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Optimal Asset Location and Allocation with Taxable and Tax-Deferred Investing

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ABSTRACT

We investigate optimal intertemporal asset allocation and location decisions for investors making taxable and tax-deferred investments. We show a strong preference for holding taxable bonds in the tax-deferred account and equity in the taxable account, reflecting the higher tax burden on taxable bonds relative to equity. For most investors, the optimal asset location policy is robust to the introduction of tax-exempt bonds and liquidity shocks. Numerical results illustrate optimal portfolio decisions as a function of age and tax-deferred wealth. Interestingly, the proportion of total wealth allocated to equity is inversely related to the fraction of total wealth in tax-deferred accounts.

A CENTRAL PROBLEM CONFRONTING INVESTORS in practice is how to efficiently invest the funds held in their taxable and tax-deferred savings accounts. The problem involves making both an optimal *asset allocation* decision (i.e., deciding how much of each asset to hold) and an optimal *asset location* decision (i.e., deciding which assets to hold in the taxable and tax-deferred accounts). Investors would like to make these decisions to reduce the tax burden of owning financial assets, while maintaining an optimally diversified portfolio over time. While only limited guidance is available to investors faced with this problem, the decision is crucial to the wealth accumulation and welfare of investors over their lifetimes.

In this paper, we examine the intertemporal portfolio problem for an investor with the opportunity to invest in both a taxable and tax-deferred savings account. In particular, we investigate how the opportunity for tax-deferred investing influences the investor's overall portfolio composition and how these asset holdings are allocated between the taxable and tax-deferred accounts. Our approach takes account of the investor's age, existing portfolio holdings, embedded capital gains, and available wealth levels in the taxable and tax-deferred

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accounts for the asset allocation and location decisions. This is in striking contrast to the traditional approach to financial planning, in which the interaction between the taxable and tax-deferred accounts is largely ignored.

The ability to invest on a tax-deferred basis is valuable to investors because it allows them to earn the pre-tax return on assets. However, because assets differ in terms of the tax liabilities they create for investors, the value of tax-deferred investing depends upon which assets are held in the tax-deferred account. For example, it is well understood that holding municipal bonds in a tax-deferred account is tax inefficient because municipal bond interest is nontaxable. Our analysis of the optimal asset location policy focuses mainly on taxable bonds and equity. We show that there is a strong locational preference for holding taxable bonds in the tax-deferred account and equity in the taxable account. This preference reflects the higher tax burden on taxable bonds relative to equity. When held in the taxable account, equity generates less ordinary income than taxable bonds, provides the investor with a valuable tax-timing option to realize capital losses and defer capital gains, and allows the investor to avoid payment of the tax on capital gains altogether at the time of death. Our analysis also examines the circumstances under which equity ownership arises in the tax-deferred account despite its greater attractiveness in the taxable account.

When investors have unrestricted borrowing opportunities in the taxable account, the optimal asset location policy involves allocating the entire tax-deferred account to taxable bonds. Investors then combine either borrowing or lending with investment in equity in the taxable account to achieve their desired risk exposure. The optimal asset location policy with unrestricted borrowing opportunities follows directly from the arbitrage arguments made by Black (1980) and Tepper (1981), who analyze the optimal investment policy for corporations with defined-benefit pension plans.[1] Investors are indifferent to the location of their asset holdings only if capital gains and losses are taxed on an accrual basis (i.e., no deferral option) and at the same tax rate that applies to ordinary income. This implies that even the most tax-inefficient equity mutual funds (i.e., those that distribute a large fraction of their capital gains each year) are better suited for the taxable account than are taxable bonds.

In a series of recent papers, Shoven (1999), Shoven and Sialm (2003), and Poterba, Shoven, and Sialm (2001) question the tax-efficiency of holding equity in the taxable account and taxable bonds in the tax-deferred account when investors have the option to invest in tax-exempt bonds. They argue that because actively managed equity mutual funds distribute a large fraction of their capital gains each year, it can be optimal to locate them in the tax-deferred account and hold tax-exempt bonds in the taxable account. Using arbitrage arguments, we show that the strategy of holding an actively managed equity mutual fund in the tax-deferred account and tax-exempt bonds in the taxable account can

[1] Black (1980) and Tepper (1981) show that it is tax efficient for corporations to fully fund their pension plans, borrowing on corporate account if necessary, and to invest the pension plan assets entirely in taxable bonds. The implications of the Black–Tepper arbitrage results for optimal asset location for individual investors were first discussed in Dammon, Spatt, and Zhang (1999) and formally illustrated by Huang (2000).

be optimal only if the actively managed equity mutual fund (1) is highly tax inefficient and (2) substantially outperforms similar tax-efficient equity investments (e.g., individual stocks, passive index funds, or exchange-traded funds) on a risk-adjusted basis. Given the well-documented underperformance of actively managed equity mutual funds, we argue that investors are better off holding tax-efficient equity investments, locating them in the taxable account, and holding taxable bonds in the tax-deferred account. Of course, highly taxed investors who wish to hold a mix of stocks and bonds in the taxable account may still find tax-exempt bonds to be a better alternative than taxable bonds.

When investors are prohibited from borrowing, the optimal asset location policy is slightly more complicated. Although investors still have a preference for holding taxable bonds in the tax-deferred account, it may not be optimal to allocate the entire tax-deferred account to taxable bonds if doing so causes the overall portfolio to be overweighted in bonds. This is because offsetting portfolio adjustments in the taxable account is no longer possible when borrowing is prohibited. In this case, investors may hold a mix of stocks and bonds in their tax-deferred accounts, but only if they hold an all-equity portfolio in their taxable accounts. Investors may still hold a mix of stocks and (taxable or tax-exempt) bonds in the taxable account, but only if they hold a portfolio composed entirely of taxable bonds in their tax-deferred accounts. Investors do not simultaneously hold a mix of stocks and bonds in both the taxable and tax-deferred accounts.

With most or all of the taxable account allocated to equity, the investor may face liquidity problems if the value of equity declines substantially. With a dramatic decline in equity, the investor may be forced to liquidate a portion of the tax-deferred account to finance consumption. For some investors, withdrawing funds from the tax-deferred account may require the payment of a penalty. In principle, this can provide an incentive to hold some additional bonds in the taxable account to reduce the risk of needing to withdraw funds from the tax-deferred account. Contrary to this intuition, we find that the tax benefit of locating taxable bonds in the tax-deferred account generally outweighs the liquidity benefit of holding taxable bonds in the taxable account. Investors are willing to shift the location of taxable bonds from the tax-deferred account to the taxable account only if catastrophic shocks to consumption (or income) are highly negatively correlated with equity returns. Even in these cases, however, the demand for bonds in the taxable account for liquidity reasons is small relative to the total holding of bonds. The risk of a liquidity shock has a much larger impact on the investor's willingness to make additional contributions to the tax-deferred account.

We investigate numerically the investor's lifetime portfolio problem by incorporating a tax-deferred investment account into the intertemporal consumption–investment model developed by Dammon, Spatt, and Zhang (2001).[2] We illustrate how the relative wealth levels in the taxable and

[2] The intertemporal consumption-investment model of Dammon, Spatt, and Zhang (2001) incorporates many realistic features of the U.S. tax code, including the taxation of capital gains upon

tax-deferred accounts influence the optimal asset location and allocation decisions. With the ability to borrow in the taxable account, the optimal overall holding of equity is relatively insensitive to the split of total wealth between the taxable and tax-deferred accounts. However, because the investor allocates 100% of the tax-deferred account to taxable bonds in this case, the proportion of the taxable account allocated to equity can exceed 100% (i.e., a levered equity position) at high levels of tax-deferred wealth. When investors are prohibited from borrowing, the holding of equity in the taxable account is capped at 100%. In this case, we find that equity can spill over into the tax-deferred account, but only at very high levels of tax-deferred wealth. However, because equity is less valuable when held in the tax-deferred account, the proportion of total wealth allocated to equity is lower at higher levels of tax-deferred wealth.

The results we derive on the optimal location of asset holdings are in sharp contrast to the financial advice that investors receive in practice. Financial advisors commonly recommend that investors hold a mix of stocks and bonds in both their taxable and tax-deferred accounts, with some financial advisors recommending that investors tilt their tax-deferred accounts toward equity. The asset location decisions made in practice mirror these recommendations, with many investors holding equity in a tax-deferred account and bonds in a taxable account. Poterba and Samwick (2003) report that 48.3% of investors who own taxable bonds in taxable accounts also own equity in tax-deferred accounts and that 41.6% of investors who own equity in tax-deferred accounts also own taxable bonds in taxable accounts. They also document that 53.1% of the owners of tax-exempt bonds also owned equity in tax-deferred accounts and that 11.3% of the owners of equity in tax-deferred accounts also owned tax-exempt bonds. Bergstresser and Poterba (2002) and Amromin (2001) report similar findings and document that a large proportion of investors have substantially more equity in their tax-deferred accounts than in their taxable accounts. We investigate the welfare costs of locating assets suboptimally between the taxable and tax-deferred accounts and find that these costs can be quite high, especially for young investors.

The paper is organized as follows. In Section I, we derive some general theoretical results regarding optimal asset location using basic arbitrage arguments. We examine the effects of borrowing and short-sale constraints, tax-exempt bonds, and liquidity shocks on the optimal asset location policy. In Section II, we present our numerical analysis of the investor's intertemporal portfolio problem, focusing on the case where there are restrictions on borrowing and short sales. Particular attention is given to the optimal asset allocation and location decisions as a function of age and the level of tax-deferred wealth. We also conduct a welfare analysis of the optimal asset location policy

realization and the forgiveness of the tax on embedded capital gains at the time of death. The impact of optimal tax timing on the realization and trading behavior of investors is also studied by Constantinides (1983, 1984), Dammon, Dunn, and Spatt (1989), Dammon and Spatt (1996), and Williams (1985). In contrast to this earlier work, the Dammon, Spatt, and Zhang (2001) model incorporates an optimal intertemporal portfolio decision, which involves a tradeoff between the diversification benefits and tax costs of trading.

and investigate the effects of exogenous liquidity shocks on the asset location and retirement contribution decisions. Section III concludes the paper.

I. Optimal Asset Location

A. No Borrowing or Short-sale Constraints

In this section, we use arbitrage arguments to derive results on the optimal location of asset holdings. Our approach extends the arbitrage approaches used by Black (1980) and Tepper (1981) to analyze corporate pension policy and by Huang (2000) to analyze the asset location decision. The arbitrage approach involves making a risk-preserving change in the location of asset holdings to determine whether the after-tax return on the investor's portfolio can be improved. The objective is to identify the asset location policy that produces the highest expected utility of after-tax wealth for the investor.

We initially assume that investors are *forced* to realize all capital gains and losses each year (i.e., no deferral option) and have *unrestricted* borrowing and short-sale opportunities in their taxable accounts. (We later relax these assumptions to see what effect they have on the optimal location decision.) We also assume that the tax rate on ordinary income (dividends and interest), τ_d, is higher than the tax rate on capital gains and losses, τ_g. Under these conditions we show that investors prefer to allocate their entire tax-deferred wealth to the asset with the *highest yield*.[3] Investors then adjust the asset holdings in their taxable accounts, borrowing or selling short if necessary, to achieve their optimal overall risk exposure. For our purposes, we define *yield* as the fraction of total asset value (price) that is distributed as either dividends or interest.

We define the random pre-tax return on asset i as $\tilde{r}_i = (1 + d_i)(1 + \tilde{g}_i) - 1$, where d_i denotes the constant pre-tax yield on asset i and $\tilde{g}_i$ denotes the random pre-tax capital gain return on asset i. For the riskless taxable bond (asset 0), we assume that $\tilde{g}_0 = 0$ and $d_0 = r$. Consider an investor in this environment who has positive holdings of both the riskless taxable bond and risky asset i in the tax-deferred account. For this investor, a shift of one *after-tax* dollar from asset i to the riskless taxable bond in the tax-deferred account, offset by a shift of x_i dollars from the riskless taxable bond (either through an outright sale or through borrowing) to asset i in the taxable account, leads to the following change in the the investor's total wealth next period:[4]

[3] Under recent tax law changes, the tax rate on dividend income is less than the tax rate on interest income. In this case, it may not be optimal to hold the asset with the highest yield in the tax-deferred account. The implications of differential tax rates on dividend and interest income are discussed later.

[4] An *after-tax* dollar in the tax-deferred account refers to a dollar owned by the investor in that account. For example, if the investor contributes *pre-tax* income to the tax-deferred account, the government taxes withdrawals from the account as ordinary income. In this case, the investor owns the fraction $(1 - \tau_d)$ of his tax-deferred account and the government owns the fraction τ_d. A one-dollar shift of the investor's wealth in the tax-deferred account would then require an actual shift of $1/(1 - \tau_d)$ of the total account balance. This is equivalent to allowing investors to contribute *after-tax* income to the tax-deferred account and imposing no tax on withdrawals (e.g., Roth IRA). In this case, the investor owns 100% of his tax-deferred account.

$$\begin{aligned}\Delta\tilde{W}_i &= \Delta\tilde{W}_i^R + \Delta\tilde{W}_i^T \\ &= \{r - [(1+\tilde{g}_i)(1+d_i) - 1]\} \\ &\quad + x_i\{[(1+\tilde{g}_i)(1+d_i(1-\tau_d)) - \tilde{g}_i\tau_g - 1] - r(1-\tau_d)\},\end{aligned} \tag{1}$$

where $\Delta\tilde{W}_i^R = \{r - [(1+\tilde{g}_i)(1+d_i) - 1]\}$ is the marginal change in tax-deferred (retirement) wealth and $\Delta\tilde{W}_i^T = x_i\{[(1+\tilde{g}_i)(1+d_i(1-\tau_d)) - \tilde{g}_i\tau_g - 1] - r(1-\tau_d)\}$ is the marginal change in taxable wealth. Letting $x_i = (1+d_i)/[1+d_i(1-\tau_d) - \tau_g]$, it is easily shown that for all values of $\tilde{g}_i$,

$$\Delta\tilde{W}_i = x_i\left[\frac{(r-d_i)(\tau_d-\tau_g)}{1+d_i}\right] = C_i. \tag{2}$$

Since C_i is independent of $\tilde{g}_i$, it represents a *risk-free* after-tax payoff that can be generated by shifting the location of asset holdings. However, because wealth in the tax-deferred account is more valuable than wealth in the taxable account, there is no guarantee that the change in the expected utility of total wealth has the same sign as C_i if the taxable and tax-deferred accounts are affected differently. To verify that the change in expected utility has the same sign as C_i, let $\tilde{U}'$ denote the marginal utility of taxable wealth and $m\tilde{U}'$ denote the marginal utility of tax-deferred wealth, where $m > 1$ is the *shadow price* of taxable wealth per dollar of tax-deferred wealth.[5] Then the change in expected utility is

$$\Delta E[\tilde{U}] = E\big[\tilde{U}'\Delta\tilde{W}_i^T\big] + mE\big[\tilde{U}'\Delta\tilde{W}_i^R\big].$$

Because the investor has unrestricted borrowing and short-sale opportunities in the taxable account, there must be indifference between bonds and stocks at the margin in this account. This implies that the first-order optimality conditions must satisfy $E[\tilde{U}'\Delta\tilde{W}_i^T] = 0$. Using $\Delta\tilde{W}_i^{\mathrm{R}} = C_i - \Delta\tilde{W}_i^T$, where C_i is given by equation (2), the change in expected utility becomes

$$\Delta E[\tilde{U}] = mC_iE[\tilde{U}'],$$

which clearly indicates that $\Delta E[\tilde{U}]$ is of the same sign as C_i.

If $C_i > 0$, then the investor is strictly better off holding taxable bonds in the tax-deferred account and asset i in the taxable account. If $C_i < 0$, then the investor is strictly better off holding taxable bonds in the taxable account and asset i in the tax-deferred account. To determine the tax benefit of shifting one

[5] Wealth is more valuable in the tax-deferred account because of the ability to earn pre-tax returns in this account. The shadow price, m, is higher for investors who have longer horizons (i.e., younger investors) over which to benefit from tax-deferred savings. When investors are prohibited from borrowing, the shadow price of tax-deferred wealth may also be a function of the split of wealth between the taxable and tax-deferred accounts. Section II.D provides numerical estimates of the shadow prices in this case.

after-tax dollar from risky asset i to risky asset j in the tax-deferred account, with an offsetting adjustment in the taxable account, one simply needs to compute the difference $(C_i - C_j)$. Only if $C_i = 0$ for all i is the investor indifferent to the location of his asset holdings.

Since x_i is strictly positive, the sign of C_i depends upon the sign of $(r - d_i)(\tau_d - \tau_g)$. If $\tau_d = \tau_g$, then $C_i = 0$ for all i and the investor is indifferent to the location of his asset holdings. This indifference result is independent of the expected returns and yields on assets and only requires that the total returns on all assets be taxed identically each year. When $\tau_d > \tau_g$, the sign of C_i depends upon the sign of $(r - d_i)$, with the value of C_i monotonically *decreasing* in d_i. Thus, when $\tau_d > \tau_g$ the investor prefers to allocate his entire tax-deferred wealth to the asset with the *highest yield*, with all other assets held in the taxable account.[6] After allocating the entire tax-deferred wealth to the asset with the highest yield, the investor then adjusts the asset holdings in the taxable account, borrowing or selling short if necessary, to achieve the desired overall risk exposure. This asset location policy provides the investor with the highest level of tax efficiency while maintaining the risk profile of his overall portfolio. The optimal asset location policy is also independent of the joint distribution of asset returns and investors' preferences.

It is widely believed that because actively managed mutual funds distribute significant capital gains each year, it can be tax-efficient to hold these funds (to the extent that they are held at all) in a tax-deferred account. Similarly, it is believed that an investor who engages in active trading should do so in a tax-deferred account to avoid the payment of capital gains taxes. Our analysis of the optimal asset location policy sheds some light on this issue. Recall that our analysis is based upon the assumption that investors are *forced* to realize all capital gains and losses each year (i.e., no deferral option). Yet, despite the inability to defer capital gains, our analysis indicates that it is still optimal to locate the asset with the *highest yield* in the tax-deferred account provided $\tau_d > \tau_g$.[7] Thus, even though actively managed mutual funds distribute most, or even all, of their capital gains each year, they should not be held in the tax-deferred account if taxable bonds have higher yields. Only in the extreme case in which the actively managed mutual fund distributes 100% of its capital gains each year, with all gains realized *short term* so that $\tau_g = \tau_d$, would the investor

[6] When the tax rate on dividend income (τ_d) is lower than the tax rate on interest income (τ_0) the sign of C_i in Equation (2) depends upon the sign of $[r(\tau_0 - \tau_g) - d_i(\tau_i - \tau_g)]$, where τ_i is equal to τ_d if d_i is dividend income or τ_0 if d_i is interest income. In this case, it is optimal to hold the asset with the highest value of $d_i(\tau_i - \tau_g)$ in the tax-deferred account. Under recent changes to U.S. tax rates, dividends and capital gain income are taxed at the same rate, while interest income is taxed at a higher rate (i.e., $\tau_0 > \tau_d = \tau_g$). This implies that it is not optimal to hold equity in the tax-deferred account, regardless of the magnitude of the dividend yield on equity.

[7] For mutual funds that distribute both long-term and short-term capital gains, the capital gains tax rate is a weighted average of the long-term and short-term tax rates, with the weights determined by the proportion of the total capital gain that is of each type. If all capital gains are realized short-term each year, then $\tau_d = \tau_g$. Otherwise, $\tau_d > \tau_g$, even for the most active of mutual funds.

be *indifferent* to holding the actively managed mutual fund or riskless taxable bond in the tax-deferred account.[8]

B. Tax-exempt Bonds

According to the above analysis, it is tax efficient to hold equity (or equity mutual funds) in the taxable account and taxable bonds in the tax-deferred account if taxable bonds have higher yields. In a recent paper, Shoven and Sialm (2003) argue that this policy can be overturned if investors have the opportunity to invest in *tax-exempt* bonds. Rather than holding taxable bonds in the tax-deferred account and equity in the taxable account, they show that it can be optimal to hold tax-exempt bonds in the taxable account and equity in the tax-deferred account. Using our framework to analyze this alternative strategy, the risk-free change in total wealth from shifting one after-tax dollar from taxable bonds to equity in the tax-deferred account and $x = (1 + d)/[1 + d(1 - \tau_d) - \tau_g]$ dollars from equity to *tax-exempt* bonds in the taxable account is

$$\hat{C} = x\left[r(\tau_d - \tau_m) - \frac{(r - d)(\tau_d - \tau_g)}{1 + d}\right], \tag{3}$$

where τ_m is the implicit tax rate reflected in the yield differential between riskless taxable bonds and riskless tax-exempt bonds.[9] Using arguments similar to those used earlier, one can show that the change in expected utility is of the same sign as $\hat{C}$. If $\tau_g > \tau_d - [r(\tau_d - \tau_m)(1 + d)]/(r - d)$, then $\hat{C} > 0$ and it is optimal for the investor to hold equity in the tax-deferred account and tax-exempt bonds in the taxable account. Assuming $r = 6\%$, $d = 2\%$, $\tau_d = 36\%$, and $\tau_m = 25\%$, $\hat{C} > 0$ for all $\tau_g > 19.17\%$.[10] For a tax-inefficient equity mutual fund that realizes 75% of its capital gains each year, two-thirds of which are short term and one-third of which are long term, the effective capital gains tax rate is $\tau_g = 23\%$. In contrast, for a tax-efficient index fund that realizes only 15% of its capital gains each year, 5% of which are short term and 95% of which

[8] When capital gain tax rates are allowed to differ across assets, the riskless taxable bond will still be held in the tax-deferred account provided it has the highest yield (i.e., $d_i < r$ for all i). However, if the dividend yields on some assets exceed the riskless taxable interest rate, then the asset with the highest yield may *not* be held in the tax-deferred account. In this case, the values of C_i in equation (2) for different assets depend upon both the yield and asset-specific capital gains tax rate.

[9] Equation (3) applies only to those investors who do not borrow in their taxable accounts. If investors do borrow, then the U.S. tax code disallows the interest deduction on an amount of borrowing equal to the tax-exempt holdings. This has the effect of setting $\tau_d = \tau_m$ in equation (3) for each dollar of borrowing that is not allowed the tax deduction. The net effect is to lower the benefit of shifting equity into the tax-deferred account for investors that hold levered equity positions.

[10] The implicit tax rate $\tau_m = 25\%$ is the 30-year average for long-term municpal bonds reported by Shoven and Sialm (2003). The implicit tax rate on short-term municipal bonds is typically closer to the statutory marginal tax rate for high-income investors. Green (1993) provides an equilibrium model of the municipal term structure that relies on clientele arguments and is broadly consistent with the empirical evidence.

are long term, the effective capital gains tax rate is only $\tau_g = 3.12\%$.[11] Clearly it can be optimal to hold equity in the tax-deferred account, and tax-exempt bonds in the taxable account, but only if the form of the equity holding is *highly* tax inefficient.

While the above analysis is instructive, it does not directly answer the question as to whether it is better to hold a tax-efficient index fund in the taxable account and taxable bonds in the tax-deferred account, or to hold an actively managed (tax-inefficient) equity mutual fund in the tax-deferred account and tax-exempt bonds in the taxable account. Assume that the two equity funds have identical dividend yields and are perfectly correlated on a pre-tax basis. Let τ_{gi} denote the effective capital gains tax rate on the tax-efficient index fund. It is straightforward to show that a shift of one after-tax dollar from taxable bonds to the actively managed equity mutual fund (asset j) in the tax-deferred account, offset by a shift of $x_{ij} = (1 + d_j)/[1 + d_j(1 - \tau_d) - \tau_{gi}]$ dollars from the index fund (asset i) to tax-exempt bonds in the taxable account, produces the following *riskless* after-tax cash flow:[12]

$$\hat{C}_{ij} = [d_j + \alpha_j(1 + d_j) - r] - x_{ij}[d_j(1 - \tau_d) - r(1 - \tau_m)], \tag{4}$$

where $\alpha_j = (\tilde{g}_j - \tilde{g}_i)$ is the *riskless* pre-tax capital gain return differential between the actively managed equity mutual fund and the tax-efficient index fund. Thus, we can interpret $\alpha_j(1 + d_j)$ as the *certainty-equivalent* pre-tax abnormal return (net of transaction costs and fees) on the actively managed equity mutual fund. Using the tax rates, dividend yields, and interest rates from above, the value of $\hat{C}_{ij}$ is strictly positive provided $\alpha_j(1 + d_j) > 0.00654$. This implies that it is optimal to hold the actively managed equity mutual fund in the tax-deferred account and municipal bonds in the taxable account (instead of taxable bonds in the tax-deferred account and the tax-efficient index fund in the taxable account) only if the actively managed equity mutual fund generates a certainty-equivalent pre-tax abnormal return (net of transaction costs and fees) of at least 65.4 basis points per year.[13] Moreover, since it is not uncommon for actively managed equity mutual funds to have expense ratios that are 100 basis points or more above that of a passive index fund, a certainty-equivalent pre-tax abnormal return (before transaction costs and fees) of 165 basis points

[11] The effective capital gains tax rate is based upon the assumption that long-term capital gains are taxed at 20%, short-term capital gains are taxed at 36%, and unrealized capital gains are untaxed by virtue of the fact that investors can defer the realization of capital gains until death, at which time the embedded tax liability is forgiven. The realization percentages used to calculate the effective capital gains tax rates for actively managed and index mutual funds are broadly consistent with those reported in Shoven and Sialm (2003).

[12] We are implicitly assuming that the financial markets are rich enough that a portfolio of securities can be constructed to match any risk and yield characteristics the investor desires. The assumption of perfect correlation implies that the return on asset j is of the following form: $\tilde{g}_j = \gamma_j \tilde{g}_i + \alpha_j$. Without loss of generality, we assume that $\gamma_j = 1$ in our analysis.

[13] If the dividend yields on both funds are zero, then the *certainty-equivalent* pre-tax abnormal return (net of transaction costs and fees) on the actively managed equity mutual fund must exceed 135.5 basis points per year.

or more may be necessary before it is beneficial to hold the actively managed equity mutual fund in the tax-deferred account. Given the well-documented underperformance of actively managed equity mutual funds (see, e.g., Gruber (1996) and Carhart (1997)), investors are more likely to benefit from holding taxable bonds in their tax-deferred accounts and tax-efficient equity investments (e.g., individual stocks, index funds, and exchange-traded funds) in their taxable accounts.

C. Borrowing Constraints and Liquidity

With unrestricted borrowing and short-sale opportunities, the investor optimally allocates his entire tax-deferred wealth to the asset with the highest yield (typically taxable bonds) and either borrows or sells short in the taxable account to achieve the desired risk exposure. If the investor faces restrictions on borrowing or selling short, then the optimal asset location policy is more complicated. In this case, the investor shifts his tax-deferred wealth into the asset with the highest yield until offsetting adjustments in the taxable account are no longer possible because of the borrowing or short-sale restrictions. The investor then begins to allocate the remaining tax-deferred wealth to the asset with the next highest yield until the restrictions again bind. The process continues with successively lower yielding assets until the investor's tax-deferred wealth has been completely allocated. Thus, with borrowing and short-sale constraints, the investor may hold a mix of taxable bonds and equity in the tax-deferred account, but only if the taxable account is invested entirely in assets with lower yields.[14]

While the optimal asset location policy maximizes the tax efficiency of the investor's overall portfolio, it also increases the risk of the taxable portfolio relative to the tax-deferred portfolio. With restrictions on borrowing and short sales, this shift in risk between the taxable and tax-deferred accounts may become important for some investors. For example, an investor with relatively little taxable wealth (relative to tax-deferred wealth) may wish to control the risk of his taxable portfolio to guarantee a minimum level of consumption. It is instructive, therefore, to investigate the extent to which liquidity considerations can affect the asset location decision.

Consider an investor who currently has all equity in the taxable account and a mix of taxable bonds and equity in the tax-deferred account. In the absence of liquidity considerations, this asset location choice is tax-efficient, as long as taxable bonds have a higher yield than equity. Now assume that the investor shifts one dollar from taxable bonds to equity in the tax-deferred account and $x = (1 + d)/[1 + d(1 - \tau_d) - \tau_g]$ dollars from equity to taxable bonds in the taxable account. As we have seen earlier, this shift in asset location is

[14] Our discussion here assumes that the tax rate on capital gains, τ_g, is identical across all risky assets. If not, then assets will be ranked on the basis of the values of $-C_i$ (with τ_{gi} replacing τ_g) in equation (2) instead of yields. Moreover, if dividends and capital gains are taxed at the same rate, while interest is taxed at a higher rate ($\tau_0 > \tau_d = \tau_g$), then dividend yields are irrelevant and the investor should never hold equity in the tax-deferred account at the same time taxable bonds are held in the taxable account.

ex ante tax-inefficient. The benefit of the shift is that it will reduce the risk of the taxable account and potentially increase the funds available to finance any unforeseen consumption shocks, thereby reducing the need to withdraw funds from the tax-deferred account. The incremental wealth in the investor's taxable account as a result of the shift from equity to taxable bonds is

$$\Delta\tilde{W}^T = x\{r(1-\tau_d) - [(1+\tilde{g})(1+d(1-\tau_d)) - \tilde{g}\tau_g - 1]\} = \tilde{z}^T. \tag{5}$$

The change in wealth in the tax-deferred account as a result of the shift from taxable bonds to equity is

$$\Delta\tilde{W}^R = \left[\frac{1-\tau_d}{1-\tau_d-p}\right]\tilde{z}^T\tilde{I} - \tilde{z}^R, \tag{6}$$

where $\tilde{z}^R = \{r - [(1+\tilde{g})(1+d) - 1]\}$, p is the penalty per dollar withdrawn from the tax-deferred account, and $\tilde{I} = 1$ if a consumption shock occurs next period that *exceeds* the investor's wealth in the taxable account and $\tilde{I} = 0$ otherwise.[15] The shift in asset location has two effects on tax-deferred wealth. The first term in equation (6) is the incremental wealth in the tax-deferred account that must be liquidated to help finance a shortfall in the taxable account resulting from a large shock to consumption. The second term, $\tilde{z}^R$, is the change in tax-deferred wealth resulting from the differential returns on bonds and stocks.

To determine whether it is beneficial for the investor to hold some bonds in the taxable account for liquidity reasons, we need to evaluate the effect of the shift in asset location on expected utility. Letting m denote the shadow price of taxable wealth per dollar of tax-deferred wealth, the change in expected utility is[16]

$$\begin{aligned}\Delta E[\tilde{U}] &= E[\tilde{U}'\Delta\tilde{W}^T] + mE[\tilde{U}'\Delta\tilde{W}^R]\\ &= E[\tilde{U}'\tilde{z}^T] + \left[\frac{m(1-\tau_d)}{1-\tau_d-p}\right]E[\tilde{U}'(\tilde{z}^T\tilde{I})],\end{aligned} \tag{7}$$

where $E[\tilde{U}'\tilde{z}^R] = 0$ by virtue of the fact that we have assumed that the investor holds both bonds and stocks in the tax-deferred account and, therefore, is indifferent between the two securities at the margin. Next, note that $(\tilde{z}^R - \tilde{z}^T)$ is equal to the positive risk-free after-tax payoff C (ignoring the i subscript) in

[15] Here we assume that the total amount of funds withdrawn from the tax-deferred account is subject to tax and penalty. In this case, the investor must withdraw $\tilde{z}^T[(1-\tau_d)/(1-\tau_d-p)]$ of his after-tax retirement account wealth to generate $\tilde{z}^T$ of incremental wealth in the taxable account after the payment of taxes and the penalty for early withdrawal. If withdrawals from the tax-deferred account are not subject to tax (e.g., Roth IRA), then the investor must withdraw $\tilde{z}^T/(1-p)$ to generate $\tilde{z}^T$ of incremental wealth in the taxable account after the payment of the penalty.

[16] Although the shadow price of taxable wealth per dollar of tax-deferred wealth, m, will depend upon the split of wealth between the two accounts when investors cannot borrow, as a first approximation we shall treat m as a constant.

equation (2). This implies that $E[\tilde{U}'\tilde{z}^T] = E[\tilde{U}'(\tilde{z}^R - C)] = -CE[\tilde{U}']$. Substituting this into the above equation, multiplying and dividing the right-hand side by $E(\tilde{U}')$, yields

$$\Delta E[\tilde{U}] = \left\{ \left[\frac{m(1-\tau_d)}{1-\tau_d-p} \right] \hat{E}[\tilde{z}^T \tilde{I}] - C \right\} E[\tilde{U}'], \tag{8}$$

where $\hat{E}(\cdot)$ is the expectation operator under the risk-neutral measure.[17] The value of $\Delta E[\tilde{U}]$ is positive provided

$$\hat{E}[\tilde{z}^T \tilde{I}] > C \left[\frac{(1-\tau_d-p)}{m(1-\tau_d)} \right]. \tag{9}$$

There are some interesting properties of the expression for $\Delta E[\tilde{U}]$. First, note that if there is no uncertainty in $\tilde{I}$ (i.e., either $\tilde{I} = 0$ or $\tilde{I} = 1$ with certainty), the value of $\Delta E[\tilde{U}]$ is strictly negative (since $\hat{E}[\tilde{z}^T] = -C$). Hence, for investors who have sufficient wealth in their taxable accounts that a consumption shock can easily be financed without having to access the tax-deferred account ($\tilde{I} = 0$ with certainty), or for investors who are certain to have consumption needs that exceed the wealth in their taxable accounts ($\tilde{I} = 1$ with certainty), it is not optimal to hold taxable bonds in the taxable account for liquidity reasons. It is only those investors who face some uncertainty about being hit with a shock to consumption that exceeds their taxable wealth for whom liquidity risk may be important. Second, note that even if $\tilde{I}$ is uncertain, but is either uncorrelated or negatively correlated with $\tilde{z}^T$ under the risk-neutral measure, the value of $\Delta E[\tilde{U}]$ will again be negative. This implies that the benefits of shifting taxable bonds into the taxable account to hedge against liquidity shocks can be optimal only when the liquidity shocks are positively correlated with $\tilde{z}^T$ (i.e., negatively correlated with equity returns) under the risk-neutral measure. The last thing to note is that the value of $\Delta E[\tilde{U}]$ can be positive even if there is no penalty for early withdrawal (i.e., if $p = 0$). This is because withdrawing funds from the tax-deferred account and foregoing the opportunity to earn pre-tax returns is costly to the investor.

The above analysis provides some useful insights regarding the conditions under which liquidity considerations can influence the asset location decision. It requires a positive probability (less than one) of a shock to consumption that exceeds the investor's resources in the taxable account (including any borrowing opportunities), combined with a sufficiently negative correlation between these shocks and equity returns. While this may be a concern for some investors, it is not likely to be a major concern for most investors. For investors who can access

[17] The derivation of the expression for $E(\tilde{U})$ is based upon the assumption that the investor holds all equity in the taxable account and a mix of bonds and equity in the tax-deferred account. If, instead, the investor holds all bonds in the tax-deferred account and a mix of bonds and equity in the taxable account, then the only change is that the shadow price, m, does not appear on the right-hand side of the expression.

their tax-deferred accounts without penalty (i.e., investors older than $59\frac{1}{2}$ or who become disabled), there is little benefit from maintaining significant liquidity in the taxable account. Many investors also receive nonfinancial (labor) income and have some ability to borrow to smooth consumption. On the whole, we do not believe that liquidity shocks alone can generate significant hedging demand for bonds in the taxable account. We investigate numerically the effect of liquidity shocks on asset location and retirement contribution decisions in Section II.E.

D. Tax-timing Considerations

While the analysis in the preceding sections highlights the importance of tax efficiency, it largely ignores the benefits of optimal tax timing. In practice, investors are not forced to realize capital gains and losses each year, but have the ability to time these realizations optimally. With the ability to realize losses and defer gains, holding equity in the taxable account can further increase tax efficiency. Not only can investors exploit the tax-timing option by realizing losses and deferring gains, but because of the reset (or step-up) provision at death, the embedded capital gain tax liability can be completely avoided through deferral. Thus, even in situations where equity generates higher ordinary income than a riskless taxable bond, the value of the tax-timing option may still be high enough to overcome the disadvantage of the higher yield.

Although the optimal asset location policy is difficult to derive analytically in the presence of tax-timing options, it is intuitive that assets with relatively lower yields and higher volatilities (typically individual stocks, index funds, and exchange-traded funds) should be held in the taxable account. However, since yields tend to increase with risk for some assets (e.g., taxable corporate bonds), it is unclear which assets are most appropriate for the tax-deferred account. Depending upon the tradeoff between yield and volatility, low-risk government bonds or high-yield corporate bonds may be found in the tax-deferred account.[18] In our numerical analysis in the next section, we incorporate the tax-deferral option on equity when investigating the interaction between the optimal asset allocation and location decisions.

II. Numerical Analysis of the Intertemporal Portfolio Problem

Section I focused on the investor's optimal asset location policy. In this section we investigate numerically how the optimal asset *allocation* decision interacts with the optimal asset *location* decision. Because the interaction between asset allocation and asset location is most pronounced when the investor faces borrowing and short-sale constraints, we focus our numerical analysis on this

[18] Constantinides and Ingersoll (1984) derive optimal tax-timing policies for taxable bonds. The benefits of optimal tax-timing for taxable bonds have been reduced by subsequent changes in the U.S. tax code that require market discounts and premiums to be amortized as ordinary income over the life of the bond. This effectively eliminates the option of treating market discounts on bonds as capital gains.

case. The model is briefly discussed in Section II.A. In Section II.B, we solve numerically for the optimal decision rules as a function of the state variables. We conduct a simulation analysis of the optimal portfolio decisions over an investor's lifetime in Section II.C. A welfare analysis is presented in Section II.D. Finally, in Section II.E, we investigate the effects of liquidity shocks on the optimal asset location and retirement contribution decisions.

A. The Model

Our model builds upon the specification in Dammon, Spatt, and Zhang (2001) by incorporating a tax-deferred (retirement) savings account together with a taxable savings account into an intertemporal model of optimal consumption and portfolio choice. Since the model itself is not the main contribution or focus of the paper, we restrict our discussion in this section to the important features of the model and refer the interested reader to the appendix for the details. The model assumes that the investor makes decisions annually starting at age 20 and lives for at most another 80 years (until age 100). The investor's annual mortality rates are calibrated to match those for the U.S. population. This allows us to directly consider the impact of the investor's age (and increasing mortality) upon the optimal location and allocation decisions.

Investors in the economy derive utility from consuming a single consumption good. We assume that investors receive annual endowment income prior to retirement at age 65. Although investors do not make an endogenous labor-leisure choice in our model, we interpret the endowment income as nonfinancial (or labor) income. Throughout the analysis we assume that pre-tax nonfinancial income is a constant fraction, l, of the investor's contemporaneous total wealth (taxable plus tax-deferred wealth) prior to retirement. This assumption is needed in our numerical analysis to keep the problem homogeneous in wealth and to limit the number of state variables. Because investors are assumed to receive nonfinanical income throughout their working years, young investors with significant future nonfinancial income will adjust the risk of their portfolios by holding slightly more equity (as a proportion of total financial wealth) than they would without nonfinancial income. Finally, the existence of nonfinancial income makes it less likely that the investor will encounter liquidity problems in financing consumption.

Investors can trade two assets in the financial markets: a riskless taxable one-period bond (equivalent to a one-year Treasury bill) and a risky stock index.[19] No transaction costs are incurred for trading these assets. The pre-tax nominal return on the taxable bond is denoted r and is assumed to be constant over time. The pre-tax nominal return on the risky stock index is $\tilde{r}_s = (1 + d)(1 + \tilde{g}) - 1$,

[19] We do not include tax-exempt bonds in our analysis because, as discussed in Section I, the existence of tax-exempt bonds does not alter the asset location decision when investors have the opportunity to invest in equity that is relatively tax efficient (e.g., exchange-traded funds, passive index mutual funds, or individual stocks). With tax-exempt bonds, the only change that would occur in our analysis is that high-tax bracket investors (those with $\tau_d > \tau_m$) would prefer to hold tax-exempt bonds instead of taxable bonds in the taxable account.

where d is the constant dividend yield and $\tilde{g}$ is the random pre-tax capital gain return. To derive numerical solutions, we assume that $\tilde{g}$ follows a binomial process with a constant mean and variance.

Investors can hold financial assets in two different types of accounts: a taxable account and a tax-deferred retirement account. We assume that investors are not allowed to borrow or sell short in either account. Nominal dividend and interest payments generated from the financial assets held in the taxable account are taxed at the ordinary tax rate of τ_d. *Realized* capital gains (and losses) on stock held in the taxable account are taxed (rebated) at a constant rate of τ_g. All *unrealized* capital gains and losses remain untaxed. To calculate the nominal capital gain, we assume that the tax basis is equal to the weighted average purchase price of all shares held by the investor at the time of sale. This modeling approach, first introduced by Dammon, Spatt, and Zhang (2001), facilitates our numerical analysis by limiting the number of state variables. The assumption that there is a single risky asset and the use of the average basis rule cause the value of the tax-timing option on equity to be understated and induce the investor to hold less equity than would be the case with multiple risky assets and separate tax bases for each asset purchase.

The treatment of the investor's retirement account is broadly consistent with practice. Prior to retirement, the investor is assumed to contribute a constant fraction k of pre-tax nonfinancial income to a retirement account each year. The investor allocates his tax-deferred wealth to the taxable bond and the risky stock index and is allowed to rebalance his portfolio holdings in the retirement account without paying capital gains taxes or transaction costs. Nominal dividends, interest, and capital gains generated from the financial assets held in the retirement account are not subject to immediate taxation, but are tax deferred. After retirement, the investor is required to withdraw the fraction h_t of the remaining tax-deferred wealth at age t, where h_t is the inverse of the investor's remaining life expectancy at age t.[20] We assume that the investor contributes the maximum to the retirement account during his working years and withdraws the minimum from the retirement account during his retirement years. Withdrawals from the retirement account are fully taxed as ordinary income at the rate τ_d. Although investors in practice are allowed to withdraw funds from their retirement accounts prior to age $59\frac{1}{2}$ with a 10% penalty, we assume that the investor is not allowed to withdraw funds from the tax-deferred account prior to retirement. We relax these assumptions in Section II.E and allow the investor to optimize the retirement contributions and withdrawals, including withdrawals prior to retirement, when analyzing liquidity shocks.

[20] Recently, the IRS has adopted a minimum withdrawal schedule that is based upon the joint life expectancy of the individual and a hypothetical beneficiary. Consequently, our withdrawal rates are somewhat higher than those required by the new regulations issued by the IRS. Although we assume that the balance in the retirement account is subject to immediate taxation at the time of the investor's death, the recently adopted IRS regulations allow the beneficiary to withdraw the remaining funds according to his own life expectancy. Consequently, our analysis somewhat understates the potential benefits of tax-deferred investing.

The investor's problem is to maximize the discounted expected utility of lifetime consumption, given the initial endowment of assets and wealth, subject to an intertemporal budget constraint. Since the investor has a positive probability of death at each date, the treatment of terminal wealth is important. We assume that at the time of death, the asset holdings in the taxable account are liquidated without incurring a capital gains tax. This is consistent with the *reset (or step-up) provision* of the current U.S. tax code, which requires the tax bases of all inherited assets to be costlessly reset to current market prices at the time of the investor's death. We also assume that the assets held in the investor's retirement account are liquidated at the time of death and that the proceeds are taxed as ordinary income. At the time of death, the investor's total wealth is liquidated and distributed as a bequest to his beneficiary. For simplicity, we assume that the investor derives utility from his bequest equal to the utility his beneficiary would derive if the bequest were used to purchase an annuity contract that provided a constant amount of *real* consumption for H periods. Higher values for H indicate a stronger bequest motive for the investor.

The value of the investor's asset holdings (i.e., the after-tax value of the retirement account plus the pre-tax value of the taxable account) serves as our measure of total wealth at each date. To eliminate total wealth as a state variable, we assume that the investor has constant relative risk-averse preferences. After normalizing by total wealth, the investor's intertemporal consumption and portfolio problem involves the following control (choice) variables: The consumption-wealth ratio, c_t; the fraction of taxable wealth allocated to equity, f_t; the fraction of taxable wealth allocated to riskless taxable bonds, b_t; and the fraction of tax-deferred wealth allocated to equity, θ_t. Given f_t and b_t, the fraction of the investor's taxable portfolio allocated to equity is $f_t/(f_t + b_t)$. The relevant state variables for the normalized optimization problem are the incoming proportion of equity in the taxable account, s_t; the basis-price ratio on the incoming equity holdings, p^*_{t-1}; the fraction of the investor's incoming total wealth that is held in the retirement account, y_t; and the investor's age, t. Because investors are allowed to rebalance their retirement account portfolios without incurring any transaction costs or taxes, the incoming asset holdings in this account are not relevant state variables for the investor's decision problem.

B. Numerical Solutions for the Optimal Policies

The *base-case* parameter values for our numerical analysis are summarized in Table I and discussed below. We assume that the nominal pre-tax interest rate on the riskless taxable bond is $r = 6\%$ per year; the nominal dividend yield on the stock index is $d = 2\%$ per year; and the annual inflation rate is $i = 3.5\%$. Inflation is relevant in our model because taxes are levied on nominal quantities. The nominal annual capital gains return on the stock index is assumed to follow a binomial process with a constant mean and standard deviation of $\bar{g} = 9\%$ and $\sigma = 20\%$, respectively. We assume that the tax rate on dividends and interest is $\tau_d = 36\%$ and that the tax rate on *realized* capital gains and

Table I

Base-case Parameter Values

The table provides the base-case parameter values that are used to conduct the numerical analysis in Section II. The bequest parameter (H) is the number of years of consumption the investor wishes to provide his beneficiary following his death. Higher values of H imply a stronger bequest motive. Labor income is assumed to be a constant proportion (l) of the investor's total wealth. The retirement contribution rate (k) is stated as a proportion of the investor's pre-tax labor income. Retirement contributions are mandatory prior to retirement. The retirement withdrawal rate (h_t) is stated as a proportion of the investor's tax-deferred wealth. Investors are not allowed to withdraw funds from their tax-deferred accounts prior to retirement.

Parameters of the Model	Notation	Base-case Value
Asset Returns:		
Riskless one-period taxable interest rate	r	6.0%
Dividend yield on equity	d	2.0%
Expected capital gain return on equity	$\bar{g}$	9.0%
Standard deviation of capital gain return	σ	20.0%
Inflation rate	i	3.5%
Tax Rates:		
Ordinary income tax rate	τ_d	36%
Capital gain tax rate	τ_g	20%
Utility and Bequest Functions:		
Utility discount factor	β	0.96
Relative risk aversion	α	3.0
Bequest parameter	H	20
Labor Income and Retirement Savings		
Labor income	l	15%
Retirement contribution rate	k	20%
Retirement withdrawal rate	h_t	1/life expectancy
Mandatory retirement age	J	65

losses is $\tau_g = 20\%$. Because the pre-tax expected return on the stock index is given by $\bar{r}_s = (1 + \bar{g})(1 + d) - 1$, the annual pre-tax equity risk premium (above the riskless interest rate) is 5.18%. While this equity risk premium is relatively low compared to the historical average equity risk premium of about 8%, Fama and French (2002) and others have argued that the expected future equity risk premium should be substantially lower than the historical average. For reasonable levels of risk aversion, the lower equity risk premium also ensures that the investor's optimal portfolio will consist of less than 100% equity.

The investor is assumed to have power utility with an annual subjective discount factor of $\beta = 0.96$ and a risk aversion parameter of $\gamma = 3.0$. We set $H = 20$ in the bequest function, indicating that the investor values the bequest as though it provided a 20-year annuity of constant real consumption for his beneficiary.[21] We assume that pre-tax nonfinancial income is a constant

[21] We also calculated the optimal decision rules for higher and lower values of H. A weaker (stronger) bequest motive increases (reduces) the optimal consumption-wealth ratio, especially at late ages, but has relatively little effect on the investor's optimal portfolio holdings across the state space.

$l = 15\%$ of the investor's total beginning-of-period wealth prior to age 65 and $l = 0\%$ thereafter. Before retirement at age 65, the investor is assumed to invest $k = 20\%$ of pre-tax nonfinancial income in the tax-deferred retirement account each year. This contribution rate is the maximum allowed in the U.S. for self-employed individuals with defined contribution plans. Although withdrawals from tax-deferred retirement accounts can be deferred until age $70\frac{1}{2}$ under current IRS rules, we assume that the investor is forced to begin withdrawing funds from the retirement account at age 65 in accordance with the withdrawal schedule h_t.

Figure 1 shows the optimal equity proportions for the taxable account (top panel), the tax-deferred retirement account (middle panel), and the overall portfolio (bottom panel). These optimal equity proportions are shown as a function of the investor's age and the fraction of beginning-of-period total wealth held in the retirement account. A two-dimensional representation of these optimal equity proportions is shown in Figure 2 for age 35 (top panel) and age 75 (bottom panel). Figures 1 and 2 are constructed using the base-case parameter values and assuming that the basis-price ratio is $p^* = 1.0$. With a basis-price ratio of $p^* = 1.0$, the investor has neither a gain nor a loss on existing equity holdings and can rebalance the portfolio in the taxable account without incurring a tax cost. We refer to these optimal equity proportions as the *zero-gain optimum* equity holdings.

As the fraction of wealth in the retirement account increases, the optimal equity proportion in the taxable account increases. This reflects the preference for holding taxable bonds in the retirement account, thus making it necessary for the investor to increase the proportion of equity in the taxable account in order to maintain an optimal overall portfolio mix. Note, however, that because of the prohibition on borrowing, the proportion of equity in the taxable account is bounded above by 100%. The top panels of Figures 1 and 2 show that prior to retirement, the investor allocates 100% of the taxable account to equity whenever the tax-deferred wealth exceeds about 30% of total wealth. In contrast, with *unrestricted borrowing* (not shown), the optimal holding of equity in the taxable account would continue to increase beyond 100% as the investor borrows to invest in equity in the taxable account. For example, with 50% of total wealth held in the retirement account, investors in their working years hold approximately 175% of their taxable wealth in equity when they are allowed to borrow.

Figures 1 (middle panel) and 2 show that the optimal equity proportion in the tax-deferred account can be nonzero when investors are prohibited from borrowing. Note, however, that the investor does not hold equity in the tax-deferred account until the tax-deferred wealth exceeds approximately 40% of total wealth. The reason the investor does not add equity to the retirement account as soon as he is constrained in the taxable account is because equity is much less valuable when held in the retirement account. For this reason, the investor holds equity in the retirement account only when the level of tax-deferred wealth is sufficiently high and refrains from holding a mix of stocks and bonds in both the taxable and tax-deferred accounts simultaneously. In fact,

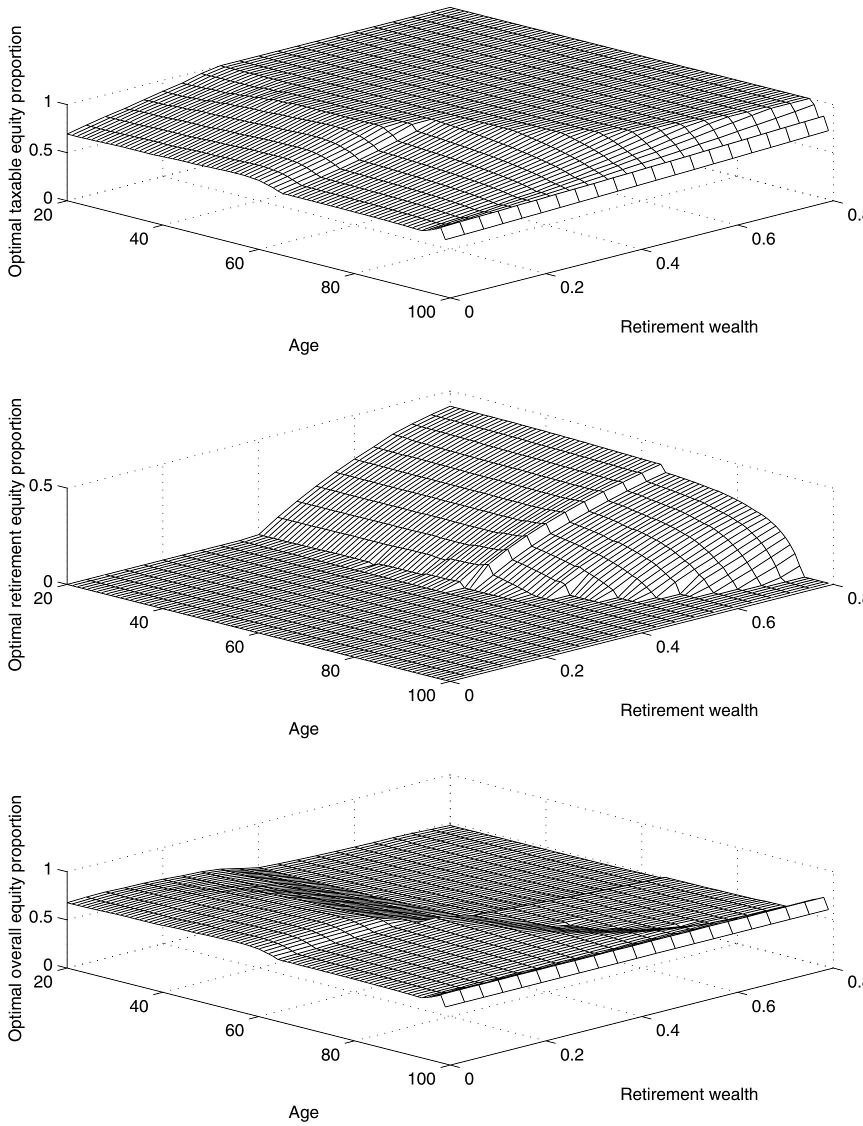

Figure 1. Optimal equity proportions as a function of retirement wealth and age. The figure shows the optimal equity proportions in the taxable account (top panel), retirement account (middle panel), and overall portfolio (bottom panel) as a function of age and the fraction of total wealth held in the retirement account. The basis-price ratio is set at $p^* = 1.0$.

over a range of tax-deferred wealth, the investor holds an all-equity portfolio in the taxable account and an all-bond portfolio in the retirement account. These findings are in contrast to the unrestricted borrowing case (not shown) in which the investor always allocates his entire tax-deferred wealth to taxable bonds (see Section I.A).

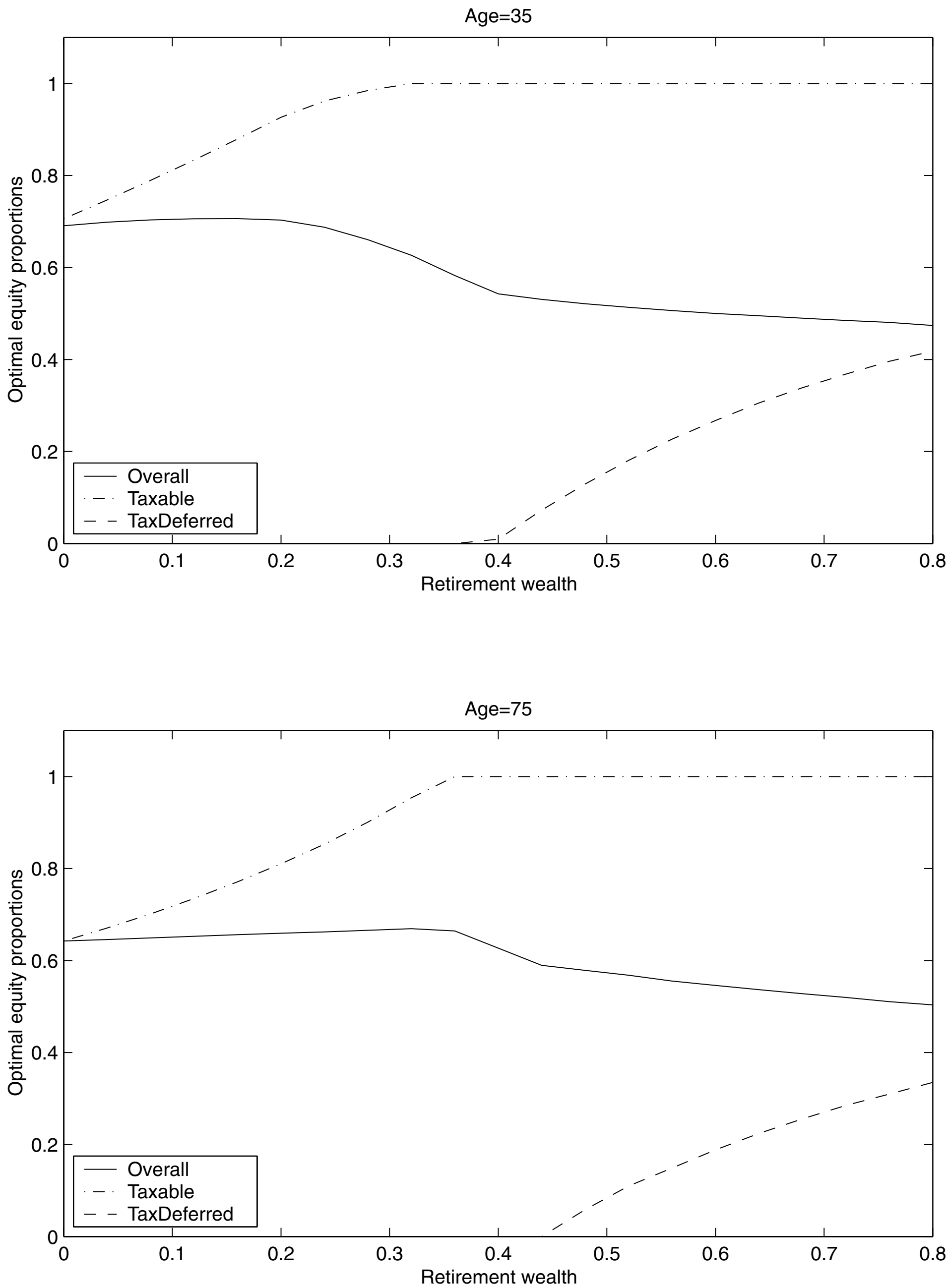

Figure 2. Optimal equity proportions at ages 35 and 75. The figure shows the optimal equity proportions in the taxable account (dash-dotted line), tax-deferred account (dashed line), and overall portfolio (solid line) at age 35 (top panel) and age 75 (bottom panel). The optimal equity proportions are shown as a function of the fraction of total wealth held in the retirement account. The basis-price ratio is set at $p^* = 1.0$.

Figures 1 (bottom panel) and 2 show the optimal proportion of *total wealth* allocated to equity for the no-borrowing case. The figures illustrate that optimal asset allocation for an investor's overall portfolio depends upon the split of wealth between the taxable and tax-deferred accounts. In fact, the optimal overall equity proportion (weakly) declines as the fraction of total wealth held in the retirement account increases. The optimal overall equity proportion is relatively flat in the level of tax-deferred wealth as long as the investor holds less than 100% equity in the taxable account. Once the investor is constrained in the taxable account, however, the optimal overall equity proportion begins to decline. This reflects the investor's reluctance to substitute equity for taxable bonds in the retirement account. At sufficiently high levels of tax-deferred wealth, the investor begins to add equity to the retirement account to avoid becoming too underweighted in equity. This causes the optimal overall equity proportion to level off once again. In contrast, when the investor has unrestricted borrowing opportunities (not shown), the optimal overall equity proportion is relatively constant across all levels of tax-deferred wealth.[22] For young investors with unrestricted borrowing opportunities, the optimal overall equity proportion is approximately 70%.

The effect of age on the overall equity proportion is also shown in the bottom panel of Figure 1. While the overall equity proportion remains relatively constant during the working years, there is a slight decline between the ages of 60 and 65. The drop in the optimal overall equity proportion at these ages reflects the anticipated loss of the relatively low-risk nonfinancial income after retirement. This effect is less pronounced at high levels of retirement wealth, where the exposure to equity has already been reduced at young ages because of the restrictions on borrowing. After retirement, the optimal overall equity proportion *increases* slightly with age. This reflects the higher value of equity for elderly investors, who because of their higher mortality rates, benefit the most from the forgiveness of capital gains taxes at death.

Figures 1 and 2 were constructed assuming that the basis-price ratio for equity held in the taxable account was $p^* = 1.0$. With an embedded capital loss (i.e., $p^* > 1.0$), the investor optimally sells his entire equity holding in the taxable account to benefit from the tax rebate and immediately rebalances to the zero-gain optimum equity holdings shown in Figure 1. With an embedded capital gain (i.e., $p^* < 1.0$), the investor's optimal equity holdings will differ from those shown in Figure 1. If the investor's taxable account is initially *underweighted* in equity, his equity holdings following optimal rebalancing will be slightly lower than the zero-gain optimum equity holdings. This is because the averaging rule used to compute the tax basis reduces the value of holding additional equity when existing shares have an embedded capital gain. The smaller

[22] With unrestricted borrowing, the optimal overall equity proportion increases slightly with the level of tax-deferred wealth. Higher levels of tax-deferred wealth allow the investor to generate higher levels of riskless tax-arbitrage profits. By investing the incremental tax-deferred wealth in bonds, and borrowing in the taxable account to invest in equity, the investor is able to increase his total wealth without incurring additional risk. The investor responds to this risk-free increase in wealth by increasing slightly his overall exposure to equity.

the embedded capital gain, the closer the optimal equity holdings are to the zero-gain optimum. If the investor's taxable account is initially *overweighted* in equity, there exists a tradeoff between the diversification benefits and tax costs of rebalancing. The willingness of the investor to realize embedded capital gains to rebalance his portfolio depends upon a number of factors. Smaller embedded capital gains, larger deviations from the zero-gain optimum equity holdings, lower mortality risk, and lower levels of future nonfinancial income increase the amount of rebalancing that is optimal. However, because rebalancing is costly in this case, the optimal equity holdings are higher than the zero-gain optimum equity holdings. The age and basis effects discussed above are similar to those derived by Dammon, Spatt, and Zhang (2001) in a model without a tax-deferred account.

Our results illustrate a preference for holding equity in the taxable account and taxable bonds in the tax-deferred account to the extent possible. We also have discussed how the investor's overall asset allocation depends upon age, the basis-price ratio, and the split of wealth between the taxable and tax-deferred accounts.[23] In the next section, we investigate the time-series profile of the investor's optimal consumption and portfolio allocation decisions using simulation analysis.

C. Simulation Analysis

Given the investor's optimal consumption and investment policies defined on the state space, we can obtain time-series profiles of optimal consumption and portfolio allocations by simulating the capital gain return on the risky stock index. Using our base-case parameter values, the simulation begins for an investor at age 20 with an initial basis-price ratio of $p^* = 1$. The investor is prohibited from borrowing and is assumed to have no tax-deferred wealth at age 20. Table II shows the age profiles for the optimal consumption, portfolio holdings, and level of retirement wealth. The values reported in the table are *averages* at each age taken across 5,000 simulation trials.

Table II shows that the investor's optimal consumption-wealth ratio slowly falls as the investor ages during his working years and then slowly rises as he ages during his retirement years. The decline in the investor's optimal consumption-wealth ratio during his working years reflects the anticipated loss of nonfinancial income after retirement. The increase in the investor's consumption-wealth ratio during his retirement years reflects the bequest motive. With $H = 20$, the bequest provides the investor the same utility as his beneficiary would receive from consuming a 20-year annuity stream. Hence, as the investor ages (and mortality risk increases), the optimal consumption-wealth ratio increases in an attempt to equate the expected marginal utility of the investor's own consumption with that of his beneficiary. With a stronger

[23] Reichenstein (2001) uses a one-period mean-variance model to generate a series of numerical examples that illustrate the interaction between the asset location and asset allocation decisions. His model, however, does not consider the impacts of age, basis-price ratio, or the split of wealth between the taxable and tax-deferred accounts on the optimal decisions.

Table II

Monte Carlo Simulation Analysis

The table summarizes the results of the Monte Carlo simulation analysis conducted in Section II.C. The numbers reported in the table are averages at each age taken over 5,000 simulation trials, starting at age 20 and ending at age 100. The simulations utilize the base-case parameter values outlined in Table I. The table reports the average consumption-wealth ratio; the fraction of total wealth in the retirement account; the overall equity proportion; the equity proportion in the taxable account; the basis-price ratio of the equity in the taxable account; the frequency of reaching 100% equity in the taxable account; the frequency of positive equity holdings in the retirement account; and the proportion of retirement wealth allocated to equity conditional on positive equity holdings in the retirement account.

Age	Consumption-Wealth Ratio	Fraction of Total Wealth Held in the Retirement Account	Overall Equity Proportion	Equity Proportion in the Taxable Account	Basis-Price Ratio	Frequency of 100% Equity in the Taxable Account	Frequency of Positive Equity in the Retirement Account	Conditional Equity Proportion in the Retirement Account
20	9.30%	1.91%	67.88%	69.35%	1.000	0.00%	0.00%	—
25	9.19%	11.11%	71.72%	81.66%	0.666	0.00%	0.00%	—
30	9.12%	19.70%	70.58%	90.08%	0.488	0.24%	0.24%	0.00%
35	8.92%	27.32%	67.13%	95.79%	0.367	3.62%	3.62%	5.19%
40	8.58%	33.93%	62.57%	98.36%	0.281	3.90%	3.90%	18.36%
45	8.10%	39.38%	58.56%	98.53%	0.218	1.78%	1.74%	29.53%
50	7.37%	43.14%	56.05%	98.12%	0.195	59.36%	55.04%	10.37%
55	6.35%	44.68%	55.37%	97.71%	0.233	64.70%	59.80%	11.90%
60	5.14%	43.50%	56.04%	96.70%	0.321	61.60%	60.64%	11.05%
65	3.86%	36.21%	53.76%	81.61%	0.379	10.90%	10.90%	20.02%
70	3.85%	28.96%	61.39%	87.27%	0.314	6.24%	6.24%	15.08%
75	3.90%	20.55%	66.27%	84.71%	0.244	1.98%	1.98%	9.19%
80	4.03%	12.19%	68.64%	78.91%	0.186	0.46%	0.18%	1.11%
85	4.24%	5.41%	70.83%	75.08%	0.141	0.00%	0.00%	—
90	4.51%	1.47%	75.70%	76.87%	0.109	0.00%	0.00%	—
95	4.88%	0.12%	86.32%	86.42%	0.084	0.00%	0.00%	—
99	5.37%	0.00%	98.99%	98.99%	0.071	0.00%	0.00%	—

bequest motive ($H = \infty$), the investor's optimal consumption-wealth ratio declines with age during retirement.

The investor contributes $k = 20\%$ of pre-tax nonfinancial income to the retirement account each year. Given the high levels of consumption, these retirement contributions represent the bulk of the investor's overall savings during his working years. As a result, the fraction of total wealth held in the retirement account increases rapidly at young ages. The fraction of total wealth held in the retirement account reaches its maximum of 45% at age 55, well before the investor reaches retirement age.[24] The decline in the fraction of total wealth held in the retirement account in the years prior to retirement reflects two things: (1) the lower fraction of total after-tax savings allocated to the retirement account at these ages and (2) the relatively low average return earned on the assets (primarily taxable bonds) held in the retirement account. Because of the mandatory distributions from the tax-deferred account during his retirement years, the investor's after-tax retirement account declines rapidly after age 65. By the time the investor reaches age 80, the after-tax wealth in the retirement account is only slightly more than 12% of total wealth on average.

The last six columns of Table II provide information about the investor's optimal lifetime portfolio choices. Because of the relatively high equity risk premium, the investor's overall demand for equity is extremely high and frequently exceeds the available resources in the taxable account. As Table II indicates, a large proportion of simulation trials result in an all-equity portfolio in the taxable account at some point during the investor's lifetime. Most (but not all) of these cases result in some positive holdings of equity in the retirement account. Since equity is less valuable when held in the retirement account, the overall holding of equity declines in these cases. While the investor has limited opportunities at early ages to realize losses on equity held in the taxable account, the profile of the average basis-price ratio in Table II indicates that he quickly becomes locked in to a capital gain. During the retirement years, the average proportion of tax-deferred wealth allocated to equity declines rapidly as the investor begins to liquidate the equity holdings in the retirement account to fund his mandatory distributions. However, as a proportion of total wealth, the investor's equity holdings continue to increase during his retirement years. This reflects the increased holdings of equity in the taxable account and the investor's reluctance to sell equity with an embedded capital gain.[25]

[24] Across the 5,000 simulations, there is considerable variation in the magnitude of tax-deferred wealth. For example, at age 55 the minimum and maximum values of tax-deferred wealth are 13% and 81%, respectively, of total wealth. The timing of when tax-deferred wealth reaches its maximum is also somewhat variable, although it typically occurs between the ages of 50 and 60.

[25] If the investor is allowed to borrow, the optimal holding of equity in the taxable account is likely to exceed 100% at late ages. This is because the investor prefers to sell bonds and borrow to finance consumption rather than selling equity with embedded capital gains. At death, the investor's equity holdings are liquidated without payment of the capital gains tax, all borrowing is repaid, and the remaining wealth is used to finance his bequest.

The results of the simulation analysis illustrate the time-series properties of the investor's optimal consumption, portfolio allocation, and asset location decisions. In the next section, we investigate the welfare benefits from following the optimal asset location policies and illustrate the value of tax-deferred investing.

D. Alternative Investment Policies and Welfare Analysis

Because many individuals in practice hold a mix of bonds and stocks in both their taxable and tax-deferred accounts, and in some cases actually tilt their retirement accounts toward equity, we want to examine the utility costs of following these suboptimal policies. We examine two alternative investment strategies. In the first, investors hold the same mix of bonds and stocks in both their taxable and tax-deferred accounts. In the second, investors first allocate their holdings of equity to the retirement account before holding equity in their taxable accounts. Given our earlier analysis, this latter policy is the worst possible choice in terms of asset location. For both of the alternative investment policies, we allow the investor to choose the optimal mix of stocks and bonds as a function of the state variables. We then conduct a welfare analysis by computing the amount of additional wealth (allocated entirely to the taxable account) that is needed to equate the investor's total expected utility under the suboptimal location policy to that under the optimal location policy. This allows us to quantify the cost of ignoring the optimal location of securities across the taxable and tax-deferred accounts.

The welfare analysis is conducted using our base-case parameter values. The one exception is that we assume that future nonfinancial income and retirement contributions are zero. This allows us to focus on the welfare costs associated with the suboptimal location policies when applied to the pre-existing wealth levels. The optimal asset location decision is unaffected by the elimination of nonfinancial income and retirement account contributions.

The top panel of Figure 3 shows the utility costs for an investor who is forced to hold the same portfolio mix in both the taxable and tax-deferred accounts. The middle panel of Figure 3 shows the utility costs for an investor who is not allowed to hold equity in the taxable account unless 100% of his tax-deferred wealth is allocated to equity. The utility costs are shown as a function of the investor's age and level of tax-deferred wealth. The figures are drawn for a basis-price ratio of $p^* = 1.0$. Since there is no embedded capital gain or loss on the investor's portfolio, the initial equity proportion in the taxable account has no effect on the optimal decision rules or utility costs.

The utility costs depicted in these two figures exhibit a strong age effect. Other things being equal, younger investors incur a higher utility cost because they have longer horizons over which the suboptimal asset location policy is in effect. The utility costs are also hump-shaped in the level of tax-deferred wealth at young and middle ages. Intuitively, the utility costs of being constrained to follow the suboptimal policy are highest when the investor's wealth is split

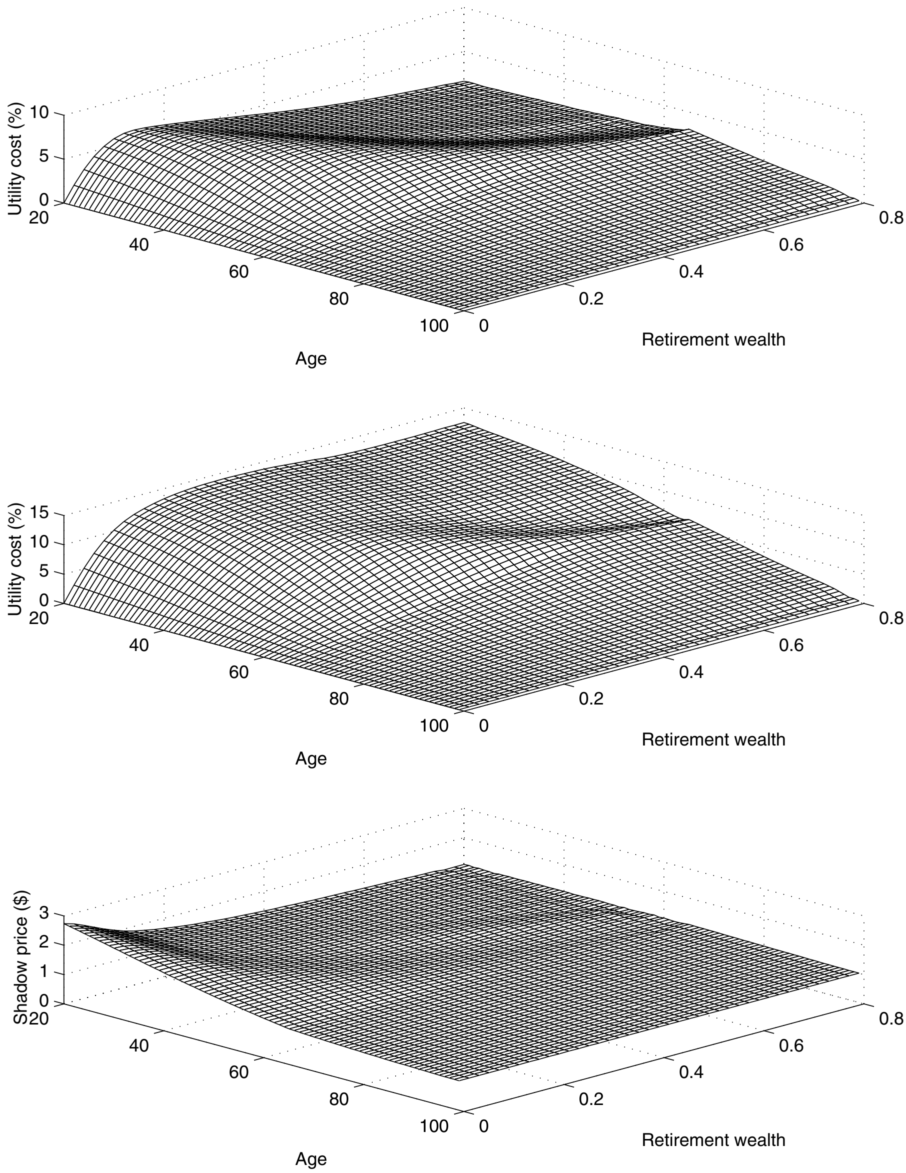

Figure 3. Utility costs and shadow prices. The top panel shows the utility costs of following the suboptimal policy of holding the same portfolio mix in both the taxable and tax-deferred accounts. The middle panel shows the utility costs of following the suboptimal policy of allocating equity to the tax-deferred account before allocating equity to the taxable account. The utility costs are measured as the percentage increase in total wealth (allocated entirely to the taxable account) needed to compensate the investor for following the suboptimal policy. The bottom panel shows the shadow prices for an additional dollar of tax-deferred wealth. The shadow price is the amount of taxable wealth the investor is willing to pay to receive an additional after-tax dollar in his retirement account. The basis-price ratio is set at $p^* = 1.0$ and future nonfinancial income and retirement contributions are assumed to be zero.

relatively evenly across the taxable and tax-deferred accounts and smallest when concentrated in one or the other of these two accounts.[26]

In the top panel of Figure 3, the utility costs are generally less than 5% across all ages, with the exception that young investors with moderate levels of retirement account wealth have slightly higher utility costs. The utility costs depicted in the middle panel of Figure 3 are higher than those depicted in the top panel. This is to be expected, since locating equity in the tax-deferred account before locating it in the taxable account is the worst possible policy for asset location. For young investors with moderate levels of tax-deferred wealth, the utility costs are nearly 15%. These utility costs would be even higher if the investor contributed additional funds to the retirement account over time. The overall impression given by these two figures is that the benefits from optimally locating equity in the taxable account and bonds in the tax-deferred account can be large, especially for young and middle-aged investors.

Our model also can be used to measure the welfare benefits of tax-deferred investing. Using the base-case parameters (without nonfinancial income and retirement contributions), and an assumed basis-price ratio of $p^* = 1.0$, the bottom panel of Figure 3 shows the shadow prices for tax-deferred wealth. The shadow price measures the amount of taxable wealth the investor is willing to give up in order to receive one additional dollar of wealth in the tax-deferred account. The shadow prices depicted in Figure 3 are all greater than 1.0, indicating that it is beneficial for the investor to have a higher fraction of total wealth in the retirement account.[27] The shadow prices are highest for young investors, who have longer horizons over which to benefit from tax-deferred investing, and for investors with lower levels of tax-deferred wealth, who can efficiently allocate their tax-deferred wealth to taxable bonds.[28] Since investors are forced to liquidate their tax-deferred accounts during their retirement years, the shadow prices for additional tax-deferred wealth decline rapidly after age 65.

E. Effects of Liquidity on Optimal Asset Location

The tax-efficient location policy involves holding equity in the taxable account and taxable bonds in the tax-deferred account, to the extent possible. While

[26] Because the split of wealth between the taxable and tax-deferred accounts is changing over time, the utility costs depend upon more than just the investor's current distribution of wealth across these two accounts. In general, it will also depend upon the investor's age, anticipated future retirement account contributions and withdrawals, level of nonfinancial income, consumption plans, and asset returns. This explains why the level of retirement account wealth that produces the highest utility cost is not equal to 0.5 and why it is not the same across all ages.

[27] Since consumption must be financed exclusively from the resources held in the taxable account, the shadow prices can be less than 1.0 for extremely high levels of tax-deferred wealth.

[28] For an investor at age 20 with no wealth in the tax-deferred account, the shadow price in Figure 3 is \$2.71. To understand the magnitude of this shadow price, consider the following simple example. Investing one dollar of tax-deferred wealth in bonds earning a pre-tax return of 6% per year will grow to \$24.65 after 55 years (the life expectancy of a 20-year-old). To produce the same terminal value after 55 years, an investor would need to invest \$3.10 of taxable wealth in bonds earning an after-tax return of 3.84% per year.

this location policy does not alter the risk of the investor's overall portfolio, it does shift most of the risk exposure to the taxable account. The higher risk exposure in the taxable account should have little or no effect on investors with substantial wealth in their taxable accounts, low labor income risk, easy access to borrowing, or penalty-free access to their tax-deferred accounts. For other investors, however, the higher risk exposure in the taxable account can create a potential liquidity problem. If equity values decline significantly, it may be necessary for these investors to incur a penalty to liquidate a portion of their tax-deferred accounts to finance consumption. In principle, this can create demand for holding taxable bonds in the taxable account to reduce liquidity risk. In Section I.C, we show that deviations from the tax-efficient asset location policy require a positive probability of a liquidity shock that exceeds the wealth in the taxable account, combined with a sufficiently negative correlation (under the risk-neutral measure) between these liquidity shocks and equity returns. In this section, we use numerical analysis to explore the effects of liquidity on the intertemporal asset location and retirement contribution decisions.

While liquidity shocks can be modeled in a number of different ways, we choose to model them as an exogenous shock to consumption. The numerical analysis of these consumption shocks is conducted using our base-case parameter values. However, unlike in our earlier analysis, we allow investors to endogenously determine their optimal contributions to the retirement account during their working years (subject to a contribution limit of $k = 20\%$) and their optimal withdrawals from the tax-deferred account during their retirement years (subject to the minimum withdrawal schedule). Investors are also allowed to withdraw funds from their tax-deferred accounts prior to retirement, but must pay ordinary income taxes and a 10% penalty on early withdrawals. Because the taxable account is the preferred location for equity, even without the benefits of optimal tax timing, we simplify our analysis by assuming that all capital gains and losses are realized each year and taxed at the rate of $\tau_g = 20\%$. This assumption is consistent with the analysis in Section I.C and allows us to drop the incoming equity proportion in the taxable account and the basis-price ratio as state variables.

We first analyze a situation in which the investor faces a *known* "consumption gulp" of 50% of total wealth at age 30.[29] The purchase of a home or college tuition are examples of the type of consumption expenditures we have in mind. At age 30, the investor withdraws funds from the retirement account and pays the

[29] Huang (2000) analyzes the asset location decision for an investor who faces a one-time tax on total wealth at a known future date. The wealth tax is a dead-weight loss for the investor in her model. In contrast, the "consumption gulp" in our model serves as a constraint on the minimum level of consumption. We model the "consumption gulp" as a fixed percentage of total wealth for simplicity. If the "consumption gulp" is a fixed dollar amount, instead of a fixed percentage of total wealth, then two complications arise. First, it is necessary to introduce wealth as an additional continuous state variable. Second, it is necessary to define a penalty function, in the event that the investor's wealth is not sufficient to meet the required "consumption gulp." In this case, the severity of the penalty function will determine the extent to which the investor hedges in the taxable account.

10% penalty only if the wealth in the taxable account is insufficient to finance the required "consumption gulp." Figure 4 illustrates the optimal holding of taxable bonds in the taxable account (top panel) and equity in the tax-deferred account (middle panel) as a function of the investor's age and the level of tax-deferred wealth. Under the tax-efficient asset location policy, positive holdings of taxable bonds in the taxable account (top panel) should not coincide with positive holdings of equity in the tax-deferred account (middle panel). Except for a few years prior to age 30, and for levels of tax-deferred wealth between about 45 to 55% of total wealth, the investor follows the tax-efficient asset location policy. Investors who deviate from the tax-efficient location policy are those for whom negative equity returns might otherwise force a liquidation of a portion of the tax-deferred account at age 30. To reduce this risk, these investors shift some taxable bonds to the taxable account and some equity to the tax-deferred account. Note, however, that the holding of taxable bonds in the taxable account is small relative to the overall holding of taxable bonds. For all other investors, the correlation between equity returns and a liquidity crisis at age 30 is not sufficiently negative to warrant a deviation from the tax-efficient asset location policy.

The bottom panel of Figure 4 shows the optimal contributions to the tax-deferred account prior to age 30. Investors with substantial tax-deferred wealth reduce (or eliminate) the contributions to the retirement account prior to age 30 to increase the wealth in the taxable account and reduce (or eliminate) the penalty for early withdrawal. The higher the level of tax-deferred wealth, the earlier the age at which the investor eliminates the retirement contributions. Note that some investors reduce (or eliminate) contributions to the retirement account, but continue to follow the optimal asset location policy. Investors older than age 30 (not shown) maximize the contributions to the retirement account during their working years and withdraw the minimum from their retirement accounts during their retirement years.

The above analysis assumes that the timing of the "consumption gulp" is known with certainty. Another approach is to assume that the investor faces a constant *per-period* probability of a liquidity shock that requires consumption to be 50% of total wealth.[30] The consumption shocks are assumed to occur with a 10% probability each period and are independent over time. The first case we consider assumes that the consumption shocks are *independent* of equity returns. In this case, the numerical results (not shown) indicate that the tax-efficient asset location policy is optimal for all investors and for all levels of tax-deferred wealth. With zero correlation between the consumption shocks and equity returns, shifting taxable bonds to the taxable account is not an effective hedge against liquidity risk and is highly tax inefficient. Rather than deviating from the tax-efficient asset location policy, investors younger than

[30] This approach is similar in spirit to the random shocks to nonfinancial income analyzed by Amromin (2001). Despite the potential for a catastrophic loss of nonfinancial income for an extended period of time, he finds that the hedging demand for taxable bonds in the taxable account is still small relative to the overall holding of taxable bonds.

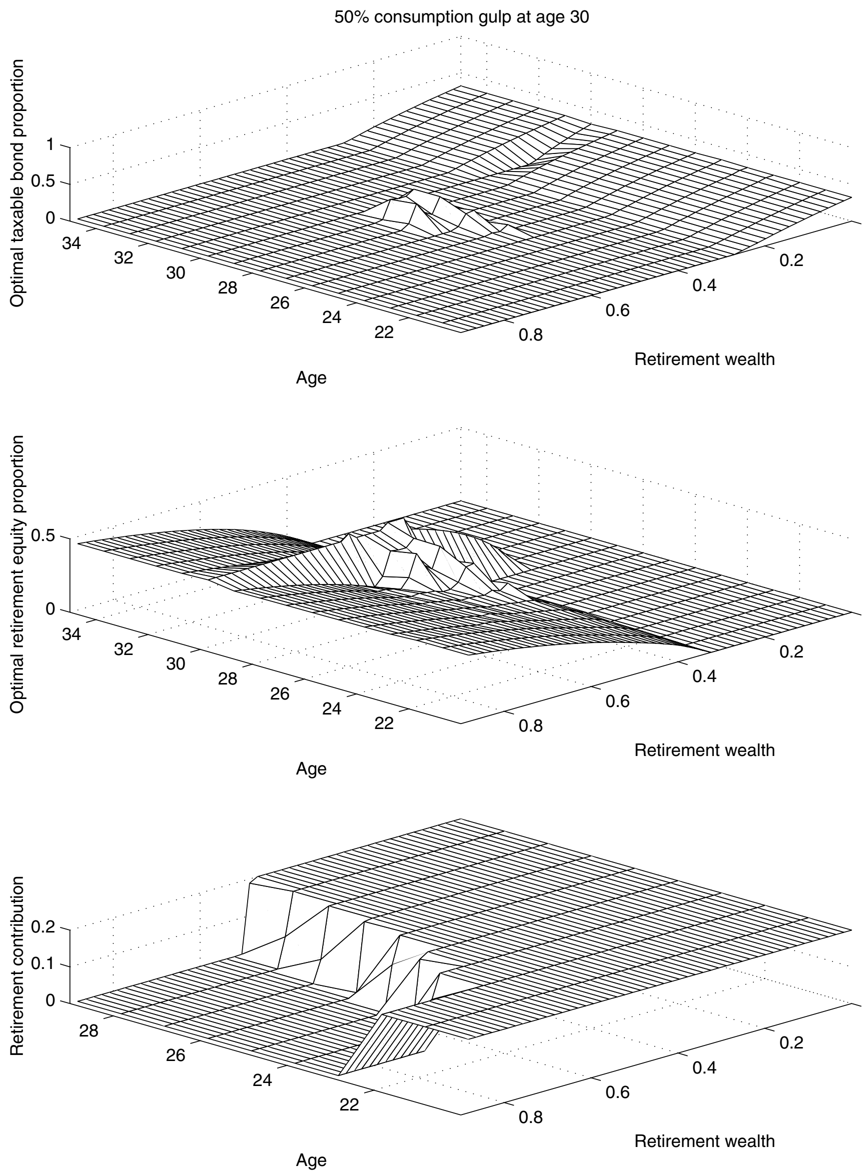

Figure 4. Optimal asset allocations and retirement contributions with a known "consumption gulp." The figure shows the optimal bond proportion in the taxable account, equity proportion in the retirement account, and retirement contributions for the case in which the investor has a known "consumption gulp" of 50% of total wealth at age 30. A 10% penalty is enforced on withdrawals from the retirement account prior to age 65. The optimal asset allocations and retirement contributions are shown as a function of age and the fraction of total wealth held in the retirement account. The top panel depicts the optimal bond proportion in the taxable account, the middle panel depicts the optimal equity proportion in the tax-deferred account, and the bottom panel depicts the optimal retirement contributions (as a percentage of pre-tax non-financial income).

retirement age hedge the risk of a liquidity shock by reducing (or eliminating) the contributions to the retirement account in order to increase the wealth in the taxable account. Since younger investors face a higher cumulative probability of a liquidity crisis prior to retirement, they contribute to the retirement account only at low levels of tax-deferred wealth.

The second case we consider assumes that consumption shocks are *negatively* correlated with equity returns. We maintain the assumption that the consumption shock requires consumption to be 50% of total wealth. Conditional on a negative equity return, the probability of a consumption shock is assumed to be 18%. Conditional on a positive equity return, the probability of a consumption shock is assumed to be 2%. Therefore, with an equal probability of a negative or positive equity return, the unconditional probability of a consumption shock is 10% per period. Figure 5 illustrates the numerical results for this case.

Figure 5 shows the optimal holding of taxable bonds in the taxable account (top panel), the optimal holding of equity in the tax-deferred account (middle panel), and the optimal contributions to the retirement account prior to age 65 (bottom panel). Because equity is riskier when its returns are negatively correlated with consumption shocks, investors hold substantially less equity (and more bonds) as a proportion of total wealth. Except for a few years prior to retirement, investors again follow the tax-efficient asset location policy. The deviations from the tax-efficient asset location policy are driven by a horizon effect; at age 65 the investor is allowed to withdraw funds from the tax-deferred account without penalty. This alters the trade-off between the liquidity benefits and tax costs of hedging for investors as they approach retirement age.

For young investors, the cumulative probability of a liquidity crisis prior to retirement is so high that deviating from the tax-efficient asset location policy is not optimal. For these investors, a shift from equity to bonds in the taxable account, offset by a shift from bonds to stocks in the tax-deferred account, involves a significant tax cost and has virtually no effect on the probability or severity of a future consumption shock. In contrast, investors approaching retirement age face a more favorable trade-off between the liquidity benefits and tax costs of hedging. For these investors, a deviation from the tax-efficient asset location policy may reduce the probability and severity of a consumption shock prior to retirement, especially if the current level of taxable wealth is only slightly below the level of the shock. The largest deviation from the tax-efficient asset location policy occurs at age 63, when the investor shifts his entire taxable account to taxable bonds if the level of taxable wealth is less than that required to meet an unforseen consumption shock next period (at age 64). Since this investor has insufficient funds in the taxable account to finance a consumption shock next period, he attempts to minimize the potential penalty for withdrawing funds from the tax-deferred account by hedging against the simultaneous occurrence of a consumption shock and negative equity returns. The pattern of retirement contributions shown in the bottom panel of Figure 5 also reflects the desire of young investors to build liquidity in their taxable accounts.

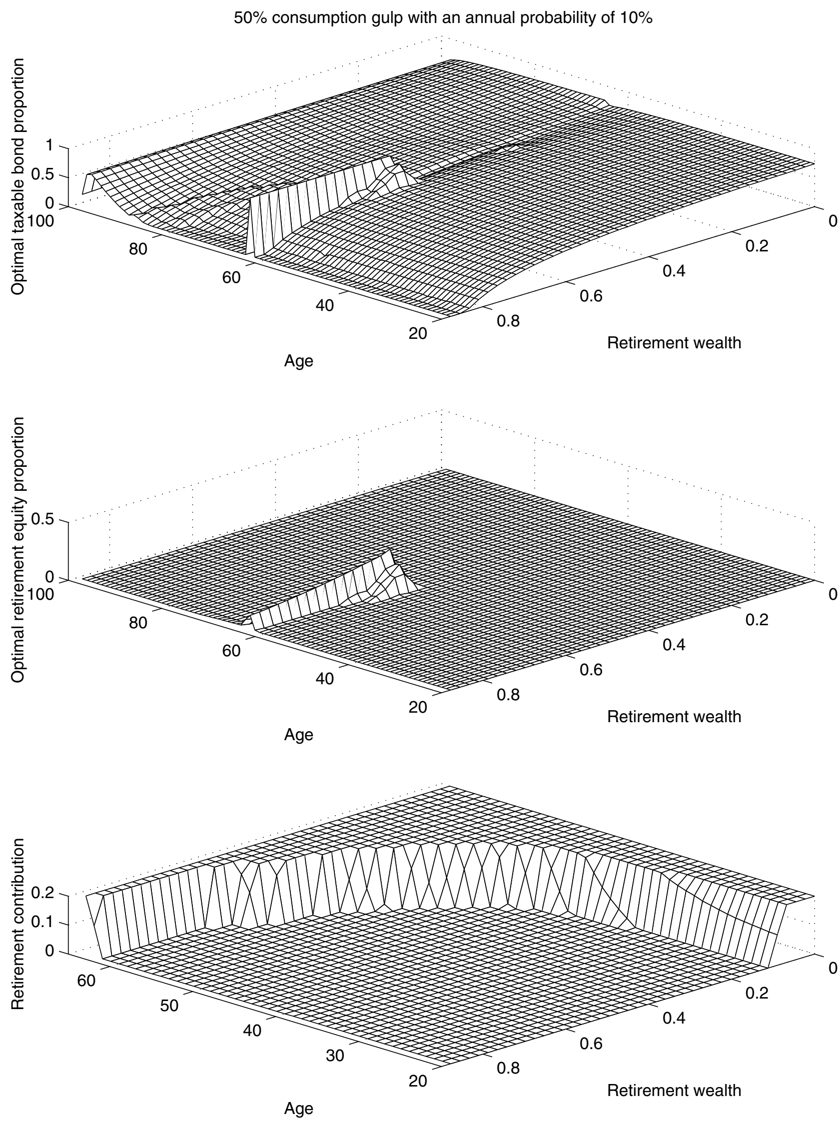

Figure 5. Optimal asset allocations and retirement contributions with a random "consumption gulp." The figure shows the optimal bond proportion in the taxable account, equity proportion in the retirement account, and retirement contributions for the case in which the investor faces a 10% probability each period of a shock to consumption requiring him to consume 50% of total wealth. The consumption shocks are assumed to be independent over time and are negatively correlated with equity returns. A 10% penalty is enforced on withdrawals from the retirement account prior to age 65. The optimal asset allocations and retirement contributions are shown as a function of age and the fraction of total wealth held in the retirement account. The top panel depicts the optimal bond proportion in the taxable account, the middle panel depicts the optimal equity proportion in the tax-deferred account, and the bottom panel depicts the optimal retirement contributions (as a percentage of pre-tax non-financial income).

III. Summary and Conclusion

In this paper, we analyze the optimal dynamic asset allocation and location decisions for an investor with both taxable and tax-deferred investment opportunities. Our results indicate that investors have a strong preference for locating taxable bonds in the tax-deferred retirement account and locating equity in the taxable account. This preference reflects the higher tax burden on taxable bonds relative to equity. When investors can borrow without restrictions in their taxable accounts, it is optimal for them to invest their entire retirement account wealth in taxable bonds and either borrow or lend in the taxable account to achieve an optimal overall portfolio mix. Moreover, the opportunity to invest in tax-exempt bonds does not alter the optimal asset location policy provided equity can be held in a relatively tax-efficient form (e.g., as individual stocks, index mutual funds, or exchange-traded funds).

When investors are prohibited from borrowing, the optimal asset location policy depends upon whether investors face liquidity shocks to consumption. In the absence of liquidity shocks, investors may optimally hold a mix of stocks and bonds in the tax-deferred account, but only if they hold an all-equity portfolio in the taxable account. In the presence of liquidity shocks, investors with insufficient resources in the taxable account to meet the potential need for liquidity tend to reduce their future contributions to the tax-deferred account. Whether investors also adjust the location of their bond holdings to reduce the risk of the taxable portfolio depends upon the magnitude and likelihood of the liquidity shocks and the correlation between these shocks and equity returns. We find that the probability and magnitude of the liquidity shocks need to be rather large and highly negatively correlated with equity returns in order to induce investors to deviate from the tax-efficient asset location policy.

Our analysis points to an important "asset location puzzle" in which the asset location decisions observed in practice deviate substantially from the predictions of our model. Amromin (2001), Poterba and Samwick (2003), Bergstresser and Poterba (2002), and Ameriks and Zeldes (2000) document that investors in practice commonly hold a mix of bonds and stocks in both their taxable and tax-deferred accounts, and in many instances tilt their tax-deferred investments toward equity. While liquidity considerations may help explain some of the observed behavior, we do not believe that liquidity concerns alone can fully account for the magnitude of the deviations that are observed in practice, especially for investors who can borrow and for elderly investors who have unrestricted access to their retirement savings. Our welfare analysis suggests that many investors would benefit considerably from shifting the location of their asset holdings to more closely conform to the tax-efficient policies derived in this paper.

Appendix: Derivation of the Model

Our model builds upon the model developed in Dammon, Spatt, and Zhang (2001) by incorporating tax-deferred investing with taxable investing. The investor's intertemporal consumption-investment problem can be stated as follows:

$$\max_{C_t, n_t, B_t, \theta_t} E\left\{\sum_{t=0}^{T} \beta^t \left[F(t)u\left(\frac{C_t}{(1+i)^t}\right) + [F(t-1) - F(t)] \sum_{k=t+1}^{t+H} \beta^{k-t} u\left(\frac{A_H W_t}{(1+i)^t}\right)\right]\right\}, \tag{A1}$$

s.t.

$$W_t = W_t^T + Y_t(1 - \tau_d), \quad t = 0, \ldots, T, \tag{A2}$$

$$W_t^T = L_t(1 - \tau_d) + n_{t-1}[1 + (1 - \tau_d)d]P_t + B_{t-1}[1 + (1 - \tau_d)r], \quad t = 0, \ldots, T, \tag{A3}$$

$$Y_t = W_{t-1}^R[\theta_{t-1}(1 + g_t)(1 + d) + (1 - \theta_{t-1})(1 + r)], \quad t = 1, \ldots, T, \tag{A4}$$

$$C_t = W_t - \tau_g G_t - n_t P_t - B_t - W_t^R(1 - \tau_d), \quad t = 0, \ldots, T - 1, \tag{A5}$$

$$W_t^R = Y_t + kL_t, \quad t = 0, \ldots, J - 1, \tag{A6}$$

$$W_t^R = Y_t(1 - h_t), \quad t = J, \ldots, T - 1, \tag{A7}$$

$$C_t \geq 0, \quad n_t \geq 0, \quad B_t \geq 0, \quad 0 \leq \theta_t \leq 1, \quad t = 0, \ldots, T - 1, \tag{A8}$$

$$n_T = 0, \quad B_T = 0, \quad W_T^R = 0, \tag{A9}$$

where t denotes time (or age); $F(t)$ is the probability of living through period t; $u(\cdot)$ denotes the investor's utility function; β is the subjective discount factor for utility; C_t is nominal consumption; n_t is the number of shares of stock held in the taxable account; B_t is the amount invested in bonds in the taxable account; θ_t is the fraction of tax-deferred wealth allocated to equity; W_t is total wealth; W_t^T is the wealth in the taxable account after payment of ordinary income taxes but prior to the payment of capital gains taxes; Y_t is the pre-tax wealth in the tax-deferred account before contributions (withdrawals) in period t; W_t^R is the wealth in the tax-deferred account after contributions (withdrawals) in period t; L_t is the pre-tax nonfinancial income; kL_t is the contribution to the retirement account in period t; $h_t Y_t$ is the withdrawal from the retirement account in period t; P_t is the per share stock price; d is the nominal dividend yield; r is the nominal riskless interest rate; g_t is the nominal pre-tax capital gain return; i is the constant rate of inflation; G_t is the total realized capital gain in period t; τ_d is the ordinary tax rate; and τ_g is the capital gains tax rate. The initial portfolio holdings, n_{-1} and B_{-1}, initial nonfinancial income, L_0, and initial tax-deferred wealth, Y_0, are assumed to be non-negative. The value of $F(t)$ in equation (A1) is given by:

$$F(t) = \exp\left(-\sum_{j=0}^{t} \lambda_j\right), \tag{A10}$$

where $\lambda_j > 0$ is the single-period hazard rate for period j with $\lambda_T = \infty$.

The expression inside the square brackets in equation (A1) is the investor's probability-weighted utility at date t. The first term measures the utility of consumption in period t weighted by the probability of living through period t, while the second term is the utility of the investor's bequest weighted by the probability of dying in period t. As written, the bequest provides the investor with a constant level of real consumption for a period of H years, where $A_H = (r^*(1+r^*)^H)/((1+r^*)^H - 1)$ is the H-period annuity factor and $r^* = [(1-\tau_d)r - i]/(1+i)$ is the real after-tax interest rate.

Equation (A2) defines the investor's total wealth as the sum of his taxable wealth and the fraction $(1-\tau_d)$ of his retirement account balance. Equations (A3) and (A4) define the wealth in the taxable and tax-deferred accounts, respectively. Equation (A5) is the investor's intertemporal budget constraint. The realized capital gain (loss) in equation (A5) is given by

$$G_t = \left\{I\left(P_{t-1}^* > P_t\right)n_{t-1} + \left[1 - I\left(P_{t-1}^* > P_t\right)\right]\max(n_{t-1} - n_t, 0)\right\}\left(P_t - P_{t-1}^*\right), \tag{A11}$$

where P_{t-1}^* is the investor's tax basis on shares held at the beginning of period t and $I(P_{t-1}^* > P_t)$ is an indicator function that takes the value of one if there is an embedded capital loss (i.e., $P_{t-1}^* > P_t$) and zero otherwise. Following Dammon, Spatt, and Zhang (2001), we assume that the tax basis is the weighted average purchase price for shares held in the taxable account. The nominal tax basis follows the law of motion:

$$P_t^* = \begin{cases} \frac{n_{t-1}P_{t-1}^* + \max(n_t - n_{t-1}, 0)P_t}{n_{t-1} + \max(n_t - n_{t-1}, 0)}, & \text{if} \quad P_{t-1}^* < P_t, \\ P_t, & \text{if} \quad P_{t-1}^* \geq P_t. \end{cases} \tag{A12}$$

This formulation takes into account that, in the absence of transaction costs, the investor optimally sells his entire holding of equity to realize a tax loss when $P_{t-1}^* \geq P_t$ and immediately repurchases equity to rebalance his portfolio. In this case, the tax basis of the newly acquired shares is the current market price, P_t. Also, when the investor has an embedded capital gain on existing shares (i.e., $P_{t-1}^* < P_t$), the tax basis is unchanged, unless the investor purchases additional shares in period t (i.e., $n_t > n_{t-1}$).

Equations (A6) and (A7) impose constraints on contributions to and withdrawals from the retirement account. Equation (A8) requires consumption to be non-negative and prohibits short sales and borrowing in the taxable and tax-deferred accounts. If investors are allowed to borrow or sell short in the taxable account, the non-negativity constraints on B_t and n_t are relaxed. Finally, equation (A9) requires the investor to liquidate his holdings at date T.

We assume that the investor's preferences can be expressed as follows

$$u\left[\frac{C_t}{(1+i)^t}\right] = \frac{\left[\frac{C_t}{(1+i)^t}\right]^{1-\gamma}}{1-\gamma}, \tag{A13}$$

where γ is the investor's relative risk aversion coefficient. Note that the summation appearing in the second term of the objective function can be rewritten as follows:

$$\sum_{k=t+1}^{t+H} \beta^{k-t} u\left[\frac{A_H W_t}{(1+i)^t}\right] = \frac{\beta(1-\beta^H)\left[\frac{A_H W_t}{(1+i)^t}\right]^{1-\gamma}}{(1-\beta)(1-\gamma)}.$$

Letting X_t denote the vector of state variables at date t, we can write the Bellman equation for the above maximization problem as follows:

$$V(X_t) = \max_{C_t, n_t, B_t, \theta_t} \left\{ \frac{e^{-\lambda_t}\left[\frac{C_t}{(1+i)^t}\right]^{1-\gamma}}{1-\gamma} + \frac{(1-e^{-\lambda_t})\beta(1-\beta^H)\left[\frac{A_H W_t}{(1+i)^t}\right]^{1-\gamma}}{(1-\beta)(1-\gamma)} + e^{-\lambda_t}\beta E_t[V(X_{t+1})] \right\} \tag{A14}$$

for $t = 0, \ldots, T-1$, subject to equations (A2)–(A9). The sufficient state variables for the investor's problem at date t are denoted by the following vector:

$$X_t = \left[P_t, P_{t-1}^*, n_{t-1}, W_t^T, Y_t, L_t\right]'. \tag{A15}$$

We simplify the investor's optimization problem by normalizing by the investor's total wealth, W_t, and assuming that the investor's nonfinancial income is a constant fraction of total wealth, $l = L_t/W_t$. Let $s_t = n_{t-1}P_t/W_t^T$ be the beginning-of-period equity proportion in the taxable account; $f_t = n_t P_t/W_t^T$ be the fraction of taxable wealth allocated to equity after trading at date t; $b_t = B_t/W_t^T$ be the fraction of taxable wealth allocated to taxable bonds after trading at date t; $y_t = Y_t(1-\tau_d)/W_t$ be the fraction of the investor's beginning-of-period total wealth that is held in the retirement account before trading at date t; $w_t^R = W_t^R/W_t$ be the fraction of the investor's beginning-of-period wealth that is held in the retirement account after trading at date t; and $p^*_{t-1} = P^*_{t-1}/P_t$ be the investor's tax basis–price ratio on the initial equity holdings in the taxable account. Then, the gross nominal rate of return on the investor's taxable portfolio from period t to $t+1$, after the tax on dividends and interest, but prior to the payment of capital gain taxes, is

$$\mu^T_{t+1} = \frac{f_t[1+(1-\tau_d)d](1+g_{t+1}) + [1+(1-\tau_d)r]b_t}{f_t + b_t}, \tag{A16}$$

and the gross rate of return on the investor's tax-deferred portfolio from period t to $t+1$ is

$$\mu^R_{t+1} = \theta_t(1+d)(1+g_{t+1}) + (1-\theta_t)(1+r). \tag{A17}$$

Using this notation, equation (A2) can be written as the following linear dynamic wealth equation:

$$W_{t+1} = \left[\mu^T_{t+1}(f_t + b_t)\frac{1-y_t}{1-l(1-\tau_d)} + \mu^R_{t+1}\frac{w^R_t(1-\tau_d)}{1-l(1-\tau_d)}\right] W_t. \tag{A18}$$

Similarly, the intertemporal budget constraint in equation (A5) can be written as follows:

$$c_t = 1 - \tau_g\delta_t(1-y_t) - (f_t+b_t)(1-y_t) - w^R_t(1-\tau_d), \tag{A19}$$

where $c_t = C_t/W_t$ is the consumption–wealth ratio for period t,

$$\delta_t = G_t/W_t = \left\{I\left(p^*_{t-1} > 1\right)s_t + \left[1 - I\left(p^*_{t-1} > 1\right)\right]\max(s_t - f_t, 0)\right\}\left(1 - p^*_{t-1}\right) \tag{A20}$$

is the fraction of beginning-of-period wealth that is taxable as *realized* capital gains in period t, and p^*_{t-1} is given by

$$p^*_{t-1} = \begin{cases} \frac{[s_{t-1}p^*_{t-2} + \max(f_{t-1}-s_{t-1},0)]/(1+g_t)}{s_{t-1} + \max(f_{t-1}-s_{t-1},0)}, & \text{if } p^*_{t-2} < 1, \\ \frac{1}{1+g_t}, & \text{if } p^*_{t-2} \geq 1. \end{cases} \tag{A21}$$

The linearity of the dynamic wealth equation and the assumption of constant relative risk-averse preferences ensure that our model has the property that the consumption and portfolio decision rules, $\{c_t, f_t, b_t, \theta_t\}$, are independent of total wealth, W_t. Furthermore, with the above normalization, the relevant state variables for the investor's problem become $x_t = \{s_t, p^*_{t-1}, y_t\}$. Defining $v(x_t) = V(X_t)/[W_t/(1+i)^t]^{1-\gamma}$ to be the normalized value function and

$\rho_{t+1} = W_{t+1}/[W_t(1+i)]$ to be one plus the *real* growth rate in wealth from period t to period $t+1$, the investor's problem can be restated as follows:

$$v(x_t) = \max_{c_t, f_t, b_t, \theta_t} \left\{ \frac{e^{-\lambda_t} c_t^{1-\gamma}}{1-\gamma} + \frac{(1-e^{-\lambda_t})\beta(1-\beta^H)A_H^{1-\gamma}}{(1-\beta)(1-\gamma)} + e^{-\lambda_t}\beta E_t\left[v(x_{t+1})\rho_{t+1}^{1-\gamma}\right] \right\},$$
$$t = 0, \ldots, T-1, \tag{A22}$$

s.t.

$$\rho_{t+1} = \left(\frac{\mu_{t+1}^T}{1+i}\right)(f_t + b_t)\frac{1-y_t}{1-l(1-\tau_d)} + \left(\frac{\mu_{t+1}^R}{1+i}\right)\frac{w_t^R(1-\tau_d)}{1-l(1-\tau_d)}, \quad t = 0, \ldots, T-1, \tag{A23}$$

$$w_t^R = \frac{y_t}{1-\tau_d} + kl, \quad t = 0, \ldots, J-1, \tag{A24}$$

$$w_t^R = \frac{y_t}{1-\tau_d}(1-h_t), \quad t = J, \ldots, T-1, \tag{A25}$$

$$c_t \geq 0, \quad f_t \geq 0, \quad 0 \leq \theta_t \leq 1, \tag{A26}$$

where c_t is given by equation (A19), δ_t is given by equation (A20), and p_{t-1}^* is given by equation (A21).

The above problem can be solved numerically using backward recursion. To do this, we discretize the lagged endogenous state variables, $x_t = \{s_t, p_{t-1}^*, y_t\}$, into a grid of $(101 \times 101 \times 21)$ over the following ranges: $s_t \in [0, 0.999]$, $p_{t-1}^* \in [0, 1.1]$, and $y_t \in [0, 0.8]$. At the terminal date T, the investor's value function takes the known value

$$v_T = \frac{\beta(1-\beta^H)A_H^{1-\gamma}}{(1-\beta)(1-\gamma)} \tag{A27}$$

at all points in the state space. The value function at date T is then used to solve for the optimal decision rules for all points on the grid at date $T-1$. The procedure is repeated recursively for each time period until the solution for date $t = 0$ is found. Tri-linear interpolation is used to calculate the value function for points in the state space that lie between the grid points.

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