# The Flow of Association and Causation in Graphs

Brady Neal

causalcourse.com

https://www.youtube.com/watch?v=Go4EkHN_PcA&list=PLoazKTcS0Rzb6bb9L508cyJ1z-U9iWkA0&index=19&fbclid=IwAR1BJa8KnN94e8TyY-Am5_KziFDL3HMIyxZ-jv5s7dEcf_JzCvhmwbdSztE

**Graph terminology**

**Bayesian networks and causal graphs**

**The basic building blocks of graphs**

**The flow of association and causation**

**Graph terminology**

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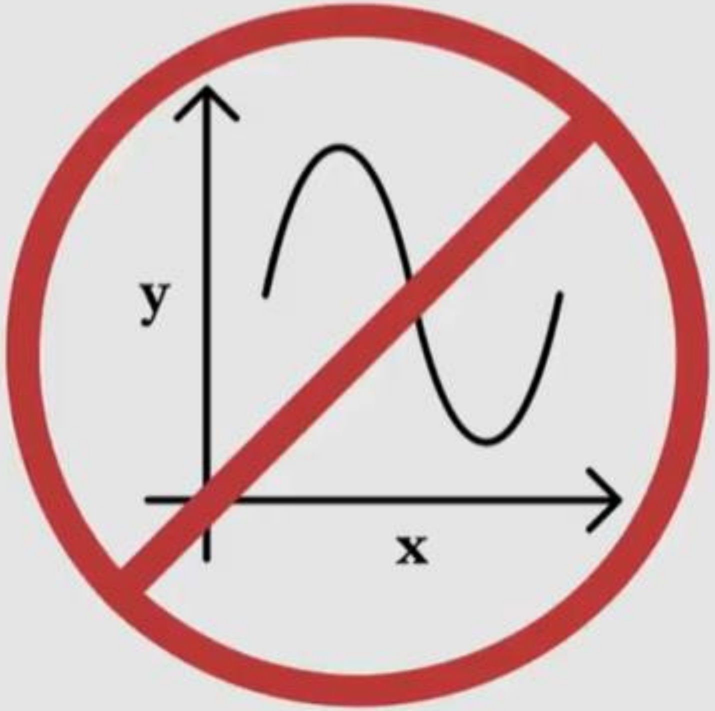
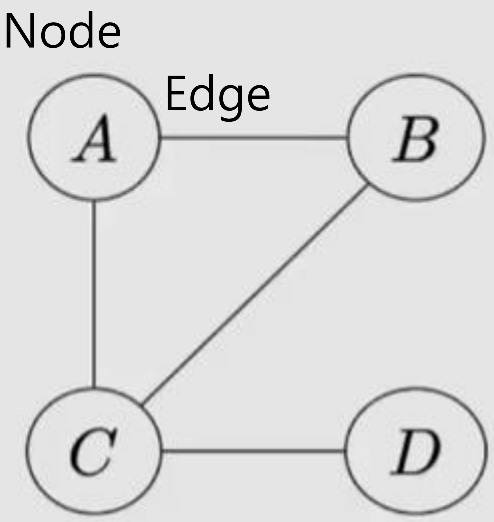
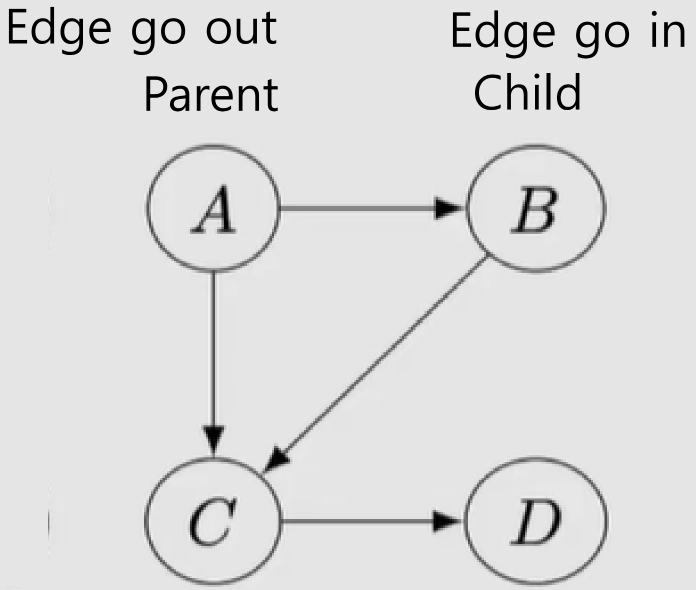
# Graph terminology: not a graph




# Graph terminology



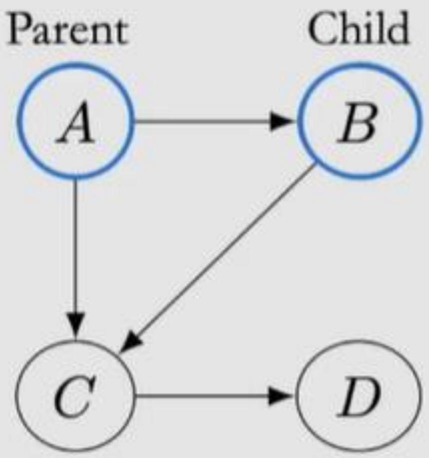

Undirected Graph



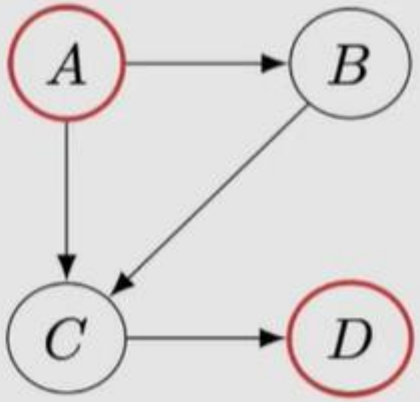

Directed Graph

# Graph terminology

Adjacent

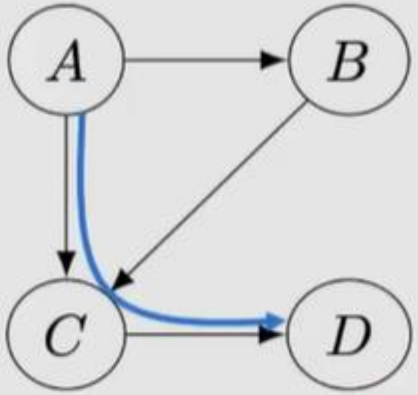



Not Adjacent



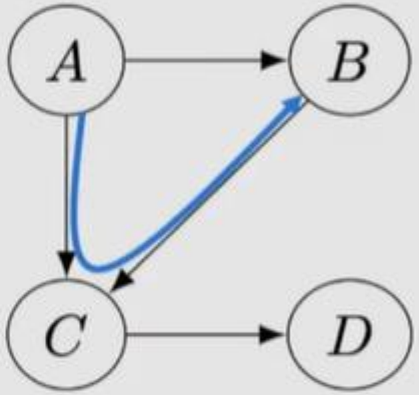
# Graph terminology

Path




Directed Path

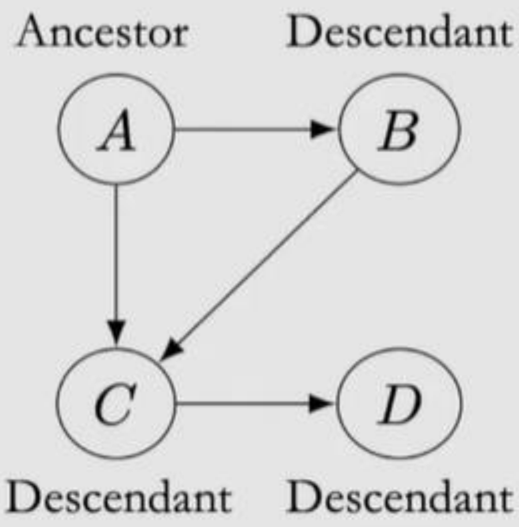
Path




Undirected Path

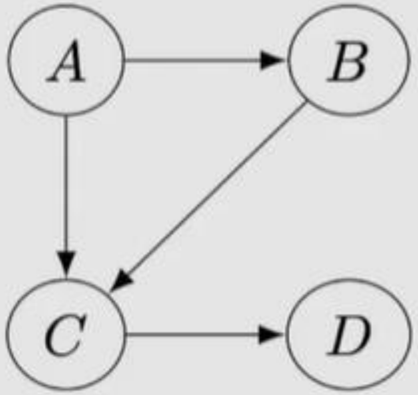
# Graph terminology

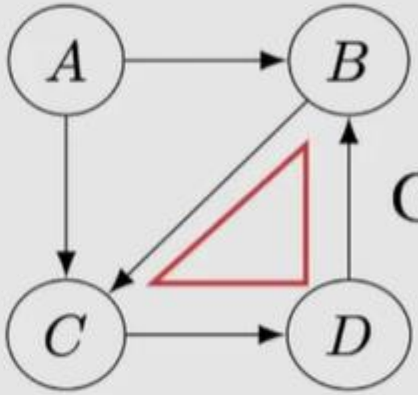
'A'로부터
Reach되는 모든 Node는
Descendant



# Graph terminology

Directed Acyclic Graph (DAG)



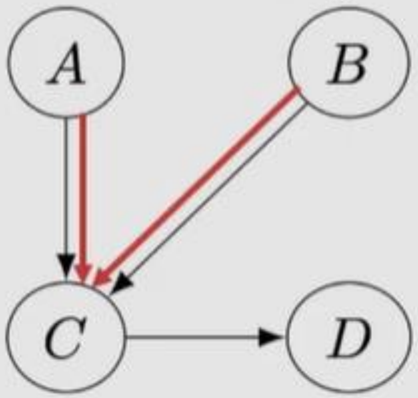




Cycle

# Graph terminology

Directed Acyclic Graph (DAG)

Immorality




두 부모노드가
하나의 자식 노드를
공유하는데,
서로 연결됭있지
않은 상태

V struct

Graph Terminology

**Bayesian networks and causal graph**

The basic building blocks of graph

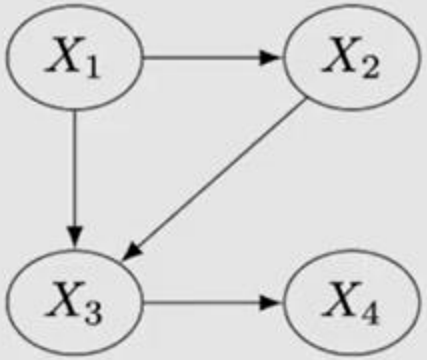
The flow of association and causation

# Naively modeling the joint distribution

Joint distribution

Statistical modeling (no causality): $P(x_1, x_2, \ldots, x_n) = P(x_1) \prod_i P(x_i \mid x_{i-1}, \ldots, x_1)$

$P(x_1, x_2, x_3, x_4) = P(x_1)\, P(x_2 \mid x_1)\, P(x_3 \mid x_2, x_1)\, P(x_4 \mid x_3, x_2, x_1)$ Chain rule




| $x_1$ | $x_2$ | $x_3$ | $P(x_4 \mid x_3, x_2, x_1)$ |
|---|---|---|---|
| 0 | 0 | 0 | $\alpha_1$ |
| 0 | 0 | 1 | $\alpha_2$ |
| 0 | 1 | 0 | $\alpha_3$ |
| 0 | 1 | 1 | $\alpha_4$ |
| 1 | 0 | 0 | $\alpha_5$ |
| 1 | 0 | 1 | $\alpha_6$ |
| 1 | 1 | 0 | $\alpha_7$ |
| 1 | 1 | 1 | $\alpha_8$ |

$2^{n-1}$ parameters!

Bayesian network로 그리면
X4가 x3에만 영향을 받아서 계산됨



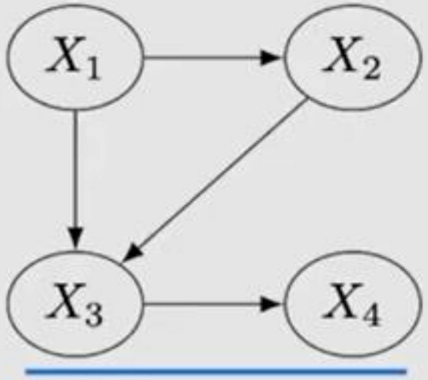
Table로 계산하면 2^(n-1) parameter

# Local Markov assumption

Given its parents in the DAG, a node X is independent of all of its non-descendants.

$$\underline{P(x_4 \mid x_3)}$$

$$P(x_1, x_2, x_3, x_4) = P(x_1)\, P(x_2 \mid x_1)\, P(x_3 \mid x_2, x_1)\, \underline{P(x_4 \mid x_3}, x_2, x_1)$$



# Local Markov assumption

Given its parents in the DAG, a node X is independent of all of its non-descendants.

$$P(x_1, x_2, x_3, x_4) = P(x_1)\, P(x_2 \mid x_1)\, P(x_3 \mid x_2, x_1)\, P(x_4 \mid x_3)$$
$$P(x_1, x_2, x_3, x_4) = P(x_1)\, P(x_2)\, P(x_3 \mid x_1)\, P(x_4 \mid x_3)$$

**Question:**
How will the factorization change now?

$$\begin{array}{ccc} X_1 & & X_2 \\ \downarrow & & \\ X_3 & \longrightarrow & X_4 \end{array}$$

# Bayesian network factorization

$$P(x_1, \ldots, x_n) = \prod_i P(x_i \mid \text{pa}_i)$$

local Markov assumption $\implies$ Bayesian network factorization

local Markov assumption $\impliedby$ Bayesian network factorization

See Chapter 3 of Koller & Friedman (2009) book for proofs

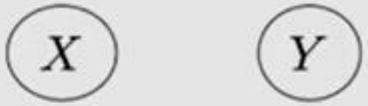
# Minimality assumption

1. Given its parents in the DAG, a node X is independent of all its non-descendants (local Markov assumption).
2. Adjacent nodes in the DAG are dependent.

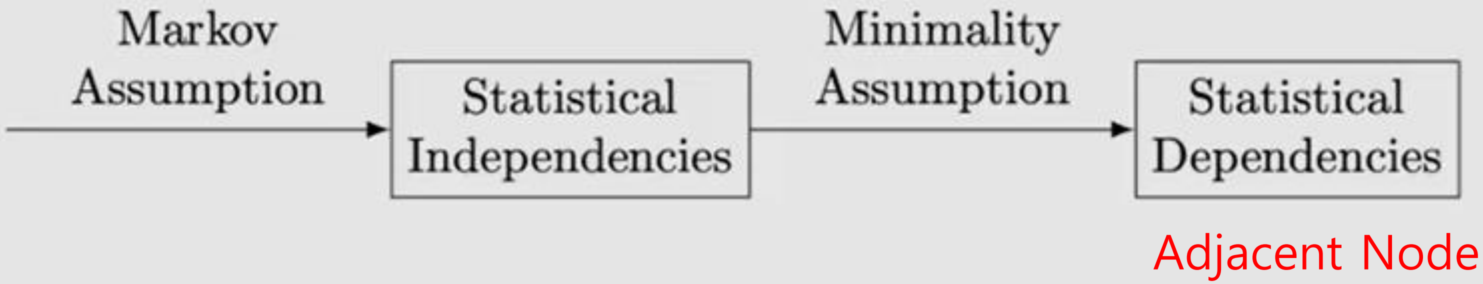

2번에 의해서 제외 (independent)

Permits distributions where $P(x, y) = P(x)\,P(y \mid x)$ ~~and also where~~

$$P(x, y) = P(x)\,P(y)$$



# Assumptions flowchart



# What is a cause?

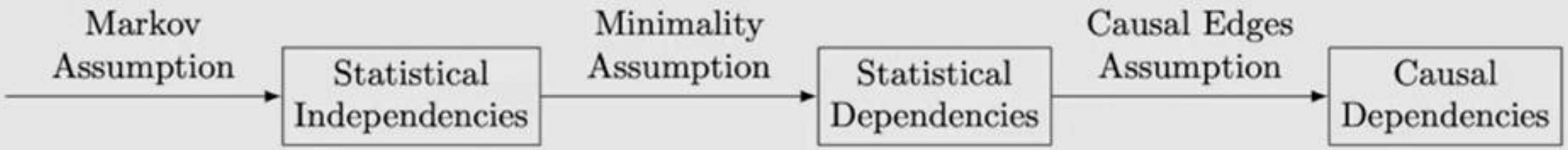
A variable X is said to be a cause of a variable Y if Y can change in response to changes in X.

Y가 X에 의해서 변화가 될 때

# Causal edges assumption

In a directed graph, every parent is a direct cause of all its children.

# Assumptions flowchart




DAG + Two assumptions to give us flow of association and causation in graphs:

1. Markov Assumption
2. Causal Edges Assumption

Minimality assumption은 2번과 동일

Graph Terminology

Bayesian networks and causal graph

**The basic building blocks of graph**

The flow of association and causation

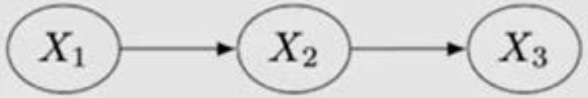
Association이 어떻게 flow하는지

# Graphical building blocks

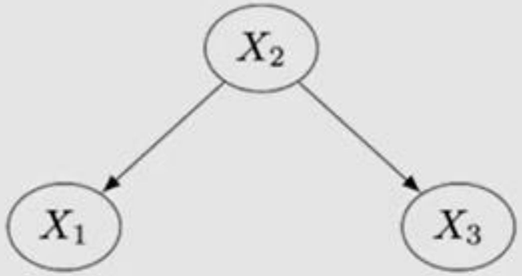
Two nodes: 

or 

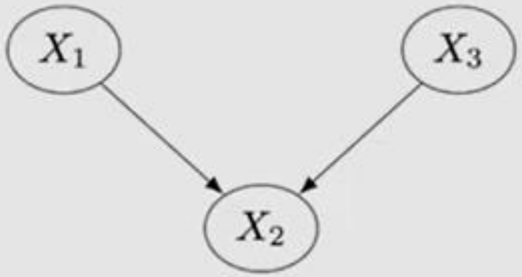

Chain



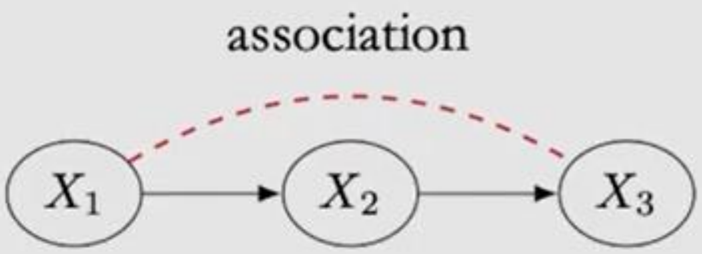

Fork



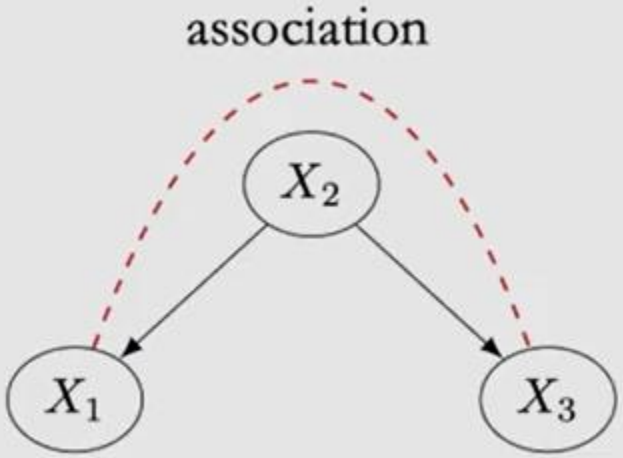

Immorality



# Chains and forks: independence




unblocked path

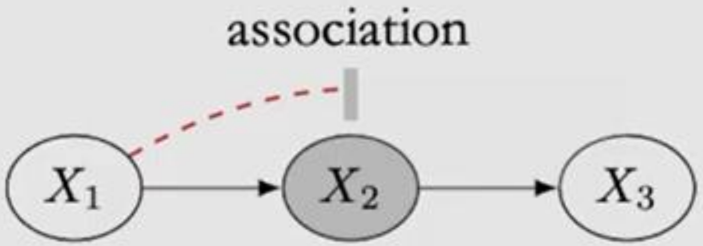



unblocked path

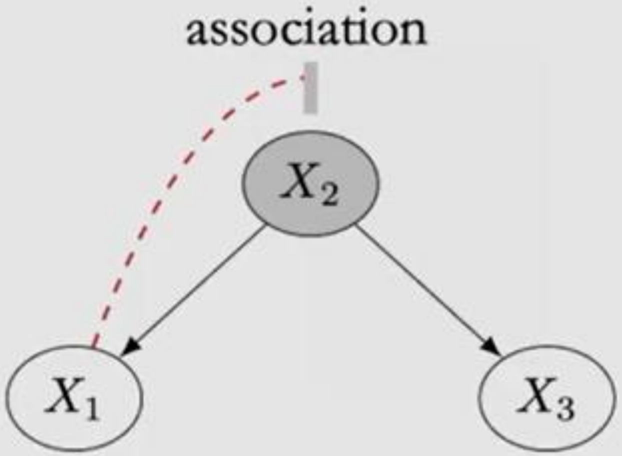
X1, x3는 association (statistical dependent)
-> flow

공통인 x2가 변하면, x1, x3가 같이 변하니까
(markov assumption; x2->x1, x2->x3)
X1, x3 association

# Chains and forks: independence




blocked path

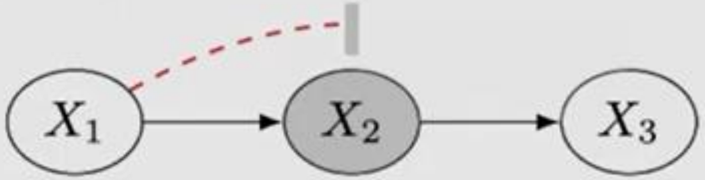



blocked path

X2가 conditioning되면
X3는 x2한테만 영향을 받으니까,
X1, x3는 conditionally independent, not association

# Proof of conditional independence in chains

Goal: show $P(x_1, x_3 \mid x_2) = \boxed{P(x_1 \mid x_2)} P(x_3 \mid x_2)$




1. Bayesian network factorization: $P(x_1, x_2, x_3) = P(x_1)\, P(x_2|x_1)\, P(x_3|x_2)$

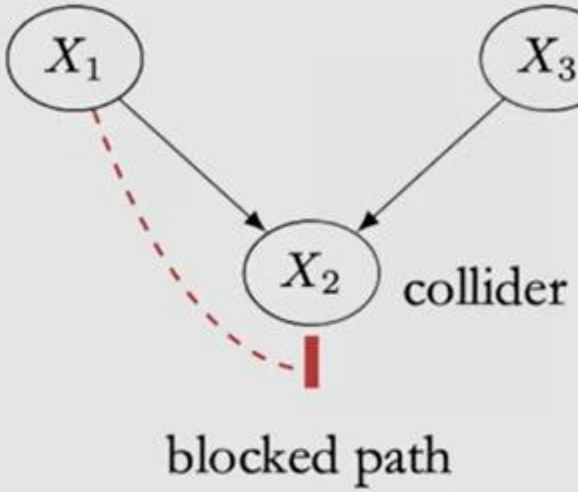
2. Apply Bayes' rule: $P(x_1, x_3 \mid x_2) = \dfrac{\boxed{P(x_1)\, P(x_2|x_1)}\, P(x_3|x_2)}{\boxed{P(x_2)}}$

2. Conditioning
-> bayes' rule

3. Apply Bayes' rule again: $P(x_1, x_3 \mid x_2) = \dfrac{P(x_1, x_2)}{P(x_2)}\, P(x_3|x_2)$

$= P(x_1|x_2)\, P(x_3|x_2)$ Conditionally independent

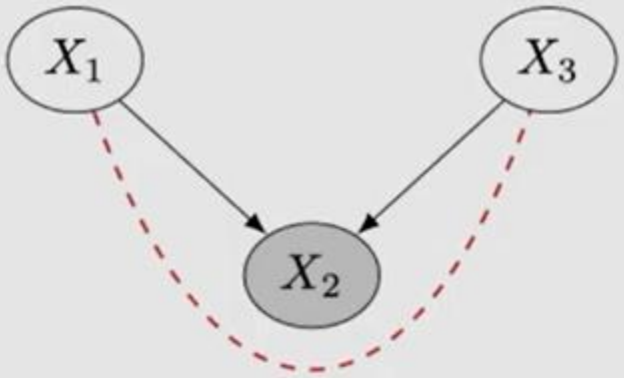
# Immoralities




$$
\begin{aligned}
P(x_1, x_3) &= \sum_{x_2} P(x_1, x_2, x_3) \\
&= \sum_{x_2} P(x_1)\, P(x_3)\, P(x_2 \mid x_1, x_3) \\
&= P(x_1)\, P(x_3) \sum_{x_2} P(x_2 \mid x_1, x_3) \\
&= P(x_1)\, P(x_3)
\end{aligned}
$$

V structure는 영향이 흐르지 않는다 (Immoralities)
X1, x3 각각 x2의 원인일 뿐, dependent하지 않다.

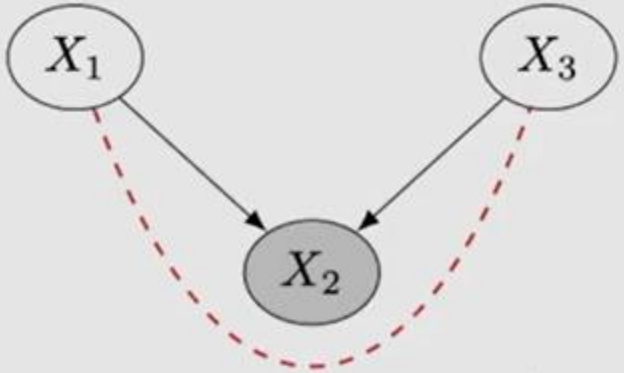
# Immoralities: conditioning on the collider




unblocked path

# Example: good-looking men are jerks

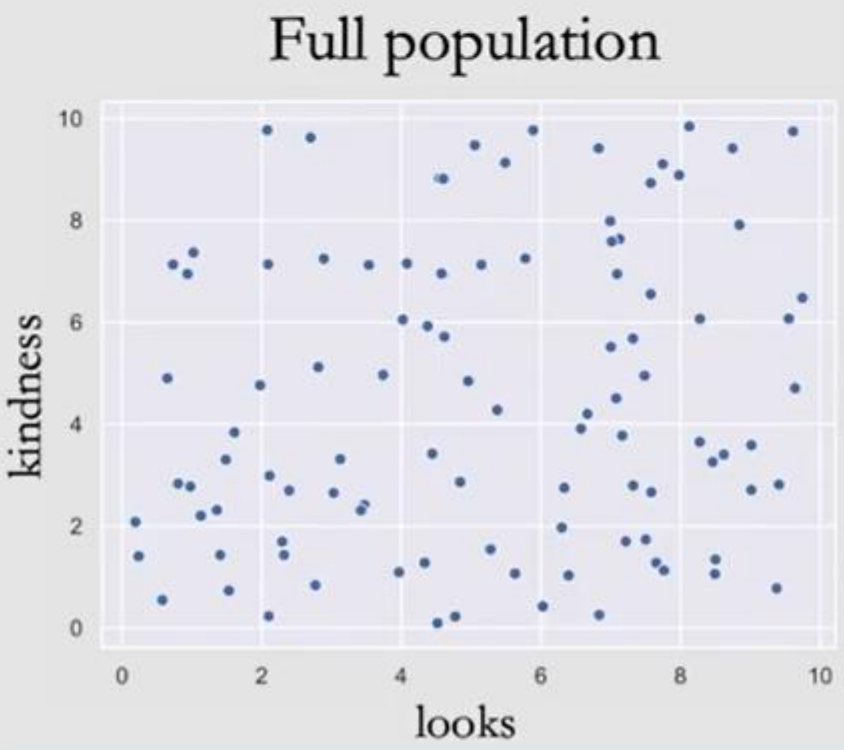
$$X_1 = \begin{cases} 1 & \text{good-looking} \\ 0 & \text{otherwise} \end{cases} \qquad X_3 = \begin{cases} 1 & \text{kind} \\ 0 & \text{jerk} \end{cases}$$

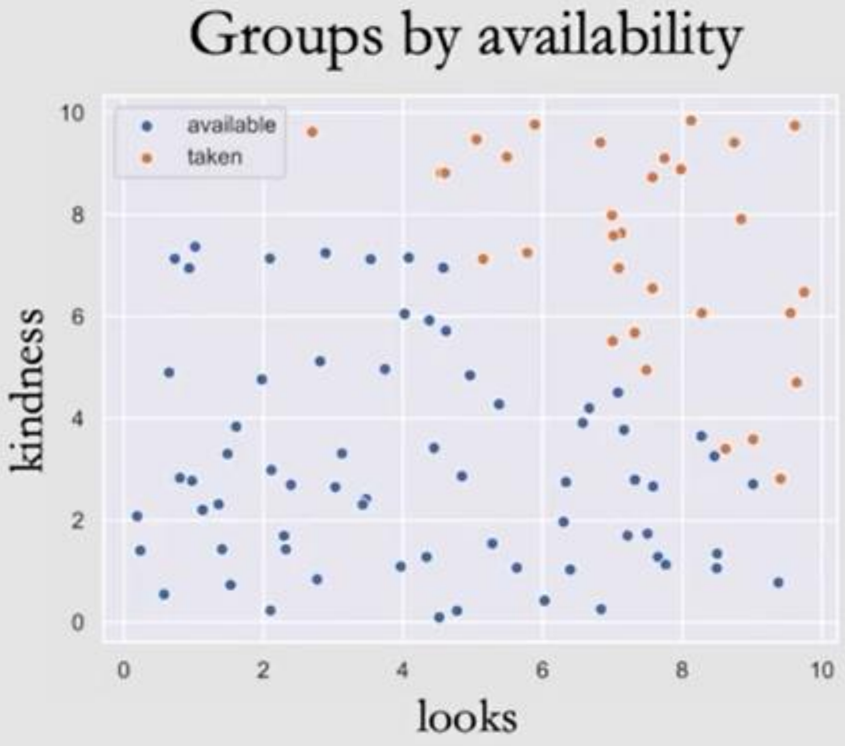



$$X_2 = X_1 \text{ AND } X_3 = \begin{cases} 1 & \text{in relationship} \\ 0 & \text{not in relationship} \end{cases}$$

X2 = remaining man이면 (condition)
X1 = 0 , x2 = 0
X1 = 1, x2 = 0
X1 = 0, x2 = 1
로 정해져 있기 때문에, x1=1이면 x2=0 (dependent)

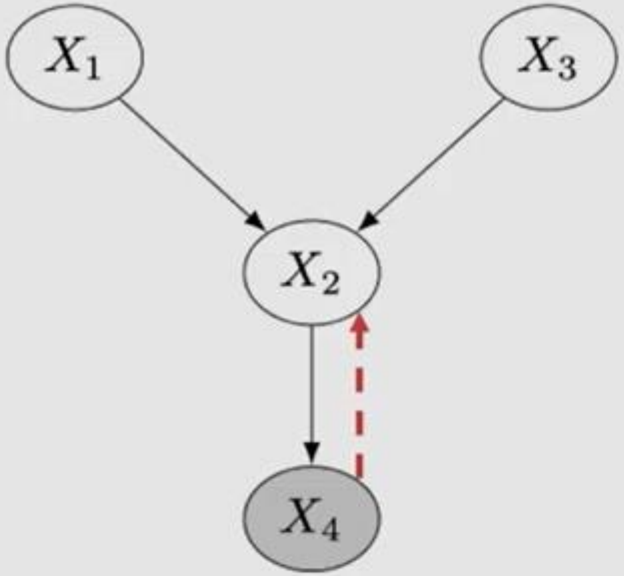
# Good-looking men are jerks scatterplot



# Conditioning on descendants of colliders

$$X_1 \not\perp\!\!\!\perp X_3 \mid X_4$$



Graph Terminology

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# Blocked path definition

A path between nodes X and Y is blocked by a (potentially empty) conditioning set Z if either of the following is true:

1. Along the path, there is a chain $\cdots \to W \to \cdots$ or a fork $\cdots \leftarrow W \to \cdots$ where $W$ is conditioned on ($W \in Z$).
2. There is a collider $W$ on the path that is not conditioned on ($W \notin Z$) and none of its descendants are conditioned on ($\mathrm{de}(W) \not\subseteq Z$).

Unblocked path: a path that is not blocked

1. chain, Fork 에서 W가 condition되거나
2. V structure에서 W가 not-condition되거나

# d-separation

Two (sets of) nodes X and Y are d-separated by a set of nodes Z if all of the paths between (any node in) X and (any node in) Y are blocked by Z.

Theorem: Given that $P$ is Markov with respect to $G$,

$$X \perp\!\!\!\perp_G Y \mid Z \implies X \perp\!\!\!\perp_P Y \mid Z$$

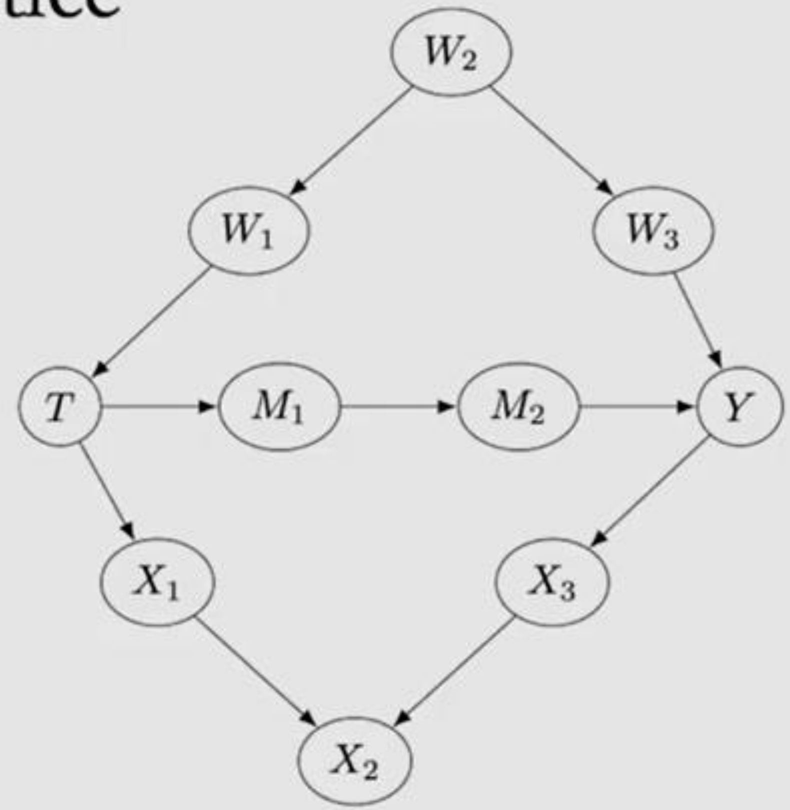
$$\underbrace{\text{local Markov assumption} \iff \text{global Markov assumption}}_{\text{Markov assumption}}$$



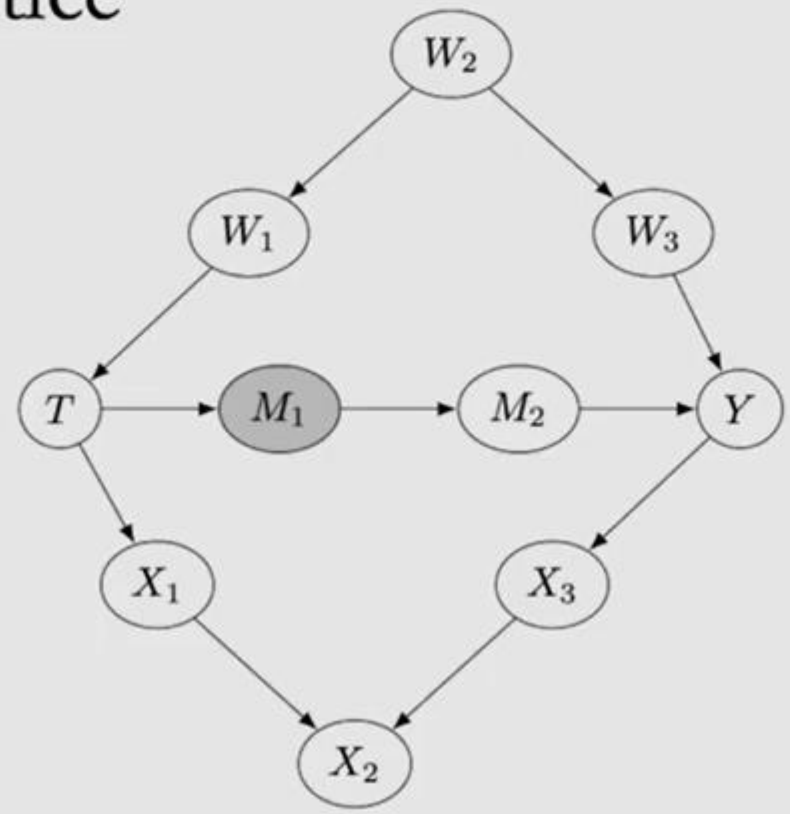
Z에 의해서 X, Y가 flow가 끊어지면 d-separation
-> probability도 conditional independent

# d-separation practice

T랑 Y가 d-seperated인가?
No, M1, M2로 Flow흐르니까



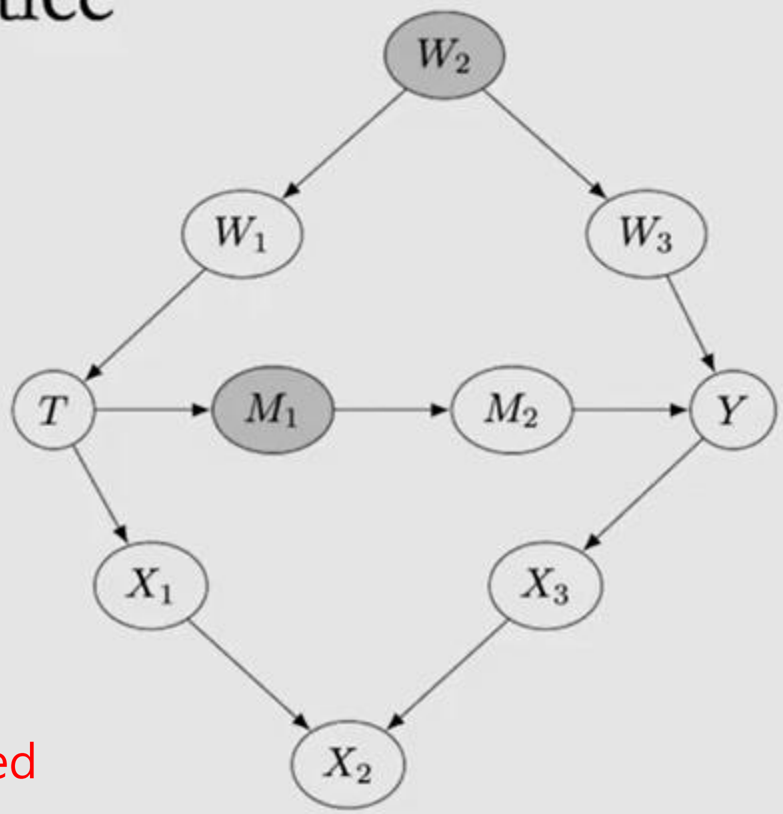
# d-separation practice




M1가 condition 되었을때,
T랑 Y가 d-seperated인가?

No, w1, w2, w3로 흐르니까

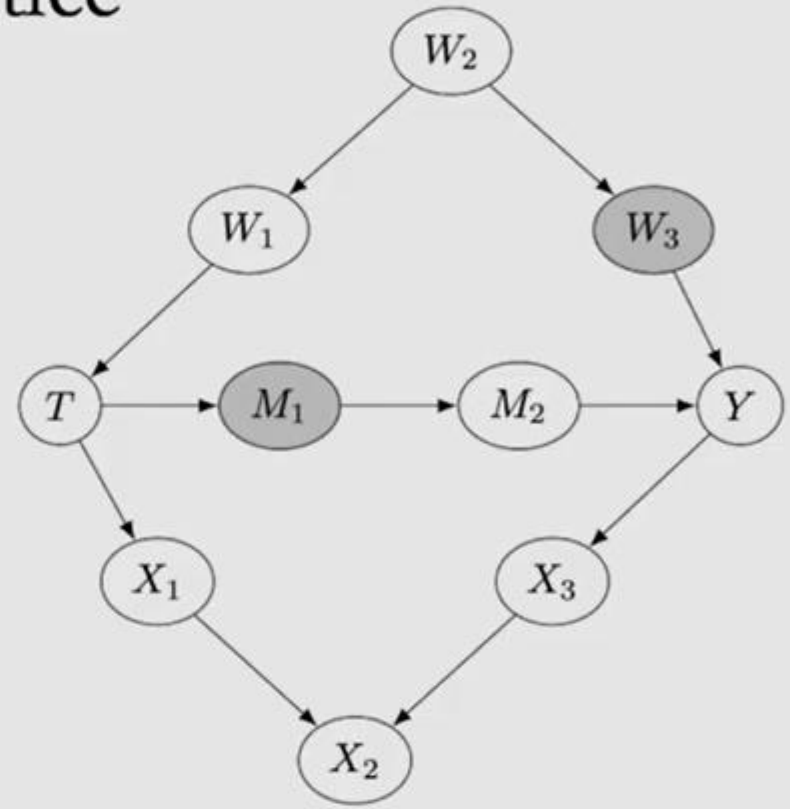
# d-separation practice




M1, W2가 condition 되었을때,
T랑 Y가 d-seperated인가?

Yes,
Chain 도 막혀 (m1, m2)
Fork도 막혀 (w1, w2, w3)
V-structure는 원래 못가 (X1, X2, X3)
이제 flow가 안흐르니까 T, Y는 de-seperated

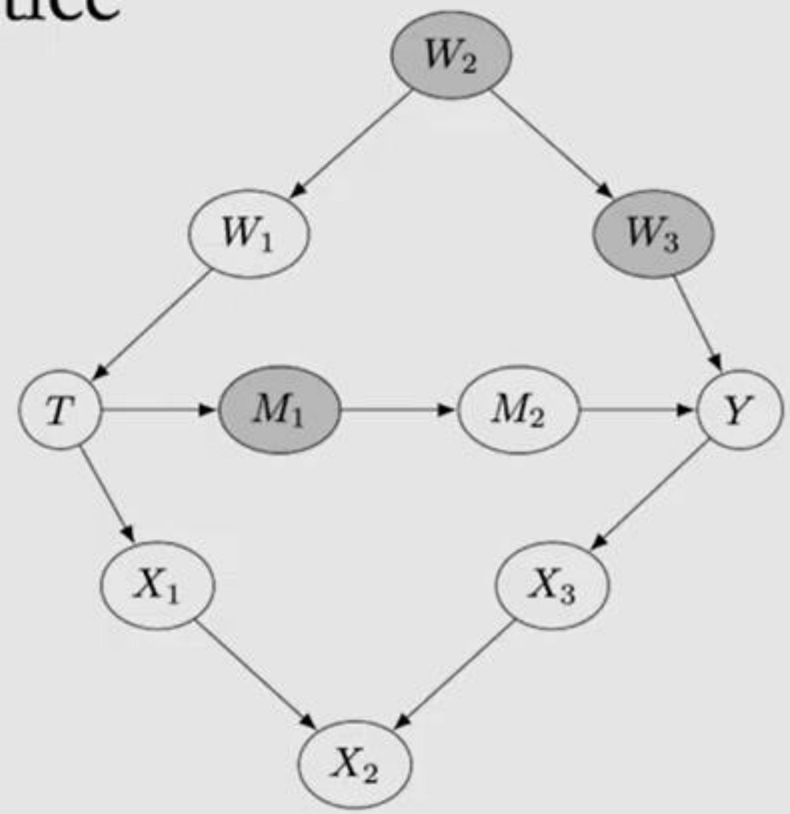
# d-separation practice




M1, W3가 condition 되었을때,
T랑 Y가 d-seperated인가?

Yes,
W2랑 동일

# d-separation practice

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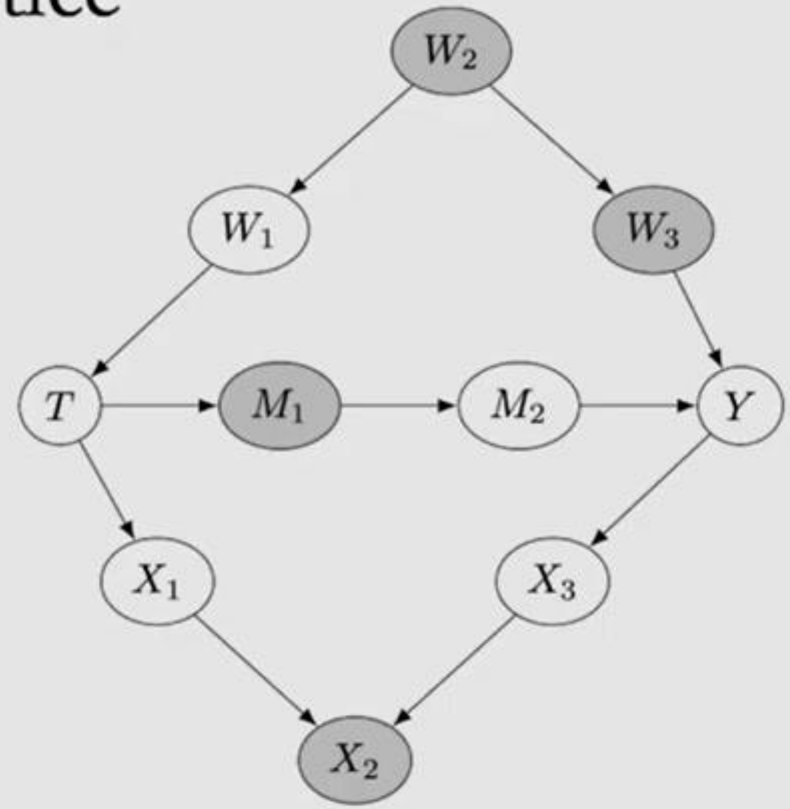
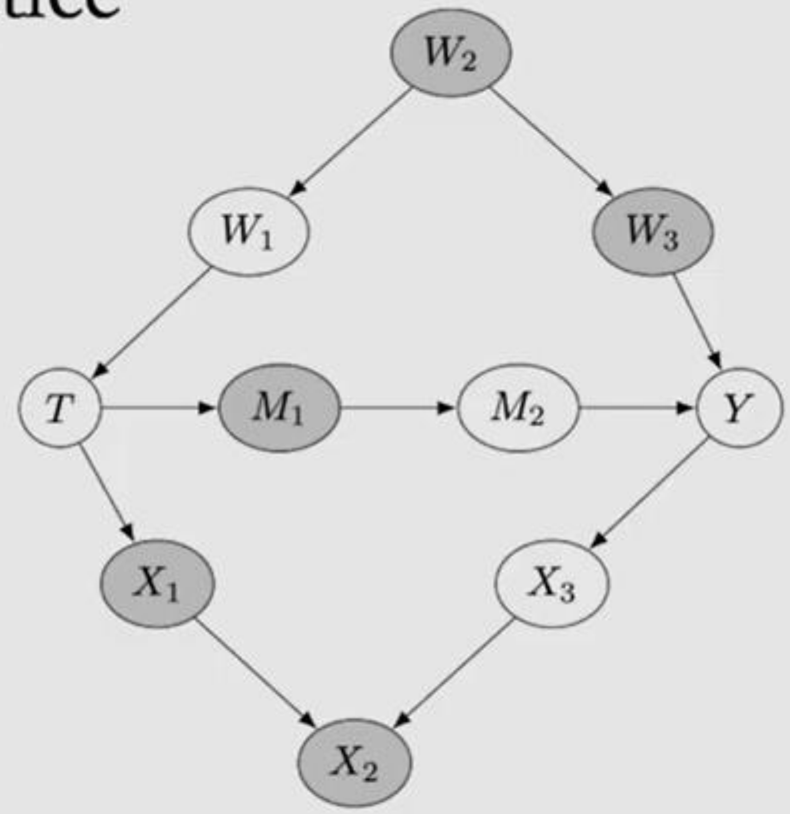
# d-separation practice




X2가 condition 되었을때,
T랑 Y가 d-seperated인가?

No,
X2가 condition되어서 flow흐르므로
T, Y는 not d-seperated

# d-separation practice

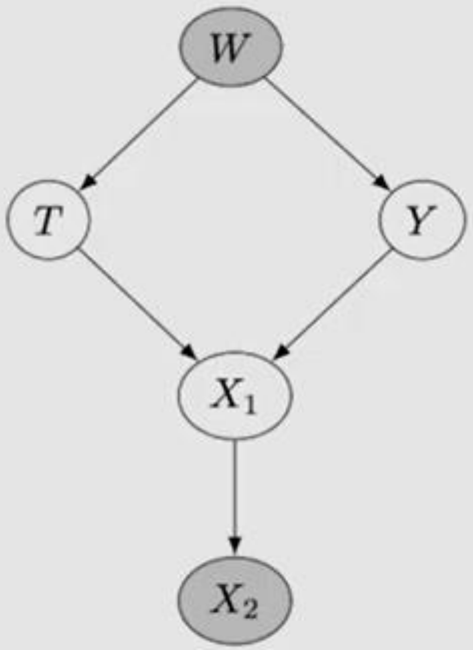


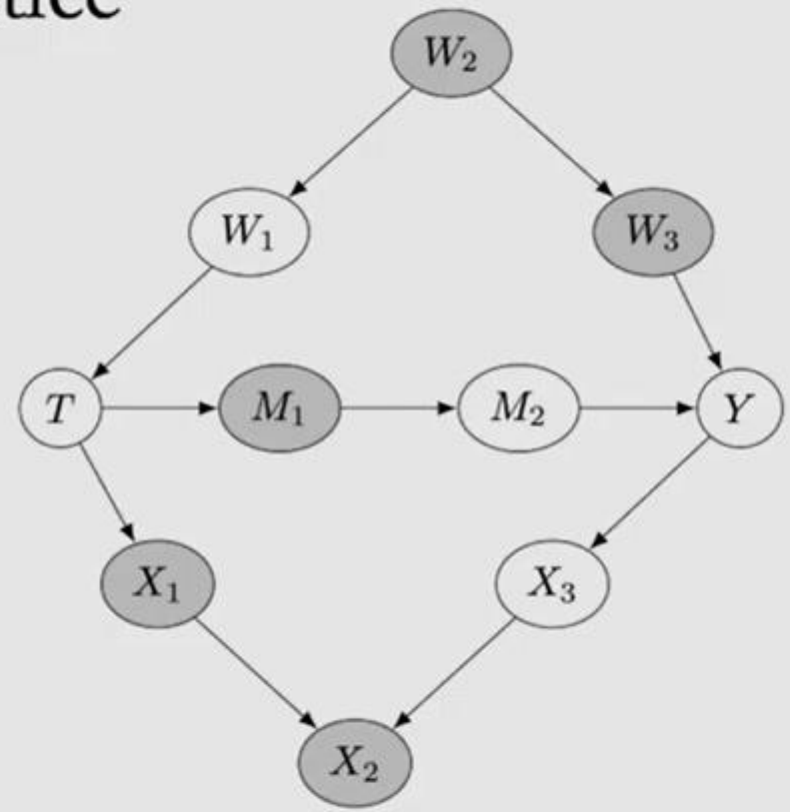

X1가 condition 되었을때,
T랑 Y가 d-seperated인가?

Yes,
X2가 흐르게 했던 것을
X1이 막아버림
(x2는 T의 영향을 못받음)

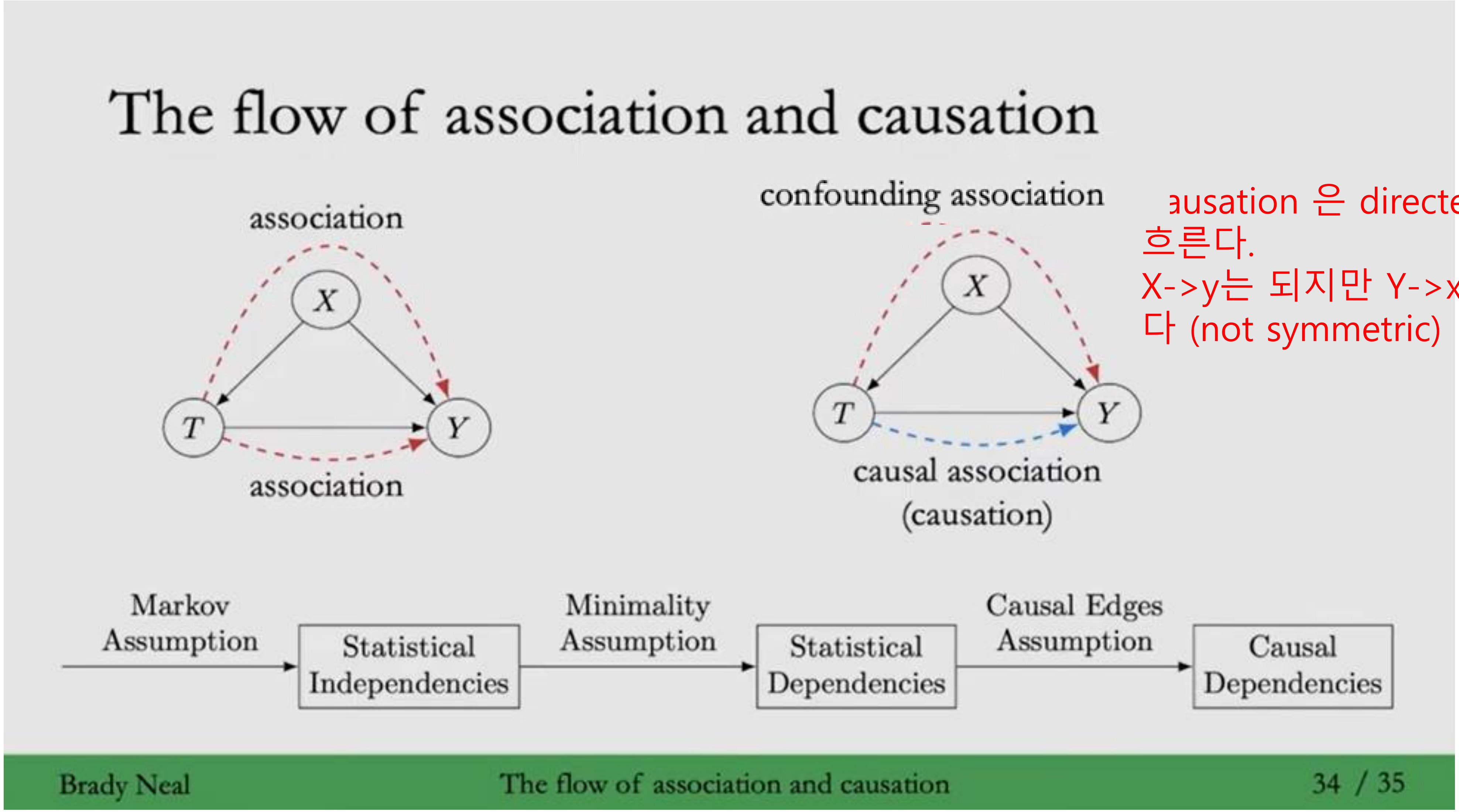
따라서, T랑 Y는 d-seperated

# d-separation practice

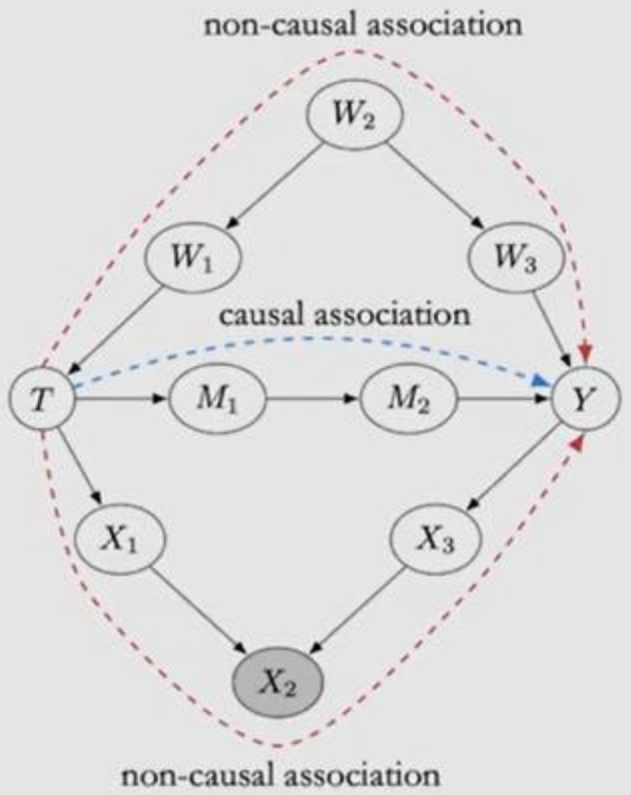



X2때문에
V-structure 가
Flow흘러서
Not d-seperation




ausation 은 directed path로 흐른다.
X->y는 되지만 Y->x는 안된다 (not symmetric)


non-causal association
$W_2$
$W_1$
$W_3$
causal association
$T$
$M_1$
$M_2$
$Y$
$X_1$
$X_3$
$X_2$
non-causal association