# CSCI 315: Data Structures Performance Analysis

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# **Analysis of Algorithms**

- Dilemma: you have two (or more) methods to solve problem, how to choose the BEST?
- One approach: implement each algorithm in C, test how long each takes to run.
- Problems:
  - Different implementations may cause an algorithm to run faster/slower
  - Some algorithms run faster on some computers
  - Algorithms may perform differently depending on data (e.g., sorting often depends on what is being sorted)

# Better Approach Step 1

- characterize performance in terms of key operation(s)
- Sorting:
  - count number of times two values compared
  - count number of times two values swapped
- Search:
  - count number of times value being searched for is compared to values in array
- Recursive function:
  - count number of recursive calls

# Better Approach Step 2

- Want to comment on the "general" performance of the algorithm
- Emperical: Measure for several examples, but what does this tell us in general?
- Analytical:
  - Instead, assess performance in an abstract manner
  - Idea: analyze performance as size of problem grows
  - Examples:
    - Sorting: how many comparisons for array of size N?
    - Searching: #comparisons for array of size N
  - May be difficult to discover a reasonable formula

# **Analsysis With Varying Results**

- Example: for some sorting algorithms, a sorting routine may require as few as N-1 comparisons and as many as  $\frac{N^2}{2}$
- Types of analyses:
  - Best-case: what is the fastest an algorithm can run for a problem of size N?
  - Average-case: on average how fast does an algorithm run for a problem of size N?
  - Worst-case: what is the longest an algorithm can run for a problem of size N?
- Computer scientists usually use worst-case analysis

# Notice: We Are Estimating

- What is often done is to approximate or estimate the performance of an algorithm
- Estimation is an important skill to learn and to use
- Example Question: How many hotdogs tall is the Empire State Building?

# Simple Example

- Simplier Question: How tall is the Empire State Building?
- Answer: The ESB is 1250 feet tall.
- Assuming that a hotdog is 6 inches from end to end, you would need, 1250 \* 2
   2500 hotdogs.



#### **Analysis**

- An objective way to evaluate the cost of an algorithm or code section.
- The cost is computed in terms of space or time, usually
- The goal is to have a meaningful way to compare algorithms based on a common measure.
- Complexity analysis has two phases,
  - Algorithm analysis
  - Complexity analysis

# **Algorithm Analysis**

- Algorithm analysis requires a set of rules to determine how operations are to be counted.
- There is no generally accepted set of rules for algorithm analysis.
- In some cases, an exact count of operations is desired; in other cases, a general approximation is sufficient.
- The rules presented that follow are typical of those intended to produce an exact count of operations.

#### Rules

- We assume an arbitrary time unit.
- Execution of one of the following operations takes time 1:
  - assignment operation
  - single I/O operations
  - 3 single Boolean operations, numeric comparisons
  - single arithmetic operations
  - function return
  - array index operations, pointer dereferences

#### More Rules

- Running time of a selection statement (if, switch) is the time for the condition evaluation + the maximum of the running times for the individual clauses in the selection.
- Loop execution time is the sum, over the number of times the loop is executed, of the body time + time for the loop check and update operations, + time for the loop setup.
- Always assume that the loop executes the maximum number of iterations possible Running time of a function call is 1 for setup + the time for any parameter calculations + the time required for the execution of the function body.

```
count = count + 1; // Cost: c1
sum = sum + count; // Cost: c2
```

Total Cost = c1 + c2.

Since we assume '+' cost 1 and assignment cost 1, the total cost is 4.

```
if (n < 0) { // Cost: c1
    absval = -n; // Cost: c2
} else {
    absval = n; // Cost: c3
}</pre>
```

- Total Cost <= c1 + max(c2,c3)</p>
- c1 is the cost of boolean evaluation. Since there is 1 evaluation (<), Cost(c1) = 1.</li>
- c2 is the cost of negating a number (1) + the cost of assignment (1). Cost(c2) = 2.
- c3 is the cost of assignment(1). Cost(c3) = 1
- Cost of the worse-case is 3.
- Cost of the best-cast is 2.
- Average case is 2.5.



```
i = 1; // Cost: c1
sum = 0; // Cost: c2
while (i <= n) { // Cost: c3
    i = i + 1; // Cost: c4
    sum = sum + i; // Cost: c5
}</pre>
```

```
i = 1; // Cost: c1
sum = 0; // Cost: c2
while (i <= n) { // Cost: c3
    i = i + 1; // Cost: c4
    sum = sum + i; // Cost: c5
}</pre>
```

• Cost(c1) = 1, Cost(c2) = 1, Cost(c3) = 1.

```
i = 1; // Cost: c1

sum = 0; // Cost: c2

while (i <= n) { // Cost: c3

i = i + 1; // Cost: c4

sum = sum + i; // Cost: c5

}
```

- Cost(c1) = 1, Cost(c2) = 1, Cost(c3) = 1.
- Cost(c4) = 1 + 1 = 2 (remember assignment and + both cost 1!).

```
i = 1; // Cost: c1
sum = 0; // Cost: c2
while (i <= n) { // Cost: c3
    i = i + 1; // Cost: c4
    sum = sum + i; // Cost: c5
}</pre>
```

- Cost(c1) = 1, Cost(c2) = 1, Cost(c3) = 1.
- Cost(c4) = 1 + 1 = 2 (remember assignment and + both cost 1!).
- Cost(c5) = 2.

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i = 1; // Cost: c1
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- Cost(c1) = 1, Cost(c2) = 1, Cost(c3) = 1.
- Cost(c4) = 1 + 1 = 2 (remember assignment and + both cost 1!).
- Cost(c5) = 2.
- How many time does the loop execute?

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i = 1; // Cost: c1
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}
```

- Cost(c1) = 1, Cost(c2) = 1, Cost(c3) = 1.
- Cost(c4) = 1 + 1 = 2 (remember assignment and + both cost 1!).
- Cost(c5) = 2.
- How many time does the loop execute?
- Loop: n times, so total cost is:

```
i = 1; // Cost: c1
sum = 0; // Cost: c2
while (i <= n) { // Cost: c3
    i = i + 1; // Cost: c4
    sum = sum + i; // Cost: c5
}</pre>
```

- Cost(c1) = 1, Cost(c2) = 1, Cost(c3) = 1.
- Cost(c4) = 1 + 1 = 2 (remember assignment and + both cost 1!).
- Cost(c5) = 2.
- How many time does the loop execute?
- Loop: n times, so total cost is:
- Total Cost = c1 + c2 + (n+1)\*c3 + n\*c4 + n\*c5 =c1 + c2 + c3 + n(c3 + c4 + c5)



```
i = 1; // Cost c1
sum = 0; // Cost c2
while (i <= n) { // Cost c3
    j = 1; // Cost c4
    while (j <= n) { // Cost c5
        sum = sum + i; // Cost c6
        j = j + 1; // Cost c7
    }
    i = i + 1; // Cost c8
}</pre>
```

```
i = 1; // Cost c1

sum = 0; // Cost c2

while (i <= n) { // Cost c3

j = 1; // Cost c4

while (j <= n) { // Cost c5

sum = sum + i; // Cost c6

j = j + 1; // Cost c7

}

i = i + 1; // Cost c8
```

```
    Cost(c1) = 1, Cost(c2) = 1, Cost(c3) = 1, Cost(c4) = 1,
    Cost(c5) = 1, Cost(c6) = 2, Cost(c7) = 2, Cost(c8) = 2
```

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i = 1; // Cost c1
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```

- Cost(c1) = 1, Cost(c2) = 1, Cost(c3) = 1, Cost(c4) = 1,
   Cost(c5) = 1, Cost(c6) = 2, Cost(c7) = 2, Cost(c8) = 2
- First (outer) while loop execution: n

```
i = 1; // Cost c1

sum = 0; // Cost c2

while (i <= n) { // Cost c3

j = 1; // Cost c4

while (j <= n) { // Cost c5

sum = sum + i; // Cost c6

j = j + 1; // Cost c7

}

i = i + 1; // Cost c8
```

- Cost(c1) = 1, Cost(c2) = 1, Cost(c3) = 1, Cost(c4) = 1,
   Cost(c5) = 1, Cost(c6) = 2, Cost(c7) = 2, Cost(c8) = 2
- First (outer) while loop execution: n
- Second (inner) while loop execution: n, total cost is:

```
i = 1; // Cost c1

sum = 0; // Cost c2

while (i <= n) { // Cost c3

j = 1; // Cost c4

while (j <= n) { // Cost c5

sum = sum + i; // Cost c6

j = j + 1; // Cost c7

}

i = i + 1; // Cost c8
```

- Cost(c1) = 1, Cost(c2) = 1, Cost(c3) = 1, Cost(c4) = 1,
   Cost(c5) = 1, Cost(c6) = 2, Cost(c7) = 2, Cost(c8) = 2
- First (outer) while loop execution: n
- Second (inner) while loop execution: n, total cost is:
- ① c1 + c2 + (n+1) \* c3 + n \* c4 + n \* (n+1) \* c5 + n \* n \* c6 + n \* n \* c7 + n \* c8 = c1 + c2 + c3 + n \* (c3 + c4 + c8) + n \* n \* c5 + n \* c5 + n \* n \* c6 + n \* n \* c7 = c1 + c2 + c3 + n \* (c3 + c4 + c5 + c8) + n \* n(c5 + c6 + c7)

```
i = 1; // Cost c1
sum = 0; // Cost c2
while (i <= n) { // Cost c3
    j = 1; // Cost c4
    while (j <= n) { // Cost c5
        sum = sum + i; // Cost c6
        j = j + 1; // Cost c7
    }
    i = i + 1; // Cost c8
}</pre>
```

- Cost(c1) = 1, Cost(c2) = 1, Cost(c3) = 1, Cost(c4) = 1,
   Cost(c5) = 1, Cost(c6) = 2, Cost(c7) = 2, Cost(c8) = 2
- First (outer) while loop execution: n
- Second (inner) while loop execution: n, total cost is:
- c1 + c2 + (n+1) \* c3 + n \* c4 + n \* (n+1) \* c5 + n \* n \* c6 + n \* n \* c7 + n \* c8 = c1 + c2 + c3 + n \* (c3 + c4 + c8) + n \* n \* c5 + n \* c5 + n \* n \* c6 + n \* n \* c7 = c1 + c2 + c3 + n \* (c3 + c4 + c5 + c8) + n \* n(c5 + c6 + c7)
- Important Note: n\*n (n²) is the highest (largest) term!



# **Comparing Algorithms**

- We measure an algorithm's time requirement as a function of the problem size.
- Problem size depends on the application: e.g. number of elements in a list for a sorting algorithm, the number disks for towers of hanoi.
- So, for instance, we say that (if the problem size is n)
  - Algorithm A requires  $5 * n^2$  time units to solve a problem of size n.
  - Algorithm B requires 7 \* n time units to solve a problem of size n.
- An algorithm's proportional time requirement is known as growth rate.
- We can compare the efficiency of two algorithms by comparing their growth rates.



- Which is better?
  - $\bullet$  50 $N^2 + 31N^3 + 24N + 15$
  - $3N^2 + N + 21 + 4 * 3^N$

- Which is better?
  - $\bullet$  50 $N^2 + 31N^3 + 24N + 15$
  - $\bullet$  3N<sup>2</sup> + N + 21 + 4 \* 3<sup>N</sup>
- Well, it depends on N:

N 
$$50N^2 + 31N^3 + 24N + 15$$
  $3N^2 + N + 21 + 4 * 3^N$ 

# What happened?

Ν	$3N^2 + N + 21 + 4 * 3^N$	4 * 3 <sup>N</sup>	% of Tota
1	37	12	32.4
2	71	36	50.7
3	159	108	67.9
4	397	324	81.6
5	1073	972	90.6
6	3051	2916	95.6
7	8923	8748	98.0
8	26465	26244	99.2
9	79005	78732	99.7
10	236527	236196	99.9

- One term dominated the others.
- This implies we really only care about the dominating (highest/largest) term.

#### As N Grows, Some Terms Dominate

Function	N=10	N=100	N=1000	N=10000	N = 100000
$log_2N$	3	6	9	13	16
N	10	100	1000	10000	100000
$N * log_2N$	30	664	9965	10 <sup>5</sup>	10 <sup>6</sup>
$N^2$	10 <sup>2</sup>	10 <sup>4</sup>	10 <sup>6</sup>	10 <sup>8</sup>	10 <sup>10</sup>
$N^3$	10 <sup>3</sup>	10 <sup>6</sup>	10 <sup>9</sup>	10 <sup>12</sup>	10 <sup>15</sup>
2 <sup>N</sup>	10 <sup>3</sup>	10 <sup>30</sup>	10 <sup>301</sup>	10 <sup>3010</sup>	10 <sup>30103</sup>

Ordering:

$$1 < log_2 N < N < N * log_2 N < N^2 < N^3 < 2^N < 3^N$$

## Big O

- If Algorithm A requires time proportional to f(n), Algorithm A is said to be order f(n), and it is denoted as O(f(n)).
- The function f(n) is called the algorithm's growth-rate function.
- Since the capital O is used in the notation, this notation is called the Big O notation.
- If Algorithm A requires time proportional to  $n^2$ , it is  $O(n^2)$ .
- If Algorithm A requires time proportional to n, it is O(n).

- If an algorithm requires  $n^2 3 * n + 10$  seconds to solve a problem size n. If constants k and  $n_0$  exist such that  $k * n^2 + n_0 > n^2 3 * n + 10$  for all n and  $n_0$ .
- Then the algorithm is order  $n^2$  (In fact, k is 3 and  $n_0$  is 2)
- Thus, the algorithm requires no more than  $k * n^2$  time units.
- So it is  $O(n^2)$

- This is actually not that difficult. It is a game of "spot the highest term!"
- $50N^2 + 31N^3 + 24N + 15 = O(N^3)$
- $3N^2 + N + 21 + 4 * 3^N = O(3^N)$
- It can get somewhat tricky:
- $N(3 + N(9 + N)) + N^2 = O(N^3)$
- $N(10 + log_2N) + N = O(N * log_2N)$

# **Growth Rate Functions Explained**

- O(1) Time requirement is constant, and it is independent of the problem's size.
- O(log<sub>2</sub>n) Time requirement for a logarithmic algorithm increases increases slowly as the problem size increases.
- O(n) Time requirement for a linear algorithm increases directly with the size of the problem.
- $O(n * log_2 n)$  Time requirement for a  $n * log_2 n$  algorithm increases more rapidly than a linear algorithm.
- $O(n^2)$  Time requirement for a quadratic algorithm increases rapidly with the size of the problem.
- O(n<sup>3</sup>) Time requirement for a cubic algorithm increases more rapidly with the size of the problem than the time requirement for a quadratic algorithm.
- O(2<sup>n</sup>) As the size of the problem increases, the time requirement is too rapid to be practical.

```
i = 1; // Cost c1
sum = 0; // Cost c2
while (i <= n) { // Cost c3
i = i + 1; // Cost c4
sum = sum + i; // Cost c5
}
```

- T(n) = c1 + c2 + (n+1)\*c3 + n\*c4 + n\*c5= (c3+c4+c5)\*n + (c1+c2+c3)= a\*n + b
- So, the growth-rate function for this algorithm is O(n)

```
i = 1; // Cost c1

sum = 0; // Cost c2

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while (j <= n) { // Cost c5

sum = sum + i; // Cost c6

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i = i + 1 // Cost c8
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i = 1; // Cost c1

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while (j <= n) { // Cost c5

sum = sum + i; // Cost c6

j = j + 1; // Cost c7

}

i = i + 1 // Cost c8

}
```

- T(n) = c1 + c2 + (n+1)\*c3 + n\*c4 + n\*(n+1)\*c5+n\*n\*c6+n\*n\*c7+n\*c8
  = (c5+c6+c7)\*n\*n + (c3+c4+c5+c8)\*n + (c1+c2+c3)
  = a\*n\*n + b\*n + c
- So, the growth-rate function for this algorithm is  $O(n^2)$

```
for (i=1; i<=n; i++) { // Cost(c1)
    for (j=1; j<=i; j++) { // Cost(c2)
        for (k=1; k<=j; k++) { // Cost(c3)
            x=x+1; // Cost(c4)
        }
    }
}</pre>
```

```
for (i=1; i<=n; i++) { // Cost(c1)
    for (j=1; j<=i; j++) { // Cost(c2)
        for (k=1; k<=j; k++) { // Cost(c3)
            x=x+1; // Cost(c4)
        }
    }
}</pre>
```

- T(n) = c1\*(n+1) + c2\*((n+1)\*(n+2)) / 2) + c3\* ( estimated: (n \* (n + 1) \* (2n + 1)) / 6) + c4\* (estimated: (n \* (n + 1) \* (2n + 1)) / 6) = a\*n3 + b\*n2 + c\*n + d
- So, the growth-rate function for this algorithm is  $O(n^3)$
- Notice: You do NOT need to know the exact number of iterations to find Big-O.

#### Unfortunately, recursive can be hard...

- By now, I hope you see that constance costs are virtually supurfulous when working with Big O.
- To find the growth-rate function for a recursive algorithm, we have to solve its recurrence relation.
- You will learn how to do this in Discrete Structures.



## Example 4 continued

- What is the cost of hanoi(n,'A','B','C')?
- when n=0 T(0) = c1
- when n>0 T(n) = c1 + c2 + T(n-1) + c3 + c4 + T(n-1)
   = 2\*T(n-1) + (c1+c2+c3+c4)
   = 2\*T(n-1) + c -> recurrence equation for the growth-rate function of hanoi-towers algorithm
- Now, we have to solve this recurrence equation to find the growth-rate function of hanoi-towers algorithm
- This turns out to be  $O(2^n)$  because for every N we make 2(n-1) calls.

### **Crash Course**

- gnuplot makes graphs
- type "gnuplot" at your terminal
- type "plot sin(x) with line"
- type "plot sin(x) with point"
- Type "set terminal postscript color"
- Type "set output "nameofplot.ps" "
- Type "replot" or "rep"

### Continued

- Type "set title "plotname" "
- Type "set ylabel "ylabel" "
- Type "set xlabel "xlabel" "
- to covert to something readable (like pdf). On the command line do:
  - \$ ps2pdf nameofplot.ps
- Then you should have nameofplot.pdf
   \$ evince nameofplot.pdf

# **Huge Time Saver!**

- The commands to gnuplot can be saved to a file and then automatically used:
- simple.plot:

```
set terminal postscript color
set output "simple.ps"
set ylabel "time (seconds)"
set xlabel "size"
```

. . .

- \$ gnuplot simple.plot
- \$ ps2pdf simple.ps
- \$ evince simple.pdf
- Will display the graph.
- This is especially useful if you put it in a makefile!

```
plot:

gnuplot simple.plot

ps2pdf simple.ps

evince simple.pdf
```

#### Perf

- Tool to find what part of the code is running slow.
- Steps:
  - ompile with debug flags! (-g)
  - run:
    - \$ sudo perf record <your-programs-name>
  - o view the report:
    - \$ sudo perf report
- Now you should see what functions are taking up all the time!

```
Event count (approx.): 2061166086
              of event 'cucles:ppp'.
Overhead
          Command
                   Shared Object
                                       Sumbo1
                                           sumOfOneTo
          a.out
                   a.out
                   1d-2.28.so
                                          0x000000000000a25a
   0.20%
                                           sumOfOneToSquared
         a.out
                   a.out
                   [kernel.kallsyms]
                                          __do_page_fault
   0.03%
                   [kernel.kallsyms]
                                          kmem_cache_alloc
         a.out
   0.00%
         perf_4. [kernel.kallsyms]
                                          native_sched_clock
   0.00%
          perf_4. [kernel.kallsyms]
                                          native_apic_mem_write
   0.00%
          perf 4.
                   [kernel.kallsums]
                                          native_write_msr
```

## **Emperical Measurement**

- While Analytical measurement of performance is important sometimes an emperical approach is most useful.
- The naive way to take timings:
  - Record time as start
  - Q Run section of code you wish to time
  - Record time as end
  - You answer is (end start).
- While there are numerous issues with this approach, it will give sufficient approximate timings for this class.

```
#include <iostream> // To print
#include <time.h> // Required for taking timings
int main(int argc, char *argv[]) { // Standard main heading.
    /* clock t is the data type for storing timing information.
     * We must make two variables, one for the start and the other to capture
     * the difference.
     */
    clock t start, diff;
    // timeAmount is used to print out the time in seconds.
    double timeAmount:
    // We want to run our algorithm over varying sizes.
    for (int i = 1000; i < 1000000; i += 1000) {
        // Capture the start clock
        start = clock():
        // This is were your algorithm should be called.
        functionCallToYouAlgorithm(i):
        // Capture the clock and subtract the start to get the total time elapsed.
        diff = clock() - start:
        // Convert clock t into seconds as a floating point number.
        timeAmount = diff * 1.0 / CLOCKS PER SEC:
        // Print out first the size (i) and then the elapsed time.
        std::cout << i << " " << timeAmount << "\n";
        // We flush to ensure the timings is printed out.
        std::cout << std::flush;
```

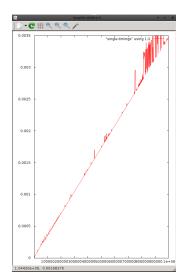
## **Example Output**

```
1000 4e-06
2000 8e-06
3000 1.2e-05
4000 2 5e-05
5000 2.9e-05
6000 2.4e-05
7000 3.5e-05
8000 2.9e-05
9000 3.2e-05
10000 3.5e-05
11000 3.9e-05
12000 4.2e-05
13000 4.5e-05
14000 5.2e-05
15000 5.6e-05
16000 6e-05
17000 6.5e-05
18000 6.8e-05
19000 7e-05
20000 7.6e-05
```

 First is the size (1000) and second is the number of seconds (pretty small.)

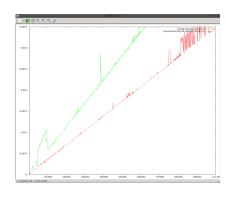
### **Plotting**

- First lets us assume we have the full listing in a file named 'single-timings'
- With gnuplot we can simply graph the timings: plot [:][:] "single-timings" using 1:2 with line



### **Multiple Data Collections**

- In addition to 'single-timings' let us assume we have another file (in the same format) named 'squaredenhanced-timings'
- With gnuplot we can graph both timings: plot [:][:] "single-timings" using 1:2 with line, "squared-enhancedtimings" using 1:2 with line
- You can append more and more data files in this
   manner

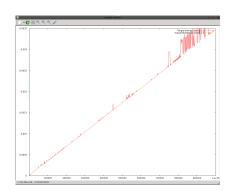


### Summary

- Hopefully you can see from today that we can:
  - Analyze algorithm analytically to predict performance.
  - Profile code to find what piece of code is the bottleneck.
  - Get & plot timings to see actual performance.
- Performance Analysis could take the whole class time.
- We stick with the basics for this class.
- I do want to alert you to a couple of things that will help!

#### Gotcha 1

 You plot the data of two algorithms, but you can see only one!



### Gotcha 1

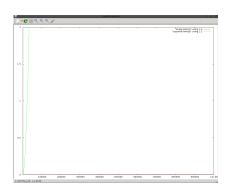
- You plot the data of two algorithms, but you can see only one!
- Check your data and axis, usually it is because it is too small to see.



#### Gotcha 1

- You plot the data of two algorithms, but you can see only one!
- Check your data and axis, usually it is because it is too small to see.
- plot [:][:] "single-timings" using 1:2 with line, "squared-timings" using 1:2 with line ->

plot [:][:2] "single-timings" using 1:2 with line,



<sup>&</sup>quot;squared-timings" using 1:2 with line

### Performance Tips

- Make sure you are working on optimizing the correct function and looking for the correct code improvements.
- From our example:

```
of event 'cycles:ppp',
                                     Event count (approx.): 2061166086
Overhead
         Command
                   Shared Obliect
                                      Sumbo1
          a.out
                   a.out
                                          0x0000000000000a25a
         a.out
                   ld-2.28.so
                                          sumOfOneToSquared
         a.out
                   a.out
         a.out [kernel.kallsums]
   0.08%
                                            do_page_fault
  0.03% a.out [kernel.kallsyms]
                                          kmem_cache_alloc
  0.00% perf_4. [kernel.kallsyms]
                                          native sched clock
                   [kernel.kallsyms]
                                          native_apic_mem_write
                   [kernel.kallsums]
          perf 4.
                                          native write msr
```

- Most of the time is spent in sumOfOneTo, so improving sumOfOneTo's performance may help.
- Improving performance of sumOfOneToSquared would not help that much.
  - Maybe we can memoize past results to use in the future or use a better data structure.

### **Summary**

- Again, there is a lot and we are scratching the surface.
- Important outcomes:
  - Be able to analytically deduce the performance of code.
  - Be able to profile code to find the hot spots.
  - Be able to emperically run programs to evaluate performance.
  - Understand there are anomalies that will not be addressed in this class.