')
9 pic_train_commons(ax[0], xx1, xx2, p)
10 pic_train_commons(ax[1], xx1, xx2, p)
11 plt.show()
12 fig_10.savefig('Probability Map (Lambda = 0.001).png')
```

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 No handles with labels found to put in legend.
 <Figure size 432x288 with 0 Axes>



```
1 p = g_func(X2,w_3)
2 p = p.reshape(50,50)
3 p = p.numpy()
4
5 # plot
6 fig_11 = plt.figure()
7 fig_11, ax = plt.subplots(1,2)
8 plt.title('lambda = 0.01')
9 pic_train_commons(ax[0], xx1, xx2, p)
10 pic_train_commons(ax[1], xx1, xx2, p)
11 plt.show()
12 fig_11.savefig('Probability Map (Lambda = 0.01).png')
```


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 <Figure size 432x288 with 0 Axes>

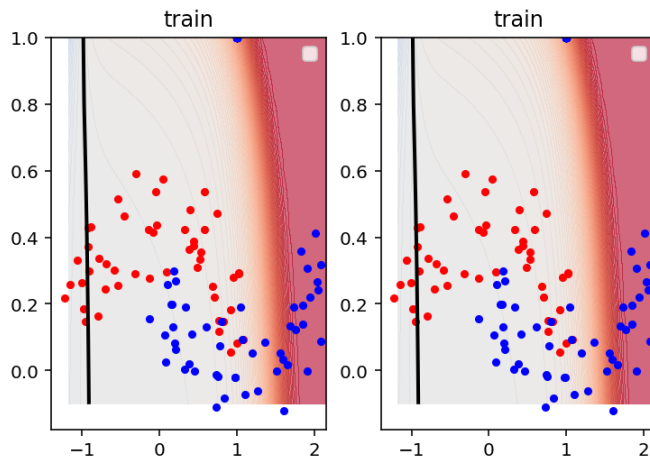


```

1 p = g_func(X2,w_4)
2 p = p.reshape(50,50)
3 p = p.numpy()
4
5 # plot
6 fig_12 = plt.figure()
7 fig_12, ax = plt.subplots(1,2)
8 plt.title('lambda = 0.1')
9 pic_train_commons(ax[0], xx1, xx2, p)
10 pic_train_commons(ax[1], xx1, xx2, p)
11 plt.show()
12 fig_12.savefig('Probability Map (Lambda = 0.1).png')

```

No handles with labels found to put in legend.
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 <Figure size 432x288 with 0 Axes>



▼ 5. Compute the classification accuracy

```

1 n = data_train.shape[0]
2 idx_class0 = (data_train[:,2]==0) # index of class0
3 idx_class1 = (data_train[:,2]==1) # index of class1
4
5 p_train_0 = g_func(X_train, w_0).numpy()
6 p_train_1 = g_func(X_train, w_1).numpy()
7 p_train_2 = g_func(X_train, w_2).numpy()
8 p_train_3 = g_func(X_train, w_3).numpy()
9 p_train_4 = g_func(X_train, w_4).numpy()
10
11 idx_train_0 = (p_train_0 < 0.5)
12 idx_train_1 = (p_train_1 < 0.5)
13 idx_train_2 = (p_train_2 < 0.5)
14 idx_train_3 = (p_train_3 < 0.5)
15 idx_train_4 = (p_train_4 < 0.5)
16

```

```

17  idx_right_0 = (idx_class0 == idx_train_0.reshape(-1))
18  idx_right_1 = (idx_class0 == idx_train_1.reshape(-1))
19  idx_right_2 = (idx_class0 == idx_train_2.reshape(-1))
20  idx_right_3 = (idx_class0 == idx_train_3.reshape(-1))
21  idx_right_4 = (idx_class0 == idx_train_4.reshape(-1))
22
23  acc_train_0 = sum(idx_right_0)/n * 100
24  acc_train_1 = sum(idx_right_1)/n * 100
25  acc_train_2 = sum(idx_right_2)/n * 100
26  acc_train_3 = sum(idx_right_3)/n * 100
27  acc_train_4 = sum(idx_right_4)/n * 100
28  acc_train = [acc_train_0, acc_train_1, acc_train_2, acc_train_3, acc_train_4]

1   n = data_test.shape[0]
2   idx_class0 = (data_test[:,2]==0) # index of class0
3   idx_class1 = (data_test[:,2]==1) # index of class1
4
5   p_0 = g_func(X_test, w_0).numpy()
6   p_1 = g_func(X_test, w_1).numpy()
7   p_2 = g_func(X_test, w_2).numpy()
8   p_3 = g_func(X_test, w_3).numpy()
9   p_4 = g_func(X_test, w_4).numpy()
10
11  idx_0 = (p_0 < 0.5)
12  idx_1 = (p_1 < 0.5)
13  idx_2 = (p_2 < 0.5)
14  idx_3 = (p_3 < 0.5)
15  idx_4 = (p_4 < 0.5)
16
17  idx_right_0 = (idx_class0 == idx_0.reshape(-1))
18  idx_right_1 = (idx_class0 == idx_1.reshape(-1))
19  idx_right_2 = (idx_class0 == idx_2.reshape(-1))
20  idx_right_3 = (idx_class0 == idx_3.reshape(-1))
21  idx_right_4 = (idx_class0 == idx_4.reshape(-1))
22
23  acc_test_0 = sum(idx_right_0)/n * 100
24  acc_test_1 = sum(idx_right_1)/n * 100
25  acc_test_2 = sum(idx_right_2)/n * 100
26  acc_test_3 = sum(idx_right_3)/n * 100
27  acc_test_4 = sum(idx_right_4)/n * 100
28  acc_test = [acc_test_0, acc_test_1, acc_test_2, acc_test_3, acc_test_4]

```

Data Frame

```

1   from pandas import Series, DataFrame
2   import pandas as pd
3
4   data = {'train_accuracy(%)' : acc_train,
5           'test_accuracy(%)' : acc_test}
6
7   train_frame = DataFrame (data, columns = ['train_accuracy(%)'],
8                             index = [0.00001, 0.0001, 0.001, 0.01, 0.1])
9
10  test_frame = DataFrame (data, columns = ['test_accuracy(%)'],
11                             index = [0.00001, 0.0001, 0.001, 0.01, 0.1])

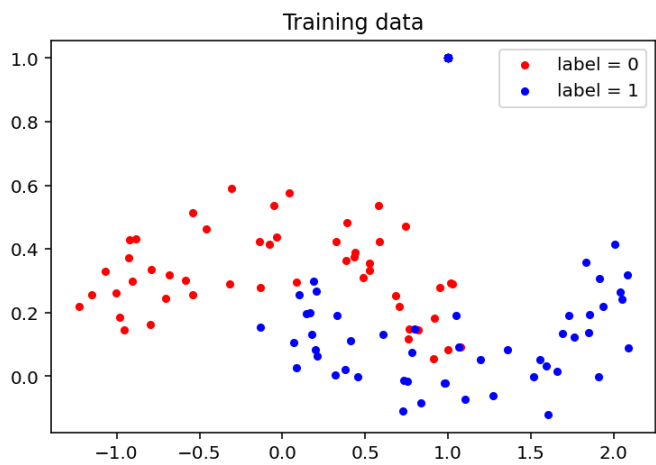
```

1

▼ OUTPUT

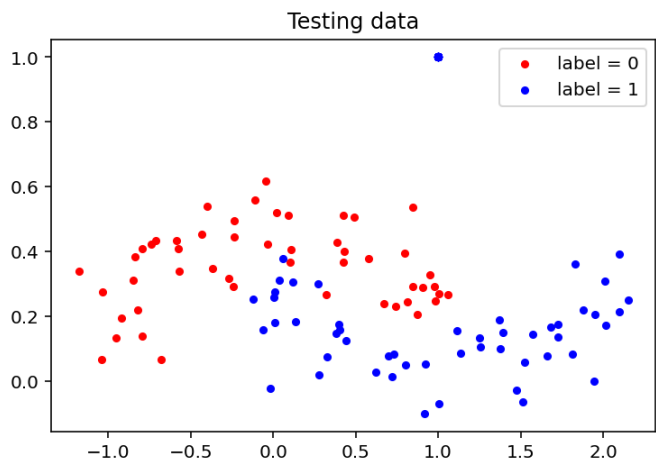
1. Plot the training data [0.5pt]

1 fig_1



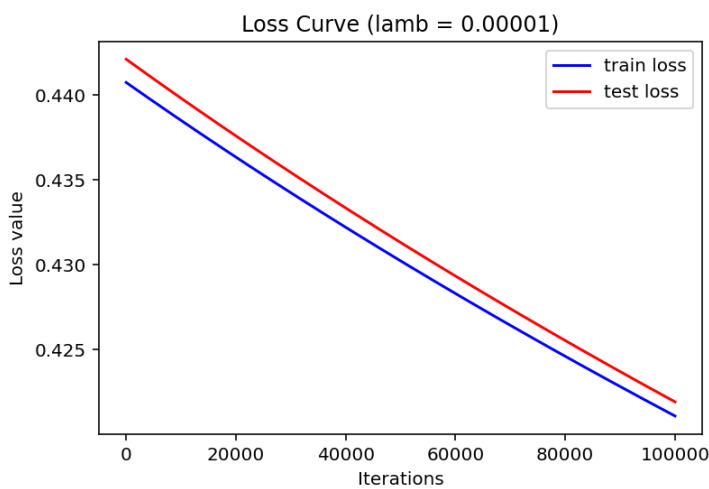
2. Plot the testing data [0.5pt]

1 fig_2



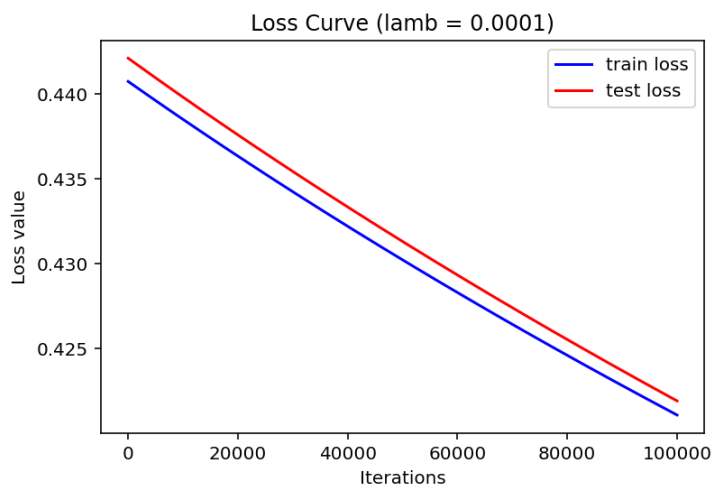
3. Plot the learning curve with $\lambda=0.00001$ [1pt]

1 fig_3



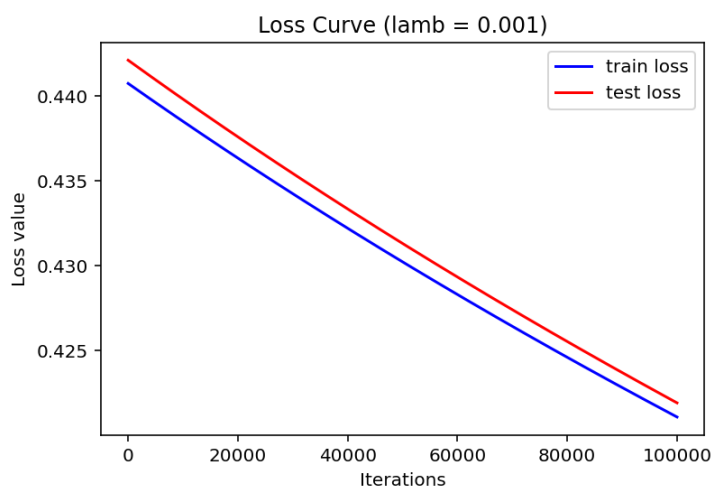
▼ 4. Plot the learning curve with $\lambda=0.0001$ [1pt]

1 fig_4



▼ 5. Plot the learning curve with $\lambda=0.001$ [1pt]

1 fig_5

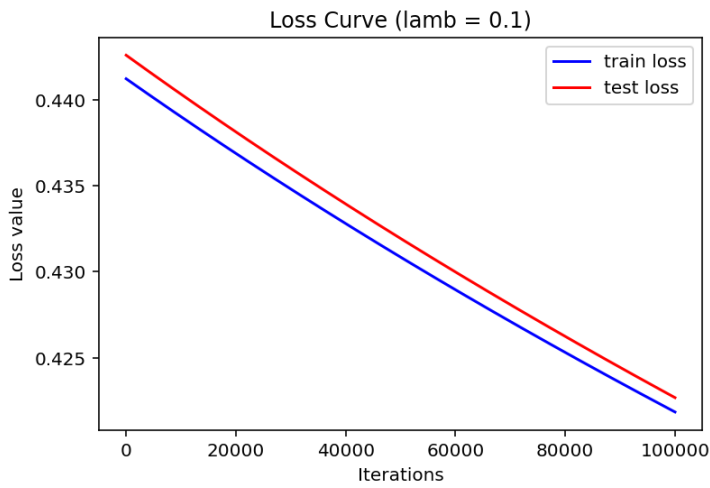


▼ 6. Plot the learning curve with $\lambda=0.01$ [1pt]

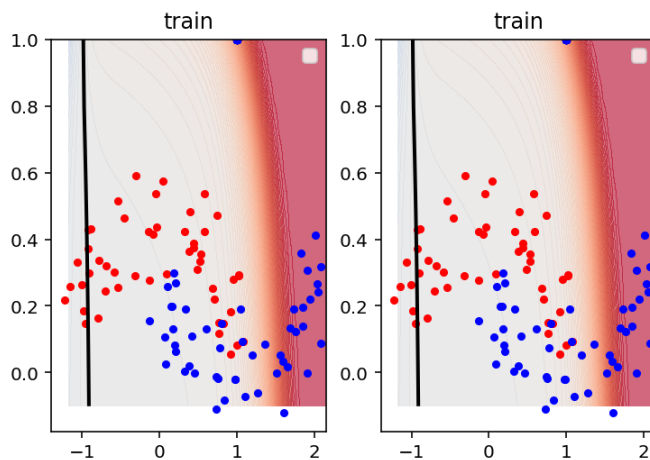
1 fig_6

▼ 7. Plot the learning curve with $\lambda=0.1$ [1pt]

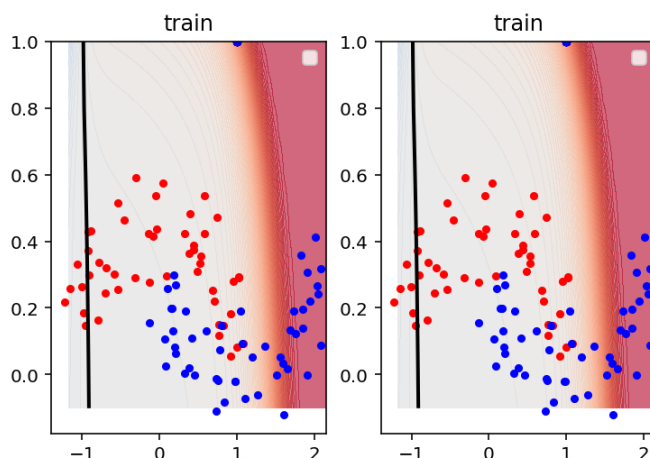
1 fig_7

▼ 8. Plot the probability map of the obtained classifier with $\lambda=0.00001$ [1pt]

1 fig_8

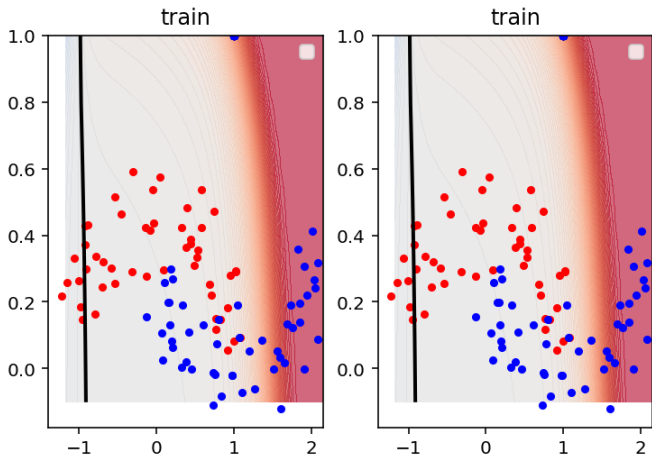
▼ 9. Plot the probability map of the obtained classifier with $\lambda=0.0001$ [1pt]

1 fig_9



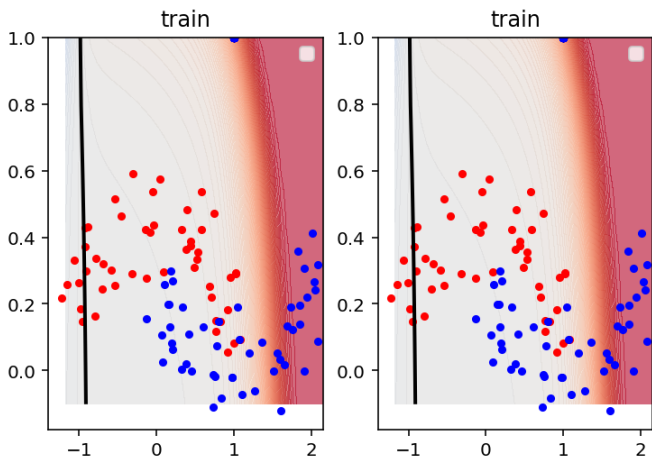
10. Plot the probability map of the obtained classifier with $\lambda=0.001$ [1pt]

1 fig_10



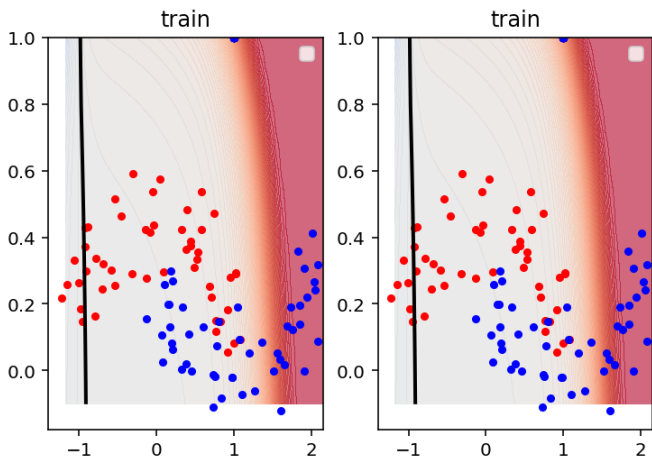
11. Plot the probability map of the obtained classifier with $\lambda=0.01$ [1pt]

1 fig_11



12. Plot the probability map of the obtained classifier with $\lambda=0.1$ [1pt]

1 fig_12



▼ 13. Print the final training accuracy with the given regularization parameters [2.5pt]

1 train_frame

train_accuracy(%)	
0.00001	78.0
0.00010	78.0
0.00100	78.0
0.01000	78.0
0.10000	78.0

▼ 14. Print the final testing accuracy with the given regularization parameters [2.5pt]

1 test_frame

test_accuracy(%)	
0.00001	77.0
0.00010	77.0
0.00100	77.0
0.01000	77.0
0.10000	77.0