Review Session

December 13, 2017

Cournot Competition

2 firms produce perfectly substitutable goods. The market price is determined by

$$P(Q) = \max\{a - Q, 0\}$$

where $Q = q_1 + q_2$. Both firm have the same cost function $c(q_i) = cq_i$ with $c < a$.

- Find a Nash equilibrium of the game.

GE under Uncertainty

3. (10 points for each part). Two farmers face the possibility that the river on which their farms lie might flood. For simplicity suppose that either of their farms might flood, but not both. The chance that either farm might flood is 1/4. Each farmer's crop will be 400 if his farm doesn't flood and 0 if it does flood. Each has a von Neumann-Morgenstern utility function with utility for the good being $u(x) = x^{\frac{1}{2}}$.

   (a) Compute the Arrow-Debreu equilibrium for this economy, where the farmers can trade contingent commodities before it is known whose farm might flood. What is the expected utility of each farmer?

Public Good

3 people. There are one private good x and one public good g. Each agent has the same utility function of the form

$$u^i(x,g) = x + \ln(g)$$

with an initial endowment $e^i(x,g) = (2,0)$. The firm producing public goods have the cost function $C(g) = g$ and $p_x = 1$.

1. Find an efficient allocation.
2. Find a symmetric competitive equilibrium of this economy.

Efficient Allocation

- For efficient allocation, we can use Samuelsonian condition,

$$\sum_i \frac{u_g^i}{u_x^i} = C'(g)$$

which means

$$\sum_i \frac{1}{y} = \frac{1}{y}\sum_i 1 = \frac{3}{y} = 1$$

And thus, the total amount of y is $y = 3$.

Competitive Eqm

- For competitive equilibrium, we cannot substitute feasibility or market clearing condition ex ante when you solve maximization problem.
- Agent 1's optimization problem is

$$\max_{x^1,y^1} u^1(x,^1 y^1 + y^2 + y^3)$$
$$s.t.\ x^1 + py^1 \leq 2$$

  FOCs:

$$\frac{1}{y^1 + y^2 + y^3} = \lambda$$
$$1 = \lambda$$

  and thus $\frac{1}{y^1+y^2+y^3} = 1$. Since we are focusing on symmetric allocation, $y^1 = y^2 = y^3 = y/3$ where y is the total amount of public goods. (This is the timing that we can use symmetric or market clearing).
- Thus, the optimal solution is $y^i = 1/3$ (total is $y = 1$).
- Check the total supply of public goods in competitive eqm is less than the efficient allocation.