UNIVERSITY OF PENNSYLVANIA
The Wharton School

Investments | Project 4: Simulation
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**Goal:** Use historical data to simulate long-horizon returns and explore implications of an endowment's asset mix, spending rate, and expected-return assumptions.

**Setting and Analysis:** The trustees of a small college have formed a committee to help them analyze long-run implications of (1) their endowment fund's investment policy, (2) the rate of spending from the endowment, and (3) their assumptions about expected returns in asset markets. The committee has asked you to assist in this analysis by performing a series of simulations. The committee is especially interested in trying to quantify the probability that the endowment would shrink in real terms, but it is also interested in knowing the probability that the endowment would fail to have real growth of at least 1% per annum. Of course, the greater the rate of spending from the endowment, the less likely it is that the fund will meet such thresholds. The fund is currently invested 60% in a U.S. stock-market index fund and 40% in a U.S. government bond fund, but it is also considering investing part of the endowment in an index fund of small-cap firms.

The committee would like you to assume two different scenarios for returns on the three asset classes (overall stock market, government bonds, small-cap stocks):

1. For the first scenario, assume that the probability distribution of future annual returns and inflation is well represented by the distribution of realized annual returns over the past 90 years (1926–2015).
2. For the second scenario, maintain the above assumption for government bond returns and inflation, but reduce the expected returns on stocks. Specifically, assume that
    a. the expected return on the overall stock market is only 3% higher than the expected return on government bonds, and
    b. the expected return on small-cap stocks is only 2% higher than the (now lower) expected return on the overall stock market.

    In the second scenario, aside from these modified expected returns, assume that all other aspects of the historical distribution of realized annual returns remain the same.

For each of the two return scenarios, use simulations of long-horizon outcomes to estimate

1. the probability of negative real growth in the endowment fund
2. the probability of less than 1% real growth in the endowment fund

Estimate these probabilities for the current asset mix as well as a more aggressive mix of 50% in the overall market index, 20% in government bonds, and 30% in small-cap stocks. For each return scenario and asset mix, estimate the probabilities for

a. horizons of 5, 10, 15, 20, 25, and 30 years
b. annual rates of spending (net of contributions) from the endowment of 1%, 2%, 3%, 4%, and 5%

Comment on the findings and summarize for the committee the nature of the tradeoffs the college faces. Also comment on which of the two return scenarios you would advise that the committee give more weight.

**Guidelines:**

1. Do not assume a standard return distribution (e.g., Normal) from which to simulate future annual returns. Instead assume that returns and inflation for any future year are drawn randomly, with replacement, from the previous 90 years. This allows whatever features of randomness are exhibited by those returns to enter the simulation, including not only volatility and correlations but also skewness and "tail" properties. Simulating by randomly re-sampling from historical data is a popular procedure in statistics commonly known as "bootstrapping." In this approach, the joint histogram of the historical series essentially becomes the random number generator, instead of a random number generator based on a standard distribution such as the Normal.

2. Changing only the expected return of an asset is easily accomplished. Before taking the random draws, simply add (X-Y) to each realized historical return on the asset, where X is the asset's desired new expected return and Y is the asset's actual average return in the data. (This latter procedure is relevant for simulating under the second scenario.)

3. The Excel file **multiyr_sim.xls** illustrates, in a single worksheet, how the bootstrap approach to simulating returns is easily accomplished using simple spreadsheet functions (without add-ins). A useful trick for randomly drawing from a list of historical returns is simply to combine Excel's VLOOKUP and RAND functions. The basic abilities to copy formulas and use relative cell addressing are then quite useful as well in setting up the spreadsheet. Of course, you are free to take an alternative approach to the computations. There is a Sampling feature in Excel's Data Analysis that can randomly draw the values, but with more work, I find. A more powerful simulation add-in is Oracle's Crystal Ball, but that costs extra and there's nontrivial start-up learning.

4. Use historical inflation rates and annual returns on the three asset classes provided in **multiyr_sim.xls**. (These data are obtained from WRDS/CRSP, except for the long-term government bond returns prior to 1942, which are from Morningstar/Ibbotson SBBI.)

5. The spreadsheet is set up to simulate 10,000 long-horizon draws, enough to give reasonably precise result in most cases. (Repeatedly redrawing the returns—hitting F9—and watching how answers change is a simple way to gain some feel for the precision: The more precise are the answers, the less they will change from one set of draws to the next.) Increasing the number of draws is a pretty straightforward modification of the spreadsheet, if you care to do that (but it should not be necessary for the project).