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Reliability analysis for series manufacturing system with imperfect inspection considering the interaction between quality and degradation

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ABSTRACT

Reliability evaluation of manufacturing system always takes machine performance and product quality as two key factors. But the imperfect inspection usually leads to unreliable quality judgments, cause the propagation of unqualified products, and aggravate the degradation of downstream machines. In this paper, considering the effect of imperfect inspection process, a new reliability model is developed based on the interaction between machine performance and product quality. Firstly, multiple degradation modes, which include continuous and discrete degradations, are modeled by different stochastic processes. Secondly, a quality reliability model is established to evaluate the satisfaction rate of production demand based on the variation estimation model of product's dimensions. Thirdly, the four states (Q-Q, U-U, Q-U, and U-Q) of quality inspection process are defined, whose effects on discrete degradation are analyzed in qualitative by referring to the quality evaluation. Finally, a system reliability evaluation model is proposed based on the analysis of minimum target requirements, and the practical case of a connecting rod illustrates the effectiveness of the proposed model. The results indicate that the abundant information existing in manufacturing system can be effectively integrated and exploited by the proposed reliability model, and the weakness of manufacturing system can also be identified easily.

1. Introduction

1.1. Problem statement

The industrial manufacturing system is one complex system consisting of many mechanical components, which may fail due to wear, shocks or other random factors during production activities. Component's failure would influence the output of a manufacturing system, as it was verified that the pass rate of stations had monotone function relation with their reliability [1]. To predict possible failures and fulfill tasks on schedule, quality improving [2], system reliability evaluation [3] and optimization [4,5] have always been the hot topics both in academic and industrial fields. Due to the complexity and discrepancy of different manufacturing systems, the reliability has been studied from various perspectives considering multiple factors.

System reliability is generally defined as the probability that a system performs its intended function under specific operating conditions for a specified period of time [6]. Recently, multiple factors have been studied, such as machine's property [6–8], pallet handling system [7], product quality [1,6,9], quality control devices, and buffers [8]. Considering the dependency between stations, a Quality-Reliability-Co-Effect (QR-Co-Effect) model was proposed to evaluate the reliability of an assembly fixture system [6,9], which only involved the interaction between quality and degradation. To evaluate the output of a serial manufacturing system, a model was established by independently considering the performance of unreliable stations, quality control devices and buffers [8]. In these researches, component's degradation, product quality, and quality control devices were considered to be independent or partially interactional. However, their systematic interaction relationships were not studied clearly, especially the important effect of quality inspection process.

In addition, product quality depends on machine's accuracy in manufacturing systems [10]. With the developments of new products, the high accuracy requirement for machines is becoming more urgent. Simultaneously, the concept of machining accuracy reliability has been proposed, which was defined as the ability to perform its specified machining accuracy under the stated conditions during a given period of time. Several factors, impacting on this performance, were taken into account, such as geometric errors, thermal errors, and tool wear [10–12]. Considering the stability or not, static factors (component weight and guide imperfection) and dynamic factors (cutting forces, motion, and acceleration) were regarded as the main factors causing

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dimensional deviations [13]. Besides, machine accuracy is influenced by multiple sources, which include not only machines' conditions, but also the cutting characteristics of the parts [14]. A complex device could also fail due to multiple failure processes induced by internal and external sources, such as corrosion, fatigue, wear, and external shocks [15]. The factors, affecting on machine accuracy, can be generally classified into two categories. The static factors (internal sources) will cause the stable continuous degradation of machining accuracy while the dynamic factors (external sources) will cause the discrete random degradation. Obviously, multi-failure-process is another characteristic for the reliability evaluation of a manufacturing system. In view of these points, a new model should be established to ensure the evaluation accuracy of system reliability, which should consider not only the complex and diverse degradation processes of machining accuracy, but also the interactions among component's degradation, product quality, and inspection process.

1.2. Literature review

Manufacturing system reliability has been paid much more attention in recent engineering literature, and several models were proposed. QR-Co-Effect model analyzed product's dimensional deviations caused by the locating errors of fixture system, and a stochastic process model of fixture wear was established [9]. Based on this research, a Quality-Reliability Chain model was constructed to describe the complex interplay between component reliability and product quality [6]. To evaluate the reliability of serial-parallel hybrid multi-operational manufacturing system, the interactions between product dimension and component's degradation were analyzed based on the linear model of product dimension and the stochastic process model of component's degradation [16]. In addition, a Quality-Reliability-Quality chain model was constructed to analyze the interactions among the system reliability, product quality, and product inherent reliability [17], in which exponential distribution was used to model the reliability of individual component.

According to these literatures, two key factors were mostly considered in the research of manufacturing system reliability: machine performance and product quality. Although interplays between them were analyzed, the degradation processes in these researches usually were estimated by one special distribution, such as lognormal distribution [9], normal distribution [16] or exponential distribution [17], which failed to consider the diversity and complexity of machine degradation. Actually, degradation process is one complex stochastic process which will be influenced by components' wear and other random shocks. Especially, as one shock, unqualified product will cause the degradation of machining accuracy. Therefore, a reliability model considering multiple degradation modes needs to be constructed for consummating the performance evaluation of manufacturing system.

According to different sources, a degradation process can be classified into the continuous degradation caused by static factors and the discrete degradation by shocks, which have been studied widely in engineering field, such as leakage current of ultra-thin gate oxides [18], micro-electro-mechanical systems [19], and sliding spool in hydraulic control systems [20]. Simultaneously, many models have been proposed to enrich researches of degradation in different systems, such as degradation models with changing degradation rate or random shocks [21–24], degradation models with mixed shocks [25] or conditional probability [26], and the model considering cumulative damage shock [27]. In these models, stochastic process theory was most frequently used in the simulation of components' degradation. For instance, the Poisson process representing the cumulative damage shock [27], the linear Wiener process depicting the jump at change point [28], Wiener diffusion processes in terms of degradation signals [29], etc. Similar to these systems, different degradation modes also can coexist in a manufacturing system, such as the continuous degradation process caused by internal factors, and the discrete degradation process caused by external shocks of unqualified products. However, multiple degradation modes were only considered in single-component systems in previous researches, which fail to depict the dynamic interplay among different stations in a manufacturing system.

To analyze multi-component systems, one reliability model was proposed in terms of the dependency between shock size and shock damage [30]. By considering the coupling between components and degradations, one dynamically updated matrix was used to depict their interactions, where degradation and shock were modeled by Gamma process and Markov renewal process respectively [31]. In addition, multiple dependent degradation processes were analyzed in another multi-component system, and the piecewise-deterministic Markov process was employed to depict the dependency between degradation and random shocks [32]. In these researches, random shocks were considered to be unrestrained, which is not real in a manufacturing system. Unlike above systems, the discrete degradations caused by shocks in a manufacturing system are conditional events, which happen only when quality inspection is imperfect. In other words, unqualified products' shocks from upstream stations will be prevented by perfect inspection process. Therefore, the inspection process is the bridge to accomplishing the interplay between machine degradation and product quality, which has not been considered in recent researches. In view of this, a novel model needs to be developed to quantify the effect of an inspection process on manufacturing system reliability.

In process industry, imperfect inspection of wall thickness measurements was considered in the analysis of maintenance decisions, and it noted that perfect inspections could increase the confidence in the estimated corrosion rate than imperfect inspections [33]. In addition, many studies have been developed to analyze the optimal maintenance policies under the consideration that imperfect inspections could cause both false alarms and undetected failures. For instance, scheduling the periodic inspection to meet the availability requirement [34], optimizing the inspection interval to improve the components' reliability [35], two inspection policies of fixed inspection interval and variable inspection frequency for improving system reliability [36], modeling maintenance policies with the periodic inspections [37], optimizing the maintenance by considering variable probabilities of inspection errors [38], etc. In these models, objects of inspections were machines, and their targets were to optimize maintenance policies. But in a manufacturing system, quality inspection process not only is the method to recognize failures, but also can be considered as one serial constitute part of stations, which intends to accomplish the process quality control. Although the dynamic reliability was also proposed by considering multilevel data of imperfect quality inspection [39], it failed to consider the interaction between product quality and machine performance. Unlike other systems, the inspection process in a manufacturing system can effectively prevent unqualified products propagating to downstream stations and reduce machine's degradation caused by random shocks from unqualified products. Namely, quality inspection process has the failure isolation effect, which has not been taken into account in recent models.

Obviously, to improve the accuracy of manufacturing system reliability evaluation, a new model should be established by considering multiple degradation modes and inspection's failure isolation effect, in which machine performance, product quality and inspection process should be taken into account overall. Machining accuracy is an important index which can depict machine's degradation and impact on product quality. This degradation is one interactive process with product's dimensional deviations, which will be modeled by two stochastic processes in this paper. Marked Point Process is used to depict the arrival intervals between unqualified products, and Wiener Process is used to depict the damage values from dimensional deviations. In addition, product quality is evaluated by a linear model, and four states of the inspection process are also defined to analyze its effect on downstream machine's degradation. The contributions of this paper are as follows.

(1) Imperfect inspection is firstly considered in the interaction between machine degradation and product quality, and four states of inspection process are defined;
(2) Discrete degradation model is proposed on the basis of unqualified products and imperfect quality inspection process, which reflects the restraints of inspection in the propagation of unqualified products;
(3) Product's dimensional deviation model is constructed based on machine's degradation, and station's quality reliability is developed to analyze the probability that quantity demand is satisfied;
(4) Reliability model of manufacturing system is established, which considers the interactions among machine degradation, unqualified quality shocks, and inspection performance.

The rest of this paper is organized as follows. In Section 2, model of a series manufacturing system with imperfect inspection (SMSII) is defined, and the interaction mechanism among three key factors is analyzed. In Section 3, the multiple degradation model, product dimensional deviation model, and inspection process model are established. In Section 4, the calculation flow of system reliability is shown. In Section 5, a case is exhibited to illustrate the effectiveness of the proposed model. Finally, conclusions are given in Section 6.

2. Series manufacturing system with imperfect inspection

2.1. System descriptions

Manufacturing system usually has four structures, namely series, parallel, series-parallel, and cycles with rework actions [7,32], which can be modeled by production lines or networks. This paper discusses the simplest structure, namely the series manufacturing system with imperfect inspection, which is one typical system in the production activities.

In a manufacturing system, station is the basic unit and has three key components: machines, inspection equipment, and products. Each component in the station will play an important role in system performance. In one station, machine and inspection equipment are connected in series, as illustrated in Fig. 1. In this block diagram, circle and rhombus respectively represent the machine M and inspection equipment I, and the inputs and outputs represent products. As shown in this diagram, performance of machine M and inspection equipment I will interact with each other and has an influence on station's effective output, namely Reliability-Quality impact (R-Q impact) which means that machine performance will impact on the judgment of inspection process. In this process, the degradation of machine accuracy will increase the dimensional variation, and affect outputs of stations by impacting the inspection performance.

In practice, not all stations contain an inspection process. If there is no inspection process in a real system, all products can pass the virtual test in the inspection process. Based on above analysis, SMSII can be constructed by a series of stations, as shown in Fig. 2. Every station includes machine M and inspection equipment I, and outputs of each station are the inputs of the next station. Besides the R-Q impact in a station, the Quality-Reliability impact (Q-R impact) between stations also exists in serial manufacturing systems. In Q-R impact, the imperfect inspection will cause unqualified products' propogatation into downstream machine, and accelerate its degradation when the error dimensions are used as positioning datum.

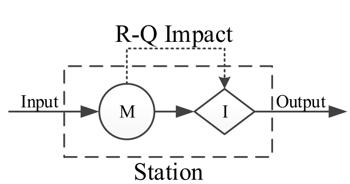


Fig. 1. Block diagram of station's structure.

2.2. Interaction mechanism of three key components

The intended function of a manufacturing system may be diverse for different research objectives, and this paper will study the probability that a manufacturing system satisfies the requirements in quantity and quality without machine failure. As described in Section 2.1, a series manufacturing system with n stations, can be decomposed into three key components: machines, inspection processes and products. Therefore, the interaction mechanism of three key components will be described in detail in this section.

As a kind of complex mechanical system, machine's reliability has been analyzed by considering several factors, such as components' wear, noise and other random factors. Referring to Archard and Dawson's research, machine's degradation closely depended on the load force, sliding distance, and the contacting surface, which could be changed by the dimensional deviations of unqualified products [40]. In SMSII, the degradation under the stable working condition can be considered as a smooth asymptotic process; but the degradation under the changeable working condition may increase abruptly due to dealing with unqualified products. Therefore, the degradation of machine performance is one saltatory asymptotic process, as shown in Fig. 3(a), where $t_{1,}$ t_2 and t_3 are different times when degradation increases abruptly due to dealing with unqualified products, and T is the time when degradation goes beyond the threshold. According to this characteristic, machine's degradation process can be studied by two equivalent processes in discrete manufacturing systems: the stable continuous degradation and the discrete random degradation, as shown in Fig. 3(b).

To illustrate the interaction among different components, a stacking diagram of reliability factors is established, as shown in Fig. 4, where factors are represented for the reliability of a manufacturing system. Like the propping relation of building blocks, blocks below are factors of blocks above, and arrows illustrate the interplay between two blocks. According to the description, the machine degradation has two modes, the stable continuous degradation and random discrete degradation. Normally, the continuous degradation is usually caused by wear, but the discrete degradation is caused by some discrete collisions of external factors such as dimensional deviations of imported products, as shown in Fig. 4.

In an inspection process, two failures may occur for different quality status of products: identifying qualified product as unqualified one (Q-U failure) or the contrary (U-Q failure). According to the analysis in Section 2.1, the degradation of machine performance will cause product's dimensional deviation; conversely, product's dimensional deviation will also lead to the discrete degradation of downstream machine, as illustrated in Fig. 4. According to the coupling relation, interaction analysis of machine degradation, dimensional deviation and inspection process will be carried out in Section 3.

3. Interaction modeling of SMSII concerning the three factors

3.1. Assumptions

To analyze the interactions in SMSII, some assumptions should be taken into account as follows.

(1) Inputs of system, namely the inputs of station M_1, are always qualified;
(2) Discrete-part manufacturing processes are considered, where the number of processing products is treated as the time index;
(3) In a station, machine's discrete degradations caused by different unqualified products are independent identically distributed;
(4) The discrete degradation and continuous degradation are independent with each other, and degradation value of machining accuracy equals to zero at $t = 0$;
(5) Time intervals between unqualified products that are propagated

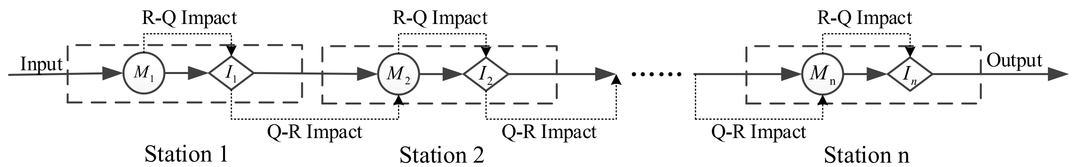


Fig. 2. Block diagram of SMSII with *n*stations.

to downstream stations are random variable and independent identically distributed;
(6) The machines have non-decreasing degradation;
(7) There are no capacity constraints for inspection process; namely the inspection equipment is available for checking in time no matter what the number of products is;
(8) Pernicious products from upstream station will be identified totally and discarded by the inspection process in the downstream station;
(9) The inspection failure is a random event; namely, unqualified products will be transmitted to the next station randomly.

3.2. Degradation modeling of machine

3.2.1. Discrete degradation process depending on product quality

Discrete degradation is the result of external factors, which has two types. One is the continuous stress that cannot lead to new degradation until its accumulation goes beyond the threshold; another is the discrete shocks that will create new random degradation once it appears. In discrete manufacturing systems, machine's degradation caused by unqualified products is the latter one. Assume that $N_i(t)$ is the frequency that unqualified products are propagated to machine i during $[0, t]$, in which the time intervals between unqualified products are independent and identically distributed. So, $N_i(t)$ is a random variable, and $\{N_i(t); t \geq 0\}$ can be considered as a renewal process, whose arrival intervals are arbitrary random variables with independent identical distribution. The calculation of $N_i(t)$ will be discussed based on the modeling of the inspection process in Section 3.3.2.

It is assumed that $X_i(t)$ is the accumulated discrete degradation of ith station's machine caused by unqualified products from station $(i-1)$ during $[0, t]$, which can be obtained by

$$X_i(t) = \sum_{j=1}^{N_i(t)} x(\delta_{ij}), \tag{1}$$

where $x(\delta_{ij})$ represents ith station's degradation increment caused by jth unqualified product from station $(i-1)$; δ_{ij} is the dimensional deviation from the limit size of jth unqualified product propagated to station i. According to above definitions, $\{X_i(t); t \geq 0\}$ can be considered as a Marked Point Process with a mark space$\{x(\delta_{ij})|j = 1, 2, \cdots, N_i(t)\}$. In addition, unqualified product's dimensional deviation δ_{ij} can be calculated for any station i by

$$\delta_{ij} = \begin{cases} \frac{d_{ij} - USL_i}{USL_i} & d_{ij} > USL_i \\ 0 & USL_i \geq d_{ij} \geq LSL_i \\ \frac{LSL_i - d_{ij}}{LSL_i} & d_{ij} < LSL_i \end{cases}, \tag{2}$$

where d_{ij} is the real size of jth unqualified product propagated to station i; USL_i and LSL_i are the upper and low dimensional specification limits, where $\delta_{ij} \in [0, +\infty)$.

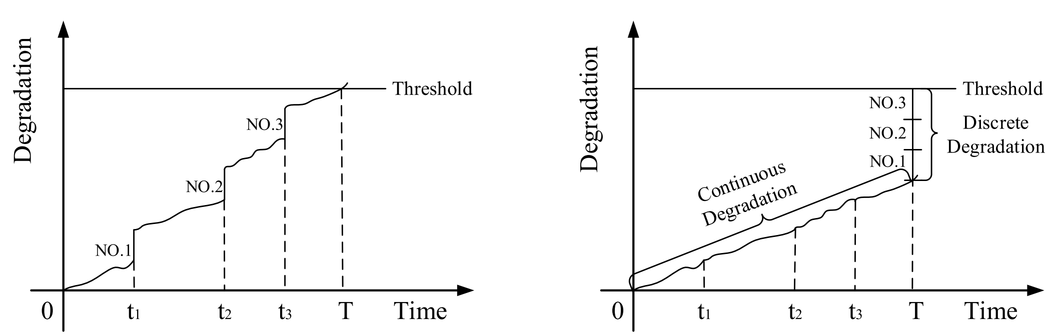
In this research, the degradation increments $x(\delta_{ij})$, $j = 1, 2, \cdots, N_i(t)$, caused by different unqualified products from station $(i-1)$, are considered to be independent. Namely, $x(\delta_{ij})$ has no relation to other degradations caused by the dimensional deviations of previous unqualified products. It is also assumed that only continuous degradation will occur when a product is qualified, and the discrete degradation value is zero. In addition, being widely used in engineering field, Wiener process is one effective tool to depict the discrete degradation, which has been verified in the degradation analysis of laser degradation problem [41], lithium-ion batteries [42] and so on. In view of these characteristics, this paper will choose the Wiener Process to depict the impact of unqualified product's dimensional deviations on degradation, which can be signed as $\{x(\delta_{ij}), \delta_{ij} \in [0, +\infty)\}$. According to the definition of Wiener Process, some conclusions can be summarized as follows

1) $x(\delta_{i0}) = 0$ when $\delta_{i0} = 0$, which is the discrete degradation caused by qualified inputs;
2) For $\forall\ \delta_{i0} < \delta_{i1} < \delta_{i2} < \ldots < \delta_{iN_i}$, stochastic variables $x(\delta_{i1}) - x(\delta_{i0})$, $x(\delta_{i2}) - x(\delta_{i1})$, …, $x(\delta_{iN_i}) - x(\delta_{iN_i-1})$are independent;
3) For $\forall \delta \geq 0$ and $\delta_{ij} > 0$, $x(\delta + \delta_{ij}) - x(\delta) \sim N(\mu_i \delta_{ij}, \sigma_i^2 \delta_{ij})$.

In addition, degradation increment $x(\delta_{ij})$ is caused by the dimensional deviation δ_{ij} of jth unqualified product, which can be considered to be independent and identical distribution according to Assumption (3). Therefore, the discrete degradation caused by any product with dimensional deviation δ_{ij} can be obtained by

$$x(\delta_{ij}) = x(\delta_{i0} + \delta_{ij}) - x(\delta_{i0}), \tag{3}$$

namely $x(\delta_{ij}) \sim N(\mu_i \delta_{ij}, \sigma_i^2 \delta_{ij})$.



(a) Original Process *(b) Equivalent Process*

Fig. 3. Saltatory asymptotic process of machine degradation.

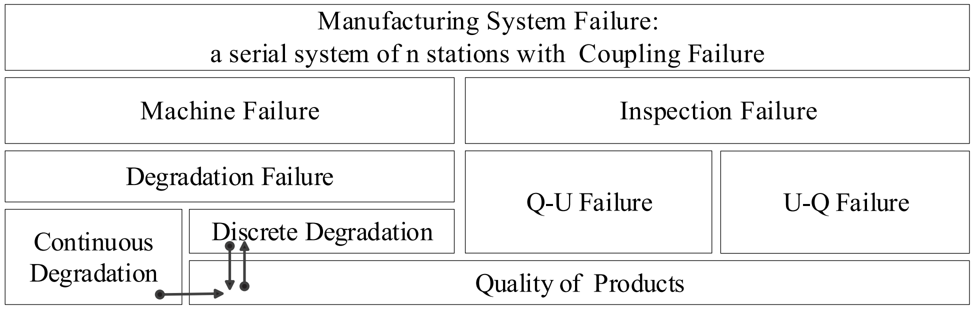


Fig. 4. Reliability factors stacking diagram of SMSII.

In practice, only in condition that the probability of a negative $x(\delta_{ij})$ is small, say $\mu_i\delta_{ij}-3\times\sqrt{\sigma_i^2\delta_{ij}}>0$, the normal assumption will be meaningful [41]. This assumption will be considered in our model. But, for extreme circumstances, this assumption will not completely avoid the occurrence of negative increments, which only make the probability much lower. Therefore, it is also should be highlighted that if the negative increment happens, the truncated normal distribution will be considered instead. The truncated normal distribution is a well-done method to solve this problem, which has been proved in the degradation analysis of laser degradation problem [41] and lithium-ion batteries [42].

In a discrete degradation process, assume that machines have a lot of degradation data $\{\boldsymbol{X}(\delta_{ij}),\ i=1,2,\ldots,n,\ j=1,2,\ldots,N_i\}$, where $\boldsymbol{X}(\delta_{ij})$ is the set of discrete degradation values caused by unqualified products with dimensional deviation δ_{ij} in station i. For any dimensional deviation δ_{ij}, the discrete degradation obeys the normal distribution $N(\mu_i\delta_{ij},\sigma_i^2\delta_{ij})$, so the mean and variance of the discrete degradation for station i should satisfy $\hat{\mu}_i\delta_{ij}=E(\boldsymbol{X}(\delta_{ij}))$ and $\hat{\sigma}_i^2\delta_{ij}=Var(\boldsymbol{X}(\delta_{ij}))$. The estimations of mean and variance can be obtained by Eqs. (4) and (5).

$$\hat{\mu}_i=\frac{1}{\delta_{ij}}E(\boldsymbol{X}(\delta_{ij})) \tag{4}$$

$$\hat{\sigma}_i^2=\frac{1}{\delta_{ij}}Var(\boldsymbol{X}(\delta_{ij})) \tag{5}$$

3.2.2. Continuous degradation process

As a complex mechanical system, machine's continuous degradation is an irreversible gradual process and increases with time monotonically, which can be depicted by a Gamma process. Gamma process can better conform to the assumption of machine's degradation and reflect the non-decreasing characteristic over time. For station i, the continuous degradation is assumed to be modeled by the Gamma process $\{Y_i(t);\ t\geq 0\}$, which has the following properties.

1) $Y_i(0)=0$,
2) For $\forall t_1<t_2<t_3<\ldots<t_k$, stochastic variables $Y_i(t_1)-Y_i(t_0)$, $Y_i(t_2)-Y_i(t_1)$, …, $Y_i(t_k)-Y_i(t_{k-1})$ are independent,
3) For $\forall t\geq s$, $Y_i(t)-Y_i(s)\sim Ga(\alpha_i(t)-\alpha_i(s),\beta_i)$,

where $\alpha_i(t)$ is the shape parameter and β_i is the scale parameter in station i. $\alpha_i(t)$ is the strictly continuous monotone non-decreasing function of time t, and $\alpha_i(0)=0$. Vector $\boldsymbol{M}_i$ is assumed to be the parameters of $\alpha_i(t)$, which can be rewritten as $\alpha_i(t;\boldsymbol{M}_i)$. According to the properties of Gamma distribution, we have $Y_i(t)\sim Ga(\alpha_i(t;\boldsymbol{M}_i),\beta_i)$ and the density function is

$$g(Y_i;t)=\frac{\beta_i^{\alpha_i(t;\boldsymbol{M}_i)}}{\Gamma(\alpha_i(t;\boldsymbol{M}_i))}Y_i^{\alpha_i(t;\boldsymbol{M}_i)-1}e^{-\beta_iY_i}I_{(0,\infty)}(Y_i), \tag{6}$$

where β_i and $\boldsymbol{M}_i$ are unknown parameters. Function $I_A(Y_i)$ means that if $Y_i\in A$, $I_A(Y_i)=1$; if not, $I_A(Y_i)=0$. $\Gamma(\alpha_i)=\int_0^\infty t^{\alpha_i-1}e^{-t}dt$ is the Gamma function, and the mean and variance of accumulated continuous degradation at time t can be obtained by Eqs. (7) and (8).

$$\mu_i(t)=E[Y_i(t)]=\frac{\alpha_i(t;\boldsymbol{M}_i)}{\beta_i} \tag{7}$$

$$\sigma_i^2(t)=Var[Y_i(t)]=\frac{\alpha_i(t;\boldsymbol{M}_i)}{\beta_i^2} \tag{8}$$

In station i, the unknown parameters β_i and $\boldsymbol{M}_i$ can also be acquired by parameter estimation. If machine i has a performance degradation data set $\{Y_{ij};\ i=1,2,\ldots;j=1,2,\ldots,num\}$ at the time set $\{t_j;\ j=1,2,\ldots,num,\ and t_1<t_2<t_3<\ldots<t_{num}\}$, and Y_{ij} is the jth accumulated degradation value of station i at time t_j. So, the estimated values $\hat{\mu}_i(t)$ and $\hat{\sigma}_i^2(t)$ at time t can be calculated, and estimated values $\hat{\beta}_i$ and $\hat{\boldsymbol{M}}_i$ can also be obtained by the regression analysis.

3.2.3. Reliability evaluation of multiple degradation modes

Referring to above researches about multiple degradation modes, machines will suffer discrete degradation from unqualified products and continuous degradation caused by wear, as illustrated in Fig. 3. Obviously, the frequency that unqualified products would be machined in station i during $[t,t+\Delta t]$ can be written as

$$\Delta N_i(\Delta t)=N_i(t+\Delta t)-N_i(t). \tag{9}$$

According to Assumptions (4) and (5), if Δt is minimum enough, $\Delta N_i(\Delta t)\leq 1$. And when $t_0=0$, there is $N_i(t_0)=0$. So, the accumulative discrete degradation in station i during $[t,t+\Delta t]$ can be calculated by

$$X_i(t+\Delta t)-X_i(t)=\sum_{j=0}^{\Delta N_i(\Delta t)}x(\delta_{ij}). \tag{10}$$

According to Assumption (3), discrete degradations caused by different unqualified products are independent identical distributed, and $x(\delta_{ij})\sim N(\mu_i\delta_{ij},\sigma_i^2\delta_{ij})$. So, it can be concluded as

$$X_i(t+\Delta t)-X_i(t)\sim N\left(\mu_i\sum_{j=0}^{\Delta N_i(\Delta t)}\delta_{ij},\ \sigma_i^2\sum_{j=0}^{\Delta N_i(\Delta t)}\delta_{ij}\right).$$

In the continuous degradation model, the accumulated continuous degradation during $[t,t+\Delta t]$ can be calculated underlying its distribution as

$$Y_i(t+\Delta t)-Y_i(t)\sim Ga(\alpha_i(t+\Delta t)-\alpha_i(t),\beta_i).$$

Based on Assumption (4), discrete degradation and continuous degradation are independent, so the total degradation in station i during $[t,t+\Delta t]$ can be obtained by

$$Z_i(t+\Delta t)-Z_i(t)=X_i(t+\Delta t)-X_i(t)+Y_i(t+\Delta t)-Y_i(t). \tag{11}$$

According to the convolution formula, the degradation in station i during $[t,t+\Delta t]$ is $Z_i=X_i+Y_i$, whose probability density function can be written as

$$f_{Z_i}(z;\Delta t) = f_{Z_i}(x, y;\Delta t) = f_{X_i}(x;\Delta t)\cdot f_{Y_i}(y;\Delta t)$$
$$= \frac{1}{\sqrt{2\pi\sigma_i^2\sum_0^{\Delta N_i(\Delta t)}\delta_{ij}}}\exp\left(-\frac{(x-\mu_i\sum_0^{\Delta N_i(\Delta t)}\delta_{ij})^2}{2\sigma_i^2\sum_0^{\Delta N_i(\Delta t)}\delta_{ij}}\right)\cdot\frac{\beta_i^{\alpha_i(t+\Delta t)-\alpha_i(t)}}{\Gamma(\alpha_i(t+\Delta t)-\alpha_i(t))}y^{\alpha_i(t+\Delta t)-\alpha_i(t)-1}\cdot \exp(-\beta_i y) \tag{12}$$

So, for any interval [0, t], the probability density function of total accumulative degradation at time t can be rewritten as

$$f_{Z_i}(z;t) = f_{Z_i}(x, y;t) = f_{X_i}(x;t)\cdot f_{Y_i}(y;t)$$
$$= \frac{1}{\sqrt{2\pi\sigma_i^2\sum_0^{N_i(t)}\delta_{ij}}}\exp\left(-\frac{(x-\mu_i\sum_0^{N_i(t)}\delta_{ij})^2}{2\sigma_i^2\sum_0^{N_i(t)}\delta_{ij}}\right)\cdot\frac{\beta_i^{\alpha_i(t)}}{\Gamma(\alpha_i(t))}y^{\alpha_i(t)-1}\exp(-\beta_i y)^{\cdot} \tag{13}$$

For station i, the distribution function of accumulative degradation failure is

$$F(Z_i;t) = P\{T\le t\} = 1 - P\{Z_i(t)\le H_{Di}\} = 1 - \iint_{x+y\le H_{Di}} f_{Z_i}(x, y;t)dxdy, \tag{14}$$

where H_D is the degradation threshold. Assume $E_{t_i}^D$ represents the event that the performance does not fail in station i during its working time [0, t_i], and reliability of the ith machine after processing the unqualified products with dimensional deviations $\boldsymbol{\delta}_{i-1}$ can be calculated by

$$R_{Di}(t) = P\{E_{t_i}^D|\delta_{i-1}\} = 1 - F(Z_i;t) = P\{Z_i(t)\le H_{Di}\} = \iint_{x+y\le H_{Di}} f_{Z_i}(x, y;t)dxdy. \tag{15}$$

3.3. Product quality and inspection process modeling

3.3.1. Product quality evaluation based on machine's degradation

Product quality is generally affected by the state of machine, which is called process-variables [6]. In addition, random noise, called the noise-variables, is another variable affecting product's quality, which is common in a machining process, such as random variations of raw materials' quality, and random environment variations. Considering the interaction between process-variables and random-variables, product's quality characteristic $\boldsymbol{D}_i(t)$ in station i can be depicted by a linear model [6], as

$$\boldsymbol{D}_i(t) = \boldsymbol{\eta}_i + \boldsymbol{A}_i^T\times\boldsymbol{Z}_i(t) + \boldsymbol{B}_i^T\times\boldsymbol{\xi}_i + \boldsymbol{Z}_i(t)^T\times\boldsymbol{\Gamma}_i\times\boldsymbol{\xi}_i, \tag{16}$$

where $\boldsymbol{D}_i(t)$ is one m_i dimensional vector, and m_i is the number of product's quality characteristics machined by station i; $\boldsymbol{\xi}_i = [\xi_{i1}, \xi_{i2}, ..., \xi_{im_i}]^T \in R^{m_i}$ is the vector of noise-variables with mean E(ξ_i) and covariance matrix $Cov(\xi_i)$ independent of time index; $\boldsymbol{\eta}_i$ is the constant vector; $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ are the vectors respectively charactering the effects of degradation $\boldsymbol{Z}_i(t)$ and noise $\boldsymbol{\xi}_i$; and $\boldsymbol{\Gamma}_i$ is a matrix characterizing the effect of interactions between degradation $\boldsymbol{Z}_i(t)$ and noise $\boldsymbol{\xi}_i$.

Assume that the machining time of each product is h_i in station i and the time axis can be divided into several continuous and uniform intervals with length h_i. Therefore, successive endpoints of the intervals can be denoted by h_i, $2h_i$, $3h_i$, ..., jh_i, ... In addition, assume that $\boldsymbol{Z}_i(t_j)$ is the degradation state of machine when it finishes machining the jth product at $t_j = j\times h$. According to the analysis in Section 2, the quality characteristics k of jth product will be impacted by the degradation state of machine, which can be rewritten as

$$D_{ijk}(t) = \eta_{ik} + A_{ik}\times Z_i(t_{j-1}) + B_{ik}\times\boldsymbol{\xi}_i + Z_i(t_{j-1})^T\times\Gamma_{ik}\times\xi_i\, t_{j-1} < t < t_j,\ k = 1, 2, ..., m_i. \tag{17}$$

As we know, product quality can be depicted by the deviation from the nominal dimension. For the given machine degradation state and specification limit USL_i and LSL_i in station i, the kth quality index of jth product can be obtained by

$$q_{ijk}(t|Z_i(t_{j-1})) = \{(|D_{ijk}(t) - D_{mid\_ik}|)|Z_i(t_{j-1})\};\ t_{j-1} < t < t_j, \tag{18}$$

where D_{mid_ik} is the average of upper and lower specification for the kth quality index in station i, which can be calculated by Eq. (19).

$$D_{mid\_ik} = (USL_i + LSL_i)/2 \tag{19}$$

For any quality index k in station i, set that the threshold is $H_{Q\_ik} = Tol_{ik}/2$, and Tol_{ik} is the dimension tolerance of kth quality characteristic; so, the event that quality indexes of jth product should satisfy the specification can be depicted as

$$E_{Qij} = \cap_{k=1}^{m_i}(q_{ijk}(t|Z_i(t_{j-1}))\le H_{Q\_ik},\ t_{j-1} < t < t_j). \tag{20}$$

Assume that Ω is a domain which subjects to

$$Z_i(t_{j-1})\in\Omega \Leftrightarrow \cap_{k=1}^{m_i}\{((|D_{ijk}(t) - D_{mid\_ik}|)|Z_i(t_{j-1}))\le H_{Q\_ik}\}.$$

Then we let

$$I_{ij} = \begin{cases}1,\ if Z_i(t_{j-1})\in\Omega\\ 0,\ else\end{cases}. \tag{21}$$

For station i, the number of qualified products Q_i and unqualified products U_i machined during [0, t] can be calculated respectively by

$$Q_i = \sum_{i=1}^{J_i} I_{ij} \tag{22}$$

$$U_i = \sum_{i=1}^{J_i}(1 - I_{ij}) \tag{23}$$

where J_i is the total number of products processed during [0, t] in station i, and $Q_i + U_i = J_i$.

Quality reliability of a station is defined as the completion rate of its minimum target, which is the lower boundary of qualified products meeting the system demand. Assume $E_{t_i}^Q$ represents the event that the target quantity of the mission is satisfied in station i during [0, t_i], and the quality reliability of station i can be calculated by

$$R_{Qi}(t) = P\left(E_{t_i}^Q|\boldsymbol{Z}_i(t)\right) = \begin{cases}1,\ if Q_i\ge Q'_i\\ \frac{Q_i}{Q'_i},\ if Q_i < Q'_i\end{cases}, \tag{24}$$

where Q_i' is the target quantity of qualified products which is the lower boundary satisfying the system demand in station i. It will be analyzed in Section 4.1.

3.3.2. Inspection process modeling

In a manufacturing system, inspection process is not always reliable, whose poor performance will cause wrong decisions about product quality and increase the risks that tasks cannot be completed on time. In conclusion, the status of an inspection process can be classified into four categories: identifying qualified product as qualified one, identifying unqualified product as unqualified one, identifying qualified product as unqualified one, and identifying unqualified product as qualified one. The former two are normal states, which are marked as Q-Q state and U-U state; the latter two are failure states, which are marked as Q-U state and U-Q state. In any station i, the states' probabilities satisfy following equations.

$$\begin{cases}P_{QQi} + P_{QUi} = 1\\ P_{UUi} + P_{UQi} = 1\end{cases} \tag{25}$$

Based on above analysis about quality evaluation, the coupling relation between machine and inspection process will lead to three kinds of outputs: effective outputs O_{ei} which are qualified products transmitted into next station, pernicious outputs O_{pi} which are unqualified products transmitted into next station, and the losses O_{li} which are judged to be unqualified and discarded. The number of each category can be calculated by

$$\begin{cases}O_{ei} = Q_i\times P_{QQi}\\ O_{pi} = U_i\times P_{UQi}\\ O_{li} = Q_i\times P_{QUi} + U_i\times P_{UUi}\end{cases}. \tag{26}$$

According to Assumption (9), pernicious outputs O_{pi} will be chosen

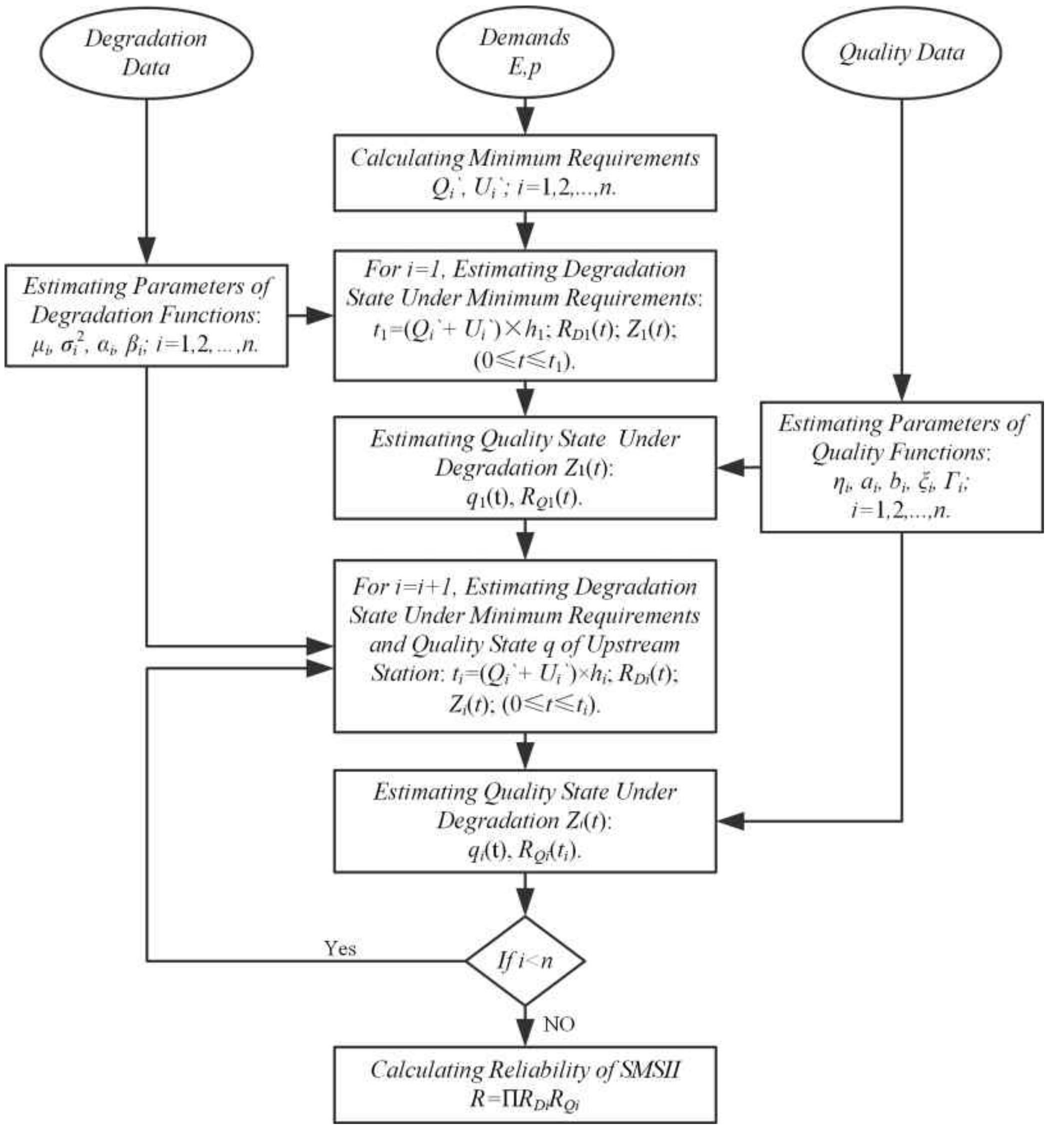


Fig. 5. Flow chart of SMSII reliability calculation.

randomly from the unqualified products U_i in any station i, so the frequency $N_i(t)$ of unqualified products processed by station i can be obtained by

$$N_i(t) = O_{pi} = \{U_i \times P_{UQi} | U_i\}. \tag{27}$$

4. Reliability analysis of SMSII

4.1. Production demand analysis

In view of the descriptions about machine degradation, product quality, and inspection process, we will discuss the reliability of series manufacturing system with n stations, as shown in Fig. 2. To produce enough products satisfying the demand E with pass-rate no less than p, the minimum target requirement of each station should be obtained. For the nth station, effective output O_{en} and pernicious outputs O_{pn} should satisfy following equations.

$$\frac{O_{en}}{O_{en} + O_{pn}} = \frac{Q_n \times P_{QQn}}{Q_n \times P_{QQn} + U_n \times P_{UQn}} \geq p \tag{28}$$

$$Q_n \times P_{QQn} + U_n \times P_{UQn} \geq E \tag{29}$$

To satisfy the demand, the minimum target requirement of nth station can be calculated when both Eqs. (28) and (29) satisfy the constraints, as

$$Q_n' = pE/P_{QQn}. \tag{30}$$

Based on the conversation law, the output of $(n-1)$th station equals the input of nth station, as

$$Q_{n-1} \cdot P_{QQn-1} + U_{n-1} \cdot P_{UQn-1} = Q_n' + U_n'. \tag{31}$$

It can be rewritten as

$$Q_{n-1} \cdot P_{QQn-1} - Q_n' = U_n' - U_{n-1} \cdot P_{UQn-1}. \tag{32}$$

According to Assumption (8), pernicious products of upstream station will be identified totally and discarded in the downstream station. So, Eq. (32) satisfies

$$Q_{n-1} \cdot P_{QQn-1} - Q_n' = U_n' - U_{n-1} \cdot P_{UQn-1} \geq 0. \tag{33}$$

Therefore, the minimum target requirement of $(n-1)$th station should satisfy the following equation.

$$Q_{n-1}' = \frac{Q'_n}{P_{QQn-1}} \tag{34}$$

In like manner, for any ith station ($i = 1,2\ldots,n-1$), the minimum target requirement for qualified products can be concluded by

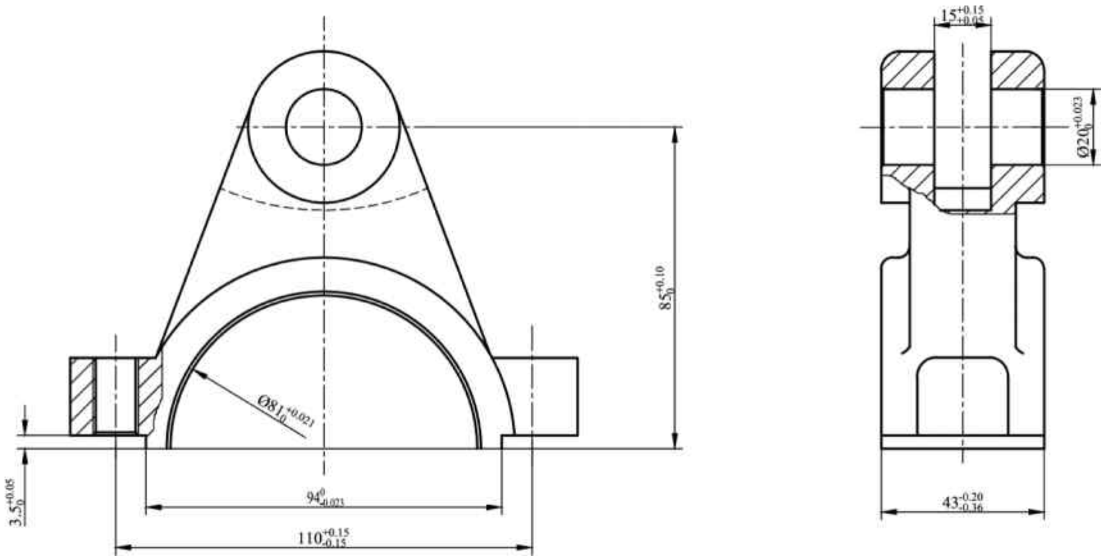


Fig. 6. Part drawing of connecting rod.

$$Q_i' = \frac{Q'_{i+1}}{P_{QQi}}. \tag{35}$$

4.2. System reliability calculation of SMSII

According to the former definition, system reliability is the probability that manufacturing system satisfies the requirements in quantity and quality without degradation failure. Therefore, the reliability of station i under the effect of quality states in station $i-1$ can be calculated by

$$\begin{aligned} R_i &= P(\text{station } i \text{ is reliable} \mid \text{quality state of station } i-1) \\ &= P\left(E_{t_i}^D \cap E_{t_i}^Q|\boldsymbol{\delta}_{i-1}\right) = P\left(E_{t_i}^D|\boldsymbol{\delta}_{i-1}\right) \times P\left(E_{t_i}^Q|\mathrm{Z}_i(t)\right), \end{aligned} \tag{36}$$

where $P(E_{t_i}^D|\boldsymbol{\delta}_{i-1})$ is the probability that station's performance does not fail when it machines unqualified products with dimensional deviations $\boldsymbol{\delta}_{i-1}$ from neighbor upstream station; $P(E_{t_i}^Q|\mathrm{Z}_i(t))$ is the probability that the target quantity is satisfied when station i has the degradation $\boldsymbol{Z}_i(t)$ at time t. So, the reliability of SMSII can be written as

$$\begin{aligned} R &= \prod_{i=1}^{n} R_i \\ &= P\left(E_{t_1}^D|\boldsymbol{\delta}_0\right) \times P\left(E_{t_1}^Q|\mathrm{Z}_1(t)\right) \times P\left(E_{t_2}^D|\boldsymbol{\delta}_1\right) \\ &\quad \times P\left(E_{t_2}^Q|\mathrm{Z}_2(t)\right) ... \times P\left(E_{t_i}^D|\boldsymbol{\delta}_{i-1}\right) \times P\left(E_{t_i}^Q|\mathrm{Z}_i(t)\right). .. \times P\left(E_{t_n}^D|\boldsymbol{\delta}_{n-1}\right) \\ &\quad \times P\left(E_{t_n}^Q|\mathrm{Z}_n(t)\right) \\ &= R_{D1}(t_1) \times R_{Q1}(t_1) \times R_{D2}(t_2) \times R_{Q2}(t_2) \times ... \times R_{Dn}(t_n) \times R_{Qn}(t_n) \\ &= \prod_{i=1}^{n} R_{Di}(t_i) \times R_{Qi}(t_i) \end{aligned} \tag{37}$$

Depending on above models illustrating the interplay behaviors among machine, inspection process and product quality, the reliability of SMSII can be calculated by following steps, as shown in Fig. 5.

Step 1: Estimate parameters of the two degradation processes based on the historical degradation and quality data, and obtain the degradation parameters of each station: μ_i, σ_i^2, α_i, β_i, $i = 2,3,\ldots,n$; according to Assumption (1), the first station will not have discrete degradation, and only α_1 and β_1 will be estimated.

Step 2: Size up parameters of quality based on the historical data, and acquire the quality parameters of each station: $\boldsymbol{\eta}_i$, $\boldsymbol{A}_i$, $\boldsymbol{B}_i$, ξ_i, $\boldsymbol{\Gamma}_i$; $i = 1,2,\ldots,n$;

Step 3: Calculate each station's target demand based on the product demands E and p, and get the minimum target requirements $Q_i'$and U_i', $i = 1, 2\ldots n$;

Step 4: For the first station, calculate the performance degradation $Z_1(t)$ and reliability $R_{D1}(t)$ under the minimum requirement time $t_1 = Q_1' \times h_1$;

Step 5: For the first station, calculate each product's quality index $q_{1jk}(t)$, $t = h_1, 2h_1, 3h_1, \ldots, t_1$; obtain the numbers Q_1 and U_1 of qualified and unqualified products, and quality reliability $R_{Q1}(t)$;

Step 6: For ith ($i > 1$) station, confirm the unqualified products from upstream station ($i-1$) by random process, and calculate the dimensional deviation from the limit size δ_{i-1} according to the dimensional variation $\boldsymbol{q}_{i-1}$; then obtain the performance degradation value $\boldsymbol{Z}_i(t)$ and the reliability $R_{Di}(t)$ under the minimum requirement time $t_i = Q_i' \times h_i$;

Step 7: For ith ($i > 1$) station, calculate each product's quality index $q_{ijk}(t)$, $t = h_i, 2h_i, 3h_i, \ldots, t_i$; get the numbers Q_i and U_i of qualified and unqualified products, and quality reliability $R_{Qi}(t)$;

Step 8: Judge whether to continue the algorithm: if $i < n$, continue step 6 and step 7; if not, go to step 9;

Step 9: Calculate the system reliability of SMSII by Eq. (37).

5. Case study

5.1. Reliability calculation

Connecting rod is a drive mechanism in engine, which is used to connect a piston and crankshaft, and drives the expanding gas pressure from the top of the piston to the crankshaft. The connecting rod is machined from a casting blank, which has 8 key product dimensions that should be finished in this manufacturing system, as shown in Fig. 6.

Referring to the process documentations, the manufacturing activities can be classified into 7 steps, which are accomplished by a system consisting of several inspection equipment and three kind of machines: milling, boring, and drilling, as shown in Fig. 7. In the station 1, the dimension $43_{-0.36}^{-0.20}$ will be finished by milling machine 1 under the noise $\xi_1 \sim N(0, 0.15)$, which is also the location dimension in the station 2. Therefore, products of station 1 will bring Q-R Impact to the station 2, like the stations 2, 3 and 6. In addition, dimensions machined in stations 2 and 3 are semifinishing dimensions of $\Phi 20_0^{+0.023}$and $\Phi 80_0^{+0.021}$, respectively. Parameters of each station are shown in Table 1.

In this manufacturing system of connecting rod, quality parameters can be assessed by the historical data collected from the machining process. Then, parameters of η_{ij}, A_{ij}, B_{ij} and $\boldsymbol{\Gamma}_{ij}$ can be obtained, where i is the number of dimensions machined in station j.

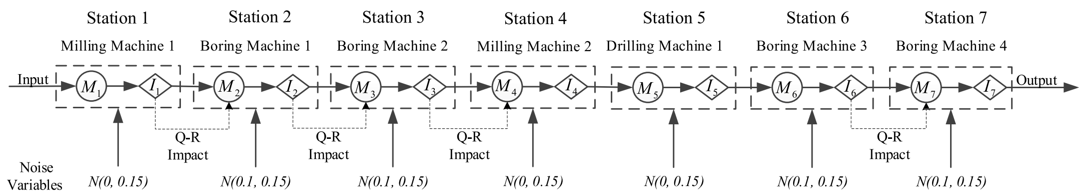


Fig. 7. Connecting rod's manufacturing system with 7 stations.

$$\eta=\begin{bmatrix}42.690 & 19.815 & 79.851 & 15.100 & 109.950 & 81.004 & 20.004\\ 0 & 0 & 0 & 85.050 & 0 & 0 & 0\\ 0 & 0 & 0 & 93.980 & 0 & 0 & 0\\ 0 & 0 & 0 & 3.525 & 0 & 0 & 0\end{bmatrix}$$

$$A=\begin{bmatrix}0.112 & 0.173 & 0.258 & 0.147 & 0.296 & 0.279 & 0.269\\ 0 & 0 & 0 & 0.238 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.203 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.186 & 0 & 0 & 0\end{bmatrix}$$

$$B=\begin{bmatrix}0.098 & 0.106 & 0.249 & 0.105 & 0.195 & 0.032 & 0.029\\ 0 & 0 & 0 & 0.141 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.034 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.072 & 0 & 0 & 0\end{bmatrix}$$

$$\Gamma=\begin{bmatrix}0.275 & 0.158 & 0.137 & 0.296 & 0.383 & 0.105 & 0.127\\ 0 & 0 & 0 & 0.152 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.132 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.143 & 0 & 0 & 0\end{bmatrix}$$

In a degradation process, assume that the shape parameter follows $\alpha(t)=a\cdot t^b$. According to the proposed methodology, the degradation parameters can be estimated by the historical degradation data.

$$\mu=[0\quad 0.1030\quad 1.5486\quad 0.0934\quad 0\quad 0\quad 1.0025]$$

$$\sigma^2=[0\quad 0.0153\quad 0.0148\quad 0.0163\quad 0\quad 0\quad 0.0027]$$

$$a=[0.3351\quad 0.1822\quad 0.2879\quad 0.1156\quad 0.5668\quad 0.2070\quad 0.2230]$$

$$b=[0.7129\quad 0.7080\quad 0.7290\quad 0.7015\quad 0.8180\quad 0.6900\quad 0.7180]$$

$$\beta=[0.1453\quad 0.3626\quad 0.3114\quad 1.0500\quad 1.1991\quad 3.2387\quad 4.4284]\times 10^3$$

Additionally, assume that the demand of the connecting rods is $E = 800$ with the qualified rate no less than 99.8%. So every station's minimum target requirement $\boldsymbol{Q'}$ can be calculated by Eqs. (30) and (35).

$$\boldsymbol{Q'}=[824\quad 821\quad 820\quad 812\quad 810\quad 806\quad 801]$$

According to calculating steps of SMSII reliability, each station's performance degradation reliability and quality reliability can be obtained from Eqs. (15) and (24).

$$\boldsymbol{R}_D=[0.9993\quad 0.6575\quad 0.9974\quad 0.7434\quad 0.9802\quad 0.9917\quad 0.9935]$$

$$\boldsymbol{R}_Q=[0.9915\quad 0.9805\quad 0.9795\quad 0.9175\quad 0.9173\quad 0.8809\quad 0.8777]$$

Then the reliability of SMSII, namely the probability that each machine works without degradation failure and the qualified products satisfy the minimum target quantity, can be calculated by Eq. (37).

$$R=\prod_{i=1}^{7} R_{Di}\times R_{Qi}=0.3045$$

5.2. Results analysis

According to the degradation reliability R_D in each station, it is obvious that stations 2 and 4 have the relatively lower reliability level than others. This illustrates that their machining accuracy will have the higher probability to go beyond the threshold during processing products. In addition, quality reliability R_Q will reduce progressively in each station because of the increased accumulative product losses. This also leads to the lower quality reliability in stations 6 and 7, which is also due to the less input from upstream stations. On the other hand, it is clear that the abrupt decrease of quality reliability occurs in stations 2 and 4, which is also consistent with their lower degradation reliability.

The performance degradation of each machine is illustrated in Fig. 8. Obviously, the degradations are smooth asymptotic processes in stations 1, 5 and 6, but the degradations are saltatory asymptotic processes in stations 2, 3, 4 and 7, which are affected by the shocks of unqualified inputs from upstream stations. In addition, the randomicity of discrete degradation values is also illustrated in Fig. 8, which verifies the effectiveness of this discrete degradation model. In addition, continuous degradation processes without considering shocks are also depicted in Fig. 8, and their degradation values are underestimated obviously. Out of question, system reliability will be overrated when ignoring the shocks of unqualified inputs.

In addition, scatter diagrams of quality characteristics are illustrated in Fig. 9 for each station. According to the stochastic analysis, frequencies of unqualified products in each 100 products interval are obtained, as shown in Fig. 10(a). Obviously, the frequencies of stations 2 and 4 are much higher than that of other stations because of the violent degradation. It can also be concluded that the number of products overranging dimensional tolerance will raise with the degradation increasing. Undoubtedly, this will increase the occurrence that unqualified products are transmitted into downstream stations, and cause more discrete degradations.

For the scatter diagrams, each dimension categories are standardized by the following equation as

$$D_{st.}=(D-D_{mid})/Tol,$$

Table 1
Parameters of stations in connecting rod's manufacturing system.

Station#	1	2	3	4	5	6	7
P_{QQ}	0.9968	0.9989	0.9911	0.9977	0.9956	0.9948	0.9974
P_{UU}	0.9921	0.9930	0.9924	0.9911	0.9951	0.9943	0.9947
Initial precision	0.020	0.030	0.020	0.010	0.10	0.010	0.010
Precision threshold	0.560	0.084	0.220	0.030	0.300	0.021	0.023
Piece time /h	1.35	0.775	1.05	2.18	0.48	1.55	1.23
Dimensions machined /mm	$43_{-0.36}^{-0.20}$	$\Phi 19.8_{0}^{+0.084}$	$\Phi 79.8_{0}^{+0.22}$	$15_{-0.15}^{+0.15}$ $85_{0}^{+0.1}$ $94_{-0.03}^{0}$ $3.5_{0}^{+0.05}$	$110_{-0.15}^{+0.15}$	$\Phi 80_{0}^{+0.021}$	$\Phi 20_{0}^{+0.023}$

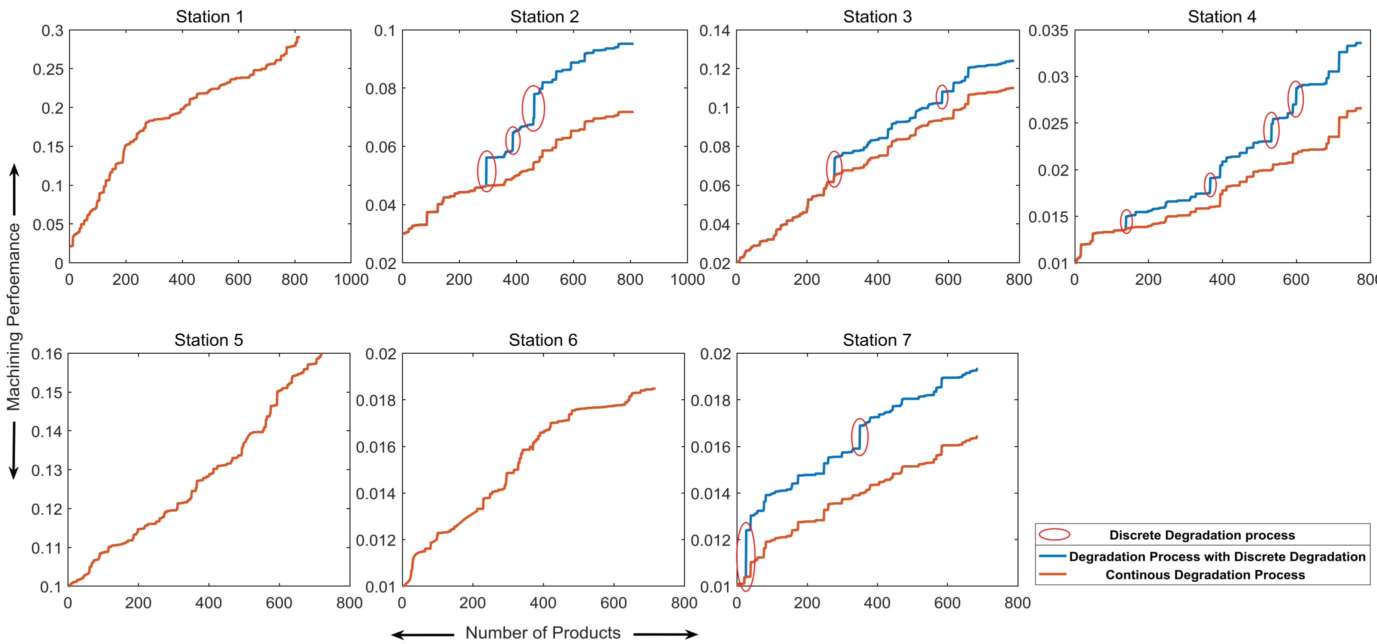


Fig. 8. Degradation curves of stations' machining performance.

where $D_{st.}$ is the standardized variable; D is the real dimension; D_{mid} is the average of the upper and lower specification; Tol is the dimensional tolerance. According to this equation, variances of standardized variables can be computed for each station, as shown in Fig. 10(b). It also illustrates that stations 2 and 4 have much higher variances because of the violent degradation, which is consistent with the frequency analysis of unqualified products.

6. Conclusions

As a complex system, reliability evaluation of manufacturing systems is always a challenging topic due to the multiple factors and interactions among them. In this paper, multiple factors and their interaction models are investigated to evaluate the reliability of a system mission. Considering the effect on machine performance and product quality, machining accuracy is used as the performance index to analyze machine's degradation. In view of the fact that quality inspection process plays a constructive role in the propagation of unqualified products and the discrete degradation of downstream machines, imperfect inspection is firstly considered in our reliability model by analyzing its effect on the interplay between machine degradation and product quality.

Firstly, machine's performance degradation is studied in the viewpoint of continuous and discrete degradation processes. The continuous degradation is modeled by Gamma Process in this paper, which can better show its non-decreasing property. The discrete degradation is depicted by Marked Point Process which simulates the arrival interval between unqualified products and Wiener Process which simulates the damage increment caused by dimensional deviations shocks. Secondly, a linear model is used to estimate product's dimensional variation under the impact of machine's degradation. According to this model, a quality reliability model is constructed to evaluate the proportion that the

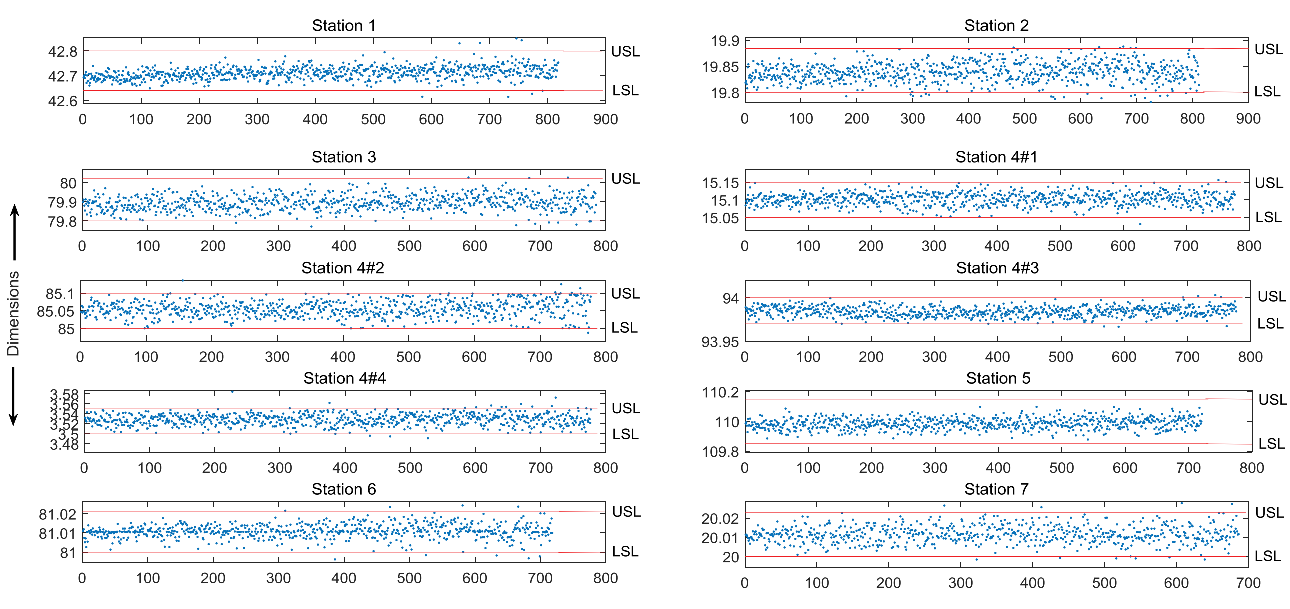
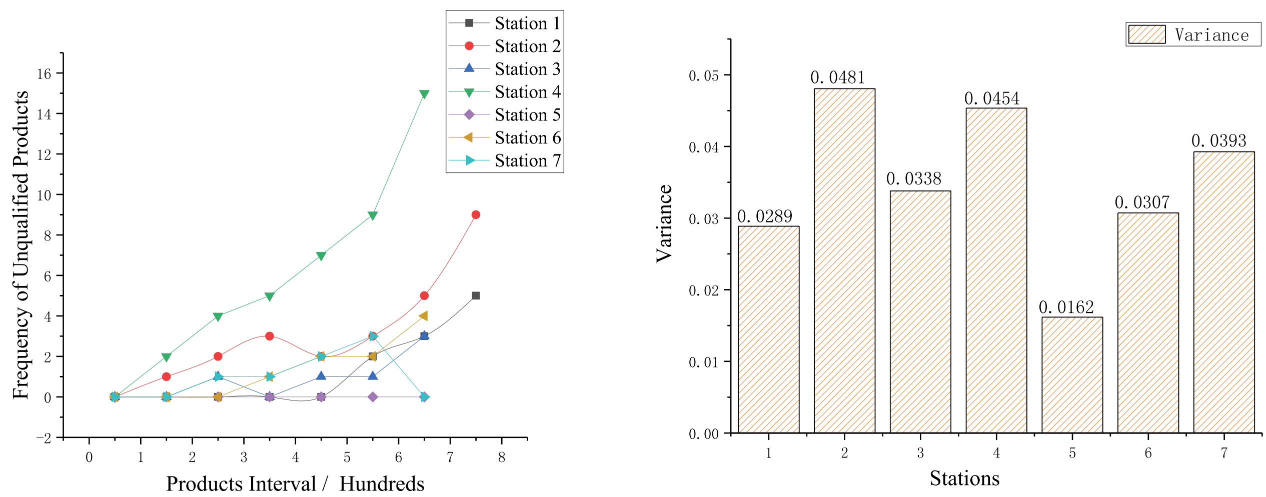


Fig. 9. Quality characteristcs of dimensions machined in stations.



(a) Frequency of Unqualified Prod (b) Variances of Standard DimensionVariables

Fig. 10. Statistical analysis of dimensional deviations.

demands are satisfied in each station. Thirdly, quality inspection performance is analyzed, and four states are defined to illustrate its effect on machine's discrete degradation process. At last, based on these models, the new reliability model is established by considering the minimum target requirements of each station, and its algorithm is illustrated in this paper.

In addition, a case is studied in this paper, which certifies the advantage and effectiveness of the proposed model. According to this new model, the weak processes to finish the target demand can be recognized. The comparison study indicates that system reliability will be overrated when ignoring the shocks of unqualified inputs. And the statistical analysis of dimensional deviations also illustrates the relevance between degradation and product quality. In addition, the abundant information existing in manufacturing system can be integrated and exploited effectively by the proposed reliability model. This can not only help to identify the weakness in manufacturing system, but also improve the veracity of reliability evaluation.

Acknowledgements

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