- layout: posttitle: "Matrix Analysis and Applications"subtitle: "This is a subtitle"date: 2022-11-06author: "夜雨声烦"header-img: "img/bg2.png"header-mask: 0.2catalog: truetags: []
- Chapter 5 Norms, Inner Products and Orthogonality
  - 5.1 Vector Norms
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Chapter 5 Norms, Inner Products and Orthogonality

5.1 Vector Norms

Euclidean Vector Norm

For a vector $x_{n\times 1}$, the ***euclidean norm*** of x is defined to be:

- $\|x\| = (\sum_{i=1}^{n} x_i^2)^{\frac{1}{2}} = \sqrt{x^T x} \quad \text{whenever}\ x \in \mathrm{R}^{n\times 1}$
- $\|x\| = (\sum_{i=1}^{n} |x_i|^2)^{\frac{1}{2}} = \sqrt{x^* x} \quad \text{whenever}\ x \in \mathrm{C}^{n\times 1}$

Several points to note:

1. Recall that if $z = a + \mathrm{i}b$, then $\bar{z} = a - \mathrm{i}b$, and the magnitude of z is $|z| = \sqrt{z\bar{z}} = \sqrt{a^2 + b^2}$.

2. The definition of euclidean norm guarantees that for all scalars α,

   - $\|x\| \geq 0$
   - $\|x\| = 0 \Leftrightarrow x = 0$
   - $\|\alpha x\| = |\alpha| \|x\|$

3. Given a vector $x = 0$, we ***normalize*** x by setting $u = \frac{x}{\|x\|}$ to have another vector that points in the same direction as x, it's easy to see that

$$\|u\| = \| \frac{x}{\|x\|} \| = \frac{1}{\|x\|} \|x\| = 1$$

4. For vectors in R^n and C^n, the ***distance*** between u and v is naturally defined to be $\|u - v\|$.

Standard Inner Product

The scalar terms defined by

$$x^T y = \sum_{i=1}^{n} x_i y_i \in \mathrm{R} \quad \text{and} \quad x^* y \sum_{i=1}^{n} \bar{x}_i y_i \in \mathrm{C}$$

are called the ***standard inner products*** for R^n and C^n, respectively.