Causal Inference

Week 5 Randomized Experiments and Identification

https://www.youtube.com/watch?v=z91LnTDyhtI&list=PLoazKTcS0Rzb6bb9L508cyJ1z-U9iWkA0&index=37

전은주

Contents

이전시간까지 Identification하는 방법으로 Backdoor adjustment를 활용하는 것을 배웠다. 이번 시간에는 그 외의 Causal inference를 계산할 수 있는 방법들에 대해 배운다.

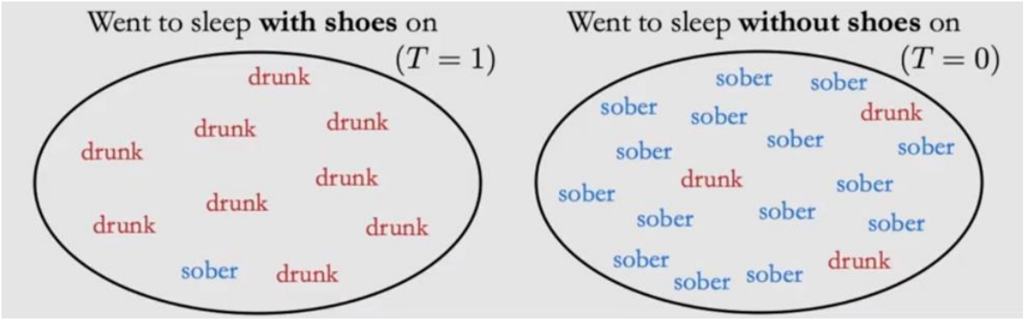
- 5.1 Randomized Experiments and Identification (Intro and Outline)
- 5.2 The Magic of Randomized Experiments
- 5.3 The Frontdoor Adjustment
- 5.4 Pearl's do-calculus
- 5.5 Determining Identifiability form the Graph

5.2 The Magic of Randomized Experiments

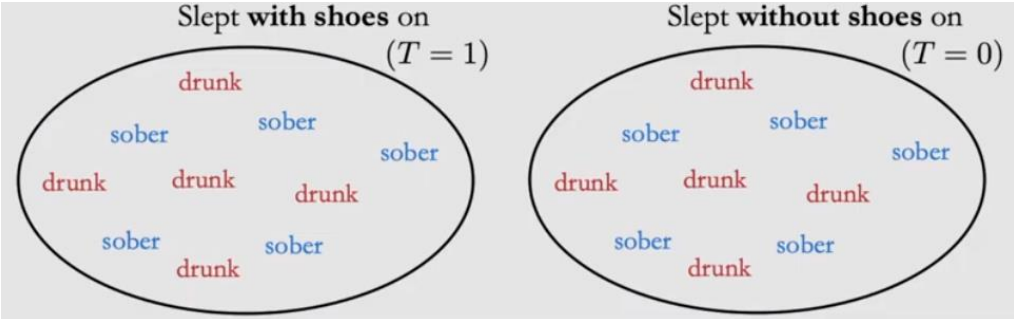
Confound facto나 Backdoor criteria에 만족하는지 정확히 알 수 없을 때,

RCT(randomize control trial)를 적용하면

No unobserved confounding



관찰만 했을 때: Drinking -> confounder



Coin 던져서 (random) T-1, T=0를 선택해서 배분하면, drunk 비율은 두 그룹에서 같아진다

1) Comparability and covariate balance: intuition

Treatment외의 모든 변수에 대한 covariate가 Treatment그룹과 control그룹 같다.
두 그룹간 차이가 있다면 'Treatment'때문이다 (Causation)

Covariate balance definition

We have covariate balance if the distribution of covariates X is the same across treatment groups. More formally,

X: Treatment외의 x변수들 , d = dimension

$$P(\boxed{X} \mid T=1) \overset{d}{=} P(X \mid T=0)$$

Randomization implies covariate balance

Because T is not at all determined by X (solely by a coin flip), $T \perp\!\!\!\perp X$

T는 X에 의해서 선택되는 것이 아니므로, T와 X는 independent

$$P(X \mid T=1) \overset{d}{=} P(X)$$

$$P(X \mid T=0) \overset{d}{=} P(X)$$

따라서, T가 conditioning되어도 P(X)와 같다.
결국 P(X|T=1)과 P(X|T=0)은 P(X)로 서로 같다.

$$\boxed{P(X \mid T=1) \overset{d}{=} P(X \mid T=0)}$$

Covariate balance implies association is causation

RCT일 때, T와 X가 독립적이고 모든 변수x가 covariate same

$$P(X \mid T=1) \stackrel{d}{=} P(X \mid T=0) \implies T \perp\!\!\!\perp X$$

Let X be a sufficient adjustment set

X가 sufficient adjustment set이면 backdoor criteria 사용

$$P(y \mid do(t)) = \sum_x P(y \mid t, x)P(x)$$

분자, 분모에 P(t|x) 둘 다 곱하고

$$= \sum_x \frac{P(y \mid t, x)P(t \mid x)P(x)}{P(t \mid x)}$$

Chain rule에 의해서 조건부 확률을 joint distribution로 변환
P(y|t,x)P(t|x)P(x) -> P(y,t,x)

$$= \sum_x \frac{P(y, t, x)}{P(t \mid x)}$$

T와 x는 서로 independent하므로 P(t|x) -> P(t)

$$= \sum_x \frac{P(y, t, x)}{P(t)}$$

BayesRule: P(y,t,x)/P(t) -> P(y,x|t)

$$= \sum_x P(y, x \mid t)$$

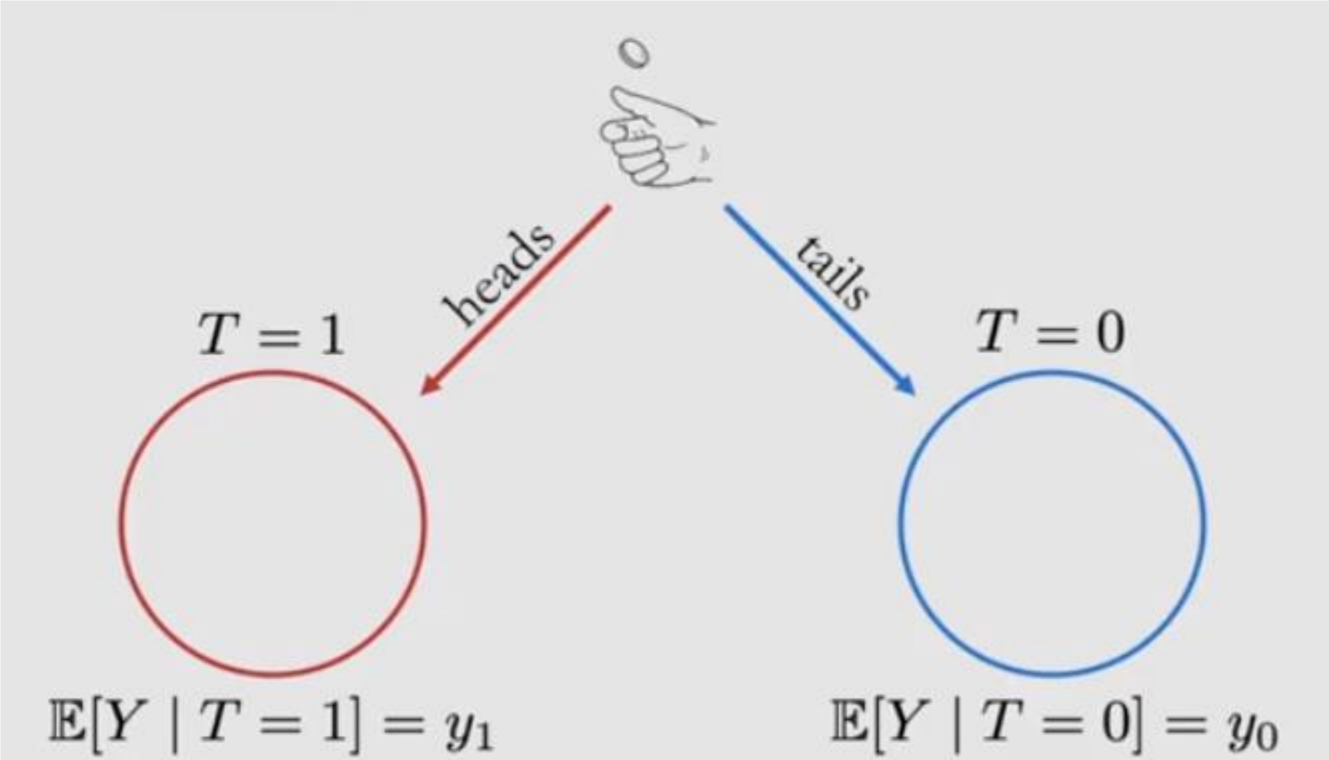
마지막으로 Sum x이므로 P(y|t)로 marginalize

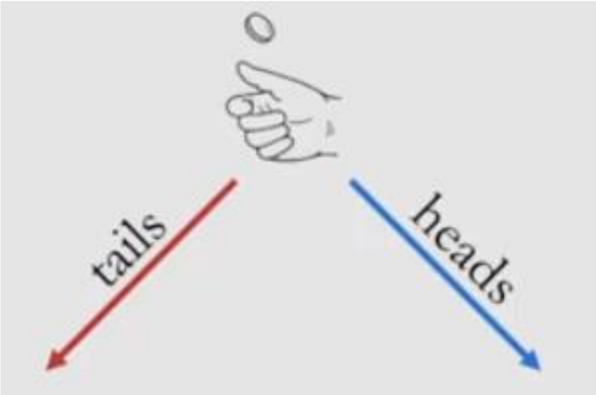
$$= P(y \mid t)$$

따라서, RCT하에서 P(y|do(t)) == P(y|t) causation 과 같다

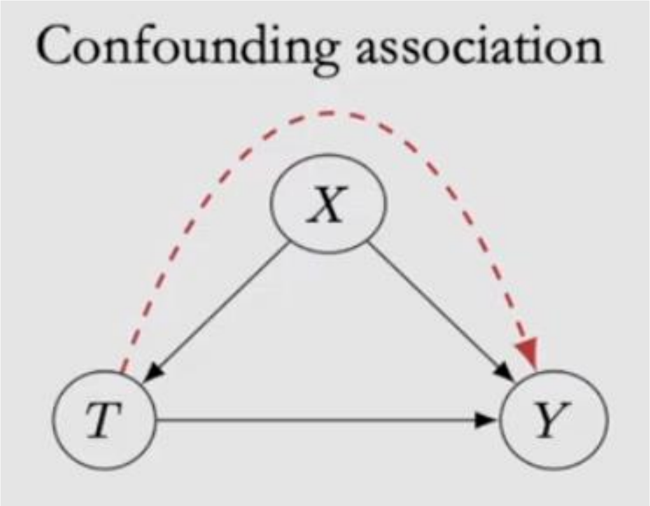
2) Exchangeability

Coin toss에 의해서 T=1과 T=0을 바꿔도 두 T=0, T=1간의 차이 (Average Outcome) 은 같다.
T에 의한 결과일 뿐 다른 영향은 없다

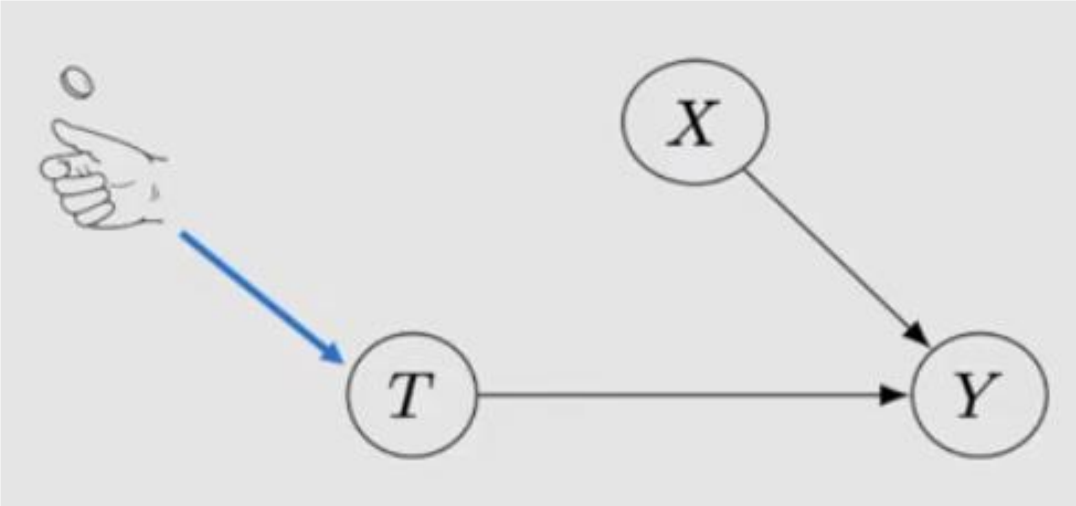




3) No backdoor paths



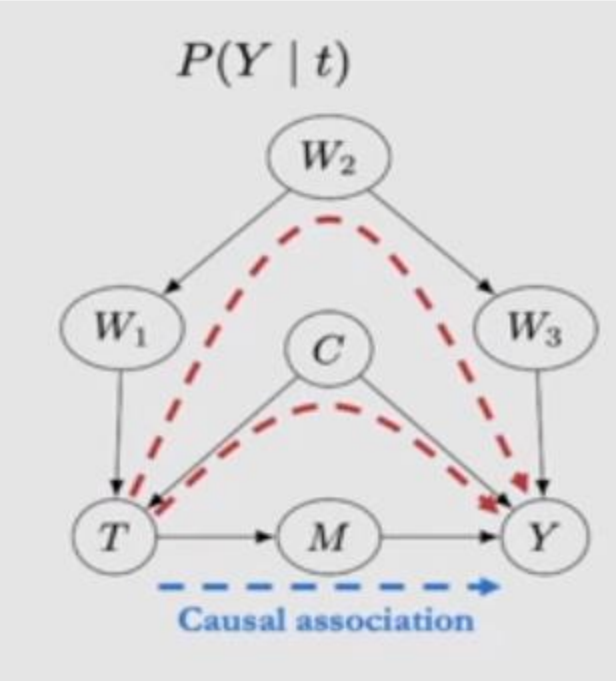
T가 coin toss에만 영향을 받기 때문에 (X와 T는 독립)
관찰 못한 X (교란변수, confounder)가 측정되더라도 T->Y에 영향을 미치지 않는다

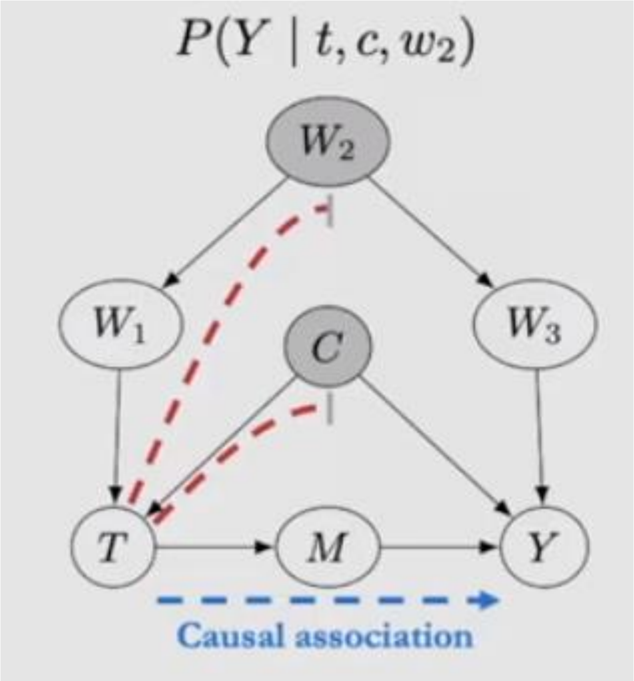


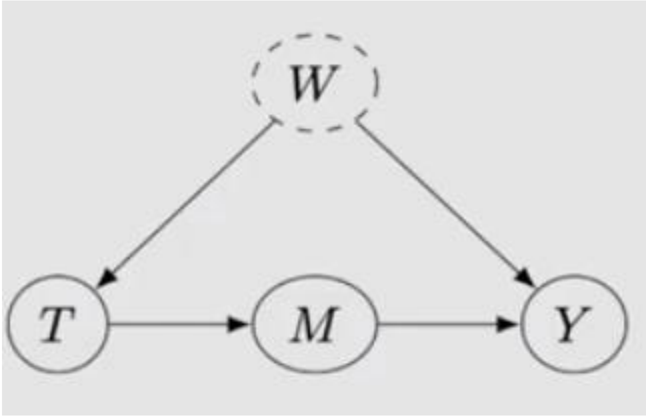
5.3 Frontdoor adjustment

Backdoor adjustment

다음과 같이 non-causal association이 흐를 때, W2, C에 conditioning하여 흐름을 막는다



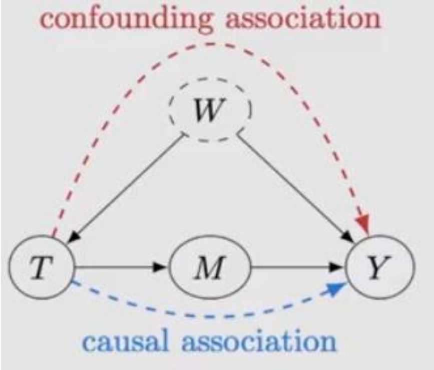


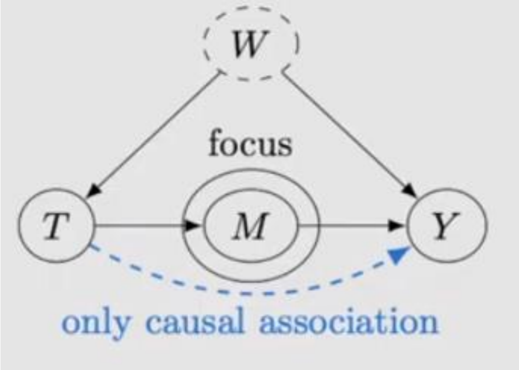


근데 만약에 W를 관찰을 못한다면?

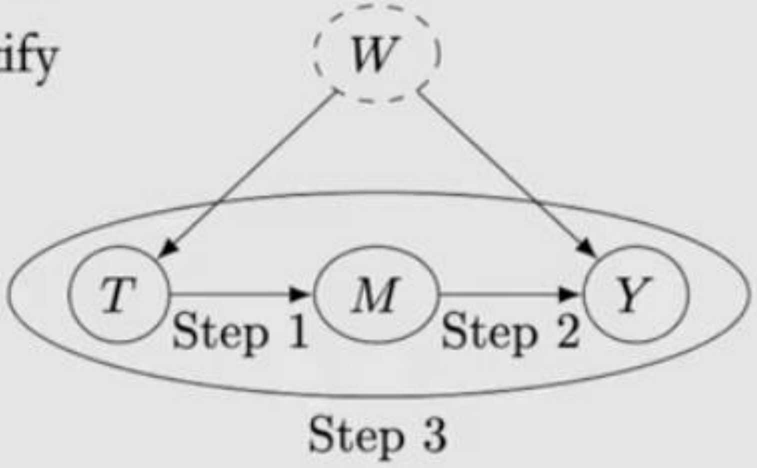
5.3 Frontdoor adjustment: big picture

W를 알 수 없을 때, M에 집중해서 막아보자





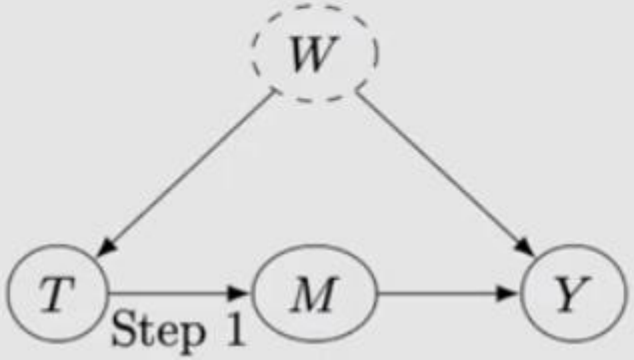
1. Identify the causal effect of T on M
2. Identify the causal effect of M on Y
3. Combine the above steps to identify the causal effect of T on Y



5.3 Frontdoor adjustment: step1

Identify the causal effect of T on M

$$P(m \mid do(t)) = P(m \mid t)$$



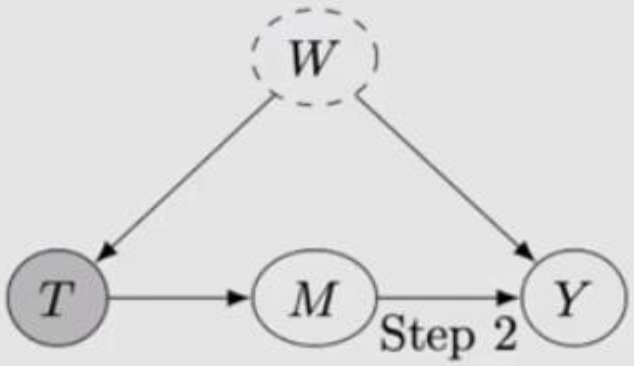
T->W->Y->M으로 가는 path는 Y가 collider로 막아버리니까

T에서 M으로 가는 backdoor path가 없으므로 Do(t)가 |t로 변화

5.3 Frontdoor adjustment: step 2

Identify the causal effect of M on Y

$$P(y \mid do(m)) = \sum_{t} P(y \mid m, t)\, P(t)$$



M->T->W->Y 로의 path는 T를 conditioning함으로써 막을 수 있음. 따라서

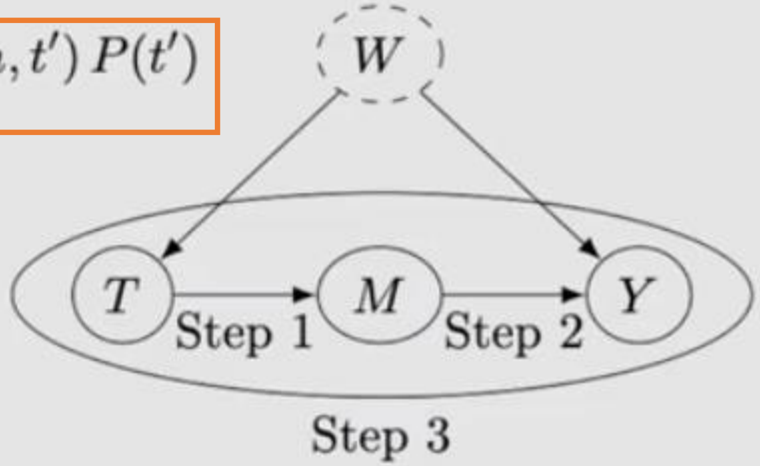
5.3 Frontdoor adjustment: step 3

Combine steps 1 and 2 to identify the causal effect of T on Y

Goal

$$P(y \mid do(t)) = \sum_m \overset{\text{STEP1}}{\boxed{P(m \mid do(t))}}\ \overset{\text{STEP2}}{\boxed{P(y \mid do(m))}}$$

$$= \sum_m \boxed{P(m \mid t)}\ \boxed{\sum_{t'} P(y \mid m, t')\, P(t')}$$



5.3 Frontdoor adjustment and criterion

If (T, M, Y) satisfy the frontdoor criterion, and we have positivity, then

$$P(y \mid do(t)) = \sum_m P(m \mid t) \sum_{t'} P(y \mid m, t')\, P(t')$$

A set of variables M satisfies the **frontdoor criterion** relative to T and Y if the following are true:

1. M completely mediates the effect of T on Y (i.e. all causal paths from T to Y go through M).
2. There is no unblocked backdoor path from T to M.
3. All backdoor paths from M to Y are blocked by T.

조건1. M은 full mediator여야 한다 (T->M->Y 100프로)
조건2. T->M으로의 backdoor는 막혀야 함 (step 1)
조건3. M->Y로의 path는 T conditioning으로 막혀야 함 (step 2)

5.4 Peal's do-calculus

Backdoor 혹은 frontdoor criterion 둘 다 만족 못할 때도,
causal effect를 identify할 수 있냐? **Yes, do-calculus**

Peal's do-calculus로 어떤 causal quantity등 identify할 수 있다.

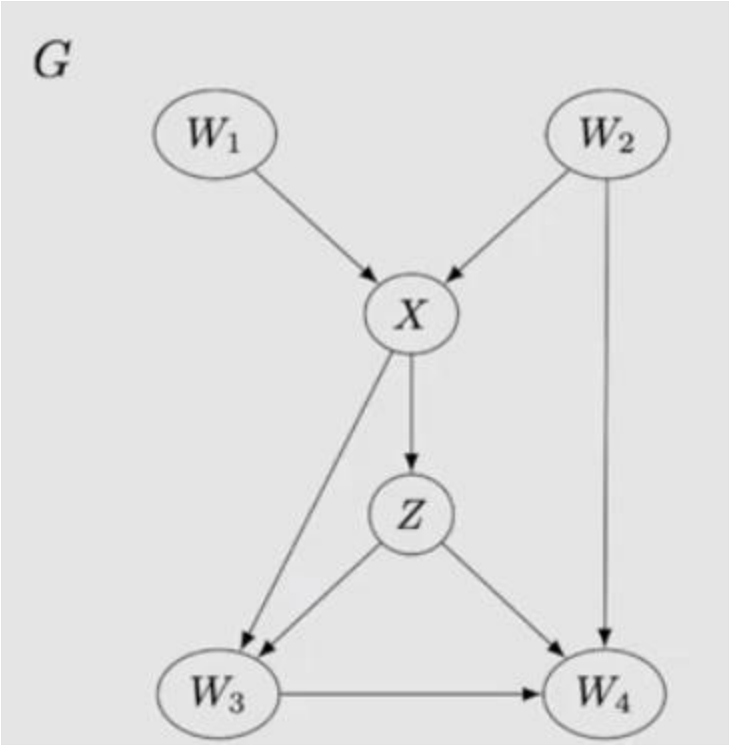
Will allow us to identify any causal quantity that is identifiable

$$P(Y \mid do(T = t), X = x)$$

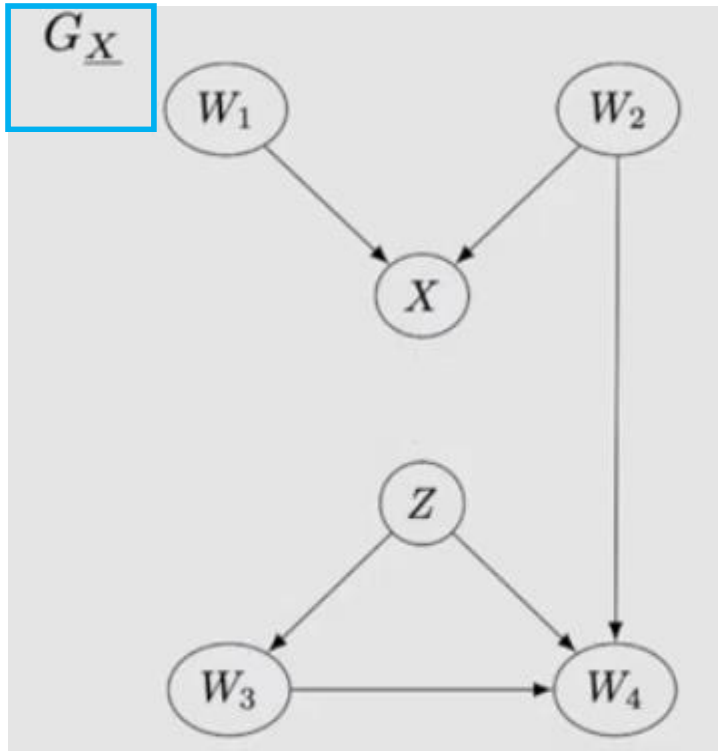
where Y, T, and X are arbitrary sets

Multiple treatments and/or multiple outcomes

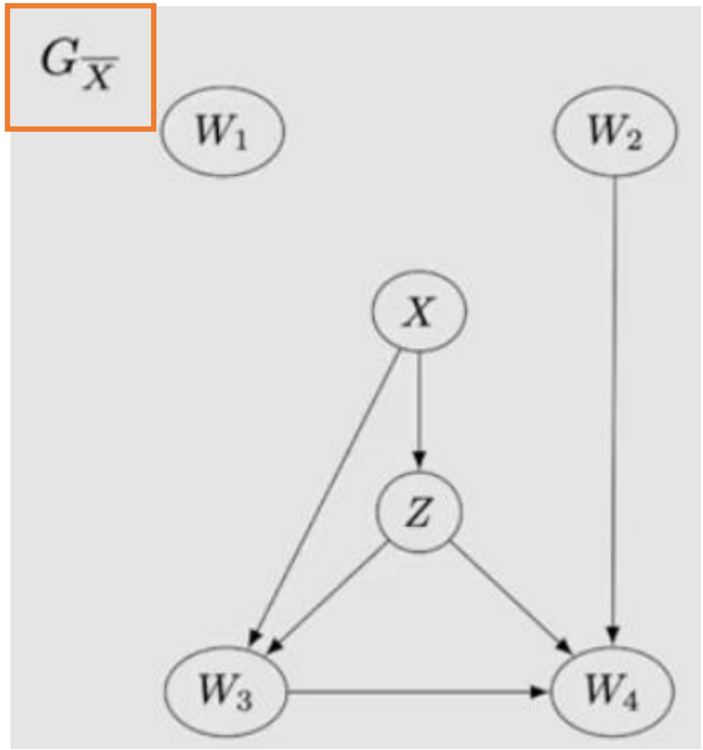
Notation for Pearl's *do*-calculus



X로부터 나가는 모든 edge 모두 삭제



X로부터 들어오는 모든 edge 삭제



Rules for Pearl's *do*-calculus

Graph에서 T의 incoming edge를 삭제했을 때 (T는 coin toss니까)
Y가 T, W가 conditioning 되었을 때 Z랑 de-separate면 (흐르지 않으면)

Rule 1 of *do*-calculus

$$P(y \mid \cancel{do(t)}, z, w) = P(y \mid \cancel{do(t)}, w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\cancel{\overline{T}}}} Z \mid T, W$$

Question: What concept does this remind you of?

T가 empty set이면, 아래와 같이 그냥 일반적인 d-separation 식

Rule 1 with do(t) removed:

$$P(y \mid z, w) = P(y \mid w) \quad \text{if } Y \perp\!\!\!\perp_{G} Z \mid W$$

Generalization of d-separation to interventional distributions

Rules for Pearl's *do*-calculus

Rule 2 of *do*-calculus

$$P(y \mid \cancel{do(t)}, do(z), w) = P(y \mid \cancel{do(t)}, z, w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\cancel{\overline{T}}, \underline{Z}}} Z \mid T, W$$

Question: What concept does this remind you of?

Rule 2 with do(t) removed:

$$P(y \mid do(z), w) = P(y \mid z, w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\underline{Z}}} Z \mid W$$

Backdoor adjustment 의 확장

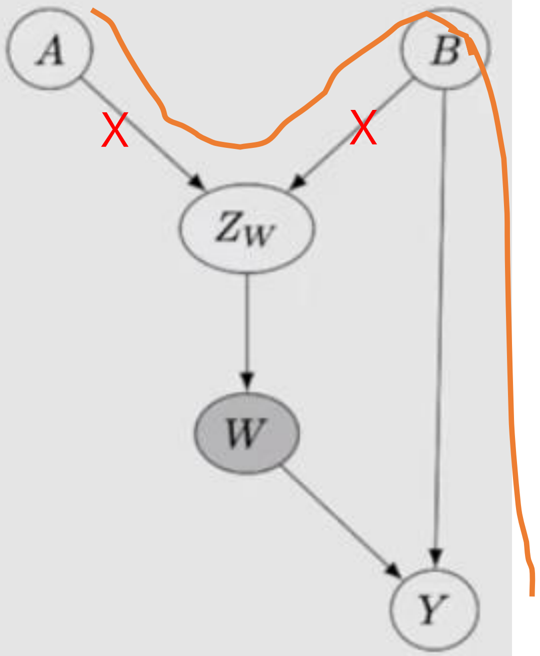
Generalization of backdoor adjustment/criterion

Rules for Pearl's *do*-calculus

Rule 3 of *do*-calculus

$$P(y \mid do(t), do(z), w) = P(y \mid do(t), w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\overline{T}, \overline{Z(W)}}} Z \mid T, W$$

where $Z(W)$ denotes the set of nodes of Z that aren't ancestors of any node of W in $G_{\overline{T}}$



Rule 3 with do(t) removed:

$$P(y \mid do(z), w) = P(y \mid w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\overline{Z(W)}}} Z \mid W$$

$$P(y \mid do(z), w) = P(y \mid w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\overline{Z}}} Z \mid W$$

Do(z)를 없애고 싶다 -> Y랑 Z랑 graph상에서 연결이 없으면 된다.
그럼, Z로 들어오는 incoming edge만 삭제하면 될까? 아니
W의 부모인 Z의 incoming edge를 삭제 해야 한다 Z(W)
W가 conditioning되면 Zw가 collider가 되어서 A->Z(W)->B->Y로 흐르게 됨으로. Z(W)의 incoming edge삭제

Rules for Pearl's *do*-calculus

The rules of *do*-calculus

Rule 1: $P(y \mid do(t), z, w) = P(y \mid do(t), w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\overline{T}}} Z \mid T, W$

Rule 2: $P(y \mid do(t), do(z), w) = P(y \mid do(t), z, w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\overline{T},\underline{Z}}} Z \mid T, W$

Rule 3: $P(y \mid do(t), do(z), w) = P(y \mid do(t), w) \quad \text{if } Y \perp\!\!\!\perp_{G_{\overline{T},\overline{Z(W)}}} Z \mid T, W$

Completeness of *do*-calculus

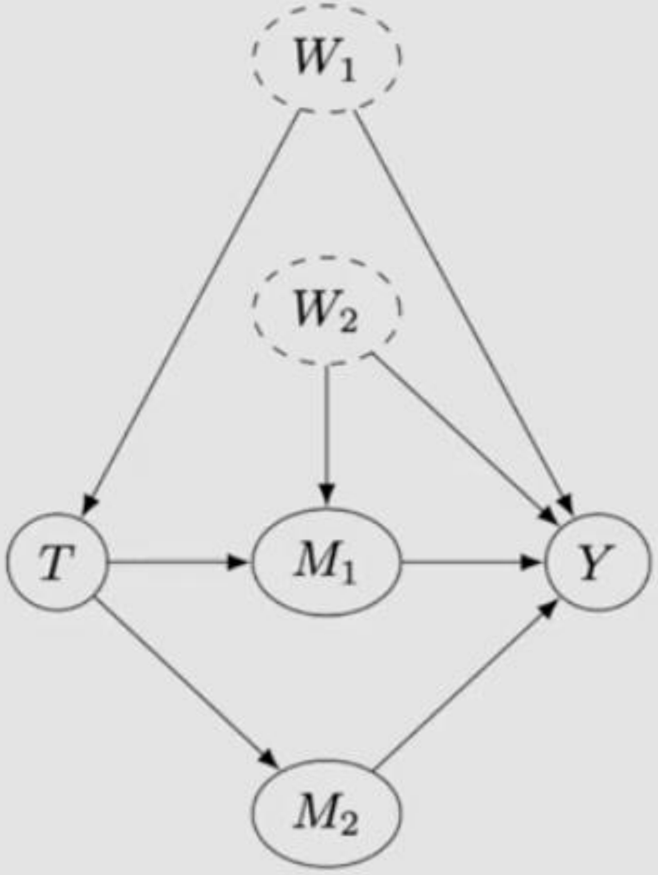
Completeness of *do*-calculus

Maybe there are some identifiable causal estimands that can't be identified using the rules of *do*-calculus

Fortunately, not, as *do*-calculus is complete (Shpitser & Pearl, 2006a; Huang & Valtorta, 2006; Shpitser & Pearl, 2006b)

5.5 Determining Identifiability form the Graph

Question:
In this graph, is the backdoor criterion satisfied?



ator여야 한다 (T->M->Y 100프로)
ackdoor는 막혀야 함 (step 1)
th는 T conditioning으로 막혀야 함 (step 2)

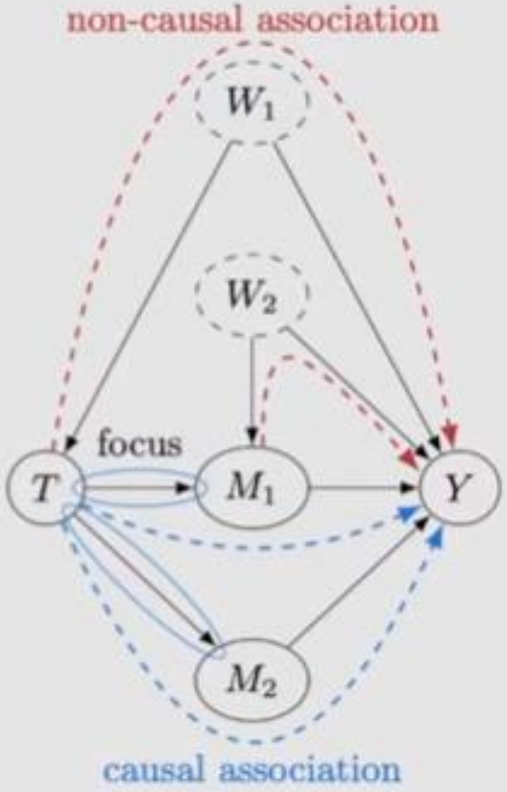
5.5 Determining Identifiability form the Graph

Unconfounded children criterion

This criterion is satisfied if it is possible to block all backdoor paths from the treatment variable T to all of its children that are ancestors of Y with a single conditioning set (Tian & Pearl, 2002).

T에서 나오는 모든 자식 노드이며 Y의 부모 노드인 것들을 막으면 Unconfounded children criterion

Sufficient condition for identifiability when T is a single variable



Backdoor: (W1 관찰 안 되서 block)
T->W1->Y

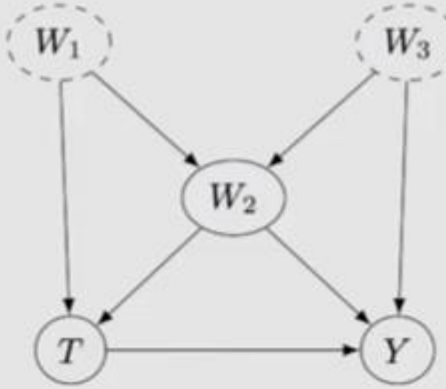
Frontdoor: (W2 관찰 안 되서 block)
M1->W2->Y

T의 자식노드이자 Y의 부모노드인 (T->M1, T->M2)만 남기고 다 막으면 된다.

(Backdoor, frontdoor보다 좀 더 general한 컨셉)

Necessary condition for identifiability

For each backdoor path from T to any child M of T that is an ancestor of Y, it is possible to block that path (Pearl, 2009, p. 92).



T의 자식이자, Y의 부모노드인 M에 영향을 미치는 path block 가능하다

Backdoor

1) T->W1->W2->W3->Y : blocked by conditioning W2

2) T->W2->Y: blocked by conditioning W2

근데, W2 conditioning하면 두개를 동시에 막을 수 없음 (collider라서)

즉, single conditioning으로 모든 path를 막을 수 있는 상황은
Unconfounded children criterion: T의 자식이며 Y의 부모 노드에 대해서만 가능

지금 상황에서 W2는 T의 자식이 아님. Y의 부모는 맞지만

Necessary condition for identifiability

Recall: identification with the rules *do*-calculus is necessary and sufficient (Shpitser & Pearl, 2006a; Huang & Valtorta, 2006; Shpitser & Pearl, 2006b)

For graphical criterion, see Shpitser & Pearl, 2006a, 2006b: hedge criterion