Economics 700 **Steven A. Matthews**
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Exam 1

60 points, 60 minutes. Closed books, notes, calculators.
Show your reasoning. ***Read all before answering any.***

ANSWER ONLY THREE QUESTIONS!

1. (20 pts) The primitives of the preference-based decision theory that we studied were a set X and a complete and transitive binary relation on X. We could instead have started with X and a binary relation $\succ$ on X satisfying

   (Asymmetry) For all x and y, if $x \succ y$ then not $y \succ x$, and

   (Negative Transitivity) For all x, y, and z : not $x \succ y$ and not $y \succ z \Rightarrow$ not $x \succ z$.

   The two approaches are equivalent. Show one direction of this equivalence by proving the following:

   Proposition 1 *Each asymmetric and negatively transitive $\succ$ is the strict preference relation derived from some complete and transitive $\succeq$.*

2. (20 pts) A consumer in a two-good world demands $x = (1, 2)$ at $(p, m) = (2, 4, 10)$, and he demands $x' = (2, 1)$ at $(p', m') = (6, 3, 15)$. Is he maximizing a locally nonsatiated utility function? Explain.

3. (20 pts) A consumer's preferences are strictly convex, locally nonsatiated, and give rise to a C^1 Marshallian demand function $x : \mathbb{R}^{L+1}_{++} \to \mathbb{R}^L_+$.

   (a) (10 pts) Fix $(p, m) \in \mathbb{R}^{L+1}_{++}$. Under what further assumptions, if any, is it true that for any differentiable utility function representing the consumer's preferences, her marginal utility of income must be positive at (p, m)?

   (b) (10 pts) Suppose $u(\cdot)$ represents the consumer's preferences, and the corresponding expenditure function satisfies $\partial^2 e / \partial p_1 \partial u > 0$ for all (p, u) at which it is well defined. What does this tell us about her demand function for good 1?

4. (20 pts) In a two-good world, consider the following possible expenditure function, where a and b are positive exponents:

$$e(p, u) = \left(\tfrac{1}{2} p_1^a + \tfrac{1}{2} p_2^b + \sqrt{p_1 p_2} \right) u.$$

   (a) (8 pts) For what values of (a, b) is e truly an expenditure function? Explain.

   For (b) and (c), assume a and b satisfy the restrictions you just identified.

   (b) (4 pts) Find the corresponding Hicksian demand functions.

   (c) (8 pts) Find a utility function for which e is the expenditure function.