

# Geometry Basics

Study Guide by Yevgen Yampolskiy

## **Axiom (postulate)**

A statement accepted as true without proof; used as a starting point for logical reasoning in geometry.

Example: 'Through any two (distinct) points, there is exactly one line.'

## **Theorem**

A statement that has been proven true using axioms, definitions, and previously established theorems.

Example: The sum of the interior angles of any triangle is  $180^\circ$ .  
This can be proven using parallel lines and alternate interior angles.

## **Conjecture**

An unproven statement believed to be true based on observations or patterns; requires proof or a counterexample.

Consider the expression  $f(n) = n^2 + n + 41$ .  
For  $n = 0, 1, 2, \dots, 10$ , the values of  $f(n)$  are:  
41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151 — all prime numbers.  
This leads to the conjecture:  
 $f(n)$  is a prime number for every natural number  $n$ .  
This is a conjecture until someone proves or disproves it.  
Try to prove or disprove this conjecture — without using AI or Google search!

## **Point**

An undefined term representing an exact location in space; has no length, width, or thickness.

## **Line**

An undefined term: a straight path extending infinitely in both directions with no thickness.



## **Plane**

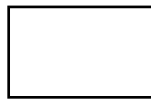
An undefined term: a flat surface extending infinitely in all directions.

## Space

The three-dimensional setting in which geometric objects exist.

## Finite plane

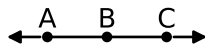
A bounded portion of a plane; for example, the interior region enclosed by a polygon or a circle.



bounded region

## Collinear points

Points that lie on the same line.



## Coplanar points (or lines)

Points or lines that lie in the same plane.



## Segment

A part of a line bounded by two distinct endpoints.



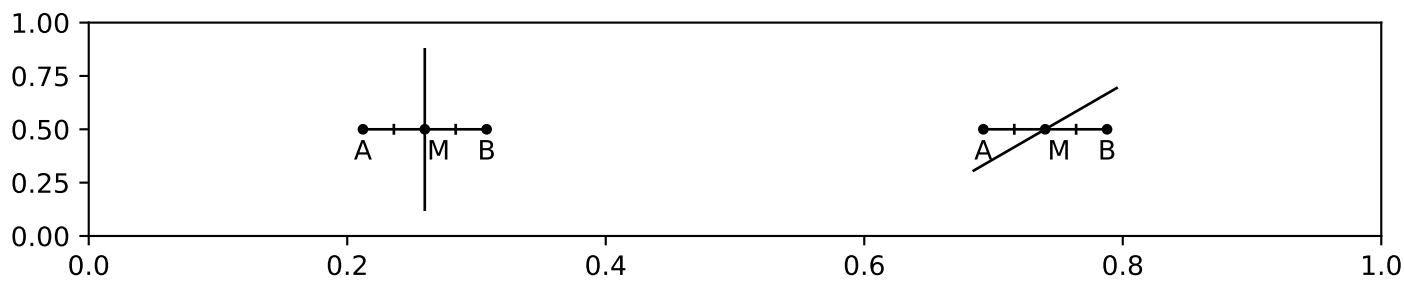
## Midpoint

The point on a segment that is equidistant from both endpoints.



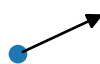
# Segment bisector

A line, ray, or segment that passes through the midpoint of a segment, dividing it into two equal parts.



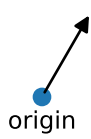
# Ray

A part of a line that starts at an endpoint (origin) and extends infinitely in one direction.



# Origin of a ray

The common endpoint from which a ray begins.



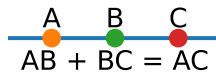
## Opposite rays

Two rays with the same origin that lie on the same line but extend in opposite directions.



## Betweenness of points

For distinct collinear points A, B, and C, point B is between A and C iff  $AB + BC = AC$ .



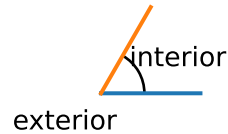
## Angle

A figure formed by two rays (sides) sharing a common endpoint (the vertex).



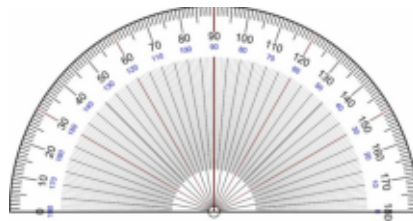
## Interior and exterior of an angle

The plane regions defined by an angle: the interior lies between the sides; the exterior is outside that region.



## Degree measure of an angle

A measure of rotation based on dividing a full turn into 360 equal parts (degrees).



## Acute angle

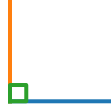
An angle whose measure is less than  $90^\circ$ .





## Right angle

An angle whose measure is exactly  $90^\circ$ .



## Obtuse angle

An angle whose measure is greater than  $90^\circ$  but less than  $180^\circ$ .



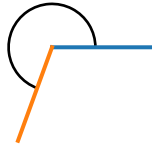
## Straight angle

An angle whose measure is exactly  $180^\circ$ .



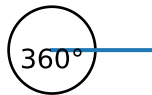
## Reflex angle

An angle whose measure is greater than  $180^\circ$  and less than  $360^\circ$ .



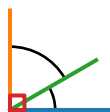
## Full angle (complete angle)

An angle equal to a full turn ( $360^\circ$ ).



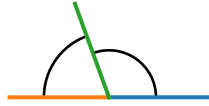
## Complementary angles

Two angles whose measures add to  $90^\circ$ .



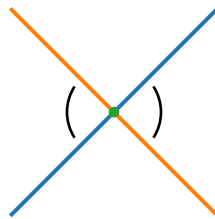
## Supplementary angles

Two angles whose measures add to  $180^\circ$ .



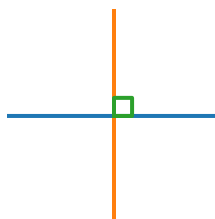
## Vertical angles

A pair of nonadjacent angles formed by two intersecting lines; they are congruent.



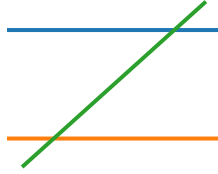
## Perpendicular lines

Lines that intersect to form right angles.



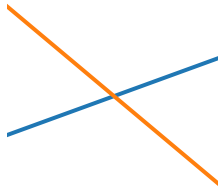
## Parallel Lines with Transversal

Two parallel lines cut by a third line (a transversal), creating special angle pairs (corresponding, alternate interior, etc.).



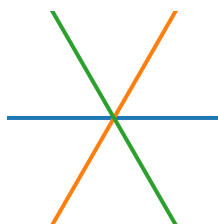
## Intersection

The set of points common to two or more geometric objects (e.g., lines, segments, or planes).



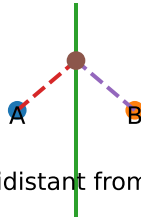
## Concurrent lines

Three or more lines that intersect at a single point.



## Equidistant

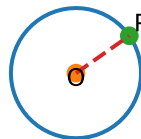
At the same distance from two or more objects or points.



The line is equidistant from points A and B

## Locus

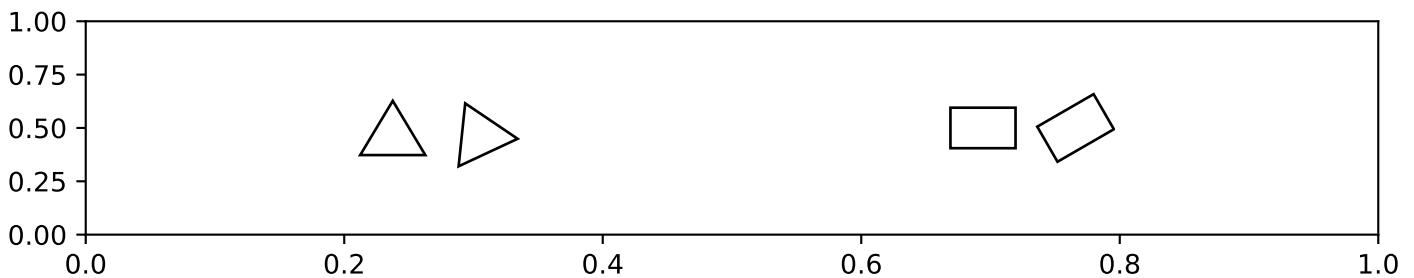
The set of all points that satisfy a given condition or rule.



Condition: all points P with  $PO = r$  (fixed distance  $r$  from O)

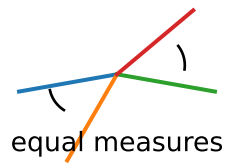
## Rigid transformation

A transformation (translation, rotation, reflection) that preserves distances and angle measures.



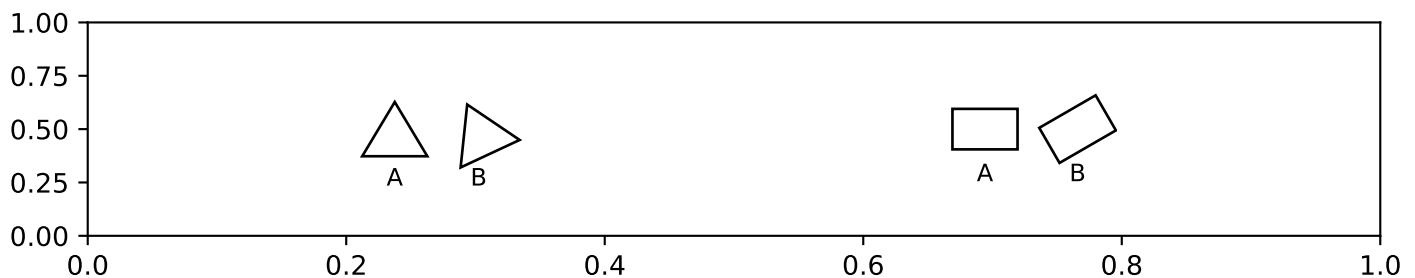
## Congruent angles

Angles that have equal measure.



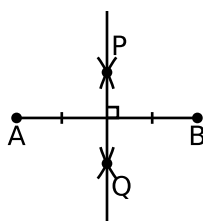
## Congruent figures

Figures that are the same size and shape; one can be mapped to the other by a rigid transformation.



## Construction

A precise drawing made using only allowed tools (typically compass and straightedge).



## Angle bisector

A ray or segment that divides an angle into two congruent angles.



## Euclidean axioms (Pogorelov-Hilbert system)

A modern, formal set of axioms for Euclidean geometry as presented by A.V. Pogorelov. This system was inspired by Hilbert's work.

- Axioms of Incidence:
  1. Through any two distinct points, there exists exactly one line.
  2. Every line contains at least two points.
  3. There exist at least three non-collinear points.
- Axioms of Order:
  4. If point B lies between A and C, then A, B, C are distinct and collinear, and B lies between C and A.
  5. Of any three collinear points, one and only one lies between the other two.
- Axioms of Congruence:
  6. Every segment is congruent to itself.
  7. If  $AB \cong CD$  and  $CD \cong EF$ , then  $AB \cong EF$  (transitivity).
  8. Congruent segments and angles can be copied from one location to another.
- Axiom of Parallelism:
  9. Through any point not on a given line, there is exactly one line parallel to the given line.