

RBA

S. Fischer - Biosys - MAIAGE

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1 Principles underlying algorithm

Figure 1 shows how the fluxes of molecules are described by RBA and where the constraint are placed.

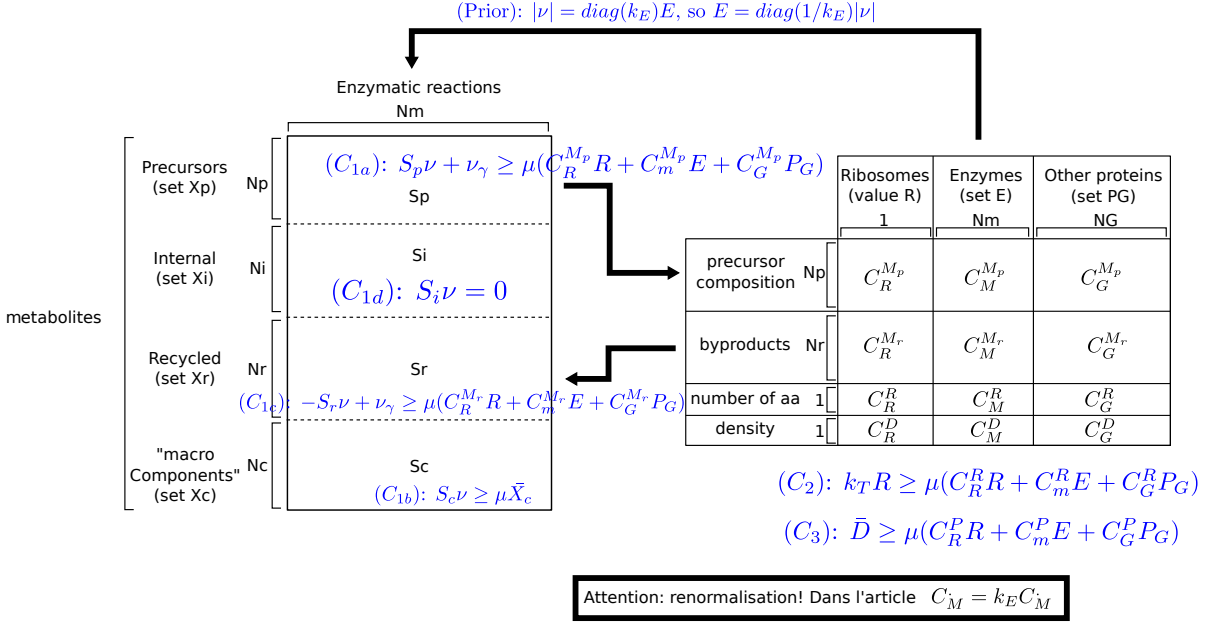


Figure 1: Description of cell by RBA.

2 Analysis of existing algorithm (RBAV01)

2.1 Structure of RBAV01

The original program is built around a multitude of functions and structures displayed on Figure 2. Figure 3 shows how matrices are built in the original algorithm.

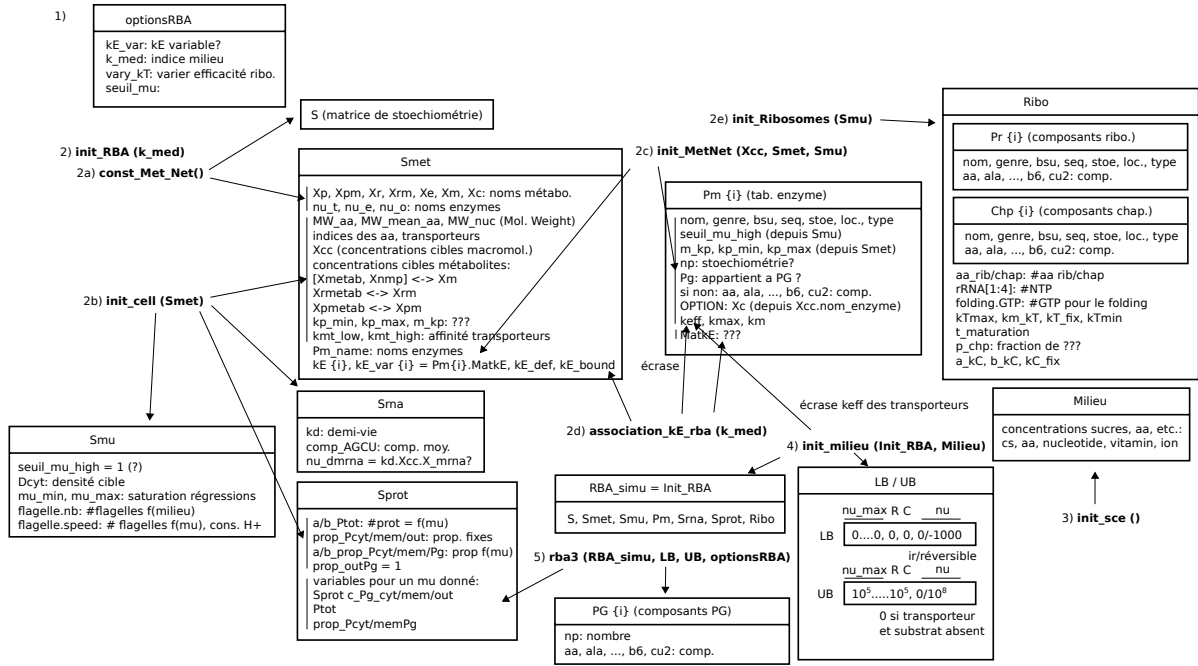


Figure 2: Algorithm used in RBAV01.

$$\begin{aligned}
 & \mathbf{f} \text{ (fonction objective)} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \\
 & \mathbf{D} \text{ (densité cytosol)} \quad \begin{bmatrix} \text{CD_M} & \text{CD_R} & \text{CD_C} \end{bmatrix} \\
 & \mathbf{M} \text{ (densité membrane)} \quad \begin{bmatrix} \text{CD_M} \end{bmatrix} \\
 & \text{taille Nm (flux)} \quad \begin{bmatrix} -\text{Pm.keff} & 1 \end{bmatrix} \\
 & \text{taille Nm (flux)} \quad \begin{bmatrix} -\text{Pm.keff} & -1 \end{bmatrix} \\
 & \text{ATP maintenance} \quad \begin{bmatrix} \text{nu_ATP} = -1 \end{bmatrix} \\
 & \mathbf{A} = \begin{bmatrix} \text{muCR_M} & \text{muCR_R} & \text{muCR_C} \\ \text{muCC_M} & \text{muCC_R} & \text{muCC_C} \\ \text{muCmp_M} & \text{muCmp_R} & \text{muCmp_C} \\ \text{muCmr_M} & \text{muCmr_R} & \text{muCmr_C} \\ \text{muX_M} & \text{muX_R} & \text{muX_C} \\ \text{muX_M} & \text{muX_R} & \text{muX_C} \\ \text{muX_M} & \text{muX_R} & \text{muX_C} \\ \text{muX_M} & \text{muX_R} & \text{muX_C} \end{bmatrix} \\
 & \mathbf{b} = \begin{bmatrix} \text{D - CD_G.C_PGcyt} \\ \text{Ptot.Pmem - CD_G.C_PGmem} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
 & \mathbf{Aeq} = \mathbf{beq} = \begin{bmatrix} -\text{CR_G.mu.C_PG} \\ -\text{CR_G.pch.mu.C_PG} \\ 0 \\ -\text{muCmp_G} \\ -\text{muCmr_G} \\ -\text{muXc} \\ -\text{muX_metab} \\ -\text{muX_nmp} \\ -\text{h_flagelle} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
 & \mathbf{mRNA} = \begin{bmatrix} -\text{nu_dmrna (X_H2O)} \\ \text{nu_dmrna (X_H+)} \\ -\text{kd.Xc (X_mrna)} \\ \text{comp_AGCU.nu_dmrna} \end{bmatrix}
 \end{aligned}$$

Figure 3: Matrices used in RBAV01.

2.2 Orientations for improving RBA algorithm

It would be more elegant and flexible to not name any metabolite or flux sets. These sets should be defined by data, not the program. The existence of $X_{e/c/r/rm/p/pm/m}$ is unnecessary. Note that there are even more subsets than defined in theory, we probably need to avoid proliferation of such subsets.

We need to find clear abstraction that can represent RBA entities as generically as possible. All macromolecules should be representable with a composition matrix that would help build the final matrices rapidly. We also need to express the second member as data, in terms of processes.

Final objectives are

- Completely separate data from code: Create data files that are meaningful to users.
- Make code more compact and easier to read (and potentially quicker): by using a full matrix formalism (avoiding loops). This can only be achieved by finding the right abstraction to represent RBA elements.

3 Rewriting RBAs in full matrix form

Composition vector for macromolecules In the following, each macromolecule is described by its **composition vector**. It is a column vector containing the metabolites necessary to build it, with a **minus** sign for metabolites consumed and a **plus** sign for byproducts generated.

Metabolism constraint The metabolism constraint (C_1) can be rewritten

$$\underbrace{S\nu}_{\substack{\text{metabolic flux} \\ \text{generated by} \\ \text{metabolism}}} + \underbrace{\mu[C_E, C_R, C_C][E; R; C]}_{\substack{\text{precursors} \\ \text{used/byproducts} \\ \text{generated by produc-} \\ \text{ing new molecules}}} = \underbrace{-\mu[C_G, C_{X_c}][P_G; X_c]}_{\substack{\text{precursors} \\ \text{used/byproducts} \\ \text{generated by produc-} \\ \text{ing new molecules}}}$$

$$-\text{diag}(k_E) E \leq \nu \leq \text{diag}(k_E) E$$

where C_i are composition matrices as defined above.

Written this way, it actually does not matter to which group a metabolite belongs. Its group is only dictated by the structures of the composition matrices. Based on this constraint only it does not make sense to create metabolic groups *in the program*.

In full matrix form, the above equations become

$$\left\{ \begin{array}{l} [S, \mu C_E, \mu C_R, \mu C_C][\nu; E; R; C] = -[\mu C_G, \mu C_{X_c}][P_G; X_c] \\ \begin{bmatrix} I & -\text{diag}(k_E) & 0 & 0 \\ -I & -\text{diag}(k_E) & 0 & 0 \end{bmatrix} [\nu; E; R; C] \leq 0 \end{array} \right.$$

The most important here is the composition matrix. In the data, the user gives the list of metabolites used for synthesis and the list of byproducts. The program then has to figure out how to reorder terms to build the matrix.

Ribosome/chaperone constraints We can rewrite the ribosome constraint as follows:

$$k_T R + \mu[C_E^R, C_R^R, C_C^R][E; R; C] = -\mu C_G^R P_G$$

In full form:

$$[0, \mu C_E^R, k_T + \mu C_R^R, \mu C_C^R][\nu; E; R; C] = [-\mu C_G^R, 0][P_G; X_c]$$

The number of aas needed to build a protein can be included in the description of molecule sets. The same applies for folding by chaperones.

These additionnal constraints have the form

$$a[\nu; E; R; C] = b_0 + b_1[P_G; X_c]$$

this means they can be appended to C_1 by **simple line concatenation**.

RNA degradation/replication In this type of constraints, we simply add new fluxes of metabolites. These fluxes are simply added up on the right hand side of (C_1)

$$\begin{aligned} \dots &= -\mu[C_G, C_{X_c}][P_G; X_c] - \mu^d C_{process} X_{process} \\ &= -[\mu C_G, \mu C_{X_c}, \mu^d C_{process}][P_G; X_c; X_{process}] \end{aligned}$$

where $d = 0$ when the target flux is an absolute flux and $d = 1$ when it compensates dilution.

Density constraints The density constraint writes

$$[0, C_E^D, C_R^D, C_C^D][\nu; E; R; C] \leq \overline{D} - [C_G^D, C_G^c][P_G; X_c]$$

In general, these constraints can be expected to be of the form

$$a[\nu; E; R; C] \leq b_0 + b_1[P_G; X_c]$$

Maintenance ATP constraint (metabolite specific constraints) It is defined as

$$\mathbf{1}_{\nu_{constraint}}[\nu; E; R; C] = b$$

where $\mathbf{1}_{\nu_{constraint}}$ is an indicator matrix selecting the reaction associated with the production of maintenance ATP/flagella fuel. Generally this means a specific reaction has to be added in S , as well as specific metabolites like ATP maintenance or protons dedicated to fuelling flagella.

Summary

$$\begin{array}{ll} [S, \mu C_E, \mu C_R, \mu C_C] & [\nu; E; R; C] = -\sum \mu^{d_i} C_i X_i \quad (\text{Base metabolism and process production}) \\ a & [\nu; E; R; C] = b_0 + b_1[\{X_i\}] \quad (\text{Process capacity}) \\ \mathbf{1}_{\nu_{constraint}} & [\nu; E; R; C] = b \quad (\text{Metabolite constraints}) \\ \left[\begin{array}{cccc} I, & -\text{diag}(k_E), & 0, & 0 \\ -I, & -\text{diag}(k_E), & 0, & 0 \end{array} \right] & [\nu; E; R; C] \leq 0 \quad (\text{Enzymatic fluxes}) \\ a & [\nu; E; R; C] \leq b_0 + b_1[\{X_i\}] \quad (\text{Density constraints}) \end{array}$$

A more graphic representation of the matrix is provide in Figure 4.

$$\begin{array}{l}
\mathbf{f} \text{ (objective function)} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \\
\mathbf{Metabolism} \begin{bmatrix} S \\ \mu C_E \\ \mu C_R \\ \mu C_C \end{bmatrix} \\
\mathbf{Ribosomes} \begin{bmatrix} \mu CR_M \\ \mu CR_R \\ \mu CR_C \end{bmatrix} \\
\mathbf{Chaperones} \begin{bmatrix} \mu CC_M \\ \mu CC_R \\ \mu CC_C \end{bmatrix} \\
\mathbf{x} \begin{bmatrix} \nu & \mathbf{E} & \mathbf{R} & \mathbf{C} \end{bmatrix}
\end{array}
= \mathbf{Aeq} = \mathbf{beq} = - \begin{array}{l}
\begin{bmatrix} \mu C_G & \mu C_{Xc} & \mu C_{Xm} & C_{mrna} \\ \mu CR_G \\ \mu CC_G \end{bmatrix} \\
\mathbf{x} \begin{bmatrix} C_{PG} & Xc & Xmetab & Xmrna \end{bmatrix}
\end{array}$$

$$\begin{array}{l}
\mathbf{Flux} \begin{bmatrix} I \\ -I \end{bmatrix} \begin{bmatrix} -Pm.keff \\ -Pm.keff \end{bmatrix} \\
\mathbf{Flux} \begin{bmatrix} -I \end{bmatrix} \begin{bmatrix} -Pm.keff \end{bmatrix} \\
\mathbf{D} \text{ (densité cytosol)} \begin{bmatrix} CD_M \\ CD_R \\ CD_C \end{bmatrix} \\
\mathbf{M} \text{ (densité membrane)} \begin{bmatrix} CD_M \end{bmatrix} \\
\mathbf{ATP maintenance} \begin{bmatrix} \nu_{ATP} = -1 \end{bmatrix} \\
\mathbf{x} \begin{bmatrix} \nu & \mathbf{E} & \mathbf{R} & \mathbf{C} \end{bmatrix}
\end{array}
= \mathbf{A} \leq \mathbf{b} = \begin{array}{l}
\begin{bmatrix} D_c \\ D_m \\ -atp_m \end{bmatrix} - \begin{bmatrix} \mu CD_G \\ \mu CD_G \end{bmatrix} \\
\mathbf{x} \begin{bmatrix} C_{PG} & Xc & Xmetab & Xmrna \end{bmatrix}
\end{array}$$

Figure 4: Structure of matrices. Note that the left hand side and the right hand side have a similar structure (except for the ν submatrix). Each column represents a set of molecules: on the right hand side, the production flux of the set has to be determined, on the left hand side it is already given *a priori*.

4 New formalism

The new formalism must allow simple interactions between user readable data and efficient construction of the RBA problem. The data is going to be written in XML format, inspired by SBML standards.

4.1 Final differences between RBAv01 and new formalism

	RBAv01 mat12	RBAv01 mat15	RBAnew mat12	RBAnew mat15
Input files	Custom files + partially hard coded		XML files (nothing hard coded)	
Code length (commented)	$\simeq 3500$ ($\simeq 1000$)		$\simeq 1500$ ($\simeq 500$)	
Parsing	15s	8s	12s	12s
Solving (23 rounds)	15s	5s	1.5-2s	1.5-2s
1 matrix update	580ms	100ms	3-10ms	3-10ms
1 CPLEX round	50ms	58ms	50ms	58ms

Table 1: Comparison between original algorithm (RBAv01) and new formalism (RBAnew). Because performance for the old algorithm depended on matlab version, we included performance for matlab R2012a (mat12) and R2015b (mat15).

Table 1 shows the main differences between RBAv01 and the new algorithm. The new algorithm uses generic XML files and is thus easier to modify for the end user. Parsing files is less efficient than the original algorithm but this is not very important as it is only done once. More important is the solving time, which is significantly lower than the original algorithm.

The new algorithm could be improved further by handling block-allocation more efficiently for sparse matrices, but current performance seems pretty good.

4.2 Input data

```
--<sbml level="2" version="4">
- <model id="maiage_bsub_01" name="Metabolism of B. subtilis">
+ <listOfCompartments></listOfCompartments>
+ <listOfSpecies></listOfSpecies>
+ <listOfReactions></listOfReactions>
</model>
</sbml>
```

Standard SBML file, see SBML specifications. Compartments are needed for density constraints. Species and reactions are needed to build the stoichiometry matrix.

Figure 5: Metabolism file is a standard SBML file.

ters for the efficiency models. Finally, transporters have a **transporterEfficiency** that modulates the enzymatic activity depending on substrate and cofactor availability.

Process file The process file is an XML file containing a **listOfProcesses** and a **listOfComponentMaps**. Each **process** can contain up to 3 subsections. The **capacityConstraint** defines a machinery used by the process that has a limited capacity. The **operatingCosts** defines which macromolecules the process produces/degrades/modifies and the cost associated with these operations. The **targets** are set fluxes that a process must maintain in order for the cell to work properly. Target fluxes can apply to metabolites (**targetValue**) and reactions (**targetReaction**). Target fluxes scale with μ if they contribute to **dilution_compensation** or if they are defined using a μ -dependent user-function. Finally **componentMaps** are used to compute the costs in the **operatingCosts** section.

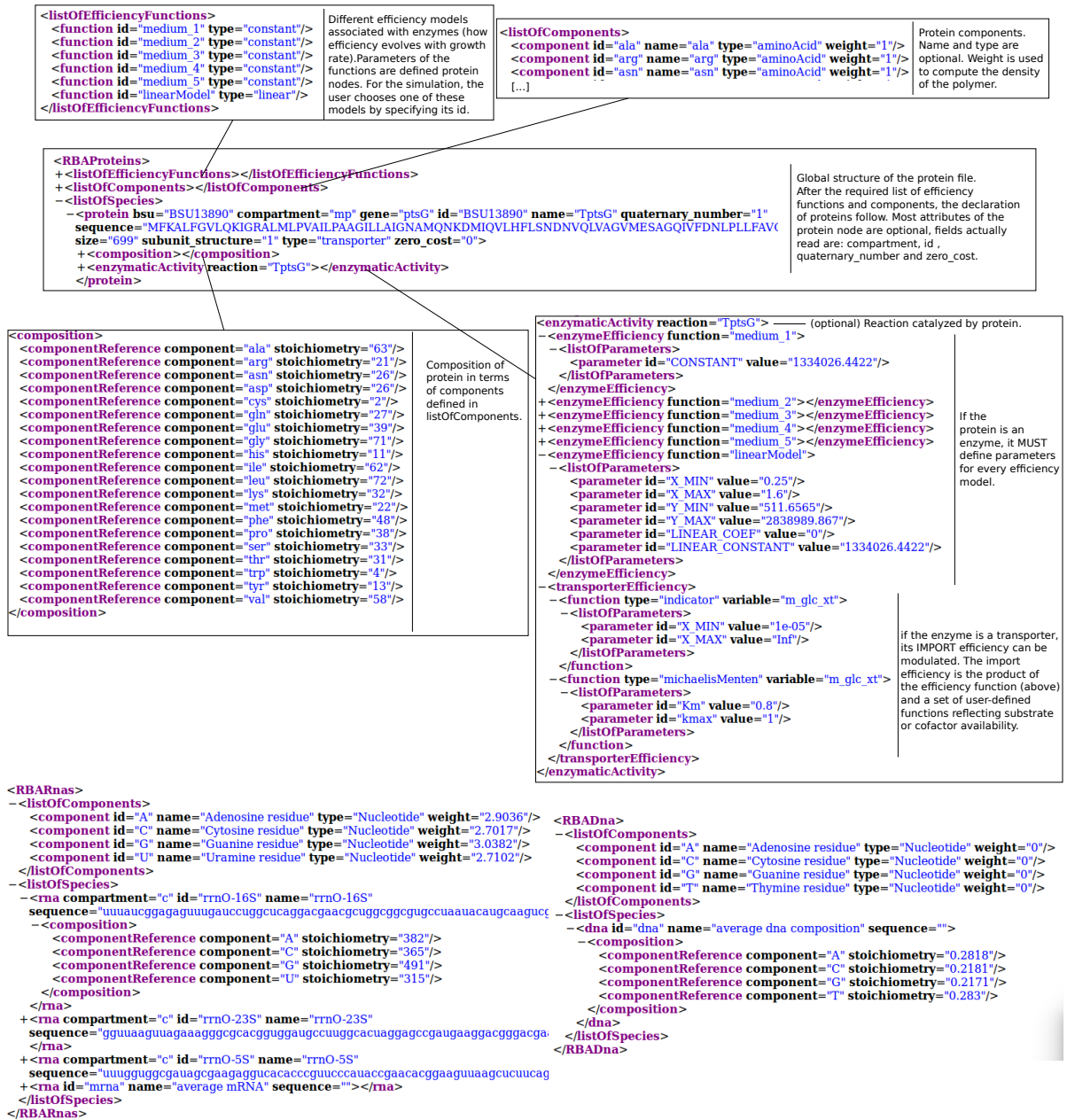


Figure 7: Structure of protein, RNA and DNA file.



Figure 8: Structure of process file.

4.3 From data to structures

Algorithm overview Figure 9 shows how the XML files are transformed into matlab structures. Note that a lot of elements are already in matrix form.

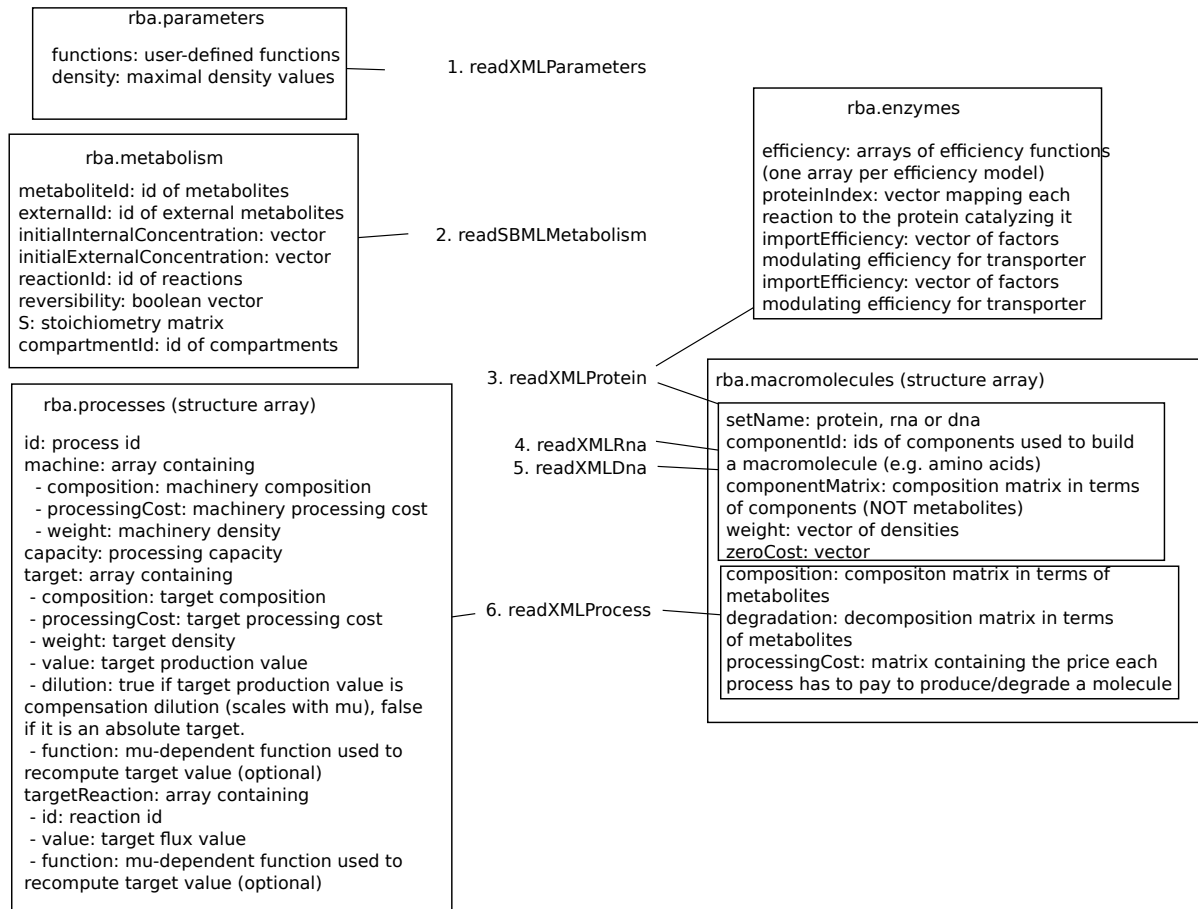


Figure 9: Algorithm used to read data in the new formalism.

Composition matrices Figure 10 shows how the matrices stored in `rba.macromolecules` are built.

Process-related matrices Figure 11 shows how the matrices stored in `rba.processes` are built.

4.4 Building final matrices

Figure 12 shows how the optimization matrices stored are built.

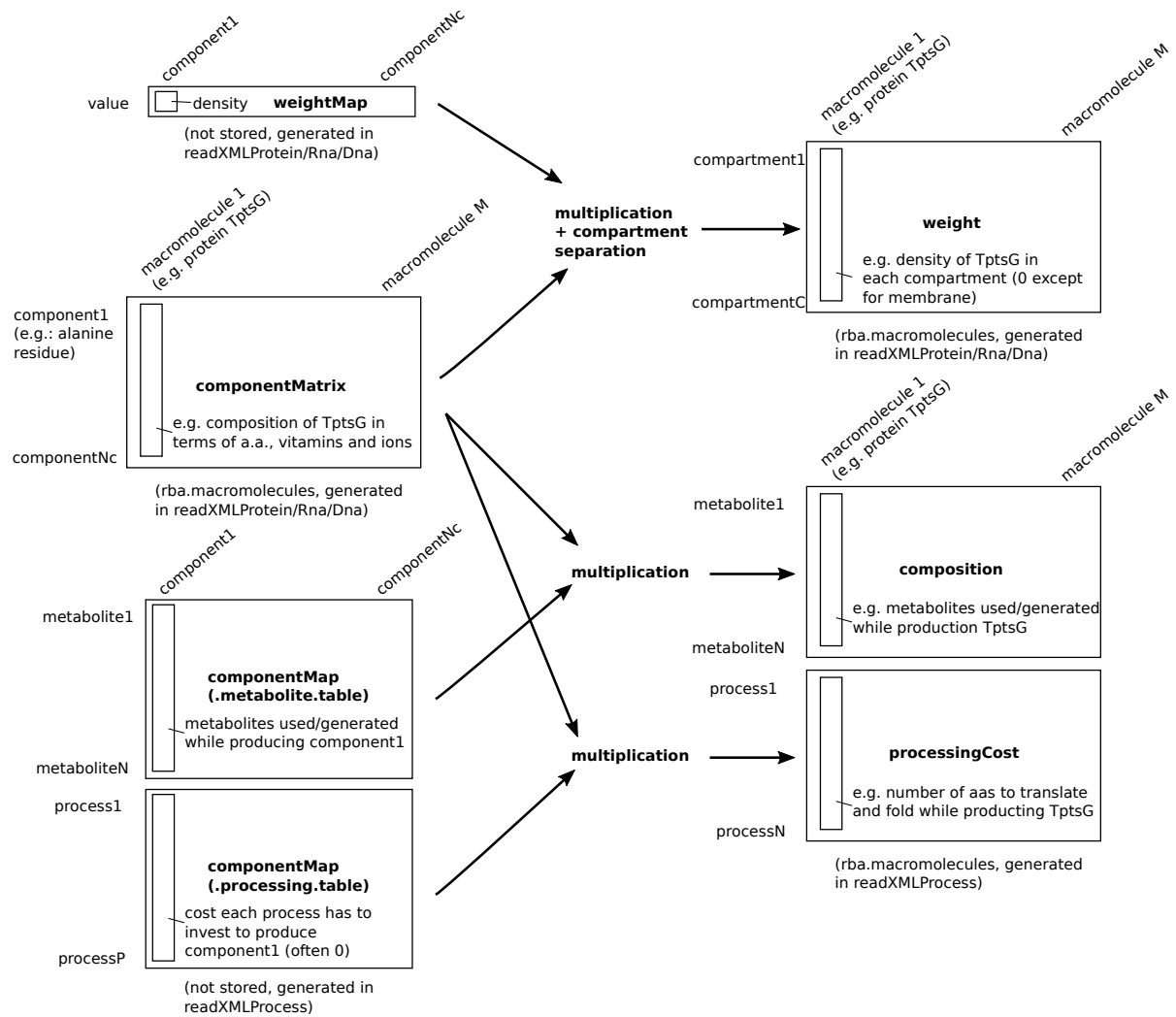


Figure 10: Macromolecule matrices.

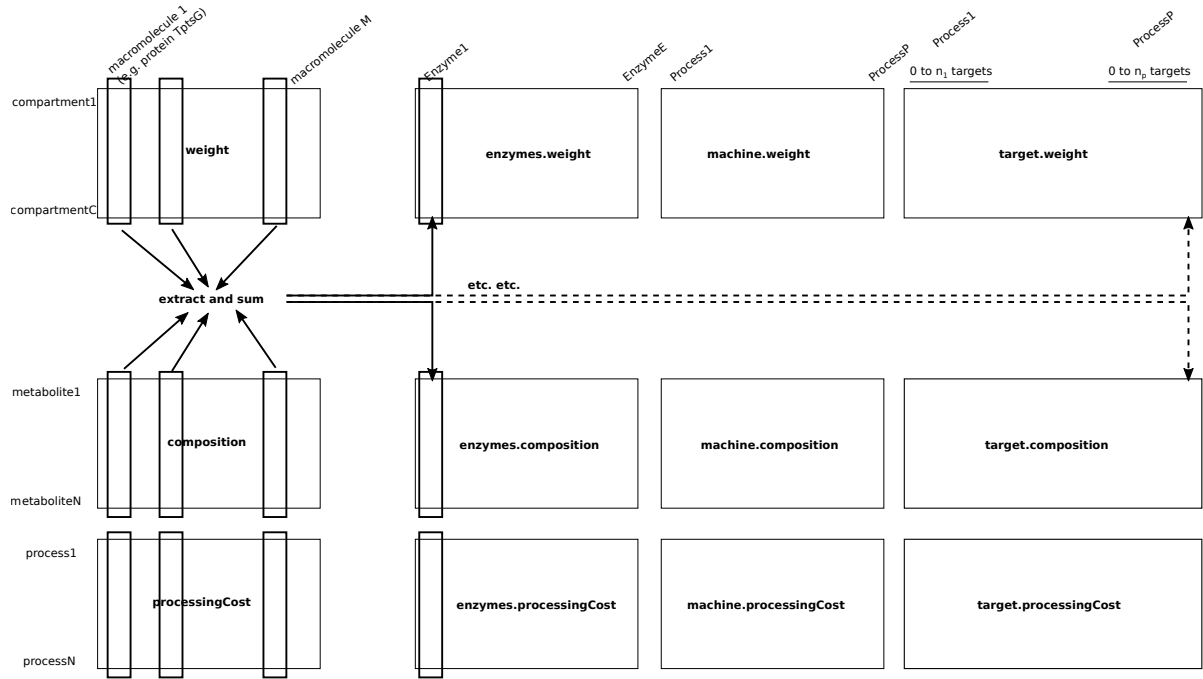


Figure 11: Process-related matrices.

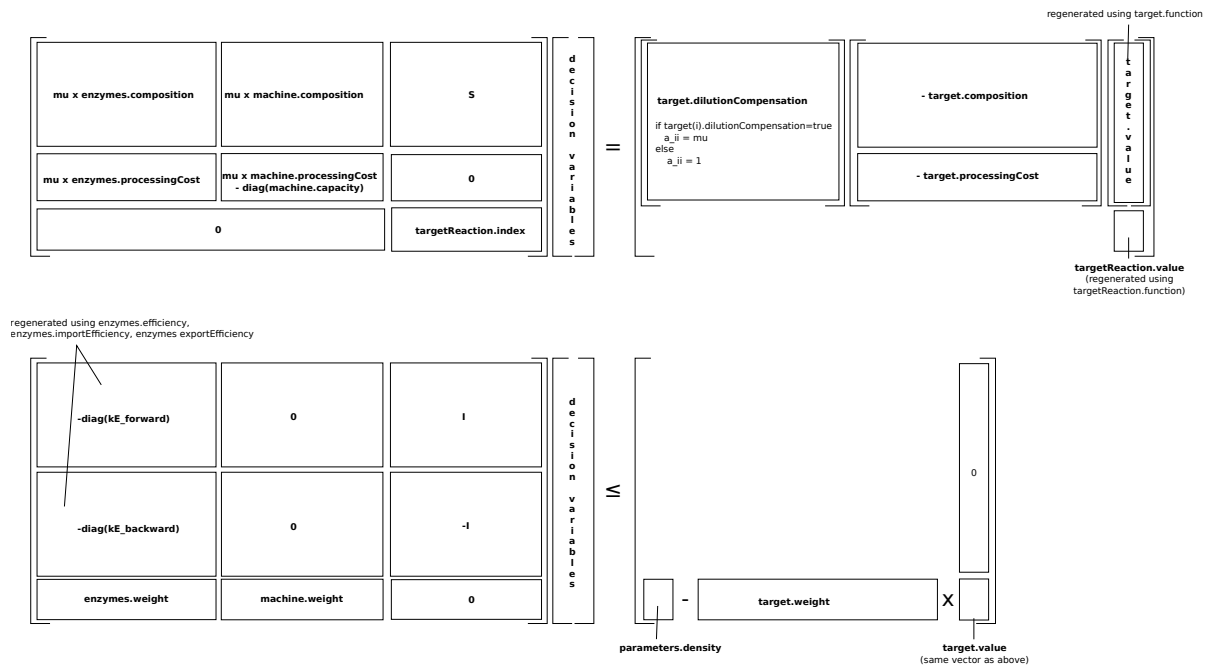


Figure 12: Matrices used to solve the optimization problem.