MILP Model for Hydraulic Fracturing Wastewater Management

Introduction

Hydraulic Fracturing, or “fracking”, is an important process in oil and gas for the development of wells. This process involves injecting water and other materials at high pressure into the ground to increase the amount of oil or gas that flows from to the well. However, fracking has been a subject for debate by environmentalists and politicians because it can be detrimental for the environment if its harmful effects are not properly mitigated. Therefore, it is important that engineers and scientists come up with ways to manage and control the dangerous effects of fracking to ensure that wastewater from fracking does not contaminate groundwater in the local environment.

In this paper, we will examine a way to minimize costs associated with wastewater fracking in an oil and gas pipeline. We have several sites where wastewater is generated, and we want to decide what to do with the wastewater. At each node, we have two main options. We can either treat the water to injection standards and inject it underground or we can treat the water to drinking standards and sell it to consumers. We demonstrate that we can use a Mixed Integer Linear Program (MILP) model to solve this optimization problem.

Assumptions

This model makes several assumptions to simplify the problem. These assumptions are listed explicitly below:

1. Wastewater is only generated at production nodes. The locations of production nodes are given to the model.
2. Wastewater can be managed in two ways: underground injection and by selling to consumers.
3. For wastewater to be sent to underground injection, it must undergo a treatment process first. This treatment happens at injection treatment facilities. This treatment process mitigates the harmful chemicals that would otherwise be sent into the environment.
4. Injection only happens at injection nodes. The wastewater must be transported from the treatment node to an injection node. Location of injection nodes are given to the model.
5. For wastewater to be sent to consumer, it must undergo a treatment process first. This treatment only happens at drinking treatment facilities. This treatment process turns the wastewater into drinking water suitable for consumers.
6. Wastewater is only sold at customer nodes. The wastewater must be transported from the treatment node to a consumer node. Location of the consumer nodes are given to the model. Each consumer node is only willing to buy up to some maximum amount of water.
7. An injection node can only be used to inject water underground if an injection well is built there. Each injection well has some maximum injection volume. However, paying more for a larger injection well comes with the smaller increases in the maximum injection volume for that well.
8. Like injection wells, paying more for a larger treatment facility will come with smaller increases in the maximum capacity for that treatment facility.

9. There are separate transportation modes for each form of water that can be invested in.
10. Drinking water is sold at the same price at every city.
11. There is no maximum capacity for the quantity of water that can be transported from one node to another. While there is a maximum capacity of water that can be transported by pipes, an unlimited quantity can be transported by trucks if we purchase as many trucks as necessary.
12. We are modeling the system in equilibrium. Everything happens at a constant rate, and we do not have to worry about how anything works as a function of time. All liquids are only generated at production nodes and leave the system only at consumer nodes and injection nodes. The water is not stored anywhere within the system unless it is in the process of being transported or in the process of being treated.

Variables

It is useful to define several variables for this model. The units of the variable are given in parentheses after the definition.

Parameters

Nodes

1. Q: Set of all production nodes.
2. R: Set of all injection nodes.
3. S: Set of all consumer nodes.
4. T: Set of potential injection treatment nodes
5. U: Set of potential drinking treatment nodes

Fixed Costs

1. F_{ij}^{P}: Fixed costs of transporting liquids from node i to node j by pipe. These costs differ by the type of liquid being transported (ie: drinking water, wastewater, injection water). ($/L)
2. F_{ij}^{T}: Fixed cost of transporting liquids from node i to node j by truck. These costs differ by the type of liquid being transported (ie: drinking water, wastewater, injection water). These costs are higher than the operating costs of transporting by truck. ($/L)
3. F_{ls}^{I}: Fixed cost of an injection treatment facility at location l of size s. Subject to economies of scale. ($)
4. F_{ls}^{D}: Fixed cost of a drinking treatment facility at location l of size s. Subject to economies of scale. ($)
5. F_{i}^{W}: Fixed cost of building an injection well at injection node i. ($)

Operating Costs

1. O_{ij}^{P}: Operating costs of transporting liquids from node i to node j by pipe. These costs differ by the type of liquid being transported (ie: drinking water, wastewater, injection water). ($/L)
2. O_{ij}^{T}: Operating costs of transporting liquids from node i to node j by truck. These costs differ by the type of liquid being transported (ie: drinking water, wastewater, injection water). These costs are lower than the operating costs of transporting by pipe. ($/L)
3. O_{ls}^{I}: Operating cost of an injection treatment facility at location l of size s. Subject to economies of scale ($)

4. O_{ls}^{D}: Operating cost of a drinking treatment facility at location l of size s. Subject to economies of scale. ($/L)
5. O_{i}^{W}: Operating cost of an injection well at injection node i. ($/L)

Capacities

1. C_{ls}^{I}: Maximum capacity of an injection treatment facility at location l and size s. ($)
2. C_{ls}^{D}: Maximum capacity of a drinking treatment facility at location l and size s. (L)
3. C_{ls}^{W}: Maximum capacity of an injection well l of size s. (L)

Other Parameters

1. W_i: Wastewater produced at production node i. (L)
2. P: Price drinking water is sold for. ($)
3. V_i: Maximum volume of drinking water a consumer node i is willing to accept. (L)
4. A_i: Maximum volume of injection water an injection node i is willing to accept. (L)
5. B_{ij}: Maximum volume of water allowed from pipes between node i and node j. (L)

Decision Variables

1. x_{ij}^{P}: binary variable 1 if any water is sent from node i to node j by pipe.
2. y_{ij}^{P}: amount of water sent from node i to node j by pipe. (L)
3. x_{ij}^{T}: binary variable 1 if any water is sent from node i to node j by truck.
4. y_{ij}^{T}: amount of water sent from node i to node j by truck. (L)
5. x_{ls}^{I}: binary variable 1 if the model decides to build an injection treatment facility at location l of size s.
6. y_{ls}^{I}: amount of water sent to injection treatment facility at location l of size s. (L)
7. x_{ls}^{D}: binary variable 1 if the model decides to build a drinking treatment facility at location l of size s.
8. y_{ls}^{D}: amount of water sent to drinking treatment facility at location l of size s. (L)
9. x_{ls}^{W}: binary variable 1 if any water is sent to injection well l of size s.
10. y_{ls}^{W}: amount of water sent to injection well l of size s. (L)

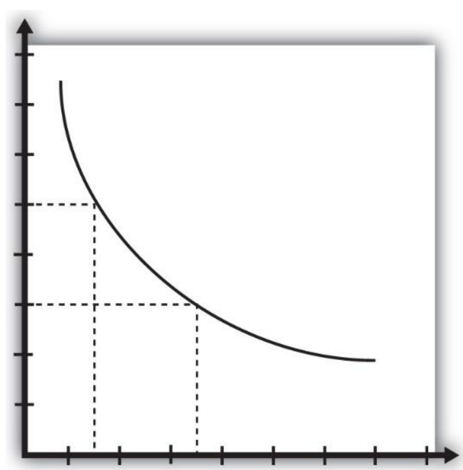

Figure 1: Generalized functional form for variables subject to economies of scale (cost/unit vs size).

Model

Objective

The objective function of this model is to minimize the total costs minus the revenue of drinking water sold. This is given by the following function:

Min: $\sum_i \sum_j F_{ij}^P \cdot x_{ij}^P + \sum_i \sum_j O_{ij}^P \cdot y_{ij}^P + \sum_i \sum_j F_{ij}^T \cdot x_{ij}^T + \sum_i \sum_j O_{ij}^T \cdot y_{ij}^T + \sum_l \sum_s F_{ls}^I \cdot x_{ls}^I + \sum_l \sum_s O_{ls}^I \cdot y_{ls}^I + \sum_l \sum_s F_{ls}^D \cdot x_{ls}^D + \sum_l \sum_s O_{ls}^D \cdot y_{ls}^D + \sum_{i\epsilon R} F_i^W \cdot x_i^W + \sum_{i\epsilon R} O_i^W \cdot y_i^W - \sum_l \sum_s y_{ls}^D \cdot P$.

In this objective function:

- $\sum_i \sum_j F_{ij}^P \cdot x_{ij}^P + \sum_i \sum_j O_{ij}^P \cdot y_{ij}^P$: represents the total costs (fixed + operating) of all pipeline transportation. Sum over all possible starting and ending nodes of the pipeline.
- $\sum_i \sum_j F_{ij}^T \cdot x_{ij}^T + \sum_i \sum_j O_{ij}^T \cdot y_{ij}^T$ represents the total costs (fixed + operating) of all trucking transportation. Sum over all possible starting and ending nodes of the truck route.
- $\sum_l \sum_s F_{ls}^I \cdot x_{ls}^I + \sum_l \sum_s O_{ls}^I \cdot y_{ls}^I$ represents the total costs (fixed + operating) of all injection treatment. Sum over all possible locations and all possible sizes of injection treatment facilities.
- $\sum_l \sum_s F_{ls}^D \cdot x_{ls}^D + \sum_l \sum_s O_{ls}^D \cdot y_{ls}^D$ represents the total costs (fixed + operating of all drinking treatment. Sum over all possible locations and all possible sizes of drinking treatment facilities.
- $\sum_{i\epsilon R} F_i^W \cdot x_i^W + \sum_{i\epsilon R} O_i^W \cdot y_i^W$ represents the total costs of all injection at an injection well
- $\sum_l \sum_s y_{ls}^D \cdot P$ represents the total amount earned from selling drinking water. Sum over all possible locations and all possible sizes of the drinking treatment facilities. There is a model constraint that ensures all water that comes into drinking treatment facilities gets sold.

Constraints

Several mathematical constraints must be added to the model to represent our assumptions.

1. $y_{ij}^P \leq M \cdot x_{ij}^P \forall i, j$
2. $y_{ij}^T \leq M \cdot x_{ij}^T \forall i, j$
3. $y_{ls}^I \leq M \cdot x_{ls}^I \forall l, s$
4. $y_{ls}^D \leq M \cdot x_{ls}^D \forall l, s$
5. $y_{ij}^W \leq M \cdot x_{ij}^W \forall i, j$

Constraints 1-5 are used to ensure that the y values are only allowed to be non-zero if the x value is set to 1. The value of M is some arbitrarily large number such that y will not exceed it.

6. $\sum_s x_{ls}^I \leq 1 \forall l$
7. $\sum_s x_{ls}^D \leq 1 \forall l$
8. $\sum_s x_{ls}^W \leq 1 \forall l$

Constraints 6-8 are used to ensure that for every location l, no more than one size is chosen for the injection treatment facilities, the drinking treatment facilities, and the injection wells. This summation is also allowed to be zero if no facility is built for that location.

9. $\sum_l \sum_s x_{ls}^I \geq 1$
10. $\sum_l \sum_s x_{ls}^D \geq 1$
11. $\sum_l \sum_s x_{ls}^W \geq 1$

Constraints 9-11 ensure that at least 1 of each facility (injection treatment, drinking treatment, or injection well) must be built.

12. $W_i = \sum_{j\epsilon T,U}(y_{ij}^P + y_{ij}^T)\forall i$

Constraint 12 states that all water coming out of a production node must go to either an injection treatment facility or a drinking treatment facility.

13. $\sum_s y_{ls}^I = \sum_{j\epsilon R}(y_{lj}^P + y_{lj}^T) \forall\ l\epsilon T$
14. $\sum_s y_{ls}^D = \sum_{j\epsilon R}(y_{lj}^P + y_{lj}^T) + \sum_{j\epsilon S}(y_{lj}^P + y_{lj}^T)\ \forall\ l\epsilon U$

Constraints 13 states that for all injection treatment facilities, all outputs must go to an injection node. Constraint 14 states that for all drinking treatment facilities, all outputs must go to either a consumer node or an injection node.

15. $\sum_i(y_{ij}^P + y_{ij}^T) \leq V_i \forall j\epsilon S$

Constraint 15 requires that each consumer node can only buy up to some maximum amount of drinking water. This is the demand of that municipality.

16. $\sum_i(y_{ij}^P + y_{ij}^T) \leq \sum_s x_{js}^I \cdot C_{js}^I \forall j\epsilon T$
17. $\sum_i(y_{ij}^P + y_{ij}^T) \leq \sum_s x_{js}^D \cdot C_{js}^D \forall j\epsilon U$
18. $\sum_i(y_{ij}^P + y_{ij}^T) \leq \sum_s x_{js}^W \cdot C_{js}^W \forall j\epsilon R$
19. $y_{ij}^P \leq B_{ij}$

Constraints 16-18 are the capacity constraints. These constraints state that for each of facility we choose to build (i.e., injection treatment facilities, drinking treatment facilities, or injection wells), we do not exceed the maximum capacity of that facility. Constraint 19 simply states that the amount flowing through a pipe does not exceed the capacity of the pipe.

20. $\sum_s y_{js}^W \cdot C_{js}^W \leq A_j \forall\ j\epsilon R$

Constraint 20 ensures that we do not build more injection well space than the maximum available injection node volume.

21. $\sum_s y_{ls}^I = \sum_j(y_{il}^P + y_{il}^T) \forall\ l\epsilon T$
22. $\sum_s y_{ls}^D = \sum_j(y_{il}^P + y_{il}^T) \forall\ l\epsilon U$
23. $\sum_s y_{ls}^W = \sum_j(y_{il}^P + y_{il}^T) \forall\ l\epsilon R$

Constraints 21-23 ensure that the amounts going into each of the facilities are all coming from trucks or pipes from anywhere else.

24. $x_{ij}^P \epsilon\{0,1\}$
25. $y_{ij}^P \geq 0$
26. $x_{ij}^T \epsilon\{0,1\}$
27. $y_{ij}^T \geq 0$
28. $x_{ls}^I \epsilon\{0,1\}$
29. $y_{ls}^I \geq 0$

30. $x_{ls}^{D} \epsilon\{0,1\}$
31. $y_{ls}^{D} \geq 0$
32. $x_{ls}^{W} \epsilon\{0,1\}$
33. $y_{ls}^{W} \geq 0$

And finally, constraints 24-33 are common constraints for MILP problems. The constraints on the x values are binary constraints, and the constraints on the y values are non-negative constraints.

Discussion

Figure 2 depicts the different flows in the model. In this figure, Q = {1], R = {2}, S = {3}, T = {4,5}, and U = {6,7}. Each arrow in the figure represents the sum of the transportation modes between those nodes, $F_{ij}^{P} + F_{ij}^{T}$. Note that any variable with a subscript ij is only defined where (i.) flow occurs from a production node i to a treatment node j, (ii.) flow occurs from a treatment node i to an injection node j, or (iii.) flow occurs from a treatment node i to a consumer node j. Defining the variables with this exception greatly decreases the total number of variables in our model. If we really wanted to, we could define these variables and set them to 0.

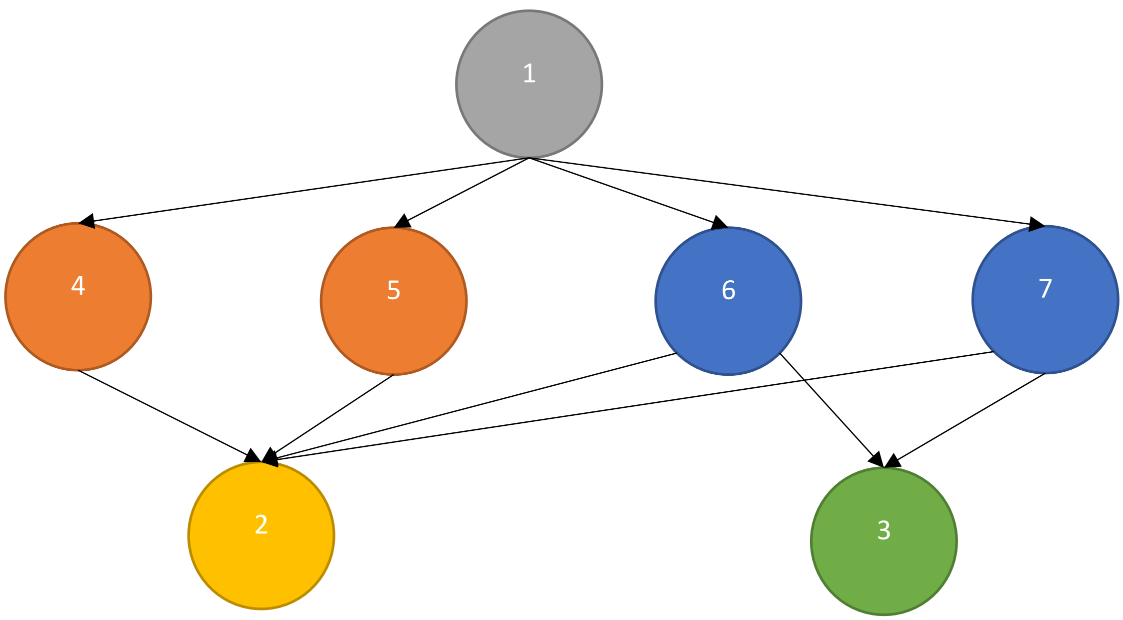

Figure 2: Depiction of a simple model with flows.

In this paper, we have presented a model for the optimization of investments in facilities for managing hydraulic fracturing-produced wastewater. We presented it as a minimize cost model, but "minimize costs" is an intentional misnomer in this scenario because we will consider the buying price of the drinking water as negative costs. This is still different from maximizing revenue because we are not considering the revenue generated from the oil production or other sources which are considerably more important revenue sources than the drinking water.

Because of the size of the problem, we have left it unsolved. With a real data source, it would be interesting to find out what tradeoffs are more important than others in the optimization. Solving a problem of this size would be exceedingly difficult because of its size. Notice that there are ten defined decision variables, each with two subscripts. Depending on how granular of a solution you would like to find, this could mean over a thousand decision variables.

One interesting aspect of this model formulation is the optimization of non-linear functions. The functions modeling varying costs of different sizes of infrastructure investments are all non-linear. Recall the functional form of those variables in Figure 1. However, we approached this formulation by parameterizing the non-linear function into discrete data. For example, F^I_{ls}, the fixed cost of an injection well could be broken up into F^I_{11}, F^I_{12}, and F^I_{13}, for the fixed costs of an injection well of size one, two, or three at location 1, respectively. While the associated capacity's, $C^I_{11}, C^I_{12}, and\ C^I_{13}$ increase linearly, the fixed costs should be set such that $F^I_{12} - F^I_{11} > F^I_{13} - F^I_{12}$. Thus, this non-linearity is taken care of in the data.

One limitation of the model is that the price that water sells for is constant in different consumer nodes. In actuality, the price that water sells for may change from city to city. Another limitation is the number of parameters that need to be defined. If the same problem could be modeled with fewer parameters, it would be much simpler to define. Similarly, if the same problem could be modeled with fewer decision variables, it would be much more realistic to solve.