Reliability Analysis of Manufacturing Machine with Degradation and Low-quality Feedstocks

Zhenggeng Ye，Zhiqiang Cai
Department of Industrial Engineering
School of Mechanical Engineering, Northwestern Polytechnical University
Xi'an, China
yezhenggeng@outlook.com;
caizhiqiang@nwpu.edu.cn

Hui Yang
The Harold and Inge Marcus Department of Industrial and Manufacturing Engineering
The Pennsylvania State University (University Park)
State College, USA
huy25@psu.edu

***Abstract*—Machine reliability is one major concern in manufacturing industries, which is affected by interior degradation and outside shocks simultaneously. Low-quality feedstocks, as one typical kind of shocks, may arrive randomly during the operation of a machine. However, due to the instability of environment and manufacturing factors, low-quality feedstocks may arrive in clusters in some specific batches. In this paper, we first proposed a reliability evaluation model for the repairable machine, accounting for machine degradation and shocks caused by low-quality feedstocks. Then, the cluster arrival of low-quality feedstocks is modeled by the Hawkes point process with the properties of self-exciting and history dependent. Moreover, considering the degradation and shocks, the mixture failure rate of a machine is modeled. Further, the expectation of remaining lifetime is derived. Finally, the simulation experiment is implemented to compare the performance between the machine when the arrival intervals of low-quality feedstocks follow the Poisson point process and the Hawkes point process. Experimental results show the effectiveness of the proposed model for machine reliability analysis.**

Keywords- feedstocks quality; failure rate; remaining lifetime; shock; cluster; Hawkes process.

The authors (ZY and ZC) thanks research support from the National Natural Science Foundation of China (Nos. 71871181, 71631001), the 111 Project (No. B13044), the Innovation Foundation for Doctor Dissertation of Northwestern Polytechnical University (No. CX202028). The author (HY) thanks Harold and Inge Marcus Professorship for financial support. (Corresponding author: Zhiqiang Cai, caizhiqiang@nwpu.edu.cn)

Z. Ye, and Z. Cai are with School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an, 710072, P.R. China. They are also with the Ministry of Industry and Information Technology Key Laboratory of Industrial Engineering and Intelligent Manufacturing, Northwestern Polytechnical University, Xi'an, 710072, P.R. China. Also, Z. Ye is a visiting scholar at Complex Systems Monitoring, Modeling and Control Laboratory in the Harold and Inge Marcus Department of Industrial and Manufacturing Engineering, Pennsylvania State University (University Park), State College, PA 16802-1401, USA

H. Yang is with the Department of Industrial and Manufacturing Engineering, Pennsylvania State University (University Park), State College, PA 16802-1401, USA.

I. Introduction

Machine reliability and product quality are both major concerns in manufacturing industries [1]. The performance of manufacturing systems is always strongly coupled with machine reliability, quality of feedstocks, and inspection performance [2]. In the engineering field, failure and degradation processes caused by multiple factors have been studied by the common failure analysis method [3], which is also used in the performance analysis of manufacturing systems [4, 5]. Machine degradation also will be affected by multiple factors, such as the wear and tear of parts or other shocks. As one kind of shocks, low-quality feedstocks of a machine have been considered in some researches [2, 5]. In general, feedstocks will be in various quality conditions, such as high-quality, marginal, or low-quality. When feedstocks satisfy the standard well, the machine will operate with normal degradation in a stable environment. But when processing low-quality or unqualified feedstocks, the machine may operate with the more intense degradation, which can be denoted as one random shock of the machine.

Due to environmental instability and other factors, production processes in real life are heterogeneous [6]. In general, the arrival interval of low-quality feedstocks may have different properties. First, if the performance of a machine providing materials is stable, low-quality feedstocks will arrive randomly, which can be depicted by one Poisson process. On the other hand, the cluster arrival of low-quality feedstocks will also occur due to quality issues in some specific batches. In mathematics and engineering fields, these phenomena are usually modeled by Hawkes Process. In mathematics, after first introduced by Hawkes in [7, 8], Hawkes Process has been greatly enriched by developing many new theories, such as the partial self-exciting point process [9], the numerical method for means of linear Hawkes processes [10], and the system theory for the truncated Hawkes process [11]. In engineering, Hawkes Process has been used to evaluate system performance with cluster, self-exciting, or

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history-dependent phenomena, such as the quantitative measure on the self-healing effect on shock model [12] and the traffic accident model based on self-exciting process [13].

Most populations of manufactured items in real life are heterogeneous [5][14], leading to the difference among low-quality feedstocks. Therefore, in addition to the arrival interval, the impact of low-quality feedstocks also shows heterogenicity. In reliability engineering, to evaluate the performance of a machine, the mixture failure rate has been proposed to analyze the degradation and shock caused by heterogeneous factors, such as a mixture of decreasing failure rate distributions or increasing failure rate distributions [15], and the discrete mixture failure rates [16]. There is also an urgent need to model the impact of low-quality feedstocks, which will bring different shocks to a machine.

Considering the broad heterogeneity during the operation of a manufacturing machine, this paper proposed one mathematic model to depict machine reliability by the mixture failure rate evaluation. In conclusion, contributions of this paper can be summarized as follows.

(1) The cluster arrival model of low-quality feedstocks is proposed and modeled by the Hawkes process.

(2) The mathematical expression of the remaining lifetime of a machine after one shock is derived when considering the Exponential lifetime distribution.

The rest of this paper is structured as follows: Section II gives the system statement and mathematic models of low-quality feedstocks with a totally random point process and cluster point process. In Section III, the remaining lifetime of a machine is analyzed by the mixture failure rate model. Sections IV and V illustrate the simulation experiment and conclusion.

II. Research Methodology

A. System Statement

One manufacturing system with a single machine is considered, which is consisted of inputs, one machine, one inspection station, and outputs, as shown in Figure 1. Among the inputs represented by circles, low-quality feedstocks will arrive by the Poisson process or Hawkes process, represented by black solid circles.

When processing low-quality feedstocks, the machine will have a more intense degradation process than the one processing high-quality feedstocks. Due to feedstock heterogeneity, random degradation values will be caused by different low-quality feedstocks. Also, assume that the arrival of low-quality feedstocks follows one counting process $\{N(t); t\geq 0\}$, which has two modes: the totally random point process and cluster point process.

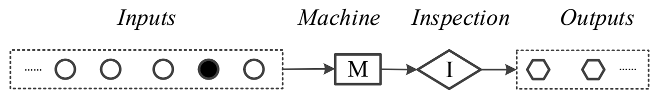


Figure 1. The system with a single machine

B. Totally Random Point Process

The simplest point process is the one occurring 'totally random', which can be depicted by one Poisson process [6]. Here, we think that low-quality feedstocks arrive randomly, and the counting process $\{N(t); t\geq 0\}$ is one Poisson Process with following probability distribution function.

$$P\{N(t+\Delta t)-N(t)=n\}=\frac{e^{-\lambda t}\cdot(\lambda t)^n}{n!}, \tag{1}$$

where the intensity function $\lambda(t)=c$ is one constant. In Figure 2, a possible arrival path of low-quality feedstocks by the Poisson Process is provided, where the intensity function $\lambda(t)=0.5$.

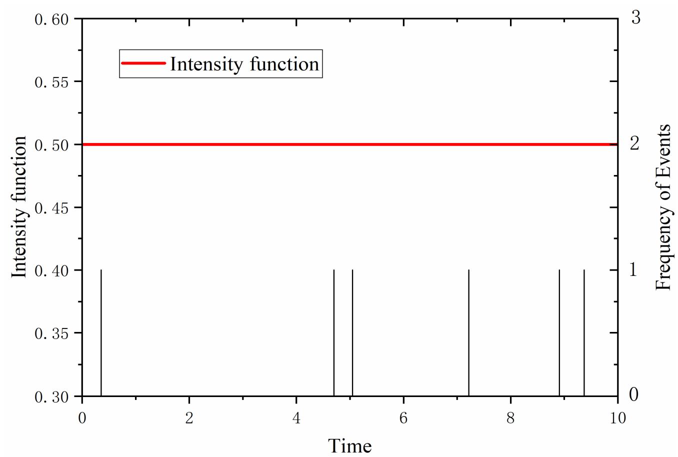


Figure 2. A possible arrival path of low-quality feedstocks by Poisson Process ($\lambda(t)=0.5$)

C. Cluster Point Process

We think that low-quality feedstocks may cluster in some specific batches, and the counting process $\{N(t); t\geq 0\}$ is one linear Hawkes process with the following intensity function.

$$\lambda(t)=v(t)+\int_0^t h(t-u)dN(u), \tag{2}$$

where $h(t)$ is the kernel function ($h(t)>0$ for $t\geq 0$, $h(t)=0$ for $t<0$) [10], as shown in Function (3).

$$h(t)=\begin{cases}\alpha e^{-\beta t}, & t\geq 0\\ 0, & t<0\end{cases}, \tag{3}$$

where $\alpha>0$ and $\beta>0$. In Figure 3, a possible arrival path of low-quality feedstocks by Hawkes Process is provided, where the intensity function is $\lambda(t)=0.5+\sum_{i=1}^{N(t)}0.1\times e^{-(t-t_i)}$.

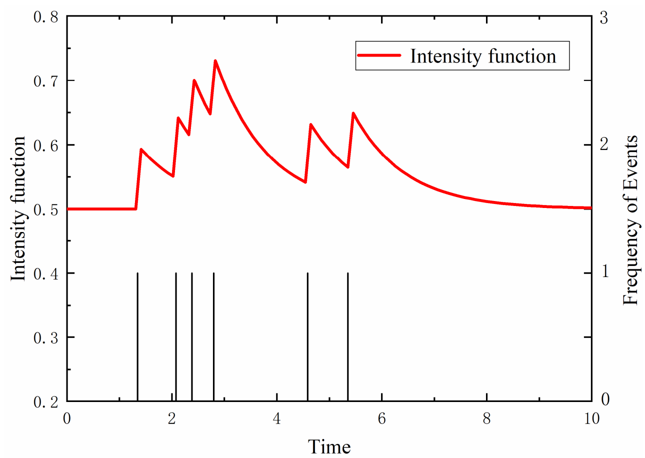


Figure 3. A possible arrival path of low-quality feedstocks by Hawkes Process (v(t)=0.5, α=0.1, β=1)

III. Machine Reliability Analysis

Because of the provision of good approximations to machine failure [17, 18], Exponential distribution becomes one popular model used in the analysis of manufacturing systems. In this paper, we assume that the lifetime T and repair time R of one machine are random variables, which follow Exponential distributions with failure rate r and repair rate μ.

Due to non-decreasing degradation, a machine will have an increasing failure rate. Also, due to the presence of machine degradation and shocks from low-quality feedstocks, the machine will have one mixture failure rate, which is caused by heterogeneous factors. Here, the shock of low-quality feedstocks will result in random additive damage (wear) to a machine, which is often described in terms of the cumulative shock model [6]. Therefore, after the shock, the failure rate r' will have a random increment Δr because of the heterogeneous low-quality feedstocks, namely

$$r' - r = \Delta r\,, \tag{4}$$

where the increment Δr is one random variable locating in $[0, \infty)$ [5]. According to Finkelstein [15], different specific cases can be used as the mixture failure rate, and the additive model is one of the effective models, as

$$r'(t) = r(t) + \Delta r\,, \tag{5}$$

where $r(t)$ is a deterministic, continuous, and positive function for $t \in [0, \infty]$, which can be viewed as the baseline failure rate. In this paper, assume that, after the shock, the lifetime of an Exponential machine still follows one Exponential distribution, but with a worse failure rate r'.

Hence, the failure rate of a machine is the mixture of two exponential failure rates r and r'. Assume that there is one shock of low-quality feedstocks at time x, and the remaining lifetime T_x is one random variable, as shown in Figure 4.

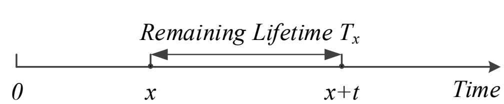


Figure 4. The remaining lifetime of a machine after one shock

Here, we can obtain the cumulative distribution function $F_x(t)$ of the remaining lifetime T_x by the law of conditional probability (on condition that it is operable at x) [6] as

$$\begin{aligned} F_x(t) &= P\{T_x \le t\} \\ &= \frac{P\{x < T \le x+t\}}{P\{T > x\}} = \frac{F(x+t) - F(x)}{\overline{F}(x)}\,, \end{aligned} \tag{6}$$

where $F(x) = 1 - e^{(-rx)}$, $\overline{F}(x) = e^{-rx}$. And we can obtain $F(x+t)$ by the following Function (7).

$$\begin{aligned} F(x+t) &= \int_0^x f(u)du + \int_x^{x+t} f'(u)du \\ &= \int_0^x re^{-ru}du + \int_x^{x+t} r'e^{-r'u}du \\ &= 1 - e^{-rx} + e^{-r'x} - e^{-r'(x+t)} \end{aligned} \;. \tag{7}$$

Therefore, the Function (6) can be written as

$$\begin{aligned} F_x(t) &= \frac{F(x+t) - F(x)}{\overline{F}(x)} \\ &= \frac{1 - e^{-rx} + e^{-r'x} - e^{-r'(x+t)} - (1 - e^{-rx})}{e^{-rx}} \\ &= \frac{e^{-r'x} - e^{-r'(x+t)}}{e^{-rx}} = \frac{e^{-r'x}(1 - e^{-r't})}{e^{-rx}} \\ &= e^{-(r'-r)x}(1 - e^{-r't}) = e^{-\Delta rx}(1 - e^{-r't}) \end{aligned} \;. \tag{8}$$

Differentiating this distribution function, we can obtain its probability density function $f_x(t)$ and expectation $E(T_x)$ of the remaining lifetime as

$$f_x(t) = e^{-\Delta rx} \cdot r'e^{-r't}\,, \tag{9}$$

$$\begin{aligned} E(T_x) &= \int_{-\infty}^{+\infty} t \cdot f_x(t)dt = \int_{-\infty}^{+\infty} t \cdot e^{-\Delta rx} \cdot r'e^{-r't}dt \\ &= e^{-\Delta rx} \cdot \int_{-\infty}^{+\infty} t \cdot r'e^{-r't}dt = e^{-\Delta rx} \cdot \frac{1}{r'} \end{aligned} \;. \tag{10}$$

Here, $1/r'$ is the expectation of machine lifetime under the new Exponential distribution. Therefore, after the shock of one low-quality feedstock, we can obtain the expected remaining lifetime of a machine by its arrival time x and the expectation of machine lifetime under the new Exponential distribution.

IV. Experimental Design and Results

A. Experiment Design

In this section, we design and implement a simulation experiment by MATLAB to evaluate and validate the proposed model. In this simulation, the increment Δr of failure rate is considered to follow one Beta distribution, $B(a, b)$. The simulation time is T=1000. Relative parameters are shown in Tables I.

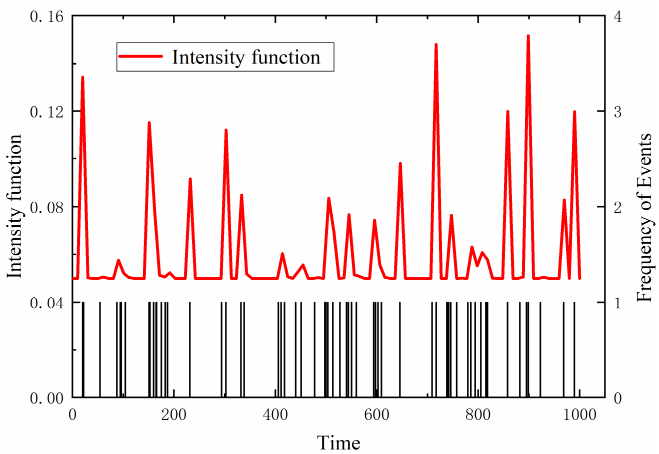
Here, when there are no low-quality feedstocks at time T=0, the arrivals of low-quality feedstocks have the same initial intensity for Hawkes and Poisson processes, namely $\lambda_H(0) = \lambda_P(0)$.

TABLE I. SIMULATION PARAMETERS

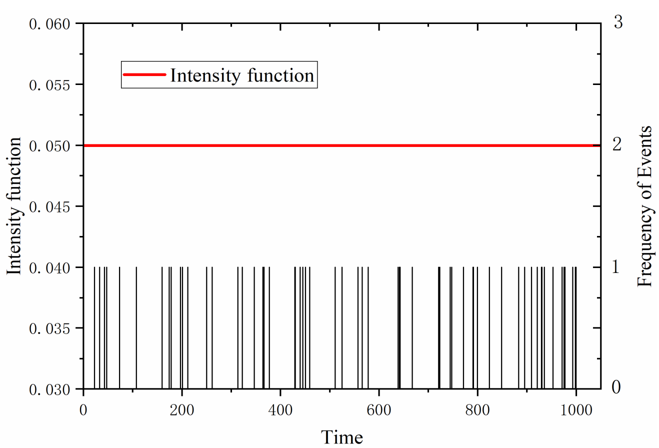
The arrival of low-quality feedstocks			
Hawkes			Poisson
$v(t)$	α	β	$\lambda(t)$
0.05	0.1	0.5	0.05
Machine reliability parameters			
Failure rate			Repair rate
r	a	b	μ
0.01	0.15	$7.3*10^{-3}$	0.1

B. *Results Analysis*

The possible arrival paths of low-quality feedstocks are illustrated in Figure 5. Obviously, with the same initial intensity, Hawkes process will have more shocks from low-quality feedstocks than the Poisson process due to the cluster ($NH(t)$=63, $NP(t)$=53).



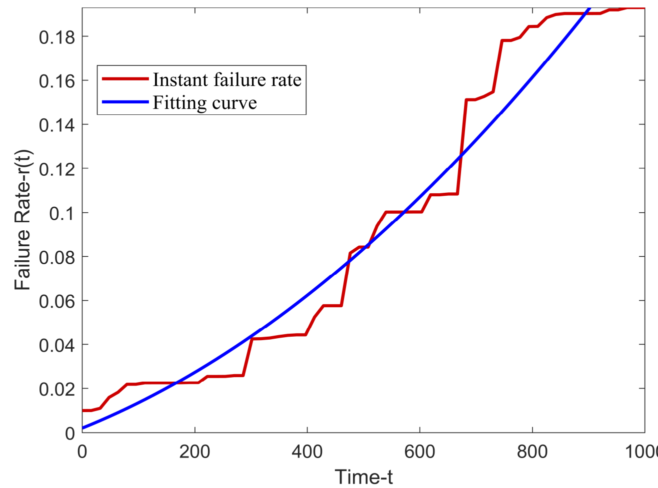
a. Hawkes Process



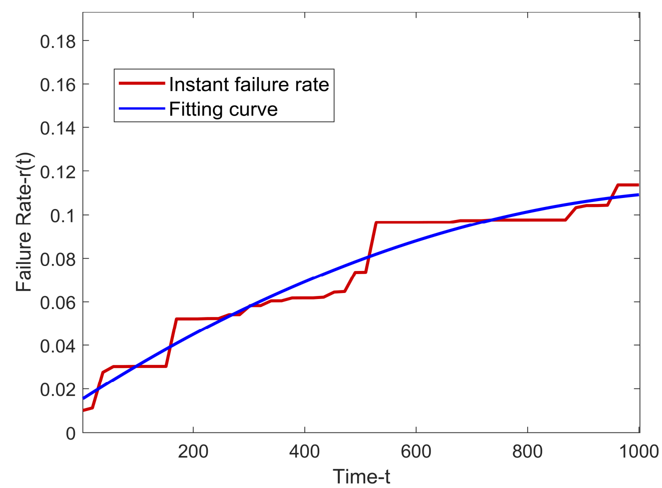
b. Poisson Process

Figure 5. Possible paths of the arrival of low-quality feedstocks

Under these shocks from low-quality feedstocks, the instant failure rates $r'(t)$ are obtained, as shown in Figure 6. According to the discrete data of instant failure rates, we also provide the second-order polynomial fitting curve. In this experiment, due to the cluster phenomenon of low-quality feedstocks, the degradation of a machine will be aggravated under Hawkes shocks, leading to a greater addition of instant failure rate than the Poisson shock. Additionally, fitting curves by second-order polynomial also show that the increased speed of Hawkes shock is much faster than the Poisson shock. An important reason is that the cluster property of Hawkes process increases the occurrence probability of low-quality feedstocks during the follow-up period, leading to more shocks from low-quality feedstocks.



a. Hawkes Process

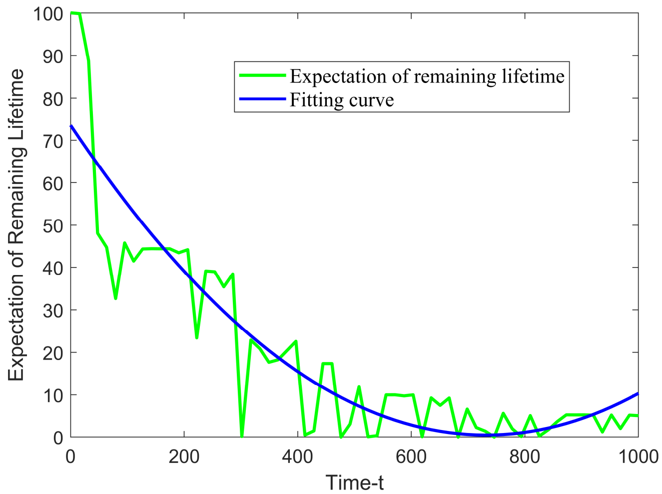


b. Poisson Process

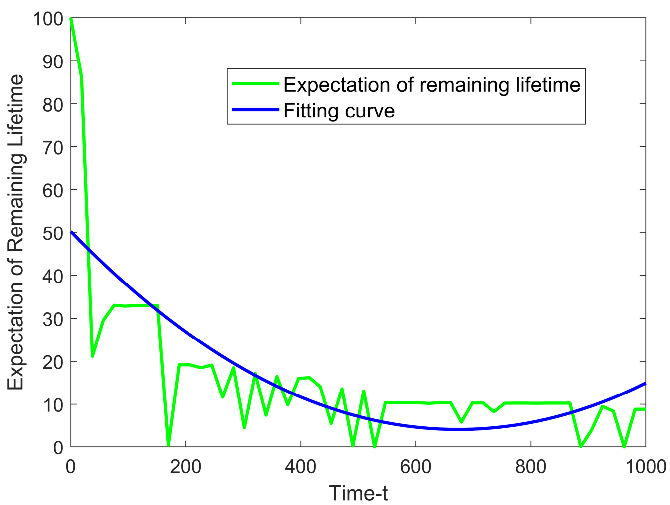
Figure 6. The instant failure rates under Hawkes and Poisson shocks

Also, we simulated expectations $E(T_x)$ of the remaining lifetime under Hawkes and Poisson shocks, as shown in Figure 7. In the same way, second-order polynomial fitting curves are also provided based on the discrete data of expectations of the remaining lifetime. The machine shows decreasing expectations of remaining lifetimes under both Hawkes and Poisson shocks, which will stabilize at a lower value when the simulation time is long enough. Similar to the instant failure rates, the decreasing speed of remaining lifetime expectation under Hawkes shocks is also much faster than the one under the Poisson process, which is also due to more shocks and the aggravated degradation caused by the cluster of Hawkes shocks.

According to the simulation experiment, the study of the shock model is further expanded in this paper. Besides the self-healing effect and damage amount of one shock which are considered in Reference [12], this simulation proved that the arriving mode of shocks will also play a significant role in the reliability of a system.



a. Hawkes Process



b. Poisson Process

Figure 7. Expectations of the remaining lifetime under Hawkes and Poisson shocks

V. Conclusions

This paper proposed a novel model to evaluate the reliability of a manufacturing machine under both self-degradation and the shock from low-quality feedstocks. The cluster shock from low-quality feedstocks is firstly modeled with the Hawkes process in this paper, which can effectively illustrate the cluster arrival of low-quality feedstocks caused by quality issues in some specific batches. Considering the heterogeneity of low-quality feedstocks, a mixture failure rate model is proposed and the expectation of remaining lifetime under shocks is obtained. Moreover, the simulation experiment illustrated the effectiveness of the proposed model. Future research can focus on the extension of proposed models in this paper to the system level. For example, the propagation characteristic of low-quality feedstocks in manufacturing systems with different structures can be investigated, and the dynamic effect of low-quality feedstocks on different machines when flowing in manufacturing systems.

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