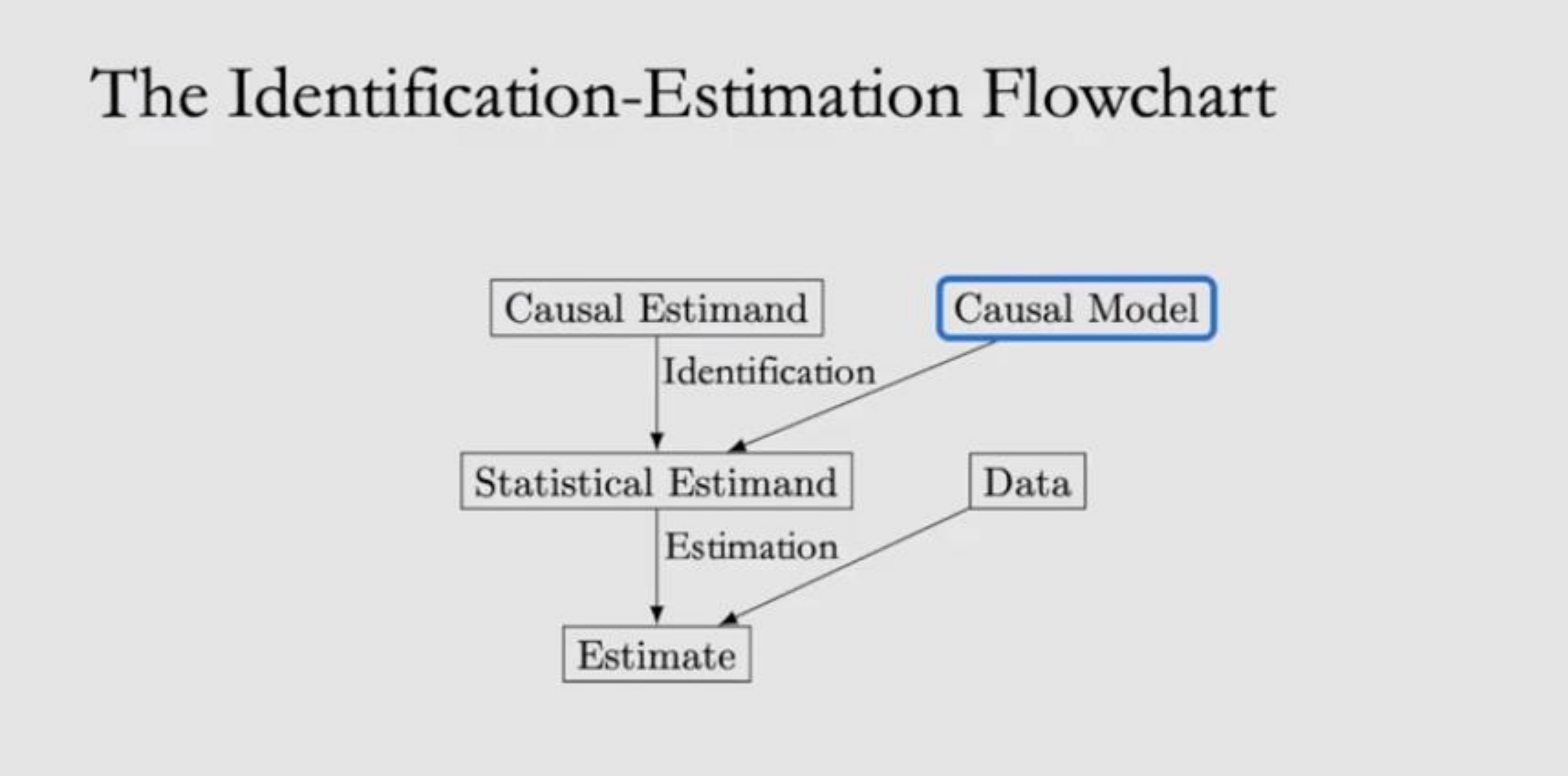
Causal Inference
Lecture 4

210428 전은주

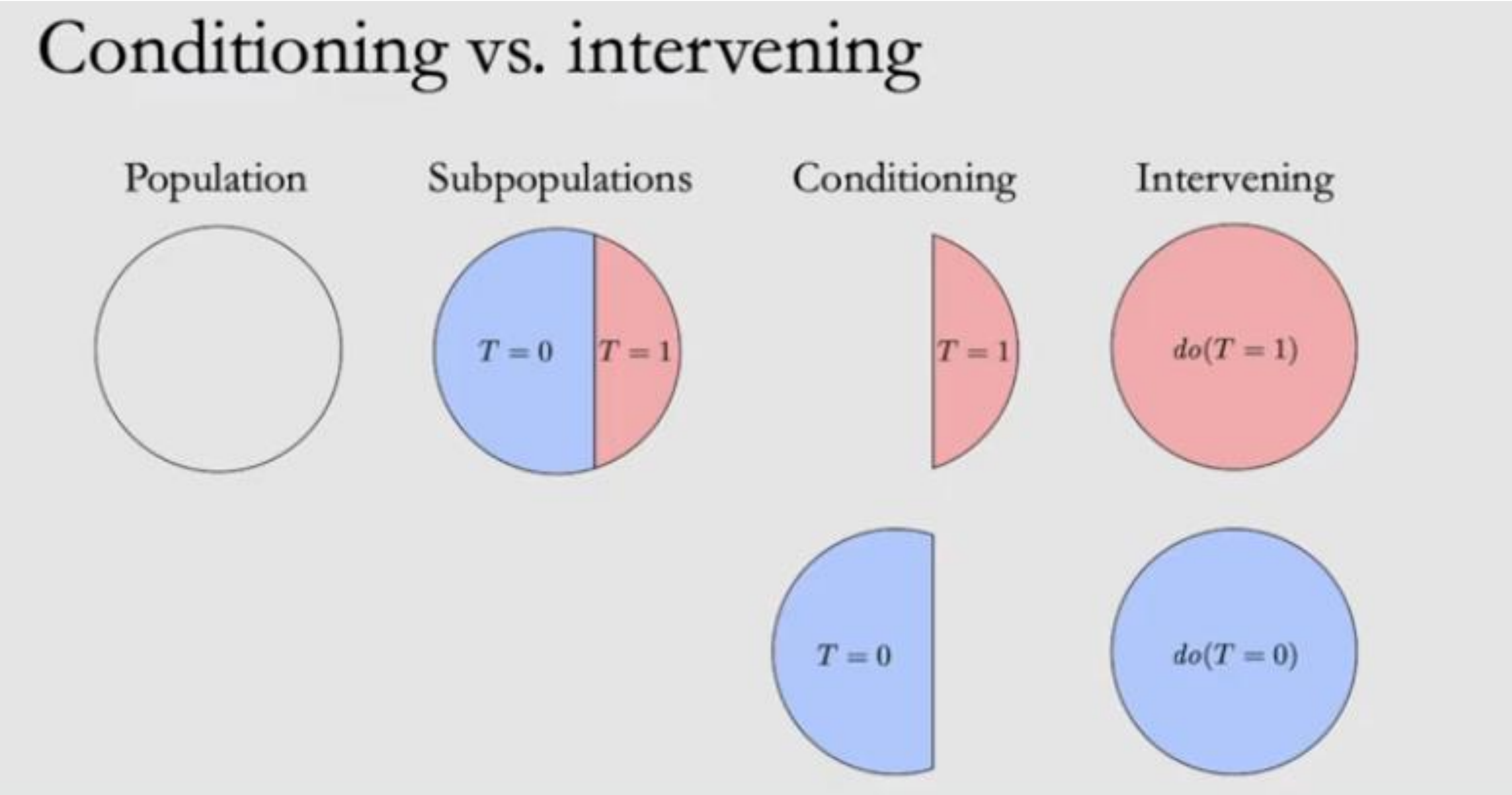
Contents

4.1 The Identification Estimation Flowchart and Lecture Intro/Outline



Lecture 4에서는 Causal Model (Graph Model)을 통한 Statistical Estimation에 대해서 논의하겠다.

4.2 Intervening, the do-operator, and identifiability



Conditioning은 조건부 확률, 즉 t=1, t=0에 대한 데이터를 나눠서 (sub-set)으로 계산
Intervening은 do(T=1)으로 전체 Population에 적용

4.2 Intervening, the do-operator, and identifiability

Some notation and terminology

Interventional distributions:

$$P(Y(t) = y) \triangleq P(Y = y \mid do(T = t)) \triangleq P(y \mid do(t))$$

같은 식

Average treatment effect (ATE):

$$\mathbb{E}[Y \mid do(T = 1)] - \mathbb{E}[Y \mid do(T = 0)]$$

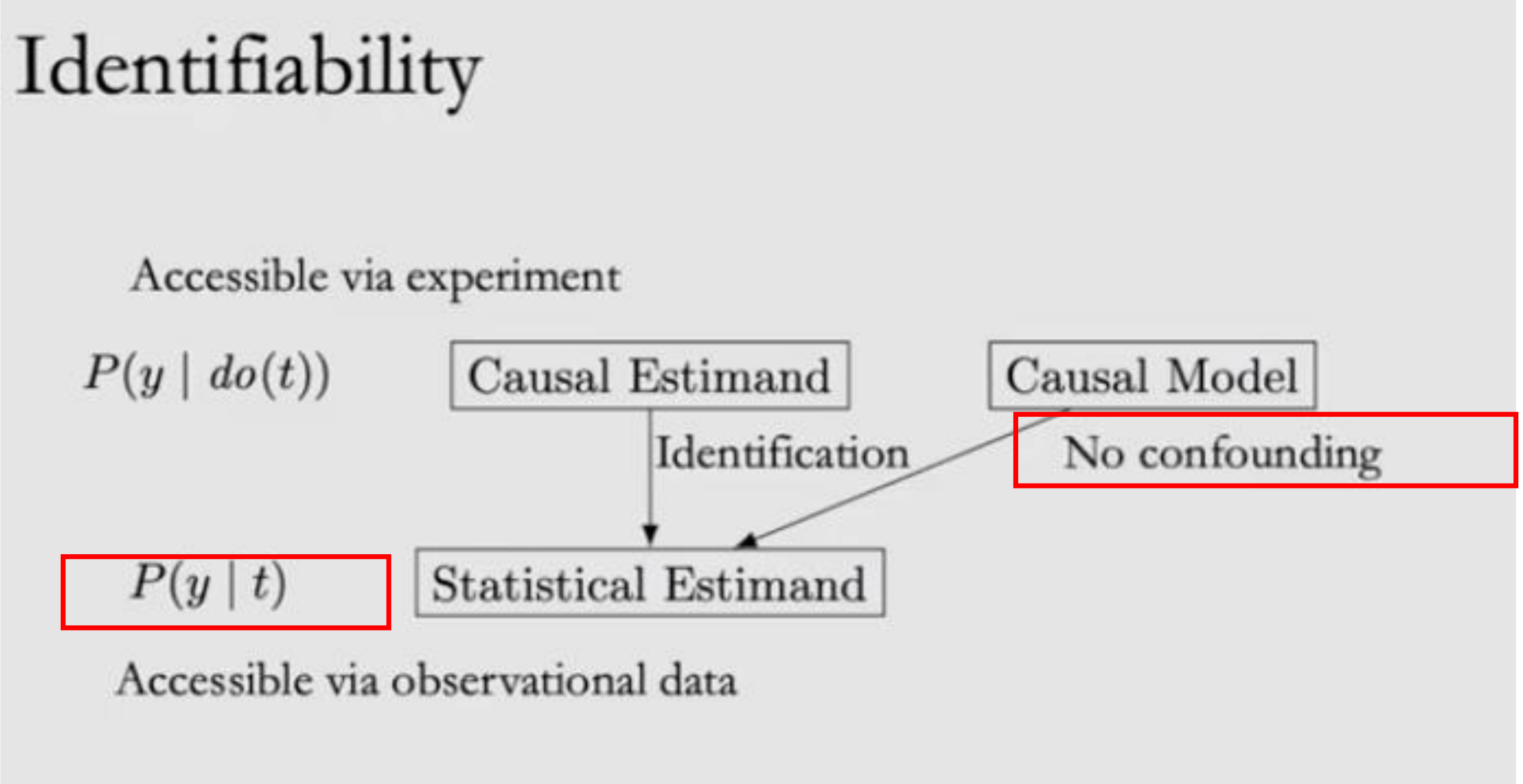
두개의 다른 intervention의 평균 차이

Observational	Interventional
$P(Y, T, X)$	$P(Y \mid do(T = t))$
$P(Y \mid T = t)$	$P(Y \mid do(T = t), X = x)$

Joint distribution (조건부 확률)
=> data가지고 뽑아낼 수 있음

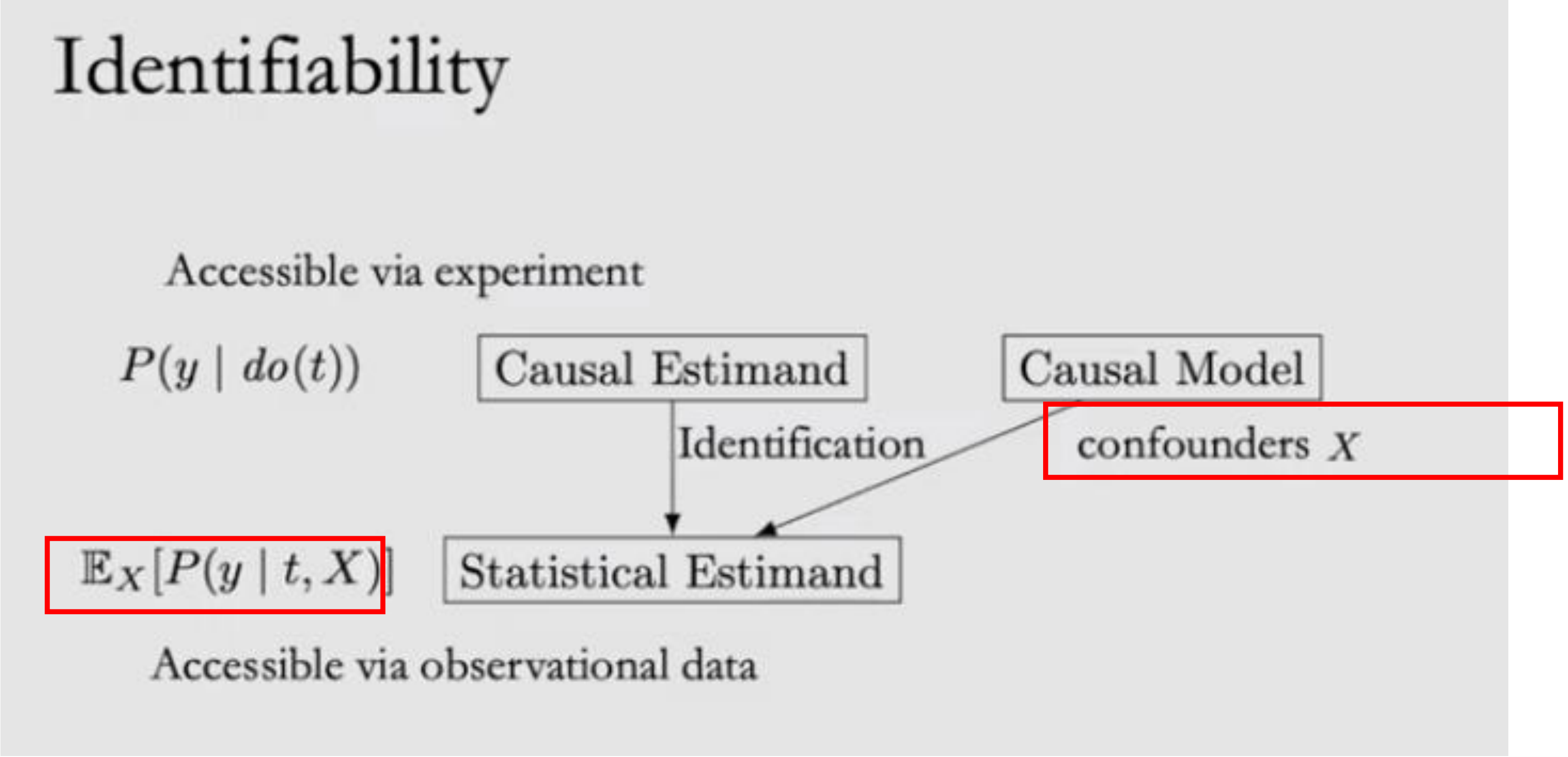
Interventional distribution
X => covariance에 대한 conditioning
=> 실험이 필요함

4.2 Intervening, the do-operator, and identifiability



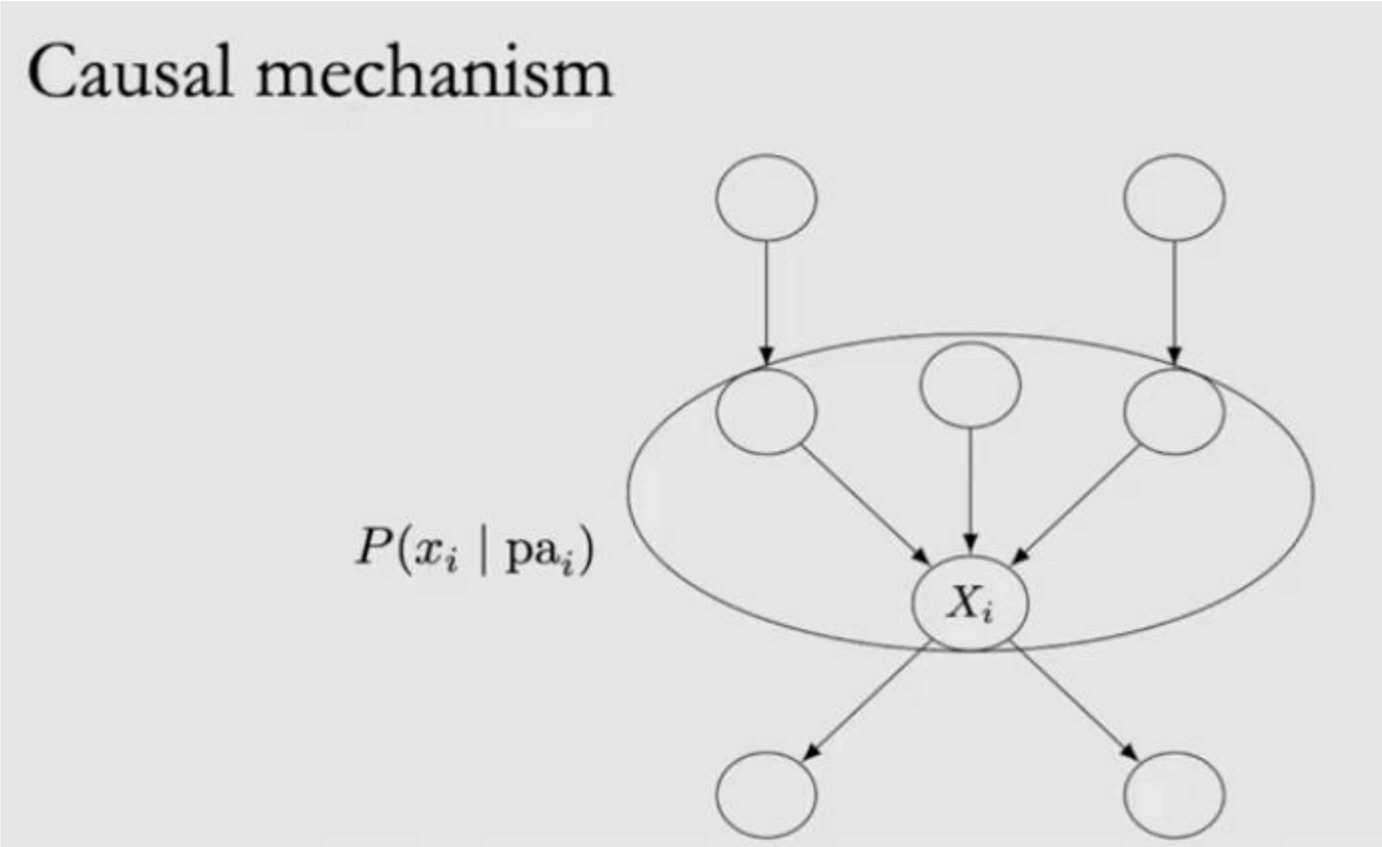
Identification: do operation에서 조건부 확률로 넘어갈 수 있을 때 (causal model로 confound 제거 후)

4.2 Intervening, the do-operator, and identifiability



Causal Model 확인했을 때 Y랑 T에 둘 다 영향을 미치는 confounder X가 있다면
조건부 확률은, t와 X가 주어졌을 때의 기대값으로 변경된다

4.3 Causal Mechanisms and the Modularity assumption



부모 노드의 변화에 의해서 x가 변함

4.3 Causal Mechanisms and the Modularity assumption

Modularity assumption

If we intervene on a node X_i, then only the mechanism $P(x_i \mid \mathrm{pa}_i)$ changes. All other mechanisms $P(x_j \mid \mathrm{pa}_j)$ where $i \neq j$ remain unchanged.

In other words, the causal mechanisms are **modular**.

Many names: independent mechanisms, autonomy, invariance, etc.

Modularity: 변화를 준 곳만 변화하고, 다른 곳에는 영향을 미치지 않는다

4.3 Causal Mechanisms and the Modularity assumption

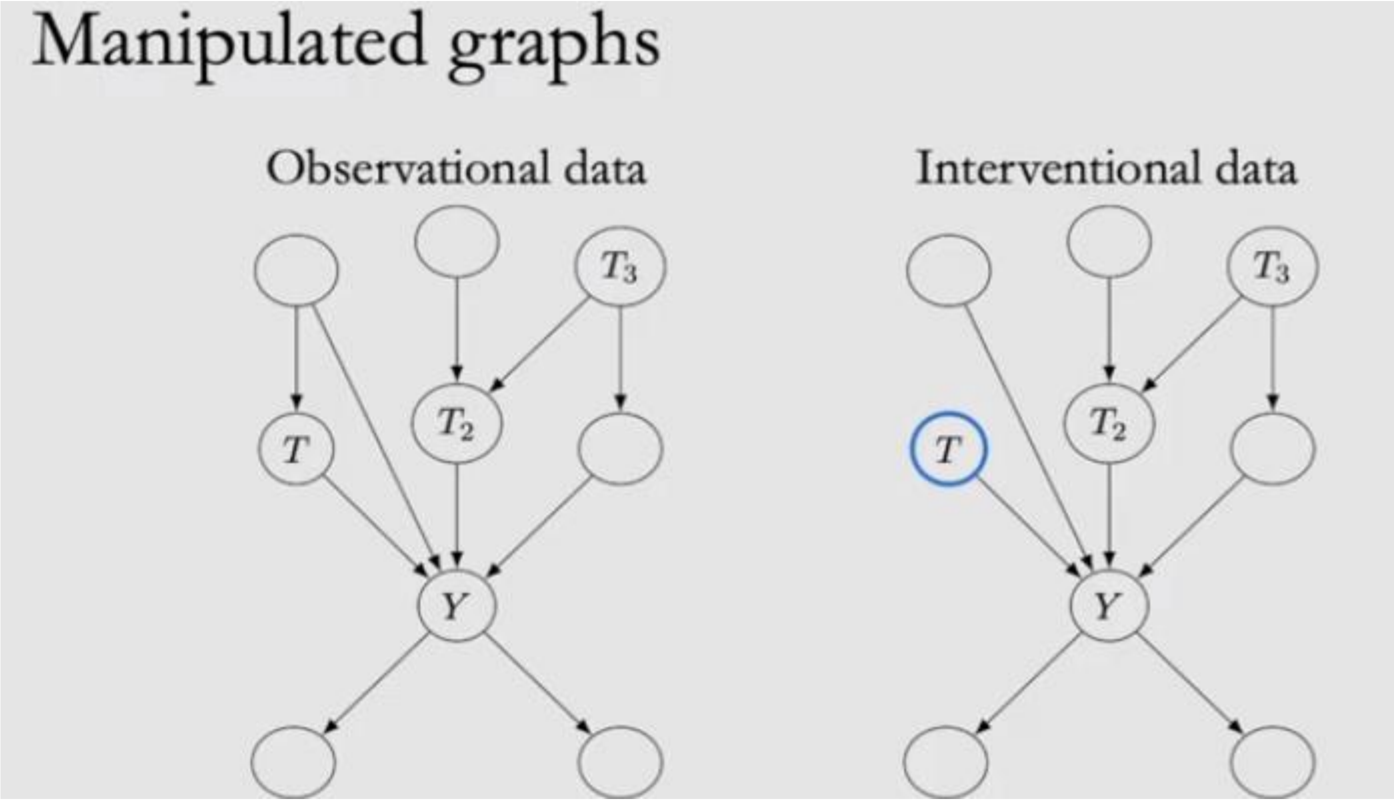
Modularity assumption: more formal

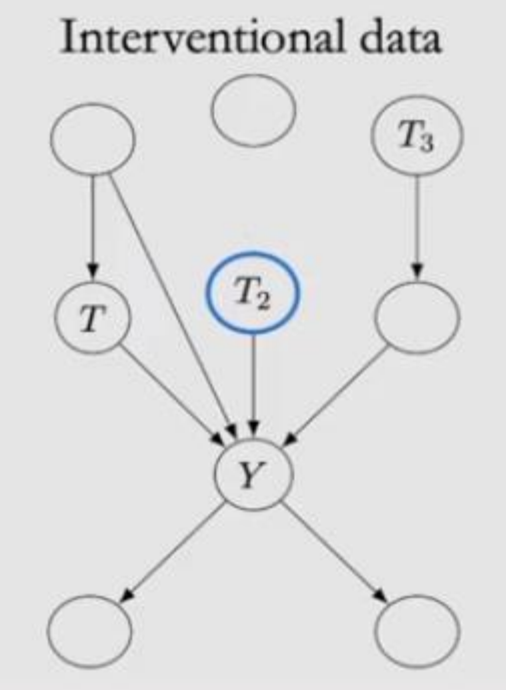
If we intervene on a set of nodes $S \subseteq [n]$, setting them to constants, then for all i, we have the following:

1. If $i \notin S$, then $P(x_i \mid \mathrm{pa}_i)$ remains unchanged.
2. If $i \in S$, then $P(x_i \mid \mathrm{pa}_i) = 1$ if $\underbrace{x_i \text{ is the value that } X_i \text{ was set to}}_{\text{consistent with the intervention}}$ by the intervention; otherwise, $P(x_i \mid \mathrm{pa}_i) = 0$.

I가 intervention set S에 안 속해 있으면, 원래 확률 그대로
I가 intervention set S이고, intervention하고 consistent하면1, inconsistent이면 0

4.3 Causal Mechanisms and the Modularity assumption





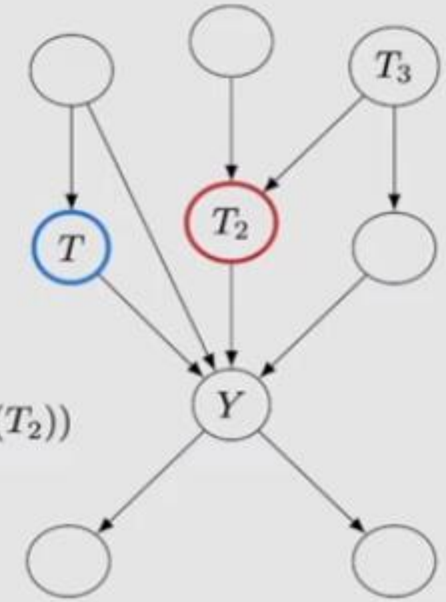
T만 변하고, t2, t3는 동일
T로 들어오는 edge삭제 됨 -> manipulated graph

4.3 Causal Mechanisms and the Modularity assumption

What would it mean if modularity is violated?

Intervention on T not only changes $P(T \mid \text{pa}(T))$

but also changes other mechanisms such as $P(T_2 \mid \text{pa}(T_2))$



Modularity가 적용되지 않는다면?? T1, T2 적용했을때 다 변화한다면 Intervention은 not local

4.4 The Truncated Factorization

Truncated factorization

Recall the Bayesian network factorization:

$$P(x_1, \ldots, x_n) = \prod_i P(x_i \mid \mathrm{pa}_i)$$

Truncated factorization:

$$P(x_1, \ldots, x_n \mid do(S = s)) = \prod_{i \notin S} P(x_i \mid \mathrm{pa}_i)$$

if x is consistent with the intervention.

Otherwise,

$$P(x_1, \ldots, x_n \mid do(S = s)) = 0$$

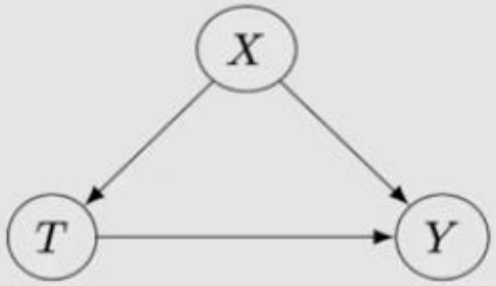
Bayesian Factorization

Truncated factorization -> I in intervention set S에 속할 때

4.4 The Truncated Factorization

Simple identification via truncated factorization

Goal: identify $P(y \mid do(t))$



Bayesian network factorization: $P(y, t, x) = P(x)\, P(t \mid x)\, P(y \mid t, x)$

Truncated factorization: $P(y, x \mid do(t)) = P(x)\, P(y \mid t, x)$

Marginalize: $P(y \mid do(t)) = \sum_x P(y \mid t, x)\, P(x)$

X는 Confounder일때

Bayesian factorization: 그래프 보고

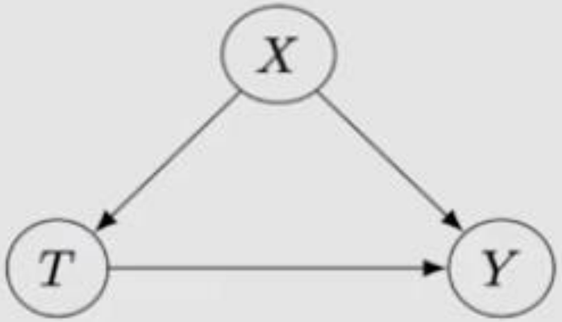
Truncated : do(t)

4.5 Another perspective on "Association is not Causation"

Association vs. causation revisited

$$\underline{P(y \mid do(t))} = \sum_x P(y \mid t, x)\, \underline{P(x)}$$

$$\underline{P(y \mid do(t))} \neq \underline{P(y \mid t)}$$

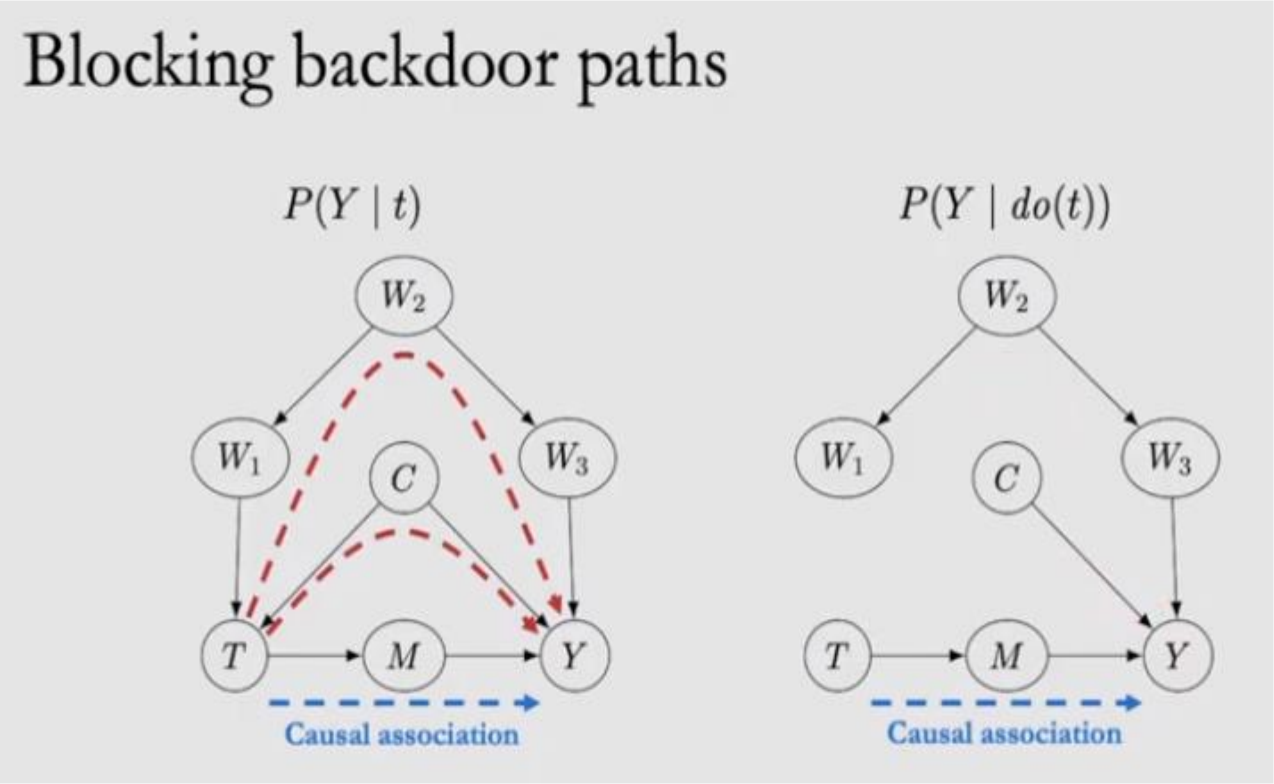


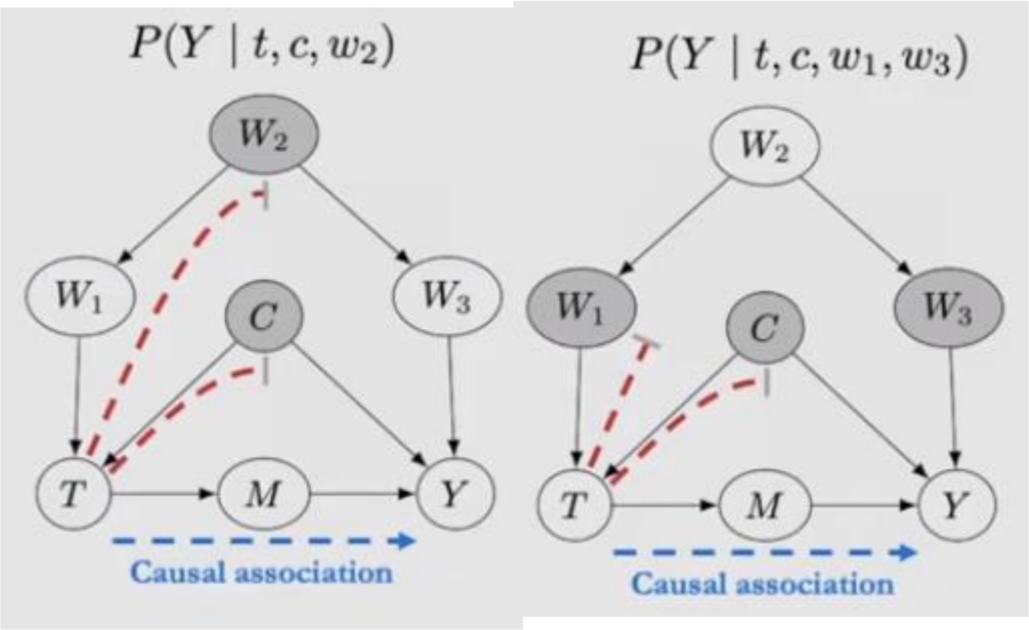
파란색 (Associatioin), 적색(Causation)
이므로 둘이 다름

$$\sum_x P(y \mid t, x)\, \underline{P(x \mid t)} = \sum_x P(y, x \mid t)$$

$$= \underline{P(y \mid t)}$$

4.6 The Backdoor Adjustment





Identification에 대한 generalization
파란: causal path , 적색: backdoor path
Intervention distribution: T로 들어가는 모든 Edge를 막아서 실험
측정데이터 가지고 쉽게 하려면? W2, C에 conditioning

4.6 The Backdoor Adjustment

Backdoor criterion and backdoor adjustment

A set of variables W satisfies the backdoor criterion relative to T and Y if the following are true:

1. W blocks all backdoor paths from T to Y
2. W does not contain any descendants of T

Given the modularity assumption and that W satisfies the backdoor criterion, we can identify the causal effect of T on Y:

$$P(y \mid do(t)) = \sum_{w} P(y \mid t, w)\, P(w)$$

1. W에 conditioning하면 T, Y로 가는 path막는다
2. 하지만 W의 자녀들을 막진 않는다

W가 backdoor criteria에 포함된다면, **W는 sufficient adjust set** 이라고 부른다

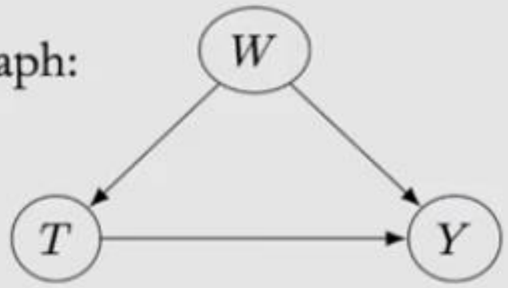
: W is sufficient to adjust for to get the causal effect of T on Y

4.6 The Backdoor Adjustment

Proof of backdoor adjustment

$$
\begin{aligned}
P(y \mid do(t)) &= \sum_w P(y \mid do(t), w)\, P(w \mid do(t)) \\
&= \sum_w P(y \mid t, w)\, P(w \mid do(t)) \\
&= \sum_w P(y \mid t, w)\, P(w)
\end{aligned}
$$

Example graph:

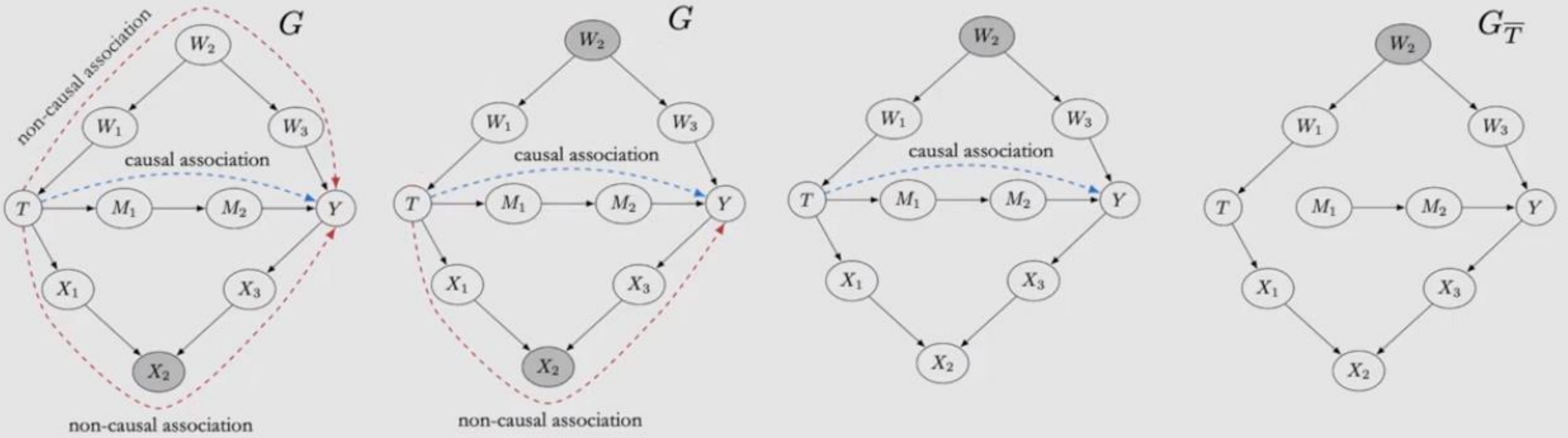


W가 T에 대해서 backdoor를 막으므로 (Y->W->t 못감) / T->Y만 남음 (causal path)

1. W blocks all backdoor paths from T to Y
2. W does not contain any descendants of T

Backdoor criterion as d-separation

$$Y \perp\!\!\!\perp_{G_{\overline{T}}} T \mid W$$



D-separation: flow가 흐르지 않게 한다!
1. W가 T->Y로 가는 backdoor path를 막고
2. W는 T의 자녀 노드가 아니고
3. T에서 나가는 out path가 제거 된다면,

T와 Y는 d-seperation이다

4.7 Structural Causal Models SCMs

Structural equations

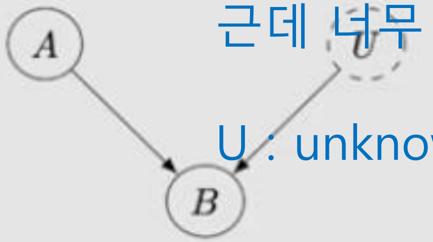
The equals sign does not convey any causal information.

$$B = A \quad \text{means the same thing as} \quad A = B$$

Structural equation for A as a cause of B:

$$B := f(A)$$

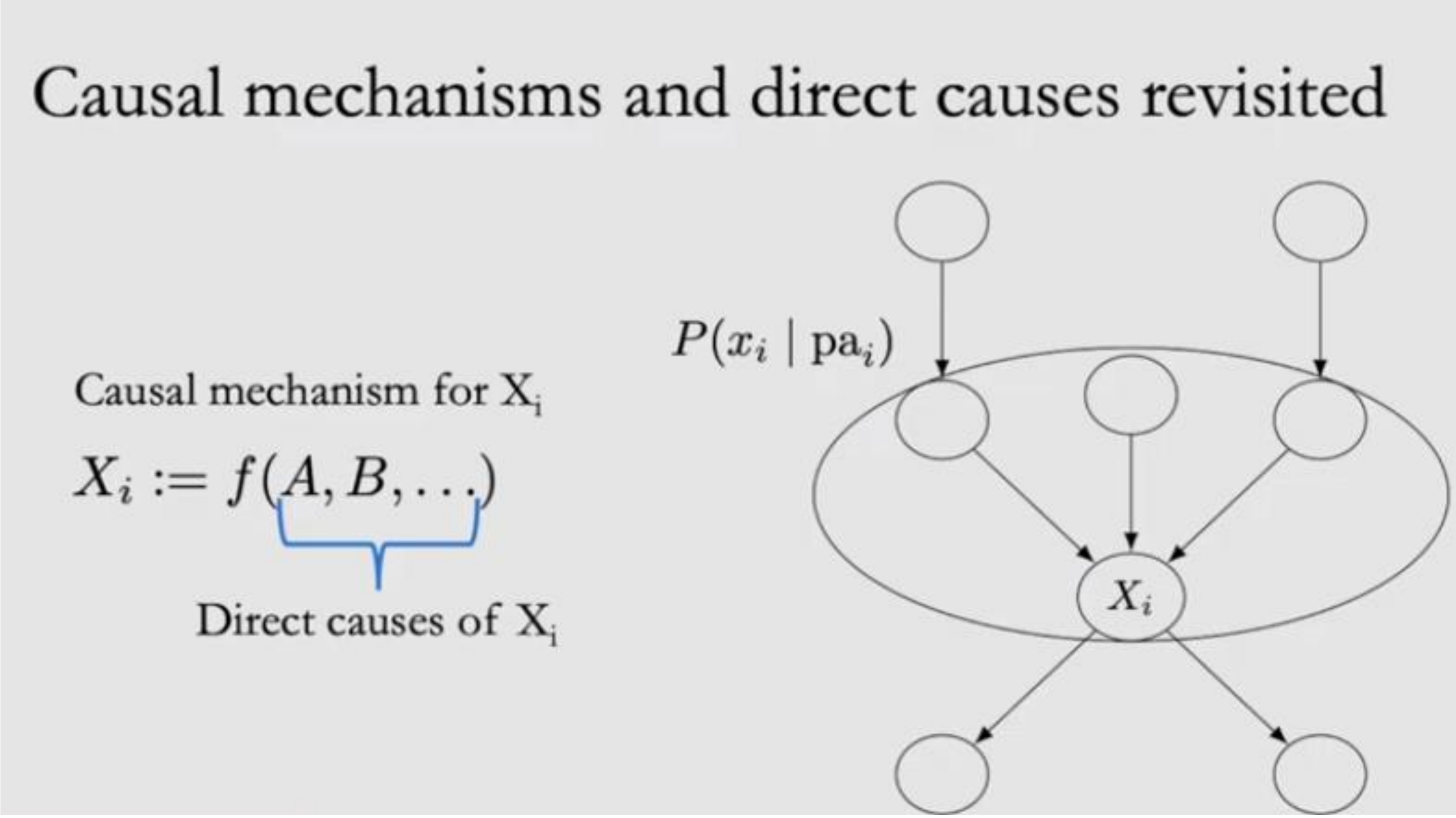
$$B := f(A, U)$$



Causal effect표현을 위한 식, =(등호)랑 비슷
근데 너무 deterministic 하니까

U : unknown bias 를 추가

4.7 Structural Causal Models SCMs



4.7 Structural Causal Models SCMs

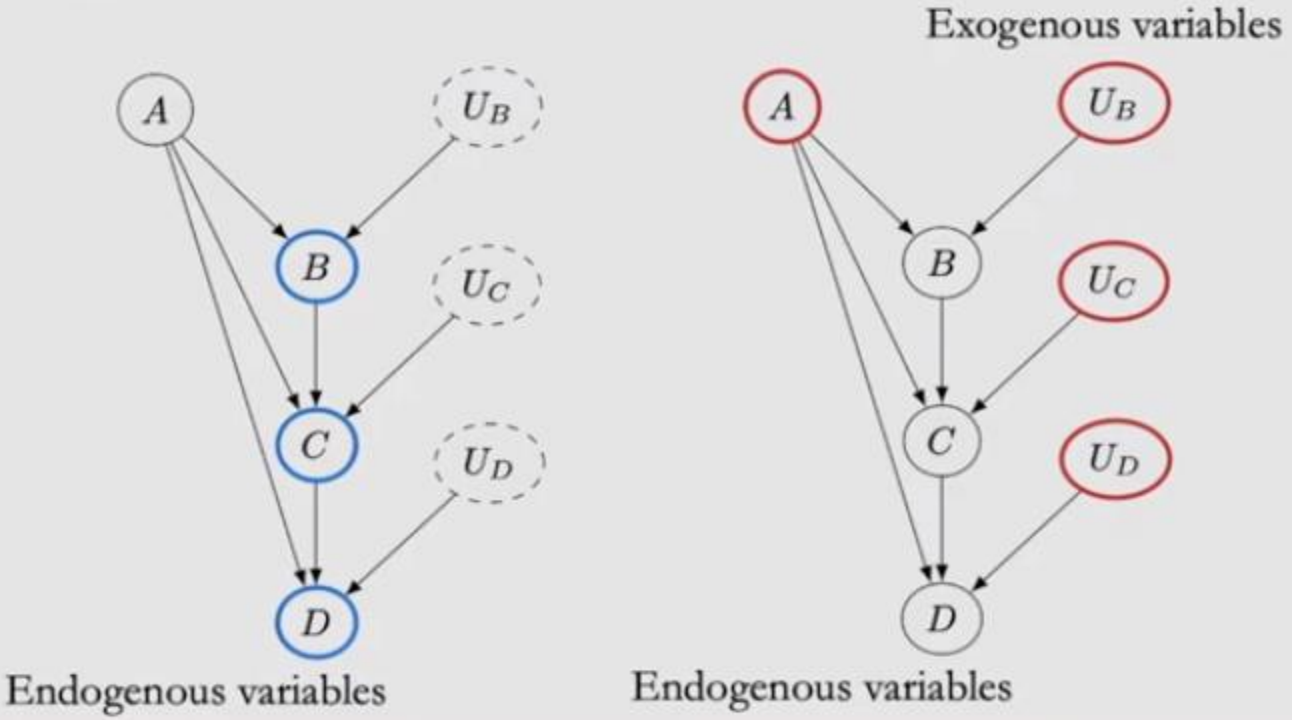
Structural causal models (SCMs)

$$M: \quad \begin{aligned} B &:= f_B(A, U_B) \\ C &:= f_C(A, B, U_C) \\ D &:= f_D(A, C, U_D) \end{aligned}$$

SCM Definition

A tuple of the following sets:

1. A set of endogenous variables
2. A set of exogenous variables
3. A set of functions, one to generate each endogenous variable as a function of the other variables

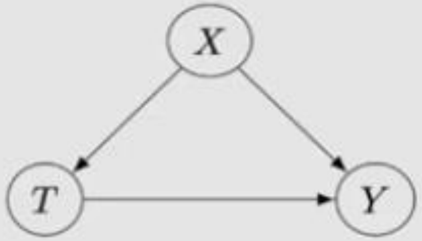


4.8 Interventions and Modularity in SCMs

Interventions

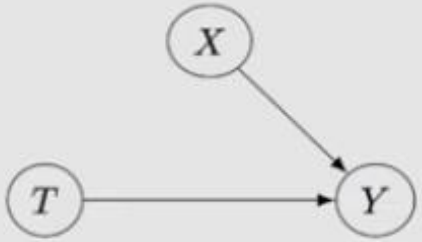
SCM (model)

$$M: \quad \begin{aligned} T &:= f_T(X, U_T) \\ Y &:= f_Y(X, T, U_Y) \end{aligned}$$



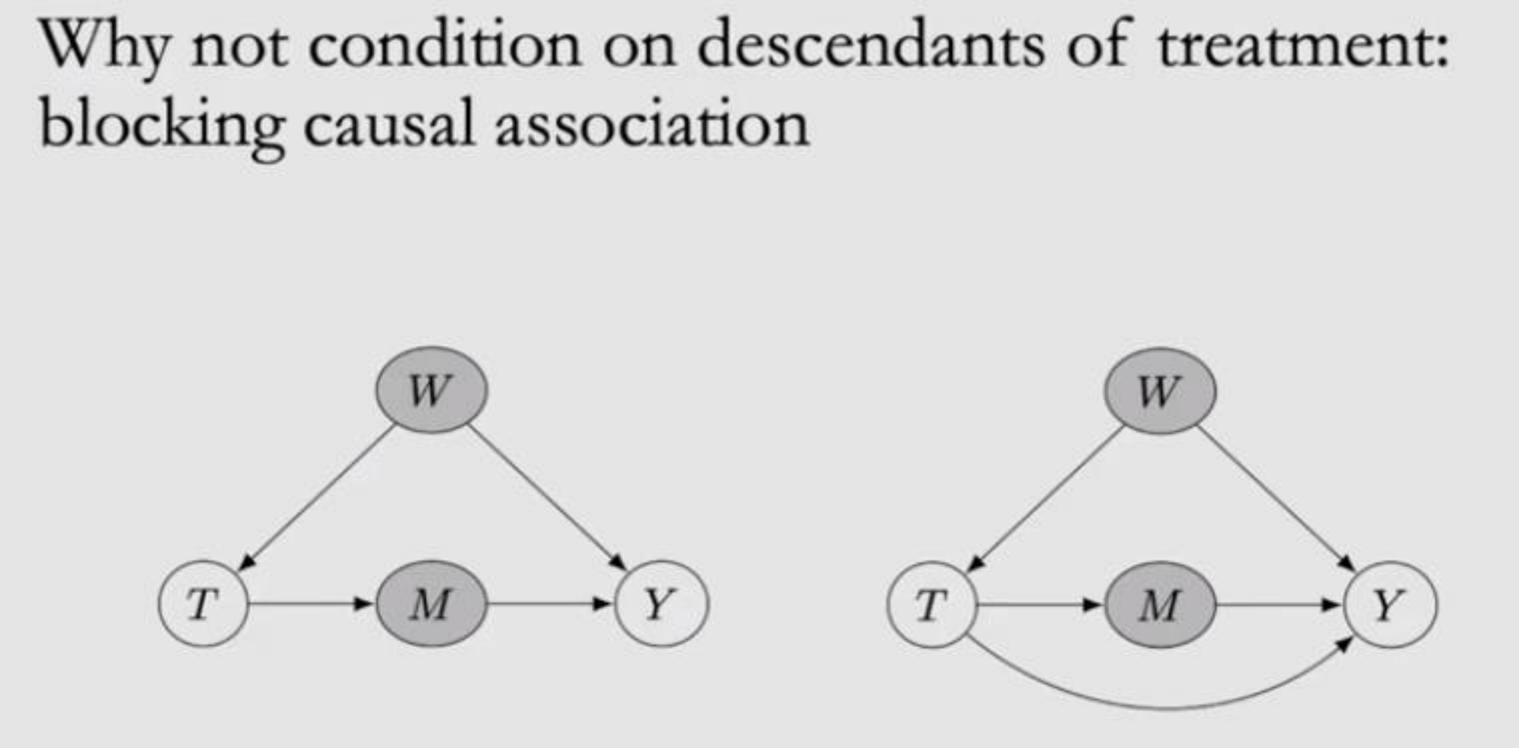
Interventional SCM (submodel)

$$\boxed{M_t :} \quad \begin{aligned} T &:= t \\ Y &:= f_Y(X, T, U_Y) \end{aligned}$$



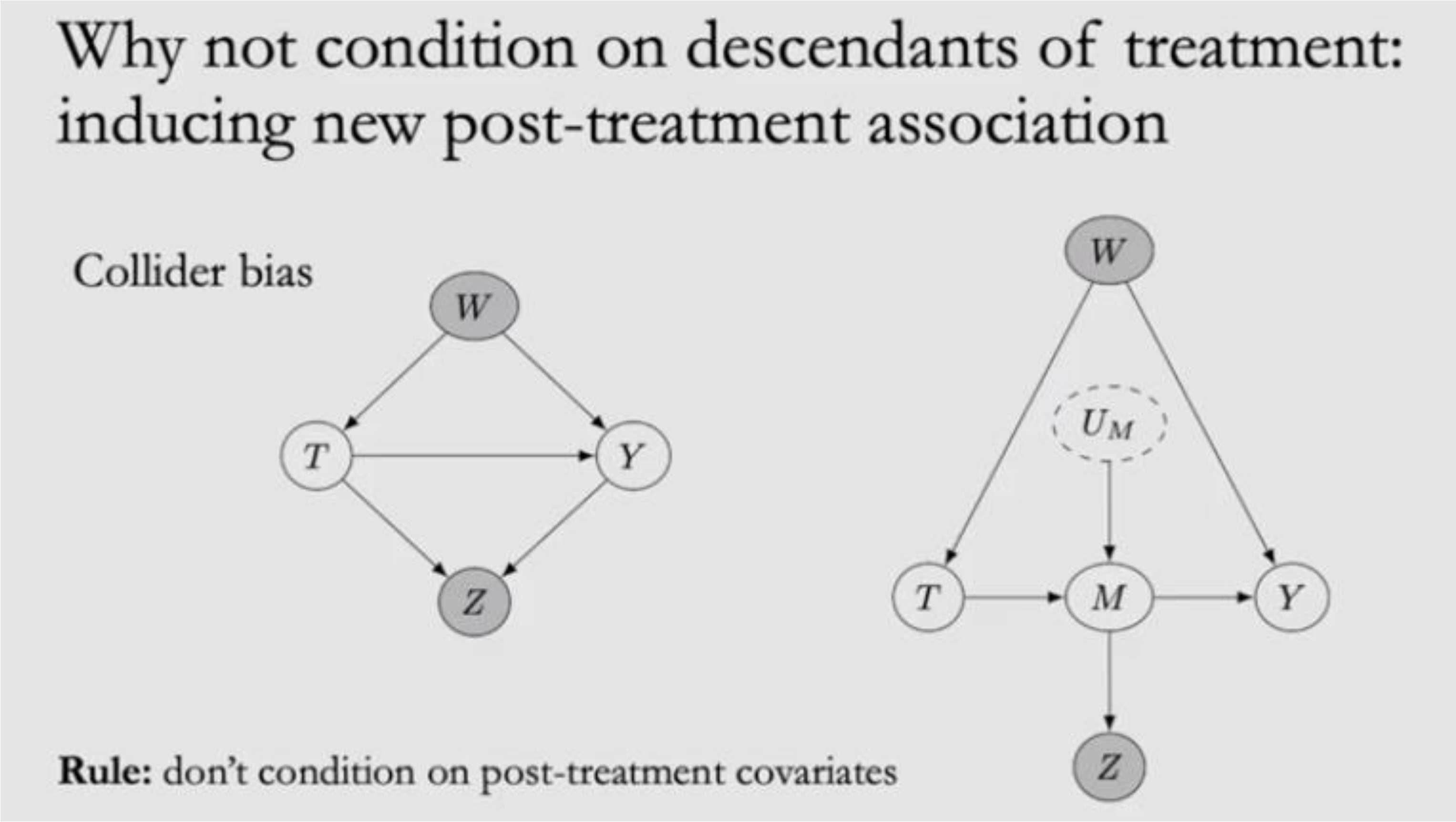
Consider an SCM M and an interventional SCM M_t that we get by performing the intervention *do*(T = t). The modularity assumption states that M and M_t share all of their structural equations except the structural equation for T, which is T := t in M_t.

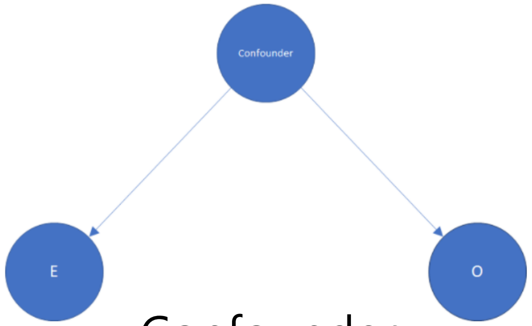
4.9 M-Bias and Conditioning on Descendants of Treatment



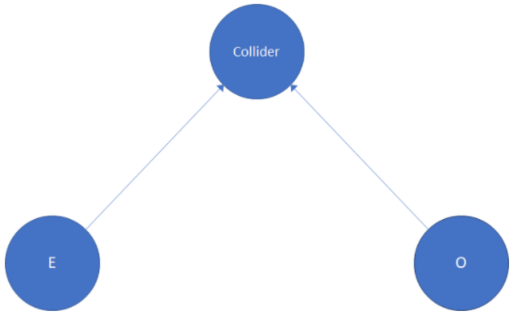
왜 T의 자녀노드를 막으면 안되는가?
T의 자녀를 M으로 막으면, T->Y로 가는 Flow를 M이 막음

4.9 M-Bias and Conditioning on Descendants of Treatment

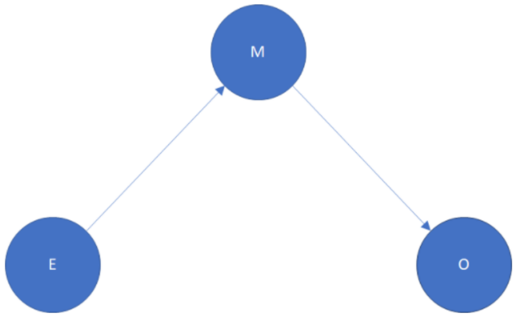




Confounder

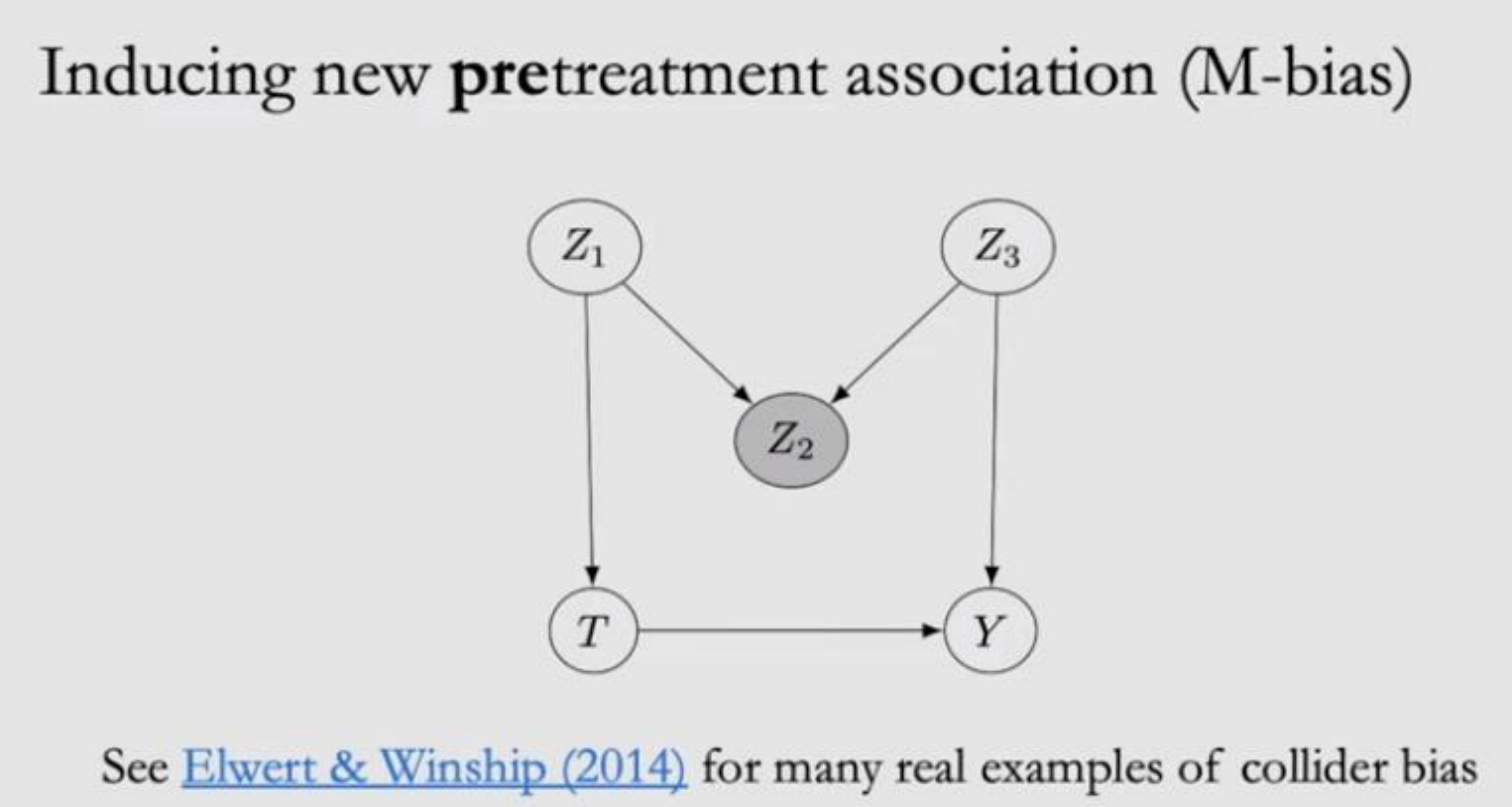


Collider



Mediator

4.9 M-Bias and Conditioning on Descendants of Treatment



T가 아니라 T직전의 PRE-TREATMENT의 (Z1)의 Collider factor를 막아도 M bias를 일으키긴 함

4.10 A Complete Graphical Example with Estimation

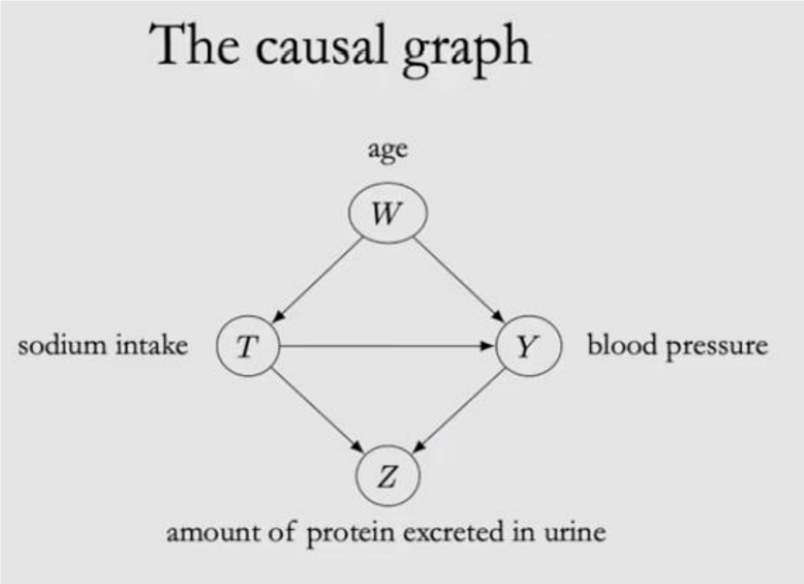
Problem: effect of sodium intake on blood pressure

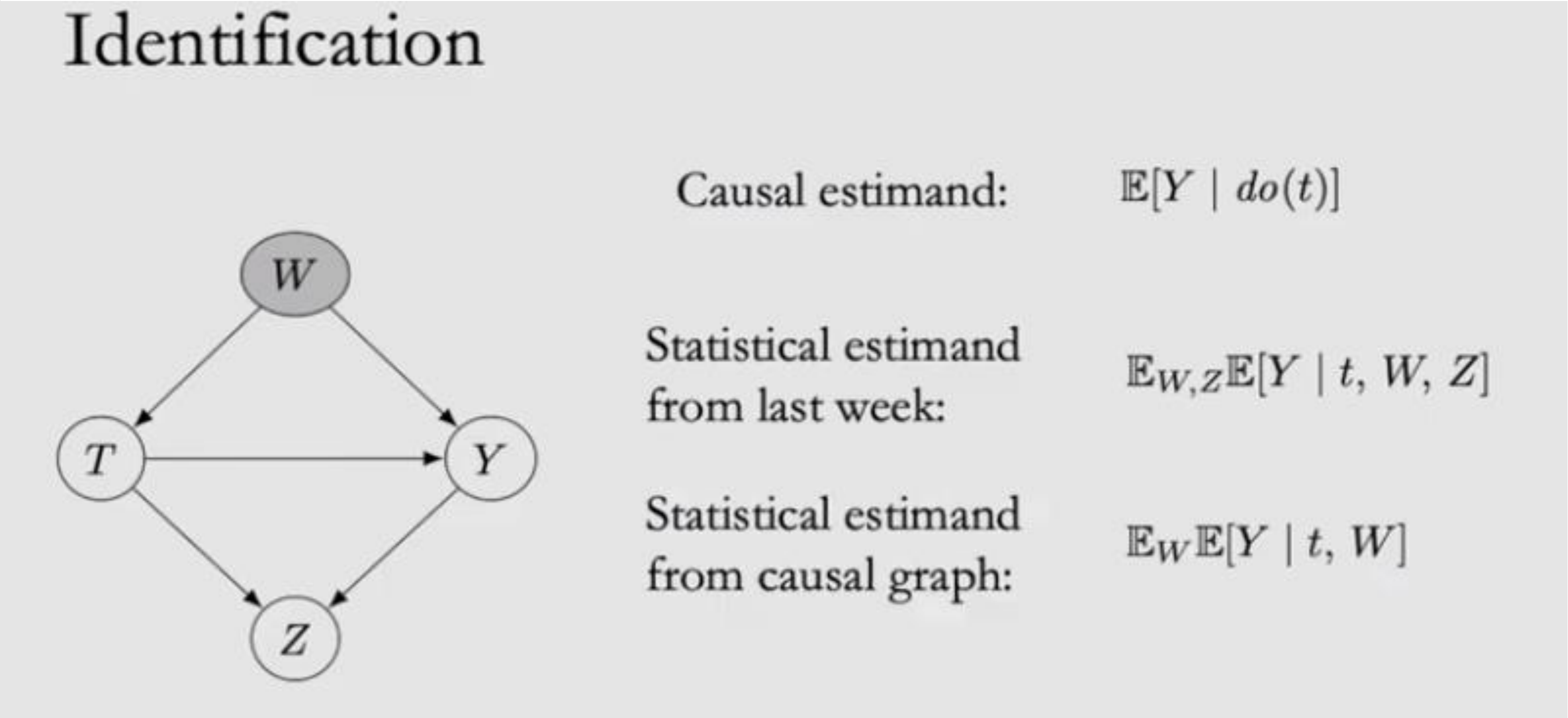
Motivation: 46% of Americans have high blood pressure and high blood pressure is associated with increased risk of mortality

Data:

- Epidemiological example taken from Luque-Fernandez et al. (2018)
- Outcome Y: (systolic) blood pressure (continuous)
- Treatment T: sodium intake (1 if above 3.5 mg and 0 if below)
- Covariates
  - W age
  - Z amount of protein excreted in urine
- Simulation: so we know the "true" ATE is 1.05

4.10 A Complete Graphical Example with Estimation





4.10 A Complete Graphical Example with Estimation

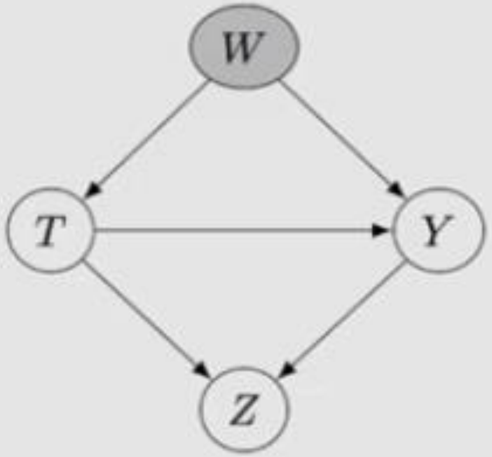
Estimation of ATE

True ATE: $\mathbb{E}[Y(1) - Y(0)] = 1.05$

Identification: $\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}_X \left[\mathbb{E}[Y \mid T = 1, X] - \mathbb{E}[Y \mid T = 0, X]\right]$

Estimation: $\frac{1}{n} \sum_i \left[\underbrace{\mathbb{E}[Y \mid T = 1, X = x_i]} - \underbrace{\mathbb{E}[Y \mid T = 0, X = x_i]}\right]$ Model을 ML, DL로 바꿔서 estimation해서 활용할 수 있다

Model (linear regression)



Estimates:

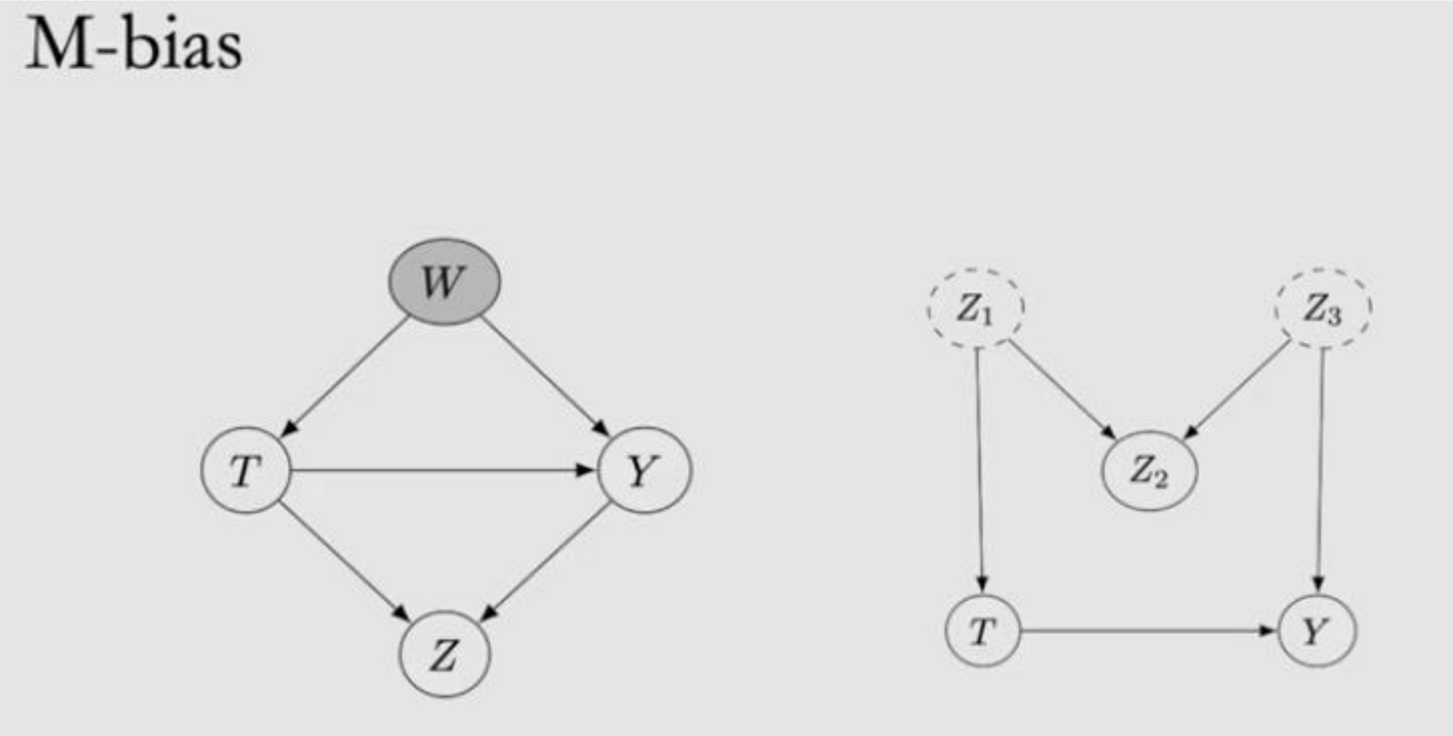
$X = \{\}$ (naive): 5.33 $\quad \frac{|5.33 - 1.05|}{1.05} \times 100\% = 407\%$ error

$X = \{W, Z\}$ (last week): 0.85 $\quad$ 19% error

$X = \{W\}$ (unbiased): 1.0502 $\quad$ 0.02% error

W,z 둘다 포함하면 collider bias때문에 19% error
W만 포함하면 0.02%

4.10 A Complete Graphical Example with Estimation



Z는 T이후의 post-treatment factor라서 막으면 안된다
Z2는? M bias가 생기는 Causal model에서는 Z2를 conditioning 하면 안된다