Geometry Basics

Study Guide — 3 cards per page

Axiom (postulate)

A statement accepted as true without proof; used as a starting point for logical reasoning in geometry.

Example: 'Through any two points, there is exactly one line.'

Theorem

A statement that has been proven true using axioms, definitions, and previously established theorems.

Conjecture

An unproven statement believed to be true based on observations or patterns; requires proof or a counterexample.

Point

An undefined term representing an exact location in space; has no length, width, or thickness.

Line

An undefined term: a straight path extending infinitely in both directions with no thickness.

Plane

An undefined term: a flat surface extending infinitely in all directions.

Space

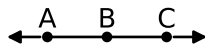The three-dimensional setting in which geometric objects exist.

Finite plane

A bounded portion of a plane; for example, the interior region enclosed by a polygon or a circle.

bounded region

Collinear points

Points that lie on the same line.

A   B   C

Coplanar points (or lines)

Points or lines that lie in the same plane.

Segment
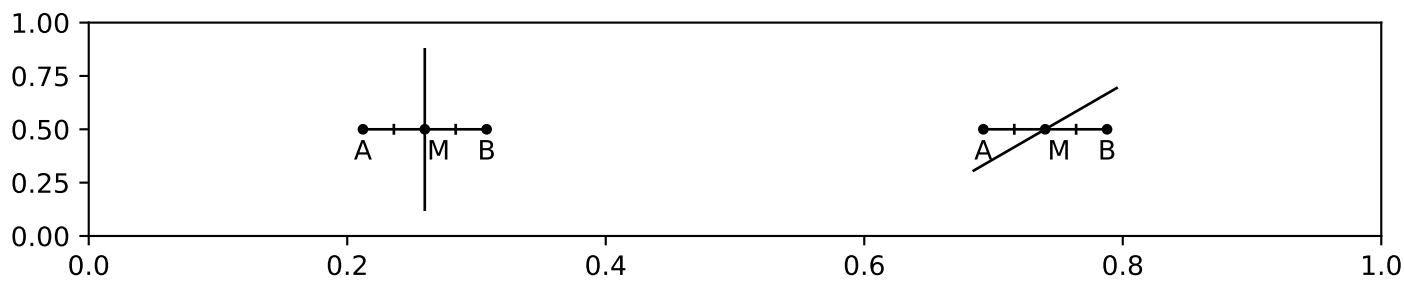
A part of a line bounded by two distinct endpoints.

Midpoint

The point on a segment that is equidistant from both endpoints.
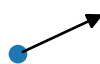
midpoint

Segment bisector

A line, ray, or segment that passes through the midpoint of a segment,
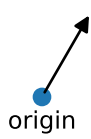dividing it into two equal parts.

Ray

A part of a line that starts at an endpoint (origin) and extends infinitely in
one direction.

Origin of a ray
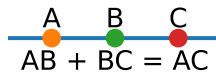
The common endpoint from which a ray begins.

origin

Opposite rays

Two rays with the same origin that lie on the same line but extend in opposite directions.

Betweenness of points

For distinct collinear points A, B, and C, point B is between A and C iff AB + BC = AC.
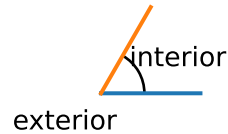
A   B   C
AB + BC = AC

Angle

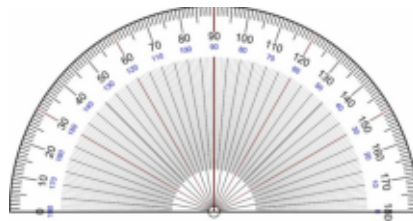A figure formed by two rays (sides) sharing a common endpoint (the vertex).

Interior and exterior of an angle

The plane regions defined by an angle: the interior lies between the sides; the exterior is outside that region.

interior

exterior

Degree measure of an angle

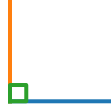A measure of rotation based on dividing a full turn into 360 equal parts (degrees).

Acute angle

An angle whose measure is less than 90°.

Right angle

An angle whose measure is exactly 90°.

Obtuse angle

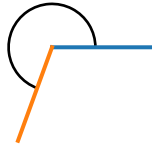An angle whose measure is greater than 90° but less than 180°.

Straight angle

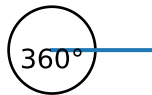An angle whose measure is exactly 180°.

Reflex angle

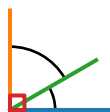An angle whose measure is greater than 180° and less than 360°.

Full angle (complete angle)
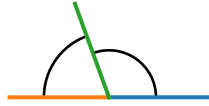
An angle equal to a full turn (360°).

360°

Complementary angles
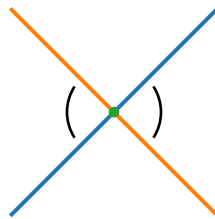
Two angles whose measures add to 90°.

Supplementary angles
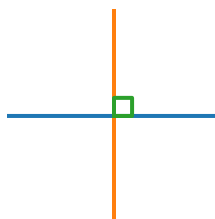
Two angles whose measures add to 180°.

Vertical angles

A pair of nonadjacent angles formed by two intersecting lines; they are congruent.
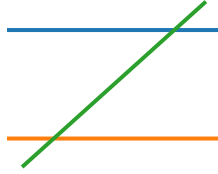
Perpendicular lines

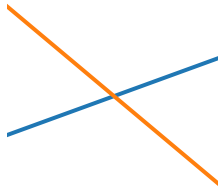Lines that intersect to form right angles.

Parallel Lines with Transversal

Two parallel lines cut by a third line (a transversal), creating special angle pairs (corresponding, alternate interior, etc.).
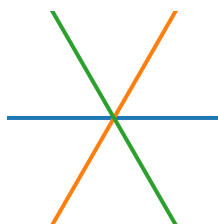
Intersection

The set of points common to two or more geometric objects (e.g., lines, segments, or planes).
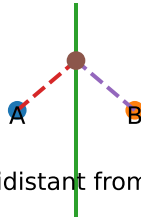
Concurrent lines

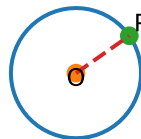Three or more lines that intersect at a single point.

Equidistant

At the same distance from two or more objects or points.

The line is equidistant from points A and B
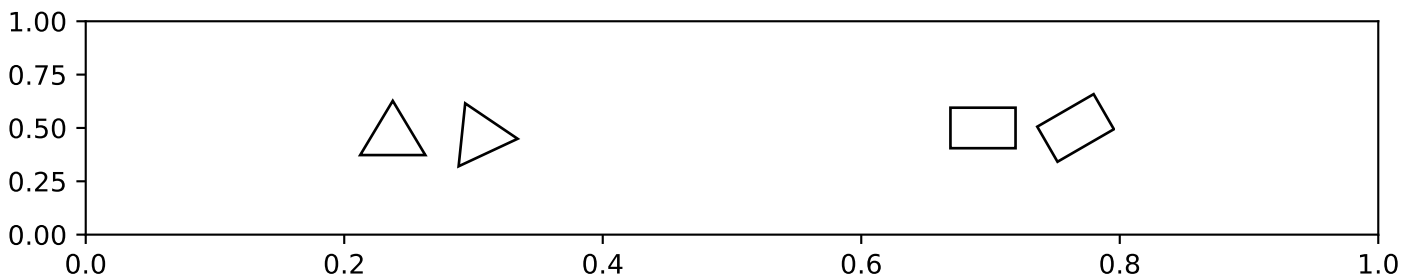
Locus

The set of all points that satisfy a given condition or rule.

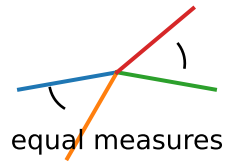Condition: all points P with PO = r (fixed distance r from O)

Rigid transformation

A transformation (translation, rotation, reflection) that preserves distances and angle measures.
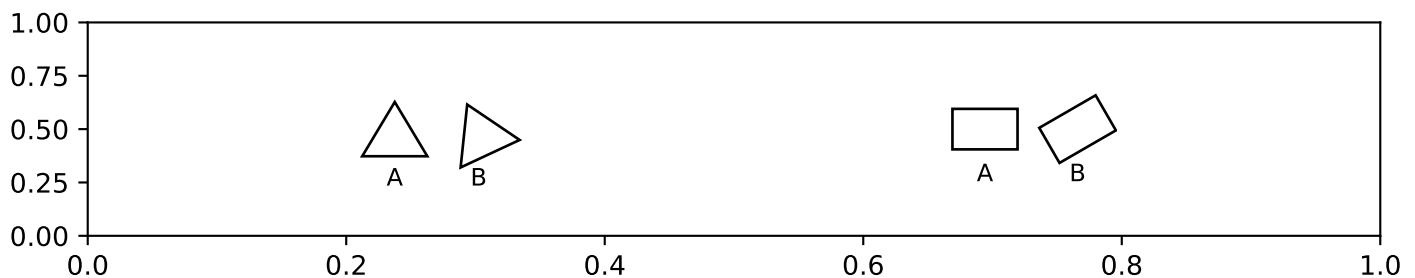
Congruent angles
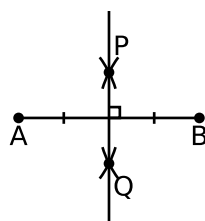
Angles that have equal measure.

equal measures

Congruent figures

Figures that are the same size and shape; one can be mapped to the other by a rigid transformation.

Construction

A precise drawing made using only allowed tools (typically compass and straightedge).

Angle bisector

A ray or segment that divides an angle into two congruent angles.

Euclidean axioms (Pogorelov–Hilbert system)

A modern, formal set of axioms for Euclidean geometry as presented by A.V. Pogorelov. This system was inspired by Hilbert's work.

• Axioms of Incidence:
  1. Through any two distinct points, there exists exactly one line.
  2. Every line contains at least two points.
  3. There exist at least three non-collinear points.
• Axioms of Order:
  4. If point B lies between A and C, then A, B, C are distinct and collinear, and B lies between C and A.
  5. Of any three collinear points, one and only one lies between the other two.
• Axioms of Congruence:
  6. Every segment is congruent to itself.
  7. If AB ≅ CD and CD ≅ EF, then AB ≅ EF (transitivity).
  8. Congruent segments and angles can be copied from one location to another.
• Axiom of Parallelism:
  9. Through any point not on a given line, there is exactly one line parallel to the given line.