Labor Supply Problem

Consider a consumer solving the following maximization problem

$$c^\star, \ell^\star \quad = \quad \arg\max_{c,\ell} \log(c) - \nu \frac{\ell^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} \tag{1}$$

$$\text{s.t.}$$

$$x \quad = \quad m + w\ell - [\tau_0 w\ell + \tau_1 \max\{w\ell - \kappa, 0\}]$$

$$c \quad \in \quad [0, x]$$

$$\ell \quad \in \quad [0, 1],$$

where c is consumption, ℓ is labor supply, m is cash-on-hand, w is the wage rate, τ_0 is the standard labor income tax, τ_1 is the top bracket labor income tax, κ is the cut-off for the top labor income bracket, x is total resources, ν scales the disutility of labor, and ε is the Frisch elasticity of labor supply.

Note that utility is monotonically increasing in consumption. This implies that

$$c^\star = x. \tag{2}$$

Questions

1) Construct a function which solves eq. (1) given the parameters.

We choose the following parameter values

$$m = 1, \ \nu = 10, \ \varepsilon = 0.3, \ \tau_0 = 0.4, \ \tau_1 = 0.1, \ \kappa = 0.4$$

2) Plot $\ell^\star$ and $c^\star$ as functions of w in the range 0.5 to 1.5.

Consider a population with $N = 10,000$ individuals indexed by i.
Assume the distribution of wages is uniform such that

$$w_i \sim \mathcal{U}(0.5, 1.5).$$

Denote the optimal choices of individual i by $\ell_i^\star$ and $c_i^\star$.

3) Calculate the total tax revenue given by

$$T = \sum_{i=1}^{N} \left[\tau_0 w_i \ell_i^\star + \tau_1 \max\{w_i \ell_i^\star - \kappa, 0\} \right].$$

4) What would the tax revenue be if instead $\varepsilon = 0.1$?

Consider a politician who wishes to maximize the tax revenue.

5) Which τ_0, τ_1 and κ would you suggest her to implement?
   Report the tax revenue you expect to obtain.