305

DISCRETE RANDOM VARIABLES

A small café sells 0, 1, 2, or 3 muffins per customer.

x	0	1	2	3
p(x)	0.1	0.4	0.3	0.2

The probability that a customer buys at least two muffins:

$P(x \geq 2) = 0.3 + 0.2 = 0.5$

0	x < 0	SS = {}
0.1	0 ≤ x < 1	SS = {0}
0.1 + 0.4 = 0.5	1 ≤ x < 2	SS = {0, 1}
0.5 + 0.3 = 0.8	2 ≤ x < 3	SS = {0, 1, 2}
0.7 + 0.2 = 1	x ≤ 3	SS = {0, 1, 2, 3}

The probability that the customer buys fewer than 2 muffins

$P(x < 2) = F(2) = 0.5$

A startup tracks the number of sales calls made per day

x	0	1	2	3	4
p(x)	0.05	0.15	0.4	0.3	0.1

Find the expected number of calls per day.

$E(X) = 0(0.05) + 1(0.15) + 2(0.4) + 3(0.3) + 4(0.1) = 2.25$

Find the variance of the number of calls per day.

$E(X^2) = 0^2(0.05) + 1^2(0.15) + 2^2(0.4) + 3^2(0.3) + 4^2(0.1) = 0.15 + 4(0.4) + 9(0.3) + 16(0.1) = 6.05$

$Var(X) = E(X^2) - (E(X)^2) = 6.05 - 2.25^2 = 6.05 - 5.0625 = 0.9875$

Find the standard deviation of the number of calls per day.

$SD(X) = \sqrt{Var(X)} = \sqrt{(0.9875)} = 0.994$

A manager wants to decide whether to stock 2 or 3 units of a product daily.

x	0	1	2	3
p(x)	0.1	0.3	0.4	0.2

Find the expected demand.

$E(X) = 0(0.1) + 1(0.3) + 2(0.4) + 3(0.2) = 1.70$

Recommend a stocking level (2 or 3 units) that will minimise the expected number of unsold units.

Case 1: Stocking 2 units

Demand = 0	Unsold = 2
Demand = 1	Unsold = 1
Demand = 2	Unsold = 0
Demand = 3	Unsold = 0

$E(X_{2\ Units}) = 2(0.1) + 1(0.3) = 0.5$

Case 2: Stocking 3 units

Demand = 0	Unsold = 3
Demand = 1	Unsold = 2
Demand = 2	Unsold = 1
Demand = 3	Unsold = 0

$E(X_{3\ Units}) = 3(0.1) + 2(0.3) + 1(0.4) = 1.3$

The recommendation is to stock 2 units as there is less expected unsold units.

CONTINUOUS RANDOM VARIABLES

A user spends time on a website uniformly distributed between 0 and 10 minutes

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } x > b \end{cases}$$

What is the probability a user spends more than 5 minutes on the website?

P(x > 5) = F(5) = 5/10 = 0.5

What is the probability that the user spends at most 3 minutes on the website?

P(x ≤ 3) = F(3) = 3/10 = 0.3

What is the probability that the user spends between 3 and 5 minutes on the website?

P(3 ≤ x ≤ 5)
= F(3) = 3/10 = 0.3
= F(5) = 5/10 = 0.5
= F(5) - F(3) = 0.5 - 0.3 = 0.2

Calls arrive to a call center at a rate of 10 customers an hour on average. The time between calls is exponentially distributed.

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1 - e^{-10x}, & \text{if } x \geq 0. \end{cases}$$

What is the probability that the second call is received at most 10 minutes after the first call?

The rate of calls was given in calls per hour, so 10 per hour = 10/60 = ⅙ hours

P(Wait time ≤ ⅙)

= $F(1/6) = 1 - e^{-10(1/6)}$

= $1 - e^{-1.666}$

= 1 - 0.18900 = 0.8111

What is the probability that there is more than an hour between two calls?

P(Wait time > 1 hour)

= $e^{-10(1)}$

= e^{-10}

= 0.00005

CONDITIONAL PROBABILITY

At a coffee shop, 60% of customers order coffee, and 40% order tea. Of those who order coffee, 25% also order a muffin

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

What is the probability that a randomly selected customer orders coffee and a muffin?
P(Coffee and Muffin) = 0.6 * 0.25 = 0.15

Given that a customer ordered coffee, what is the probability they also ordered a muffin?
P(Muffin|Coffee)
= P(Muffin and Coffee) / P(Coffee)
= 0.15 / 0.6 = 0.25

A company sends out promotional emails. 70% of recipients open the email. Of those who open it, 20% click on the link

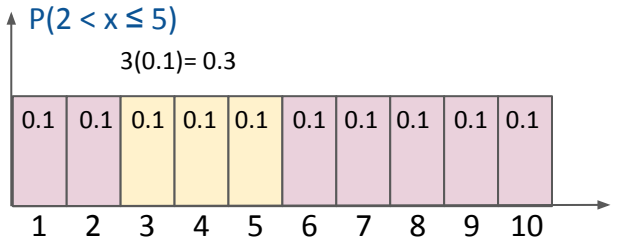
What is the probability that a recipient opens the email and clicks the link?
P(Opens email and click link) = 0.7 * 0.2 = 0.14

What is the probability a recipient clicks the link given they opened the email?
P(Click link | Opens email)
= P(Click link and Open email) / P(Opens email)
= 0.14 / 0.7 = 0.20

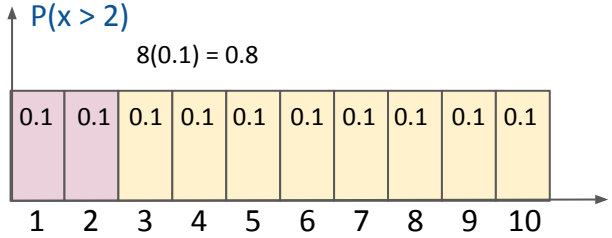
The time a user spends on a website is uniformly distributed between 0 and 10 minutes.

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \le x \le b \\ 1 & \text{if } x > b \end{cases} \qquad P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

What is the probability a user spends more than 5 minutes?
P(x > 5) = 5/10 = 0.5



Given that a user has spent more than 2 minutes on the website,
what is the probability they spend at most 5 minutes (total) on the website?
P(x ≤ 5 | x > 2)
= P(2 < x ≤ 5) / P(x > 2)
= 0.3 / 0.8 = 0.375



Given that a user has already spent 2 minutes on the website,
what it the probability that they spend an additional 5 minutes on the website?
P(x > 7 | x > 2)
= P(x > 7) / P(x > 2)
= 0.3 / 0.8 = 0.375

The time it takes to help customers are a service desk is exponentially distributed with rate 5 customers per hour (or 1/12 customers per minute)

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1 - e^{-10x}, & \text{if } x \geq 0. \end{cases} \qquad P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

What is the probability it takes more than 6 minutes to help a customer.

$P(x > 6) = F(6) = e^{-1/12(6)} = 0.60653$

Given that an employee has already been helping a customer for 3 minutes, what is the probability they spend more than 6 minutes (total)?

$P(x > 6 \mid x > 3)$
$= P(x > 6) / P(x > 3)$
$= 0.60653 / F(3)$
$= 0.60653 / e^{-1/12(3)}$
$= 0.60653 / 0.77880 = 0.7788$

Given that an employee has already been helping a customer for 6 minutes, what it the probability that they spend at least additional 6 minutes on the website?

$P(x > 12 \mid x > 6)$
$= P(x > 12) / P(x > 6)$
$= F(12) = 1 - e^{-1/12(12)} = 0.36788$
$= F(6) = 1 - e^{-1/12(6)} = 0.60653$
$= 0.36788 / 0.60653 = 0.6065$

A cafe manager is analysing customer purchasing behaviour. They have noticed that 90% of customers purchase a drink, and 60% of customers purchase food. 45% of customers purchase both food and drink.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Given a particular customer orders a drink, what it the probability that they will also purchase food?

P(Purchase Food| Order Drink)
P(Food and Drink) = 0.45
P(Orders Drink) = 0.9
0.45/0.9 = 0.5