



## **MECH0064 MSc Group Design Project**

# **A Compact Continuum Robotics Manipulator for Applications in Therapeutic Ultrasound**

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## Abstract

The robotic manipulator plays a crucial role in mitigating the labor for people across different fields. This project designed a specialized 4-unit fishbone continuum manipulator intended for medical applications. The end effector of this manipulator is going to be equipped with a Focused Ultrasound (FUS) transducer, enabling the eradication of tumor tissues within the human body. The project methodology is divided into three parts, including modeling and strain analysis, derivation of forward and inverse kinematics, and actuation control strategies.

Through this approach, a viable design solution for the continuum manipulator has been crafted. By using 8 motors to control the 8 cables of the manipulator separately, we achieved an 8 DOF configuration with a working space of 300x300x300mm. The resulting manipulator boasts user-friendly operation, affordability, and remarkable precision. At the same time, there are still some improvements to be made to optimize the design, namely the closed-loop feedback systems and the structural strength analysis.

Looking ahead, the project should focus on the transition toward the practical manufacturing phase, where the simulation result and parameters can be verified by producing a real manipulator prototype.

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**Key Words:** *Continuum Robotic, Therapeutic Ultrasound, Robot Kinematics, Strain Analysis, Arduino*

# Contents

	<b>Page</b>
<b>Abstract</b>	<b>I</b>
<b>List of Figures</b>	<b>II</b>
<b>List of Tables</b>	<b>III</b>
1. Introduction . . . . .	1
1.1 Background . . . . .	1
1.2 Motivation . . . . .	1
2. Literature Review . . . . .	3
2.1 Manipulators in Biomedical Applications . . . . .	3
2.2 Types of Manipulator . . . . .	3
2.3 Types of Continuum Robots . . . . .	5
2.3.1 Tendon-Driven Robots . . . . .	5
2.3.2 Fishbone Robots . . . . .	6
2.3.3 Concentric Tube Continuum Robots . . . . .	7
3. Design . . . . .	8
3.1 Structural Design . . . . .	8
3.2 Designation and Parameter Definition of Manipulator . . . . .	8
3.3 Material Selection . . . . .	9
3.4 Methodology . . . . .	12
3.4.1 Strain Analysis . . . . .	12
3.4.2 Forward Kinematics . . . . .	14
3.4.3 Inverse Kinematics . . . . .	16
3.4.4 Angular Conversion . . . . .	18
3.5 Electronic Control . . . . .	18
3.5.1 Actuation Control . . . . .	18

3.5.2	Parameter Input by Serial Monitor . . . . .	22
3.5.3	Verification of MPU 6050 . . . . .	22
3.5.4	Using HC-SR04 to measure distance . . . . .	23
3.5.5	SSD 1306 OLED . . . . .	24
4.	Result and Discussion . . . . .	25
4.1	Strength Analysis . . . . .	25
4.1.1	The Result of Strength Analysis . . . . .	25
4.1.2	Limitations and Solutions . . . . .	26
4.2	Error Analysis of Cross-Shaped Sheets . . . . .	28
4.3	Manipulator Workspace Analysis . . . . .	29
4.4	Inverse Kinematics . . . . .	32
4.4.1	Posture Solution of Single Module Bending . . . . .	32
4.4.2	Trajectory Replication . . . . .	34
4.4.3	Limitation and Improvement . . . . .	37
4.5	Electronic Control . . . . .	38
4.5.1	Actuation Control . . . . .	38
4.5.2	Information Acquisition and Display . . . . .	41
5.	Evaluation . . . . .	43
5.1	Sustainability . . . . .	43
6.	Conclusion . . . . .	45

<b>References</b>	i
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<b>A. The Appendix of Tables</b>	viii
<b>B. The Code Display</b>	xiii
<b>C. Figures for Illustration</b>	xvi
<b>D. Equations and Derivations</b>	xx

# List of Figures

1	A three-tendons continuum robot with one segment . . . . .	6
2	The cable-driven fishbone continuum robot . . . . .	6
3	An example concentric tube continuum robot . . . . .	7
4	The main components of proposed manipulator and cables arrangement . . . . .	8
5	The simplified model of manipulator . . . . .	12
6	The mesh independence table . . . . .	13
7	The mesh model of manipulator . . . . .	13
8	The kinematics model of manipulator at the initial position . . . . .	14
9	The right angle bending postures of single modules . . . . .	15
10	The flow chart of the IK algorithm . . . . .	17
11	The flow chart of Arduino step motor control programme . . . . .	21
12	The MPU6050 chip with ancillary Arduino circuit . . . . .	23
13	The wiring diagram and simulation of HC-SR04 . . . . .	24
14	The displacement analysis of manipulator . . . . .	25
15	The stress analysis of manipulator . . . . .	26
16	The simulation results of errors against the number of sheets . . . . .	28
17	The entire workspace with different random indices . . . . .	29
18	The cubic workspace with different segmentation . . . . .	30
19	The top view of the workspace and the labels of detection points . . . . .	30
20	The kinematics model of manipulator with respective bending modules . . . . .	33
21	The trajectory replications with and without genetic method . . . . .	35
22	The cross-shaped trajectory and its replication by IK algorithm . . . . .	36
23	The closed trajectory and its replication by IK algorithm . . . . .	36
24	The circuit connection of Arduino and stepper motors . . . . .	39

# List of Tables

1	The Characteristics of Different Manipulators. . . . .	4
2	The Parameters of Manipulators. . . . .	9
3	The Reachability of Detection Points with Different H. . . . .	31
4	The Average Errors of the replicated trajectories. . . . .	35
5	The Pin Assignment of Stepper Motors. . . . .	38

# 1 Introduction

## 1.1 Background

Robotic manipulators refer to devices that help operators perform repetitive tasks without direct physical contact at a very fast speed [1]. In the early stages of robotic manipulator development, rigid joint manipulators dominated the field and were mainly used in industrial production to perform simple tasks previously done manually, or being applied in hazardous environments to avoid human casualties. As this technology progressed, various types of manipulators were developed and utilized in different fields. In specific fields such as healthcare [2], precision manufacturing, and searching and rescuing, there are strict requirements for the precision, motion trajectory, and morphology of robotic arms. Thus, continuum manipulators emerged. Their flexible structure allows them to attain any desired posture to reach target positions, making them suitable to operate in complex environments [3]. In recent years, continuum manipulators have been widely applied, saving time and labor costs while reducing potential risks brought about by inaccuracy.

## 1.2 Motivation

Continuum robots have wide applications in the medical field, especially in surgeries requiring extremely high precision and specific postures to complete tasks. They are smaller, more flexible, and can increase the success rate of surgeries while minimizing patient discomfort.

Among the various surgical operations that continuum manipulators can perform, there is one certain surgery that requires the application of a Focused Ultrasound (FUS) transducer to emit ultrasound waves to penetrate the human skin, then bombard and destroy tumor tissues buried beneath the skin [4]. In the past, manipulators capable of performing this surgery existed, but they were very expensive [5] and not easy for doctors to operate.

This project aims to design a continuum manipulator to manipulate FUS transducers for surgical operations on the skin area of patients. At the same time, it is expected to solve the challenges that previous manipulators were facing, namely: Cost: The manipulator

designed in this project needs to be manufacturable at an extremely low cost and be easy to mass-produce. Reliability: This manipulator needs to operate with high precision, have a wide range of workspace, and be able to handle certain mass loads without causing any damage to itself. Simplicity: The manipulator needs to be easy to use.

Objectives To design a 4-unit fish-bone continuum manipulator that can be applied in FUS applications, having a workspace of 300x300x300mm. The test and verification are based on Ansys, Matlab, and Arduino simulations. The project starts on 10th Nov. 2023 and will finish on 15th Mar. 2024.

## 2 Literature Review

In the last two decades, significant progress in electronic and computer technologies has led to remarkable growth in the field of manipulator robotics. Manipulators developed by various institutions have been integrated into the industrial sector to autonomously or semi-autonomously perform repetitive tasks. Simultaneously, manipulators are utilized for tasks with stringent precision requirements to minimize errors. Additionally, essential tasks are undertaken by manipulators to substitute for humans in challenging environments.

### 2.1 Manipulators in Biomedical Applications

With a growing emphasis on the field of biology, manipulators have also been introduced to assist. In the field of medicine, manipulators have been utilized since the end of the last century [1–5]. At the beginning of the 21st century, a novel robotic system, da Vinci robotic surgery system, was designed to facilitate more intricate surgical procedures [6]. Moreover, manipulators can be leveraged in the field of biological physics as a viable approach for biological experiments. In the HIFU system developed by An et al., the SCARA (self compliant automatic robot assembly) robot was employed as manipulator, incorporating an ultrasound probe to scan biological tissues [7]. The robotic system FUSBOTs (Focal Ultrasound Surgery RoBOTs) was proposed and upgraded to accomplish more precise targeted treatment with multiple DoF (Degrees of Freedom) manipulators [8–10]. Despite the various utilization of manipulator platforms designed for accommodating ultrasonic transducers [11–13], certain limitations persist. A comparison between different types of manipulators is necessary in the selection of appropriate type manipulator for integrating ultrasonic transducer.

### 2.2 Types of Manipulator

From the perspective of geometry, rigid-body manipulators can be categorized into two main types: parallel mechanisms and serial links [14]. The control of parallel mechanisms is relatively complex. Meanwhile, serial link manipulators can be further divided into five types: Cartesian (PPP), articulated (RRR), cylindrical (RPP), spherical (RRP), and SCARA (RRP). Additionally, with the advancement of robotics technology, two other types

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of manipulators, namely biomimetic and anthropomorphic, have demonstrated their advantages [15, 16]. The comparative analysis will be conducted to highlight the distinctive features of different manipulators presented in Table 1, leading to the identification of the most suitable type for specific applications.

**Table 1:** The Characteristics of Different Manipulators.

Manipulators	Types	DoF	Features	Applications
Stewart Platform [17]	Series-Parallel	6	series-parallel duality, mature algorithm.	flight simulation [18]. robocrane [19]
Cartesian	PPP	3	technological maturity, low complexity, cost-effective, high payload.	3D-Printing framework [20] warehousing & hoisting [21] agricultural machinery [22]
Articulated [23]	RRR	6	factory automation, programmable, high precision and payload.	milling operations [24] material handling [25]
Cylindrical [26]	RPP	3	good precision, limited workspace.	assembling industries [27]
Spherical [28]	RRP	3	collision avoidance, low precision, long reach, light weight.	low-precision semi-automated tasks [24, 25]
SCARA [26]	RRP	3	high accuracy and fast operation.	surgical applications [29] medical rehabilitation [30]
Continuum/Soft [31, 32]	Biomimetic	$\infty$	ability to handle fatigue objects, environmental adaptability.	tendon driven [33–35] fibre-reinforced [36, 37] fluid-elastic drive [38–40] bionics materials [41–43]
Humanoid	anthropomorphic	7	high efficiency, flexibility.	humanoid robots driven by pneumatic artificial muscles [44]

The distinctive structure of the continuum manipulator imparts to the system with theoretically infinite DoF. The characteristic enables the continuum manipulator to navigate through confined spaces or manipulate objects along specific trajectories within the workspace [31, 32, 45]. Simultaneously, the continuum robotic manipulator

demonstrates suboptimal performance in the context of payload, making it challenging to meet the payload capacities commonly observed in industrial robotic systems. Nevertheless, within the realm of biomedical applications, the emphasis shifts towards precision and operability for manipulators. In this report, the manipulator platform integrated with the ultrasonic transducer module necessitates a design. The selection of the continuum manipulator was predicated upon its outstanding flexibility and the lightweight attributes of the ultrasonic transducer module. An appropriate type of continuum robot will be selected to serve as the centrepiece for the manipulator platform.

## 2.3 Types of Continuum Robots

Nowadays, a variety of continuum robots exist, each exhibiting unique structures and functions. They serve in different fields such as medicine, construction, and exploration. In this section, some of the most popular continuum robots will be introduced, providing basic insights into their structures as well as discussing their merits and drawbacks.

### 2.3.1 Tendon-Driven Robots

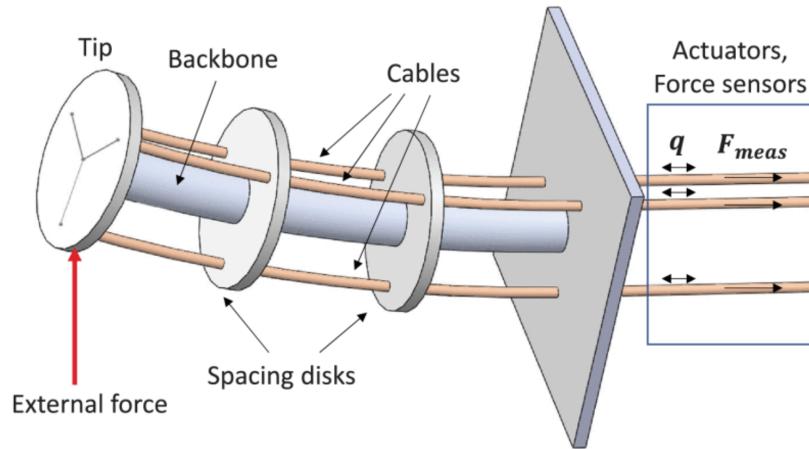
The arm of the Tendon-Driven robot consists of a backbone, several tendons and disks. The backbone defines the structure and posture of the entire robot arm, while disks define the diameter and divide the robot arm into segments, and tendons are stretched to create deformation and movements of different directions for the robot arm.

Figure 1 shows only the simplest tendon-driven robots. In practical applications, there may be more than one backbone, and the disks are not necessarily parallel.

Compared with other continuum robots, one of the most significant advantages of tendon-driven robots is their flexibility. This advantage makes it more effective in performing tasks in complex and restrictive spaces. In addition, due to the simple components needed to construct the robot, it is easier to meet lightweight design specifications. Moreover, like the concentric-tube continuum robots, tendon-driven continuum robots can be built and designed on a small scale with diameters of below 10 mm [46].

However, due to its simple actuating principle, more complex algorithms are needed to control it more accurately. Also, the tendon-driven robots actuate by pulling the tendon, which makes the friction between the tendon and other components inevitable, which will

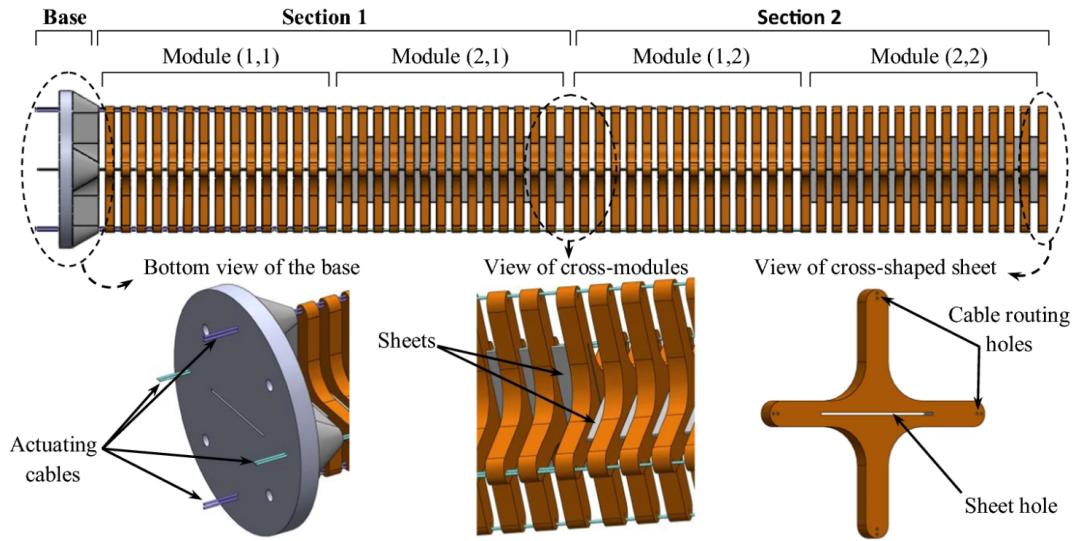
accelerate the wearing speed of the tendon-driven robots.



**Figure 1: A three-tendons continuum robot with one segment [47].**

### 2.3.2 Fishbone Robots

Fishbone robots are inspired by fish bones. It consists of several "fishbone modules", which are composed of a number of rigid cross-shaped plates, and a flexible elastic plate backbone embedded in the middle, forming a rigid-soft coupling structure[48].



**Figure 2: The cable-driven fishbone continuum robot with cable arrangement [49].**

According to Figure 2, when the different modules are connected, the backbones of the bionic fishbone modules are perpendicular to each other, and finally form a complete main frame of the fishbone robot. Like the tendon-driven robot, the fishbone robot is also controlled by cable, but it is worth noting that each module is controlled by two separate

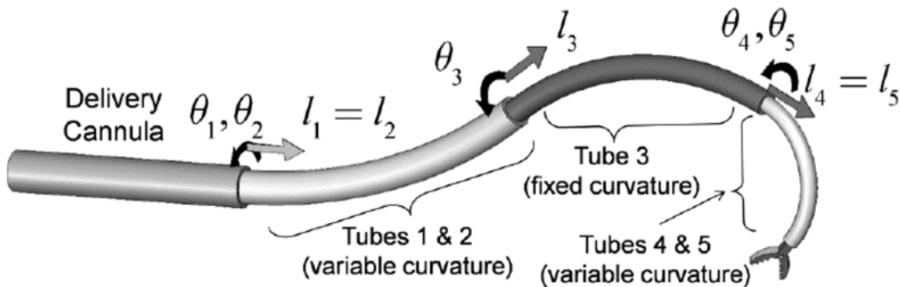
cables. This means that for a fishbone robot composed of  $n$  fishbone modules, there are a total of  $2n$  cables controlling the motion of the whole robot. In addition, the planes formed by the two ropes that control each module are perpendicular to each other, so that each section rotates along a plane perpendicular to each other.

The Figure 2 shows that because of the numbers of rigid cross-shaped sheets formed a large diameter frame, the structural stability of this kind of robot is very strong. Also, each fishbone unit can provide one DoF, making it easy to reach a very high total DoF. However, due to the multiple-curves deformation trajectory, the fishbone robots are not suitable for situations where strict trajectory is required[48].

### 2.3.3 Concentric Tube Continuum Robots

The concentric tube robots, shaped like retractable walking sticks, consist of many tubes with decreasing diameters. Each tube is nested on top of the previous wider tube.

The concentric robots are made of two parts: tubes and coaxial actuation units. The tubes are the main structural element of this robot and act as the backbone. The coaxial actuation unit consists of two motors which are responsible for rotation and translation movement respectively. Each tube is actuated by an independent coaxial actuation unit.



**Figure 3: An example concentric tube continuum robot [50].**

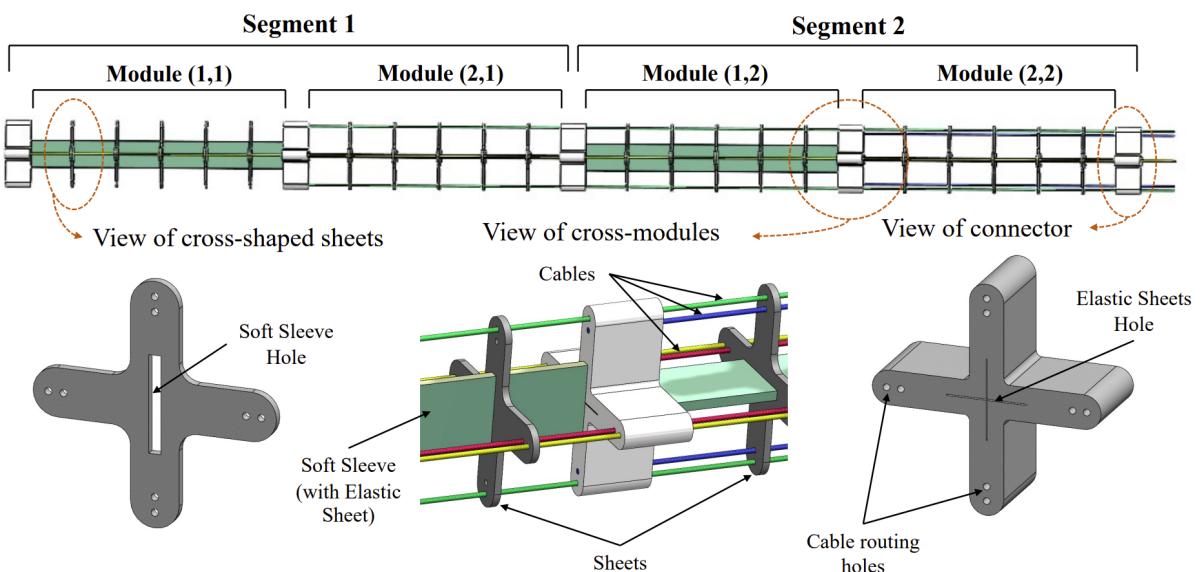
The most significant advantage of this kind of robots is that of all the continuum robots, concentric tube robots have the smallest possible outer diameter and are best suited to work in confined and narrow spaces. Therefore, it is the ideal choice for surgical operations.

Their disadvantages, on the other hand, are also very evident. Since each tube requires an independent actuation unit, the overall length of the robot cannot be very long, because longer lengths will lead to more tubes, and will lead to more tip position errors.

### 3 Design

#### 3.1 Structural Design

The novel Continuous Six-degree-of-freedom Manipulator introduced in this paper, which corresponds to the scheme design shown in Figure 2, belongs to the class of bio-inspired continuum robots [48]. In the design process, a nylon cable drive mechanism is employed as a substitute for the complex musculature of fish, enabling it to maintain stability while undergoing deformation movements, thus achieving effective deformation motion and controllability. As depicted in Figure 4, the rigid cross-shaped sheets is utilized to mimic the spine and ribs, uniformly arranging these sheets on a rubber pure soft sleeve to replace the spinal joints.



**Figure 4: The main components of proposed manipulator and cables arrangement.**

#### 3.2 Designation and Parameter Definition of Manipulator

The manipulator is composed of four identical modules, each of which can be independently controlled to bend within a two-dimensional space. Each module consists of an elastic sheet and five uniformly arranged cross-shaped sheets, where the rubber pure soft sleeve outside the elastic sheet is interference-fitted with the cross-shaped sheets. The connector end of each module features two symmetrically arranged inextensible cables that pass through the cross-shaped sheets and are anchored to the

connector, driving the end of each manipulator module. By applying tension to one of these cables, the elastic sheet can be deflected, thereby achieving a single degree of freedom planar motion at the end of the manipulator module. Since the elastic sheets of the modules are transversely mounted, the deflection of two or more sheets simultaneously allows for spatial motion at the end of the manipulator, as shown in Figure 4. Overall, the proposed manipulator comprises two segments, each maintaining a constant length of 675 mm and a weight of 88 grams. Each module contains five cross-shaped sheets with a circumferential radius of 20 mm and a thickness of 1.5 mm. Both the head and the tail of each segment are connected by connectors, which have the same circumferential radius as the cross-shaped sheet and a thickness of 15 mm.

The definition of manipulator parameters is presented in Table 2. The subsequent sections will adhere to the corresponding naming conventions.

**Table 2:** The Parameters of Manipulators.

Paramter	Definition	Value (mm)
$Sr_i$	the length of elastic sheet in Module i	$Sr_{1,2,3,4} = 150$
$d_i$	the thickness of the cross-shaped connector	$d_{1,2,3,4,5} = 15$
$N$	the number of cross-shaped sheet in a module	$N = 0 \sim 15$
$r_i$	the distance between the centroid of connector and cable routing hole	$r_{1,2} = 17.5$ $r_{3,4} = 15$
$\Delta S_i$	the change volume of cable, cable <sub>2<i>i</i>-1</sub> and cable <sub>2<i>i</i></sub> corresponds to Module i	
$\Delta S$	a series of change volume of cables	$\Delta S_1 \sim \Delta S_8$
$\alpha_i$	the bending angle of module i	
$\alpha$	a series of bending angles about four modules	$[\alpha_1, \alpha_2, \alpha_3, \alpha_4]$
$\theta$	a series of inverse kinematics solutions	$[\theta_1, \theta_2, \theta_3, \theta_4]$
$\epsilon$	threshold of the error in FABRIKc algorithm	$\epsilon = 0.02$

### 3.3 Material Selection

The fabrication of the Continuous Six-Degree-of-Freedom Manipulator primarily employs the following methods: Shape Deposition Manufacturing (SDM)[51], Casting[52, 53], and Multi-material 3D Printing Technologies[54, 55]. By integrating SDM

with Casting techniques, it is possible to utilize rigid, flexible, and soft materials, and integrate components such as sensors and circuits into the structure, which is particularly crucial for the integrated manufacturing of sensors and actuators. On the other hand, Multi-material 3D Printing technology supports the unified manufacturing of soft and hard materials, enabling the creation of structures with more complex geometries and adjustable material hardness according to specific requirements. This approach results in structures that are not only more scientifically arranged and exhibit a more rational combination of rigidity and flexibility but also possess higher stability and superior performance. Moreover, Multi-material 3D Printing enhances manufacturing efficiency. Given these advantages, 3D Printing technology emerges as the preferred method for constructing continuum robots.

The materials for the cross-shaped sheets and connectors can be fabricated in an integrated manner using Multi-material 3D Printers, which avoids the impact of imperfect assembly on deformation and achieves a compact design. The material chosen simulates engineering plastics (Mainly digital material with a Poisson's ratio of 0.394, a modulus of elasticity of 2.2 GPa and a Young's modulus of 2.5 GPa). The soft sleeve material is made of silicone rubber, characterized by a Poisson's ratio of 0.47. Furthermore, a 65Mn alloy steel (with a Poisson's ratio of 0.244 and a Young's modulus of 0.2 GPa) elastic sheet (flexible) embedded within the soft sleeve forms a bio-inspired sheath, creating a bio-inspired fishbone structural unit. This structural unit is a hybrid of rigidity, flexibility, and softness, not only exhibiting excellent constant curvature characteristics but also higher structural stability. The bio-inspired fishbone unit also provides a constraint mechanism for the synchronous bending of the driving cables, effectively reducing the impact of external mechanical collisions on the cables [48]. Therefore, this unique structural design is a key foundation for achieving high-precision modelling of continuum robots.

In this design, the elastic sheet is combined with the connector by embedding it into a 0.3 mm wide slot on the connector and utilizing an interference fit, a method that eliminates the need for additional fasteners and ensures the structural integrity of the continuum robot during standard operations. Nonetheless, to prevent the robot from separating under extreme overload conditions, special fixing holes were designed on both the elastic sheet and the connector. The installation of screws through these holes enhances the robustness of the continuum robot in harsh environments. Leveraging the

modular design characteristic of the continuum robot, it can be constructed by serially connecting multiple similar motion modules. The example demonstrated in this study is comprised of two such motion modules, as shown in Figure 4, whereby altering the length of the driving cables enables the continuum robot to achieve multi-degree of freedom movements.

Figure 4 illustrates the cable configuration scheme proposed for the continuum robot in this paper. Each bio-inspired fishbone structural unit is actuated by two symmetrically arranged cables, which traverse through the guide holes of the connectors and cross-shaped sheets, with each module being controlled by two motors, totaling eight motors. The guide holes in the cross-shaped sheets ensure that the cables maintain an arc shape within their bending region. In the actual design process, the maximum number of cross-shaped sheets is determined by the maximum bending angle preset for the bio-inspired fishbone unit. Through simulation analysis, we have concluded that the bending performance of the elastic sheet is optimal when the bending angle reaches 73°.

The design proposal offers the following advantages:

- Lightweight: Utilizing cross-shaped sheets results in a lighter weight compared to traditional cylindrical manipulators.
- Stable Deformation Motion: The deformation motion is stable, offering an improvement over traditional cylindrical manipulator.
- Rigidity: The adoption of a stacked skeleton structure effectively eliminates bending deformation.
- Energy Consumption [56]: The proposed robot design facilitates a reduction in the energy required to achieve spatial movement; that is, each module employs two actuation motors instead of three, as is the case with continuum robots possessing a cylindrical backbone.

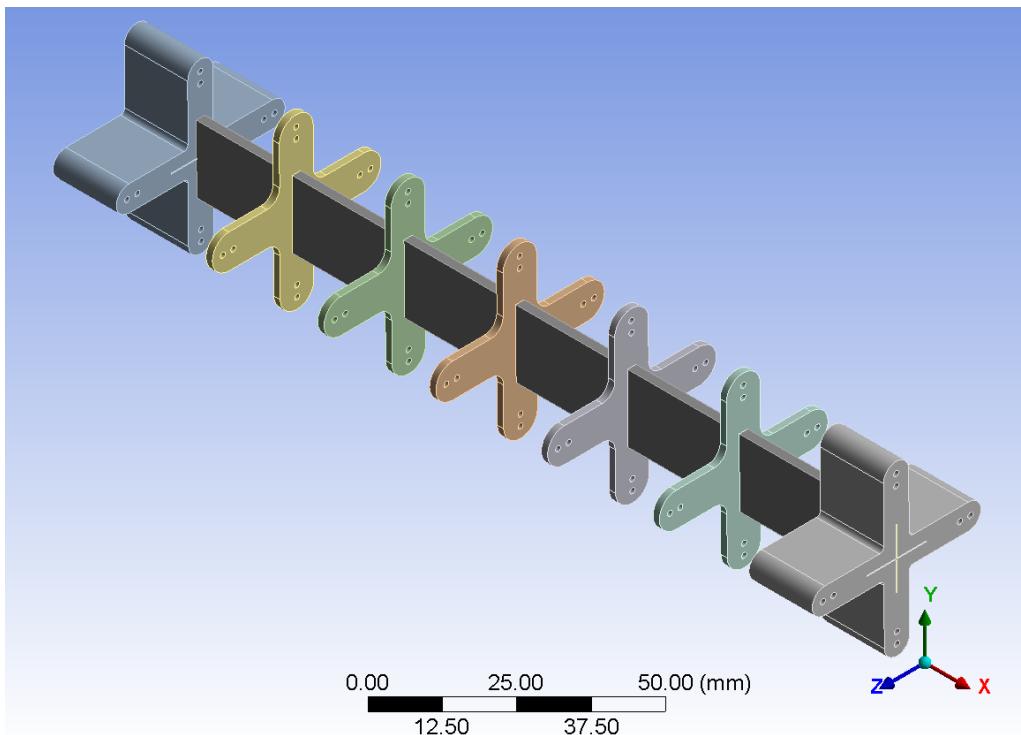
Like all continuum robots, the sole drawback of this design is the complexity involved in modeling.

## 3.4 Methodology

### 3.4.1 Strain Analysis

The objective of the analysis is to determine the mechanical boundaries and failure risks of the manipulator to improve its design for better durability and performance. Using ANSYS, the manipulator was evaluated at its maximum operational angle to understand its behavior under extreme conditions. This revealed specific strength levels of the model, aiding in verifying the arm's structural soundness for its intended use.

As the four modules of the model are identical, it was reasonable to simplify the model to a single module to reduce computational resources and analysis time while maintaining a high level of accuracy in the results. The simplified model is shown in Figure 5.

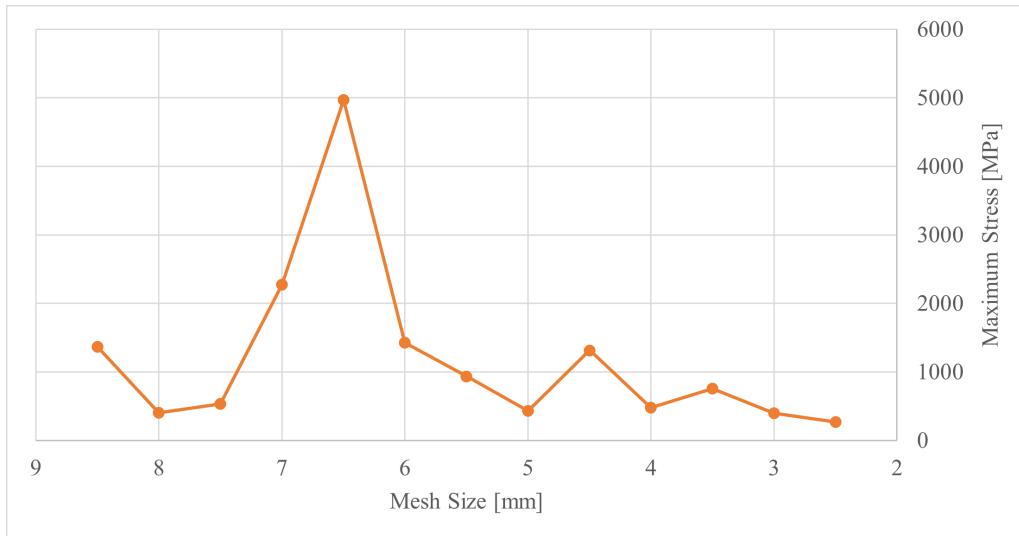


**Figure 5: The simplified model of manipulator.**

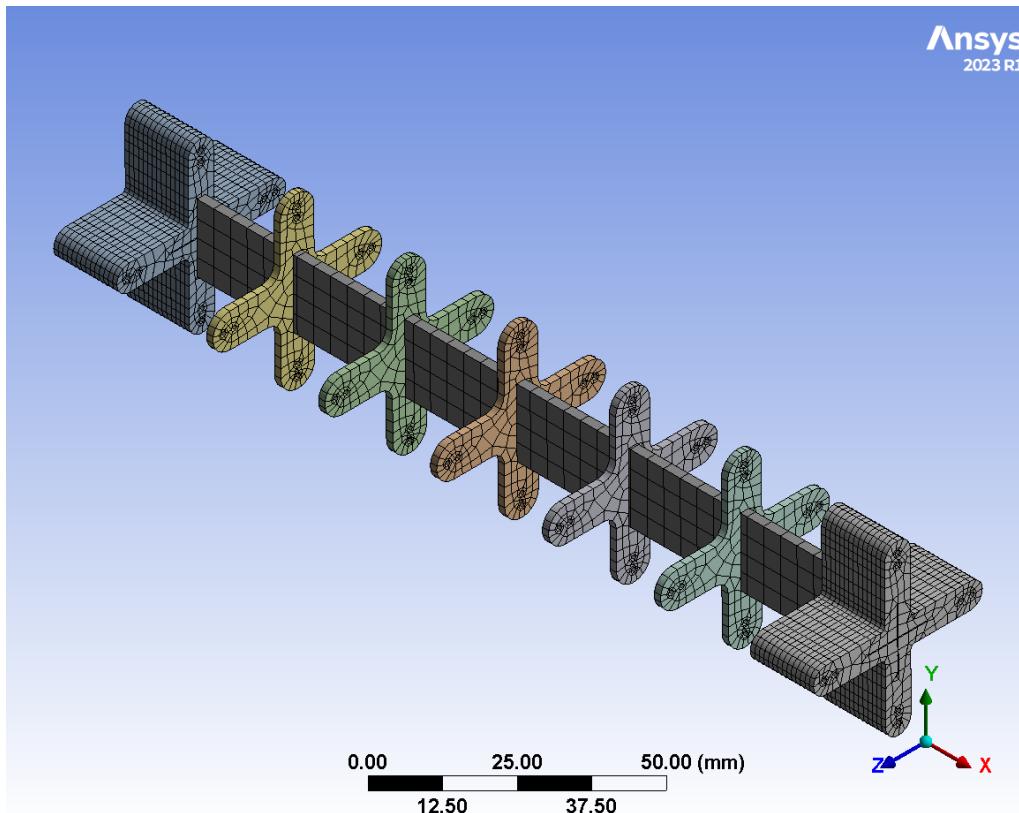
Load it into ANSYS and use the Static Analysis Module to evaluate the stresses and strains at a maximum bending angle of 73 degrees.

To ensure that the results of the simulation are independent of the mesh size, a mesh-independent study is required. Finite element analyses were performed with mesh sizes ranging from 8.5 mm down to 2.5 mm. As illustrated in Figure 6 a significant peak in stress at a mesh size of 6.5 mm suggests possible numerical artifacts or unresolved stress

concentrations. Subsequent refinement below this threshold showed the stress values approaching a constant level, indicating the potential achievement of mesh independence.



**Figure 6: The mesh independenc table.**



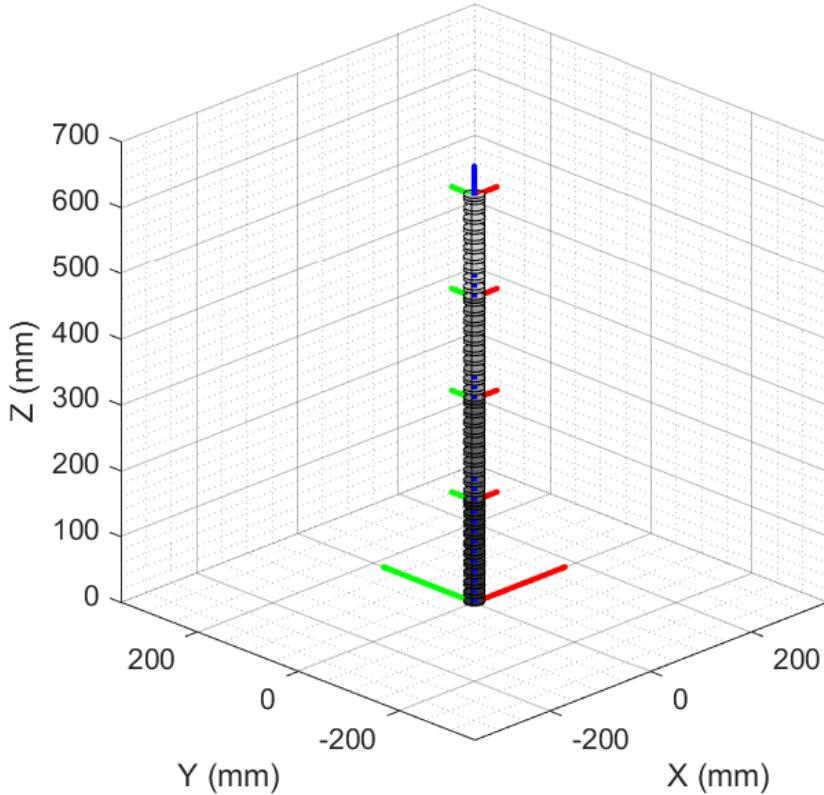
**Figure 7: The mesh model of manipulator.**

The trend toward constant stress values for mesh sizes finer than 4 mm suggests that the mesh is sufficiently detailed to capture the stress distribution within the component

accurately. This finding establishes that a mesh size of 4 mm provides a balance between computational efficiency and the accuracy of the stress predictions. Based on these outcomes, a mesh size of 4 mm is used for further analysis, ensuring that the results are both computationally economical and reliable for engineering judgments. Figure 7 shows the mesh of the model.

### 3.4.2 Forward Kinematics

The forward kinematics (FK) provided by Zhou et al. [48] were solely focused on a single segment with two modules. However, utilizing the FK algorithm for calculation became complex while there were four modules in the manipulator. The FK algorithm was rederived to generalize the workspace of the manipulator.

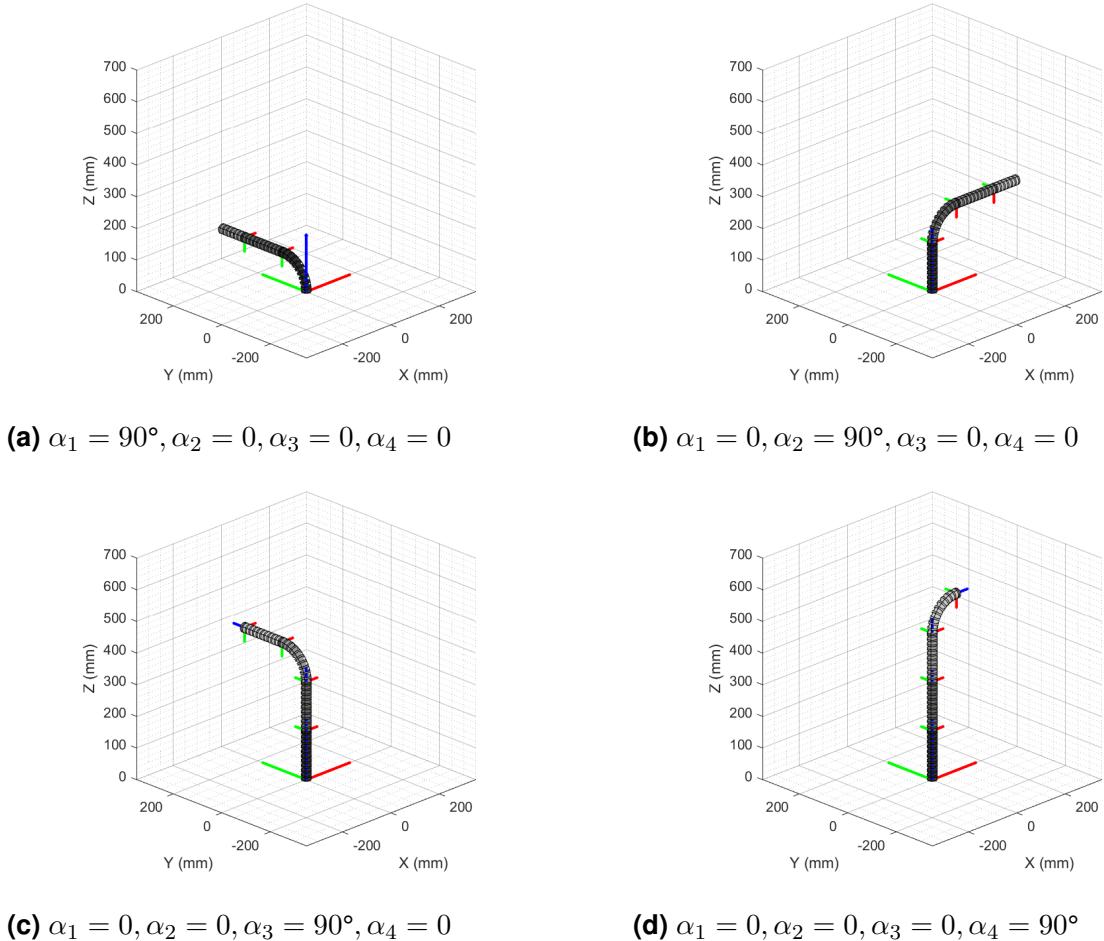


**Figure 8: The kinematics model of manipulator at initial position.**

The base coordinate system (CS) was established with the centroid of base connector upper surface serving as the origin. The backbones of Module 1 and 3, which are Module (2,2) and (2,1), were parallel to the X-axis of base CS, while the backbones of Module 2

and 4, which are Modules (1,2) and (1,1), were parallel to the Y-axis of base CS. The upper surface centroids of connectors were designated as  $node_1$ ,  $node_2$ ,  $node_3$ ,  $node_4$ , and  $node_5$ . The initial posture with CSs originated at  $node_1 \sim node_5$  are shown in Figure 8.

The Module 1 was restricted to bending in Y-Z plane of CS whose origin is  $node_1$ , while the Module 2 was restricted to bending in X-Z plane of CS whose origin is  $node_2$ . Similarly, the Module 3 and 4 were subject to the same constraints. The right angle bending postures of single modules are illustrated in Figure 9.



**Figure 9: The right angle bending postures of single modules.**

The bending angle  $\alpha$  was defined to be positive while the module bent toward positive direction of the X-axis or Y-axis. Owing to the distinct properties of the four modules, diverse calculation methods were utilized. The Module  $i$  has a base node  $node_i$  and an end effector node  $node_{i+1}$ . The position of  $node_{i+1}$  in CS whose origin is  $node_{i+1}$  was

calculated in Equation 1. The detailed derivations are listed in Section D.

$$\mathbf{P}_{i+1}^{base} = \mathbf{R}_i \times \mathbf{P}_{i+1}^i + \mathbf{P}_i^{base} \quad (1)$$

$\mathbf{P}_{i+1}^{base}$ : The absolute position matrix (APM) of  $node_{i+1}$  in the base CS.

$\mathbf{R}_i$ : The rotational matrix transforms the base CS into CS  $i$  whose origin is  $node_i$ .

$\mathbf{P}_{i+1}^i$ : The relative position matrix (RPM) of  $node_{i+1}$  in CS  $i$ .

$\mathbf{P}_i^{base}$ : The absolute position matrix (APM) of  $node_i$  in the base CS.

Applying the corresponding transformations to the Equation 1 revealed the patterns shown in Equations 2 and 3.

$$\mathbf{P}_{i+1}^{base} - \mathbf{P}_i^{base} = \prod_{n=1}^i \mathbf{R}_n \times \mathbf{P}_{i+1}^i \quad (2)$$

$$\mathbf{P}_{i+1}^{base} = \sum_{m=1}^i \left[ \prod_{n=1}^m \mathbf{R}_n \times \mathbf{P}_{m+1}^m \right] \quad (\mathbf{P}_1^{base} = 0) \quad (3)$$

### 3.4.3 Inverse Kinematics

The inverse kinematics (IK) algorithm was inspired by a rigid manipulator IK solver called FABRIK [57]. W. Zhang et al.[58] expanded this solver to the continuum manipulator domain. Considering the distinctive nature of the manipulator, adaptations were implemented to the FABRIKc solver, transforming it into a tailored IK algorithm for this particular application. For further elucidation, the IK derivation was demonstrated using a target whose angles  $target = [0,0,90,0]$  °. The comprehensive process of the first epoch will be elaborated as follows. Simultaneously, the flowchart of IK algorithm was shown in Figure 10.

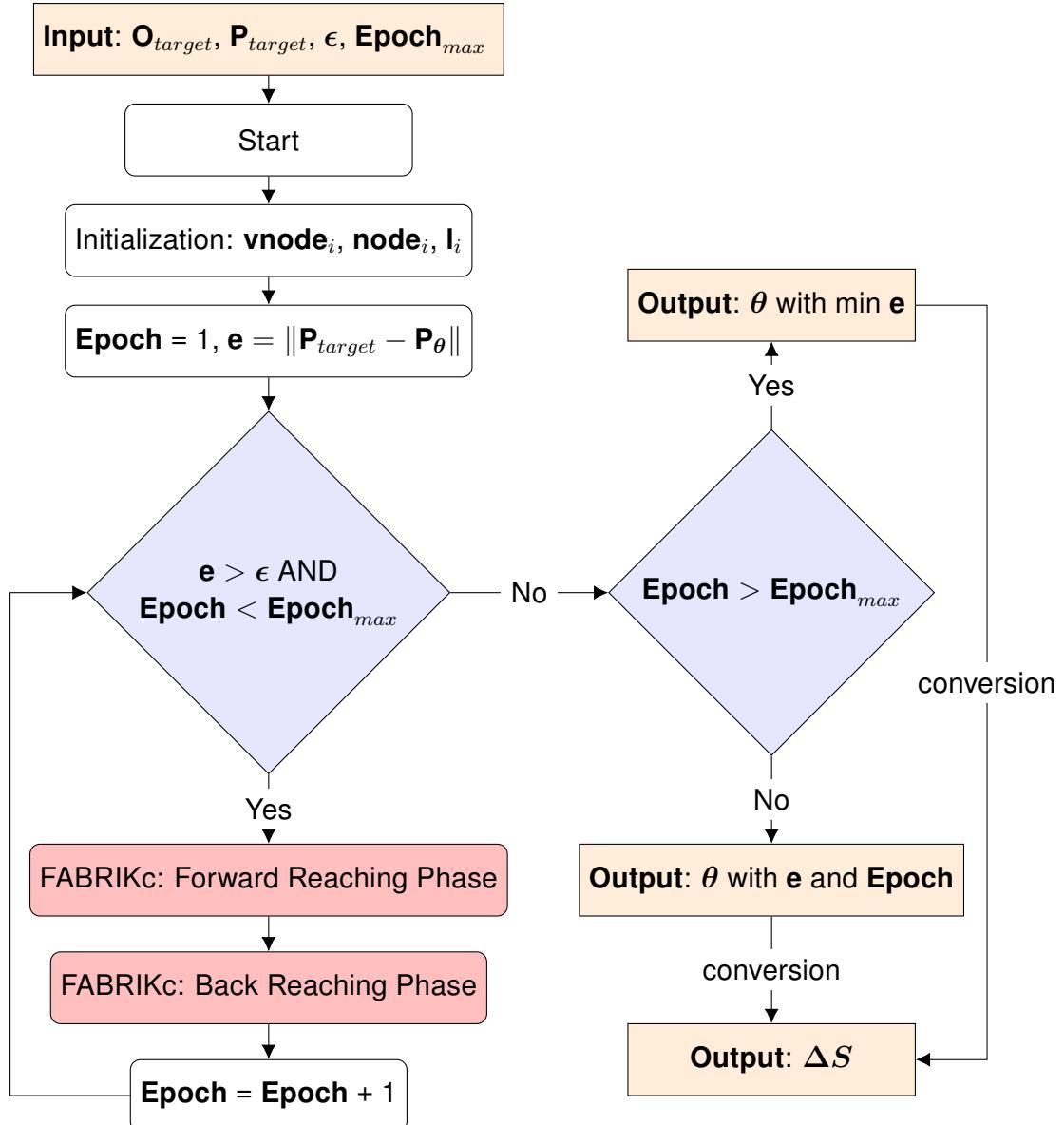


Figure 10: The flow chart of the IK algorithm.

The orientation and position matrices of the target were derived in Equation 4. These matrices were utilized as the input of the IK algorithm. The output of the algorithm was a series of bending angles  $\theta$ .

$$\mathbf{O}_{target} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad \mathbf{P}_{target} = \begin{bmatrix} 0 \\ 245.493 \\ 395.493 \end{bmatrix} \quad (4)$$

Additionally, it is essential to introduce a novel concept called "virtual node". This raised the fact that FABRIK algorithm was originally designed for rigid manipulators, which implied that continuum manipulators need to be abstracted for modeling purpose. In this context,

the module of manipulator were abstracted as a rigid manipulator with two prismatic joints and one revolute joint. The location of the revolute joint served as the virtual node.

Subsequently, the forward reaching phase of the FABRIKc algorithm can commence. The calculation which involves four iterations was specified in Section D.

Ultimately, the FABRIKc algorithm initiated the backward reaching phase, which is the FK algorithm. The FABRIKc algorithm inherently guaranteed the orientation of manipulator's end effector, with the necessity to account solely for positional errors. The error was calculated by Equation 5.

$$\mathbf{e} = \|\mathbf{P}_{target} - \mathbf{P}_{theta}\| \quad (5)$$

For the first epoch, the results of IK algorithm were  $\theta = [6.71, -0.0, 83.29, -0.0]^\circ$ , The error of the first epoch was 33.77506. The results about IK algorithm with the target angle  $\alpha = [0, 0, 90, 0]$  are shown in Table A.1 in Appendix A. The algorithm was configured to execute a maximum of 200 epochs, with the provision to pause when the error reaches a sufficiently low level, thereby conserving computational resources. The maximum epoch number was defined as  $\text{Epoch}_{max}$ .

### 3.4.4 Angular Conversion

The Arduino programme utilized  $\Delta S$  as input to control the motors. However, the solutions of IK algorithm are  $\theta$ , which are angles. It is vital to convert angles  $\theta$  into  $\Delta S$ . The variables  $R_1$ ,  $R_2$ , and  $R_3$  were mentioned in Equations D.1, D.4, and D.7, while  $R_4 = Sr_4/\alpha_4$ . The relationship between  $\theta$  and  $\Delta S$  was shown in Equation D.42. The relationship can be further generalized into matrix in Equation D.43 for Python programme.

## 3.5 Electronic Control

### 3.5.1 Actuation Control

As mentioned above, the manipulator is divided into four units in total, with each unit being controlled for its bending movements by two cables. This means that the entire manipulator is controlled by a total of 8 cables. How to control the extension and retraction of these 8 cable ropes is the key to the Control part.

In this project, 8 stepper motors are employed to control the 8 individual cables. The

planned design involves wrapping the cables around the rotor of each motor, and the stepper motors are programmed to extend or retract the ropes by stepping a certain number of steps in either a counterclockwise or clockwise direction. These 8 motors are connected to an Arduino board together, and the user can control the programme by a keyboard, and a serial monitor.

- Actuation Principle

In this programme, the user inputs the desired length changes for 8 cables  $\Delta S_1$   $\Delta S_8$  ( $\Delta S_{1,2,3,4}$  ranging from -36.65 mm to 36.65 mm,  $\Delta S_{5,6,7,8}$  ranging from -68.07 to 68.07 mm). After conversion, it outputs the corresponding step numbers  $Step_1$   $Step_8$  for the eight stepper motors. Based on this, the 8-stepper motors will step correspondingly. The relationship between the changes in cable length and the motor steps can be defined by Equation 6.

$$Step_i = \frac{(\Delta S_i - \Delta S_{i(prev)}) \cdot N}{\pi \cdot d}, \quad i \in [1, 8] \quad (6)$$

$\Delta S_{i(prev)}$ : The length change for  $i^{th}$  cable in the previous location.

$d$ : The diameter of the rotor. A rotor of diameter 10 mm is used currently.

$N$ : Number of steps/rev. 2048 for 28YBG-48 motor.

- Component Selection

To achieve a successful and precise control functionality, the first thing to do is to select appropriate components. Below are the main electronic components chosen:

- Arduino Mega 2560 board:

Because the project requires connecting 8 motors at the same time, the board should have more pins on the board as well as stronger processing capabilities compared to ordinary boards. Therefore, the Arduino Mega 2560 has been selected. It boasts up to 54 I/O pins and a more powerful processor to allow for better performance in multitasking operations [59].

- 28YBJ-48 stepper motor:

The 28YBJ-48 stepper motor is currently one of the most commonly used stepper motors on the market. It operates in two modes: half-step and full-step, corresponding to 4096 steps/rev and 2048 steps/rev respectively. This

high-resolution configuration allows the manipulator to perform high-precision operations. Additionally, at its rated voltage, it can provide a torque of up to 34Nm, which is more than sufficient for pulling the cables of a lightweight mechanical arm.

- ULN 2003A motor control chip:

The motor control chip acts as the brain of the motor, responsible for translating instructions from the Arduino board into operations that the motor can execute. The ULN2003A chip is specifically matched with the 28YBJ-48 motor, ensuring seamless coordination and operation between the chip and the motor.

- 9V power supply:

The rated operating voltage of the stepper motor is 5-12V. Here, a 9V power supply is chosen to allow losses due to component resistance.

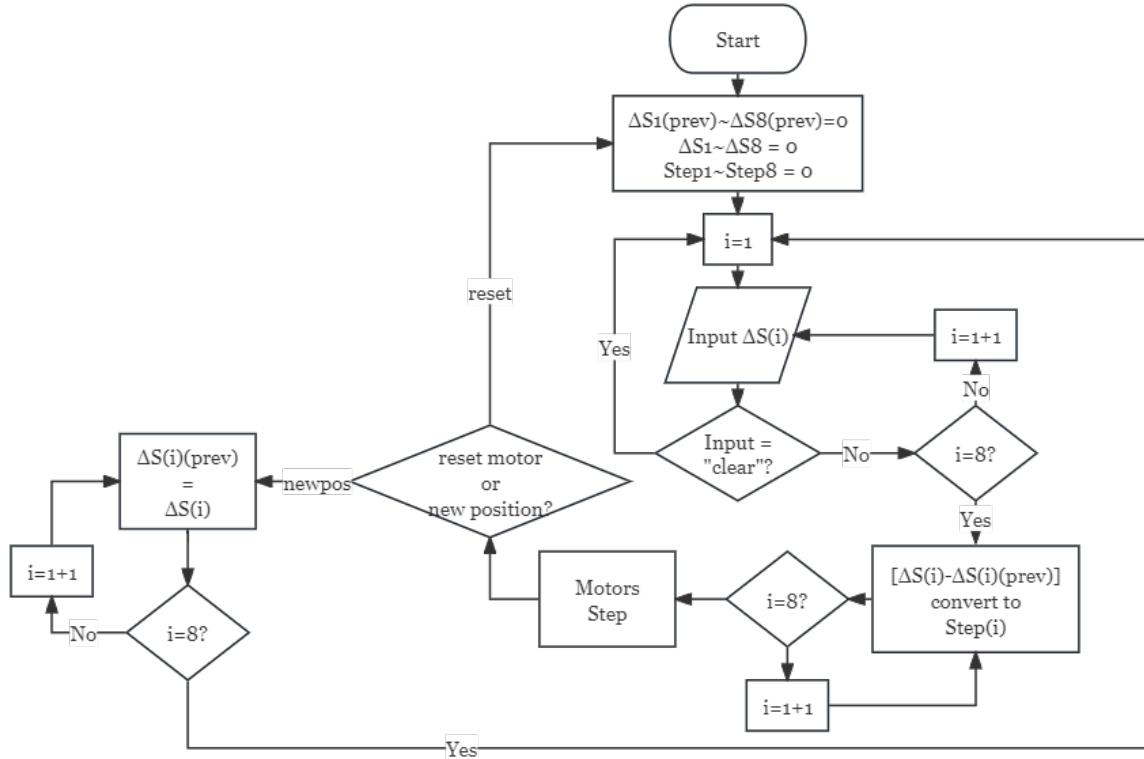
- Circuit Schematics

The schematic of the overall circuit for the manipulator actuation control part is shown in Figure C.1 in Appendix C.

The 8 motors use pin 23-52 on the Arduino Mega 2560 board, and are powered by a 9V battery. The power supply is in a parallel configuration on a breadboard. The Arduino board is connected to a serial monitor and a keyboard for state inspection and manual input.

- Programme development

The main logic framework of the programme has been illustrated by the flowchart shown in Figure 11.



**Figure 11: The flow chart of Arduino step motor control programme.**

After the programme starts, the first thing is to declare the variables.  $\Delta S_{i(\text{prev})}$ ,  $\Delta S_i$  and  $Step_i$  is all set to 0 initially. Then the programme asks the users to input 8 numbers representing the length change for 8 cables. The 8 values will be assigned to variable  $\Delta S_1 \dots \Delta S_8$  in order. During the input process, the user can input 'clear' instead of numbers to clear all the previous input and start over again from  $\Delta S_1$ .

Once the input process is complete, the programme converts the  $\Delta S_1 \dots \Delta S_8$  to  $Step_1 \dots Step_8$  based on Equation 6, with the  $d$  set to 10 mm, N set to 2048 steps/rev. The motors will start to step immediately after the conversion is finished.

During the stepping process, the motors controlling the first and second unit (the first 4 motors) are set to have a stepping speed of 300 steps/second, and the motors controlling the third and fourth unit (the last 4 motors) are set to have  $\frac{300}{17.5} \cdot (17.5 + 15) \approx 557$  steps/second. This is because the moving speed for the last two units should be  $\frac{17.5+15}{17.5}$  times faster than the first two units so that the 4 units can finish moving approximately at the same time.

The programme shall wait until the motors finished stepping. Once the stepping process is finished, the programme asks the user what to do next: reset the motors

to initial position or move from the current position to the next position. If the user chooses to reset the motors, all the motors rotate backwards for same steps. However, If the user chooses to let the motors move to a new position, the values for current  $\Delta S_1 \dots \Delta S_8$  will be stored to variable  $\Delta S_{1(prev)} \dots \Delta S_{8(prev)}$ . The programme then asks for a new set of  $\Delta S_1 \dots \Delta S_8$ , then calculate the new  $Step_1 \dots Step_8$ . Hence, the loop of the programme is closed.

### 3.5.2 Parameter Input by Serial Monitor

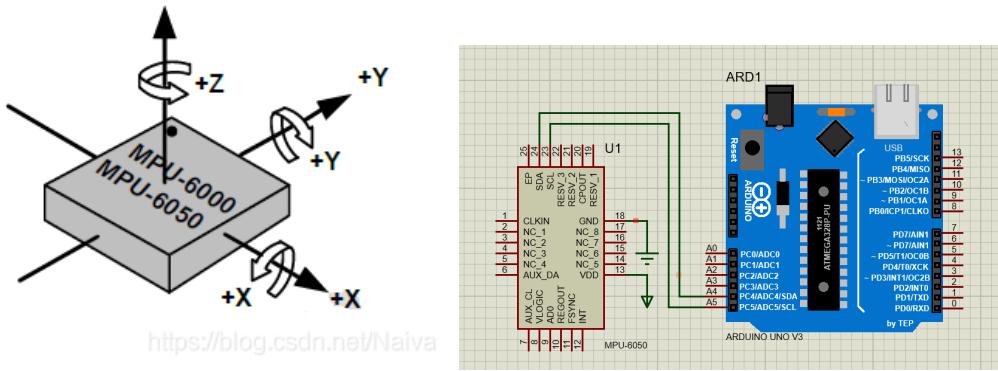
In the operating logic, the control parameters are inputted through the keyboard to control the motion of the multiple stepper motors. The basic function is to input the steps into the serial monitor through the keyboard of the host computer and the input string will be read by the microcontroller and converted into data, which are used as the number of steps of the stepper motor movement. For easy operation and display, the input parameters need to be displayed as text on the screen of the LCD IIC. LCD IIC will be in the form of a 2-line, 16-bit display to show more content.

The logic flowchart of the code to control the motor and LCD IIC by inputting parameters with the serial monitor of the host computer is shown in Figure C.2a in Appendix C.

### 3.5.3 Verification of MPU 6050

According to the project content, adding sensors to the end effector to measure its orientation angle and distance from the contact plane can assist the user to carry out more accurate operation. The function of detecting the orientation angle is realised by the MPU6050.

The Orientation angle (Euler angle) corresponding to the MPU6050 chip is shown in Figure 12a. Pitch is rotation around the X-axis. Yaw is rotation around the Y-axis. Roll is rotation around the Z-axis.



(a) The MPU6050 chip corresponding to the Euler angle  
(b) The wiring diagram of MPU6050 and microcontroller

**Figure 12: The MPU6050 chip with ancillary Arduino circuit.**

The MPU6050 requires an external 5V power supply and GND. SCL is the Clock Pin of the IIC when the MPU6050 is used as a slave, and SDA is the Data Pin of the IIC when the MPU6050 is used as a slave. The wiring diagram is shown in Figure 12b.

The logic flowchart of the code that controls the MPU 6050 to display acceleration and direction angle is shown in Figure C.2b in Appendix C. In this project, the acceleration parameters will not be acquired and displayed.

Since Proteus does not support dynamic simulation of the MPU6050, the verification of the MPU6050 has been carried out by physical parts and Arduino Mega while the project is in progress. The validation results have an acceptable accuracy and are able to display the corresponding orientation angle on the serial port and OLED screen.

### 3.5.4 Using HC-SR04 to measure distance

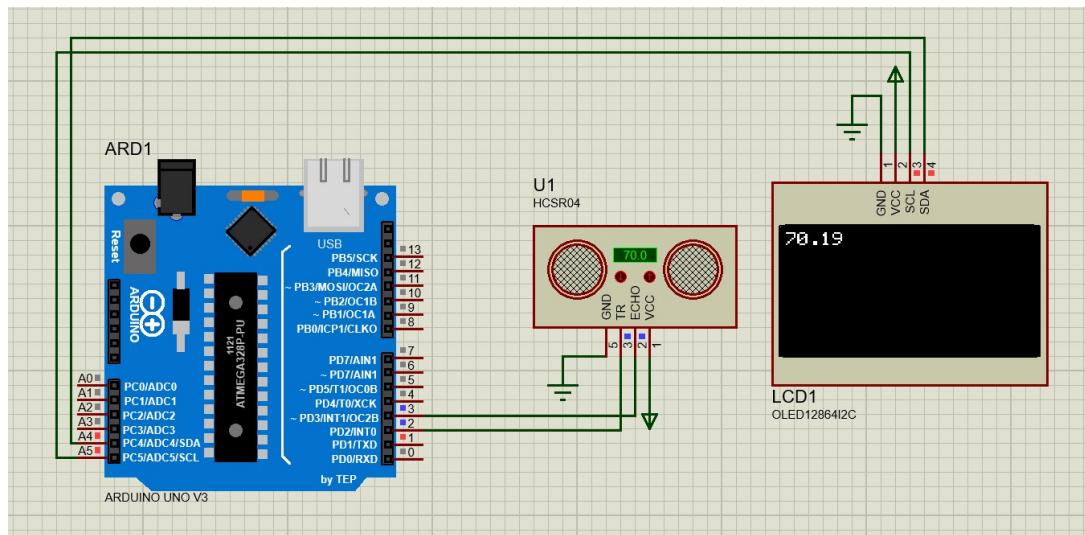
The HC-SR04 Ultrasonic Sensor uses sonar to determine the distance to an object. It has an ultrasonic transmitter and a receiver module which provides excellent non-contact range detection, high accuracy and stable readings. The measurement range is from 2 to 400 cm. The resolution is 0.3 cm and the measurement angle is within 30°. It operates in a process unaffected by sunlight or ferrous materials.

The sensor is triggered by sending a HIGH pulse of 10 microseconds. Previously, the speaker gives a short low-level pulse to ensure that a clean HIGH pulse is obtained. The basic principle of distance measurement is Time-of-Flight (ToF). The formula is as follows.

$$distance = (duration/2) \times v_{sound} \quad (7)$$

$$(v_{sound} = 343 \text{ m/s} = 1/29.1 \text{ cm/us})$$

The wiring diagram and simulation results are shown in Figure 13. An SSD 1306 OLED screen is used to display the data in the simulation. The logic flowchart of the code that controls the HC-SR04 to measure distance and display acceleration by OLED screen is shown in Figure C.3a in Appendix C.



**Figure 13: The wiring diagram and simulation of HC-SR04.**

### 3.5.5 SSD 1306 OLED

Using SSD 1306 OLED requires calling two libraries, `<Adafruit_SSD1306.h>` and `<Adafruit_GFX.h>`. `<Adafruit_SSD1306.h>` is a dedicated display library for SSD1306 OLED screens. `<Adafruit_GFX.h>` library is the common parent graphics library for LCD and OLED screens.

The resolution of SSD1306 is 128x64 pixel dot array, SCK is the clock pin, SDA is the data pin, and external power supply and GND are required. Figure C.3b in Appendix C is the logic flowchart of the code to control the OLED to display the text content.

## 4 Result and Discussion

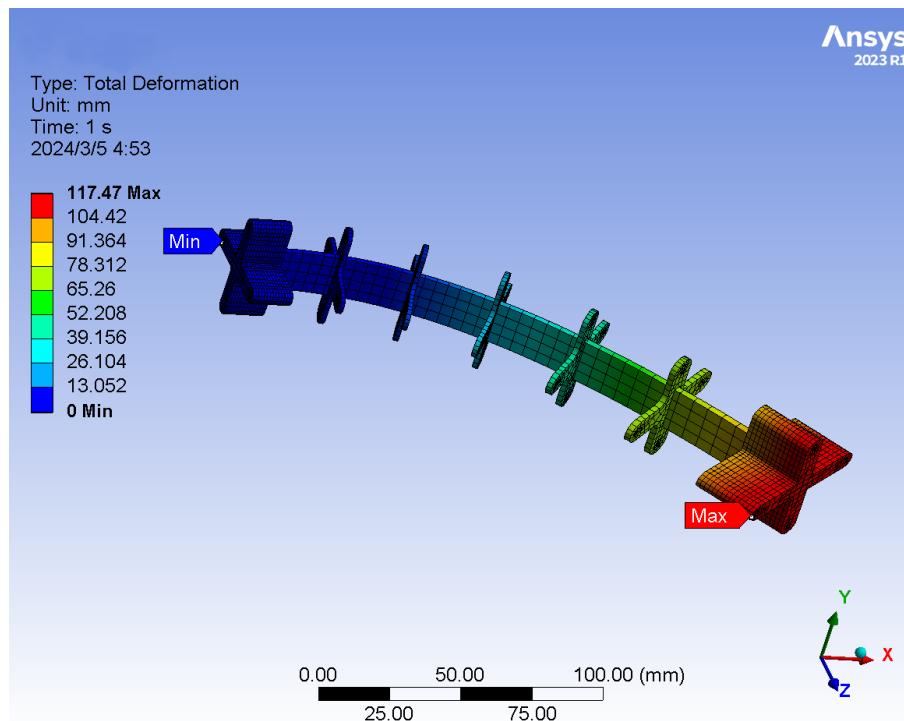
### 4.1 Strength Analysis

#### 4.1.1 The Result of Strength Analysis

The strength analysis conducted using ANSYS has yielded the following results.

- Displacement

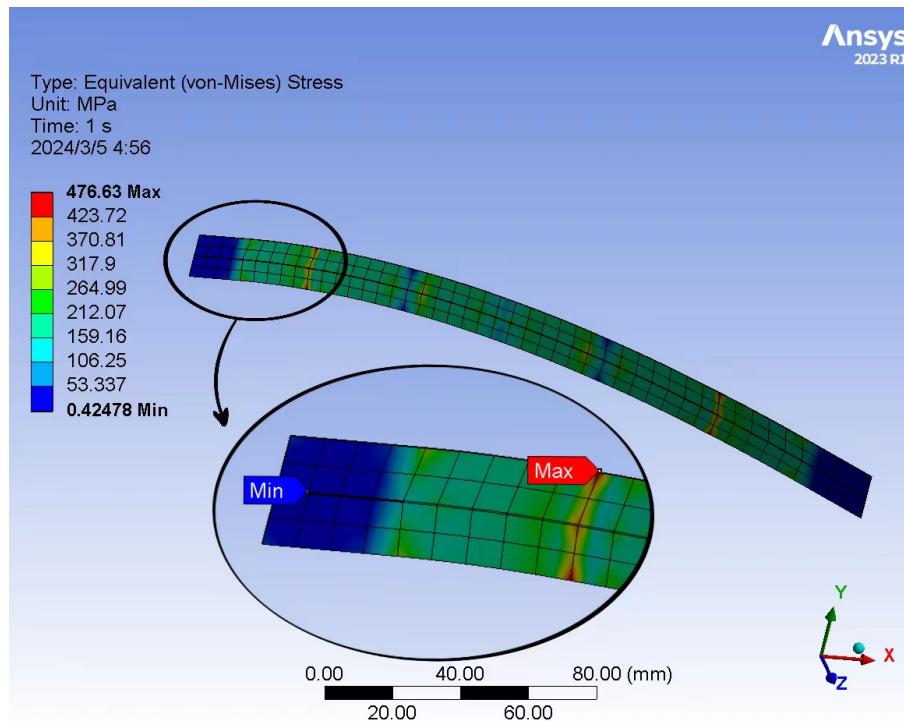
The maximum displacement observed in the manipulator was 117.44 mm, located at the end-effector, which is shown in Figure 14.



**Figure 14: The displacement analysis of manipulator.**

- Stress

As shown in Figure 15, the maximum stress was found to be 476.63 MPa, occurring at the base of elastic sheet where the manipulator experiences the most significant load.



**Figure 15: The stress analysis of manipulator.**

The results indicate that Maximum stress is concentrated on the elastic sheet of the manipulator near the connector, which suggests that this is the most likely location for failure under the current design and loading conditions. The maximum von Mises stress did not exceed the typical yield strength of 65Mn alloy steel, indicating that the arm would return to its original shape after the removal of the load and there is no potential risk of plastic deformation or failure. This enhances the arm's reliability and extends its operational lifespan.

Given that the material remains within its elastic limit, the design could potentially be optimized further to reduce weight or increase load capacity. Future work may explore these possibilities while maintaining the safety factors within the design criteria.

#### 4.1.2 Limitations and Solutions

- Simplification of the Model

The initial analysis simplified the complex structure of the manipulator by focusing on a single module, potentially overlooking critical interactions between various modules that contribute to the overall strength and performance of the system. This simplification, while beneficial for computational efficiency, may not accurately

reflect the integrated behavior of the manipulator in real-world scenarios where interactions between modules can significantly influence outcomes. *Solution:* Future analyses could incorporate the interactions between all modules to provide a more comprehensive understanding of the manipulator's behavior under various conditions. Such an approach will ensure a more accurate representation of the manipulator's behavior under various operational conditions, leading to more reliable performance predictions and informed design decisions.

- Failure Point Concentration

Stress concentration at the base of the elastic sheet indicates a potential failure point but does not account for fatigue or long-term wear. This oversight could underestimate the long-term durability of the manipulator, potentially leading to unexpected maintenance issues or failure.

*Solution:* To address this gap, incorporating a detailed fatigue analysis into future work is crucial. This analysis should evaluate the impact of repeated stress cycles on the manipulator's components over extended periods. By understanding these effects, design modifications can be implemented to enhance the durability and operational lifespan of the manipulator. The modifications could include optimizing material selection, redesigning stress-prone areas to reduce concentration points, and introducing preventive maintenance schedules based on predicted failure.

- Impact of cables

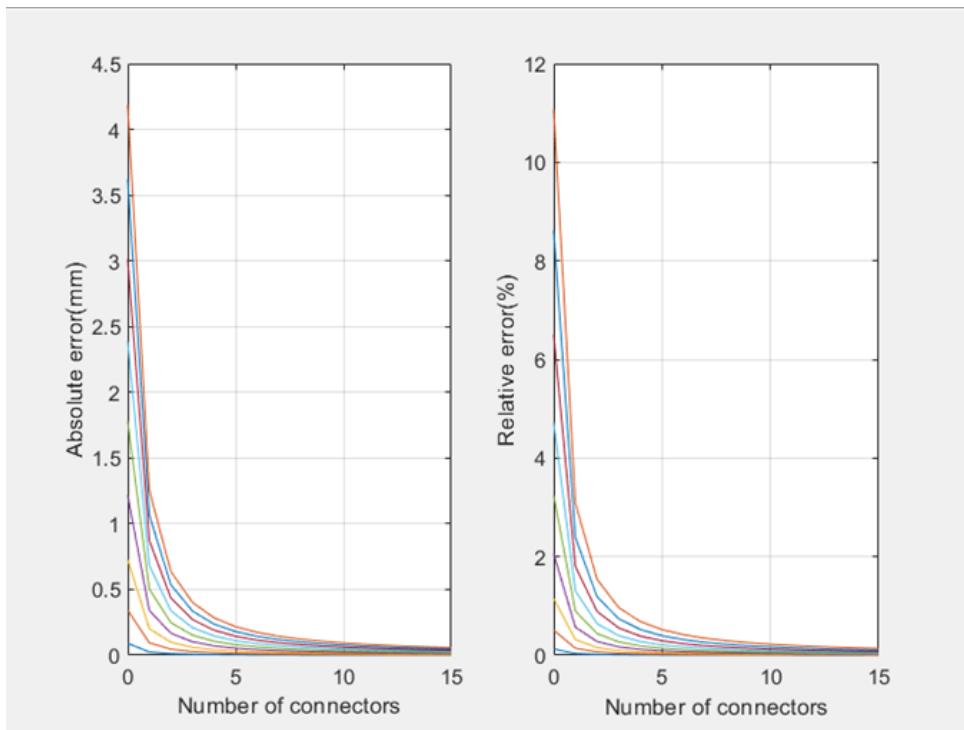
The analysis did not consider the impact of cable tension on the manipulator's strength. Cable tension can significantly affect the manipulator's mechanical behavior. Neglecting this factor may lead to an incomplete understanding of the manipulator's operational capabilities and potential failure modes.

*Solution:* Future analyses should integrate the effects of cable tension to achieve a more accurate assessment of the manipulator's performance. This could involve modifying the existing model to include boundary conditions that accurately represent the forces exerted by cables under various load scenarios. Implementing dynamic simulations that account for variations in tension forces during different stages of operation will further enhance the fidelity of the model. These improvements will enable a more nuanced understanding of the manipulator's

capabilities, assisting in the design of a more robust and reliable system capable of withstanding the complexities of real-world applications.

## 4.2 Error Analysis of Cross-Shaped Sheets

At the early stage of the project, given that the number of cross-shape sheets in each module have an influence on the maximum bending angle  $\alpha_{max}$ , which is an essential parameter of those which affect the workspace of the manipulator, hence the number of sheets should be determined by deriving the functions of both absolute error and relative error [48] against the number of sheets. The simulation results are shown in Figure 16.



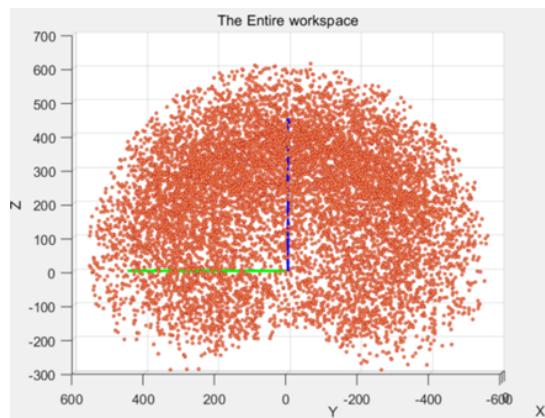
**Figure 16: The simulation results of errors against the number of sheets.**

As illustrated in Figure 16, the changing trends of both absolute and relative errors are almost identical since the relative error is proportional to the absolute error, and the different lines with different colors represent the data with different bending angles whose range is from 10 degrees to 90 degrees in both directions (positive or negative). It is obvious that both errors decrease with the increase in the number of connecting sheets, hence theoretically, the number of sheets should be as large as possible to make both errors could be negligible. However, the number of sheets also has an influence on the size of simulation space, more the number of sheets less the size of simulation space

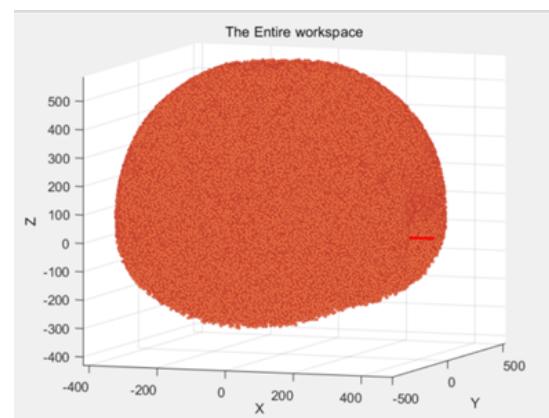
given that if the length of manipulator is much longer, the space near to the basement of manipulator could not be reached due to the characteristics of the material. Hence, the number of sheets could be assumed to be 10 rather than 15, even though both the errors are smaller when the number of sheets is 15.

### 4.3 Manipulator Workspace Analysis

The workspace simulation of the manipulator has been illustrated since it is important to determine the variables of joints based on the required workspace. The segmented workspace is affected by several aspects, which include the simulation index and the range within the three-dimensional coordinates. The effective workspace of the manipulator is a cubic whose dimension is 300x300x300 (mm). The specific workspace has a fixed range on the x and y axes from -150 to 150 mm. Therefore, it is only necessary to determine the range of the workspace along the z-axis. The parameter H, representing the height of the basement of specific workspace, is utilized to determine the range of the workspace along the z-axis. The impacts of different indices and parameter H will be discussed separately, aiming to segment the appropriate cubic workspace of the manipulator.



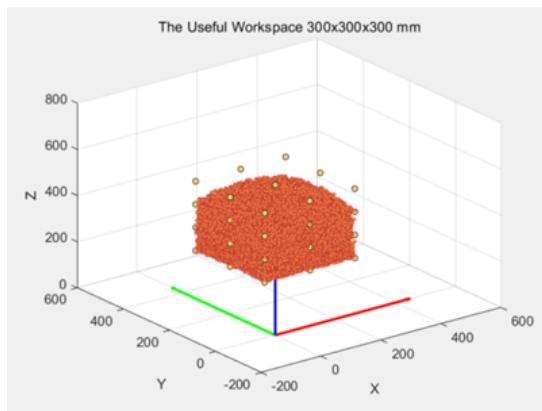
(a) The entire workspace where simulation index=10000



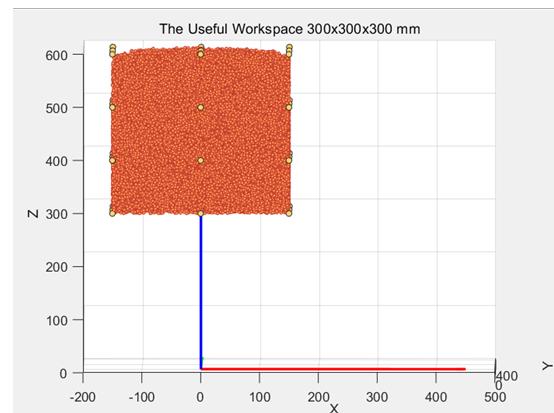
(b) The entire workspace where simulation index=1000000

**Figure 17: The entire workspace with different simulation indices.**

As shown in Figure 17a, the working space of manipulator forms a spherical shell-shaped point cloud consisting of 10000 floating points since the simulation index is 10000, which is insufficient to fully occupied the theoretical workspace. To further generalize the theoretical workspace, it is necessary to increase the simulation index. The appropriate workspace should be divided from the workspace whose index is 1000000.



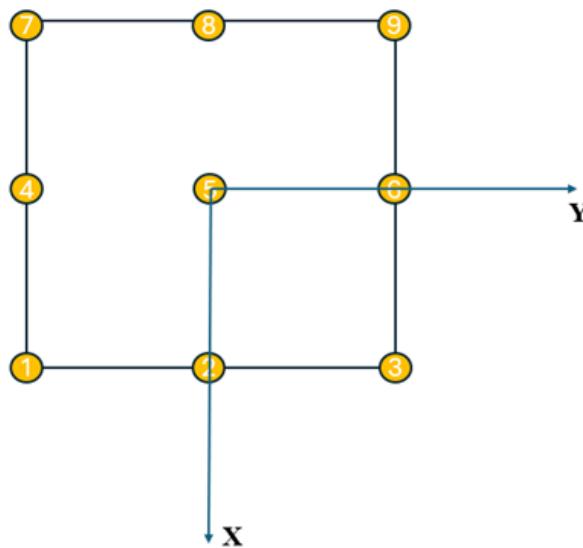
(a) The cubic workspace of manipulator with index = 1000000 H=350-650 mm



(b) index = 1000000 H=300-600 mm

**Figure 18: The cubic workspace with different segmentation.**

Comparing the simulation results illustrated in Figure 18, it can be observed that the upper area of the segmented cubic workspace whose H=350mm failed to be occupied. However, the segmented cubic workspace whose H=300mm is almost fully occupied. To determine threshold of parameter H, a series of yellow detection points are utilized to test whether specific locations in the workspace can be reached. A programme is utilized to calculate the distance between the detection points and the simulated point cloud and identify the minimum distances. The simulation results whose index=1000000 and H=300mm are shown in Figure B.6 in Appendix B. The top view of the cubic workspace in Figure 19 associates labels with detection points, facilitating subsequent explanations.



**Figure 19: The top view of the workspace and the labels of detection points.**

While H=300 mm, The heights of the first, second, third, and fourth layers are 300 mm, 400 mm, 500 mm, and 600 mm, respectively. As the second and third layers are situated in

the middle, theoretically all detection points on these two layers are reachable. Therefore, the mean and variance of distances to these points can be calculated. Combined with a normal distribution whose  $\mu = 0$ , a threshold is set to be 99.7% of the data are within 3 standard deviations of the mean can be derived to determine whether detection points on the first and fourth layers are reachable. The threshold  $\tau$  can be calculated in Equation 8.

$$\begin{aligned}\bar{d}_{2,3} &= \sum_{i=2}^3 \sum_{j=1}^9 d_{ij,min} \\ \sigma_{2,3} &= \sqrt{\frac{1}{n} \times \sum_{i=1}^n (d_i - \bar{d})^2} \\ \tau &= \bar{d}_{2,3} + 3 \times \sigma_{2,3}\end{aligned}\tag{8}$$

The segmentation of the effective workspace for other values of H, specifically H=240, 260, and 280 mm, can also be performed using the same approach. The numerical values corresponding to different H parameters are illustrated in Figures B.3, B.4, B.5 in Appendix B. The results and reachability for different H are presented in Table 3.

**Table 3:** The Reachability of Detection Points with Different H.

H (mm)	Layer	$\bar{d}_{2,3}$ (mm)	$\sigma_{2,3}$ (mm)	Threshold $\tau$ (mm)	Reachability
300	1	5.79	2.50	13.29	9
	4	5.79	2.50	13.29	3
280	1	4.87	2.02	10.94	9
	4	4.87	2.02	10.94	4
260	1	5.37	2.01	11.41	7
	4	5.37	2.01	11.41	7
240	1	5.35	2.48	12.78	8
	4	5.35	2.48	12.78	9

As shown in Table 3, setting H to 240 mm achieves better reachability, which is more reasonable compared to other scenarios. Additionally, to obtain a more accurate threshold, multiple tests or experimenting with a greater number of simulation indices can be undertaken to reduce random and systematic errors.

## 4.4 Inverse Kinematics

This section will demonstrate the posture solution of single modules and trajectory replication results obtained through the IK algorithm. An in-depth analysis of accuracy and computational speed will be conducted, along with a discussion of its advantages and limitations. Using angles as input for the programme testing was preferred due to the complexity involved in specifying 12 indices of  $\mathbf{O}_{target}$  and  $\mathbf{P}_{target}$ .

### 4.4.1 Posture Solution of Single Module Bending

Firstly, the bending of individual modules was tested as the target for the IK solution. The IK algorithm was tested for different angles of bending for Modules 1, 2, 3, and 4, and the specific results are presented in Table A.2 in Appendix A. The solutions of the Modules 3 and 4 were highly satisfactory, with Module 3 even converging to an error of 0.02 mm in 6 epochs.

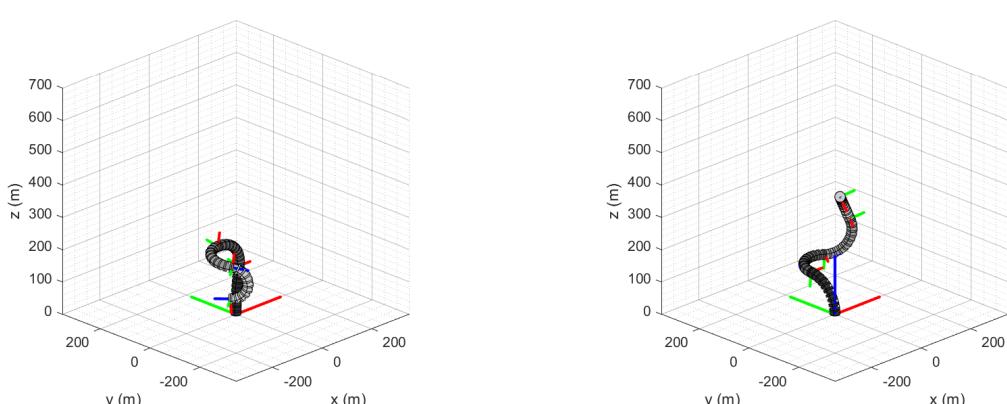
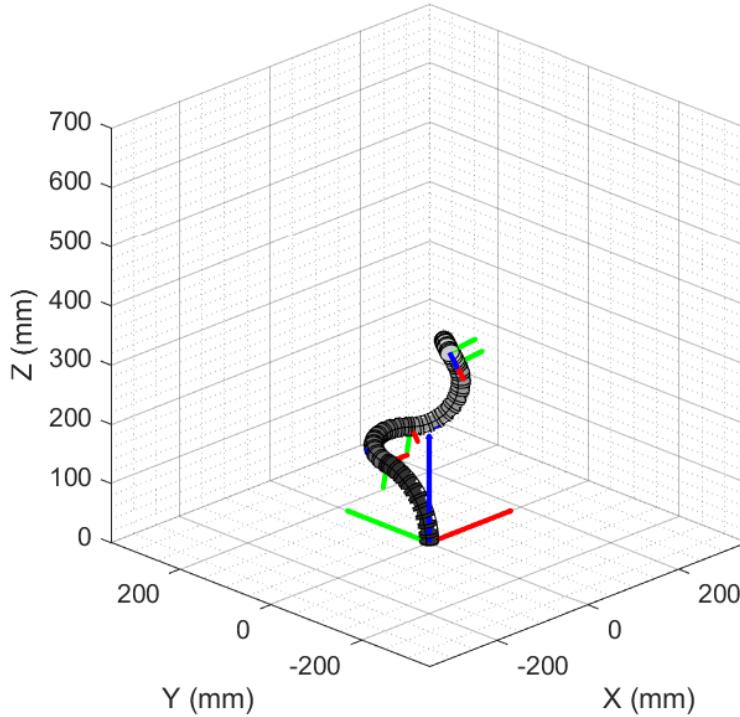
However, the results of the Modules 1 and 2 were unsatisfactory, which were caused by inappropriate initialization. According to the flowchart in Figure 10, the IK algorithm requires an initialization to start iterations. Take the example of the target with  $\alpha = [0, 90, 0, 0]^\circ$ , after initializing with the initial posture in Figure 8 and applying the IK algorithm for iterations, the first iteration yielded a posture with  $\theta = [-0.0, 104.04, -0.0, -22.0]^\circ$ . Moreover, since each iteration starts from Module 4, the error in Module 4 cannot be eliminated.

To mitigate this effect, selecting a more suitable posture for initialization is crucial. If the posture used for initialization with  $\alpha = [0, 100, 0, 0]^\circ$ , the solution of the IK algorithm would change to  $\theta = [-0.0, 81.77, -0.0, 9.3]^\circ$ . Nevertheless, due to errors generated by the iteration sequence, complete elimination remains challenging, and efforts were made to minimize the error to the greatest extent possible.

Afterwards, the complex bending angles of modules was utilized as the target to investigate the influences of initialization. The target with angles  $\alpha = [80, 120, -120, 90]^\circ$  was selected because its solution is complex, and it lies within the workspace of manipulator. The posture of the manipulator is shown in Figure 20a. This scenario is likely to occur in practical applications and necessitates resolution through relevant methods.

With the initialization using the initial posture, the solution was  $\theta = [-8.88, -89.99, -126.81, -169.73]^\circ$ , which is shown in Figure 20b. However, the

solution of IK algorithm was  $\theta = [84.98, 122.93, -114.9, 63.01]^\circ$ , which is significantly close to the target while the angles for initialization were  $[85, 125, -115, 55]^\circ$ . The solution of the IK algorithm with more appropriate initialization is shown in Figure 20c. This signifies that if the manipulator is presently in a posture similar to the target configuration, it can accurately determine the corresponding bending angles  $\theta$  through the IK algorithm. This proved to be highly beneficial for trajectory replication.



**(b)** initialization:  $\alpha = [0, 0, 0, 0]^\circ$ ;  
 $\theta = [-8.88, -89.99, -126.81, -169.73]^\circ$

**(c)** initialization:  $\alpha = [85, 125, -115, 55]^\circ$ ;  
 $\theta = [84.98, 122.93, -114.9, 63.01]^\circ$

**Figure 20: The inverse kinematics algorithm with different initialization.**

With the initialization using the initial posture, the solution was  $\theta = [-8.88, -89.99, -$

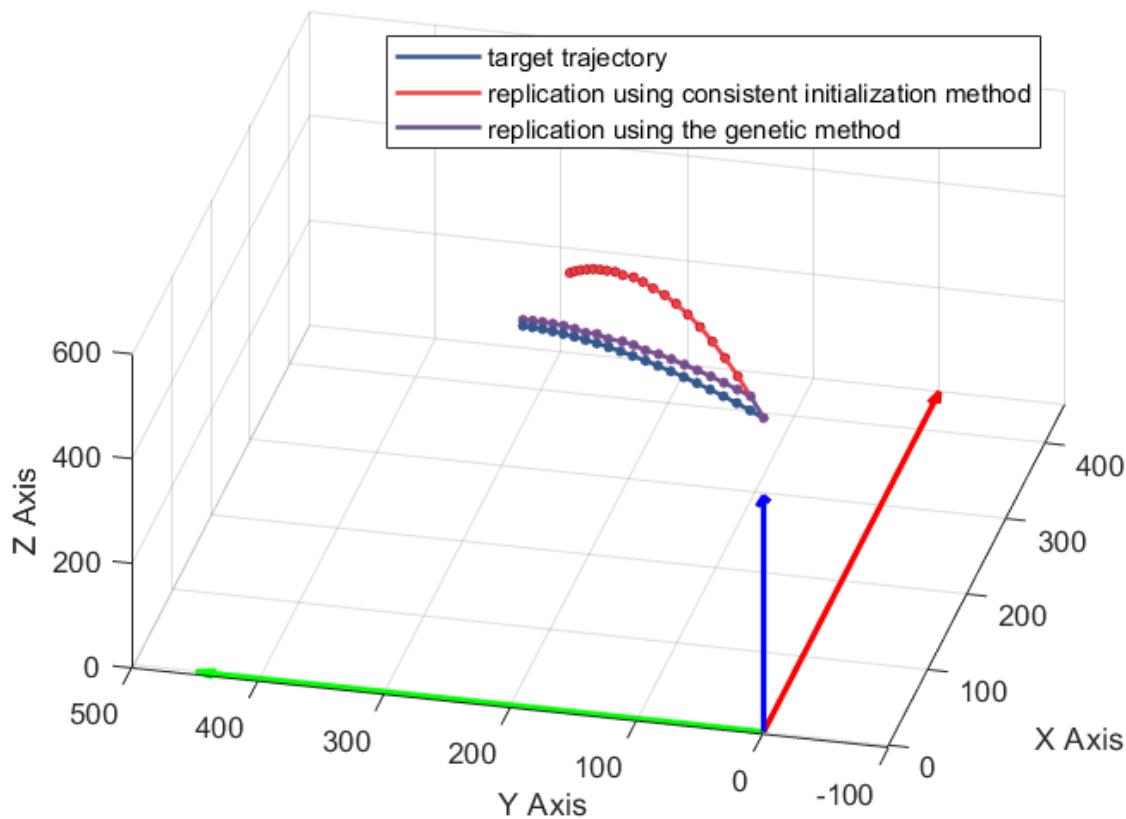
$126.81, -169.73]^\circ$ , which is shown in Figure 20b. However, the solution of IK algorithm was  $\theta = [84.98, 122.93, -114.9, 63.01]^\circ$ , which is significantly close to the target while the angles for initialization were  $[85, 125, -115, 55]^\circ$ . The solution of the IK algorithm with more appropriate initialization is shown in Figure 20c. This signifies that if the manipulator is presently in a posture similar to the target configuration, it can accurately determine the corresponding bending angles  $\theta$  through the IK algorithm. This proved to be highly beneficial for trajectory replication.

Ultimately, The efficient computational capability of algorithm is one of its strengths. The algorithm only took 1.905 seconds to complete 10,000 epochs updating. In practical applications, it requires only 200 epochs to determine convergence. In comparison to traditional methods like inverse Jacobian [60], which involve matrix transformations and derivatives, this approach provides potential solutions in a short time, addressing the issue of singular points. However, this method has its limitations. In scenarios with multiple segments, the algorithm may introduce significant errors due to the iteration order, and these errors were difficult to be eliminated. The only viable solution is to enhance the algorithm's performance through suitable initialization methods.

#### 4.4.2 Trajectory Replication

The preceding discussion has highlighted genetic method, which is utilizing the current posture for initialization in trajectory replication. In this phase, a comparison of different initializations aimed to highlight the benefits of incorporating genetic method in trajectory replication and planning for the IK algorithm. Subsequently, the genetic method and IK algorithm are employed to replicate two types of trajectories: arc segments and closed paths. This section will analyze the genetic method and these two trajectories, elucidating the strengths and weaknesses of inverse kinematics algorithm.

The blue trajectory in Figure 21 is the target trajectory, which bending from  $\alpha = [0, 0, 0, 0]$  to  $\alpha = [20, 20, 20, 20]$  equally in 20 steps. The red and purple trajectories are the replications using consistent initialization method and the genetic initialization method, respectively. The trajectory replication using the genetic method was comparatively better aligned with the target. The corrective effectiveness of the genetic algorithm was validated for other targets. The data of trajectories in Figure 21 is listed in Table A.3.

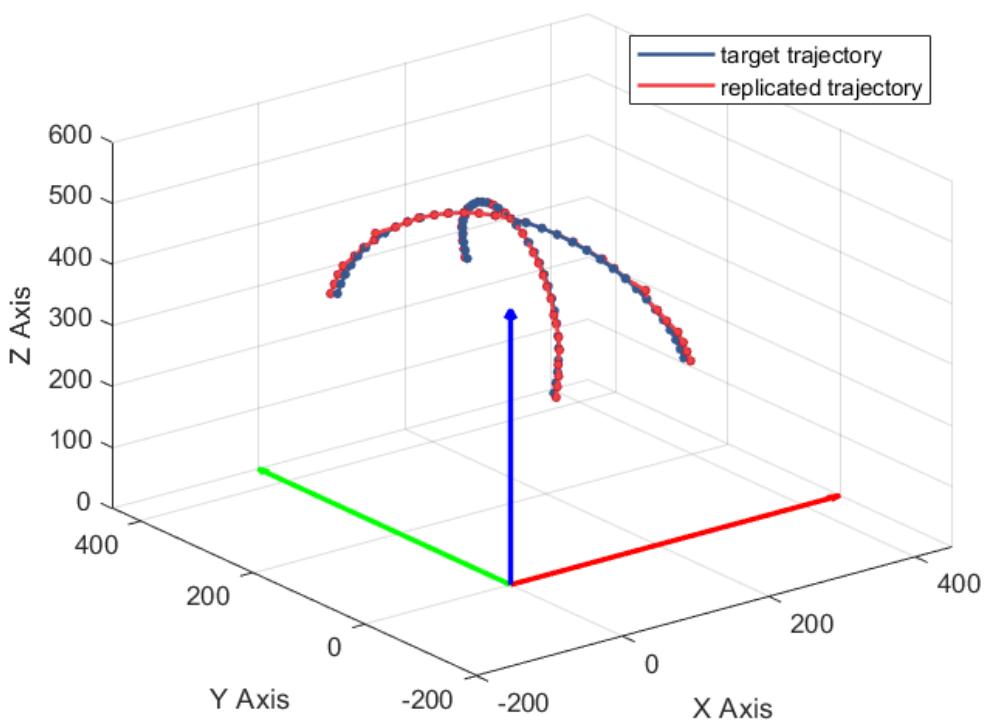


**Figure 21: The trajectory replications with and without genetic method.**

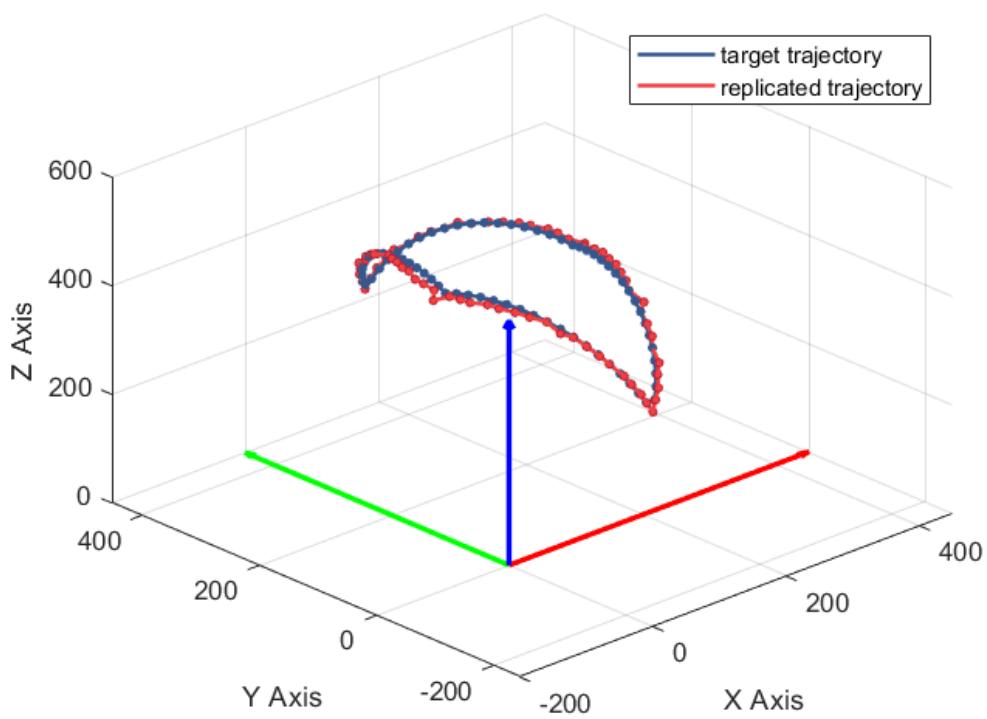
The replicated trajectories of both target trajectories closely align, but there exists a certain degree of error. This discrepancy was attributed to the inclusion of the error threshold  $\epsilon$  set at 0.02 in the programme, allowing for computational efficiency while maintaining accuracy. However, when dealing with significantly larger angular deviations, instances may arise where the replicated trajectories failed to precisely match the target trajectories. This situation was overlooked as it fell beyond the defined workspace. The average errors about the trajectory replication were calculated in Table 4.

**Table 4:** The Average Errors of the replicated trajectories.

Replicated Trajectory	$e_{average}$ (mm)	$e_{average}$ (mm)
<b>total</b>	3.576157	6.415941
<b>in workspace</b>	1.032255	2.642474



**Figure 22: The cross-shaped trajectory and its replication.**



**Figure 23: The closed trajectory and its replication.**

#### 4.4.3 Limitation and Improvement

However, it is challenging to calculate certain postures with IK algorithm. A method has been proposed involving continuous posture updating through a factor  $f_u$ . Nevertheless, this approach lacks systematic validation and is currently feasible only in specific circumstances. Therefore, it is mentioned here solely as a potential solution.

In practical applications, the initialization can be started with a relatively similar posture. In this context, the target where  $\theta_{target} = [45, 45, -45, -45]^\circ$  and the initialization with  $\alpha_0 = [20, 20, -20, -20]^\circ$  were utilized to exemplify the corresponding calculations. The *target* is the position and orientation of the manipulator end effector.

The current posture with  $\alpha_0 = [20, 20, -20, -20]^\circ$  was used as an initialization for calculations through inverse kinematics algorithm, resulting in the solution  $\theta_1$ . The difference between  $\alpha_0$  and  $\theta_1$  was calculated by  $f_u = 0.5$  to acquire  $\alpha_1$ , which is shown in Equation 9.

$$\begin{aligned}\theta_1 &= \text{FABRIKc}(\text{target}, \alpha_0) = [42.38, 73.25, -69.59, -62.85]^\circ \\ \alpha_1 &= \alpha_0 + f_u \cdot (\theta_1 - \alpha_0) = [31.19, 46.63, -34.80, -41.43]^\circ\end{aligned}\quad (9)$$

The  $\alpha_i$  can be continuously updated according to the method, which the results yield in Equations 10, 11, 12, 13, and 14.

$$\begin{aligned}\theta_2 &= \text{FABRIKc}(\text{target}, \alpha_1) = [56.95, 34.48, -56.40, -38.96]^\circ \\ \alpha_2 &= \alpha_1 + f_u \cdot (\theta_2 - \alpha_1) = [44.07, 40.56, -45.60, -40.20]^\circ\end{aligned}\quad (10)$$

$$\begin{aligned}\theta_3 &= \text{FABRIKc}(\text{target}, \alpha_2) = [43.64, 51.49, -45.97, -48.84]^\circ \\ \alpha_3 &= \alpha_2 + f_u \cdot (\theta_3 - \alpha_2) = [43.56, 46.03, -45.79, -44.52]^\circ\end{aligned}\quad (11)$$

$$\begin{aligned}\theta_4 &= \text{FABRIKc}(\text{target}, \alpha_3) = [46.22, 42.71, -45.9, -43.88]^\circ \\ \alpha_4 &= \alpha_3 + f_u \cdot (\theta_4 - \alpha_3) = [44.89, 44.37, -45.85, -44.20]^\circ\end{aligned}\quad (12)$$

$$\begin{aligned}\theta_5 &= \text{FABRIKc}(\text{target}, \alpha_4) = [44.87, 45.23, -45.03, -45.30]^\circ \\ \alpha_5 &= \alpha_4 + f_u \cdot (\theta_5 - \alpha_4) = [44.88, 44.80, -45.85, -44.20]^\circ\end{aligned}\quad (13)$$

$$\begin{aligned}\theta_6 &= \text{FABRIKc}(\text{target}, \alpha_5) = [45.08, 44.91, -45.09, -44.96]^\circ \\ \alpha_6 &= \alpha_5 + f_u \cdot (\theta_6 - \alpha_5) = [44.94, 44.86, -45.47, -44.58]^\circ\end{aligned}\quad (14)$$

In addition, another approach involves manually adjusting the manipulator's posture through observation, which is relatively straightforward but lacks precision. It is important to note that due to orientation constraints, the sum of angles for modules 1 and 3 and modules 2 and 4 should remain constant during the adjustment process.

## 4.5 Electronic Control

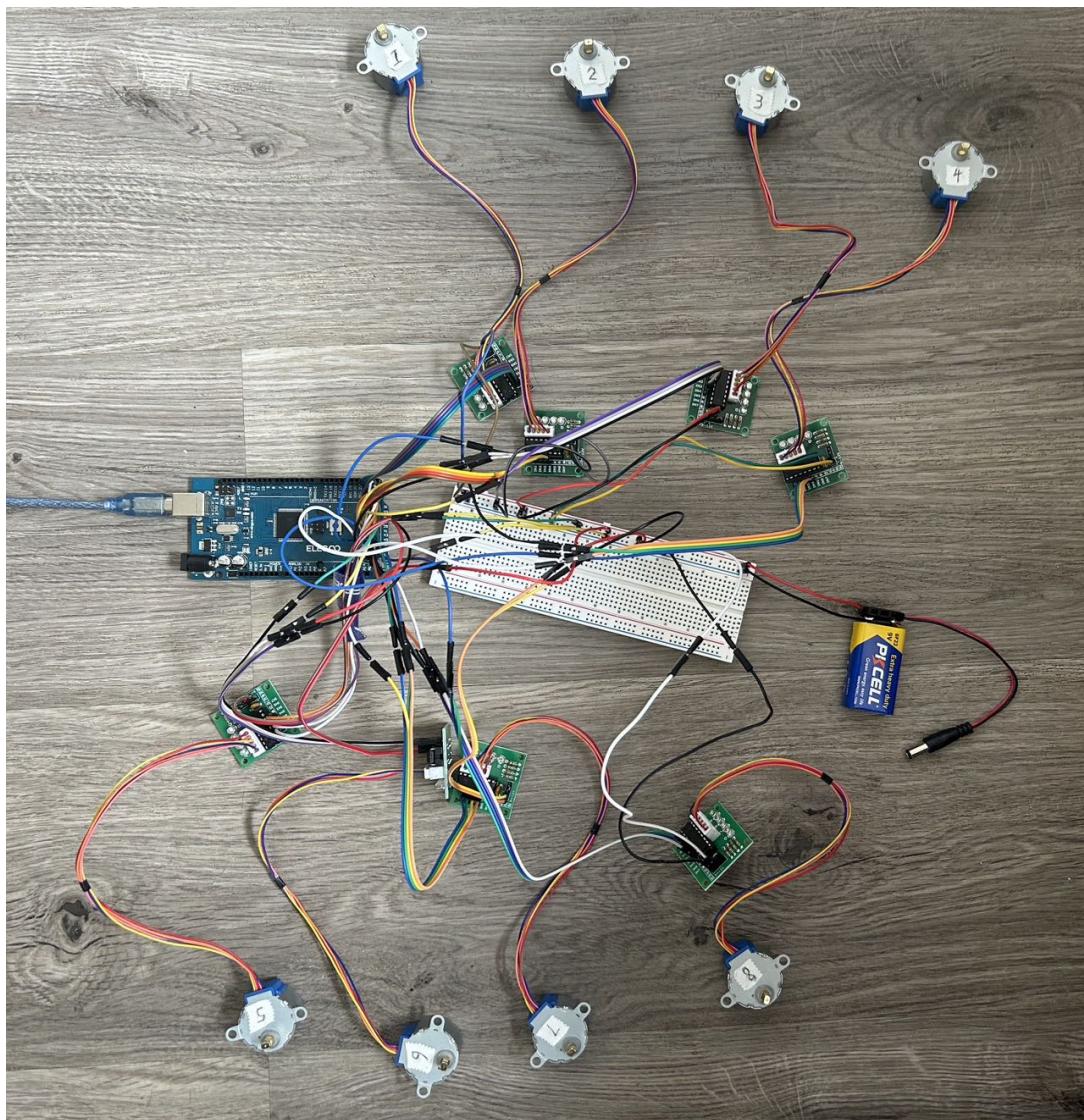
### 4.5.1 Actuation Control

Initially, the testing of the results was supposed to be carried out through Proteus simulation. However, it was discovered that during the simulation process in Proteus, the motor components did not function normally, with frequent occurrences of stepping errors. Meanwhile, the simulation could not be completed in real-time due to the heavy load on the computer CPU. After consideration, the team members purchased a complete set of real components for testing. In the real circuit, the pin that each motor occupies are listed in Table 5 to reduce the complexity of wiring.

**Table 5:** The Pin Assignment of Stepper Motors.

Stepper Motor	1	2	3	4	5	6	7	8
Pin	22	30	38	46	23	31	39	47
Pin	24	32	40	48	25	33	41	49
Pin	26	34	42	50	27	35	43	51
Pin	28	36	44	52	29	37	45	53

After connecting the Arduino to the stepper motors as depicted in Figure 24, the programme is ready for execution.



**Figure 24: The circuit connection of Arduino and stepper motors.**

Users are required to sequentially input eight values of  $\Delta S_1 \sim \Delta S_8$  to initiate the corresponding operation of the stepper motor. If the input is less than eight, the control system will not be executed.

According to Figure B.2 in Appendix B, the change volume of cables are input into the Arduino program for execution, with the input data series being  $\Delta S = [0, 0, -8, 8, -15, 15, 65, -65]$  (mm). The value of  $\Delta S_i$  is always in pairs, e.g, two cables controlling the same unit should always have the opposite  $\Delta S_i$  value, this is decided by the working principle of the manipulator. After conversion, the corresponding  $Step_1 \sim Step_8$  is  $[0, 0, -521, 521, -978, 978, 4237, -4237]$  (steps), where the “-” sign represents counterclockwise steps. Afterwards, the stepping can be finished in certain time. The function `Serial.print(motor.currentPosition())` can be utilized to check the steps of motors.

According to Figure B.1a in Appendix B, result of first testing is demonstrated, with all the motors are working appropriately. In this programme, all floating-point precision is taken to the  $10^{-4}$  millimeter while the result printed on the monitor is  $10^{-2}$  millimeter, which means that the accuracy of the result is acceptable.

Then, reset the motors, the motors went back to initial condition, which is  $Step_1 \sim Step_8 = 0$ . Repeat the motor stepping process, but this time, the motors are starting from a previous location from  $\Delta S_{now} = [0, 0, -8, 8, -15, 15, 65, -65]$  (mm) to  $\Delta S_{next} = [33, -33, 16, -16, 0, 0, 10, -10]$  (mm). The difference  $\Delta S_{diff} = [33, -33, 24, -24, 15, -15, -55, 55]$  (mm), while the steps for motors are [2151, -2151, 1565, -1565, 978, -978, -3585, 3585] (steps). The result can be verified by checking how many revolutions each motor rotated in Figure B.1b in Appendix B. The results are aligned with the calculation. Hence, the actuation control system runs correctly. The Arduino programme is updated in GitHub repository in Arduino Simulation.

Overall, the method of using Arduino to control the stepper motor and drive the manipulator is a feasible solution. Although the initial idea of using simulation software failed to yield satisfying results, the testing outcomes with real components solved this problem.

Additionally, from the perspective of test results, it can be observed that the precision of the stepper motors are very high, which is a great advantage for an open-loop (no feedback) system as it minimizes errors to the greatest extent. In the 28YBJ-48 motor parameter settings, full stepping configuration implies that the motor is divided into 2048 steps per revolution. For a rotor driving a cable with a circumference of 31.416 mm, this means a resolution of 0.01534 mm per step, which is enough to provide the accuracy required.

However, at the same time, there are some limitations to consider in this aspect. The most apparent limitation is that when cables wrap around the rotor, it effectively increases the diameter of the rotor, leading to some errors. For a cable with a total length of 68 mm, it can wrap around the rotor twice. Assuming the cable has a thickness of 1 mm, this would introduce a maximum error of approximately  $31.416/(31.416+2) = 0.05$ . Although in practice, it is unlikely that two layers of cable would perfectly stack on top of each other, this should be considered, because it's a factor that will cause errors. To solve this, an equation relating the length change of cables and the change in diameters should be deduced and applied in the program to minimize the potential error, but due to limited time, this part was

not finished.

Furthermore, in this programme, only a rough setting for the step speed of the motors has been set. However, for achieving synchronization in starting/ending movement of each unit of the manipulator, more precise mathematical derivations for the speed of motors are required. In the following development process, it would be helpful to associate the speed of stepper motors with the distance to the target position, thereby dynamically setting a speed value for each motor to accomplish this synchronization.

Ultimately, the input parameters of the control system are  $\Delta S$ , which consist of eight indices. It is inconvenient for users to input a series of indices. The Arduino programme can be optimized to take the angles  $\theta$  as input parameters, which only require the users to input four parameters. The Python version programme is uploaded in the GitHub and ready to be converted into Arduino programme.

#### 4.5.2 Information Acquisition and Display

In the design of the sensing and control section, the following functionalities have been implemented.

- Keyboard Input Control for Stepper Motor

The system allows users to input commands via an upper computer keyboard to drive the stepper motor by specifying the number of steps. The motor parameters are displayed on an LCD screen.

- MPU6050 Sensor for End-Effector Orientation

The MPU6050 sensor collects the orientation angles of the manipulator's end effector. The real-time data is displayed on an OLED screen to assist users. The accuracy is controlled within 0.1°, meeting precision requirements. However, prolonged sensor operation leads to increased internal chip temperature, resulting in noticeable noise and yaw zero-point drift issues.

- HC-SR04 Ultrasonic Sensor for Distance Measurement

The HC-SR04 ultrasonic sensor measures the distance from the manipulator's end effector to the target object in the polar coordinate system. The collected data is updated in real-time on the OLED screen to aid user operations. The accuracy is effectively controlled within 3 mm, meeting precision requirements.

Due to the lack of a proper spatial coordinate detection system for the manipulator's end effector, a closed-loop system has not been established in this project. The primary improvement task is to construct a closed-loop system, with the core technology being the determination of the end effector's three-dimensional coordinates. Feasible solutions include:

- Integration of Acceleration Data from MPU6050

The MPU6050 sensor detects acceleration along three axes. Integrating acceleration twice over time theoretically yields displacement along these axes, which can further be used to derive spatial position coordinates. However, this approach has limitations. Despite compensating for errors and drift, the results from double integration will rapidly accumulate over time. Thus, relying solely on MPU6050 cannot provide accurate coordinate information for precise feedback in a closed-loop system.

- Adding a Two-Degree-of-Freedom Gimbal at the End-Effector

By attaching a gimbal with two degrees of freedom to the end-effector, the ultrasonic sensor can rotate vertically to face three mutually perpendicular walls in the operating environment. The gimbal's motor angles will be derived based on the MPU6050's orientation angles, ensuring that the ultrasonic sensor measures distances along the coordinate axes. However, this approach assumes a workspace shaped like a rectangular prism with at least three perpendicular solid walls.

- Visual Processing Using Computer Vision

Employing visual processing techniques, such as using cameras to track the sensor's end effector, can derive distances and further calculate coordinates based on phase focusing algorithms.

Among the three methods, the second approach has relatively low costs and minimal implementation difficulty. Once the end effector's coordinates are obtained, combining them with the MPU6050's orientation data allows for closed-loop control feedback. Additionally, considering that the MPU6050 performs best in low-temperature environments, measures such as chip cooling are necessary.

## 5 Evaluation

### 5.1 Sustainability

Sustainability in the context of compact continuum robotics manipulators, particularly those used in therapeutic ultrasound applications, encapsulates not only the environmental impact of manufacturing and using such devices but also their role in promoting sustainable healthcare practices. The novel design and application of continuum robots, as discussed in the article, offer significant advancements in minimally invasive surgeries and therapeutic treatments. These robots' high precision, flexibility, and ability to navigate complex anatomical structures reduce the need for large, invasive surgical interventions, leading to shorter hospital stays and faster recovery times for patients. This directly contributes to reducing the overall environmental footprint of healthcare by minimizing energy consumption, medical waste, and the use of disposable medical supplies.

Moreover, the materials selection and manufacturing methods for these robots focus on durability and efficiency. Utilizing advanced fabrication techniques such as multi-material 3D printing not only allows for the creation of more complex and tailored designs but also potentially reduces waste during the production process. The selection of materials that balance rigidity, flexibility, and durability contributes to the longevity of the devices, thereby reducing the need for frequent replacements and the associated environmental impact.

The research and development of continuum robots also open pathways for innovations in renewable energy and environmental monitoring applications, further extending their sustainability impact. For instance, their flexible and adaptable design could be employed in underwater environments for repairing or maintaining renewable energy installations, such as tidal or wind turbines, without the need for large and invasive machinery. Additionally, their capacity for precise operation in delicate environments makes them suitable for environmental monitoring and data collection, aiding in the conservation of ecosystems and biodiversity.

In conclusion, the integration of compact continuum robotics manipulators in therapeutic ultrasound and beyond holds promising potential for enhancing sustainability within healthcare and other sectors. By reducing the environmental impact of medical

procedures, improving the efficiency of renewable energy maintenance, and contributing to environmental conservation efforts, these innovative robotic systems embody a step forward in the pursuit of sustainable technology solutions.

## 6 Conclusion

This is the Conclusion of the final report.

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# Appendix A

## The Appendix of Tables

**Table A.1:** The results of FABRIKc algorithm with target angle  $\alpha = [0 \ 0 \ 90 \ 0]$ .

Epoch	$\theta_1$ (°)	$\theta_2$ (°)	$\theta_3$ (°)	$\theta_4$ (°)	Error (mm)
1	6.71	-0.0	83.29	-0.0	33.77506
2	6.77	-0.0	83.23	-0.0	34.07841
3	6.50	-0.0	83.50	-0.0	32.69662
4	6.37	-0.0	83.63	-0.0	32.07427
5	6.32	-0.0	83.68	-0.0	31.79337
6	6.29	-0.0	83.71	-0.0	31.66917
7	5.94	-0.0	84.06	-0.0	29.88369
8	5.74	-0.0	84.26	-0.0	28.91057
9	5.66	-0.0	84.34	-0.0	28.4723
10	5.62	-0.0	84.38	-0.0	28.2791
11	5.6	-0.0	84.4	-0.0	28.19218
12	5.25	-0.0	84.75	-0.0	26.43303
13	5.1	-0.0	84.9	-0.0	25.64898
14	5.03	-0.0	84.97	-0.0	25.29275
15	5.0	-0.0	85.0	-0.0	25.13415
16	4.98	-0.0	85.02	-0.0	25.06414
17	4.63	-0.0	85.37	-0.0	23.31238
18	4.48	-0.0	85.52	-0.0	22.52919
19	4.41	-0.0	85.59	-0.0	22.17736

Continued on next page

**Table A.1 – continued from previous page**

<b>Epoch</b>	$\theta_1$ (°)	$\theta_2$ (°)	$\theta_3$ (°)	$\theta_4$ (°)	<b>Error (mm)</b>
20	4.38	-0.0	85.62	-0.0	22.01754
21	4.36	-0.0	85.64	-0.0	21.94852
22	4.01	-0.0	85.99	-0.0	20.19744
23	3.84	-0.0	86.16	-0.0	19.29379
24	3.76	-0.0	86.24	-0.0	18.8911
25	3.72	-0.0	86.28	-0.0	18.71167
26	3.7	-0.0	86.3	-0.0	18.63148
27	3.7	-0.0	86.3	-0.0	18.59747
28	3.35	-0.0	86.65	-0.0	16.86972
29	3.2	-0.0	86.8	-0.0	16.09738
30	3.13	-0.0	86.87	-0.0	15.7551
31	3.1	-0.0	86.9	-0.0	15.59885
32	3.09	-0.0	86.91	-0.0	15.53487
33	3.08	-0.0	86.92	-0.0	15.50114
34	3.08	-0.0	86.92	-0.0	15.48938
35	2.74	-0.0	87.26	-0.0	13.77029
36	2.59	-0.0	87.41	-0.0	13.00123
37	2.52	-0.0	87.48	-0.0	12.65954
38	2.47	-0.0	87.53	-0.0	12.43367
39	2.45	-0.0	87.55	-0.0	12.33092
40	2.44	-0.0	87.56	-0.0	12.28876
41	2.44	-0.0	87.56	-0.0	12.26754
42	2.44	-0.0	87.56	-0.0	12.25842
43	2.1	-0.0	87.9	-0.0	10.55236
44	1.95	-0.0	88.05	-0.0	9.79285
45	1.88	-0.0	88.12	-0.0	9.45857
46	1.85	-0.0	88.15	-0.0	9.3098
47	1.84	-0.0	88.16	-0.0	9.24442
48	1.83	-0.0	88.17	-0.0	9.21438
49	1.83	-0.0	88.17	-0.0	9.19955
50	1.83	-0.0	88.17	-0.0	9.19355

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**Table A.1 – continued from previous page**

<b>Epoch</b>	$\theta_1$ (°)	$\theta_2$ (°)	$\theta_3$ (°)	$\theta_4$ (°)	<b>Error (mm)</b>
51	1.49	-0.0	88.51	-0.0	7.48619
52	1.34	-0.0	88.66	-0.0	6.73267
53	1.27	-0.0	88.73	-0.0	6.39321
54	1.24	-0.0	88.76	-0.0	6.24293
55	1.23	-0.0	88.77	-0.0	6.17847
56	1.22	-0.0	88.78	-0.0	6.14882
57	1.22	-0.0	88.78	-0.0	6.13682
58	1.22	-0.0	88.78	-0.0	6.1309
59	1.22	-0.0	88.78	-0.0	6.12797
60	1.22	-0.0	88.78	-0.0	6.12521
61	0.88	-0.0	89.12	-0.0	4.42272
62	0.73	-0.0	89.27	-0.0	3.66632
63	0.66	-0.0	89.34	-0.0	3.31015
64	0.63	-0.0	89.37	-0.0	3.1559
65	0.61	-0.0	89.39	-0.0	3.08583
66	0.61	-0.0	89.39	-0.0	3.05386
67	0.61	-0.0	89.39	-0.0	3.03937
68	0.6	-0.0	89.4	-0.0	3.03352
69	0.6	-0.0	89.4	-0.0	3.03063
70	0.6	-0.0	89.4	-0.0	3.0305
71	0.6	-0.0	89.4	-0.0	3.03044
72	0.6	-0.0	89.4	-0.0	3.03041
73	0.6	-0.0	89.4	-0.0	3.0304
74	0.27	-0.0	89.73	-0.0	1.33635
75	0.12	-0.0	89.88	-0.0	0.59092
76	0.05	-0.0	89.95	-0.0	0.25938
77	0.02	-0.0	89.98	-0.0	0.11515
78	0.01	-0.0	89.99	-0.0	0.05169
79	0.0	-0.0	90.0	-0.0	0.02296

**Table A.2:** The Singular Posture Solution by FABRIKc start with initial posture.

$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	Converge Epoch	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	Error
0	0	0	90	None	-0.0	3.34	-0.0	89.09	18.6954
0	0	0	60	None	-0.0	1.61	-0.0	59.58	9.5398
0	0	0	30	None	-0.0	0.49	-0.0	29.67	2.77844
0	90	0	0	79	0.0	-0.0	90.0	-0.0	0.02296
0	60	0	0	36	0.01	-0.0	59.99	-0.0	0.03408
0	30	0	0	6	0.01	-0.0	29.99	-0.0	0.02928
0	0	90	0	None	-0.0	89.84	-0.0	0.28	0.66891
0	0	60	0	None	-0.0	76.97	-0.0	-22.73	79.01594
0	0	30	0	None	-0.0	52.28	-0.0	-22.87	114.34898
90	0	0	0	None	106.26	-0.0	-16.26	-0.0	80.98985
60	0	0	0	None	82.04	-0.0	-22.04	-0.0	111.99102
30	0	0	0	None	52.72	-0.0	-22.72	-0.0	117.37369

**Table A.3:** The Singular Posture Solution by FABRIKc start with initial posture.

Step	Target			Replication			Replication (Genetic)		
0	0.00	600.00	0.00	0.00	600.00	0.00	0.00	0.00	600.00
1	7.85	13.09	599.74	26.09	18.29	598.85	26.09	18.29	598.85
2	15.70	26.15	598.96	51.83	36.48	595.42	33.92	31.32	597.63
3	23.53	39.18	597.67	76.79	54.33	589.87	41.60	44.30	595.92
4	31.34	52.15	595.86	100.65	71.73	582.38	49.32	57.20	593.68
5	39.12	65.04	593.55	123.25	88.55	573.19	56.99	70.09	590.94
6	46.87	77.83	590.73	144.35	104.70	562.58	64.50	82.78	587.73
7	54.57	90.50	587.41	163.86	120.13	550.84	72.01	95.39	584.01
8	62.22	103.03	583.59	181.74	134.70	538.26	79.41	107.80	579.82
9	69.82	115.41	579.28	196.54	148.46	525.92	85.14	120.05	575.50
10	77.35	127.61	574.49	212.76	161.50	511.45	94.01	132.20	570.00
11	84.82	139.63	569.23	226.25	173.16	497.69	100.42	143.05	564.81
12	92.20	151.43	563.51	235.36	184.43	486.09	105.37	155.90	559.14

Continued on next page

**Table A.3 – continued from previous page**

<b>Step</b>	<b>Target</b>			<b>Replication</b>			<b>Replication (Genetic)</b>		
13	99.50	163.00	557.33	248.94	194.70	470.21	114.85	167.77	551.86
14	106.72	174.34	550.72	257.19	203.79	458.15	118.44	177.76	546.33
15	113.84	185.41	543.67	266.79	212.39	444.19	127.41	189.24	538.21
16	120.85	196.21	536.20	275.39	220.38	430.47	135.17	200.44	529.96
17	127.76	206.71	528.33	282.41	227.69	417.87	141.56	210.90	521.89
18	134.55	216.92	520.07	288.70	234.38	405.67	147.84	221.06	513.45
19	141.22	226.81	511.43	294.33	240.59	393.81	154.01	230.89	504.66
20	147.77	236.36	502.43	299.41	246.28	382.29	159.37	240.06	495.95

# **Appendix B**

## **The Code Display**

Movement Complete!

The steps motor 1 took is: 0, corresponding to a cable length change of: 0.00mm  
The steps motor 2 took is: 0, corresponding to a cable length change of: 0.00mm  
The steps motor 3 took is: -521, corresponding to a cable length change of: -8.00mm  
The steps motor 4 took is: 521, corresponding to a cable length change of: 8.00mm  
The steps motor 5 took is: -978, corresponding to a cable length change of:-15.00mm  
The steps motor 6 took is: 978, corresponding to a cable length change of:15.00mm  
The steps motor 7 took is: 4237, corresponding to a cable length change of:64.99mm  
The steps motor 8 took is: -4237, corresponding to a cable length change of:-64.99mm

**(a) the result of first stepping test**

Movement Complete!

The steps motor 1 took is: 2151, corresponding to a cable length change of: 33.00mm  
The steps motor 2 took is: -2151, corresponding to a cable length change of: -33.00  
The steps motor 3 took is: 1565, corresponding to a cable length change of: 24.00mm  
The steps motor 4 took is: -1565, corresponding to a cable length change of: -24.00mm  
The steps motor 5 took is: 978, corresponding to a cable length change of:15.00mm  
The steps motor 6 took is: -978, corresponding to a cable length change of:-15.00mm  
The steps motor 7 took is: -3585, corresponding to a cable length change of:-54.99mm  
The steps motor 8 took is: 3585, corresponding to a cable length change of:54.99mm

**(b) the result of second stepping test**

**Figure B.1: The Display of Arduino Control System.**

Welcome to Arduino control panel!  
Please input the 8 angles in order  
type 'clear' to re-input

---

Now, please input the 1st angle:

delta s1 is:0.00

Now, please input the 2nd delta s:

delta s2 is:0.00

Now, please input the 3rd delta s:

delta s3 is:-8.00

Now, please input the 4th delta s:

delta s4 is:8.00

Now, please input the 5th delta s:

delta s5 is:-15.00

Now, please input the 6th delta s:

delta s6 is:15.00

Now, please input the 7th delta:

delta s7 is:65.00

Now, please input the 8th delta s:

delta s8 is:-65.00

Input complete!

**Figure B.2: The input parameters for first stepping test.**

```

Distance to the nearest point:
Height:240 mm
    10.1916   5.9839   9.6846   8.9962   25.3592   5.7040   7.1835   6.2378   4.1283

Height:340 mm
    5.9085   3.2570   5.4114   4.1478   5.1970   2.1388   5.5338   4.1535   6.4524

Height:440 mm
    5.8770   2.5728  11.3366   4.0684   3.2286   2.8352   8.7876   5.1880  10.1459

Height:540 mm
    7.8421   8.8544  10.5260   8.6281   5.5724   7.2780   7.5941   6.2517  11.6773

```

**Figure B.3: The detection result while index=1000000 and H=240.**

```

Iteration: 1000000
Distance to the nearest point:
Height:260 mm
    7.5602   6.4772   7.0509   2.4059  19.3812   4.0904  11.5647   7.4664   6.8634

Height:360 mm
    4.0825   5.6066   5.5248   1.9642   3.8820   3.0379   7.8672   3.1782   7.8525

Height:460 mm
    7.2979   5.3467   2.2831   7.4993   3.6660   6.2619   7.4820   5.4839   8.3480

Height:560 mm
    8.1052  12.6313   6.9750   9.3641   3.6025   5.9004  17.1690   4.9512   7.4249

```

**Figure B.4: The detection result while index=1000000 and H=260.**

```

Distance to the nearest point:
Height:280 mm
    6.5362   4.0102   7.9380   2.6343  10.0980   4.1573   6.8283   3.2028   4.8241

Height:380 mm
    6.0912   1.5101   4.3438   4.4031   3.4151   2.6523   5.5229   2.3801   3.9658

Height:480 mm
    8.7091   5.0989   3.5441   8.3705   3.1214   8.0341   4.9664   4.7573   6.8580

Height:580 mm
    12.0959   7.0461  13.6910   7.1938   7.6483  11.4657  18.2672   8.7651  11.4104

```

**Figure B.5: The detection result while index=1000000 and H=280.**

```

Height:300 mm
    7.0696   3.3457   5.8420   5.1691   6.9859   9.6064  12.0161   3.1330   8.0543

Height:400 mm
    3.2065   5.6137   6.2471   1.9011   3.2564   6.4037   7.3970   5.9186   7.0688

Height:500 mm
    10.3989   3.3835   9.2886   7.4610   3.5080   5.3177   9.7733   6.1874   1.8141

Height:600 mm
    33.8095  13.3727  30.6118   6.0586   9.5076   8.8049  30.4824  16.8214  32.5093

```

**Figure B.6: The detection result while index=1000000 and H=300.**

## **Appendix C**

### **Figures for Illustration**

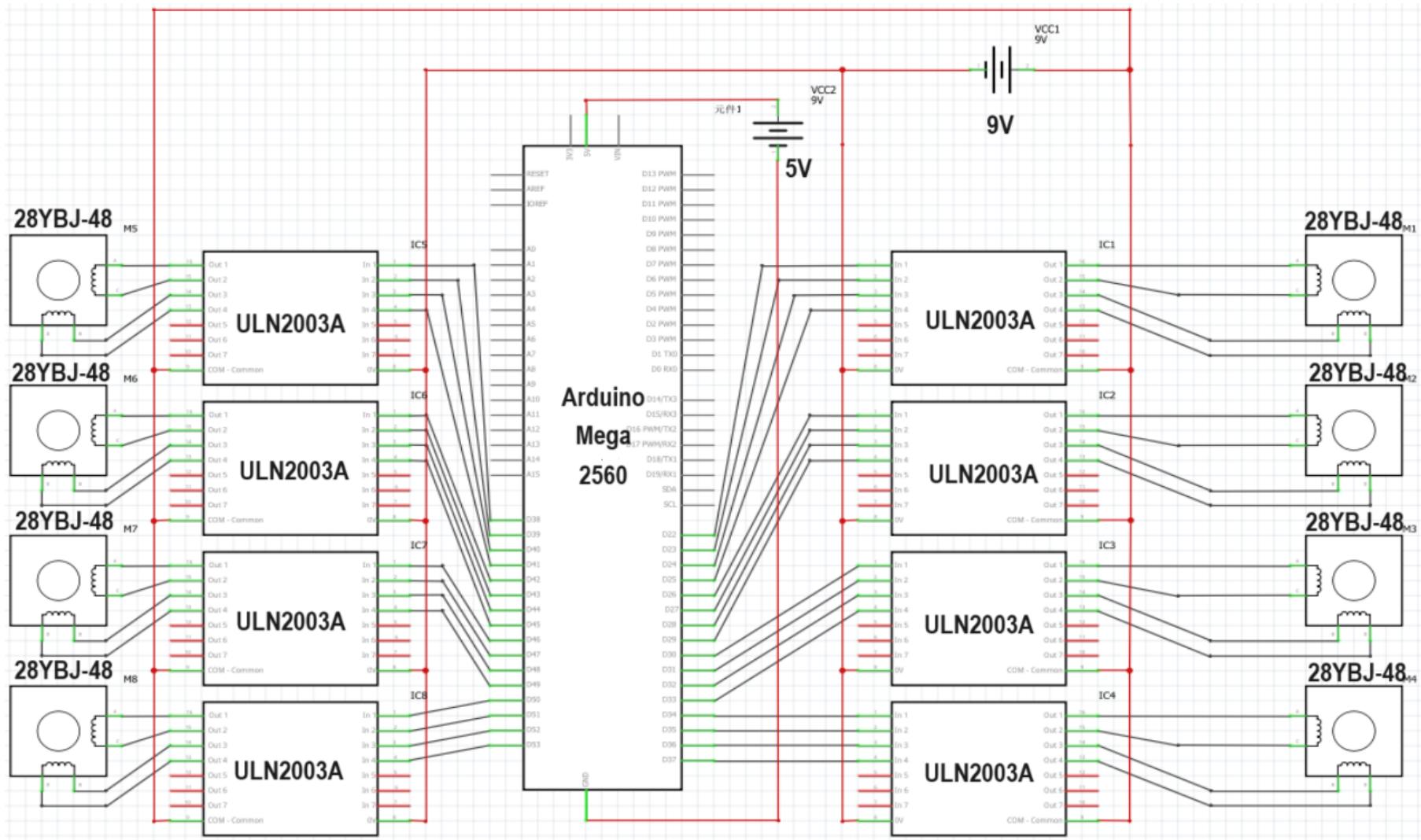
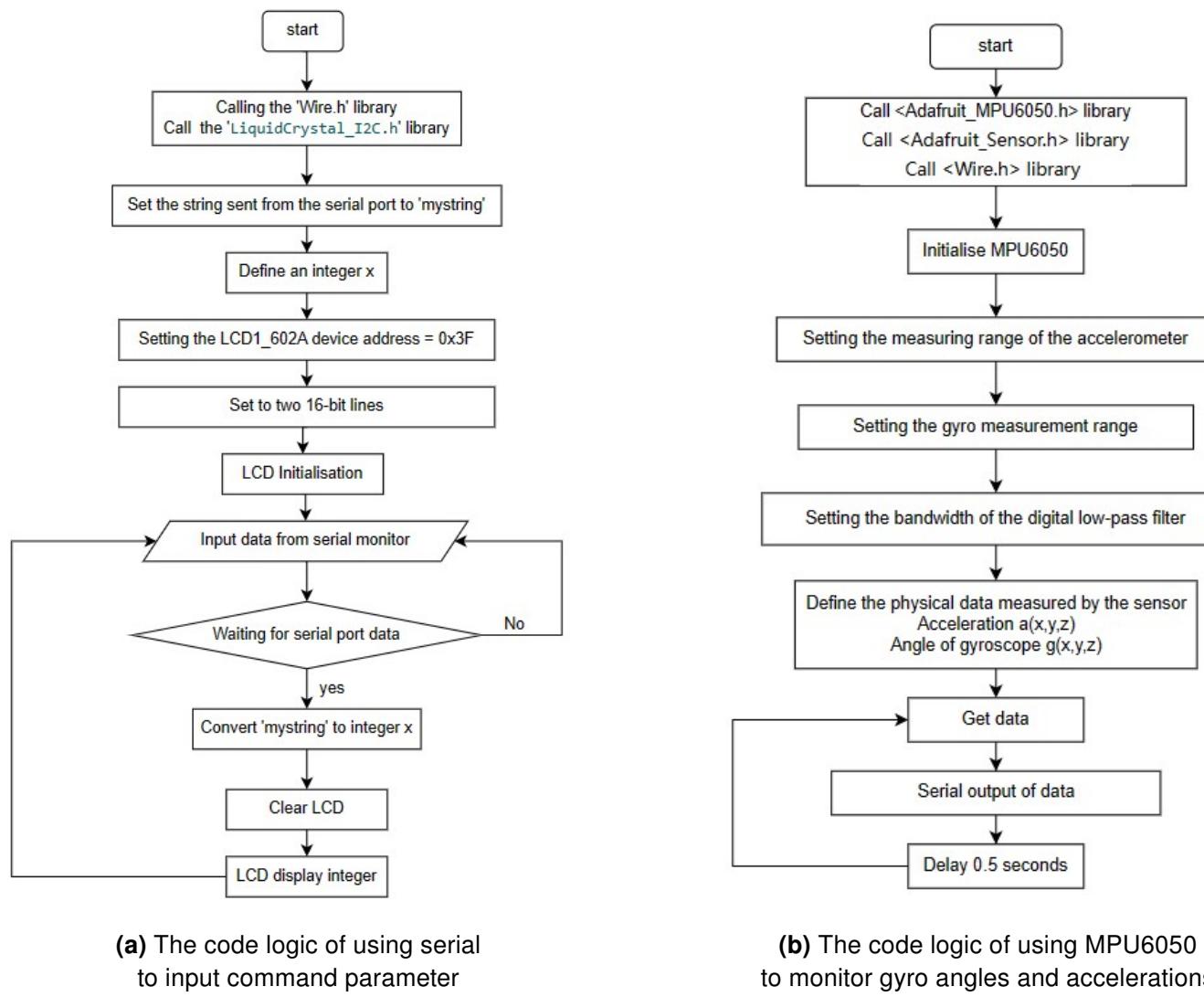
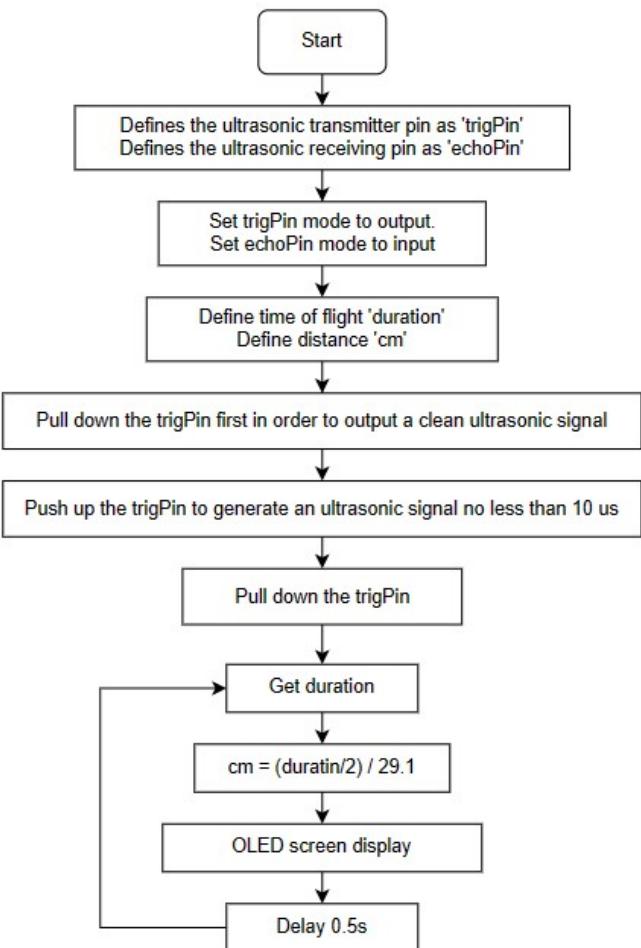


Figure C.1: The circuit layout of Arduino step motor control system.

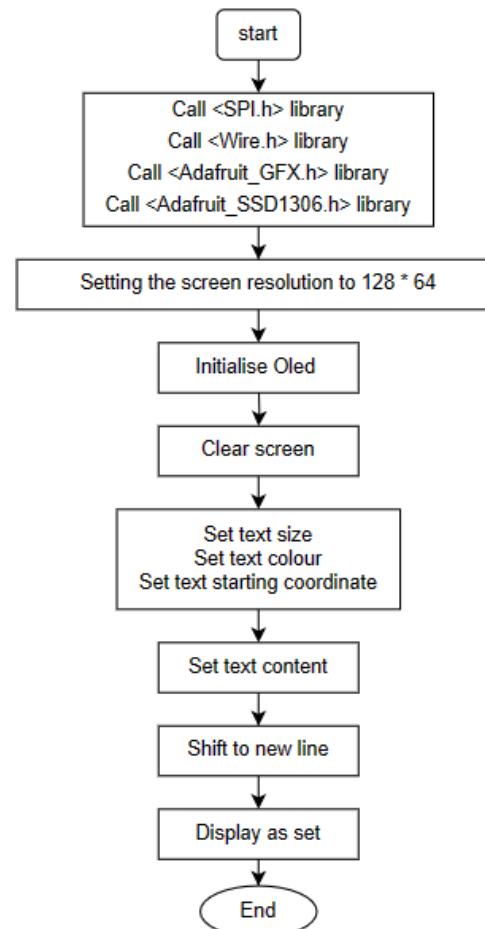


**Figure C.2: The code logic of input parameters and MPU6050.**

XX



(a) The code logic of using HC-SR04  
to measure distance



(b) The code logic of using SSD  
1306 OLED to display data

Figure C.3: The code logic of HC-SR04 and SSD 1306 OLED.

# Appendix D

## Equations and Derivations

### The Equations and Derivations of FK

- Module 1

For Module 1, the RPM  $\mathbf{P}_2^1$  with  $\alpha_1$  was shown in Equation D.1.  $R_1$  was the radius of the Module 1 bending curve. The rotational matrix  $\mathbf{R}_1$  and the APM of *node<sub>1</sub>*  $\mathbf{P}_1^{base}$  were shown in Equation D.2.

$$\mathbf{P}_2^1 = \begin{bmatrix} 0 \\ (R_1 \cdot (1 - \cos(\alpha_1)) + d_2 \cdot \sin(\alpha_1)) \\ (R_1 \cdot \sin(\alpha_1) + d_2 \cdot \cos(\alpha_1)) \end{bmatrix} \quad (\text{D.1})$$

*(hint :  $R_1 = Sr_1/\alpha_1$ )*

$$\mathbf{R}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{P}_1^{base} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (\text{D.2})$$

According to the Equations D.1 and D.2, the APM  $\mathbf{P}_2^{base}$  was calculated in Equation D.3.

$$\mathbf{P}_2^{base} = \mathbf{R}_1 \times \mathbf{P}_2^1 + \mathbf{P}_1^{base} \quad (\text{D.3})$$

- Module 2

For Module 2, the RPM  $\mathbf{P}_3^2$  with  $\alpha_2$  was shown in Equation D.4.  $R_2$  was the radius of the Module 2 bending curve. The rotational matrix  $\mathbf{R}_2$  and the APM  $\mathbf{P}_2^{base}$  were shown

in Equations D.5 and D.3.

$$\mathbf{P}_3^2 = \begin{bmatrix} (R_2 \cdot (1 - \cos(\alpha_2)) + d_3 \cdot \sin(\alpha_2)) \\ 0 \\ (R_2 \cdot \sin(\alpha_2) + d_3 \cdot \cos(\alpha_2)) \end{bmatrix} \quad (\text{D.4})$$

(hint :  $R_2 = Sr_2/\alpha_2$ )

$$\mathbf{R}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha_1) & \sin(\alpha_1) \\ 0 & -\sin(\alpha_1) & \cos(\alpha_1) \end{bmatrix} \quad (\text{D.5})$$

According to the Equations D.3, D.4, and D.5, the APM  $\mathbf{P}_3^{base}$  was calculated in Equation D.6.

$$\mathbf{P}_3^{base} = \mathbf{R}_1 \times \mathbf{R}_2 \times \mathbf{P}_3^2 + \mathbf{P}_2^{base} \quad (\text{D.6})$$

- Module 3

For Module 3, the RPM  $\mathbf{P}_4^3$  with  $\alpha_3$  was shown in Equation D.7.  $R_3$  was the radius of the Module 3 bending curve. The rotational matrix  $\mathbf{R}_3$  and the APM  $\mathbf{P}_3^{base}$  were shown in Equations D.8 and D.6.

$$\mathbf{P}_4^3 = \begin{bmatrix} 0 \\ (R_3 \cdot (1 - \cos(\alpha_3)) + d_4 \cdot \sin(\alpha_3)) \\ (R_3 \cdot \sin(\alpha_3) + d_4 \cdot \cos(\alpha_3)) \end{bmatrix} \quad (\text{D.7})$$

(hint :  $R_3 = Sr_3/\alpha_3$ )

$$\mathbf{R}_3 = \begin{bmatrix} \cos(\alpha_2) & 0 & \sin(\alpha_2) \\ 0 & 1 & 0 \\ -\sin(\alpha_2) & 0 & \cos(\alpha_2) \end{bmatrix} \quad (\text{D.8})$$

According to the Equations D.6, D.7, and D.8, the APM  $\mathbf{P}_4^{base}$  was calculated in Equation D.9.

$$\mathbf{P}_4^{base} = \mathbf{R}_1 \times \mathbf{R}_2 \times \mathbf{R}_3 \times \mathbf{P}_4^3 + \mathbf{P}_3^{base} \quad (\text{D.9})$$

- Module 4

In summary, the APM  $\mathbf{P}_5^{base}$  was calculated in Equation D.10. The calculation of  $\times \mathbf{R}_4$  and  $\mathbf{P}_4^4$  was similar to Equations D.5 and D.3.

$$\mathbf{P}_5^{base} = \mathbf{R}_1 \times \mathbf{R}_2 \times \mathbf{R}_3 \times \mathbf{R}_4 \times \mathbf{P}_5^4 + \mathbf{P}_4^{base} \quad (\text{D.10})$$

# The Equations and Derivations of IK

- Initialization

The manipulator was estimated to stay at the initial position, which is shown in Figure 8. Under this circumstance, the nodes  $\mathbf{node}_i$  and virtual nodes  $\mathbf{vnode}_i$  of the manipulator were listed in Equations D.11 and D.12. The connector thickness of manipulator was not taken into account in this demonstration, despite being considered in the programme. The virtual length  $\mathbf{l}_i$  for Module i, which is the distance from  $\mathbf{vnode}_i$  to  $\mathbf{node}_i$ , was derived in Equation D.13.

$$\mathbf{node}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{node}_2 = \begin{bmatrix} 0 \\ 0 \\ 150 \end{bmatrix} \quad \mathbf{node}_3 = \begin{bmatrix} 0 \\ 0 \\ 300 \end{bmatrix} \quad \mathbf{node}_4 = \begin{bmatrix} 0 \\ 0 \\ 450 \end{bmatrix} \quad \mathbf{node}_5 = \begin{bmatrix} 0 \\ 0 \\ 600 \end{bmatrix} \quad (\text{D.11})$$

$$\mathbf{vnode}_1 = \begin{bmatrix} 0 \\ 0 \\ 75 \end{bmatrix} \quad \mathbf{vnode}_2 = \begin{bmatrix} 0 \\ 0 \\ 225 \end{bmatrix} \quad \mathbf{vnode}_3 = \begin{bmatrix} 0 \\ 0 \\ 375 \end{bmatrix} \quad \mathbf{vnode}_4 = \begin{bmatrix} 0 \\ 0 \\ 525 \end{bmatrix} \quad (\text{D.12})$$

$$\mathbf{l}_i = \frac{Sr_i}{\theta_i} \cdot \tan(\theta_i) \quad (\text{hint : } \mathbf{l}_i = Sr_i/2 \text{ while } \theta_i = 0) \quad (\text{D.13})$$

$$\mathbf{l}_1 = \mathbf{l}_2 = \mathbf{l}_3 = \mathbf{l}_4 = 75 \text{ (unit : mm)}$$

- Iteration 1

The virtual node of Module 4 was derived by  $\mathbf{O}_{target}$  and  $\mathbf{P}_{target}$  in Equation 4. The virtual node  $\mathbf{vnode}_4$  was derived by Equation D.14.

$$\mathbf{vnode}'_4 = \mathbf{P}_{target} - \mathbf{l}_4 \cdot \mathbf{O}_{target}^z \quad (\text{hint : } z \text{ is } z\text{-axis orientation}) \quad (\text{D.14})$$

Afterwards, the vector from virtual node of Module 3 to virtual node of Module 4 was derived based on the positions of  $\mathbf{vnode}_3$  and  $\mathbf{vnode}'_4$ . The coordinate of the vector in the orientation of  $\mathbf{vnode}'_4$  was calculated in Equation D.15.

$$\mathbf{vector}_4 = \mathbf{O}_{target} \times (\mathbf{vnode}'_4 - \mathbf{vnode}_3) \quad (\text{D.15})$$

The bending angle of Module 4  $\theta_4$  was derived in Equation D.16. The Y-axis directional component was neglected because Module 4 can only bend in the X-Z

plane. The updated length  $\mathbf{l}_4^{update}$  was calculated according to Equation D.17. Meanwhile, the updated  $\mathbf{vnode}_4^{update}$  was rederive using  $\mathbf{l}_4^{update}$  in Equation D.18. The  $\mathbf{node}_4$  was determined based on the orientation of  $\mathbf{vnode}_4^{update}$ , which is  $\mathbf{O}_{vnode_4}$ , in Equation D.20.

$$\theta_4 = -\arctan2(\mathbf{vector}_4^x, \mathbf{vector}_4^z) \quad (\text{D.16})$$

$$\mathbf{l}_4^{update} = \frac{Sr_4}{\theta_4} \cdot \tan(\theta_4) \quad (\text{D.17})$$

$$\mathbf{vnode}_4^{update} = \mathbf{P}_{target} - \mathbf{l}_4^{update} \cdot \mathbf{O}_{target}^z \quad (\text{D.18})$$

$$\mathbf{O}_{vnode_4} = \begin{bmatrix} \cos(\theta_4) & 0 & \sin(\theta_4) \\ 0 & 1 & 0 \\ -\sin(\theta_4) & 0 & \cos(\theta_4) \end{bmatrix} \times \mathbf{O}_{target} \quad (\text{D.19})$$

$$\mathbf{node}_4 = \mathbf{vnode}_4^{update} - \mathbf{l}_4^{update} \cdot \mathbf{O}_{vnode_4}^z \quad (\text{D.20})$$

- Iteration 2

The virtual node of Module 3 was derived by  $\mathbf{O}_{vnode_4}$  and  $\mathbf{node}_4$  in Equations D.19 and D.20. The virtual node  $\mathbf{vnode}_3$  was derived by Equation D.21.

$$\mathbf{vnode}'_3 = \mathbf{node}_4 - \mathbf{l}_3 \cdot \mathbf{O}_{vnode_4}^z \quad (\text{D.21})$$

Afterwards, the vector from virtual node of Module 2 to virtual node of Module 3 was derived based on the positions of  $\mathbf{vnode}_2$  and  $\mathbf{vnode}'_3$ . The coordinate of the vector in the orientation of  $\mathbf{vnode}'_3$  was calculated in Equation D.22.

$$\mathbf{vector}_3 = \mathbf{O}_{vnode_3} \times (\mathbf{vnode}'_3 - \mathbf{vnode}_2) \quad (\text{D.22})$$

The bending angle of Module 3  $\theta_3$  can be derived in Equation D.23. The X-axis directional component was neglected because Module 3 can only bend in the Y-Z plane. The updated length  $\mathbf{l}_3^{update}$  was calculated according to Equation D.24. Meanwhile, the updated  $\mathbf{vnode}_3^{update}$  was rederived using  $\mathbf{l}_3^{update}$  in Equation D.25. The  $\mathbf{node}_3$  was determined based on the orientation of  $\mathbf{vnode}_3^{update}$ , which is  $\mathbf{O}_{vnode_3}$ , in Equation D.27.

$$\theta_3 = -\arctan2(\mathbf{vector}_3^y, \mathbf{vector}_3^z) \quad (\text{D.23})$$

$$\mathbf{l}_3^{update} = \frac{Sr_3}{\theta_3} \cdot \tan(\theta_3) \quad (\text{D.24})$$

$$\mathbf{vnode}_3^{update} = \mathbf{node}_4 - \mathbf{l}_3^{update} \cdot \mathbf{O}_{vnode_4}^z \quad (\text{D.25})$$

$$\mathbf{O}_{vnode_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_3) & \sin(\theta_3) \\ 0 & -\sin(\theta_3) & \cos(\theta_3) \end{bmatrix} \times \mathbf{O}_{vnode_4} \quad (D.26)$$

$$\mathbf{node}_3 = \mathbf{vnode}_3^{update} - \mathbf{l}_3^{update} \cdot \mathbf{O}_{vnode_3}^z \quad (D.27)$$

- Iteration 3

The virtual node of Module 2 was derived by  $\mathbf{O}_{vnode_3}$  and  $\mathbf{node}_3$  in Equations D.26 and D.27. The virtual node  $\mathbf{vnode}_2$  was derived by Equation D.28.

$$\mathbf{vnode}'_2 = \mathbf{node}_3 - \mathbf{l}_2 \cdot \mathbf{O}_{vnode_3}^z \quad (D.28)$$

Afterwards, the vector from virtual node of Module 1 to virtual node of Module 2 was derived based on the positions of  $\mathbf{vnode}_1$  and  $\mathbf{vnode}'_2$ . The coordinate of the vector in the orientation of  $\mathbf{vnode}'_2$  was calculated in Equation D.29.

$$\mathbf{vector}_2 = \mathbf{O}_{vnode_2} \times (\mathbf{vnode}'_2 - \mathbf{vnode}_1) \quad (D.29)$$

The bending angle of Module 2  $\theta_2$  was derived in Equation D.30. The Y-axis directional component was neglected because Module 2 can only bend in the X-Z plane. The updated length  $\mathbf{l}_2^{update}$  was calculated according to Equation D.31. Meanwhile, the updated  $\mathbf{vnode}_2^{update}$  was rederived using  $\mathbf{l}_2^{update}$  in Equation D.32. The  $\mathbf{node}_2$  was determined based on the orientation of  $\mathbf{vnode}_2^{update}$ , which is  $\mathbf{O}_{vnode_2}$ , in Equation D.34.

$$\theta_2 = -\arctan2(\mathbf{vector}_2^x, \mathbf{vector}_2^z) \quad (D.30)$$

$$\mathbf{l}_2^{update} = \frac{Sr_2}{\theta_2} \cdot \tan(\theta_2) \quad (D.31)$$

$$\mathbf{vnode}_2^{update} = \mathbf{node}_3 - \mathbf{l}_2^{update} \cdot \mathbf{O}_{vnode_3}^z \quad (D.32)$$

$$\mathbf{O}_{vnode_2} = \begin{bmatrix} \cos(\theta_2) & 0 & \sin(\theta_2) \\ 0 & 1 & 0 \\ -\sin(\theta_2) & 0 & \cos(\theta_2) \end{bmatrix} \times \mathbf{O}_{vnode_3} \quad (D.33)$$

$$\mathbf{node}_2 = \mathbf{vnode}_2^{update} - \mathbf{l}_2^{update} \cdot \mathbf{O}_{vnode_2}^z \quad (D.34)$$

- Iteration 4

The virtual node of Module 1 was derived by  $\mathbf{O}_{vnode_2}$  and  $\mathbf{node}_2$  in Equations D.33 and D.34. The virtual node  $\mathbf{vnode}_1$  was derived by Equation D.35.

$$\mathbf{vnode}'_1 = \mathbf{node}_2 - \mathbf{l}_1 \cdot \mathbf{O}_{vnode_2}^z \quad (\text{D.35})$$

Afterwards, the coordinate of the vector in the orientation of  $\mathbf{vnode}'_1$  was directly calculated in Equation D.36. This was because the  $\mathbf{O}_{node_1}^z$  must be oriented vertically upward.

$$\mathbf{vector}_2 = \mathbf{O}_{vnode_1} \times \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad (\text{D.36})$$

The bending angle of Module 1  $\theta_1$  was derived in Equation D.37. The X-axis directional component was neglected because Module 1 can only bend in the Y-Z plane. The updated length  $\mathbf{l}_1^{update}$  was calculated according to Equation D.38. Meanwhile, the updated  $\mathbf{vnode}_1^{update}$  was rederived using  $\mathbf{l}_1^{update}$  in Equation D.39. The  $\mathbf{node}_1$  was determined based on the orientation of  $\mathbf{vnode}_1^{update}$ , which is  $\mathbf{O}_{vnode_1}$ , in Equation D.41.

$$\theta_1 = -\arctan2(\mathbf{vector}_1^y, \mathbf{vector}_1^z) \quad (\text{D.37})$$

$$\mathbf{l}_1^{update} = \frac{Sr_1}{\theta_1} \cdot \tan(\theta_1) \quad (\text{D.38})$$

$$\mathbf{vnode}_1^{update} = \mathbf{node}_2 - \mathbf{l}_1^{update} \cdot \mathbf{O}_{vnode_2}^z \quad (\text{D.39})$$

$$\mathbf{O}_{vnode_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_1) & \sin(\theta_1) \\ 0 & -\sin(\theta_1) & \sin(\theta_1) \end{bmatrix} \times \mathbf{O}_{vnode_2} \quad (\text{D.40})$$

$$\mathbf{node}_1 = \mathbf{vnode}_1^{update} - \mathbf{l}_1^{update} \cdot \mathbf{O}_{vnode_1}^z \quad (\text{D.41})$$

## The Angular Conversion

$$\Delta S_1 = (R_1 + r_1) \cdot \alpha_1 - Sr_1 = \alpha_1 \cdot r_1$$

$$\Delta S_2 = (R_1 - r_1) \cdot \alpha_1 - Sr_1 = -\alpha_1 \cdot r_1$$

$$\Delta S_3 = (R_2 + r_2) \cdot \alpha_2 - Sr_2 = \alpha_2 \cdot r_2$$

$$\Delta S_4 = (R_2 - r_2) \cdot \alpha_2 - Sr_2 = -\alpha_2 \cdot r_2$$

$$\Delta S_5 = \Delta S_1 + (R_3 + r_3) \cdot \alpha_3 - Sr_3 = \Delta S_1 + \alpha_3 \cdot r_3$$

$$\begin{aligned}
\Delta S_6 &= \Delta S_2 + (R_3 - r_3) \cdot \alpha_3 - S r_3 = \Delta S_2 - \alpha_3 \cdot r_3 \\
\Delta S_7 &= \Delta S_3 + (R_4 + r_4) \cdot \alpha_4 - S r_4 = \Delta S_3 + \alpha_4 \cdot r_4 \\
\Delta S_8 &= \Delta S_4 + (R_4 - r_4) \cdot \alpha_4 - S r_4 = \Delta S_4 - \alpha_4 \cdot r_4
\end{aligned} \tag{D.42}$$

$$\Delta S = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ -r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & -r_2 & 0 & 0 \\ r_1 & 0 & r_3 & 0 \\ -r_1 & 0 & -r_3 & 0 \\ 0 & r_2 & 0 & r_4 \\ 0 & -r_2 & 0 & -r_4 \end{bmatrix} \times \boldsymbol{\alpha} \tag{D.43}$$