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## Binary decision diagram-based reliability modeling of phased-mission manufacturing system processing multi-type products

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#### **ABSTRACT**

The development of science and technology drives phased-mission manufacturing systems to be increasingly complex, which poses significant challenges to system reliability analysis. Although it has been considered that machine degradation is the key factor affecting system reliability, very little has been done to explore the effect of multi-type products on machine degradation, where the dependency of phases processing multitype products further increases the difficulty in reliability modeling. Focusing on the phased-mission manufacturing system processing multitype products, this paper presents a novel machine reliability model to investigate the mixed degradation process caused by multi-type products, which characterizes the dynamics of machine reliability. Further, given the proposed dynamic machine reliability model dealing with phase dependency, a binary-decision-diagram-model-based algorithm is proposed to analyze the reliability of phased-mission manufacturing systems when processing multi-type products. Finally, the experimental result verifies the effectiveness of the proposed method, and the accuracy is validated by the traditional Monte Carlo method. Also, the proposed method shows great potential to provide decision support for the operation management of phased-mission manufacturing systems.

#### ARTICLE HISTORY

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#### **KEYWORDS**

System reliability; phasedmission system; binary decision diagram; machine degradation; multi-type product

#### 1. Introduction

Phased-mission system (PMS) is a complex system that performs specific missions composed of multiple consecutive and non-overlapping phases (Bazdar et al., 2017; Huang et al., 2019; Li et al., 2020). Unlike other systems, PMS completes different tasks in distinct phases. For example, the flight of an aircraft is a typical PMS (Yu et al., 2020; Zhai et al., 2018), which includes taxi, take-off, ascent, level-flight, descent, and landing phases. The successful completion of the flight relies on the continuous and reliable operation of various critical components, including but not limited to the engines, throughout each phase. As a specific form in the manufacturing field, phased-mission manufacturing systems are widely used in some manufacturing industries such as aerospace products, putting more strict requirements on reliability. Therefore, it is vital to explore an effective reliability analysis method for phased-mission manufacturing systems to maintain their operation status and reduce unnecessary production costs.

System reliability is generally defined as the probability that a system performs its intended function under specified operating conditions for a specified period (Ye et al., 2019; Zuo, 2021). However, the influence of multi-type products is crucial for a comprehensive understanding of manufacturing systems in real-world scenarios (He et al., 2020; Qian et al., 2021), which introduces great challenges to the reliability model and optimization (Si et al., 2020). The impact of products on machine performance has been focused on. He et al. (2017) proposed a Quality State Task Network integrating task requirements, material quality, and machine performance. Lu and Zhou (2019) proposed a quality and reliability-oriented condition-based maintenance policy for serial multistage systems to evaluate the quality loss of products and system failure rate. Considering the loss function caused by the product quality, Mao et al. (2021) proposed a hidden quality cost-production cost reliability prediction based on a quadratic exponential quality loss function. However, while existing research mainly focused on the influence of product quality, it has often overlooked the impact of multi-type products on machine reliability in practical scenarios.

In addition, the diversification of products requires a flexible manufacturing system, which contains different phases to finish the processing of multi-type products. In this circumstance, the increasing flexibility will lead to significant uncertainties and dynamics for the phased-mission manufacturing system. Considering the continuity of the process, there exists a close dependency between different phases. Current reliability researches usually focus on simple structures, such as the series manufacturing system evaluation based on threshold model (Broek et al., 2020; Qiu et al., 2019) and quality-reliability chain model (Zhu et al., 2020), series-parallel manufacturing system reliability analysis (Jin et al., 2022; Xu et al., 2018). Although the quality-reliability dependency is considered, these existing researches are also not well-suited for effectively dealing with phase dependency, which is urgent to seek a new modeling method.

The simulation method has great universality in system representation, but only approximate results could be obtained in reliability analysis (Du et al., 2020; Soltanali et al., 2020). However, the modeling method achieves the ideal combination of the analytical solution and flexibility of system representation. For example, the novel method based on the shot-noise process (Qiu et al., 2019), and the stochastic failure process (Qiu et al., 2019) are proposed. Qiu and Cui (2018) proposed the dependent two-stage failure process model with the random hazard rate increment. To solve the challenges posed by multiple phases, a common method is to construct differentiated reliability models considering the differences in different phases (Li et al., 2021; Zhao et al., 2019), such as the Markov chain model (Jia et al., 2020; Sheng et al., 2019; Wu & Cui, 2020), reliability block diagram (Catelani et al., 2019), combination model (Jafary et al., 2020; Tan & Xie, 2019), and fault tree (Ammar et al., 2019; Kozjek et al., 2017). However, these above methods cannot visualize intuitively the logic structure of the PMS. In contrast, the decision diagram is effective to deal with the reliability analysis of PMS where phase difference exists (Khosravi et al., 2017; Wang et al., 2020). Zhou et al. (2020) evaluated system reliability from the binary decision diagram (BDD). Focusing on a two-state phased-mission manufacturing system, BDD demonstrates a notable advantage due to its ability to efficiently represent the system failure behaviors. However, current BDD-based manufacturing systems reliability analysis does not take into account the effect of multi-type products. Therefore, considering the processing of one product as a phased mission with a timevarying state provides a novel idea to study the effect of multi-type products on the manufacturing

Based on the research gaps mentioned above, this paper proposes a novel approach to evaluate the reliability of phased-mission manufacturing systems under the mixed degradation process caused by multi-type products. First, we present relevant assumptions and establish a machine reliability model to characterize the mixed degradation process. Then, the BDD-based reliability model for the mixed degradation process is constructed to evaluate system reliability. At last, we develop an algorithm to compute system reliability, whose effectiveness is verified by experiment. The significant contributions of this paper are summarised as follows.



- (1) The machine reliability model based on the mixed degradation process is proposed, which accounts for the influence of multi-type products through a random degradation increment.
- (2) The BDD-based system reliability model is established to effectively capture operation dynamics in the phased-mission manufacturing system, especially in the failure mechanisms with processing multi-type products.
- (3) Based on the proposed model, a novel algorithm is developed to evaluate system reliability under the mixed degradation process, which is verified by a comparative experiment.

The rest of this paper is structured as follows: Section 2 presents the structure and properties of the phased-mission manufacturing system. Section 3 constructs the system reliability model based on BDD with the mixed degradation process. Section 4 introduces the modeling details and evaluation algorithm. Section 5 shows an experiment to illustrate the effectiveness compared with the Monte Carlo simulation method. Conclusions and future work are discussed in Section 6.

#### 2. Phased-mission manufacturing system

#### 2.1 System description

The phased-mission manufacturing system is composed of many machines that are connected in series to accomplish different kinds of products at different phases, where each machine's outputs serve as inputs for the next machine. As shown in Figure 1, products can be considered as inputs of the system, and different colored lines represent the processing paths of different products. Considering the impact on mission execution, the product plays a critical role in the manufacturing system as it not only represents the mission but also serves as a carrier for propagated failures. Therefore, besides machines, products (including their types) are equally essential constituents in evaluating the manufacturing system's performance.

Focusing on a two-state manufacturing system, namely, machines and systems only have two states: work state (0) and failure state (1). The reliability of a phased-mission manufacturing system refers to the probability that no machine failure occurs as it performs a task consisting of multi-type products. In a manufacturing system with N machines to handle M products, the

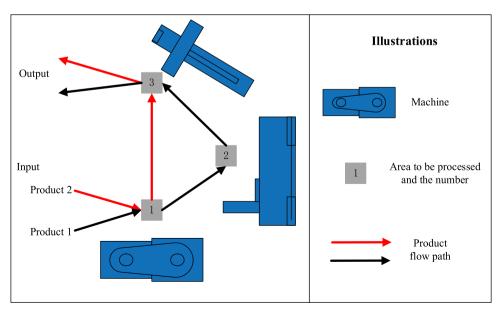


Figure 1. A phased-mission manufacturing system.

products are processed sequentially through machines 1 to N, and the process of finishing one type of product is defined as a phase. Therefore, a mission with M product types to be processed is divided into M phases and the system reliability can be evaluated based on the reliability of all phases.

#### 2.2 System properties

Compared with single-phase systems, the properties of phased-mission manufacturing systems that contribute to the complexity of reliability analysis are as follows.

#### (1) Property 1: dynamic states.

When PMS executes different phases, PMS will respond to different stresses, environmental conditions, and reliability requirements, which increases the dynamic of manufacturing systems and the difficulty of its reliability assessment. The probability of machine state transitions is not stationary but dynamic. This dynamic behavior stems from product diversity, which poses a significant challenge to reliability analysis.

#### (2) Property 2: phase dependency.

Due to the continuity of processing time, the final state of machines in the current phase becomes the initial state of the next phase, resulting in phase dependency, which leads to the difficulty in reliability analysis that must strictly follow the phase sequence. Besides, as multi-type products flow through the manufacturing system, machine degradation caused by different products accumulates from the initial phase to the final phase due to its non-decreasing nature, which requires attention to phase dependency.

### 3. Reliability modeling of phased-mission manufacturing system processing multi-type products

#### 3.1 Machine degradation process

The reliability of the manufacturing system can be regarded as the joint characteristic of machine reliability. Machine reliability is a crucial influencing factor, and even a slight change in machine reliability can have a significant impact on the overall system reliability. To address the varying degradation behaviors, this paper defines two degradation processes under different conditions: the basic degradation process and the machined degradation process.

The basic degradation process refers to the process in which machine reliability is in a stable and continuous degradation process no matter in the process of processing products or not, subjecting to the machine's characteristic degeneration. The machined degradation process refers to the process in which machine performance is disturbed by the impact of processing multi-type products. The random degradation increment is related to the product types, processing time, etc. In real-world engineering, machines often experience a mixed degradation process, presenting a significant challenge for reliability analysis.

#### 3.2 Machine reliability model based on mixed degradation

According to the description in Section 2, machine reliability is defined as the probability of the machine being in the work state. As an expression of machine reliability, the failure rate denotes the conditional probability that failure occurs per unit of time in the future when the machine is in the work state at the current moment, expressed by Equation (1).

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{p(t < T < t + \Delta t | T > t)}{\Delta t},$$
(1)

where *T* is the machine life.

In practical engineering, machine reliability exhibits high sensitivity to processing time, and its degradation degree is intricately connected to the product types. Therefore, the machine shows the cumulative characteristics of continuously increasing failure rate and failure-rate increment, as shown in Figure 2.

In the basic degradation process, the continuously increasing failure rate is directly correlated with processing time and monotonically grows, which can be effectively represented by using the Weibull distribution, as shown in Equation (2).

$$r_a(t) = \frac{\beta_a}{\alpha_a} \left(\frac{t}{\alpha_a}\right)^{\beta_a - 1},\tag{2}$$

where  $\alpha_a$ ,  $\beta_a$  are the scale parameter and shape parameter of the Weibull distribution showing the continuously increasing failure rate respectively, and  $\beta_a > 1$ .

In the machined degradation process, each product introduces a random failure rate increment that varies with processing time and product type. Therefore, the failure rate increment is the sum of increments generated by all processed products, as shown in Equation (3).

$$r_b(t) = \sum_{k=1}^{N(t)} \Delta r_k,\tag{3}$$

where N(t) is denoted as the number of products up to time t,  $\Delta r_k$  is denoted as the failure rate increment generated by the kth product, shown in Equation (4).

$$\Delta r_k = \frac{\beta_b}{\alpha_b} \left( \frac{\varepsilon_k t_k}{\alpha_b} \right)^{\beta_b - 1},\tag{4}$$

where  $\alpha_b$ ,  $\beta_b$  are the scale parameter and shape parameter of the Weibull distribution with continuous failure rate increment respectively, and  $\beta_h > 1$ . Here,  $t_k$  represents the processing time of the kth product. The coefficient  $\varepsilon_k$  denotes the influence degree of product types on machine degradation and  $\varepsilon_k$  is set to be non-negative, where  $\varepsilon_k = 0$  denotes that the machine is not affected in the processing phase. Therefore, a greater effect of product types on machine degradation will occur when  $\varepsilon_k$  rises.

Based on the above definitions, the machine failure rate based on mixed degradation can be obtained by Equation (5).

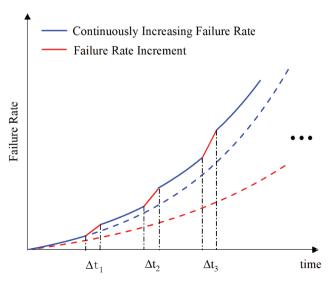


Figure 2. Continuously increasing failure rate and failure rate increment.

$$r(t) = r_a(t) + r_b(t) = \frac{\beta_a}{\alpha_a} \left(\frac{t}{\alpha_a}\right)^{\beta_a - 1} + \sum_{k=1}^{N(t)} \frac{\beta_b}{\alpha_b} \left(\frac{\varepsilon_k t_k}{\alpha_b}\right)^{\beta_b - 1}.$$
 (5)

The failure rate is known to be the ratio of the failure probability density function to the reliability r(t) = f(t)/R(t). And failure probability density function is the derivative of the failure probability distribution function f(t) = dF(t)/dt. Then, the failure probability distribution is obtained by Equation (6).

$$F_n(t) = 1 - e^{-\left(\frac{t}{a_a}\right)^{\beta_a}} \prod_{k=1}^{N(t)} e^{-\frac{1}{\epsilon_k} \left(\frac{\epsilon_k t_k}{a_b}\right)^{\beta_b}}.$$
 (6)

Finally, the machine reliability model based on mixed degradation at time t is obtained by Equation (7).

$$R(t) = e^{-\left(\frac{t}{\alpha_a}\right)^{\beta_a}} \prod_{k=1}^{N(t)} e^{-\frac{1}{\varepsilon_k} \left(\frac{\varepsilon_k t_k}{\alpha_b}\right)^{\beta_b}}.$$
 (7)

#### 3.3 System reliability model based on binary decision diagram

In a manufacturing system S(N, M) with N machines and M product types to be processed, each phase represents a specific goal for processing one type of product in the execution of the mission, usually consisting of a series of machine activities. The phase state  $X_h$  is defined as the combination of all machine states in the system during executing phase h, denoted as  $X_h = [S_{h1} \cdots S_{hi} \cdots S_{hi}]$ , where  $S_{hi}$  is the state of machine i in phase h whose work state and failure state are represented by 0 and 1, respectively. The cumulative-phase state  $G_h$  refers to the union of each phase until the current phase h, characterizing possible state trajectories of all machines during the mission. The initial-phase state means that all machines have the optimal performance of work state without any degradation effect, namely  $G_0 = X_0 = [0 \cdots 0 \cdots 0]$ . The system state  $M_s$  is mission completed or failure, which is represented by 0 and 1, respectively. In any following phase  $h(1 \le h \le M)$ , the cumulative-phase state  $G_h$  can be represented by a  $(h+1) \times N$  dimensional matrix, and derived from the cumulative phase state  $G_{h-1}$  in phase h-1, as shown in Equation (8).

$$G_h = \begin{bmatrix} G_{h-1} \\ X_h \end{bmatrix} = \begin{bmatrix} S_{01} & \cdots & S_{0N} \\ \vdots & \ddots & \vdots \\ S_{h1} & \cdots & S_{hN} \end{bmatrix}. \tag{8}$$

Based on the binary state of all machines and multiple states of phases, the Multi-State Multi-Valued Decision Diagrams (MMDD), consisting of several non-sink nodes and sink nodes, can be used to represent the cumulative-phase states in a phased-mission manufacturing system. Considering that the phase state  $X_{h-1}$  contains N machines with binary state in phase  $h-1(2 \le h \le M)$ , the node  $G_{h-1}$  has  $2^N$  outgoing edges as shown in Figure 3. In these outgoing edges, only one will be connected to the node  $G_h$  that will generate multiple edges to connect to the next cumulative-phase state, which represents that the system still works successfully at the end of phase h. While  $2^N-1$  outgoing edges where at least one machine is in failure state will be connected to the node  $G_h$  that is connected directly to the sink node 'mission failure'.

To explain the machine state in each phase clearly, the outgoing edge  $X_h = [0 \cdots 0 \cdots 0]$  can be refined into a binary decision diagram. The outgoing edge '0' of the node  $S_{hi}$  that represents the machine in work state is connected with the next node  $S_{h(i+1)}$  and follows this rule until the node  $G_h$ 

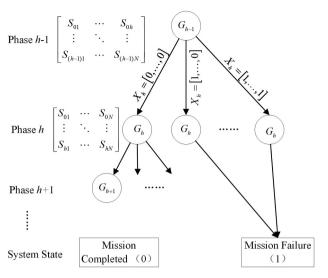


Figure 3. Connection pattern of nodes $G_h$  in decision diagram.

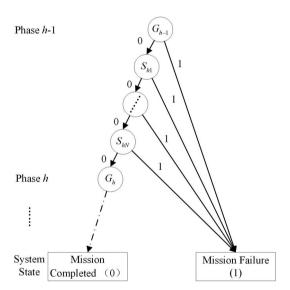


Figure 4. Rule of building BDD model.

is connected. Then, the BDD model is shown in Figure 4, and system reliability in phase h can be calculated by Equation (9).

$$P(G_h) = \prod_{i=1}^{N} P(S_{hi} = 0).$$
(9)

#### 4. Performance analysis of the phased-mission manufacturing system

#### 4.1 Binary decision diagram generation method

Based on the above method, this section will give the detailed algorithm to generate the BDD and evaluate the system reliability as follows.

**Step 1**: Defining the phase and building the fault tree of each phase. First, the manufacturing system is divided into M phases according to the M product types to be processed. Second, it is important to obtain the processing time  $t_{ij}$  of product j in machine i where i is taken as  $1, 2, \ldots, N$  and j is taken as  $1, 2, \ldots, M$ . Then, the processing time of the jth phase is obtained by  $t_j = \sum_{i=1}^N t_{ij}$ . As shown in Figure 5, the top event is the case that the manufacturing system completes the task of the jth phase successfully and the bottom event is the machine state. According to the system mechanism that all machines keep in work state can ensure the completion of the mission, the 'and' logic gate serves as the logical relationship that connects all machines with the top event sequentially, and the fault tree model is established, where  $A_{ij}$  denotes that the state variable of the ith machine in jth phase.

**Step 2**: Transforming the fault tree into phase BDD model. The machine state that is illustrated by a Boolean variable is the node, and the outgoing edge '0' indicates the machine is in the work state, while '1' indicates its failure state. The sink node 'phase work' indicates that the manufacturing system will complete the phase mission, while the node 'phase fail' does not. Considering the importance of sorting the state variables in the fault tree before building the BDD model, it assumes that the ranking of state variables is consistent with the processing path, namely,  $A_{1j} < A_{2j} < \cdots < A_{jj} < \cdots < A_{Nj}$ . Then, the BDD in the *j*th phase mission is shown in Figure 6.

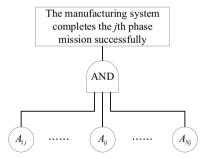


Figure 5. The fault tree of the jth phase mission.

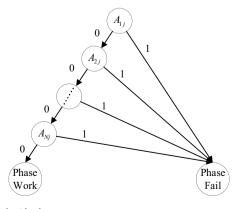


Figure 6. The phase BDD model of the jth phase.

Combining with Equation (7),  $R_i(t)$  denotes the reliability of the *i*th machine at time t in Equation (10).

$$R_i(t) = e^{-\left(\frac{t}{\alpha_{ai}}\right)^{\beta_{ai}}} \prod_{l=1}^j e^{-\left(\frac{\varepsilon_{il}t_{il}}{\alpha_{bi}}\right)^{\beta_{bi}-1}}, \tag{10}$$

where  $\alpha_{ai}$ ,  $\beta_{ai}$  and  $\alpha_{bi}$ ,  $\beta_{bi}$  are the parameters of the *i*th machine in the basic degradation process and machined degradation process respectively, and  $\varepsilon_{il}$  denotes the influence coefficient of the *l*th product on the *i*th machine.

**Step 3**: Building the system BDD model. According to the phased-mission characteristics and the operating rules of 'and' logic gates, the system BDD model can be obtained by connecting the sink node 'phase work' in the previous phase BDD model with the root node in the later phase BDD model in sequence. Besides, the sink node '0' indicates that the manufacturing system completes the mission successfully, while node '1' does not. All nodes 'phase fail' from all phase BDD models are directly connected to the sink node '1' and only the node 'phase work' in the *M*th phase is connected to the sink node '0', shown in Figure 7.

As shown in Figure 8, the simplified system BDD model is finally obtained by dealing with the redundant structure, which can be simplified by the following rules.

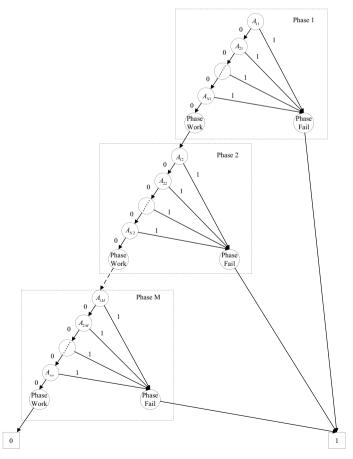


Figure 7. The system BDD model.

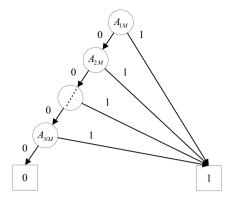


Figure 8. The simplified system BDD model.

- (1) The work state of the manufacturing system at phase i and phase j is equal to the work state at phase i, if i > j.
- (2) The failure state of the manufacturing system at phase i and phase j are equal to the failure state at phase j, if i > j.

**Step 4**: Reliability evaluation of the phased-mission manufacturing system. All paths to the sink node '0' in the simplified system BDD model are listed by enumeration, namely,  $A_{1M} < A_{2M} < \cdots < A_{iM} < \cdots < A_{NM}$ . Therefore, the system reliability is equal to the product of all machine reliability in Equation (11).

$$R_s(t) = \prod_{i=1}^{N} R_i(t).$$
 (11)

#### 4.2 BDD-based evaluation algorithm

By discretizing total processing time T to simplify the evaluation process, the flowchart presents the reliability evaluation algorithm of the system S(N, M) underlying the proposed model shown in Figure 9, which contains six steps as follows.

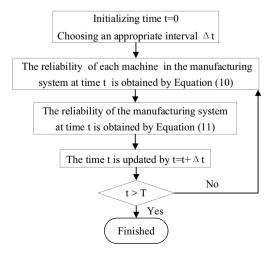


Figure 9. The flowchart of the evaluation algorithm.



- **Step 1**: Initializing time t = 0 and other parameters.
- **Step 2**: Choosing an appropriate time interval  $\Delta t$ .
- **Step 3**: According to the phase BDD model at time t, the outgoing edge '0' of the node  $A_{iM(t)}$  representing that machine state in M(t) phase is connected to the next node, and the probability of edge '0' can be described as the machine reliability, namely,  $R_i(t)$ ,  $i = 1, \ldots, N$  by Equation (10).
- **Step 4**: Based on the system BDD model, all paths from the root node to sink node '0' are listed and the probabilities of all edges '0' can be obtained. Then, the system reliability  $R_s(t)$  can be calculated by Equation (11).
  - **Step 5**: Updating the processing time *t* by  $t = t + \Delta t$ .
- **Step 6**: By judging t > T, it can be determined whether the current processing time overflows the total processing time T. If so, the experiment is finished. If not, return to step 3.

#### 5. Case study

The aviation industry is a typical multi-specification small batch manufacturing, where the production of similar parts with different specifications could be finished in a flexible manufacturing line. It is a typical phased-mission manufacturing system when it performs the task of processing different types of products. Based on the investigation of a real factory in the aviation industry, a simplified phased-mission manufacturing system is given to verify the effectiveness of our proposed methods. In detail, the manufacturing system is composed of four machines with eight product types to be processed, and the processing time is given in Table 1.

It is assumed that all machines have the same parameters: continuously increasing failure rate  $\alpha_a = 8200$ ,  $\beta_a = 3.32$  and failure rate increment  $\alpha_b = 6500$ ,  $\beta_b = 2.17$ . Assuming that the influence intensity of the same product on all machines follows the same distribution, and the influence coefficient of each product type on machine degradation is set as  $\varepsilon = [1, 5, 8, 10, 12, 15, 18, 20]$ .

#### 5.1 Reliability analysis of machine and system

After completing the mission, the probabilities of all machines being in the work state are 0.941, 0.9601, 0.9649, and 0.9682 respectively in Figure 10, indicating that the machines maintain relatively high reliability. At the beginning of the mission, all machines are in work states and the failure rates are equal to 0. It can be observed that the machine reliability almost is a constant in the early phase, and gradually decreases as the processing time increases, indicating a sustained and non-decreasing degradation effect of products on machine reliability. During the degradation process, the machine reliability decline rate increases with the processing time, which is determined by the Weibull distribution of the basic degradation process and is affected by the influence coefficient. According to the production schedule in Table 1, the influence coefficients of different products processed by machines 1 to 4 decrease sequentially at the same moment, and the gradients of corresponding reliability curves are decreasing in Figure 10, indicating that the increase of the

Table 1. The processing time of all machines in the case (units: minute).

Phase	Start Time	End Time	Processing Time			
			Machine 1	Machine 2	Machine 3	Machine 4
1	0	61	16.69	21.95	19.26	0.1
2	61	122	34.77	19.69	4.88	1.65
3	122	164	7.09	17.42	11.13	6.36
4	164	210	10.59	9.48	14.95	10.98
5	210	252	4.81	8.33	15.87	12.99
6	252	294	12.88	16.11	11.07	1.95
7	294	336	13.53	17.13	9.69	1.65
8	336	382	2.66	18.74	6.67	17.94

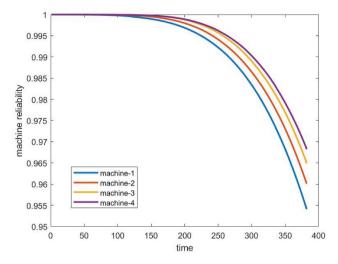


Figure 10. The reliability curve of each machine.

influence coefficient will aggravate the decline rate of machine reliabilities. Besides, the increment of the failure rate is mainly affected by the product type and processing time, leading to the difference in reliability degradation. For example, the processing time of machine 1 is longer than other machines according to Table 1, leading to a more serious degradation of machine 1 when other parameters are consistent.

As shown in Figure 11, system reliability decreases continuously with processing time, and the decline rate increases continuously. Upon completing the specified mission, the system reliability closes to 0.8559. Compared with the machine reliability curve, the decrease of system reliability during the mission is much faster than the ones of machines, demonstrating that system reliability is vulnerable to machine reliability and even minor declines in machine reliability can be magnified in the degradation of system reliability. In production management, maintenance for a good machine reliability level is very important for the completion of a mission with multi-type products, even when machines still have favorable performance. Before starting the mission, the machine and

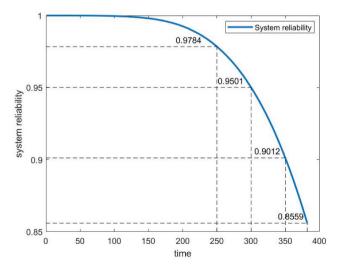


Figure 11. The reliability curve of the manufacturing system.

system reliability can be judged by our proposed model, to identify risk machines and assist in the formulation of maintenance policies. Therefore, the proposed model could enhance transparency in industrial production, accelerate the development of informatization, and make intelligent maintenance decisions for manufacturing systems feasible.

#### 5.2 Reliability analysis with different influence coefficient

In addition, to compare the influence degree of products on machine degradation, three different levels of the influence coefficient are set as shown in Table 2.

As shown in Figure 12, the degradation rate of system reliability accelerates with the increase of the influence coefficient, which conforms to the production practice and verifies the rationality and effectiveness of the proposed method. Besides, the reliability differences in any neighbouring two curves at any three moments are illustrated, such as 250, 300, and 350. When the influence coefficient increases from 'weak' to 'medium', the degradation degrees of reliability levels are 0.0262, 0.0589, and 0.1009. However, when the influence coefficient increases from 'medium' to 'strong', the degradation degrees of reliability levels are 0.0566, 0.1195, and 0.2006, respectively. In summary, the influence intensity in Table 2 shows a linear increase from 'weak' to 'medium' and then to 'strong', but the degradation degrees of two adjacent influence intensities are non-linear, indicating that there is a nonlinear relationship between the influence coefficient and system reliability. Moreover, the system reliability degradation under the 'strong' influence is larger than that under the 'weak' influence after completing the mission, indicating that the influence intensity seriously affects the system reliability degradation.

At the technical level, previous models only considered the inherent machine degradation patterns. This experiment illustrates the influence of multi-type products on machine reliability degradation, revealing the complex machine degradation process. In industrial applications, due to

Table 2. The different levels of the influence coefficient.

Tuble 2. The different levels of the limitence coefficient.					
Influence Intensity	ε				
Strong	[4, 20, 32, 40, 48, 60, 72, 80]				
Medium	[2, 10, 16, 20, 24, 30, 36, 40]				
Week	[1, 5, 8, 10, 12, 15, 18, 20]				

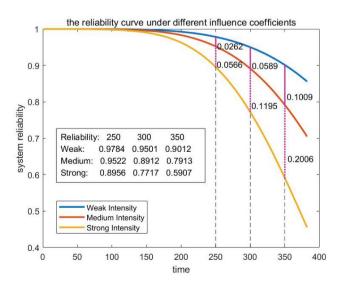


Figure 12. The reliability curve under different influence coefficient.

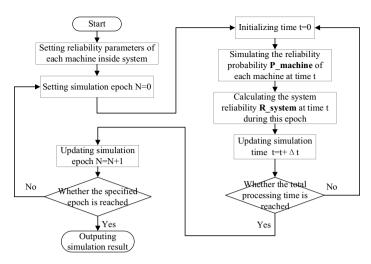


Figure 13. The flowchart of the Monte Carlo method.

the nonlinear relationship between the influence coefficient of products and system reliability degradation, the managers should explore the product types with worse influence intensity, formulate reasonable strategies to reduce their impact on degradation, and keep their degradation influences at a low level as far as possible.

#### 5.3 Evaluation and comparison with the Monte Carlo method

In reliability engineering, the Monte Carlo method is widely used and plays an important role in practices, which is a beneficial tool for reliability assessment. Thus, the Monte Carlo method is applied to verify the accuracy of the proposed model, as illustrated in Figure 13.

Setting simulation epoch N = 10000 and time interval  $\Delta t = 50$ , the reliability comparison result is shown in Figure 14. The Monte Carlo simulation method yields reliability that fluctuates up and down around the system reliability curve obtained from the proposed method without extremely abnormal conditions. The consistency between the results obtained from the two methods is evident, which shows the proposed model can accurately and effectively evaluate the system reliability.

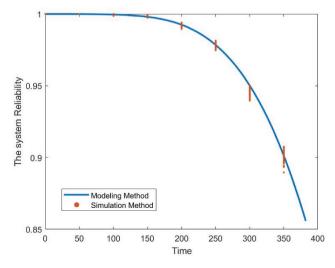


Figure 14. The reliability comparison chart.



Table 3. The reliability comparison result.

Time	Reliability obtained by model	The average obtained by simulation	Error	Error ratio
0	1	1	0	0
50	0.999964	1	0.000036	0.003600%
100	0.999626	0.999686	0.000060	0.006020%
150	0.998048	0.997837	0.000211	0.021146%
200	0.992522	0.993014	0.000492	0.049546%
250	0.978438	0.980008	0.001570	0.160203%
300	0.950050	0.947635	0.002415	0.254845%
350	0.901161	0.895037	0.006124	0.684218%

The absolute value of the difference between the model and simulation results is defined as 'Error', while the 'Error Ratio' represents the ratio of the error to the average of the simulation results. In Table 3, it can be observed that the Error and Error Ratio are small enough to ignore the difference between the two methods, showing the modeling results closely align with the simulation results. This comparison result serves as validation for the accuracy of the proposed method.

Upon comparing the two methods, it is evident the Monte Carlo method faces challenges in effectively representing the intricate phased-mission characteristics. In contrast, the modeling method offers the capability to accurately solve it and effectively characterize the influence of multitype products on machine degradation. Moreover, the model demonstrates its versatility by accommodating reliability analysis in both discrete and continuous time domains, providing substantial advantages for analyzing systems with varying time scales. Additionally, its ability to derive exact solutions, as opposed to the approximate solutions obtained by the simulation method, enhances the precision of reliability evaluations. In summary, the modeling method exhibits remarkable innovation and practicability, rendering it a valuable tool for the reliability analysis of phased-mission manufacturing systems.

#### 6. Conclusion and future work

Previous studies have indeed identified the dynamic relationship between products and machines in manufacturing systems, but such phenomena have received limited attention in phased-mission manufacturing systems. Consequently, a novel model is proposed and makes significant progress, aiming to assess phased-mission manufacturing system reliability by incorporating machine reliability under mixed degradation conditions.

The proposed method offers several advantages. First, the machine reliability model is established based on the mixed degradation process, where the influence of multiple-type products on machine reliability is characterized by a random degradation increment. The proposed model provides more detailed information about the machine state and is generally applicable to various types of machine life distributions. Second, the system reliability model, which integrates the machine reliability model with the binary decision diagram model, facilitates a more intuitive understanding of the operation mechanism of phased-mission manufacturing systems. Third, an evaluation algorithm based on the system reliability model is proposed, which achieves accurate reliability assessments compared with the Monte Carlo simulation method.

One potential limitation is the assumption that only product type affects machine degradation. In practical scenarios, other characteristics of products, such as their quality, can also significantly influence machine reliability, which merits further investigation. Thus, future research should explore the relationships between product failure, machine degradation, and the presence of multiple machine states to refine the accuracy of reliability models and provide a more comprehensive representation. While our model implements some



simplifications to address computational efficiency concerns, it is essential to acknowledge that computational complexity may still increase significantly with larger mission scales and machine quantities. As part of future research, exploring parallel computing implementations, developing more efficient algorithms, and exploring more management experience will be crucial.

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