Journal of Property Finance

Computing Markowitz Efficient Frontiers Using a Spreadsheet Optimizer

Peter Byrne, Stephen Lee,

Journal of
Property
Finance
5,1

Computing Markowitz Efficient Frontiers Using a Spreadsheet Optimizer

Peter Byrne and Stephen Lee

Introduction

Using the basic premiss that investors want higher rather than lower returns, and prefer lower risk to higher risk, Markowitz[1] showed that assets can be combined to produce an "efficient" portfolio that will give the highest level of portfolio return for any level of portfolio risk, as measured by the variance or standard deviation. These portfolios can then be connected to generate what is termed an "efficient frontier". The efficient frontier (EF) represents the boundary of the risk/return set of asset combinations (portfolios). An inefficient portfolio is then one which has a higher risk level for a given return, or one which has a lower return at a particular risk level. All such portfolios lie inside the EF (see below for example).

The mathematics necessary to achieve this are regarded as complex and difficult. Furthermore, until recently such calculations required considerable computing power and sophisticated and expensive, specialist software[2].

In this article we discuss the calculation of the EF for combinations of assets, using a spreadsheet optimizer. The spreadsheet can now, in principle, deal with this kind of problem using matrix methods for the basic portfolio calculations (see Appendix). Problems of machine power may still arise however, especially memory requirements, when the number of assets which might be included in the portfolio is large.

Example

To illustrate the derivation of the EF, we use the data from the Investment Property Databank (IPD) Long Term Index of Investment Returns for the period 1971 to 1993. The data set for the three asset classes – property, equities and gilts – is shown in Table I. The summary statistics for the asset classes are shown in Table II.

As can be seen in Table II, equities offer the highest level of return, but also have the highest level of risk, such that the risk/return trade-off of equities is considerably larger than either of the other asset classes. Property in comparison, gives the lowest level of return and the lowest level of risk. It should be noted also that the risk for property is lower than the average level of return for property, a characteristic which the other asset classes do not share. This may well be due to the use of appraisals to derive property returns in this

Journal of Property Finance,
Vol. 5 No. 1, 1994, pp. 58-66.
MCB University Press, 0958-868X

Year	Property	Equities	Gilts
1971	16.4	45.2	35.3
1972	24.1	16.3	–10.0
1973	22.4	–28.6	–12.3
1974	–13.0	–51.7	–22.2
1975	10.2	151.0	39.0
1976	7.0	–0.8	15.0
1977	23.1	49.0	45.0
1978	22.8	6.9	–1.5
1979	21.4	8.0	4.9
1980	15.8	33.3	20.7
1981	15.0	14.3	2.4
1982	7.5	30.7	52.6
1983	7.6	28.4	16.2
1984	9.4	29.8	10.4
1985	9.3	19.8	12.6
1986	11.7	25.9	12.5
1987	26.1	7.1	16.4
1988	27.9	10.4	8.2
1989	15.1	36.0	7.4
1990	–9.3	–9.8	7.9
1991	–4.9	20.0	18.2
1992	–3.5	20.8	19.1
1993	18.1	27.9	25.1

Table I. IPD Long Term Index – Investment Return Percentages

		Nominal percentage	
	Property	Equities	Gilts
Mean	12.18	21.30	14.04
SD	11.30	36.22	17.74
Risk/return	0.93	1.70	1.26

Table II. Statistics of the IPD Long Term Index of Investment Returns 1971-1993

case, rather than the transaction prices used for equities and gilts. The question of whether property returns are or are not truly representative of the market is beyond the scope of this article, but such issues could be of considerable significance in interpreting the place of property in a multi-asset portfolio[3].

Deriving the Efficient Frontier

There are a number of alternative approaches to enable the EF to be traced (see for example[4,5]). Here, using SOLVER, the optimizing tool in Microsoft

EXCEL, a very simple heuristic is applied in order to record the EF. The method is as follows:

(1) Find the maximum return portfolio – compute the risk for this (σ_1).

(2) Find the minimum risk ((σ_2) portfolio – compute the return for this.

(These two combinations represent the end points of the EF.)

(3) Compute the difference between σ_1 and σ_2 ($\sigma_2 - \sigma_1$) and divide this into a sufficient number of points to produce a reasonable graph.

(4) Solve the maximum return combination for each of these risk levels.

(5) Tabulate, or better, graph, these returns against the risks.

To make the calculation of the return for each level of risk, SOLVER, has to be invoked a reasonable number of times. This repeat computation can be carried out manually, by changing the risk level in SOLVER and then storing each of the results in successive cells. Given the nature of the problem, however, manual adjustments of this kind are tedious and to be avoided to say the least. It is therefore much more efficient to automate the repetition, which may have to be carried out a fairly large number of times, by using a Macro.

A new Macro language is now available in EXCEL version 5, VISUAL BASIC for Applications. This allows a complete application to be programmed into the EXCEL environment using a structured modification of the well known BASIC programming language. The example presented here has programmed the computation of the EF and the graphical output. The calculation of covariances, correlations, portfolio means and variances has not at this stage been automated, but this would be relatively trivial.

Table III shows the mixtures of the three assets which arise as the risk/return combinations change, as we move from one end of the EF to the other. This is

Table III. Asset Mix Changes along the Efficient Frontier

	Proportion of:			
Risk	Property	Equities	Gilts	Return
36.22	0	100	0	21.30
33.57	7.67	92.24	0	20.57
30.92	15.59	84.41	0	19.84
28.27	23.53	76.47	0	19.11
25.62	31.6	68.4	0	18.37
22.97	39.88	60.12	0	17.63
20.32	48.47	51.53	0	16.86
17.67	52.28	41.21	6.51	16.06
15.02	56.02	30.17	13.81	15.19
12.37	60.29	17.75	21.96	14.15
9.72	71.31	0	28.69	12.72

shown for nine portfolios between the maximum return and minimum risk portfolios, but could be set for any reasonable number.

Figure 1 shows the efficient frontier. The shape of the EF depends on the number of assets, the asset mix, and of course the quality and quantity of the basic data from which the asset measurements (means, variances) are derived. Typically, as the EF moves away from the maximum return portfolio, the risk level changes rapidly, without a correspondingly large reduction in return, illustrating the (desirable) portfolio effect, where large returns can be sustained with quite large reductions in risk. At some point along the EF, however, the low return/low risk assets start to become dominant and the reduction in risk is more than matched by the fall in return. In this case, at point A in Figure 1, as the weight in gilts begins to dominate, the rate of change in the fall in portfolio return starts to accelerate, a process that increases as the weighting in the higher risk, higher return asset, equities, decreases by comparison with the lower risk, lower return assets, gilts and property. Figure 1 also shows the benefits of diversification in comparison with holdings of individual assets. For example, although property individually offers the lowest level of risk, if it was to be combined with the two assets, not only can return be improved, but risk can also be reduced. This is illustrated by the case of the minimum risk portfolio which has a return of 12.72 per cent, higher than for property individually, but with a risk of 9.72 per cent, considerably below that for property, the lowest risk asset. This is the “portfolio effect” of combining assets with low correlations, leading to a reduction in portfolio risk without affecting returns, and in many cases enhancing returns as shown by the portfolios on the EF.

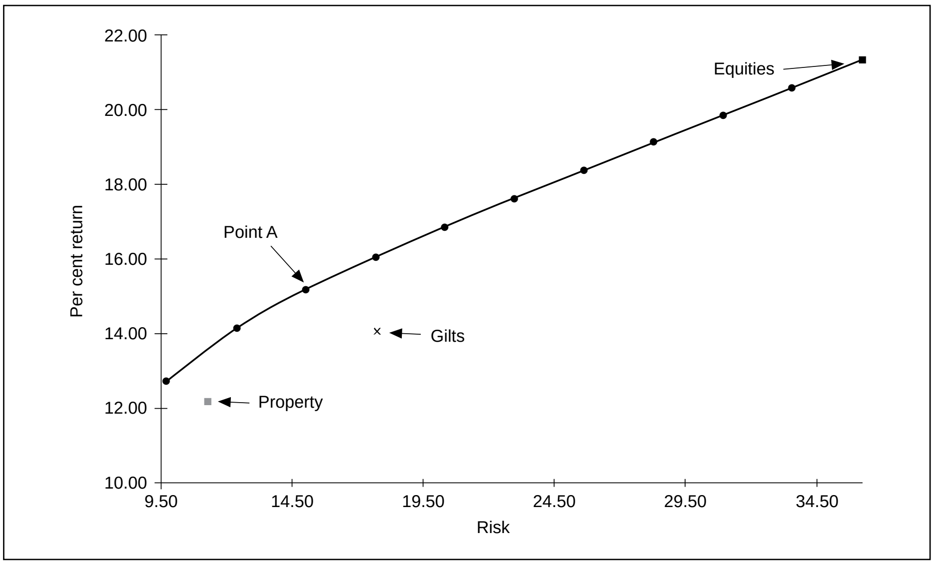

Figure 1.
Asset Risk/Return and the Efficient Frontier

Inefficient Portfolios

The simplest portfolio is the so-called naïve, equal weighted, case. However, in most situations, such a portfolio is inefficient in the Markowitz sense; that is, for the same level of risk, there is another portfolio with a higher level of return which lies on the EF. For example, Table IV compares the characteristics of the naïve portfolio with the Markowitz efficient portfolio at the same level of risk.

For a risk level of 17.46 per cent per annum, the naïve portfolio gives an average return of 15.84 per cent. In comparison, the Markowitz efficient portfolio, for the same level of risk, gives a return of 16.00 per cent, an increase of 16 basis points over the naïve portfolio (see Figure 2). To achieve this, the asset mix changes from an equal holding of one-third in each asset to a heavy concentration of property (53 per cent) and equities (40 per cent), with now, only a small holding in gilts. Such a heavy concentration in a few assets is again typical of the Markowitz approach, where the only constraint on holding an asset is that it must be positive. This may be unacceptable to many investors, who might require a certain specific level of holding or a restriction on holdings in at least some of the assets. Such additional constraints may be readily incorporated in the model to generate a constrained EF with upper and/or lower bounds. This can then be compared with the unconstrained EF to see whether the reduction in return is acceptable.

Table IV.
Comparison of Naïve and Markowitz Efficient Portfolios

Portfolio	Property	Equities	Gilts	Return	Risk
Naïve	33.33	33.33	33.33	15.84	17.46
Efficient	52.53	40.39	7.08	16.00	17.46

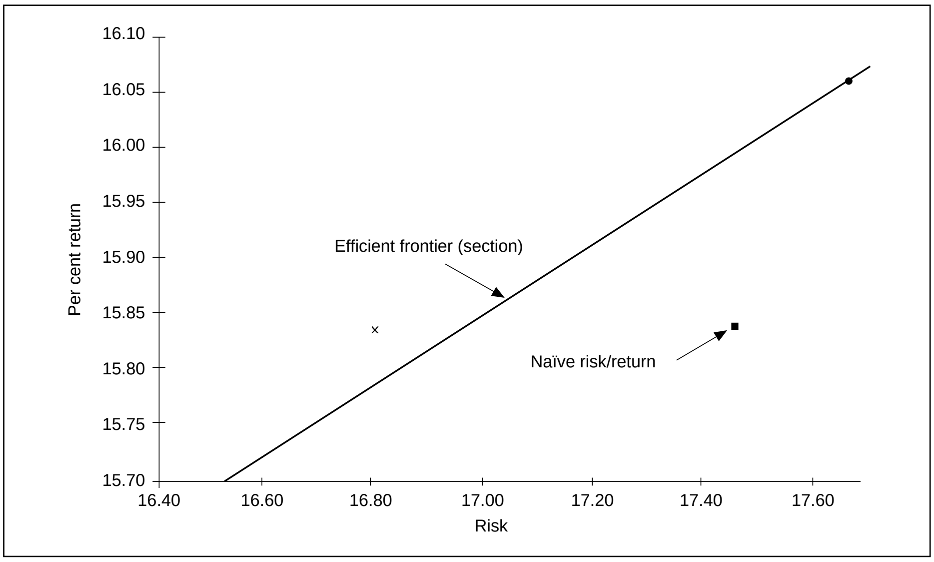

Figure 2.
Naïve Portfolio Relative to the Efficient Frontier

Constrained Portfolios

To see the effect that these additional constraints may have, we adopt a fairly typical pension fund profile. Here we assume that no more than 20 per cent of the total may be held in property. This is known as an upper bound constraint. Secondly, between 40 per cent and 60 per cent must be in equities. These constraints are called lower and upper bounds and both must be specified separately in SOLVER. Any remaining holding must be of gilts.

Once again, the end portfolios of the EF are computed first, and others defined within these limits. The results are shown in Table V.

As can be seen in Table V, the effect of imposing these upper and lower bounds on asset holdings is to constrain severely the extent of the EF in terms of both risk and returns. The maximum return and minimum risk values of the two EFs are compared in Table VI.

Figure 3 shows that initially, the constrained EF moves towards the unconstrained EF, as more is allocated to property. This is to be expected, as the constrained portfolio takes on the asset characteristics of the unconstrained portfolio. However, as the (20 per cent) upper bound constraint on property is reached, the constrained EF veers away from the unconstrained EF quite markedly. Nevertheless, within the boundaries of the maximum and minimum constrained portfolios, this constrained EF is not substantially different in risk/return performance from the unconstrained EF. This may be more by luck

		Proportion of:		
Risk	Property	Equities	Gilts	Return
26.88	0.00	60.00	40.00	18.40
26.22	5.55	60.00	34.45	18.29
25.56	11.40	60.00	28.60	18.18
24.90	17.61	60.00	22.39	18.07
24.24	20.00	58.18	21.82	17.89
23.58	20.00	55.27	24.26	17.68
22.92	20.00	52.23	27.67	17.47
22.26	20.00	49.34	30.66	17.25
21.59	20.00	46.29	33.71	17.03
20.93	20.00	43.18	36.82	16.80
20.27	20.00	40.00	40.00	16.57

Table V. Asset Mix Changes along the Constrained Efficient Frontier

	Maximum return	Minimum risk
Unconstrained	21.30	9.72
Constrained	18.40	20.27

Table VI. Constrained and Unconstrained Portfolios, Maximum Return-Minimum Risk Measures

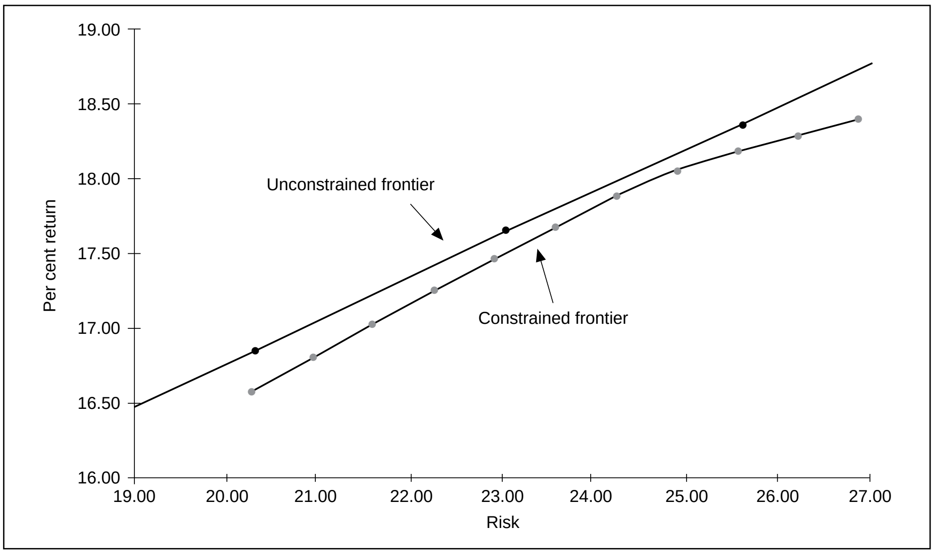

Figure 3. Constrained and Unconstrained Efficient Frontiers

than design, and more research into the actual portfolio mixes of institutional investors who may invest in property is needed before a definitive statement can be made as to the efficiency of institutional investors in portfolios with these assets!

Conclusion

Academics and property professionals have become increasingly interested in the case for property as part of a multi-asset portfolio. This has resulted in concern about the adequacy of appraisal based property data when combined with transaction based data for equities and gilts. Studying the validity of such combinations and analysing the effect on portfolio performance has also been limited by the difficulty and expense of using quadratic optimization software.

In this article we have shown that it is now relatively easy to use the optimizer available in at least one spreadsheet to calculate efficient portfolios for various levels of risk and return, both constrained and unconstrained, so as to be able to generate a number of efficient frontiers. Such an approach can readily incorporate changes in the asset means, variances and covariances to test various assumptions about property market data. This will be the basis of a future article.

References

1. Markowitz, H.M., "Portfolio Selection", *Journal of Finance*, Vol. 7 No. 1, 1952, pp. 77-91
2. Sharpe, W.F., "Asset Allocation Systems", *Financial Analysts Journal*, May 1985, pp. 10-11, 75
3. MacGregor, B.D. and Nanthakumaran, N., "The Allocation to Property in the Multi-asset Portfolio: The Evidence and Theory Reconsidered", *Journal of Property Research*, Vol. 9, 1992, pp. 5-32.

4. Findlay, M.C., Messner, S.D., Tarantello, R.A. and Yormark, J.S., "Optimal Real Estate Portfolios", *AREUEA Journal*, Vol. 7, 1979, pp. 298-317.
5. Markowitz, H.M., "The Optimization of a Quadratic Function Subject to Linear Constraints", *Naval Research Logistics Quarterly*, Vol. 3, March/June 1956, pp. 111-117.

Appendix 1. Matrix Calculations in Microsoft EXCEL

The calculation of the portfolio variance in the Markowitz portfolio allocation model is tedious, even if the number of assets which may be allocated to the portfolio is small.

Fortunately, it is possible, using the EXCEL spreadsheet, to carry out *matrix* and *array* operations to calculate the main values required for the optimizer. This will be demonstrated here using the naïve portfolio variance example from the main text.

Triangual covariance output			
With	Property	Equities	Gilts
Property	127.72	–	–
Equities	69.16	1311.54	–
Gilts	11.54	421.70	300.88
Required form for matrix calculations			
With	127.72	69.16	11.54
Equities	69.16	1311.54	421.70
Gilts	11.54	421.70	300.88

Table AI.

The first step is the calculation of the covariances between the asset classes. This may be accomplished using the data analysis tool, covariance, which will make the asset covariance calculations in almost a single step. The tool produces a triangular output. This must then be converted to a square array or matrix by copying values into opposite cells. The square matrix will have as many rows and columns as there are assets (here 3×3). These variants are shown in Table AI.

In formal, matrix type, notation, the portfolio variance is found by:

$$\sigma_p^2 = WCW^T$$

where:

W is the matrix of asset weights;

C is the asset covariance matrix;

W^T is the transpose of the asset weight matrix.

Using EXCEL, the portfolio variance can then be calculated in three steps:

(1) *Pre*-multiply the covariance matrix (C) by the matrix of weights. The matrix of weights (W) in this example is a 1×3 matrix: 0.33, 0.33, 0.33.

This is done using the EXCEL function MMULT (Matrix MULTiplication). The order of the matrices entering the multiplication is important, both here and later. The result of this MMULT (W;C) is an intermediate 1×3 matrix (In): 69.47, 600.80, 44.71.

(2) The matrix of weights has to be transposed. The matrix is transposed using the EXCEL function TRANSPOSE. The Transpose of a one row, three column matrix, is a one column, three row matrix (W^T).

(3) The intermediate matrix (In) is *post*-multiplied by (W^T), MMULT (In; W^T). The result is a single value, the portfolio variance. The portfolio standard deviation is then the square root of this number:

portfolio variance = 304.99;

portfolio standard deviation = 17. 46.

(Peter Byrne is a Senior Lecturer and Stephen Lee is a Lecturer in the Department of Land Management and Development at the University of Reading, UK.)

This article has been cited by:

1. S. Prashant Mahasagara, Andry Alamsyah, Brady Rikumahu. Indonesia infrastructure and consumer stock portfolio prediction using artificial neural network backpropagation 1-4. [Crossref]
2. TajaniFrancesco, Francesco Tajani, MoranoPierluigi, Pierluigi Morano. 2017. Evaluation of vacant and redundant public properties and risk control. *Journal of Property Investment & Finance* **35**:1, 75-100. [Abstract] [Full Text] [PDF]
3. Cortez AlejandroKlender, Klender Cortez Alejandro, Rodríguez GarcíaMartha del Pilar, Martha del Pilar Rodríguez García. 2017. An assessment of OECD sustainable portfolios with a multi-criteria approach under uncertainty. *Kybernetes* **46**:1, 67-84. [Abstract] [Full Text] [PDF]
4. Joseph B. Oyedele, Alastair Adair, Stanley McGreal. 2014. Performance of global listed infrastructure investment in a mixed asset portfolio. *Journal of Property Research* **31**:1, 1-25. [Crossref]
5. Joseph B. Oyedele, Stanley McGreal, Alastair Adair, Peter Ogedengbe. 2013. Performance and role of European listed infrastructure in a mixed-asset portfolio. *Journal of Financial Management of Property and Construction* **18**:2, 160-183. [Abstract] [Full Text] [PDF]
6. James R. Brown. 2010. Managing the retail format portfolio: An application of modern portfolio theory. *Journal of Retailing and Consumer Services* **17**:1, 19-28. [Crossref]
7. Gianluca Marcato, Tony Key. 2007. Smoothing and Implications for Asset Allocation Choices. *The Journal of Portfolio Management* **33**:5, 85-99. [Crossref]
8. Peter Byrne, Stephen Lee. 2004. Different risk measures: different portfolio compositions?. *Journal of Property Investment & Finance* **22**:6, 501-511. [Abstract] [Full Text] [PDF]
9. Gabriel A. Petersen, A. J. Singh, Atul Sheel. 2003. Hotel Real Estate in a Property Investment Portfolio: An Examination of Results from 1992 to 2001. *The Journal of Hospitality Financial Management* **11**:1, 11-25. [Crossref]
10. Stephen Lee, Simon Stevenson. 2003. Time weighted portfolio optimisation. *Journal of Property Investment & Finance* **21**:3, 233-249. [Abstract] [Full Text] [PDF]
11. Stephen Lee, Peter Byrne, Nick French. 1996. Assessing the future of property in the multi-asset portfolio: The case of UK pension funds. *Journal of Property Research* **13**:3, 197-209. [Crossref]
12. Peter Byrne, Stephen Lee. 1995. Is there a place for property in the multi-asset portfolio?. *Journal of Property Finance* **6**:3, 60-83. [Abstract] [Full Text] [PDF]
13. Peter Byrne, Stephen Lee. 1994. Real Estate Portfolio Analysis Using a Spreadsheet Optimizer. *Journal of Property Finance* **5**:4, 19-31. [Abstract] [Full Text] [PDF]