Economics 701A **Steven A. Matthews**
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Problem Set 1
Due Wednesday, Sep. 13

1. Prove the following Lemma stated in class.

   Lemma 1. If $\succsim$ is complete and transitive, then

   (a) $\succ$ is transitive;

   (b) $\sim$ is transitive;

   (c) $x \succ y$ and $y \succsim z \Rightarrow x \succ z$; and

   (d) $x \succsim y$ and $y \succ z \Rightarrow x \succ z$.

2. Let $\succsim$ be a rational preference relation. Show that if $x \in A \subseteq B$ and $x \in C^*(B, \succsim)$, then $x \in C^*(A, \succsim)$. Is this implication true if $\succsim$ is not rational?

3. Suppose that a choice structure $\langle \mathfrak{B}, C \rangle$ satisfies WARP. Consider two binary relations, $\succ^\#$ and $\succ^{**}$, defined as follows:

   - $x \succ^\# y \Leftrightarrow$ there is some $B \in \mathfrak{B}$ such that $x, y \in B$, $x \in C(B)$, and $y \notin C(B)$;
   - $x \succ^{**} y \Leftrightarrow x \succsim^* y$ but not $y \succsim^* x$, where $\succsim^*$ is the standard revealed preference relation defined in MWG and page 13 of slide set 1.

   Show that:

   (a) $\succ^\#$ and $\succ^{**}$ are the same binary relation: $x \succ^\# y \Leftrightarrow x \succsim^{**} y$ for all $x, y \in X$.

   (b) If $\mathfrak{B}$ includes all three-element subsets of X, then $\succ^\#$ is transitive.

4. Consider a choice structure $\langle \mathfrak{B}, C \rangle$ for which $\mathfrak{B}$ contains all size 2 subsets of X. Show that $\langle \mathfrak{B}, C \rangle$ satisfies WARP if and only if the following two properties suggested by Amartya Sen are satisfied:

   (a) If $x \in B \subseteq A$ and $x \in C(A)$, then $x \in C(B)$.

   (b) If $y \in B \subseteq A$ and $y \in C(A)$, then $C(B) \subseteq C(A)$.