Diversification Returns and Asset Contributions

David G. Booth and Eugene F. Fama / FAJ 1992

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Booth David, G., and F. Fama Eugene. "Diversification returns and asset contributions." Financial Analysts Journal 48.3 (1992).

Overview

- Key Ideas
  - The portfolio’s compound return is greater than the weighted average of the compound returns on the assets in the portfolio.
  - The incremental return is due to diversification.

- Terminology
  - Return Contribution: Contribution of an asset to a portfolio’s compound return
  - Asset-Class Investing: Investing to capture the returns of an asset class without regard to stock selection – index fund is an example
  - Diversification returns: Difference between the portfolio’s return and the weighted average of each asset’s compound return (will see later why this is the case)

Demystifying Compound Return

$$\boldsymbol{C_j \approx \ln[1 + E(R_j)] - \frac{s_j^2}{2[1 + E(R_j)]^2}}$$

- Let us denote simple return as R_j. We can express the simple return as a function of r_j, which is the continuously compounded return:

  $$R_j = \lim_{n \to \infty} \left(1 + \frac{r_j}{n}\right)^n - 1 = e^{r_j} - 1$$

  Rearranging to express r_j in terms of R_j, we have $r_j = \ln[1 + R_j]$.

- The variable of interest to us is $E[r_j] = E[\ln[1 + R_j]]$ which we can obtain using Taylor expansion around the mean. Recall that the Taylor expansion for a logarithm of X is given as:

  $$\log X = \log \mu - \frac{X - \mu}{\mu} - \frac{(X - \mu)^2}{2\mu^2} + \cdots$$

  We can directly apply the expansion to $E[r_j]$:

  $$E[\ln[1 + R_j]] = \ln(1 + E[R_j]) - \frac{E\left[\left(R_j - E[R_j]\right)^2\right]}{2\left(1 + E[R_j]\right)^2} + \frac{E\left[\left(R_j - E[R_j]\right)^3\right]}{3\left(1 + E[R_j]\right)^3} \cdots$$

Interpreting Compound Return (1/2)

$$\boldsymbol{C_j \approx \ln\left[1 + E\left(R_j\right)\right] - \frac{s_j^2}{2\left[1 + E\left(R_j\right)\right]^2}}$$

- The higher the variance of the returns $\left(s_j^2\right)$, the lower the compound returns
- This is exactly why the asset's compound return understates the contribution of the asset to the compound return of the portfolio – the risk (variance) of an asset in a diversified portfolio is less than the risk (variance) of the asset's returns by itself.
- Let's follow Booth and Fama a little more. Recall that an asset i's beta to the portfolio p can be expressed as

$$\beta_j = \frac{Cov\left(R_j, R_p\right)}{Var\left(R_p\right)} = \frac{Cov\left(R_j, R_p\right)}{s_p^2}$$

Since the portfolio's beta to itself is 1 and betas can be linearly summed, we can further assert that:

$$1 = \beta_p = \sum_{j=1}^{N} w_j \beta_j = \sum_{j=1}^{N} \frac{w_j Cov\left(R_j, R_p\right)}{s_p^2} \rightarrow s_p^2 = \sum_{j=1}^{N} w_j Cov\left(R_j, R_p\right)$$

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Interpreting Compound Return (2/2)

$$\boldsymbol{C_j \approx \ln\left[1 + E\left(R_j\right)\right] - \frac{s_j^2}{2\left[1 + E\left(R_j\right)\right]^2}}$$

- The last result from the previous page implies that the covariance of each asset in the portfolio and the portfolio, weighted by the asset's representation in the portfolio, will sum to the portfolio's variance.
- Using this property, an estimate of asset j's contribution to compound return of portfolio p can thus be written as

$$D_j \approx \ln\left[1 + E\left(R_j\right)\right] - \frac{\beta_j \cdot s_p^2}{2\left[1 + E\left(R_j\right)\right]^2}$$

- Comparison of the equations for C_j and D_j:
  - The difference between the two terms is the difference between $\beta_j s_p^2$ and s_j^2.
  - In other words, the return contribution of asset j is greater than its compound return because the contribution of asset j to the variance of the return of portfolio ($\beta_j s_p^2$) is less than the variance of the return on asset j (s_j^2)

Empirical Results

- Booth and Fama estimate return contributions of seven asset classes for different time periods and portfolios
  - Betas are estimated from quarterly data
  - Average returns and standard deviations are estimated from monthly data
- Incremental returns due to diversification are greater for small-cap stocks than for the other assets.
  - Substantially increases the premium of small-cap stocks over the S&P 500
  - Because small stocks are not highly correlated with other assets, their risk in a diversified portfolio is much less than their return variance. In other words, they bring a lot of diversification benefits.
- Diversification Returns vs. Investment Uncertainty
  - After running simulations, authors find that the average standard deviation for the random timing portfolio is equal to that of a constant-mix portfolio invested 53% in the S&P500 and 47% in bonds
  - Shifting weights not only reduces compounded return but also introduces uncertainty
  - Punchline: investors need a large premium to be willing to incur the additional uncertainty of active management