University of Pennsylvania
The Wharton School

Professor Stambaugh | Investment Management

SAMPLE EXAM QUESTIONS

1. A pension plan sponsor would like to invest \$50 million in some combination of cash and an index fund (no borrowing or short sales). These assets have the following expected annual returns and volatilities.

| Asset | Expected Return | Standard Deviation |
|---|---|---|
| Index Fund | 10% | 25% |
| Cash | 4 | 0 |

   Assume that the return on the index fund obeys a Normal distribution (a table of the standard Normal distribution is attached).

   The sponsor is considering two alternative investment objectives and would like to know, for each objective, the asset mix that would be appropriate. For each objective, provide (i) the recommended allocations to cash and to the index fund and (ii) the standard deviation of the return on the recommended portfolio.

   a. *Objective 1:* Achieve an expected rate of return over the next year of at least 8% with the lowest possible volatility.

   b. *Objective 2:* Achieve the highest expected rate of return with no more than a 30% probability that the portfolio will experience a negative rate of return over the next year.

2. The Sharpe ratio of a portfolio $p$, $S_p$, is the ratio of the portfolio's expected excess return to its volatility:

$$S_p = \frac{E_p - R_f}{\sigma_p}, \tag{1}$$

   where $E_p$ and $\sigma_p$ are the mean and standard deviation of the portfolio's return, and $R_f$ is the riskless rate of interest. Assume that returns on risky assets obey a Normal distribution. If a portfolio has $S_p = 0.5$ based on monthly returns, what is the probability that such a portfolio will return less than the rate on riskless cash in a given month? (A table of the normal distribution is attached at the end of the exam.)

3. A client with \$1 billion dollars has asked you to recommend an allocation between cash and an index fund. The overall portfolio should have the highest possible expected return over the next month, subject to the requirement that there not be more than a 33% probability of a loss (negative rate of return) over the next month. Assume that the monthly return on the index fund obeys a Normal distribution, with expected return equal to 0.01 and standard deviation equal to 0.06. (A table of the *standard* Normal distribution is attached.) Assume that cash (riskless) returns 0.004 per month.

4. You manage a \$100 million fund that currently has its assets invested 20% in (riskless) cash and 80% in an S&P 500 index portfolio. You would like to convert your fund to a new one that delivers double exposure to the S&P, meaning that the new fund's return would (1) be perfectly correlated with the S&P return and (2) have twice the standard deviation of the S&P return.

   a. How could you accomplish this by taking a position in a return swap (or futures) while not changing your holdings of cash and the S&P index portfolio? Include a numerical value for the size of the swap (or futures) position in your answer.

   b. If you estimate the standard deviation of your original fund's return to be 20% per year, what is your estimate of the standard deviation of the return on your new fund?

5. You have a \$2 million stock portfolio to which you would like to add a short position in S&P index futures so that the new portfolio is market-neutral (zero-beta). Assume

| | Standard Deviation | Beta |
|---|---|---|
| Original portfolio | 0.20 | 0.60 |
| S&P Index | 0.15 | 1.00 |

   Also assume that the S&P index is the market portfolio used to define betas and that the futures position requires no initial investment.

   a. What is the size (dollar amount) of your short futures position?

   b. What is the standard deviation of the return on the new market-neutral portfolio?

6. You have temporarily acquired a \$1 million long position in the stock of XYZ Corporation. You plan to liquidate this position in several weeks, but in the meantime you would like to hedge away the risk attributable to fluctuations in the overall stock market by taking a short position in S&P 500 index futures. To determine the appropriate futures position, you have estimated the following "characteristic line" regression relation between the excess weekly return on the stock, $R_{i,t}$, and the excess weekly return on the S&P, $R_{M,t}$,

$$R_{i,t} = 0.0002 + 1.20 R_{M,t} + \epsilon_{i,t},$$

   where $\epsilon_{i,t}$ is the regression residual (having zero expected value). The regression's goodness of fit, measured by R-squared, is 50%. You have also estimated the standard deviation of weekly returns on the S&P to be 0.03.

   a. What is the size of your short futures position?

   b. What is your estimate of the standard deviation of the weekly dollar profit on your overall hedged position (combining stock and futures)?

7. Consider the following portfolio of two assets, along with each asset's parameters in a characteristic-line regression of the asset's monthly excess return ($R_{i,t}$) on the S&P 500 portfolio's excess return ($R_{M,t}$),

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + \epsilon_{i,t}.$$

| Asset (i) | Amount held | $\alpha_i$ | $\beta_i$ | std. dev. of $\epsilon_{i,t}$ |
|---|---|---|---|---|
| 1 | \$10 mil. | .005 | 1.2 | 0.030 |
| 2 | \$20 mil. | .002 | 1.5 | 0.015 |

Also assume that the correlation between $\epsilon_{1,t}$ and $\epsilon_{2,t}$ equals zero.

To this portfolio, you wish to add a short position in futures on the S&P 500 that will hedge away the risk attributable to market exposure.

a. What is the size (dollar amount) of the short futures position?

b. What is the standard deviation of the dollar profit per month on the overall hedged portfolio?

8. Hal and Stu have started a small fund, investing \$1 million of their own capital, with hopes of establishing a track record that will later help them to attract money from outside investors. Their quantitative strategy has identified 100 stocks to buy, and they plan to buy equal dollar amounts of each stock. They would like to hedge their long position in the 100 stocks with a short position in S&P 500 futures, so as to make their overall portfolio market-neutral (zero-beta).

   For each stock, the two partners have estimated characteristic line regressions of the form,

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + \epsilon_{i,t}$$

   where $R_{i,t}$ is a stock's return in month $t$, and $R_{M,t}$ is the return on the S&P 500. (Both $R_{i,t}$ and $R_{M,t}$ denote returns in excess of the riskless interest rate.) The partners estimate that, across the 100 stocks, the average $\alpha_i$ equals 0.02, and the average $\beta_i$ equals 0.75. Based on historical estimates, they are also assuming that

   - the standard deviation of $\epsilon_{i,t}$ equals 0.10 for each individual stock from the list of 100
   - the correlation between $\epsilon_{i,t}$ and $\epsilon_{j,t}$ equals 0.25 for every possible pair of stocks from the list of 100
   - the expected excess monthly return on the market equals 0.005
   - the standard deviation of the monthly market return is 0.06

   **a.** What is the dollar amount of the short futures position they should establish?

   **b.** What dollar profit on the overall position should the partners expect over the next month?

   **c.** What is the standard deviation of the dollar profit on the overall position over the next month? (Hint: treating 100 stocks as an infinite number of stocks will deliver an acceptably close approximation.)

9. For each of the three statements below, briefly evaluate whether each statement's final sentence follows correctly from those that precede it.

a. "Some have argued that the CAPM fails to price certain kinds of stocks, but I think it works fine on well-diversified portfolios. I've been examining the behavior of well-diversified portfolios, generally containing at least one hundred securities selected from a variety of industries. When I run a regression of such a portfolio's monthly returns (dependent variable) on the returns on a broad market index (independent variable), the regression's R-squared is typically 90% or higher. Therefore, the CAPM does a pretty good job for the well-diversified portfolios I've been examining."

b. "I don't understand all of the talk recently about how 'beta is dead.' I've noticed that when the market goes up a lot, the stocks with higher betas are often those whose prices rise the most. Similarly, I've noticed that, in bear markets, the high-beta stocks are often those with the biggest price drops. Therefore, the CAPM is still useful."

c. "I've been managing a market-neutral fund, which has a zero-beta with respect to the S&P. My fund's Sharpe ratio is definitely positive, but I admit it's substantially less than the Sharpe ratio of the S&P. Nevertheless, any investor who (i) agrees with my numbers, (ii) wants a high Sharpe ratio, and (iii) is currently invested solely in the S&P should want to transfer some amount to my fund."

10. Consider the following strategy:

| **(Assumes $100 Investment)** | |
|---|---|
| Long equity portfolio A | $100 |
| Short equity portfolio B | $100 |
| Gross exposure | $200 |
| Net exposure | $0 |

Portfolios A and B both have betas of 1 with respect to the market index portfolio. The returns on each of the portfolios have the same variance, which is significantly higher than the variance of the market index return. The covariance between the returns on portfolios A and B is equal to the variance of the market index return.

Jack and Sally disagree about which exposure better reflects overall risk—gross exposure or net exposure. Jack believes gross exposure is better, since there is $100 at risk in each of the long and short legs. Sally thinks net exposure is better, given that the risks in the long and short legs go in opposite directions. Mel tells Jack and Sally that, if overall risk is measured as the standard deviation of total profit, both of them are partly right but partly wrong. That is, net exposure is correct for handling one source of the risk, but gross exposure comes closer for another source of the risk. In clarifying the latter statement about gross exposure, Mel adds that $141 is actually a more relevant number than $200.

Can you make sense of what Mel is saying, or do you instead agree with either Jack or Sally? Explain.

11. You have identified 20 stocks whose returns (in excess of a riskless rate) you believe to obey characteristic-line regressions of the form,

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + e_{i,t},$$

where, in any period $t$, $R_{i,t}$ is the excess return on stock $i$, $R_{m,t}$ is the excess return on a market index, and $e_{i,t}$ for each stock has zero mean and standard deviation equal to 30%. Assume $e_{i,t}$ is uncorrelated with $e_{j,t}$ for all $j \neq i$. Assume that $R_{m,t}$ has a mean of 6% and a standard deviation of 20%. Suppose each of the 20 stocks has $\beta_i = 1.0$, and you buy \$1 million of an equally weighted portfolio containing 10 of the stocks while going short \$1 million of an equally weighted portfolio containing the other 10 stocks. What is the standard deviation of your dollar profit?

12. Consider the following recommended allocations (in percentages that add to 100) made by investment advisors from four different Wall Street firms. Assume each advisor believes that stocks have greater volatility and greater expected return than do bonds, and that bonds have greater expected return than the (riskless) return on cash. Assume that each advisor is prohibited from recommend borrowing or short positions.

| | Objective | Stocks | Bonds | Cash |
|---|---|---|---|---|
| Advisor A | Conservative | 20 | 30 | 50 |
| | Aggressive | ? | ? | 10 |
| Advisor B | Conservative | 30 | 40 | 30 |
| | Aggressive | 40 | 40 | 20 |
| Advisor C | Conservative | 20 | 40 | 40 |
| | Aggressive | 50 | 50 | 0 |
| Advisor D | Conservative | 60 | 20 | 20 |
| | Aggressive | 50 | 50 | 0 |

a. If Advisor A attempts to recommend efficient portfolios (those having the greatest expected return for their level of volatility), what should be his "Aggressive" stock and bond allocations?

For each of the other three advisors, viewed one at a time, state whether that advisor's set of allocations is consistent with his recommending efficient portfolios. ("Consistent" means that you cannot exclude the possibility that the advisor is recommending efficient portfolios. "Inconsistent" means you can exclude that possibility.) Explain your reasoning in each case.

b. Advisor B: consistent inconsistent (circle one)

c. Advisor C: consistent inconsistent (circle one)

d. Advisor D: consistent inconsistent (circle one)

13. Below are the portfolio proportions recommended by four different financial advisors. Each advisor offers recommended blends for three different levels of portfolio volatility.

| | Advisor A | | | Advisor B | | | Advisor C | | | Advisor D | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Volatility | Cash | Bonds | Stock | Cash | Bonds | Stock | Cash | Bonds | Stock | Cash | Bonds | Stock |
| Low | 0.50 | 0.25 | 0.25 | 0.60 | 0.10 | 0.30 | 0.50 | 0.40 | 0.10 | 0.40 | 0.40 | 0.20 |
| Medium | 0.30 | 0.35 | 0.35 | 0.00 | 0.50 | 0.50 | 0.00 | 0.50 | 0.50 | 0.30 | 0.35 | 0.35 |
| High | 0.10 | 0.45 | 0.45 | 0.00 | 0.70 | 0.30 | 0.00 | 0.20 | 0.80 | 0.20 | 0.20 | 0.60 |

"Volatility" refers to the standard deviation of the return. Assume that borrowing and short positions are not allowed. Also assume that all advisors believe that (i) stocks have both a higher expected return and a higher volatility than do bonds and (ii) bonds have a higher expected return than does riskless cash.

Classify the advisors according to whether their advice is consistent or inconsistent with recommending mean-variance efficient portfolios. ("Consistent" means you cannot rule out the possibility that all of the advisor's portfolios are efficient.) Briefly explain the reasoning in each case.

a. Advisor A: consistent inconsistent (circle one)

b. Advisor B: consistent inconsistent (circle one)

c. Advisor C: consistent inconsistent (circle one)

d. Advisor D: consistent inconsistent (circle one)

14. You are advising several clients on asset allocation. You have just recommended the following allocation to Client A:

| | |
|---|---|
| Stocks | 60% |
| Bonds | 30% |
| Cash | 10% |
| Total | 100% |

You believe that this portfolio's monthly return has a standard deviation of 4% and is expected to exceed the riskless cash rate by 0.4%. You also believe that this portfolio offers the highest possible expected return for its level of standard deviation.

a. Client B would like you to recommend a combination of the same three asset classes having the highest expected return for a monthly return standard deviation of 2%. What allocation would you recommend?

| | |
|---|---|
| Stocks | __________ |
| Bonds | __________ |
| Cash | __________ |
| Total | 100% |

What is this portfolio's expected monthly return in excess of the cash rate?

b. Client C would like you to recommend a combination of the same three asset classes that maximizes an objective function of the form $U = \mathrm{E}(R) - \frac{1}{2}A\mathrm{Var}(R)$ for $A = 10$. What allocation would you recommend?

| | |
|---|---|
| Stocks | ________ |
| Bonds | ________ |
| Cash | ________ |
| Total | 100% |

15. Consider the following four alternative allocations to a stock portfolio, a bond portfolio, and cash.

| | A | B | C | D |
|---|---|---|---|---|
| stocks | .4 | .5 | .4 | .6 |
| bonds | .4 | .5 | .6 | .4 |
| cash | .2 | 0 | 0 | 0 |

Assume:
- the stock and bond portfolios are identical across the allocations
- borrowing (a negative cash position) is prohibited
- stocks have a higher expected return than bonds
- stocks have a higher standard deviation of return than bonds
- bonds have an expected return greater than the riskless return on cash

If three of the four allocations above are mean-variance efficient, which allocation is not mean-variance efficient? Explain.

16. You are advising a client whom you assume has an objective function of the form,

$$E(R_p) - \frac{A}{2}\mathrm{var}(R_p),$$

where $R_p$ is the return on her portfolio, and $A$ is her coefficient of risk aversion. Until very recently, she has chosen to hold the following portfolio, having relied on your assumptions about expected returns and standard deviations given below:

| Asset | Allocation | Expected return | Standard Deviation |
|---|---|---|---|
| S&P 500 index fund | 60% | 8% | 20% |
| Money market fund | 40% | 2% | 0 |

Recently you told her that, although you are still assuming a 20% stock-market volatility, you are now temporarily making different expected-return assumptions:

| Asset | Expected return |
|---|---|
| S&P 500 index fund | 10% |
| Money market fund | 1% |

She tells you that, given those new assumptions, she should probably change her stock-market exposure, but she does not want to disturb her holdings in either the S&P 500 index fund or the money market fund. She is willing to have you open a stock-index futures position on her behalf, however. She currently has a total of $4 million invested.

a. What size (in dollars) futures position you would recommend for her? Indicate whether the futures position should be long or short.

b. Once that futures overlay is included, what is the standard deviation (in dollars) of the profit/loss on her entire portfolio over the next period?

17. Consider the following four alternative allocations to a stock portfolio, a bond portfolio, and cash.

| | A | B | C | D |
|---|---|---|---|---|
| Stocks | .4 | .5 | .5 | .7 |
| Bonds | .4 | .3 | .5 | .3 |
| Cash | .2 | .2 | 0 | 0 |
| Standard Deviation of Overall Return | 0.117 | 0.136 | 0.146 | 0.185 |

Assume:

- the compositions within the stock and bond portfolios are identical across the allocations
- borrowing (a negative cash position) is prohibited
- stocks have a higher expected return than bonds
- stocks have a higher standard deviation of return than bonds
- bonds have an expected return greater than the riskless return on cash

a. Three of the four allocations above *are* mean-variance efficient. Which allocation is then *not* mean-variance efficient? Explain briefly.

b. Give the allocations for the mean-variance efficient portfolio whose standard deviation of return is equal to 0.05.

Stocks __________

Bonds __________

Cash __________

Total 100%

18. You are advising clients about investment allocations in their employer-sponsored retirement accounts. You believe each client desires to maximize an objective of the form,

$$E(r_p) - \frac{A}{2}\text{var}(r_p),$$

where $r_p$ is the return over the next period on the client's portfolio, and $A$ is the client's coefficient of risk aversion. (Each client's value of $A$ is generally different from that of other clients.) Currently you are deciding what to advise two clients, Alice and Bob, both of whom have taken new jobs.

Alice's previous employer offered each employee the ability to allocate his/her retirement account among three funds, all from the ABC mutual fund company: the Market Fund, the Growth Fund, and the Money Fund. Alice's allocation to these funds has been as follows:

| | |
|---|---|
| Market Fund | 40% |
| Growth Fund | 40% |
| Money Fund | 20% |

You had recommended this allocation to her, after consulting with her and assessing her risk tolerance. She has recently changed jobs. Her new employer's retirement plan also offers ABC funds, but just the Market Fund and the Money Fund. You must now recommend a new allocation to Alice that includes just those two funds.

Bob is in the opposite situation. His previous employer offered just two ABC funds, the Market Fund and the Money Fund, but his new employer offers those two funds plus the Growth Fund. Bob's previous allocation, which you had also recommended to him, was as follows:

| | |
|---|---|
| Market Fund | 70% |
| Money Fund | 30% |

You must now recommend a new allocation to Bob that includes all three funds.

In advising both clients, you had been assuming the following values, and you continue to maintain these assumptions:

| Fund | Expected Return | Standard Deviation | Correlations: Market | Correlations: Growth | Correlations: Money |
|---|---|---|---|---|---|
| Market | 0.10 | 0.20 | 1.00 | 0.75 | – |
| Growth | 0.14 | 0.30 | | 1.00 | – |
| Money | 0.03 | 0 | | | – |

**a.** What new allocation would you now recommend to Alice?

**b.** What allocation would you now recommend to Bob?

19. Assume

- your "normal" combination of stocks and bonds has a Sharpe ratio of 0.40 and a standard deviation of 0.15
- the standard deviations of the returns on stocks and bonds are 20% and 10%, respectively
- the correlation between the returns on stocks and your normal portfolio is 0.80
- the correlation between the returns on bonds and your normal portfolio is 0.50

a. What are the "neutral" expected returns on stocks and bonds (in excess of the riskless cash return) that are consistent with the belief that your normal portfolio has the highest Sharpe ratio among all possible stock-bond combinations?

b. Suppose your total portfolio consists of an 80% investment in the above normal portfolio and 20% in cash, and you believe that this total portfolio maximizes an objective function of the form $E(R_p) - \frac{1}{2}A\text{var}(R_p)$, where $R_p$ is the return on your total portfolio. Suppose another investor has the same objective function (with the same $A$) and agrees with all of the above assumptions and calculations. If, however, that investor can invest only in cash and stocks (no bonds), what percentage of that investor's portfolio would you recommend be allocated to stocks?

20. The following table gives some characteristics of returns on three assets—stocks, bonds, and cash. Also shown are weights in portfolio $P$, which is composed of those three assets. Assume that portfolio $P$ is mean-variance efficient, i.e., it provides the highest expected return for its level of variance.

| Asset | Weight in Portfolio $P$ | Expected Return | Standard Deviation | Correlations: Stocks | Bonds | Cash |
|---|---|---|---|---|---|---|
| Stocks | 0.35 | ? | 0.20 | 1.00 | 0.30 | – |
| Bonds | 0.35 | ? | 0.10 | | 1.00 | – |
| Cash | 0.30 | 0.03 | 0 | | | – |

a. What are the weights in the mean-variance efficient portfolio whose standard deviation of return is equal to *one half* of the standard deviation of portfolio $P$?

b. What is the standard deviation of the return on portfolio $P$?

c. If the expected rate of return on portfolio $P$ is equal to 0.058, what are the expected rates of return on stocks and bonds?

21. You are advising an endowment that presents you with the following information about their overall portfolio $P$:

| Asset class | Allocation | Correlation of return with return on portfolio $P$ | Standard deviation of return | Expected return |
|---|---|---|---|---|
| Large-cap stocks | 50% | 0.80 | 0.25 | ? |
| Small-cap stocks | 20% | 0.60 | 0.40 | ? |
| Fixed income | 20% | 0.20 | 0.10 | 0.02 |
| Cash | 10% | 0.00 | 0.00 | 0.01 |
| Portfolio $P$ | 100% | 1.00 | 0.15 | 0.08 |

If the endowment's managers believe that portfolio $P$'s current allocation is mean-variance efficient, what must those managers believe to be the expected rates of return on large-cap stocks and small-cap stocks?

| | Expected return |
|---|---|
| Large-cap stocks | ____________ |
| Small-cap stocks | ____________ |

22. The “policy” portfolio of an endowment fund is given below, along with some characteristics of the assets in the fund.

| Asset | Weight in Policy Portfolio | Expected Return | Standard Deviation |
|---|---|---|---|
| Large-cap stocks | 0.30 | ? | 0.20 |
| Small-cap stocks | 0.10 | ? | 0.40 |
| Cash | 0.60 | 0.02 | 0 |
| Policy portfolio | | 0.08 | 0.0963 |

The covariance between the returns on large-cap stocks and small-cap stocks is 0.068.

**a.** If the policy portfolio is mean-variance efficient, what are the weights in the mean-variance efficient portfolio whose standard deviation of return is equal to *twice* the standard deviation of the policy portfolio?

**b.** If the policy portfolio is mean-variance efficient, what is the expected return on large-cap stocks?

**c.** If the policy portfolio is mean-variance efficient, what is the expected return on small-cap stocks?

**d.** A member of the endowment’s investment committee has been analyzing small-cap stocks and believes they are currently underpriced to a substantial degree. (He has not analyzed large-cap stocks.) He recommends that the assumed expected return on small-cap stocks be raised significantly above the answer in part c. He also recommends that an optimizer be run with that new assumption, keeping the expected return on large-cap stocks at the answer in part b. Do you believe his recommendation makes sense? Explain.

23. Consider two investors who are deciding whether to invest in the Greatbuys mutual fund. Mr. Smith is looking for a single fund in which to concentrate his entire portfolio, and he is considering Greatbuys as a candidate. Ms. Jones currently has her entire portfolio invested in a market index fund, and she is considering moving some modest fraction of her wealth to Greatbuys.

a. (8 points) Which investor is likely to be more interested in Greatbuy’s Sharpe ratio? Explain briefly.

b. (7 points) Suggest an alternative performance measure that the other investor might find more useful. Explain briefly.

24. You are asked to analyze the performance of an endowment whose portfolio each year consists of stocks and bonds selected by the managers within each class. An asset-allocation consultant advises the endowment on its stock-bond allocation each year. A twenty-year record of the endowments returns and allocations to stocks and bonds is provided below. Also shown are returns on passive stock and bond indexes, and the policy benchmark against which manager is compared is a mix of 60 percent stock and 40 percent bonds. Using the information provided, evaluate the endowment’s overall performance as compared to its

benchmark, and give the amounts of this overall performance attributable to (i) asset allocation to stocks versus bonds and (ii) security selection within each of those two asset classes. How would you interpret the difference between the total performance relative to the benchmark and the sum of the asset allocation and security selection components. Explain clearly how you compute your answers.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| | Manager's Portfolio | | | | | Passive Index Returns | | Products of Columns | |
| | Allocations | | Returns | | | | | | |
| Year | Stocks | Bonds | Stocks | Bonds | Overall | Stocks | Bonds | (1) × (6) | (2) × (7) |
| 1 | 0.3 | 0.7 | 0.13 | 0.08 | 0.095 | 0.12 | 0.07 | 0.036 | 0.049 |
| 2 | 0.4 | 0.6 | -0.02 | 0.05 | 0.022 | -0.03 | 0.06 | -0.012 | 0.036 |
| 3 | 0.7 | 0.3 | 0.26 | 0.04 | 0.194 | 0.23 | 0.05 | 0.161 | 0.015 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 19 | 0.5 | 0.5 | 0.12 | 0.10 | 0.110 | 0.08 | 0.14 | 0.040 | 0.070 |
| 20 | 0.9 | 0.1 | 0.25 | 0.10 | 0.235 | 0.20 | 0.10 | 0.180 | 0.010 |
| Column Average | 0.6 | 0.4 | 0.11 | 0.05 | 0.0969 | 0.08 | 0.06 | 0.0564 | 0.0246 |

25. Let $R_{P,t}$ denote the return in month $t$ on a managed portfolio $P$. Consider the regressions

$$R_{P,t} - R_{F,t} = \alpha + \beta(R_{M,t} - R_{F,t}) + \epsilon_{P,t} \qquad \text{(I)}$$

$$R_{P,t} - R_{F,t} = \mathrm{a} + \mathrm{b}(R_{M,t} - R_{F,t}) + \mathrm{c}(R_{M,t} - R_{F,t})D_t + e_{P,t}\ , \qquad \text{(II)}$$

where $R_{M,t}$ is the return on the S&P index, $R_{F,t}$ is a riskless rate of interest, and

$$D_t = \begin{cases} 1 & \text{if } R_{M,t} > R_{F,t} \\ 0 & \text{otherwise.} \end{cases}$$

Consider the following results obtained when the above regressions are estimated for two actively managed portfolios, $A$ and $B$, using ten years of monthly data. Each estimated coefficient is shown (as a decimal, not a percent) along with its $t$-statistic (below in parentheses).

| | Regression I | | Regression II | | |
|---|---|---|---|---|---|
| | $\alpha$ | $\beta$ | a | b | c |
| Portfolio A | 0.030 | 1.11 | -0.010 | 1.03 | 0.55 |
| | (3.50) | (31.5) | (0.70) | (30.7) | (3.85) |
| Portfolio B | 0.015 | 1.11 | 0.013 | 1.12 | 0.15 |
| | (2.60) | (32.7) | (2.10) | (31.4) | (0.50) |

a What is "market timing"? How can one test for its presence using one of the above regressions (I or II). Which of portfolios A and B, if either, appears to exhibit significant market timing ability?

b In performance evaluation, what is the usual interpretation of a finding that the $\alpha$ (Jensen measure) in regression I is significantly greater than zero?

c. Given the above regression evidence, is the usual interpretation of $\alpha$ that you gave in part b. more appropriate for portfolio A or for portfolio B (choose one)? Explain your reasoning.

26. You are asked to analyze the performance of an endowment whose twenty-year record of returns and allocations to stocks and bonds is provided below. Also shown are returns on passive stock and bond indexes. The passive policy benchmark against which the endowment is compared contains a constant mix of 60 percent stock and 40 percent bonds.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| | Manager's Portfolio | | | | | Passive Index Returns | | Products of Columns | |
| | Returns | | | Allocations | | | | | |
| Year | Stocks | Bonds | Overall | Stocks | Bonds | Stocks | Bonds | (4) × (6) | (5) × (7) |
| 1 | 0.13 | 0.08 | 0.095 | 0.3 | 0.7 | 0.12 | 0.07 | 0.036 | 0.049 |
| 2 | -0.02 | 0.05 | 0.022 | 0.4 | 0.6 | -0.03 | 0.06 | -0.012 | 0.036 |
| 3 | 0.26 | 0.04 | 0.194 | 0.7 | 0.3 | 0.23 | 0.05 | 0.161 | 0.015 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 19 | 0.12 | 0.10 | 0.110 | 0.5 | 0.5 | 0.08 | 0.14 | 0.040 | 0.070 |
| 20 | 0.25 | 0.10 | 0.235 | 0.9 | 0.1 | 0.20 | 0.10 | 0.180 | 0.010 |
| Column Average | 0.121 | 0.051 | 0.112 | 0.62 | 0.38 | 0.105 | 0.045 | 0.061 | 0.030 |

Using the information provided above, evaluate the endowment's overall performance as compared to its policy benchmark, and give the amounts of this overall performance attributable to (i) asset allocation to stocks versus bonds, (ii) security selection within each of those two asset classes, and (iii) the interaction between asset allocation and security selection.

a. Overall performance in excess of benchmark __________

b. Asset allocation __________

c. Security selection __________

d. Interaction __________

27. You are asked to analyze the performance of a managed portfolio over the most recent 12-month period relative to the S&P 500. You are given the weights in each of 20 economic sectors at the beginning of the period, for both the portfolio and the index, as well as the rate of return on each sector earned by the managed portfolio and the S&P. This information is summarized below.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| | S&P 500 | | Managed Portfolio | | Products of Columns | | | |
| Sector | Weight | Return | Weight | Return | (1) × (2) | (1) × (4) | (2) × (3) | (3) × (4) |
| Commercial Services | 0.01 | 0.08 | 0.02 | 0.09 | 0.0008 | 0.0009 | 0.0016 | 0.0018 |
| Communications | 0.04 | 0.05 | 0.03 | 0.05 | 0.0020 | 0.0020 | 0.0015 | 0.0015 |
| Consumer Durables | 0.01 | 0.04 | 0.04 | 0.07 | 0.0004 | 0.0007 | 0.0016 | 0.0028 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| Transportation | 0.02 | 0.10 | 0.12 | 0.05 | 0.0020 | 0.0010 | 0.0120 | 0.0060 |
| Utilities | 0.04 | 0.10 | 0.08 | 0.09 | 0.0040 | 0.0036 | 0.0080 | 0.0072 |
| Column Total | 1.00 | 1.60 | 1.00 | 1.50 | 0.0900 | 0.0700 | 0.1200 | 0.1100 |

Compute the portfolio's overall performance as compared to the S&P, and give the amounts of this overall performance attributable to (i) asset allocation across sectors, (ii) security selection within each sector and (iii) the interaction between asset allocation and security selection.

a. Overall performance in excess of S&P ____________

b. Asset allocation ____________

c. Security selection ____________

d. Interaction ____________

28. You are asked to analyze the performance of a managed portfolio over the most recent 12-month period relative to the S&P 500. You are given the weights in each of 20 economic sectors at the beginning of the period, for both the portfolio and the index, as well as the rate of return on each sector earned by the managed portfolio and the S&P. This information is summarized below.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| | Managed Portfolio | | S&P 500 | | Products of Columns | | | |
| Sector | Weight | Return | Weight | Return | (1) × (2) | (1) × (4) | (3) × (4) | (2) × (3) |
| Commercial Services | 0.02 | 0.09 | 0.01 | 0.08 | 0.0018 | 0.0016 | 0.0008 | 0.0009 |
| Communications | 0.03 | 0.05 | 0.04 | 0.05 | 0.0015 | 0.0015 | 0.0020 | 0.0020 |
| Consumer Durables | 0.04 | 0.07 | 0.01 | 0.04 | 0.0028 | 0.0016 | 0.0004 | 0.0007 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| Transportation | 0.12 | 0.05 | 0.02 | 0.10 | 0.0060 | 0.0120 | 0.0020 | 0.0010 |
| Utilities | 0.08 | 0.09 | 0.04 | 0.10 | 0.0072 | 0.0080 | 0.0040 | 0.0036 |
| Column Total | 1.00 | | 1.00 | | 0.1300 | 0.1100 | 0.0900 | 0.0800 |

Compute the portfolio's overall performance as compared to the S&P, and give the amounts of this overall performance attributable to (i) asset allocation across sectors, (ii) security selection within each sector and (iii) the interaction between asset allocation and security selection.

a. Overall return minus the S&P return ____________

b. Asset allocation ____________

c. Security selection ____________

d. Interaction ____________

29. Consider a characteristic-line regression,

$$R_{p,t} = \alpha_p + \beta_i R_{M,t} + \epsilon_{p,t}$$

where $R_{p,t}$ is the return in month $t$ on a portfolio of stocks with high book-to-market ratios, and $R_{M,t}$ is the return on the S&P 500 portfolio. (Both $R_{p,t}$ and $R_{M,t}$ denote returns in excess of the riskless interest rate.) Historical estimates indicate that $\alpha_p$ is positive, to a statistically reliable degree.

**a.** Can a positive $\alpha_p$ be consistent with mean-variance efficiency of the S&P 500 portfolio? Explain.

**b.** Can a positive $\alpha_p$ be consistent with informational efficiency of the stock market—a market in which prices correctly reflect available information? Explain.

**c.** If the stock market is not informationally efficient, why might one expect a portfolio of high book-to-market stocks to have a positive $\alpha_p$?

30. The rate of return on a "market-neutral" strategy—a strategy with zero market beta—is uncorrelated with the return on the market portfolio. Cash (earning a riskless interest rate) is often proposed as an appropriate benchmark against which to compare the average return on a market-neutral strategy.

a. (7 points) What basic argument can justify the use of cash as a benchmark?

b. (8 points) Suppose you learn that, in a particular market-neutral strategy, stocks of "value" firms are held predominantly in long positions (as opposed to sold short). Explain why, given the empirical evidence on value-stock returns, cash might not be the appropriate benchmark return for judging the skill of the manager of that market-neutral strategy.

31. The board of a small foundation recently attended a presentation by an investment firm that specializes in offering index-style (passive) small-cap value portfolios (with "value" stocks being those with high ratios of book value of equity to market value of equity). The foundation's investment fund currently has some of its assets in "cash" (short-term money market) and the remainder in an S&P index fund. You have been retained to advise the board on whether to add a small-cap value portfolio to its asset mix.

a. What would you give as the main argument(s) in favor of a reallocation toward small-cap value?

b. What could you raise as the potential drawback(s) to a reallocation toward small-cap value?

c. One member of the board remarked that he would like to see the foundation's portfolio exposed to the lowest possible volatility, subject to an expected-return target, and he worries that small-cap value stocks could be substantially more volatile than the S&P. How would you address his concern?

32. Consider a portfolio manager whose holdings concentrate on "value" stocks of companies with ratios of book value of equity to market value of equity that are relatively high. Suppose you have analyzed the returns that such a manager has produced historically and have concluded that, to a reliable degree of statistical precision, the manager's portfolio has a positive intercept $\alpha_p$ in a regression of the portfolio's excess return ($R_{p,t}$) on the excess return on the S&P 500 ($R_{M,t}$),

$$R_{p,t} = \alpha_p + \beta_p R_{M,t} + \epsilon_{p,t}.$$

a. Under what assumption(s) would the positive $\alpha_p$ lead you to conclude that the portfolio contained stocks that were priced too low by the market when the manager purchased them?

b. Provide an argument against interpreting a positive $\alpha_p$ as indicating that the manager can select stocks that are priced too low.

33. Consider a hedge fund for which a significant fraction of the portfolio is invested in less liquid assets. If the hedge-fund claims to have a low beta, and this claim is supported by a regression of monthly returns of the fund on the monthly returns of the market, why might you be suspicious that the true beta could be higher? How would you investigate that suspicion?

34. An endowment that is currently invested 100% in an S&P 500 index fund is considering reallocating some of its portfolio to *one* of two hedge funds. The foundation's objective in selecting the hedge fund, and the amount to allocate to it, is to obtain the highest Sharpe ratio for the foundation's overall portfolio. You have been asked to advise the foundation, and thus far you have estimated, means, standard deviations, and the following characteristic-line regression for each fund,

$$R_{P,t} = \alpha_P + \beta_P R_{M,t} + \epsilon_{P,t},$$

where $R_{P,t}$ is the annual return (in excess of cash) on the fund and $R_{M,t}$ is the excess annual return on the S&P 500. Your estimates include the following:

| Fund ($P$) | $E(R_P)$ | std.dev.($R_P$) | $\beta_P$ |
|---|---|---|---|
| A | 0.13 | 0.40 | 1.0 |
| B | 0.07 | 0.20 | 0.5 |
| M | 0.10 | 0.20 | 1.0 |

a. Of the two hedge funds, which is the better one to receive a portion of the foundation's overall allocation?

b. In the new overall portfolio, for each dollar allocated to the S&P, how much would you allocate to the hedge fund?

c. What is the Sharpe ratio of the new overall portfolio?

35. A foundation that is currently invested 100% in an S&P 500 index fund is considering reallocating some of its portfolio to *one* of two hedge funds. The foundation's objective is to obtain the highest Sharpe ratio for its overall portfolio. (A portfolio's Sharpe ratio is its expected excess return divided by the standard deviation of its excess return.) You have been asked to advise the foundation, and thus far you have estimated the following characteristic-line regression for each fund,

$$R_{P,t} = \alpha_P + \beta_P R_{M,t} + \epsilon_{P,t},$$

where $R_{P,t}$ is the monthly return (in excess of cash) on the fund and $R_{M,t}$ is the excess monthly return on the S&P 500. Your estimates of the parameters in the regression for each fund are as follows:

| Fund ($P$) | $\alpha_P$ | $\beta_P$ | std.dev.($\epsilon_P$) |
|---|---|---|---|
| A | 0.02 | 0.5 | 0.11 |
| B | 0.01 | 0.0 | 0.05 |

Also, according to your estimates, the excess monthly return on the S&P has a mean of 0.02 and a standard deviation of 0.05.

a. Of the two hedge funds, which is the better one to receive a portion of the foundation's overall allocation?

b. In the new overall portfolio, for each dollar allocated to the S&P, how much would you allocate to the hedge fund chosen in part (a)?

c. What is the Sharpe ratio of the new overall portfolio?

d. The monthly hedge fund returns you used in your analysis are those reported by each fund's management. You later learn that (1) the hedge fund you did not select above holds relatively liquid equities, but (2) the fund you selected has the bulk of its investments in relatively illiquid assets that are difficult to value precisely on a month-to-month basis. *Assume you would still maintain the objective of achieving the highest overall Sharpe ratio.* How could your new knowledge about asset liquidity potentially cause you to change your fund selection? Explain. (Qualitative answer - no numerical calculations.)

36. A foundation that is currently invested 100% in an S&P 500 index fund is considering reallocating some of its portfolio to *one or both* of two actively managed funds. The foundation's objective is to obtain the highest Sharpe ratio for its overall portfolio. (A portfolio's Sharpe ratio is its expected excess return divided by the standard deviation of its excess return.) You have been asked to advise the foundation, and thus far you have estimated the following characteristic-line regression for each fund,

$$R_{P,t} = \alpha_P + \beta_P R_{M,t} + \epsilon_{P,t},$$

where $R_{P,t}$ is the monthly return (in excess of cash) on the fund and $R_{M,t}$ is the excess monthly return on the S&P 500. Your estimates of the parameters in the regression for each fund are as follows:

| Fund ($P$) | $\alpha_P$ | $\beta_P$ | std.dev.($\epsilon_P$) |
|---|---|---|---|
| A | 0.02 | 0.4 | 0.14 |
| B | 0.02 | 0.0 | 0.14 |

You have also concluded that the non-market returns ($\epsilon_{P,t}$'s) are uncorrelated with each other (i.e., correlation($\epsilon_{A,t}$, $\epsilon_{B,t}$) = 0). Finally, according to your estimates, the excess monthly return on the S&P has a mean of 0.02 and a standard deviation of 0.05.

a. In the new overall portfolio, for each dollar allocated to the S&P, how much would you allocate to each fund?

b. What is the Sharpe ratio of the new overall portfolio?

37. A hedge fund claims to be market neutral. Using a history of monthly returns, you have estimated the correlation between the fund's reported return each month $t$ and the market's return in month $t$. Your estimate of that correlation is very close to zero, consistent with the market-neutral claim. While conducting your analysis, however, you also noticed that the correlation between the fund's reported return in month $t+1$ with the market's return in month $t$ is significantly positive. How would you interpret this result? Would it affect your view as to whether the fund is indeed market neutral?

38. A foundation that is currently invested 100% in an S&P 500 index fund is considering reallocating some of its portfolio to *one or both* of two actively managed long-short equity strategies, denoted A and B. The foundation's objective is to obtain the highest Sharpe ratio for its overall portfolio. (The Sharpe ratio is expected excess return divided by the standard deviation of excess return.) You have been asked to advise the foundation, and thus far you have estimated the following properties of each strategy. Also shown are the same properties for the excess market return ($r_M - r_f$) and the riskless rate ($r_f$).

| | Mean | Std. Dev. | Beta |
|---|---|---|---|
| $r_{A,L} - r_{A,S}$ | 0.025 | 0.10 | 0.30 |
| $r_{B,L} - r_{B,S}$ | 0.020 | 0.10 | 0.40 |
| $r_M - r_f$ | 0.050 | 0.20 | 1.00 |
| $r_f$ | 0 | 0 | 0 |

You have also concluded that the non-market returns of strategies A and B are uncorrelated with each other, i.e., correlation($\epsilon_{A,t}$, $\epsilon_{B,t}$) = 0.

Note: $r_{A,L}$ denotes the return on the equity portfolio held long in strategy A; $r_{A,S}$ denotes the return on the equity portfolio held short in strategy A. The returns $r_{B,L}$ and $r_{B,S}$ are defined similarly for strategy B.

a. Would you recommending allocating a non-zero amount

to strategy A ? (yes or no) ____________

to strategy B ? (yes or no) ____________

b. In the new overall portfolio, for each dollar allocated to the S&P, how much would you allocate

to strategy A ? (dollar amount) ____________

to strategy B ? (dollar amount) ____________

c. What is the Sharpe ratio of the new overall portfolio?

39. a. Some observers of the hedge-fund industry have said that much of the apparent alpha produced by hedge funds is "just beta waiting to be discovered." Provide an explanation of this statement. Does alpha necessarily reflect skill to identify mispriced assets? If you suspect that a given manager's estimated alpha doesn't reflect such skill, how could you confirm your suspicion? That is, what data would you want, and what empirical test would you run? (Postpone any consideration of illiquidity until part b.)

    b. How can illiquidity of some hedge-fund investments give rise to apparent alpha that is really just beta? How can this problem be detected?

40. Psychologists tell us that overconfidence is a commonly possessed behavioral property. Suppose overconfidence is sufficiently strong and widespread to affect prices in the stock market. Describe a strategy designed to profit from such overconfidence, and explain why overconfidence should lead to the strategy's being profitable.

41. Proponents of behavioral finance often claim that individuals exhibit "loss aversion"—briefly described as tending to seek pride and avoid regret.

    a. Is there evidence in financial markets of loss aversion? Explain.

    b. Give an example of an investment strategy that one might expect to profit from effects of loss aversion on stock prices, and explain why one might expect the strategy to be profitable.

42. Empirical evidence on mutual fund performance indicates short-run persistence in the relative performance of mutual funds. Even after adjusting for exposures to the market return as well as size and book-to-market factors, ranking funds by last year's performance has some power to predict this year's fund rankings. One possible reason for this is that, at least in the short run, skill levels differ across managers. Provide another possible explanation for short-run persistence in rankings that does not involve differences in skill levels across managers.

43. A quantitative investment firm has estimated one-way trading costs (in basis points) on two of their portfolios:

| | Large-Cap Value | Small-Cap Growth |
|---|---|---|
| Executed Orders | | |
| Brokerage Commissions | 15 | 12 |
| Bid-Offer Spread | 34 | 60 |
| Market Impact | -23 | 110 |
| Cost of Actual Transactions | 26 | 182 |
| Unexecuted Orders | | |
| Opportunity Cost | 220 | 425 |
| Total Transactions Costs | 55 | 201 |
| Fraction of Orders Executed | 85% | 92% |

The Small Cap Growth portfolio primarily implements momentum strategies based on analyst revisions and earnings surprises.

**a.** What is "market impact"? How would you measure it?

**b.** Do the market-impact values, especially their difference between the two portfolios, make sense? Explain.

**c.** How would you measure the opportunity cost of unexecuted orders? If this cost were ignored, explain how the firm could overestimate a strategy's expected return.

44. A foundation that is currently invested 100% in an S&P 500 index fund is considering reallocating some of its portfolio to *one or both* of two actively managed long-short equity strategies, denoted A and B. The foundation's objective is to obtain the highest Sharpe ratio for its overall portfolio. (The Sharpe ratio is expected excess return divided by the standard deviation of excess return.) You have been asked to advise the foundation, and thus far you have estimated the following properties of each strategy and the excess market return $(r_M - r_f)$, where $r_f$ denotes the riskless interest rate.

| | Mean(%) | Std. Dev.(%) | Beta |
|---|---|---|---|
| $r_{A,L} - r_{A,S}$ | 0.8 | 5 | 0.5 |
| $r_{B,L} - r_{B,S}$ | 0.3 | 4 | 0 |
| $r_M - r_f$ | 1.0 | 6 | 1 |
| $r_f$ | 0.1 | 0 | 0 |

You have also concluded that the correlation between the non-market returns of strategies A and B is equal to $-0.20$, i.e., correlation$(\epsilon_{A,t}, \epsilon_{B,t}) = -0.20$.

Note: $r_{A,L}$ denotes the return on the equity portfolio held long in strategy A; $r_{A,S}$ denotes the return on the equity portfolio held short in strategy A, and $\epsilon_A$ is the residual in the characteristic-line regression,

$$(r_{A,L} - r_{A,S})_t = \alpha_A + \beta_A (r_M - r_f)_t + \epsilon_{A,t}.$$

The corresponding quantities for strategy B are defined similarly.

**a.** Would you recommend that less than 100% be allocated to the S&P index fund? (Calculations beyond what is required to answer yes or no are not required for this part.) Explain.

**b.** For each dollar allocated to the S&P, how much would you allocate

to strategy A ? (dollar amount) ____________

to strategy B ? (dollar amount) ____________

**c.** What is the Sharpe ratio of the new overall portfolio?

**d.** Give an example of two long-short strategies discussed in the course whose non-market returns are typically negatively correlated. Does the negative correlation make sense for these strategies? Explain.

**e.** What are the benefits to "slicing" a strategy, so that only a fraction of the total amount devoted to the strategy is subject to rebalancing at any given time. Why might it make sense for one strategy to have more slices than another?

45. Explain the difference between a momentum strategy based on lagged returns and an earnings-based momentum strategy. Give an example of each, and provide a behavioral-based rationale for the profitability of each strategy.

46. a. Suppose you are asked to analyze the trading costs of an investment manager. Explain how you would construct a measure of the "market-impact" component of trading costs to assist in this analysis. What data are required to compute your measure?

    b. For which type of strategy is your your measure of market-impact costs likely to be higher, (1) a value-oriented strategy or (2) a momentum strategy? Explain.

47. a. Briefly describe an example of a simple quantitative rule for constructing a "value" long-short stragegy (i.e., long value, short growth).

    b. Briefly describe an example of a simple quantitative rule for constructing a "momentum" long-short stragegy.

    c. Ron believes that the concepts behind momentum and value seem rather opposite to each other, and therefore running both long-short strategies simultaneously would work at cross purposes—the two strategies would tend to cancel each other's profits. He therefore argues that it makes more sense for any given investment manager to run just one or the other of these strategies, not both at once. To what extent do you agree with Ron? To what extent do you disagree?

48. A portfolio of value stocks is often constructed by sorting on a simple ratio such as book-to-market (B/M) or earnings-price (E/P), where value stocks would be those with high values of these ratios. Describe an example of another approach to identifying value stocks, and explain what data/assumptions are required to implement the strategy.

49. Compare the relative merits of three different approaches to estimating the current market equity premium (expected return in excess of a riskless rate):

a. the historical average of the market's realized returns in excess of the riskless rate

b. the current dividend yield *plus* a forecast of real GDP growth *minus* the current real interest rate

c. the current earnings-price ratio for the market *minus* the current real interest rate

50. Jim Bidwell, a portfolio manager for a major Wall Street firm, believes that stocks offer only about a 3 percent expected return over bonds for the next few decades. Jim reasons that the price-earnings (P/E) ratio on the overall market is about 20 (once earnings are properly measured) and that 1/20 = 0.05, implying an expected real return on stocks of 5% per year. The yield on long-term inflation-adjusted U.S. Government bonds is about 2%, thereby leaving an equity premium of 3%.

    a. Suppose you accept Jim's view that the P/E is indeed 20. What other assumption(s) are needed to support his argument that the equity premium is 3%?

    Ken Adams, another portfolio manager at the firm, believes that Jim's estimate of the equity premium is too low and notes that, looking back over the past fifty years, stocks have outperformed bonds by substantially more than 3% per year on average.

    b. What reasonable response(s) could Jim offer to Ken's argument?

51. a. Bill believes that the expected real rate of return on stocks over the long run is 6 percent, which he computes by adding the market's current dividend yield of 2 percent to his estimate of 4 percent for the expected real growth rate in dividends per share. Bill also believes that the volatility of stocks is 30 percent per year.

    You would like to use Bill's estimates to assess the probability that, at the end of a 10-year horizon, a current investment of $1 million in stocks (with dividends reinvested) will be worth less than an investment of $1 million in an inflation-indexed riskless bond, returning 2.00 percent per year (or 1.98 percent continuously compounded—you may round that to 2). You are willing to assume that (i) returns are uncorrelated from year to year and (ii) the continuously compounded rate of return on stocks is normally distributed. What is your estimate of the probability? (A table of the normal distribution is attached to the end of the exam.)

    b. Another colleague has noted that the historical average of stock returns is nearly twice as high as Bill's estimate. You have confronted Bill with this point, and he responded "I know. If anything, that high past performance makes me more confident that the expected return is now fairly low." Can you make sense of Bill's response? Explain.

52. a. You have asked Jack for his projected expected real annual rate of return on the stock market. He says that, since the current price-earnings (P/E) ratio for the market is about 20, the expected real return is simply 5%. What assumption(s) can justify Jack's answer?

    b. Jill thinks Jack's approach is reasonable. She also believes that the market's P/E ratio is mean-reverting, in that it fluctuates but tends to revert back toward its long-run mean. Jill argues that this mean reversion helps makes target-date funds a sensible investment vehicle. Do you agree with her conclusion? Explain. ("Target-date" funds are also known as "target-retirement" funds or "life-cycle" funds.)

53. Tom, Dick, and Harry work for a large bank and have been asked to supply an estimate of the current equity premium for the U.S. stock market to use in the firm's asset-allocation models. So far they have assembled the following data:

| | |
|---|---|
| Current level (share price) of the S&P 500 | \$1160 |
| Next year's forecasted earnings per share on the S&P | \$25 |
| Next year's forecasted dividends per share on the S&P | \$17 |
| Long-run forecasted real GDP growth | 2.5% |
| Long-term TIPS yield | 1.6% |
| Historical (arithmetic) average equity premium | 7% |

**a.** Tom believes that corporate earnings, in real terms, will grow at the same rate as GDP, and that dividends as a fraction of earnings will be stable at the current level. What is a reasonable value of the equity premium for Tom to recommend? Explain.

**b.** Dick does not necessarily believe Tom's assumptions. Dick does believe that capital markets are in equilibrium in the sense that a dollar invested in the composite basket of real corporate assets is expected to earn the same rate of return as a dollar invested in the S&P stock portfolio. What is a reasonable value of the equity premium for Dick to recommend? Explain.

**c.** Harry agrees with neither Tom's nor Dick's assumptions. He prefers to rely instead on the historical average premium. Suppose Dick is right. What can contribute to the difference between his estimate and Harry's?

54. Your corporation is considering making a one-time payment to an insurance company for the purpose of covering future health-care benefits promised to retirees. Specifically, the insurance company will be paid \$10 million in five years. As treasurer, you have been asked to determine the amount of funds necessary to invest today such that the value of that investment in five years has at least an 80% probability of exceeding the required \$10 million payment. The funds would be invested in a portfolio whose continuously compounded annual returns are normally distributed with a mean of 8% and a standard deviation of 15%. What is the amount necessary to invest? (A table of the Normal distribution is attached at the end of the exam.)

55. A pension fund has consulted you regarding the risk of future shortfalls. The current portfolio of the fund consists of \$2 million invested in an S&P index portfolio, and the continuously compounded return on that portfolio is assumed to be normally distributed with a mean of 10% per year and a standard deviation of 20% per year. The current value of the pension liability is also \$2 million, and the continuously compounded growth rate of the liabilities is assumed to be normally distributed with a mean of 4% per year and a standard deviation of 8% per year. The correlation between the return on the S&P and the growth rate of the liabilities is assumed to be 0.3. What is the probability of the plan being underfunded in five years if the portfolio experiences no additions or withdrawals? (A table of the normal distribution is attached at the end of the exam.)

56. A pension fund has consulted you regarding its investment funding. The pension committee would like the fund's portfolio to be completely indexed, so that its rate of return in any period is equal to that of the S&P 500. The continuously compounded annual return on the S&P is assumed to be normally distributed with a mean of 6% and a standard deviation of 20%. The current value of the pension liability is $30 million, and the continuously compounded annual growth rate of the liabilities is assumed to be normally distributed with a mean of 3% per year and a standard deviation of 10% per year. The correlation between the S&P return and the growth rate in the liabilities is 0.2. The pension committee would to have no more than a 20% probability that the portfolio value will be less than the liability in five-years (assuming no contributions or withdrawals are made during that period). What is the minimum amount that you recommend the fund currently have invested? [Note: Prob($z < -0.84) = 0.20$, where $z$ has a standard Normal distribution.]

57. A publicly traded corporation is considering how to invest funds set aside to meet a fixed obligation to pay a single lump sum of $100 million in 20 years. To begin its analysis of the problem, the corporation is considering two alternatives. The first alternative is to invest at a riskless rate of 5% per year, continuously compounded. The second alternative is to invest in a diversified stock portfolio whose continuously compounded annual return is Normally distributed, with an expected value of 10% and a standard deviation of 20%.

    a. To fund the obligation under the first alternative, the corporation would invest the amount

    $$\$100 \text{ million} \times e^{-20(.05)} = \$36.79 \text{ million}$$

    If the same initial amount of $36.79 million is instead invested in the stock portfolio, what is the probability that the value of that stock investment in 20 years will be greater than the $100 million obligation? (A table for the Normal distribution is attached to the end of the exam.)

    b. Does your answer in part a. support an argument that investing in the stock portfolio is the superior alternative? If it is fairly likely that the stock investment will outperform the riskless investment over 20 years, does that suggest stocks are the better alternative for the corporation? Explain. (To simplify things, ignore tax considerations and the possibility of corporate bankruptcy.)

58. A small college currently has its $100 million endowment invested in a portfolio that replicates the S&P 500, whose continuously compounded annual return is normally distributed with a mean of 10% and a standard deviation of 20%. The college is considering a proposal to contribute these assets to a trust and in turn receive two classes of shares, Common shares and Preference shares. The trust would be managed as an S&P 500 index fund and would have a 30-year life. At the end of year 30, the Common shares would receive any trust assets in excess of the "redemption" value. The Preference shares would receive either the redemption value or the total assets in the trust, whichever is less. The redemption value would be the initial trust contribution grown at a continuously compounded annual rate of 3%, or $e^{(30)0.03}$ times $100 million.

    a. What is the probability that the Preference shares would receive the full redemption value? (A table of the normal distribution is attached at the end of the exam.)

b. After being shown the above probability in part (a), some college administrators believe the trust is a good idea. They argue that the probability in part (a) implies that the college should be able to sell the Preference shares immediately for close to $100 million. The college could retain the Common shares, preserving up-side exposure to the S&P, while raising cash nearly equal to the current value of the endowment. Do you agree with this reasoning? Explain.

59. "Target-date" funds typically allocate a smaller fraction to stocks and a greater fraction to bonds as the investor's target retirement date draws closer.

    a. How could you justify this investment strategy?

    b. What considerations, for some investors, could weigh against such a strategy?

60. Jack works for a mutual fund company that is considering adding target-date funds to its product offerings. (Such funds are also known as target-retirement funds, age-based funds, or life-cycle funds.) He has been asked to prepare a document explaining why such funds are desirable for investors. Thus far, Jack is trying to focus on the dispersion in equity returns for investment horizons of various lengths. For a given horizon length of $N$ years, Jack obtains real equity returns over numerous $N$-year periods using a long historical sample. For each $N$-year period, he computes the annualized return—he divides the sum of the $N$ (continuously compounded) annual returns by $N$. He then analyzes the frequency distribution of the resulting annualized returns, and in particular he computes the distribution's 10th and 90th percentiles as well as its standard deviation. The above procedure is repeated for different values of $N$, yielding the following results:

| | Percentiles of annualized returns | | Standard deviation of |
|---|---|---|---|
| Investment horizon ($N$) | 10th percentile | 90th percentile | annualized return |
| 5 years | -6.5% | 16.5% | 8.94% |
| 10 years | -3.1% | 13.1% | 6.32% |
| 20 years | -0.7% | 10.7% | 4.47% |

Based on this evidence, Jack argues that equity returns are less disperse for long-horizon investors than for investors with shorter horizons, and thus an equity allocation that declines with age, as in a target-date fund, makes sense.

**a.** Do you agree with Jack's reasoning? Explain.

**b.** Can you suggest an alternative measure of volatility that Jack should examine in making an argument for target-date funds?

**c.** What pattern of results for your suggested volatility measure in part b would Jack hope to find, if we wants to support a case for target-date funds? Explain.

**d.** Based on Jack's calculations reported above, do you think he will he find the pattern you describe in part c? Explain.

**e.** Give Jack another argument, not directly involving volatility calculations, that supports the age-related equity allocation patterns used by target-date funds.

61. Briefly evaluate the validity of the following argument made by the chief financial officer of a corporation with a *defined benefit* pension plan.

> We invest the bulk of our pension fund in equities. In the long run, equities are more likely to offer the highest compound rates of growth and thereby provide a greater likelihood of future surpluses in the pension fund. Such surpluses, which would allow us to reduce future contributions to the fund, are not possible under a fixed-income stategy that is simply "immunized" to match future benefit obligations. Thus, we believe that equities provide a superior investment vehicle for the assets of our pension fund.

62. a. Summarize a reasonable and concise case for investing the assets of a defined-benefit (DB) pension plan in bonds.

    b. Summarize a reasonable and concise case for investing the assets of a DB plan in stocks.

    c. To what extent is the fact that stocks have higher expected returns on bonds a reasonable justification for investing the assets of a DB plan in stocks? Explain briefly.

63. Alan works for a corporation that has estimated it will have to pay a pension obligation in 15 years of \$50 million, adjusted for inflation. The yield on a 15-year TIPS (Treasury Inflation-Protected Security) is 1% per year, so the amount of current investment necessary to fund that future obligation is \$43.068 million ($= 50/(1.01)^{15}$). Alan is evaluating two investment alternatives for funding the required amount:

    Alternative I. invest \$43.068 million in 15-year TIPS

    Alternative II. invest \$43.068 million in an S&P 500 index portfolio

    Note returns on pension-fund assets are not taxed. To begin his analysis, Alan has obtained a history of annual real rates of return (simple, not continuously compounded) on the S&P. From them he has computed the following statistics:

| | |
|---|---|
| Geometric mean | 05% |
| Standard deviation | 20% |

**a.** Alan would like to use the above information to estimate the probability that Alternative II will provide an amount in 15 years that exceeds the \$50 million obligation. Provide Alan with an estimated probability. In doing so, assume that continuously compounded annual real returns are normally distributed and that (for simplicity) they have the same standard deviation as the simple real returns. (A table of the standard Normal distribution is attached to the exam.)

**b.** If Alan would like to recommend the alternative that results in greater current value of the corporation, comment on the relevance of each of the following to Alan's decision:

**i.** the probability in part a

**ii.** the tax-deductibility of interest payments by corporations

**iii.** the limited liability of corporations

ANSWERS

1. a. Let $R_p$ denote the return on the overall portfolio that combines cash and the index fund. For any proportion $x$ placed in the index fund, the expected return and volatility of the fund's return are given by

$$E_p = x(.10) + (1-x).04 \tag{2}$$
$$\sigma_p = x(.25) \tag{3}$$

Use (2) and solve for $x$ such that

$$.08 = x(.10) + (1-x)(.04) \quad \Longrightarrow \quad x = .667, \tag{4}$$

or an allocation of \$33 million in the index fund and \$17 million in cash. Using (3), the volatility of this portfolio is

$$\sigma_p = (.667)(.25) = .167.$$

[This is the lowest volatility possible for an expected return of 8%, since a lower volatility would require a lower $x$ and, therefore, an expected return less than 8%.]

b. By standardizing the return,

$$\text{Prob}\{R_p < 0\} = \text{Prob}\{z < \frac{0 - E_p}{\sigma_p}\}. \tag{5}$$

From the table of the standard Normal distribution, we see that

$$\text{Prob}\{z < -.524\} = .30. \tag{6}$$

Therefore, the portfolio's mean and volatility should satisfy

$$\frac{E_p}{\sigma_p} = .524. \tag{7}$$

and combining this with (2) and (3) gives

$$\frac{x(.10) + (1-x).04}{x(.25)} = .524 \quad \Longrightarrow \quad x = .563, \tag{8}$$

or an allocation of \$28 million in the index fund and \$22 million in cash. [Any other portfolio with a higher expected return also has a probability greater than 30% of a negative return: the ratio $E_p/\sigma_p$ is a decreasing function of $x$, so increasing $x$ would then increase the probability in (5).] Using (3), the volatility of this portfolio is

$$\sigma_p = (.563)(.25) = .14.$$

2. The probability of the portfolio's returning less than cash is computed as

$$\begin{aligned}
\text{Prob}\{R_p < R_f\} &= \text{Prob}\{z < \frac{R_f - E_p}{\sigma_p}\} \\
&= \text{Prob}\{z < -S_p\} \\
&= \text{Prob}\{z < -0.5\} = 0.309
\end{aligned}$$

3. Let $R_p$, $E_p$, and $\sigma_p$ denote the portfolio's return, expected return, and standard deviation of return.

$$\begin{aligned}
\text{Prob}\{R_p < 0\} &= \text{Prob}\{\frac{R_p - E_p}{\sigma_p} < \frac{0 - E_p}{\sigma_p}\} \\
&= \text{Prob}\{z < \frac{-E_p}{\sigma_p}\},
\end{aligned}$$

where $z$ is a standard Normal variable. From the Normal table, we see that

$$\text{Prob}\{z < -.44\} = 0.33,$$

so we must have

$$-.44 = \frac{-E_p}{\sigma_p} \text{ or } E_p = .44\sigma_p. \tag{9}$$

If $x$ is the weight to be placed in the index fund, then

$$E_p = x(.01) + (1 - x)(.004) \tag{10}$$

$$\sigma_p = x(.06). \tag{11}$$

Substituting the right-hand sides of (10) and (11) for $E_p$ and $\sigma_p$ in (9) gives

$$\begin{aligned}
x(.01) + (1 - x)(.004) &= .44x(.06) \\
x &= .196,
\end{aligned}$$

or 19.6% of the portfolio invested in the index fund, and thus 80.4% invested in cash.

4. (a) You would want to take a position in a return swap whose payoff is the difference between the S&P return and an interest rate, times the size of the position (the "notional" value of the swap). Equivalently, you could take a long position in futures, whose payoff is essentially the same thing. You want to change your effective position on the capital allocation line for the S&P. Currently that position has an allocation of $y = 0.8$ to the risky asset (S&P), whereas you want an effective allocation of $y^* = 2$, requiring that your ratio of position size to invested wealth $X/W$ satisfy

$$\begin{aligned}
y_* &= y + \frac{X}{W} \\
&= 0.8 + \frac{X}{\$100 \text{ mil.}},
\end{aligned}$$

implying a position size of $X = \$120$ million.

(b) If $\sigma_p$ denotes the standard deviation of the S&P return, the original 0.20 volatility ($\sigma_c$) must satisfy

$$\sigma_c = y\sigma_p,$$

whereas the new volatility ($\sigma_c^*$) must satisfy

$$\sigma_c^* = y^*\sigma_p,$$

or

$$\sigma_c^* = \frac{y^*}{y}\sigma_c = \frac{2.0}{0.8}(0.20) = 0.50.$$

5.

a. The rate of return on the new portfolio, $R_{P,t}$, is

$$\begin{aligned} R_{P,t} &= \frac{L \cdot R_O - S \cdot (R_{M,t} - R_{F,t})}{L} \\ &= R_{O,T} - \frac{S}{L}(R_{M,T} - R_{F,T}) \\ &= [R_{F,t} + \alpha_O + \beta_O(R_{M,t} - R_{F,t}) + \epsilon_{O,t}] - \frac{S}{L}(R_{M,t} - R_{F,t}), \end{aligned} \tag{12}$$

where $O$ denotes the original portfolio, $L$ denotes the long position ($2 million), and $S$ denotes the short futures position. Setting the hedge ratio $\frac{S}{L}$ equal to $\beta_O$ gives

$$R_{P,t} = R_{F,t} + \alpha_O + \epsilon_{O,t}, \tag{13}$$

which has a beta of zero. So, the size of the short position is

$$S = L \cdot \beta_O = \$\ 2 \text{ million} \cdot 0.6 = \$\ 1.2 \text{ million}.$$

b. The random portion of the return, $\epsilon_{O,t}$, has variance

$$\begin{aligned} \text{var}\{\epsilon_{O,t}\} &= \text{var}\{R_{O,t}\} - \beta_O^2 \text{var}\{R_{M,t}\} \\ &= (.20)^2 - (0.6)^2(.15)^2 = 0.0319, \end{aligned} \tag{14}$$

so the standard deviation of the new portfolio's return is 0.179.

6. a. The hedge ratio is $\beta_i$, the slope coefficient in the regression, so the size of the required short futures position is

$$(\$1 \text{ million})(1.20) = \$1.2 \text{ million}$$

b. The market-related variance of the stock's return is

$$\beta_i^2 \text{Var}(R_{M,t}) = (1.2)^2(0.03)^2 = 0.00130$$

and, since this is half the variance of the total stock return (given the R-squared of 50%), the above value is also the non-market related variance that remains once the hedge is in place. The standard deviation of the hedged weekly profit is thus

$$(\$1 \text{ million})(\sqrt{0.00130}) = \$36,000.$$

7. (a) The beta of the portfolio is given by

$$\beta_p = (1/3)(1.2) + (2/3)(1.5) = 1.4,$$

so the size of the futures position should be

$$(1.4)(\$30 \text{ million}) = \$42 \text{ million}.$$

(Equivalently, the same amount can be calculated as (1.2)($10 million) + (1.5)(($20 million).)

(b) The non-market return of the two-stock portfolio ($\epsilon_{p,t}$) is given by

$$\epsilon_{p,t} = (1/3)\epsilon_{1,t} + (2/3)\epsilon_{2,t},$$

and this is the risky component of the hedged portfolio's rate of return. Since $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are uncorrelated, the variance of the return is

$$\begin{aligned}
\text{Var}(\epsilon_{p,t}) &= (1/3)^2\text{Var}(\epsilon_{1,t}) + (2/3)^2\text{Var}(\epsilon_{2,t}) \\
&= (1/3)^2(.03)^2 + (2/3)^2(.015)^2 \\
&= [(1/3)(.03)]^2 + [(2/3)(.015)]^2 \\
&= 2[.01]^2,
\end{aligned}$$

and thus the standard deviation of the return is $\sqrt{2}(.01)$. Multiplying this standard deviation by the size of the portfolio gives the standard deviation of the profit as

$$\sqrt{2}(.01)(\$30 \text{ million}) = \$424,264.$$

8. (a) Since the beta of the equally weighted portfolio of the 100 stocks is 0.75, the dollar size of the short position should be 0.75 × $1 mil. = $750,000.

(b) With market exposure removed, the expected return (in excess of the cash rate) on the overall position is the alpha on the portfolio of 100 stocks, which is 0.02. Thus, the expected dollar profit over the next month is 0.02 × $1 mil. = $20,000, plus $ 1 mil. times the riskless cash rate.

(c) With market exposure removed, the variance of the return on the overall position is the variance of the non-market return

$$\epsilon_{p,t} = \frac{1}{100}\sum_{i=1}^{1} 00\epsilon_{i,t}.$$

A basic property of diversification is that, as the number of stocks increases to infinity, the portfolio's non-market variance $\sigma^2(\epsilon_p)$ will approach the limit

$$\sigma^2(\epsilon_p) \to \rho(\epsilon_i, \epsilon_j)\sigma^2(\epsilon_i),$$

where $\rho(\epsilon_i, \epsilon_j)$ is the correlation between $\epsilon_{i,t}$ and $\epsilon_{j,t}$. This limit provides a very close approximation by the time the number of stock reaches 100 (as advised by the hint). Thus, in this case,

$$\sigma^2(\epsilon_p) \to 0.25(0.10^2) = 0.0025,$$

giving the standard deviation of return equal to $\sigma(\epsilon_p) = 0.05$. The standard deviation of the dollar profit over the next month is then 0.05 × $1 mil. = $50,000.

9. a. It doesn't follow. The high R-squared reflects the fact that much non-market risk is being diversified away in the large portfolios being examined. This is not really a prediction of the CAPM. The CAPM is a statement about *expected* returns. The R-squared values say nothing about whether the average returns on these portfolios conform to the predictions of the CAPM.

   b. It doesn't follow. The CAPM would predict that, when one averages across both the up and down markets, the average returns should be increasing in betas. The fact that average returns display the stated ordering when the up and down markets are separated doesn't say much about the CAPM. It does say that a substantial portion of the volatility in stock returns is market-related, and thus the stocks with higher market sensitivities (betas) are likely to be those whose returns are most extreme in whatever direction the market moves.

   c. It does follow. Since the market-neutral fund's covariance with the market is zero, adding a small amount of it makes no incremental contribution to the variance of a portfolio whose risky part is just the S&P. At the same time, since its expected excess return is positive (it's Sharpe ratio is positive), adding a small amount of it does make a positive contribution to the portfolio's excess return. This is essentially why cash is used as a benchmark for market-neutral strategies: such a strategy shouldn't beat cash on average if the market (S&P) is mean-variance efficient (as in the CAPM).

10. The two sources of risk Mel has in mind are market risk and non-market risk. To see how these risks enter, first note that total profit in period $t$, $\Pi_t$ is

$$\Pi_t = \$100 r_{A,t} - \$100(r_{B,t} - r_f), \tag{15}$$

where $r_{A,t}$ is the total rate of return on portfolio $A$, $r_{B,t}$ is the total rate of return on portfolio $B$, and $r_f$ is the (riskless) interest rate. Substituting the characteristic-line representations of the returns on the equity portfolios gives

$$\Pi = \$100(r_f + \alpha_A + \beta_A R_{M,t} + \epsilon_{A,t}) - \$100(\alpha_B + \beta_B R_{M,t} + \epsilon_{B,t}), \tag{16}$$

where $R_{M,t}$ is the excess market return. The risky portion of $\Pi$ in which market risk enters is

$$\$100\beta_A R_{M,t} - \$100\beta_B R_{M,t},$$

from which it is clear that there is no market risk if $\beta_A = \beta_B$. Thus, for market risk, the net exposure value of 0 is appropriate. The risky portion of $\Pi$ in which non-market risk enters is

$$\$100\epsilon_{A,t} - \$100\epsilon_{B,t}.$$

We know that $\epsilon_{A,t}$ and $\epsilon_{B,t}$ have the same variances, since $r_{A,t}$ and $r_{B,t}$ have equal variances and $\beta_A = \beta_B$. If $\epsilon_{A,t}$ and $\epsilon_{B,t}$ are uncorrelated with each other as well, then the variance of the non-market risk is

$$\$100^2\text{Var}(\epsilon_{A,t}) + \$100^2\text{Var}(\epsilon_{B,t}) = 2(\$100^2)\text{Var}(\epsilon_{A,t}) = \$141 \cdot \text{std. dev.}(\epsilon_{A,t}).$$

The standard deviation of non-market risk in either the long or short position separately is $\$100 \cdot \text{std. dev.}(\epsilon_{A,t})$. We thus see that a gross concept is more relevant for non-market risk, except that diversification makes \$141, rather than \$200, the effective amount exposed to the non-market standard deviation in either of the separate \$100 positions.

We know that $\epsilon_{A,t}$ and $\epsilon_{B,t}$ must in fact be uncorrelated, since in general

$$\text{Cov}(r_{A,t}, r_{B,t}) = \beta_A \beta_B \text{Var}(r_{M,t}) + \text{Cov}(\epsilon_{A,t}, \epsilon_{B,t}),$$

which implies $\text{Cov}(\epsilon_{A,t}, \epsilon_{B,t}) = 0$ given that, in this case, $\text{Cov}(r_{A,t}, r_{B,t}) = \text{Var}(r_{M,t})$ and $\beta_A = \beta_B = 1$.

11. Since the $\beta$'s on the long and short legs of the position are the same, the market-related component of return drops out of the overall position. Therefore, the dollar profit on the spread is given by

$$\begin{aligned} P &= (\$1 \text{ million})\frac{1}{10}\sum_{i=1}^{10}(R_{i,t} + r_{f,t}) - (\$1 \text{ million})\frac{1}{10}\sum_{i=11}^{20} R_{i,t} \\ &= (\$1 \text{ million})\left(r_{f,t} + \frac{1}{10}\sum_{i=1}^{10}\alpha_i + \frac{1}{10}\sum_{i=1}^{10}\epsilon_{i,t} - \frac{1}{10}\sum_{i=11}^{20}\alpha_i - \frac{1}{10}\sum_{i=11}^{20}\epsilon_{i,t}\right). \end{aligned}$$

Note that $r_f$ and the $\alpha_i$'s are constants and thus don't contribute to variance. Using the assumption that the $\epsilon_{i,t}$'s are uncorrelated across stocks, the variance of the profit is

$$\begin{aligned} \sigma^2(P) &= (\$1 \text{ million})^2\left(\frac{1}{10^2}(10)(0.30)^2 + \frac{1}{10^2}(10)(0.30)^2\right) \\ &= (\$1 \text{ million})^2\frac{2}{10}(.30)^2, \end{aligned}$$

so the standard deviation of the profit is

$$\begin{aligned} \sigma(P) &= (\$1 \text{ million})\sqrt{\frac{2}{10}}(0.30) \\ &= 134,164. \end{aligned}$$

12.

a. The aggressive allocation should be 36 in stocks, 54 in bonds: the stock-bond ratio should be the same in any portfolio that contains cash.

b. Inconsistent: this advisor's recommendations violate the property used in part a.

c. Consistent: At levels of expected return and volatility above those of the tangent portfolio, efficient portfolios contain no cash (since borrowing is precluded). Those efficient portfolios simply trace out the (curved) opportunity set of stock-bond combinations. With stocks having higher expected return and volatility than bonds, the weight in stock will increase as one moves further up the efficient set.

d. Inconsistent: For the reason described above, the stock-bond ratio for efficient no-cash portfolios cannot be less than the stock-bond ratio in the tangent portfolio.

13.

a. Consistent. In general, for all portfolios that contain cash, the stock-bond ratio should be the same, since efficient portfolios then combine the same risky "tangent" portfolio with riskless cash. Of course, the fraction in cash drops as volatility increases.

b. Inconsistent. There is only one portfolio containing cash, so the condition in part a. is not violated. Note, however, that the stock-bond ratio falls as portfolio volatility increases, which is inconsistent with the assumptions about the expected returns and volatilities of the individual assets. Given those assumptions, as one moves to portfolios with higher expected return and volatilities, the stock bond ratio must first stay the same (when cash is present) and then increase.

c. Consistent. This is like part b, except that the stock-bond ratio now goes in the right direction as volatility increases.

d. Inconsistent. All three portfolios contain cash, but the stock-bond ratio changes, thereby violating the condition stated in part a.

14. a. To achieve a standard deviation half as large, this portfolio should have half the exposure to the risky assets as Client A's portfolio. The proportion of stock relative to bonds should stay at the same 2-to-1 ratio, though, in order to obtain a portfolio that is also mean-variance efficient. Thus, the new weights are

| | |
|---|---|
| Stocks | 30% |
| Bonds | 15% |
| Cash | 55% |
| Total | 100% |

The expected return in excess of cash is, 0.2%, half that of Client A's portfolio.

b. The weight an investor with risk aversion $A$ places in the risky portfolio $p$, when allocating between cash and $p$, is given by

$$y = \frac{E_p}{A\sigma_p^2}$$

where $E_p$ is the expected excess return on $p$ and $\sigma_p^2$ is the variance of $p$'s return. The easiest application of this relation is to treat Client A's overall portfolio as $p$, giving

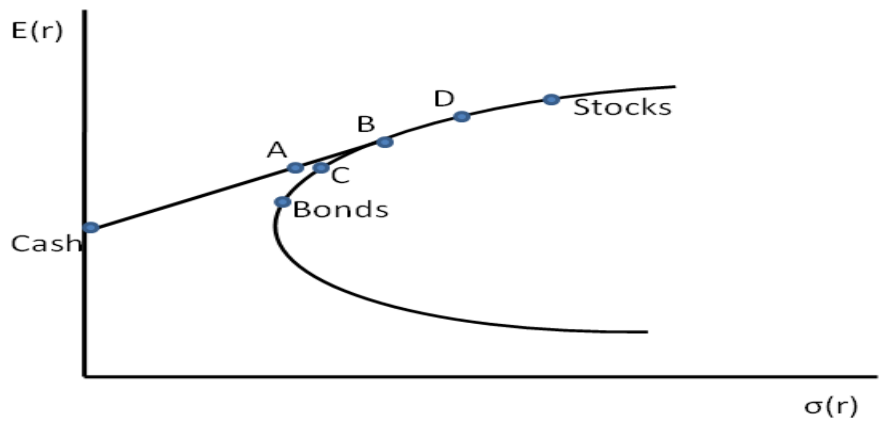
$$y = \frac{0.004}{10 \cdot 0.04^2} = 0.25$$

so Client C would allocate 0.25 to Client A's overall portfolio and 0.75 to cash. This then translates to the following weights:

| | |
|---|---|
| Stocks | 15% |
| Bonds | 7.5% |
| Cash | 77.5% |
| Total | 100% |

One could instead arrive at the same values by first using Client A's portfolio to compute the weights, expected excess return, and volatility of the tangent portfolio containing only stocks and bonds. Then, that tangent portfolio could be used as portfolio $p$ in above equation, and the resulting $y$ would then be the weight in the tangent portfolio.

15. Allocation C must be the one that is not mean-variance efficient. A and B have the same stock-bond ratio, so either both are inefficient or both are efficient. It must be the latter, since three of the four are efficient. Since B is efficient, C cannot be, because it lies to the left of B and would thus by dominated by a combination of B and cash. (See sketch below.)




16. (a) The value of $A$ implied by the 60% stock allocation ($y$) given the previous assumptions is

$$\begin{aligned} A &= \frac{E_s - r_f}{y\sigma_s^2} \\ &= \frac{0.08 - 0.02}{(0.6)0.20^2} \\ &= 2.5 \end{aligned}$$

Based on the new return assumptions, the desired new stock allocation ($y'$) is

$$\begin{aligned} y' &= \frac{E_s - r_f}{A\sigma_s^2} \\ &= \frac{0.10 - 0.01}{(2.5)0.20^2} \\ &= 0.90 \end{aligned}$$

With a futures position of size $X$ and total invested wealth $W$,

$$\begin{aligned} y' &= y + \frac{X}{W} \\ 0.90 &= 0.60 + \frac{X}{\$4{,}000{,}000}, \end{aligned}$$

giving a long futures position of $X = \$1{,}200{,}000$.

(b) The standard deviation of $r_p$, the rate of return on the overall portfolio, is

$$\begin{aligned} \sigma_p &= y'\sigma_s \\ &= 0.9(0.20) \\ &= 0.18 \end{aligned}$$

The profit/loss is $Wr_p$, which has standard deviation $W\sigma_p = \$4,000,000(0.18) = \$720,000$.

17. (a) The inefficient portfolio must be either A or B, since both contain cash but have different stock-bond ratios. Therefore portfolios C and D must both be efficient. If portfolio B were efficient, however, then C would have to be as well. C's lower stock-bond ratio, compared to B, would place C on the stock-bond curve to the left of the tangent portfolio in B and thus below that tangent line. Therefore B cannot contain the tangent portfolio and must be inefficient.

(b) Since C is the tangent portfolio (same stock-bond ratio as A), we need to allocate the fraction $w_C$ of the total amount invested to that portfolio and the fraction $1 - w_C$ to cash. The standard deviation of the overall portfolio is then given by

$$\begin{aligned} \sigma_p &= w_c\sigma_C \\ 0.05 &= w_c 0.146, \end{aligned}$$

which implies $w_C = 0.3425$. This gives an allocation to each of stocks and bonds equal to $0.3425(0.5) = 0.171$, while the allocation to cash is $1 - 0.3425 = 0.658$.

18. (a) The tangent portfolio, which is a 50-50 combination of the Market and Growth funds, has expected return and variance given by

$$\begin{aligned} E_T &= 0.5E_{MKT} + 0.5E_{GRO} \\ &= 0.5(0.10 + 0.14) \\ &= 0.12 \\ \sigma_T^2 &= 0.5^2\sigma_{MKT}^2 + 0.5^2\sigma_{GRO}^2 + 2(0.5)(0.5)\sigma_{MKT}\sigma_{GRO}\rho_{MKT,GRO} \\ &= 0.5^2\left[0.2^2 + 0.3^2 + 2(0.2)(0.3)(0.75)\right] \\ &= 0.25(0.22) \\ &= 0.055 \end{aligned}$$

Since Alice had previously allocated the fraction $y = 0.8$ of her overall portfolio to the tangent portfolio, her risk aversion coefficient is given by

$$\begin{aligned} A &= \frac{E_T - r_f}{y\sigma_T^2} \\ &= \frac{0.12 - 0.03}{(0.8)0.055} \\ &= 2.0455 \end{aligned}$$

When the Market Fund is the only risky investment available, her new allocation $y_{MKT}$ to that fund should be

$$\begin{aligned} y_{MKT} &= \frac{E_{MKT} - r_f}{A\sigma_{MKT}^2} \\ &= \frac{0.10 - 0.03}{(2.0455)0.20^2} \\ &= 0.856, \end{aligned}$$

with the remaining fraction of 0.144 in the Money Fund.

(b) Bob's risk aversion coefficient is given by

$$\begin{aligned} A &= \frac{E_{MKT} - r_f}{y\sigma_{MKT}^2} \\ &= \frac{0.10 - 0.03}{(0.7)0.20^2} \\ &= 2.484 \end{aligned}$$

His allocation to the tangent portfolio is given by

$$\begin{aligned} y_T &= \frac{E_T - r_f}{A\sigma_T^2} \\ &= \frac{0.12 - 0.03}{(2.484)0.055} \\ &= 0.6588, \end{aligned}$$

which implies a weight of $1 - 0.6588 = 0.341$ in the Money Fund and weights of $0.6588/2 = 0.329$ in both the Market Fund and the Growth Fund.

19.

a. If the normal portfolio $p$ has the highest Sharpe ratio, then the expected returns on the assets in that portfolio obey the relation

$$\begin{aligned} E_i - R_f &= \frac{\sigma_{ip}}{\sigma_p^2}(E_p - R_f) \\ &= \sigma_i \left(\frac{\sigma_{ip}}{\sigma_i \sigma_p}\right)\left(\frac{(E_p - R_f)}{\sigma_p}\right) \\ &= \sigma_i \rho_{ip} S_p. \end{aligned} \tag{17}$$

Evaluating (17) for stocks and bonds gives

$$E_S - R_f = 0.20 \cdot 0.80 \cdot 0.40 = 0.064$$

and

$$E_S - R_f = 0.10 \cdot 0.50 \cdot 0.40 = 0.020.$$

b. The allocation $y$ to the risky portfolio $p$ must satisfy

$$\begin{aligned} y &= \frac{E_p - R_f}{A\sigma_p^2} && (18) \\ &= \left(\frac{E_p - R_f}{\sigma_p}\right)\left(\frac{1}{A \cdot \sigma_p}\right) = S_p \cdot \frac{1}{A \cdot \sigma_p} && (19) \end{aligned}$$

so

$$0.8 = 0.4\frac{1}{A \cdot 0.15}$$

and $A = 3.333$. Applying (18) to the stocks-only case then gives the fraction in stocks as

$$y_S = \frac{E_S - R_f}{A \cdot \sigma_S^2} = \frac{0.064}{3.333 \cdot 0.20^2} = 0.48.$$

20. a. To achieve half the volatility, this portfolio must invest only half as much in the tangent portfolio of risky assets as does portfolio $P$. Therefore, this portfolio's weights are 0.175, 0.175, and 0.65.

b. The standard deviation of the tangent portfolio $T$, a 50-50 combination of stocks and bonds, is

$$\begin{aligned} \sigma_T &= [w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_Sw_B\sigma_S\sigma_B\rho_{SB}]^{1/2} \\ &= [(.5^2)(.2^2) + (.5^2)(.1^2) + 2(.5)(.5)(.2)(.1)(.3)]^{1/2} \\ &= .5[.2^2 + .1^2 + 2(.2)(.1)(.3)]^{1/2} = .1245 && (20) \end{aligned}$$

Since portfolio $P$ invests 70% in the tangent portfolio, its standard deviation is $(.7)(.1245) = .087$.

c. To get the expected return on stocks, $E_S$, we can use the condition that, because $T$ is the tangent portfolio,

$$E_S = R_F + \frac{\text{cov}\{R_S, R_T\}}{\text{var}\{R_T\}}[E_T - R_F] \quad (21)$$

(Same form as the security market line, where the market portfolio $M$ is identified as the tangent portfolio $T$.) Since,

$$E_P = (0.3)(0.03) + (0.7)E_T = 0.058, \quad (22)$$

we get $E_T = 0.07$. The covariance between $S$ and $T$ is

$$\begin{aligned} \text{cov}\{R_S, R_T\} &= \text{cov}\{R_S, .5R_S + .5R_B\} \\ &= .5\text{var}\{R_S\} + .5\text{cov}\{R_S, R_B\} \\ &= .5(.2^2) + .5(.2)(.1)(.3) = 0.023. && (23) \end{aligned}$$

Therefore, (21) gives

$$E_S = .03 + \frac{0.023}{0.1245^2}[0.07 - 0.03] = .089. \tag{24}$$

The expected return on bonds, $E_B$ could have been obtained in the same manner. Once either $E_S$ or $E_B$ is computed, the other is easily obtained from the fact that

$$E_T = 0.5E_S + .5E_B. \tag{25}$$

Thus,

$$0.07 = .5(.089) + .5E_B, \tag{26}$$

or $E_B = 0.051$.

21. If portfolio $P$ is efficient, then each asset's alpha with respect to that portfolio must be zero. That is, the expected excess return on each asset is its beta with respect to $P$ times the expected excess return on $P$. This is the calculation performed, for example, to obtain the neutral expected returns in the Black-Litterman model. With large-cap stocks ($LC$),

$$\begin{aligned} \mathrm{E}(r_{LC}) &= r_f + \frac{\mathrm{Cov}(r_{LC}, r_P)}{\sigma_P^2}\left[\mathrm{E}(r_P) - r_f\right] \\ &= r_f + \frac{\sigma_{LC}}{\sigma_P}\mathrm{Corr}(r_{LC}, r_P)\left[\mathrm{E}(r_P) - r_f\right] \\ &= 0.01 + \frac{.25}{.15}(0.8)[.08 - .01] \\ &= 0.1033. \end{aligned} \tag{27}$$

Similarly, for small-cap stocks ($SC$),

$$\begin{aligned} \mathrm{E}(r_{SC}) &= r_f + \frac{\mathrm{Cov}(r_{SC}, r_P)}{\sigma_P^2}\left[\mathrm{E}(r_P) - r_f\right] \\ &= r_f + \frac{\sigma_{SC}}{\sigma_P}\mathrm{Corr}(r_{SC}, r_P)\left[\mathrm{E}(r_P) - r_f\right] \\ &= 0.01 + \frac{.40}{.15}(0.6)[.08 - .01] \\ &= 0.1220. \end{aligned} \tag{28}$$

22. (a) To achieve twice the standard deviation of the policy portfolio, in this case one can simply invest twice as much in the tangent portfolio that combines large-cap and small-cap in a 3-1 ratio. This still leaves 20% in cash (so an alternative ratio of large-cap to small-cap need not be considered). The allocations to large-cap and small-cap would simply double to 0.60 and 0.20, respectively.

(b) Mean variance efficiency requires that an asset's expected return be linearly related to the covariance of its return with that of the tangent portfolio. Specifically, for each asset $i$ in the tangent portfolio,

$$E_i = r_f + \frac{\mathrm{Cov}(r_i, r_T)}{\sigma_T^2}(E_T - r_f)$$

The expected return on the tangent portfolio must be such that a 40-60 combination of it with cash gives the expected return on the policy portfolio, so

$$0.4E_T + 0.6(0.02) = 0.08,$$

or $E_T = 0.17$. Since 40% is invested in the tangent portfolio, its standard deviation must be such that

$$0.0963 = 0.4\sigma_T,$$

or $\sigma_T = 0.2408$. The covariance between the return on large-cap (LC) stocks and the return on the tangent portfolio (T) can be computed as

$$\begin{aligned}
\mathrm{Cov}(r_{LC}, r_T) &= \mathrm{Cov}(r_{LC}, 0.75r_{LC} + 0.25r_{SC}) \\
&= 0.75\mathrm{Cov}(r_{LC}, r_{LC}) + 0.25\mathrm{Cov}(r_{LC}, r_{SC}) \\
&= 0.75\mathrm{Var}(r_{LC}) + 0.25\mathrm{Cov}(r_{LC}, r_{SC}) \\
&= 0.75(0.2^2) + 0.25(0.068) \\
&= 0.047
\end{aligned} \tag{29}$$

We can now compute

$$\begin{aligned}
E_{LC} &= r_f + \frac{\mathrm{Cov}(r_{LC}, r_T)}{\mathrm{Var}(r_T)}(E_T - r_f) \\
&= 0.02 + \frac{.047}{0.2408^2}(0.17 - 0.02) \\
&= 0.1416
\end{aligned} \tag{30}$$

(c) Given that the expected return on the policy portfolio must satisfy

$$0.08 = 0.3E_{LC} + 0.1E_{SC} + 0.6(0.02),$$

and given the previous solution $E_{LC} = 0.1416$, we obtain

$$E_{SC} = \frac{1}{0.1}[0.08 - 0.3(0.1416) - 0.6(0.02)],$$

or $E_{SC} = 0.2552$.

(d) Adjusting only the small-cap expected return, while leaving the large-cap assumption unchanged, is probably not a good idea. Even though he has analyzed only small-cap stocks, the committee member's view about that asset class really includes an implicit view about the other asset class, since small-cap stocks and large-cap stocks are rather highly correlated. (In this case, their correlation is $0.068/(0.2 \cdot 0.4) = 0.85$.) Probably more sensible would be to adjust the large-cap expected return upward to some degree as well, using the Black-Litterman approach. In the absence of such adjustments, the resulting output of an optimization is likely to be less useful, as the resulting weights are likely to imply an extreme shift in allocations.

23. a. Probably Mr. Smith. Ms. Jones might be interested in the Sharpe ratio of her overall portfolio, but the Sharpe ratio on just a fraction of that portfolio doesn't indicate the extent to which Greatbuys can contribute to that.

b. Ms. Jones would probably be more interested in the fund's alpha—the intercept in a regression of its excess return on the market's excess return. If the alpha is positive, then Ms. Jones can improve her overall Sharpe ratio by shifting *some* of her portfolio into Greatbuys. Even more useful would be the ratio of alpha to the standard deviation of the non-market (residual) return—the information ratio (IR).

24. a. The average return on the benchmark, $\bar{R}_P$, is obtained by combining the average returns on the passive indexes (columns 6 and 7) using the 60-40 benchmark weights:

$$\bar{R}_P = 0.6(0.08) + 0.4(0.06) = 0.072. \tag{31}$$

This gives overall performance equal to $0.0969 - 0.072 = 0.0249$.

b. Asset allocation is the excess of $\bar{R}_A$ over $\bar{R}_P$, where $\bar{R}_A$ combines the index returns (columns 6 and 7) using the actual allocations (columns 1 and 2). In other words, $\bar{R}_A$ is simply the sum of the averages of columns 8 and 9, so asset allocation is given by

$$\bar{R}_A - \bar{R}_P = (0.0564 + 0.0246) - 0.072 = 0.009. \tag{32}$$

c. Security selection is the excess of $\bar{R}_S$ over $\bar{R}_P$, $\bar{R}_S$ combines the endowment's returns (columns 3 and 4) and the 60-40 benchmark allocations, so security selection is given by

$$\bar{R}_S - \bar{R}_P = [.6(.11) + .4(.05)] - 0.072 = 0.086 - 0.072 = 0.014. \tag{33}$$

d. The difference between the overall performance (0.0249) and the sum of the asset-allocation and security-selection components (0.023) is 0.0019. This "interaction" term reflects some positive ability to allocate more to an asset class in years when the security-selection within that class is relatively better.

25. a. Timing is the ability of a manager to raise the portfolio's beta in anticipation of a positive excess market return. One can test for its presence by regressing the manager's return simultaneously on the market and a convex function of the market, such as $(R_{M,t} - R_{F,t})D_t$ (or $(R_{M,t} - R_{F,t})^2$). A positive coefficient on the latter independent variable indicates timing ability, and Portfolio A indicates such ability in the regressions reported.

b. A positive alpha is interpreted as an ability of the manager to pick underpriced securities.

c. It's more appropriate for Portfolio B, since that portfolio exhibits no significant timing ability. Once timing ability is identified, the simple Jensen alpha is no longer interpretable as in part b. (The presence of timing can create a significant non-zero estimated alpha even in the absence of stock-picking ability.)

26. (a) The average return on the benchmark, $R_p$, is obtained by combining the average returns on the passive indexes (column 6 and 7) using the 60-40 benchmark weights:

$$\bar{R}_p = 0.6(0.105) + 0.4(0.045) = 0.081$$

This gives overall performance in excess of benchmark to be 0.112-0.081=0.031.

(b) Asset allocation is the Excess of $\bar{R}_A$ over $\bar{R}_p$, where $\bar{R}_A$ combines the index returns (column 6 and 7) using the actual allocations (columns 1 and 2). In other words, $\bar{R}_A$ is simply the sum of the averages of columns 8 and 9, so asset allocation is given by

$$\bar{R}_A - \bar{R}_P = (0.061 + 0.030) - 0.081 = 0.01$$

(c) Security selection is the excess of $\bar{R}_S$ over $\bar{R}_P$, $\bar{R}_S$ combines the endowment's returns (column 1 and 2) and the 60 - 40 benchmark allocations, so security selection is given by

$$\bar{R}_S - \bar{R}_P = [0.6(0.121) + 0.4(0.051)] - 0.081 = 0.012$$

(d) Interaction term is the difference between overall performance (0.031) and the sum of asset allocation (0.01) and security selection (0.012), which is 0.009.

27. (a) Overall performance in excess of the S&P is col. 8 minus col. 5: $0.11 - 0.09 = 0.02$

(b) Asset allocation is col. 7 minus col. 5: $0.12 - 0.09 = 0.03$

(c) Security selection is col. 6 minus col. 5: $0.07 - 0.09 = -0.02$

(d) Interaction is the answer in (a) minus the sum of the answers in (b) and (c): $0.02 - (0.03 - 0.02) = 0.01$

28. (a) Overall performance in excess of the S&P is col. 5 minus col. 7: $0.13 - 0.09 = 0.04$

(b) Asset allocation is col. 6 minus col. 7: $0.11 - 0.09 = 0.02$

(c) Security selection is col. 8 minus col. 7: $0.08 - 0.09 = -0.01$

(d) Interaction is the answer in (a) minus the sum of the answers in (b) and (c): $0.04 - (0.02 - 0.01) = 0.03$

29. (a) No, if the S&P portfolio is mean-variance efficient, then $\alpha_p$ must equal zero. A positive $\alpha_p$ implies that there exists a combination of portfolio $p$ and the S&P portfolio that has a higher Sharpe ratio than just the latter.

(b) Yes. The positive value of $\alpha_p$ can reflect compensation for additional sources of risk that are (i) different from overall stock-market risk and (ii) cannot be diversified away. Given the latter, someone must be compensated to bear those risks, and it is reasonable that those who do would expect to earn a higher return as compensation. Risk-based explanations have been offered for the historically observed "value premium" earned by high book-to-market stocks. One example discussed in the course was that of Fama and French, who suggest that these are stocks of firms that are typically more "distressed."

(c) The basic idea is that one potential reason for a relatively high ratio of book value to market value is that the market value is too low, due to underpricing of the firm's shares. Controlling by book value simply helps abstract from cross-sectional differences in market value that reflect differences in the real scale/size of firms, instead of mispricing. Various behavioral theories have been advanced as to why one might expect value stocks to be underpriced. One is recency, where investors project recent poor performance—typical of value stocks—too far into the future. Another is overconfidence, causing investors to devote too little attention to firms for which relevant information is often less prominent than it is for growth/glamour stocks.

30. a. The CAPM, which says that an investment is expected to earn the same as the cash rate if its return is uncorrelated with the return on the market. That is, as long as the strategy is market neutral, its risk does not command an expected-return premium, essentially because its risk does not contribute to the variance of the typical investor's portfolio (the market).

    b. Even though the fund is market neutral, it may not be neutral with respect to other risks that are priced—if the CAPM doesn't hold. One such possible risk is an exposure to a "value-versus-growth" factor. Empirical evidence suggests that, over fairly long historical periods, a diversified portfolio of value stocks outperforms, on average, a diversified portfolio of growth stocks. One would therefore expect a similar premium, not attributable to the manager's skill, in a market-neutral strategy that is long value.

31. a. The main argument in favor is largely based on empirical evidence. The well known studies of Fama and French, for example, document higher average returns for small firms as opposed to large firms and, especially, for high book-to-market firms (value) as opposed to low book-to-market firms (growth). Standard market betas in a CAPM framework do not account for these differences, so the empirical evidence indicates that a portfolio concentrated in small-cap value stocks earns a positive alpha. Thus, reallocating some of the foundation's stock portfolio away from the market and toward small-cap value would provide a higher Sharpe ratio.

    b. One potential drawback is that the higher average returns on small-cap value firms could simply reflect compensation for risk common to such stocks. For example, these stocks could have substantially greater exposure to a worsening of economic conditions that increase financial distress. Such a dimension of risk is not necessarily captured by overall return volatility, so taking on more such risk, even if the Sharpe ratio is higher, need not appeal to all investors. Another potential drawback could simply be that, in the absence of a compelling and well supported economic explanation for why the average return differences have occurred historically, one might worry that they will not continue in the future.

    c. For an investor interested in the lowest volatility for a given target expected return, obtaining a higher Sharpe ratio should be the right objective. The higher volatility of the small-cap value firms need not require a higher volatility of the foundation's overall portfolio: the allocation to cash could be increased and the overall exposure to equities could be decreased. Choosing portfolios along a capital allocation line with a higher slope (Sharpe ratio) would mean that the foundation could actually achieve a lower standard deviation for any target level of expected return.

32. (a) Essentially an assumption that the market attempts to set prices according to the standard CAPM. In other words, an assumption that investors care only about beta as a measure of risk. In that case, the positive alpha indicates expected returns that are too high, and thus prices that are too low. The CAPM implies that prices should be set to make the market portfolio mean-variance efficient, whereas a positive alpha indicates that the market portfolio is not mean-variance efficient.

    (b) Value stocks as a group could possess an additional dimension of risk, in addition to overall market exposure, that cannot be diversified away. One example could be greater risk of financial distress. If so, investors would require compensation for bearing such risk, thereby pricing value stocks lower and producing a positive alpha in the characteristic line containing just the market excess return. But the prices would not be too low—they would be appropriate given that additional risk.

33. When the fund holds positions with relatively low liquidity, the returns reported by the fund on these positions for a given month can be somewhat arbitrary and exhibit less correlation with the market than exists in the true underlying returns. One approach to detecting this hidden market-related risk is to regress the fund's return not only on the contemporaneous market return, as in standard estimation of beta, but to include additional independent variables consisting of market returns in lagged months. Positive slope coefficients on the lagged terms reveals additional market-related risk. A common approach to estimating the fund's beta is simply to add up the slope coefficients on the contemporaneous and lagged market returns. Another approach to detecting the true market-related risk would be to lengthen the return period used to estimate beta, e.g., annual as opposed to monthly, but this approach typically confronts the problem of short sample histories.

34. a. The alpha of each fund is its expected return minus beta times the expected market return:

$$\begin{aligned} \alpha_A &= 0.13 - (1.0)(0.10) = 0.03 \\ \alpha_B &= 0.07 - (0.5)(0.10) = 0.02 \end{aligned} \tag{34}$$

The variance of $\epsilon_P$ for each fund is its total variance minus beta squared times the market variance:

$$\begin{aligned} \sigma^2(\epsilon_A) &= (0.40)^2 - (1)^2(0.20)^2 = 0.12 \\ \sigma(\epsilon_A) &= 0.3461 \\ \sigma^2(\epsilon_B) &= (0.20)^2 - (0.5)^2(0.20)^2 = 0.03 \\ \sigma(\epsilon_B) &= 0.1732 \end{aligned}$$

The information ratios for each fund:

$$IR_A = \alpha_A/\sigma(\epsilon_A) = 0.03/0.3461 = 0.0867$$
$$IR_B = \alpha_B/\sigma(\epsilon_B) = 0.02/0.1732 = 0.1155$$

Thus, fund B, with the higher information ratio, is the better choice.

b. The weight in the hedge fund is

$$w^* = \frac{w_0}{1 + (1 - \beta_B) w_0}, \quad w_0 = \left(\frac{IR_B}{S(M)}\right)\left(\frac{\sigma(R_M)}{\sigma(\epsilon_B)}\right),$$

which when evaluated give

$$w_0 = \left(\frac{0.1155}{0.5}\right)\left(\frac{0.20}{0.1732}\right) = 0.2667,$$

and

$$w^* = \frac{0.2667}{1 + (1 - 0.5)0.2667} = 0.2353,$$

so the amount invested in the hedge fund, per dollar in the S&P, is $0.2353/(1 - 0.2353) = 0.307$.

c. The new squared Sharpe ratio, $S(Q)^2$, is the original one, $S(M)^2$, plus the hedge fund's squared information ratio:

$$\begin{aligned} S(Q)^2 &= S(M)^2 + IR_B^2 \\ &= (0.5)^2 + (0.1155)^2 \\ &= 0.2633 \\ S(Q) &= 0.513 \end{aligned} \tag{35}$$

35. (a) The information ratios for each fund:

$$\begin{aligned} IR_A &= \alpha_A/\sigma(\epsilon_A) = 0.02/0.11 = 0.182 \\ IR_B &= \alpha_B/\sigma(\epsilon_B) = 0.01/0.05 = 0.2 \end{aligned}$$

Thus, fund B, with the higher information ratio, is the better choice.

(b) The weight in the hedge fund is

$$w_0 = \left(\frac{IR_B}{S(M)}\right)\left(\frac{\sigma(R_M)}{\sigma(\epsilon_B)}\right), \; w^* = \frac{w_0}{1 + (1 - \beta_B) w_0}$$

which when evaluated given

$$w_0 = \left(\frac{0.2}{0.02/0.05}\right)\left(\frac{0.05}{0.05}\right) = 0.5$$

and

$$w^* = \frac{0.5}{1 + (1 - 0)0.5} = \frac{1}{3},$$

so the amount invested in the hedge fund , per dollar in the S&P, is (1/3)/(1-1/3)=$0.5.

(c) The new squared Sharpe ratio, $S(Q)^2$, is the original one, $S(M)^2$, plus the hedge fund's squared information ratio:

$$\begin{aligned} S(Q)^2 &= S(M)^2 + IR_B^2 \\ &= (0.4)^2 + 0.2^2 \\ &= 0.2 \\ S(Q) &= 0.4472 \end{aligned}$$

(d) If fund B consists of illiquid assets, the regression coefficients $(\alpha_B, \beta_B)$ may be misspecified and most likely underestimated, which makes sense as $\beta_B$ is 0. One way to get better estimate of $\beta_B^*$ is to run a regression on lagged market returns or use longer periods of returns for the estimation. If true $\beta_B^*$ is not 0, then the true $\alpha_B^*$ would be

$$\alpha_B^* = \alpha_B - \beta_B^* \bar{R}_{M,t} = \alpha_B - \beta_B^* 0.02$$

which could be larger than $\alpha_B$ if $\beta_B^*$ is negative, and smaller than $\alpha_B$ if $\beta_B^*$ is positive. Since we are still maintaining the objective of achieving the highest overall Sharpe ratio, we look at the Information Ratios. If $\beta_B^*$ is positive and is sufficiently large, $IR_B^*$ would be smaller than $IR_A$ since $\alpha_B^* < \alpha_B$, so you would want to switch to fund A.

36. (a) Since both funds have the same $\alpha_P$ (0.02) as well as the same standard deviation of non-market return (0.14), a 50-50 combination—portfolio C—has the highest information ratio of any combination of funds A and B. Any combination of the two has the same alpha, and a 50-50 combination has the lowest standard deviation of non-market return. The latter value is

$$\sigma(\epsilon_C) = \sqrt{2(.5)^2(.14)^2} = 0.099,$$

so its information ratio is

$$IR_C = \alpha_C/\sigma(\epsilon_C) = 0.02/0.099 = 0.202.$$

The weight in portfolio C is computed as

$$w^* = \frac{w_0}{1 + (1 - \beta_C)w_0}, \quad w_0 = \left(\frac{IR_C}{S(M)}\right)\left(\frac{\sigma(R_M)}{\sigma(\epsilon_C)}\right),$$

which when evaluated gives (noting $\beta_C = 0.2$ with a 50-50 combination)

$$w_0 = \left(\frac{0.202}{0.02/0.05}\right)\left(\frac{0.05}{0.099}\right) = 0.2551$$

and

$$w^* = \frac{0.2551}{1 + (1 - 0.2)0.2551} = 0.2119.$$

Therefore, the amount invested in C, per dollar in the S&P, is (0.2119)/(1-0.2119) = \$0.269, implying that \$0.134 is invested in each of funds A and B.

(b) The new squared Sharpe ratio, $S(Q)^2$, is the original one, $S(M)^2$, plus portfolio C's squared information ratio:

$$\begin{aligned} S(Q)^2 &= S(M)^2 + IR_C^2 \\ &= (0.4)^2 + 0.202^2 \\ &= 0.2008 \\ S(Q) &= 0.448. \end{aligned}$$

37. A likely interpretation is that the fund holds somewhat illiquid assets whose true current values at the end of month $t$ are not reflected in the fund's return reported for month $t$. In other words, the recorded prices of those assets, and thus the fund's reported returns, tend to lag their true underlying values. The fund's true return contains market sensitivity, but the lag in reported values causes market movements in month $t$ to show up in the fund's reported return in month $t+1$. The latter positive correlation probability indicates that the fund isn't really market neutral. In other words, over a longer period in which the market incurs substantial losses, one would expect the fund to incur losses as well.

38. (a) The alphas on both strategies:

$$\begin{aligned} \alpha_A &= \mathrm{E}(r_{A,L} - r_{A,S}) - \beta_A \mathrm{E}(r_M - r_f) = 0.025 - 0.3(0.05) = 0.01 \\ \alpha_B &= \mathrm{E}(r_{B,L} - r_{B,S}) - \beta_B \mathrm{E}(r_M - r_f) = 0.020 - 0.4(0.05) = 0.00 \end{aligned}$$

You would allocate a non-zero (positive) amount to asset A, because it has a positive alpha. You would allocate nothing to strategy B. Its zero alpha implies that it won't help improve the Sharpe ratio.

(b) The standard deviation of strategy A's non-market return is

$$\sigma(\epsilon_A) = \sqrt{\sigma_A^2 - \beta_A^2 \sigma_M^2} = \sqrt{0.10^2 - 0.3^2 0.2^2} = 0.08,$$

so its information ratio is

$$IR_A = \alpha_A / \sigma(\epsilon_A) = 0.01/0.08 = 0.125.$$

The weight in portfolio A is computed as

$$w^* = \frac{w_0}{1 + (1 - \beta_A) w_0}, \quad w_0 = \left(\frac{IR_A}{S(M)}\right)\left(\frac{\sigma(R_M)}{\sigma(\epsilon_A)}\right),$$

which gives

$$w_0 = \left(\frac{0.125}{0.05/0.20}\right)\left(\frac{0.20}{0.08}\right) = 1.25$$

and

$$w^* = \frac{1.25}{1 + (1 - 0.3)1.25} = 0.667.$$

Therefore, the amount invested in A, per dollar in the S&P, is (0.667)/(1-0.667) = \$2.00.

(c) The new squared Sharpe ratio, $S(Q)^2$, is the original one, $S(M)^2$, plus portfolio A's squared information ratio:

$$\begin{aligned} S(Q)^2 &= S(M)^2 + IR_A^2 \\ &= (0.25)^2 + 0.125^2 \\ &= 0.078125 \\ S(Q) &= 0.280. \end{aligned}$$

39. (a) The term "alpha" must always be defined with respect to a given set of one or more "factors." That is, a fund's alpha is the intercept in a regression of the form

$$r_{A,t} - r_{f,t} = \alpha_A + \sum_{j=1}^{K} \beta_{j,A} F_{j,t} + \epsilon_{A,t}, \tag{36}$$

where $r_{A,t} - r_{f,t}$ is the fund's excess return and $F_{j,t}$ is the realization of factor $j$ in period $t$. The set of $K$ factors must always be specified. A traditional choice, still common, is to use just a single market factor, so that $K = 1$ and $F_{1,t} = r_{M,t} - r_{f,t}$. Additional factors, such as return spreads on size and value strategies (e.g., the Fama-French model) are increasingly popular. In general, the intent of including additional factors is to capture returns that can be earned through passive or mechanical "no-skill" strategies, so that what remains as $\alpha_A$ reflects skill. If part of $r_{A,t}$ reflects compensation for the fund's exposure to a $(K+1)$-st factor that is omitted from the set of $K$ used in (36), then the $\alpha_A$ in (36) could simply reflect that "beta" ($\beta_{K+1,A}$) rather than skill.

To confirm your suspicion that a fund's alpha is really beta waiting to be discovered, you would want to try to include one or more additional factors in (36) that represent mechanical "no-skill" versions of the fund's investment style. For a merger-arb fund, for example, you could include a factor that represents a mechanical long-short return on all prospective deals, without any skill or discretion applied to sort out the good from the bad. If including such a factor drops the value of $\alpha_A$ significantly, your suspicion would be confirmed.

Alternative answer:

A positive alpha can reflect skill in timing the market rather than skill in identifying mispriced securities. Market timing creates a convex relation between the excess market return (x axis) and the excess return on the managed portfolio. That relation can pass through the origin, but a single line fitted through the entire scatter of points can have a positive intercept. To confirm a suspicion that this is the case, include an additional independent variable to pick up the convexity, such as a variable that equals the excess market return when that value is positive but zero otherwise. A significantly positive coefficient on that variable would confirm the presence of timing.

(b) If a fund's assets are illiquid, the valuation numbers used to compute the fund's reported return in a given period can be stale. In that case, it becomes more likely that some portion of the realized market return (or realizations of additional factors) in a given period $t$ do not become fully reflected in the fund's reported return until period

$t + 1$ or later. In that case, a regression of the fund's period-$t$ return on just period-$t$ factor returns tends to miss some of the fund's true exposure to the factors. As a result, a portion of the fund's expected return that should properly be attributed to factor exposure instead shows up as alpha. That is, the risk adjustment becomes insufficient. The most common approach to detecting (and correcting for) this effect is to add lagged factor returns (from periods $t - 1$ and earlier) to the set of independent variables in the regression in (36).

40. Overconfidence can be used in several contexts as a behavioral justification for different trading strategies. In the case of an earnings-based momentum strategy, which exploits the fact that stocks which have experienced positive earning surprises tend to outperform those which have received negative earnings surprises, we can justify the strategy by saying that analyst's overconfidence makes them slow to revise their views as quickly as they should, and that this view is passed on to investors. Likewise a return based momentum strategy can also be explained by investor's overconfidence making them slow to revise their views of past winners and losers, leading to a difference in their performance.

    Overconfidence can also be used as a justification for a value strategy if you believe that investor's overconfidence in their own stock picking ability leads them to overvalue companies with easily observable strong fundamentals, while ignoring, and therefore undervaluing, relatively cheap stocks.

41. (a) The evidence most often cited is the "disposition" effect, which is the observed tendency for investors to sell stocks on which they've had gains but not to sell stocks on which they've had losses. The idea is that, by not selling their losing stocks, investors avoid the regret they would feel if those losses were actually realized. Conversely, selling their winners allows investors to experience greater pride than if the gains were not actually realized.

    (b) One example would be a momentum strategy, which buys stocks that have experience recent gains and shorts stocks that have had recent losses (over, say, the past six to twelve months). The idea is that investors' sales of recent winners temporarily keep the prices of such stocks from going up as much as they should, thereby making them attractive buys. At the same time, investors' reluctance to sell losers temporarily keeps the prices of those stocks from falling as much as they should, thereby making those stocks relatively attractive shorts.

42. A likely alternative explanation is the momentum effect in stock returns. Winning stocks outperform losing stocks, on average, even after they are ranked as such based solely on prior performance. In a large cross-section of funds, some funds are likely to find themselves holding a portfolio with a high number of last year's winning stocks, even in the absence of any managerial skill. With momentum in stock returns, those same funds are therefore more likely to continue to experience above-average performance. Note that this explanation does not require that any mutual funds actually employ momentum strategies.

43. (a) A market-impact cost is incurred when the price moves in response to an attempt to trade the stock. When the fund tries to buy a significant quantity of a stock, for example,

the fund might have to pay a price higher than what is currently prevailing in order to induce those who own the stock to sell it now. One way to measure market-impact cost is to compare the offer price at the time the fund initiates the attempt to buy the stock to the price at which the purchase ultimately occurs. For sales, the comparison is between the initial bid price and the ultimate execution price. (This is the method described in the Numeric Investors case.) Other common methods include comparing the execution price to the "VWAP" (volume-weighted average price) for all trades in the stock on that same day.

(b) The substantial market-impact cost for the momentum-driven strategy reflects the lower liquidity of small-cap stocks as well as the fact that a strategy based on analysis revisions and earnings surprises is using information that is relatively time-sensitive. The latter points to a trading approach in which relatively prompt execution is sought, thereby requiring greater price concessions—higher market impact—to induce others to take the other side of the desired trades. A stock's classification as "value" typically doesn't change in the short run. Thus, a more patient trading strategy can be pursued, often even supplying liquidity to other parties who initiate the trades, thereby earning the price concession rather than paying it. The negative market-impact cost for the large-cap value strategy is likely a reflection of that patience.

(c) One way to measure the cost of unexecuted orders is to compare the price at the time the fund initiates the attempt to trade to the price at the time the order is cancelled. If the latter price exceeds the former, then the difference is a positive cost if a purchase was intended (the opposite if a sale was intended). Although this cost does not represents a payment incurred by the fund, it does represent the return that was foregone by failing to execute at the initial prevailing price. When a fund investigates a strategy's average profitability using a "backtest," where typically the returns are computed assuming immediate execution at prevailing prices, then failing to include this "opportunity" cost as part of overall trading costs will tend to overstate actual profitability.

44. (a) Yes. As long as the alpha is positive on one or both funds, a higher Sharpe ratio can be obtained. The existence of a positive alpha is seen most readily for strategy $B$, given that its beta is zero and its expected excess return, and this its alpha, is 0.3%.

(b) First note that strategy A has the same alpha and non-market volatility as strategy B:

$$\begin{aligned} \alpha_A &= 0.008 - 0.5(0.01) = 0.003, \\ \sigma(\epsilon_A) &= \left[(.05)^2 - (.5)^2(.06)^2\right]^{1/2} = 0.04\,. \end{aligned}$$

Any combination C of strategies A and B will therefore have the same alpha, denoted as $\alpha_C = 0.003$. Denote the correlation between non-market returns on strategies A and B as $\rho = -.20$, define $\sigma = \sigma(\epsilon_A) = \sigma(\epsilon_B)$, and let $w$ denote the weight placed in strategy A when combining both strategies A and B into strategy C. The non-market volatility of strategy C is given by

$$\begin{aligned} \sigma(\epsilon_C) &= \left[w^2\sigma^2 + (1-w)^2\sigma^2 + 2w(1-w)\rho\sigma^2\right]^{1/2} \\ &= \sigma\left[1 - 2w(1-w)(1-\rho)\right]^{1/2}, \end{aligned}$$

which is minimized when $w(1-w)$ is as large as possible, i.e. when $w = 0.5$. (Placing equal amounts in equal-variance assets minimizes the resulting variance.) Substituting then gives

$$\sigma(\epsilon_C) = 0.04[1 - 2(.5)^2(1.2)]^{1/2} = 0.0253.$$

The beta of strategy C is given by $\beta_C = 0.5(0.5) + 0.5(0) = 0.25$, and the information ratio of strategy C is given by

$$IR_C = \frac{\alpha_C}{\sigma_C} = \frac{0.003}{0.0253} = 0.1186.$$

The overall portfolio with the highest Sharpe ratio will place weight $w_C$ in strategy C and weight $(1 - w_C)$ in the S&P. Let $S_M$ and $\sigma_M$ denote the S&P's Sharpe ratio and volatility. We then have

$$\begin{aligned}
w_0 &= \left(\frac{IR_C}{S_M}\right)\left(\frac{\sigma_M}{\sigma(\epsilon_C)}\right) \\
&= \left(\frac{0.1186}{1/6}\right)\left(\frac{.06}{.0253}\right) \\
&= 1.6875 \\
w_C &= \frac{w_0}{1 + (1 - \beta_C)w_0} \\
&= \frac{1.6875}{1 + (1 - 0.25)1.6875} \\
&= 0.7448,
\end{aligned}$$

implying an overall weight in each of strategies A and B equal to 0.7448/2 = 0.3724. Therefore, per dollar invested in the S&P, the amount invested in strategy A, as well as in strategy B, is equal to $0.3724/(1 - 2 \cdot 0.3724) = 1.4595$ dollars.

(c) The Sharpe ratio of the overall portfolio is given by

$$\begin{aligned}
S^* &= [S_M^2 + IR_C^2]^{1/2} \\
&= [(1/6)^2 + 0.1186^2]^{1/2} \\
&= 0.2045
\end{aligned}$$

(d) The clearest example of the negative correlation is between value and momentum (as noted in both the Numeric Investor and GMO cases). The negative correlation makes sense, in that a stock classified as a buy in a value strategy has typically underperformed other stocks, whereas a stock classified as a buy in a momentum strategy has outperformed other stocks.

(e) The benefits are explained in the second paragraph of slicing on page 5 of the GMO case. The less frequently a strategys rankings turn over, the more slices it can have. With $n$ slices, a given slice is rebalanced once every $n$ periods, so $n$ can be greater for a strategy whose rankings are more persistent.

45. Returns-based momentum is the tendency for stock portfolios sorted on past returns, typically over the previous year, to earn future returns that spread in the same direction (past winners outperforming past losers). Earnings momentum is the tendency for stock portfolios sorted by some measure of past earnings surprises or analyst earning-forecast revisions to earn different average returns in the future. In particular, the stocks that have had unexpectedly good earnings news tend to outperform the stocks that have experienced poor news. An example of one strategy is to rank stocks based on an ”SUE” (standardized unexpected earnings), which is simply the difference between a firm’s most recently announced quarterly earnings and the quarterly earnings a year earlier, divided by the historical volatility of these quarterly differences. Then go long, say, the top decile and go short the bottom decile. Another strategy is to rank stocks based on the average past revision in analysts’ forecasted earnings. The behavioral explanation for returns momentum often centers on “overconfidence” by investors, where overconfidence makes investors slow to revise their views as quickly as they should. The earnings-momentum strategies share a similar behavioral motivation, except that the overconfidence resulting slow responses is often attributed to analysts, who then influence the valuations investors place on the stocks when determining prices.

46. (a) With data that include that fund’s planned transactions, one can compare the stock’s observed price at the time the decision to trade the stock is made (before that decision is implemented) to the actual price (or prices) paid to acquire or sell the stock. The value of such differences, averaged across transactions, gives an estimate of the market-impact costs of the fund’s trading.

    (b) Momentum strategies tend to be very time-sensitive and therefore involve trades that need to be executed relatively quickly. Because of this time-sensitivity, the trader is in the position of demanding liquidity, leading to relatively high market-impact costs. Conversely, value strategies are much more sensitive to price than time, and tend to involve trades carefully executed over a longer horizon, which can lead to a lower (possibly even negative) level of market-impact costs.

47. (a) One example is to sort stocks base on their ratios of book value per share to share price, and then go long stocks with high book/market ratios and go short those with low ones. Additional possibilities: sort on other valuation ratios (earnings/price, cashflow/price, etc.), or even sort on residuals in a cross-sectional regression of share prices on ratios and other explanatory variables, going long the negative residhals and short the positive ones (e.g., numeric investors case).

    (b) One example, exploiting “price” momentum, is to sort stocks on their returns over the past six to twelve months (typically excluding the most recent month), and then go long stocks with the highest returns and short those with the lowest. Another example, exploiting “earnings” momentum, is to sort based on revisions in analysts’ earnings forecasts and/or unexpected realized earnings, and then go long (short) the stocks with the highest (lowest) revisions and/or unexpected earnings.

    (c) Ron is correct in that the profits on value and momentum stragegies tend to be negatively correlated. A stock that has done poorly is often a value stock, tending to make it a short in a momentum strategy but a long in a value strategy. He is incorrect in concluding that this negative correlation should lead one to run one strategy or the other, but not both. If the expected profit on each by itself is positive, the expected profit on doing them in combination is simply the weighted average—also positive. Moreover, the negative correlation serves to diversify risk and make the combination strategy less risky, with a higher Sharpe ratio, than either strategy alone.

48. One example would be the approach described in the case *Grantham, Mayo, Van Otterloo & Co. 2001* (page 4, Exhibits 4a/b). That method is based on a dividend-discount valuation model and uses firm-specific variables to predict the rate at which profitability (ROE) and dividend-payout will revert to average levels for a firm's industry. Value firms are then those with a relatively low ratio of actual price to "fair value" produced by the valuation model. Another example would be the regression-based approach described in the case *numeric investors l.p.* (page 5). There, a large cross section of stocks is used to estimate a multiple regression in which an observation of the dependent variable is a given firm's stock price, and the corresponding observations of the independent variables are values of ratios and other characteristics for that firm. Value firms are then those with a relatively low ratio of actual price to the fitted value of price from the regression.

49. Method (c) has the virtue that no numerical estimates or forecasts need be supplied. Its drawback is that it's also the most restrictive in that, not only does it rest on the usual assumptions of the standard Gordon model—constant expected dividend growth and constant discount rate—but it essentially assumes that the market has no internal "growth options." That is, taking the market as a large composite firm, that firm's return on equity (ROE) is assumed to be equal to the discount rate—the expected return on that firm's stock. Method (b) can be justified by the Gordon model without invoking the latter assumption. It does require, however, an assumed value for the expected long-run growth rate of dividends. Method (a) is in many respects the least restrictive, in that valuation-based modeling need not be invoked. The implicit assumption is essentially just that the equity premium is relatively stable through time and, thus, average realized returns give an unbiased estimate of expected returns. One drawback to that method is simply lack of precision—the volatility in realized stock returns makes their average fairly imprecise as an estimate of the expected return. Another drawback to method (a) occurs when the equity premium changes. For example, if the expected return on equity drops, as valuation-based approaches suggest it has over the last half century, then average realized returns over that period will be especially upward biased as an estimate of the new lower expected return, because unanticipated drops in expected future returns are accompanied by higher-than-expected realized returns.

50. (a) In order for Jim's method of calculating the equity risk premium to be correct, we need to assume the standard Gordon Growth Model holds, with constant expected dividend growth and constant discount rate, and we need to assume that the market has no internal growth options. That is, taking the market as a large composite firm, that firm's return on equity (ROE) is assumed to be equal to the discount rate, the expected return on that firm's stock.

    (b) In order for Ken's method to be correct, we need to assume that the equity premium is stable through time. For example, if the expected return on equity drops, as valuation-based approaches suggest it has over the last half century, then average realized returns over that period will be especially upward biased as an estimate of the new lower expected return, because unanticipated drops in expected future returns are accompanied by higher-than-expected realized returns. Another drawback to his method is simply lack of precision; the volatility in realized stock returns makes their average fairly imprecise as an estimate of the expected return.

51. (a) Since $\mathrm{E}(R_{S,t}) = 0.06$ is the expected rate with simple compounding (evident from its underlying calculation), the expected continuously compounded rate of return in any year $t$, $\mathrm{E}(r_{S,t})$, is given by

$$\mathrm{E}(r_{S,t}) = \mathrm{E}(R_{S,t}) - \frac{1}{2}\sigma^2(R_{S,t}) = 0.06 - \frac{1}{2}(0.30)^2 = 0.015.$$

The probability in question is then

$$\begin{aligned}
\mathrm{Prob}\left\{\sum_{t=1}^{10} r_{S,t} < 10(0.0198)\right\} &= \mathrm{Prob}\left\{z < \frac{10(0.0198 - 0.015)}{\sqrt{10(0.30)^2}}\right\} \\
&= \mathrm{Prob}\{z < 0.053\} \\
&= 0.52.
\end{aligned}$$

(b) If the expected return has dropped over time in order to arrive at a level that "is now fairly low," then such a drop would probably have contributed to prices moving up. That is, a drop in the expected return—the rate at which future expected payments to stockholders are discounted—would tend to raise the stock price. Such price increases would have contributed to making historical realized returns higher.

52. (a) The basic assumption is that, when the market is viewed as essentially one big composite "firm," the expected rate of return earned by that firm on funds invested internally—the market's ROE—is the same as the expected rate of return than can be earned by anyone who invests in the stock market—$E(r_S)$. In other words, the expected rates of return on internal and external funds are the same. The reasoning is simply that, if this were not the case, money would flow either into firms (if ROE $> E(r_S)$) or out of firms (if ROE $< E(r_S)$) until ROE $= E(r_S)$ in equilibrium. A secondary assumption is that the reported accounting earnings represent true economic earnings; otherwise some adjustments to the reported earnings could be taken (e.g., the Deutsche Bank case).

(b) A P/E that is currently above its long-run mean is more likely when prices have risen recently. If the P/E is more likely to fall subsequently, i.e. revert toward its long-run mean, then so are prices more likely to fall. Thus, high market returns tend to follow low market returns, and vice versa. This property makes stocks, on a per-year basis, less volatile over longer holding periods than over shorter ones. Investors with a longer horizon can therefore tolerate a greater exposure to stocks. Target-date funds give greater exposure to stocks the longer is an investor's horizon, so mean reversion in the P/E ratio makes target date funds a sensible investment (than they would be without such mean reversion).

53. (a) With his assumed constant payout rate, Tom believes dividends, like earnings, will grow at the constant rate of 2.5%. He can then apply the simple Gordon growth model, which values the stock as

$$P = \frac{D}{k-g},$$

where $P$ is the stock price, $D$ is the expected dividend for the next period, $k$ is the discount rate, and $g$ is the constant growth rate. The discount rate $k$ is also the expected return on the stock, which is then obtained as

$$\begin{aligned} k &= \frac{D}{P} + g \\ &= \frac{17}{1160} + .025 \\ &= 0.0397, \end{aligned}$$

giving an estimated equity premium of $3.97\% - 1.6\% = 2.37\%$.

(b) Given Dick's assumption, the expected rate of return on the stock ($k$) is the same as the expected rate at which current corporate assets will produce earnings over the next period. With corporate assets valued at their current market price, $P$, and expected earnings denoted as $E$, we simply have

$$k = \frac{E}{P} = \frac{25}{1160} = 0.0216,$$

giving an estimated equity premium of $2.16\% - 1.6\% = 0.56\%$.

(c) The expected equity premium going forward could be lower than what it was in the past. Thus, even if past realized returns were exactly *equal* to what they were expected to be, those average realized returns would be higher than what is expected going forward. In addition, however, if the equity premium dropped, it is then likely that past returns were actually *higher* than what they were expected to be. That is, an unanticipated decline in expected return—the rate used to discount future expected payoffs—would likely have caused an unanticipated capital gain during the period in which the decline occurred, thereby driving an even bigger wedge between Dick's estimate and Harry's.

Another possible explanation is that there could have been a lot of unanticipated good news in the past about corporate profits, leading to unexpectedly high stock returns. One would not, of course, forecast unanticipated news going forward to be positive on balance, so this simpler distinction between a sample average and a population mean could also drive a wedge between Dick's estimate and Harry's.

54. Let $I_5$ denote the final value of an investment of $X$ dollars today, and let $V_5$ denote the final value per \$1 invested today. Then

$$\begin{aligned} \text{Prob}\{I_5 < 10\} &= \text{Prob}\{V_5 < \frac{10}{X}\} \\ &= \text{Prob}\left\{\ln V_5 < \ln\left(\frac{10}{X}\right)\right\} \\ &= \text{Prob}\left\{z < \frac{\ln\left(\frac{10}{X}\right) - 5(.08)}{\sqrt{5}(.15)}\right\}. \end{aligned}$$

Since, from the Normal table,

$$\text{Prob}\{z < -.84\} = 0.20,$$

a 20% shortfall probability implies that the initial investment $X$ must satisfy

$$\frac{\ln\left(\frac{10}{X}\right) - 5(.08)}{\sqrt{5}(.15)} = -.84.$$

Solving this for the initial investment $X$:

$$\begin{aligned} \ln\left(\frac{10}{X}\right) &= \sqrt{5}(.15)(-.84) + 5(.08) \\ &= 0.1183. \end{aligned} \tag{37}$$

Taking the exponential function of both sides gives

$$\begin{aligned} \frac{10}{X} &= e^{.1183} \\ &= 1.1256, \end{aligned}$$

so $X = 10/1.1256 = 8.88$.

55. Let $V_S$ denote the final value of a one-dollar current investment in the S&P and let $V_L$ denote the final value of a current one dollar of the pension liability. Let $r_{S,t}$ and $r_{L,t}$ denote the continuously compounded stock return and liabilility growth rate in year $t$. Then The probability of a shortfall is given by

$$\text{Prob}\{V_S/V_L < 1\} = \text{Prob}\{\ln V_S - \ln V_L < 0\}.$$

Observe that

$$\begin{aligned} \ln V_S - \ln V_L &= \sum_{t=1}^{5} r_{S,t} - \sum_{t=1}^{5} r_{L,t} \\ &= \sum_{t=1}^{5} (r_{S,t} - r_{L,t}), \end{aligned}$$

The quantity $(\ln V_S - \ln V_L)$ is normally distributed with mean

$$\begin{aligned} \text{E}(\ln V_S - \ln V_L) &= 5 \cdot \text{E}(r_{S,t} - r_{L,t}) \\ &= 5(0.10 - 0.04) = .30, \end{aligned}$$

and variance

$$\begin{aligned} \text{Var}(\ln V_S - \ln V_L) &= 5 \cdot \text{Var}(r_{S,t} - r_{L,t}) \\ &= 5[(0.20)^2 + (0.08)^2 - 2(0.30)(0.20)(0.08)] = (0.429)^2. \end{aligned}$$

Thus, the probability of a shortfall is

$$\begin{aligned} \text{Prob}\{\ln V_S - \ln V_L < 0\} &= \text{Prob}\{z < \frac{0 - 0.30}{0.429}\} \\ &= .242 \end{aligned}$$

56. Let $V_S$ and $V_L$ denote the values in five years of an initial dollar of assets and liabilities, respectively. If $X$ denotes the minimum amount (in millions) to be invested, then $X$ must satisfy

$$\text{Prob}(X V_S < 30 V_L) = 0.20,$$

which can be rewritten, taking logs, as

$$\text{Prob}(\Delta < \ln(30/X)) = 0.20,$$

where $\Delta = \ln V_S - \ln V_L$. Standardizing by the mean and standard deviation of $\Delta$ then gives

$$\text{Prob}(z < \frac{\ln(30/X) - \text{E}(\Delta)}{\text{std. dev.}(\Delta)}) = 0.20,$$

so $X$ must solve

$$\frac{\ln(30/X) - \text{E}(\Delta)}{\text{std. dev.}(\Delta)} = -0.84,$$

or

$$X = \exp[\ln(30) + 0.84 \cdot \text{std. dev.}(\Delta) - \text{E}(\Delta)].$$

The mean, $\text{E}(\Delta)$, is equal to $5(0.06 - 0.03) = 0.15$, and the standard deviation is the square root of $\text{Var}(\Delta) = 5[.2^2 + .1^2 - 2(.2)(.2)(.1)] = (.4583)^2$. Substituting those values into the above equation gives $X = \$37.94$ million.

57. a. If $V_{20}$ denotes the value in 20 years of one dollar initially invested in stocks, and $r_t$ denotes the continuously compounded stock return in year $t$, the probability that the stock investment will exceed the riskless investment over 20 years is then

$$\begin{aligned}
\text{Prob}[36.79 \cdot V_{20} > 100] &= \text{Prob}[V_{20} > \frac{100}{36.79}] \\
&= \text{Prob}[\ln(V_{20}) > \ln\left(\frac{100}{36.79}\right)] \\
&= \text{Prob}[r_1 + r_2 + \cdots + r_t + \cdots + r_{20} > 20(.05)] \\
&= \text{Prob}[z > \frac{20(.05) - 20(.10)}{\sqrt{20}(.20)} \\
&= \text{Prob}[z > -1.18] \\
&= 0.87
\end{aligned}$$

b. The stock investment isn't necessarily the superior alternative simply because it has a high probability of beating cash. Even though the possibility of falling short of cash is small, the cost of insuring against that possibility—essentially the price of a put option—is nontrivial. Moreover, the likely surplus (in excess of the \$100 million) that stocks provide has a current value equal to the price of a call option, and that call price is equal to the price of the put option. (This follows from an application of put-call parity.) Thus, if the corporation's current value reflects both the potential cost of having to make up the less likely shortfall as well the potential benefit of the more likely surplus, there is no net advantage, in terms of corporate valuation, to the stock investment.

58. (a) The Preference shares return the full redemption value if

$$(\$100 \text{ million})e^{r_1+r_2+\cdots+r_{30}} \geq (\$100 \text{ million})e^{30(.03)},$$

or if

$$\sum_{t=1}^{30} r_t \geq 30(.03).$$

The probability in question is then

$$\begin{aligned}
\text{Prob}\left\{\sum_{t=1}^{30} r_t \geq 30(0.03)\right\} &= \text{Prob}\left\{z \geq \frac{30(0.03-0.10)}{\sqrt{30(0.20)^2}}\right\} \\
&= \text{Prob}\left\{z < -\frac{30(0.03-0.10)}{\sqrt{30(0.20)^2}}\right\} \\
&= \text{Prob}\{z < 1.917\} \\
&= 0.972.
\end{aligned}$$

(b) Even though the Preference shares will receive full redemption value with probability 0.972, that does not imply that they should sell for close to \$100 million. The Preference shares can be viewed as a combination of a riskless bond that delivers the redemption value for certain plus a short position in a put option that insures against any shortfall. There is less than a 3% chance of the put's finishing in the money; the most the put could ever pay off is the full redemption value; and the present value of that maximum payoff is \$100 million. These facts might lead one to conclude, thinking in actuarial terms (pricing insurance as payoffs times probabilities), that the put could at most be worth 0.03× \$100 million. Such is not the case. The no-arbitrage price of the put depends on the volatility of terminal stock value, which grows with the horizon. Especially at long horizons, the price of the put can be much higher. (It will approach \$100 million as the horizon grows to infinity.) The price of the preference shares, \$100 million minus the put price, can therefore be substantially less than \$100 million.

59.

a. One argument in favor comes from considering human capital as part of one's overall wealth and to view the wage income produced by that capital as more bond-like in its characteristics, at least for many workers. That is, wages are likely to be a fairly steady stream, more like bond coupons than equity payoffs. For a younger employee, human capital is likely to be a bigger fraction of overall wealth, so a lower allocation to bonds is thereby justified. Another argument in favor is evidence that stock returns exhibit a degree of mean reversion—the variance of long-run returns is lower than if stock returns were independent from year to year. The lower long-run variance of equities can then also justify a higher allocation to stocks for the investor with a more distant target retirement date.

b. The argument that human capital is more bond-like might not apply to some workers, such as those whose wages or compensation have a strong pro-cyclical or market-sensitive component. Risk tolerance could also play an important role: a young worker with low risk tolerance might prefer less equity than an older worker with more risk tolerance.

60. (a) No. As $N$ increases, the outcomes for the annualized return become less disperse, but that is simply a property of an average—it becomes less volatile as more values are used to compute the average. This fact does not, in a meaningful way, lower the volatility faced by a long-term investor.

    (b) An alternative measure would be the variance of the N-period return, expressed on a per-period basis. That is, Jack could examine the annualized variance of returns.

    (c) If the annualized variance is lower for larger $N$, this is evidence of mean reversion: a positive return in one period is somewhat more likely to be followed by negative returns in future periods (and vice versa). This partial offsetting of returns is not helpful to a short-run investor but lowers volatility for a long-run investor.

    (d) No. The standard deviations of the annualized returns simply drop at rate $\sqrt{N}$. For example, $4.47/8.94 = \sqrt{5/20} = 0.5$. The annualized variances will be the same across $N$. This is simply what one would expect if returns are independent from year to year, with a constant variance each year.

    (e) Human capital represents a larger fraction of wealth for younger investors. Since the returns on human capital (wages) are not very volatile for most workers, the human-capital component of wealth can be viewed more like bonds than equity. That high effective bond allocation in the overall wealth portfolio of younger investors justifies a higher equity allocation in their financial-wealth portfolios. Older workers, even if their overall risk tolerance is similar to that of the young, have a smaller human-capital bond-like component in their overall wealth portfolio, and thus desire a smaller equity allocation in their financial portfolios. Consistent with this argument, target-date funds decrease the equity allocation as an investor ages.

61. It is probably true that, with stocks in the pension fund, a surplus is more likely than a deficit. At the same time, however, the the corporation's promise to make up a possible deficit, which can be valued as a put option, is equal to the potential surplus, which can be valued as a call option. Put-call parity implies that these values offset, even if a surplus is much more likely than a deficit. In that sense, there is no advantage to investing in equities. Moreover, if the sponsoring corporation would make up any deficit completely but capture less than 100% of any surplus, then the equity-investment policy is suboptimal.

62. (a) One simple argument involves a corporation's fiduciary responsibility to the pension plan. If the promised benefits of the DB plan are largely known, bonds provide a closer match between those future commitments and the value of the assets in the pension fund, thereby increasing the security of the promised benefits. Another argument is that having bonds in the pension fund should generally lower the volatility of future pension expenses and shareholder risk. The reduced risk essentially lowers the effective leverage of the combined entity (corporation plus pension fund), thereby allowing

greater leverage on the corporate side that could capture additional advantages of the interest tax shield on debt.

(b) One might argue that stocks provide a hedge against future increases in wages, and promised benefits are pegged to employees' future wages (typically wages in the few years preceding retirement). Also, given the limited liability of corporations, having equity in the pension fund essentially provides the corporation's stockholders with an implicit put option that allows them to default on their pension obligation and avoid paying the full amount of a potential shortfall, while standing to gain fully from a potential surplus in the fund. The option essentially lets the stockholders "put" the assets of the pension fund plus the corporation to the DB plan (or the government, if the plan is insured) at a "strike price" equal to the total value of the promised benefits.

(c) This justification doesn't seem very strong. While it is true that a high expected return can make a surplus more likely than a shortfall, the current values of those two possibilities are comparable. The potential surplus essentially represents a long position in a call option on the plan's assets, while the potential shortfall represents a short position in a put option on the plan's assets. For a fully funded plan—one whose plan assets have current value equal to the present value of accumulated pension liabilities—put-call parity implies equal values for the two options, regardless of the expected return on the plan assets.

63. (a) Let $R_t$ and $r_t$ denote the period-$t$ simple and continuously compounded returns, respectively. First note that the geometric mean $R^G$ is computed as

$$1 + R^G = [(1+R_1)(1+R_2)\cdots(1+R_T)]^{\frac{1}{T}},$$

so

$$\begin{aligned} r^G = \ln\{1+R^G\} &= \frac{1}{T}[\ln(1+R_1) + \ln(1+R_2) + \cdots + (1+R_T)] \\ &= \frac{1}{T}(r_1 + r_2 + \cdots + r_T). \end{aligned}$$

In other words, the geometric mean return, expressed as a continuously compounded rate, is simply the arithmetic average of the continuously compounded returns. The latter average is the estimate of $\mu = \mathrm{E}\{r_t\}$. In this case, therefore, $\mu = \ln(1.05) = 0.0488$. (Using $\mu = 0.05$ instead doesn't change the answer a lot and will not be penalized significantly.) If $V_{15}$ denotes the amount in 15 years produced by a current one-dollar investment in the S&P, the probability in question is as follows:

$$\begin{aligned} \mathrm{Prob}\{V_T(43.068) > 50\} &= \mathrm{Prob}\{V_T > (1.01)^{15}\} \\ &= \mathrm{Prob}\{\ln V_T > 15(\ln 1.01)\} \\ &= \mathrm{Prob}\left\{z > \frac{15(\ln 1.01) - 15(0.0488)}{\sqrt{15}(.20)}\right\} \\ &= \mathrm{Prob}\{z > -0.752\} = 0.774. \end{aligned}$$

(b)
   i. The probability in part a is not very relevant. Even though a potential surplus is more likely than a potential shortfall, the current values of each are the same, when recognized as essentially just put and call options on the fund's S&P investment. That is, the potential shortfall represents a short position in a put, and the potential surplus represents a long position in a call. Both options have the same "strike" price—$50 million—and since the present value of that strike price is the current value of the stock, put-call parity implies the options have the same values.
   ii. Tax deductibility of interest payments gives an incentive for a corporation to have debt, rather than equity, in its capital structure. That incentive is countered by costs of leverage arising from potential financial distress. Those distress costs lessen as corporate earnings become less volatile, and having bonds in the pension fund contributes to lower volatility of earnings, by reducing or eliminating volatility associated with future pension expenses. Thus, switching from equity to bonds in the pension fund essentially raises the corporation's debt capacity and potential tax-shield value.
   iii. Limited liability creates the potential for corporate shareholders to capture all of a potential surplus but avoid paying all of a potential shortfall when the corporation lacks sufficient assets to do so. This creates an asymmetry between the current value of the potential surplus and shortfall, making the call in part (i) worth more than the current value of the potential shortfall. The latter can be viewed as a short position in the put in part (i), plus a long position in a put—also with a $50 million strike—on the sum of corporate and pension-fund assets.