# FMCW Radar Signal Processing

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## 2 Definition

### 2.1 LFM signal in time domain

Instantaneous frequency of the signal

$$f(t) = f_c + Kt \tag{1}$$

$$K = \frac{B}{T_{fast}} \tag{2}$$

where $f_c$ is central frequency, $T_{fast}$ is fast time, $B$ is bandwidth, $K$ is the slope, $t$ is time within fast time.

Transmitted signal

$$s_t(t) = exp\{-j2\pi f(t)\} = exp\{-j2\pi(f_c + \frac{K}{2}t)t\} \tag{3}$$

Received signal

$$s_r(t) = s_t(t - \tau_0) = exp\{-j2\pi * (f_c + \frac{K}{2} * (t - \tau_0)) * (t - \tau_0)\} \tag{4}$$

$$\tau_0 = \frac{2R_0}{c} \tag{5}$$

where $\tau_0$ is initial time delay, $R_0$ is initial distance of target, $c$ is light speed.

Conjugate of the transmitted signal (filter signal in the mixer)

$$h(t) = s_t^*(t) = exp\{j2\pi(f_c + \frac{K}{2}t)t\} \tag{6}$$

# 3 Range: 1D FFT for LFM

Mixing (4) and (6) gives the dechirped signal (baseband signal)

$$s_{mix}(t) = s_r(t)h(t) = exp\{j2\pi(f_c\tau_0 + Kt\tau_0 - \frac{K}{2}\tau_0^2)\} \tag{7}$$

Assuming one target staying at a certain distance, introducing (5) to (7) gives

$$s_{mix}(t) = exp\{j2\pi(f_c\tau_0 + K\tau_0 t - \frac{K}{2}\tau_0^2)\} \tag{8}$$

where the first and third terms are constant. Applying FFT to (8) to a signal series along t-axis measured within one fast time gives the coefficient of $t$, i.e., range $R_0$.

# 4 Range, Doppler: 2D FFT for LFM

Assuming one target moving along range axis, (5) becomes

$$\tau_0 = \frac{2R_0 + 2vT_{int}n + 2vt}{c} \tag{9}$$

where $v$ is radial velocity along range, $T_{int}$ is time interval of one chirp, $n$ is slow time index. Introducing (9) to (7) gives

$$s_{mix}(t) = exp\{j2\pi(f_c\frac{2R_0 + 2vT_{int}n + 2vt}{c} + Kt\frac{2R_0 + 2vT_{int}n + 2vt}{c} - \frac{K}{2}(\frac{2R_0 + 2vT_{int}n + 2vt}{c})^2)\} \tag{10}$$

The third term in (10) is much smaller than the other, because it is a fraction with denominator $c$ squared. So we drop it out, and then

$$s_{mix}(t) = exp\{j2\pi(\frac{2R_0f_c}{c} + \frac{2Kv}{c}t^2 + (\frac{2vf_c}{c} + \frac{2KR_0}{c})t + (\frac{2f_cvT_{int}}{c} + \frac{2KvT_{int}t}{c})n\} \tag{11}$$

Taking simple notation

$$f_d = \frac{2vf_c}{c} \tag{12}$$

and (5) gives

$$s_{mix}(t) = exp\{j2\pi(f_c\tau_0 + \frac{2Kv}{c}t^2 + f_dt + K\tau_0t + f_dT_{int}n + \frac{2KvT_{int}t}{c}n\} \tag{13}$$

where the second term is negligible. The third term is negligible, because we assume the target does not move within one fast time interval, and thus only the fourth term plays the main role in range processing. The sixth term is negligible. Now we get

$$s_{mix}(t) = exp\{j2\pi(f_c\tau_0 + K\tau_0t + f_dT_{int}n\} \tag{14}$$

Applying 2D FFT to (14) to a signal matrix, first along fast time axis and then slow time axis, gives the coefficient of $t$, i.e., range $R_0$ (or $\tau_0$) and the coefficient of $n$, i.e., range $v$ (or $f_d$).

## 5 Range + Doppler + Azimuth: 2D FFT for LFM + 1D FFT in space domain

The estimation of range and Doppler information relies on the fast time and slow time of LFM/FMCW waveform. Detection of angle (DOA) relies on the spatial distribution of antenna arrays.

Assuming a target in the far field of an uniform linear array (ULA), Figure 1 shows the coordinate in space domain. $R$ is the distance of the target from the phase centre, $\theta$ is the azimuth angle in $XOY$ horizontal plane, $d$ is the distance between neighbouring antenna units (usually designed as half wavelength $\lambda/2$ ), $l$ is the index of antenna units.

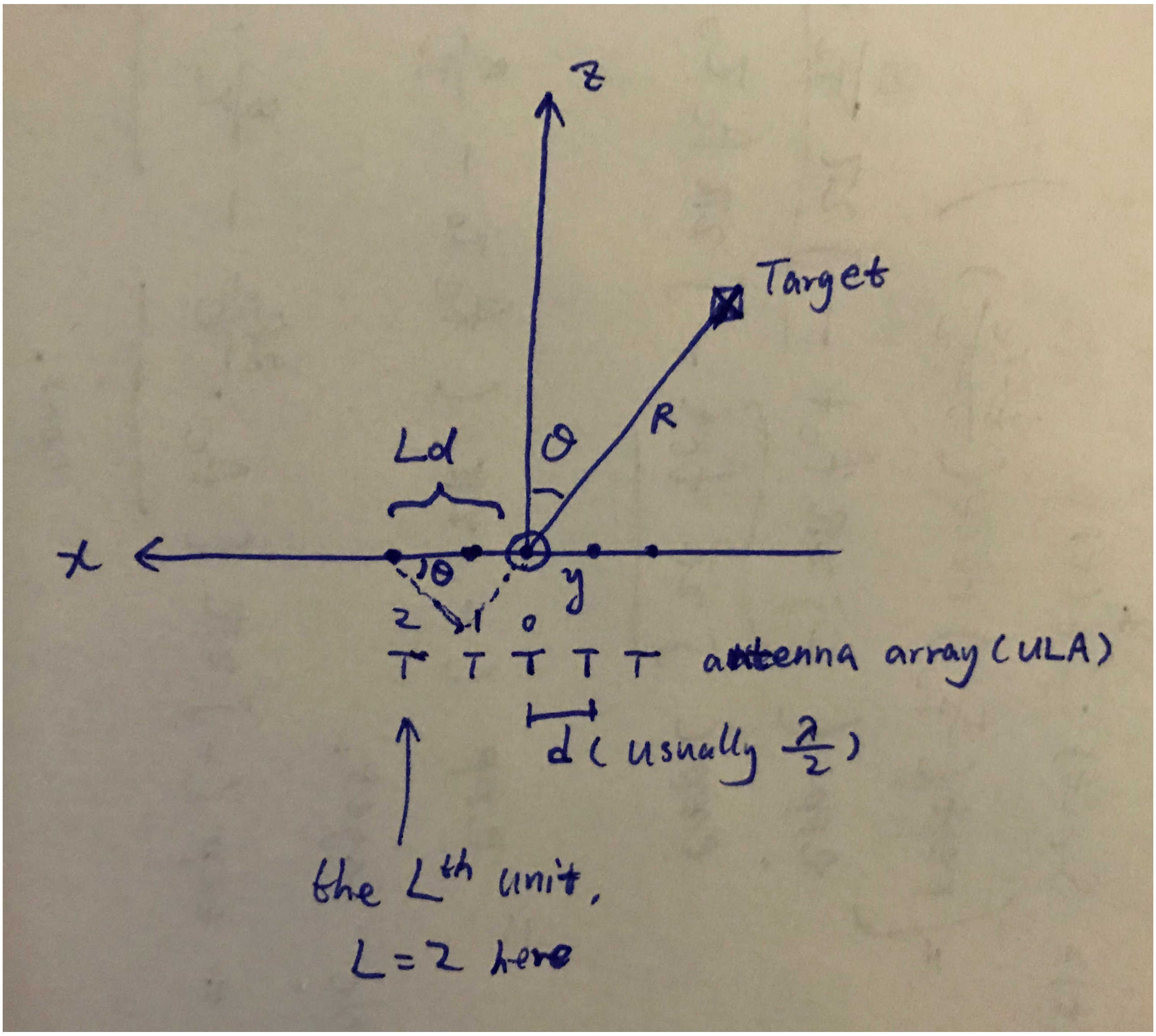


Figure 1:

Assuming a master antenna unit transmitting and all units receiving, considering the time delay of the receiving signal at the $l^{th}$ unit, (5) becomes

$$\tau_0 = \frac{2(R_0 + vT_{int}n + vt) + ld\sin\theta}{c} \tag{15}$$

Introducing (15) to (7), dropping out some terms as in (13) and (14), gives

$$s_{mix}(t) = exp\{j2\pi(f_c\tau_0 + K\tau_0 t + f_d T_{int} n + \frac{f_c l d \sin\theta}{c})\} \quad (16)$$

Applying 3D FFT to (16) to a signal cube, first along fast time axis, then slow time axis and then azimuth axis, gives the coefficient of $t$, i.e., range $R_0$ (or $\tau_0$), the coefficient of $n$, i.e., range $v$ (or $f_d$) and the coefficient of $l$, i.e., $\theta$ (or $\sin\theta$).

# 6 Range + Doppler + Azimuth + Elevation: 2D FFT for LFM + 2D FFT in space domain