

# MATH5.1EL HW1

## Deadline: 18 Nov, 2023

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Name, class, class no.: \_\_\_\_\_  
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### 1 Introductory Problems

**Problem 1.1.** A teacher wants to divide a class of 18 students into 3 groups, each of 6 students, to do 3 different statistical projects.

- (a) In how many ways can the students be grouped?
- (b) If there are 3 girls in the class, find the number of ways that students can be grouped so that one girl is in each group?

**Problem 1.2.** An 8-digit phone number is formed by a permutation of 2, 3, 4, 5, 6, 7, 8 and 9 .

- (a) How many different 8-digit phone numbers can be formed?
- (b) If the first digit and the last digit of an 8-digit phone number are odd numbers, how many different eight digit phone numbers can be formed?

**Problem 1.3.** Jack climbs along a cubical framework from a corner  $A$  to meet Jill at the opposite corner  $B$ . The framework, shown in the figure, is formed by joining bars of equal length. Jack chooses randomly a path of the shortest length to meet Jill. An example of such a path is also shown in Figure 1.

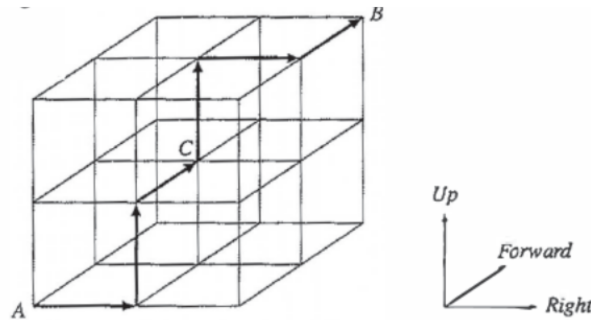


Figure 1: Right-Up-Forward-Up-Right-Forward Movement

- (a) Find the number of shortest paths from  $A$  to  $B$ .
- (b) If there is a trap at the centre  $C$  of the framework which catches anyone passing through it,
  - (i) find the number of shortest paths from  $A$  to  $C$ ,

(ii) hence find the probability that Jack will be caught by the trap on his way to  $B$ .

**Problem 1.4.** 3 students are randomly selected from 10 students of different weights. Find the probability that

- (a) the heaviest student is in the selection,
- (b) the heaviest one out of the 3 selected students is the 4 th heaviest among the 10 students,
- (c) the 2 heaviest students are not both selected.

**Problem 1.5.** Given that  $x_1, x_2, x_3, x_4$  are **positive distinct** integers.

- (a) Find the number of solution to  $x_1 + x_2 + x_3 + x_4 = 10$ .
- (b) Find the number of solution to  $x_1 + x_2 + x_3 + x_4 = 11$ .
- (c) Find the number of solution to  $x_1 + x_2 + x_3 + x_4 = 12$ .

## 2 Advanced Problems

**Problem 2.1.** Determine the number of ways to choose 5 numbers from the first 18 positive integers such that no two numbers are consecutive.

**Problem 2.2.** There are 50 rods of lengths  $1, 3, 5, 7, \dots, 99$ . How many ways are there to pick three of these rods to form a triangle? (Consider the triangle inequality).

**Problem 2.3.** A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

**Problem 2.4.** The increasing sequence  $1, 3, 4, 9, 10, 12, 13, \dots$  consists of all those positive integers which are powers of 3 or sums of distinct powers of 3. Find the 100<sup>th</sup> term of this sequence (where 1 is the 1<sup>st</sup> term, 3 is the 2<sup>nd</sup> term, and so on).