CPS 290.4/590.4
Crowdsourcing Societal Tradeoffs


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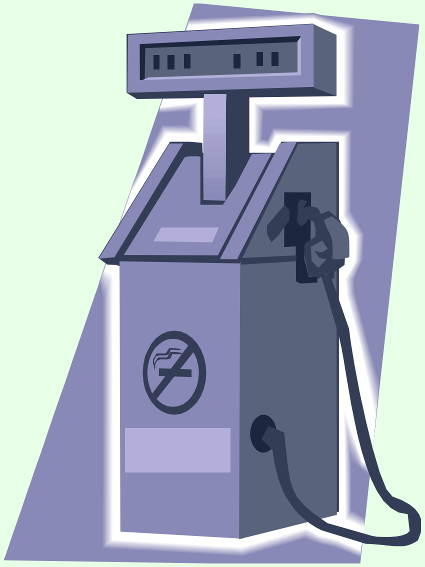
Motivating question



1 bag of landfill trash *is as bad as* using *x* gallons of gasoline

How to determine x?

- Other examples: clearing an acre of forest, fishing a ton of bluefin tuna, causing the average person to sit in front of a screen for another 5 minutes a day, …

Example decision scenario

- Benevolent government would like to get old inefficient cars off the road
- But disposing of a car and building a new car has its own energy and other costs
  - Perhaps also benefits to the economy
- Which cars should the government aim to get off the road?
  - Even energy costs are not directly comparable, e.g., perhaps gasoline contributes to energy dependence, coal does not

Inconsistent tradeoffs can result in inefficiency

- Agent 1: 1 gallon = 3 bags = -1 util
  - I.e., agent 1 feels she should be willing to sacrifice up to1 util to reduce trash by 3, but no more
- Agent 2: 1.5 gallons = 1.5 bags = -1 util
- Agent 3: 3 gallons = 1 bag = -1 util

- Cost of reducing gasoline by x is x^2 utils for each
- Cost of reducing trash by y is y^2 for each

- What will they do individually? (A little calculus…)

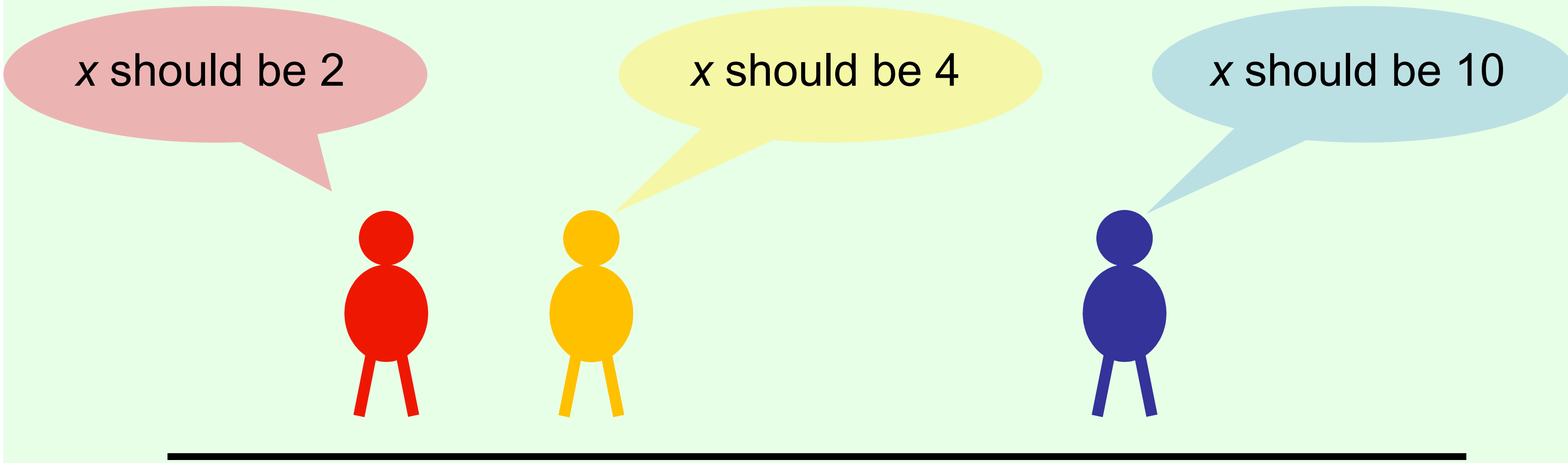
- What would be a better solution for all involved?

Pigovian taxes

Arthur Cecil Pigou

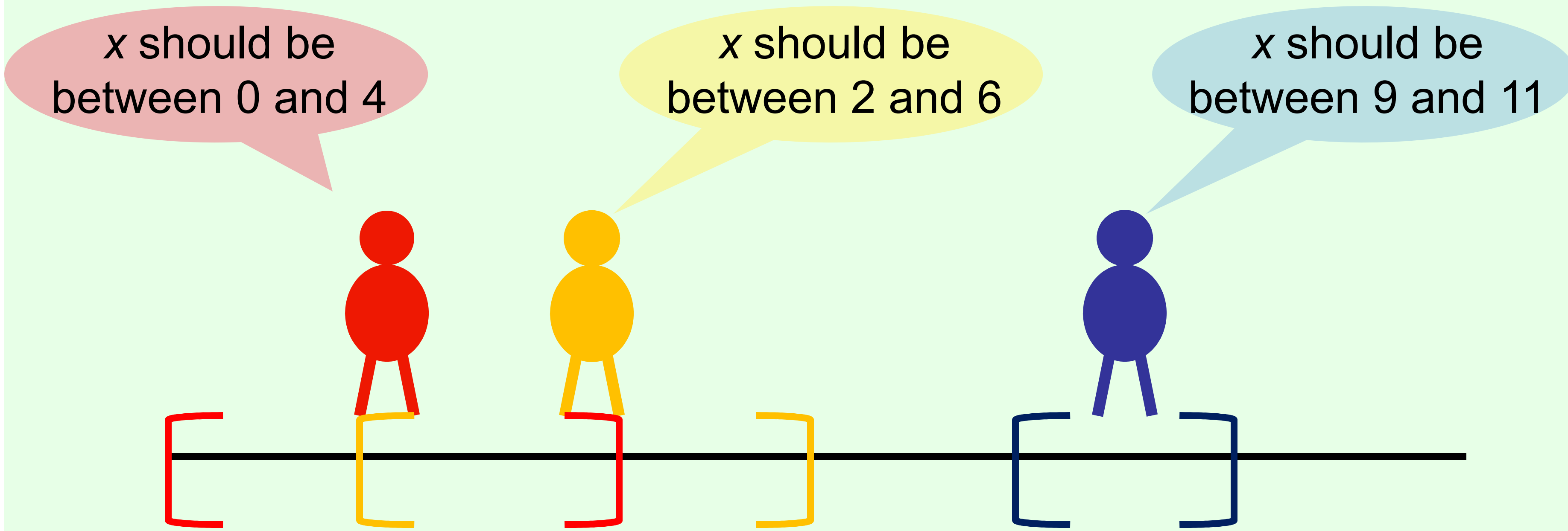
- Taxes intended to discourage undesirable behavior
- More precisely: pay the cost your activity imposes on society (the externality of your activity)
- If we decided using 1 gallon of gasoline came at a cost of $*x* to society, so we charged a tax of $*x* on each gallon…
- … then people would only use a gallon if they gained at least $*x* from doing so (net of purchase cost), which is societally ideal
- Isn't this better than (say) sales tax?
- But where would we get *x*?

One approach: let's vote!



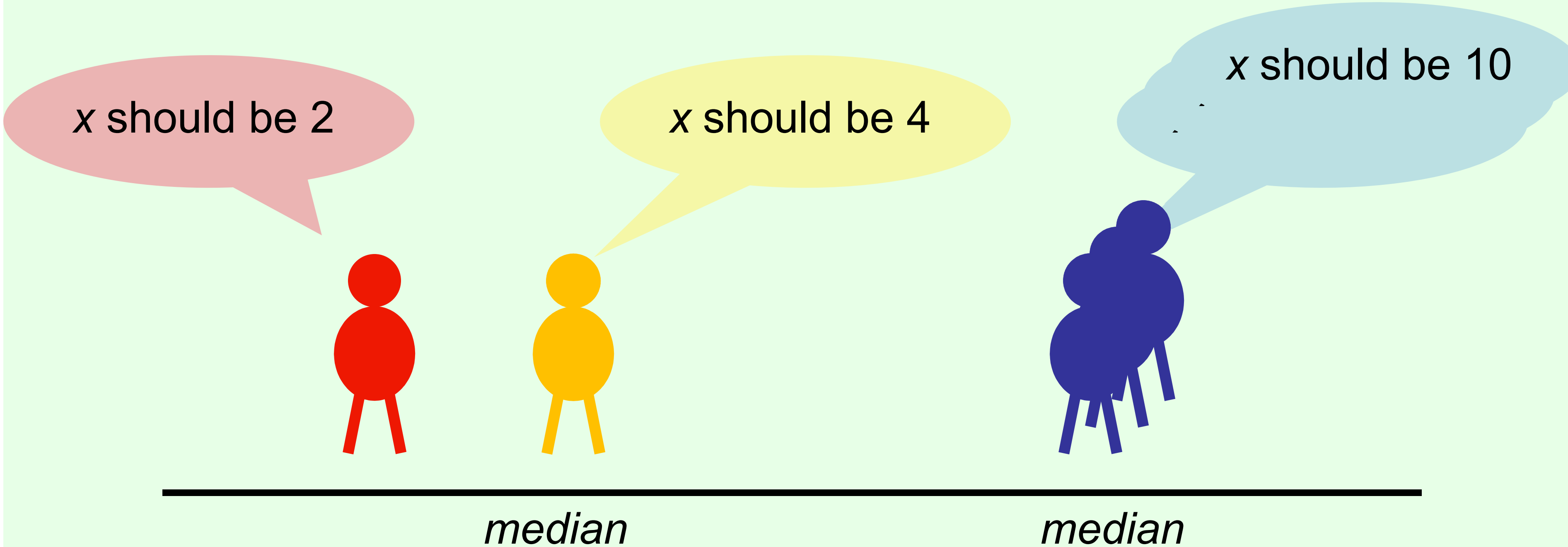
- So what should the outcome be…? Average? Median?
  - Note average(2,4,10)=5.33 but average(1/2,1/4,1/10)=.2833 (not 1/5.33)
- Is this the right way to vote?
- Who even gets to vote? Where/how?

Perhaps more reasonable…



- E.g., due to missing information (what kind of trash?) or plain uncertainty
- How should we aggregate these?

Voting more than once…



How to prevent this manipulation?

Consistency (for an individual)

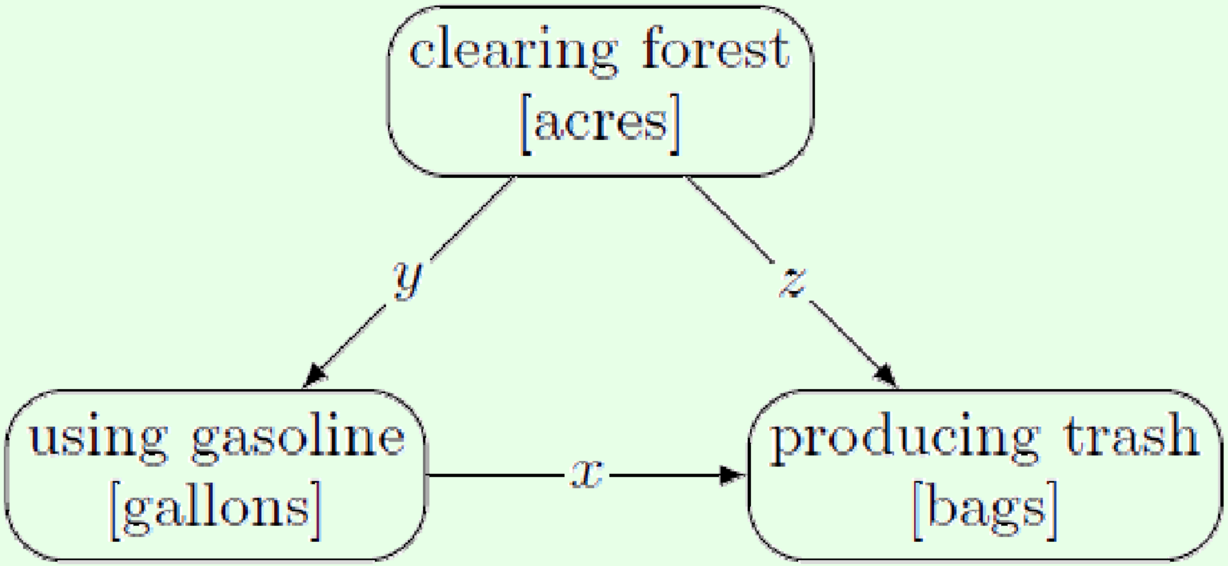


Figure 1: Weighted graph representing the numerical tradeoffs chosen by an individual voter. An arrow from activity A to activity B with weight w represents that one unit of A is considered as bad as w units of B. The tradeoff is consistent if $z = x \cdot y$.

A paradox: individual consistency may lead to societal inconsistency

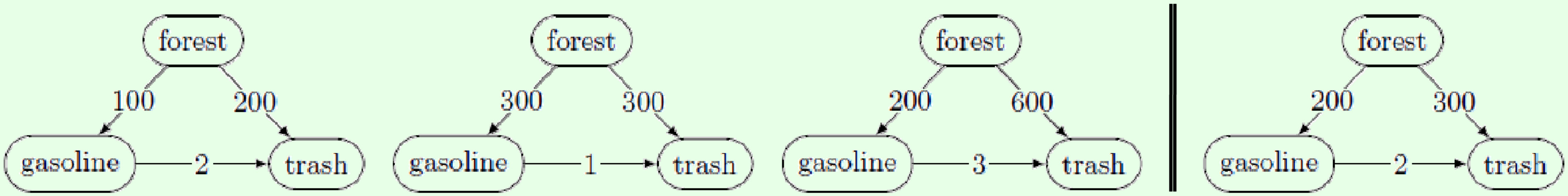
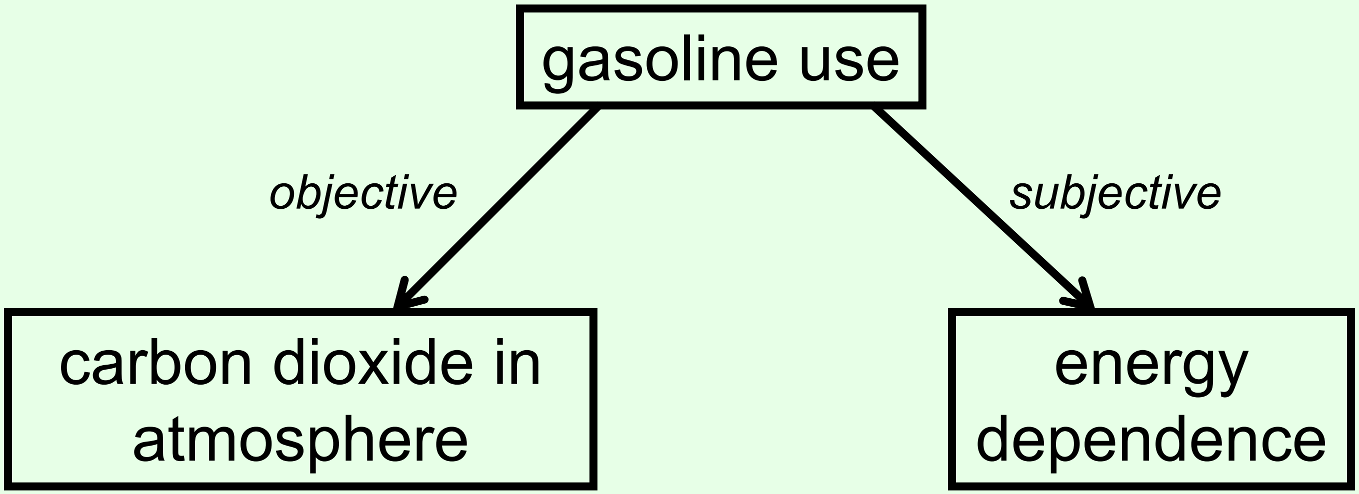


Figure 2: Example illustrating that the pairwise median rule can lead to inconsistent outcomes even when each individual voter is consistent. The left three graphs each illustrate the consistent preferences of a single voter, but the rightmost graph, which is inconsistent because $300 \neq 2 \cdot 200$, results from taking the median on each edge.

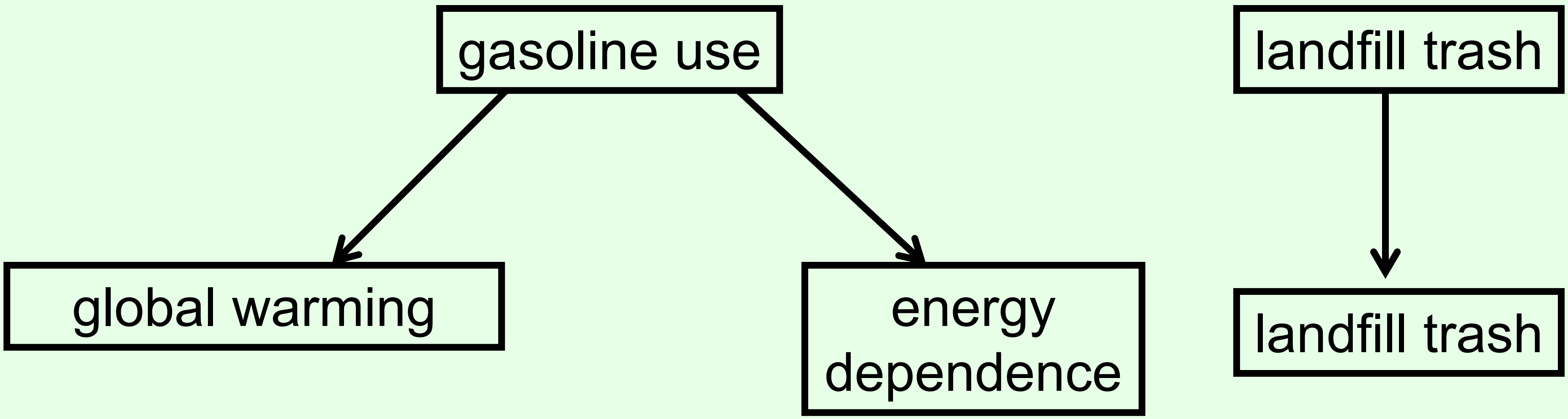
But we want our societal tradeoffs to be consistent… Otherwise we'll get inefficiency or incomprehensibility

Decomposition



Maybe we should reason about these components directly… How? How do we determine the components?

Another paradox

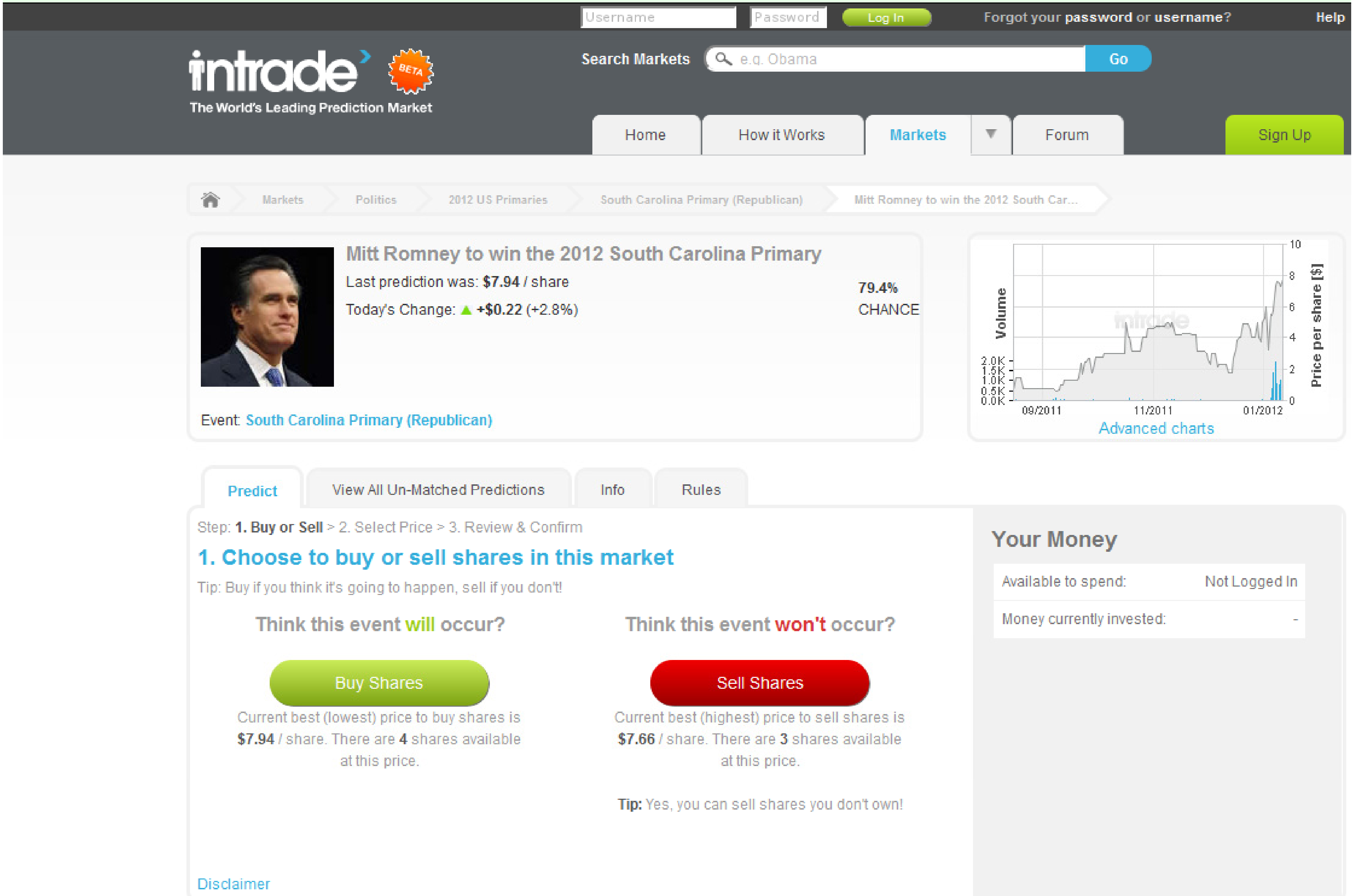


- Everyone agrees GU contributes 1 to GW, 1 to ED
- Voter 1: 1 GW = 2 LT, 1 ED = 1 LT (so 1 GU = 3 LT)
- Voter 2: 1 GW = 1 LT, 1 ED = 2 LT (so 1 GU = 3 LT)
- Voter 3: 1 GW = 1 LT, 1 ED = 1 L (so 1 GU = 2 LT)
- Median on attributes would conclude 1 GW = 1 ED = 1 LT (so 1 GU = 2 LT), but median directly gives 1 GU = 3 LT

Objective vs. subjective tradeoffs

- Some tradeoffs seem objective…
  - Measuring carbon dioxide emissions
- … others subjective…
  - Global warming vs. energy dependence
- Separate process? Who determines which is which?
- How to bring expert knowledge to bear?

Prediction markets



Locality

- Should we always obtain a single global tradeoff?

- Right tradeoff may depend on where we are
  - … though then arguably they’re not the same things being traded off

- Different entities (say, countries) may wish to reach their tradeoffs independently

- I may care only about the opinions of my neighbors in my social network

Some topics to cover

- Social choice theory
  - Voting
  - Judgment aggregation
  - Voting in pursuit of the “truth”
- Strategic agents
  - Game theory
  - Mechanism design
- Prediction markets
- Preference elicitation

Voting and social choice

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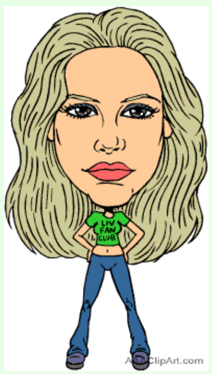
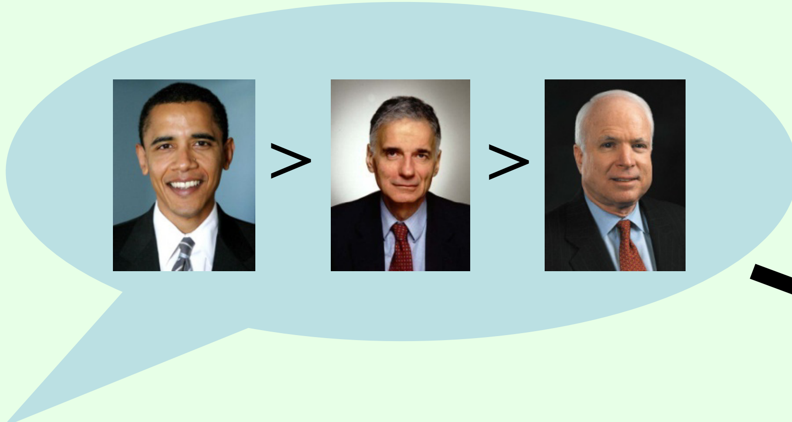
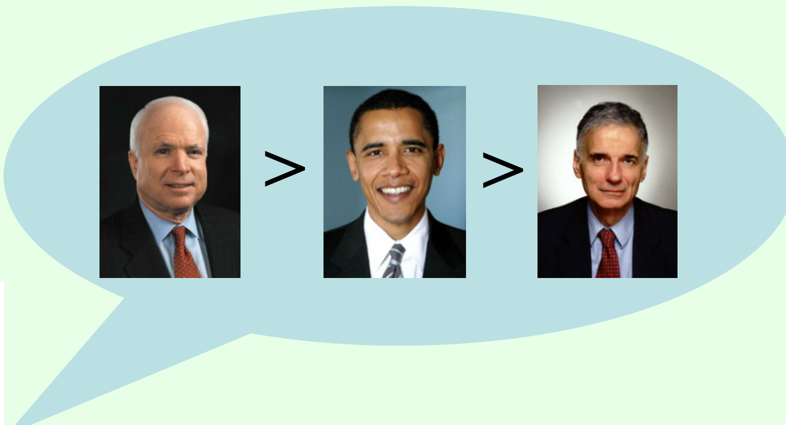
Voting over alternatives







voting rule
(mechanism)
determines winner
based on votes

- Can vote over other things too
  - Where to go for dinner tonight, other joint plans, …

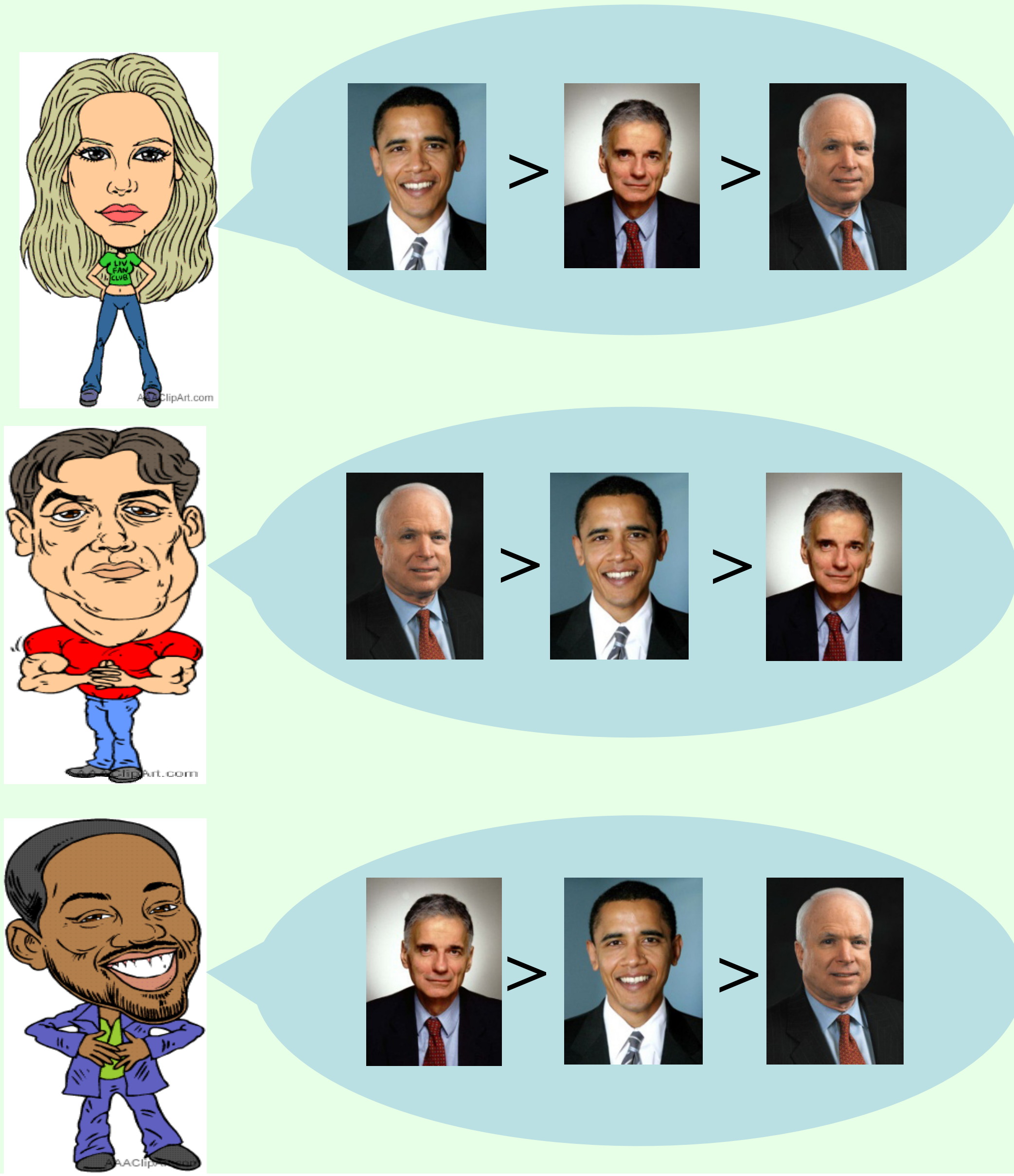
Voting (rank aggregation)

- Set of m candidates (aka. alternatives, outcomes)
- n voters; each voter ranks all the candidates
  - E.g., for set of candidates {a, b, c, d}, one possible vote is b > a > d > c
  - Submitted ranking is called a vote
- A voting rule takes as input a vector of votes (submitted by the voters), and as output produces either:
  - the winning candidate, or
  - an aggregate ranking of all candidates
- Can vote over just about anything
  - political representatives, award nominees, where to go for dinner tonight, joint plans, allocations of tasks/resources, …
  - Also can consider other applications: e.g., aggregating search engines' rankings into a single ranking

Example voting rules

- Scoring rules are defined by a vector $(a_1, a_2, \ldots, a_m)$; being ranked ith in a vote gives the candidate a_i points
  - Plurality is defined by (1, 0, 0, …, 0) (winner is candidate that is ranked first most often)
  - Veto (or anti-plurality) is defined by (1, 1, …, 1, 0) (winner is candidate that is ranked last the least often)
  - Borda is defined by (m-1, m-2, …, 0)
- Plurality with (2-candidate) runoff: top two candidates in terms of plurality score proceed to runoff; whichever is ranked higher than the other by more voters, wins
- Single Transferable Vote (STV, aka. Instant Runoff): candidate with lowest plurality score drops out; if you voted for that candidate, your vote transfers to the next (live) candidate on your list; repeat until one candidate remains
- Similar runoffs can be defined for rules other than plurality

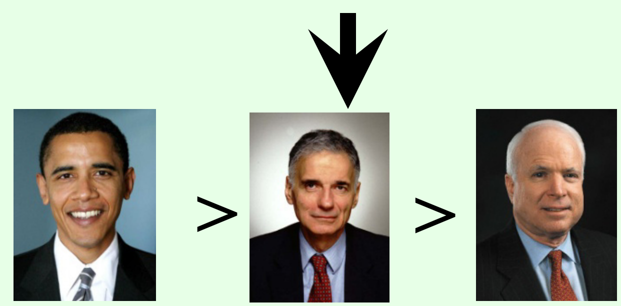
Pairwise elections



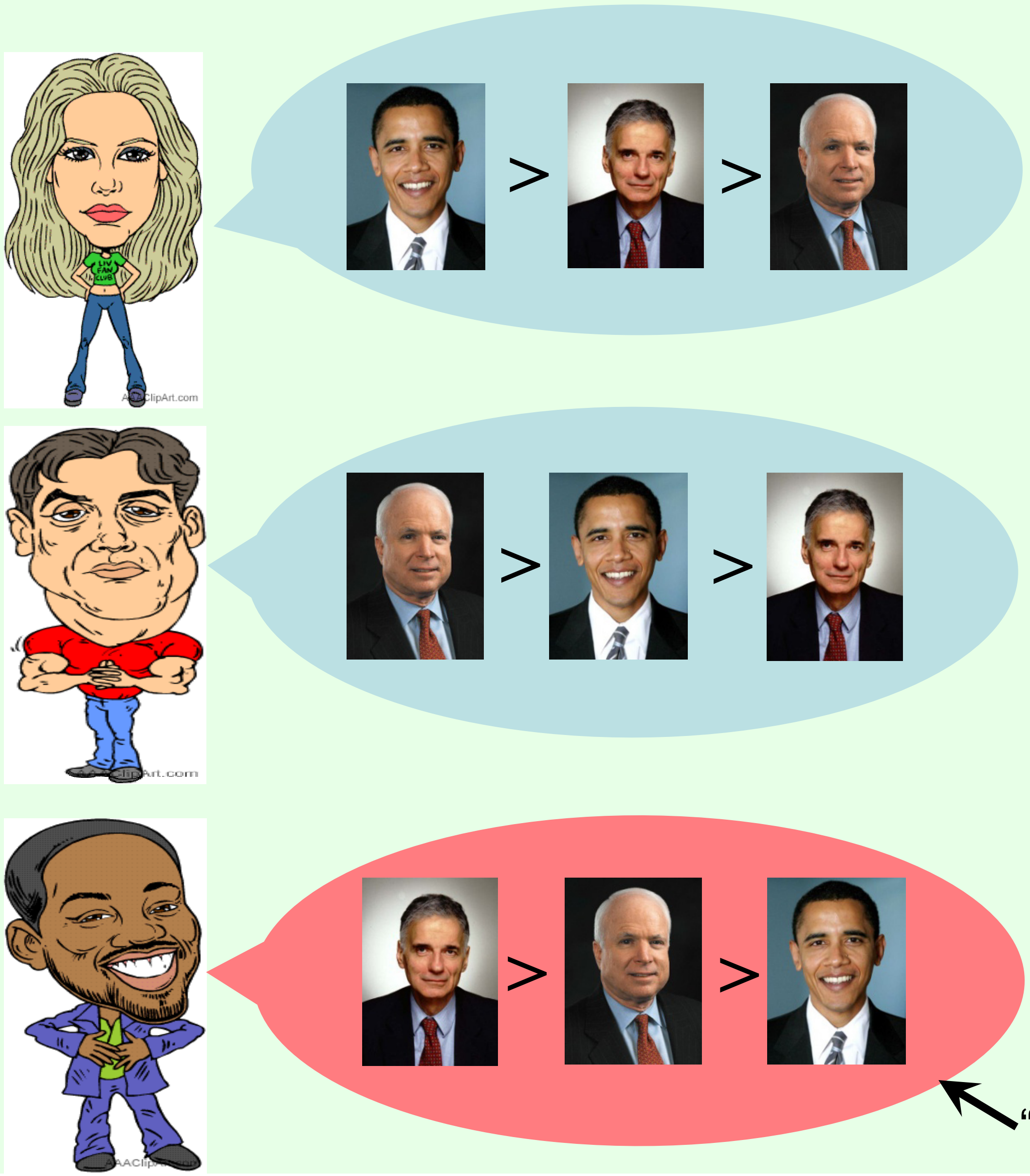
two votes prefer Obama to McCain

two votes prefer Obama to Nader

two votes prefer Nader to McCain

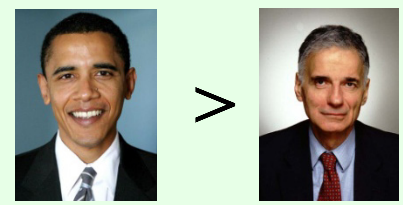


Condorcet cycles



two votes prefer McCain to Obama

two votes prefer Obama to Nader



two votes prefer Nader to McCain

?

"weird" preferences

Voting rules based on pairwise elections

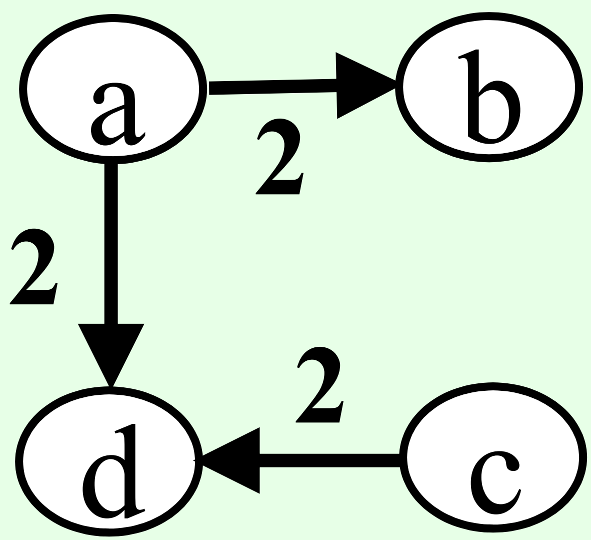
- Copeland: candidate gets two points for each pairwise election it wins, one point for each pairwise election it ties
- Maximin (aka. Simpson): candidate whose worst pairwise result is the best wins
- Slater: create an overall ranking of the candidates that is inconsistent with as few pairwise elections as possible
  - NP-hard!
- Cup/pairwise elimination: pair candidates, losers of pairwise elections drop out, repeat

Even more voting rules…

- Kemeny: create an overall ranking of the candidates that has as few *disagreements* as possible (where a disagreement is with a vote on a pair of candidates)
  - NP-hard!
- Bucklin: start with k=1 and increase k gradually until some candidate is among the top k candidates in more than half the votes; that candidate wins
- Approval (not a ranking-based rule): every voter labels each candidate as approved or disapproved, candidate with the most approvals wins

Pairwise election graphs

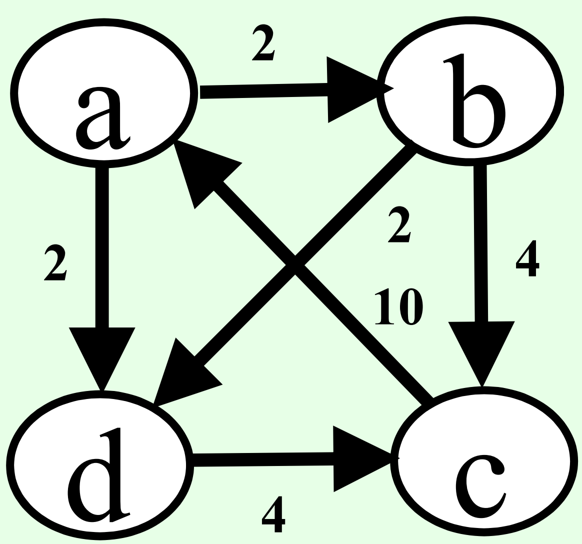
- Pairwise election between *a* and *b*: compare how often *a* is ranked above *b* vs. how often *b* is ranked above *a*
- Graph representation: edge from winner to loser (no edge if tie), weight = margin of victory
- E.g., for votes *a* > *b* > *c* > *d*, *c* > *a* > *d* > *b* this gives



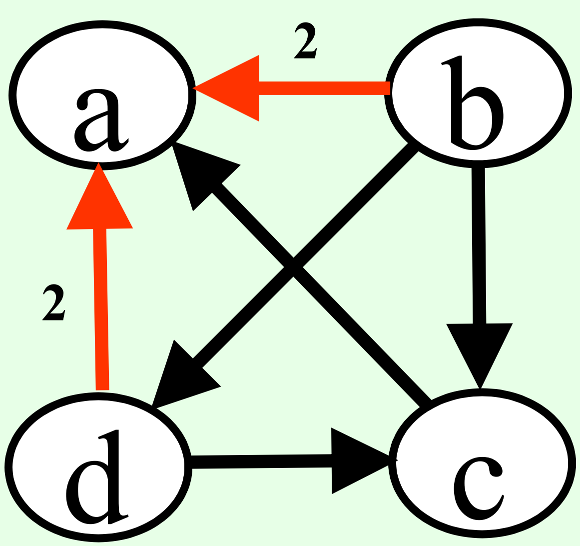
Kemeny on pairwise election graphs

- Final ranking = acyclic tournament graph
  - Edge (a, b) means a ranked above b
  - Acyclic = no cycles, tournament = edge between every pair
- Kemeny ranking seeks to minimize the total weight of the inverted edges

pairwise election graph



Kemeny ranking

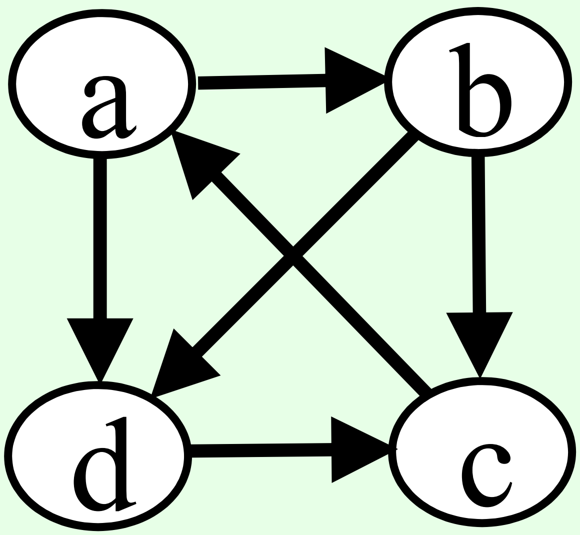


(b > d > c > a)

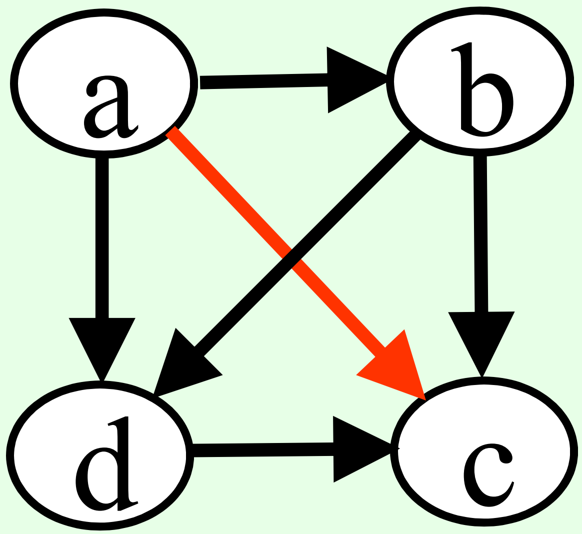
Slater on pairwise election graphs

- Final ranking = acyclic tournament graph
- Slater ranking seeks to minimize the number of inverted edges

pairwise election graph



Slater ranking



(a > b > d > c)

Choosing a rule

- How do we choose a rule from all of these rules?
- How do we know that there does not exist another, “perfect” rule?
- Let us look at some criteria that we would like our voting rule to satisfy

Condorcet criterion

- A candidate is the Condorcet winner if it wins all of its pairwise elections
- Does not always exist…
- … but the Condorcet criterion says that if it does exist, it should win

- Many rules do not satisfy this
- E.g. for plurality:
  - b > a > c > d
  - c > a > b > d
  - d > a > b > c
- a is the Condorcet winner, but it does not win under plurality

Majority criterion

- If a candidate is ranked first by most votes, that candidate should win
  - Relationship to Condorcet criterion?
- Some rules do not even satisfy this
- E.g. Borda:
  - a > b > c > d > e
  - a > b > c > d > e
  - c > b > d > e > a
- a is the majority winner, but it does not win under Borda

Monotonicity criteria

- Informally, monotonicity means that "ranking a candidate higher should help that candidate," but there are multiple nonequivalent definitions
- A weak monotonicity requirement: if
  - candidate w wins for the current votes,
  - we then improve the position of w in some of the votes and leave everything else the same,

  then w should still win.
- E.g., STV does not satisfy this:
  - 7 votes b > c > a
  - 7 votes a > b > c
  - 6 votes c > a > b
- c drops out first, its votes transfer to a, a wins
- But if 2 votes b > c > a change to a > b > c, b drops out first, its 5 votes transfer to c, and c wins

Monotonicity criteria…

- A strong monotonicity requirement: if
  - candidate w wins for the current votes,
  - we then change the votes in such a way that for each vote, if a candidate c was ranked below w originally, c is still ranked below w in the new vote

  then w should still win.
- Note the other candidates can jump around in the vote, as long as they don't jump ahead of w
- None of our rules satisfy this

Independence of irrelevant alternatives

- Independence of irrelevant alternatives criterion: if
  - the rule ranks a above b for the current votes,
  - we then change the votes but do not change which is ahead between a and b in each vote

  then a should still be ranked ahead of b.
- None of our rules satisfy this

Arrow's impossibility theorem [1951]

- Suppose there are at least 3 candidates
- Then there exists no rule that is simultaneously:
  - Pareto efficient (if all votes rank a above b, then the rule ranks a above b),
  - nondictatorial (there does not exist a voter such that the rule simply always copies that voter's ranking), and
  - independent of irrelevant alternatives

Muller-Satterthwaite impossibility theorem [1977]

- Suppose there are at least 3 candidates
- Then there exists no rule that simultaneously:
  - satisfies unanimity (if all votes rank a first, then a should win),
  - is nondictatorial (there does not exist a voter such that the rule simply always selects that voter’s first candidate as the winner), and
  - is monotone (in the strong sense).

Manipulability

- Sometimes, a voter is better off revealing her preferences insincerely, aka. manipulating
- E.g. plurality
  - Suppose a voter prefers a > b > c
  - Also suppose she knows that the other votes are
    - 2 times b > c > a
    - 2 times c > a > b
  - Voting truthfully will lead to a tie between b and c
  - She would be better off voting e.g. b > a > c, guaranteeing b wins

- All our rules are (sometimes) manipulable

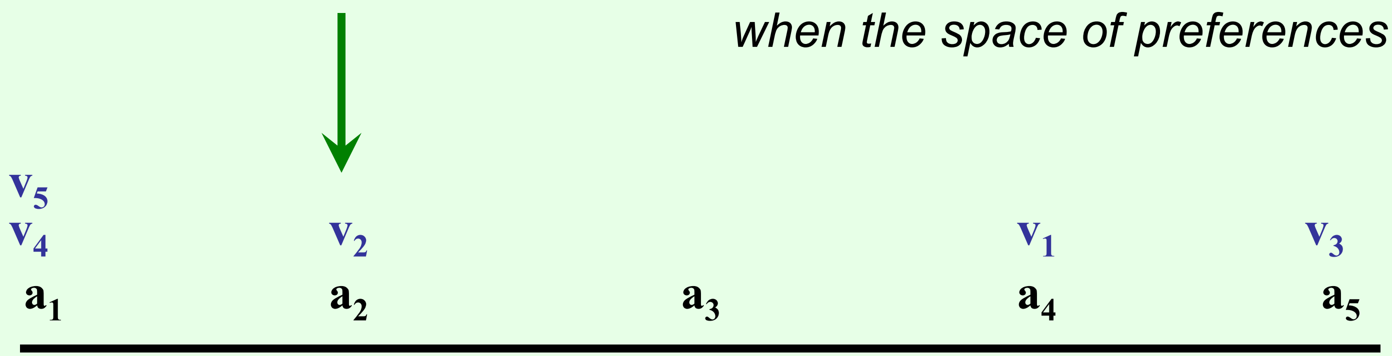
Gibbard-Satterthwaite impossibility theorem

- Suppose there are at least 3 candidates
- There exists no rule that is simultaneously:
  - onto (for every candidate, there are some votes that would make that candidate win),
  - nondictatorial (there does not exist a voter such that the rule simply always selects that voter’s first candidate as the winner), and
  - nonmanipulable

Single-peaked preferences

- Suppose candidates are ordered on a line
- Every voter prefers candidates that are closer to her most preferred candidate
- Let every voter report only her most preferred candidate (“peak”)
- Choose the median voter’s peak as the winner
  - This will also be the Condorcet winner
- Nonmanipulable!

Impossibility results do not necessarily hold when the space of preferences is restricted



Some computational issues in social choice

- Sometimes computing the winner/aggregate ranking is hard
  - E.g. for Kemeny and Slater rules this is NP-hard
- For some rules (e.g., STV), computing a successful manipulation is NP-hard
  - Manipulation being hard is a **good** thing (circumventing Gibbard-Satterthwaite?)… But would like something stronger than NP-hardness
  - Also: work on the complexity of controlling the outcome of an election by influencing the list of candidates/schedule of the Cup rule/etc.
- Preference elicitation:
  - We may not want to force each voter to rank **all** candidates;
  - Rather, we can selectively query voters for parts of their ranking, according to some algorithm, to obtain a good aggregate outcome
- Combinatorial alternative spaces:
  - Suppose there are multiple interrelated issues that each need a decision
  - Exponentially sized alternative spaces
- Different models such as ranking webpages (pages “vote” on each other by linking)

An integer program for computing Kemeny/Slater rankings

$y_{(a, b)}$ is 1 if a is ranked below b, 0 otherwise

$w_{(a, b)}$ is the weight on edge (a, b) (if it exists)

in the case of Slater, weights are always 1

minimize: $\Sigma_{e \in E} w_e y_e$

subject to:

for all $a, b \in V$, $y_{(a, b)} + y_{(b, a)} = 1$

for all $a, b, c \in V$, $y_{(a, b)} + y_{(b, c)} + y_{(c, a)} \geq 1$

Brief introduction to linear and mixed integer programming

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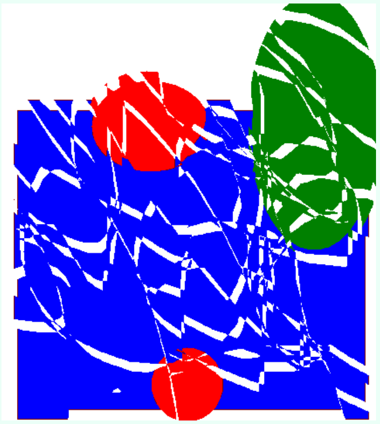
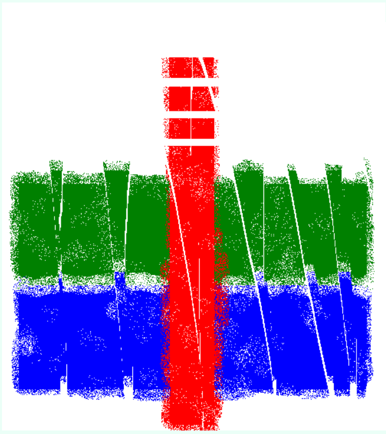
Linear programs: example

- We make reproductions of two paintings





- Painting 1 sells for \$30, painting 2 sells for \$20
- Painting 1 requires 4 units of blue, 1 green, 1 red
- Painting 2 requires 2 blue, 2 green, 1 red
- We have 16 units blue, 8 green, 5 red

maximize $3x + 2y$

subject to

$4x + 2y \leq 16$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$

Solving the linear program graphically

maximize $3x + 2y$

subject to

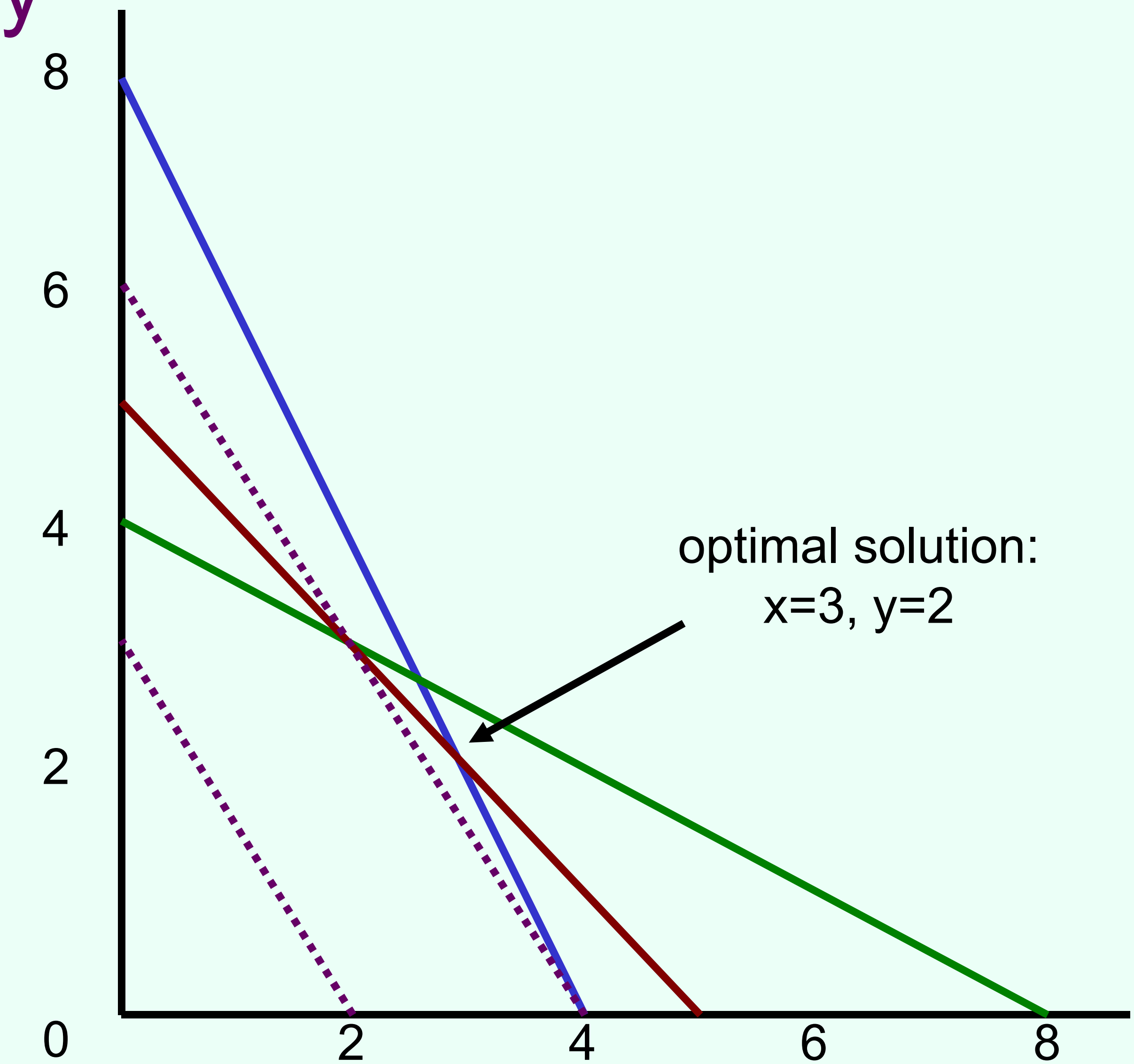
$4x + 2y \leq 16$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$



Modified LP

maximize $3x + 2y$

subject to

$4x + 2y \leq 15$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$

Optimal solution: $x = 2.5$, $y = 2.5$

Solution value $= 7.5 + 5 = 12.5$

Half paintings?

Integer (linear) program

maximize $3x + 2y$

subject to

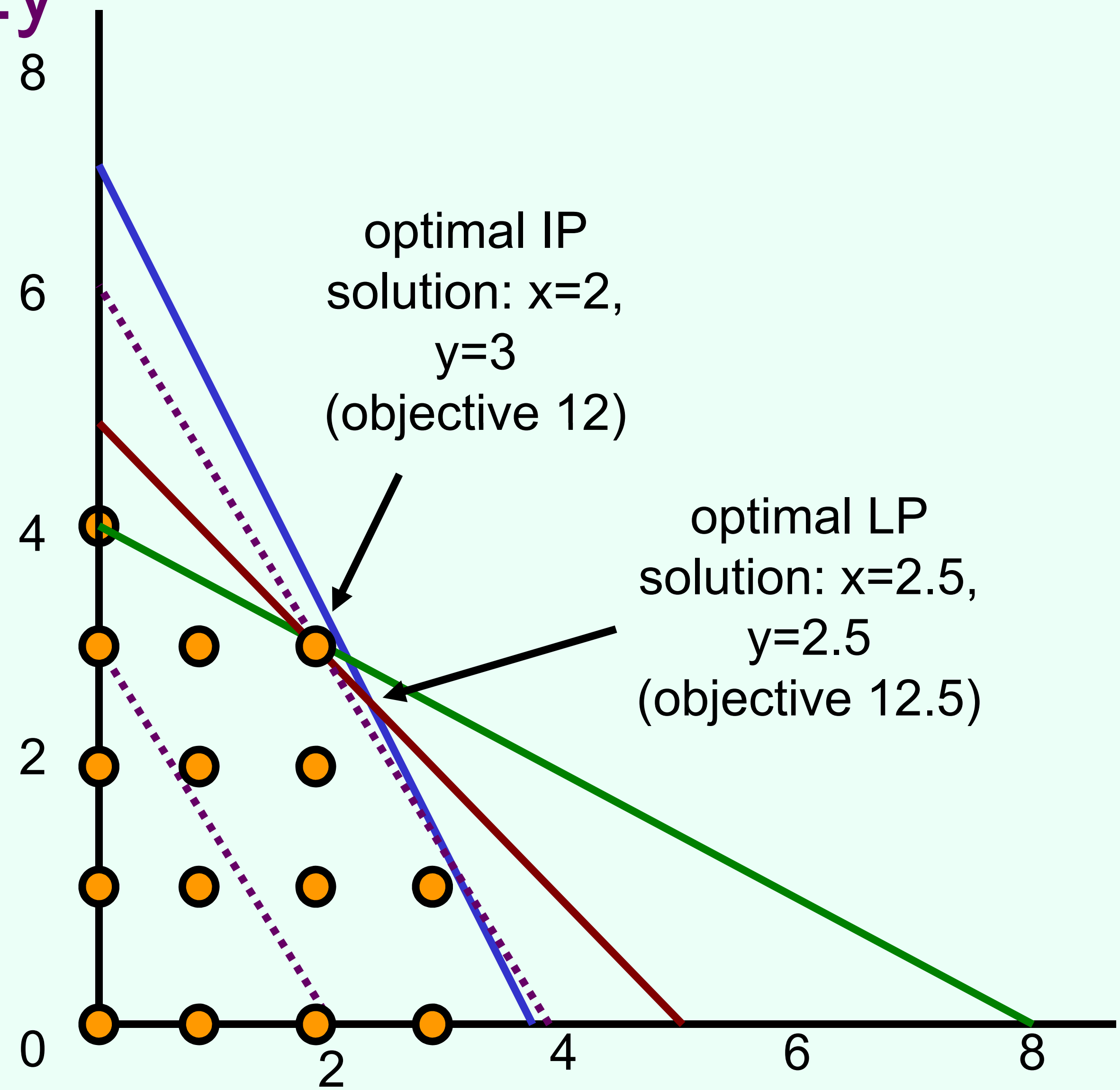
$4x + 2y \leq 15$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$, integer

$y \geq 0$, integer



Mixed integer (linear) program

maximize 3x + 2y

subject to

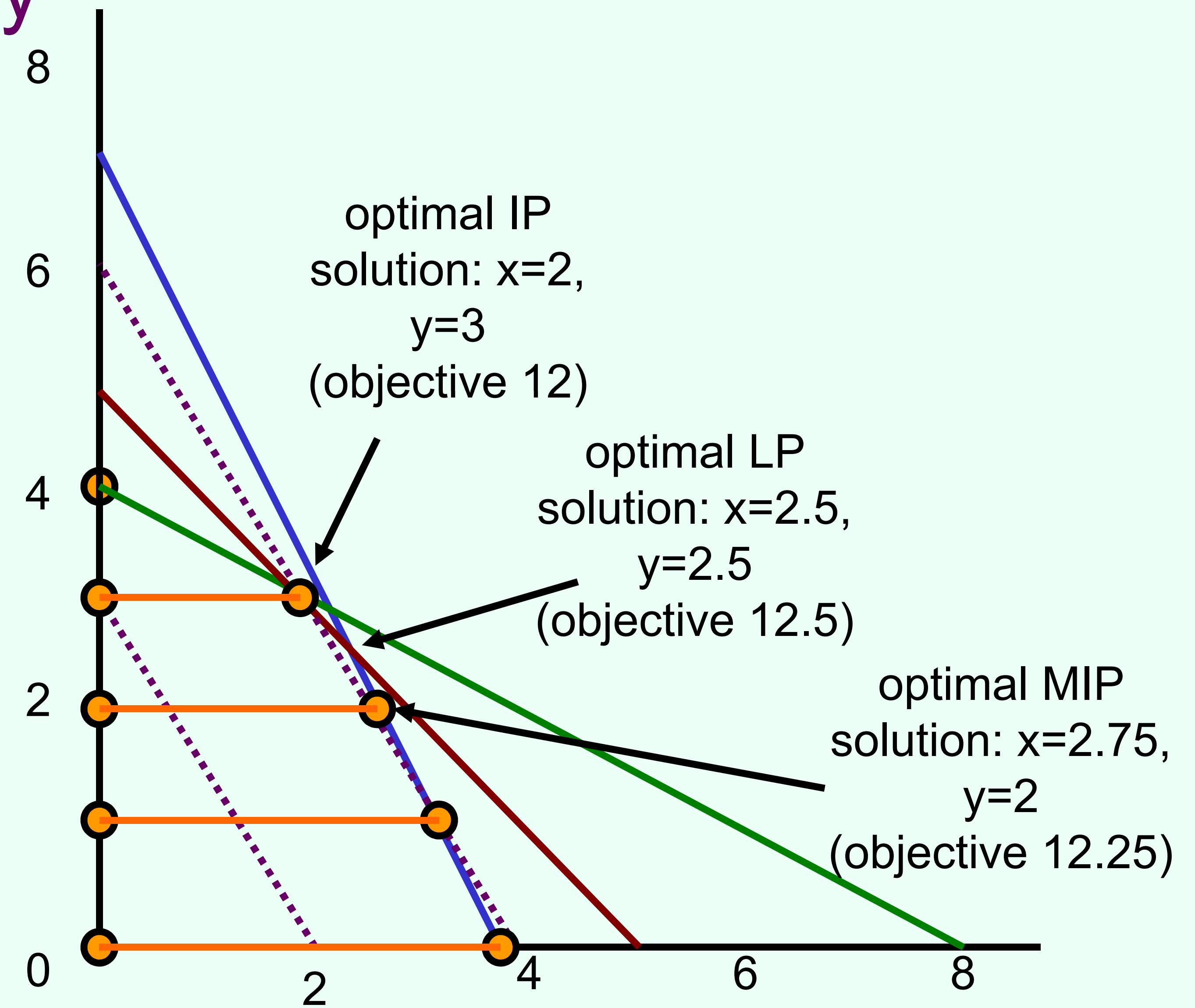
4x + 2y ≤ 15

x + 2y ≤ 8

x + y ≤ 5

x ≥ 0

y ≥ 0, integer



Solving linear/integer programs

- Linear programs can be solved efficiently
  - Simplex, ellipsoid, interior point methods…
- (Mixed) integer programs are NP-hard to solve
  - Quite easy to model many standard NP-complete problems as integer programs (try it!)
  - Search type algorithms such as branch and bound
- Standard packages for solving these
  - GNU Linear Programming Kit, CPLEX, …
- LP relaxation of (M)IP: remove integrality constraints
  - Gives upper bound on MIP (~admissible heuristic)

Exercise in modeling: knapsack-type problem

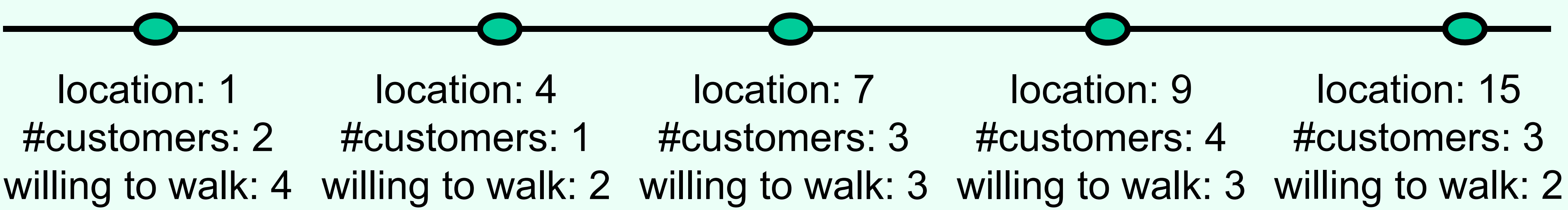
- We arrive in a room full of precious objects
- Can carry only 30kg out of the room
- Can carry only 20 liters out of the room
- Want to maximize our total value
- Unit of object A: 16kg, 3 liters, sells for \$11
  - There are 3 units available
- Unit of object B: 4kg, 4 liters, sells for \$4
  - There are 4 units available
- Unit of object C: 6kg, 3 liters, sells for \$9
  - Only 1 unit available
- What should we take?

Exercise in modeling: cell phones (set cover)

- We want to have a working phone in every continent (besides Antarctica)
- … but we want to have as few phones as possible
- Phone A works in NA, SA, Af
- Phone B works in E, Af, As
- Phone C works in NA, Au, E
- Phone D works in SA, As, E
- Phone E works in Af, As, Au
- Phone F works in NA, E

Exercise in modeling: hot-dog stands

- We have two hot-dog stands to be placed in somewhere along the beach
- We know where the people that like hot dogs are, how far they are willing to walk
- Where do we put our stands to maximize #hot dogs sold? (price is fixed)



Judgment Aggregation

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Based on *Tutorial on Judgment Aggregation* by **Ulle Endriss**
https://staff.fnwi.uva.nl/u.endriss/teaching/aamas-2013/

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Stiftung/Foundation

Doctrinal Paradox

- Three judges have to decide on a case of an alleged breach of contract
- The need to decide whether (a) the contract is **valid** and whether (b) the contract has been **breached**.
- Legal doctrine stipulates that the defendant is **liable** if and only if (a) and (b) hold.

	Valid?	*Breach?*	*Liable?*
Judge 1	**Yes**	**Yes**	**Yes**
Judge 2	**Yes**	**No**	**No**
Judge 3	**No**	**Yes**	**No**

Discursive Dilemma

- Instead of expressing **judgments on atoms** (valid, breached, liable) and imposing **constraints** (liable iff valid and breached), we could allow **judgments on compound formulas**:

	p	q	$p \wedge q$
Judge 1	**Yes**	**Yes**	**Yes**
Judge 2	**Yes**	**No**	**No**
Judge 3	**No**	**Yes**	**No**
Majority	**Yes**	**Yes**	**No**

	p	q	$r \leftrightarrow p \wedge q$	r
Judge 1	**Yes**	**Yes**	**Yes**	**Yes**
Judge 2	**Yes**	**No**	**Yes**	**No**
Judge 3	**No**	**Yes**	**Yes**	**No**
Majority	**Yes**	**Yes**	**Yes**	**No**

Why Paradox?

	p	q	$p \wedge q$
Judge 1	Yes	Yes	Yes
Judge 2	Yes	No	No
Judge 3	No	Yes	No
Majority	Yes	Yes	No

- **Reason 1: Premise-based procedure** and **conclusion-based procedure** produce different outcomes.
- **Reason 2:** Even though each individual judgment is logically consistent, the **majority outcome** is not.

Outline

- Formal framework
- Axioms and impossibilities
- Aggregation Rules
  - quota-based rules
  - distance-based rules
  - premise-based rules
- Computational aspects
- Strategic aspects

Formal Framework

- An **agenda** A is a finite nonempty subset of propositional formulas closed under complementation (i.e., $\phi \in A \Rightarrow \neg\phi \in A$)
- A **judgment set** J on an agenda A is a subset of A
  - J is **complete** if $\phi \in J$ or $\neg\phi \in J$ for all $\phi \in A$
  - J is **consistent** if there exists an assignment satisfying all formulas in A
- A finite set of **individuals** $N=\{1,\ldots,n\}$ express judgements on the formulas in A, producing a **profile** $J=(J_1,\ldots,J_n)$
  - we assume that $n \geq 3$ and n is odd
- An **aggregation rule** maps a profile of complete and consistent individual judgement sets to a single collective judgment set

Example: Majority Rule

- $N = \{1,2,3\}$
- $A = \{p, \neg p, q, \neg q, (p \vee q), \neg(p \vee q)\}$

	p	q	$p \vee q$
Agent 1	Yes	Yes	Yes
Agent 2	Yes	No	Yes
Agent 3	No	No	No

- **Majority rule** returns all propositions accepted by $> n/2$ agents
  - in the example: $F_{maj}(J) = \{p, \neg q, p \vee q\}$ (complete and consistent!)

Relation to Voting

- Preference aggregation can be **embedded into JA** as follows:
  - Suppose the set of alternatives is given by *{a,b,c}*
  - Take atomic propositions to be $p_{a>b}$, $p_{a>c}$, etc.
  - All individuals accept the following propositions:
    - $p_{a>b} \vee p_{b>a}$ etc. (completeness)
    - $p_{a>b} \wedge p_{b>c} \longrightarrow p_{a>c}$ etc. (transitivity)
- The **Condorcet paradox** in JA language:

	$p_{a>b}$	$p_{a>c}$	$p_{b>c}$	corresponding order
Agent 1	**Yes**	**Yes**	**Yes**	a>b>c
Agent 2	**No**	**No**	**Yes**	b>c>a
Agent 3	**Yes**	**No**	**No**	c>a>b
Majority	**Yes**	**No**	**Yes**	**not a linear order**

Axioms

- **Anonymity**: treat all individuals symmetrically
- **Neutrality**: treat all propositions symmetrically
  - if two formulas are accepted by the same judges, then the collective must accept either both or neither of them
- **Independence**: only the “pattern of acceptance” should matter
  - if a formula is accepted in some profile, then it is accepted in all profiles that have the same judgements on that formula
- **Monotonicity**: if an collectively accepted formula is accepted by an additional agent, it should still get collectively accepted
- **Completeness**: the collective judgement is always complete
- **Consistency**: the collective judgement is always consistent

Impossibility

- Majority rule violates consistency
- Question: Are there "reasonable" **consistent aggregation rules**?
- **Theorem** (List and Pettit, 2002): There is no aggregation rule that simultaneously satisfies **anonymity**, **neutrality**, **independence**, **completeness**, and **consistency**.
  - assuming the agenda is sufficiently rich (e.g., $\{p,q,p\wedge q\}\subseteq A$) and n>1
  - proof sketch: the first three properties implies that acceptance of a formula only depends on the number of judges that accept it
- What now? Weaken axioms? Which axioms?

Quota-based Rules

- A **quota rule** F_q is defined by a function $q: A \rightarrow \{0,1,...,n+1\}$ and accepts a formula ϕ iff ϕ is accepted by at least $q(\phi)$ individuals
  - a quota rule is called **uniform** if there is a k s.t. $q(\phi)=k$ for all $\phi \in A$
- Examples of uniform quota rules:
  - the **constant rule** F_0 (F_{n+1}) accepts all (no) formulas
  - the **intersection rule** F_n accepts ϕ iff everybody does
  - the **(strict) majority rule** F_{maj} has quota $q = \lceil (n+1)/2 \rceil$
- What are the advantages of high/low quotas?

Axiomatic Characterizations

- **Proposition** (Dietrich and List, 2007): An aggregation rule is anonymous, independent and monotonic iff it is a **quota rule**.

Clearly, a quota rule is neutral iff it is uniform. Therefore:

- **Corollary:** An aggregation rule is anonymous, neutral, independent and monotonic (= **ANIM**) iff it is a **uniform quota rule**.
- **Corollary:** An aggregation rule is ANIM, complete and complement-free iff it is the **(strict) majority rule**.
    - a rule is **complement-free** if it never accepts both ϕ and $\neg\phi$
    - this holds for odd n; for even n, no rule satisfies these properties

Distance-based Rules

- **Idea:** Find a **consistent** judgment set that minimizes the "distance" to the profile
- **Hamming distance** between two judgment sets is given by the number of disagreements
  - $H(J_i,J_i') = 1/2 * |J_i \Delta J_i'|$
  - distance to a profile given by sum of distances to individual judgment sets in the profile

p	*q*	*r*
No	**No**	**Yes**
Yes	**Yes**	**Yes**

- Two ways to define aggregation rule based on Hamming distance:
  - minimize Hamming distance to profile ← **generalized Kemeny rule**
  - minimize Hamming distance to majority outcome ← **generalized Slater rule**

Example

ϕ_1 and ϕ_2 are both equivalent to
$p \vee (q_1 \wedge q_2) \vee (r_1 \wedge r_2 \wedge r_3)$

	p	q_1	q_2	r_1	r_2	r_3	ϕ_1	ϕ_2
1 agent	Yes	No	No	No	No	No	Yes	Yes
10 agents	No	Yes	Yes	No	No	No	Yes	Yes
10 agents	No	No	No	Yes	Yes	Yes	Yes	Yes

Kemeny	No	Yes	Yes	No	No	No	Yes	Yes
Slater	Yes	No	No	No	No	No	Yes	Yes

Premise-based Rules (1)

- Idea of premise-based rules (PBRs): Divide formulas into **premises** and **conclusions**, apply **majority rule on premises**, and accept conclusions that **logically follow** from accepted premises
- A PBR yields complete and **consistent outcomes** if
  - the set of premises is the set of literals
  - the agenda is closed under propositional letters

	p	q	$p \wedge q$
Judge 1	Yes	Yes	Yes
Judge 2	Yes	No	No
Judge 3	No	Yes	No
PBR	Yes	Yes	Yes

Premise-based Rules (2)

- A PBR might violate **unanimity**
  - … and "composition-consistency"

	$p \vee q$	r	$(p \vee q) \vee r$
Judge 1	**Yes**	**No**	**Yes**
Judge 2	**Yes**	**No**	**Yes**
Judge 3	**No**	**Yes**	**Yes**
PBR	**Yes**	**No**	**Yes**

	p	q	r	$p \vee q \vee r$
Judge 1	**Yes**	**No**	**No**	**Yes**
Judge 2	**No**	**Yes**	**No**	**Yes**
Judge 3	**No**	**No**	**Yes**	**Yes**
PBR	**No**	**No**	**No**	**No**

Winner Determination

- How hard is it to **compute the outcome** of an aggregation rule?
- **Fact:** Winner determination for quota-based rules is in P.
- **Fact:** Winner determination for premise-based rule is in P.
  - Proof: counting (for premises) + model checking (for conclusions)
- **Theorem** (Endriss et al., 2012): The winner determination problems of the **generalized Kemeny rule** and the **generalized Slater rule** are both NP-hard.
  - Proof: reduction from corresponding voting problem
- Efficiently computable variants of distance-based rules: **representative-voter rules**
  - only search through judgment sets proposed by individuals
  - find "most representative voter"

Strategic Manipulation

- Do aggregation rules **incentivise** judges to report their judgments **truthfully**?
  - what are the **preferences** of a judge?
- **Example:** Hamming preferences and the PBR
  - **Hamming preferences:** prefer outcomes closer to own judgment set

	p	q	r	$p \vee q$	$p \vee r$
Judge 1	**No**	**No**	**No**	**No**	**No**
Judge 2	**Yes**	**No**	**No**	**Yes**	**Yes**
Judge 3	**No**	**Yes**	**Yes**	**Yes**	**Yes**

Preventing Manipulation

Sometimes manipulation is impossible.

- **Proposition** (Dietrich and List, 2007): An aggregation rule is **immune to manipulation** if and only if it is both **independent** and **monotonic**.
  - but: independent and monotonic rules are not very attractive…

Sometimes it is hard.

- **Proposition** (Endriss et al., 2012): **Manipulating** the premise-based rule with Hamming preferences is **NP-complete.**
  - but: this is only a worst-case result…

Summary

- JA provides a framework to aggregate binary opinions
- Motivated by **paradoxes** and **impossibilities**
- Concrete **aggregation rules** violate at least some **axioms**
- **Computational** and **strategic** considerations are important
- Further reading:
  - U. Endriss. Judgment aggregation. In *Handbook of Computational Social Choice*, chapter 17. Cambridge University Press, 2015.
  - C. List. The Theory of Judgment Aggregation: An Introductory Review. *Synthese*, 187(1):179–207, 2012.
  - U. Endriss, U. Grandi, and D. Porello. Complexity of Judgment Aggregation. *Journal of Artificial Intelligence Research*, 45:481–514, 2012.
- **Discussion:** Which rules/axioms are appealing? How is this material relevant to our goal of deriving societal tradeoffs? …

Voting over combinatorial domains

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Centralized decision making (rank aggregation)

- Several agents judge a set of items
- A collective decision has to be made
- From the individuals' rankings to a single ranking
- Respecting the judges' preferences as much as possible

						Aggregation		
Rank	Judge A	Judge B	Judge C	Judge D	Judge E	sum	average	rank
item 1	5	6	2	9	1	23	4.6	4
item 2	7	3	6	5	7	28	5.6	8
item 3	5	3	2	3	3	16	3.2	2
item 4	2	9	6	9	3	29	5.8	9
item 5	7	3	9	1	7	27	5.4	6
item 6	1	9	6	5	6	27	5.4	6
item 7	2	8	1	5	7	23	4.6	4
item 8	2	6	9	4	10	31	6.2	10
item 9	10	1	2	5	1	19	3.8	3
item 10	7	1	2	1	3	14	2.8	1

- Environment: judges' preferences (or polls)
- Goal: collective preference ranking
- Actions: choose an aggregation method
- Uncertainty: over the rankings
- Centralized approach to achieve consensus or compromise

- Examples:
  - Web search engines ranking web pages
  - Cameras ranking the plausible interpretations of an object
  - A hiring committee
  - Recommender systems
  - Friends deciding where to go for dinner
  - Political elections

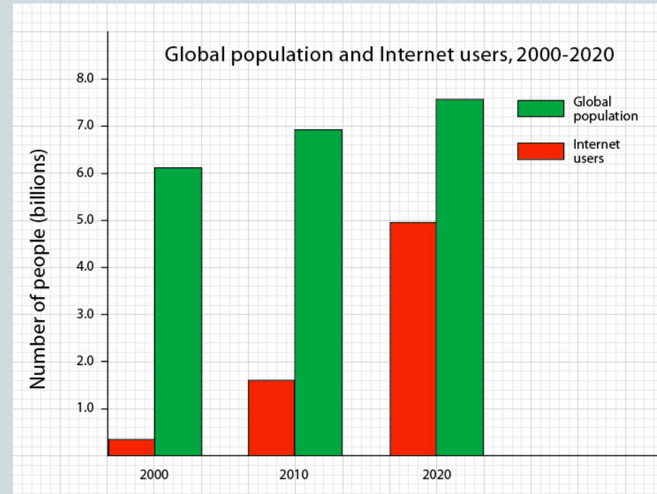
A simple example: friends choosing a dinner

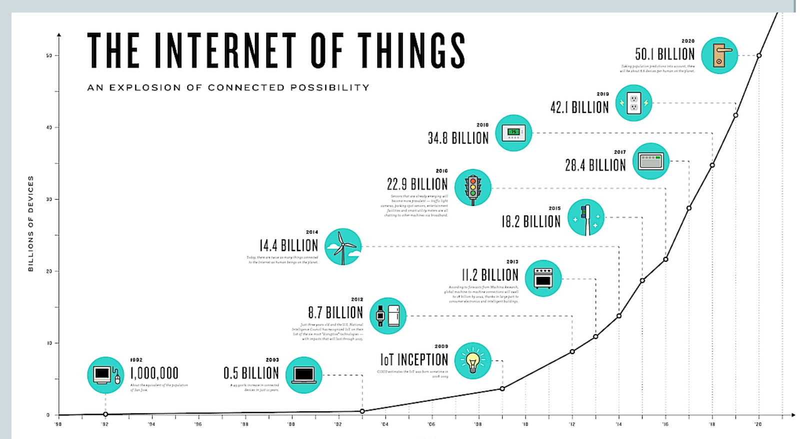
- They want to eat the same meal
  - Pasta, main dish, dessert, drink
- 5 options for each

  ➔ $5^4 = 625$ possible dinners
- Each friend has his own preferences over the possible dinners
- We need to choose one of them
- How do we model their preferences?
- How do we choose the single meal they will all eat?

These scenarios are more and more frequent

- People are connected and want to take decisions with their friends or peers
- Online social networks
- Influences among agents
- Manipulations
- Not just people, also “things”
  - 50 billion connected by 2020





More formal setting

- From
  - M candidates
  - N agents
  - For each agent, a preference order over the candidates
- we want to get
  - A single ordering of the candidates (social welfare function)
  - Or at least a “winning” candidate (social choice function)
- such that
  - the preferences of the agents are “considered”
- We can exploit concepts and techniques from voting theory (social choice)
  - After all, it looks like an election!

Voting rules (social choice functions)

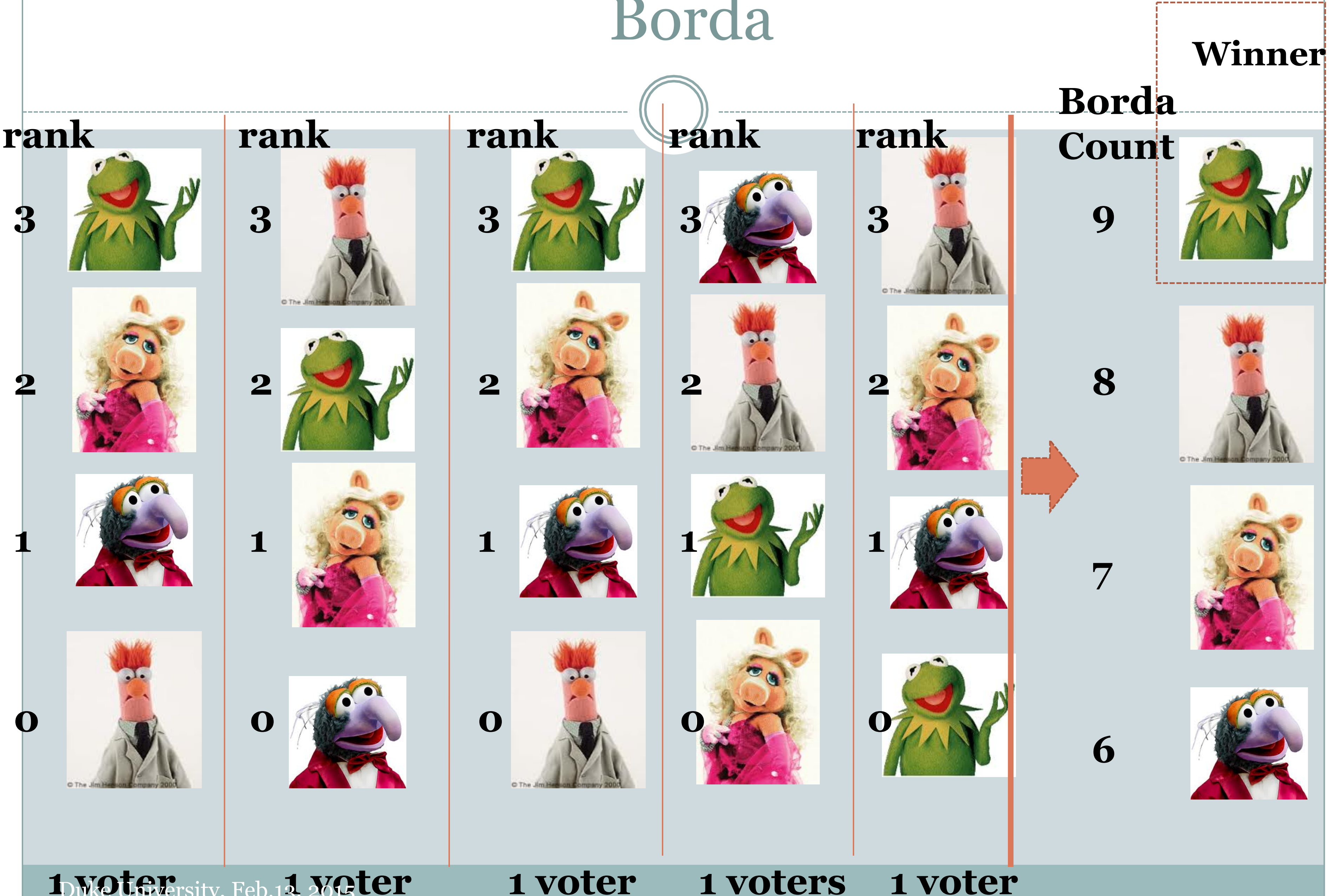
- Plurality: one vote for each agent, score is number of votes
- Borda: each agent give full ranking, score depend on the position on the ranking
- Approval: each agent approves some candidates, score is number of approvals
- K-approval: each agent approves k candidates
- Also many others
  - Kemeny, Single Transferrable Vote, Veto, Copeland, Minimax, Range, Schulze, Banks, Slater, Bucklin, Dogson, …

Plurality

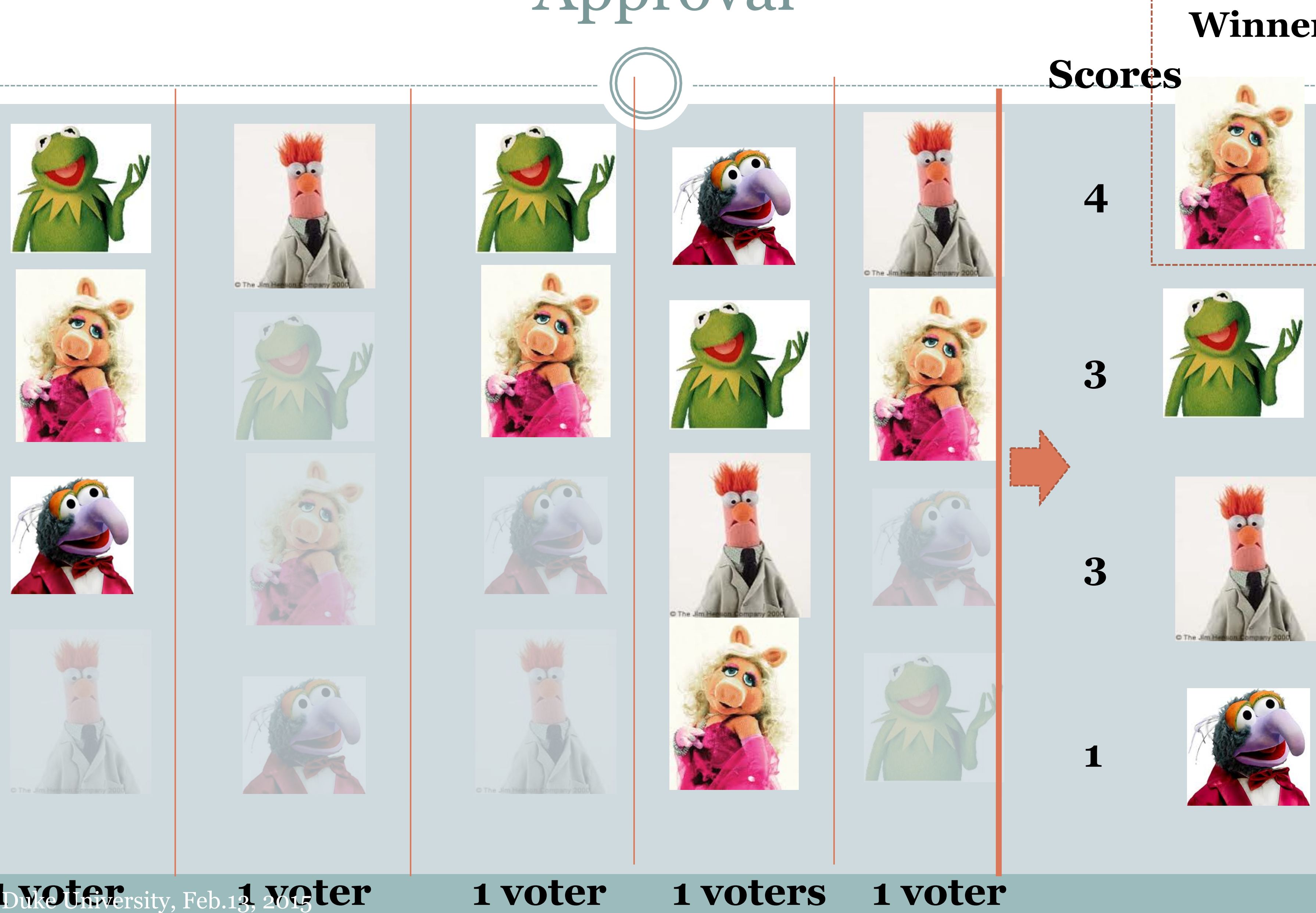
- Vote: 1 candidate
- Result: candidate(s) with the most vote(s)
- Example:
  - 6 voters
  - Candidates:

Winner

Borda


Winner
Borda Count
rank
3
2
1
0
rank
3
2
1
0
rank
3
2
1
0
rank
3
2
1
0
rank
3
2
1
0
9
8
7
6
1 voter
1 voter
1 voter
1 voters
1 voter

Approval



2-approval

Which social choice function should we choose?

- With 2 candidates, easy: majority!
- With more than 2 candidates, look at their properties
- Examples of properties:
  - We don't want one friend to be always deciding the dinner, independently of what the other friends say (non-dictatorship)
  - If everybody says that dinner A is preferred to dinner B, we don't want B to be chosen (Pareto-efficiency)
  - Between A and B, the fact that A is chosen, or B, or none, should depend only on how the friends ordered A and B (independence of irrelevant alternatives)

Which social choice function should we choose?

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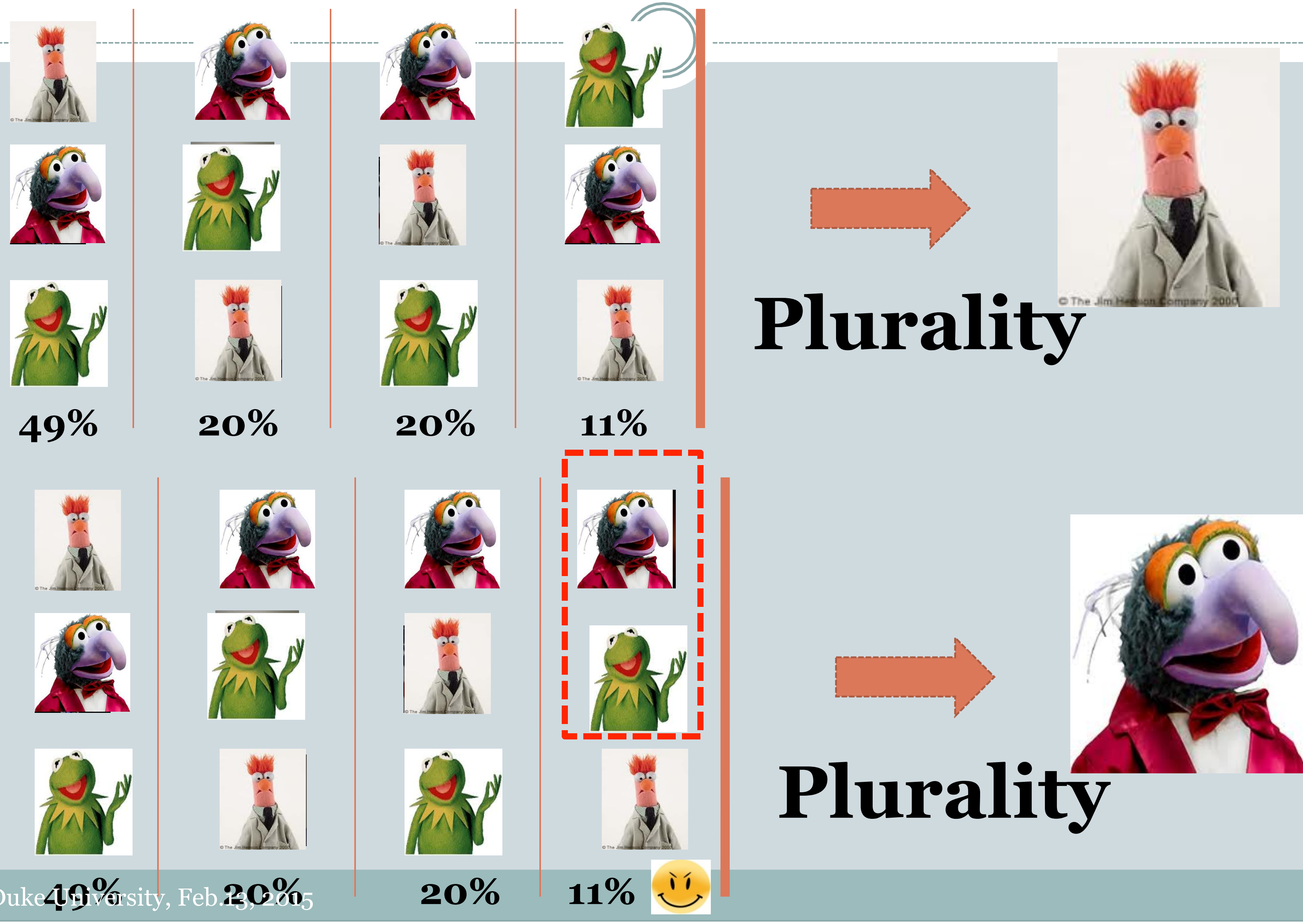
  - Between A and B, the fact that A is chosen, or B, or none, should depend only on how the friends ordered A and B (independence of irrelevant alternatives)
- Unfortunately …. No voting rule has these three properties!
  - Kenneth Arrow, Nobel in economics, 1972

Another desirable property: strategy-proofness

- A social choice function is strategy-proof if agents cannot manipulate the result
- Manipulation: misreporting his preferences in order to get a better result

Manipulation

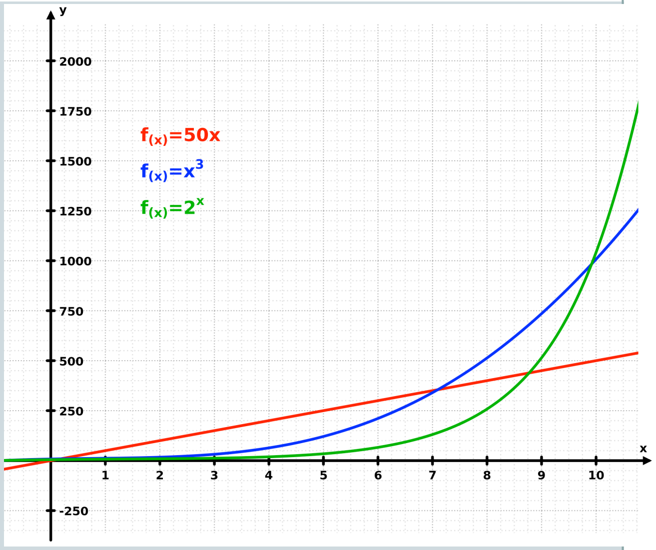


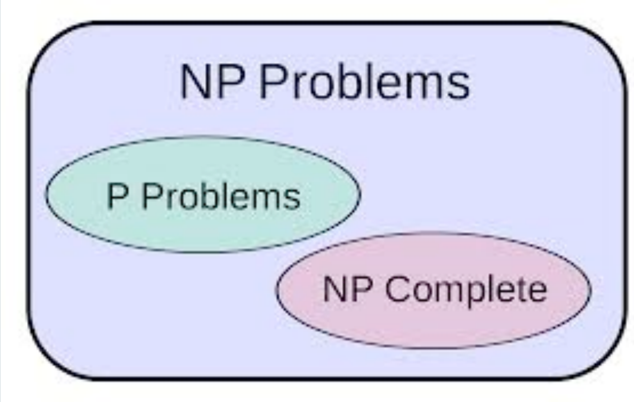
Again not possible

- Unfortunately .... all social choice functions are manipulable!
  - Unless we accept dictatoriality or some candidate that can never win
  - Gibbard-Sattherwite theorems, 1975

Is it difficult to (check if we can) manipulate?

- We consider computational difficulty
  - Exponential time to figure out how to manipulate, knowing how the other agents will vote, in the worst case
  - Easy when it takes always polynomial time
- It depends on the voting rule
  - Easy for Plurality, Borda, and Approval
  - Difficult for other rules, like STV
- We could use voting rules where it is computationally difficult to (check if we can) manipulate!
  - So we mitigate the impossibility result





Is it computationally easy to compute the winner?

- Yes, for most of the rules
  - Plurality, Borda, approval, …
- But not for all
  - Example: Kemeny rule

Multiple issues

- Until now we have considered voting over one issue only
- Now we consider several issues
- Example:
  - 3 referendum (yes/no)
  - Each voter has to give his preferences over triples of yes and no
  - Such as: YYY>NNN>YNY>YNN>etc.
- With k issues, k-tuples (2^k if binary issues)

Paradox of multiple elections

- 13 voters are asked to each vote yes or no on 3 issues:
  - 3 voters each vote for YNN, NYN, NNY
  - 1 voter votes for YYY, YYN, YNY, NYY
  - No voter votes for NNN
- Majority on each issue: the winner is NNN!
  - Each issue has 7 out of 13 votes for no

➔ not reasonable to vote independently over the features

Plurality on combinations

- Ask each voter for her most preferred combination and apply plurality
  - Avoids the paradox, computationally light
  - Almost random decisions
  - Example: 10 binary issues, 20 voters ➔ $2^{10} = 1024$ combinations to vote for but only 20 voters, so very high probability that no combination receives more than one vote ➔ tie-breaking rule decides everything
- Similar also for voting rules that use only a small part of the voters' preferences (ex.: k-approval with small k)

Other rules on combinations

- Vote on combinations and use other voting rules that use the whole preference ordering on combinations
- Avoids the arbitrariness problem of plurality
- Not feasible when there are large domains
- Example:
  - Borda (needs the whole preference ordering)
  - 6 binary issues ➔ 2^6=64 possible combinations ➔ each voter has to choose amongst 64! possible ballots

Sequential voting

- Vote separately on each issue, but do so sequentially
- This gives voters the opportunity to make their vote for one issue depend on the decisions on previous issues

Condorcet losers

- Condorcet loser (CL): candidate that loses against any other candidate in a pairwise contest
- Electing a CL is very bad, but Plurality sometimes elects it
- Example:
  - 2 votes for X > Y > Z
  - 2 votes for Y > X > Z
  - 3 votes for Z > X > Y
  - Z is the Plurality winner and the Condorcet loser

Sequential voting and Condorcet losers

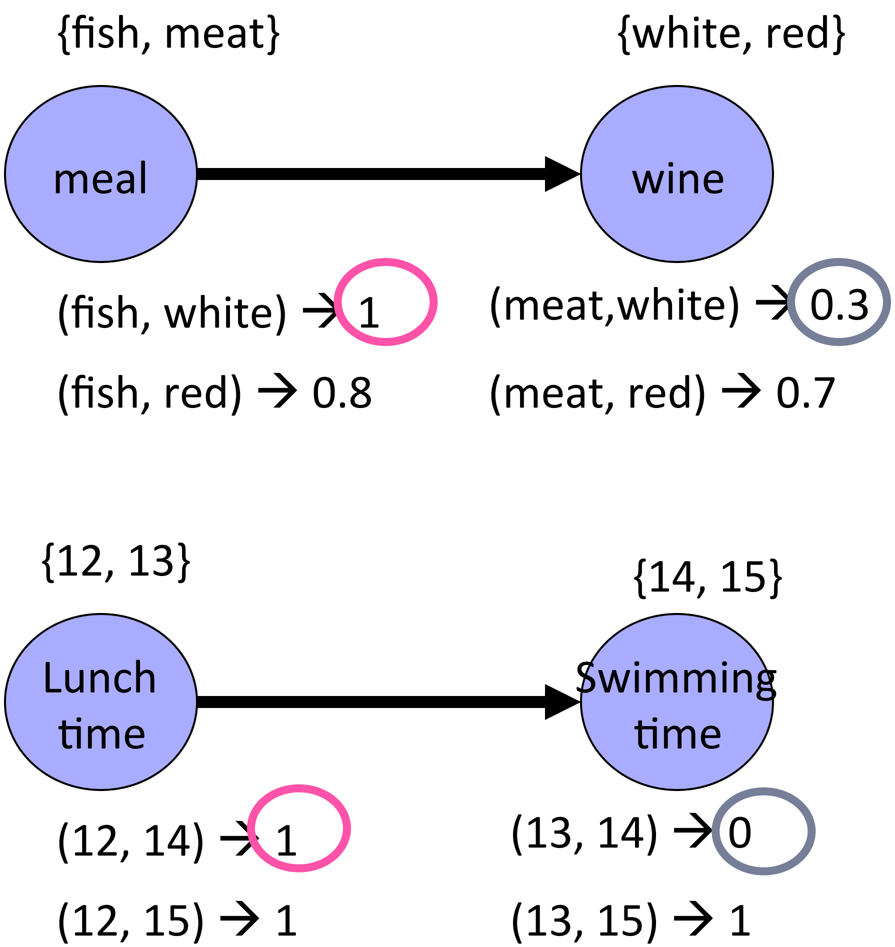
- Sequential voting avoids the problem of electing Condorcet losers
- Thm.: Sequential plurality voting over binary issues never elects a Condorcet loser
  - Proof: Consider the election for the final issue. The winning combination cannot be a CL, since it wins at least against the other combination that was still possible after the penultimate election
  - [Lacy, Niou, J. of Theoretical Politics, 2000]
- But no guarantee that sequential voting elects the Condorcet winner (Condorcet consistency)

How to express preferences compactly over a set of related items?

- When there are many candidates, unfeasible to rank them all
  - Think of the dinner example: 625 possible dinners!
- We need a compact way to say what we prefer
- Two main options
  - Quantitative (ex. Soft constraints)
    - Decide on a scale, ex. from 0 to 5, where higher number means more preferred
    - Evaluate some parts of the dinner over this scale
  - Qualitative (ex. CP-nets)
    - Dependence between items, total order on the options for each item

Example: fuzzy constraints

Preference of a decision: minimal preference of its parts
Aim: to find a decision with maximal preference
Preference values: between 0 and 1



Decision *A*	
Lunch time=	13
Meal =	meat
Wine =	white
Swimming time=	14
pref(A)=min(0.3,0)=0	

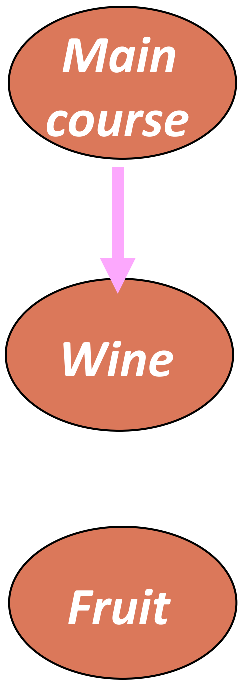
Decision B	
Lunch time =	12
Meal =	fish
Wine =	white
Swimming time =	14
pref(B)=min(1,1)=1	

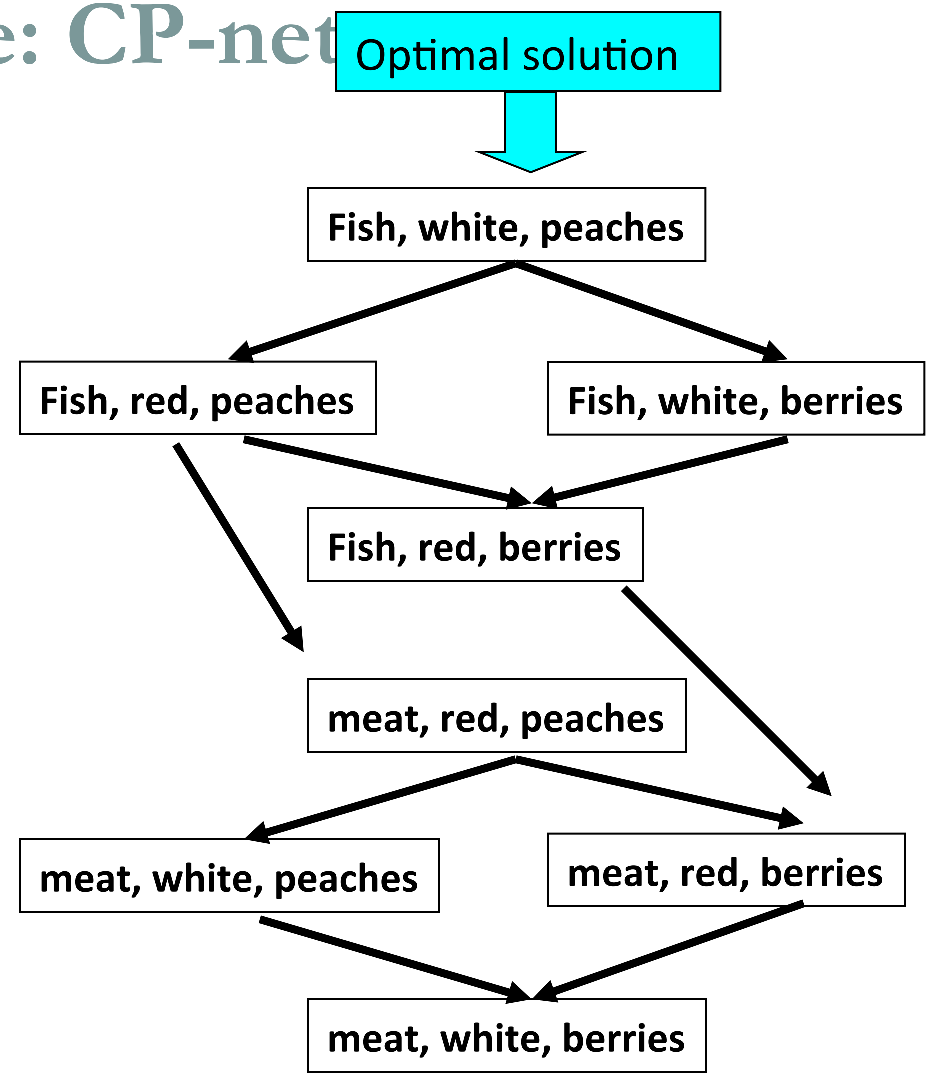
Example: CP-net

fish>meat

Main course	*Wine*
fish	white > red
meat	red > white

peaches > strawberries





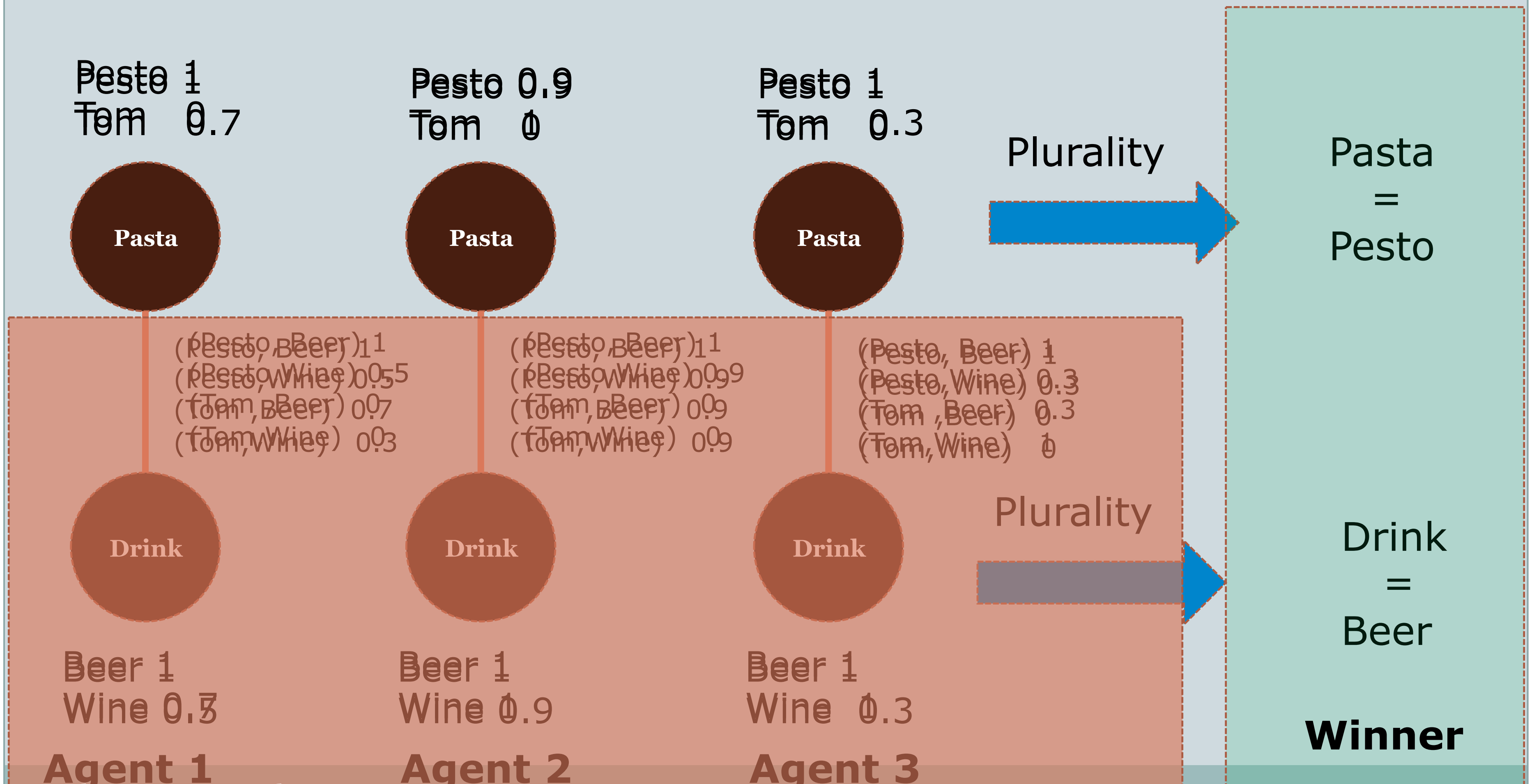
Soft constraints vs. CP-nets

	Soft CSPs	Tree-like soft CSPS	CP-nets	Acyclic CP-nets
Preference orderings	all	all	some	some
Find an optimal decision	difficult	easy	difficult	easy
Compare two decisions	easy	easy	difficult	difficult
Find the next best Decision	difficult	difficult for weighted, easy for fuzzy	difficult	easy
Check if a decision is optimal	difficult	easy	easy	easy

Sequential voting with soft constraints

- Agents expressing preferences via soft constraints
- Over a common set of decisions/options
  - options = complete variable assignments
- Same vars and var domains for all agents, different soft constraints
- Profile = preferences of all agents
  - Explicit profile: preference orderings are given
  - Implicit profile: compact representation of the preferences
- We will select a decision using a voting rule
  - Decision = solution for the agents soft constraint satisfaction problems (sof CSP)
  - Voting rule: function from an explicit profile to a decision
- In the dinner example:
  - Each friend has his own soft CSP to express the preferences over the dinners
  - We need to select one dinner over the 625 possible ones

Dinner example using plurality



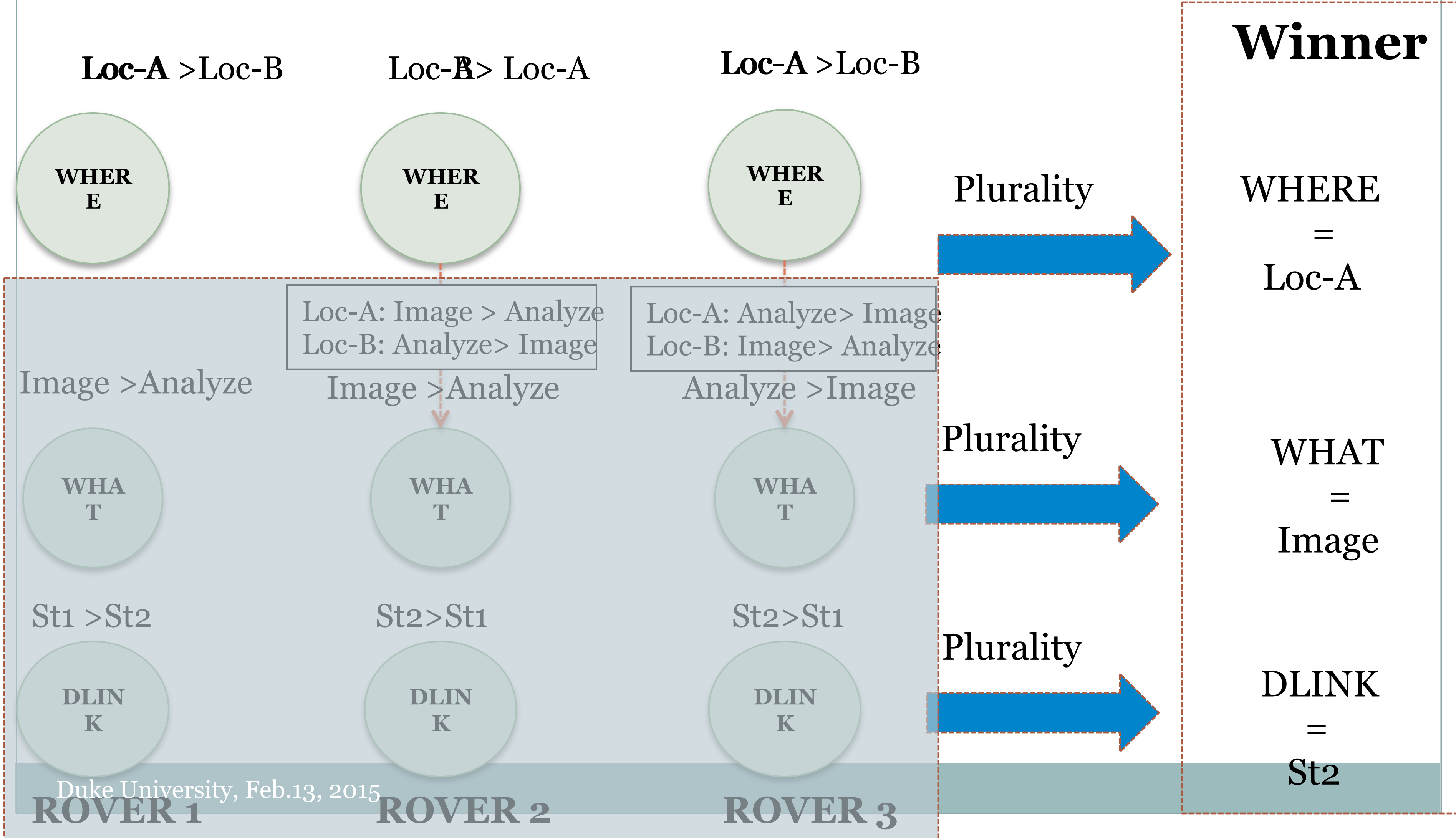
Sequential voting with CP-nets

- n voters, voting by giving a CP-net each
  - Same variables, different dependency graph and CP tables
- Compatible CP-nets: there exists a linear order on the variables that is compatible with the dependency graph of all CP-nets (that is, it completes the DAG)
- Then vote sequentially in this order
- Thm.: Under these assumptions, sequential voting is Condorcet consistent if all local voting rules are
  - (Lang and Xia, Math. Social Sciences, 2009)

Example

3 Rovers must decide:

- Where to go: Location A or Location B
- What to do: Analyze a rock or Take a picture
- Which station to downlink the data to: Station 1 or Station 2



Sequential voting: soft constraints vs. CP-nets

- With soft constraints
  - Solving and projecting at each step
  - Tractable in some case (ex. Tree)
  - But no requirement on compatibility among SCSPs
- With CP-nets
  - Directional dependency links
  - Compatibility is required
  - No solving/projecting effort

Voting in pursuit of the truth

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Objectives of voting

- **OBJ1:** Compromise among subjective preferences

- **OBJ2:** Reveal the "truth"

A model with two alternatives

- One of the two alternatives {R, B} is the "correct" winner; this is not directly observed
- Each voter votes for the correct winner with probability $p > ½$, for the other with $1-p$ (i.i.d.)
- The probability of a given profile in which r is the number of votes for R and b that for B ($r+b=n$)...
  - ... given that R is the correct winner is $p^r(1-p)^b$
  - ... given that B is the correct winner is $p^b(1-p)^r$

Condorcet Jury Theorem [1785]

- **Theorem.** In the model on the previous slide, if we use the majority rule, the probability that the correct winner is chosen goes to 1 as the number of votes goes to infinity.
- *Proof*: by law of large numbers the fraction of votes for the correct winner will converge to p

- Lots of variants: voters of unequal skill, correlated voters, strategic voters, …

Statistical estimation

- We have some parameters of our statistical model with unknown values
  - for example, the mean and variance of people's height
  - … in our case, which alternative is the correct winner
- We have some data
  - for example, some people's height
  - … in our case, the votes
- Want to estimate the values of the parameters

Maximum likelihood estimation

- Choose the parameter values *q* that maximize the likelihood of the data

- I.e., choose arg max P(data | *q*)

Example of MLE (**not** our main model)

- p of the voters vote for R; we don't know p, try to estimate it
- From a poll, we observe votes for R, B, B, R, B
- Let's call our estimate q
- If q were correct, the probability of our profile would be $q^2(1-q)^3$
- Differentiating with respect to q:

  $2q(1-q)^3 - 3q^2(1-q)^2 = 0$ or $2(1-q)-3q=0$ or $q=2/5$
- ... which is the sample fraction of R (generally true)

Back to our main model

- One of the two alternatives {R, B} is the "correct" winner; this is not directly observed
- Each voter votes for the correct winner with probability $p > ½$, for the other with $1-p$ (i.i.d.)
- The probability of a given profile in which r is the number of votes for R and b that for B ($r+b=n$)...
  - ... given that R is the correct winner is $p^r(1-p)^b$
  - ... given that B is the correct winner is $p^b(1-p)^r$
- Maximum likelihood estimate: correct winner = whichever has more votes (majority rule)

What if voters have different skill?

[Nitzan & Paroush 1982, Shapley and Grofman 1984]

- Each voter i votes for the correct winner with probability $p_i > ½$, for the other with $1\text{-}p_i$ (independently)
- The probability of a given profile in which r is the number of votes for R and b that for B (r+b=n)...
  - ... given that R is the correct winner is

    $(\prod_{\text{i for R}} p_i)(\prod_{\text{i for B}} (1\text{-}p_i))$ →**take log**→ $\Sigma_{\text{i for R}} \log p_i + \Sigma_{\text{i for B}} \log(1\text{-}p_i)$
    $= \Sigma_{\text{i for R}} (\log p_i - \log(1\text{-}p_i)) + \Sigma_{\text{i in } \textbf{all votes}} \log(1\text{-}p_i)$
  - ... given that B is the correct winner is

    $(\prod_{\text{i for B}} p_i)(\prod_{\text{i for R}} (1\text{-}p_i))$ →**take log**→ $\Sigma_{\text{i for B}} \log p_i + \Sigma_{\text{i for R}} \log(1\text{-}p_i)$
    $= \Sigma_{\text{i for B}} (\log p_i - \log(1\text{-}p_i)) + \Sigma_{\text{i in } \textbf{all votes}} \log(1\text{-}p_i)$
- Hence if we do weighted majority with weight $\log p_i - \log(1\text{-}p_i) = \log [p_i / (1\text{-}p_i)]$ we will get the MLE!

A bit more about probability

- Example: roll two dice
- Random variables:
  - X = value of die 1
  - Y = value of die 2
- Outcome is represented by an ordered pair of values (x, y)
  - E.g., (6, 1): X=6, Y=1
  - Atomic event or sample point tells us the **complete** state of the world, i.e., values of **all** random variables
- Exactly one atomic event will happen; each atomic event has a ≥0 probability; sum to 1
  - E.g., P(X=1 and Y=6) = 1/36

Y						
6	1/36	1/36	1/36	1/36	1/36	1/36
5	1/36	1/36	1/36	1/36	1/36	1/36
4	1/36	1/36	1/36	1/36	1/36	1/36
3	1/36	1/36	1/36	1/36	1/36	1/36
2	1/36	1/36	1/36	1/36	1/36	1/36
1	1/36	1/36	1/36	1/36	1/36	1/36
	1	2	3	4	5	6
						X

- An event is a proposition about the state (=subset of states)
  - X+Y = 7
- Probability of event = sum of probabilities of atomic events where event is true

Conditional probability

- We might know something about the world – e.g., "X+Y=6 or X+Y=7" – given this (and **only** this), what is the probability of Y=5?
- Part of the sample space is eliminated; probabilities are renormalized to sum to 1

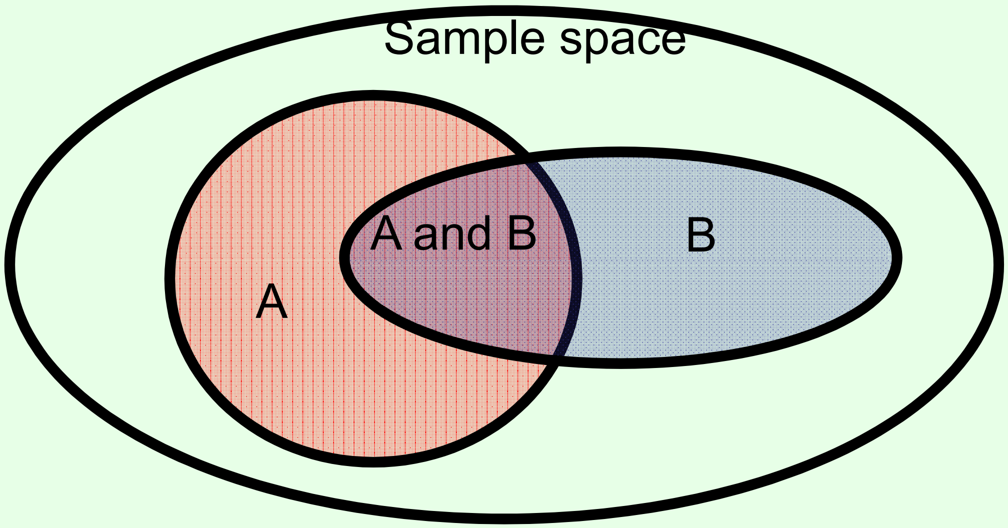
Y						
6	1/36	1/36	1/36	1/36	1/36	1/36
5	1/36	1/36	1/36	1/36	1/36	1/36
4	1/36	1/36	1/36	1/36	1/36	1/36
3	1/36	1/36	1/36	1/36	1/36	1/36
2	1/36	1/36	1/36	1/36	1/36	1/36
1	1/36	1/36	1/36	1/36	1/36	1/36
X	1	2	3	4	5	6

Y						
6	1/11	0	0	0	0	0
5	1/11	1/11	0	0	0	0
4	0	1/11	1/11	0	0	0
3	0	0	1/11	1/11	0	0
2	0	0	0	1/11	1/11	0
1	0	0	0	0	1/11	1/11
X	1	2	3	4	5	6

- P(Y=5 | (X+Y=6) or (X+Y=7)) = 2/11

Facts about conditional probability

- $P(A \mid B) = P(A \text{ and } B) / P(B)$



- $P(A \mid B)P(B) = P(A \text{ and } B) = P(B \mid A)P(A)$
- $P(A \mid B) = P(B \mid A)P(A)/P(B)$
  - Bayes' rule

Maximum a posteriori

- Maybe what we really want is
  arg max P(q | data)

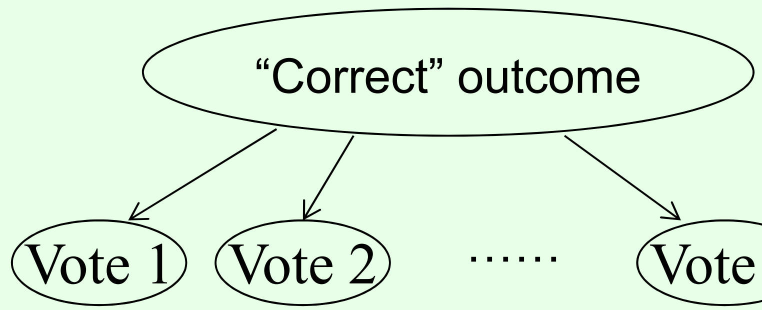
- But by Bayes' rule,
  P(q | data) = P(data | q) P(q) / P(data)

- So **if** P(q) is uniform,
  arg max P(q | data) = arg max P(data | q)
  (MLE)

More generally: The MLE approach to voting

[can be said to date back to Condorcet 1785]

- Given the "correct outcome" c
  - each vote is drawn conditionally independently given c, according to $\Pr(V|c)$



- The MLE rule: For any profile $P = (V_1, \ldots, V_n)$,
  - The likelihood of P given c: $L(P|c)=\Pr(P|c)=\prod_{V\in P}\Pr(V|c)$
  - The MLE as a rule is defined as

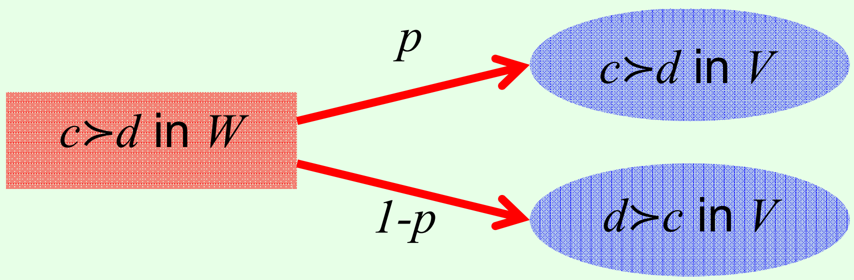
$$\text{MLE}_{\Pr}(P)=\text{argmax}_c \prod_{V\in P}\Pr(V|c)$$

- Body of work characterizing these with >2 alternatives [Young '88, '95, C. & Sandholm '05, C., Rognlie, Xia '09, Xia & C. '11, Procaccia, Reddi, Shah '12 …]

A noise model for >2 alternatives

[dating back to Condorcet 1785]

- Correct outcome is a ranking W , p>1/2



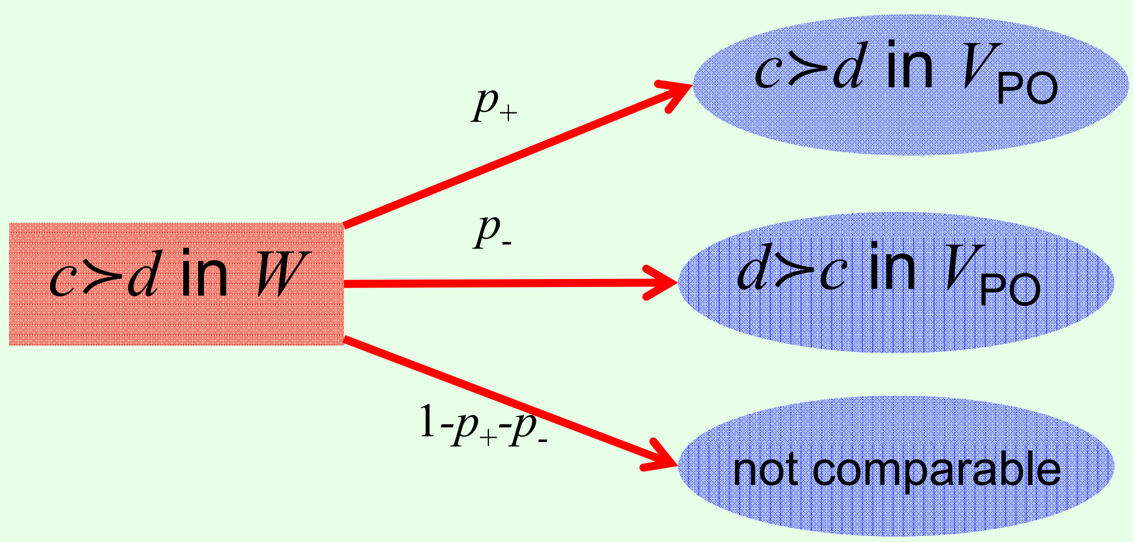
$$\Pr(\ b \succ c \succ a \mid a \succ b \succ c\ ) = p(1-p)^2$$

- MLE = Kemeny rule [Young '88, '95]
  - $\Pr(P|W) = p^{nm(m\text{-}1)/2\text{-}\mathrm{K}(P,W)}\,(1\text{-}p)^{\mathrm{K}(P,W)} = p^{nm(m-1)/2}\left(\dfrac{1-p}{p}\right)^{K(P,W)}$
  - The winning rankings are insensitive to the choice of p (>1/2)

A variant for partial orders

[Xia & C. IJCAI-11]

- Parameterized by $p_+ > p_- \geq 0$ $(p_+ + p_- \leq 1)$
- Given the “correct” ranking W, generate pairwise comparisons in a vote V_{PO} independently



MLE for partial orders…

[Xia & C. IJCAI-11]

- In the variant of Condorcet's model
  - Let T denote the number of pairwise comparisons in P_{PO}
  - $\Pr(P_{\text{PO}}|W) = (p_+)^{T\text{-K}(P_{\text{PO}},W)} (p_-)^{\text{K}(P_{\text{PO}},W)} (1\text{-}p_+\text{-}p_-)^{\text{nm(m-1)/2-}T}$

$$= (1 - p_+ - p_-)^{nm(m-1)/2 - T} (p_+)^T \left(\frac{p_-}{p_+}\right)^{K(P_{PO},W)}$$

  - The winner is $\text{argmin}_W \text{K}(P_{\text{PO}},W)$

Which other common rules are MLEs for some noise model?

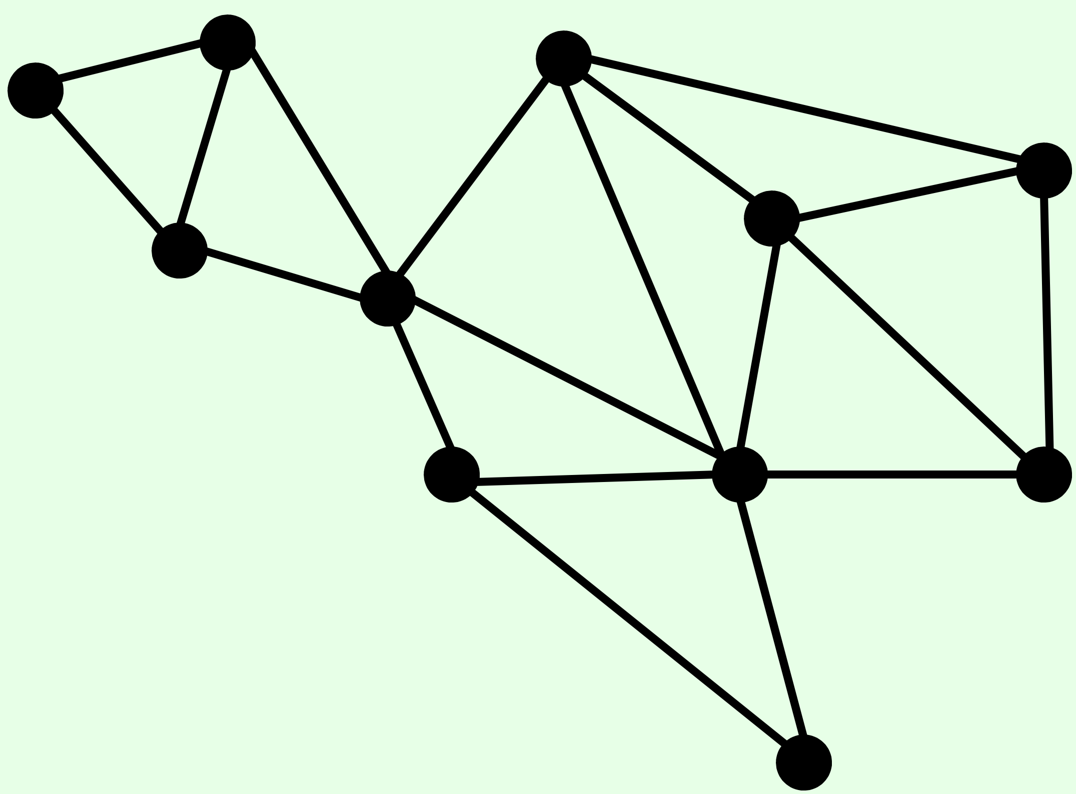
[C. & Sandholm UAI'05; C., Rognlie, Xia IJCAI'09]

- Positional scoring rules
- STV - kind of…
- Other common rules are provably not
- Reinforcement: if $f(V_1) \cap f(V_2) \neq \emptyset$ then $f(V_1+V_2) = f(V_1) \cap f(V_2)$ (f returns rankings)
- *Every MLE rule must satisfy reinforcement!* (why?)
- Incidentally: Kemeny uniquely satisfies neutrality, reinforcement, and Condorcet property [Young & Levenglick 78]

Correct **alternatives**

- Suppose the ground truth outcome is a correct alternative (instead of a ranking)
- Positional scoring rules are still MLEs
- Reinforcement: if $f(V_1) \cap f(V_2) \neq \emptyset$ then $f(V_1+V_2) = f(V_1) \cap f(V_2)$ (but now f produces a winner)
- Positional scoring rules* are the only voting rules that satisfy anonymity, neutrality, and reinforcement! [Smith '73, Young '75]
  - * Can also break ties with another scoring rule, etc.
- Similar characterization using reinforcement for ranking?

Independence assumption ignores social network structure



Voters are likely to vote similarly to their neighbors!

What should we do if we know the social network?

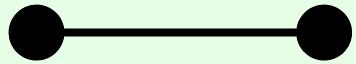
- Argument 1: *"Well-connected voters benefit from the insight of others so they are more likely to get the answer right. They should be weighed more heavily."*
- Argument 2: *"Well-connected voters do not give the issue much independent thought; the reasons for their votes are already reflected in their neighbors' votes. They should be weighed less heavily."*
- Argument 3: *"We need to do something a little more sophisticated than merely weigh the votes (maybe some loose variant of districting, electoral college, or something else...)."*

Factored distribution

- Let A_v be v's vote, N(v) the neighbors of v
- Associate a function $f_v(A_v, A_{N(v)} \mid c)$ with node v (for c as the correct winner)
- Given correct winner c, the probability of the profile is $\Pi_v\ f_v(A_v, A_{N(v)} \mid c)$
- **Assume:**

  $f_v(A_v, A_{N(v)} \mid c) = g_v(A_v \mid c)\ h_v(A_v, A_{N(v)})$
  - Interaction effect is independent of correct winner

Example
(2 alternatives, 2 connected voters)



- $g_v(A_v{=}c \mid c) = .7$, $g_v(A_v{=}-c \mid c) = .3$
- $h_{vv'}(A_v{=}c, A_{v'}{=}c) = 1.142$,

  $h_{vv'}(A_v{=}c, A_{v'}{=}-c) = .762$
- $P(A_v{=}c \mid c) =$

  $P(A_v{=}c, A_{v'}{=}c \mid c) + P(A_v{=}c, A_{v'}{=}-c \mid c) =$
  $(.7*1.142*.7*1.142 + .7*.762*.3*.762) = .761$

- (No interaction: $h=1$, so that $P(A_v{=}c \mid c) = .7$)

Social network structure does not matter!

- **Theorem.** The maximum likelihood winner does not depend on the social network structure. (So for two alternatives, majority remains optimal.)
- *Proof.*

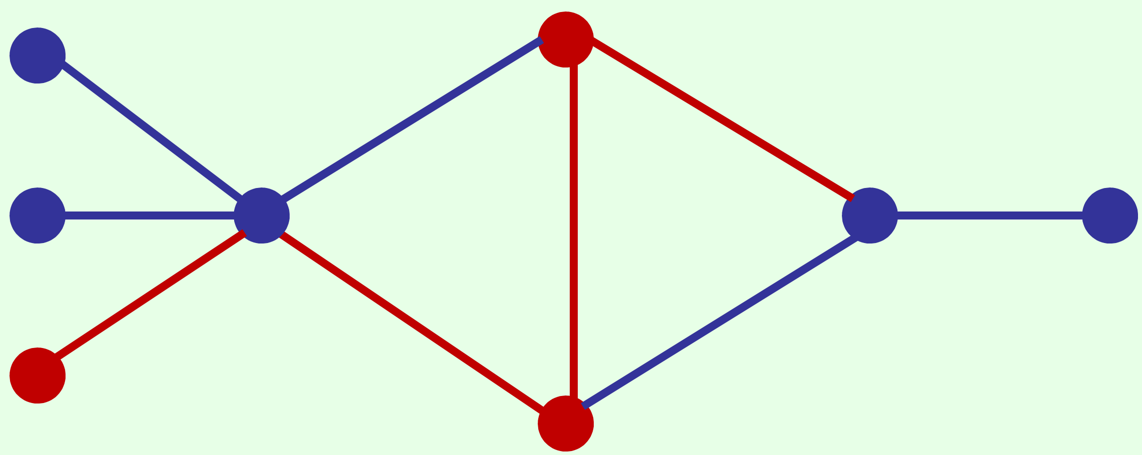
  $\arg\max_c \Pi_v f_v(A_v, A_{N(v)} \mid c) =$

  $\arg\max_c \Pi_v g_v(A_v \mid c)\, h_v(A_v, A_{N(v)}) =$

  $\arg\max_c \Pi_v g_v(A_v \mid c)$.

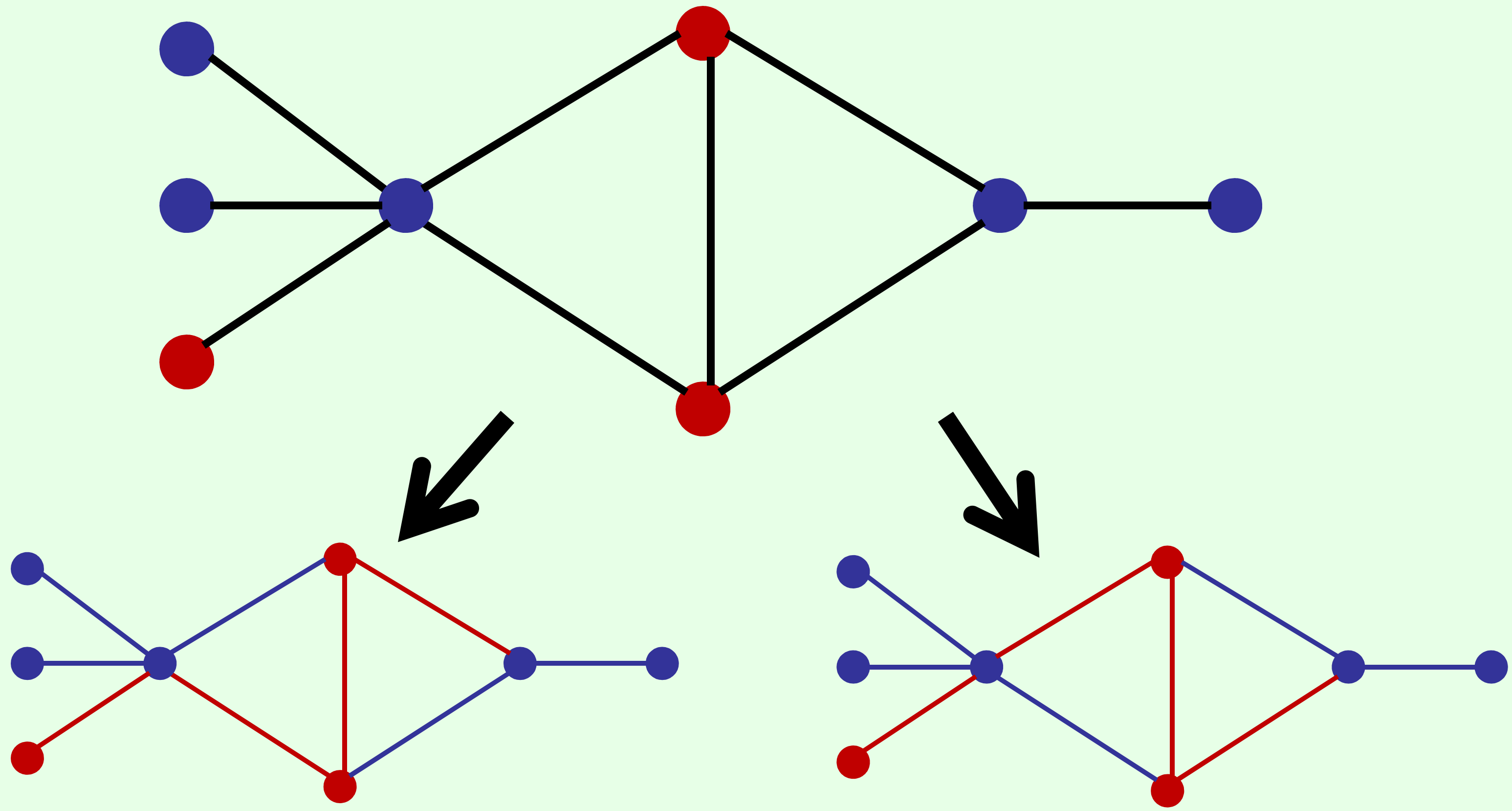
The independent conversations model

- Edges are associated with alternatives
  - i.i.d., $p>0.5$ of correct alternative
- Voters go with the majority of their edges



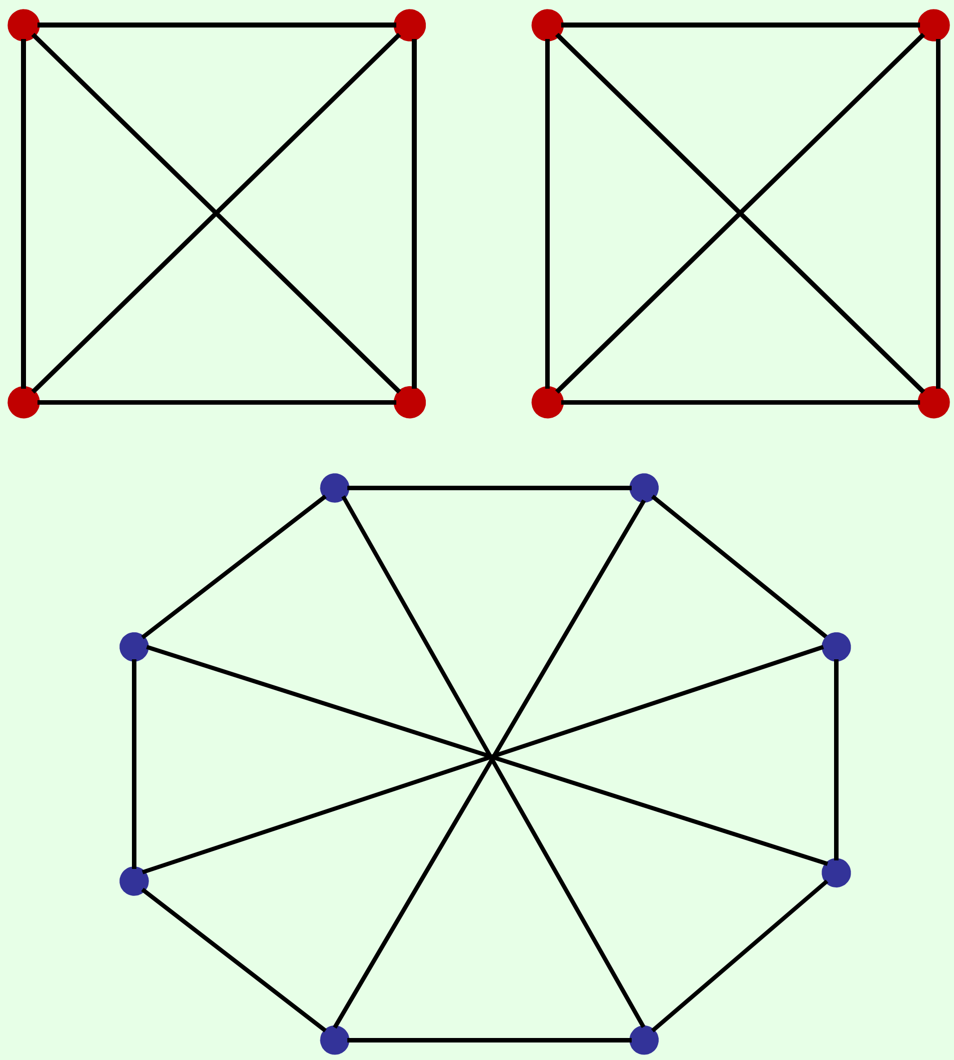
- If blue is correct, the probability of this configuration is $p^5(1-p)^4$; if red is correct, it is $p^4(1-p)^5$
- ... but we don’t actually know the edge colors!

Attempt #2



If blue is correct, the probability of the votes is $2p^5(1-p)^4$; if red is correct, it is $2p^4(1-p)^5$

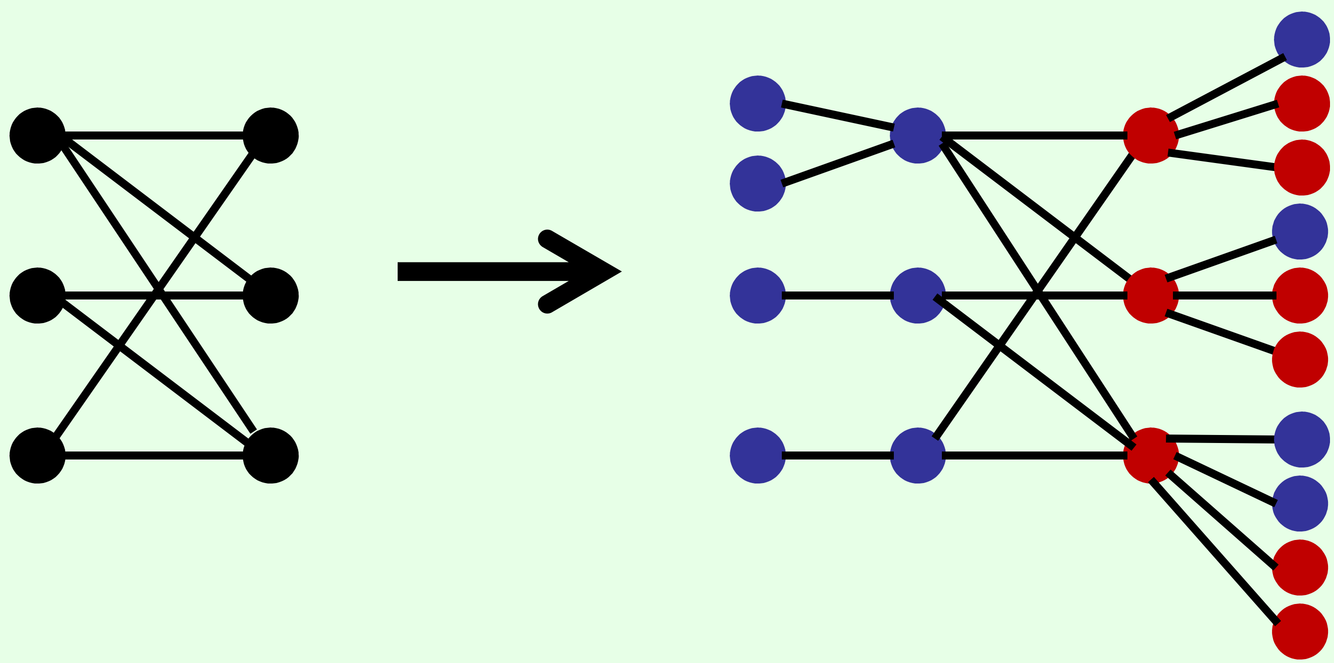
How about this one?



- Opposite colored edges must be a matching
- Matchings for red part: 1 of 0, 12 of 1, 42 of 2, 36 of 3, 9 of 4
- Matchings for blue part: 1 of 0, 12 of 1, 42 of 2, **44** of 3, **7** of 4
- If p=0.9, blue wins
- If p=0.6, red wins!

#P-hardness

- **Theorem.** Computing $P(A_V \mid c)$ is #P-hard.
- *Proof.* Reduction from PERMANENT (counting the number of perfect bipartite matchings)



Estimating c and A_E together

- **Theorem.** An element of
  arg $\max_{c,A_E} P(A_V, A_E \mid c)$
  can be found in polynomial time (even if edges have different p_e)
- *Proof.* For given c, reduction to a general version of *b*-matching
  - Choosing an edge = it votes for c
  - Each vertex has lower and upper bounds on number of matches
  - Weight on e is $\log p_e - \log(1-p_e)$
    - Goal is to max $\Sigma_{e \text{ votes for } c} \log p_e + \Sigma_{e \text{ votes for } -c} \log(1-p_e)$

Conclusions

- In some voting contexts we might think there's a correct outcome and votes are noisy perceptions of the truth
- MLE model allows us to
  - formalize this intuition,
  - justify certain voting rules as optimal, but also:
  - identify the assumptions going into those justifications (and modify them if we don't like them)
- General methodology that can also be applied to voting in combinatorial domains, judgment aggregation, etc.

Prediction Markets

David Pennock
Microsoft Research

Thanks: Miro Dudik, Sebastien Lahaie, David Rothschild
Jake Abernethy, Rafael Frongillo, Dan Osherson, Rajiv Sethi, Rob Schapire, Jenn Wortman Vaughan, Arvid Wang, ...

An Example
Prediction Market

- A random variable, e.g.

Will Hillary Clinton win
2016 Presidential election? (Y/N)

- Turned into a financial instrument payoff = realized value of variable

I am entitled to:

$100 if Clinton wins

$0 if Other wins

Trade Elections Like Stocks

2016 Presidential Election - Next President

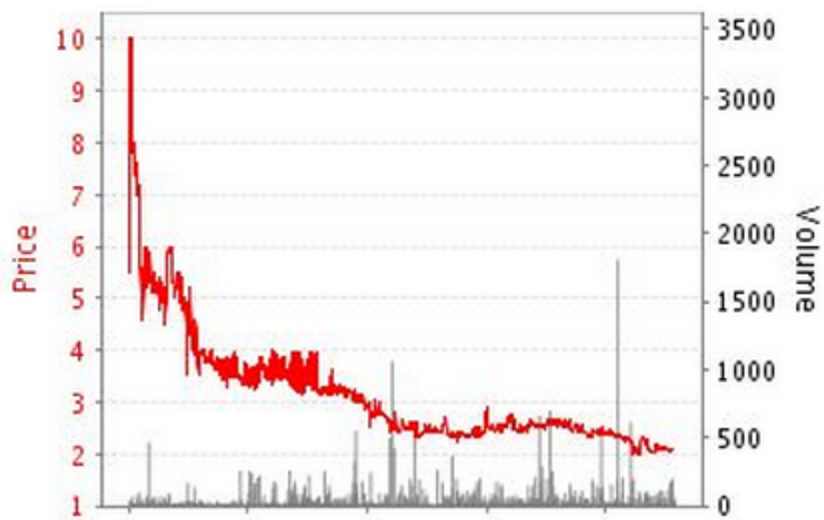
Betting on: Hillary Clinton

Total matched on this event: **£65,227**

Betting summary - Volume: **£0**

Last price matched: **2.08**

Price/Volume over time



Traded and Available

Price	To back	To lay
2.02	£230	
2.04	£25	
2.06	£870	
2.1		£101
2.2		£53
2.3		£58

Trade Elections Like Stocks

betfair

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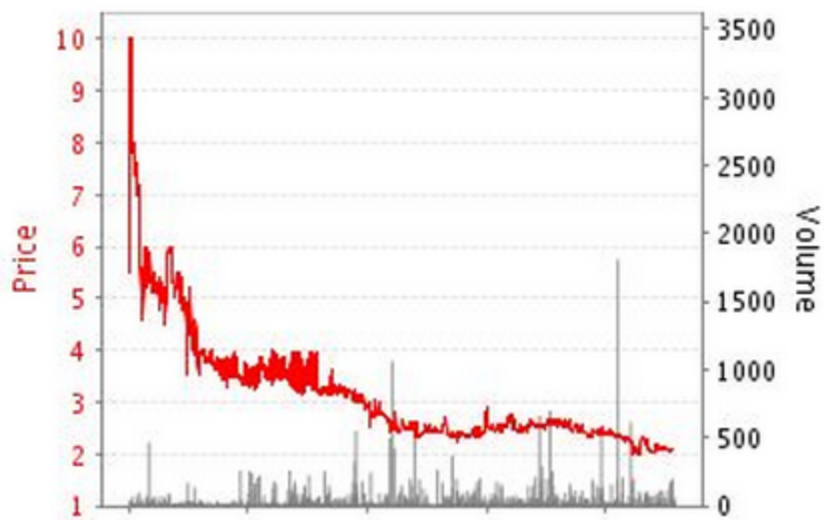


Traded and Available		
Price	**To back**	**To lay**
2.02	£230	
2.04	£25	
2.06	£870	
2.1		£101
2.2		£53
2.3		£58

Between 47.6% and 48.5% chance
2015/02/23 14:44 EST

Trade Elections Like Stocks

betfair

2016 Presidential Election - Democratic Nominee

Betting on: Hillary Clinton

Total matched on this event: **£59,140**

Betting summary - Volume: **£55,182**

Last price matched: **1.43**

Price/Volume over time

1.24	£102		
1.25	£5,122		£562
1.26	£352		£1,000
1.27	£146		£52
1.28	£77		£453
1.29	£198		£317
1.30	£78		£737
1.31	£126		£64
1.32	£51		£117
1.33	£5		£458
1.34			£145
1.35			£223
1.36			£15
1.37			£32
1.38		£17	£63
1.39		£43	£159
1.40		£51	£1,138

Between 72.5% and 75.2% chance
2015/02/24 06:30 EST

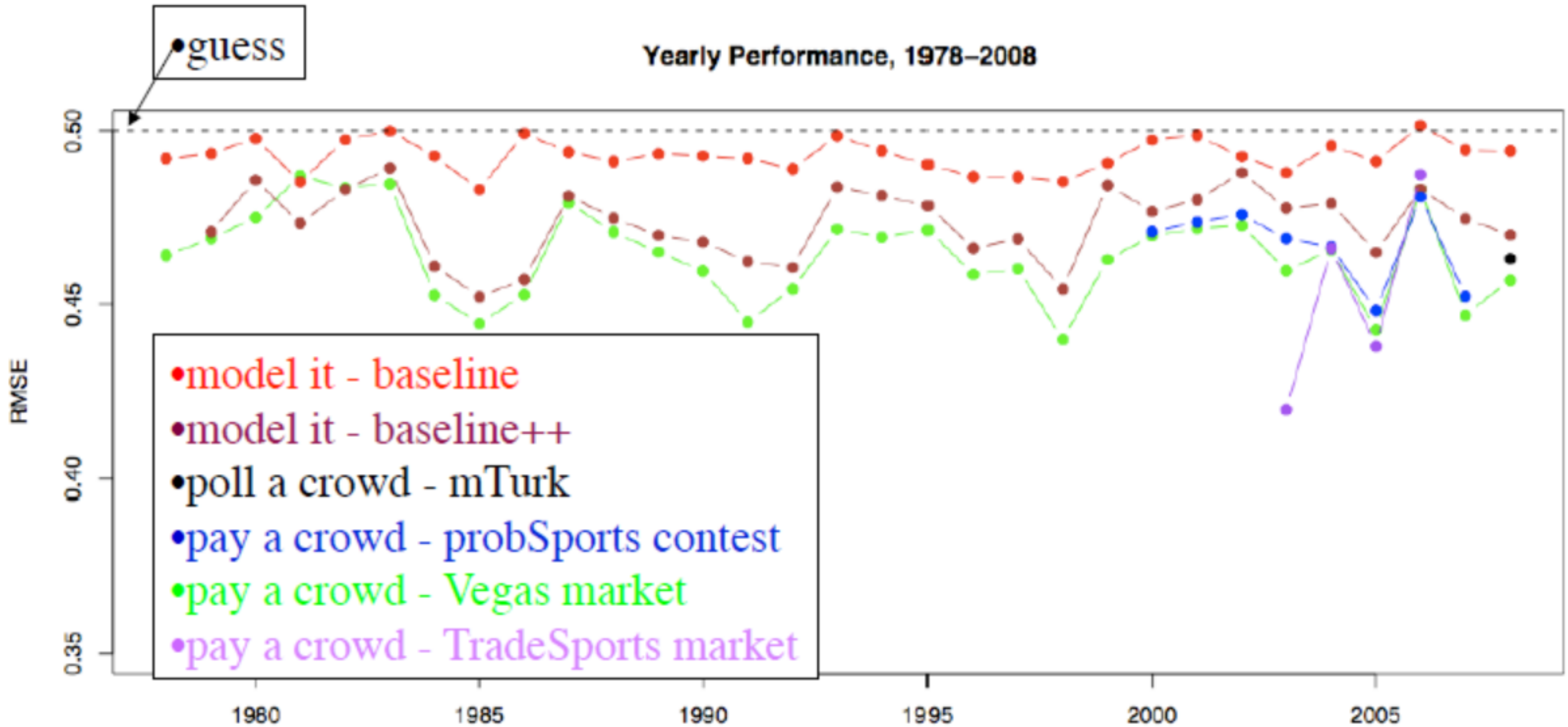
[Thanks: Yiling Chen]

Does it work?

Yes, evidence from real markets, laboratory experiments, and theory

- Racetrack odds beat track experts [Figlewski 1979]
- Orange Juice futures improve weather forecast [Roll 1984]
- I.E.M. beat political polls 451/596 [Forsythe 1992, 1999][Oliven 1995] [Rietz 1998][Berg 2001][Pennock 2002]
- HP market beat sales forecast 6/8 [Plott 2000]
- Sports betting markets provide accurate forecasts of game outcomes [Gandar 1998][Thaler 1988][Debnath EC' 03][Schmidt 2002]
- Laboratory experiments confirm information aggregation [Plott 1982;1988;1997][Forsythe 1990][Chen, EC' 01]
- Theory: "rational expectations" [Grossman 1981][Lucas 1972] Performance of Kelly bettors [Beygelzimer et al. 2012]
- Market games work [Servan-Schreiber 2004][Pennock 2001]

Does it work?



•guess
Yearly Performance, 1978–2008
RMSE
0.50
0.45
0.40
0.35
1980
1985
1990
1995
2000
2005
•model it - baseline
•model it - baseline++
•poll a crowd - mTurk
•pay a crowd - probSports contest
•pay a crowd - Vegas market
•pay a crowd - TradeSports market

Super Tuesday 2012

Prediction	Likelihood
VA: Romney	98.7%
MA: Romney	99.6%
VT: Romney	97.5%
ID: Romney	95.7%
GA: Gingrich	98.0%
OH: Romney	82.6%
OK: Santorum	88.6%
ND: Romney	72.1%
AK: Romney	82.6%
TN: Santorum	57.1%

- Predictions as of
  2:52 p.m. ET
  Monday, March 5, 2012

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OH: Romney	82.6%
OK: Santorum	88.6%
ND: Romney	72.1%
AK: Romney	82.6%
TN: Santorum	57.1%
total	872.5%
Expected num correct	**8.725**

- Predictions as of
  2:52 p.m. ET
  Monday, March 5, 2012

DESTINATION 2012 from YAHOO! NEWS

THE SIGNAL

YOUR FRIENDS' ACTIVITY

1-12 of 23

Why we're happy we got one prediction wrong on Super Tuesday

By David Pennock | The Signal – Fri, Mar 9, 2012

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On Super Tuesday, the Signal got 9 out of 10 election predictions correct, following up on an Oscar night where we got 3 of 4 award picks right. Not bad, but we really wish we didn't have those two annoying blemishes, right?

In fact, we're delighted! When the markets are functioning properly, we *expect* to get a certain percentage wrong every time. (The exact amount depends on how certain the probabilities are for each ... ts we actutally expected to get about one wrong ... have meant our confidence was too low, or what ... e, but it's actually possible to get too many

YOU ON YAHOO! NEWS

David Pennock
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- Political Search Trends: The partisanship behind popular Web queries
  19 hrs ago
- NCAA Predictions: Kentucky is favorite to win, East is wide open
- Twitter softens (slightly) on Santorum after Alabama and Mississippi wins
- Santorum's wins knock Gingrich out of the running in Louisiana
- Santorum wins Alabama and Mississippi (and you read it here first)

Conditional probability
"logistic regression"
"combinatorial prediction markets"

Oscar Sunday 2015

Picture	Director	Best Actor	Best Actress	Supporting Actor	Supporting Actress	Adapted Screenplay	Original Screenplay
Birdman 67%	Birdman (Alexandro G Inarritu) 57%	The Theory of Everything (Eddie Redmayne) 84%	Still Alice (Julianne Moore) 100%	Whiplash (J K Simmons) 100%	Boyhood (Patricia Arquette) 100%	The Imitation Game 77%	The Grand Budapest Hotel 74%
Boyhood 32%	Boyhood (Richard Linklater) 43%	Birdman (Michael Keaton) 15%	Gone Girl (Rosamund Pike) 0%	Birdman (Edward Norton) 0%	Birdman (Emma Stone) 0%	Whiplash 20%	Birdman 22%
American Sniper 1%	Foxcatcher (Bennett Miller) 0%	American Sniper (Bradley Cooper) 1%	Wild (Reese Witherspoon) 0%	Boyhood (Ethan Hawke) 0%	The Imitation Game (Keira 0%	The Theory of Everything 2%	Foxcatcher 2%
The Grand Budapest Hotel 0%	The Imitation Game (Morten 0%	The Imitation Game (Benedict 0%	Two Days, One Night (Marion 0%	Foxcatcher (Mark Ruffalo) 0%	Wild (Laura Dern) 0%	American Sniper 1%	Boyhood 1%
Other Movies 0	The Grand Budapest Hotel (Wes Anderson) 0%	Foxcatcher (Steve Carell) 0%	The Theory of Everything (Felicity Jones) 0%	The Judge (Robert Duvall) 0%	Into The Woods (Meryl Streep) 0%	Inherent Vice 0%	Nightcrawler 1%

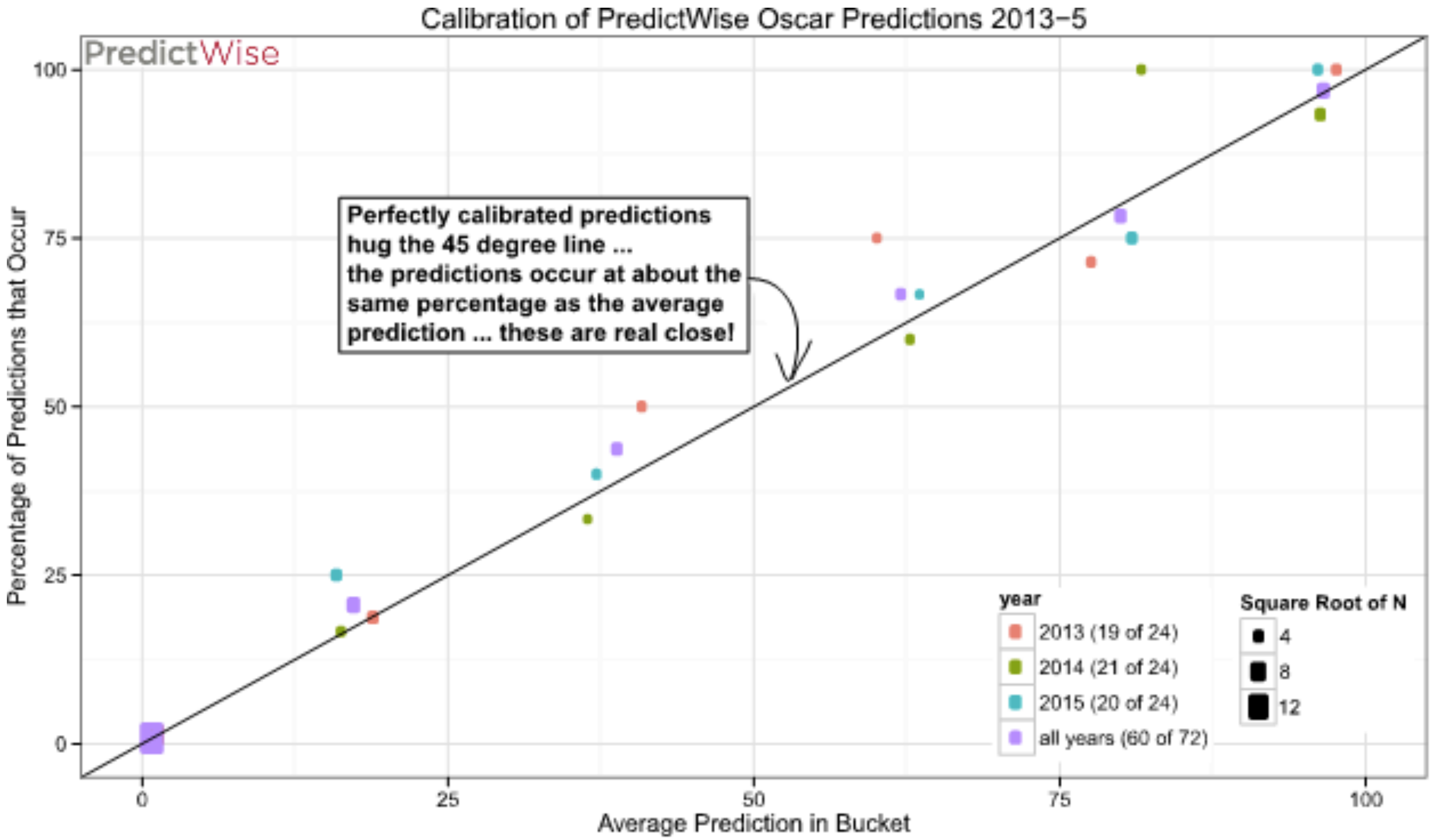
Animated Feature	Animated Short	Foreign Language	Documentary Feature	Documentary Short	Live Action Short	Original Song	Original Score
How to Train Your Dragon 2 83%	Feast 79%	Ida 80%	Citizenfour 91%	Crisis Hotline Vets Press 1 85%	The Phone Call 78%	Selma (Glory) 90%	The Theory of Everything 67%
Big Hero 6 15%	The Dam Keeper 15%	Leviathan 14%	Virunga 8%	Joanna 13%	Boogaloo and Graham 17%	The lego Movie (Everything is Awesome) 7%	The Grand Budapest Hotel 31%
The Tale of Princess Kaguya 1%	The Bigger Picture 5%	Wild Tales 5%	Finding Vivien Maier 2%	Our Curse 1%	Parvaneh 3%	Glen Campell (Im Not Gonna Miss 2%	Interstellar 1%
Song of The Sea 1%	A Single Life 1%	Tangerines 0%	Last Days of 0%	White Earth 1%	Butter Lamp 2%	Begin Again (Lost 1%	The Imitation 1%
The Boxtrolls 0%	Me and My Moulton 0%	Timbuktu 0%	The Salt of the Earth 0%	The Reaper 1%	Aya 0%	Beyond the Lights (Grateful) 0%	Mr Turner 0%

Cinematography	Costume Design	Film Editing	Makeup and Hairstyling	Production Design	Sound Editing	Sound Mixing	Visual Effects
Birdman 96%	The Grand Budapest Hotel 98%	Boyhood 78%	The Grand Budapest Hotel 82%	The Grand Budapest Hotel 96%	American Sniper 85%	Whiplash 43%	Interstellar 88%
The Grand Budapest Hotel 3%	Into The Woods 2%	Whiplash 21%	Guardians of the Galaxy 17%	The Imitation Game 2%	Interstellar 8%	American Sniper 37%	Dawn of the Planet of the Apes 10%
Mr Turner 0%	Maleficent 0%	American Sniper 1%	Foxcatcher 1%	Mr Turner 1%	Birdman 7%	Interstellar 12%	Guardians of the 2%
Unbroken 0%	Mr Turner 0%	[illegible]	[illegible]	[illegible]	[illegible] 1%	Birdman 8%	Captain America Winter Soldier 0%
Ida 0%	Inherent Vice 0%	[illegible]	[illegible]	[illegible]	[illegible] 0%	Unbroken 0%	X-Men Days of Future Past 0%

Feb 22, 2015: Expected **82%**
Feb 23: Got 20/24 right = **83%**

[Source: David Rothschild]

Oscar Sunday 2015

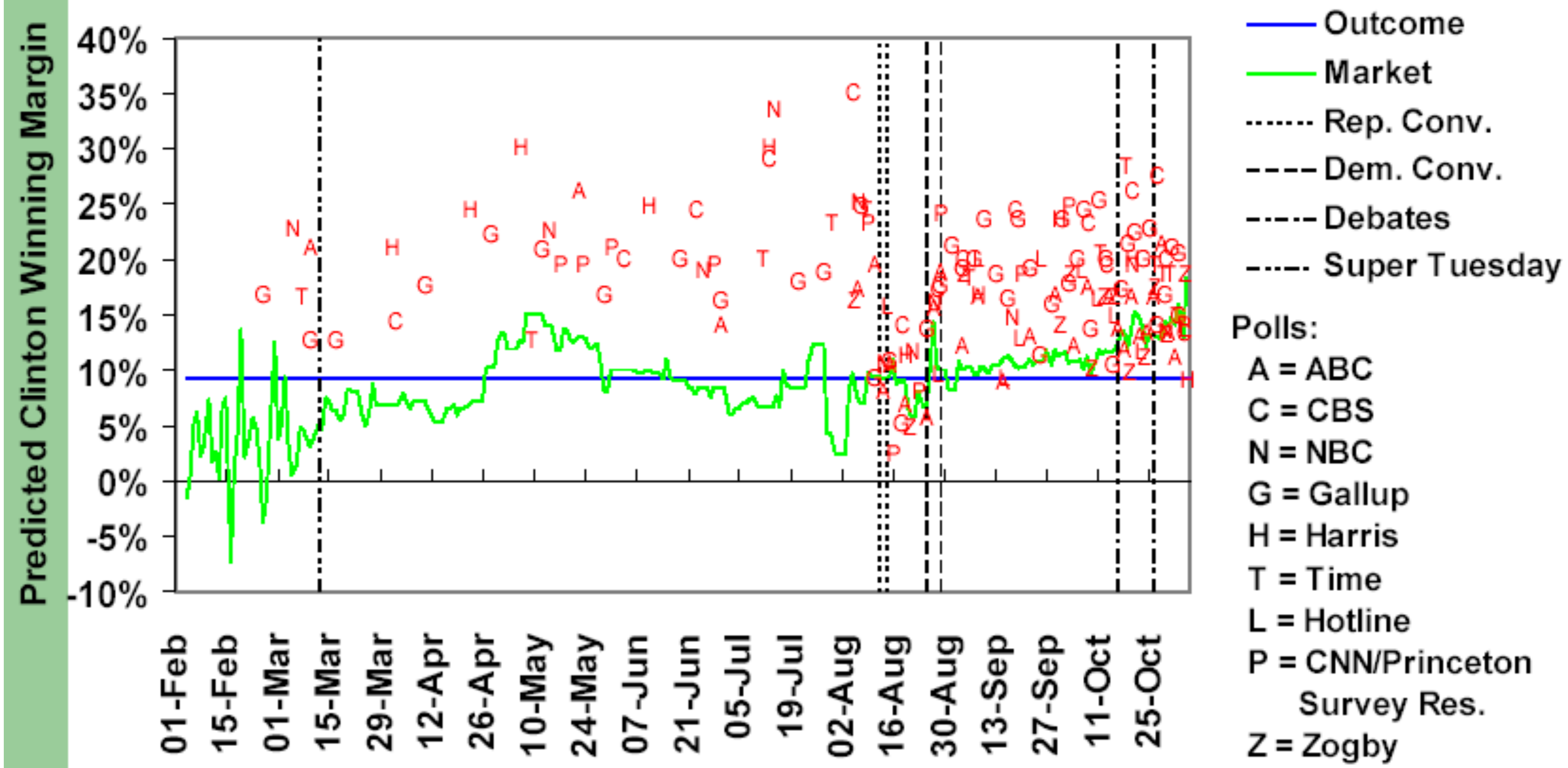


[Source: David Rothschild]

[Source: Berg, DARPA Workshop, 2002]

IEM versus Polls: 1996

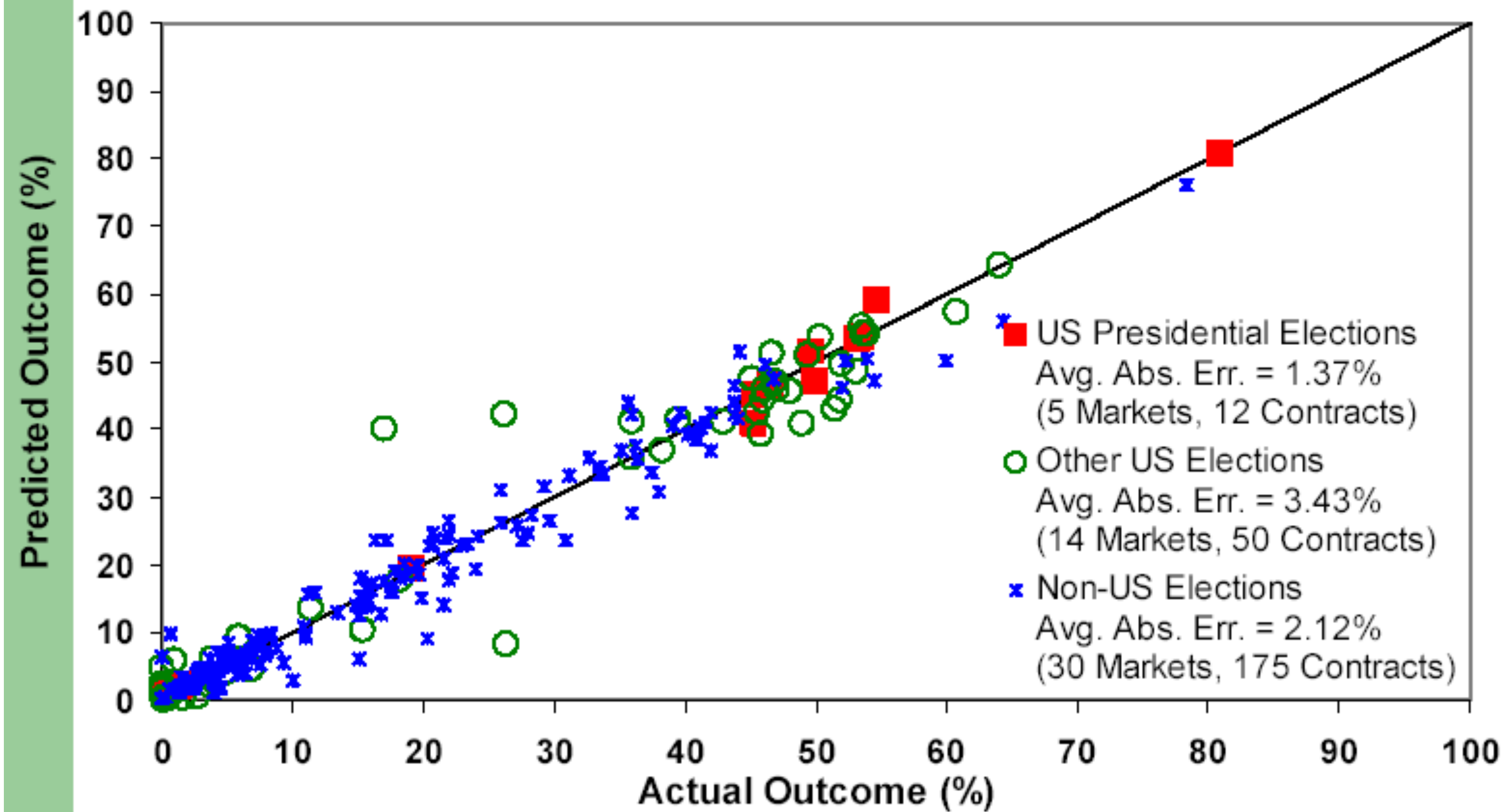
(Berg, Nelson and Rietz, 2001)



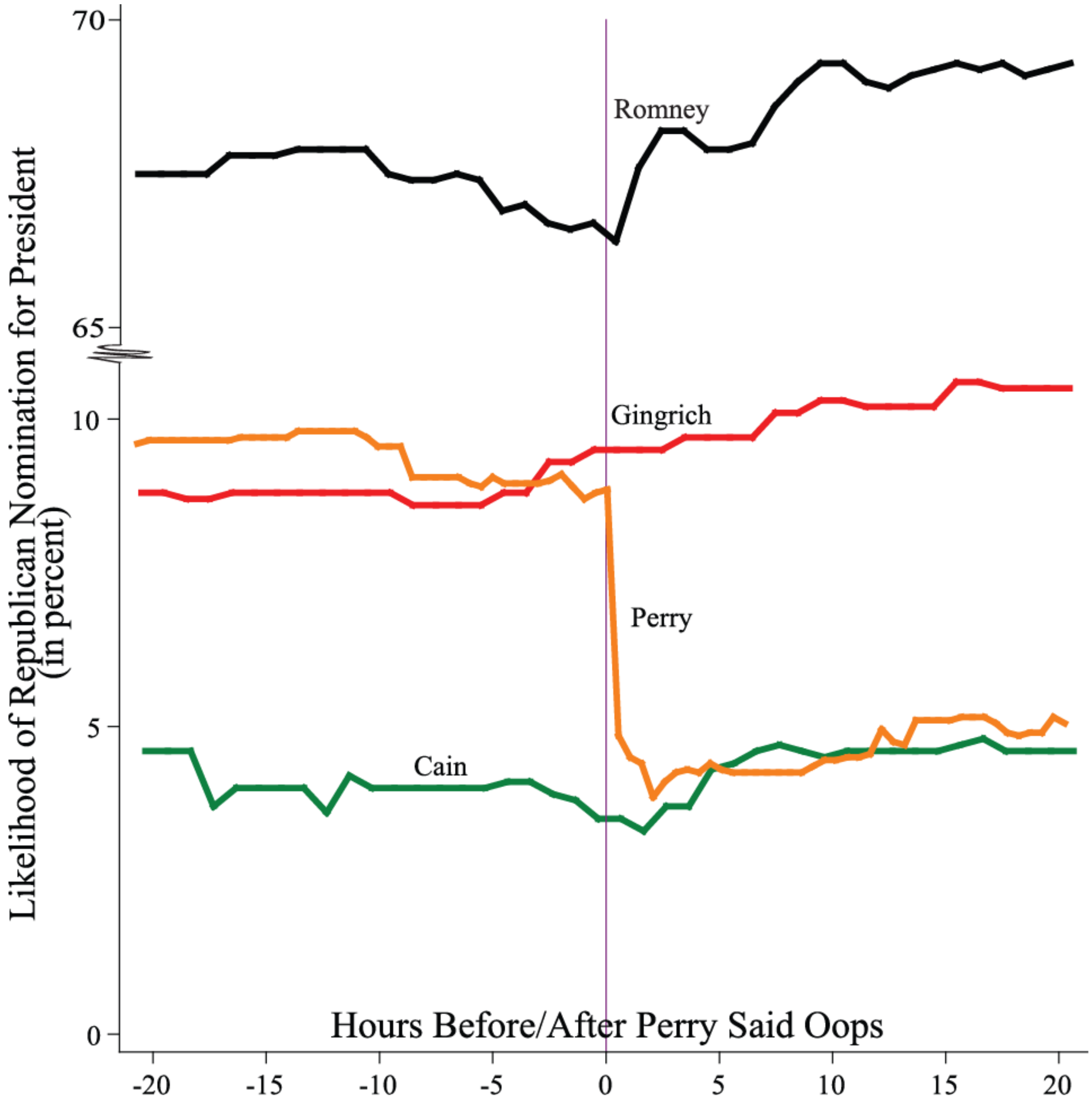
[Source: Berg, DARPA Workshop, 2002]

Predictive Accuracy

Berg, Forsythe, Nelson and Rietz (2001)



Real Time



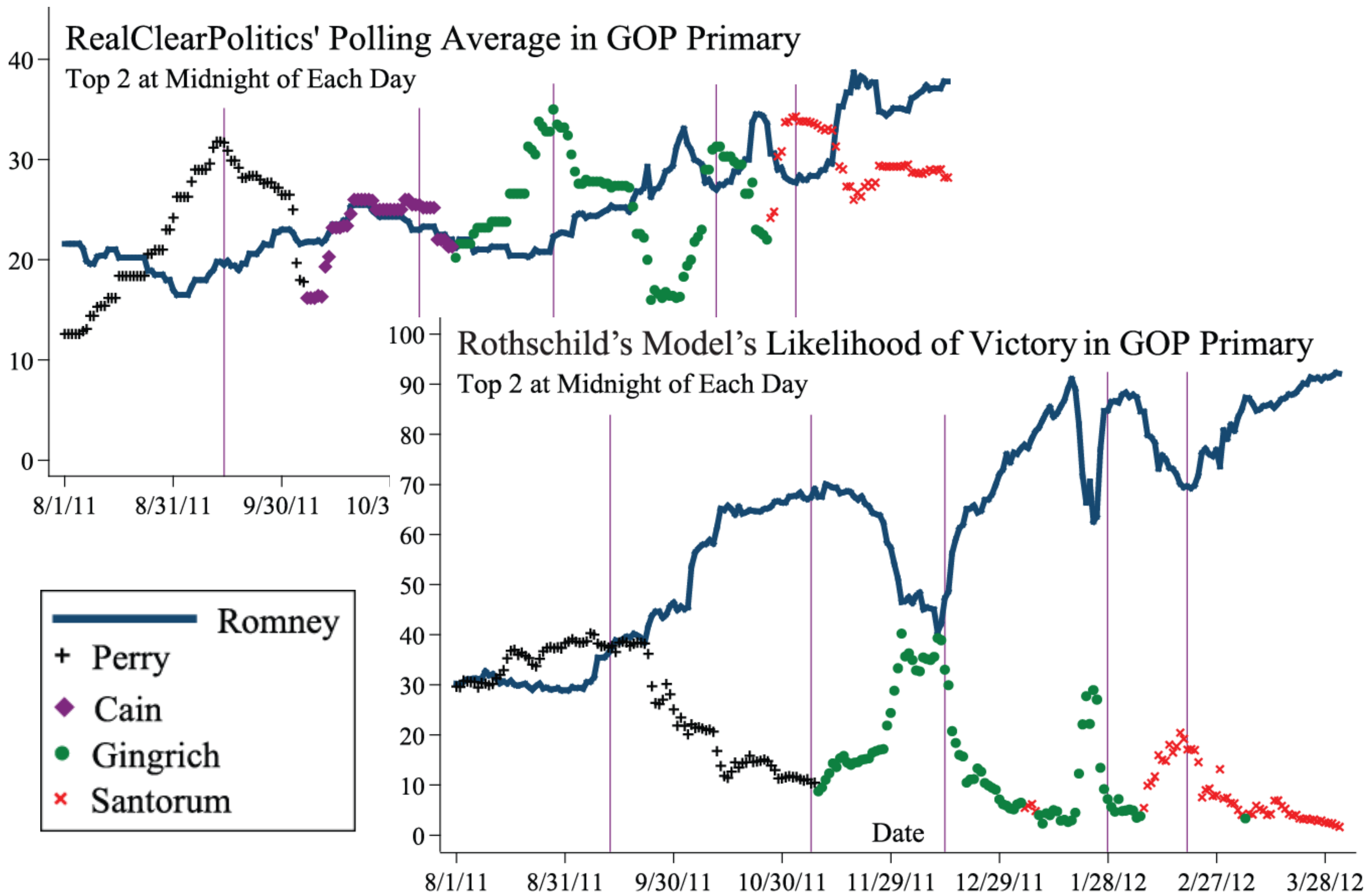
Likelihood of Republican Nomination for President
(in percent)
70
65
10
5
0
Romney
Gingrich
Perry
Cain
Hours Before/After Perry Said Oops
-20
-15
-10
-5
0
5
10
15
20

Real Time

Prediction	Likelihood
VA: Romney	98.7%
MA: Romney	99.6%
VT: Romney	97.5%
ID: Romney	95.7%
GA: Gingrich	98.0%
OH: Romney	82.6%
OK: Santorum	88.6%
ND: Romney	72.1%
AK: Romney	82.6%
TN: Santorum	57.1%
total	872.5%
Expected num correct	**8.725**

- Prior to MI primary, Santorum favored in Ohio: leading polls by wide margin
- Within hours of Romney winning MI, *OH* prediction market swung to Romney
- New York Times Nate Silver's poll-based forecast a couple days behind at least

GOP Primary: Polls vs Markets



Answering ?-mark headlines with data

After Rick Santorum's Sweep, How Worried Should Mitt Romney Be?

By George Stephanopoulos | ABC OTUS News – Wed, Feb 8, 2012

Markets still lukewarm on Santorum after candidate's three-state sweep

By David Rothschild | The Signal – Wed, Feb 8, 2012

[Santorum's] odds of winning the nomination rose slightly, to a non-negligible 11.3 percent likelihood to Romney's 79.5

- Reuters: "The dominant narrative, the day after the presidential election, is the **triumph of the quants**."
- Mashable: "here is the absolute, undoubted winner of this election: Nate Silver and his running mate, big data."
- ReadWrite: "This is about the **triumph of machines and software over gut instinct**"

Conditionals

- Pr(Clinton wins election)
  ≈ Pr(Clinton wins Dem nomination & election)
  = 47.6 - 48.5%
- Pr(Clinton wins Dem nomination)
  = 72.5 - 75.2%
- Pr(election|nomination) =elec&nom/nom
  = **Probability she wins election *if* nominated**
  = 0.48/0.74
  = **63.3 - 66.9%**

Conditionals

- Causes of win
  - Probability she wins election *if* nominated
  - Probability she wins if unemployment <7%
  - Probability she wins if ISIS controls Mosul
- *Effects* of win
  - Unemployment rate if she wins
  - War casualties if she wins
  - Tax rates, S&P, GDP, currency, global temps, consumer confidence, inflation, deficit, ...

Conditionals

– Cancer rates if landfill reduced
– Cancer rates if carbon emissions reduced
– GDP if carbon tax
– global temps if carbon tax
– ...
– “happiness” if X

Decision Markets

- Condition on a *decision* [Hanson 1999]
- Example 1: election
- Example 2: fire CEO?
  Microsoft stock price if CEO fired
  >? =? <?
  Microsoft stock price if CEO not fired
- Guidance to actually make the decision

Correlation ≠ Causation

- In a world where CEO is fired, company likely doing poorly
- In a world where Ron Paul wins Republican nomination, some new development (e.g., Snowden II) may make his general election candidacy especially strong. Different than simply anointing him the nominee.

Thought Experiment = Causation

- If CEO firing is random, conditional does reflect (beliefs about) causation
- If CEO firing is deterministic based on market, incentives get distorted
- As long as CEO firing is partially random, can recover (beliefs about) causation
[Chen et al. 2011]

Prediction Markets

Work for events that are

- Objective
- Verifiable
- Completely describable ("clarity test")
  Harder than it seems
  - Reform party nom
  - Bush v. Gore
  - North Korean missile

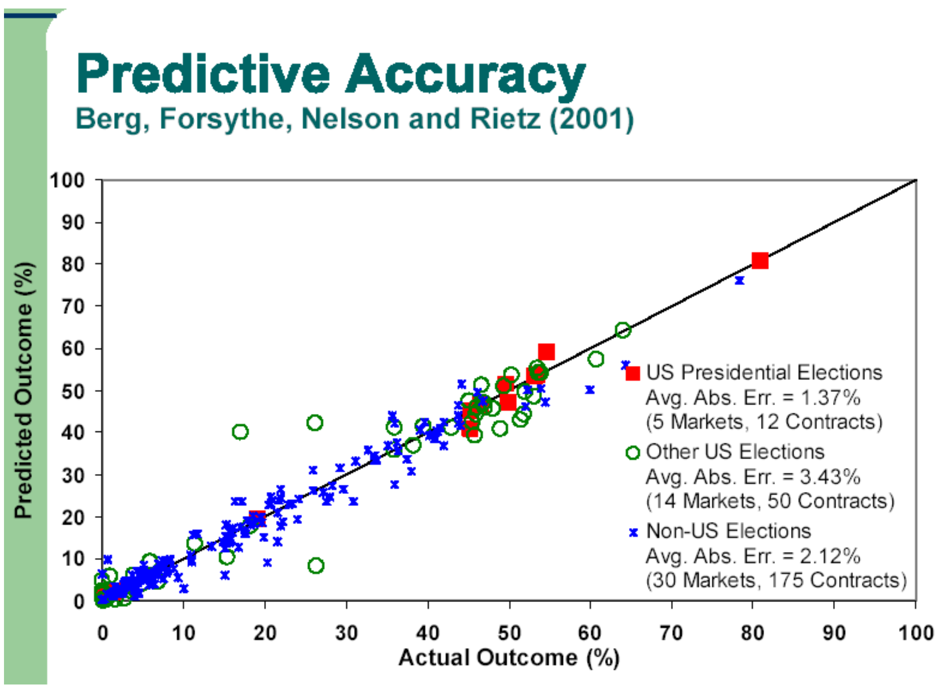
Don't Work for events that are

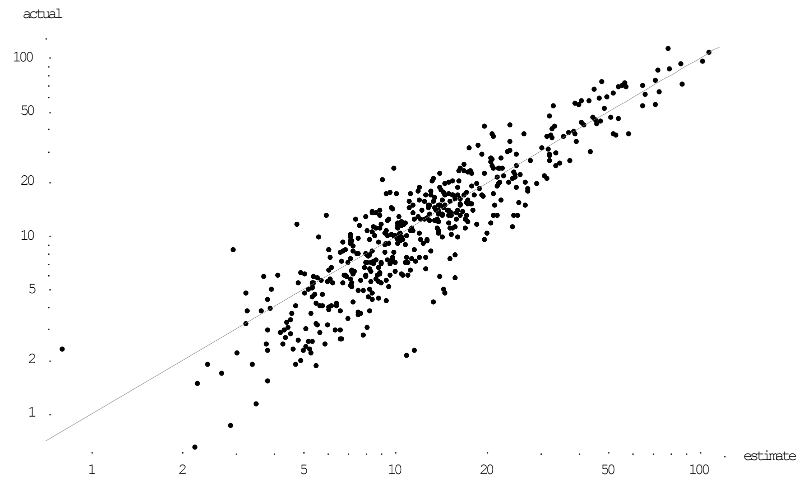
- Subjective (movie, restaurant)
- End of the world
- e.g. "First female US Pres: 2016? 2024? 3000? Never"
- Unverifiable
- Hard to describe
- See: *Peer prediction*
  "Bayesian truth serum"
  And be skeptical

Pay a crowd

With *money* | With *points*

IEM: 237 Candidates | HSX: 489 Movies

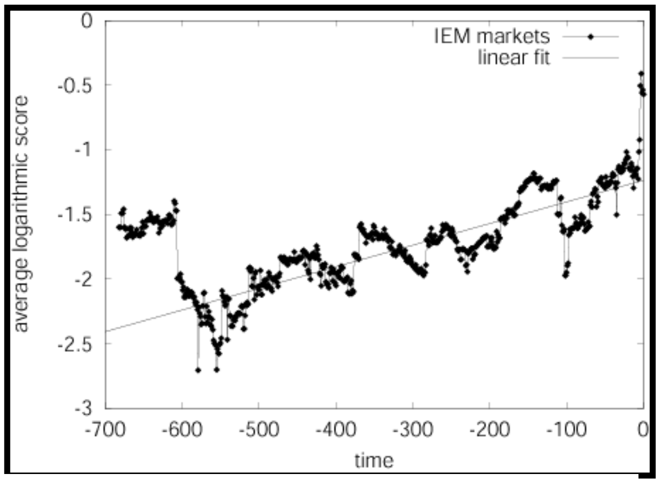


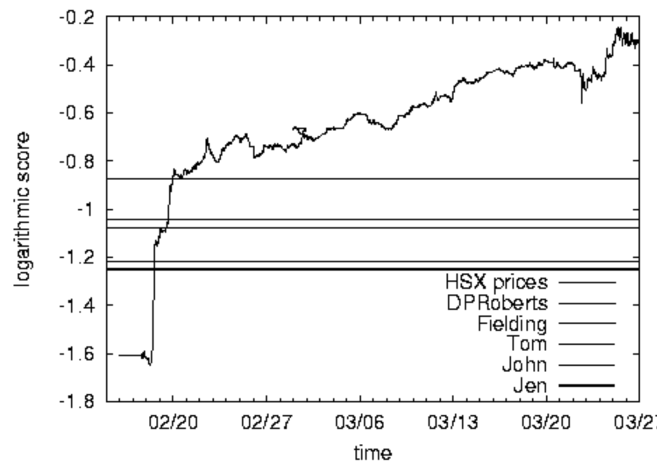


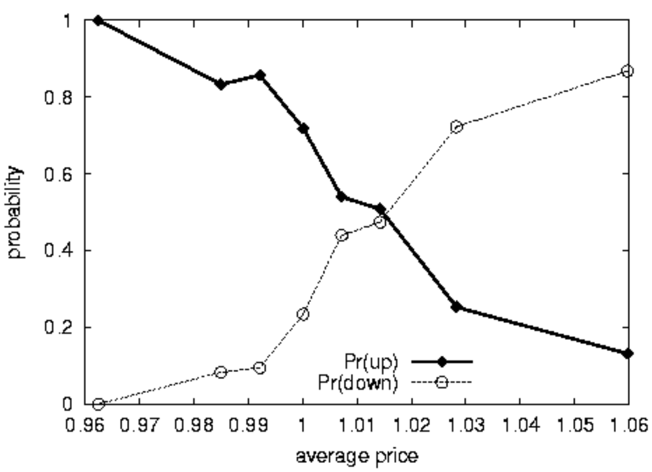
Foresight Exchange, HSX.com, InklingMarkets.com, Microsoft Prediction Lab, CrowdCast, Yahoo! Predictalot, Newsfutures.com, CasualObserver.net, FTPredict.com, ProTrade.com, StorageMarkets.com, TheSimExchange.com, TheWSX.com, Alexadex, Celebdaq, Cenimar, BetBubble, Betocracy, CrowdIQ, MediaMammon,Owise, PublicGyan, RIMDEX, Smarkets, Trendio, TwoCrowds, ...

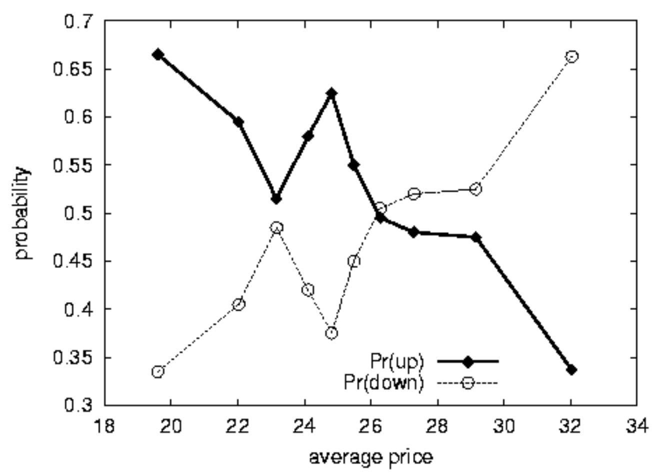
Pay a crowd

With *money* With *points*





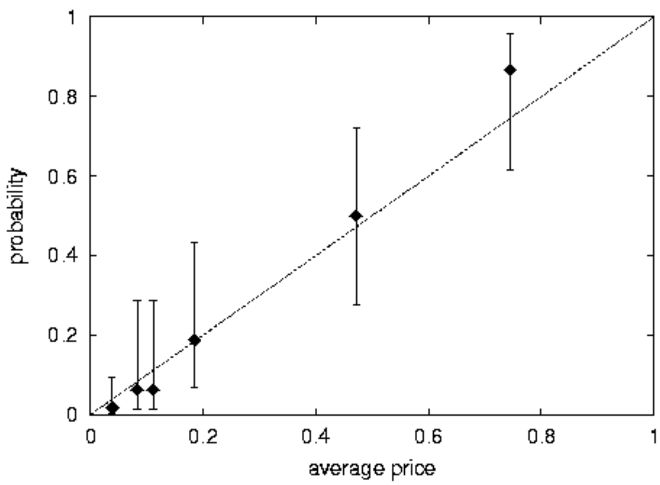




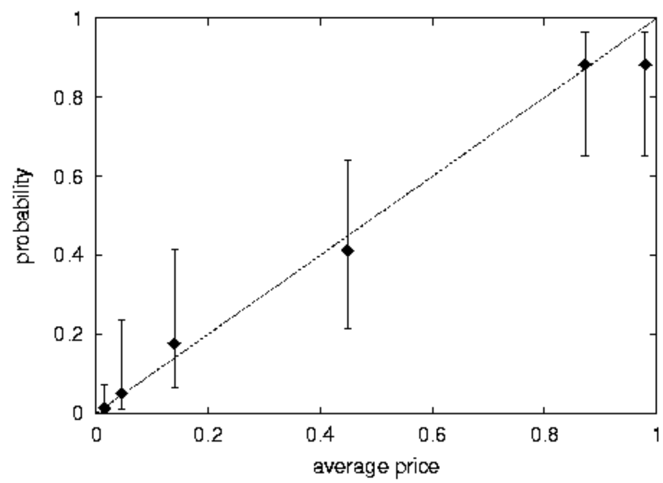
Real markets vs. market games

probabilistic forecasts

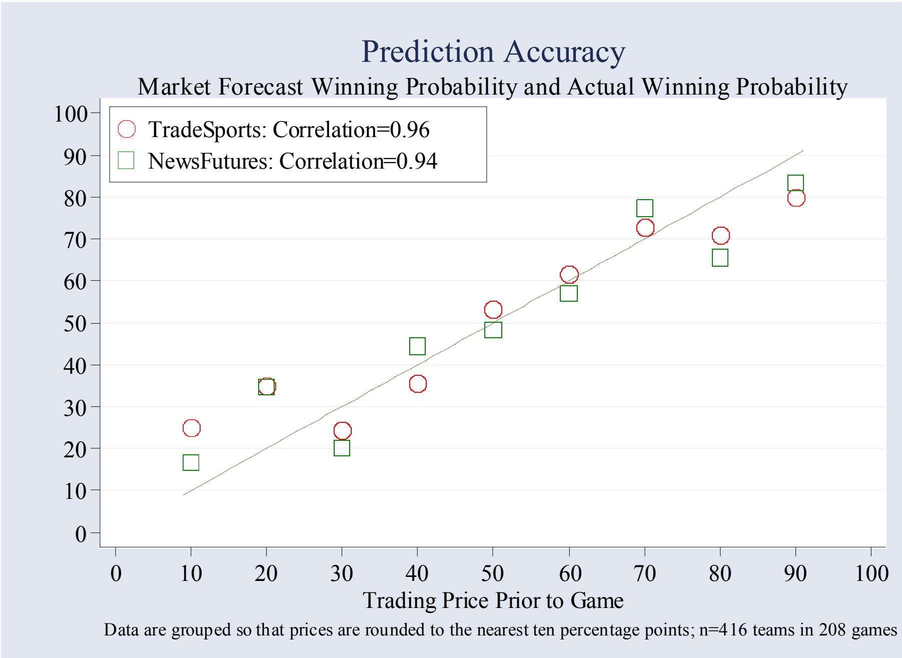
HSX

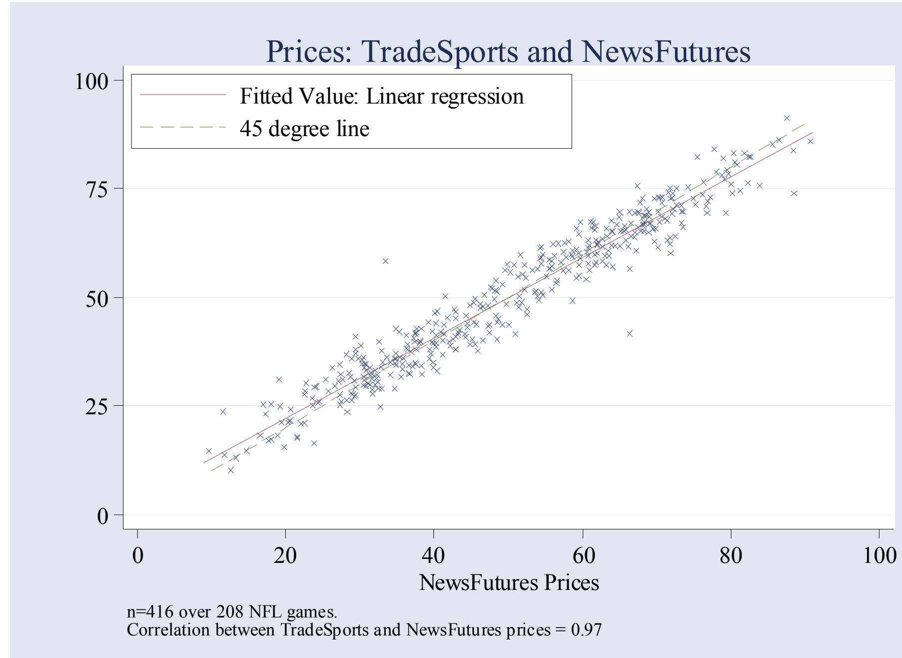


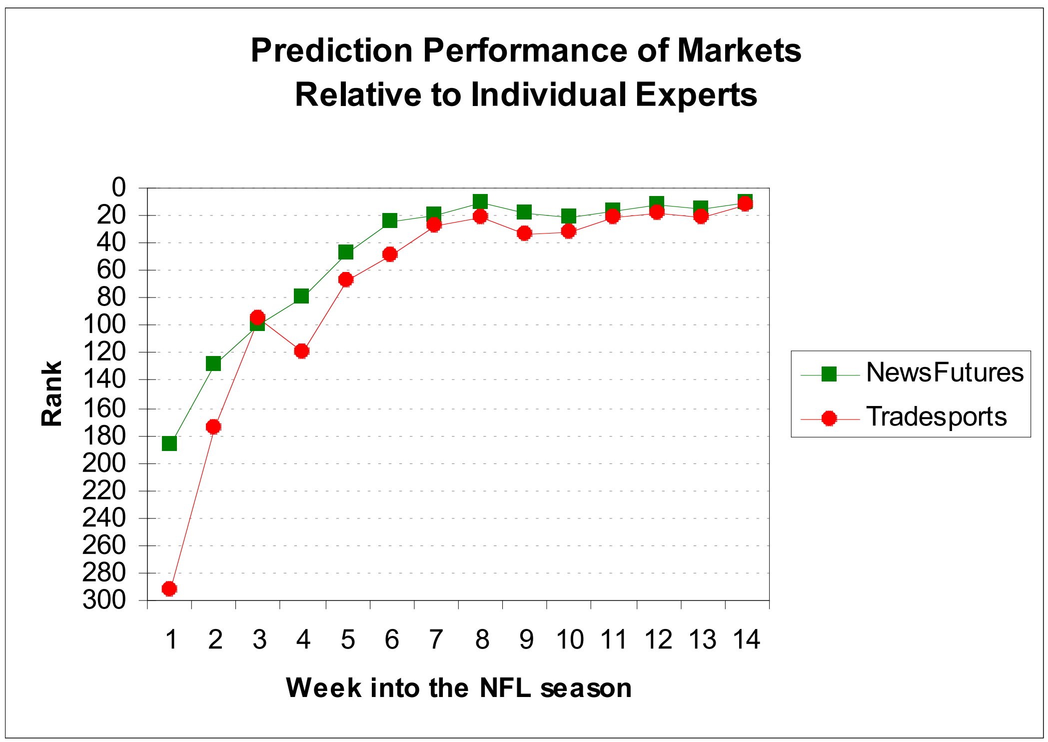
FX, F1P6



forecast source	avg log score
F1P6 linear scoring	**-1.84**
F1P6 F1-style scoring	**-1.82**
betting odds	-1.86
F1P6 flat scoring	-2.03
F1P6 winner scoring	-2.32


Prediction Accuracy
Market Forecast Winning Probability and Actual Winning Probability
TradeSports: Correlation=0.96
NewsFutures: Correlation=0.94
0 10 20 30 40 50 60 70 80 90 100
Trading Price Prior to Game
Data are grouped so that prices are rounded to the nearest ten percentage points; n=416 teams in 208 games


Prices: TradeSports and NewsFutures
Fitted Value: Linear regression
45 degree line
0 25 50 75 100
0 20 40 60 80 100
NewsFutures Prices
n=416 over 208 NFL games.
Correlation between TradeSports and NewsFutures prices = 0.97


Prediction Performance of Markets
Relative to Individual Experts
Rank
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Week into the NFL season
NewsFutures
Tradesports

Combinatorial Predictions

- 22% chance Romney will win in Iowa but Obama will win the national election
- 75.7% chance the same party will win both Michigan and Ohio
- 48.3% chance Obama gets 300 or more Electoral College votes
- 12.3% chance Obama will win between 6 and 8 states that begin with the letter M
- Path of blue from Canada to Mexico

Some Counting

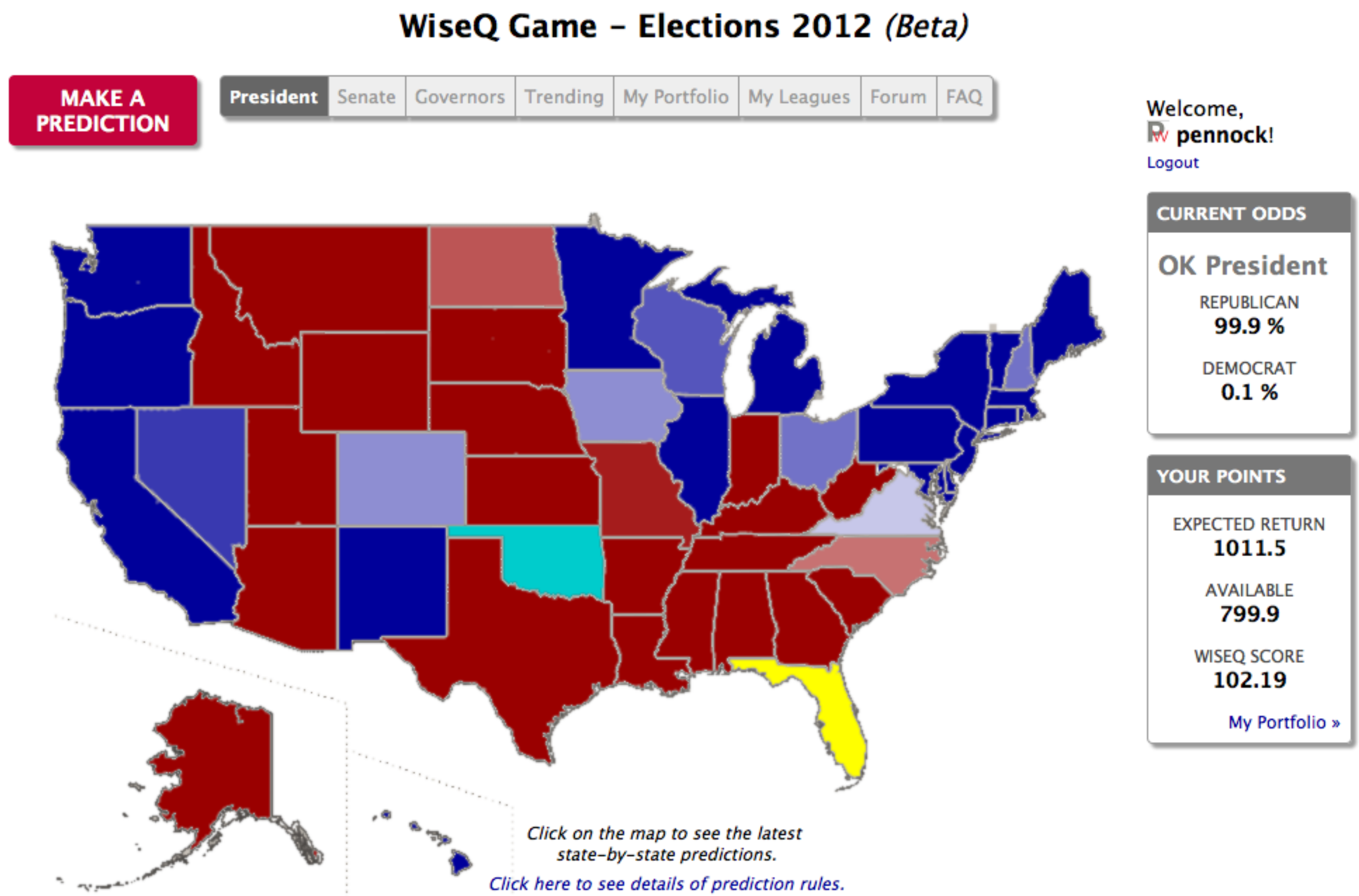
- 54 “states”: 48 + DC + Maine (2), Nebraska (3)
- 2^{54} = 18 quadrillion possible outcomes
- $2^{2^{54}} \sim 10^{18008915383333485}$ distinct predictions
  More than a googol, less than a googolplex
- NOT independent

Real Conditionals:
How pivotal was Ohio?

- Day after 1st debate (October 4)
- If Obama wins Ohio, 90% he'll win the election
- If Obama wins Florida, 95%; Virginia, 92%
- If Romney wins Ohio, 41% he'll win election; Florida, 32%; Virginia 31%

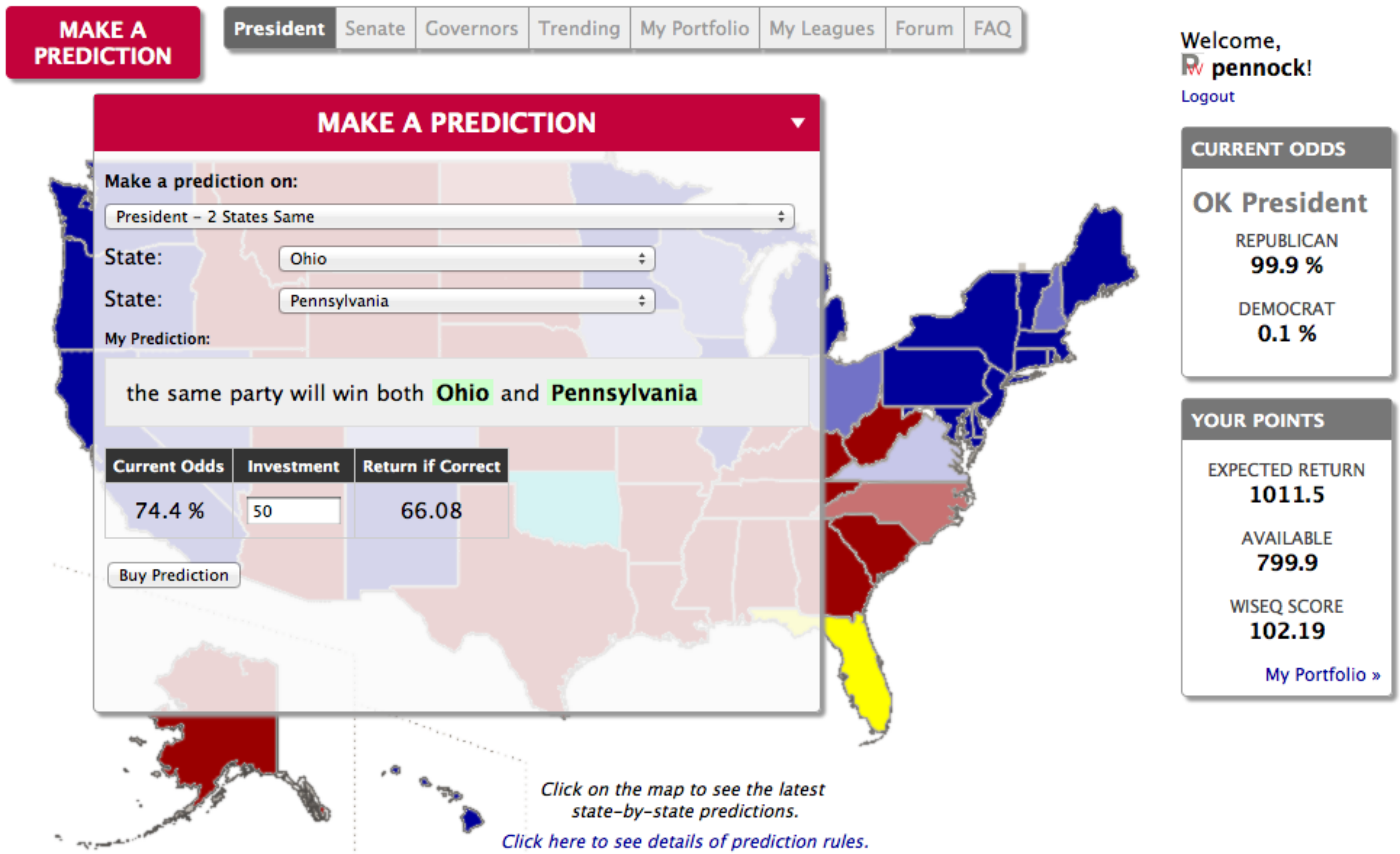
PredictWiseQ

- In theory, combo PM yields billions more predictions "for free". What about in practice?
- A combinatorial prediction market for the election: **http://PredictWiseQ.com**
- Fully working beta & field test
  - Sep 16 – Nov 6, 2012
  - 680 users; 437 made at least one trade
  - 3137 trades in all 23 possible prediction categories
  - 514 distinct security types were bought
    261 of these were traded by a unique user
  - Empirical paper in ACM EC'13
  - Market maker algorithm details in ACM EC'12



http://PredictWiseQ.com

WiseQ Game – Elections 2012 *(Beta)*



http://PredictWiseQ.com

Why Expressiveness?

- Dem Pres, Dem Senate, Dem House
  Dem Pres, Dem Senate, GOP House
  Dem Pres, GOP Senate, Dem House
  Dem Pres, GOP Senate, GOP House...
- Dem Pres
  Dem House
  Dem wins >=270 electoral votes
  Dem wins >=280 electoral votes...

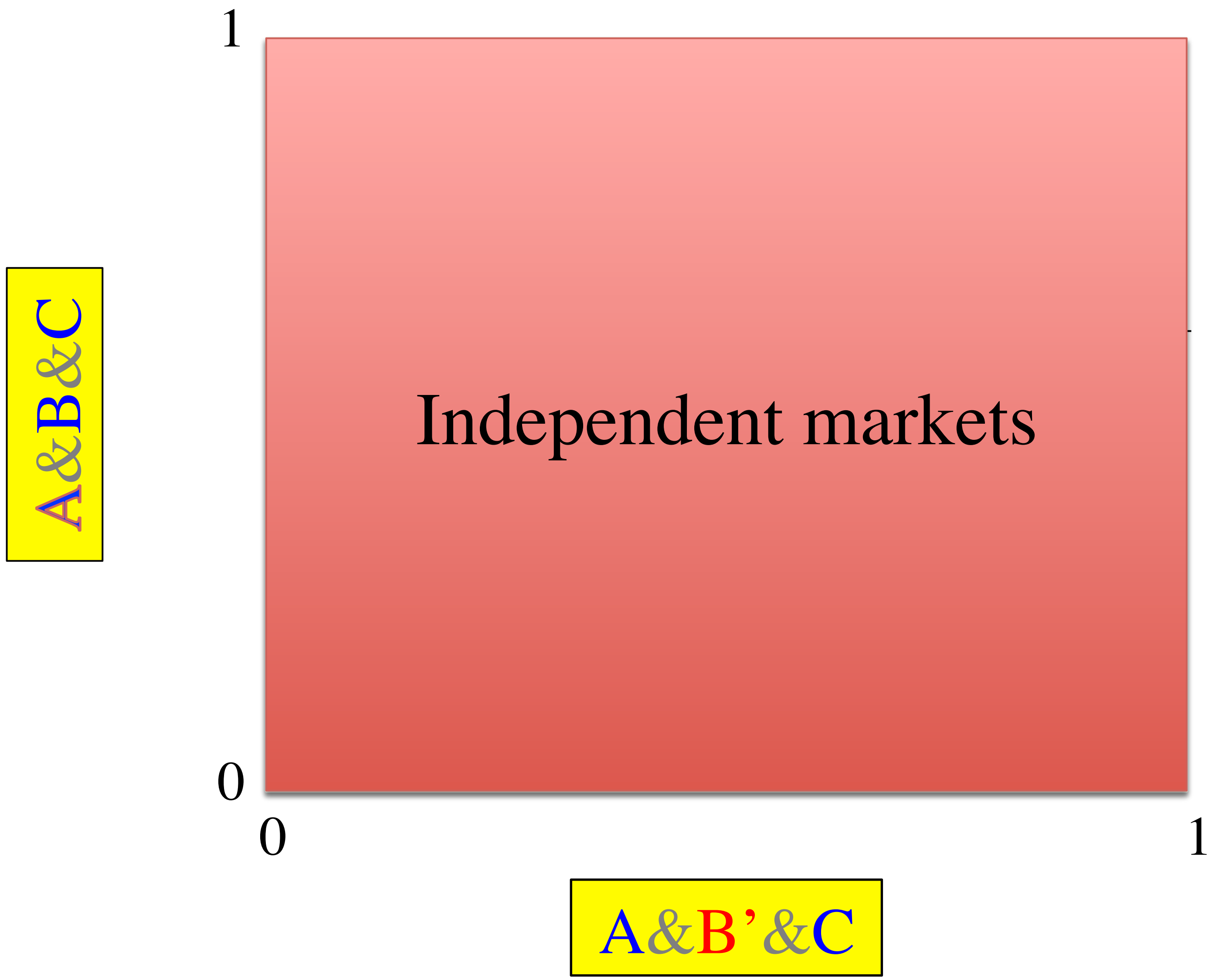
Standard vs Combinatorial

- Industry standard: Ignore relationships
  Treat them as independent markets
- Las Vegas sports betting
  Kentucky horseracing
  Wall Street stock options
  High Street spread betting

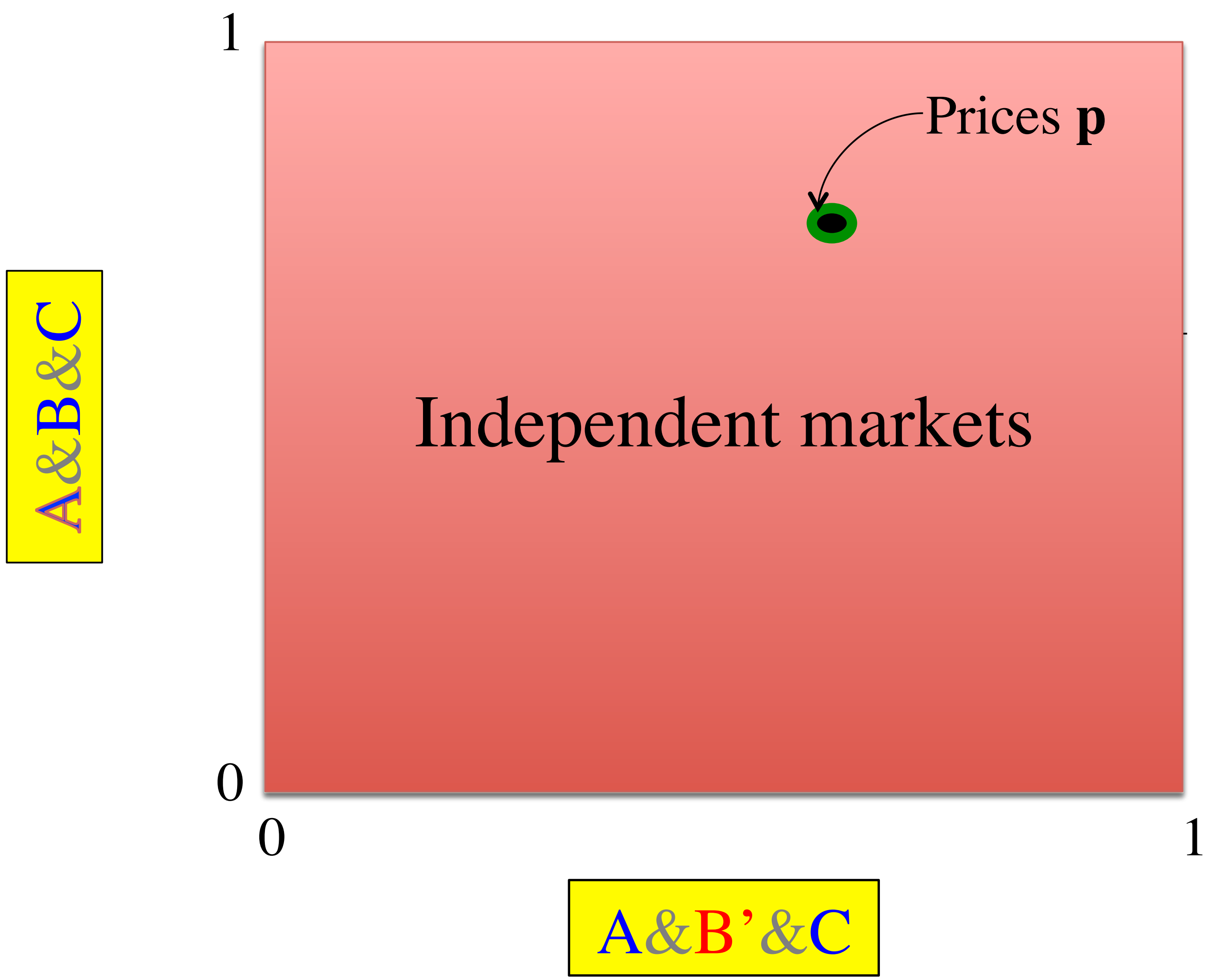
Las Vegas	sports betting
Kentucky	horseracing
Wall Street	stock options
High Street	spread betting

- Combinatorial
  - Automatic information propagation: reward traders for information, not computational horsepower

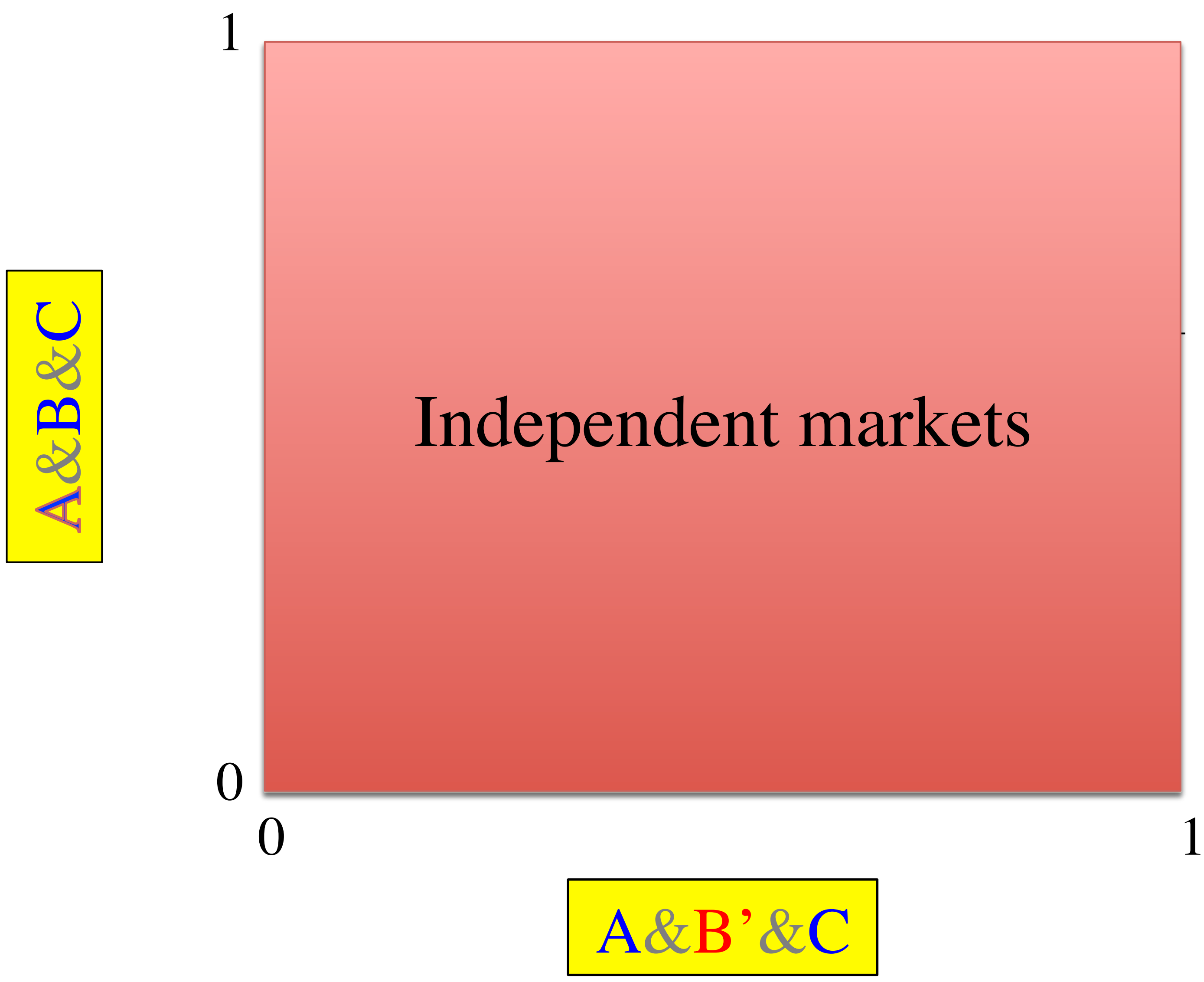
Consistent pricing



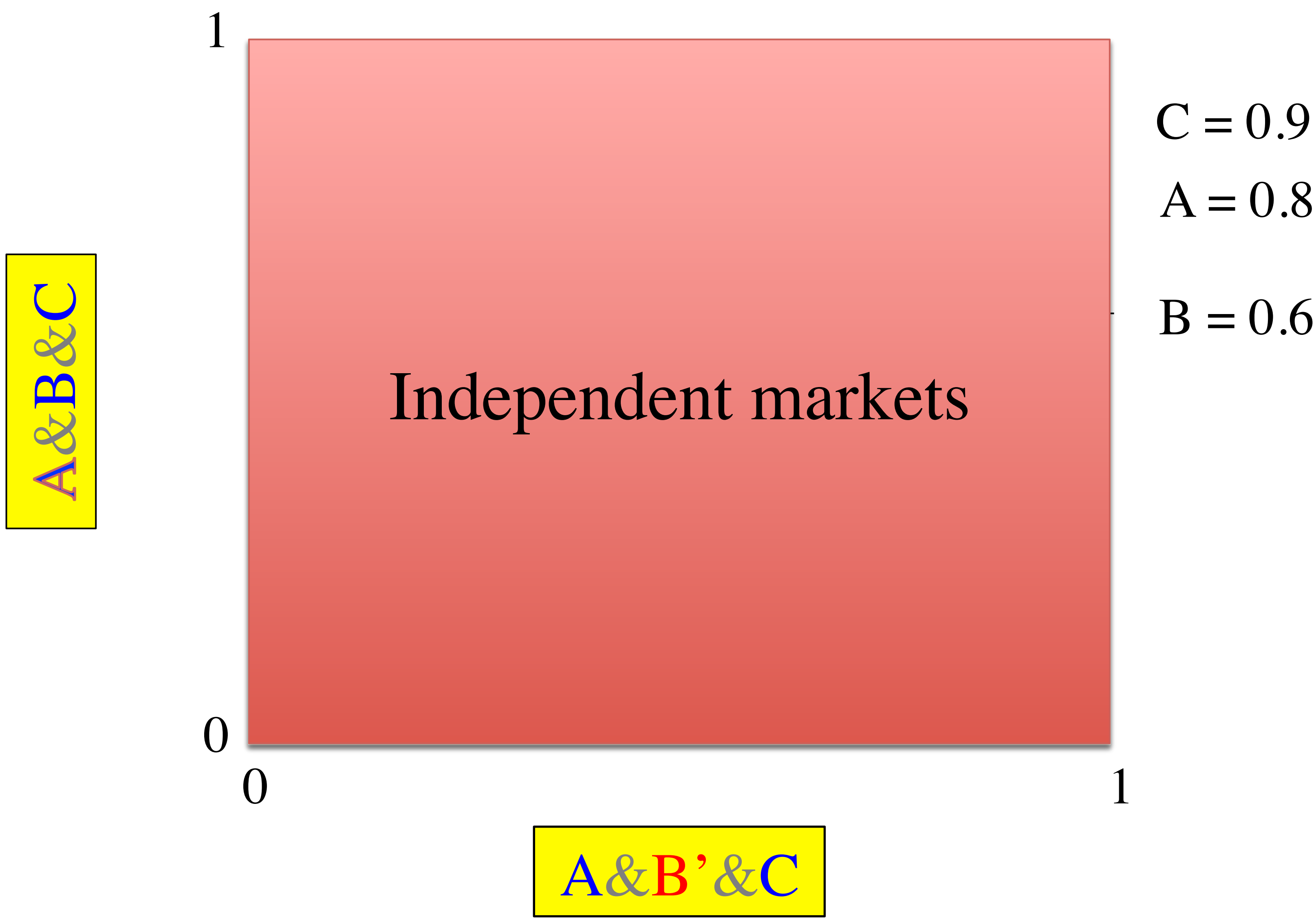
Consistent pricing



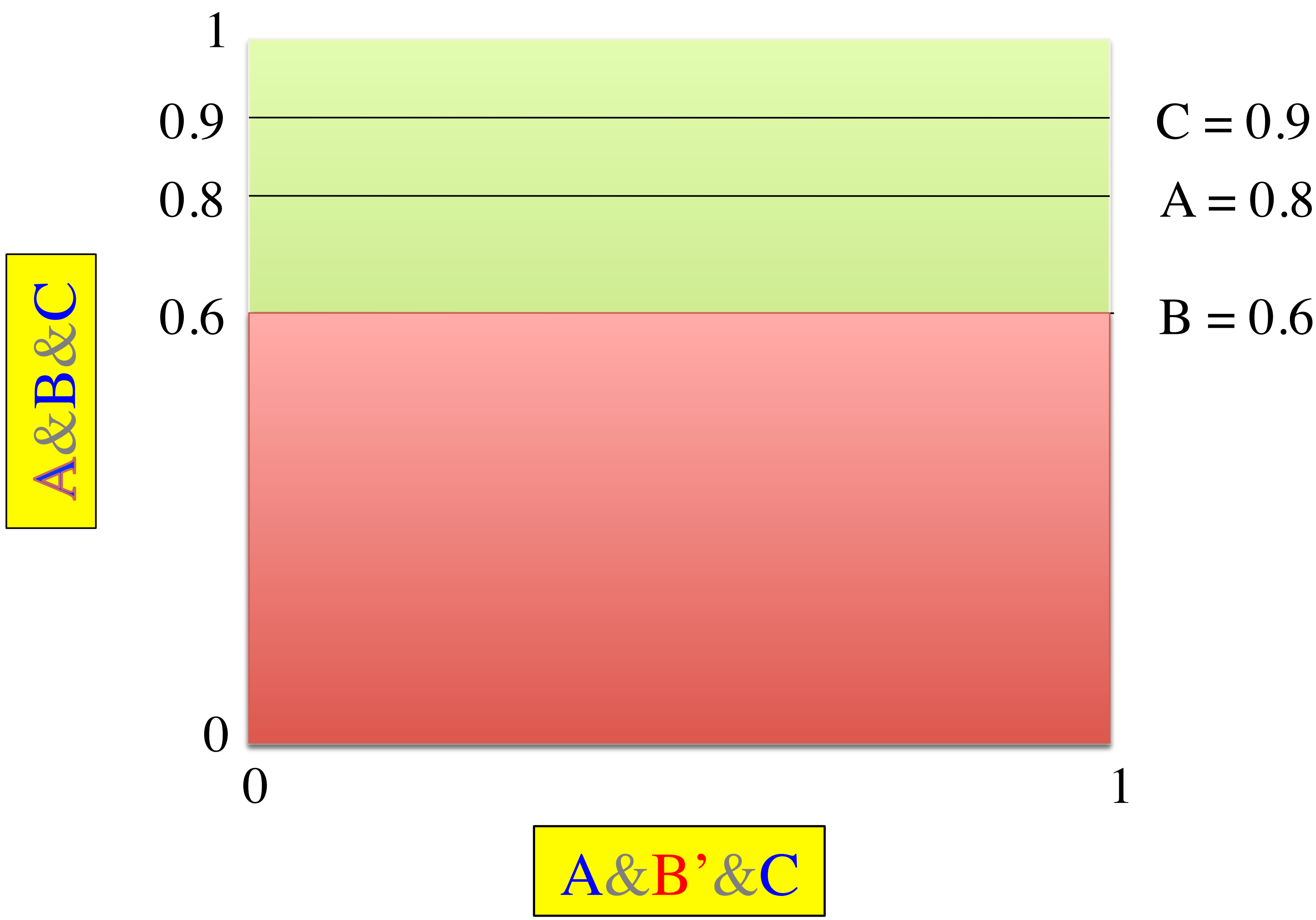
Consistent pricing



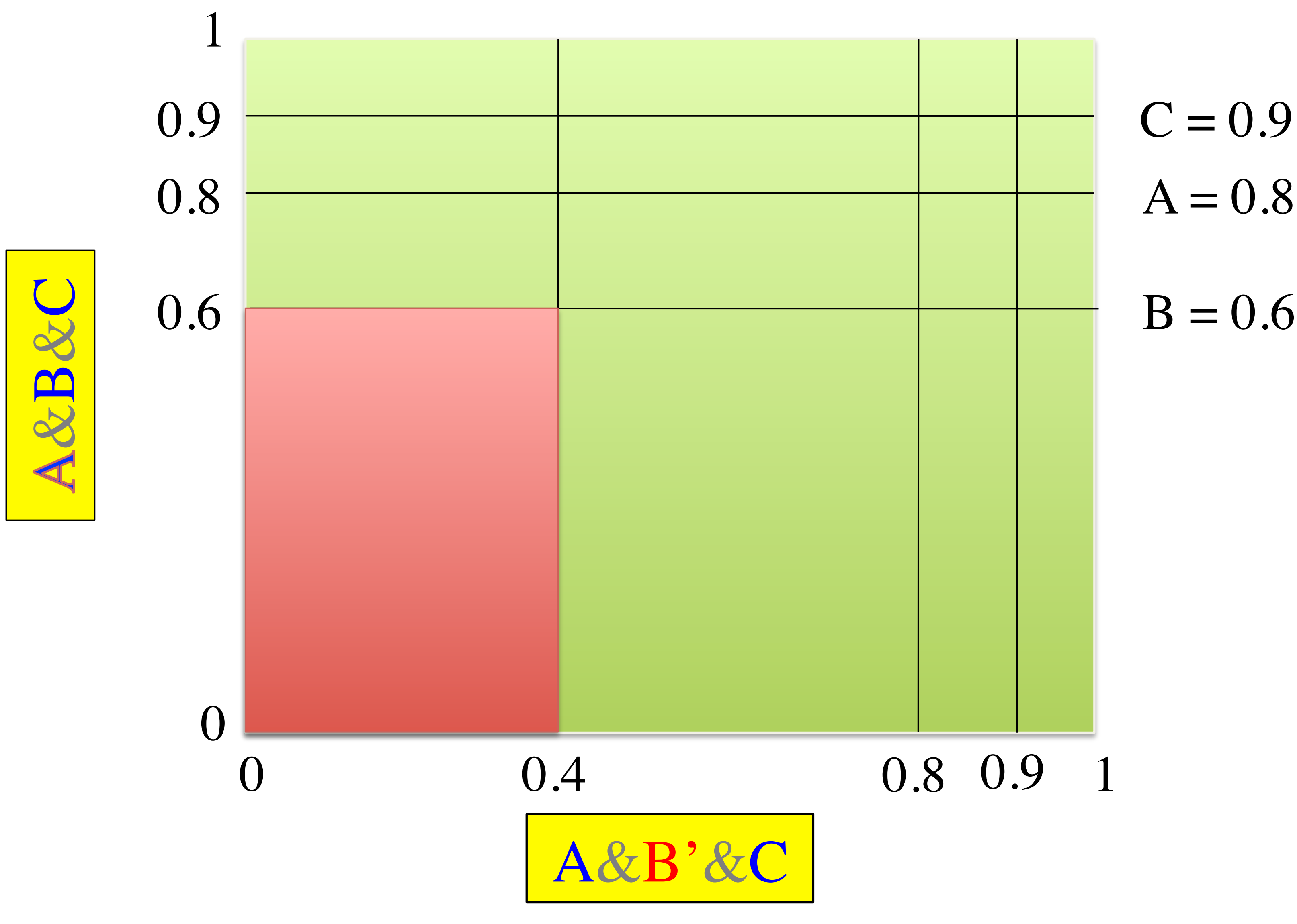
Consistent pricing



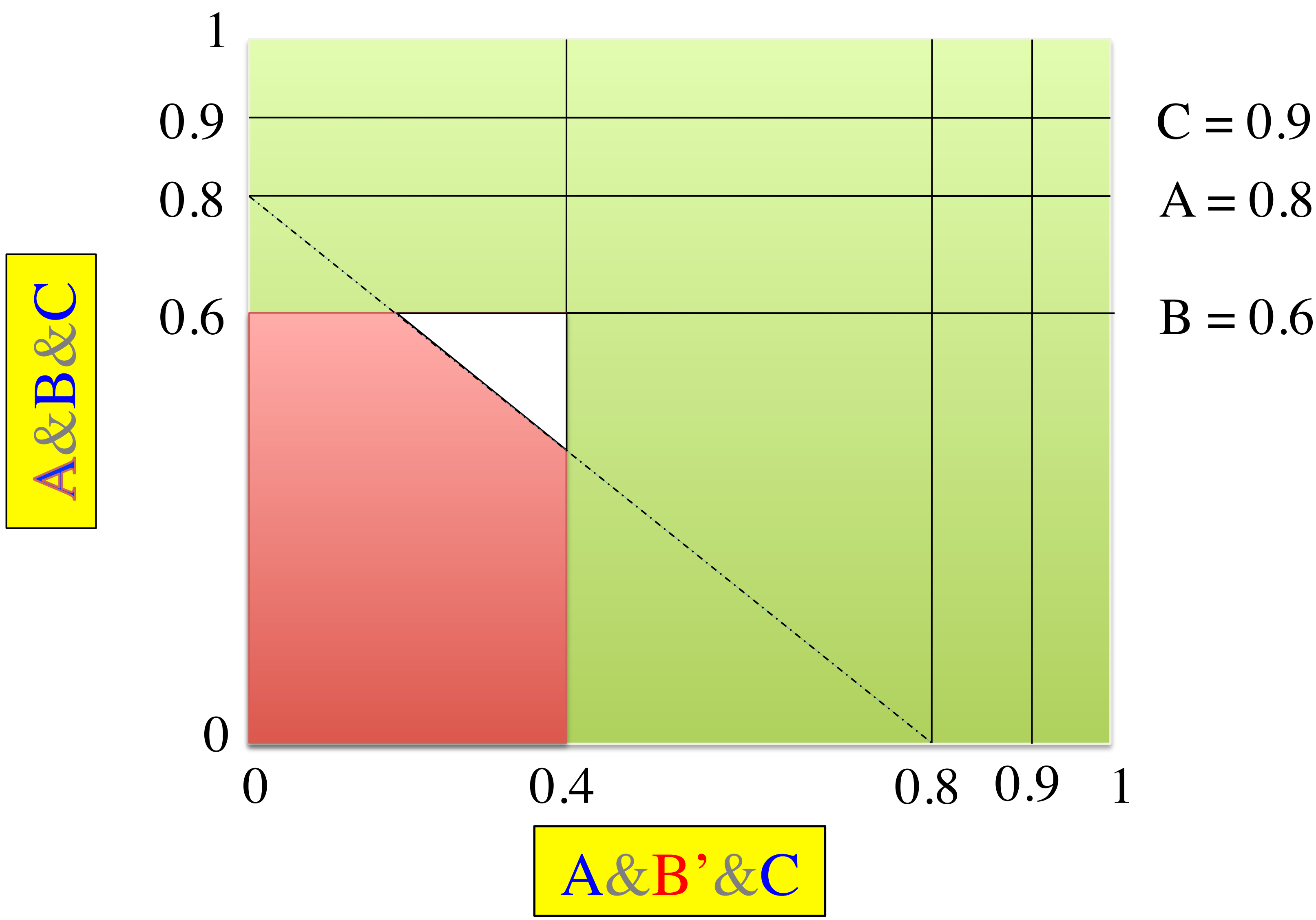
Consistent pricing



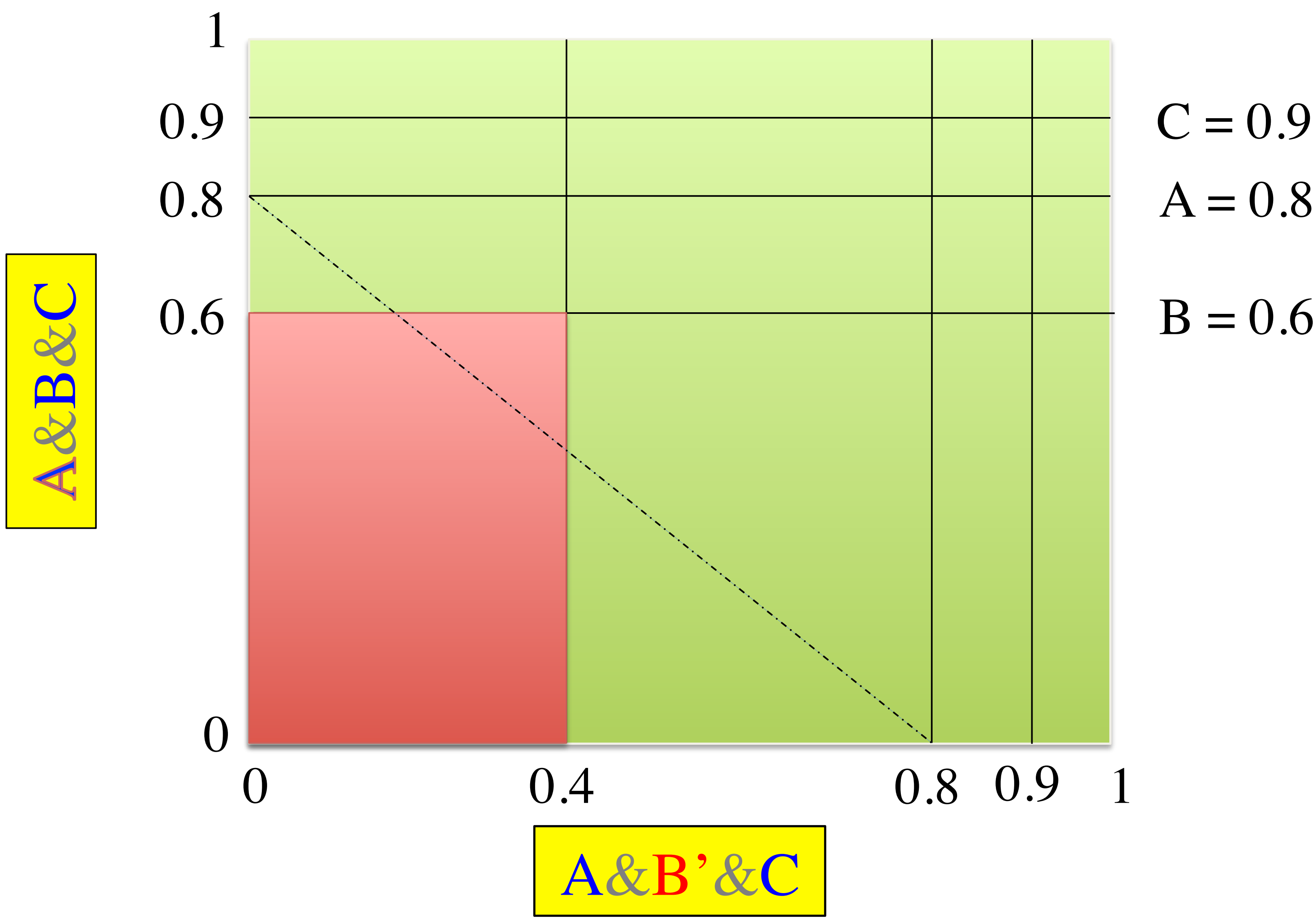
Consistent pricing



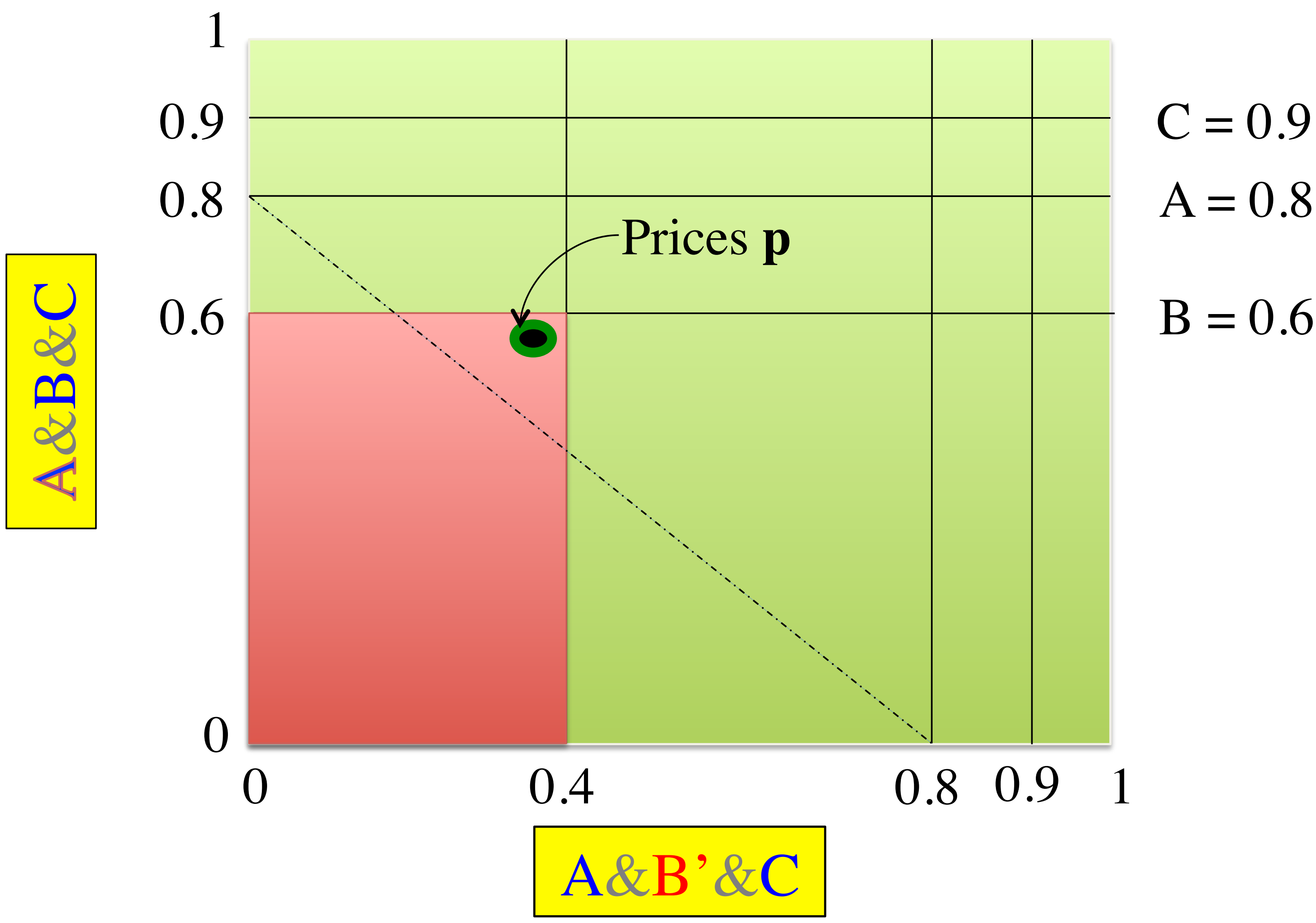
Consistent pricing



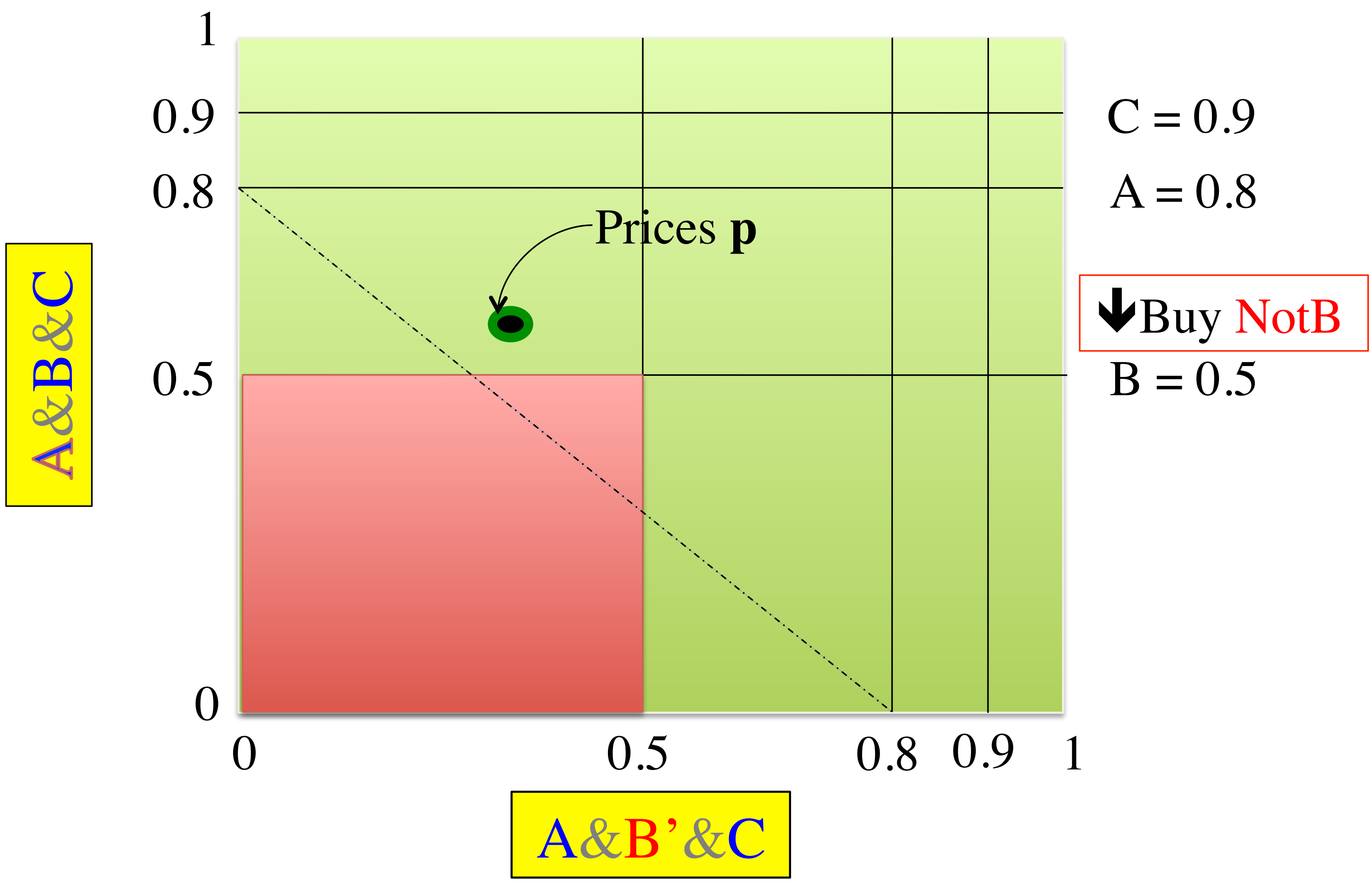
Approximate pricing



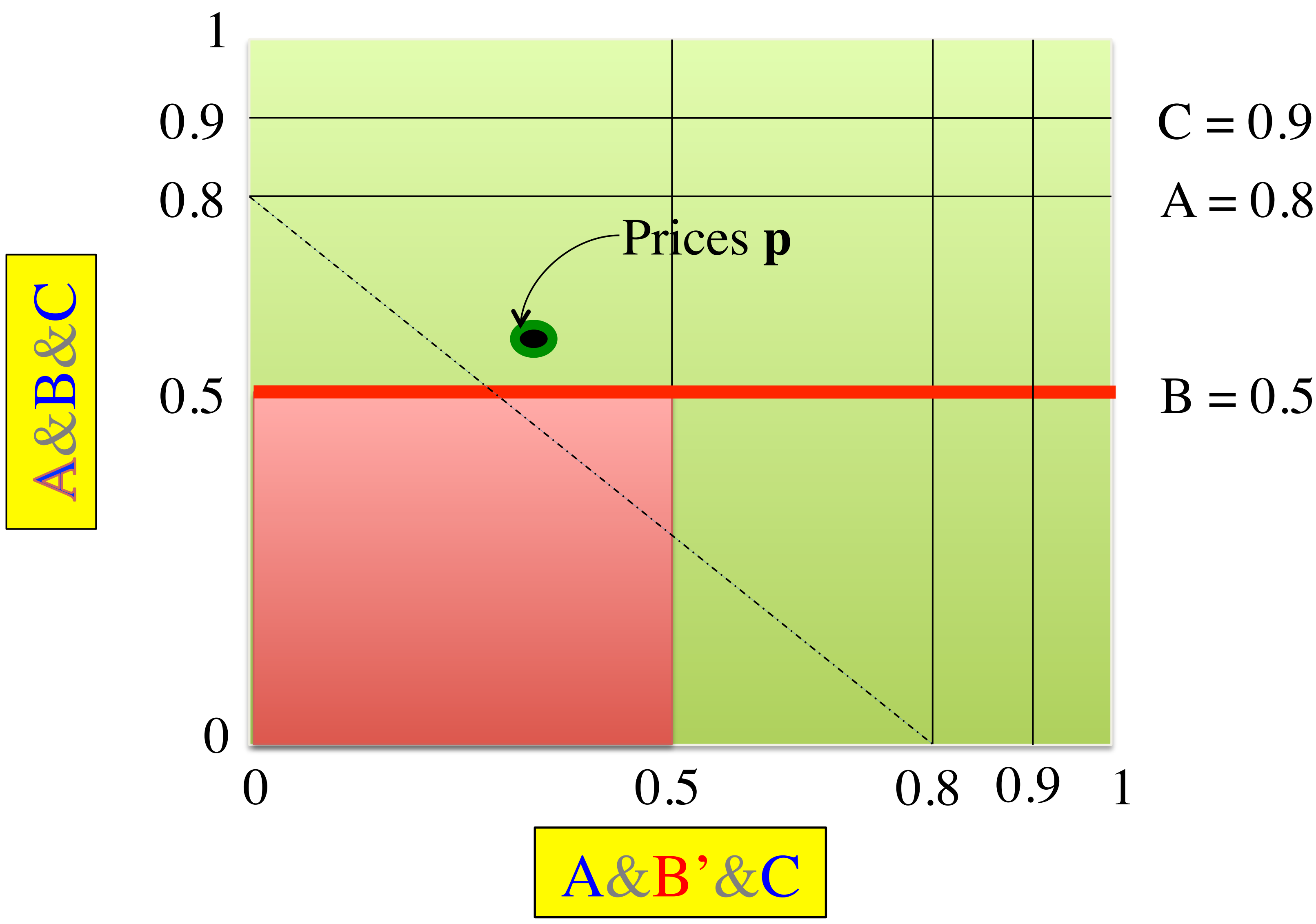
Approximate pricing



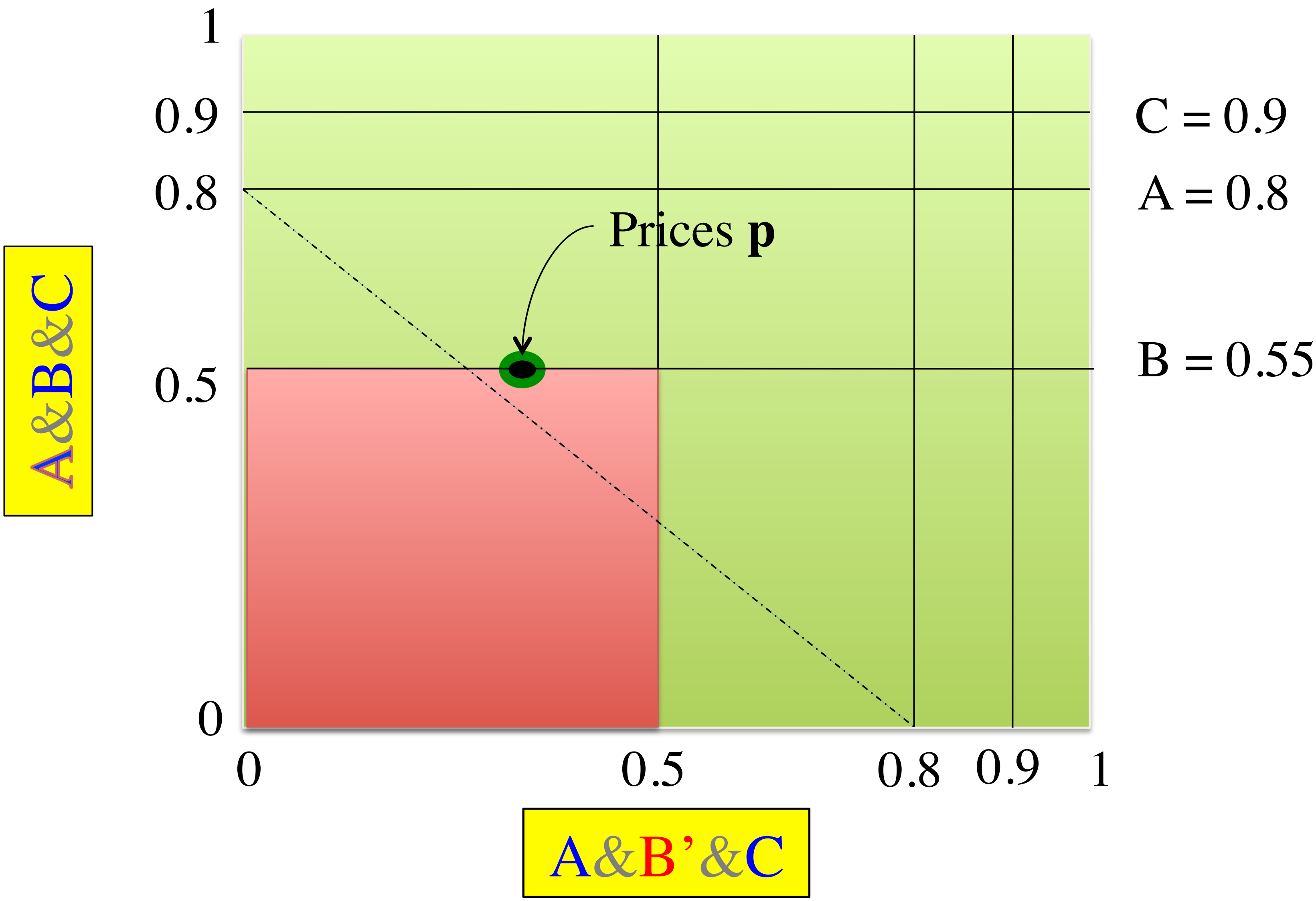
Approximate pricing



Approximate pricing

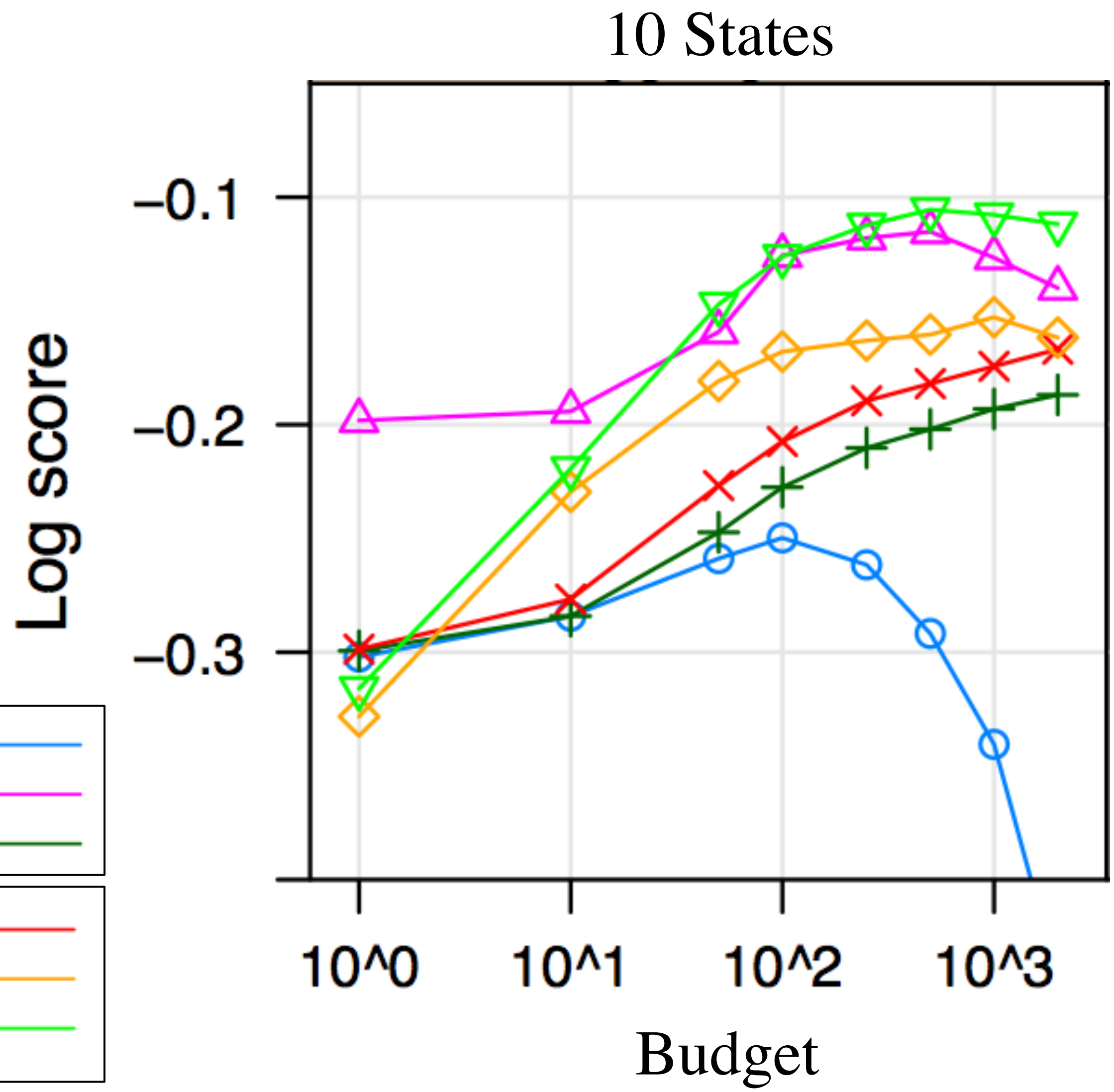


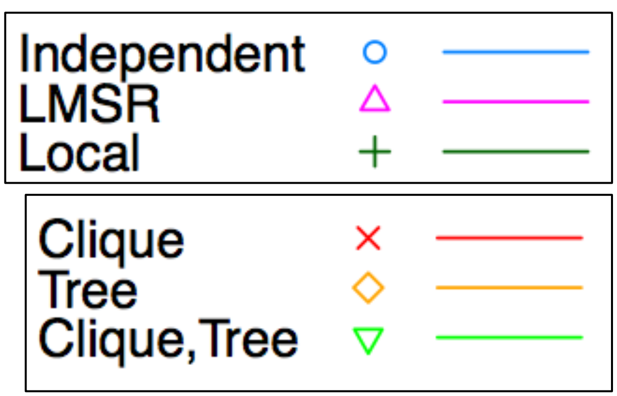
Approximate pricing



Does it work?

Tested on over 300K complex predictions from Princeton study





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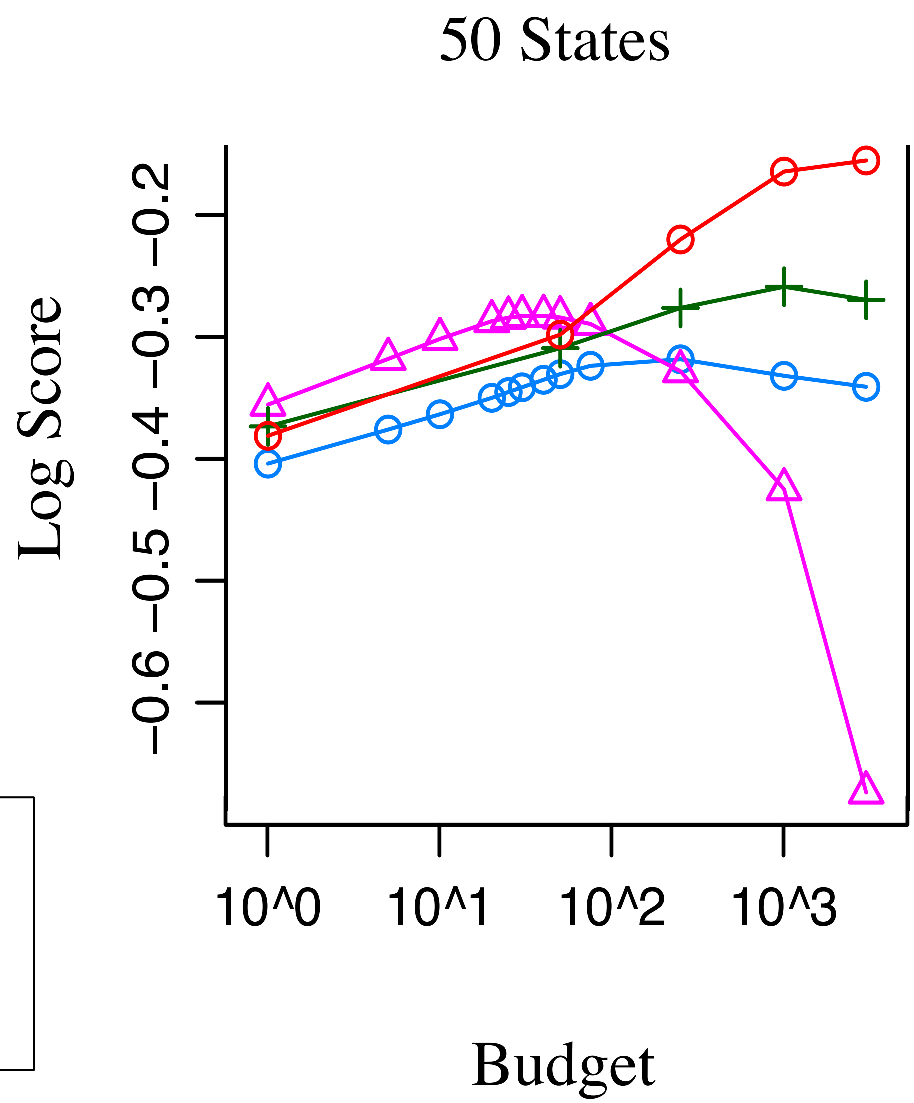
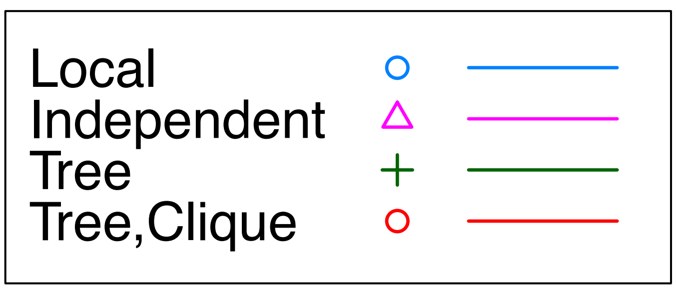
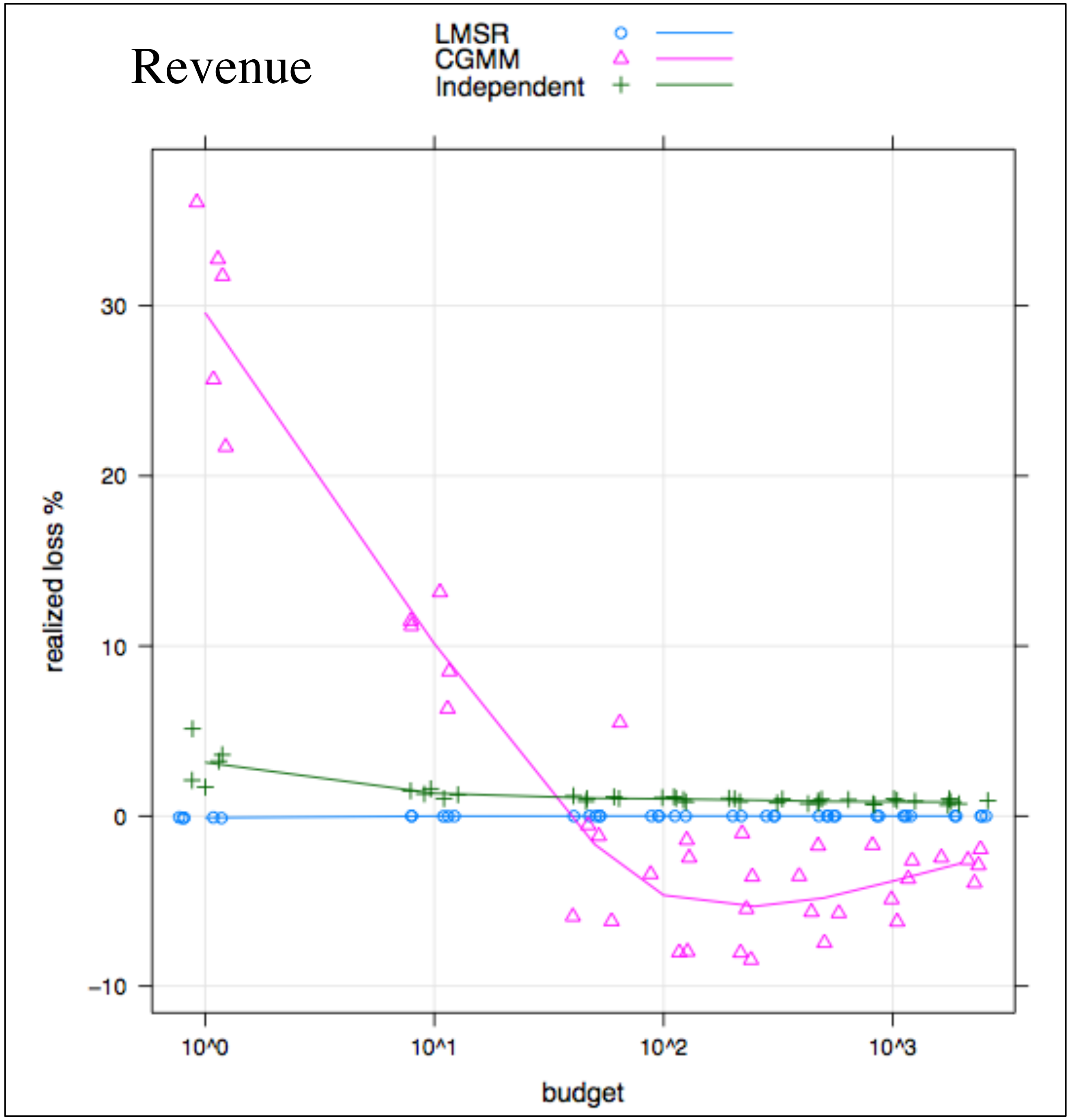


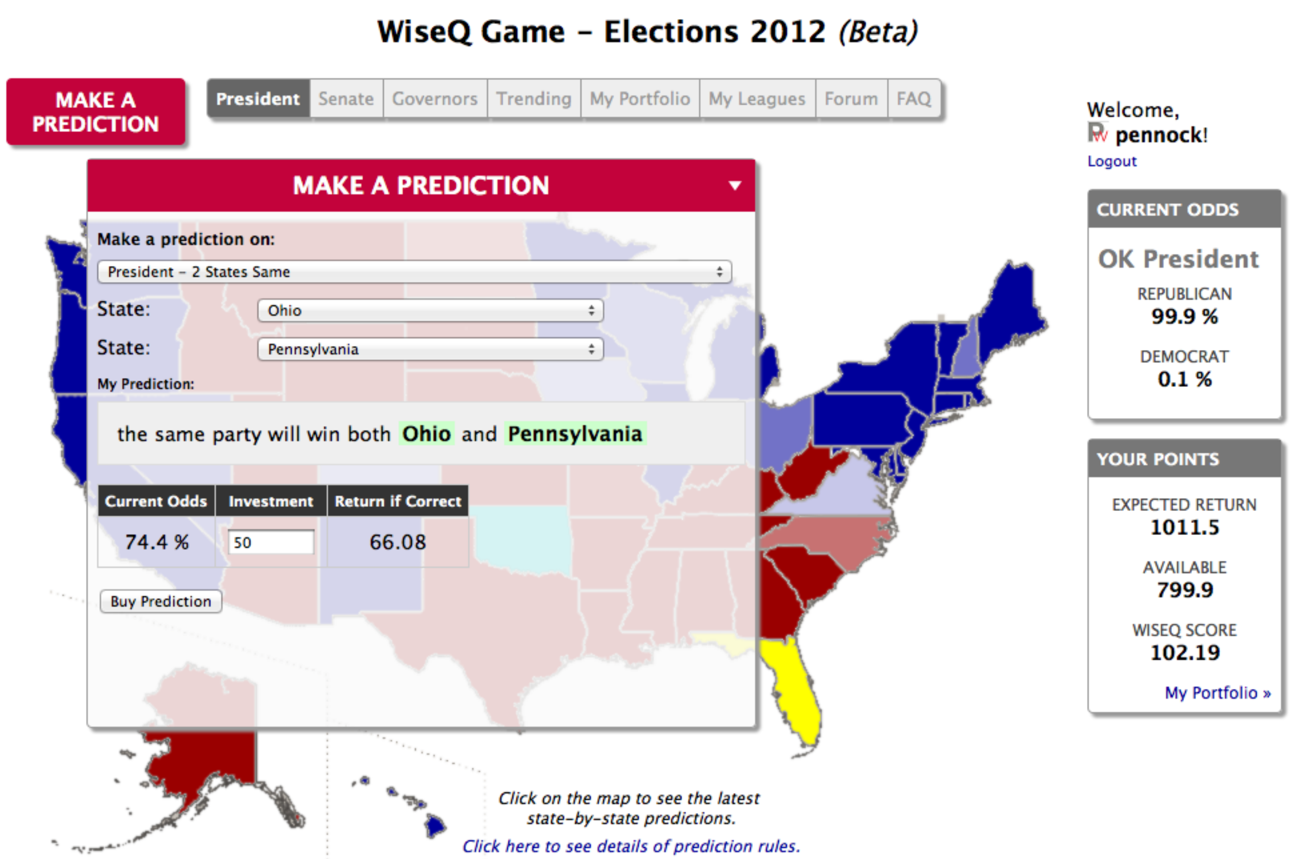


Tested on over 300K complex predictions from Princeton study

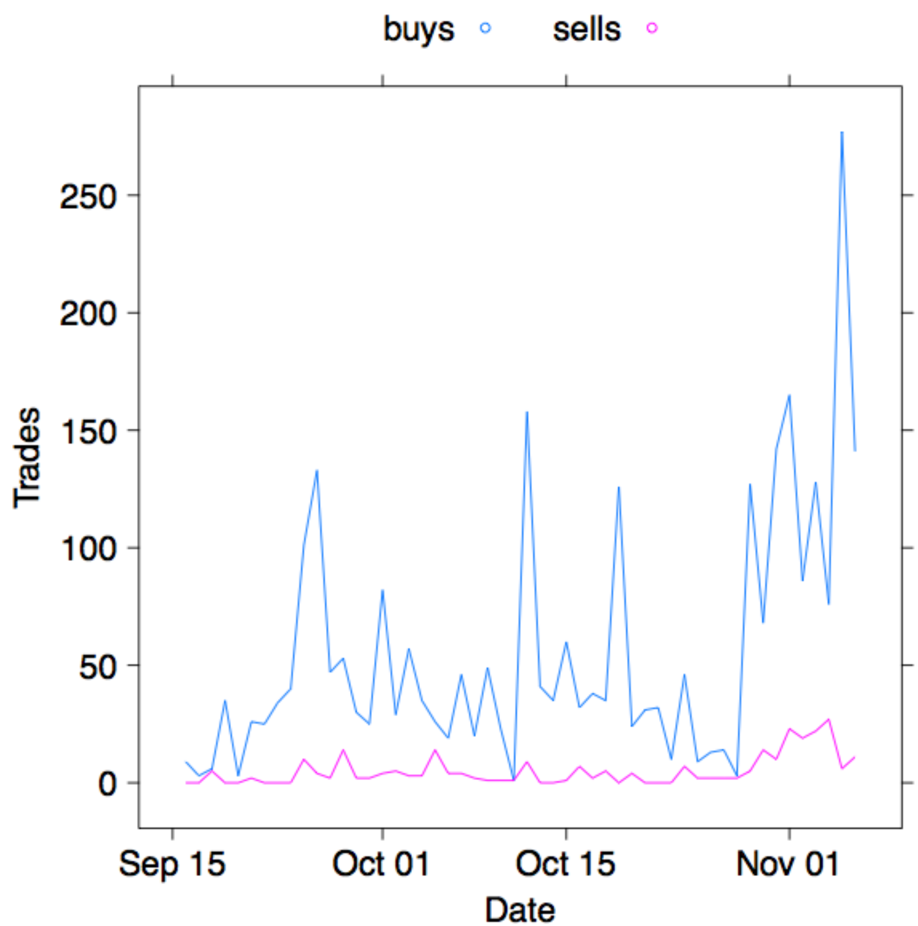


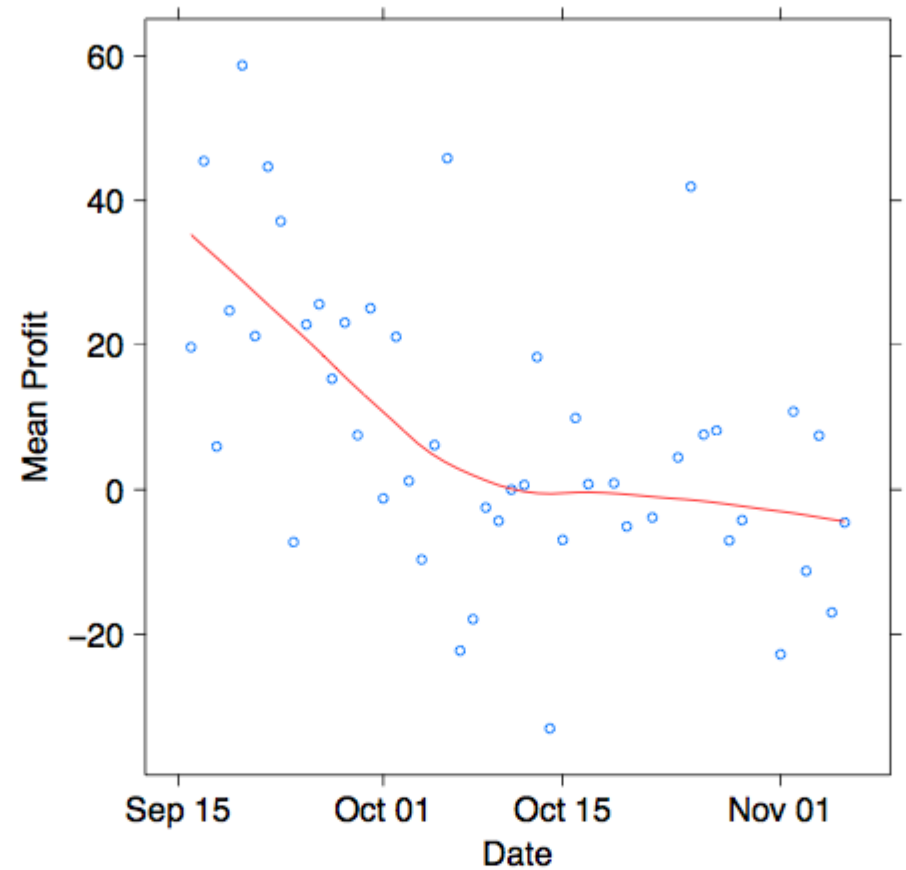
No really, does it work?

- http://PredictWiseQ.com



User activity, Mean daily profit





Focal predictions

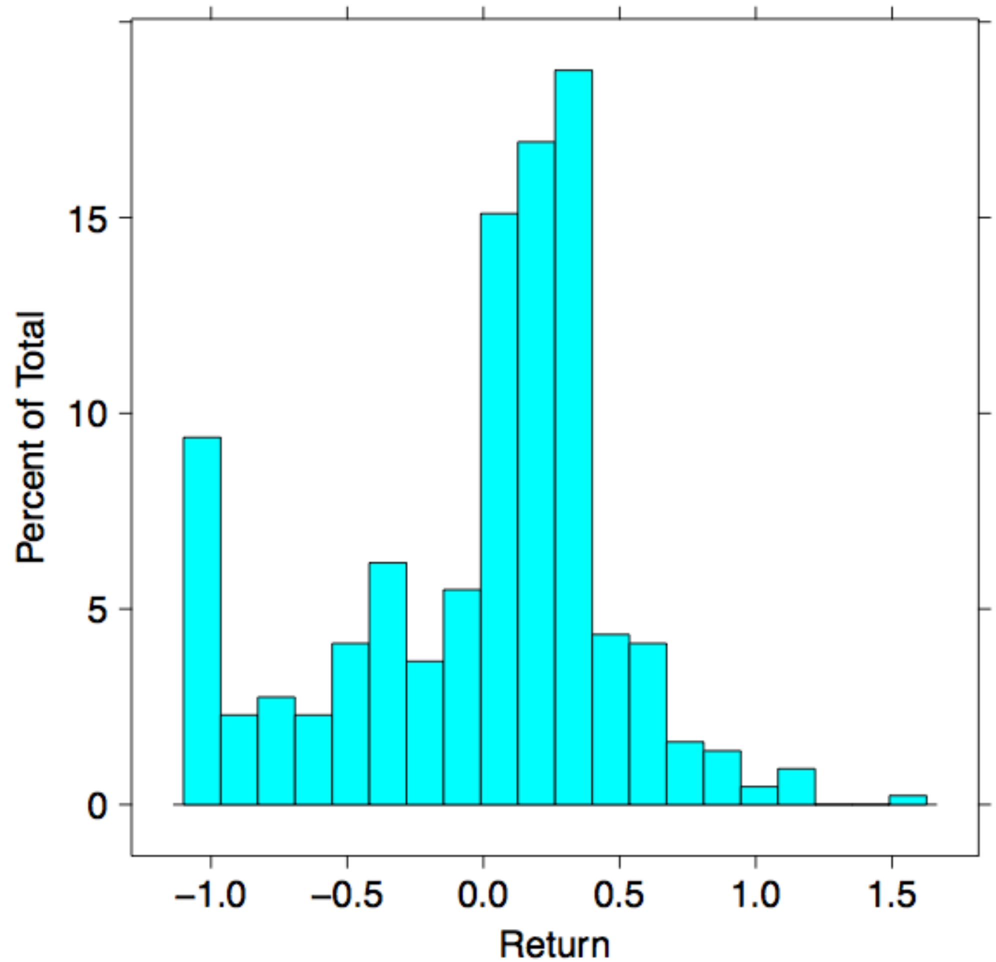
Top singleton securities

- Federal, 43K
- FL, 26K
- OH, 17K
- VA, 15K
- CO, 12K
- Electoral votes, 10K
- NC, 10K
- Senate, 8K

Top combinatorial securities

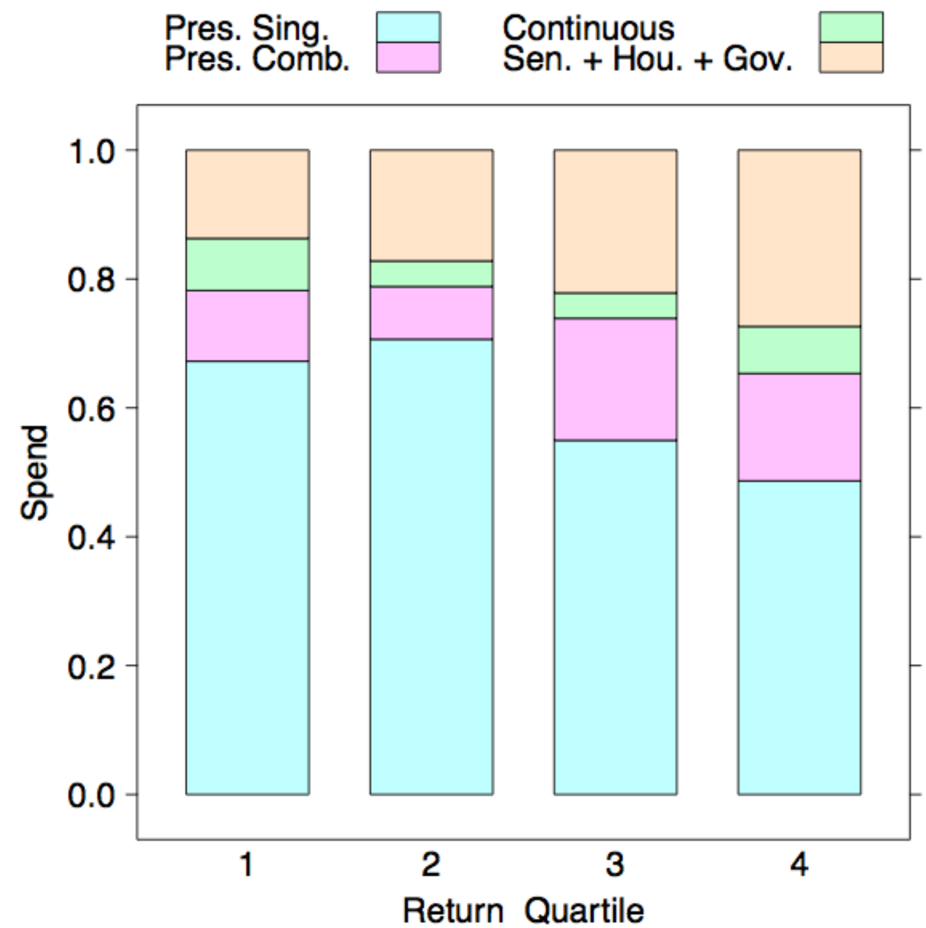
- VA, WI = Dem
- CO, IN, NH, OH, VA = Dem
- Fed, OH = Rep
- OH, VA = Dem
- FL=Rep, Fed=Dem
- FL, NC = Rep
- FL, Fed, OH = Dem
- NH, NV, OH, VA, WI = Dem

Return



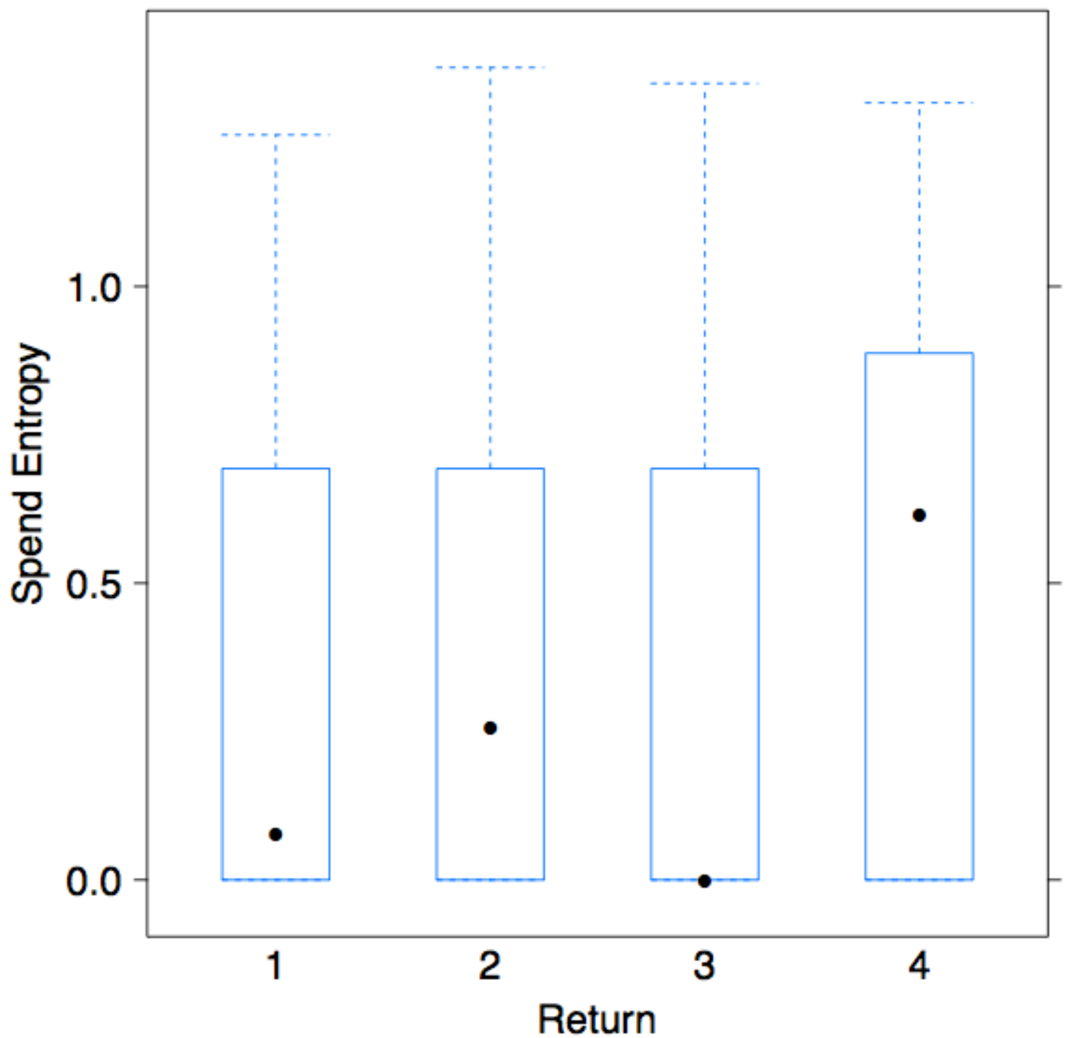
Percentile	Revenue	Spend	Profit	Return	Buys	Sells
0%	0.00	1.97	-1400.00	-1.00	1	0
25%	60.83	100.00	-50.07	-0.34	2	0
50%	238.47	350.02	9.93	0.12	4	0
75%	1006.14	1199.97	95.29	0.29	9	0
100%	5041.55	4505.96	1841.55	1.53	64	37

Types of securities by user return

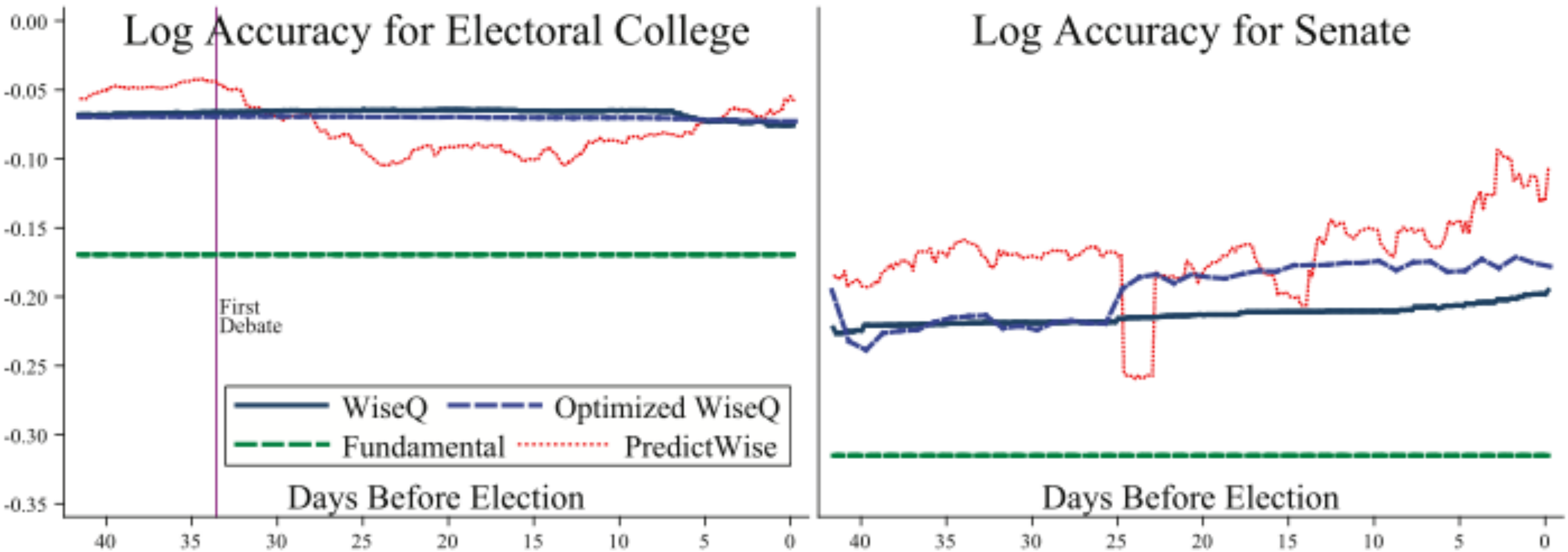


Prediction Type	Unique Users	Trades per User
Presidential—Singleton	413	3.87
Senate and House—Singleton	176	3.12
Presidential—Combinatorial	150	3.12
Electoral Votes	63	1.79
Governor	47	1.49
Economic Indicators	37	1.54
Senate and House—Combinatorial	12	1.50

“Diversification” by user return

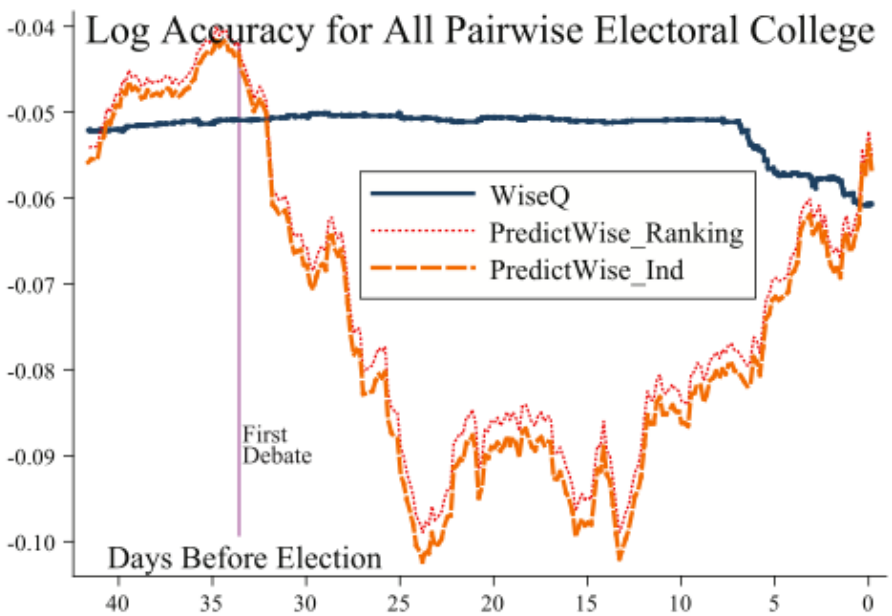


Accuracy

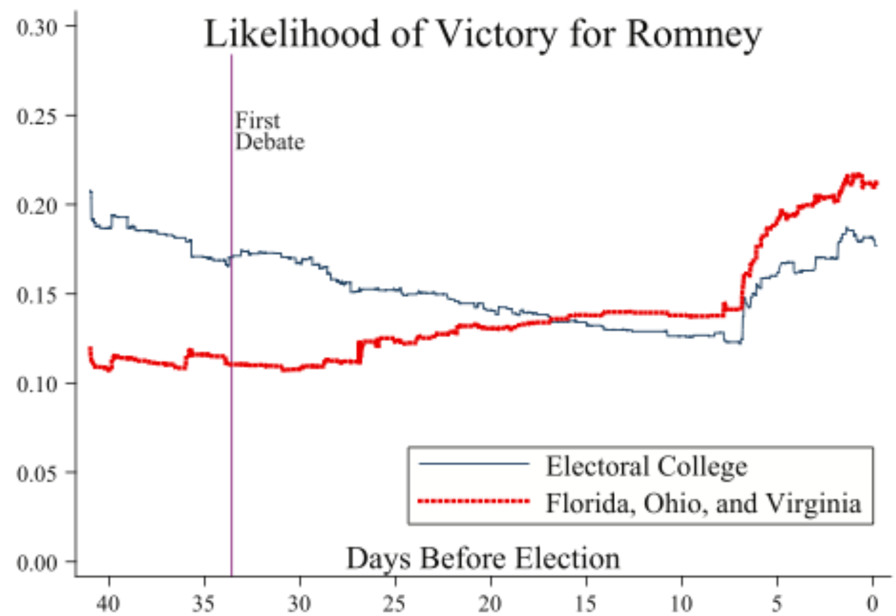


Log Accuracy for Electoral College
Log Accuracy for Senate
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
First Debate
WiseQ
Optimized WiseQ
Fundamental
PredictWise
Days Before Election
40
35
30
25
20
15
10
5
0
Days Before Election

Accuracy

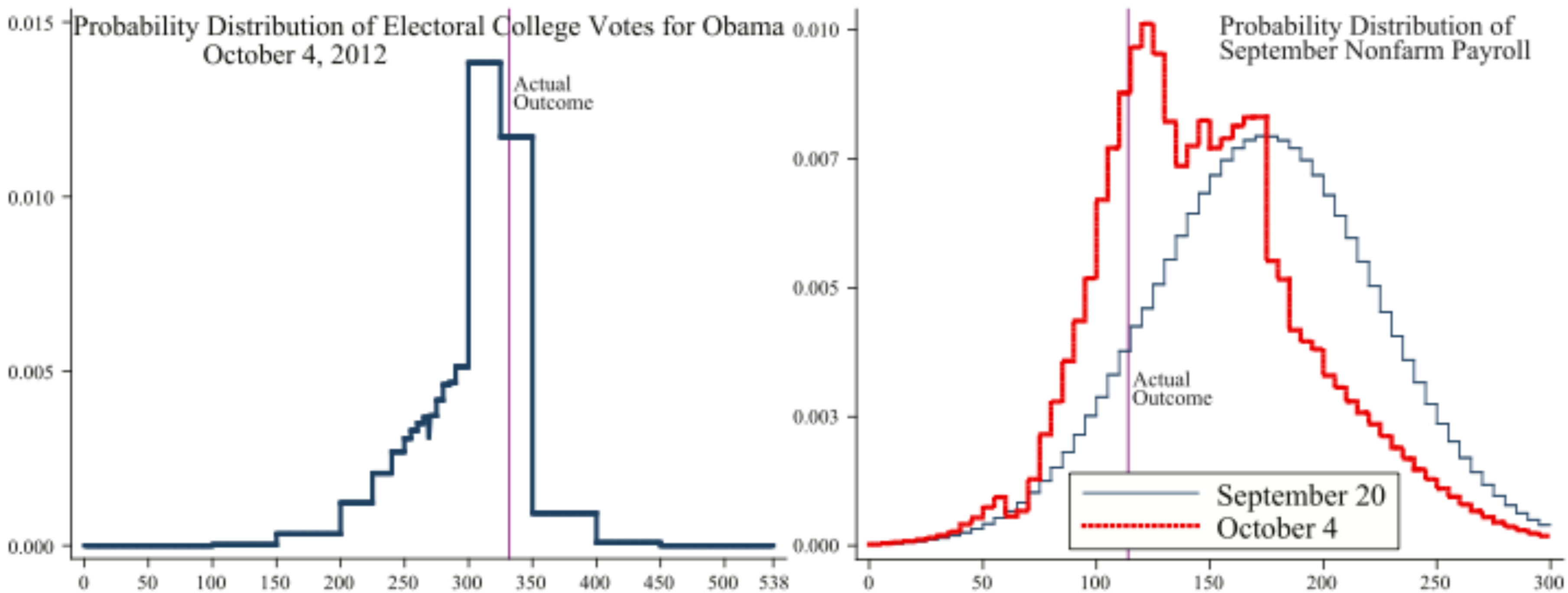


Log Accuracy for All Pairwise Electoral College
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.10
WiseQ
PredictWise_Ranking
PredictWise_Ind
First
Debate
Days Before Election
40
35
30
25
20
15
10
5
0

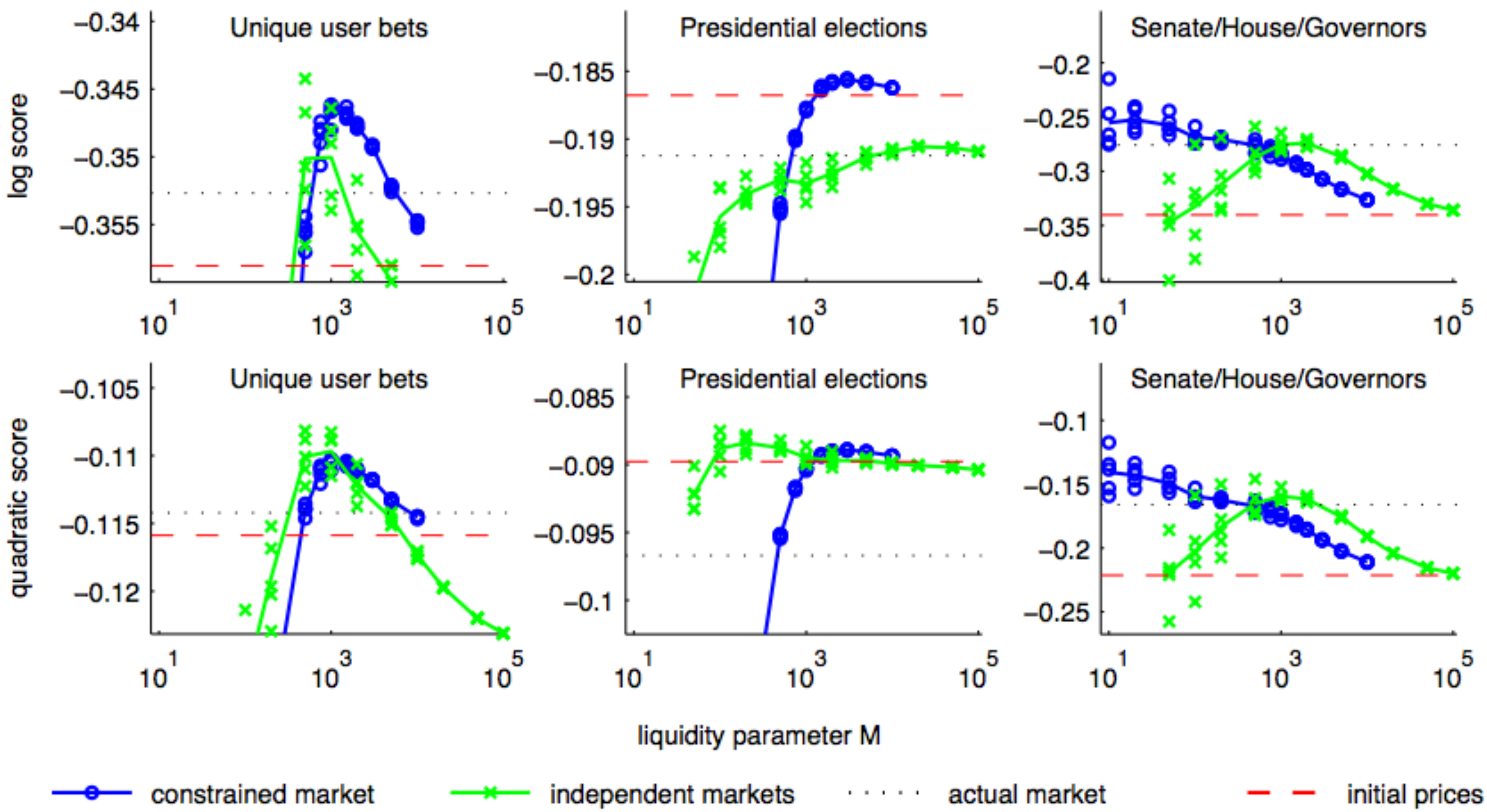


Likelihood of Victory for Romney
0.30
0.25
0.20
0.15
0.10
0.05
0.00
First
Debate
Electoral College
Florida, Ohio, and Virginia
Days Before Election
40
35
30
25
20
15
10
5
0

Full distributions



Counterfactuals



Further reading

Our paper in **ACM EC'13**
http://research.microsoft.com/apps/pubs/default.aspx?id=192965

Our paper in **ACM EC'12**
http://research.microsoft.com/apps/pubs/default.aspx?id=167977

Blog post on **PredictWiseQ**
http://blog.oddhead.com/2012/10/06/predictwiseq/

Blog on gory details: **What is (and what good is) a combinatorial prediction market?**
http://bit.ly/combopm

Guest post on **Freakonomics**
http://bit.ly/combopmfreak

NYSE 1926

http://online.wsj.com/article/SB10001424052748704858404576134372454343538.html

NYSE 1987

http://online.wsj.com/article/SB10001424052748704858404576134372454343538.html

NYSE 2006-2011


Mark Lennihan/Associated Press

- 2011 Deutsche Börse AG
- 2007 Euronext
- 2006 Archipelago, ipo

NYSE 7pm Sep 10, 2012

Exxon Mobil Corporation (XOM) - NYSE

89.48 ↓0.44 (0.49%) 4:00PM EDT | After Hours

Order Book

Top of Book

Bid		Ask	
Price	Size	Price	Size
88.60	100	89.98	100
88.59	1,000	90.31	700
		94.41	50

New markets, same old CDA

Intrade

Order Book			
2012.PRES.OBAMA			
Bid		Offer	
Qty	Price	Price	Qty
1	56.2	56.9	12
51	56.1	57.0	494
156	56.0	57.1	21
132	55.9	57.4	6
987	55.8	57.5	523
36	55.7	57.9	45
348	55.6	58.0	104
1050	55.5	58.5	133
600	55.4	59.0	100
500	55.3	59.3	60
500	55.2	59.5	100
625	55.1	59.8	100
1124	55.0	59.9	215
100	54.9	60.0	200
104	54.8	60.2	207

Betfair

1.53	£130		£110,216
1.54	£992		£44,612
1.55	£6,573		£58,305
1.56	£3,499		£85,048
1.57			£122,367
1.58		£7,630	£63,837
1.59		£747	£64,479
1.60		£539	£70,414
1.61		£78	£63,195
1.62		£121	£55,921
1.63		£95	£58,138
1.64		£127	£86,808
1.65		£1,598	£92,735
1.66		£2,673	£29,193
1.67		£182	£21,888
1.68		£228	£30,840
1.69		£50	£41,203
1.70		£129	£25,480
1.71		£114	£16,005
1.72		£50	£17,674
1.73		£2	£54,606
1.74		£1,525	£71,804
1.75		£263	£62,116

Figure 7: The order books for Obama to win the presidency, Intrade left and Betfair right, at 1:45 PM ET on August 16, 2012

A Better Way
(Or,... Bringing trading into digital age)

- Expressiveness
  - Linear programming
  - Bossaerts, Fine, Ledyard: Combined Value Trading
    Fortnow et al.: Betting Boolean Style
  - `http://bit.ly/multipm`
- Expressiveness + Liquidity
  - Automated market maker
  - Always quote a price on *anything*
  - Downside: requires subsidy/risk

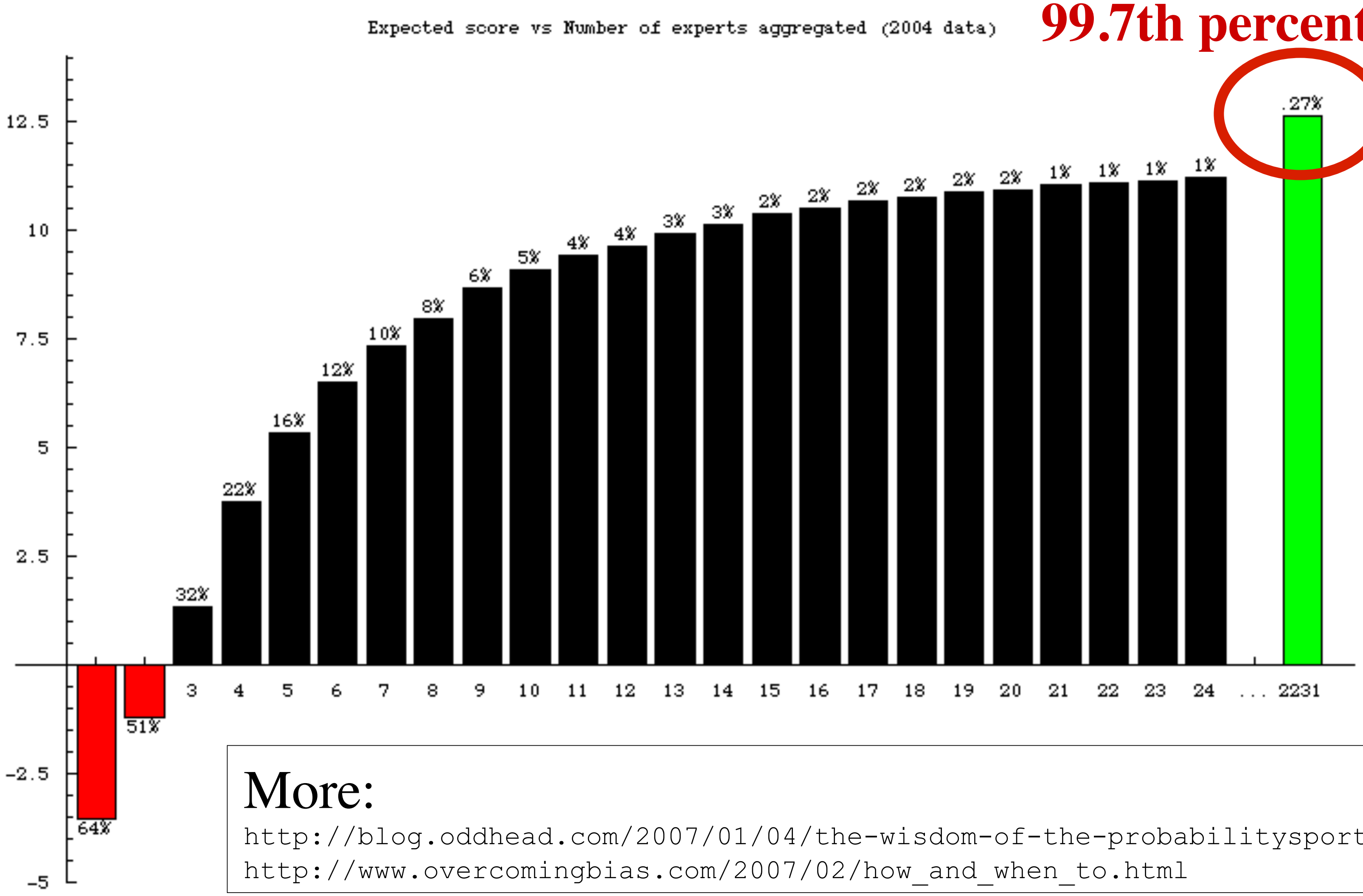
Performance of Prediction Markets with Kelly Bettors

David Pennock, Microsoft*
John Langford, Microsoft*
Alina Beygelzimer (IBM)

*work conducted at Yahoo!

Wisdom of crowds

When to guess: if you're in the 99.7th percentile



More:
http://blog.oddhead.com/2007/01/04/the-wisdom-of-the-probabilitysports-crowd/
http://www.overcomingbias.com/2007/02/how_and_when_to.html

Can a prediction market learn?

Typical

- Mkt >> avg expert
- Static

This paper

- Mkt >? best expert
- Dynamic/Learning (experts algos)
- Assumption: Agents optimize growth rate (Kelly)

Kelly betting

- You have \$1000
- Market price mp is 0.05
- You think probability p is 0.10
- Q: How much should you bet?

Kelly betting

- You have \$1000
- Market price mp is 0.05
- You think probability p is 0.10
- Q: How much should you bet?
- Wrong A: \$1000
  P(bankruptcy)$\rightarrow$1

Kelly betting

- You have \$1000
- Market price mp is 0.05
- You think probability p is 0.10
- Q: How much should you bet?
- Wrong A: fixed \$1
  No compounding magic

Kelly betting

- You have \$1000
- Market price mp is 0.05
- You think probability p is 0.10
- Q: How much should you bet?
- Optimal A: **\$52.63**
  or f* fraction of your wealth
  where f* = (p-mp)/(1-mp) "edge/odds"

Why?

- Kelly betting ≡ maximizing log utility
  - Maximizes compounding growth rate
    Maximizes geometric mean of wealth
  - Minimizes expected doubling time
    Minimizes exp time to reach, say, $1M
  - Does not maximize expected wealth
    ("All in" does, but ensures bankruptcy)

Wealth-weighted average

- Assume all agents optimize via Kelly and are "price takers"
- Then $mp = \sum w_i \, p_i$

Fractional Kelly betting

- Bet λf^* fraction of your wealth, $\lambda \in [0,1]$
- Why? Ad hoc reasons
  - Full Kelly is too risky (finite horizon)
  - I'm not confident of p (2nd-order belief)
- Our (new) reason
  - λ fraction Kelly behaviorally equiv to full Kelly with revised belief $\lambda p+(1-\lambda)mp$
  - Has Bayesian justification

Wealth-weighted average

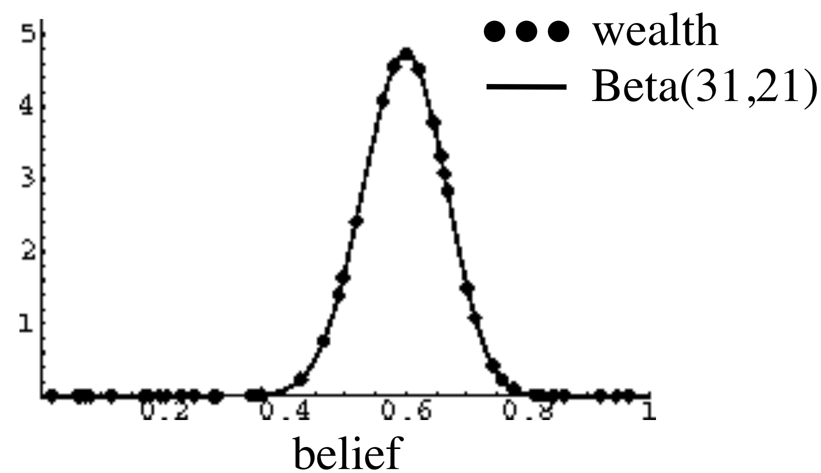
- Assume all agents optimize via Kelly and are "price takers"
- Then $mp = \sum w_i p_i$
- Fractional Kelly: $mp = \sum \lambda_i w_i p_i$

Wealth dynamics

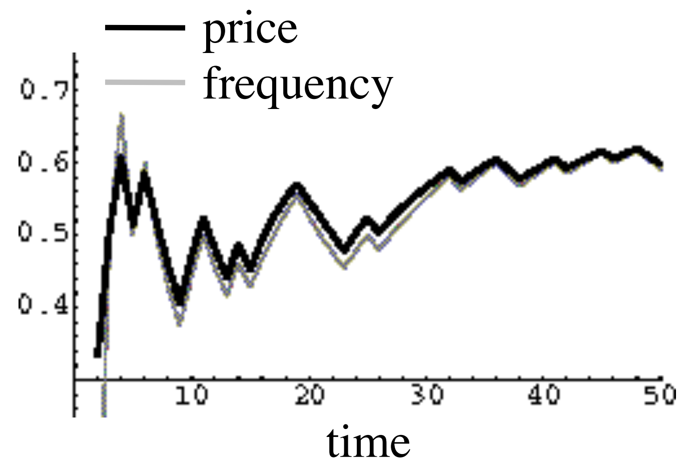
- Agents trade in a binary pred market
  Event outcome is revealed
  Wealth is redistrib. (losers pay winners)
  Repeat
- After each round
  If event happens: $w_i \leftarrow w_i\, p_i/pm$
  If event doesn't: $w_i \leftarrow w_i(1-p_i)/(1-pm)$
- Wealth is redistributed like Bayes rule!

Wealth dynamics

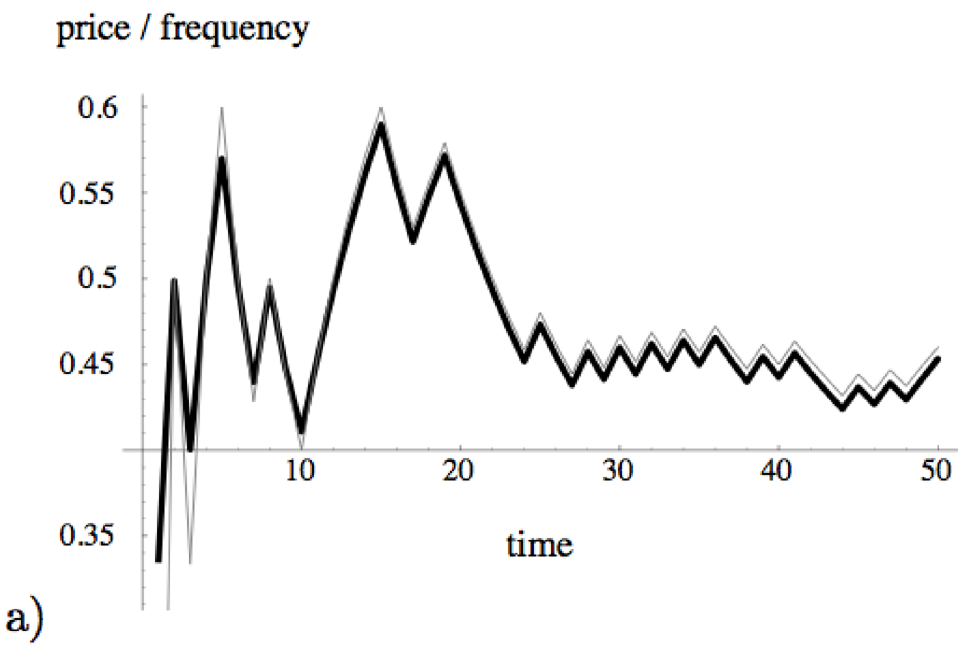
- Single security
- Multiperiod market
- Agents with log util
- Fixed beliefs



successes:30 trials:50



Wealth dynamics



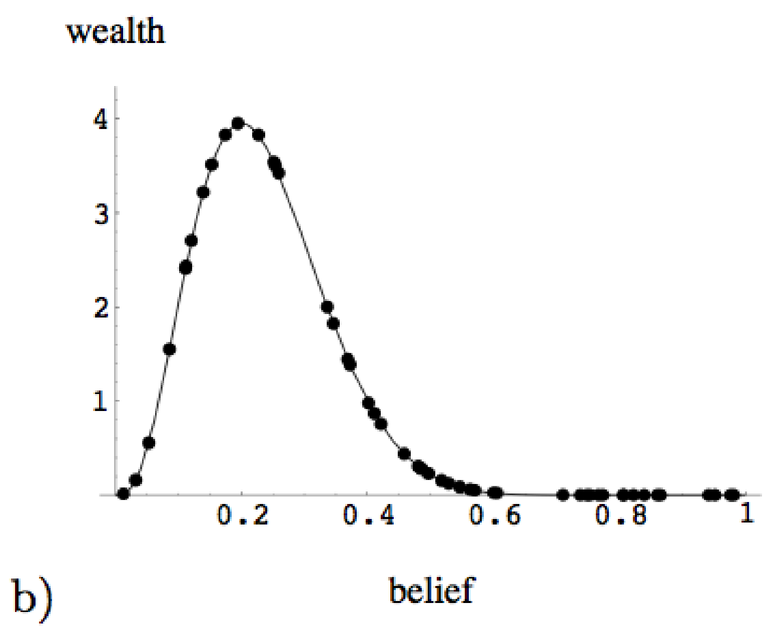


Fig. 1. a) Price (black line) versus the observed frequency (gray line) of the event over fifty time periods. The market consists of one hundred full-Kelly agents with initial wealth $w_i = 1$. b)Wealth after fifteen time periods versus belief for fifty Kelly agents. The event has occurred in three of the fifteen trials. The solid line is the posterior Beta distribution consistent with the observation of three successes in fifteen independent Bernoulli trials.

Wealth dynamics: Regret

- Theorem:

Mkt log loss < $\min_i$ agent i log loss - $\ln w_i$

- Applies for all agents, all outcomes, even adversarial
- Same bound as experts algorithms
  $\Rightarrow$ No "price of anarchy"

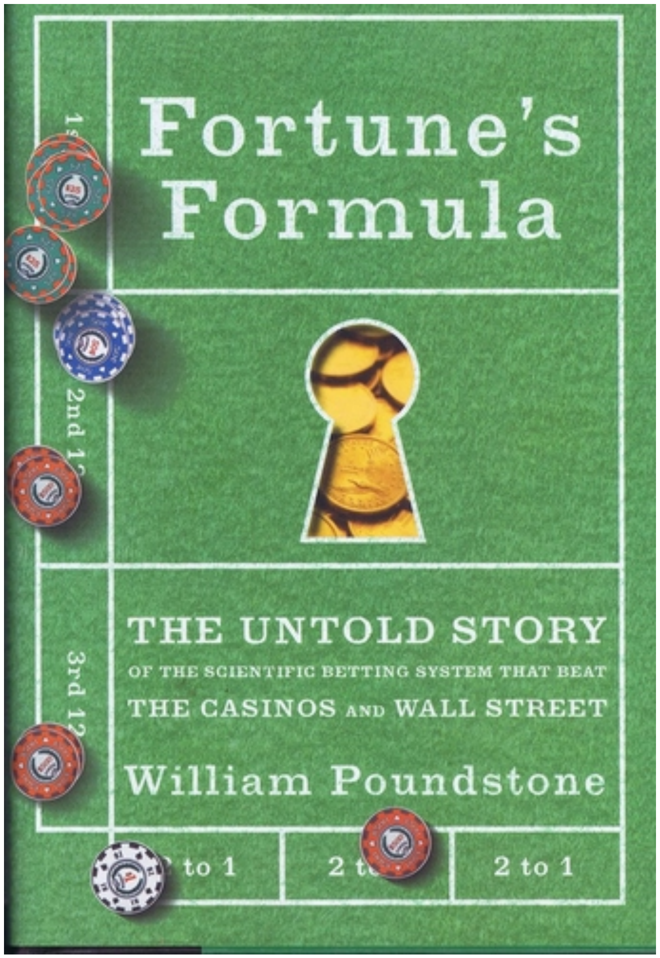
Learning the Kelly fraction

- Competitive equilibrium: $\lambda = 1$
- Rational expectations equil: $\lambda = 0$
- Practical (wisdom of crowds): $\lambda = \varepsilon$
- A proposal: Fractional Kelly as an experts algorithm btw yourself and the market
  - Start with $\lambda = 0.5$
  - If right, increase λ; If wrong, decrease
  - Won't do much worse than market (0)
    Won't do much worse than original prior p

Summary and Questions

- Self-interested Kelly bettors implement Bayes' rule, minimize market's log loss
- If mkt & agents care about log loss: win-win
- Questions
  - Can non-Kelly bettors be induced to minimize log loss?
  - Can Kelly bettors be induced to minimize squared loss, 0/1 loss, etc.?
  - Can/should we encourage Kelly betting?

Recommended Read



- Great science writing (Pulitzer prize winner)
- Fun read
- Accessible yet accurate
- Well researched
- Mob meets politics meets Time Warner meets Wall Street meets Vegas meets Bell Labs scientists

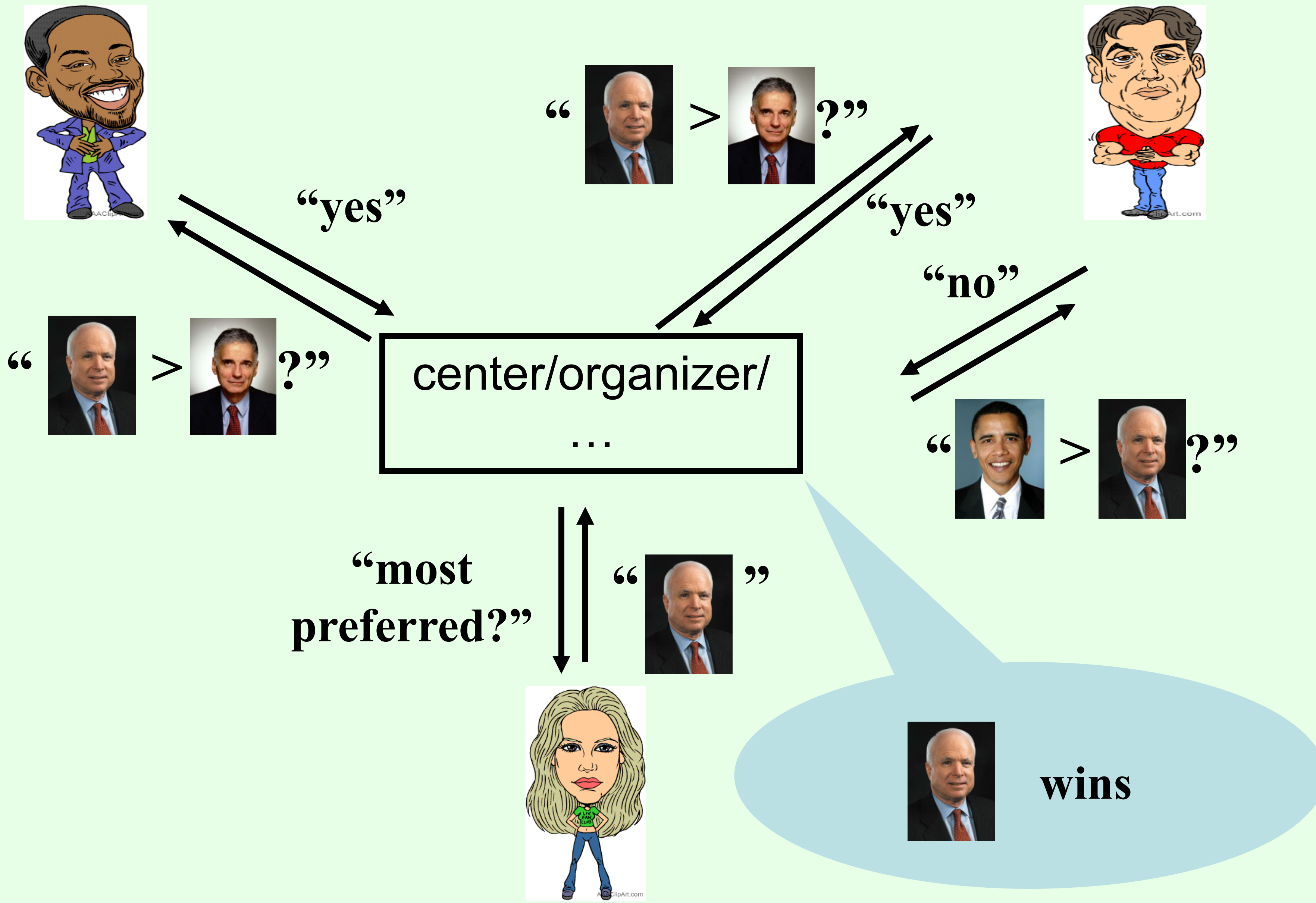
Preference elicitation

Vincent Conitzer
conitzer@cs.duke.edu

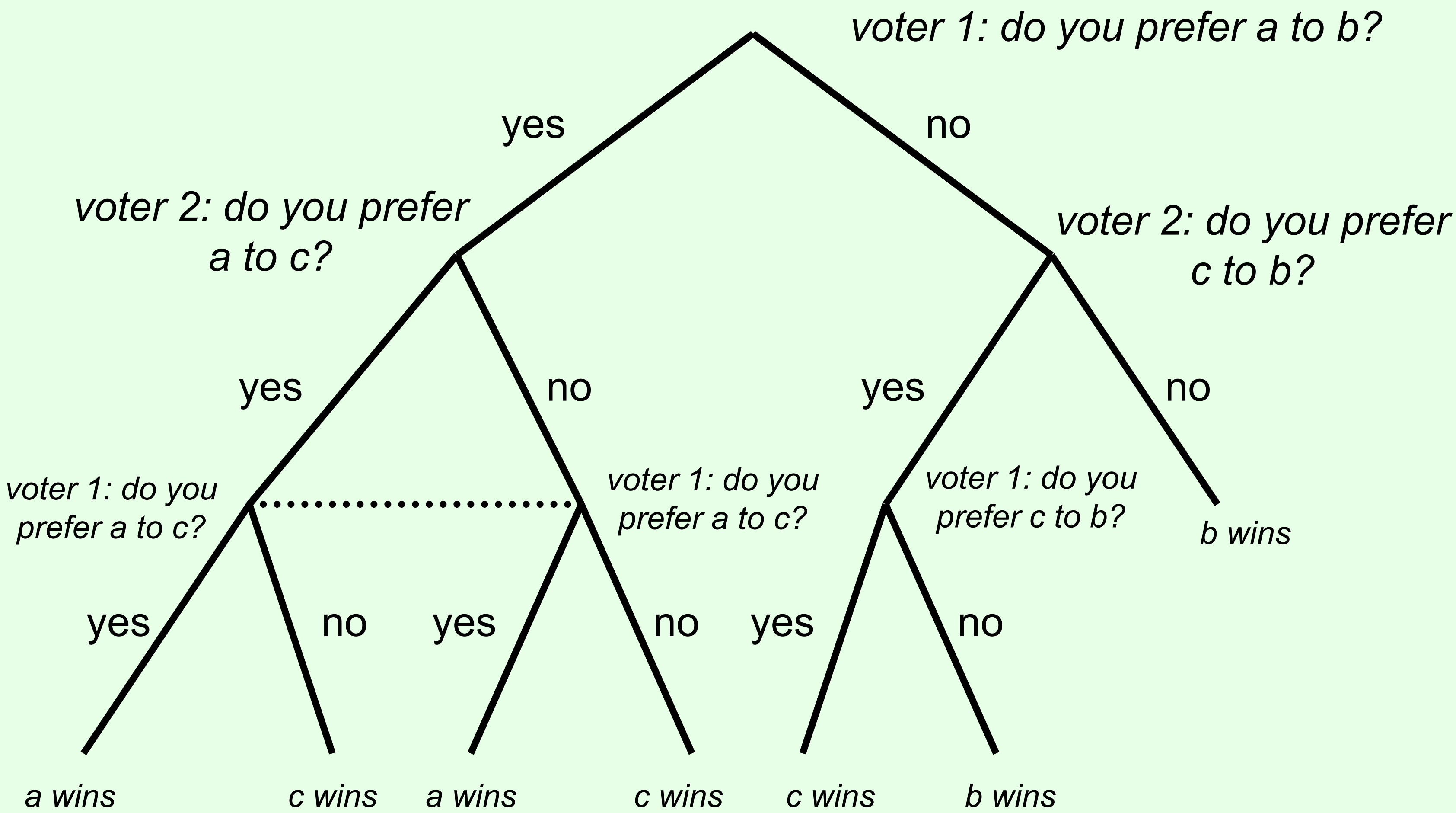
Excessive communication

- So far we have focused on revealing full preferences; often not practical
  - Rate/rank all the songs you've ever heard
  - Rate/rank all possible budget allocations
  - …
- Some ways to address this:
  - Directly model underlying structure
    - Directly express preferences over *aspects* of song (genre, instruments), *individual items* on the budget, …
    - Need to worry about interactions (voting in combinatorial domains)
  - Preference elicitation: ask voters selected queries about preferences, avoid asking for irrelevant information

Preference elicitation (elections)



Representing elicitation protocols



- Can choose to hide information from agents, but only insofar as it is not implied by queries we ask of them
- (What's inefficient about the above protocol?)

Strategic issues

- Consider the following protocol for eliciting a plurality winner with three voters

- Ask voter 1 for her most-preferred alternative t_1
- Ask voter 2 for his most-preferred alternative t_2
- If $t_1 = t_2$ then declare it the winner
- Otherwise, ask voter 3 if one of t_1 or t_2 is her most-preferred alternative
  - If yes, then declare that one the winner
  - If no, then toss a coin to decide between t_1 and t_2

- What opportunity for strategic manipulation does this provide voter 3?

Two possible goals

(Assume truthful reporting for the rest of this lecture)

1. Ask queries until we know the true winner under a given common voting rule (e.g., plurality)

2. Keep asking queries until we've found an alternative that is "good enough"

(allows us to get away with fewer queries)

(Note: 2 will still correspond to some voting rule)

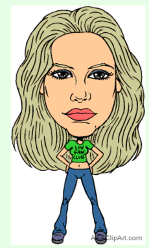
Elicitation algorithms for given rules

- Design elicitation algorithm to minimize queries for given rule
- What is a good elicitation algorithm for STV?
- What about Bucklin?

An elicitation algorithm for the Bucklin voting rule based on binary search

[C. & Sandholm 05]

- Alternatives: A B C D E F G H


• Top 4?	{A B C D}	{A B F G}	{A C E H}
• Top 2?	{A D}	{B F}	{C H}
• Top 3?	{A C D}	{B F G}	{C E H}

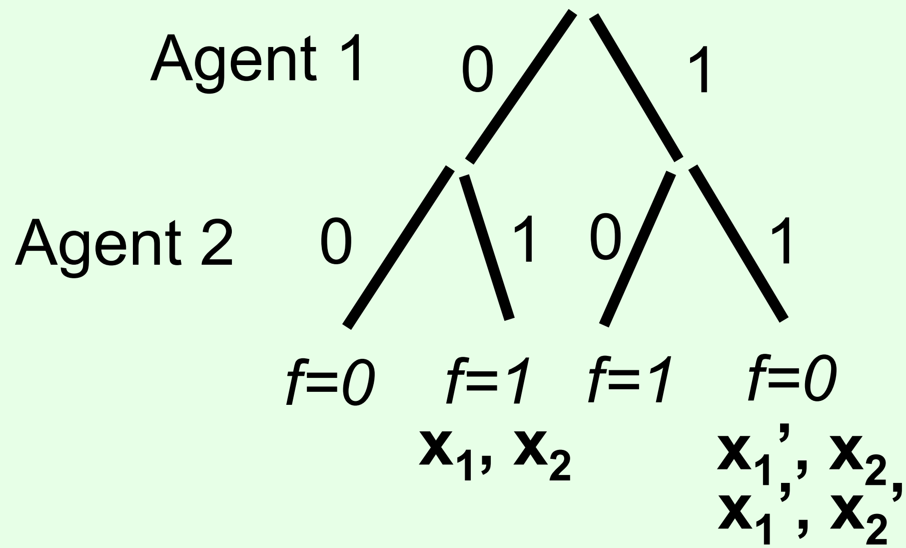
Total communication is nm + nm/2 + nm/4 + … ≤ 2nm bits
(n number of voters, m number of candidates)

Lower bounds on communication

- Communication complexity theory can be used to show lower bounds
  - "Any elicitation algorithm for rule r requires communication of at least N bits (in the worst case)"
- Voting [C. & Sandholm 05]
  - Bucklin requires at least on the order of nm bits
  - STV requires at least on the order of n log m bits
    - Natural algorithm uses on the order of $n(\log m)^2$ bits

How do we know that we have found the best elicitation protocol for a mechanism?

- Communication complexity theory: agent i holds input x_i, agents must communicate enough information to compute some $f(x_1, x_2, \ldots, x_n)$
- Consider the tree of all possible communications:
- Every input vector goes to some leaf



- If $x_1, \ldots, x_n$ goes to same leaf as $x_1', \ldots, x_n'$ then so must any mix of them (e.g., $x_1, x_2', x_3, \ldots, x_n'$)
- Only possible if f is same in all 2^n cases
- Suppose we have a fooling set of t input vectors that all give the same function value f_0, but for any two of them, there is a mix that gives a different value
- Then all vectors must go to different leaves $\Rightarrow$ tree depth must be $\geq \log(t)$

Example: plurality

- A fooling set:
- *a, a, b, b, c, c, a*
- *a, a, c, c, b, b, a*
- *b, b, a, a, c, c, a*
- *b, b, c, c, a, a, a*
- *c, c, a, a, b, b, a*
- *c, c, b, b, a, a, a*

any two can be mixed to get a different winner, e.g., *a, a, b, b, b, b, a*

a wins on all of these

can be extended to show an order *n* log *m* lower bound (best possible)

- Not a fooling set:
- *a, a, a, b*
- *c, a, a, a*

Finding a winner that is "good enough"

- If we have elicited enough information to determine that an alternative is a necessary winner, we can stop
- But what if we want to stop earlier?
- Suppose we have some measure of welfare (e.g., Borda score) and partially elicited preferences
- Max regret of an alternative *a*:

  How much worse than the best alternative could *a*'s welfare still be?
- Minimax regret alternative: an alternative that minimizes maximum regret
- Then ask queries to try to quickly reduce minimax regret
- [Lu & Boutilier 2011]

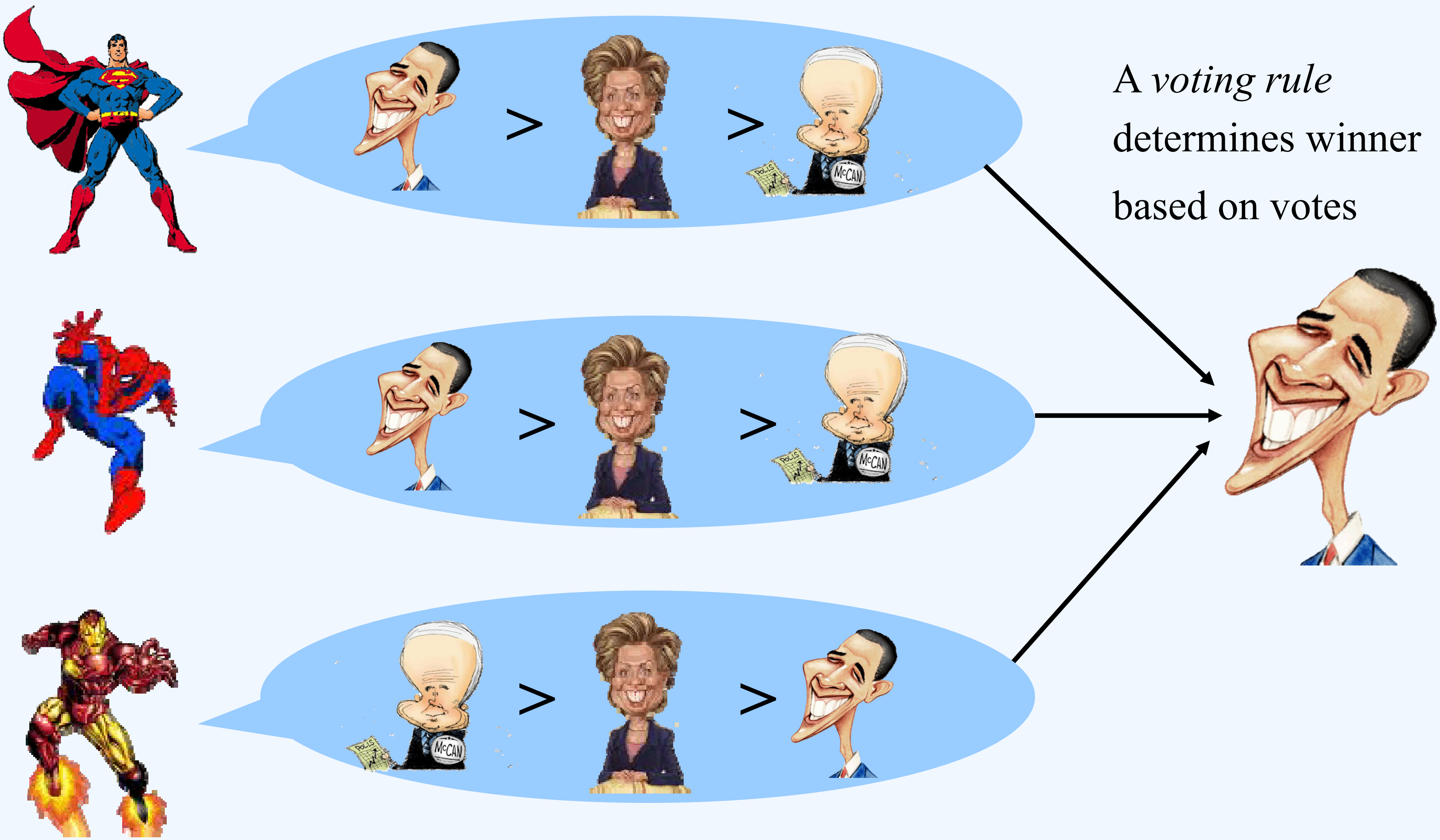
Example (Borda)

- Partial profile:
- Voter 1: *a*>*b*>*c*
- Voter 2: *a*>*b*
- Voter 3: *c*>*a*
- Max regret for *a*: 1 (against *c*, if vote 2 is *c*>*a*>*b* and vote 3 is *c*>*b*>*a*)
- Max regret for b: 4 (against *a*, if vote 2 is *a*>*c*>*b* and vote 3 is *c*>*a*>*b*)
- Max regret for *c*: 3 (against *a*, if vote 2 is *a*>*b*>*c* and vote 3 is *c*>*a*>*b* or *b*>*c*>*a*; or against *b*, if vote 2 is *a*>*b*>*c* and vote 3 is *b*>*c*>*a*)
- *a* is the minimax regret alternative

Strategic voting

with thanks to: **Lirong Xia** **Jérôme Lang**

Let’s vote!

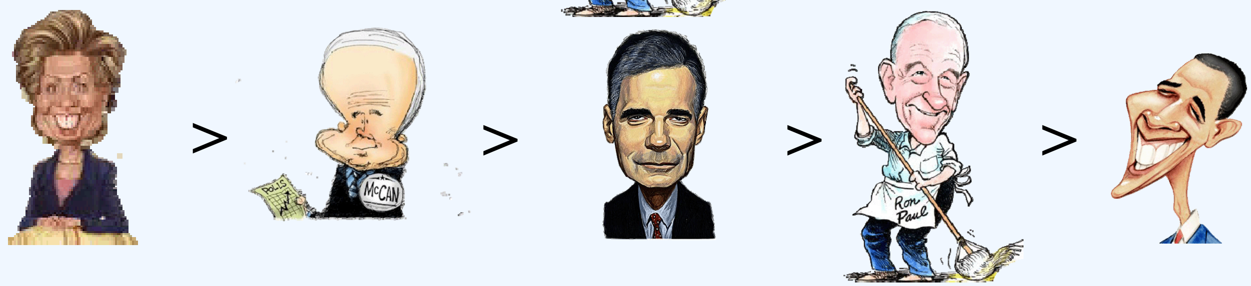


Voting: Plurality rule

Superman

:

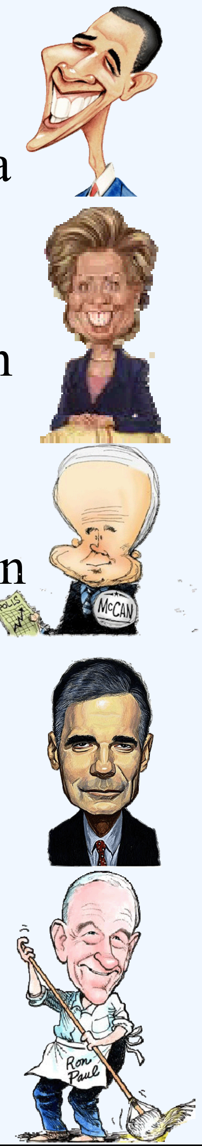
:



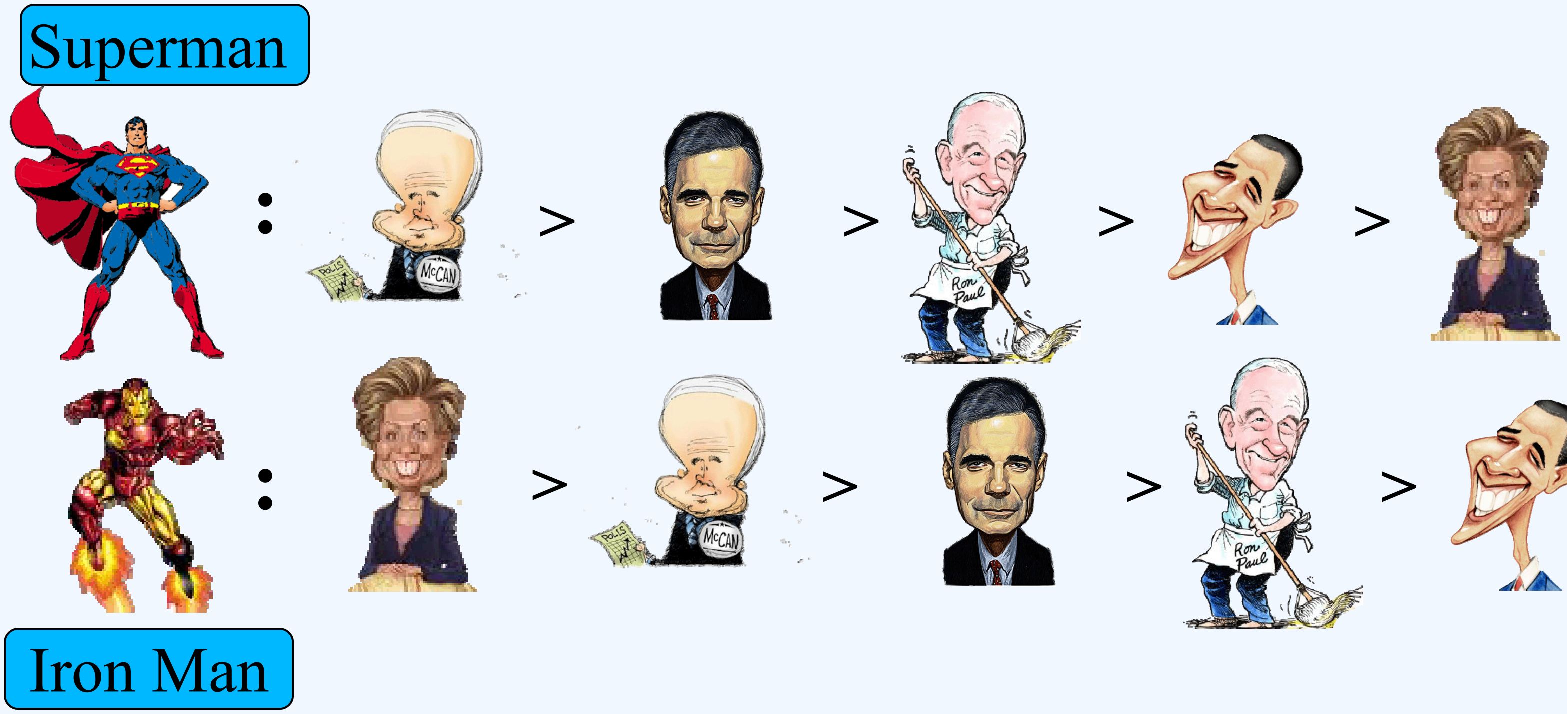
Iron Man

Plurality rule, with ties broken as follows:

Obama
V
Clinton
V
McCain
V
Nader
V
Paul



Voting: Borda rule



Simultaneous-move voting games

- *Players:* Voters $1,\ldots,n$
- *Preferences:* Linear orders over alternatives
- *Strategies / reports:* Linear orders over alternatives
- *Rule:* $r(P')$, where P' is the reported profile
- *Nash equilibrium:* A profile P' so that no individual has an incentive to change her vote (with respect to the true profile P)

Many bad Nash equilibria…

- Majority election between alternatives a and b
  - Even if everyone prefers a to b, everyone voting for b is an equilibrium
  - Though, everyone has a weakly dominant strategy
- Plurality election among alternatives a, b, c
  - In equilibrium everyone might be voting for b or c, even though everyone prefers a!
- Equilibrium selection problem

Voters voting sequentially

Duke CS TGIF* Movie Night

Do you plan to attend the next movie night?

Yes, count me in!

Yes, count me in! (Vegetarian)

Current count: 29

Current top films:

1. Inception
2. Eternal Sunshine of the Spotless Mind
3. Pulp Fiction

4 vote(s) *You have voted for this film.*

Title: Willow

Description: [link] This epic Lucasfilm fantasy serves up enough magical adventure to satisfy fans of the genre, though it treads familiar territory. With abundant parallels to Star Wars, the story (by George Lucas) follows the exploits of the little farmer Willow (Warwick Davis), an aspiring sorcerer appointed to deliver an infant princess from the evil queen (Jean Marsh) to whom the child is a crucial threat. Val Kilmer plays the warrior who joins Willow's campaign with the evil queen's daughter (Joanne Whalley, who later married Kilmer). Impressive production values, stunning locations (in England, Wales, and New Zealand) and dazzling special effects energize the routine fantasy plot, which alternates between rousing action and cute sentiment while failing to engage the viewer's emotions. A parental warning is appropriate: director Ron Howard has a light touch aimed at younger viewers, but doesn't shy away from grisly swordplay and at least one monster (a wicked two-headed dragon) that could induce nightmares.

Trailer: http://matttrailer.com/willow_1988

10 vote(s) *You have voted for this film.*

Title: Pulp Fiction

Description: [link] http://www.youtube.com/watch?v=OAHETuK70Sc The lives of two mob hit men, a boxer, a gangster's wife, and a pair of diner bandits intertwine in four tales of violence and redemption

3 vote(s) *You have voted for this film.*

Title: Tom yum goong

Description: [link] In Bangkok, the young Kham was raised by his father in the jungle with elephants as members of their family. When his old elephant and the baby Kern are stolen by criminals, Kham finds that the animals were sent to Sidney. He travels to Australia, where he locates the baby elephant in a restaurant owned by the evil Madame Rose, the leader of an international Thai mafia. With the support of the efficient Thai sergeant Mark, who was involved in a conspiracy, Kham fights to rescue the animal from the mobsters.

2 vote(s) *You have voted for this*

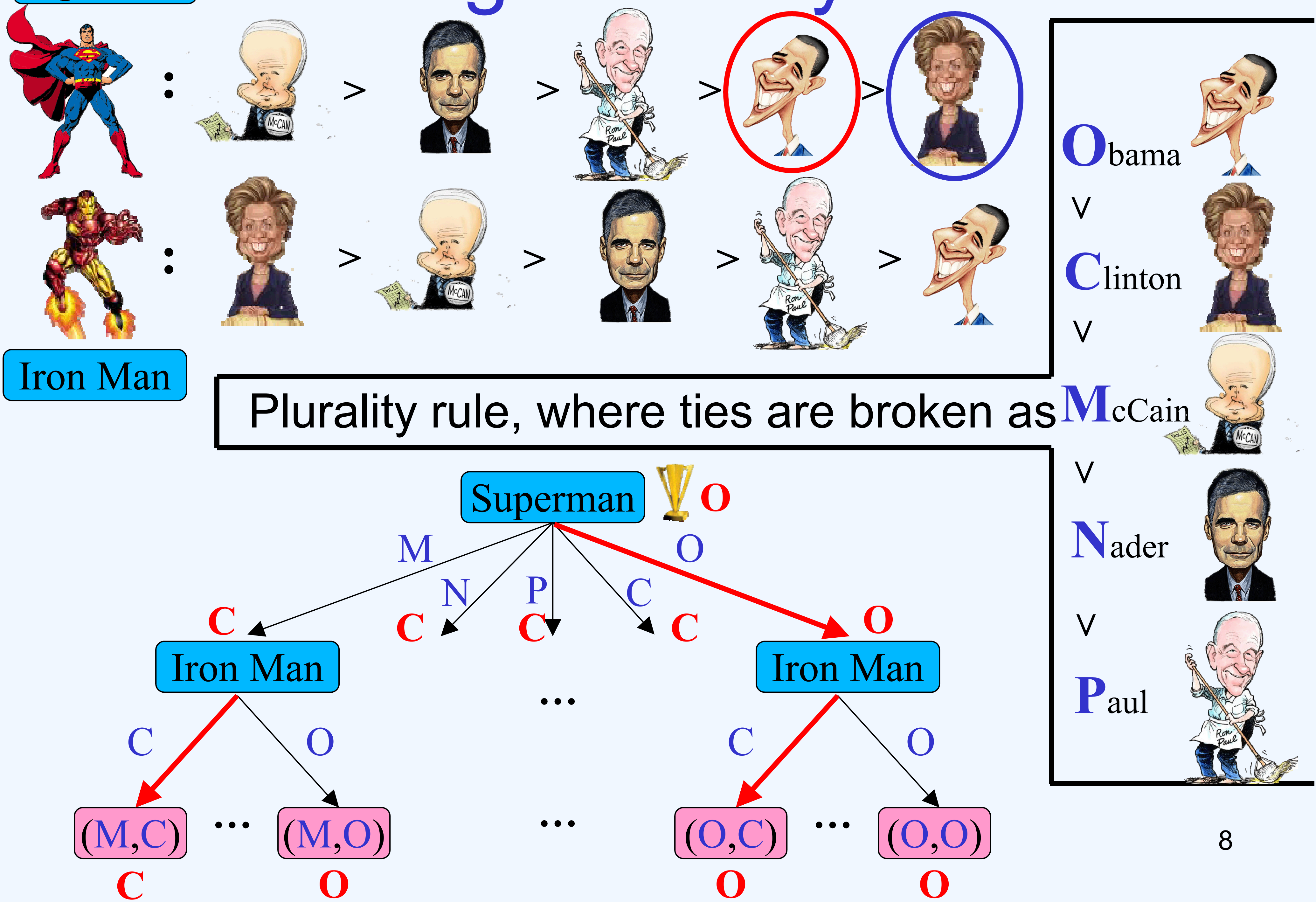
Title: Dogville

Description: [link] Dogville is a 2003 philosophical drama written and directed by Lars von Trier, and starring Nicole Kidman. It is a parable that uses an extremely minimal, stage-like set to tell the story of Grace Mulligan (Kidman), a woman hiding from mobsters, who arrives in the small mountain town of Dogville and is provided refuge in return for physical labor. Because she has to win and keep the acceptance of every single one of the inhabitants of the town to be allowed to stay, any attempt by her to do things her own way or to put a limit on her service risks driving her back out into the

Our setting

- Voters vote sequentially and strategically
  - voter $1 \rightarrow$ voter $2 \rightarrow$ voter $3 \rightarrow \ldots$ etc
  - states in stage i: all possible profiles of voters $1,\ldots,i$-1
  - any terminal state is associated with the winner under rule r
- At any stage, the current voter knows
  - the order of voters
  - previous voters' votes
  - true preferences of the later voters (complete information)
  - rule r used in the end to select the winner
- We call this a Stackelberg voting game
  - Unique winner in subgame perfect Nash equilibrium (not unique SPNE)
  - the subgame-perfect winner is denoted by $\boldsymbol{SG_r(P)}$, where P consists of the true preferences of the voters

Voting: Plurality rule



Literature

- Voting games where voters cast votes one after another
  - [Sloth GEB-93, Dekel and Piccione JPE-00, Battaglini GEB-05, Desmedt & Elkind EC-10]

Key questions

- How can we compute the backward-induction winner efficiently (for general voting rules)?
- How good/bad is the backward-induction winner?

Computing $SG_r(P)$

- Backward induction:
  - A state in stage i corresponds to a profile for voters 1, …, i-1
  - For each state (starting from the terminal states), we compute the winner if we reach that point
- Making the computation more efficient:
  - depending on r, some states are equivalent
  - can merge these into a single state
  - drastically speeds up computation

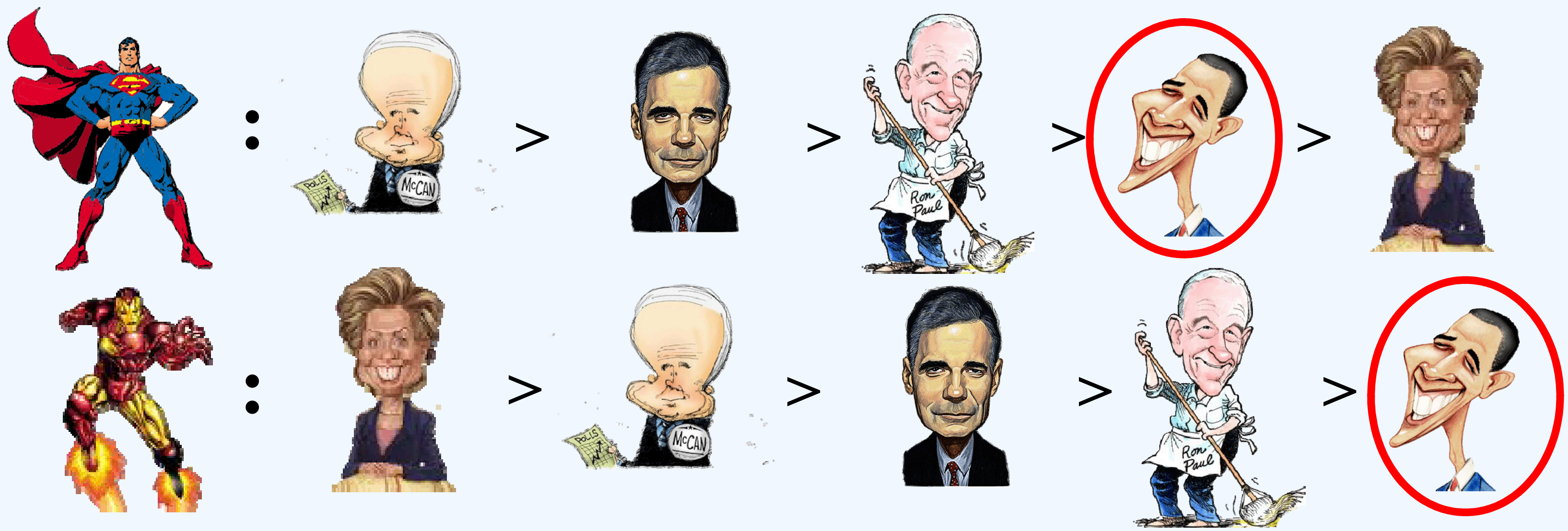
An equivalence relationship between profiles

- The plurality rule
- 160 voters have cast their votes, 20 voters remaining

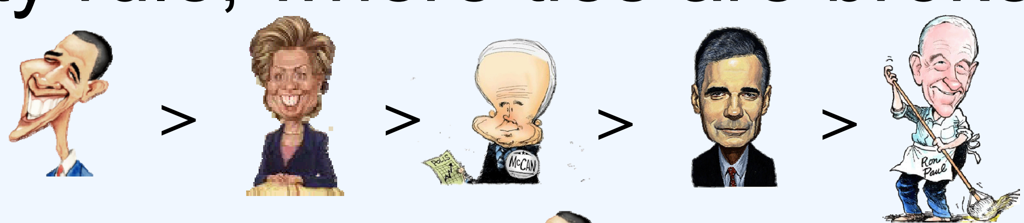
50 votes $x>y>z$		31 votes $x>y>z$
30 votes $x>z>y$	$=$	21 votes $y>z>x$
70 votes $y>x>z$		0 votes $z>y>x$
10 votes $z>x>y$		
(80, 70, 10)		(31, 21, 0)
x y z		x y z

- This equivalence relationship is captured in a concept called *compilation complexity* [Chevaleyre et al. IJCAI-09, Xia & C. AAAI-10]

Paradoxes



- Plurality rule, where ties are broken according to



- The SG_{Plu} winner is 
- Paradox: the SG_{Plu} winner is ranked almost in the bottom position in all voters' true preferences

What causes the paradox?

- Q: Is it due to the bad nature of the plurality rule / tiebreaking, or it is because of the strategic behavior?
- A: The strategic behavior!
  - by showing a ubiquitous paradox

Domination index

- For any voting rule r, the **domination index** of r when there are n voters, denoted by $\mathrm{DI}_r(n)$, is:
- the smallest number k such that for any alternative c, any coalition of $n/2+k$ voters can guarantee that c wins.
  - The DI of any **majority consistent** rule r is 1, including any Condorcet-consistent rule, plurality, plurality with runoff, Bucklin, and STV
  - The DI of any positional scoring rule is no more than

$$n/2\text{-}n/m$$

  - Defined for a voting rule (not for the voting game using the rule)
  - Closely related to the **anonymous veto function** [Moulin 91]

Main theorem (ubiquity of paradox)

- **Theorem 1**: For any voting rule r and any n, there exists an n-profile P such that:
  - (*many voters are miserable*) $SG_r(P)$ is ranked somewhere in the bottom two positions in the true preferences of n-2$\cdot\mathrm{DI}_r(n)$ voters
  - (*almost Condorcet loser*) if $\mathrm{DI}_r(n) < n/4$, then $SG_r(P)$ loses to all but one alternative in pairwise elections.

Proof

- **Lemma:** Let P be a profile. An alternative d is **not** the winner $SG_r(P)$ if there exists another alternative c and a subprofile $P_k = (V_{i_1}, \ldots, V_{i_k})$ of P that satisfies the following conditions: (1) $k \geq \lfloor n/2 \rfloor + \mathrm{DI}_r(n)$, (2) $c>d$ in each vote in P_k, (3) for any $1 \leq x < y \leq k$, $\mathrm{Up}(V_{i_x}, c) \supseteq \mathrm{Up}(V_{i_y}, c)$, where $\mathrm{Up}(V_{i_x}, c)$ is the set of alternatives ranked higher than c in V_{i_x}

$$V_1 = \ldots = V_{\lfloor n/2 \rfloor - \mathrm{DI}_r(n)} = [c_3 > \ldots > c_m > c_1 > c_2]$$

$$V_{\lfloor n/2 \rfloor - \mathrm{DI}_r(n)+1} = \ldots = V_{\lfloor n/2 \rfloor + \mathrm{DI}_r(n)} = [c_1 > c_2 > c_3 > \ldots > c_m]$$

$$V_{\lfloor n/2 \rfloor + \mathrm{DI}_r(n)+1} = \ldots = V_n = [c_2 > c_3 > \ldots > c_m > c_1]$$

- c_2 is not a winner (letting $c = c_1$ and $d = c_2$ in the lemma)

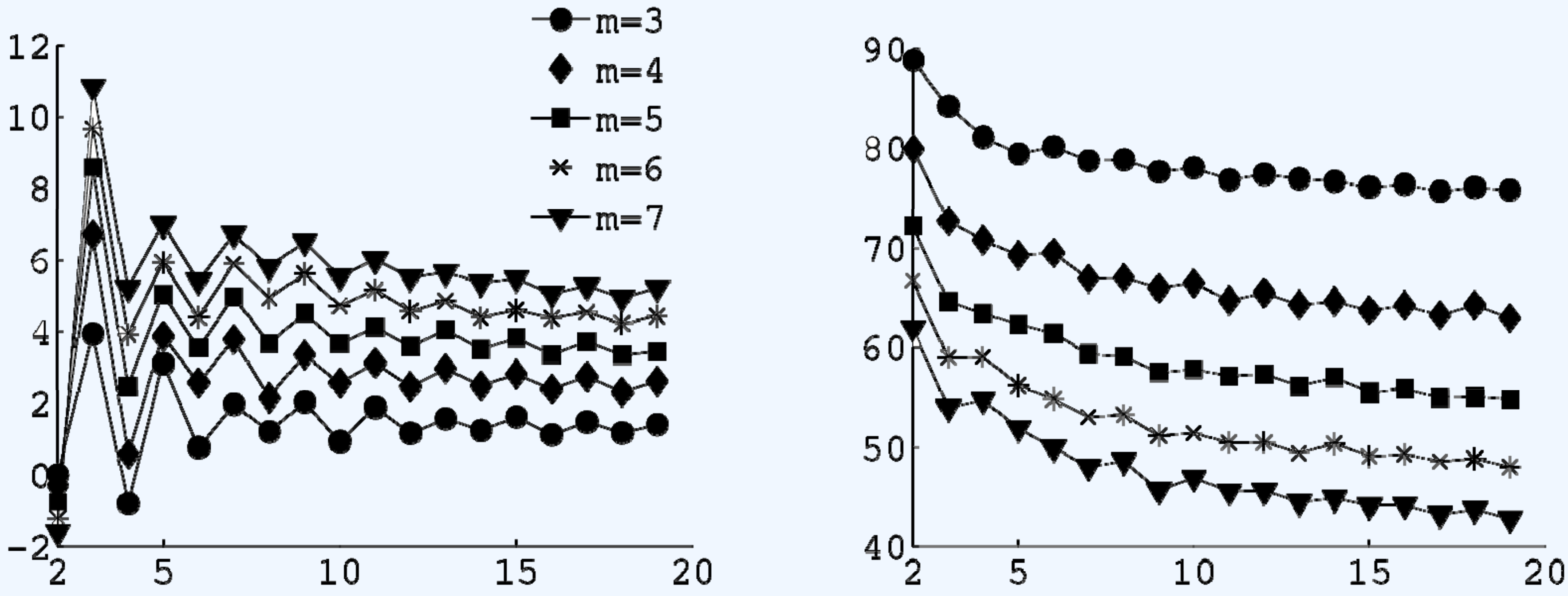
- For any $i \geq 3$, c_i is not a winner (letting $c = c_2$ and $d = c_i$ in the lemma)

What do these paradoxes mean?

- These paradoxes state that for any rule r that has a low domination index, *sometimes* the backward-induction outcome of the Stackelberg voting game is undesirable
  - the DI of any majority consistent rule is 1
- Worst-case result
- Surprisingly, on average (by simulation)
  - # { voters who prefer the SG_r winner to the truthful r winner}

    > # { voters who prefer the truthful r winner to the SG_r winner}

Simulation results



(a) (b)

- Simulations for the plurality rule (25000 profiles uniformly at random)
  - x-axis is #voters, y-axis is the percentage of voters
  - (a) percentage of voters where $SG_r(P) > r(P)$ minus percentage of voters where $r(P) > SG_r(P)$
  - (b) percentage of profiles where the $SG_r(P) = r(P)$
- SG_r winner is preferred to the truthful r winner by more voters than vice versa
  - Whether this means that SG_r is “better” is debatable

Interesting questions

- How can we compute the winner or ranking more efficiently?
- How can we communicate the voters' preferences more efficiently?
- How can we use computational complexity as a barrier against manipulation and control?
- How can we analyze agents' strategic behavior from a game-theoretic perspective?
- How can we aggregate voters' preferences when the set of alternatives has a combinatorial structure?

Outline

- Stackelberg Voting Games: Computational Aspects and Paradoxes

TOPIC CHANGE!

- Strategic Sequential Voting in Multi-Issue Domains and Multiple-Election Paradoxes

Voting over joint plans

[Brams, Kilgour & Zwicker SCW 98]

- The citizens of LA county vote to directly determine a government plan
- Plan composed of multiple sub-plans for several issues
  - E.g.,

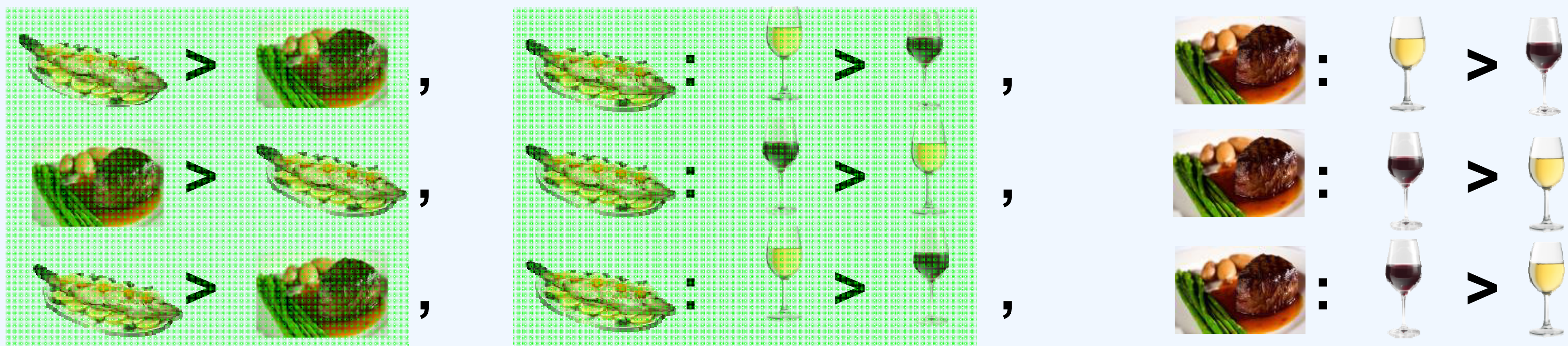
- # of candidates is exponential in the # of issues

Combinatorial voting: Multi-issue domains

- The set of candidates can be uniquely characterized by multiple issues
- Let $I=\{x_1,\ldots,x_p\}$ be the set of p issues
- Let D_i be the set of values that the i-th issue can take, then $C=D_1\times\ldots\times D_p$
- Example:
  - Issues={ Main course, Wine }
  - Candidates={   } $\times$ {   }

Sequential rule: an example

- Issues: main course, wine
- Order: main course > wine
- Local rules are majority rules
- V_1:
- V_2:
- V_3:



- **Step 1:** 
- **Step 2: given**  **,**  **is the winner for wine**
- **Winner: (**  **,**  **)**

Strategic sequential voting (SSP)

- Binary issues (two possible values each)
- Voters vote simultaneously on issues, one issue after another according to O
- For each issue, the majority rule is used to determine the value of that issue
- Game-theoretic aspects:
  - A complete-information extensive-form game
  - The winner is unique (computed via backward induction)

Strategic sequential voting: Example

S **T**

$V_1 : st > \bar{s}t > s\bar{t} > \bar{s}\bar{t}$

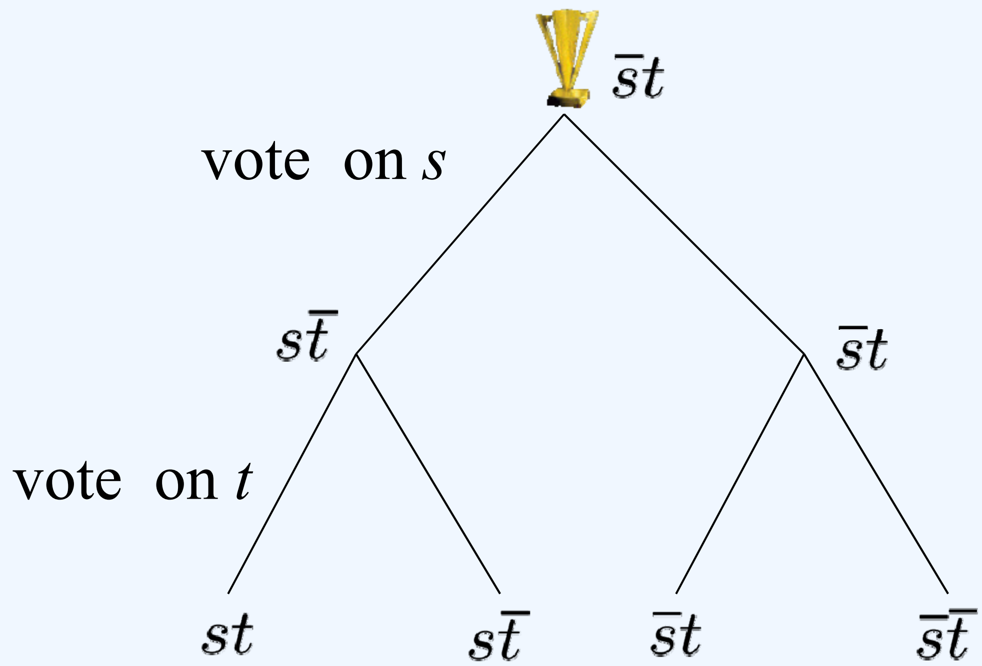
$V_2 : s\bar{t} > st > \bar{s}t > \bar{s}\bar{t}$

$V_3 : \bar{s}t > \bar{s}\bar{t} > s\bar{t} > st$

- In the first stage, the voters vote simultaneously to determine **S**; then, in the second stage, the voters vote simultaneously to determine **T**
- If **S** is built, then in the second step $t > \bar{t},\ \bar{t} > t,\ \bar{t} > t$ so the winner is $\boxed{s\bar{t}}$
- If **S** is **not** built, then in the 2nd step $t > \bar{t},\ t > \bar{t},\ t > \bar{t}$ so the winner is $\boxed{\bar{s}t}$
- In the first step, the voters are effectively comparing $s\bar{t}$ and $\bar{s}t$, so the votes are $\bar{s} > s,\ s > \bar{s},\ \bar{s} > s$, and the final winner is $\bar{s}t$

Voting tree

- The winner is the same as the (truthful) winner of the following *voting tree*



- "Within-state-dominant-strategy-backward-induction"
- Similar relationships between backward induction and voting trees have been observed previously [McKelvey&Niemi JET 78], [Moulin Econometrica 79], [Gretlein IJGT 83], [Dutta & Sen SCW 93]

Paradoxes: overview

- Strong paradoxes for strategic sequential voting (SSP)
- Slightly weaker paradoxes for SSP that hold for any O (the order in which issues are voted on)
- Restricting voters' preferences to escape paradoxes

Multiple-election paradoxes for SSP

- **Main theorem** (informally). For any $p \geq 2$ and any $n \geq 2p^2 + 1$, there exists an n-profile such that the SSP winner is
  - Pareto dominated by almost every other candidate
  - ranked almost at the bottom (exponentially low positions) in every vote
  - an almost Condorcet loser
- Other multiple-election paradoxes:

  [Brams, Kilgour & Zwicker SCW 98], [Scarsini SCW 98], [Lacy & Niou JTP 00], [Saari & Sieberg 01 APSR], [Lang & Xia MSS 09]

Is there any better choice of the order O?

- **Theorem** (informally). For any $p \geq 2$ and $n \geq 2^{p+1}$, there exists an n-profile such that for **any** order O over $\{x_1, \ldots, x_p\}$, the SSP_O winner is ranked somewhere in the bottom $p+2$ positions.
  - The winner is ranked almost at the bottom in every vote
  - The winner is still an almost Condorcet loser
  - I.e., at least some of the paradoxes cannot be avoided by a better choice of O

Getting rid of the paradoxes

- **Theorem(s)** (informally)
  - Restricting the preferences to be separable or lexicographic gets rid of the paradoxes
  - Restricting the preferences to be O-legal does not get rid of the paradoxes

Paradoxes for other voting rules

- **Theorem(s)** (informally) When voters vote truthfully, there are no multiple-election paradoxes for dictatorships, plurality with runoff, STV, Copeland, Borda, Bucklin, k-approval, and ranked pairs

Agenda control

- **Theorem.** For any $p \geq 4$, there exists a profile P such that any alternative can be made to win under this profile by changing the order O over issues
  - When p=1, 2 or 3, all p! different alternatives can be made to win
  - The chair has full power over the outcome by agenda control (for this profile)

Summary of SSP

- We analyze voters' strategic behavior when they vote on binary issues sequentially
- The strategic outcome coincides with the truthful winner of a specific voting tree
  - cf. [McKelvey&Niemi JET 78], [Moulin Econometrica 79], [Gretlein IJGT 83], [Dutta & Sen SCW 93]
- We illustrated several types of multiple-election paradoxes to show the cost of the strategic behavior
- We further show a contrast with the truthful common voting rules; this provides more evidence that the paradoxes come from the strategic behavior
- Combinatorial voting is a promising and challenging direction!

Conclusion

- “Sequential” voting games (either voters or issues sequential) avoid equilibrium selection issues
- Paradoxes: Outcomes can be bad (in the worst case)

Thank you for your attention!

Proper Scoring Rules & Peer Prediction

Jens Witkowski
University of Pennsylvania

CPS 290.4/590.4: Crowdsourcing Societal Tradeoffs

Duke University

April 3, 2015

Overview

1. Proper Scoring Rules
   - Bonus: Prediction Polls
2. Classical Peer Prediction

CS-Econ Seminar (another bonus): Robust Peer Prediction

Proper Scoring Rules

Proper Scoring Rules [Brier, 1950]

Truthfully elicit beliefs about publicly observable events.

The Good Judgment Project™

#1244 Will India and/or Brazil become a permanent member of the U.N. Security Council before March 1 ?

A: Yes

B: No

1. Agent reports belief $y \in [0, 1]$ of event occurring.
2. March 1: pay $R(y, \omega)$, where $\omega = \begin{cases} 1, & \text{if event occurs} \\ 0, & \text{if not.} \end{cases}$

Naive Approach: Linear Scoring Rule

Linear Scoring Rule

$$R_l(y,\omega) = \begin{cases} y, & \text{if } \omega = 1 \\ 1-y, & \text{if } \omega = 0 \end{cases}$$

True belief $p = 0.6$.

Expected score: $0.6 \cdot y + 0.4 \cdot (1-y) = 0.4 + 0.2y$

$\Rightarrow y = 1 \neq p$ maximizes expected score.

Linear Rule not proper!

Quadratic Scoring Rule

Quadratic Scoring Rule

$R_q(y, \omega) = 1 - (y - \omega)^2$

True belief: $p = 0.6$.

Expected score:

$$
\begin{aligned}
& p \cdot R_q(y, 1) + (1 - p) \cdot R_q(y, 0) \\
= \; & 0.6 \cdot \left(1 - (y - 1)^2\right) + 0.4 \cdot \left(1 - (y - 0)^2\right) \\
= \; & -y^2 + 1.2y + 0.4
\end{aligned}
$$

Derive and set to 0: $\quad -2y + 1.2 := 0 \Leftrightarrow y = 0.6$

Quadratic Rule is proper: $y = p$ maximizes expected score!

Bonus: Prediction Polls^{TM}

Good Judgment Project (GJP)

- Forecasting tournament for geo-political questions.
- ~10,000 active forecasters.
- ~140 questions / year.
- Prediction markets and proper scoring rules.
- (Mostly) play money (leaderboard).

Probability elicitation in real world:

- Many forecasters: aggregation?
- Not one-shot: beliefs are continuously updated.
- Not every forecaster reports on every question.
- Not every question has same duration.

How do you translate Proper Scoring Rules into the real world?

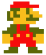
Forecast Aggregation in GJP Prediction Polls

1. Take weighted average:
    - Current score (closed questions): previous accuracy.
    - Frequency of updates: ~effort.
    - Only k most recent forecasts: robustness vs novelty.
2. Extremize:
    - If average $< 0.5 \Rightarrow$ push towards 0.
    - If average $> 0.5 \Rightarrow$ push towards 1.

Extremizing: Intuition

Probability of Heads (H) for biased ?

- Before observing flip: $p(H) = 0.5$
- Two forecasters observe flip and report:

 $p_1(H) = 0.7$  $p_2(H) = 0.7$

- Aggregated forecast:

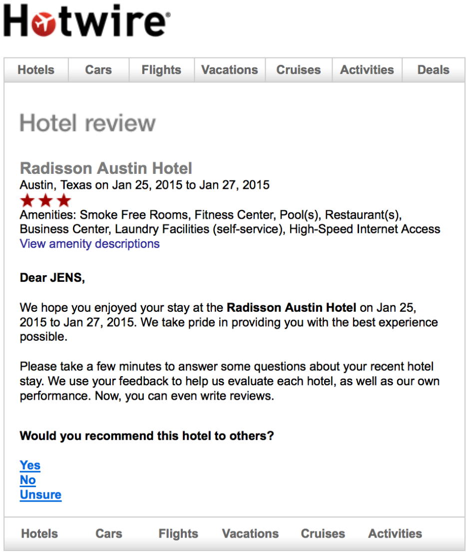
  Same coin flip $\Rightarrow p_{1,2}(H) = 0.7$

  Same coin, different flips $\Rightarrow p_{1,2}(H) > 0.7$

Less information overlap $\Rightarrow$ more extremizing!

Peer Prediction

Motivation: Information Elicitation



Hotwire

Hotels | Cars | Flights | Vacations | Cruises | Activities | Deals

Hotel review

Radisson Austin Hotel
Austin, Texas on Jan 25, 2015 to Jan 27, 2015
★★★
Amenities: Smoke Free Rooms, Fitness Center, Pool(s), Restaurant(s), Business Center, Laundry Facilities (self-service), High-Speed Internet Access
View amenity descriptions

Dear JENS,

We hope you enjoyed your stay at the **Radisson Austin Hotel** on Jan 25, 2015 to Jan 27, 2015. We take pride in providing you with the best experience possible.

Please take a few minutes to answer some questions about your recent hotel stay. We use your feedback to help us evaluate each hotel, as well as our own performance. Now, you can even write reviews.

Would you recommend this hotel to others?

Yes
No
Unsure

Hotels | Cars | Flights | Vacations | Cruises | Activities

Motivation: Information Elicitation

The Good Judgment Project™

#1244 Will India and/or Brazil become a permanent member of the U.N. Security Council before March 1 ?

A: Yes

B: No

Comment ID: 225814, assigned: 09/01/14

Unless the Presidential winner is resolved quickly, there is no chance the Security Agreement will be signed in Wales at the September 4-5 NATO meeting. Audit results are now delayed until at least Sep. 10 and both sides have pulled their observers out of the audit process. Searches for a unity government agreement have also gone nowhere and are unlikely to be successful anyhow. Chances of signing before November 1 seem low. Efforts are under way to find a way to keep American forces in Afghanistan beyond year-end without a security agreement and that would seems to rapidly be becoming the only option short of pulling out troops.

How useful is this comment?

- 1. Not at all Useful (No use of CHAMPS KNOW)
- 2. Slightly Useful
- 3. Useful
- 4. Very Useful
- 5. Extremely Useful (Great Integration of CHAMPS KNOW)

Submit

Motivation: Information Elicitation

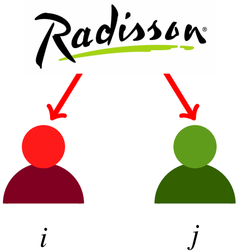
Research Questions

1. How can opinions or experiences be elicited truthfully?
2. How can we incentivize effort for information acquisition?

Basic Setup

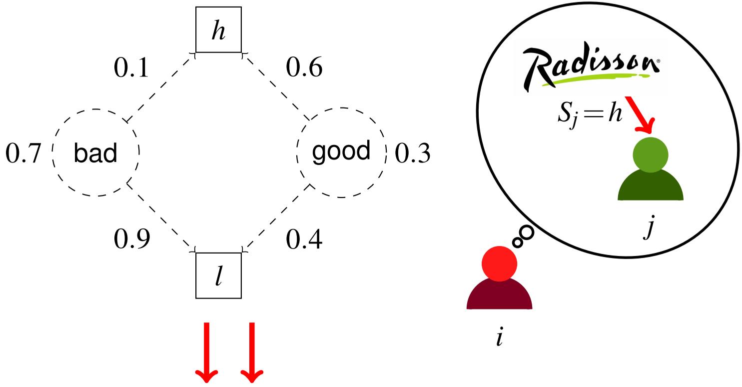
- Elicit informative signal (e.g. “high” or “low” experience).
- Ground truth never observed (e.g. true quality of hotel).
- Allow for payments.

Agents experience same environment:



Key assumption: signals are correlated!

Belief Model (Common Knowledge)



Agent i's belief that agent j observes h:

$p(h|l) = 0.18$

$p(h|h) = 0.46$

Minority Opinions

Agent i's belief that agent j observes h:

$p(h|l) = 0.18$

$p(h|h) = 0.46 \leftarrow$ minority opinion: $p(h|h) < p(l|h)$

Is Chicago capital of Illinois? [Prelec and Seung, 2006]

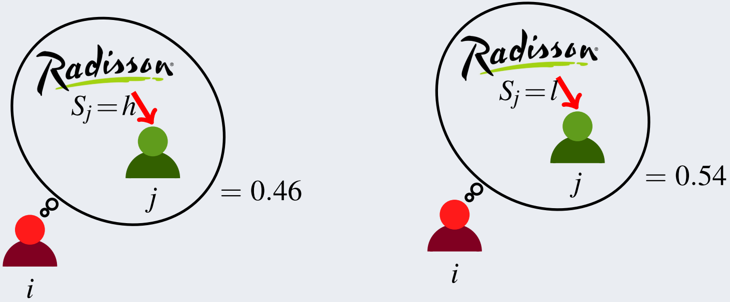
People who know it's not, still believe they're in the minority.

Peer prediction mechanisms elicit minority opinions truthfully!

Output Agreement

Compare two agents' reports and pay: $\begin{cases} \$2 & \text{if reports agree,} \\ \$0 & \text{otherwise.} \end{cases}$

Example with $S_i = h$

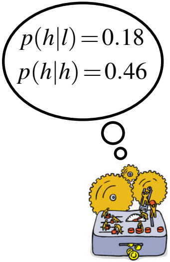


$\mathbf{E}\left[\text{payment}\right]$ reporting h: \$0.92 $\mathbf{E}\left[\text{payment}\right]$ reporting l: \$1.08

Output Agreement not truthful!

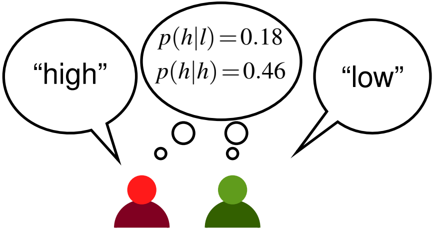
Classical Peer Prediction [Miller et al., 2005]

Mechanism



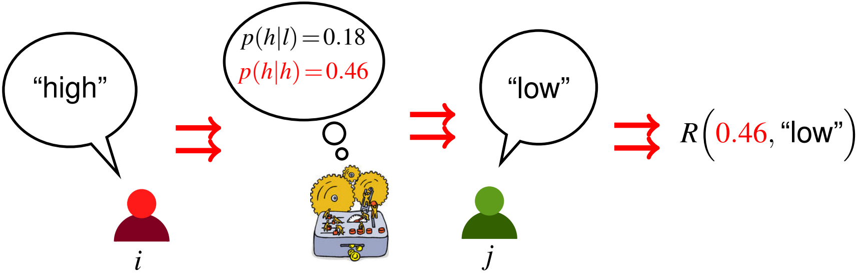
- Knows belief model.

Agents



- Share same belief model.
- Report: Signal.

Classical Peer Prediction: Mechanism



Intuition

1. Define agent j's signal report as event.
2. Restrict possible belief reports to possible posteriors.

Crucial: mechanism knows how to transform signal to belief!

Subsequent Work in Peer Prediction

- Linear Programming formulation [Jurca and Faltings, 2006]
- Collusion-inhibiting mechanisms [Jurca and Faltings, 2009]
- Multiple equilibria unavoidable [Waggoner and Chen, 2014]
- Mechanisms not needing to know belief model [Prelec, 2004, W. and Parkes, 2012a, Radanovic and Faltings, 2013]
- Mechanisms for subjective prior beliefs [W. and Parkes, 2012b, 2013]
- Effort incentives [W. et al, 2013]

Summary

1. Proper Scoring Rules: elicit probabilistic forecasts.
2. Prediction Polls: aggregate forecasts in real-world system.
3. Peer Prediction: elicit opinions, experiences, or ratings.

CS-Econ: Peer Prediction with relaxed common knowledge!

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(S)electing subsets

CPS 290.4/590.4: Crowdsourcing Societal Tradeoffs

April 8, 2015

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Representation

- limited resources:
  - single representative / choice
  - multiple representatives / set of choices

- “optimal” resource allocation:
  - objective(s):
    - winner-takes-all
    - maximizing total welfare/value
    - minimizing regret/loss/cost
    - fairness?

Representation

- parliaments, assemblies
- committees, company boards, project teams
- product assortment, product bundles

Stylized (simplistic) setting:

- m candidates: $a, b, c, \ldots$
- n single-minded choices $n = n_a + n_b + n_c + \ldots$

  (w.l.o.g. assume $n_a \geq n_b \geq n_c \geq \ldots$)
- k slots to be assigned to m candidates

Proportional Representation

Total of n choices for k slots: n/k choices per slot

Quota:

Assign candidate *j* either

$LQ = INT(n_j/(n/k))$ or $UQ = 1 + INT(n_j/(n/k))$

- **The method of largest reminders**
  (a.k.a., Hamilton's method, Vinton's method)

  1. assign each candidate j: $LQ = INT(n_j/(n/k))$ slots

  2. assign remaining $n - \sum INT(n_j/(n/k))$ slots to candidates with largest reminder values

  $$n_j/(n/k) - INT(n_j/(n/k))$$

US Constitution Article I, Section 2

- …the House Representatives be *apportioned* among the several States according to their respective populations;
- …the number of Representatives shall not exceed one for every 30,000 persons;
- …each State shall have at least one Representative;
- …the reapportionment shall occur once every 10 years as a result of the decennial census.

Apportionment of the seats
in the US House of Representatives

Fairness of the largest reminders approach?

- The Alabama Paradox

- The New State Paradox

- The Population Paradox

The Alabama Paradox

Apportionment based on 1880 Census:

With 299 seats: Alabama gets 8 seats (quota of 7.646)

With 300 seats: Alabama gets 7 seats (quota of 7.671)

A candidate (state) can lose a slot (seat) if the number of slots increases.

- other reminders become larger
- could construct smaller examples (with three candidates)

The New State Paradox

Oklahoma became a state in 1907. The house had 386 seats. Given Oklahoma population adding to the US population total at that time, Oklahoma should get 5 seats and the House should increase to 386+5=391 seats.

When the method of largest reminders was reapplied, Maine moved from 3 to 4 seats, while New York moved from 38 to 37 seats.

Adding a new candidate and additional choices supporting the candidate (without changing support for others), slot assignments of existing candidates can change.

The Population Paradox

According to 1900 Census, Virginia had a significantly higher (absolute and percentage) population growth than Maine.

	1900	1901 (est.)
Virginia	1,854,184	1,873,951 (1.07% increase)
Maine	694,466	699,114 (0.67% increase)
Virginia	**10** (9.599 quota)	**9** (9.509 quota)
Maine	**3** (3.595 quota)	**4** (3.548 quota)

Candidate whose support increases at a faster (absolute and percentage) rate could lose slots to a candidate whose support increases at a slower rate.

An Impossibility Theorem

Theorem (Balinski & Young 1982)
There exists no method that allocates k slots to $m>2$ candidates and
(1) satisfies Quota,
(2) avoids the Alabama Paradox,
(3) avoids the New State Paradox,
(4) avoids the Population Paradox.

- In fact the theorem holds with (1) and (4) only (Also, with (1) and (3) only.)
- If one can choose $k' \geq k$, then a method can be constructed
- Plenty of methods satisfying (2),(3) and (4).

Divisor methods

- Instead of specifying n/k rate per slot, choose rate d

1. Pick a rounding rule r: $\boldsymbol{N} \rightarrow \boldsymbol{R}$,

   if $x < r(INT(x))$ then round down to $INT(x)$

   if $x \geq r(INT(x))$ then round up to $1+INT(x)$

2. Pick divisor d

3. Assign slots to each candidate j according to $r(INT(n_j/d))$

- Pick divisor d so that exactly k slots are assigned.

   (not unique)

- Divisor methods avoid all three paradoxes by construction
- For any divisor method there is an instance that violates quota

Webster's method

(a.k.a. Sainte-Laguë, major fractions)

1. Use a standard rounding rule r: $\boldsymbol{N} \rightarrow \boldsymbol{R}$,

if $x < INT(x)+0.5$ then round down to $INT(x)$

if $x \geq INT(x)+0.5$ then round up to $1+INT(x)$

2. Pick divisor d

3. Assign slots to each candidate j according to $r(INT(n_j/d))$

Example: k=4; $n_a= 61$, $n_b= 14$, $n_c= 13$, $n_a= 12$.

Use $d=26$ (but could also use $d=25$)

$a \rightarrow 61/26 = 2.35 \rightarrow 2$ slots,

$b \rightarrow 14/26 = 0.54 \rightarrow 1$ slot,

$c \rightarrow 13/26 = 0.50 \rightarrow 1$ slot,

$d \rightarrow 12/26 = 0.46 \rightarrow 0$ slots.

Jefferson's method

(a.k.a. d'Hondt, greatest divisors)

1. Always round down r: $\boldsymbol{N} \to \boldsymbol{R}$,

  if $x < INT(x)+1$ then round down to $INT(x)$

  if $x \geq INT(x)+1$ then round up to $1+INT(x)$

2. Pick divisor d

3. Assign slots to each candidate j according to $r(INT(n_j/d))$

Example: k=4; $n_a = 61$, $n_b = 14$, $n_c = 13$, $n_a = 12$.

  Use $d=15$

  $a \to 61/15 = 4.03 \to 4$ slots,

  $b \to 14/15 = 0.93 \to 0$ slots,

  $c \to 13/15 = 0.86 \to 0$ slots,

  $d \to 12/15 = 0.8 \to 0$ slots.

Huntington-Hill's method

(a.k.a. equal proportions)

1. Geomean-based rounding rule: r: $\boldsymbol{N} \rightarrow \boldsymbol{R}$,

if $x < SQRT((INT(x)*(INT(x)+1))$ then round down to $INT(x)$

if $x \geq SQRT((INT(x)*(INT(x)+1))$ then round up to $1+INT(x)$

2. Pick divisor d

3. Assign slots to each candidate j according to $r(INT(n_j/d))$

Example: k=4; $n_a = 61$, $n_b = 14$, $n_c = 13$, $n_a = 12$.

Use $d=50$

$a \rightarrow 61/50 = 1.22\ (r=1.41) \rightarrow 1$ slot,

$b \rightarrow 14/50 = 0.28\ (r=0) \rightarrow 1$ slot,

$c \rightarrow 13/50 = 0.26\ (r=0) \rightarrow 1$ slot,

$d \rightarrow 12/50 = 0.24\ (r=0) \rightarrow 1$ slot.

Turbulent US history

- All four methods have been used for apportionment of US House seats at different times
- Cause of the very first presidential veto (1791)
- Cause of the incorrect result of the presidential election in 1876 (Hayes vs. Tilden, due to incorrect apportionment four years earlier)
- Cause of a direct violation of the constitution in 1921
- Huntington-Hill used since 1941, survived the constitutional challenge (upheld by the Supreme Court in 1992)

Divisor methods

- Jefferson's (d'Hondt) favors large n_j
  - desirable property for seat assignment in parliamentary elections? (majority building), used in many countries
- Webster (Sainte-Laguë) is claimed to introduce least bias, used in many countries (e.g., Germany)
- Huntington-Hill:
  -guarantees a slot to each candidate
  -minimizes pairwise discrepancies in the number of votes per slot among candidates
- Intense debate (and theorems) on
  Webster vs. Huntington-Hill
  (arithmetic vs. geometric mean)

Other Proportional Representation Methods

- Winner-takes-all: assign all k slots to candidate a
- Generate n_a, n_b, n_c, ... by other methods: scoring, weighted voting, approval, etc.
- Minimize misrepresentation (Monroe 1995, Chamberlain and Courant 1983):
  - Assign voters to each of k slots.
  - For each voter calculate an individual degree of misrepresentation given candidate allocated that slot
  - Aggregate total misrepresentation for a slot (sum of individual misrepresentations, weighted by #voters assigned to a slot)
- Can be described as a huge IP (Potthoff and Brams, 1998), NP-complete (Proccacia et al, 2008)

Subset (S)election

- Proportional representation:

  k slots (seats) > m candidates (parties, states)

- **What if $k < m$?**

  Standard (s)election/choice arguments/methods impossibility results apply.

  Can think of all possible 2^m subsets (or just allowed ones, e.g., all subsets of prespecified size k) as available alternatives and elicit preferences over those.

  $\Rightarrow$ Exponential blow-up

  However, new combinatorial issues emerge.

Subset (S)election

⇒ choosing a subset **S**
from the set of ***m*** available alternatives

⇒ choosing a feasible (admissible) subset **S**

- **social choice**
- **voting**
- **multi-criteria decision-making**
- **consumer choice (?)**

Social Choice

There are ***n*** individuals, each having preferences over ***m*** alternatives.

- **How to aggregate preferences into a "consensus" preference structure?**

- **Arrow's Impossibility Theorem**

  - Independence of Irrelevant Alternatives (IIA)

  - Inherent multidimensionality (single-peaked prefs.)

Choosing a subset?

Voting

There are ***n*** voters, each having preferences over ***m*** candidates.

- **How to aggregate preferences and determine the winner?**

- **Gibbard-Satterhwaite Theorem**

  - IIA implying Arrow's result

  or

  - manipulability

Choosing a subset?

Multi-Criteria Decision-Making (MCDM)

There are **n** criteria, each defining a preference structure over **m** alternatives

- **How to aggregate preferences and determine the "consensus" preference structure over alternatives?**

Choosing a subset?

Consumer Choice?

- **Behavioral aspects dominate**

(Normative approach: multicriteria decision-making)

BUT…

Automated consumer choice suggestions

- pagerank in search results, ad placement, etc.

- suggested product (e.g., credit cards, computers)

….

Choosing a subset?

Consumer Choice?

- **buying a product bundle**

  - related products (e.g., media system components)

  - subset of extra options (cars)

  - features of a highly customized commodity-like products (laptops, smartphones, software,…)

- **multiple criteria** (functionality, looks, safety, price, …)

Choosing a single alternative

Information requirement on voters' preferences

- SWF $\Rightarrow$ rankings
- plurality $\Rightarrow$ top choice
- prediction, betting, scoring rules $\Rightarrow$ constrained cardinal utility (IIA???)
- **approval voting $\Rightarrow$ subset choice**

CHOSING A SINGLE ALTERNATIVE

	V1	V2	V3	V4	V5	V6	V7	V8	V9
1	2	8	3	6	1	7	8	5	5
2	1	7	8	1	3	1	7	1	6
3	7	1	5	4	2	8	5	2	2
4	5	4	1	8	8	3	4	8	7
5	4	3	6	3	7	4	3	3	4
6	6	5	7	2	6	5	1	4	8
7	8	2	2	7	5	6	2	6	1
8	3	6	4	5	4	2	6	7	3

CHOSING A SINGLE ALTERNATIVE

APPROVAL

	V1	V2	V3	V4	V5	V6	V7	V8	V9
1	2	8	3	6	1	7	8	5	5
2	1	7	8	1	3	1	7	1	6
3	7	1	5	4	2	8	5	2	2
4	5	4	1	8	8	3	4	8	7
5	4	3	6	3	7	4	3	3	4
6	6	5	7	2	6	5	1	4	8
7	8	2	2	7	5	6	2	6	1
8	3	6	4	5	4	2	6	7	3

CHOSING A SINGLE ALTERNATIVE

APPROVAL

	V1	V2	V3	V4	V5	V6	V7	V8	V9
1		✓				✓	✓		
2		✓	✓				✓		
3	✓					✓			
4				✓	✓			✓	✓
5					✓				
6		✓							✓
7	✓			✓					
8		✓						✓	

CHOSING A SINGLE ALTERNATIVE

APPROVAL

	V1	V2	V3	V4	V5	V6	V7	V8	V9	
1	**0**	**1**	**0**	**0**	**0**	**1**	**1**	**0**	**0**	**3**
2	**0**	**1**	**1**	**0**	**0**	**0**	**1**	**0**	**0**	**3**
3	**1**	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**2**
4	**0**	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**	**4**
5	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**1**
6	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**2**
7	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**2**
8	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**2**

APPROVAL VOTE PROFILE

	V1	V2	V3	V4	V5	V6	V7	V8	V9
1		✓				✓	✓		
2		✓	✓				✓		
3	✓					✓			
4				✓	✓			✓	✓
5					✓				
6		✓							✓
7	✓			✓					
8		✓						✓	

37, 1268, 2 , 47 , 45 , 13 , 12 , 48 , 46

APPROVAL VOTE PROFILE

	V1	V2	V3	V4	V5	V6	V7	V8	V9	
V=(	**37,**	**1268,**	**2**	**, 47 ,**	**45 ,**	**13 ,**	**12 ,**	**48 ,**	**46**	**)**

CHOSING A SUBSET

Information requirement on voters' preferences

- SWF $\Rightarrow$ rankings on all 2^m subsets
- plurality $\Rightarrow$ top choice among all 2^m subsets
- scoring rules $\Rightarrow$ constrained card. utility on all 2^m subsets
- approval voting $\Rightarrow$ subset choice on all 2^m subsets

CHOSING A SUBSET

- using “consensus” ranking of alternatives

  but

  for all voters: 1>2>3 or 2>1>3

  AND

  13>23>12 or 23>13>12

- divide and conquer:

  break into several separate singleton choices

- proportional representation

IGNORING INTERDEPENDANCIES
(substitutability and complementarity)

CHOSING A SUBSET

- Barbera et al. (ECA91): impossibility

A manageable scheme that accounts for interdependencies?

Proposal: Approval Voting with modified subset count.

Threshold Approach:

- define $t(S)$ for every feasible S

- $AC_t(S)$= # of voters i such that $|Vi \cap S| \geq t(S)$

AV THRESHOLD APPROACH

Define t(S) for every feasible S

$$AC_t(S) = \#\text{ of voters } i \text{ such that } V_iS = |Vi \cap S| \geq t(S)$$

Threshold functions (TF):

- t(S)=1 (minimal representation)
- t(S) = |S|/2 (majority)
- t(S) = (S|+1)/2 (strict majority)
- t(S) = |S| (unanimity)

….

COMPLEXITY of AVCT

- If X= the set of all feasible subsets, is part of the input then computing AVCT winner is polynomial in $mn+|X|$

Theorem.

If X is predetermined (not part of the input), then computing AVCT winner is NP-complete at best.

Proof: choosing a k-set, $t\equiv 1$. Suppose $|V_i|=2$ for all i.

Note: alternatives ~ vertices of a graph

V_i ~ edges of a graph

k-set approved by all voters ~ vertex cover of size k

Vertex Cover is a fundamental NP-complete problem.

COMPLEXITY cont'd

- not as problematic as it seems.

Theorem. (Garey-Johnson)

If X is predetermined (not part of the input), then computing

$$\mathbf{max}_{S \text{ in} X}\ \mathbf{sum}_{i \text{ in} S}\ \mathbf{score(i)}$$

is NP-complete.

LARGER IS NOT BETTER

Example: m=8, n=12, strict majority TF: $t(S)=(|S|+1)/2$

V= (123,15,1578,16,278,23,24,34,347,46,567,568)

- 1-set (AC):

1,2,3,4,5,6,7 all approved by <u>4 voters</u> (8 is approved by 3 voters)

- 2-set:

15,23,34,56,57,58,78 all approved by <u>2 voters</u>

- 3-set: 234 approved by <u>5 voters</u>
- 4-set: 5678 approved by <u>3 voters</u>
- 5-set: 15678 approved by <u>4 voters</u>

TOP INDIVIDUAL NOT IN A TOP TEAM

Example: m=5, n=6, majority TF: t(S)=|S|/2

V= (123,124,135,145,25,34)

- Top individual:

1 approved by <u>4 voters</u> (all other alternatives approved by 3 voters)

- Top team

2345 is the only team approved by all <u>5 voters</u>

- could generalize examples for almost any TF

- could generalize to top k individuals

THRESHOLD SENSITIVITY

Theorem

For any $K>1$, there exist n,m and a corresponding V such that AVCT winner S_k (where X is the set of all K-sets), $k=1,\ldots,K$ are mutually disjoint.

ANY GOOD PROPERTIES?

P1. Nullity.

If every vote is the empty set, any choice is good.

P2. Anonymity.

If U is a permutation of V, the choices for U and V are identical.

P3. Partition Consistency.

If S is chosen in two voter disjoint elections, then S would be chosen in the joint election.)

P4. Partition Inclusivity.

If no S is chosen by a single voter and in an election of the remaining n-1 voters, then any choice would also be chosen in an election w/o one of the voters.

SINGLE VOTER PROPERTIES

$$\tau(S) = \min\{AS: S \text{ is a choice for } A\}$$

P5. For every choice S, there exists votes A and B such that A is a choice for S but not for B.

P6. Let S be a choice for vote A that does not choose everyone. If BS>AS then S is a choice for B

P7. For every S, there is an A such that $AS= \tau(S) -1$

P8. Suppose vote B chooses every committee. For all A_1, A_2 and for all choices S, T: If $A_1S= \tau(S)$, $A_2T= \tau(T)$, then $BS>A_1S$ implies $BT>A_2T$

Characterization Theorem

Theorem (Fishburn and P.)

If P1-8 hold, then the subset choice function is the AVCT.

AV THRESHOLD APPROACH

- low informational burden

- simplicity

- takes into account subset preferences

Results:

- properties of TFs, axiomatic characterization
- complexity
- robustness properties: theorems show what is possible and not what is probable

Need:

-Comparison with other methods, data validation
- strategic considerations

(S)electing subsets: Summary

n voters, *m* candidates, *k* slots

Proportional representation (m<k)

-quota, paradoxes, impossibility theorems

-multiple methods used in practice

Subset selection (m>k)

-standard voting and social choice results extend

-new combinatorial issues

-approval voting is a natural method (since it is subset-based)

-threshold approach reduces informational burden

-open field for other methods

A better rule for aggregating societal tradeoffs

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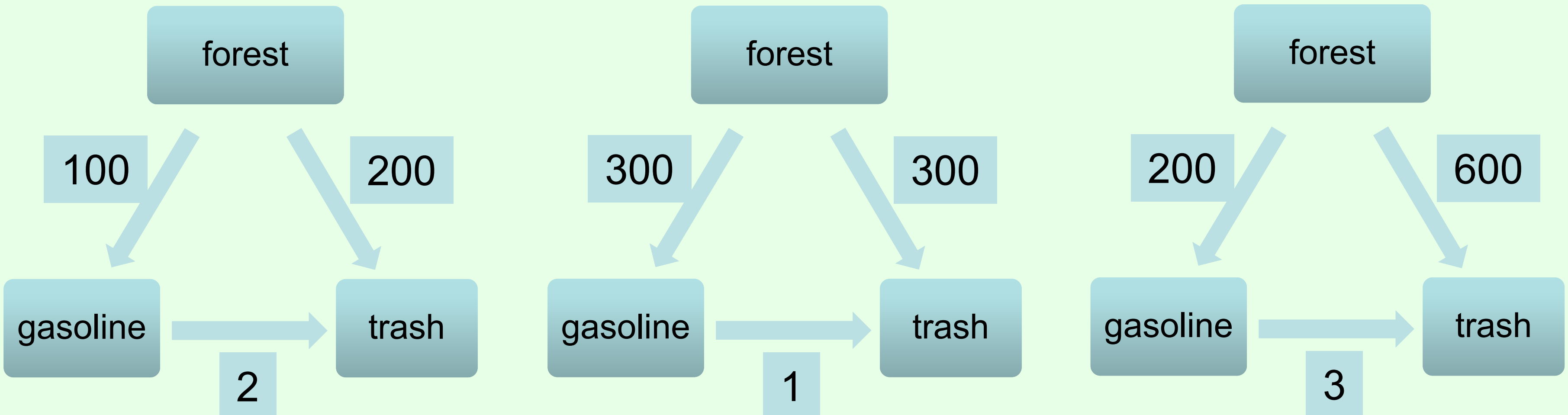
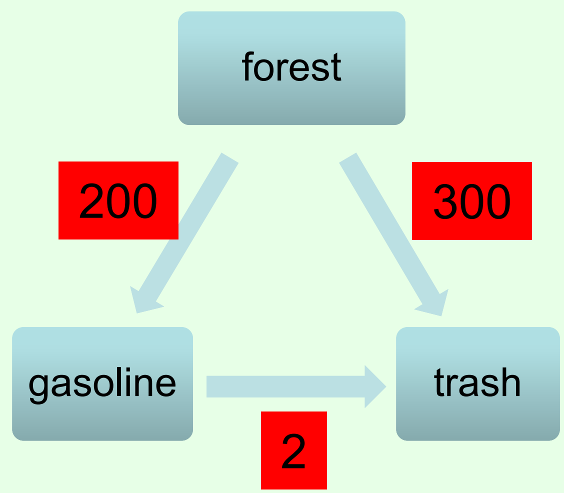
Recall our motivating example



Just taking medians pairwise results in inconsistency



Recall the rule from the midterm

- Let $t_{a,b,i}$ be voter i's tradeoff between a and b
- Tradeoff profile t has score

  $\Sigma_i \Sigma_{a,b} \mid t_{a,b} - t_{a,b,i} \mid$
- Upsides:
  - Coincides with median for 2 activities
- Downsides:
  - Dependence on choice of units:

    $\mid t_{a,b} - t_{a,b,i} \mid \neq \mid 2t_{a,b} - 2t_{a,b,i} \mid$
  - Dependence on direction of edges:

    $\mid t_{a,b} - t_{a,b,i} \mid \neq \mid 1/t_{a,b} - 1/t_{a,b,i} \mid$
  - We don't have a general algorithm

A generalization

- Let $t_{a,b,i}$ be voter i's tradeoff between a and b
- Let f be a monotone increasing function – say, $f(x) = x^2$
- Tradeoff profile t has score

  $\Sigma_i \Sigma_{a,b} \mid f(t_{a,b}) - f(t_{a,b,i}) \mid$
- Still coincides with median for 2 activities!

$t_{a,b}$ 1 **2** 3

$f(t_{a,b})$ 1 **4** 9

An MLE justification

- Suppose probability of tradeoff profile $\{t_i\}$ given true tradeoff t is

  $\prod_i \prod_{a,b} \exp\{-| f(t_{a,b}) - f(t_{a,b,i}) |\}$

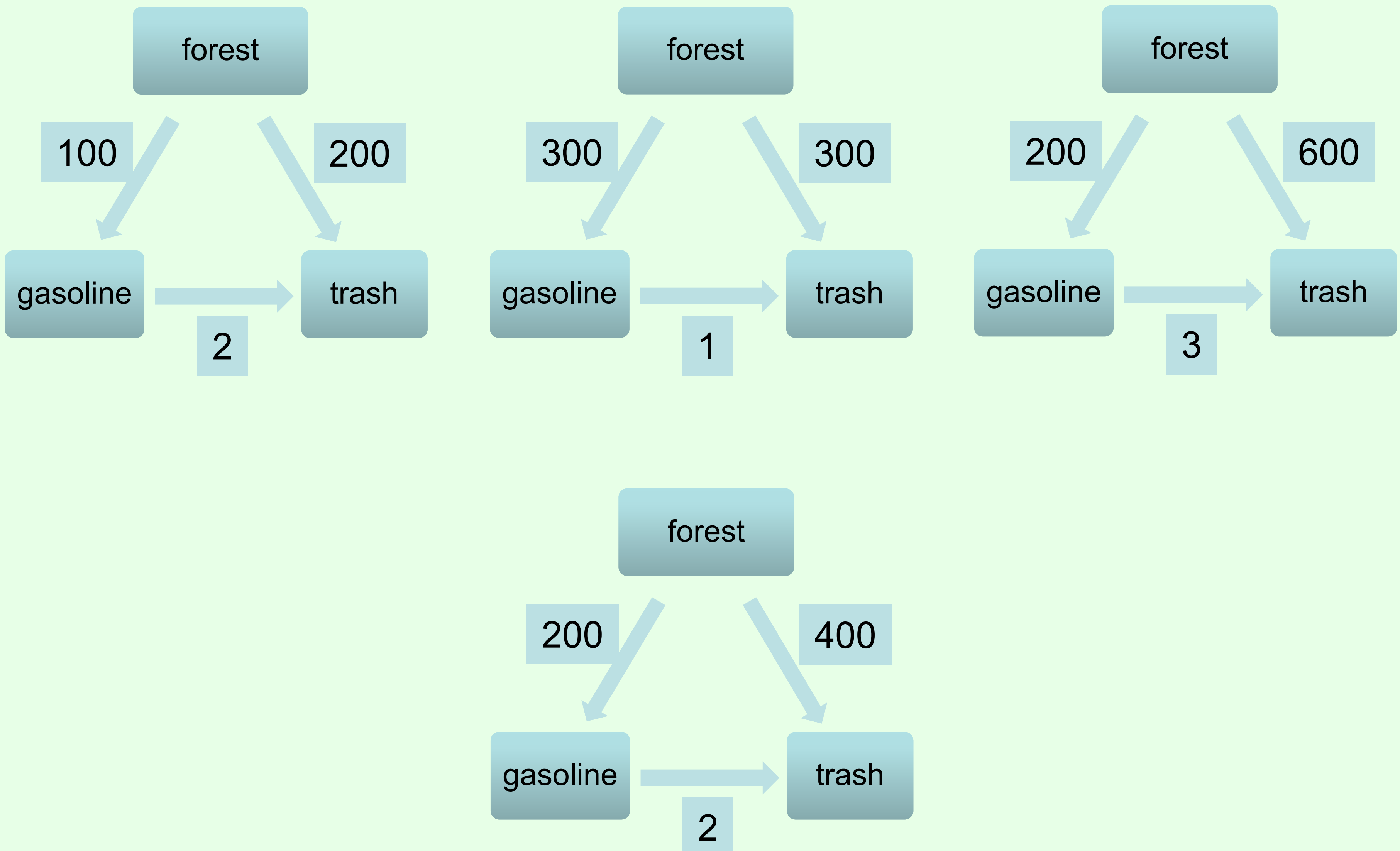
- Then $\arg\max_t \prod_i \prod_{a,b} \exp\{-| f(t_{a,b}) - f(t_{a,b,i}) |\} =$
  $\arg\max_t \log \prod_i \prod_{a,b} \exp\{-| f(t_{a,b}) - f(t_{a,b,i}) |\} =$
  $\arg\max_t \Sigma_i \Sigma_{a,b} -| f(t_{a,b}) - f(t_{a,b,i}) | =$
  $\arg\min_t \Sigma_i \Sigma_{a,b} | f(t_{a,b}) - f(t_{a,b,i}) |$
  which is our rule!

So what's a good f?

- Intuition: Is the difference between tradeoffs of 1 and 2 the same as between 1000 and 1001, or as between 1000 and 2000?
- So how about $f(x)=\log(x)$?
  - (Say, base e – remember $\log_a(x)=\log_b(x)/\log_b(a)$)

$t_{a,b}$	**1 2**		**1000**	**2000**
$\ln(t_{a,b})$	**ln(1)**	**ln(2)**	**ln(1000)**	**ln(2000)**
	0	0.69	6.91	7.60

On our example



Properties

- Independence of units

  $| \log(1) - \log(2) | = | \log(1/2) | =$

  $| \log(1000/2000) | = | \log(1000) - \log(2000) |$

  More generally:

  $| \log(ax) - \log(ay) | = | \log(x) - \log(y) |$

- Independence of edge direction

  $| \log(x) - \log(y) | = | \log(1/y) - \log(1/x) | =$

  $| \log(1/x) - \log(1/y) |$

Consistency constraint becomes additive

$xy = z$

is equivalent to

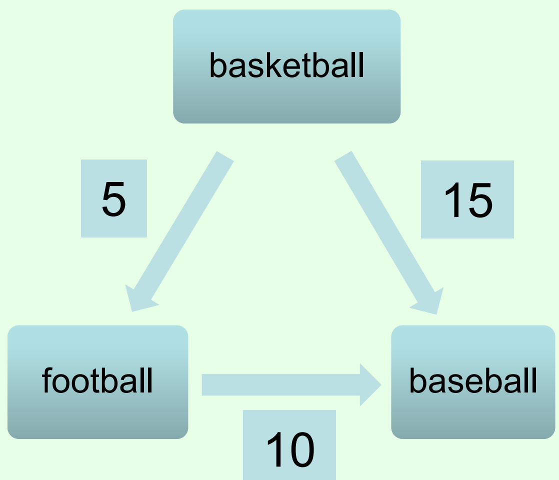
$\log(xy) = \log(z)$

is equivalent to

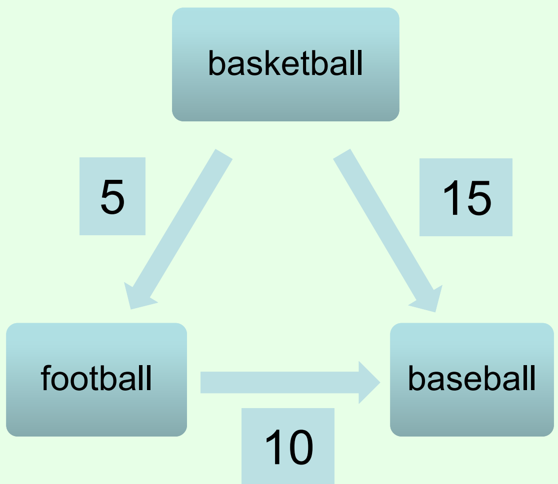
$\log(x) + \log(y) = \log(z)$

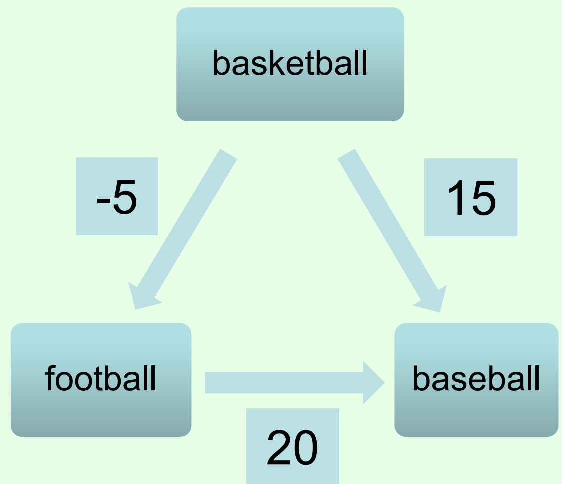
An additive variant

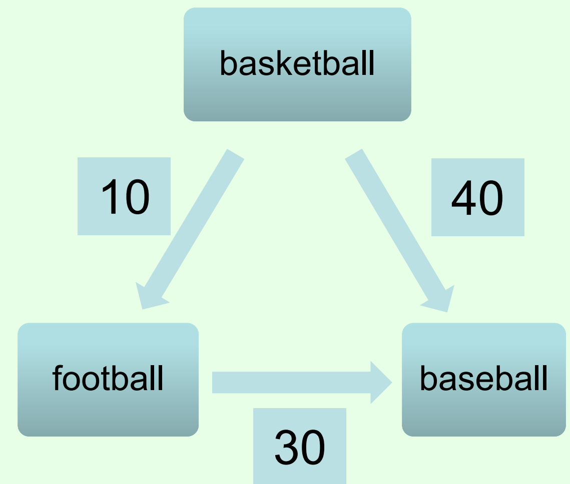
- “I think basketball is 5 units more fun than football, which in turn is 10 units more fun than baseball”



Aggregation in the additive variant

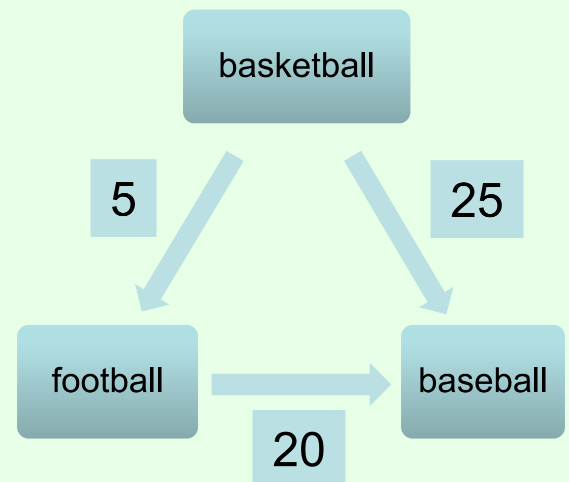






Natural objective:

minimize $\Sigma_i \Sigma_{a,b} d_{a,b,i}$ where $d_{a,b,i} = | t_{a,b} - t_{a,b,i} |$ is the distance between the aggregate difference $t_{a,b}$ and the subjective difference $t_{a,b,i}$



objective value 70 (optimal)

A linear program for the additive variant

q_a: aggregate assessment of quality of activity a (we're really interested in $q_a - q_b = t_{a,b}$)

$d_{a,b,i}$: how far is i's preferred difference $t_{a,b,i}$ from aggregate $q_a - q_b$, i.e., $d_{a,b,i} = |q_a - q_b - t_{a,b,i}|$

minimize $\Sigma_i \Sigma_{a,b} d_{a,b,i}$

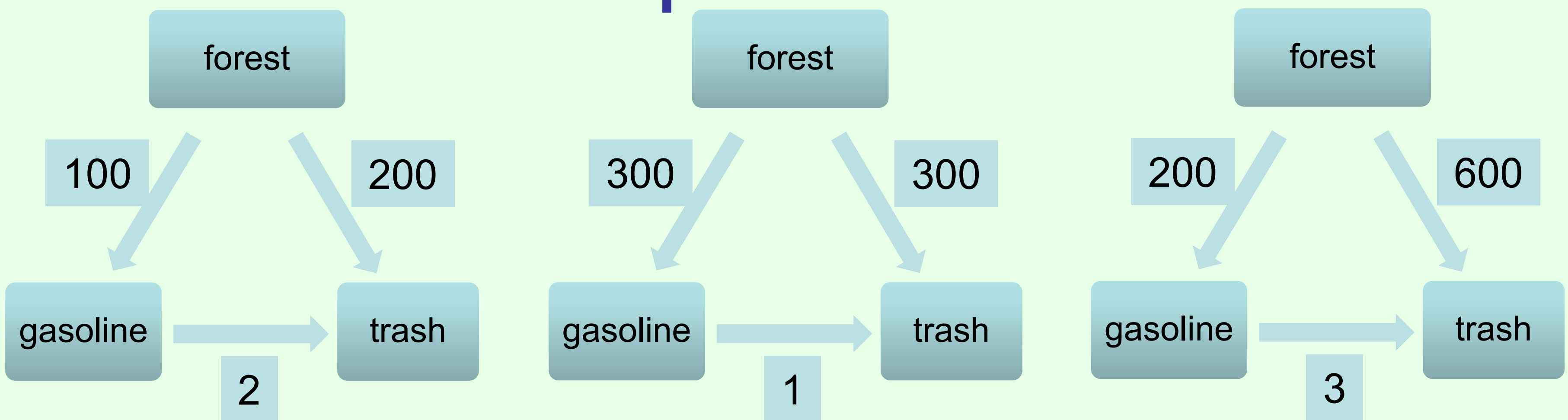
subject to

for all a,b,i: $d_{a,b,i} \geq q_a - q_b - t_{a,b,i}$

for all a,b,i: $d_{a,b,i} \geq t_{a,b,i} - q_a + q_b$

(Can arbitrarily set one of the q variables to 0)

Applying this to the logarithmic rule in the multiplicative variant



Just take logarithms on the edges, solve the additive variant, and exponentiate back