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Performance evaluation of serial-parallel manufacturing systems based on the impact of heterogeneous feedstocks on machine degradation

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ABSTRACT

Heterogeneity of feedstock quality can bring disturbances to machine degradation and product quality. Therefore, machine reliability modeling and machining quality analysis considering the flow of heterogeneous feedstocks are very essential in the performance evaluation of manufacturing systems. This paper presents a new mixture degradation model to evaluate the reliability of manufacturing machines that accounts for the flow and impact of heterogeneous feedstocks in serial-parallel manufacturing systems. Specifically, this mixture model leverages two Weibull distributions to describe machine degradation processes under the conditions of high-quality and low-quality feedstocks. Then, the flow of low-quality feedstocks is modeled by the non-homogeneous Poisson process, so that an interacting chain of quality and reliability is formulated for the serial-parallel manufacturing system. Based on the proposed model, an evaluation framework for the performance of serial-parallel manufacturing systems is provided. Finally, simulation experiments are implemented to analyze the operation status and quality loss of the system. The results showed the effectiveness of the proposed method in performance modeling of serial-parallel manufacturing systems with mixture machine degradation. The proposed approach shows strong potentials for general applications in the performance analysis of complex-structure manufacturing systems.

1. Introduction

The safe, reliable, and efficient production system is indispensable for manufacturing industries, whose actualization greatly depends on machine reliability and product quality. Therefore, an effective integrated evaluation of machine reliability and product quality is greatly required for improving the performance of a manufacturing system. In actual production, however, the instability of production processes, leading to the erratic trajectories of machine degradation and quality status, poses great challenges to the formulation and implementation of evaluation methods. On the other hand, to enhance the production capacity or increase routing flexibilities, the parallel structure has been widely used in various manufacturing systems [1,2], leading to a great rise in the complexity of system configurations and uncertainty of feedstock flowing. This brings another level of difficulty to analyze the interaction between machine reliability and product quality.

The performance of manufacturing systems is strongly coupled with machine reliability, which has been broadly considered in the research on manufacturing systems, such as the performance evaluation of production systems by sensor data [3] or real-time machine degradation signals [4]. In the traditional evaluation of machine reliability, the influence of geometric errors, thermal errors, and tool wear has been investigated [5–7], which are concluded as machine conditions and modeled by the degradation model, such as Gamma process [8], Weibull distribution [9], and the multi-stage degradation model [10], which considering the degradation of different machines are independent. In practical engineering, however, machine degradation will be influenced by multiple sources, consisting of not only machine conditions but also manufactured objects [11,12]. In a manufacturing system, after being processed by the previous machine, the product will become feedstocks with different quality status, such as high-quality, marginal, or low-quality, and flow to a subsequent refining process. Among them, the

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low-quality or unqualified feedstock, engendered by previous unreliable machines, is an important source of abnormal machine wear and cannot be ignored. When processing these feedstocks with quality issues, the machine in a subsequent refining process may operate with more intense degradation. As a result, tracking the performance of an interactive set of machines will become more difficult under this circumstance.

As the significant attribute of engineering systems, time-varying and dynamic behaviors may lead to unstable states of systems [13,14], which have been widely studied, such as Gaussian process models depicting time-varying degradation rates [15,16] and degradation model considering time-varying motion and loading working conditions [17]. Specifically, the dynamic degradation process of machines caused by the heterogeneity of feedstocks is the typical dynamic behavior in a manufacturing system [18]. Considering the infrequency of homogeneous populations in real life, the mixture model has been demonstrated to be an effective tool for modeling lifetimes in heterogeneous populations in reliability applications [19,20]. For example, the Gaussian mixture model [21–23], the exponential mixture model [24], and the Weibull mixture model [9,25,26] have been proposed to depict various engineering systems. Also, as one popular method, the mixed-effects model can depict the investigated subject with a more exquisite degradation process, containing both fixed impacts that are common for all populations and random impacts that represent some individual item's properties [27]. Therefore, the mixture model is one effective method to represent unstable machine degradation under the impact of heterogeneous feedstock quality.

Also, product quality is always an important constituent of the performance of manufacturing systems in the modern industry [28], which has been evidenced by many studies, such as the quality analysis in ultraprecision machining and additive manufacturing [29,30], the mission reliability investigation based on operational quality data [31], and the integrated analysis of multi-stage manufacturing systems by considering the quality and machine degradation [32]. Therefore, any more caution for product quality will not be redundant. In recent researches on the performance of manufacturing systems, however, the interaction between product quality and machine reliability in complex system configurations has not received enough attention. Further, when considering the heterogeneity of feedstock quality, the unreliability of inspection processes, which will lead to the flow of low-quality feedstocks, is also an incentive to the interactive behaviors between machines in manufacturing systems.

The manufacturing system, as a subset of the production system, was considered as the arrangement and operation of various elements [33]. To study the interaction between machine reliability and feedstock quality, this paper simplifies the normal manufacturing system by focusing on the key factors of machines, feedstocks, and inspections during system analysis. Then we proposed an integrated model of these three elements to analyze the dynamic performance of the serial-parallel manufacturing system. Through analytical models and experimental simulations, the impact of heterogeneous feedstocks on machine degradation and machining performance is studied. Further, the dynamic evolutions of machine reliability and product quality are prognosticated, and the operation status and quality loss of the serial-parallel manufacturing system are analyzed. In conclusion, the contributions of this paper are summarized as follows.

(1) A reliability model based on the mixture of two Weibull distributions is proposed, providing an effective method to analyze machine reliability under heterogeneous feedstocks.
(2) The stochastic production and dissemination models of low-quality products and feedstocks are constructed when considering machine reliabilities, which are critical for analyzing the interaction between machine reliability and feedstock quality in a manufacturing system.
(3) An effective computational framework is proposed to analyze system performance under mixture machine degradation.

The rest of this paper is structured as follows: Section 2 depicts the machine degradation when processing low-quality feedstocks and the flow of low-quality feedstocks in a serial-parallel manufacturing system. In Section 3, the mixture degradation model is constructed to depict machine reliability when processing heterogeneous feedstocks. In Sections 4, we establish a stochastic model for the flow of low-quality products when considering the process capability of a machine. In Section 5, the evaluation framework of system performance is proposed. In Section 6, the simulation experiment is designed and performed to analyze the performance of the serial-parallel manufacturing system. Section 7 gives the conclusions about this work.

2. Serial-parallel manufacturing systems with low-quality feedstocks

2.1. Machine degradation when processing low-quality feedstocks

As one mechanical system, machine degradation is affected by component wear, noise, and other random factors. According to Archard and Dawson's research, machine degradation closely depended on the load force, the sliding distance, and the contacting surface [34], which would be changed by dimensional deviations of low-quality feedstocks. One typical component impacted by feedstocks is the fixture system that is extensively used in manufacturing machines. As shown in Fig. 1(a), it is a 3–2–1 fixture system equipped for locating manufactured parts. During its operation, the two locating pins (P_1 and P_2) constrain the part rotation and translation in the X-Y plane, and the three locating blocks (B_1, B_2, B_3), used as locators working with clamps, constrain the part movement in the Z direction [35].

In the 3–2–1 fixture system, for one low-quality feedstock using holes as the location datum, unreliable locating will occur due to its dimensional deviations, as shown in Fig. 1(b). It may be noted that these dimensional deviations will lead to the vibration of low-quality feedstocks when being processed, increasing relative slippages between the feedstock and fixture system. Therefore, the rising wear of fixture systems will appear, so that the machine will operate with a more severe degradation. In summary, the low-quality feedstock plays a large part in disturbing load force, sliding distance, or contacting surface in a machine, all of which may further aggravate machine degradation. Also, the unstable production process may trigger additional noises to other components in a machine, and aggravate its degradation in a severe environment. This process is defined as the degradation under the severe environment, where the machine deteriorates more quickly than the baseline environment of processing high-quality feedstocks. Therefore, the quality status of feedstocks, as one important factor affecting machine degradation, cannot be ignored in the analysis of manufacturing system performance.

2.2. Flow of low-quality feedstocks in serial-parallel manufacturing systems

In a manufacturing system with serial-parallel configurations, the flow of low-quality feedstocks among machines connected by different structures will determine which machine will suffer severe degradation. As shown in Fig. 2, the serial and parallel configurations are basic structures in a manufacturing system. In general, the deterministic flow between serial machines and stochastic flow among parallel machines can be summarized.

In a serial-parallel manufacturing system, the deterministic flow of feedstocks only occurs between serial machines, as shown in Fig. 2(a). Here, all feedstocks from one previous machine will flow to its subsequent machine. Between serial machines, the flow and impact of low-quality feedstocks will have two stages: firstly, degradation of the previous machine will impact dimensional deviations of manufactured products, increasing the frequency of low-quality products; secondly, low-quality products become low-quality feedstocks of the subsequent

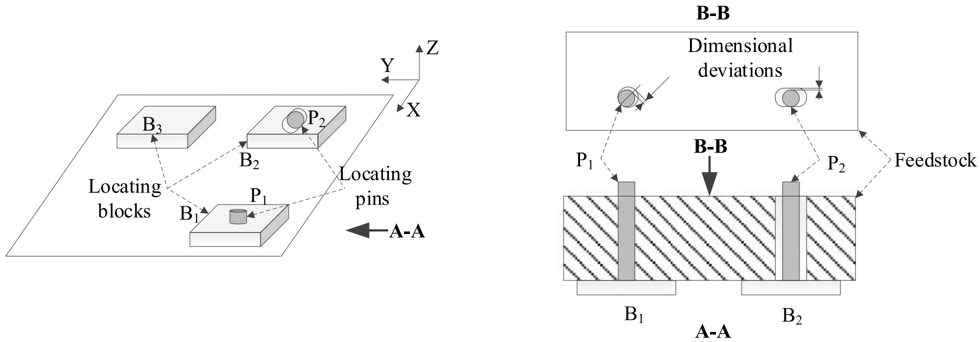

Fig. 1. Impacts of low-quality feedstocks on machine degradation.

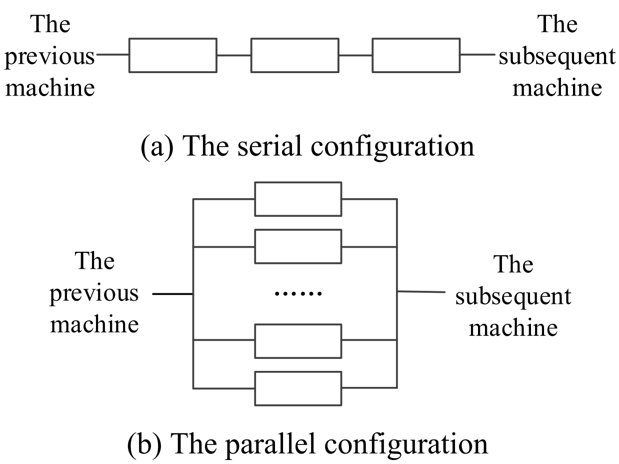

Fig. 2. The basic configurations in the manufacturing system.

machine, and play a role in the degradation of the subsequent machine. Note that low-quality feedstocks will deterministically flow to the subsequent machine and the distributary does not occur here.

Among parallel machines, there will be uncertainty about the flow of low-quality feedstocks. This uncertainty has two styles: one is that machine degradation will be impacted by multiple sources of low-quality feedstocks, which is called multi-sources uncertainty, such as the subsequent machine with multi-sources from the parallel machines, shown in Fig. 2(b); another is that low-quality products produced by the previous machine will be transported into different subsequent machines, which is called the multi-termini uncertainty, such as the previous machine with multi-termini to the parallel machines in Fig. 2(b). All these will increase the randomicity of the flow of low-quality feedstocks in a serial-parallel manufacturing system.

To depict the flow of feedstocks in a serial-parallel manufacturing system with S machines, an $S \times S$ transmission matrix G is constructed as

$$G = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1S} \\ g_{21} & g_{22} & \cdots & g_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ g_{S1} & g_{S2} & \cdots & g_{SS} \end{bmatrix}, \tag{1}$$

where $g_{ij} \in [0, 1]$ with the following meanings:

$g_{ij} \in (0, 1)$: means that the machine i will randomly provide the percentage g_{ij} of its products to the machine j as the feedstocks, and the machine j has at least one parallel machine.
$g_{ij}=0$: means that no feedstock flows from machine i to machine j, representing that there is no directed connection from the machine i to machine j in the current system.
$g_{ij}=1$: means that all products exported by the machine i will become the feedstocks of the machine j, namely the machine i and j have the serial configuration.

For each machine i owning subsequent machines, it satisfies $\sum_{j=1}^{S} g_{ij} = 1$. Also, for each machine i, denote $\Omega_i = \{j \mid g_{ij} \neq 0 \text{ for } j \in [1, S]\}$ as the set of machines connected after the machine i. If using $|\Omega_i|$ to represent the number of elements in set Ω_i, then we have:

when $|\Omega_i|=1$, the machine i and j have a serial configuration.
when $|\Omega_i|>1$, the machines in Ω_i are parallel after the machine i.
when $|\Omega_i|=0$, it means that the machine i is one ending machine having no subsequent machines.

The modeling assumptions in the present study are listed as follows.

(1) Product transmission between stations are reliable, and the transmission time is not considered.
(2) If there is no inspection process after a real-world manufacturing machine, all products can be considered to pass the test, and the inspection process is virtual.
(3) Original system feedstocks (feedstocks of the first machine in the system) are always high-quality. In other words, raw materials are always qualified.
(4) The productivity of each machine is assumed to be stable, which will not change with time.
(5) The low-quality feedstock is the random sample in all feedstocks, so that each feedstock, no matter low-quality or high-quality ones, has the same probability flowing into each subsequent machine.

3. Machine reliability modeling under the impact of heterogeneous feedstocks

3.1. Mixture failure rate of the machine

A machine operates in a baseline environment when processing the high-quality or qualified feedstocks, but operates in a severe environment when processing the low-quality or unqualified feedstocks. As shown in Fig. 3, when processing high-quality feedstocks during $[0, t_1]$, $[t_1+h_1, t_2]$, $[t_2+h_2, t_3]$, and so on, the machine will follow the baseline-degradation trajectory; however, when processing low-quality feedstocks during $[t_1, t_1+h_1]$, $[t_2, t_2+h_2]$, $[t_3, t_3+h_3]$, and so on, the machine will follow the severe-degradation trajectory. Therefore, it is reasonable to assume that the machine has a mixture degradation process when suffering both baseline and severe degradations.

Regardless of baseline and severe degradations, when considering the practical situation that all machines suffer wear and tear and deteriorate with an increase in age, an effective reliability model with an increasing failure rate is necessary for analyzing machine performance.

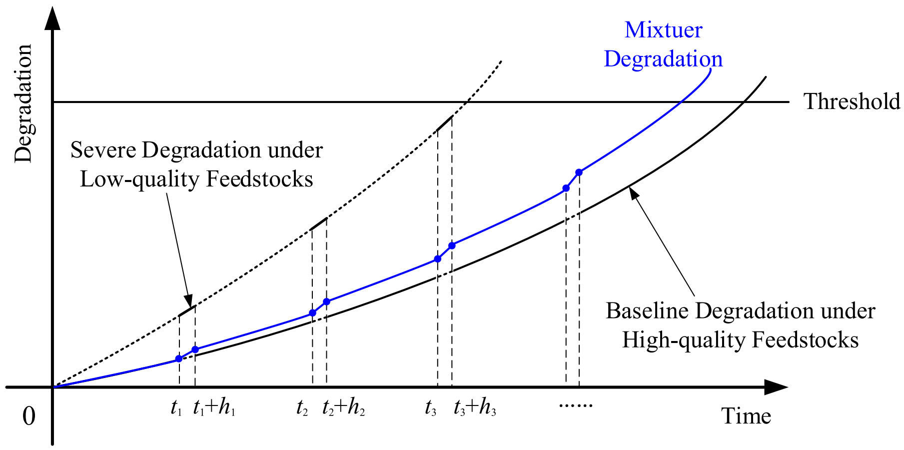

Fig. 3. Degradation trajectories of a machine with heterogeneous feedstocks.

Due to the flexibility to characterize various failure patterns [36], Weibull distribution has become one popular distribution used in the analysis of machine reliability. Consider an aging machine that operates in a baseline condition, denotes the corresponding cumulative distribution function (CDF) of time to failure by $F_b(t)$, which obeys one Weibull distribution. Assume this machine will operate in a severe environment with the CDF of time to failure denoted by $F_s(t)$, obeying another Weibull distribution, as shown in Functions (2) and (3).

$$F_b(t) = 1 - e^{-(t/\alpha)^{\beta}}, \tag{2}$$

$$F_s(t) = 1 - e^{-(t/\omega)^{\pi}}, \tag{3}$$

where $\alpha \in (0, +\infty)$ and $\omega \in (0, +\infty)$ are scale parameters, and $\beta \in (0, +\infty)$ and $\pi \in (0, +\infty)$ are shape parameters. When modeling the degradation with an increasing failure rate, the shape parameters satisfy $\beta > 1$ and $\pi > 1$.

The failure rate $r(t)$ is an effective index depicting machine failure over time, which characterizes the conditional probability of failure occurred at the interval $(t, t+\Delta t)$ when the item does not fail before time t [37], namely its lifetime T satisfies the probability function: $P(t<T<t+\Delta t \mid T>t) = r(t) \cdot \Delta t$. Denote the failure rates in the baseline environment by $r_b(t)$ and in the severe environment by $r_s(t)$, obtained by the Function (4) and (5),

$$r_b(t) = \frac{f_b(t)}{R_b(t)} = \frac{\beta}{\alpha} \cdot \left(\frac{t}{\alpha}\right)^{\beta-1}, \tag{4}$$

$$r_s(t) = \frac{f_s(t)}{R_s(t)} = \frac{\pi}{\omega} \cdot \left(\frac{t}{\omega}\right)^{\pi-1}, \tag{5}$$

where $f_b(t) = F_b'(t)$ and $f_s(t) = F_s'(t)$ are the probability density functions in baseline and severe environments, and $R_b(t)$=1-$F_b(t)$ and $R_s(t)$=1-$F_b(t)$ are the reliability functions.

According to the analysis in Section 2.1, it is reasonable to assume that degradation in the severe environment is more intensive, so that the time to accumulate the same amount of degradation or wear is shorter than in the baseline one [18]. Namely, Inequality (6) exists:

$$\overline{F_s}(t) < \overline{F_b}(t). \tag{6}$$

Further, according to Inequality (6), the scale and shape parameters satisfy that:

$$\frac{\omega^{\pi}}{\alpha^{\beta}} \cdot t^{\beta-\pi} < 1. \tag{7}$$

In actual engineering applications, this inequality can be used to verify the validity of parameters obtained by the parameter estimation.

Assume that the cumulative distribution function (CDF), the probability density function (PDF), and the failure rate under the mixture degradation process are denoted by $F_m(t)$, $f_m(t)$, and $r_m(t)$. Here, we define the CDF and PDF under the mixture condition as follows,

$$F_m(t) = \tau F_s(t) + (1-\tau) F_b(t), \tag{8}$$

$$f_m(t) = \tau f_s(t) + (1-\tau) f_b(t), \tag{9}$$

where τ is the weight of degradation under the intensive working condition, which can be obtained by the proportion of low-quality feedstocks processed in the machine. And, the mixture failure rate of the machine suffering mixture degradation is calculated by:

$$\begin{aligned} r_m(t) &= \frac{f_m(t)}{\overline{F}_m(t)} = \frac{\tau f_s(t) + (1-\tau) f_b(t)}{1 - \tau F_s(t) - (1-\tau) F_b(t)} \\ &= \frac{\tau f_s(t) + (1-\tau) f_b(t)}{\tau \overline{F}_s(t) + (1-\tau) \overline{F}_b(t)} \\ &= \Psi(t) \cdot r_s(t) + (1 - \Psi(t)) \cdot r_b(t) \end{aligned}, \tag{10}$$

where $\Psi(t) = \frac{\tau \overline{F_s}(t)}{\tau \overline{F_s}(t) + (1-\tau) \overline{F_b}(t)}$.

3.2. Process capability of the machine

The process capability of a machine is depicted as the probability that a feedstock will be transformed into a qualified product, which is impacted by machine degradation. Therefore, one machine will have a time-varying probability to produce a qualified product, denoted by $P_c(t)$, and a relevant probability to produce an unqualified product, denoted by $P_b(t)$=1-$P_c(t)$. Similar to the failure rate of a machine, the index $P_b(t)$, being affected by machine degradation, is also one conditional probability on condition that the machine does not fail before time t, namely $P_b(t)$=P(*The feedstock processed at time t will become an unqualified product* | $T>t$), where t is less than the lifetime T of the machine. Here, we constitute one exponential representation to depict the process capability affected by the mixture failure rate, and the function of index $P_b(t)$ is defined by the Function (11):

$$P_b(t) = \begin{cases} 1 - \varepsilon \cdot e^{-\sigma \cdot r_m(t)} & \text{When} T > t \\ 1 & \text{When} T \le t \end{cases}, \tag{11}$$

where $\varepsilon \in (0,1]$, and $\sigma \in (0, \infty)$. On condition that the machine does not fail ($T>t$), when the mixture failure rate $r_m(t)$ increases, the index $P_b(t)$ will also increase and be close to 1 from the left side; conversely, when $r_m(t)$ decreases to be zero, $P_b(t)$=1-ε becomes a constant, representing the process capability under the perfect condition. Then, on condition that the machine fails before time t ($T \le t$), $P_b(t)$ equals to 1, meaning that the machine will fail and cannot produce any qualified product.

4. Product quality under the mixture degradation of machines

4.1. Stochastic modeling for manufacturing low-quality products

Product quality is affected by the process capability of the machine, whose degradation will cause more low-quality products. In real production processes, the existence of random fluctuations and small sustained shifts may lead to the constant changing of the process capability of machines. Accordingly, the manufacturing of low-quality products in a given period has a time-varying probability. In engineering fields, non-homogeneous Poisson process (NHPP) is one mathematical model to monitor the number of nonconformities or defects in a unit from a repetitive process, expressing the time-varying probability of a given number of nonconformities occurring in a fixed interval of time with a time-dependent arrival rate $\lambda(t)$ [37,38]. Also, with the attributes of tractability and simple form of the rate, NHPP has been proved to be effective to depict engineering phenomena, such as the external shocks with the changing rate in time environment [39], the breakdown process of a product under the time-dependent tendency [40], and the fatal shocks of accidental production errors with the degradation or tool wear process [41]. Therefore, considering the nonstationary process capability of machines, NHPP is one appropriate model to depict the number of defects occurring in a fixed interval of time.

In this paper, a random variable $M(t)$ obeying the NHPP is used to depict the number of low-quality products produced by a machine until time t. According to the properties of NHPP, the model is depicted as follows:

(1) $M(0)=0$, meaning that no low-quality product is produced at time $t=0$;
(2) The process has independent increments, namely $M(t_2)$-$M(t_1)$ and $M(t_4)$-$M(t_3)$ are independent for any $t_1<t_2<t_3<t_4$, representing that the number of low-quality products only depends on the process capability of machines in the current period;
(3) $P\{M(t+\Delta t)-M(t)\geq 2\}=o(\Delta t)$, representing that the number of low-quality products manufactured by a machine is not more than 1 within the unit processing time Δt;
(4) $P\{M(t+\Delta t)-M(t)=1\}=\lambda(t)\cdot\Delta t+o(\Delta t)$, namely the probability that a low-quality product is manufactured within the unit processing time Δt.

where Δt is the processing time of unit feedstocks; $\lambda(t)$ is the intensity function determined by the process capability, which is proportional to the index $P_b(t)$ of a machine. With the aggravation of machine degradation, the probability of low-quality products occurring will increase. For a machine with productivity P_d and index $P_b(t)$ of process capability, the intensity function can be calculated by the Function (12).

$$\lambda(t) = P_d \cdot P_b(t). \tag{12}$$

Here, the intensity function satisfies that: $\lambda(t) < P_d \cdot 1 = P_d < \infty$, representing that the intensity of outputting low-quality products does not exceed the productivity of the machine. Also, we denote the average low-quality products until time t as $E[M(t)]=m(t)$, which can be obtained by the integral of intensity function,

$$m(t) = \int_0^t \lambda(s)ds. \tag{13}$$

Then, we can obtain the probability function that n_m low-quality products are produced in a machine during any interval $[t, t+h]$,

$$P\{M(t+h) - M(t) = n_m\} = e^{\{-(m(t+h)-m(t))\}}\frac{[m(t+h)-m(t)]^{n_m}}{n_m!}, \tag{14}$$

where,$n_m \geq 0$ is the number of low-quality products processed by the machine. Namely, $M(t+h)$-$M(t)$ obeys the Poisson distribution with mean $m(t+h)$-$m(t)$ [42].

Note that, in a manufacturing system, the inspection process can effectively prevent low-quality products flowing to the subsequent machine and reduce the occurrence of intensive degradation [12]. However, the inspection process does not always work successfully, which may give a wrong decision to pass the low-quality product (i.e., false negative). Assume that random variable Y is the result of the judgment of an inspection process for one low-quality product. Obviously, for each low-quality product, the inspection process only has two states, i.e., pass or no pass in the judgment, which obeys one Bernoulli distribution, as shown in Function (15).

$$\begin{cases} P\{Y=1\} = p \\ P\{Y=0\} = 1-p \end{cases}, \tag{15}$$

where $Y=1$ represents that the inspection process has a wrong judgment for the low-quality product. Then, the number of low-quality products flowing and not flowing to the subsequent machine can be depicted by two different stochastic variables $\{N(t+h)$-$N(t), t\geq 0, h\geq 0\}$ and $\{N'(t+h)$-$N'(t), t\geq 0, h\geq 0\}$, as follows:

$$N(t+h) - N(t) = \sum_{i=1}^{M(t+h)-M(t)} Y_i, \tag{16}$$

$$N'(t+h) - N'(t) = \sum_{i=1}^{M(t+h)-M(t)} (1-Y_i). \tag{17}$$

Assume that n ($n \leq n_m$) low-quality products will flow to subsequent machines and m ($m \leq n_m$) low-quality products will be discovered by the inspection process and do not flow during $[t, t+h]$, where, $n+m=n_m$. So, we can obtain the joint probability shown in Function (18).

$$\begin{aligned}
&P\{N(t+h) - N(t) = n, N'(t+h) - N'(t) = m\} \\
&= P\{N(t+h) - N(t) = n, N'(t+h) - N'(t) = m | M(t+h) - M(t) = n_m\}\cdot P\{M(t+h) - M(t) = n_m\} \\
&= C_{n_m}^n p^n (1-p)^{n_m-n}\cdot e^{\{-(m(t+h)-m(t))\}}\frac{[m(t+h)-m(t)]^{n_m}}{n_m!} \\
&= \frac{n_m!}{n!(n_m-n)!}p^n(1-p)^{n_m-n}\cdot e^{\{-(m(t+h)-m(t))\}}\frac{[m(t+h)-m(t)]^{n_m}}{n_m!} \\
&= \frac{n_m!}{n!m!}p^n(1-p)^m\cdot e^{\{-(m(t+h)-m(t))\}}\frac{[m(t+h)-m(t)]^{n_m}}{n_m!} \\
&= e^{\{-p\cdot(m(t+h)-m(t))\}}\frac{[p\cdot(m(t+h)-m(t))]^n}{n!}\cdot e^{\{-(1-p)\cdot(m(t+h)-m(t))\}}\frac{[(1-p)\cdot(m(t+h)-m(t))]^m}{m!}
\end{aligned} \tag{18}$$

Obviously, $N(t+h)$-$N(t)$ and $N'(t+h)$-$N'(t)$ can be considered as two random variables obeying NHPP with means $p(m(t+h)-m(t))$ and $(1-p)(m(t+h)-m(t))$ respectively [42]. Therefore, the probability that there are n low-quality products flowing to subsequent machines during $[t, t+h]$ can be obtained, as shown in Function (19).

$$P\{N(t+h)-N(t)=n\} = e^{\{-p\cdot(m(t+h)-m(t))\}}\frac{[p\cdot(m(t+h)-m(t))]^n}{n!}. \tag{19}$$

4.2. Stochastic modeling for flowing of low-quality feedstocks

After inspection, the product processed by the previous machine will become feedstocks of the subsequent machine. Between serial machines, all these undetected low-quality feedstocks will be transported to the subsequent machine and the distributary does not occur. Therefore, the number of low-quality feedstocks flowing to the subsequent machine can be depicted by the Function (19) directly.

In parallel configurations, the flow of low-quality feedstocks is more complicated. In the multi-termini situation, feedstocks from the previous machine i will be assigned to different subsequent machines j with probabilities g_{ij}. According to the Assumption (5), as the sample of feedstocks, low-quality feedstocks $N(t+h)$-$N(t)$ will also have the same assignation property with the population of feedstocks, namely the same transmission matrix G. Assume that the random variable $N_{rij}(t+h)$-$N_{rij}(t)$ represents the number of low-quality feedstocks imported to subsequent machines j from the previous machine i during $[t, t+h]$. From the generalization of Function (19), we can get that all $N_{rij}(t+h)$-$N_{rij}(t)$ are independent for each subsequent machine j, which obeys the non-homogeneous Poisson process with the mean $pg_{ij}(m(t+h)-m(t))$ [42]. Therefore, for each previous machine i with the multi-termini transmission, the probability that n_{rij} low-quality feedstocks are assigned to the subsequent machines j can be obtained:

$$\begin{aligned} &P\{N_{rij}(t+h)-N_{rij}(t)=n_{rij}\} \\ &= e^{\{-p_i g_{ij}\cdot(m(t+h)-m(t))\}}\frac{[p_i g_{ij}\cdot(m(t+h)-m(t))]^{n_{rij}}}{n_{rij}!}. \end{aligned} \tag{20}$$

In the multi-source situation, the total number of low-quality feedstocks imported to the subsequent machine j can be obtained by the accumulation of each source machine, namely the accumulation of several non-homogeneous Poisson processes. Therefore, for each subsequent machine j with the multi-source transmission, the number of imported low-quality feedstocks can be obtained by the Eq. (21):

$$N_{imported\ to\ j} = \sum_i (N_{rij}(t+h)-N_{rij}(t)). \tag{21}$$

5. Evaluation framework of system performance

The generation of random numbers is the key step for pertinent computations about the non-homogeneous Poisson process [43]. In this paper, the sparse method is used to generate random numbers obeying the non-homogeneous Poisson process. In our model, the intensity function satisfies $\lambda(t)=P_d \cdot P_b(t) \le P_d = \lambda_0$, and according to Assumption (4), the productivity P_d of each machine is a known constant. Therefore, in the time interval $[t, t+h]$, arrival instants obeying the non-homogeneous Poisson process can be generated by following steps of the sparse method:

- Generate arrival instants $s_1, s_2, \ldots s_k \in [t, t+h]$, obeying homogeneous Poisson process with parameter λ_0.
- Generate the random numbers $x_1, x_2, \ldots x_k$ locating in [0, 1]. If $x_i \le \lambda(s_i)/\lambda_0$, keep s_i; else, abandon s_i.
- All reserved s_i constitute the arrival instant obeying non-homogeneous Poisson process with intensity function $\lambda(t)$.

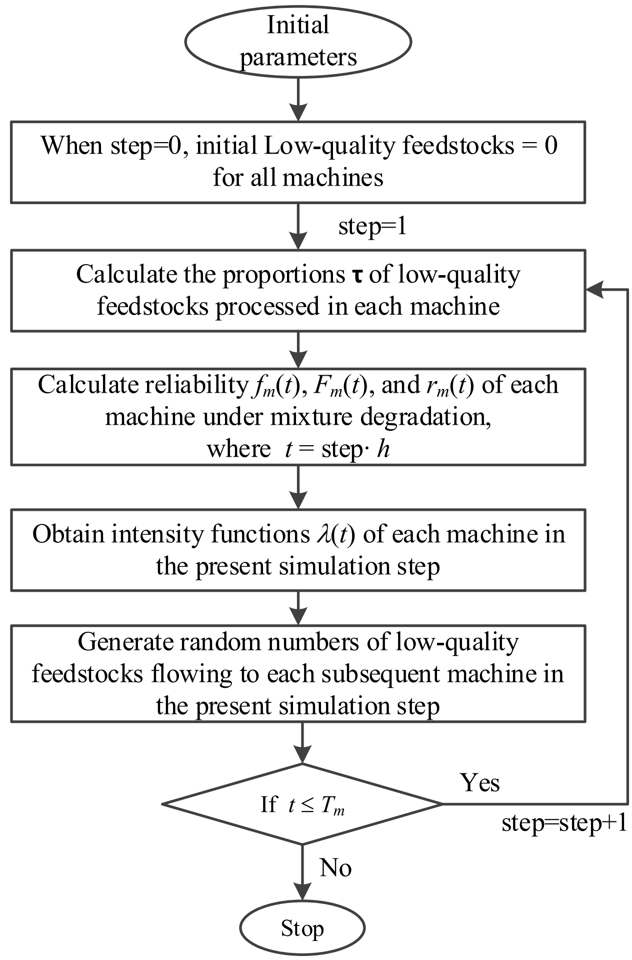

Fig. 4. The evaluation framework.

According to the sparse method, the simulation framework is proposed to implement the evaluation, as shown in Fig. 4. To prognosticate system performance until time T_m, the simulation is implemented by step size h. According to the Assumption (3), raw materials of the manufacturing system are supposed to be perfect, so that feedstocks of the first machine are always high-quality. And the initial feedstock of each machine at step=0 is also high-quality. In each interval $[t, t+h]$, four circulated procedures should be finished according to our proposed models, as follows.

Operation 1: Considering all assigned low-quality products in the last simulation step, calculate the proportion τ of accumulative low-quality feedstocks processed in each machine until the present simulation step.

Operation 2: Based on the Functions (8), (9), and (10), calculate the mixture reliability indexes $f_m(t)$, $F_m(t)$, and $r_m(t)$ for each machine with the mixture degradation, where $t = step \cdot h$.

Operation 3: Through the Functions (11) and (12), calculate the intensity function $\lambda(t)$ for producing low-quality products.

Operation 4: According to the non-homogeneous Poisson processes and sparse method, generate random numbers of low-quality products produced and flowing in each machine in the present simulation step.

6. Simulation experiments

6.1. Experimental design

Considering the serial-parallel manufacturing system in Fig. 5, we design a simulation experiment with stochastic parameters as shown in Table 1 for the proposed model, where the scale and shape parameters of

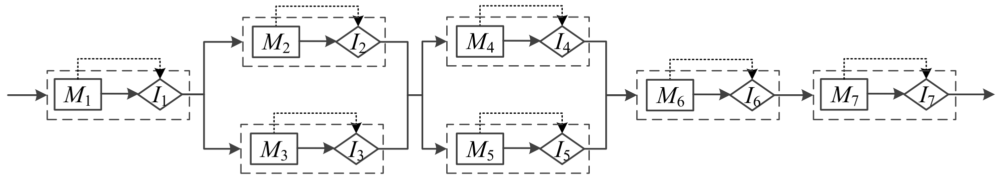

Fig. 5. One manufacturing system with serial-parallel structure.

Table 1
Simulation parameters.

Machines	M_1	M_2	M_3	M_4	M_5	M_6	M_7
P_d	4	2	2	2	2	4	4
p	0.0850	0.0729	0.0448	0.0651	0.0170	0.0531	0.0634
α	7143	3070	2185	4010	2118	8367	8593
β	1.07	1.51	1.62	1.73	1.23	1.02	1.14
ω	3162.22	1623.36	1125.55	1963.43	1055.75	3411.02	3381.37
π	1.17	1.64	1.77	1.90	1.35	1.13	1.27
ε	0.95	0.99	0.93	0.98	0.92	0.95	0.95
σ	12.08	9.85	8.27	15.71	14.22	17.21	18.73

Weibull distributions must satisfy the Inequality (7).

Here, we consider a time-balance serial-parallel manufacturing system, where the productivity of each serial machine equals the sum of parallel machines. For example, Machines 1, 2, and 3 in Fig. 5 satisfy P_{d1}=P_{d2} +P_{d3}. Also, the simulation step size h is set as the integer multiple of the max processing time per product in each machine, namely the integer multiple of 0.5 ($1/P_{di,}$ $i = 2,3,4,5$). Assume that each machine has the same probability with its parallel machines to get feedstocks from their previous machines, and the transmission matrix G is shown in Eq. (22).

$$G = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{22}$$

6.2. Analysis of experimental results

6.2.1. The effectiveness of simulation experiments

To analyze the effectiveness of the simulation framework, four simulation experiments for the same T_m=15,000 with different step sizes are carried out. We define the accumulative quality loss of a machine (AQLM) as the percentage of accumulative low-quality products produced by it. As shown in Table 2, the quality loss of each machine is obtained by the simulation experiment, and their means and variances are also calculated. The results show that four simulation experiments do not have any significant difference in the variance of quality loss in each machine. Therefore, the proposed simulation framework can be judged to have the ability to stably and effectively describe the process capability of machines suffering from mixture degradation.

6.2.2. Machine reliability and process capability

According to the Assumption (3) that all original system feedstocks are high-quality, Machine 1 is assumed to operate only in the baseline environment. Here, only simulation analysis of other machines with mixture degradation will be presented. Considering the coherence of simulation experiments with different step sizes, the one with $h = 1.5$ is chosen to analyze the dynamic performance of machines suffering from the mixture degradation. As shown in Fig. 6, the accumulative distribution function of machine failure under mixture degradation is simulated. With step size $h = 1.5$, the simulation also gets the difference of accumulative distribution functions between the mixture and baseline conditions ($F_{difference}$=$F_m(t)$-$F_b(t)$) and obtains their polynomial fitting curves. The coefficients of determinations (R^2) are shown in the first row in Table 3, demonstrating a well-fitting for all curves.

Note that whatever the degradation rate is (Machines 2, 3, 4, 5 have higher degradation rates than Machines 6 and 7), each machine suffering from mixture degradation has a higher failure probability than the baseline condition, which is illustrated by the non-negative difference $F_{difference}$ in Fig. 6. Due to the accumulation of imported low-quality feedstocks, the difference in failure probability will increase with time. But when machines have a high failure probability, this difference will become weak. This suggests that the mixture degradation analysis is more important for the early machine operation than its later operation. Although there is a difference in failure probability between mixture and baseline conditions, it is not outstanding because the baseline condition dominates during machine operation. This fits the theory that the mixture failure rate is the one describing the corresponding limiting behavior, where the mixture failure rate asymptotically tends to the failure rate of the strongest subpopulation [19].

Based on the process capability of machines, the intensity function for producing low-quality products, their differences between mixture and baseline conditions ($\lambda_{difference}$=$\lambda_m(t)$- $\lambda_b(t)$), and the polynomial fitting curves of differences are also simulated with step size $h = 1.5$, as

Table 2
Accumulative quality loss of each machine with different simulation steps.

Machines	1	2	3	4	5	6	7
$h = 0.5$	0.0947	0.0180	0.0796	0.0278	0.0856	0.1058	0.1137
$h = 1$	0.0966	0.0170	0.0798	0.0279	0.0915	0.1054	0.1187
$h = 1.5$	0.0941	0.0185	0.0847	0.0272	0.0893	0.1047	0.1136
$h = 3$	0.0977	0.0195	0.0804	0.0281	0.0902	0.1048	0.1151
Mean	0.0958	0.0183	0.0811	0.0278	0.0892	0.1052	0.1153
Variance	2.11E-06	8.12E-07	4.43E-06	1.19E-07	4.83E-06	2.1E-07	4.23E-06

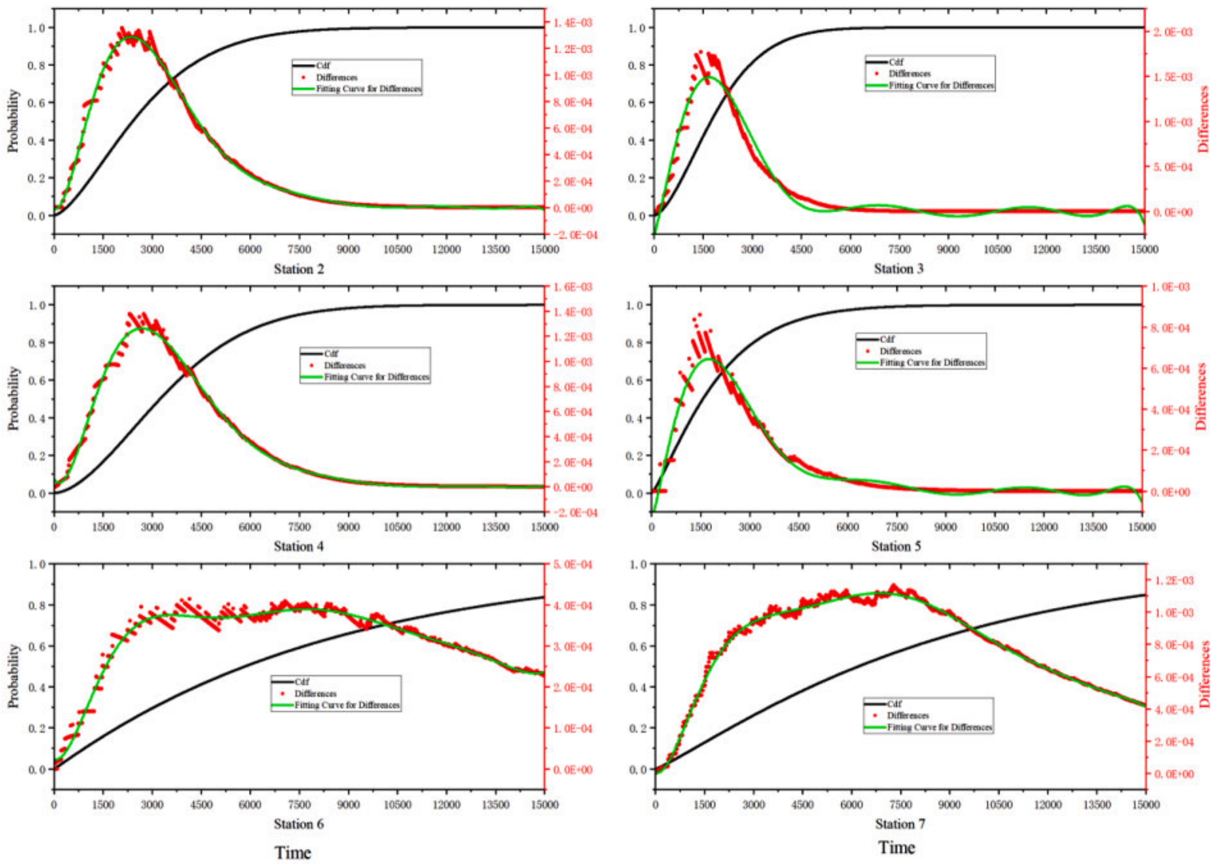

Fig. 6. Accumulative distribution functions under mixture machine degradation and differences to baseline conditions.

Table 3
Coefficient of determination of fitting curves.

Machines	Coefficient of determination (R^2) 2	3	4	5	6	7
$F_m(t)$	0.9964	0.9708	0.9948	0.9709	0.9805	0.9946
$\lambda(t)$	0.9872	0.9739	0.9878	0.9158	0.9232	0.9818

shown in Fig. 7. The coefficients of determinations (R^2) are shown in the second row in Table 3, which demonstrate good fitting performance. Due to the rise of machine failure rates, the intensity function of low-quality products will also increase with time. Similar to the failure probability, the machine under mixture degradation will have a larger intensity function to produce low-quality products than the baseline condition, as shown in the non-negative difference $\lambda_{difference}$ in Fig. 7. This difference will disappear when machine performance gets worse enough; in other words, the difference will become zero during the later time. This also proves that the mixture degradation analysis is more important for the early machine operation than its later operation.

6.2.3. System performance

For this serial-parallel manufacturing system suffering mixture machine degradation, the multi-state performance of the system will be analyzed. Three states are defined to depict system performance: the operation state with full or low productivity, and the failed state. In the operation state with full productivity, no machine fails; in the operation state with low productivity, some but not all parallel machines fail, and the productivity of systems will reduce; in the failed state, some serial machines or all parallel machines fail, and the system cannot finish the whole manufacturing process. According to the system configuration in Fig. 5, the probability function of three states are established in Functions (23).

$$\begin{cases} R_{full}(t) = \prod_{i=1}^{7} R_i(t) \\ R_{low}(t) = \sum_{i=2,3,4,5} \left(\overline{R}_i(t) \cdot \prod_{A-i} R_i(t) \right) + \sum_{\substack{i=2,3 \\ j=4,5}} \left(\overline{R}_i(t) \cdot \overline{R}_j(t) \cdot \prod_{A-i-j} R_i(t) \right), \\ R_{failed}(t) = 1 - R_{full}(t) - R_{low}(t) \end{cases} \tag{23}$$

where $R_i(t)$ is the reliability function of the Machine i, namely $R_i(t)$=1-$F_i(t)$. A represents the set of all machines in the serial-parallel manufacturing system, and A-i represents the set without machine i. Based on the simulation with step size $h = 1.5$, the probability curves of system states and their differences ($R_{difference}$=$R_m(t)$-$R_b(t)$) to the baseline conditions are obtained, as shown in Fig. 8. During the early time, the manufacturing system has a high probability to operate in the full state and the low probability to operate in the failed state. But after operating for enough time, the system will have a gradually-rising probability to operate in the failed state.

Comparing with the baseline condition, the difference of probabilities among the three states also show vastly distinct characteristics. In the early time, the difference in probability of states with full productivity is always negative value, representing that the system under the mixture condition always has a lower probability of operating in the state with full productivity than the baseline condition. However, it is inverse for the probability of the failed state. In the later operation, differences between two states in different conditions will disappear, further proving the importance of mixture degradation analysis in early system operation. It might also be noted that the difference in the

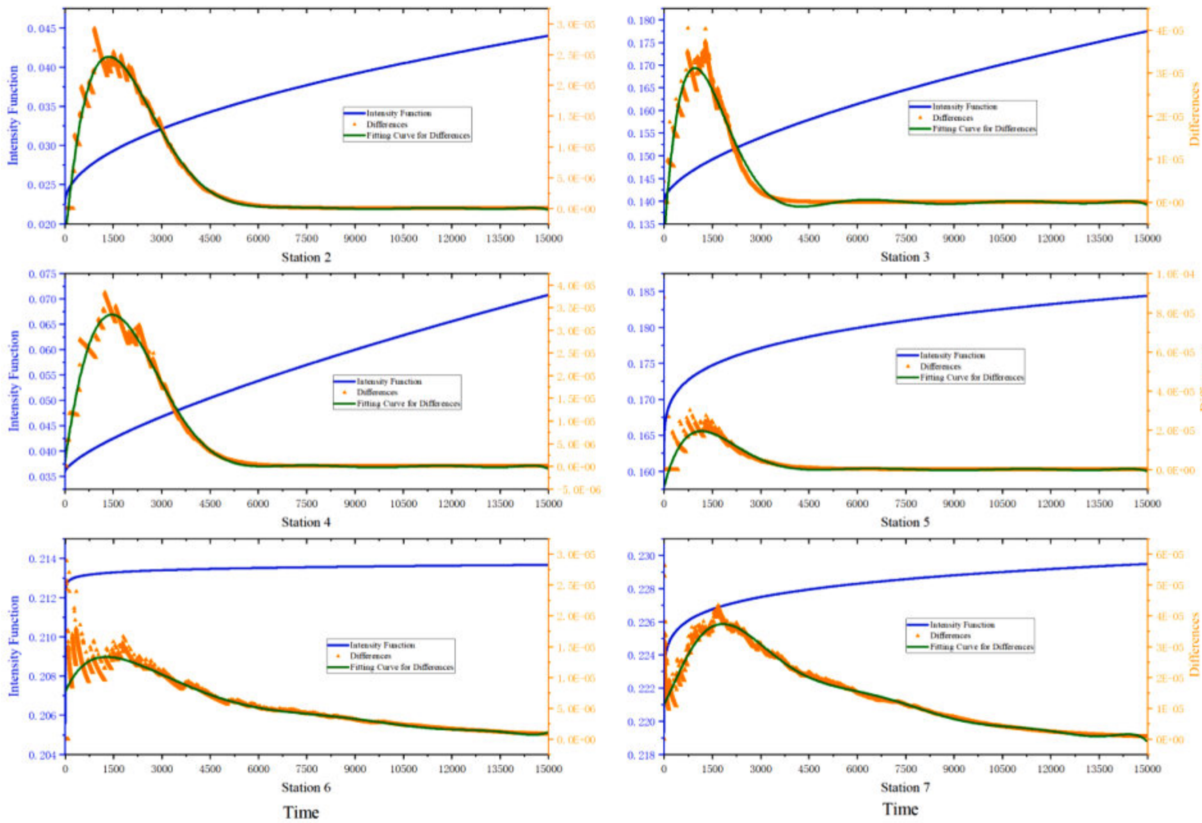

Fig. 7. Intensity functions under mixture machine degradation and differences to baseline conditions.

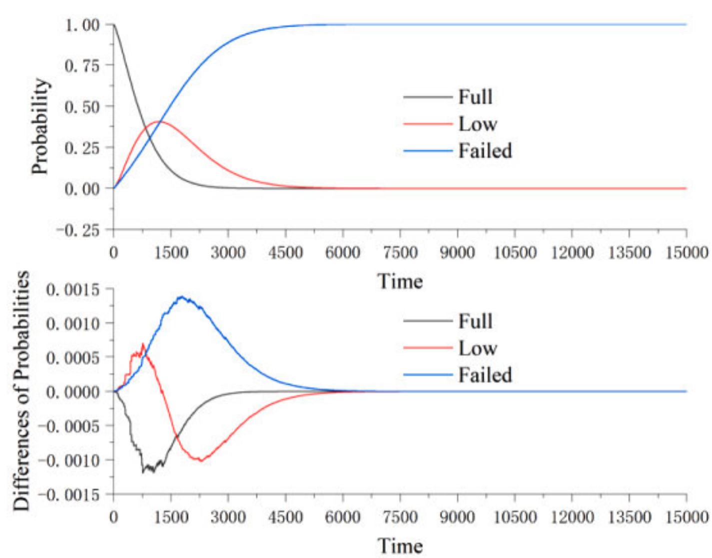

Fig. 8. The probability curves of system states (Full, Low, Failed) and their differences to the baseline conditions.

probability of states with low productivity can be negative or positive values during different operation periods, as shown in Fig. 8. Therefore, when only considering the baseline condition, the probability in states with low productivity will be undervalued during the early operation and overestimated during the later operation.

In the view of product quality, we also implement statistical analysis for the quality loss of the whole system in the simulation with step size h = 1.5. According to productivities P_d = [422224 4], the total number of products being processed in the manufacturing system in one step can be obtained:

$$Num = h\cdot \sum P_d = 1.5 \times 20 = 30. \tag{24}$$

Among the 30 processed products in each simulation step, the percentage of low-quality products is defined as the instant quality loss (IQL) of the system. Also, define the $Y_\tau=\{y_{\tau 1}, y_{\tau 2}, y_{\tau 3}, \dots\}$ as the ordered sequence of simulation steps in which the IQL τ occurs, where $y_{\tau i}$ represents that the percentage τ of products produced in the step $y_{\tau i}$ are low-quality. Then we get all sequences of simulation steps for different levels

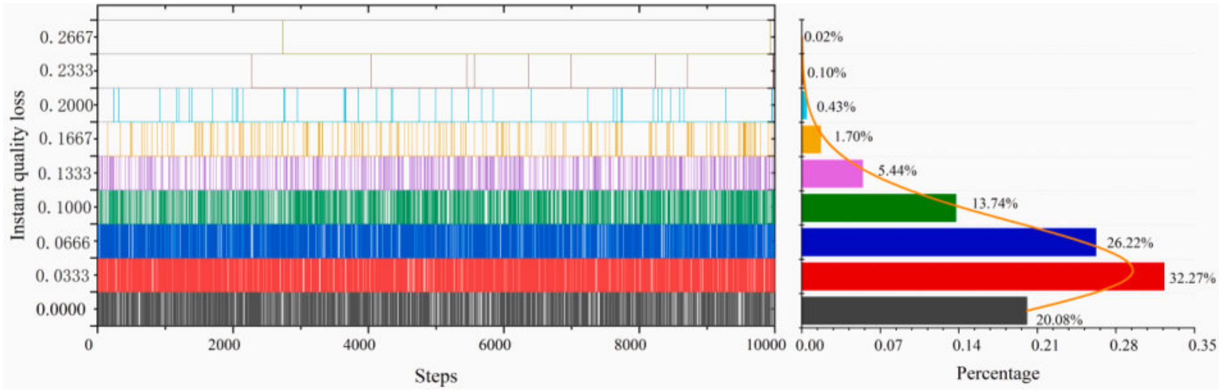

Fig. 9. The sequence of IQLs and statistical analysis.

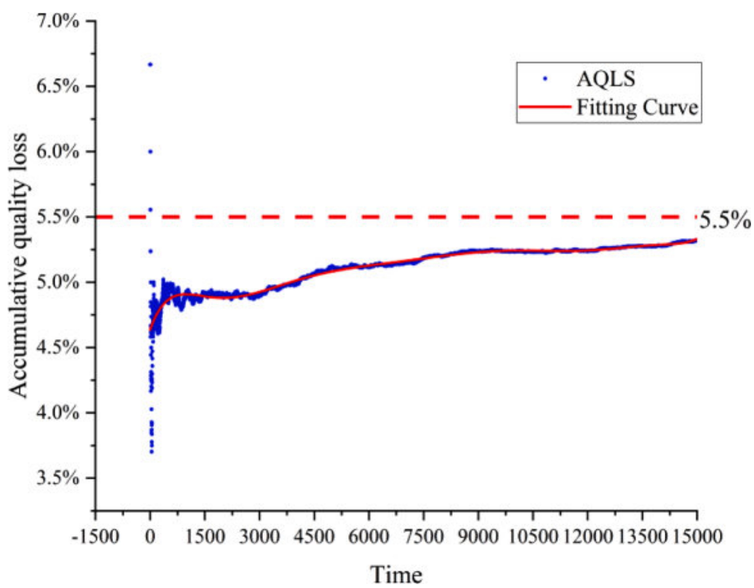

Fig. 10. The dynamic accumulative quality loss of the system.

of IQL, as shown in the left of Fig. 9. In the right of Fig. 9, statistical analysis of IQL is presented, revealing the similarity between the distribution of instant quality loss and the normal distribution. This also proves the effectiveness of our proposed model. Here, the IQL level with the largest probability is 3.33%.

Furthermore, the accumulative quality loss of the whole manufacturing system (AQLS) is counted, as shown in Fig. 10. Because of the insufficiency of samples, the accumulative percentage of low-qualified products presents instability at the beginning of the simulation. Considering enough samples in the whole simulation time, the polynomial fitting curve of the AQLS is also obtained, which will increase with time as shown in Fig. 10. This shows that the quality loss of the whole system will also increase with the aggravation of mixture machine degradation.

7. Conclusions

In this paper, we specifically consider the heterogeneity of feedstock quality, and further investigate the impact on the performance of serial-parallel manufacturing systems. A complete theoretical model is constructed to characterize the mixture machine degradation, random production of low-quality products, and stochastic flow of low-quality feedstocks. According to the proposed evaluation framework, the operation status and quality loss of serial-parallel manufacturing systems can be effectively prognosticated. Experimental results show that it is critical to account for the mixture degradation analysis, which is more necessary for the early machine operation than the later operation. Furthermore, the proposed method is also strongly indicative to improve the predictability of industrial production, which is indispensable to production planning, control, and the realization of digital manufacturing.

In general, the proposed model can be applicable and extended in the industrial field. First, the Weibull distribution helps greatly improve the usability of the mixture model in analyzing machine degradation with different failure patterns. Second, stochastic models of multi-sources and multi-termini flows of low-quality feedstocks can further be extended to study the performance of complex manufacturing networks. Finally, there are future works that can be done to further improve the model. For example, by Assumption (3), this paper proposed a control variable method to study system performance when only considering the inner interactions in the serial-parallel manufacturing system. In other words, only the quality issues of feedstocks inside the system are considered. This helps avoid confusing the outside impact of raw materials with internal causes in the manufacturing system. In future work, further investigations can be focused on quality issues from the outside, e.g., various arrival patterns of low-quality feedstocks in the forms of random or clustered arrivals.

CRediT authorship contribution statement

Zhenggeng Ye: Conceptualization, Methodology, Software, Writing - original draft. **Hui Yang:** Resources, Writing - review & editing. **Zhiqiang Cai:** Methodology, Validation, Supervision, Writing - review & editing, Project administration. **Shubin Si:** Supervision, Writing - review & editing. **Fuli Zhou:** Investigation, Writing - review & editing.

Declaration of Competing Interest

I would like to declare on behalf of my co-authors that the work described was original research that has not been published previously, and not under consideration for publication elsewhere, in whole or in part. No conflict of interest exits in the submission of this manuscript, and all the authors listed have approved the manuscript that is enclosed.

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