Share Repurchases and Stock Valuation Models

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Today's high levels of share repurchases should heighten investor interest about the impact of repurchases on stock valuations and about any implication for using traditional valuation methods. Total repurchases and net repurchases (repurchases minus new issues) are at unprecedented levels compared to cash dividends. In the face of high repurchases, the opinions about the usefulness of traditional valuation models, such as dividend discount models (DDMs), often fall into one of two extremes:

- *DDMs are obsolete.* Because DDMs do not incorporate the cash flows to investors from repurchases, they systematically understate the present value of future cash flows from a stock and underestimate the true rates of return on equity. Total cash flow models (based on dividends and repurchases combined) should replace DDMs for firms that repurchase shares.
- *Ignore repurchases and use DDMs anyway.* The logic behind the DDM is so basic and fundamental that it is impossible for the model to be wrong.

The complicating factor with respect to DDMs and share repurchases is accurately specifying the share-reduction effect associated with share repurchases. We describe an alternative discounted cash flow model that includes both dividends and repurchases; we refer to this model as a Total Cash Flow Model—TCFM. Done carefully, valuations and rate of return estimates from DDMs and TCFMs are consistent.

This article presents two conclusions about the effect of repurchases on valuation models. First, traditional DDMs that do not explicitly incorporate repurchases are valid (given correctly specified inputs) and can still be used to establish unbiased valuations and required rate of return estimates for common stocks. Second, TCFMs that include both dividends and repurchases also result in unbiased estimates of value and required rates of return. In fact, valuation estimates from correctly specified DDMs and TCFMs will be identical.

The article is organized as follows. We first present background on the scope and significance of repurchases, a brief restatement of traditional DDMs, and a review of prior uses of TCF in valuation models. The following section presents the general form of the TCFM and establishes the logical consistency between DDMs and TCFMs. We then present the special case of the constant-growth TCFM (analogous to the constant-growth DDM) and discuss issues in using TCFMs. We conclude that correctly specified DDMs and TCFMs will identically value common shares, although analysts may have a preference between the two models in specific applications.

BACKGROUND

In this section, we first summarize the contemporary economic significance of share repurchases. Although a large published literature on repurchases exists, very little of it focuses on the role of repurchases in valuation models. After reviewing the facts about the traditional DDM, we will present some analyst commentary about the implications of repurchases for valuation models. As we will try to make clear, the guidance regarding these implications can be incomplete or inaccurate.

Economic Significance of Repurchases

Repurchases have grabbed the market's attention in recent years because of their unprecedented scale. Payout policy is also topical in the U.S. because the Jobs and Growth Tax Reconciliation Act of 2003 lowered the maximum tax rate on qualifying dividends to 15%, the same tax rate that applies to realized long-term capital gains. For the first 70 years of the twentieth century, repurchases represented an almost insignificant portion of firm payout. During the 1990s, repurchases grew rapidly compared to dividend distributions, and repurchases were increasingly substituted for dividends as the method to distribute earnings.

The aggregate levels of repurchases and dividends are given in Exhibit 1 for the 20-year period 1987–2006. Two sets of data—S&P 500 companies (on the left side of the exhibit) and all companies in the S&P Compustat database (on the right side)—are presented. The first set is the S&P 500 members as of early 2007. The second set includes all publicly traded U.S. companies in the Compustat active and research files. In both sets, the amounts of repurchases relative to dividends have grown substantially over this time period, with the ratio of repurchases to cash dividends roughly doubling in the second half of the period. Consider first the S&P 500 companies. For the first half of the period (1987–1996), the ratio of repurchases to dividends is highly variable, averaging 55%, and for the second half of the period (1997–2006), it averages 117%. Net repurchases (repurchases minus sales of shares) averaged 20% of dividends for the first half of the period and 69% for the second half. For the S&P 500 firms, repurchases exceeded dividends for 7 of the last 10 years and were dramatically larger in 2005 and 2006. For all Compustat companies, the second set of data, the trends are similar. The ratio of repurchases to dividends for the Compustat companies tends to be about one-third lower than for the S&P 500 companies, and it also has doubled in the last half of the period. Relative to dividends, the set of all Compustat companies has engaged in more stock sales, although these companies changed from net sellers of shares to net repurchasers of shares between the two halves of the time period. It is clear that repurchases and net repurchases currently are at unprecedented levels.

Not only is repurchase activity at record levels, but variations in total yields (repurchases and dividends) now appear to be more valuation-relevant than variations in dividend yields alone. Boudoukh, Michaely, Richardson, and Roberts [2007] found that payout yields (including repurchases and dividends) that were once positively correlated to dividend yields are no longer significantly related. They also found that payout yields have significant time-series predictability for stock returns and that dividend yields do not. Skinner [2008] found that a small set of large companies is responsible for the majority of total payouts, dividend policy is becoming more conservative, and dividends are less related to earnings than are repurchases. He finds a declining propensity to pay dividends, particularly outside of large companies, and suggests that repurchases are becoming the dominant form of payout.

Our concern in this article is how share repurchases should be viewed when doing valuations. Most of the published literature on share repurchases has not focused on valuation, focusing instead on institutional features and other important economic issues, such as the agency, signaling, and tax effects of repurchases. Some practical considerations that might favor the use of repurchases include favorable tax treatment, the flexibility they provide for firms in managing earnings per share, and as signals of good news. Considerations that might argue against the use of repurchases are their lack of transparency, potential unequal treatment of sellers and nonsellers, and their lesser ability (compared to cash dividends) to discipline managers. For extensive recent surveys of the literature on dividend policy and share repurchases, see Bratton [2005] and Vermaelen [2005].

EXHIBIT 1
Repurchases, Dividends, and Sales, 1987–2006

	S&P 500 Companies			All Compustat Companies		
Year	Repurchases	Dividends	Sales	Repurchases	Dividends	Sales
2006	440.8	233.9	94.6	712.5	584.8	292.7
2005	320.2	244.2	101.7	548.2	607.3	292.2
2004	204.6	181.0	94.6	376.2	518.0	290.7
2003	123.4	156.3	62.4	228.4	431.2	199.4
2002	124.4	137.2	57.2	214.9	369.8	175.9
2001	129.7	132.0	87.6	226.0	363.2	267.1
2000	149.2	124.5	66.1	244.8	355.0	396.0
1999	131.1	113.5	66.0	247.9	341.9	311.8
1998	121.1	103.6	43.7	244.1	309.1	217.3
1997	100.2	87.4	40.0	185.2	293.2	180.9
1996	64.7	79.1	32.2	126.1	271.6	192.9
1995	58.6	73.8	22.4	98.3	242.8	124.2
1994	33.3	68.0	20.0	60.6	214.6	117.4
1993	26.3	65.3	32.3	53.1	198.9	156.0
1992	24.4	61.3	28.3	46.2	191.7	119.9
1991	16.3	57.0	23.6	36.6	184.7	97.4
1990	24.6	55.8	12.2	56.9	186.7	54.4
1989	29.4	51.4	18.5	72.5	176.8	77.5
1988	28.0	48.9	10.3	68.1	168.2	47.4
1987	32.5	44.3	14.6	74.4	154.2	97.3

Note: Repurchases, dividends, and sales are in U.S.$ billions.

Traditional DDMs

We briefly restate here the familiar DDMs that some analysts consider obsolete. The traditional dividend discount models for finite and indefinite holding periods are, respectively,

$$P_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n} \qquad (1)$$

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} \qquad (2)$$

Gordon [1962] incorporated the assumption that dividends are expected to grow at a constant rate, $D_t = D_{t-1}(1+g)$, which, when used for an indefinite holding period, resulted in the familiar Gordon growth model,

$$P_0 = \frac{D_0(1+g)}{r-g} = \frac{D_1}{r-g} \qquad (3)$$

The discount rate r that makes the present value of expected future dividends (and expected future stock price) equal the current stock price is an estimate of the stock's expected rate of return. The Gordon growth model estimate of expected return is

$$r = \frac{D_1}{P_0} + g = d + g \qquad (4)$$

where the equilibrium rate of return has two components—the dividend yield and the growth rate. In the Gordon model, the dividend grows at rate g, which implies that the stock price P_t is also expected to grow at rate g. The expected values of two return components are also constant through time, such that $d = D_t/P_{t-1}$ and $g = (P_t - P_{t-1})/P_{t-1}$. The basic Gordon model also assumes that earnings grow at the same rate and that the payout ratio is fixed.

As a numerical example of the Gordon model, assume that D_1 is \$1.00, $g = 5\%$, and $r = 9\%$. The stock price is $P_0 = 1.00/(0.09 - 0.05) = \$25.00$. The expected return of 9% is composed of an expected dividend yield of 4% and an expected capital gains yield of 5%. Both dividends and the stock price are expected to grow at $g = 5\%$.

Repurchases and Total Cash Flow Analysis

Some analysts have explored the valuation implications of repurchases and, in our opinion, reached inappropriate conclusions. In a series of articles, Lamdin addresses the repurchase issue, using numerical examples to tackle some of the difficulties inherent in valuing companies whose shares are being repurchased. Lamdin [2001c, p. 486] stated that "[a]t an intuitive level, the discounted dividend model fails because cash flows to shareholders fall through the cracks... . To correctly estimate the cost of equity with a discounted dividend model requires that appropriate adjustments be made." Lamdin [2001b, p. 62] also stated that "users of the DDM are well advised to be aware of the repurchasing policy of corporations. Those that regularly repurchase will have its [sic] cost of equity underestimated by the standard DDM." Regarding the discounted dividend valuation model, Lamdin [2000, p. 252] argued that "[u]nless the analyst adds repurchases to the model, however, it will undervalue corporations that use share repurchases, in addition to dividends, to distribute cash to investors." Echoing Lamdin's call for DDM revision, Randall [2000] concluded that "[o]ver time, the usefulness of the model, as it is normally presented, seems to have diminished" and stated that the DDM should be revised, asserting that "[t]he solution to the problem is a modification of the traditional model as it is normally presented. Essentially, one must discount the total value distributed as dividends and share repurchases."

By clarifying the relationship between the two models, we clarify the effects of repurchases on DDMs and TCFMs.

TOTAL CASH FLOW VALUATION

In this section, we will describe the logic of the general total cash flow model (TCFM), demonstrate the logical equivalence of the DDMs and TCFMs, and mention a basic mistake that is easily committed when implementing a TCFM.

The Logic of TCFMs

A TCFM values expected dividends and expected share repurchases. In general, for an n-year holding period, the present value of the expected cash flows is

$$P_0 = \sum_{t=1}^{n-1} \frac{F_t D_t + f_t P_t}{(1+r)^t} + \frac{F_n D_n + F_n P_n}{(1+r)^n} \tag{5}$$

In a TCFM, the total cash flow in period t is the expected dividend and expected repurchase, $F_t D_t + f_t P_t$, where

F_t = fraction of the *original* shares that has not been repurchased prior to period t,
f_t = fraction of the *original* shares that is repurchased at time t,
$F_t D_t$ = expected amount of cash dividends received at t per each share owned today, which reflects the effect of share repurchases prior to time t, and
$f_t P_t$ = expected proceeds from (payout to) share repurchases per each share owned today.

The proportion of shares outstanding, F_t, is reduced by the amounts repurchased each year, f_t, where $F_t = 1 - \Sigma_{t=1}^{t-1} f_t$ for $t > 1$. For $t = 1$, we have $F_1 = 1$. It is also convenient to use the relationship $F_t = F_{t-1} - f_t$.

For an indefinite holding period, the present value of expected cash flows with a TCFM is

$$P_0 = \sum_{t=1}^{\infty} \frac{F_t D_t + f_t P_t}{(1+r)^t} \tag{6}$$

Each year, the expected cash flow is the expected amount received from dividends (adjusted for prior-period share repurchases) plus the expected amount received from repurchases at t, per share owned today. This model contrasts with the general DDM, Equation (2), for which the expected cash flow each year is the expected dividend on shares then outstanding, D_t.

These cash flows can be visualized with the use of Exhibit 2, which shows the cash flows for various holding periods along the rows and the expected cash flows for the TCFM in the bottom row.

Logical Consistency of DDMs and TCFMs

Given the dubious argument that share repurchases make DDMs obsolete and require the use of a TCFM, we wish to assert their logical equivalence. Consider the dividends and share prices laid out in Exhibit 2. For DDMs, such as Equations (1) and (2), the value of a share given a particular holding period can be found by applying the DDM to value the cash flows in the exhibit row corresponding to that holding period. For a TCFM, the expected cash flows for each year (in each column) are estimated and then discounted, as in Equation (6), to find their total present value. Instead of finding their expected values and then discounting them, we can reverse the process. We can discount the cash flows (for each holding period, or row) in Exhibit 2, and then find their expected values. Doing this results in

$$P_0 = \sum_{i=1}^{\infty} f_i \left(\sum_{t=1}^{i} \frac{D_t}{(1+r)^t} + \frac{P_i}{(1+r)^i} \right) \tag{7}$$

Equation (7) is equivalent to Equation (6); the only difference is the *order* in which the calculations are performed. In fact, the TCFM estimate of value is nothing but a weighted average of DDM estimates of value. The components of a TCFM are the same as those of a DDM, and the two approaches are logically equivalent.

Accounting for Share Reduction in Using TCFMs

Accurately estimating the expected cash flows is fundamental to any DCF model. An underlying mistake, easily made and appearing in different guises, is failing to account for the reduction in the number of shares due to repurchases. Someone making this mistake will overestimate the cash flows that occur with a TCFM.

To illustrate this point, consider the simple example given earlier for the Gordon model. Assume that a stock selling for \$25.00 pays a \$1.00 dividend in year one and a \$1.05 dividend in year two. The stock has a 9% required rate of return and should sell for \$26.25 at the end of year

EXHIBIT 2
Dividends and Share Prices for Various Holding Periods

Holding Period	Fraction of Original Shares Repurchased	Cash Flow at Time t					
		1	2	3	...	n	...
1	f_1	D_1+P_1					
2	f_2	D_1	D_2+P_2				
3	f_3	D_1	D_2	D_3+P_3			
⋮	⋮	⋮	⋮	⋮			
n	f_n	D_1	D_2	D_3	...	D_n+P_n	
⋮	⋮	⋮	⋮	⋮		⋮	...
	Sum = 1.0	Column Expected Values					
		$D_1+f_1P_1$	$F_2D_2+f_2P_2$	$F_3D_3+f_3P_3$	...	$F_nD_n+f_nP_n$	

one and will be sold for $27.56 at the end of year two. The firm repurchases some shares in year one and the investor can elect to sell some shares. We show the stock valued correctly with a DDM and a TCFM, but then valued incorrectly by double counting the cash flow.

For the DDM, these cash flows are as follows:

DDM cash flows

Cash flow	25	1.00	(1.05 + 27.56) = 28.61
	+---------+---------+---------→		
Time	0	1	2

In the DDM case, the investor does not sell back any shares at $t = 1$. For this cash flow stream, the present value is $25 and the rate of return is 9%.

In the TCFM case, the investor sells back some shares at $t = 1$. If the firm repurchases $2.00 of its stock from the investor in the first period, then at the end of Time 1 the investor will receive both the $1.00 dividend and the expected $2.00 of repurchases. The ex dividend price at Time 1 will be $26.25 (= 25.00 × 1.09 − 1.00), and the investor sells back 2.00/26.25 shares. Thus, for the TCFM, the expected cash flows for the investor who sells back $2.00 of shares at the end of the first period are the following:

TCFM cash flows

Cash flow	25	1.00 + 2.00 = 3.00	(1 − 2.00/26.25) × (1.05 + 27.56) = 26.43
	+--------+------------+----------→		
Time	0	1	2

For this cash flow stream, the present value is $25 and the rate of return is, again, 9%.

If the investor tries to use the TCFM approach and does not account for the reduction of the number of shares caused by the repurchase, the investor can double count the cash flows as follows:

Double-counted cash flows

Cash flow	25	1.00 + 2.00 = 3.00	(1.05 + 27.56) = 28.61
	+---------+-------------+--------→		
Time	0	1	2

For the double-counted cash flows, the present value (discounted at 9%) is $26.83 and the rate of return is 13.14%; the investor is clearly double counting some cash flows. Compared to the DDM, this investor is assuming an extra $2.00 at Time 1 and the same cash flow at Time 2. Compared to the correctly specified TCFM, this investor is assuming the same cash flow at Time 1 and an extra $2.18 at Time 2. The investor essentially trades off $2.18 at Time 2 for $2.00 at Time 1 when he sells part of his shares. But the investor cannot enjoy both outcomes—the investor cannot have his cake and eat it too.[1]

By accurately tracking the number of shares and the associated dividends and share repurchases the investor receives, both a DDM and a correctly specified TCFM will result in the same valuations and rates of return for a stock investment with any pattern of dividends and repurchases. When analysts find different valuations, they are failing to accurately account for the share-reduction effect that results when the investor sells back some shares—trading the future dividends and stock proceeds for the repurchase price.

CONSTANT-GROWTH TCFM

Just as in the case of DDMs, one can develop single-stage (constant-growth) and multistage TCFMs. In this section, we will present a constant-growth TCFM that is the analog of the constant-growth DDM. As in the constant-growth DDM, total earnings of the firm grow at g_T, and the firm's retention and payout rates are held constant.

Assume the firm pays out a dividend and repurchase amount (per each currently outstanding share) of D_1 and B_1, respectively, at the end of the first period, and that total distributions grow at total cash flow growth rate g_T. With $d = D_1/P_0$ and $b = B_1/P_0$, the value per share is

$$P_0 = \frac{D_1 + B_1}{r - g_T} \quad \left(\text{and also } P_0 = \frac{D_1 + B_1}{d + b}\right) \tag{8}$$

In Equation (8), the investor's return can be expressed as the sum of a dividend yield and an appreciation yield. The appreciation yield has two parts, $g = b + g_T$, where g_T is the total cash flow growth rate and the appreciation that would be realized with no repurchases, and b is the repurchases yield, or the extra growth rate for dividends and share price due to repurchases,

$$r = \frac{D_1}{P_0} + \frac{B_1}{P_0} + g_T = d + b + g_T$$

In the constant-growth model the dividend per share and the price per share will grow at rate $g = g_T + b$, while the total market value and total cash dividends will grow at rate g_T. The lower growth rate for total cash flows (and total market value and total dividends) occurs because a fraction of the shares is being repurchased each year. The proportion of outstanding shares repurchased each year is $f = b/(1 + r - d)$.

Exhibit 3 summarizes a worked example. A constant fraction f of the beginning-of-year outstanding shares is repurchased each year. In Year 1 in the example, the firm pays a dividend of \$0.60 and repurchases \$0.40 of stock, for a total payout of \$1.00. Total distributions will grow at 5% annually, and the required rate of return is 9%. The value of a share, found with Equation (8), is

$$P_0 = (D_1 + B_1)/(r - g_T) = (0.60 + 0.40)/(0.09 - 0.05) = \$25$$

The dividend yield is $d = 2.4\%$, the repurchase yield is $b = 1.6\%$, and the total cash flow growth rate is $g_T = 5\%$. Dividends per share (and share prices) will grow at $g = 6.6\%$. The difference between g and g_T is equal to the buyback yield of 1.6%. The difference between the two growth rates also reflects the rate of decline in outstanding shares due to repurchases. The fraction of the shares outstanding at the beginning of a period that is repurchased is $f = b/(1 + r - d) = 0.016/(1 + 0.09 - 0.024) = 0.016/1.066 = 0.015009$, or 1.5%.

The constant-growth DDM applies to the same scenario as the TCFM. The initial dividend of \$0.60 per share grows at 6.6%, so the value with the constant-growth DDM is

$$P_0 = D_1/(r - g) = 0.60/(0.09 - 0.066) = \$25$$

Essentially, the DDM discounts a cash flow stream of \$0.60 (Year 1 dividends) growing at 6.6% annually, while the TCFM discounts a cash flow stream of \$1.00 (Year 1 dividends and repurchases) growing at 5.0%. With the TCFM, the numerator includes an extra initial cash flow from repurchases, but the effect on value is offset by the lower total cash flow growth rate.

An examination of the rows and columns of Exhibit 3 helps explain the interrelationships between the DDM and TCFM approaches. For a given time horizon, the entries in the rows in Exhibit 3 (except the last, which pertains to the TCFM) are the expected per share cash flows for shares being repurchased at the respective horizon. Moving across from one time period to the next, both expected dividends per share and price per share

EXHIBIT 3
Numerical Example of Constant-Growth TCFM

Holding Period	Fraction of Original Shares Repurchased $f_t = f(1-f)^{t-1}$	Cash Flow at Time t					
		1	2	3	...	n	...
1	0.016/1.066 = 0.015	0.60 + 26.65					
2	(0.016/1.066)(1.05/1.066) = 0.0148	0.60	0.64 + 28.41				
3	(0.016/1.066)(1.05/1.066)^2 = 0.0146	0.60	0.64	0.68 + 30.28			
⋮	⋮	⋮	⋮	⋮			
n	(0.016/1.066)(1.05/1.066)$^{n-1}$	0.60	0.64	0.68	...	$D_n + P_n$	
⋮	⋮	⋮	⋮	⋮		⋮	...
	Sum = 1.0	Column Expected Values					
		1.00	1.05	1.1025	...		

grow at 6.6%. The DDM value at a given horizon is found as the present value of the row of expected cash flows. In the constant-growth TCFM, the entries in the second column (the repurchase fractions) sum to one. Thus, the TCFM value for the $t = 1$ cash flow, at the bottom of the column for $t = 1$, is simply the expected value of cash flows shown for $t = 1$. Similarly, for the bottom row, the expected value for the cash flow is given for $t = 2$, and so on. Moving across the bottom row of expected cash flows for the TCFM, the cash flows grow at 5%. The TCFM value is found as the present value of those expected cash flows.

In the TCFM, total distributions are what matter. If a company is going to make distributions of, say, 4% of the firm's value each period, it does not matter whether the firm's distribution is 100% dividends, 100% buybacks, or anything in between. It is also critical that the growth rate of the total distributions (in the previous example, 5%) is known. Cash dividends and cash repurchases are substitutes in this model. Moreover, when varying the mix of dividends and repurchases, the growth rates for dividends per share and total cash flows will differ except for the special case in which no repurchases are made.

To recap, Exhibit 4 reports the DDM and TCFM valuations a constant-growth firm with and without repurchases. In Case 1, a firm pays a dividend that grows at a constant rate. In Case 2, a firm pays out the same amount as in Case 1, but the payment is partially in the form of a dividend and partially in the form of share repurchases.

The two cases are firms with the same total payouts, but with a different payout mix. The two cases have the same valuations whether a DDM or TCFM is used. The price and the dividend per share will grow faster (6.6% instead of 5.0%) in Case 2, which reflects the lower dividend and the roughly 1.5% annual reduction in shares because of the repurchases.

In the constant-growth TCFM, it is possible to analyze repurchases incorrectly. If the analyst erroneously uses the growth rate of dividends per share (6.6%) in the TCFM, the estimate of value will be too high,

$$P_0 = \frac{D_1 + f_1 P_1}{r - g} = \frac{0.60 + 0.40}{0.09 - 0.066} = \frac{1.00}{0.024} = \$41.67$$

Because the actual growth rate of total cash flow is 5%, the correct valuation is \$25. Whether using a DDM or TCFM, the appropriate growth rate must be employed.

Consistent expected rates of return are also derivable from both a DDM and a TCFM. For the constant-growth DDM,

$$r = \frac{D_1}{P_0} + g = d + g$$

In the constant-growth DDM, it is inappropriate to look independently at the dividend yield or the growth rate. Similarly, for the constant-growth TCFM, where the expected rate of return is

$$r = \frac{D_1}{P_0} + \frac{fP_1}{P_0} + g_T = d + b + g_T$$

it would be inappropriate to look independently at the dividend yield, repurchase yield, or total cash flow growth rate. In Case 2 versus Case 1, investors are accepting a lower dividend yield and receiving a higher rate of appreciation (specifically 6.6%, which can be decomposed into a repurchase component of 1.6% and a total cash flow component of 5.0%). Perhaps one of the more egregious examples of misinterpreting these yields occurred when some analysts in the late 1990s justified high stock market valuations by the fact that total cash flow yields ($d + b$) were much higher than dividend yields (d alone). The benefit of the higher total cash flow yield is offset by a lower total cash flow growth rate.

DISCUSSION AND CONCLUSION

Do share repurchases make DDMs obsolete? The answer is unequivocal—absolutely not. The basic logic remains elegant and relevant. The key to effective use is properly accounting for the effect of repurchases on the dividend growth rate or rates. Moreover, it is possible to use another class of models, that is, TCFMs, which incorporate both cash dividends and share repurchases. If applied correctly, TCFMs result in the same valuations and estimates of the cost of equity as DDMs.

For some companies, total cash distributions (dividends and repurchases) may be a more practical measure of return to investors than dividends alone. This is particularly the case for a nondividend-paying company that is repurchasing shares. Companies that pay little or no dividends might be appropriately analyzed with a TCFM if they are distributing economically meaningful amounts

EXHIBIT 4
Comparison of the DDM and TCFM Valuations of a Constant-Growth Firm With and Without Repurchases

	Case 1 Constant-Growth Firm with No Repurchases	Case 2 Constant-Growth Firm with Repurchases
D_1, initial dividend	$D_1 = 1.00$	$D_1 = 0.60$
B_1, initial repurchase	$B_1 = 0.00$	$B_1 = 0.40$
r, required return	$r = 9\%$	$r = 9\%$
g, growth rate of dividends per share	$g = 5\%$	$g = 6.6\%$
g_T, growth rate of total payouts	$g_T = 5\%$	$g_T = 5\%$
$P_0 = \dfrac{D_1}{r-g}$ (DDM)	$P_0 = \dfrac{1.00}{0.09-0.05} = 25.00$	$P_0 = \dfrac{0.60}{0.09-0.066} = 25.00$
$P_0 = \dfrac{D_1+B_1}{r-g_T}$ (TCFM)	$P_0 = \dfrac{1.00+0.00}{0.09-0.05} = 25.00$	$P_0 = \dfrac{0.60+0.40}{0.09-0.05} = 25.00$
d, dividend yield	$d = 4\%$	$d = 2.4\%$
b, repurchase yield	$b = 0\%$	$b = 1.6\%$
$f = b/(1+r-d)$, repurchase fraction	$f = 0/1.05 = 0\%$	$f = 0.016/1.066 = 0.015009 = 1.5\%$

of cash through repurchases. If total payout is more valuation-relevant, as Boudoukh et al. [2007] and Skinner [2008] believed, such that total distributions align with free cash flow better than dividends alone, a TCFM may be preferable to a DDM.

An important insight is that the TCFM works for both repurchases and sales of shares. Although the examples in this article were for repurchases, where $f_i > 0$ in Equation (5) or (6), the TCFM can also incorporate sales of shares with a repurchase fraction less than zero, $f_i < 0$. Another important insight is that the general DDMs and TCFMs—Equations (1) and (2) or Equations (5) and (6)—do not require the assumption that future stock prices are at their equilibrium values. When a company repurchases its overvalued shares, it dilutes the shares of remaining shareholders and shifts wealth from long-term holders to sellers. Conversely, repurchasing shares when they are undervalued benefits long-term holders at the expense of sellers. Although share repurchases/sales are a financial transaction that may not create value per se, wealth transfers do occur.

Some of the detail provided in this article was necessary to establish the equivalence between DDMs and TCFMs. Someone applying the TCFM could use total distributions at the firm level instead of Equation (5) or (6) as

$$\text{Firm PV} = \sum_{t=1}^{\infty} \frac{\text{Total div} + \text{Total repurch at time } t}{(1+r)^t} = \sum_{t=1}^{\infty} \frac{\text{Total distributions at time } t}{(1+r)^t}$$

where total distributions at time t are cash dividends paid plus net share repurchases (repurchases minus sales) for the firm as a whole. Firm PV is simply divided by the number of outstanding shares to get the value per share. Applying the TCFM in this manner may help users avoid the accounting mistakes that have sometimes occurred.

Analysts should be comfortable using traditional DDMs when those models are appropriate, but should also be facile at valuing total distributions with a TCFM if that approach better fits the circumstances. Both approaches are valid and, when correctly applied, mutually consistent.

ENDNOTE

[1]The issue of double counting cash flows can also be illustrated with a loan analogy. Bonds, mortgages, and loans also include cash flows that are a mix of interest payments and principal payments. The value of a debt instrument is the present value of its future cash flows. A simple example is a three-year \$100 loan with 10% interest for the first two years. The cash flows for the loan for years 1, 2, and 3, respectively, would be \$10, \$10, and \$110. If the issuer repurchases half of the debt at $t = 1$ for \$50 (or the borrower prepays \$50 at $t = 1$), the cash flows for years 1, 2, and 3, respectively, would be \$60, \$5, and \$55. The value of the cash flows at $t = 0$ remains \$100 and the return is still 10%. The value of the loan was not increased by the extra cash flow at $t = 1$ because it was associated with smaller cash flows in the future. Similarly, a shareholder selling his shares accepts a cash flow in exchange for smaller cash flows in the future. Of course, if a borrower has the option to prepay a loan when yields drop, this option has the potential to shift wealth from lenders to borrowers. Conversely, if lenders can call a loan when yields increase, this option potentially shifts wealth to the lenders. The expected costs of these options should be reflected in the pricing of the loan at $t = 0$.

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