DISZKRÉT MATEMATIKA

1. feladatsor

1. Bizonyítsa be, hogy minden $n \geq 1$ természetes számra teljesül:

a) $1+3+5+\ldots+(2n-1)=n^2$ **1** **2**

b) $1+2+3+\ldots+n=\dfrac{n(n+1)}{2}$ **1** **2**

c) $\dfrac{1}{1.2}+\dfrac{1}{2.3}+\dfrac{1}{3.4}+\ldots+\dfrac{1}{n(n+1)}=\dfrac{n}{n+1}$ **1** **2**

d) $1^2+2^2+3^2+\ldots+n^2=\dfrac{n(n+1)(2n+1)}{6}$ **1** **2**

e) $1^2+3^2+5^2+\ldots+(2n-1)^2=\dfrac{n(4n^2-1)}{3}$ **1** **2**

f) $1.2+2.3+3.4+\ldots+n(n+1)=\dfrac{n(n+1)(n+2)}{3}$ **1** **2**

g) $1^3+2^3+3^3+\ldots+n^3=\left(\dfrac{n(n+1)}{2}\right)^2$

h) $1.4+2.7+3.10+\ldots+n(3n+1)=n(n+1)^2$

i) $(1+1).\left(1+\dfrac{1}{2}\right).\left(1+\dfrac{1}{3}\right).\ldots.\left(1+\dfrac{1}{n}\right)=n+1$

j) $\dfrac{1}{2.3.4}+\dfrac{2}{3.4.5}+\dfrac{3}{4.5.6}+\ldots+\dfrac{n}{(n+1)(n+2)(n+3)}=\dfrac{n(n+1)}{4(n+2)(n+3)}$

k) $2^0+2^1+2^2+2^3+\ldots+2^{n-1}=2^n-1$

l) $\dfrac{1}{2^2-1}+\dfrac{1}{4^2-1}+\dfrac{1}{6^2-1}+\ldots+\dfrac{1}{(2n)^2-1}=\dfrac{n}{2n+1}$ **2**

m) $1-\dfrac{1}{2!}-\dfrac{2}{3!}-\dfrac{3}{4!}-\ldots-\dfrac{n-1}{n!}=\dfrac{1}{n!}$ **1**

n) $\left(1-\dfrac{1}{4}\right).\left(1-\dfrac{1}{9}\right).\ \ldots\ .\left(1-\dfrac{1}{(n+1)^2}\right)=\dfrac{n+2}{2n+2}$

o) $1.2^0+2.2^1+3.2^2+4.2^3+\ldots+n2^{n-1}=(n-1)2^n+1$ **1**

p) $3^0+3^1+3^2+3^3+\ldots+3^{n-1}=\dfrac{3^n-1}{2}$

q) $1^2-2^2+3^2-\ldots+(-1)^{n-1}n^2=(-1)^{n-1}\dfrac{n(n+1)}{2}$

r) $1.1!+2.2!+3.3!+4.4!+\ldots+n.n!=(n+1)!-1$ **2**