THE LEAST COST DISPATCH PROBLEM

Introduction

The least cost dispatch problem is a problem all electric companies face when trying to meet demand. With changing needs for demands across the world, especially in developing countries, there is a need for electric companies to provide more and more electricity while keeping its carbon footprint as low as possible. However, for an individual company, it is not always economically feasible from their perspective to keep its carbon footprint low. Therefore, some governments may try to step in and impose a carbon tax that forces companies to decrease their carbon footprint if they want to optimize their profit.

In this paper, we will look from the point of view of an electric company who wants to provide electricity for their customers with the lowest possible cost. Our electric company has historically had several options for providing power: solar panels, wind turbines, a gas plant, and a coal plant. However, the company now has the option to add a battery to their inventory. The battery could act as a load balancer by storing energy that was generated earlier and using it later. Theoretically, this would mean in times where renewable energy is being produced and captured at high rates, the battery could be charging up. Thus, when renewable energy is being produced at lower rates, the battery could be used rather than using up non-renewable, carbon depleting resources.

Assumptions

The model makes several assumptions to simplify the problem. These assumptions are listed below:

1. The company already owns all the hardware it needs to generate the electricity. In essence, the gas plant, wind turbines, solar panels, and coal plant all already exist, and there is no need to include the decision to purchase them in our model as they are sunk costs.
2. The demand load for a given hour can be forecasted with a high degree of confidence. Optimization can then be done around these predicted demands.
3. Availability of non-renewable resources vary throughout the day based on environmental conditions. For example, solar energy that can be produced throughout the day will depend on the sun, and therefor will not be abundant outside of hours where there is ample sunlight. Wind energy will also change throughout the day.
4. Gas and coal plants have a constant capacity rate throughout the day.
5. There are no costs associated with wind and solar energy. The variable costs associated with gas and coal are constant.
6. There is no carbon produced when using wind and solar energy. The carbon factors of gas and coal production are constant.
7. The battery cannot hold electricity indefinitely. The battery will lose its storage over time at a constant rate of 0.02.

8. We will run two cases for the company. In the first case, there will be no carbon tax imposed at all. In the second case, we assume a constant carbon tax of $50 per ton of carbon produced.
9. There is an unlimited amount of battery capacity that is available to us for rent. We pay for the rental of battery capacity in terms of dollars per kilowatt hour-day.
10. The battery rental will be a one-time decision. We cannot change the amount of battery capacity we are renting throughout the day.
11. The battery initially starts with no stored energy.

Variables

It is useful to define several variables for this model. The units of the variable are given in parentheses after the definition.

Parameters

1. L_t: Load at time t (GW)
2. C_{it}: Capacity Factor for electricity type i at time t.
3. I_{it}: Capacity for electricity type i at time t (GW).
4. A_{it}: Available capacity for electricity type i at time t (GW).
    - $A_{it} = C_{it} I_{it} \; \forall \; i, t$
5. V_i: Variable costs for electricity type i that is assumed to be constant ($/kWh).
6. N_i: Carbon factor for electricity type i that is assumed to be constant (lbs./kWh).
7. T: Carbon tax. Assumed to be 0 in the no tax model and fifty in the tax model ($/ton).
8. B: Fraction of battery lost every hour.
9. R: Cost to rent a battery ($/kWh).

Decision Variables

1. P_{it}: Power produced by electricity type i at time t (GW).
2. Y_t: Power produced by battery at time t (GW).
3. H_t: Power charged and stored in battery at time t (GW).
4. E: Battery storage capacity (GW).

Endogenous Variables

1. c_{it}: total variable costs of electricity type i at time t ($).
    - $c_{it} = P_{it} V_i \cdot 10^6 \; \forall \; i, t$
2. p_t: Total power produced at time t (GW).
    - $p_t = \sum_i P_{it} \;\; \forall \; t$
3. e_t: Excess power produced at time t (GW).
    - $e_t = p_t - l_t \; \forall \; t$
4. o_t: Total output at time t (GW).
    - $o_t = p_t + Y_t \; \forall \; t$
5. n_{it}: Carbon produced by electricity type i at time t (lbs.).
    - $n_{it} = N_i P_{it} \; \forall \; i, t$
6. x_t: Total carbon tax at time t ($).
    - $x_t = T \sum_i n_{it} \;\; \forall \; t$
7. b_t: Battery remaining capacity at beginning of hour t (GW).

- $b_t = b_{t-1}(1-B) + H_{t-1} - Y_{t-1} \; \forall \, t > 0$
- Essentially, this is the amount stored in a battery at the beginning of a time, t, is the stored capacity from the last hour multiplied by a loss factor plus the amount that was added into the battery last period minus the amount that was used by the battery last period.
- $b_0 = 0$

8. s: Battery storage capacity cost ($/day).
    - $s = ER \cdot 10^6$

Model

Objective

The objective of this model is to minimize the costs required to service demand to all customers at all times of the day. This is represented by the following equation: $\sum_i \sum_t (c_{it}) + \sum_t (x_t) + s$

Where:

- $\sum_i \sum_t (c_{it})$ represents the total variable costs across all electricity's i and all times t
- $\sum_t (x_t)$ represents the total cost paid in carbon taxes
- s is the battery storage capacity cost

Constraints

Several mathematical constraints must be added to the model to represent our assumptions.

1. $P_{it} \leq A_{it} \; \forall \, i, t$
    - The amount of each electricity type i that is produced at time t must be less than or equal to what is available for that electricity type.
2. $H_t \leq e_t \; \forall \, t$
    - The amount of electricity stored in the battery at time t can not be more than the amount of electricity that was created in excess for that time
3. $Y_t \leq b_t \; \forall \, t$
    - The amount of electricity produced by the battery at time t cannot be more than what was already stored in the battery at the period before
4. $o_t \geq l_t \; \forall \, t$
    - The output at time t must be greater than the given load at time t to continually meet customer demand.
5. $E \geq b_t \; \forall \, t$
    - The capacity of the battery we rent for the day must be greater than all the storage that we need at any individual time t
6. $Y_t \leq E \; \forall \, t$
    - The power we produce from the battery must be less than what we rent for the day.
7. $H_t \leq E \; \forall \, t$
    - The power we store in the battery must be less than what we rent for the day.

Discussions

Figure 1 shows the hourly dispatch of the no tax case, and Figure 2 shows the hourly dispatch of the case where the carbon tax is included.

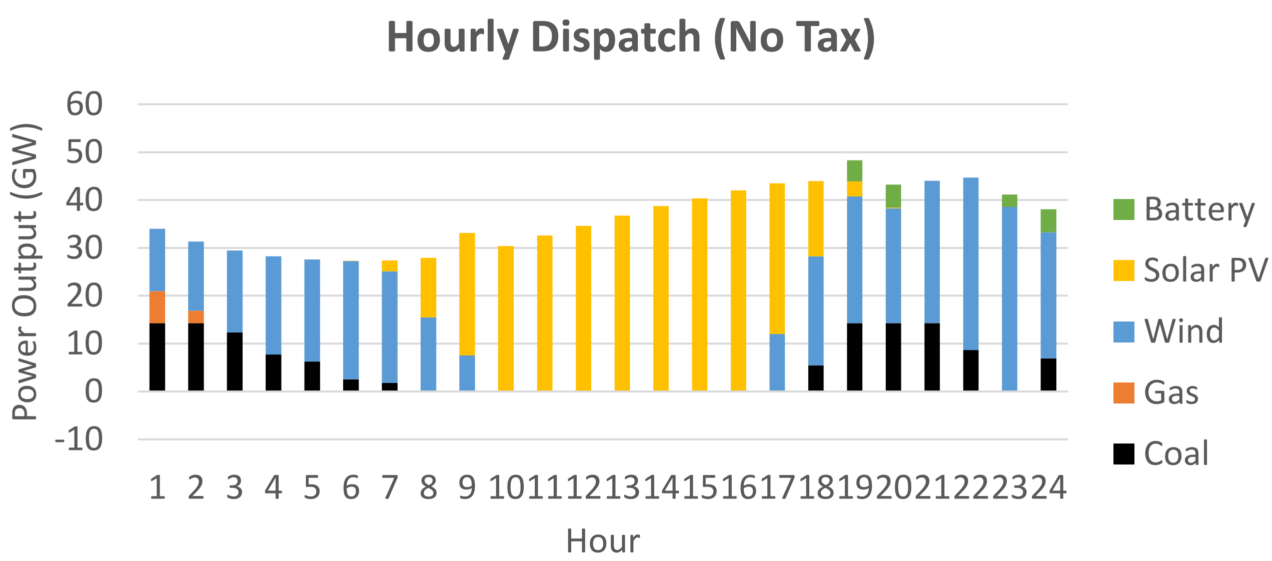

Figure 1: Optimized Hourly Dispatch for the Case where No Tax is Imposed

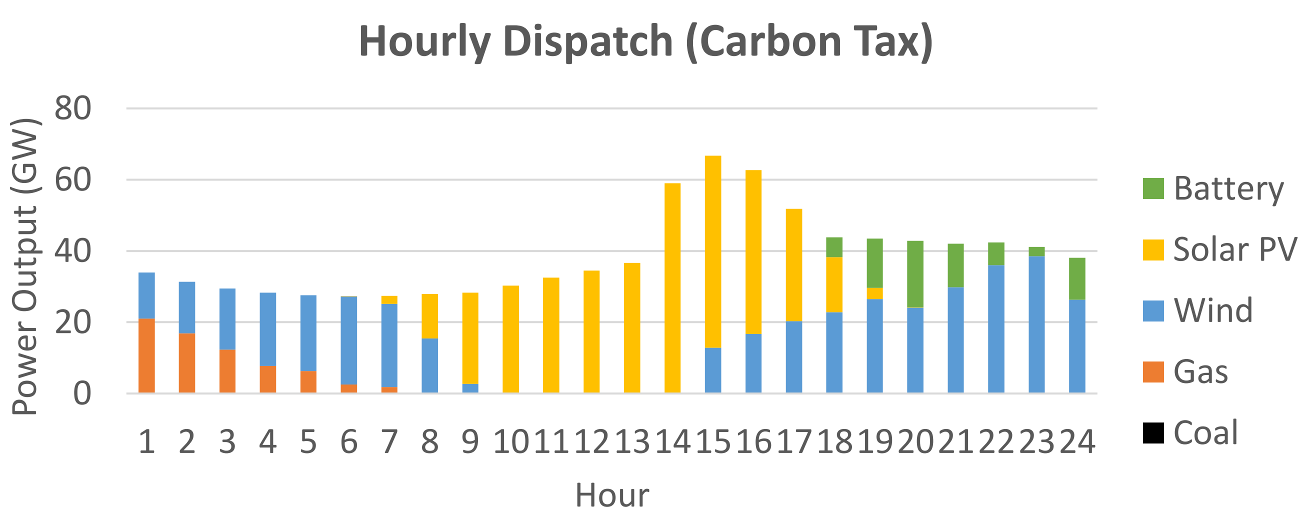

Figure 2: Optimized Hourly Dispatch for the Case where a $50 Carbon Tax is Imposed

In both cases, the problems were solved to optimality using Excel Solver and a linear programming solution. All constraints were satisfied. The objective function of Case I was solved to $2.8M, and the objective function of Case II was solved to $5.4M. This behavior is expected, as an additional cost was added in the second case.

Perhaps the most interesting result we get from this analysis is related to the "duck curve." Notice how in Figure 1, the use of carbon-utilizing resources starts in the first few hours of the day, diminish in the middle hours, and rise again at the end of the day. In Figure 2, however, the usage of carbon-utilizing resources never picks back up at the end of the day. This is because the excess solar energy, collected in the middle of the day when the sun was out, could be used to meet the demands of the later hours. This is one of the benefits of paying for the battery.

If you look closely, you can see that this flattening out of the duck curve appears in Figure I as well. This effect is less pronounced in this case, because although the battery saves the company's carbon footprint, it is still not optimal for companies to use this route effectively with the expensive cost of the battery storage. The reason this becomes more pronounced in Figure 2 is because of the additional carbon tax. The additional carbon tax makes it relatively a better decision to use the battery more often. This is one of the major benefits (to the government) of imposing the carbon tax.

Another effect of the carbon tax was the switch for this company from using coal powered electricity to gas powered electricity when resorting to carbon-based resources. This is expected because the carbon factor of coal, N_{coal}, is significantly higher than the carbon factor of gas. Thus, while it makes economic sense to use coal when it is not taxed, it no longer makes economic sense to use coal when there are relatively heavier taxes on it than there are on gas.

The effect of the carbon tax on the amount of carbon that the company utilized dropped the total used carbon from 257 million lbs. down to 83.6 million lbs. The total costs for the company increased from $2.8M to $5.4M. Thus, by doubling the cost that the company paid the government was able to cut the company's carbon emissions into one third.

References

Leibowicz, D. B. (2021, February 25). Systems Modeling. *Electricity Storage: Electricity*.