COMP5920M SCHEDULING MAY/JUNE 2019 EXAM

Question 1

Under the Generate and Select approach for train crew scheduling, the Select Phase can be solved as a set covering integer linear programming (ILP) problem.

(a) Explain the principle constraints in the ILP. [2 marks]

- Each work piece must be covered by at least one selected shift
- With *m* work pieces and *n* candidate shifts, A_{ij} being the coverage coefficient matrix and x_j being the binary candidate shift selection variables

$$\sum_{j=1}^{n} A_{ij}\, x_j \geq 1 \qquad i = 1,2, \ldots, m$$

(b) One of the tasks in your coursework was to implement a greedy heuristic to reduce the given dataset DS2 from 73,517 down to a target of 200 candidate shifts before applying the ILP.

(i) Briefly describe the greedy heuristic used. [3 marks]

- Choose from the pot a random candidate shift *S*. If all the work pieces of *S* are covered by more than one candidate shifts, *S* is redundant and discarded; otherwise *S* is retained.
- This is repeated until the number of shifts remaining in the pot plus those shifts already retained equals to the target, or there is no more candidate shift remaining in the pot.
- Then, all the shifts remaining in the pot are also retained

(ii) Suppose a feasible solution schedule for DS2 requires at least 80 shifts. What would be the effect of setting a target for the greedy heuristics to reduce DS2 down to 70 candidate shifts? [2 marks]

- The heuristic ensures that it will be possible to yield a feasible solution from the retained candidate shifts at the end. When the target is set too low, the number of candidate shifts retained will be higher than the target before the iteration loop is terminated with no more candidate shifts remaining in the pot

(iii) Suggest how some candidate shifts could be prioritised for deletion thereby improving the greedy heuristic. [3 marks]

- Choose the least coverage count amongst all the work pieces of a candidate shift as the candidate shift's ranking value
- A low ranking value implies the candidate shift would be more likely to be needed for covering a certain work piece
- Candidate shifts with a higher ranking value is prioritised for deletion

(c) The ILP can be solved in two stages as discussed in this module.

(i) Outline the two stages. [5 marks]

- In stage 1, a relaxed LP is solved in which the integer requirement is ignored for all the shift variables
- In stage 2, the sum of the fractional values in the relaxed LP solution is rounded up as an upper bound target for the number of integral shifts to be used. A new constraint is added for the upper bound target.
- Branch-and-bound is used to find an all integer solution in stage 2.
- A specialised branch-and-bound algorithm is used, which exploits certain features found in the continuous relaxed LP solutions obtained in the process

(ii) Briefly explain how crew scheduling problem specific characteristics can be exploited in the second stage and give one example to illustrate.

[5 marks]

- Branching based on a problem characteristic P so that on one of the branches the candidate shifts do not exhibit characteristic P and on the other branch they do.
- Relevant candidate shifts are physically eliminated on each branch, which is more efficient than inserting many equivalent constraints into the ILP.
- For example, P may focus on a particular relief opportunity R identified as critical. One branch bans the use of R in the candidate shifts and the other branch allows R to be used.

Question 2

(a) How far ahead actual operations are scheduled has very significant implications on the scheduling process. Discuss the above statement in the context of public transport.

[8 marks]

- Very long term planning, e.g. for many years into the future in franchise or local council tender bids
    - Ball park estimates, no detailed operational schedules required, strategic
- Long term planning, for regular operations such as half-yearly timetables
    - Optimisation is critically important because the daily schedules are operated repeatedly over many months
    - Much time and effort is spent on scheduling in advance
- Short term planning, re-scheduling in advance of predictable service changes, e.g. rail track maintenance, Bank Holiday timetables
    - Minimum change from the regular long term schedules are desirable
    - Importance of schedule robustness against delays increases
- Very short term planning, re-scheduling on the day of operation due to unplanned disruptions
    - A lot of real time information needed
    - The re-scheduling process has to be very fast

(b) The table below shows a timetable for 40 university courses. For example Course 11 (highlighted) has been assigned to Room 4 in Timeslot 5, but this might not be feasible if there are more students enrolled for Course 11 than Room 4 can accommodate.

Timeslot	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Room	Course								
1		19		12	36	39	6	24	29

2	7	25	26	27	2	16		14	
3	13		30	1		4	33		35
4		3	40	34	**11**		5	32	8
5	18	20	31	17		28	15		
6	38	21	10		22		37	9	23

Many simple ‘low level heuristics’ (LLH) could be designed to try to improve the timetable. For example, the assignments in the timetable could be swapped individually or in rows or in columns.

(i) Discuss how the goodness of a potential solution timetable could be evaluated by an LLH. [3 marks]

- Quantitative measures of hard and soft constraint violations should be used in the assessment
- Weighted sum of the quantitative measures forms an overall penalty value to be minimised
- Huge weights for hard constraint violation

(ii) Outline a performance based hyper-heuristic algorithm utilising a group of LLHs, which may be applied for the above timetabling problem. [4 marks]

- Construction of a crude initial solution
- In each iteration, the hyper-heuristic controller selects one of the LLHs to be executed based on their historical performance statistics
- At the end of an LLH execution, the performance of the LLH in that iteration is measured by the improvement on the current best solution the LLH has achieved.
- Infrequently used LLH might gradually be given added priority to be selected

(c) Telescopes used in astronomy, e.g. the Hubble Space Telescope, are very expensive scarce resources. Suppose a scheduling algorithm for such telescopes is classified as
1 | clv, prec, pmtn | $\sum \boldsymbol{w_j U_j}$.
Explain the meaning of this classification [5 marks]

- Single machine (the telescope)
- Job characteristics
    - clv – predictable behaviour of the target objects to be observed
    - prec – there are precedence relationships between some jobs
    - pmtn – job pre-emption is allowed
- Minimise the weighted sum of unit penalties for all the jobs

Question 3

(a) Utilising limited resources is commonly required in scheduling. For rail transport scheduling, train units and crews are resources readily available at their arrival stations. Discuss the role of shortest path algorithms in optimising re-deployment of train units and crews at the end of the timetabled trips they have served.

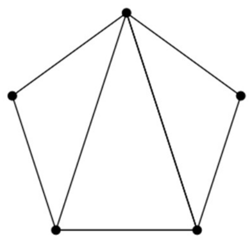
[5 marks]

- Shortest path algorithms enable the main scheduling algorithms to consider the feasibility of redistributing resources to other locations for their further deployment
    - is there sufficient time to make the connection?
    - how much does it cost to make the connection?
    - there is a trade-off between incurring idle time of keeping a resource stay put and redistribution it to other locations
- Helps when supply and demand of resources is imbalanced across the network, especially when often there is peak and off-peak traffic in opposite directions.

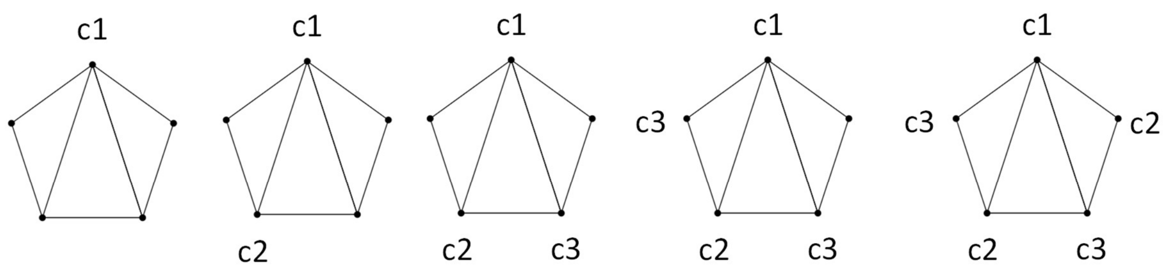
(b) Discuss how a graph can be set up for a bus timetable with the intention of using the graph's chromatic number as an estimate of the fleet size required to cover the timetable [3 marks]

- Each trip in the bus timetable is represented by a graph vertex
- Graph edges are inserted between any pair of trip vertices in which the same bus vehicle cannot serve both trips
    - e.g. the trips overlap in time or a link would require relocation but there is not enough time for the relocation; the trips require different vehicle types

(c) The graph below represents a graph colouring problem, which can be solved by the 'saturation degree ordering heuristic'. Outline the heuristic and illustrate its steps by applying it to the graph below. [5 marks]



- One uncoloured vertex is to be chosen at a time to be coloured until all vertices are coloured
    - Choice criterion: maximum saturation degree (number of differently colour labels in the adjacent vertices). When there is a tie, choose the tied vertex that has the largest number of edges connected to it; in case of a further tie, choose any one of the tied vertices
    - Assuming the colours are sequentially numbered, use the lowest feasible colour number
- Illustration using the given graph



(i) (ii) (iii) (iv) (v)

(d) Outline the modelling of train unit scheduling as a multi-commodity network flow problem (a mathematical formulation of the associated ILP is not required)

[7 marks]

- A network is set up as a directed acyclic graph (DAG)
  - Graph vertices represent train trips plus a source (from which all train units begin) and a sink (to which all train units end)
  - Graph edges are inserted to represent feasible potential links between the vertices
- Seek optimal paths from source to sink, each path is the schedule for a train unit
  - Minimising operational costs
  - Covering all the train trips in the timetable
  - Meeting passenger seat demands
  - Satisfying operational constraints
- Each train unit type is a commodity type, and each path from source to sink has an associated flow value being the number of units of a certain unit type