Matematika informatikusoknak 2 – Differenciálszámítás

3. gyakorlat

Példa

$$\lim_{n\to\infty} \frac{2n^4 - 6n^3 + 8n}{45 + 16n^3 + 8n^4} = \lim_{n\to\infty} \frac{2n^4 - 6n^3 + 8n}{45 + 16n^3 + 8n^4} = \frac{2}{8}$$

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$$\lim_{n\to\infty} \frac{2n^4 - 6n^3 + 8n}{45 + 16n^3 + 8n^4} = \lim_{n\to\infty} \frac{\frac{2n^4 - 6n^3 + 8n}{n^4}}{\frac{45 + 16n^3 + 8n^4}{n^4}}$$

$$= \lim_{n\to\infty} \frac{\frac{2n^4}{n^4} - \frac{6n^3}{n^4} + \frac{8n}{n^4}}{\frac{45}{n^4} + \frac{16n^3}{n^4} + \frac{8n^4}{n^4}} = \lim_{n\to\infty} \frac{2 - \frac{6}{n} + \frac{8}{n^3}}{\frac{45}{n^4} + \frac{16}{n} + 8} = \frac{2}{8} = \frac{1}{4}$$

$$\lim_{n\to\infty} \frac{6n^3 + 8n}{45 + 16n^3 + 8n^4} = \lim_{n\to\infty} \frac{6n^3 + 8n}{45 + 16n^3 + 8n^4} = 0$$

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$$\lim_{n\to\infty} \frac{6n^3 + 8n}{45 + 16n^3 + 8n^4} = \lim_{n\to\infty} \frac{\frac{6n^3+8n}{n^3}}{\frac{45+16n^3+8n^4}{n^3}}$$

$$= \lim_{n\to\infty} \frac{\frac{6n^3}{n^3} + \frac{8n}{n^3}}{\frac{45}{n^3} + \frac{16n^3}{n^3} + \frac{8n^4}{n^3}} = \lim_{n\to\infty} \frac{6 + \frac{8}{n^2}}{\frac{45}{n^3} + 16 + 8n} = \lim_{n\to+\infty} \frac{6}{16 + 8n} = 0$$

$$\lim_{n\to\infty} \frac{7n^2 + 5n}{3n + 9} = \lim_{n\to+\infty} \frac{7n^2 + 5n}{3n + 9} = +\infty$$

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$$\lim_{n\to\infty} \frac{7n^2+5n}{3n+9} = \lim_{n\to+\infty} \frac{\frac{7n^2+5n}{n}}{\frac{3n+9}{n}} = \lim_{n\to+\infty} \frac{\frac{7n^2}{n}+\frac{5n}{n}}{\frac{3n}{n}+\frac{9}{n}}$$

$$= \lim_{n\to+\infty} \frac{7n+5}{3+\frac{9}{n}} = \lim_{n\to+\infty} \frac{7n+5}{3} = +\infty$$

$$\lim_{n\to+\infty} \frac{(n-1)(n-2)(n-3)(n-4)}{(5n+1)^4}$$

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$$= \lim_{n\to+\infty} \frac{n-1}{5n+1} . \frac{n-2}{5n+1} . \frac{n-3}{5n+1} . \frac{n-4}{5n+1} = \frac{1}{5} . \frac{1}{5} . \frac{1}{5} . \frac{1}{5}$$

$$\lim_{n\to+\infty} \frac{2^{n+1}+3^n}{3^{n+1}+6.2^n}$$

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$$\lim_{n\to+\infty} \frac{2^{n+1}+3^n}{3^{n+1}+6.2^n} = \lim_{n\to+\infty} \frac{3^n\left(\frac{2^{n+1}}{3^n}+\frac{3^n}{3^n}\right)}{3^n\left(\frac{3^{n+1}}{3^n}+\frac{6.2^n}{3^n}\right)}$$

$$=$$

$$\lim_{n\to+\infty} \frac{2\left(\frac{2}{3}\right)^n+1}{3+6\left(\frac{2}{3}\right)^n} = \frac{1}{3}$$

$$\lim_{n\to+\infty} \frac{n+\sqrt{n^2+n}}{n+\sqrt{n^2-n}}$$

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$$= \lim_{n\to+\infty} \frac{n\left(\frac{n+\sqrt{n^2+n}}{n}\right)}{n\left(\frac{n+\sqrt{n^2-n}}{n}\right)} = \lim_{n\to+\infty} \frac{\frac{n}{n}+\frac{\sqrt{n^2+n}}{n}}{\frac{n}{n}+\frac{\sqrt{n^2-n}}{n}}$$

$$= \lim_{n\to+\infty} \frac{1+\frac{\sqrt{n^2+n}}{n}}{1+\frac{\sqrt{n^2-n}}{n}} = \lim_{n\to+\infty} \frac{1+\sqrt{\frac{n^2+n}{n^2}}}{1+\sqrt{\frac{n^2-n}{n^2}}}$$

$$= \lim_{n\to+\infty} \frac{1+\sqrt{\frac{n^2}{n^2}+\frac{n}{n^2}}}{1+\sqrt{\frac{n^2}{n^2}-\frac{n}{n^2}}} = \lim_{n\to+\infty} \frac{1+\sqrt{1+\frac{1}{n}}}{1+\sqrt{1-\frac{1}{n}}} = \frac{2}{2} = 1$$

$$\lim_{n\to+\infty}(\sqrt{n^2+18n}-n)$$

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$$=\lim_{n\to+\infty}(\sqrt{n^2+18n}-n)\frac{\sqrt{n^2+18n}+n}{\sqrt{n^2+18n}+n}$$

$$=\lim_{n\to+\infty}\frac{(n^2+18n)-n^2}{\sqrt{n^2+18n}+n}=\lim_{n\to+\infty}\frac{18n}{\sqrt{n^2+18n}+n}$$

$$=\lim_{n\to+\infty}\frac{\frac{18n}{n}}{\frac{\sqrt{n^2+18n}+n}{n}}=\lim_{n\to+\infty}\frac{18}{\sqrt{\frac{n^2+18n}{n^2}}+\frac{n}{n}}=\lim_{n\to+\infty}\frac{18}{\sqrt{\frac{n^2}{n^2}+\frac{18n}{n^2}}+1}$$

$$=\lim_{n\to+\infty}\frac{18}{\sqrt{1+\frac{18}{n}}+1}=\frac{18}{2}=9$$