More about Graphs and Functions Exercise

1. It is given that $f(x) = 2x^2 - 4x + 9$.

   (a) The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ to the right by 3 units. Find $g(x)$.

   (2 marks)

   (b) It is given that $h(x) = f(2x)$. Write down the single transformation to obtain the graph of $y = h(x)$ from the graph of $y = f(x)$.

   (1 mark)

   (c) Does the graph of $y = g(x)$ intersect with the graph of $y = h(x)$? Explain your answer.

   (3 marks)

2. The graph $C_1 : y = x(x+3)(x+k)$, where k is a constant, is translated 5 units to the left to obtain the graph $C_2 : y = g(x)$. P is a point on C_1.

   (a) If $Q(-6, 4)$ is the image of P, find the coordinates of P and the value of k.

   (3 marks)

   (b) Find $g(x)$.

   (2 marks)

   (c) If $C_2 : y = g(x)$ is reflected about x-axis and then reflected about y-axis to obtain the graph $C_3 : y = h(x)$. Find $h(x)$.

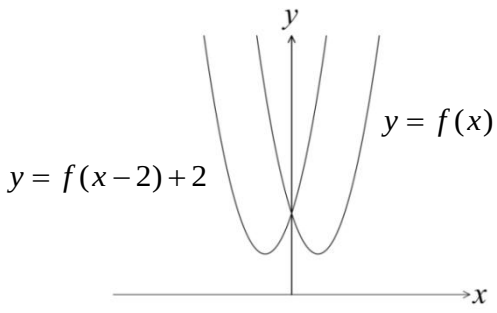
   (2 marks)

   (d) Are there any intersection point(s) of C_1 and C_3? Explain your answer.

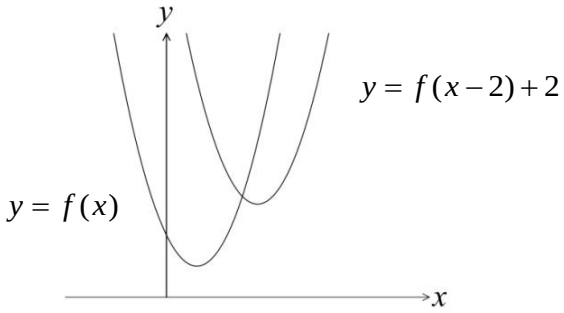
   (3 marks)

3. (MC) Which of the following may represent the graph of $y = f(x)$ and the graph of $y = f(x-2)+2$?

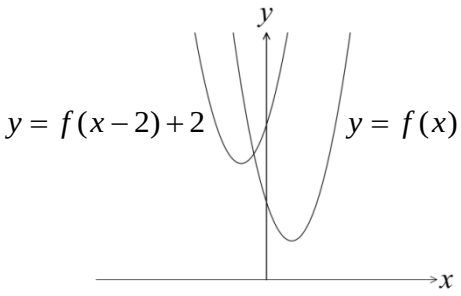
A.



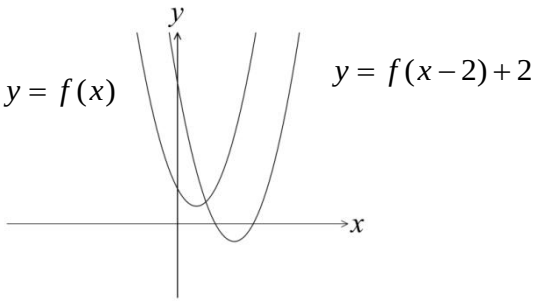
B.



C.

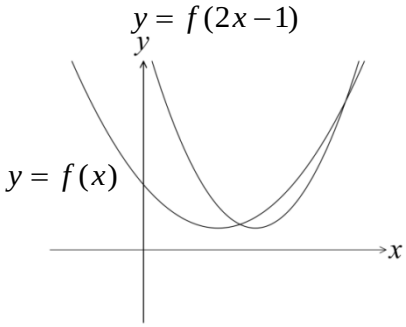


D.

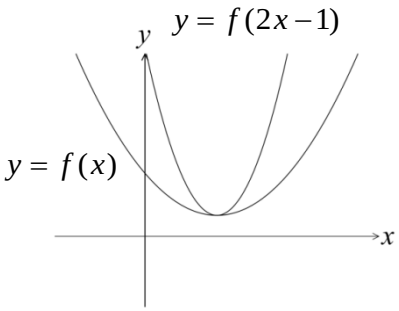


4. (MC) Which of the following may represent the graph of $y = f(x)$ and the graph of $y = f(2x-1)$?

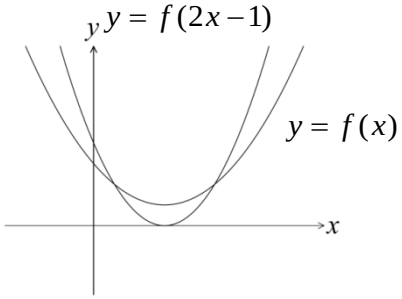
A.



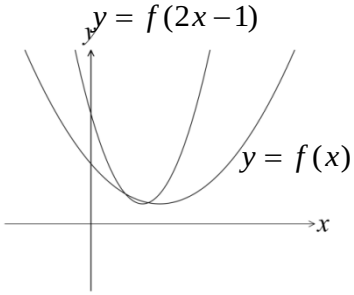
B.



C.

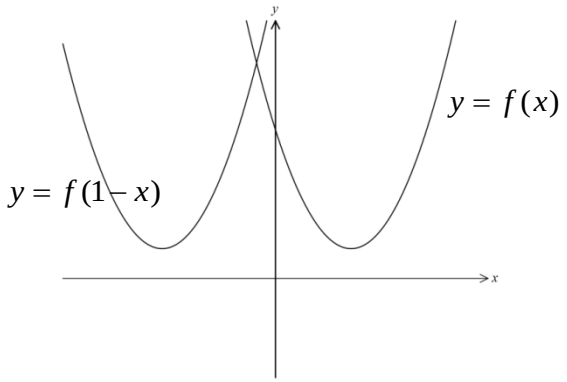


D.

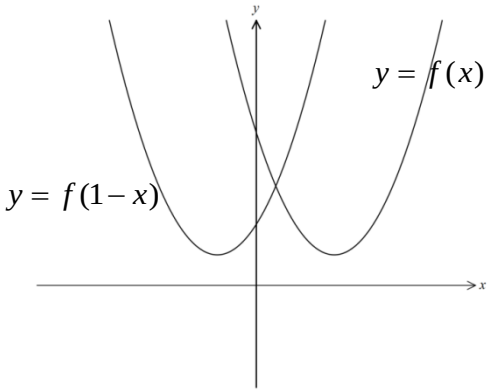


5. (MC) Which of the following may represent the graph of $y = f(x)$ and the graph of $y = f(1-x)$?

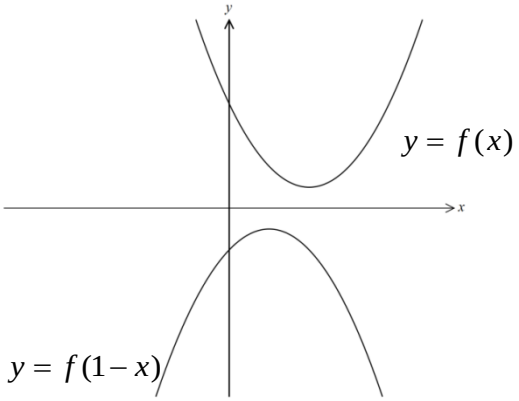
A.



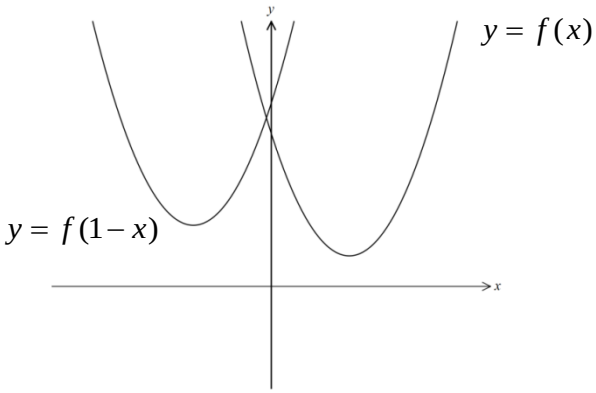
B.



C.



D.



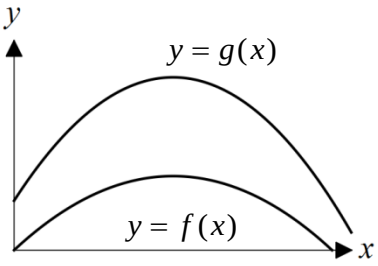
6. (MC) The graph of $y = f(x)$ is transformed to the graph of $y = g(x)$. Which of the following could be true?

A. $g(x) = f(x) + 1$

B. $g(x) = 2f(2x)$

C. $g(x) = 2f(x+1)$

D. $g(x) = 2f(x) + 1$



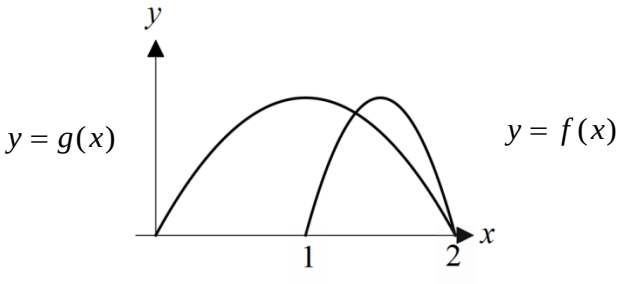
7. (MC) The graph of $y = f(x)$ is transformed to the graph of $y = g(x)$. Which of the following could be true?

A. $g(x) = f\left(\frac{x}{2}\right)$

B. $g(x) = f\left(\frac{x}{2} - 1\right)$

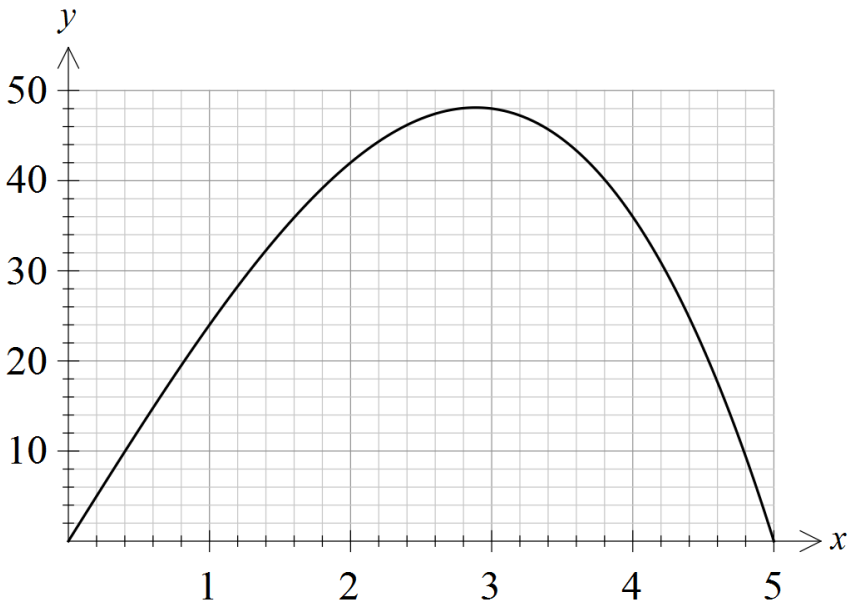
C. $g(x) = f\left(\frac{x}{2} + 2\right)$

D. $g(x) = f\left(\frac{x+2}{2}\right)$



8. (HKCEE 1980 MATH)

The figure shows the graph of $y = 25x - x^3$ for $0 \le x \le 5$.



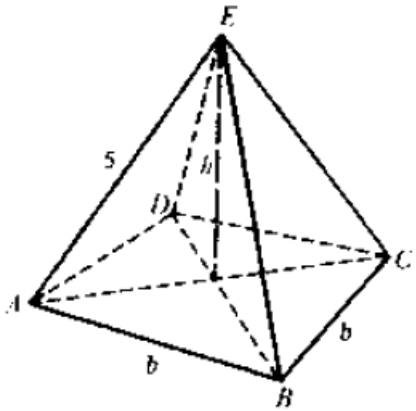
(a) By adding suitable straight line to the graph, solve the equation

$$30 = 25x - x^3,$$

where $0 \le x \le 5$. Given your answer correct to the nearest 0.1.

(2 marks)

(b) The figure below shows a right pyramid with a square base $ABCD$. $AB = b$ units and $AE = 5$ units. The height of the pyramid is h units and its volume is V cubic units.



(i) Express b in terms of h.

Hence show that $V = \frac{2}{3}(25h - h^3)$.

(3 marks)

(ii) Using (a), find the two values of h such that $V = 20$.

(Your answers should be correct to the nearest 0.1)

(2 marks)

9. (HKCEE 1983 MATH)

Equal squares of side k cm are cut from the four corners of a square sheet of paper of side 7 cm (see figure (a)). The remaining part is folded along the dotted lines to form a rectangular box as shown in figure (b).

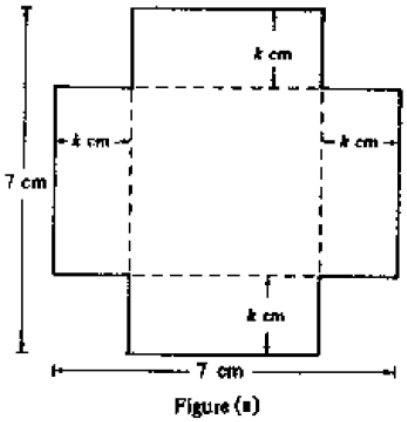


Figure (a)

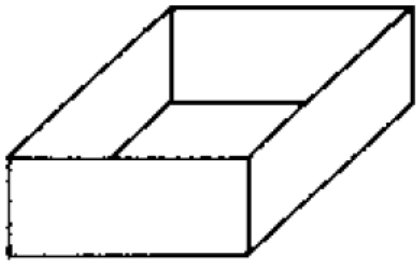


Figure (b)

(a) Show that the volume V of the rectangular box in cm^3, is $V = 4k^3 - 28k^2 + 49k$.

(3 marks)

(b) Figure (c) shows the graph of $y = 4x^3 - 28x^2 + 49x$ for $0 \le x \le 5$.
Draw a suitable straight line in figure (c) and use it to find all the possible values of x such that $4x^3 - 28x^2 + 49x - 20 = 0$. (Give the answers to 1 decimal place.)

(4 marks)

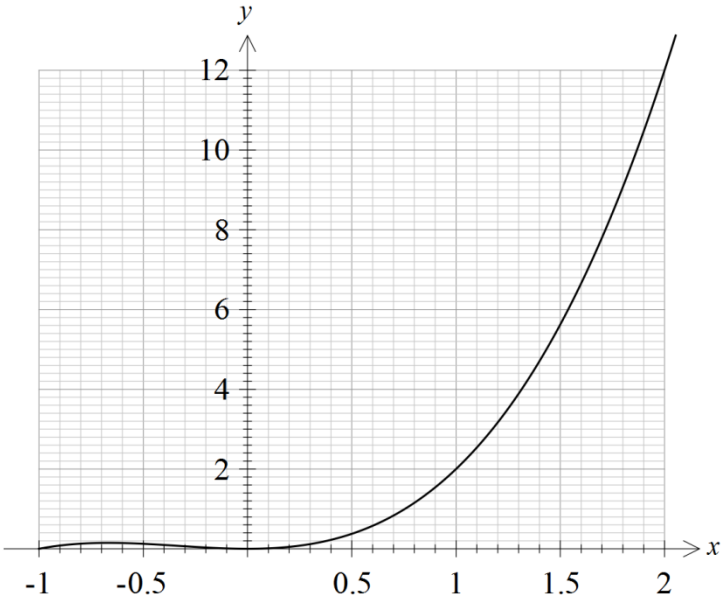
(c) Using the results of (a) and (b), deduce the values of k such that the volume of the box is $20cm^3$. (Give the answers to 1 decimal place.)

(2 marks)

Figure (c)

10. (HKCEE 1984 MATH)

(a) The figure shows the graph of $y = x^3 + x^2$ for $-1 \le x \le 2$.



Draw a suitable straight line in the figure and use it to find a root of the equation

$x^3 + x^2 + x - 4 = 0$.

(Give you answer to 1 decimal place.)

(5 marks)

(b) A bank introduces the following savings scheme in which interest is compound yearly.

If a customer deposits \$2500 on the first day of each year for three successive years, he will receive \$10 000 at the end of the third year.

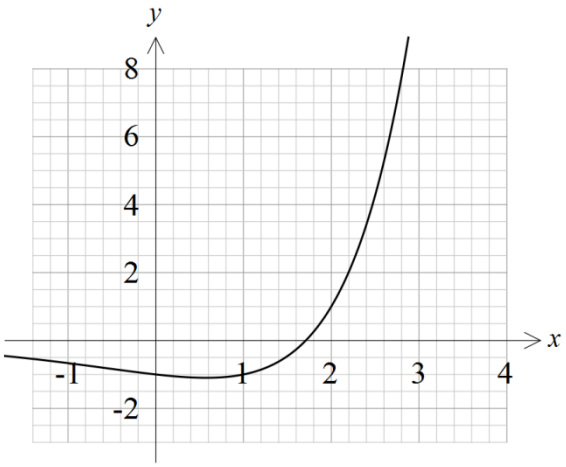
Assume that the interest rate is r% per annum.

(i) Show that $(1+r\%)^3 + (1+r\%)^2 + (1+r\%) = 4$

(ii) Find, correct to 1 decimal place, the value of r.

(5 marks)

11. The figure shows the graph of $y = a^x - b^{x+1}$ for $-1 \le x \le 3$. The graph passes through $(2,1)$ and $(0,-1)$.



(a) Find *a* and *b*.

(4 marks)

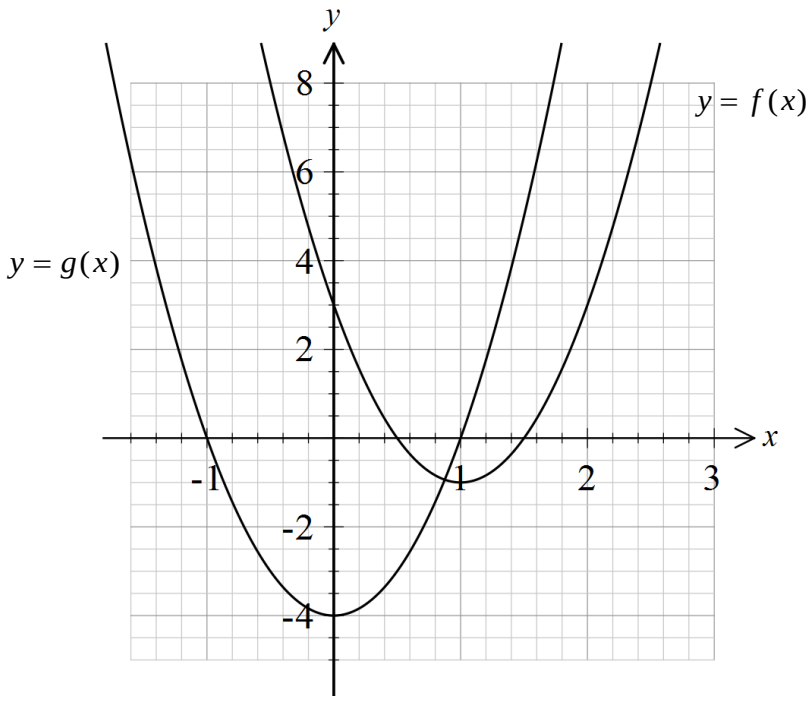
(b) By adding suitable straight line, solve the equation $\left(\frac{3}{2}\right)^x = 2 + 2\left(\frac{1}{2}\right)^x$, correct to nearest 0.1.

(3 marks)

(c) By adding suitable straight line, solve the equation $\frac{5(3^x) - 3(2^{x+1})}{2(3^x) - 2^{x+1}} = 2$, correct to nearest 0.1.

(3 marks)

12. The figure shows the graphs of $y = g(x)$ and $y = f(x)$. It is given that $f(x) = 4x^2 - 8x + 3$ and $g(x) = 4x^2 - 4$.



(a) (i) Express $f(x)$ in the form of $A(x+B)^2 + C$ where A, B and C are constants.

(3 marks)

(ii) Hence write down the transformations to obtain the graph $y = g(x)$ from the graph $y = f(x)$.

(4 marks)

(b) If the graph of $y = h(x)$ is obtained by translating the graph of $y = g(x)$ in the positive x direction by 2 units, and then enlarging it along the y direction by a factor of 2.

(i) Write down the mathematical relation between $g(x)$ and $h(x)$.

(ii) Solve the inequality $h(x) > 5$ by adding a suitable straight line on the graph above. (correct your answers to the nearest 0.2)

(6 marks)