Economics 681 **Steven A. Matthews**
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Suggested Solutions to Problem Set 3
Today's Date: October 19, 2017

1. JR Exercise 1.63 (Assume the demand function is C^1)

   Soln: We know $b = 2$, since the substitution matrix is symmetric because the demand function is C^1. We also know $Sp = 0$, and so

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{bmatrix} a & b \\ 2 & -1/2 \end{bmatrix} \begin{pmatrix} 8 \\ p_2 \end{pmatrix} = \begin{pmatrix} 8a + bp_2 \\ 16 - p_2/2 \end{pmatrix}.$$

   Solving these two equations for a and p_2 yields $a = -8$ and $p_2 = 32$. ■

2. A consumer in a three-good economy with wealth level $y > 0$ is maximizing locally non-satiated preferences on $\mathbb{R}^3$ and has demand functions for goods 1 and 2 given by:

$$\begin{aligned} x_1(p, y) &= 100 - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \delta\frac{y}{p_3} \\ x_2(p, y) &= \alpha - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \gamma\frac{y}{p_3}. \end{aligned}$$

   (a) Calculate the demand for good 3.
   (b) Verify that x_1 and x_2 are homogeneous of degree 0.
   (c) What conditions on α, β, δ and γ are implied by demand theory?

   Soln:

   (a) Calculate the demand for good 3.
   Using Walras' law,

$$x_3 = \frac{y - p_1 x_1 - p_2 x_2}{p_3}.$$

   From this and the given demand functions for x_1 and x_2 we obtain

$$x_3 = \frac{y - [100p_1 + \alpha p_2 - 5\frac{p_1^2}{p_3} + \frac{(\beta-5)p_1p_2}{p_3} + \beta\frac{p_2^2}{p_3} + \frac{y(\delta p_1 + \gamma p_2)}{p_3}]}{p_3}.$$

   (b) Verify that x_1 and x_2 are homogeneous of degree 0.
   Let $\lambda > 0$. Then:

$$\begin{aligned} x_1(\lambda p, \lambda y) &= 100 - 5\frac{\lambda p_1}{\lambda p_3} + \beta\frac{\lambda p_2}{\lambda p_3} + \delta\frac{\lambda y}{\lambda p_3} \\ &= 100 - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \delta\frac{y}{p_3} \\ &= x_1(p, y). \end{aligned}$$

   Similarly for x_2,

$$\begin{aligned} x_2(\lambda p, \lambda y) &= \alpha - 5\frac{\lambda p_1}{\lambda p_3} + \beta\frac{\lambda p_2}{\lambda p_3} + \gamma\frac{\lambda y}{\lambda p_3} \\ &= \alpha - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \gamma\frac{y}{p_3} \\ &= x_2(p, y). \end{aligned}$$

(c) What conditions on α, β, δ and γ are implied by demand theory?

From the Slutsky equation and the result that Hiksian demand curves slope down, we have

$$0 \geq \frac{\partial x_1^h}{\partial p_1} = -5\frac{1}{p_3} + \frac{\delta}{p_3}[100 - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \delta\frac{y}{p_3}].$$

By letting $p_3 = 1$ and taking p_1 and p_2 to 0, we obtain

$$-5 + 100\delta + \delta^2 y \leq 0.$$

The only way this can hold for all y is if $\delta = 0$.

Similarly:

$$0 \geq \frac{\partial x_2^h}{\partial p_2} = \frac{\beta}{p_3} + \frac{\gamma}{p_3}[\alpha - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \gamma\frac{y}{p_3}].$$

Once again, letting $p_3 = 1$ and taking p_2 and p_1 to 0 yields

$$\beta + \alpha\gamma + \gamma^2 y \leq 0.$$

So $\gamma = 0$ and $\beta \leq 0$.

The Law of Reciprocity holds here because these demand functions are C^1, and hence

$$\frac{\partial x_1^h}{\partial p_2} = \frac{\partial x_2^h}{\partial p_1}.$$

Since $\delta = 0$, we have $\frac{\partial x_1}{\partial y} = 0$, and so the Slutsky equation implies

$$\frac{\partial x_1}{\partial p_2} = \frac{\partial x_1^h}{\partial p_2} - x_2\frac{\partial x_1}{\partial y} = \frac{\partial x_1^h}{\partial p_2}.$$

Similarly, since $\gamma = 0$, we have $\frac{\partial x_2}{\partial y} = 0$, and so $\frac{\partial x_2^h}{\partial p_1} = \frac{\partial x_2}{\partial p_1}$. Hence, since Hicksian demand satisfies the Law of Reciprocity (note that the demand functions are C^1),

$$\frac{\partial x_1}{\partial p_2} = \frac{\partial x_1^h}{\partial p_2} = \frac{\partial x_2^h}{\partial p_1} = \frac{\partial x_2}{\partial p_1},$$

and so

$$\frac{\beta}{p_3} = \frac{-5}{p_3} \Rightarrow \beta = -5.$$

The parameter α is unrestricted.

■

3. Three candidates are running for mayor of Bucolia. The big issue is how will Bucolia collect revenue to pay for the operation of its new fitness center. *H. Economicus*, a citizen of Bucolia, is deciding how to cast his vote. All three candidates have proposed combinations of *mandatory* once-only membership fees (head taxes) and per-use fees. The proposals of candidates A, B, and C, respectively, are as follows:

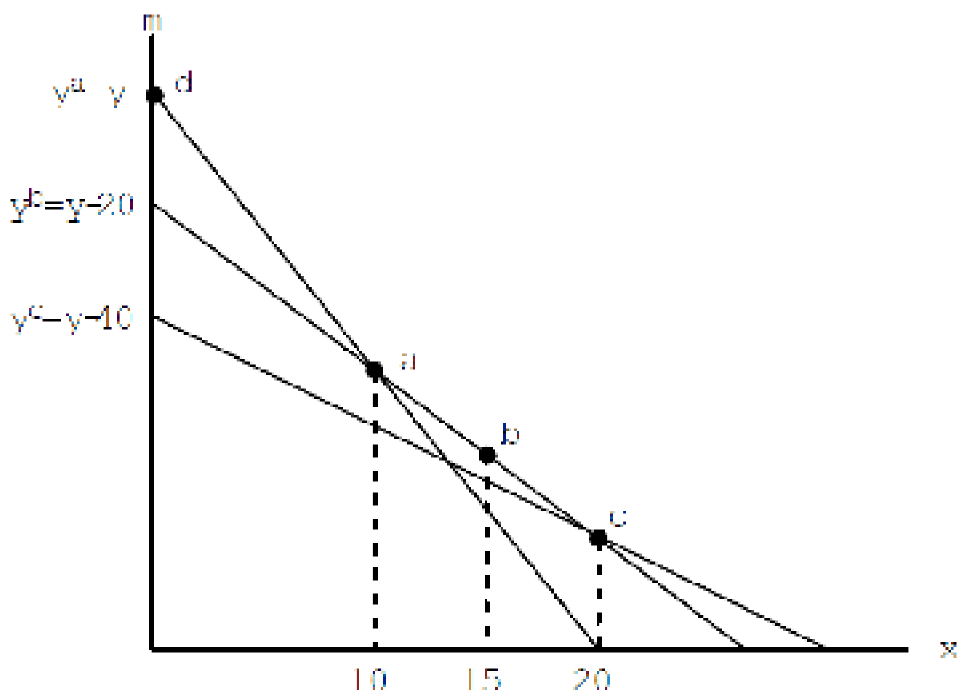
   (a) \$0 membership fee, \$5 use fee.

   (b) \$20 membership fee, \$3 use fee.

   (c) \$40 membership fee, \$2 use fee.

H. Economicus would use the facility 10 times if A wins; 15 times if B wins, and 20 times if C wins. Assuming H. Economicus is a sincere voter, for whom will he vote? How does he rank the remaining two candidates? Could any candidate make him worse off than if the facility is not provided at all? Prove your answers.

Soln: Denote H.E.'s consumption bundle as $(x_1, y - px_1 - F)$, where x_1 is the number of times he goes to the fitness center, and $y - px_1 - F$ is what he spends on other goods. Here, y is his income, p is the per-use fee and F is the mandatory membership fee. Denote his optimal bundles when candidates A, B, and C win as a, b, and c, respectively. Denote the "no provision at all" bundle as $d = (0, y)$. Simple arithmetic verifies the following picture.



Hence, b is revealed preferred to both a and c, and so H.E. will vote for B. There is not enough information given to determine his preferences between a and c. We also see that a is revealed preferred to d. Hence, by transitivity, b is preferred to d as well. Only c could possibly make H.E. worse off than d. ■

4. JR Exercise 4.20

Soln: To be clear, let's call the particular good this problem refers to as good 1, x_1, with price p_1. We have $p^0 = (1, p_{-1})$ and $p^1 = (4, p_{-1})$, where p_{-1} is the fixed vector of prices other than that of good 1. Observe that for any (p, y),

$$x(p, y) := (y/p_1, 0, \ldots, 0)$$

satisfies the budget constraint, and so the function $x(p, y)$ must be the demand function. (Any other $\hat{x}(p, y) \in \mathbb{R}^n_+$ with $\hat{x}_1(p, y) = y/p_1$ would violate the budget constraint.) Hence, $x^0 := x(p^0, 7) = (7, 0, \ldots, 0)$. Furthermore, we have $e(p^1, u^0) = 28$, since with this much money the consumer can purchase x^0 at p^1 to obtain utility $u^0 = u(x^0)$, and at any income $y \leq 28$ the following shows that the consumer cannot achieve more than this level of utility at p^1, assuming $u(x)$ increases in x_1 : for any $y < 28$ we have

$$\begin{aligned} u(\frac{y}{4}, 0, \ldots, 0) &= \max_x u(x) \text{ s.t. } p^1 \cdot x \leq y \\ &\leq u(7, 0, \ldots, 0) \\ &= u^0. \end{aligned}$$

The definition of CV now implies

$$CV(p^0, p^1, 7) = e(p^1, u^0) - 7 = 28 - 7 = 21.$$

(There are other proofs – this seems by far the simplest.) ■