

## THE PINCH DESIGN METHOD FOR HEAT EXCHANGER NETWORKS

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**Abstract**—A novel method is presented for the design of heat exchanger networks. The method is the first to combine sufficient simplicity to be used by hand with near certainty to identify “best” designs, even for large problems. “Best” designs feature the highest degree of energy recovery possible with a given number of capital items. Moreover, they feature network patterns required for good controllability, plant layout, intrinsic safety, etc.

### SCOPE

Heat exchanger network (HEN) design is a key aspect of chemical process design. Typically, 20–30% energy savings, coupled with capital savings, can be realised in state-of-the-art flowsheets by improved HEN design [12]. The task involves the placement of process and utility heat exchangers to heat and cool process streams from specified supply to specified target temperatures. The objective is to minimise total costs, i.e. capital and operating costs expressed as annual charges. Development of systematic procedures to meet this objective has been an active area of interest in the chemical engineering literature for more than ten years. A review was recently given by Nishida *et al.* [15].

As stated in this review, the approach to the problem has been radically altered by two major discoveries: network performance targets and the network temperature pinch.

The network temperature pinch represents a bottleneck to feasible heat recovery in HEN design. Its location was first described by Linnhoff *et al.* [5, 6] and Umeda *et al.* [7, 8] although its full significance for the design task was not recognised in either source. The present paper shows that by fully exploiting the properties of the pinch the task of optimal HEN design is greatly simplified.

Network performance targets exist for the minimum utility usage, the minimum overall surface area, and the minimum number of “units” (i.e. process and utility exchangers). Calculation of these targets is simple and is possible independent of design. Thus, targets can be used, first to stimulate the designer towards better designs, and second to give the designer confidence that his solution is near-optimal.

Procedures to target for minimum utility requirements were given by Hohmann [2] and by Linnhoff

and Flower [5]. Both procedures identify the best possible degree of process heat recovery as a function of the minimum temperature difference in exchangers, heaters, and coolers,  $\Delta T_{\min}$ . The procedure by Linnhoff and Flower offers more information about the availability of heat at different temperature levels.

Capital costs can be considered through targets for minimum overall surface area and minimum number of units. However, as far as surface area is concerned it is easily established that widely differing solutions for the same problem generally feature similar overall surface areas as long as their degree of energy recovery is similar. What is important in the early stage of network design is the target for the minimum number of units. Units in excess of the minimum number require more foundations, pipework, instrumentation, maintenance, etc. and lead to greatly increased capital cost [10].

Hohmann [2] observed that the minimum number of units was generally a function of the number of process streams and utilities, expressed in the simple relationship

$$U_{\min} = N - 1 \quad (1)$$

where  $U_{\min}$  = the minimum number of units;  $N$  = the number of process streams and utilities.

Later, Linnhoff *et al.* [6] explained this equation as a special case of Euler's general network theorem [16]. They also showed that the general form of Euler's theorem relates the actual number of units in a network, as opposed to the minimum number, to the number of separate components into which the network can be split and to the number of “loops” (or feedback cycles). The theory was then extended further by Linnhoff [14] and, independently, by Grimes [13] demonstrating that, in problems featuring a network pinch, minimum utility requirements and minimum number of units are often mutually incompatible. The present paper explains this situation and

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introduces a procedure for the controlled reduction of the number of units starting from a minimum utility design.

In general then, minimum cost networks feature the correct degree of energy recovery and the correct number of units. This is achieved in two stages with the design method presented in this paper. First, the method aims for a minimum energy solution, corresponding to a specified  $\Delta T_{\min}$ , with no more units than is compatible with minimum energy. This task is achieved through a thorough understanding of the pinch phenomenon, hence the method is called the pinch design method. We also show in this paper how the pinch design method can be used to quickly identify the best starting value of  $\Delta T_{\min}$ . Second, the method involves a controlled reduction in the number of units. This may require "backing-off" from minimum utility usage. Together, both stages represent the only design method published to date combining an ease and speed of application suitable for hand calculation with near-certainty of finding "best" solutions. Previously published methods were either fast but prone to failure[3, 4, 13] or dependable but cumbersome to use[5, 6].

As an added bonus, the pinch design method identifies situations where stream splitting is inevitable for a minimum utility design. A clear rationale is given clarifying whether stream splitting can be avoided or not. This represents a breakthrough in understanding. To date, many published design methods have made use of stream splitting but none could guarantee that, for the problem in question, there would not be another solution without stream splits. In other words, one could not be sure whether streams were split because of the complexity of the problem or because of the inadequacy of the method. This was an undesirable state of affairs as stream splits involve complications in pipework and control and should only be implemented if they are necessary. Just as the pinch design method itself, the stream splitting rationale follows from a better understanding of the implications of the HEN pinch.

## THE PINCH

In this section the phenomenon of the pinch itself is discussed. The discussion is in five parts. First, a brief review is given of how the pinch is located. Second, the physical significance of the pinch and its implications on utility usage are described. Third, a powerful representation for both the pinch and HEN stream data is explained. This representation will be used extensively in the paper. Fourth, the factors affecting the occurrence of pinches in industrial HEN problems are highlighted and a reason is given why the majority of industrial HEN problems have pinches. Finally, the implications of pinches on capital costs are considered.

### Locating the pinch (a review)

The task of locating the pinch and indeed the application of the pinch design method is illustrated by using an example problem for which stream data are given in Table 1. The example problem is called Test Case No. 3 (TC3) following the convention of Linnhoff and Flower[5] who introduced Test Case No. 1 and Test Case No. 2. Note that the data for all Test Cases is based on constant CP. This feature is often referred to as a shortcoming of all early work in this area. However, every practical process, with and without phase changes, can easily be described in terms of linearised (i.e. CP = constant) temperature enthalpy data[17].

The pinch location for a HEN problem, together with the minimum utility requirement, can be calculated using the problem table algorithm of Linnhoff and Flower[5] for a specified value of  $\Delta T_{\min}$ . The results of the procedure, when applied to TC3 with a  $\Delta T_{\min}$  of 20°C, are shown in Table 2.

In the table the stream data are shown on the left, divided into six temperature intervals, corresponding to "subnetworks" and therefore called SN1-SN6. These intervals are defined by process stream supply and target temperatures. For example, SN3 is defined by the target temperature of stream No. 4 and the supply temperature of stream No. 2.

Table 1. Stream data for test case No. 2

STREAM NUMBER AND TYPE	HEAT CAPACITY FLOWRATE CP (Kw / °C)	TS (°C)	TT (°C)
(1) HOT	2	150	60
(2) HOT	8	90	60
(3) COLD	2.5	20	125
(4) COLD	3.0	25	100

$$\Delta T_{\min} = 20^{\circ}\text{C}$$

Note that to ensure the feasibility of complete heat exchange hot and cold streams are separated by  $\Delta T_{\min}$ . For example, the upper boundary of SN3 is defined by cold stream No. 4 at 100°C while the hot stream at this point is at 120°C.

The feasibility of complete heat exchange between all hot and cold streams is an important feature of the problem table algorithm. It means that for each subnetwork there will either be a net heat deficit or surplus but never both. These deficit or surplus figures are shown in column 1 of Table 2. The sign convention is such that a surplus is negative and a deficit positive.

Another important feature of the problem table algorithm is the feasibility of heat transfer from higher to lower subnetworks (cascading). In other words, heat surplus from higher temperature subnetworks can be used to satisfy heat deficit of lower temperature subnetworks. The calculation of the amount of heat which can be passed on in this manner is performed in column 2 and column 3 of Table 2. It is initially assumed that the heat input from external utilities is zero. This is represented in Table 2 by a zero input for SN1 (column 2). Having made this assumption, it is an easy task to calculate the output from SN1 by simply adding the surplus to the input. This then forms the input to SN2. The procedure is repeated for all subnetworks.

To be feasible, the flow of heat from high temperature subnetworks to low temperature subnetworks must not be negative. Thus if negative values are generated in columns 2 and 3 of Table 2 the heat input to SN1 must be increased. The minimum increase is that which guarantees that all heat flows are positive or zero see columns 4 and 5. The minimum hot utility usage is then given by the input to SN1 (column 4). The minimum cold utility usage is given by the heat flow out of the coldest sub-

network (column 5). (Note in [5] "heat flows" were referred to as "heat fluxes". The term flow replaces flux in this paper so as to avoid possible confusion with the term heat flux as used in the context of heat transfer equipment.)

The results of the problem table analysis in Table 2 are shown diagrammatically in Fig. 1(a). Each subnetwork is shown with all heat flows as calculated by the problem table algorithm. Notice that the heat flow from SN3 to SN4 is zero. All other flows are positive. The point of zero heat flow represents the pinch.

In Fig. 1(b), the pinch is shown partitioning the problem into two regions, a hot and a cold end. The hot end, which comprises all streams or parts of streams hotter than the pinch temperature, requires only process exchange and utility heating. Utility cooling is not required. The cold end, which comprises all streams or parts of streams cooler than the pinch temperature, requires only process exchange and utility cooling. Utility heating is not required. There is no heat transfer across the pinch. Also, both utility requirements are the minimum achievable.

#### Significance of the pinch

We are now in a position to discuss the causes and effects of using more than the minimum utility usage. We shall use TC3, decomposed into its hot and cold ends as an example (see Fig. 2). Note that in Fig. 2 the hot end is referred to as a heat "sink" as only utility heating is required. The cold end is referred to as a heat "source" as only utility cooling is required.

First consider the effects of transferring heat across the pinch (see Fig. 2(a)). Any heat transferred must, by enthalpy balance around the sink, be supplied from hot utility in addition to the minimum requirement. Similarly, enthalpy balance around the source

Table 2. The problem table for Test Case No. 3

STREAMS AND TEMPERATURES				1	2	3	4	5
SUBNETWORK	COLD STREAMS (3) (4) :		T (°C) :	DEFICIT	ACCUMULATED		HEAT FLOWS	
					INPUT	OUTPUT	INPUT	OUTPUT
			150					
SN1		125	145	- 10	0	10	107.5	117.5
SN2		100	120	+12.5	10	-2.5	117.5	105
SN3		70	90	+ 105	-2.5	-107.5	105	0
SN4		40	60	- 135	-107.5	27.5	0	135
SN5		25		+82.5	27.5	-55	135	52.5
SN6		20		+12.5	-55	-67.5	52.5	40

shows that heat transfer across the pinch also increases the cold utility above the minimum required. In other words, heat transfer across the pinch incurs the double penalty of increased hot and cold utility requirement for the HEN design task [6].

Using an equivalent argument, the effect of utility cooling above and utility heating below the pinch can be assessed (see Figs. 2(b) and 2(c)). A removal of heat  $Y$  from the sink increases both utilities by  $Y$  (see Fig. 2(b)). A supply of heat  $Z$  to the source has the

analogous effect (see Fig. 2(c)). Thus for minimum utility usage utility cooling is not permitted above the pinch and utility heating is not permitted below.

The above relationships between the pinch and utility usage are fundamental observations of which the HEN designer ought to be aware if he is to produce optimal solutions to HEN problems. Without such knowledge, design for best energy performance is very much a case of shooting in the dark. In complex networks, exchangers and utility heaters and coolers will almost inevitably be placed in positions which violate the pinch. The result will be more utility heating and cooling than otherwise would have been required. In retrofit studies the equivalent arguments apply. The pinch highlights existing utility or process exchangers which are "at fault", i.e. violate the pinch and prevent a minimum utility design.

#### The pinch in the grid representation

It is desirable when developing a design, whether new or retrofit, to do so on a representation which shows the stream data and the pinch together. In addition, the presentation ought to be sufficiently flexible to allow easy manipulation of matches.

The grid representation, first introduced by Linnhoff and Flower [5], can be modified to achieve these objectives. To illustrate this, TC3 has been drawn in the modified grid (see Fig. 3). Hot streams are grouped together at the top and run left to right from their supply to target temperatures. Cold streams beneath run countercurrent. In contrast to [5], the pinch division is represented in the diagram by dividing the stream data at the appropriate temperatures, remembering to separate hot and cold streams by  $\Delta T_{\min}$ .

Process exchangers are represented by vertical lines and circles on the streams matched (see Fig. 4). This contrasts with the circle/triangle representation of [5] in which the orientation of triangles was used to

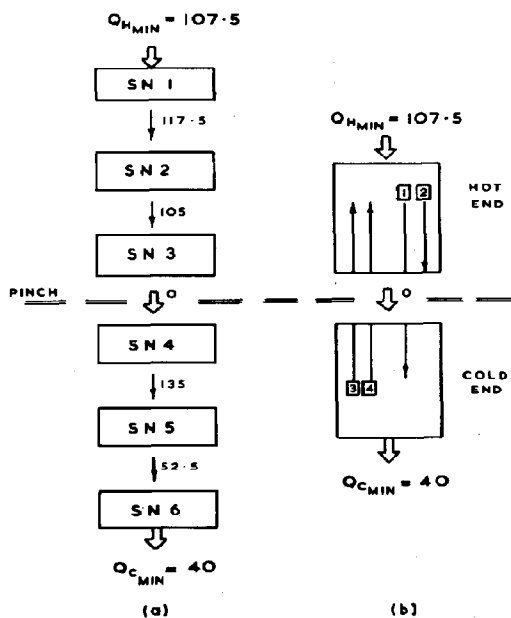


Fig. 1. (a) Subnetwork heat flow diagram for TC3. (b) Subnetworks combined into a hot and cold region.

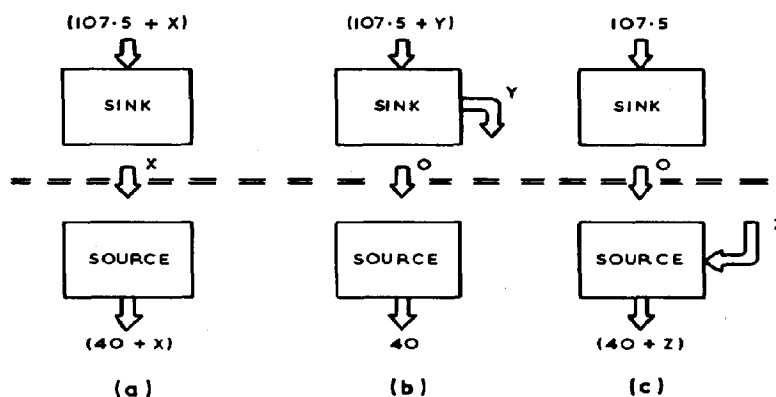


Fig. 2. (a) Effect of heat transfer across the pinch. (b) Effect of utility cooling above the pinch. (c) Effect of utility heating below the pinch.

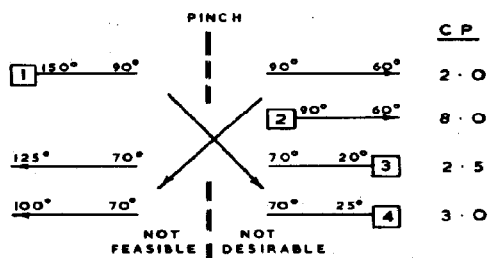


Fig. 3. Pinch division of TC3 shown in the grid.

indicate the relative magnitude of the stream CPs. This added detail is not required in the present paper. As in [5] heaters are represented in the modified grid by circles on the cold streams and coolers by circles on the hot streams (see Fig. 4). As indicated in Fig. 3, heat exchange from the cold end of the problem to the hot end is not feasible with  $\Delta T_{\min} = 20^\circ\text{C}$ . Exchange from the hot end to the cold end is not desirable as this would constitute heat transfer across the pinch incurring the energy penalties shown in Fig. 2(a). Thus, Fig. 3 provides a "grid" for two completely separate design tasks, the hot end and the cold end.

#### Threshold problems

A pinch does not occur in all HEN problems. Certain problems remain free of a pinch until the minimum allowed driving force,  $\Delta T_{\min}$ , is increased up to or beyond a threshold value  $\Delta T_{\text{thresh}}$ . It is for this reason that we call these problems "threshold problems".

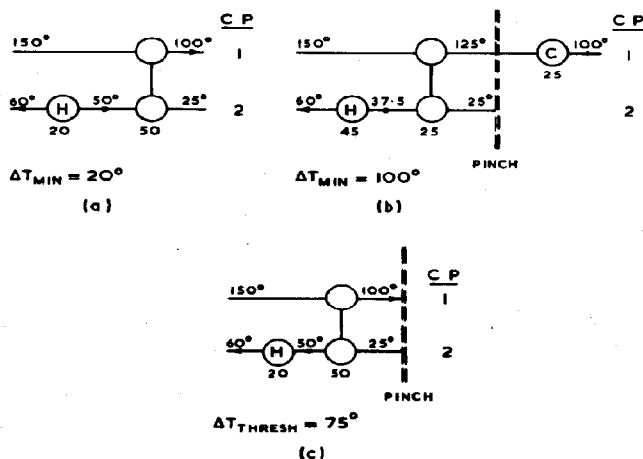
The concept of a threshold problem can be envisaged as a "very hot" hot stream matched to a "very

cold" cold stream (see Fig. 4(a)). The design for this "network" consists of a single exchanger and a utility heater. The single exchanger completely satisfies the smaller of the two stream heat loads. The utility heater is required only to achieve the enthalpy balance for the total problem.

It is apparent from Fig. 4(a) that the hot utility heat load remains constant, unaffected by any specification of  $\Delta T_{\min}$ , providing the specified  $\Delta T_{\min}$  is less than the smallest temperature driving force in the exchanger, which is  $75^\circ\text{C}$  in the example. (It is assumed, for the purposes of this example, that the hot utility supply temperature is such that the smallest  $\Delta T$  will always occur in the exchanger.) However, when the specified  $\Delta T_{\min}$  exceeds  $\Delta T_{\text{thresh}}$ , as in Fig. 4(b), the need for both utility heating and cooling is introduced. This is because complete heat exchange between the two streams is no longer feasible without violating  $\Delta T_{\min}$ . Notice also that a pinch has been introduced into the problem.

A borderline situation occurs when the specified  $\Delta T_{\min}$  equals the threshold value (see Fig. 4(c)). The problem has become pinched but the utility usage is the same as for lower values of  $\Delta T_{\min}$ . This borderline case is a general feature of a threshold problem.

Figure 5 shows the threshold behaviour in terms of a plot of utility requirements vs  $\Delta T_{\min}$  for the HEN problem "7SP2". This problem was first introduced by Masso and Rudd [1]. In Fig. 5(a), the specified  $\Delta T_{\min}$  at  $20^\circ\text{C}$ , is less than  $\Delta T_{\text{thresh}}$ . As a result there is no pinch and only utility cooling is required. When  $\Delta T_{\min}$  equals  $\Delta T_{\text{thresh}}$  a pinch is introduced into the problem (see Fig. 5(b)). As for the simple two stream problem discussed above, there is no increase in utility usage. The utility usage only increases when the minimum allowed driving force is increased above  $\Delta T_{\text{thresh}}$  (see Fig. 5(c)). Both hot and cold utilities are then required and the problem is pinched.

Fig. 4. (a) A threshold problem. (b) A pinched problem. (c) The threshold  $\Delta T_{\min}$ .

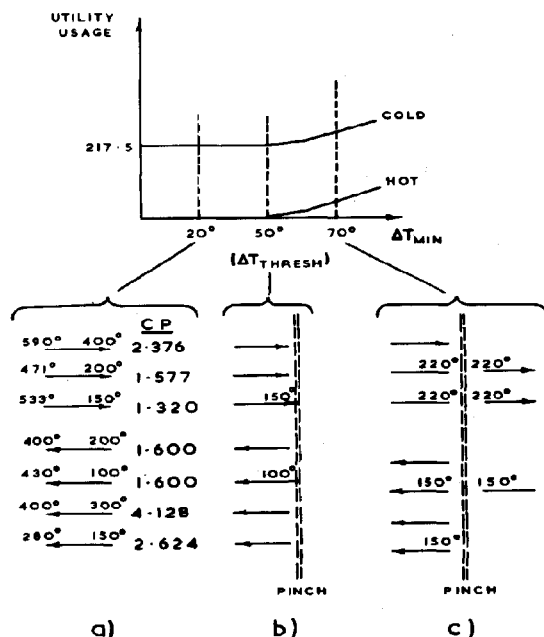


Fig. 5. (a) Data for 7SP2. (b) The threshold  $\Delta T_{\min}$  for 7SP2. (c)  $\Delta T_{\min} > \Delta T_{\text{thresh}}$  for 7SP2.

It is interesting to note that most of the early example problems in the literature on HEN design (i.e. 4SP2, 5SP1, 6SP1, 7SP1, 7SP2, 10SP1, etc.) were threshold problems while at the same time the basic distinction between pinched and threshold problems was not understood[6]. The full importance of this comment will become clear when considering that most industrial problems are pinched problems (see below). This then explains why none of the early

work in HEN design generated much impact in industrial practice.

#### The utility pinch

In applying the problem table procedure to TC3 it was assumed that the utilities were available at extreme temperatures, i.e. the hot utility was hot enough and the cold utility cold enough for all process requirements. In practice, this is rarely desirable as less extreme utilities tend to cost less, e.g. low pressure steam for process heating costs less than high pressure steam, cooling water costs less than refrigeration, etc. There is often a good cost incentive for reducing extreme temperature utility loads by the introduction of intermediate temperature utilities. The reasoning in "significance of the pinch", tells us that any new hot utility must be supplied above the pinch and any new cold utility must be supplied below the pinch. Failure to do so would incur the double penalty of increased utility heating and cooling.

In Fig. 6(a), a new hot utility supply has been introduced to the hot end of a hypothetical problem. As the heat load on this new utility increases, savings are made on the hottest utility supply (see Fig. 6(b)). There comes a point when the hottest utility load is reduced to such an extent that it just satisfies the heating requirements in the hottest region of the problem (see Fig. 6(c)). The result is a division of the hot end of the HEN design task into two separate regions, i.e. a new pinch has been created. As it is a direct consequence of the introduction of the new utility, we call it a utility pinch. The problem table algorithm is easily adapted to calculate the maximum loads on intermediate temperature utilities and the resulting utility pinch locations[14].

With this understanding it is hardly surprising to find that, in industrial HEN design, the occurrence of an unpinched or threshold problem is extremely rare! The pinched problem is the norm.

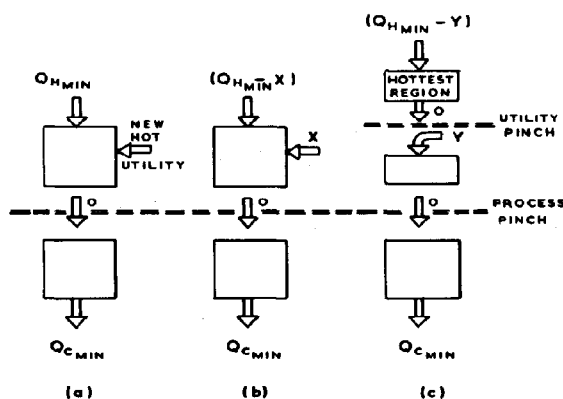


Fig. 6. (a) The correct placement of a new hot utility. (b) Distributing the minimum heating requirement. (c) The utility pinch.

*Capital cost implications of the pinch*

Linnhoff[14] and Grimes[13] independently concluded that the minimum number of units and the minimum utility usage are often mutually incompatible in some HEN problems. This incompatibility was shown by both authors to be the result of the pinch division of the problem. They showed that the "minimum number of units" targeting equation

$$U_{\min} = N - 1 \quad (1)$$

where  $U_{\min}$  = the minimum number of units;  $N$  = the number of process streams and utilities, must be applied to the hot end and the cold end separately.

Figure 7(a) shows the results when the targeting equation is applied in this manner to TC3. A target minimum of seven units is required to complete a minimum utility design, three units for the hot end, four units for the cold end. Note that by accepting the pinch division, stream Nos. 1, 3 and 4 are counted twice by the targeting equation. They exist in both the hot and cold ends.

Figure 7(b) shows the result when the targeting equation is applied singularly to the whole problem. Streams Nos. 1, 3 and 4 are no longer counted twice and the target for the minimum number of units to complete the design is now five. To achieve a five unit design in practice means that either:

- \* heat "X" must be transferred across the pinch, incurring the double penalty of increased hot and cold utility usage, see Fig. 7(b) or
- \* the  $\Delta T_{\min}$  constraint initially imposed can be relaxed in certain exchangers subject to  $\Delta T_{\min} > 0$ .

In the first case there is a cost trade-off between the number of capital items and the utility usage. In the second case there is a cost trade-off between the number of units and surface area. The exploration of these trade-offs is discussed in the section "simplifying the minimum utility design".

**THE PINCH DESIGN METHOD**

The above discussion has dealt with the relationship between the utility usage, the number of capital items and the occurrence and location of the HEN pinch. With this understanding, a totally new design method is now developed which yields, quickly, "best" solutions to HEN problems. The initial aim is to produce designs using the minimum utility usage with as few capital items, i.e. units, as is compatible.

The design method is based on a fundamentally new approach. This is discussed first under "the philosophy". The discussion is followed by details of the actual procedure, namely, the development of "feasibility criteria" which quantify the restrictions placed on design by the pinch, the use of a "tick-off heuristic" to ensure the design is steered towards the fewest possible units and the solution of the "remaining problem" allowing consideration of process constraints and other requirements.

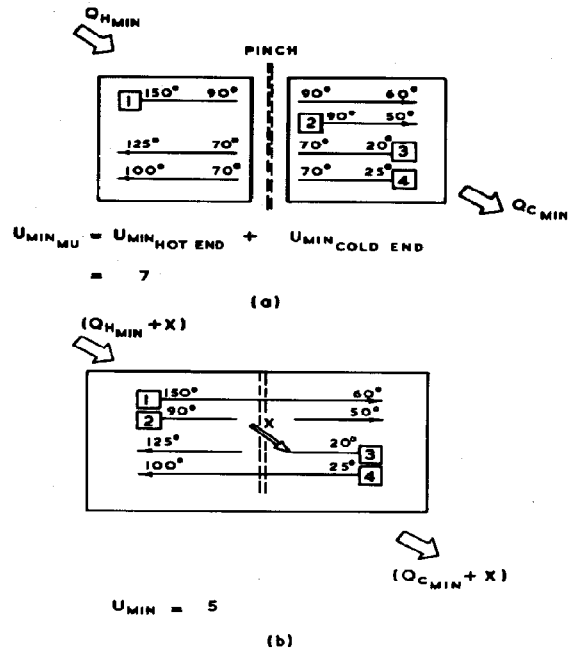


Fig. 7. (a) The number of units for maximum energy recovery. (b) The overall minimum number of units.

*The philosophy*

The pinch represents the most constrained region of a design; after all,  $\Delta T_{\min}$  exists between all hot and cold streams at the pinch. As a result the number of feasible matches in this region is severely restricted. Quite often there is a crucial or "essential" match. If this match is not made, this will result in heat transfer across the pinch and thus in increased hot and cold utility usage. The pinch design method, therefore

- \* recognises the pinch division
- \* starts the design at the pinch developing it separately into two remaining problems.

This approach is completely different from the normal intuitive approach of starting the design at the hot side and developing it towards the cold. When a design is started at the hot side, initial design decisions may later necessitate follow-up decisions which violate the pinch. On the other hand, when a design is started at the pinch, initial design decisions are made in the most constrained part of the problem and are less likely to lead to difficulties later.

Thus, commencing a design at the pinch has the distinct advantage of allowing the designer to identify essential matches or topology options in the most constrained region of the design which are in keeping with minimum utility usage. There is a further advantage, namely the designer will always have the option to violate the pinch if required with full knowledge of

the final penalties to be incurred. When a match is placed knowingly in violation of the pinch the heat flow across the pinch can be quickly established. This heat flow is equivalent to the final increase in hot and cold utility (Fig. 2(a)).

Once away from the pinch the design task is no longer so constrained hence the number of topology options usually increases. This increase in the number of options can be used to advantage by the designer. After all, the design objective is not just the identification of a cost optimal topology but also one which is safe and controllable. By discriminating between match options the designer can steer his design, using his judgement and process knowledge, towards a safe, controllable and practical network.

In developing the pinch design method this benefit has been recognised. As a result, the method does not "tell" the designer which matches to make but rather it informs him of his options. In the temperature constrained region near the pinch essential matches are identified using feasibility criteria. The same criteria will inform the designer whether there are options available at the pinch and whether stream splitting is required. When designing away from the pinch the need for feasibility criteria diminishes and the method allows the designer to choose topologies based on process requirements.

To summarise, the pinch design method incorporates two fundamentally important features. First, it realises the pinch is the most temperature constrained region. The design is started at the pinch and developed moving away. Second, it allows the designer to choose between options.

#### Feasibility criteria at the pinch

The identification of essential matches at the pinch, of available design options and of the need to split streams, is achieved by applying three feasibility criteria to the stream data at the pinch. In developing these feasibility criteria reference is made to "pinch exchangers" (sometimes called "pinch matches"). As Fig. 8 shows these are exchangers which have the minimum temperature approach,  $\Delta T_{\min}$ , on at least one side and at the pinch.

*The number of process streams and branches.* The first feasibility criterion concerns the stream population at the pinch. The population of hot and cold streams has to be such that it will allow an arrangement of exchangers compatible with minimum utility usage.

Consider a hot end design as in Fig. 9(a) (not TC3). Utility cooling above the pinch would violate the minimum utility objective. Therefore, each hot stream has to be cooled to the pinch temperature by process exchange. This is attempted in Fig. 9(a) by placing pinch matches between hot stream No. 2 and cold stream No. 4 and hot stream No. 3 and cold stream No. 5. Notice, however, that having made these matches hot stream No. 1 cannot be matched with either cold stream without violating the  $\Delta T_{\min}$  constraint! Utility cooling would now be required

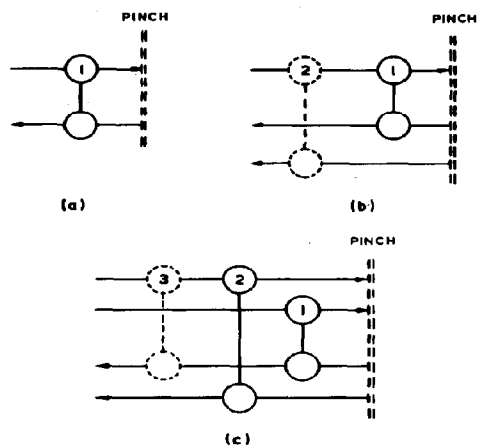


Fig. 8. (a) A pinch match. (b) Exchanger 2 is not a pinch match. (c) Exchanger 3 is not a pinch match.

above the pinch to cool stream No. 1 to the pinch temperature. In such circumstances we say the original stream data at the pinch is not compatible with a minimum utility design.

When this incompatibility occurs the streams at the pinch need "correcting" by stream splitting (see Fig. 9(b)). By splitting a cold stream an extra cold "branch" is created, allowing a pinch match with hot stream No. 1.

To summarise, the hot end stream population at the pinch is compatible with a minimum utility design only if a pinch match can be found for each hot stream. For this to occur inequality (2a) must apply

$$NH \leq NC \quad (2a)$$

where  $NH$  is the number of hot streams or branches and  $NC$  is the number of cold streams or branches. Stream splitting may be needed to ensure that the inequality is fulfilled.

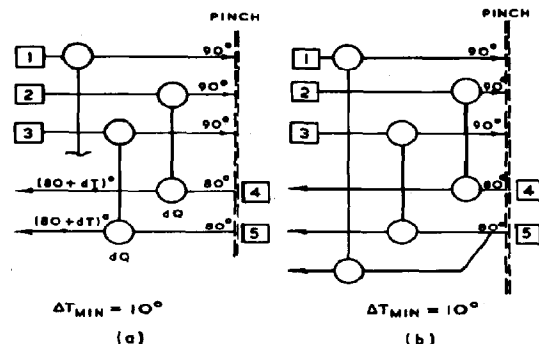


Fig. 9. (a) An infeasible hot end design at the pinch. (b) Stream splitting at the pinch.



The converse arguments apply below the pinch. To avoid utility heating each cold stream must be brought to the pinch temperature by process exchange, see Fig. 10. As a result, a pinch match is required for each cold stream at the pinch and this is possible only if inequality (2b) holds

$$NH \geq NC. \quad (2b)$$

Once again stream splitting may be necessary to ensure that the inequality is fulfilled.

*The CP inequality for individual matches.* The second feasibility criterion is concerned with temperature feasibility. As shown in Fig. 11, temperature driving force in a pinch match cannot decrease away from the pinch. For this condition to be fulfilled the following CP inequalities must apply in every pinch match

Hot end pinch match

$$CPH \leq CPC. \quad (3a)$$

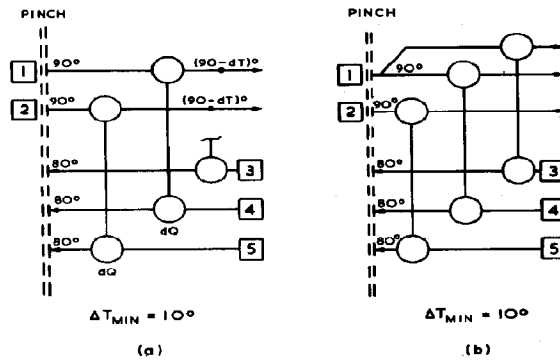


Fig. 10. (a) An infeasible cold end design at the pinch. (b) Stream splitting at the pinch.

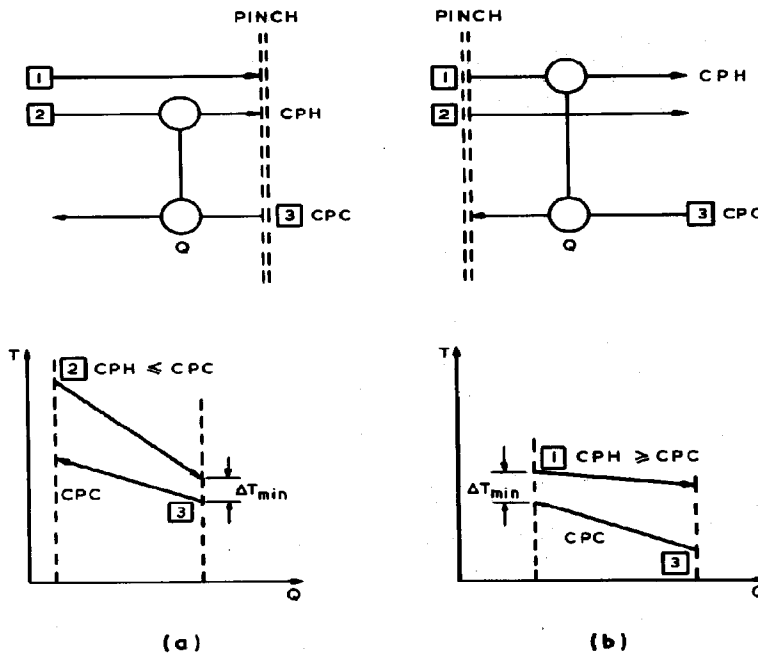


Fig. 11. (a) A feasible pinch exchanger above the pinch. (b) A feasible pinch exchanger below the pinch.

## Cold end pinch match

$$CPH \geq CPC \quad (3b)$$

where  $CPH$  is the heat capacity flowrate of a hot stream or stream branch and  $CPC$  is the heat capacity flowrate of a cold stream or stream branch.

If an arrangement of matches fulfilling these inequalities is not possible then it is necessary to change one or more CPs by stream splitting.

It should be noted that inequalities (3a) and (3b) only apply at the pinch. Away from the pinch, temperature driving forces may have increased sufficiently to allow matches in which the CP's of the streams matched violate the inequalities.

*The CP difference.* To understand the third feasibility criterion at the pinch it is convenient to define the "CP difference"

For a hot end pinch match

$$CP \text{ difference} = CPC - CPH. \quad (4a)$$

For a cold end pinch match

$$CP \text{ difference} = CPH - CPC. \quad (4b)$$

Similar equations can be written for differences in the overall sum of hot stream CPs and cold stream CPs at the pinch.

## Immediately above the pinch

$$\text{Overall CP difference} = \sum_1^{NC} CPC - \sum_1^{NH} CPH. \quad (5a)$$

## Immediately below the pinch

$$\text{Overall CP difference} = \sum_1^{NH} CPH - \sum_1^{NC} CPC. \quad (5b)$$

Figure 12 illustrates how we can use the concept of the CP difference for an early identification of matches that are feasible themselves but are not compatible with a feasible overall network. In Fig. 12(a) a case is shown where the sum of the match CP differences equals the overall difference. All streams at the pinch are involved in pinch exchangers. Figure 12(b) shows a hot end pinch design for a different problem where the pinch match CP differences amount to less than the total. In this case not all streams at the pinch are involved in pinch matches. Figure 12(c) shows a different problem again where the pinch match CP difference exceeds the total. The pinch match shown is feasible by itself (it fulfills inequality (3c) but it is incompatible with the overall CP difference. (The pinch match has a CP difference of 6 whereas the total available is only 4.) Thus, it will not be possible to complete this design. A match between the remaining hot stream and the remaining cold stream, which is required to cool the hot stream

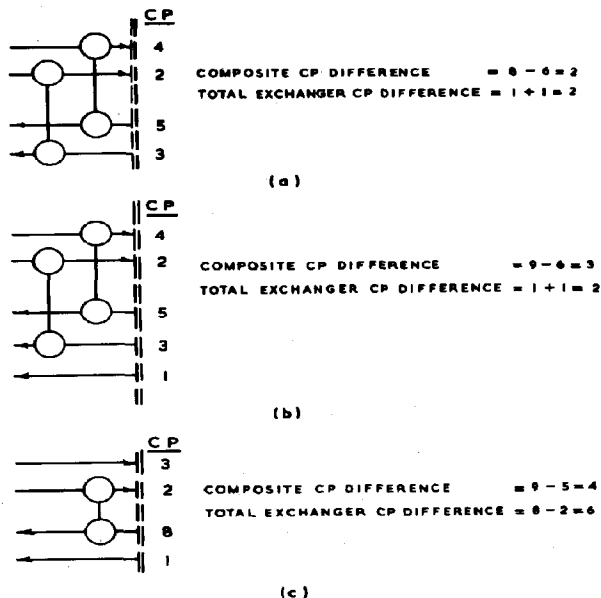


Fig. 12. (a) Two feasible pinch topologies showing that the composite CP. (b) Difference bounds the total exchanger difference. (c) An infeasible match based on the CP difference.

to the pinch temperature, is not feasible. We can generalise by saying that the CP differences of all pinch matches must always be bounded by the overall CP difference.

#### The CP table

Figures 13(a) and 13(b) show a step-by-step procedure for applying the feasibility requirements listed above. By following the sequence, the designer can

- \* identify essential matches at the pinch.
- \* identify available match options at the pinch.
- \* identify the need to split streams and generate stream splitting options at the pinch.

The procedure is aided by the use of another new concept, the "CP table". CP tables for the hot and cold ends of TC3 are shown in Figs. 14 and 16 respectively. In these tables hot and cold stream CPs at the pinch are separately listed in numerical order. The appropriate feasibility criteria are noted at the top of the table and the CPs representing streams which have to be involved in process exchange at the pinch are boxed for emphasis. A pinch match is represented in the table by pairing the CPs of a hot and a cold stream. Stream splits are represented by writing the separate branch flowrate CPs adjacent to the original CP (see Fig. 16(c)). As will be demonstrated below (see "worked example"), the step by step procedure from Fig. 13 is easily followed in the CP table.

#### The "tick-off" heuristic

Once a pinch topology has been chosen, the design of both hot and cold ends must be continued in such a manner as to keep capital costs at a minimum, i.e. the final designs ought to be steered towards the minimum number of units. This can be achieved by employing a "tick-off" heuristic to identify the heat loads on the pinch exchangers.

The heuristic results directly from the targeting equation for the minimum number of units

$$U_{\min} = N - 1. \quad (1) \text{ repeat.}$$

The equation is satisfied if every match brings one stream to its target temperature or exhausts a utility. In this case, the match is said to "tick-off" the stream or utility, i.e. the stream or utility need no longer be considered part of the remaining design task.

The pinch exchangers can usually be made to tick-off streams by choosing each exchanger load to equal the smaller heat load of the two streams matched. The CP inequalities (3a) and (3b) will guarantee the possibility of choosing pinch exchanger loads by ticking-off streams as long as the stream CP remains constant with varying temperature and as long as cold and hot stream temperature overlaps do not require an excessive number of shells for a single pinch match.

The tick-off heuristic is a "heuristic" as it can occasionally penalise the design by introducing the

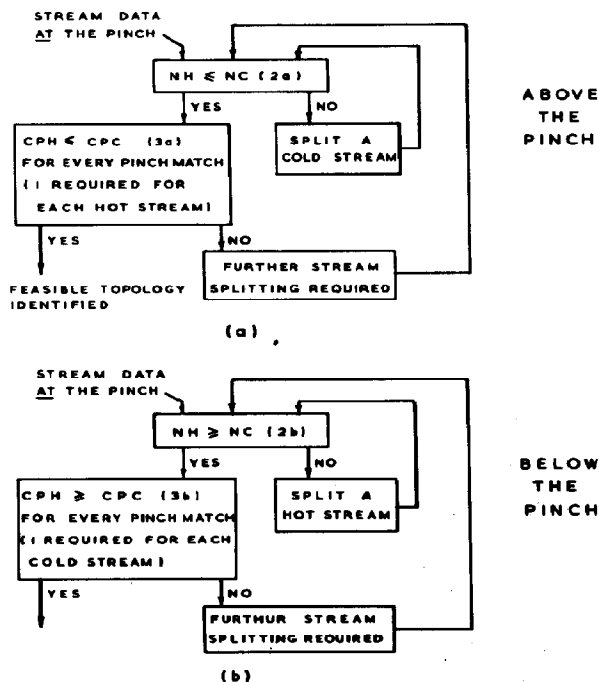


Fig. 13. (a) Hot end pinch design procedure. (b) Cold end pinch design procedure.

need for increased utility usage. Temperature driving force, essential elsewhere, may be used up excessively in pinch exchangers that are extended too far into the remaining problem. In such cases the designer can choose either to

- \* reduce the load on the offending pinch match and run the risk of needing more than the minimum number of units

- \* use another pinch topology in which the tick-off heuristic does not cause essential driving force to be used up.

The problem table algorithm can be used to check whether a maximised pinch match load is in keeping with minimum utility usage. Consider, for example, a hot end design. Application of the problem table algorithm to the complete hot end would tell us that utility heating would be required and no utility cooling, i.e. the hot end effectively constitutes a threshold problem where the  $\Delta T_{\min}$  specified is the threshold value. Now consider the situation where a pinch match has been identified and its load has been determined using the tick-off heuristic. There will exist a "remaining problem" of hot and cold streams (strictly speaking parts of streams) for which a network design is required. The problem table algorithm can be applied to this remaining problem. One of two results would then occur.

First, the algorithm may calculate that no utility cooling is required to solve the remaining problem and the utility heating predicted would be the same as before. In this case the designer knows that the pinch match load he has assigned, using the tick-off heuristic, will not penalise the design in terms of increased utility usage.

Second, the algorithm may calculate that utility cooling is required for the remaining hot end problem and that the hot utility usage would, accordingly, be increased. In this case the designer knows that the pinch match load he has assigned, using the tick-off heuristic, is not compatible with minimum utility usage.

The above technique has been called remaining problem analysis for obvious reasons. It is a dependable mechanism to check the consequences of the application of the tick-off heuristic. However, due to the effort involved it should only be used in complex situations to confirm key design steps.

#### *The remaining problem*

In accordance with the pinch design method's philosophy the designer is given a "free hand" once that temperature driving forces no longer restrict topology options.

Thus when designing away from the pinch in the "remaining problem" matches are chosen discretely by the designer to satisfy the process objectives. For example, heaters and coolers can be placed for direct control of stream target temperatures. Preferred topology requirements or materials, safety

and other constraints can be used to steer the placement of other matches.

In addition, the remaining problem analysis technique discussed above is not only restricted to checking the placement of pinch matches. It can also be used to ensure that matches placed in the remaining problem are compatible with the minimum utility objective.

#### *Design method summary*

The pinch design method incorporates five important stages. These are

- \* The HEN problem is divided at the pinch into separate problems.

- \* The design for these separate problems is started at the pinch and developed moving away from the pinch. At the pinch essential matches, match options and stream splitting requirements are identified by applying the feasibility criteria.

- \* When options exist at the pinch, the engineer is free to base his selection to suit the process requirements.

- \* The heat loads of exchangers at the pinch are determined using the stream "tick-off" heuristic. In case of difficulty (increased utility usage) a different exchanger topology at the pinch can be chosen or the load on the offending match can be reduced.

- \* Away from the pinch there is generally a "free choice" of matches. The procedure does not insist on particular matches but allows the designer to discriminate between matches based on his judgement and process knowledge.

#### **WORKED EXAMPLE: TEST CASE NO. 3**

We will now demonstrate the pinch design method using TC3 as the example problem. The calculation of the pinch temperature, the minimum utility requirements and the identification of the hot and cold end design tasks for TC3 have been discussed previously in this paper. The pinch design and completion of the remaining problem are considered below. Note that at this stage no attempt has been made to "optimise" the  $\Delta T_{\min}$  value.

#### *Hot end design*

The CP table for the hot end to TC3 is shown in Fig. 14(a). A pinch topology is identified in this table by applying the feasibility criteria according to Fig. 13(a). The first feasibility check (2a) shows that the stream population at the pinch is consistent with a design using the minimum utility usage. There are fewer hot streams than cold. Application of the second feasibility requirement (3a) shows that there are two design options, (see Figs. 14(b) and 14(c)).

In the design of an industrial HEN problem, process requirements or preferences would lead to the choice of one of these options. For the purpose of this paper, however, both options have been deemed valid and developed through to final hot end designs utilising the

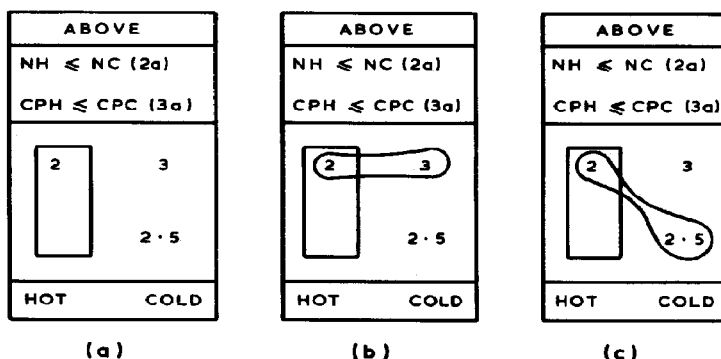


Fig. 14. (a) The CP table for TC3 hot end. (b) and (c) Feasible pinch matches identified in the CP Table.

tick-off heuristic. These two designs are shown in Figs. 15(a) and 15(b).

In Fig. 15(a), the load of the pinch match identified in Fig. 14(b) has been maximised to tick-off stream No. 1. The residual load of 17.5 units on stream No. 3 then forms a new stream in the remaining problem. Identifying a feasible topology for this remaining problem is a trivial task. There are no options, i.e. the remaining problem can only be solved by the placement of two heaters.

The other design option identified in Fig. 14(c) is developed through to a final hot end design in Fig. 15(b). The pinch match load now ticks-off stream No. 4. This leaves a residual load on stream No. 1 of 30 units with only stream No. 3 to match against. Thus the remaining problem is solved by the placement of one exchanger and one heater.

#### Cold end design

An empty CP table for the cold end of TC3 is shown in Fig. 16(a). A pinch topology is identified in this table by applying the feasibility criteria according to Fig. 13(b). The first feasibility requirement, inequality (2b), shows that the stream population at the pinch is consistent with a minimum utility design. However, an arrangement of pinch matches cannot be found since feasibility criterion (3b) cannot apply to all pinch matches simultaneously (see Fig. 16(b)). Stream splitting is required if the problem is to be solved by using the minimum utility usage and by observing the  $\Delta T_{\min}$  constraint.

One might think that in this situation it would be advisable to split a cold stream so that the cold stream CP becomes smaller. However, the situation is more complex than that. As the number of hot and cold streams in TC3's cold end pinch design are equal we know that if we were to split a cold stream we would then, of necessity, have to split a hot stream to fulfill inequality (2b). To demonstrate this Fig. 16(c) shows a feasible pinch arrangement in which the cold CP of 2.5 was first split. (Note, the rationale behind setting the split stream branch flowrates is covered in the next

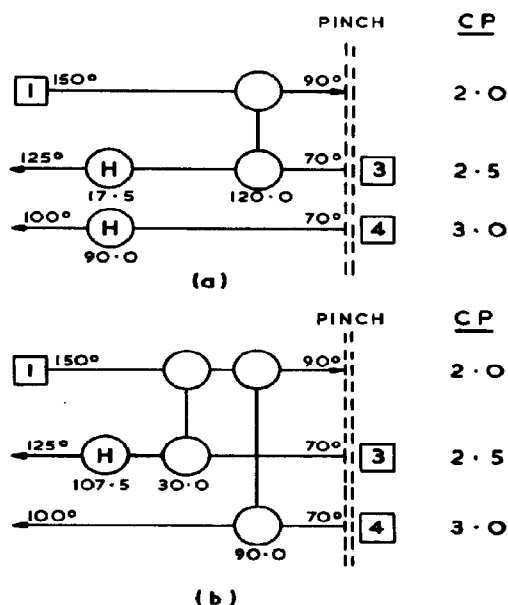


Fig. 15. (a, b) Two hot end designs to TC3.

section.) It was then necessary to split the hot CP of 8 in order to fulfil inequality (2a). Given this understanding, it is preferable to split a hot stream right away as this maintains compliance with inequality (2a) and may allow a solution to be found with only one stream split. Such a solution is shown in Fig. 16(d).

Two different cold end designs can be developed from this pinch topology depending on how the tick-off heuristic is applied. In Figs. 17(a) and 17(b), matches No. 1 and No. 2 are the essential pinch matches. The topology in Fig. 17(a) results when the load on match 1 is chosen to tick-off cold stream No. 4. The topology in Fig. 17(b) results when the load on match No. 2 is chosen to tick-off stream No. 3.



design. Figure 18(a) shows such a combination involving the hot end design from Fig. 15(a) and the cold end design from Fig. 17(a). The final overall design features minimum utility usage and has a total of seven units. This is the minimum consistent with minimum utility usage.

#### SIMPLIFYING THE MINIMUM UTILITY DESIGN

As already discussed, there will generally be scope to simplify minimum utility designs by a controlled reduction in the number of units. By transferring heat across the pinch and therefore increasing the utility usage the number of capital items can be reduced. There is a trade-off between units (capital cost) and the utility usage (energy cost).

In order to explore the scope for a controlled reduction in the number of units it is important to understand the concepts of heat load loops and heat load paths.

#### Heat load loops

Whenever a design features more than the target minimum number of units for the whole problem, ignoring the pinch, it is due to the existence of heat load loops. This was first stated as an observation by Hohmann[2] who also introduced the term "heat load loop". Later, it was confirmed by Linnhoff *et al.* [6] to be generally true, based on Euler's network theorem. There will be one loop for each extra unit. As an example, the minimum utility design to TC3 shown in Fig. 18 has two more units than the definite minimum

according to Fig. 7(b). Hence there must be two loops in the design. Figs. 18(a) and 18(b) show these loops.

An important feature of every loop is that heat loads can be shifted around the loop from one unit to another. The load is subtracted from one unit, added to the next in the loop, subtracted from the next and so on around the loop. This load shift always maintains the correct stream heat loads but the exchanger duties are changed and may cause a violation of  $\Delta T_{\min}$ . However, driving forces can be "restored" using heat load paths.

#### Heat load paths

A path is a continuous connection in the grid between a heater, heat exchangers and a cooler [14]. Figure 19(b) shows the simplest form of a path.

Load shifts along paths follow equivalent rules to load shifts around a loop. Load is added to a heater, subtracted from an exchanger, added to the next exchanger in the path, subtracted from the next, and so on along the path until it is finally added to a cooler. Stream enthalpy balance is maintained but exchanger loads and operating temperatures are changed. This last feature means that a path can be used to restore driving forces.

We will now illustrate the use of heat load loops and paths to reduce the number of units of the design in Fig. 18 from seven to six.

#### Example

It is apparent that load shifts around loops can form the basic mechanism for the reduction in the number

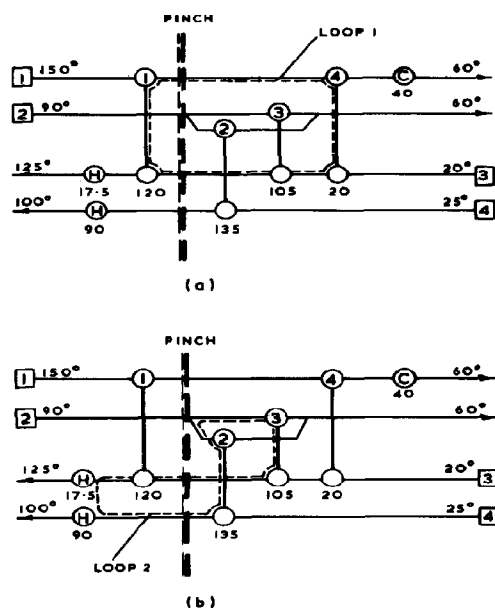


Fig. 18. (a, b) A minimum utility design to TC3 showing the two heat load loops.

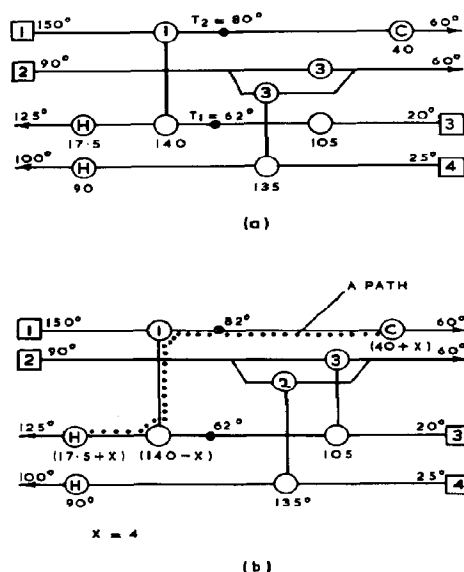


Fig. 19. (a) A six unit topology to TC3, the result of breaking loop 1, Fig. 18a. (b) Identifying a path to restore the  $\Delta T_{\min}$  constraint.

of units. When the load shift around a loop leads to a reduction in the heat load of a unit which equals the load on that unit then the unit is removed from the design and the number of units is reduced by one. (Su used this mechanism when he developed a loop breaking evolutionary method for the synthesis of HEN networks[9]).

Consider Fig. 18(a) which shows a minimum utility design to TC3 with seven units. A good choice of exchanger to remove is exchanger No. 4 as it has the smallest load and forms part of the simplest loop. Fig. 19(a) shows the topology and temperatures after the load of match No. 4 has been transferred to exchanger No. 1. The heat loads of all other units in the design are unchanged as they were not part of the original loop. There is now a small violation in  $\Delta T_{\min}$  as reflected by the difference in temperatures T1 and T2.

However,  $\Delta T_{\min}$  can be restored using the heat load path shown in Fig. 19(b). It is apparent that T1 is fixed at 62°C. It is therefore T2 which must be changed to restore  $\Delta T_{\min}$ . Requiring T2 to equal 82°C, the heat load of individual units can now be changed while the stream heat loads are maintained by using the path through exchanger No. 1. It is a trivial task to calculate the hot and cold utility increase  $X$  required. This load is 4 kW. In other words, by supplying a further 4 kW of utility heating and cooling and by reducing the heat load on exchanger No. 1 by 4 kW, the solution is brought back in line with the  $\Delta T_{\min}$ .

It is important to distinguish between the elimination of units using heat load loops and paths and the mere replacement of units by heaters and coolers. Consider again the design shown in Fig. 18(a). The number of units can obviously be reduced by substituting exchanger No. 4 with an increased heater load on stream No. 3 and an increased cooler load on stream No. 1. The removal of exchanger No. 4, in this case, would necessitate a 20 kW increase in both hot and cold utility usage. By first redistributing the exchanger load using a loop as above we found that the hot and cold utility increase was only 4 kW. This is a substantial improvement over the 20 kW penalty incurred if the exchanger is simply substituted.

Moreover, it is worth emphasising that the  $\Delta T_{\min}$  constraint should not always be restored after a load shift around a loop. Consider again Fig. 19(a) which shows a topology from TC3 after the loop load shift. Although a  $\Delta T_{\min}$  violation occurs the network is thermodynamically feasible. In the example above  $\Delta T_{\min}$  was restored for no other reason than it was part of the original problem specification. In practice there will be an optimum value of T2-T1. This optimum value correctly trades energy and capital costs, see Fig. 20.

We can now summarise this section as follows

- \* There is generally scope to reduce the number of units in a pinched problem starting from a minimum utility design.
- \* This reduction in the number of units can be

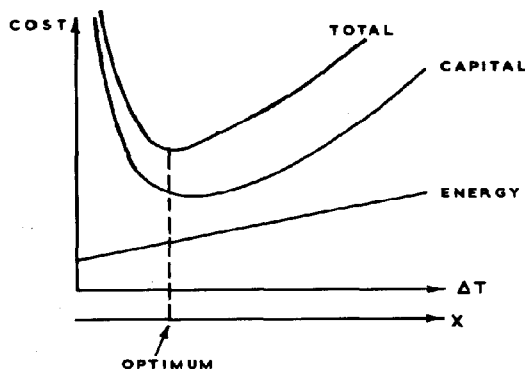


Fig. 20. Defining the optimum  $\Delta T$  after loop breaking.

achieved in a controlled manner. By this we mean that the utility penalty incurred in reducing the number of units is minimised. For example in TC3 an exchanger of load 20 kW was removed but the increase required in the utility usage to restore feasibility was only 4 kW.

- \* Not all units exist in a suitable loop or along a suitable path. Thus, the procedure would not be applicable to the "removal" of such units.

- \* There is an optimum value of utility increase which establishes the correct trade-off between utility costs and capital costs. This optimum value will nearly coincide with  $\Delta T_{\min}$  and has to be established by optimisation.

#### CHOICE OF $\Delta T_{\min}$

Consider Fig. 21 which shows qualitatively the variation of a network's annualised capital cost and annualised utility cost with variation in  $\Delta T_{\min}$ . Obviously when  $\Delta T_{\min}$  equals zero the heat exchanger network cost is infinite as infinite heat exchange surface area is required. The utility usage and hence utility cost is at a minimum. As  $\Delta T_{\min}$  is increased the network capital cost initially falls sharply but at higher values of  $\Delta T_{\min}$  "flattens out" and in certain problems may tend to rise again. Utility costs increase steadily with increase in  $\Delta T_{\min}$ .

It is evident from Fig. 21 that total network cost (annualised utility and capital costs) passes through a minimum value which corresponds to a particular utility usage and  $\Delta T_{\min}$ . It is, therefore, necessary to ensure that networks are designed using a  $\Delta T_{\min}$  in the region of the optimum. This  $\Delta T_{\min}$  is easily identified using the pinch design method by costing several designs.

#### SUMMARY

The pinch design method generates, with a high degree of certainty, low cost solutions to HEN problems by producing minimum utility designs with as few units as is compatible with minimum utility usage.



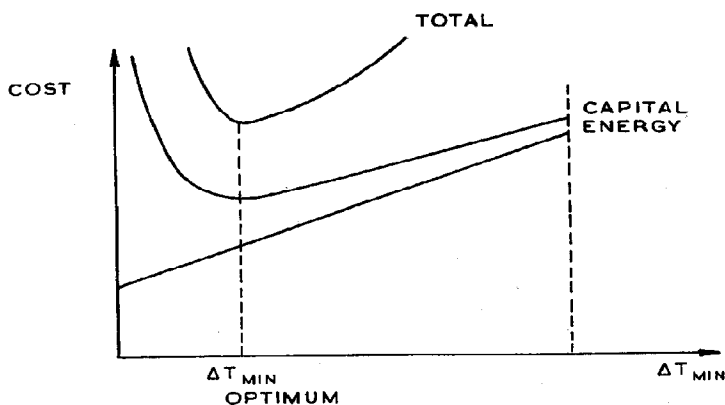


Fig. 21. Network cost as a function of  $\Delta T_{\min}$ .

The method is so simple and straight-forward that it is possible to produce such designs for large problems quickly and easily by hand. The method incorporates two fundamentally important elements.

First, the method recognises that the design objective is not just the identification of a "rigorous" cost optimal topology—after all, we are dealing with preliminary design—but one which is safe and controllable. To achieve this, the method informs the designer when topology options exist. By discriminating between these options the design can steer his design towards, controllable and practical topologies, using his judgement and process knowledge.

Second, the method recognises the fundamental importance of the HEN pinch. At the pinch the choice of matches is severely restricted. Often there is only one choice, an essential match, which if not made will penalise the design by increasing utility usage above the minimum. Thus, the design must be started at the pinch, and developed moving away from the pinch. This realisation is a completely new insight in HEN design and leads to the simple but enormously effective feasibility criteria which are summarised in Fig. 13 and form the basis of the actual procedure.

#### *The heuristic element*

The pinch design method employs the stream tick-off heuristic to speed up the procedure sufficiently for hand application and to steer the design towards minimum number of units compatible with minimum energy. This tick-off heuristic is not the same as that previously condemned by Linnhoff *et al.* [6] as being partly responsible for the failure of virtually all previous work in the area. In these previous sources, streams were ticked off by going across the pinch and therefore incurring heat transfer across the pinch. In the present paper, streams are ticked-off either side of the pinch. However, even so the heuristic can occasionally penalise the design by introducing need for excess utility. Fortunately such

occurrences are rare and when they do occur they can be rectified either by restricting the load on the offending exchanger, or by using a different pinch topology option.

The heuristic in no way interferes with the method's ability to identify essential matches and options available at the pinch compatible with a minimum utility design.

#### *Stream splitting rationale*

The pinch design method identifies situations where stream splitting is essential to achieve a minimum utility solution observing the  $\Delta T_{\min}$  constraint. The rationale is rigorous. This represents a breakthrough in understanding. To date, many published design methods have made use of stream splitting but none could guarantee that it could not perhaps have been avoided.

#### *Threshold problems*

Most of the early HEN problems presented in the literature were not pinched for the specified values of  $\Delta T_{\min}$ . We now recognise them as threshold problems because of their relationship between minimum utility usage and  $\Delta T_{\min}$ . This relationship can be summarised as follows:

$$0 \leq \Delta T_{\min} < \Delta T_{\text{thresh}}$$

Either hot or cold utility usage (but not both) is required and the problem is not pinched.

$$\Delta T_{\min} = \Delta T_{\text{thresh}}$$

The same hot or cold utility requirement as for lower values of  $\Delta T_{\min}$  is required but the problem is now pinched.

$$\Delta T_{\min} > \Delta T_{\text{thresh}}$$

Both hot and cold utilities are required. The problem is pinched.

It is thus possible to apply the pinch design method to threshold problems providing the  $\Delta T_{\min}$  is adjusted to the threshold value. Obviously in this case the designer would not be too concerned about a  $\Delta T_{\min}$  violation.

In industrial design, however, threshold problems are rare. This stems mainly from the fact that most industrial processes use more than one hot or cold utility. There are often savings to be made by utilising cheaper grade utilities. In this paper it is shown that each intermediate temperature utility, the use of which is maximised, introduces a "utility" pinch into the problem.

Moreover, even in cases where only one hot and one cold utility are available, threshold problems can only exist if the underlying process (reactors, separators, etc.) exists with large driving forces. Given today's energy costs, processes like that become less and less common.

#### *Energy-capital tradeoffs*

The minimum number of units required to complete a design can be assessed from the stream data. Due recognition must however be made of the pinch division of the HEN problem and the targeting equation relating units to streams must be applied separately above and below the pinch. Using the equation in this way gives the target minimum number of units required by a design which achieves minimum utility usage.

The target number of units obtained in this manner is usually greater than that for the whole problem undivided by the pinch. This is because some streams exist on both sides of the pinch.

Designs with fewer units than is compatible with the minimum utility usage must transfer heat across the pinch or certain exchangers must violate the  $\Delta T_{\min}$  constraint. Thus there are often trade-offs between utility usage, number of units and exchange surface area. These trade-offs can be explored by applying the properties of heat load loops and paths to a minimum utility design.

#### *Comparison with previous work*

The strategy of the pinch design method, namely dividing a HEN problem at the pinch, starting the design at the pinch and developing it into the remaining problem is a completely new approach different to all other published design methods. It is the result of recognising the fundamental importance of the HEN pinch and is the reason behind the ease of application of the method. It is this ease of application which allows the designer to quickly explore practical topologies for various  $\Delta T_{\min}$  values, which ensures that "best" solutions are always found with minimal effort.

The TI Method by Linnhoff *et al.* [5] came close to the above strategy. The subnetwork arrangement, established using the problem table algorithm, en-

sured that the pinch division was recognised. However, in [5] the insight of starting the design at the pinch was missing. In addition the method, although guaranteeing minimum utility designs, usually produced designs with more than the minimum number of units. This was despite the fact that the procedure recommended making matches in subnetworks based on the neighbouring match principle.

Grimes [13] came close too. He recognised the importance of the pinch division and the need for separate  $U_{\min}$  targets above and below the pinch. However, he selected matches in the hot and cold ends on the basis of minimum average temperature difference. The decisive insight of starting a design at the pinch was missing and with it the recognition of the feasibility criteria at the pinch. The present authors found that as a result the method quite often fails to give minimum utility designs.

All other methods published to date did not recognise the need to divide a problem at the pinch and are therefore well and truly out of date. However, this is not to say that some of these methods could not now be applied to solve the threshold problems of the hot and of the cold ends separately. This may provide many a PhD project for all those who prefer exhaustive optimisation to the simple hand-calculation based approach presented here.

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#### NOTATION

CP	the heat capacity flowrate of a stream, kW/C
CPC	the heat capacity flowrate of a cold stream or a cold stream branch at the pinch, kW/C
CPH	the heat capacity flowrate of a hot stream or a hot stream branch at the pinch, kW/C
N	the number of process streams and utilities
NC	the number of cold streams and cold stream branches at the pinch
NH	the number of hot streams and hot stream branches at the pinch
Q	heat exchanger load, kW
dQ	small change in exchanger load, kW
QH	hot utility requirement for a HEN problem, kW
QC	cold utility requirement for a HEN problem, kW
$QH_{\min}$ , $QC_{\min}$	the minimum hot and cold utility requirement
T	temperature, C
dT	small change in temperature, C
TS	stream supply temperature, C

$TT$  stream target temperature, C  
 $\Delta T$  the actual temperature difference at one point of an exchanger, heater or cooler  
 $\Delta T_{\min}$  the minimum allowed temperature approach in an exchanger, heater or cooler, C  
 $\Delta T_{\text{thresh}}$  the threshold value of  $\Delta T_{\min}$  for a HEN problem, C  
 $U_{\min}$  the minimum number of exchangers, heaters and coolers.  
 $U_{\min_{\text{ms}}}$  the minimum number of exchangers, heaters and coolers in a pinched problem which is consistent with the minimum utility usage.  
 $X, Y, Z$  heat flows, kW.

## REFERENCES

- [1] Masso A. H. and Rudd D. F., *A.I.Ch.E. J.* 1969 **15** 10.
- [2] Hohmann E. C., Optimum networks for heat exchange. Ph.D. Thesis, University of Southern California (1971).
- [3] Ponton J. W. and Donaldson R. A. B., *Chem. Engng Sci.* 1974 **29** pp. 2375.
- [4] Nishida N., Liu Y. A. and Lapidus L., *A.I.Ch.E.J.* 1977 **23** 77.
- [5] Linnhoff B. and Flower J. R., *A.I.Ch.E.J.* 1978 **24** 633; *A.I.Ch.E.J.* 1978 **24** 642.
- [6] Linnhoff B., Mason D. R. and Wardle I., *Comput. Chem. Engng* 1979 **3** 295.
- [7] Umeda T., Itoh J. and Shiroko K., *Chem. Engng Prog.* 1978 **75** 70.
- [8] Umeda T., Niida K. and Shiroko K., *A.I.Ch.E.J.* 1979 **25** 423.
- [9] Su Tow-Lih, A loop breaking evolutionary method for the synthesis of heat exchanger networks. M.Sc. Thesis, Sever Institute of Washington University, Saint Louis, Missouri (1979).
- [10] Boland D. and Linnhoff B., *Chem. Engr* April 1979 222-228.
- [11] Flower J. R. and Linnhoff B., A thermodynamic-combinatorial approach to the design of optimum heat exchanger networks. *A.I.Ch.E.J.* 1980 **26** 1.
- [12] Linnhoff B. and Turner J. A., *Chem. Engr* December 1980 742-746.
- [13] Grimes L. E., The synthesis and evolution of networks of heat exchanger that feature the minimum number of units. M.Sc. Thesis, Carnegie-Mellon Univ., Pittsburgh 1980.
- [14] Linnhoff B., Process synthesis. Undergraduate lecture notes, Imperial College 1980/81.
- [15] Nishida N., Stephanopoulos G. and Westerberg A. W., *A.I.Ch.E.J.* 1981 **27** 321.
- [16] Harary F., *Graph Theory*. Addison-Wesley, Reading, Mass. 1972.
- [17] Cerda J., Westerberg A. W., Mason D. R. and Linnhoff B., *Chem. Engng Sci.* 1983 **38** 373.