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Suggested Solutions to Problem Set 6
Today's Date: November 12, 2017

1. JR Exercise 3.55

   Soln: Given a plant size K, the firm must use an amount $F_i = y_i^2/K$ to supply the demand y_i. Hence, $F_d = 16/K$ and $F_n = 9/K$. For this firm, maximizing profit is equivalent to minimizing cost: it chooses K to minimize

   $$w_k K + w_f \left(\frac{16}{K} + \frac{9}{K} \right).$$

   The solution obtained from the FOC is $K = 5w_f^{1/2} w_k^{-1/2}$. ■

2. JR Exercise 4.3

   Soln: Denoting consumer i's Marshallian demand function for the good as $q^i(p, \mathbf{p}, y^i)$, the market demand function is

   $$Q(p, \mathbf{p}, y^1, \ldots, y^n) = \sum_{i=1}^{n} q^i(p, \mathbf{p}, y^i).$$

   The problem states that the good is a normal good for each consumer. Thus, from the Slutsky equation and the fact that each consumer's Hicksian demand function decreases for q decreases in p, each $q^i(p, \mathbf{p}, y^i)$ decreases in p. It follows that $Q(p, \mathbf{p}, y^1, \ldots, y^n)$ also decreases in p. ■

3. An industry has the demand curve $D(p) = A - p$. Each of a very large number of potential firms has the same cost function in the long run as in the short run, and it is

   $$c(q) = \begin{cases} q + q^2 + 9 & \text{if } q > 0 \\ 0 & \text{if } q = 0. \end{cases}$$

   (a) For $A = 28$, find the long-run competitive equilibrium price, output per operating firm, and number of operating firms.

   Soln: The long-run equilibrium price is equal to minimum average cost (the zero profit condition). Average cost is

   $$c^A(q) = 1 + q + \frac{9}{q},$$

   a nice convex function. So the FOC for minimizing it, $c^{A\prime}(q) = 0$, is a sufficient condition for a minimizer, yielding $q^{\min} = 3$.

   The long-run competitive equilibrium price is $p^{\min} = c^A(3) = 7$.

   Since each operating firm produces $q^{\min} = 3$, and the total output is $D(7)$, the long-run equilibrium number of operating firms is

   $$\hat{J} = \frac{D(7)}{3} = 7.$$

   ■

(b) Now the demand curve shifts up in the sense that A increases to 67. In the short run, the number of firms is fixed at the number you found in (a). Find the new short-run equilibrium price and per-firm output.

Soln: The number of firms is fixed at $\hat{J} = 7$. Profit maximization requires the price to equal marginal cost, and so

$$p = 1 + 2q. \tag{1}$$

A second equation is obtained by equating supply and demand:

$$S(p) = D(p) \iff 7q = 67 - p. \tag{2}$$

Solving (1) and (2) yields

$$p^* = \frac{47}{3}, \quad y^* = \frac{22}{3}.$$

■

(c) Now find the new long-run equilibrium for $A = 67$.

Soln: By the exact same reasoning as in (a), the long-run equilibrium price and per-firm output are $p^{\min} = 7$ and $q^{\min} = 3$. (These quantities do not depend on the demand function.) The equilibrium number of firms is now

$$\hat{J} = \frac{67 - 7}{3} = 20.$$

■

4. JR Exercise 4.7, (a) and (b), assuming $a > 0$ and $b > 0$ (not what the book says!)

Soln: (a) For the SR equilibrium we have two equations to solve:

$$\begin{aligned} p &= a + 2bq \text{ (price equal to marginal cost)}, \\ Jq &= \frac{\alpha - p}{\beta} \text{ (supply equal to demand)}. \end{aligned}$$

Solving them for p and q yields

$$p^{sr} = \frac{a\beta J + 2b\alpha}{2b + \beta J}, \quad q^{sr} = \frac{\alpha - a}{2b + \beta J}.$$

This is the answer, provided $\alpha \geq a$. If instead $\alpha < a$, then $q^{sr} = 0$, $Q^{sr} = J \cdot 0 = 0$, and any price $p^{sr} \in [\alpha, a]$ is a corresponding equilibrium price, as both supply and demand are zero at these prices. ■

Soln: (b) In the long run, profits are zero for each firm. Hence, since the average cost function is $a + bq$, we have

$$p = a + bq.$$

Each firm chooses its output to maximize profit, and so price is also equal to marginal cost:

$$p = a + 2bq.$$

Lastly, supply must equal demand:

$$Jq = \frac{\alpha - p}{\beta}.$$

The solution of the first two equations is $(q^{lr}, p^{lr}) = (0, a)$. This makes the third equation become

$$J \cdot 0 = \frac{\alpha - a}{\beta}.$$

We conclude that if $\alpha > a$, then the long-run equilibrium is

$$(q^{lr}, p^{lr}, Q^{lr}, \hat{J}) = \left(0, a, \frac{\alpha - a}{\beta}, \infty\right).$$

The interpretation of this is that each of an infinite number of firms produces an infinitesimal amount which together add up to the positive amount $(\alpha - a)/\beta$. ■

5. A monopoly has cost function $c(q) = 6q$. The output q is consumed only by consumers a and b. Their demand functions for q are $D_a(p) = 10 - p$ and $D_b(p) = 20 - p$, respectively.

   (a) Find the industry demand function $D(p)$; the inverse demand function $P(Q)$; the revenue function $R(Q)$; and the marginal revenue function $R'(Q)$.

   Soln: Note first that the actual demand functions are as stated only if the resulting amount demanded is not negative. Thus, in fact $D_a(p) = 0$ if $p > 10$, and $D_b(p) = 0$ if $p > 20$. The industry demand curve is the horizontal sum of the individual demand curves, the graph of the industry demand function given by

$$D(p) = \begin{cases} 30 - 2p & \text{if } p \leq 10 \\ 20 - p & \text{if } 10 < p \leq 20 \\ 0 & \text{if } p > 20. \end{cases}$$

   The inverse demand function is thus

$$P(Q) = \begin{cases} 20 - Q & \text{if } q \leq 10 \\ \frac{1}{2}(30 - Q) & \text{if } 10 < q \leq 30 \\ 0 & \text{if } q > 30. \end{cases}$$

   The revenue function is

$$R(Q) = P(Q)Q = \begin{cases} 20Q - Q^2 & \text{if } Q \leq 10 \\ \frac{1}{2}\left(30Q - Q^2\right) & \text{if } 10 < Q \leq 30 \\ 0 & \text{if } Q > 30, \end{cases}$$

   and so marginal revenue is

$$R'(Q) = \begin{cases} 20 - 2Q & \text{if } Q \leq 10 \\ 15 - Q & \text{if } 10 < Q \leq 30 \\ 0 & \text{if } Q > 30. \end{cases}$$

   ■

   (b) Find the monopoly output Q^M and price p^M.

   Soln: The monopoly choose Q to equate marginal revenue to marginal cost. Hence, $R'(Q) = 6$. Note that for $10 < Q \leq 30$, this equation is $15 - Q = 6$, or rather, $Q = 9$, which contradicts $10 < Q \leq 30$. For $Q \leq 10$ this equation is $20 - 2Q = 6$, or rather $Q = 7$, the true solution to $R'(Q) = 6$. Hence,

$$Q^M = 7 \text{ and } p^M = P(7) = 13.$$

   ■

(c) Suppose now that the firm can practice price discrimination, i.e., charge a price p_a to consumer a and a price p_b to consumer b. Find the firm's optimal prices, p_a^* and p_b^*, and quantitities q_a^* and q_b^*. Who is better off, and who is worse off, relative to the uniform-price solution in (b)?

Soln: The firm will now equate marginal revenue from customer a with marginal cost for customer a, and likewise for customer b. For $q < 10$ we have

$$\begin{aligned} P_a(q) &= 10 - q, \\ R_a(q) &= P_a(q)q = 10q - q^2, \\ R_a'(q) &= 10 - 2q. \end{aligned}$$

Equating marginal revenue from a with marginal cost yields $10 - 2q = 6$, and solving yields

$$q_a^* = 2 \text{ and } p_a^* = P_a(2) = 8.$$

Similar calculations for consumer b yield

$$q_b^* = 7 \text{ and } p_b^* = P_b(7) = 13.$$

The monopoly must be better off when allowed to price discriminate, since it could choose $p_a = p_b = p^M$ and achieve exactly the same profit as in the uniform-price case. Indeed, it is easy to check that its profit when optimally discriminating is

$$q_a^*(p_a^* - 6) + q_b^*(p_b^* - 6) = 53,$$

and in the uniform pricing case it is the lower amount

$$q^M(p^M - 6) = 49.$$

Consumer a is also better off in the price discrimination case, since it results in a lower price for her: $p_a^* = 8$ as opposed to $p^M = 13$. Consumer b is exactly as well off in both cases, since $p_b^* = p^M$. ■