Уилкинсон Дж.Х., Райнш С. Справочник алгоритмов на языке Алгол. Линейная алгебра

М.: Машиностроение, 1976. — 390 с.

В книге приведены алгоритмы решения всех основных задач линейной алгебры, реализованные в виде процедур на языке Алгол-

60. Для специалистов по теории управления представляют интерес алгоритмы решения проблемы собственных значений для произвольных матриц

page 37, (53)

$$A53\_o := \begin{pmatrix} 360360 & 180180 & 120120 & 90090 & 72072 & 60060 & 51480 \\ 180180 & 120120 & 90090 & 72072 & 60060 & 51480 & 45045 \\ 120120 & 90090 & 72072 & 60060 & 51480 & 45045 & 40040 \\ 90090 & 72072 & 60060 & 51480 & 45045 & 40040 & 36036 \\ 72072 & 60060 & 51480 & 45045 & 40040 & 36036 & 32760 \\ 60060 & 51480 & 45045 & 40040 & 36036 & 32760 & 30030 \\ 51480 & 45045 & 40040 & 36036 & 32760 & 30030 & 27720 \end{pmatrix} \qquad Gilbert7 := A53\_o$$

$$det\_A53\_o := 381614277072600 \qquad det\_A53\_o2 := 8.47353913\cdot 10^{-2}\cdot 2^{52}$$

$$det\_A53\_o2 - det\_A53\_o = -3.888\times 10^{5}$$

$$det\_A53 := |A53\_o| \qquad \delta\_A53 := \frac{|det\_A53 - det\_A53\_o|}{det\_A53\_o} \qquad \delta\_A53 = 5.289\times 10^{-10}$$

$$invA53\_t := \left(A53\_o^{-1}\right)$$

$$invA53\_t = \begin{pmatrix} 1.36\times 10^{-4} & -3.263\times 10^{-3} & 0.024 & -0.082 & 0.135 & -0.108 & 0.033 \\ -3.263\times 10^{-3} & 0.104 & -0.881 & 3.133 & -5.385 & 4.431 & -1.4 \\ 0.024 & -0.881 & 7.93 & -29.371 & 51.923 & -43.615 & 14 \\ -0.082 & 3.133 & -29.371 & 111.888 & -201.923 & 172.308 & -56 \\ 0.135 & -5.385 & 51.923 & -201.923 & 370.192 & -319.846 & 105 \\ -0.108 & 4.431 & -43.615 & 172.308 & -319.846 & 279.138 & -92.4 \\ 0.033 & -1.4 & 14 & -56 & 105 & -92.4 & 30.8 \end{pmatrix}$$

$$invA53\_o := \begin{pmatrix} 1.35971314283\cdot10^{-4} & -3.26325113753\cdot10^{-3} & 2.44740603295\cdot10^{-2} & -8.15793917156\cdot10^{-2} & 1 \\ -3.26325113738\cdot10^{-3} & 1.04422850066\cdot10^{-1} & -8.81060620311\cdot10^{-1} & 3.13264074649\cdot10^{0} & - \\ 2.44740603601\cdot10^{-2} & -8.81060620282\cdot10^{-1} & 7.92950934930\cdot10^{0} & -2.93684512777\cdot10^{1} & \\ -8.15793917875\cdot10^{-2} & 3.13264074692\cdot10^{0} & -2.93684512768\cdot10^{1} & 1.11879649598\cdot10^{2} & - \\ 1.346049544738\cdot10^{-1} & -5.38420043662\cdot10^{0} & 5.19190848981\cdot10^{1} & -2.01907567906\cdot10^{2} & \\ -1.07683295031\cdot10^{-1} & 4.43041069139\cdot10^{0} & -4.36119354458\cdot10^{1} & 1.72294292795\cdot10^{2} & - \\ 3.33303738252\cdot10^{-2} & -1.39988227242\cdot10^{0} & 1.39988675007\cdot10^{1} & -5.59956005459\cdot10^{1} & \end{pmatrix}$$

$$invA53\_o - invA53\_t = \begin{pmatrix} -3.822\times10^{-9} & 1.521\times10^{-7} & -1.464\times10^{-6} & 5.69\times10^{-6} & -1.043\times10^{-5} & 9.013 \\ 1.521\times10^{-7} & -6.054\times10^{-6} & 5.826\times10^{-5} & -2.264\times10^{-4} & 4.15\times10^{-4} & -3.585 \\ -1.464\times10^{-6} & 5.826\times10^{-5} & -5.606\times10^{-4} & 2.178\times10^{-3} & -3.992\times10^{-3} & 3.449 \\ 5.69\times10^{-6} & -2.264\times10^{-4} & 2.178\times10^{-3} & -8.462\times10^{-3} & 0.016 & -0 \\ -1.043\times10^{-5} & 4.15\times10^{-4} & -3.992\times10^{-3} & 0.016 & -0.028 & 0. \\ 9.013\times10^{-6} & -3.585\times10^{-4} & 3.449\times10^{-3} & -0.013 & 0.025 & -0 \\ -2.96\times10^{-6} & 1.177\times10^{-4} & -1.133\times10^{-3} & 4.399\times10^{-3} & -8.062\times10^{-3} & 6.965 \end{pmatrix}$$

$$matVariation(invA53\_t, invA53\_o) = \begin{pmatrix} 2.811\times10^{-5} & 4.662\times10^{-5} & 5.983\times10^{-5} & 6.975\times10^{-5} & 7.749\times10^{-5} & 8 \\ 4.662\times10^{-5} & 5.798\times10^{-5} & 6.613\times10^{-5} & 7.227\times10^{-5} & 7.707\times10^{-5} & 8 \\ 5.982\times10^{-5} & 6.613\times10^{-5} & 7.07\times10^{-5} & 7.417\times10^{-5} & 7.689\times10^{-5} & 7 \\ 6.975\times10^{-5} & 7.227\times10^{-5} & 7.417\times10^{-5} & 7.564\times10^{-5} & 7.681\times10^{-5} & 7 \\ 7.749\times10^{-5} & 7.707\times10^{-5} & 7.689\times10^{-5} & 7.681\times10^{-5} & 7.678\times10^{-5} & 7 \\ 8.37\times10^{-5} & 8.093\times10^{-5} & 7.909\times10^{-5} & 7.777\times10^{-5} & 7.678\times10^{-5} & 7 \\ 8.879\times10^{-5} & 8.41\times10^{-5} & 8.09\times10^{-5} & 7.857\times10^{-5} & 7.679\times10^{-5} & 7 \end{pmatrix}$$

$$minmax(matVariation(invA53\_t, invA53\_o)) = \begin{pmatrix} 2.811\times10^{-5} & 8.879\times10^{-5} \end{pmatrix}$$

$$mkVec\_e(size, index) := \left| \begin{array}{l} R_{size-1} \leftarrow 0 \\ R_{index} \leftarrow 1 \\ \text{return } R \end{array} \right.$$

page 49 b_p47_o := mkVec_e(7, 0)

d_p47_t := lsolve(A53_o, b_p47_o)

$$d\_p47\_o := \begin{pmatrix} 1.35975135069 \cdot 10^{-4} \\ -3.26340327676 \cdot 10^{-3} \\ 2.44755247690 \cdot 10^{-2} \\ -8.1585082940 \cdot 10^{-2} \\ 1.34615387384 \cdot 10^{-1} \\ -1.07692310159 \cdot 10^{-1} \\ 3.33333341518 \cdot 10^{-2} \end{pmatrix}$$

$$d\_p47\_t^{T} = \begin{pmatrix} 1.36 \times 10^{-4} & -3.263 \times 10^{-3} & 0.024 & -0.082 & 0.135 & -0.108 & 0.033 \end{pmatrix}$$

$$matVariation(d\_p47\_t, d\_p47\_o) = \begin{pmatrix} 6.418 \times 10^{-9} \\ 4.468 \times 10^{-9} \\ 1.244 \times 10^{-8} \\ 1.711 \times 10^{-8} \\ 2.111 \times 10^{-8} \\ 2.348 \times 10^{-8} \\ 2.515 \times 10^{-8} \end{pmatrix}$$

page56???LU-разложение LU decomposition

https://en.wikipedia.org/wiki/LU_decomposition

Substituting these values into the LU decomposition above yields

$$\begin{bmatrix} 4 & 3 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1.5 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 0 & -1.5 \end{bmatrix}.$$

$$M\_forLU1 := \begin{pmatrix} 4 & 3 \\ 6 & 3 \end{pmatrix}$$

$$lu(M\_forLU1) = \begin{pmatrix} 0 & 1 & 1 & 0 & 6 & 3 \\ 1 & 0 & 0.667 & 1 & 0 & 1 \end{pmatrix}$$

```
__digToVec(d, Vec) := | n ← rows(Vec) − 1
                      | for i ∈ 0..n
                      |   Vec_i ← d
                      | return Vec
```

mkVecByDig(

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```
genGdiag_A61 := | size ← 40
                | R ← mkVecByDig(6, size)
                | R_0 ← 5
                | R_size−1 ← 5
                | return R
```

$genGdiag\_A61^{T} =$

	0	1	2	3	4	5	6
0	5	6	6	6	6	6	

$$ftDiag\_A61 := \begin{array}{|l} size \leftarrow 40 - 1 \\ R \leftarrow mkVecByDig(-4, size) \\ return\ R \end{array}$$

$ftDiag\_A61^T =$

	0	1	2	3	4	5	6
0	-4	-4	-4	-4	-4	-4	-

$$sdDiag\_A61 := \begin{array}{|l} size \leftarrow 40 - 2 \\ R \leftarrow mkVecByDig(1, size) \\ return\ R \end{array}$$

$sdDiag\_A61^T =$

	0	1	2	3	4	5	6
0	1	1	1	1	1	1	

$fForGenDiag(i) := (i\ \ i)$

$$fForFtDiag(i) := \begin{pmatrix} i & i+1 \\ i+1 & i \end{pmatrix}$$

fillMatr(Vec, Func

$$fForSdDiag(i) := \begin{pmatrix} i & i+2 \\ i+2 & i \end{pmatrix}$$

$$fForTrdDiag(i) := \begin{pmatrix} i & i+3 \\ i+3 & i \end{pmatrix}$$

$$mk3DiagMatr(genDiag, ftDiag, sdDiag) := \begin{array}{|l} n \leftarrow rows(genDiag) - 1 \\ R_{n,n} \leftarrow 0 \\ R \leftarrow fillMatr(genDiag, fForGenDiag, R) \\ R \leftarrow fillMatr(ftDiag, fForFtDiag, R) \\ R \leftarrow fillMatr(sdDiag, fForSdDiag, R) \\ return\ R \end{array}$$

$A\_p61\_o := mk3DiagMatr(genGdiag\_A61, ftDiag\_A61, sdDiag\_A61)$

$det\_A\_p61\_o := 4.10400390 \cdot 10^{-1} \cdot 2^{12}$ $\quad det\_A\_p61\_o = 1.681 \times 10^{3}$

$det\_A\_p61\_t := |A\_p61\_o|$ $\quad det\_A\_p61\_t = 1.681 \times 10^{3}$

$$d\_A\_p61 := \frac{|det\_A\_p61\_t - det\_A\_p61\_o|}{det\_A\_p61\_o}$$

$d\_A\_p61 = 1.524 \times 10^{-9}$

$b\_p62 := mkVec\_e(40, 0)$

$A\_p61\_o =$

	0
0	5
1	-4
2	1
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

$x\_p62\_t := \mathrm{lsolve}(A\_p61\_o, b\_p62)$

x_p62_t =

	0
0	13.171
1	25.366
2	36.61
3	46.927
4	56.341
5	64.878
6	72.561
7	79.415
8	85.463
9	90.732
10	95.244
11	99.024
12	102.098
13	104.488
14	106.22
15	...

$$x\_p62\_part1\_o := \begin{pmatrix} 1.3170731707\cdot 10^{1} \\ 2.5365853659\cdot 10^{1} \\ 3.6609756098\cdot 10^{1} \\ 4.6926829268\cdot 10^{1} \\ 5.6341463415\cdot 10^{1} \\ 6.4878048780\cdot 10^{1} \\ 7.2560975610\cdot 10^{1} \\ 7.9414634146\cdot 10^{1} \\ 8.5463414634\cdot 10^{1} \\ 9.0731707317\cdot 10^{1} \\ 9.5243902439\cdot 10^{1} \\ 9.9024390244\cdot 10^{1} \\ 1.0209756098\cdot 10^{2} \\ 1.0448780488\cdot 10^{2} \\ 1.0621951220\cdot 10^{2} \\ 1.0731707317\cdot 10^{2} \\ 1.07804878049\cdot 10^{2} \\ 1.0770731707\cdot 10^{2} \\ 1.0704878049\cdot 10^{2} \\ 1.0585365854\cdot 10^{2} \end{pmatrix}$$

$$x\_p62\_part2\_o := \begin{pmatrix} 1.0 \\ 1.0 \\ 9.9 \\ 9.6 \\ 9.2 \\ 8.8 \\ 8.4 \\ 7.9 \\ 7.4 \\ 6.9 \\ 6.4 \\ 5.85 \\ 5.2 \\ 4.6 \\ 4.0 \\ 3.3 \\ 2.7 \\ 2.0 \\ 1.3 \\ 6.8 \end{pmatrix}$$

$x\_p62\_o := \mathrm{stack}(x\_p62\_part1\_o, x\_p62\_part2\_o)$

$\mathrm{minmax}(\mathrm{matVariation}(x\_p62\_o, x\_p62\_t)) = \begin{pmatrix} 2.158 \times 10^{-13} & 4.647 \times 10^{-11} \end{pmatrix}$

page 88 $\quad size\_p88 := 7$

$$genGdiag\_A\_p88 := \left| \begin{array}{l} R \leftarrow \mathrm{mkVecByDig}(6, size\_p88) \\ R_{0} \leftarrow 5 \\ R_{size\_p88-1} \leftarrow 5 \\ \mathrm{return}\ \ R \end{array} \right.$$

$ftDiag\_A\_p88 := \mathrm{mkVecByDig}(-4, size\_p88 - 1)$

$sdDiag\_A\_p88 := \mathrm{mkVecByDig}(1, size\_p88 - 2)$

$A\_p88\_o := \mathrm{mk3DiagMatr}(genGdiag\_A\_p88, ftDiag\_A\_p88, sdDiag\_A\_p88)$

$$A\_p88\_o = \begin{pmatrix} 5 & -4 & 1 & 0 & 0 & 0 & 0 \\ -4 & 6 & -4 & 1 & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 1 & -4 & 6 & -4 \\ 0 & 0 & 0 & 0 & 1 & -4 & 5 \end{pmatrix} \qquad b\_p88 := mkVec\_e(size\_p88, 4-1) \qquad b\_p88 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x\_p88\_t := lsolve(A\_p88\_o, b\_p88)$$

$$x\_p88\_t = \begin{pmatrix} 4 \\ 7.5 \\ 10 \\ 11 \\ 10 \\ 7.5 \\ 4 \end{pmatrix} \qquad x\_p88\_o := \begin{pmatrix} 4.0 \\ 7.5 \\ 10.0 \\ 11.0 \\ 10.0 \\ 7.5 \\ 4.0 \end{pmatrix}$$

$$minmax(matVariation(x\_p88\_t, x\_p88\_o)) = \begin{pmatrix} 0 & 1.998 \times 10^{-15} \end{pmatrix}$$

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$$b\_p105 := mkVec\_e(size\_p88, 5-1) \cdot 360360 \qquad b\_p105 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 3.604 \times \\ 0 \\ 0 \end{pmatrix}$$

$$x\_p105\_t := lsolve(Gilbert7, b\_p105)$$

$$x\_p105\_t = \begin{pmatrix} 4.851 \times 10^{4} \\ -1.94 \times 10^{6} \\ 1.871 \times 10^{7} \\ -7.276 \times 10^{7} \\ 1.334 \times 10^{8} \\ -1.153 \times 10^{8} \\ 3.784 \times 10^{7} \end{pmatrix} \qquad x\_p105\_o := \begin{pmatrix} 4.85100315047 \cdot 10^{4} \\ -1.94040128440 \cdot 10^{6} \\ 1.87110125732 \cdot 10^{7} \\ -7.27650495356 \cdot 10^{7} \\ 1.33402591837 \cdot 10^{8} \\ -1.15259840116 \cdot 10^{8} \\ 3.78378265214 \cdot 10^{7} \end{pmatrix}$$

$$minmax(matVariation(x\_p105\_t, x\_p105\_o)) = \begin{pmatrix} 6.501 \times 10^{-7} & 7.014 \times 10^{-7} \end{pmatrix}$$

$$complMul\_p105 := \begin{pmatrix} 1+i & 1-i & 1+2i & 1-2i & 1+3i & 1-3i & 1+4i \end{pmatrix}^{T}$$

$G7complex\_p105 := Gilbert7 \cdot diag(complMul\_p105)$

$det\_p105\_cmplx\_t := |G7complex\_p105|$

$det\_p105\_cmplx\_t = 3.816 \times 10^{16} + 1.526i \times 10^{17}$

$det\_p105\_cmplx\_o := \left(3.30997622 \cdot 10^{-2} + i \cdot 1.32399048 \cdot 10^{-1}\right) \cdot 2^{60}$

$$G7complex\_p105 = \begin{pmatrix} 3.604 \times 10^{5} + 3.604i \times 10^{5} & 1.8(\\ 1.802 \times 10^{5} + 1.802i \times 10^{5} & 1.2(\\ 1.201 \times 10^{5} + 1.201i \times 10^{5} & 9.0(\\ 9.009 \times 10^{4} + 9.009i \times 10^{4} & 7.2(\\ 7.207 \times 10^{4} + 7.207i \times 10^{4} & 6.0(\\ 6.006 \times 10^{4} + 6.006i \times 10^{4} & 5.1\cdot \\ 5.148 \times 10^{4} + 5.148i \times 10^{4} & 4.5(\end{pmatrix}$$

$det\_p105\_cmplx\_o - det\_p105\_cmplx\_t = -4.719 \times 10^{7} - 1.125i \times 10^{9}$

$3.816 \times 10^{16} + 1.526i \times 10^{17}$

$$lsolve(G7complex\_p105, mkVec\_e(size\_p88, 0)) = \begin{pmatrix} 6.799 \times 10^{-5} - 6.799i \times 10^{-5} \\ -1.632 \times 10^{-3} - 1.632i \times 10^{-3} \\ 4.895 \times 10^{-3} - 9.79i \times 10^{-3} \\ -0.016 - 0.033i \\ 0.013 - 0.04i \\ -0.011 - 0.032i \\ 1.961 \times 10^{-3} - 7.843i \times 10^{-3} \end{pmatrix}$$

????????

page 188 $size\_p188 := 44$

$$genGdiag\_B\_p188 := \left| \begin{array}{l} R \leftarrow mkVecByDig(6, size\_p188) \\ R_0 \leftarrow 5 \\ R_{size\_p188-1} \leftarrow 5 \\ \text{return } R \end{array} \right.$$

$$ftDiag\_B\_p188 := \left| \begin{array}{l} R \leftarrow mkVecByDig(3, size\_p188 - 1) \\ R_0 \leftarrow 2 \\ R_{size\_p188-2} \leftarrow 2 \\ \text{return } R \end{array} \right.$$

$sdDiag\_B\_p188 := mkVecByDig(1, size\_p188 - 2)$

$trdDiag\_B\_p188 := mkVecByDig(1, size\_p188 - 3)$

mk4DiagMatr(genDiag, ftDiag, sdDiag, trdDiag) :=
$n \leftarrow \text{rows(genDiag)} - 1$
$R_{n,n} \leftarrow 0$
$R \leftarrow \text{fillMatr(genDiag, fForGenDiag, R)}$
$R \leftarrow \text{fillMatr(ftDiag, fForFtDiag, R)}$
$R \leftarrow \text{fillMatr(sdDiag, fForSdDiag, R)}$
$R \leftarrow \text{fillMatr(trdDiag, fForTrdDiag, R)}$
return R

B_p188_o := mk4DiagMatr(genGdiag_B_p188, ftDiag_B_p188, sdDiag_B_p188, trdDiag_B_p188)

B_p188_o =

	0	1	2	3	4	5	6	7	8	9
0	5	2	1	1	0	0	0	0	0	0
1	2	6	3	1	1	0	0	0	0	0
2	1	3	6	3	1	1	0	0	0	0
3	1	1	3	6	3	1	1	0	0	0
4	0	1	1	3	6	3	1	1	0	0
5	0	0	1	1	3	6	3	1	1	0
6	0	0	0	1	1	3	6	3	1	1
7	0	0	0	0	1	1	3	6	3	1
8	0	0	0	0	0	1	1	3	6	3
9	0	0	0	0	0	0	1	1	3	6
10	0	0	0	0	0	0	0	1	1	3
11	0	0	0	0	0	0	0	0	1	1
12	0	0	0	0	0	0	0	0	0	1
13	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	...

eigenval_B_p188_t := sort(eigenvals(B_p188_o))

eigenval_B_p188_t =

	0
0	0.039
1	0.154
2	0.34
3	0.59
4	0.894
5	1.239
6	1.612
7	1.998
8	2.382
9	2.751
10	3.092

eigenvec_for6 := eigenvec$\left(\text{B_p188_o}, \text{eigenval_B_p188_t}_{14}\right)$

	0

10	3.092
11	3.394
12	3.649
13	3.852
14	4
15	...

eigenvec_for6 =

0	-0.183
1	0.183
2	0
3	-0.183
4	0.183
5	0
6	-0.183
7	0.183
8	0
9	-0.183
10	0.183
11	0
12	-0.183
13	0.183
14	0
15	...

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$$A\_p200 := \begin{pmatrix} 10 & 1 & 2 & 3 & 4 \\ 1 & 9 & -1 & 2 & -3 \\ 2 & -1 & 7 & 3 & -5 \\ 3 & 2 & 3 & 12 & -1 \\ 4 & -3 & -5 & -1 & 15 \end{pmatrix} \qquad B\_p200 := \begin{pmatrix} 5 & 1 & -2 & 0 & -2 & 5 \\ 1 & 6 & -3 & 2 & 0 & 6 \\ -2 & -3 & 8 & -5 & -6 & 0 \\ 0 & 2 & -5 & 5 & 1 & -2 \\ -2 & 0 & -6 & 1 & 6 & -3 \\ 5 & 6 & 0 & -2 & -3 & 8 \end{pmatrix}$$

eigenvals_Ap200_t1 := eigenvals(A_p200)

$$eigenvals\_Ap200\_t1 = \begin{pmatrix} 1.655 \\ 6.995 \\ 9.366 \\ 15.809 \\ 19.175 \end{pmatrix} \qquad vals\_A\_p200\_o := \begin{pmatrix} 1.65526620792\cdot 10^{0} \\ 6.99483783061\cdot 10^{0} \\ 9.36555492014\cdot 10^{0} \\ 1.58089207644\cdot 10^{1} \\ 1.91754202773\cdot 10^{1} \end{pmatrix}$$

$$minmax(matVariation(eigenvals\_Ap200\_t1, vals\_A\_p200\_o)) = \begin{pmatrix} 6.014 \times 10^{-13} & 1.165 \times 10^{-10} \end{pmatrix}$$

eigenvals_Bp200_t1 := eigenvals(B_p200)

$$\text{eigenvals\_Bp200\_t1} = \begin{pmatrix} -1.599 \\ -1.599 \\ 4.456 \\ 4.456 \\ 16.143 \\ 16.143 \end{pmatrix}$$

$$\text{vals\_B\_p200\_o} := \begin{pmatrix} -1.59873429360\cdot 10^{0} \\ -1.59873429360\cdot 10^{0} \\ 4.45598963849\cdot 10^{0} \\ 4.45598963849\cdot 10^{0} \\ 1.61427446551\cdot 10^{1} \\ 1.61427446551\cdot 10^{1} \end{pmatrix}$$

$$\text{minmax(matVariation(eigenvals\_Bp200\_t1, vals\_B\_p200\_o))} = \begin{pmatrix} 1.362\times 10^{-12} & 1.166\times 10^{-11} \end{pmatrix}$$

$$\text{eigenVecss\_Ap200\_t1} := \text{eigenvecs(A\_p200)}$$

$$\text{eigenVecss\_Ap200\_t1} = \begin{pmatrix} -0.387 & -0.654 & 0.052 & 0.624 & 0.175 \\ 0.366 & -0.2 & -0.86 & 0.159 & -0.247 \\ 0.704 & -0.257 & 0.506 & 0.227 & -0.362 \\ -0.119 & 0.66 & 2.012\times 10^{-4} & 0.693 & -0.264 \\ 0.453 & 0.174 & -0.046 & 0.233 & 0.841 \end{pmatrix}$$

$$\text{vecs\_A\_p200\_o}^{\langle 0\rangle} := \begin{pmatrix} 3.87296874886\cdot 10^{-1} \\ -3.66221021131\cdot 10^{-1} \\ -7.04377266220\cdot 10^{-1} \\ 1.18926222076\cdot 10^{-1} \\ -4.53423108037\cdot 10^{-1} \end{pmatrix}$$

$$\text{vecs\_A\_p200\_o}^{\langle 1\rangle} := \begin{pmatrix} 6.54082984085\cdot 10^{-1} \\ 1.99681268959\cdot 10^{-1} \\ 2.56510456336\cdot 10^{-1} \\ -6.60402722389\cdot 10^{-1} \\ -1.74279863500\cdot 10^{-1} \end{pmatrix}$$

$$\text{vecs\_A\_p200\_o}^{\langle 2\rangle} := \begin{pmatrix} 5.21511178463\cdot 10^{-2} \\ -8.59963866689\cdot 10^{-1} \\ 5.05575072575\cdot 10^{-1} \\ 2.01166650650\cdot 10^{-4} \\ -4.62191996239\cdot 10^{-2} \end{pmatrix}$$

$$\text{vecs\_A\_p200\_o}^{\langle 3\rangle} := \begin{pmatrix} -6.23702499852\cdot 10^{-1} \\ -1.59101120870\cdot 10^{-1} \\ -2.27297494237\cdot 10^{-1} \\ -6.92684385756\cdot 10^{-1} \\ -2.32822283880\cdot 10^{-1} \end{pmatrix}$$

$$\text{vecs\_A\_p200\_o}^{\langle 4\rangle} := \begin{pmatrix} 1.74505109459\cdot 10^{-1} \\ -2.47302518851\cdot 10^{-1} \\ -3.61641739446\cdot 10^{-1} \\ -2.64410853099\cdot 10^{-1} \\ 8.41244069212\cdot 10^{-1} \end{pmatrix}$$

$$vecs\_A\_p200\_o = \begin{pmatrix} 0.387 & 0.654 & 0.052 & -0.624 & 0.175 \\ -0.366 & 0.2 & -0.86 & -0.159 & -0.247 \\ -0.704 & 0.257 & 0.506 & -0.227 & -0.362 \\ 0.119 & -0.66 & 2.012\times 10^{-4} & -0.693 & -0.264 \\ -0.453 & -0.174 & -0.046 & -0.233 & 0.841 \end{pmatrix}$$

$$normVec(v) := \text{return } \sum \overrightarrow{|v|}$$

$$normVec\left(vecs\_A\_p200\_o^{\langle 0\rangle}\right) = 2.03$$

$$VariationEigenVecs(A,B) := \left| \begin{array}{l} n \leftarrow rows(A) - 1 \\ \text{for } i \in 0..n \\ \quad \left| \begin{array}{l} mult \leftarrow \dfrac{A_{0,i}}{B_{0,i}} \\ R^{\langle i\rangle} \leftarrow A^{\langle i\rangle} - mult \cdot B^{\langle i\rangle} \end{array} \right. \\ \text{return } R \end{array} \right.$$

$$deltaEigenVecs\_A\_p200 := VariationEigenVecs(vecs\_A\_p200\_o, eigenVecss\_Ap200\_t1)$$

$$deltaEigenVecs\_A\_p200 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1.439\times 10^{-12} & -1.56\times 10^{-11} & -1.035\times 10^{-10} & -6.502\times 10^{-12} & 5.741\times 10^{-12} \\ 2.327\times 10^{-12} & 1.027\times 10^{-11} & 5.396\times 10^{-11} & 3.32\times 10^{-12} & 1.243\times 10^{-11} \\ 5.545\times 10^{-12} & -1.222\times 10^{-11} & 1.722\times 10^{-11} & 1.104\times 10^{-11} & 1.164\times 10^{-11} \\ -2.715\times 10^{-12} & -4.011\times 10^{-12} & -2.551\times 10^{-12} & 6.824\times 10^{-12} & -1.913\times 10^{-11} \end{pmatrix}$$

$$minmax(deltaEigenVecs\_A\_p200) = \begin{pmatrix} -1.035\times 10^{-10} & 5.396\times 10^{-11} \end{pmatrix}$$

$$eigenVecss\_Bp200\_t1 := eigenvecs(B\_p200)$$?????

$$eigenVecss\_Bp200\_t1 = \begin{pmatrix} 0.44 & 0.259 & -0.557 & 0.499 & -0.075 & 0.419 \\ 0.265 & 0.499 & 0.509 & -0.352 & -0.304 & 0.453 \\ 0.621 & -0.185 & 0.224 & -0.092 & 0.722 & 0 \\ 0.259 & -0.44 & 0.499 & 0.557 & -0.419 & -0.075 \\ 0.499 & -0.265 & -0.352 & -0.509 & -0.453 & -0.304 \\ -0.185 & -0.621 & -0.092 & -0.224 & 0 & 0.722 \end{pmatrix}$$

$$vecs\_B\_p200\_o^{\langle 0\rangle} := \begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix}$$

$$eigenvals\_Bp200\_t1 = \begin{pmatrix} -1.599 \\ -1.599 \\ 4.456 \\ 4.456 \\ 16.143 \\ 16.143 \end{pmatrix}$$

???????

$$\text{eigenvec}\left(B\_p200, \text{eigenvals\_Bp200\_t1}_1\right) = \begin{pmatrix} 0.259 \\ -0.011 \\ 0.633 \\ 0.44 \\ 0.565 \\ 0.14 \end{pmatrix}$$

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```
genGdiag_A_p227 := | size ← 30
                   | for i ∈ 0.. size − 1
                   |   R_i ← (i + 1)^4
                   | return R
```

```
ftDiag_A_p227 := | size ← 30 − 1
                 | for i ∈ 0.. size − 1
                 |   R_i ← (i + 1) − 1
                 | return R
```

```
mkDiagMatr(genDiag, ftDiag) := | n ← rows(genDiag) − 1
                               | R_{n,n} ← 0
                               | R ← fillMatr(genDiag, fForGenDiag, R)
                               | R ← fillMatr(ftDiag, fForFtDiag, R)
                               | return R
```

A_p227 := mkDiagMatr(genGdiag_A_p227, ftDiag_A_p227)

vals_A_p227_t := eigenvals(A_p227)   vals_A_p227_approx_o := genGdiag_A_p227

vals_A_p227_t =

	0
0	1
1	15.985
2	80.993
3	255.998
4	625.001
5	$1.296 \cdot 10^3$
6	$2.401 \cdot 10^3$
7	$4.096 \cdot 10^3$
8	$6.561 \cdot 10^3$
9	$1 \cdot 10^4$
10	$1.464 \cdot 10^4$

11	$2.074\cdot10^4$
12	$2.856\cdot10^4$
13	$3.842\cdot10^4$
14	$5.063\cdot10^4$
15	...

chole

minmax(matVariation(vals_A_p227_t, vals_A_p227_approx_o)) = $\begin{pmatrix} 0 & 9.616\times10^{-4} \end{pmatrix}$

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reA_p343 := $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 0 & 2 \\ 0 & -1 & 3 \end{pmatrix}$ imA_p343 := $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

A_p343 := reA_p343 + i·imA_p343

A_p343 = $\begin{pmatrix} 1+i & -1-i & 2+2i \\ 0 & i & 2 \\ 0 & -1 & 3+i \end{pmatrix}$

eigenvals(A_p343) = $\begin{pmatrix} 1+i \\ 1+i \\ 2+i \end{pmatrix}$

eigenvecs(A_p343) = $\begin{pmatrix} 1 & -0.236-0.236i & 0.707 \\ 0 & 0.843 & 0.354-0.354i \\ 0 & 0.422 & 0.354-0.354i \end{pmatrix}$

?????

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$$\mathrm{matVariation}(A,B) := \left|\begin{array}{l} n \leftarrow \mathrm{rows}(A) - 1 \\ m \leftarrow \mathrm{cols}(A) - 1 \\ R_{n,m} \leftarrow 0 \\ \text{for } i \in 0..n \\ \quad \text{for } j \in 0..m \\ \qquad R_{i,j} \leftarrow \dfrac{\left|A_{i,j} - B_{i,j}\right|}{\left|B_{i,j}\right|} \\ \text{return } R \end{array}\right.$$

$$\mathrm{minmax}(M) := \left|\begin{array}{l} R \leftarrow (\, \mathrm{min}(M) \quad \mathrm{max}(M)\,) \\ \text{return } R \end{array}\right.$$

$$\left.\begin{matrix} .34604954657\cdot 10^{-1} & -1.07683294804\cdot 10^{-1} & 3.33303738787\cdot 10^{-2} \\ 5.38420043660\cdot 10^{0} & 4.43041069058\cdot 10^{0} & -1.39988227277\cdot 10^{0} \\ 5.19190848985\cdot 10^{1} & -4.36119354451\cdot 10^{1} & 1.39988675014\cdot 10^{1} \\ 2.01907567907\cdot 10^{2} & 1.72294292794\cdot 10^{2} & -5.59956005457\cdot 10^{1} \\ 3.70163885040\cdot 10^{2} & -3.19821597912\cdot 10^{2} & 1.04991937762\cdot 10^{2} \\ 3.19821597914\cdot 10^{2} & 2.79117246791\cdot 10^{2} & -9.23930348819\cdot 10^{1} \\ 1.04991937763\cdot 10^{2} & -9.23930348819\cdot 10^{1} & 3.07977132849\cdot 10^{1} \end{matrix}\right)$$

$$\left.\begin{matrix} \times 10^{-6} & -2.959\times 10^{-6} \\ \times 10^{-4} & 1.177\times 10^{-4} \\ \times 10^{-3} & -1.133\times 10^{-3} \\ .013 & 4.399\times 10^{-3} \\ 025 & -8.062\times 10^{-3} \\ .021 & 6.965\times 10^{-3} \\ \times 10^{-3} & -2.287\times 10^{-3} \end{matrix}\right)$$

$$\left.\begin{matrix} 3.37\times 10^{-5} & 8.879\times 10^{-5} \\ .093\times 10^{-5} & 8.41\times 10^{-5} \\ .909\times 10^{-5} & 8.09\times 10^{-5} \\ .777\times 10^{-5} & 7.857\times 10^{-5} \\ .678\times 10^{-5} & 7.679\times 10^{-5} \\ .601\times 10^{-5} & 7.539\times 10^{-5} \\ .539\times 10^{-5} & 7.425\times 10^{-5} \end{matrix}\right)$$

```
(dig, size) := | n ← size − 1
               | R_n ← 0
               | for  i ∈ 0 .. n
               |   R_i ← dig
               | return  R
```

6	7	8	9
6	6	6	...

	7	8	9
4	-4	-4	...

	7	8	9
1	1	1	...

```
t, Matr) := | n ← rows(Vec) − 1
            | for  i ∈ 0 .. n
            |    | indx ← Funct(i)
            |    | for  j ∈ 0 .. rows(indx) − 1
            |    |    | r ← indx_{j, 0}
            |    |    | c ← indx_{j, 1}
            |    |    | Matr_{r, c} ← Vec_i
            | return  Matr
```

1	2	3	4	5	6	7	8	9
-4	1	0	0	0	0	0	0	0
6	-4	1	0	0	0	0	0	0
-4	6	-4	1	0	0	0	0	0
1	-4	6	-4	1	0	0	0	0
0	1	-4	6	-4	1	0	0	0
0	0	1	-4	6	-4	1	0	0
0	0	0	1	-4	6	-4	1	0
0	0	0	0	1	-4	6	-4	1
0	0	0	0	0	1	-4	6	-4
0	0	0	0	0	0	1	-4	6
0	0	0	0	0	0	0	1	-4
0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	...

$$\left.\begin{matrix}
414634146\cdot 10^{2} \\
195121951\cdot 10^{2} \\
292682927\cdot 10^{1} \\
195121951\cdot 10^{1} \\
682926829\cdot 10^{1} \\
780487805\cdot 10^{1} \\
512195122\cdot 10^{1} \\
902439024\cdot 10^{1} \\
975609756\cdot 10^{1} \\
756097561\cdot 10^{1} \\
268292683\cdot 10^{1} \\
36585365854\cdot 10^{1} \\
585365854\cdot 10^{1} \\
439024390\cdot 10^{1} \\
121951220\cdot 10^{1} \\
658536585\cdot 10^{1} \\
073170732\cdot 10^{1} \\
390243902\cdot 10^{1} \\
634146341\cdot 10^{1} \\
292682927\cdot 10^{0}
\end{matrix}\right)$$

10^5

$2 \times 10^{5} - 1.802i \times 10^{5}$	$1.201 \times 10^{5} + 2.402i \times 10^{5}$	$9.009 \times 10^{4} - 1.802i \times 10^{5}$	$7.207 \times 10^{4} + 2.162i \times 10^{5}$	$6.006 \times$
$1 \times 10^{5} - 1.201i \times 10^{5}$	$9.009 \times 10^{4} + 1.802i \times 10^{5}$	$7.207 \times 10^{4} - 1.441i \times 10^{5}$	$6.006 \times 10^{4} + 1.802i \times 10^{5}$	$5.148 \times$
$9 \times 10^{4} - 9.009i \times 10^{4}$	$7.207 \times 10^{4} + 1.441i \times 10^{5}$	$6.006 \times 10^{4} - 1.201i \times 10^{5}$	$5.148 \times 10^{4} + 1.544i \times 10^{5}$	$4.505 \times$
$7 \times 10^{4} - 7.207i \times 10^{4}$	$6.006 \times 10^{4} + 1.201i \times 10^{5}$	$5.148 \times 10^{4} - 1.03i \times 10^{5}$	$4.505 \times 10^{4} + 1.351i \times 10^{5}$	$4.004 \times$
$6 \times 10^{4} - 6.006i \times 10^{4}$	$5.148 \times 10^{4} + 1.03i \times 10^{5}$	$4.505 \times 10^{4} - 9.009i \times 10^{4}$	$4.004 \times 10^{4} + 1.201i \times 10^{5}$	$3.604 \times$
$48 \times 10^{4} - 5.148i \times 10^{4}$	$4.505 \times 10^{4} + 9.009i \times 10^{4}$	$4.004 \times 10^{4} - 8.008i \times 10^{4}$	$3.604 \times 10^{4} + 1.081i \times 10^{5}$	$3.276 \times$
$5 \times 10^{4} - 4.505i \times 10^{4}$	$4.004 \times 10^{4} + 8.008i \times 10^{4}$	$3.604 \times 10^{4} - 7.207i \times 10^{4}$	$3.276 \times 10^{4} + 9.828i \times 10^{4}$	$3.003 \times$

$$
\left.\begin{matrix}
10^{4} - 1.802i \times 10^{5} & 5.148 \times 10^{4} + 2.059i \times 10^{5} \\
10^{4} - 1.544i \times 10^{5} & 4.505 \times 10^{4} + 1.802i \times 10^{5} \\
10^{4} - 1.351i \times 10^{5} & 4.004 \times 10^{4} + 1.602i \times 10^{5} \\
10^{4} - 1.201i \times 10^{5} & 3.604 \times 10^{4} + 1.441i \times 10^{5} \\
10^{4} - 1.081i \times 10^{5} & 3.276 \times 10^{4} + 1.31i \times 10^{5} \\
10^{4} - 9.828i \times 10^{4} & 3.003 \times 10^{4} + 1.201i \times 10^{5} \\
10^{4} - 9.009i \times 10^{4} & 2.772 \times 10^{4} + 1.109i \times 10^{5}
\end{matrix}\right)
$$