Economics 681 **Steven A. Matthews**
Fall 2017 **University of Pennsylvania**

Solutions for Midterm 2

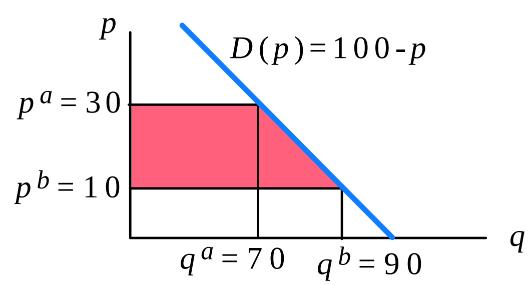
75 minutes, – points. Closed books, notes, calculators.
Indicate your reasoning, using clearly written words as well as math.

1. (20 ponts) In the town Chico, the price of electricity is now $p^a = 30$. If a dam is built on the nearby river, that price will fall to $p^b = 10$, without affecting any other prices. All Chico citizens have the same preferences and income. In the past, the price of electricity has varied enough for the town econometrician to estimate a Chico citizen's demand for electricity as a function of price:

$$D(p) = 100 - p.$$

However, as incomes have never changed, she has no estimate of how demand depends on income. But she knows electricity is a normal good for every citizen.

(a) (10 points) If the government builds the dam without raising taxes, what will be the change, ΔCS, in a Chico citizen's consumer surplus?
Soln:



$$\Delta CS = \int_{p^a}^{p^b} D(p)dp = -\int_{10}^{30} D(p)dp$$
$$= q^a \cdot \Delta p + \frac{1}{2} \cdot \Delta p \cdot \Delta q = 70 \cdot 20 + \frac{1}{2} \cdot 20 \cdot 20 = 1600.$$

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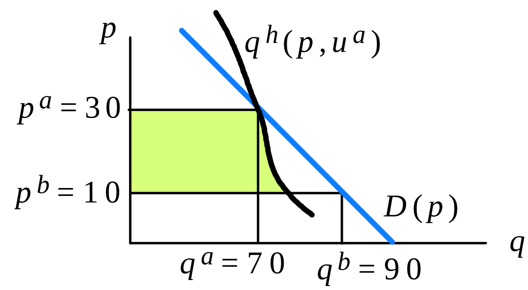
The cost of building the dam is c dollars per citizen of Chico. Suppose now that each citizen will be taxed, as a lump-sum income tax, c dollars if the dam is built.

(b) (10 points) If $c < \Delta CS$, is it necessarily true that the citizens will be better off if the dam is built? If instead $c > \Delta CS$, is it necessarily true that the citizens will be worse off if the dam is built? Explain your answers.
Soln: The citizens will be better off if $c < |CV|$, and worse off if $c > |CV|$, where CV is the compensating variation for the price change:

$$CV = \int_{p^a}^{p^b} q^h(p, u^a)dp,$$

(Note that I'm not writing in the prices of the other goods simply because they are held constant in this problem.) Because electricity is a normal good, the Hicksian demand curve is steeper at (q^a, p^a) than is the ordinary demand curve $D(p)$:



The indicated area is $|CV|$, and so we have $|CV| < \Delta CS$. Thus:

$$c < \Delta CS \implies c \gtreqless |CV| \implies \text{citizens could be better or worse off,}$$

$$c > \Delta CS \implies c > |CV| \implies \text{citizens definitely worse off.}$$

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2. (20 pts) Use a revealed preference argument to prove that a competitive firm's long-run supply function is weakly increasing.

Soln: Let $c(y)$ be the firm's LR cost function. (I've neglected to write the input prices because they will be held fixed). Let p_1 and $p_2 > p_1$ be two different output prices, and let y_1 and y_2 be the corresponding profit-maximizing outputs. We must show that $y_2 \geq y_1$.

Since the firm could have chosen y_2 when the price was p_1, and it could have chosen y_1 when the price was p_2, we have two "revealed preference" inequalities:

$$\begin{aligned} p_1 y_1 - c(y_1) &\geq p_1 y_2 - c(y_2), \\ p_2 y_2 - c(y_2) &\geq p_2 y_1 - c(y_1). \end{aligned}$$

Summing these inequalities yields

$$p_1 y_1 - c(y_1) + p_2 y_2 - c(y_2) \geq p_1 y_2 - c(y_2) + p_2 y_1 - c(y_1).$$

Canceling terms and rearranging yields

$$(p_2 - p_1)(y_2 - y_1) \geq 0.$$

Since $p_2 - p_1 > 0$, we conclude that $y_2 - y_1 \geq 0$. ■

3. (20 pts) A competitive firm has a strictly increasing concave production function $f(x_1, x_2)$ that gives rise to the cost function

$$c(w, q) = \frac{w_1 w_2 q^2}{2w_1 + w_2}.$$

(a) (5 pts) Find the firm's supply function, $q(p, w)$.
Soln: $q(p, w)$ is the q that maximizes the firm's profit:

$$q(p, w) \in \arg\max_q pq - \frac{w_1 w_2 q^2}{2w_1 + w_2}.$$

From the FOC we obtain

$$q(p,w) = \frac{2w_1 + w_2}{2w_1 w_2} p.$$

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(b) (5 pts) Find the conditional input demand functions.

Soln: Use Shepard's lemma:

$$x_1(w,q) = \frac{\partial c(w,q)}{\partial w_1} = \frac{w_2^2 q^2}{(2w_1 + w_2)^2},$$

$$x_2(w,q) = \frac{\partial c(w,q)}{\partial w_2} = \frac{2w_1^2 q^2}{(2w_1 + w_2)^2}.$$

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(c) (10 pts) Find the production function $f(x)$.

Soln: Letting $r = w_1/w_2$, we can (because $x(w,q)$ is homogeneous in w) write the conditional input demand functions in (b) as two equations in two unknowns, r and q :

$$x_1 = \frac{q^2}{(2r+1)^2}, \qquad x_2 = \frac{2r^2 q^2}{(2r+1)^2}.$$

Solving this system of equations for q gives us $f(x)$. There are various ways of doing so, yielding

$$q = \sqrt{x_1} + \sqrt{2x_2} =: f(x).$$

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4. (20 pts) Andy owns an asset that will generate a random monetary payoff $\tilde{x}$. Let g denote the gamble (probability distribution) for $\tilde{x}$, and assume it is nondegenerate. Andy's vNM utility function, u is strictly increasing, and he has no initial wealth. Thus, if he keeps the asset his expected utility is $u(g) = \mathbb{E}_g u(\tilde{x})$. To him the asset is worth having, i.e., $u(g) > u(0)$.

   Before the asset payoff is realized, Andy meets Beth, who has no asset, no wealth, and the same vNM utility function u.

   Assume u exhibits *Increasing* Absolute Risk Aversion (IARA). Prove whether or not a price p exists at which Andy is willing to sell the asset, and Beth is willing to buy the asset.

   Soln: Such a price *does* exist.

   Proof. Let s be the minimum price at which Andy is willing to sell the asset, and b the maximum price at which Beth is willing to buy it. Then they are both better off from trade at a price p iff $s < p < b$. Thus, such a mutually benificial trade exists iff $s < b$.

   Now, note that s is the certainty equivalent of the asset:

$$s = c(g,u).$$

The buy price b satisfies $\mathbb{E}_g u(\tilde{x} - b) = u(0)$, and so b is the certainty equivalent of the asset for the utility function $u_{-b}(x) := u(x - b)$:

$$b = c(g, u_{-b}).$$

Now, because $\mathbb{E}_F u(\tilde{x}) > u(0)$, we see that $b > 0$. So IARA implies u_{-b} is strictly less risk averse than u, and by Pratt's Theorem we have

$$s = c(g, u) < c(g, u_{-b}) = b.$$

(The inequality is strict because g is nondegenerate.) Thus, there are prices satisfying $s < p < b$, and both Andy and Beth will be better off if Andy sells the asset to Beth for such a price. ■

5. (20 pts) Each potential firm in an industry has the production function

$$f(x_1, x_2) = \sqrt{x_1 x_2},$$

and the input prices are fixed at $w_1 = w_2 = 2$. The industry demand function is $D(p) = A - p$, where A is a large positive number.

(a) (10 pts) In the short run, each firm's amount of input 2 is fixed at $\bar{x}_2 = 1$, and the number of firms is $J = 4$. Find the short-run equilibrium price, and per-firm and total outputs.

Soln: A firm needs $x_1 = q^2$ in order to produce an output q. So its short-run cost function is

$$c^s(q) = w_1 q^2 + w_2 \bar{x}_1 = 2q^2 + 2.$$

Its marginal cost function is $c^{s\prime}(q) = 4q$, which is an increasing function that is greater than the average variable cost function, $c^{av}(q) = 2q$, for all $q > 0$. Hence, a firm produces positive output given any $p > 0$, and its short-run supply function is just the inverse of the marginal cost function: $q^s(p) = p/4$. The market demand function is $S^s(p) = Jq^s(p) = p$. Setting this equal to $D(p) = A - p$ and solving for p gives us the equilibrium price,

$$p^s = \tfrac{1}{2}A.$$

The per-firm and total equilibrum outputs are

$$q^s = q^s(p^s) = \tfrac{1}{8}A, \quad Q^s = Jq^s = \tfrac{1}{2}A.$$

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(b) (10 pts) Find the long-run equilibrium price, per-firm and total outputs, and number of firms.

Soln: A firm's long-run cost function is obtained by solving two equations for $(x_1, x_2) : \sqrt{x_1 x_2} = q$, and the tangency condition, $f_1(x)/f_2(x) = w_1/w_2$, which for our f and $w_1 = w_2$ amounts to $x_1 = x_2$. Thus, the input demands are $x_1 = x_2 = q$, and so the long-run cost function is

$$c(q) = w_1 q + w_2 q = 4q.$$

The marginal cost function is thus constant, $c'(q) = 4$. The zero-profit condition therefore requires the price to equal 4:

$$p^L = 4.$$

Total output is

$$Q^L = D(p^L) = A - 4.$$

Since $p^L = c'(q)$ for every q, every $q \geq 0$ maximizes a firm's profit (which is zero). Thus, the per-firm equilibrium outputs are indeterminate, except they must sum to Q^L. For any integer $J > 0$ and positive numbers $q_1, \ldots, q_J$ that sum to Q^L, there is a long-run equilibrium with J firms in which firm j produces q_j for $j = 1, \ldots, J$. ■