Market and Non-Market Return

- $R_{i,t}$ and $R_{m,t}$: returns in excess of cash ($R_{i,t} = r_{i,t} - r_{f,t}$, $R_{m,t} = r_{m,t} - r_{f,t}$)
- "Characteristic line" regression of asset i's excess return on the market excess return:

$$R_{i,t} = \underbrace{\alpha_i + \beta_i R_{m,t}}_{\text{regression line}} + \underbrace{\epsilon_{i,t}}_{\text{residual}} \tag{1}$$

- Random: $R_{i,t}$, $R_{m,t}$, $\epsilon_{i,t}$; constant: α_i, β_i
- By construction,

$$\mathrm{cov}\,(\epsilon_{i,t}, R_{m,t}) = 0 \quad \text{and} \quad E(\epsilon_{i,t}) = 0 \tag{2}$$

- Return decomposition:
  - $\beta_i R_{m,t}$: market-related *realized* return
  - $\epsilon_{i,t}$: non-market-related *realized* return
  - α_i: part of *expected* return

$$\alpha_i = E(R_{i,t}) - \beta_i E(R_{m,t}) \tag{3}$$

Market and Non-Market Risk

- Variance of a sum: x and y random; a and b constants

$$\text{Var}(ax + by) = a^2\text{Var}(x) + b^2\text{Var}(y) + 2ab\text{Cov}(x, y) \tag{4}$$

- Applying this rule to the regression:

$$\begin{aligned} \text{Var}(R_{i,t}) &= \text{Var}(\alpha_i + \beta_i R_{m,t} + \epsilon_{i,t}) \\ &= \underbrace{\beta_i^2\text{Var}(R_{m,t})}_{\text{market-related risk}} + \underbrace{\text{Var}(\epsilon_{i,t})}_{\text{non-market risk}} \end{aligned} \tag{5}$$

- The R^2 ("R-squared") gives the fraction of the asset's total variance that can be attributed to its market-related variance:

$$R^2 = \frac{\beta_i^2\text{Var}(R_{m,t})}{\text{Var}(R_{i,t})} \tag{6}$$

- Non-market variance:

$$\text{Var}(\epsilon_{i,t}) = (1 - R^2)\text{Var}(R_{i,t}) \tag{7}$$

Beta and R-Squared

- Slope coefficient is the ratio of the covariance of the "dependent" and "independent" variables to the variance of the independent variable:

$$\beta_i = \frac{\text{Cov}(R_{i,t}, R_{m,t})}{\sigma^2(R_{m,t})} \tag{8}$$

- Beta can also be expressed in terms of correlation and standard deviations:

$$\beta_i = \frac{\sigma(R_i)}{\sigma(R_m)}\rho(R_i, R_m), \tag{9}$$

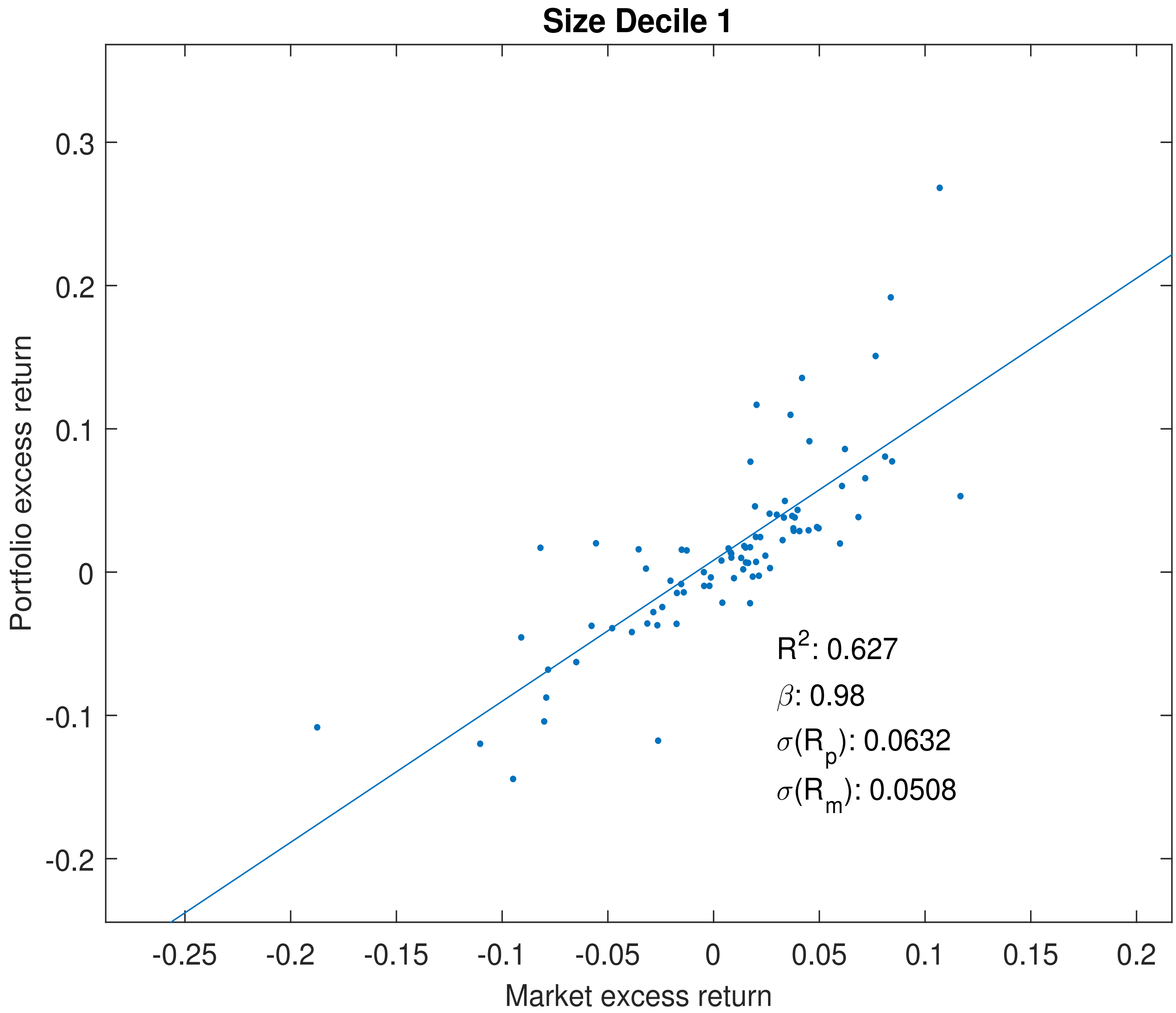
since correlation (denoted ρ) is defined as

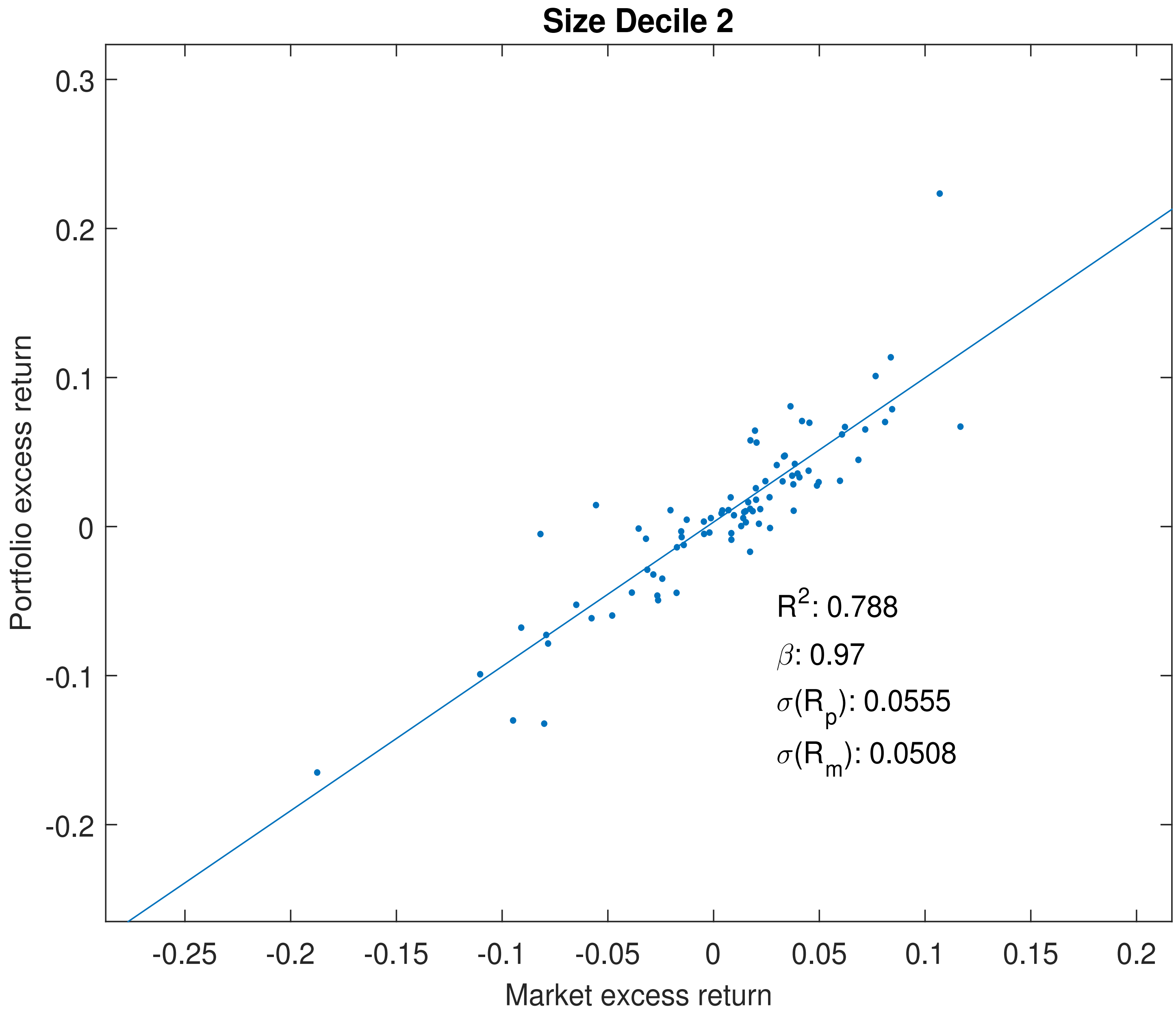
$$\rho(R_i, R_m) = \frac{\text{Cov}(R_{i,t}, R_{m,t})}{\sigma(R_i)\sigma(R_m)}. \tag{10}$$

- In a simple regression (one independent variable), R-squared is squared correlation:

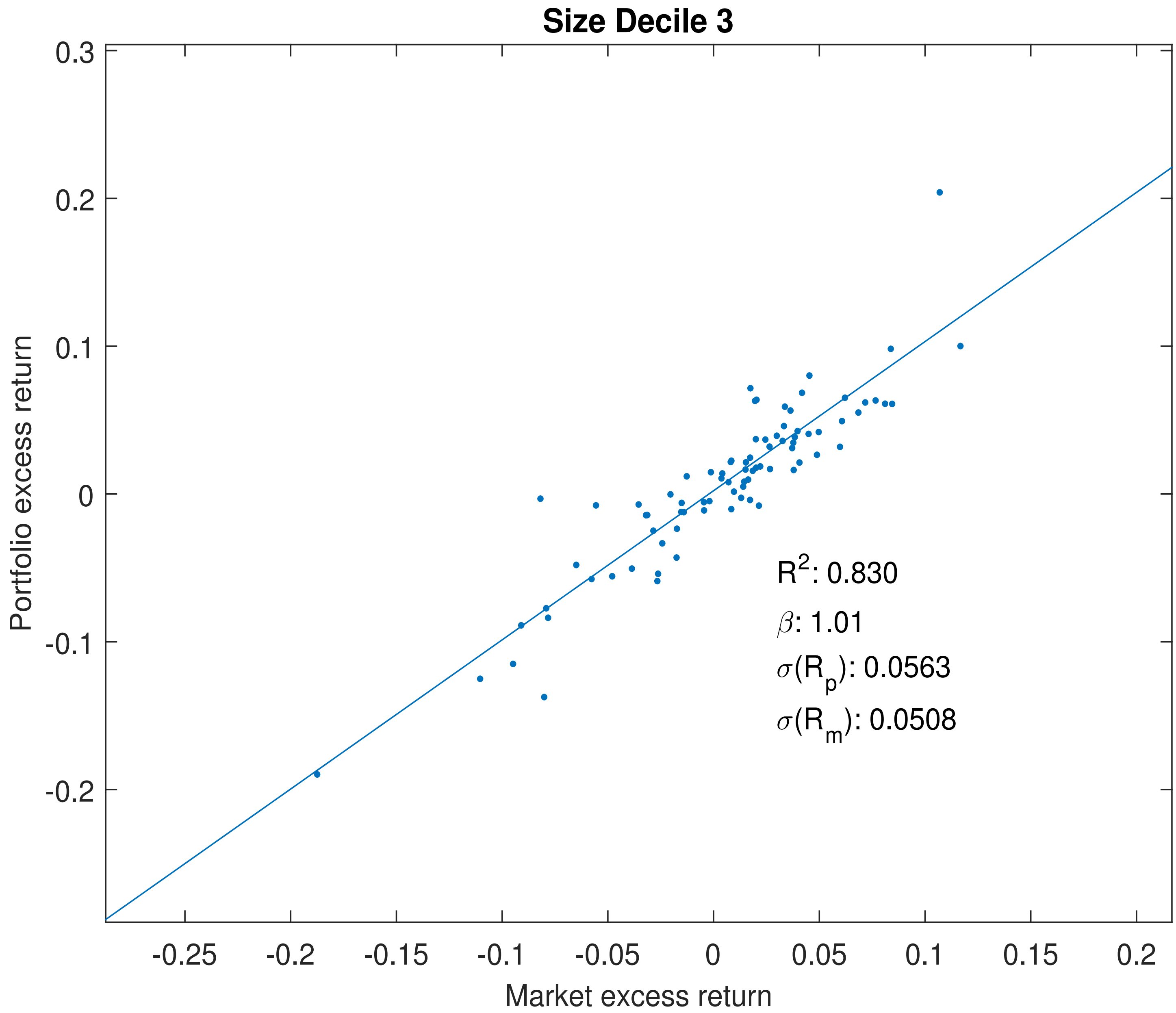
$$R^2 = [\rho(R_i, R_m)]^2. \tag{11}$$

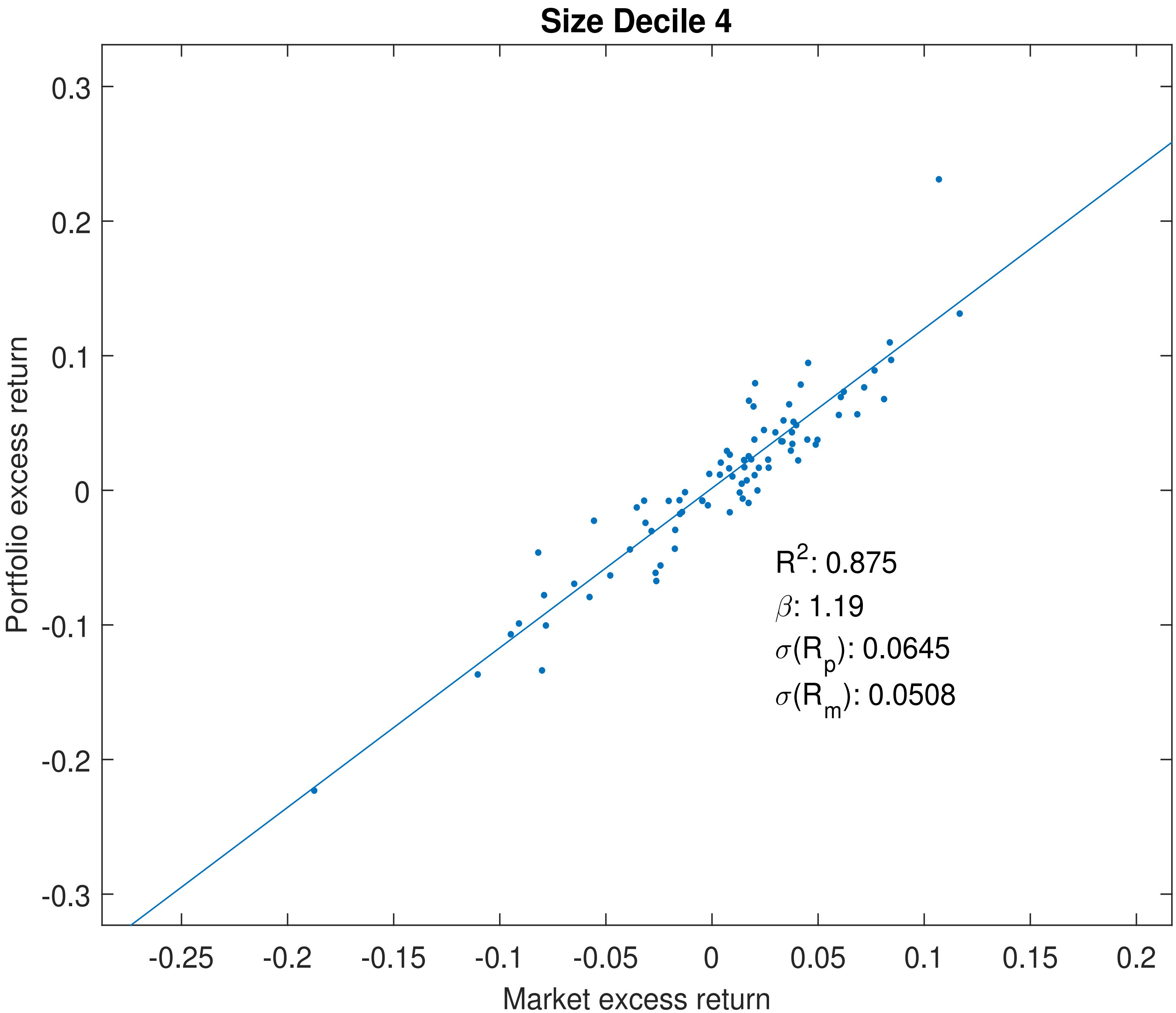
- Important: β_i and R^2 describe different properties


Size Decile 1
Portfolio excess return
Market excess return
0.3
0.2
0.1
0
-0.1
-0.2
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
R^2: 0.627
β: 0.98
$\sigma(R_p)$: 0.0632
$\sigma(R_m)$: 0.0508

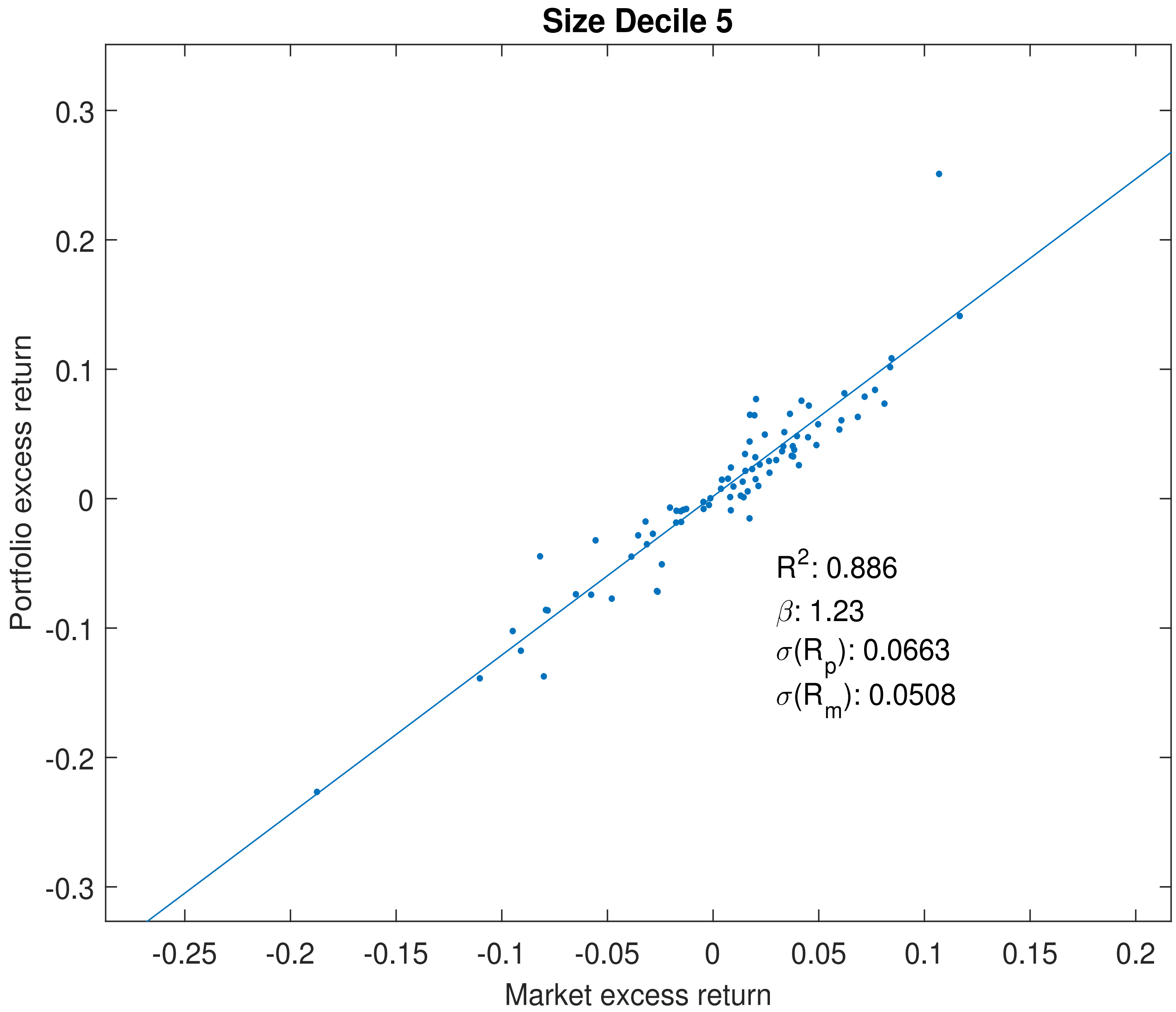


Size Decile 2
0.3
0.2
0.1
0
-0.1
-0.2
Portfolio excess return
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Market excess return
R2: 0.788
β: 0.97
σ(Rp): 0.0555
σ(Rm): 0.0508

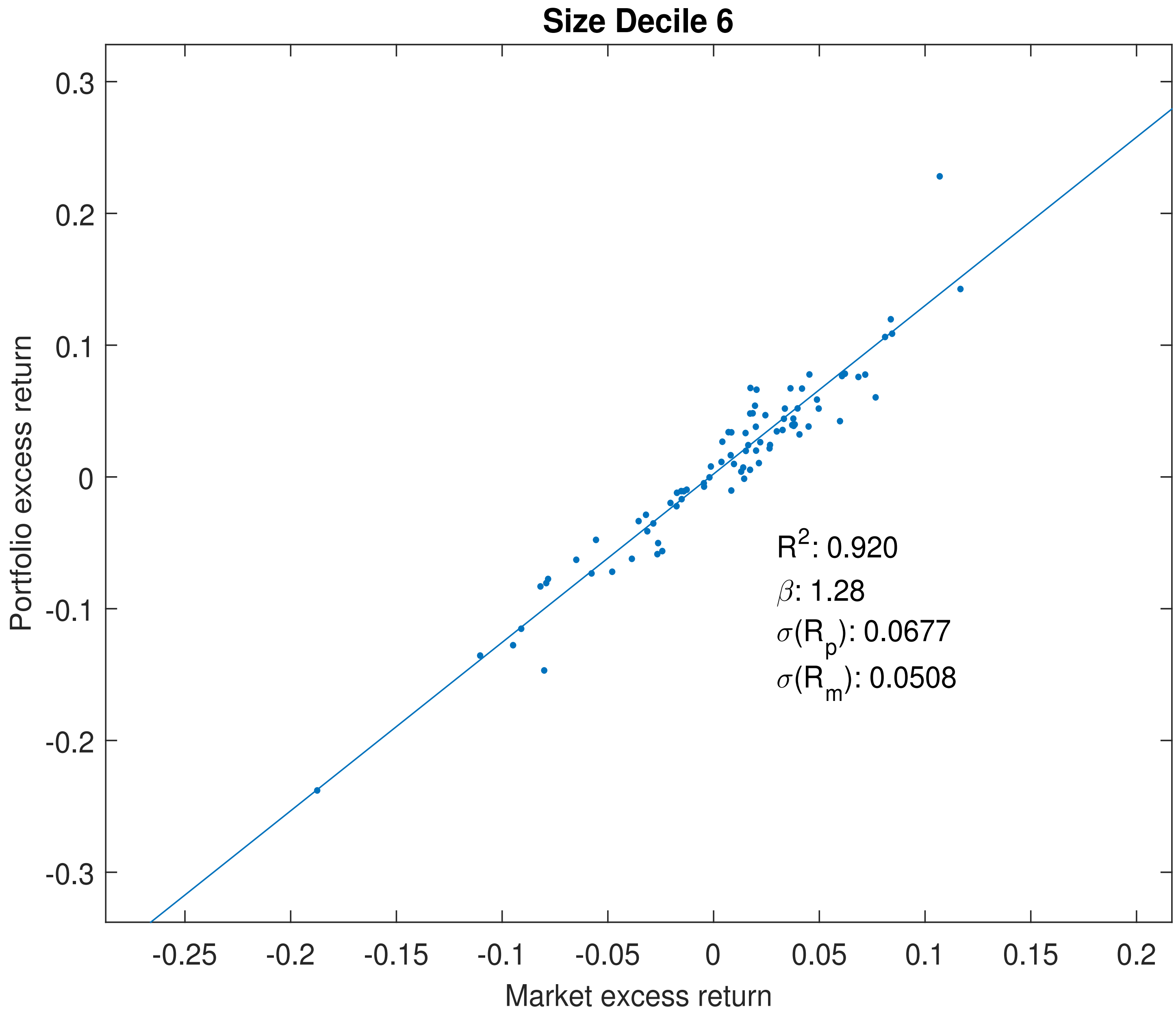

Size Decile 3
Portfolio excess return
Market excess return
0.3
0.2
0.1
0
-0.1
-0.2
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
R^2: 0.830
β: 1.01
$\sigma(R_p)$: 0.0563
$\sigma(R_m)$: 0.0508



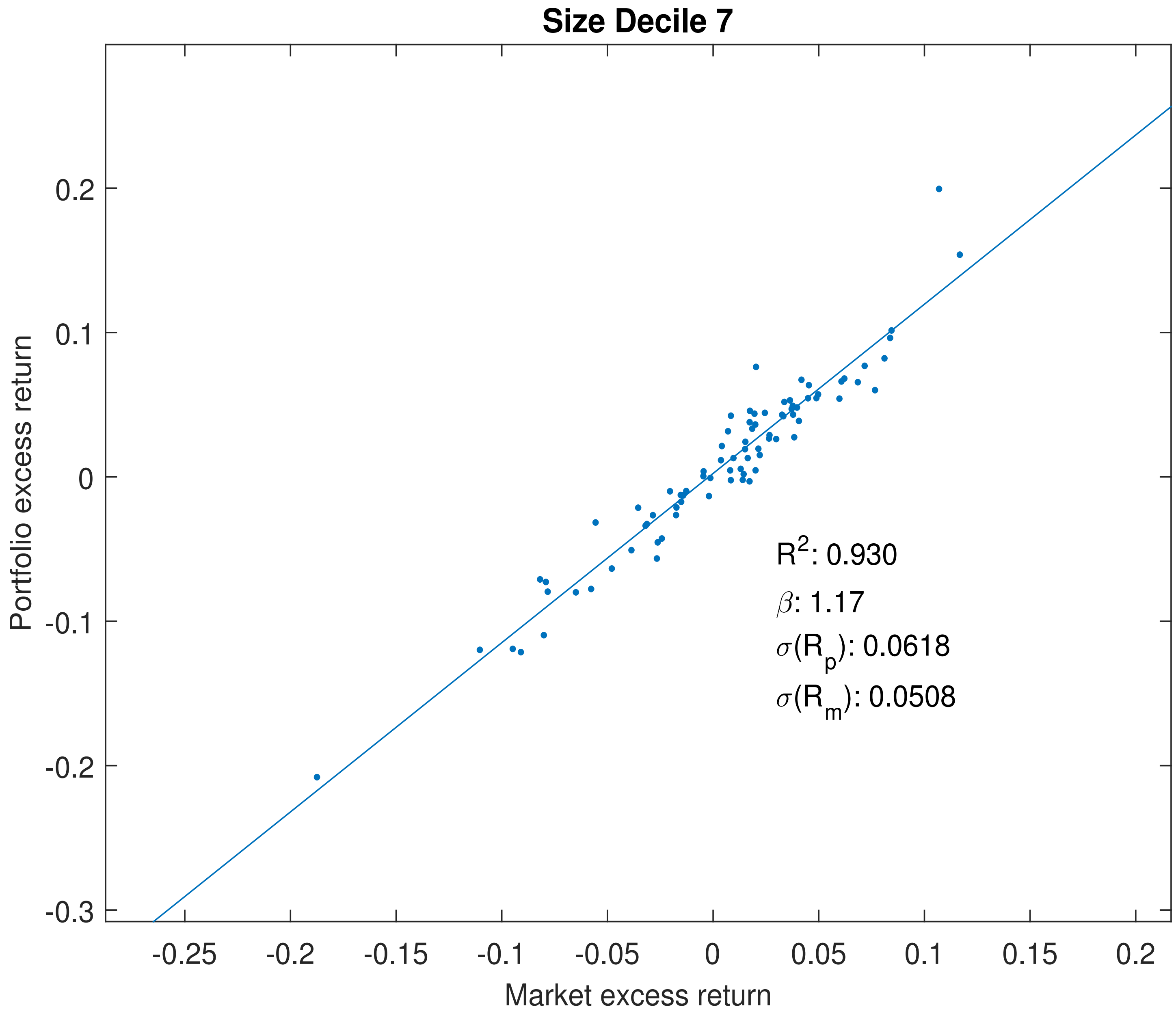
Size Decile 4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
Portfolio excess return
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Market excess return
R^2: 0.875
β: 1.19
$\sigma(R_p)$: 0.0645
$\sigma(R_m)$: 0.0508



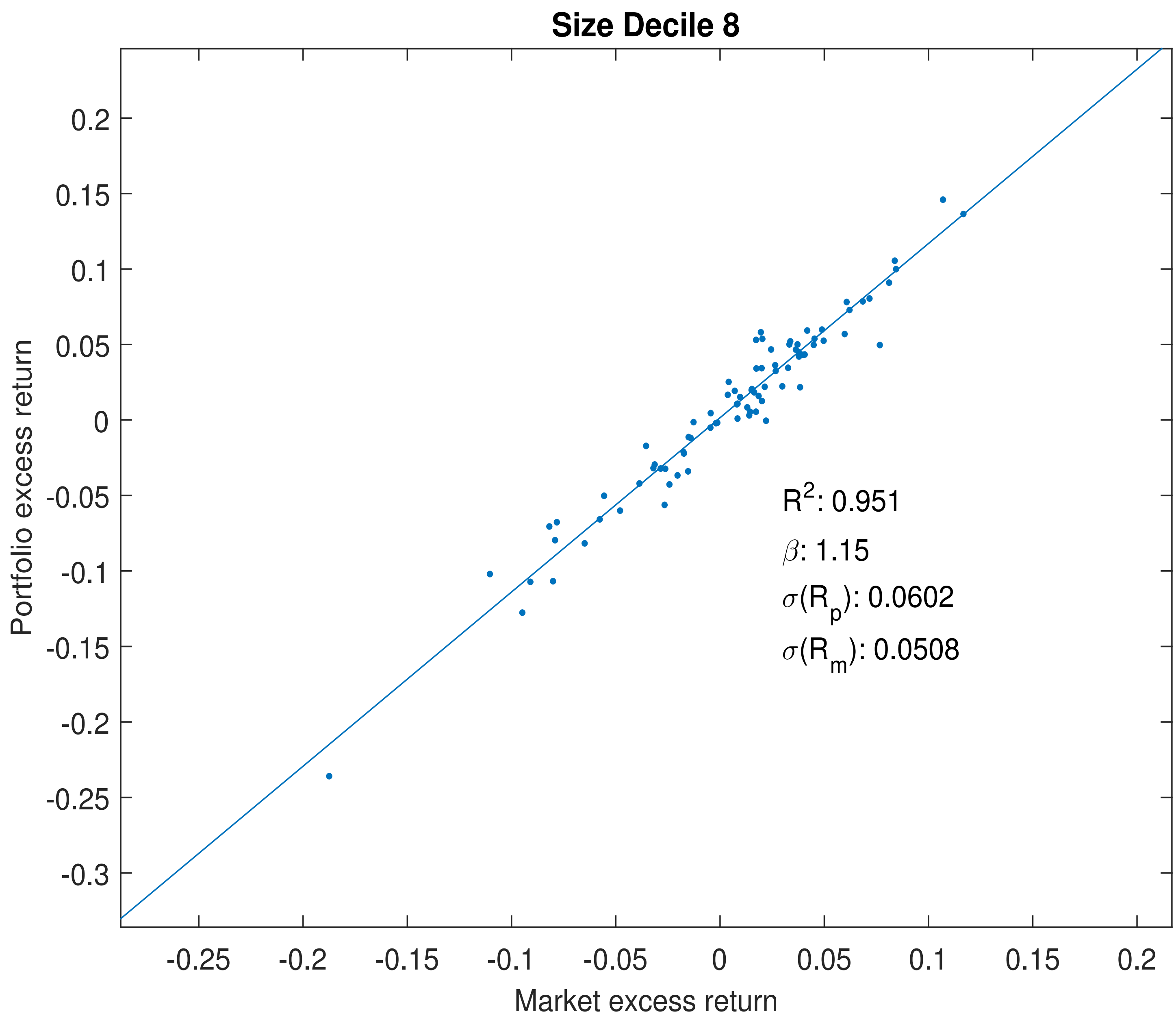
Size Decile 5
R^2: 0.886
β: 1.23
$\sigma(R_p)$: 0.0663
$\sigma(R_m)$: 0.0508
Portfolio excess return
Market excess return
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2



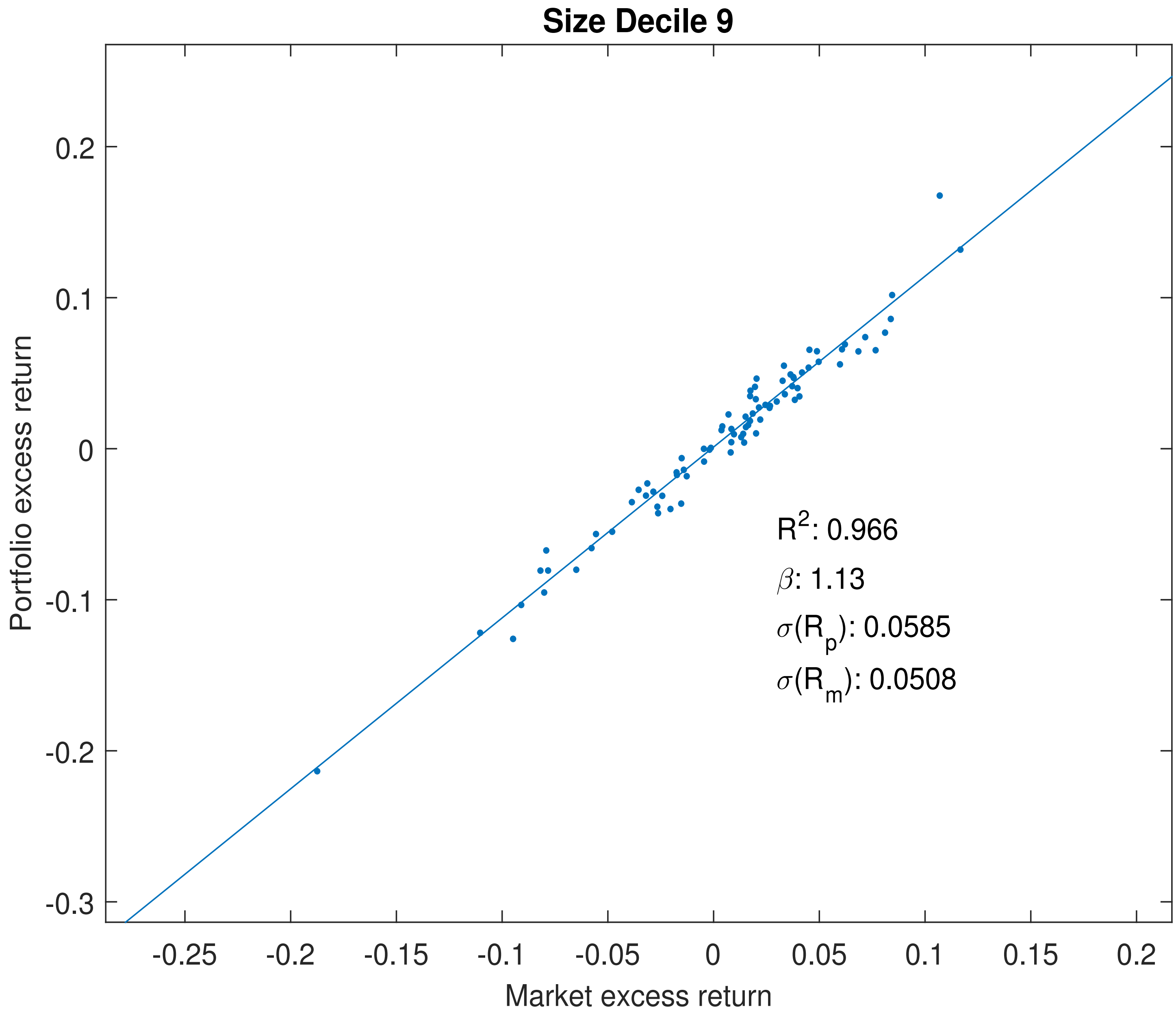
Size Decile 6
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
Portfolio excess return
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Market excess return
R^2: 0.920
β: 1.28
$\sigma(R_p)$: 0.0677
$\sigma(R_m)$: 0.0508



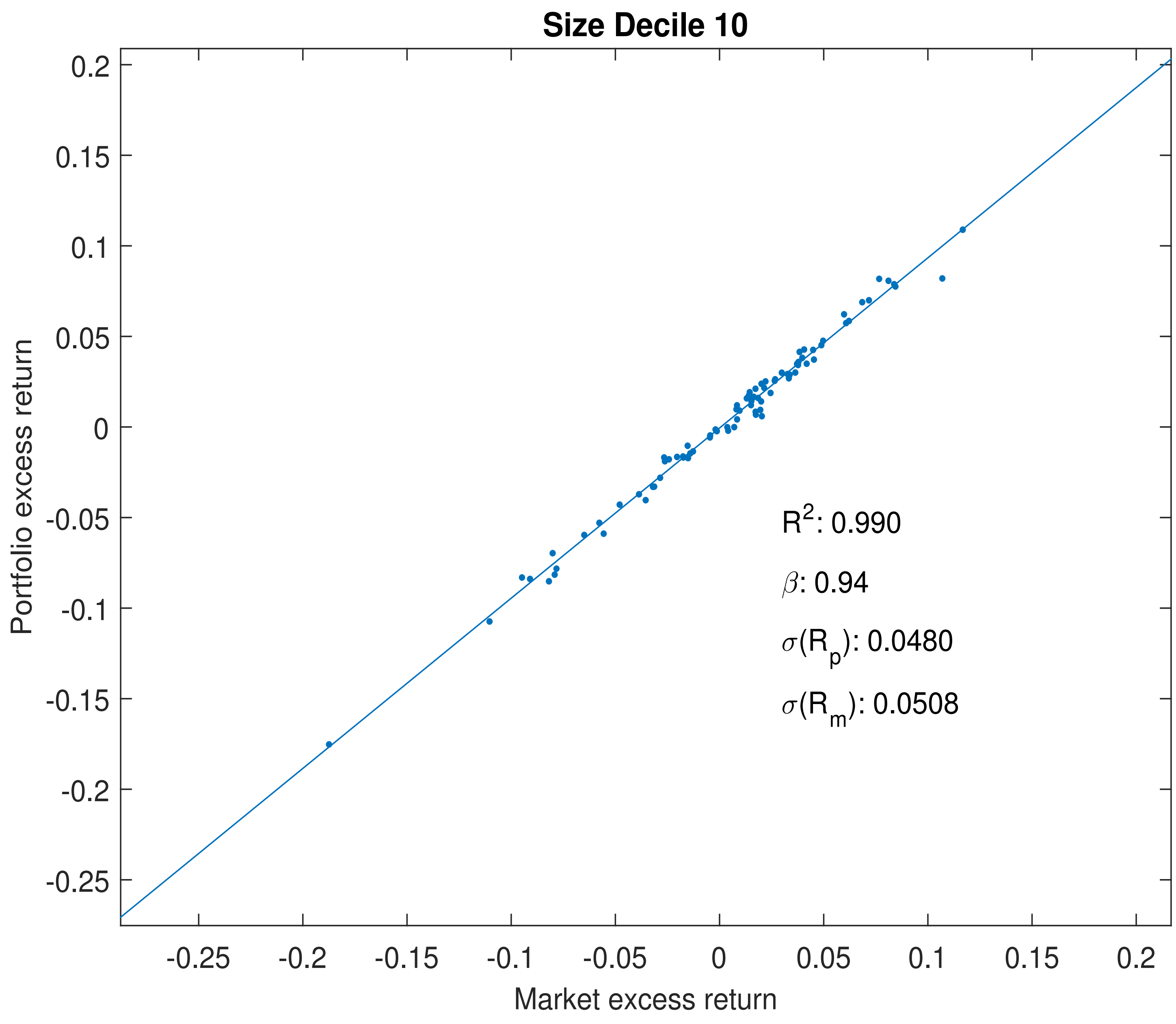
Size Decile 7
0.2
0.1
0
-0.1
-0.2
-0.3
Portfolio excess return
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Market excess return
R^2: 0.930
β: 1.17
$\sigma(R_p)$: 0.0618
$\sigma(R_m)$: 0.0508



Size Decile 8
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
Portfolio excess return
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Market excess return
R^2: 0.951
β: 1.15
$\sigma(R_p)$: 0.0602
$\sigma(R_m)$: 0.0508



Size Decile 9
Portfolio excess return
Market excess return
0.2
0.1
0
-0.1
-0.2
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
R^2: 0.966
β: 1.13
$\sigma(R_p)$: 0.0585
$\sigma(R_m)$: 0.0508



Size Decile 10
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
Portfolio excess return
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Market excess return
R^2: 0.990
β: 0.94
$\sigma(R_p)$: 0.0480
$\sigma(R_m)$: 0.0508

Futures Overlay

- Recall that profit on a market-index futures position of size X_{t-1} is $X_{t-1}R_{m,t}$
  (note $R_{m,t}$ denotes the return in excess of cash)
- Consider an overall investment that combines
  - zero-investment futures position of size X_{t-1}
  - positive investment of size W_{t-1} in asset i
- Overall return (in excess of cash return $r_{f,t}$):

$$
\begin{aligned}
R_{C,t} &= \frac{\text{change in wealth from time } t-1 \text{ to } t}{\text{initial wealth at time } t-1} - r_{f,t} \\
&= \frac{W_{t-1}(R_{i,t} + r_{f,t}) + X_{t-1}R_{m,t}}{W_{t-1}} - r_{f,t} \\
&= R_{i,t} + \frac{X_{t-1}}{W_{t-1}}R_{m,t} \qquad (12)
\end{aligned}
$$

Neutralizing Market Risk Using Futures

- Using the characteristic line for asset i;

$$\begin{aligned} R_{C,t} &= R_{i,t} + \frac{X_{t-1}}{W_{t-1}} R_{m,t} \\ &= (\alpha_i + \beta_i R_{m,t} + \epsilon_{i,t}) + \frac{X_{t-1}}{W_{t-1}} R_{m,t} \\ &= \alpha_i + \left(\beta_i + \frac{X_{t-1}}{W_{t-1}}\right) R_{m,t} + \epsilon_{i,t} \end{aligned} \tag{13}$$

- To achieve an overall market-neutral investment, choose X_{t-1} so that

$$\left(\beta_i + \frac{X_{t-1}}{W_{t-1}}\right) = 0 \tag{14}$$

or

$$X_{t-1} = -\beta_i W_{t-1}. \tag{15}$$

- Gives

$$R_{C,t} = \underbrace{\alpha_i}_{\text{expected}} + \underbrace{\epsilon_{i,t}}_{\text{unexpected}} \tag{16}$$

Example

- \$20 million portfolio
  - standard deviation: 0.40
  - beta: 0.8
  - R-square: 0.75
- What is the market-neutralizing futures overlay?
- What is the volatility of the neutralized investment return?

Example

- \$20 million portfolio
  - standard deviation: 0.40
  - beta: 0.8
  - R-square: 0.75
- What is the market-neutralizing futures overlay?
- What is the volatility of the neutralized investment return?

$$\begin{aligned} X_{t-1} &= -\beta_i W_{t-1} \\ &= -0.8 \cdot (\$20 \text{ million}) \\ &= -\$16 \text{ million} \end{aligned}$$

Example

- \$20 million portfolio
  - standard deviation: 0.40
  - beta: 0.8
  - R-square: 0.75
- What is the market-neutralizing futures overlay?
- What is the volatility of the neutralized investment return?

$$
\begin{aligned}
X_{t-1} &= -\beta_i W_{t-1} \\
&= -0.8 \cdot (\$20 \text{ million}) \\
&= -\$16 \text{ million}
\end{aligned}
$$

$$
\begin{aligned}
\sigma(R_{C,t}) &= [\sigma^2(\epsilon_{i,t})]^{1/2} \\
&= [(1 - R^2)\sigma^2(R_{i,t})]^{1/2} \\
&= \sqrt{0.25 \times 0.40^2} \\
&= 0.20
\end{aligned}
$$

Estimating Betas

- The OLS regression estimator $\hat{\beta}_i$ often referred to as "historical beta."
- The standard error of $\hat{\beta}_i$ is given (approximately) by

$$s(\hat{\beta}_i) = \sqrt{\frac{(1 - R^2)}{T} \frac{\sigma^2(R_{i,t})}{\sigma^2(R_{m,t})}} \tag{17}$$

- Thus, the precision of an OLS beta estimate is higher with
  - a larger number of observations (T)
  - a portfolio instead of an individual stock—higher R^2 and lower $\sigma^2(R_{i,t})$
- With monthly data, 5-7 years seems to give good tradeoff between
  - greater accuracy arising from more observations
  - lower accuracy from changes in true beta.
- Increasing T by using higher frequency returns, e.g., daily
  - allows a shorter window in calendar time
  - can be problematic for less liquid stocks (more on this later)

BAC US $ ↓ 15.925 +.225 N15.92 / 15.93N 890 x 1000

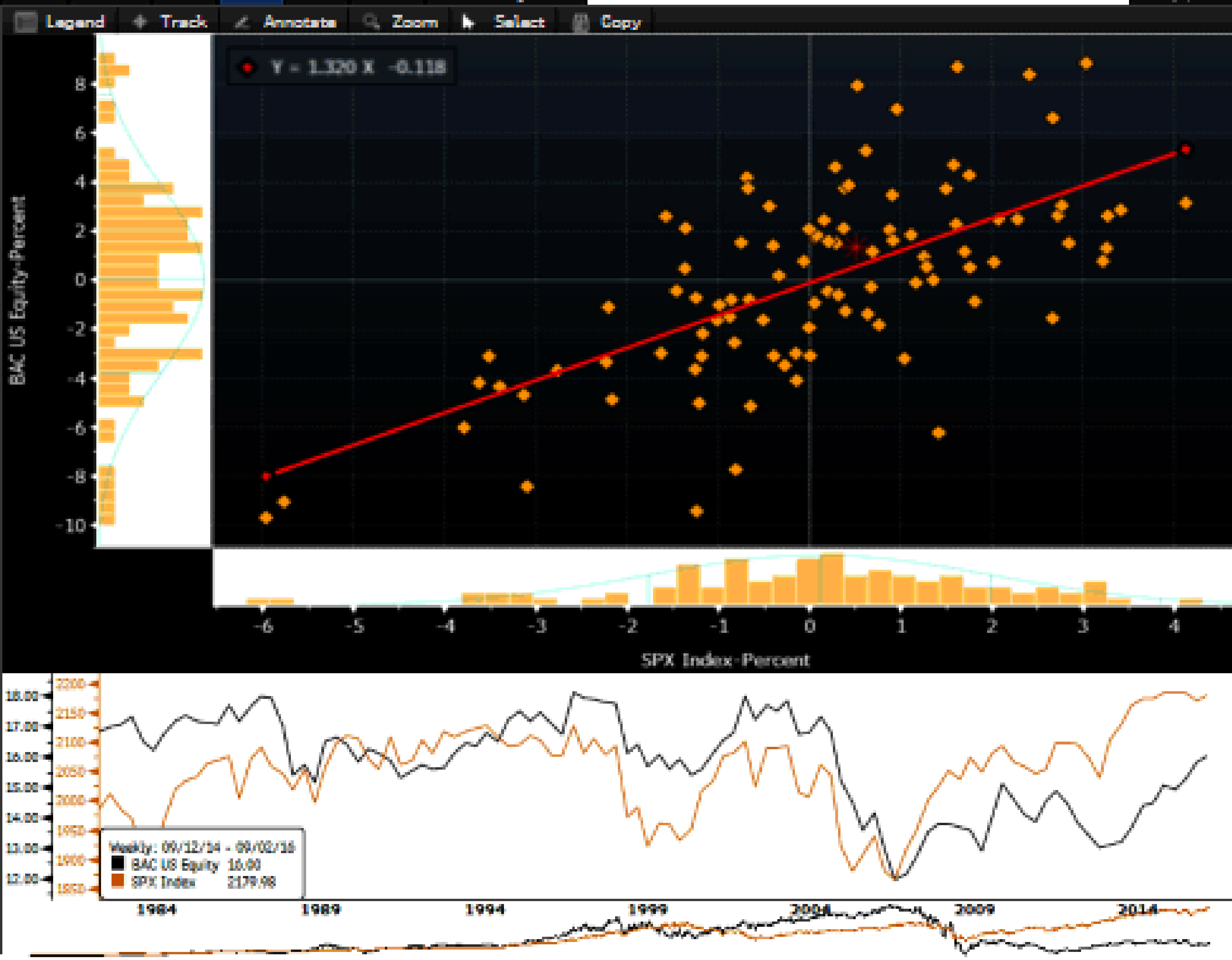
At 12:57 d Vol 40,070,702 O 15.77K H 15.94D L 15.68N Val 633.618M

BAC US Equity | Relative Index | SPX Index | 96) Actions | 97) Edit | Historical Beta

Data Last Price | Data Last Price | Wkly | Linear | Beta +/- | Non-Param | Reg On Percent

09/09/2014 - 09/08/2016 | 09/09/2013 - 09/09/2015 | Lag 0 | Winsorize 2 Std Dev | Local

6M YTD 1Y 2Y 5Y Max Weekly | Statistics | Transformations



Y = BANK OF AMERICA CORP

X = S&P 500 INDEX

Linear Beta	Range 1
Raw BETA	1.320
Adjusted BETA	1.213
ALPHA (Intercept)	-0.118
R^2 (Correlation^2)	0.428
R (Correlation)	0.654
Std Dev of Error	2.883
Std Error of ALPHA	0.285
Std Error of BETA	0.152
t-Test	8.699
Significance	0.000
Last T-Value	0.270
Last P-Value	0.606
Number of Points	103
Last Spread	2163.98
Last Ratio	0.007

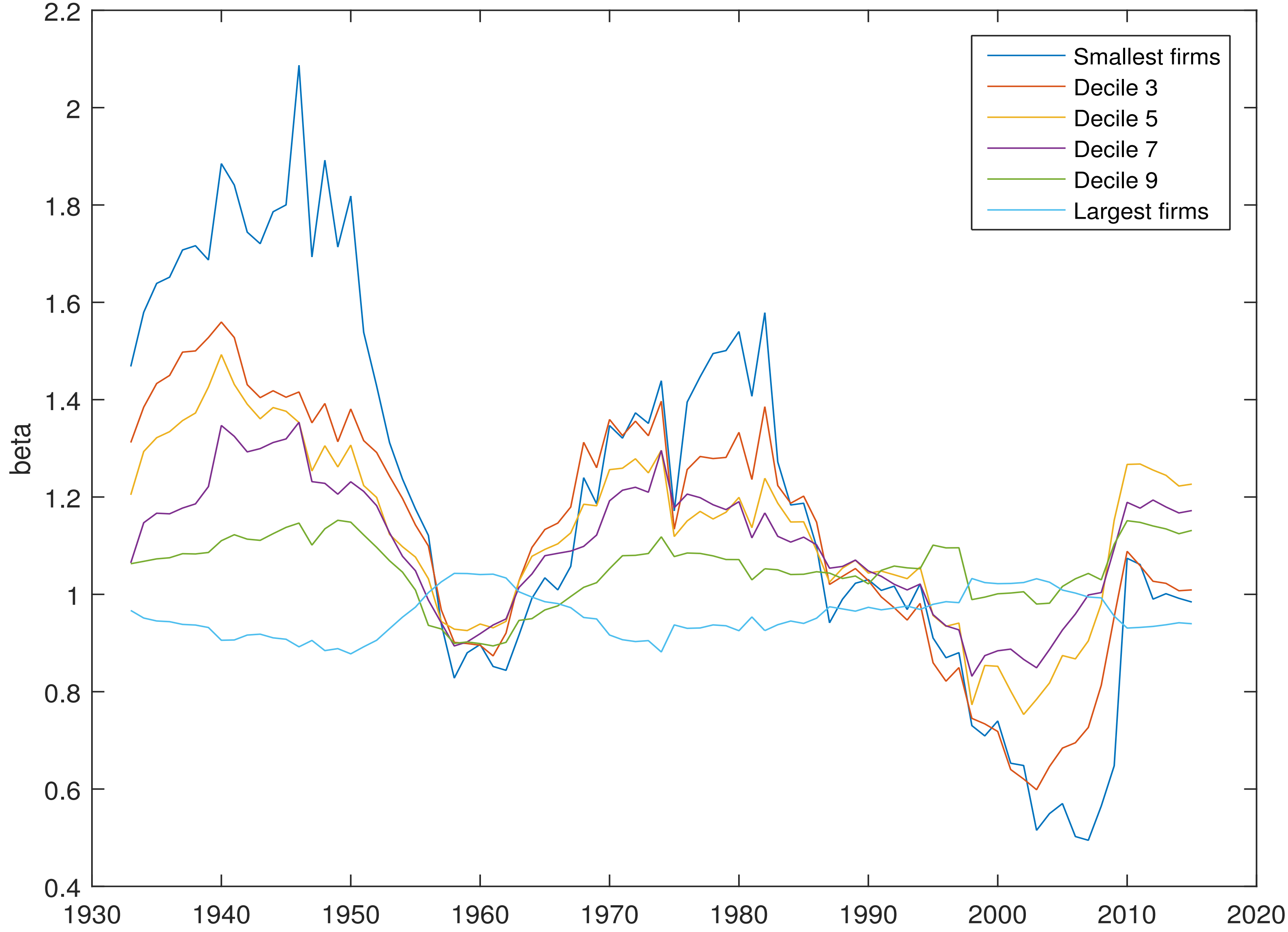
Adjusting Historical Betas on Individual Stocks

- Historical betas tend to “regress to the mean” from one period to the next.
- Two potential reasons:
  - economic: firms become larger and more “typical” as they mature
  - statistical: historical betas contain errors
- Second reason probably more important
- For any quantity estimated with error
  - Highest values in the cross section are probably too high
  - Lowest values in the cross section are probably too low
- “Raw” historical betas on individual stocks are often adjusted toward 1:

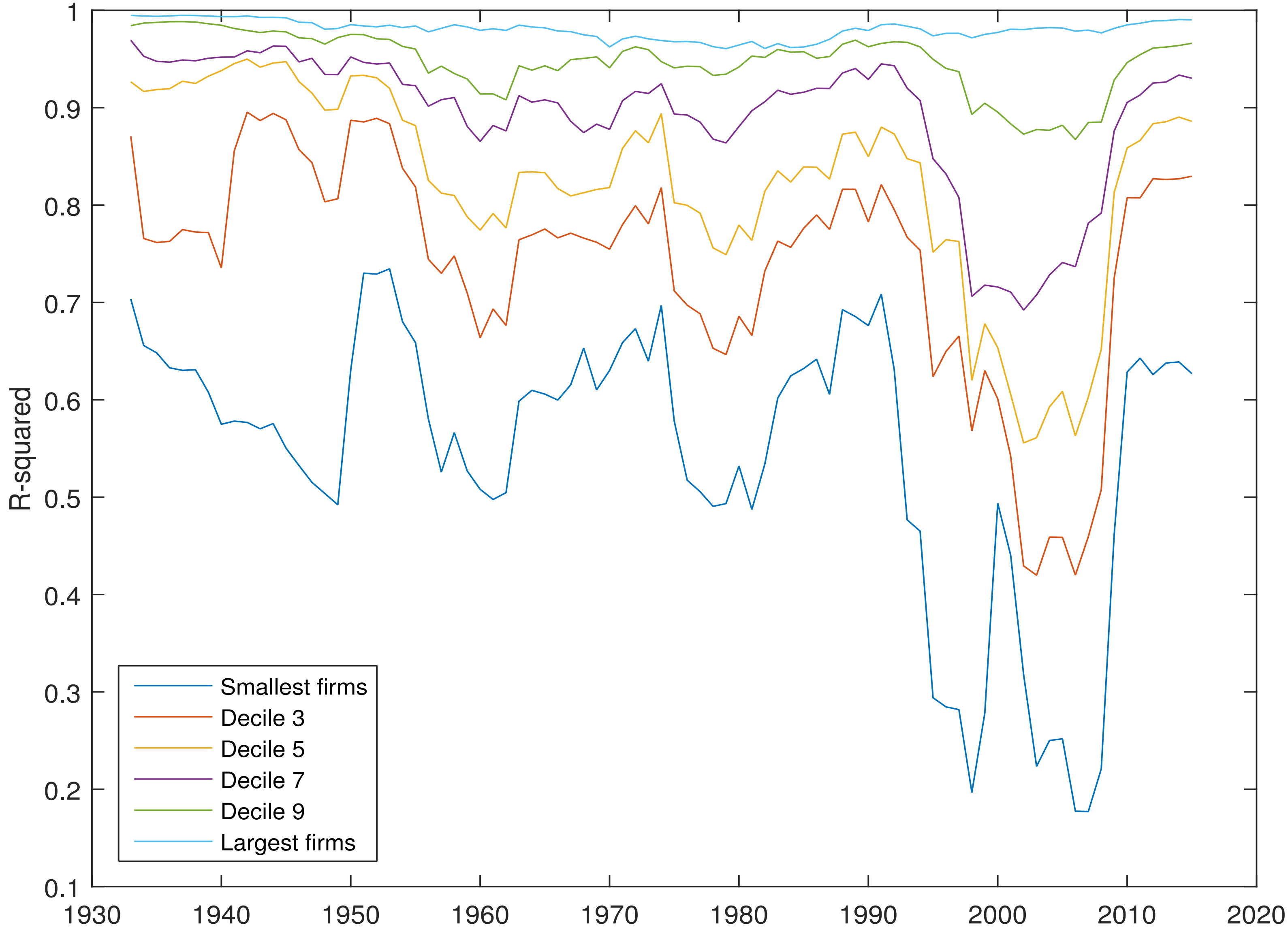
$$\hat{\beta}_{ADJ,i} = w \cdot 1 + (1 - w) \cdot \hat{\beta}_i \,. \tag{18}$$

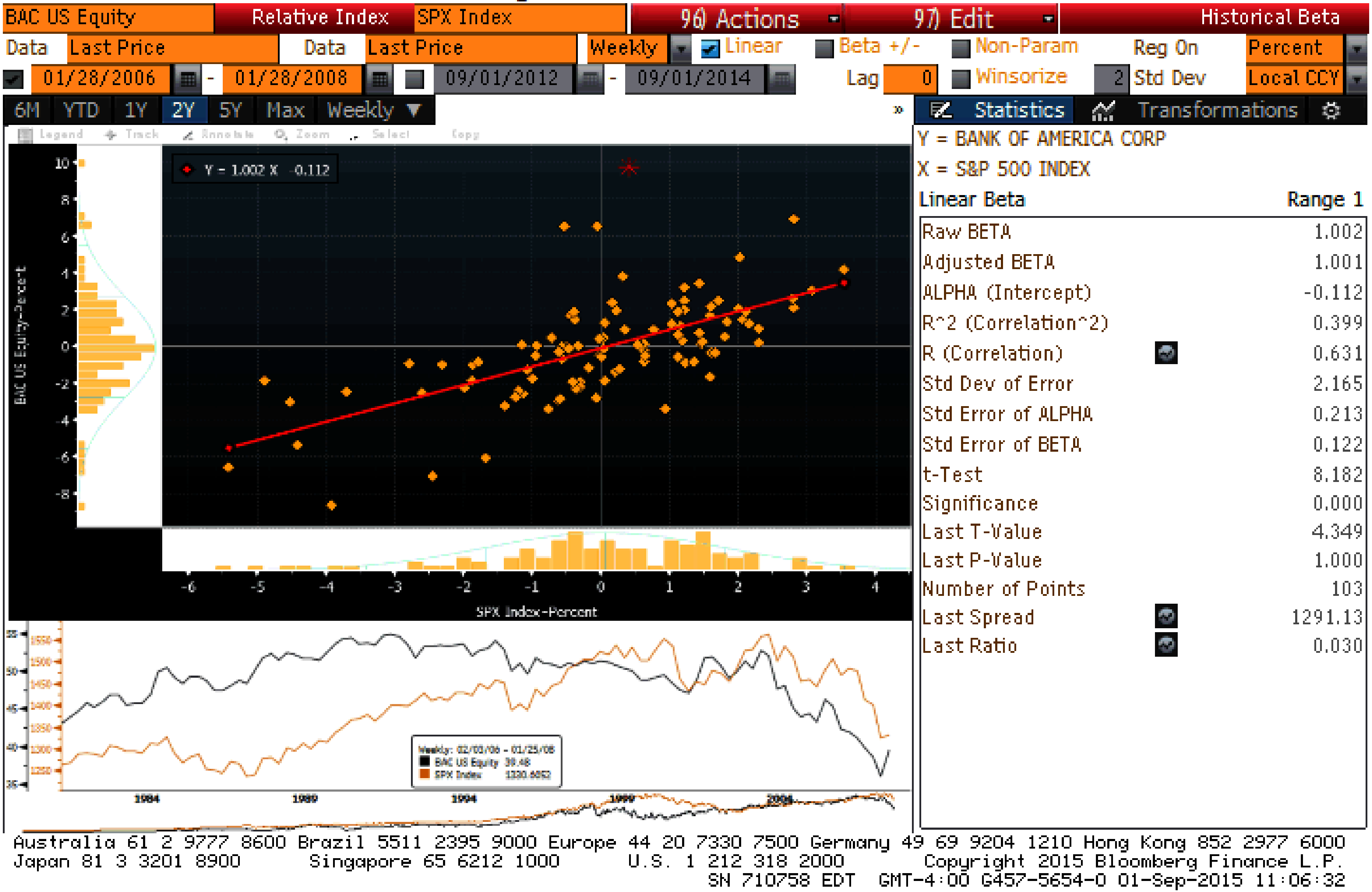
- Common specification: $w = 1/3$.
- Historical betas on portfolios are typically not adjusted.

Beta Changes Over Time ...



As Does R-Squared ...



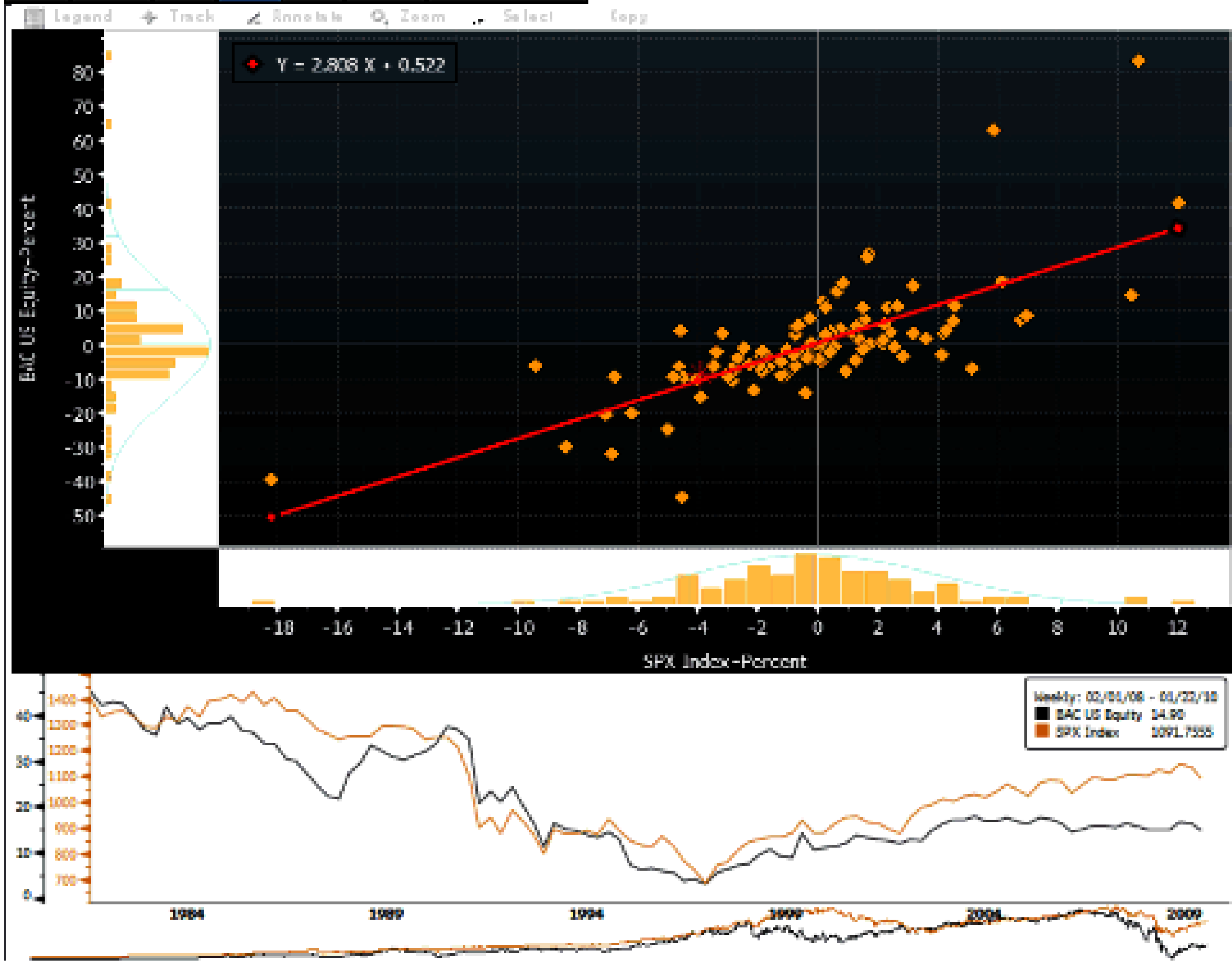

BAC US Equity
Relative Index
SPX Index
96) Actions
97) Edit
Historical Beta
Data
Last Price
Data
Last Price
Weekly
Linear
Beta +/-
Non-Param
Reg On
Percent
01/28/2006
01/28/2008
09/01/2012
09/01/2014
Lag
0
Winsorize
2
Std Dev
Local CCY
6M
YTD
1Y
2Y
5Y
Max
Weekly
Statistics
Transformations
Y = 1.002 X -0.112
BAC US Equity-Percent
SPX Index-Percent
Y = BANK OF AMERICA CORP
X = S&P 500 INDEX
Linear Beta
Range 1
Raw BETA 1.002
Adjusted BETA 1.001
ALPHA (Intercept) -0.112
R^2 (Correlation^2) 0.399
R (Correlation) 0.631
Std Dev of Error 2.165
Std Error of ALPHA 0.213
Std Error of BETA 0.122
t-Test 8.182
Significance 0.000
Last T-Value 4.349
Last P-Value 1.000
Number of Points 103
Last Spread 1291.13
Last Ratio 0.030
Weekly: 02/03/06 - 01/25/08
BAC US Equity 38.48
SPX Index 1330.6052
1984
1989
1994
1999
2004
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2015 Bloomberg Finance L.P.
SN 710758 EDT GMT-4:00 G457-5654-0 01-Sep-2015 11:06:32

BAC US Equity | Relative Index | SPX Index | 96) Actions | 97) Edit | Historical Beta

Data: Last Price | Data: Last Price | Weekly | Linear | Beta +/- | Non-Param | Reg On: Percent

01/28/2008 - 01/28/2010 | 09/01/2012 - 09/01/2014 | Lag 0 | Winsorize | 2 Std Dev | Local CCY

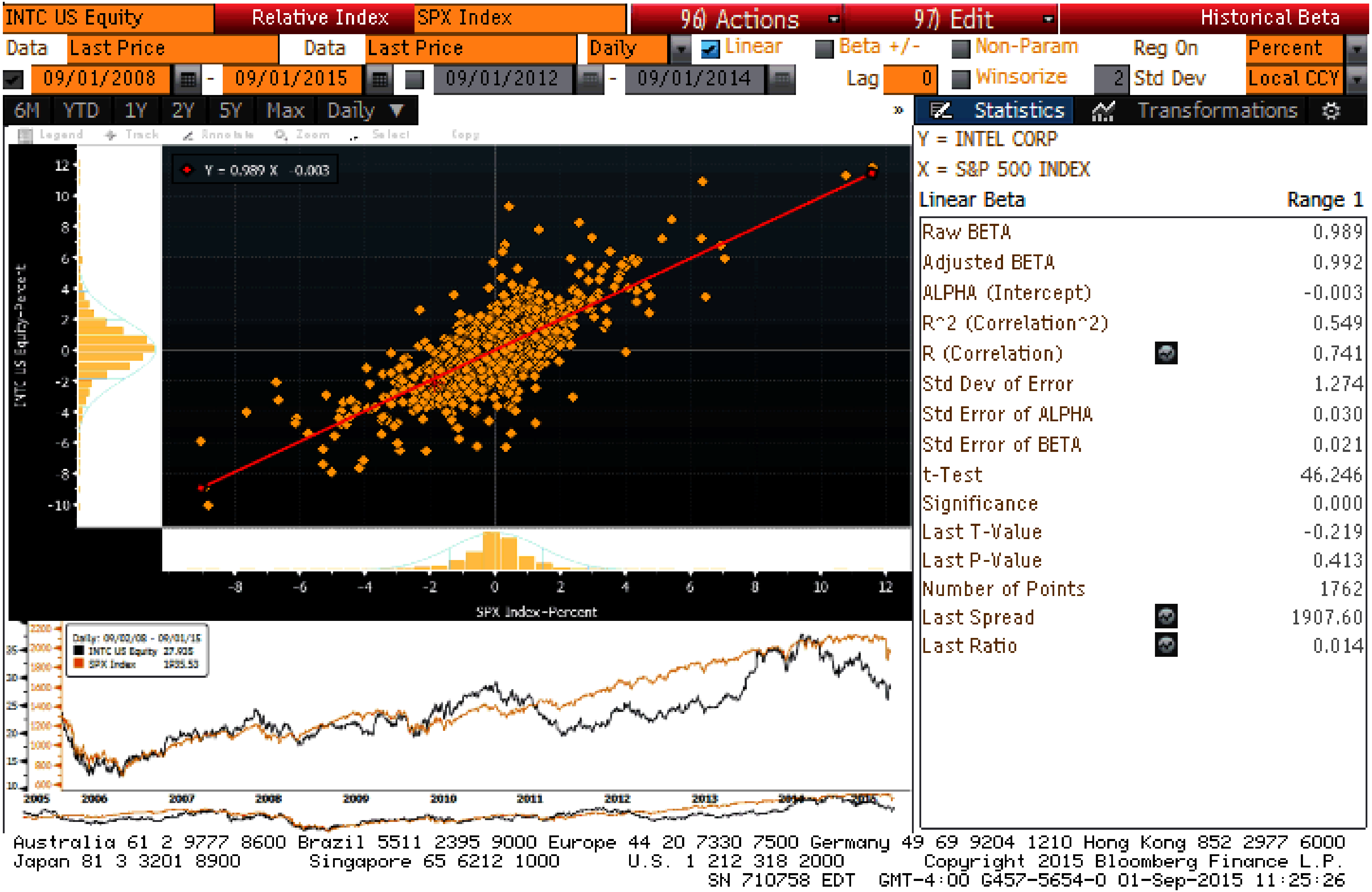
6M | YTD | 1Y | 2Y | 5Y | Max | Weekly | Statistics | Transformations

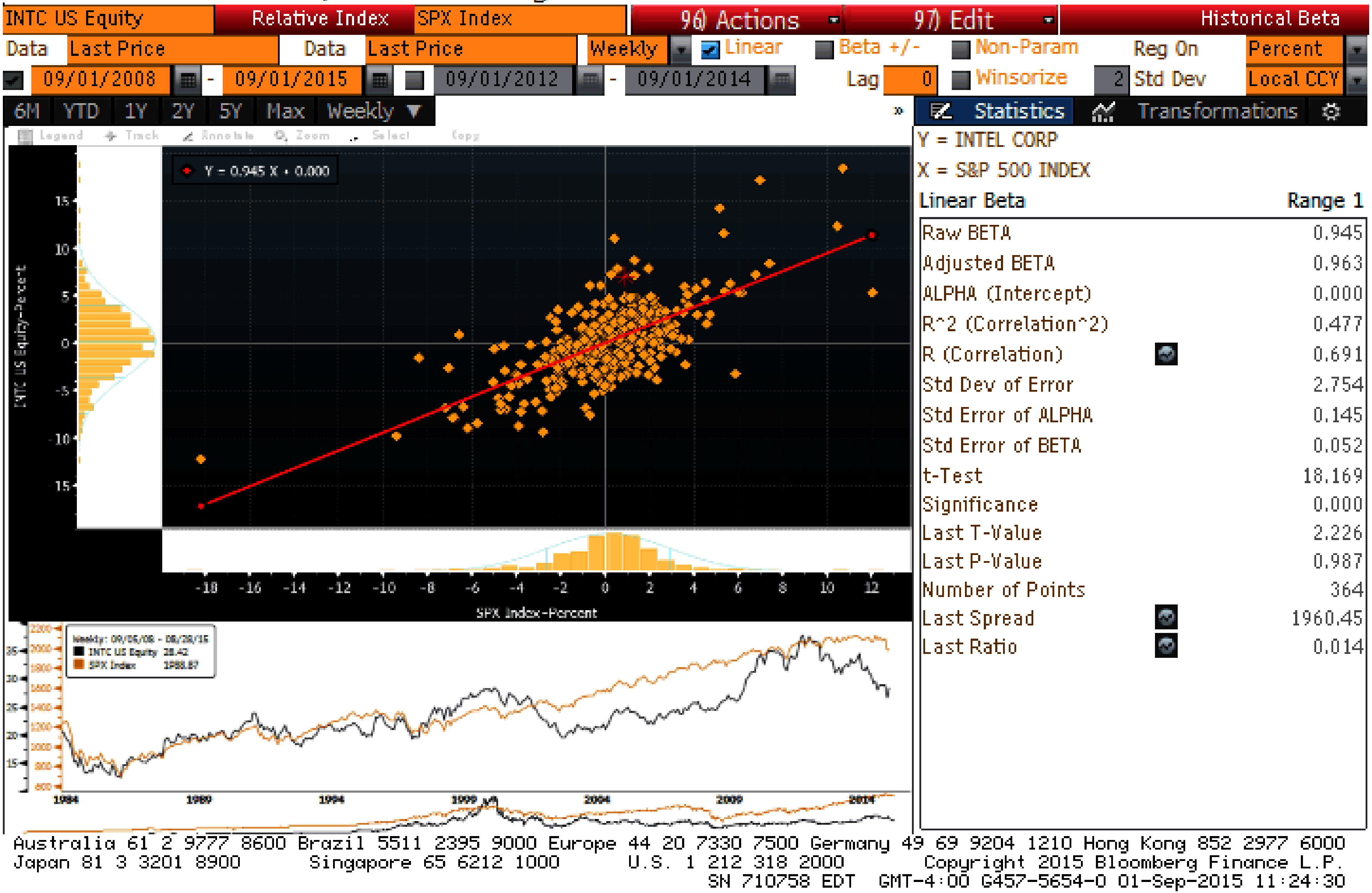


Y = BANK OF AMERICA CORP

X = S&P 500 INDEX

Linear Beta	Range 1
Raw BETA	2.808
Adjusted BETA	2.205
ALPHA (Intercept)	0.522
R^2 (Correlation^2)	0.534
R (Correlation)	0.731
Std Dev of Error	10.965
Std Error of ALPHA	1.081
Std Error of BETA	0.261
t-Test	10.755
Significance	0.000
Last T-Value	0.186
Last P-Value	0.574
Number of Points	103
Last Spread	1076.86
Last Ratio	0.014


INTC US Equity
Relative Index
SPX Index
96) Actions
97) Edit
Historical Beta
Data
Last Price
Data
Last Price
Daily
Linear
Beta +/-
Non-Param
Reg On
Percent
09/01/2008 - 09/01/2015
09/01/2012 - 09/01/2014
Lag 0
Winsorize
2 Std Dev
Local CCY
6M YTD 1Y 2Y 5Y Max Daily
Statistics
Transformations
Y = 0.989 X -0.003
INTC US Equity-Percent
SPX Index-Percent
Y = INTEL CORP
X = S&P 500 INDEX
Linear Beta
Range 1
Raw BETA 0.989
Adjusted BETA 0.992
ALPHA (Intercept) -0.003
R^2 (Correlation^2) 0.549
R (Correlation) 0.741
Std Dev of Error 1.274
Std Error of ALPHA 0.030
Std Error of BETA 0.021
t-Test 46.246
Significance 0.000
Last T-Value -0.219
Last P-Value 0.413
Number of Points 1762
Last Spread 1907.60
Last Ratio 0.014
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2015 Bloomberg Finance L.P.
SN 710758 EDT GMT-4:00 G457-5654-0 01-Sep-2015 11:25:26



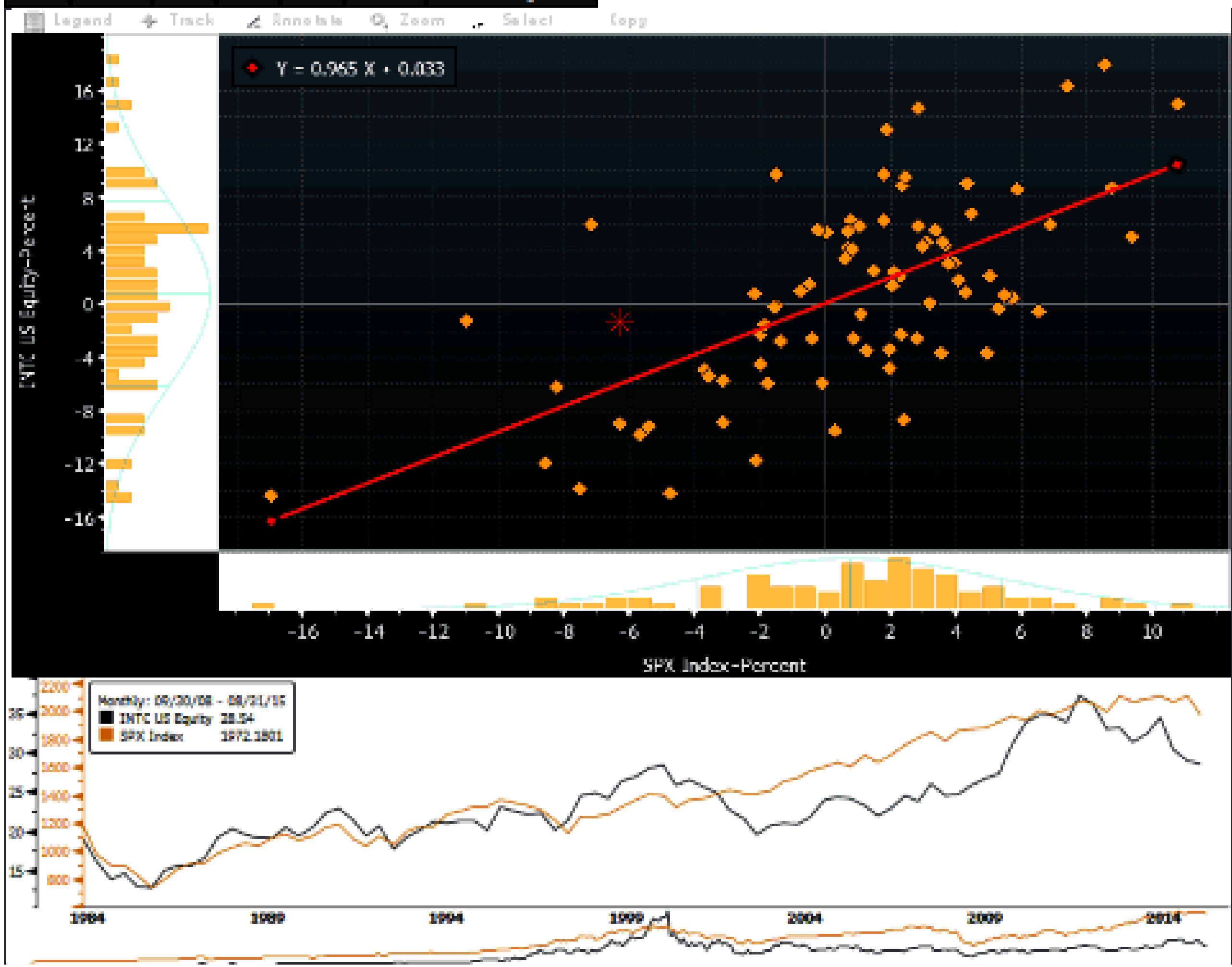
INTC US Equity
Relative Index
SPX Index
96) Actions
97) Edit
Historical Beta
Data
Last Price
Data
Last Price
Weekly
Linear
Beta +/-
Non-Param
Reg On
Percent
09/01/2008 - 09/01/2015
09/01/2012 - 09/01/2014
Lag 0
Winsorize
2 Std Dev
Local CCY
6M YTD 1Y 2Y 5Y Max Weekly
Statistics
Transformations
Y = 0.945 X + 0.000
INTC US Equity-Percent
SPX Index-Percent
Y = INTEL CORP
X = S&P 500 INDEX
Linear Beta
Range 1
Raw BETA 0.945
Adjusted BETA 0.963
ALPHA (Intercept) 0.000
R^2 (Correlation^2) 0.477
R (Correlation) 0.691
Std Dev of Error 2.754
Std Error of ALPHA 0.145
Std Error of BETA 0.052
t-Test 18.169
Significance 0.000
Last T-Value 2.226
Last P-Value 0.987
Number of Points 364
Last Spread 1960.45
Last Ratio 0.014
Weekly: 09/05/08 - 08/28/15
INTC US Equity 28.42
SPX Index 1988.87
1984
1989
1994
1999
2004
2009
2014
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2015 Bloomberg Finance L.P.
SN 710758 EDT GMT-4:00 G457-5654-0 01-Sep-2015 11:24:30

INTC US Equity | Relative Index | SPX Index | 96) Actions | 97) Edit | Historical Beta

Data Last Price | Data Last Price | Monthly | Linear | Beta +/- | Non-Param | Reg On | Percent

09/01/2008 - 09/01/2015 | 09/01/2012 - 09/01/2014 | Lag 0 | Winsorize | 2 Std Dev | Local CCY

6M YTD 1Y 2Y 5Y Max Monthly

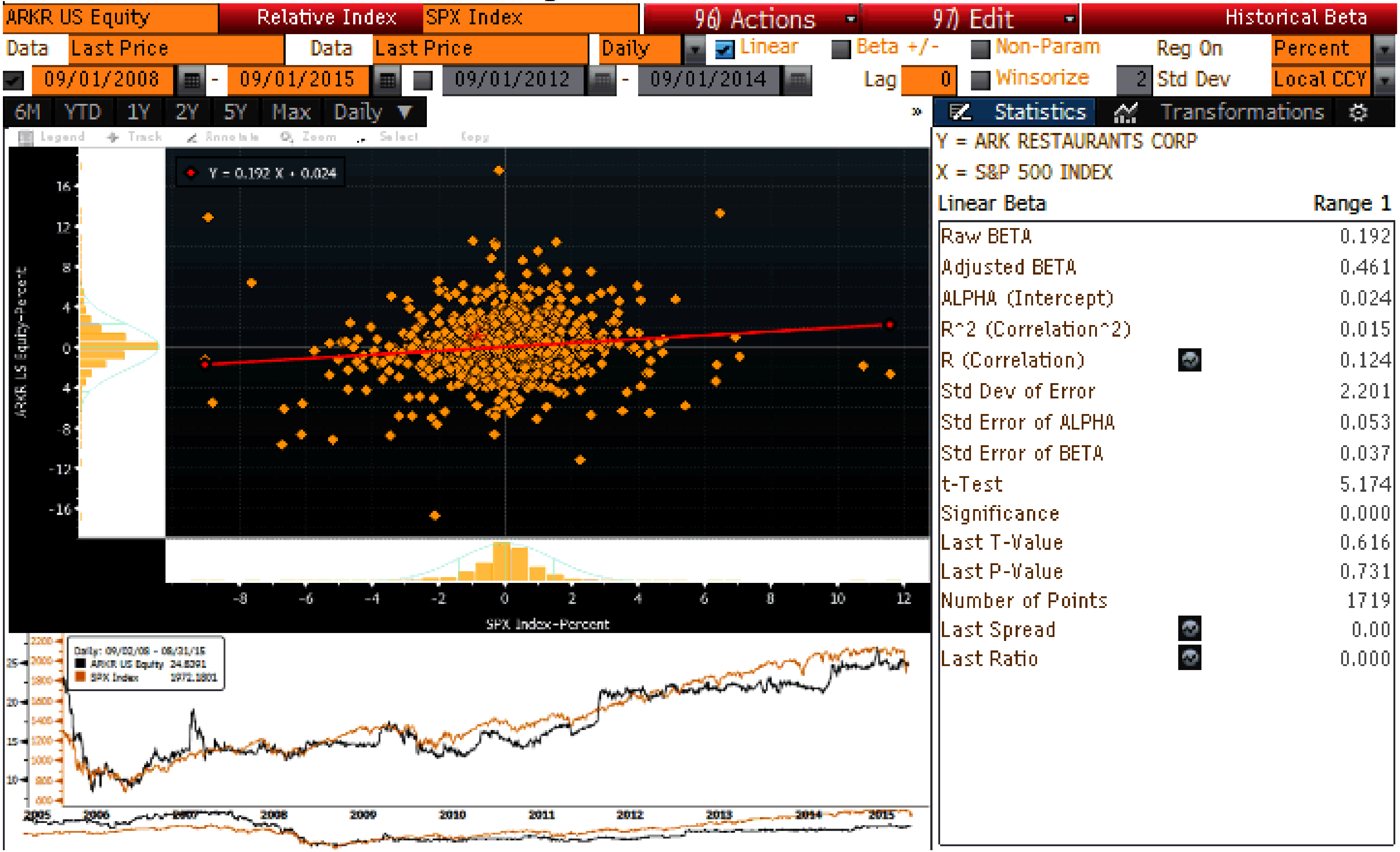


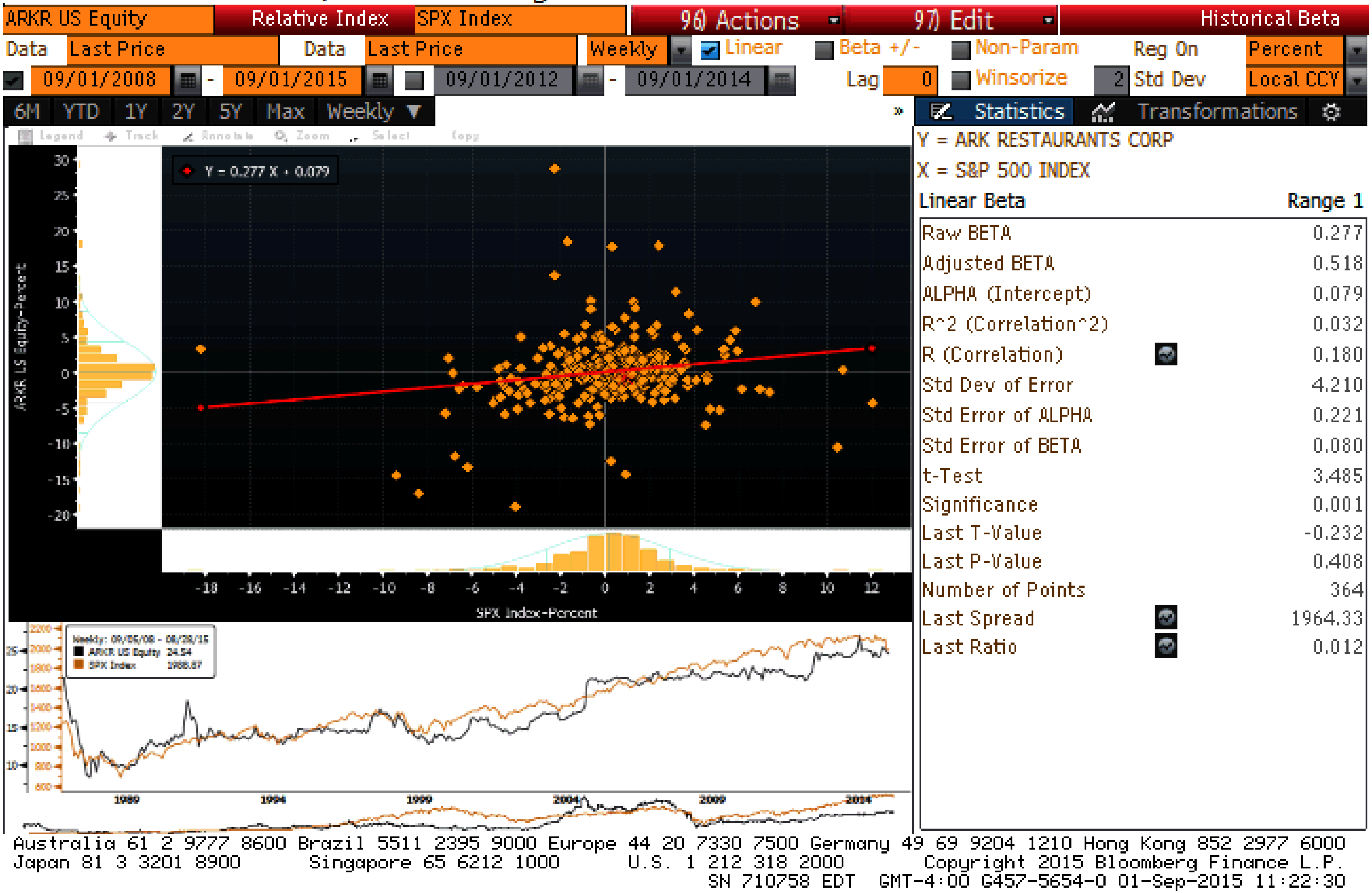
Statistics | Transformations

Y = INTEL CORP

X = S&P 500 INDEX

Linear Beta	Range 1
Raw BETA	0.965
Adjusted BETA	0.977
ALPHA (Intercept)	0.033
R^2 (Correlation^2)	0.412
R (Correlation)	0.642
Std Dev of Error	5.427
Std Error of ALPHA	0.603
Std Error of BETA	0.128
t-Test	7.539
Significance	0.000
Last T-Value	0.830
Last P-Value	0.795
Number of Points	83
Last Spread	1943.64
Last Ratio	0.014


ARKR US Equity
Relative Index
SPX Index
96) Actions
97) Edit
Historical Beta
Data
Last Price
Data
Last Price
Daily
Linear
Beta +/-
Non-Param
Reg On
Percent
09/01/2008 - 09/01/2015
09/01/2012 - 09/01/2014
Lag 0
Winsorize
2 Std Dev
Local CCY
6M YTD 1Y 2Y 5Y Max Daily
Statistics
Transformations
Y = 0.192 X + 0.024
ARKR US Equity-Percent
SPX Index-Percent
Y = ARK RESTAURANTS CORP
X = S&P 500 INDEX
Linear Beta
Range 1
Raw BETA 0.192
Adjusted BETA 0.461
ALPHA (Intercept) 0.024
R^2 (Correlation^2) 0.015
R (Correlation) 0.124
Std Dev of Error 2.201
Std Error of ALPHA 0.053
Std Error of BETA 0.037
t-Test 5.174
Significance 0.000
Last T-Value 0.616
Last P-Value 0.731
Number of Points 1719
Last Spread 0.00
Last Ratio 0.000


ARKR US Equity
Relative Index
SPX Index
96) Actions
97) Edit
Historical Beta
Data
Last Price
Data
Last Price
Weekly
Linear
Beta +/-
Non-Param
Reg On
Percent
09/01/2008 - 09/01/2015
09/01/2012 - 09/01/2014
Lag 0
Winsorize
2 Std Dev
Local CCY
6M YTD 1Y 2Y 5Y Max Weekly
Statistics
Transformations
Y = 0.277 X + 0.079
ARKR US Equity-Percent
SPX Index-Percent
Y = ARK RESTAURANTS CORP
X = S&P 500 INDEX
Linear Beta
Range 1
Raw BETA 0.277
Adjusted BETA 0.518
ALPHA (Intercept) 0.079
R^2 (Correlation^2) 0.032
R (Correlation) 0.180
Std Dev of Error 4.210
Std Error of ALPHA 0.221
Std Error of BETA 0.080
t-Test 3.485
Significance 0.001
Last T-Value -0.232
Last P-Value 0.408
Number of Points 364
Last Spread 1964.33
Last Ratio 0.012
Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2015 Bloomberg Finance L.P.
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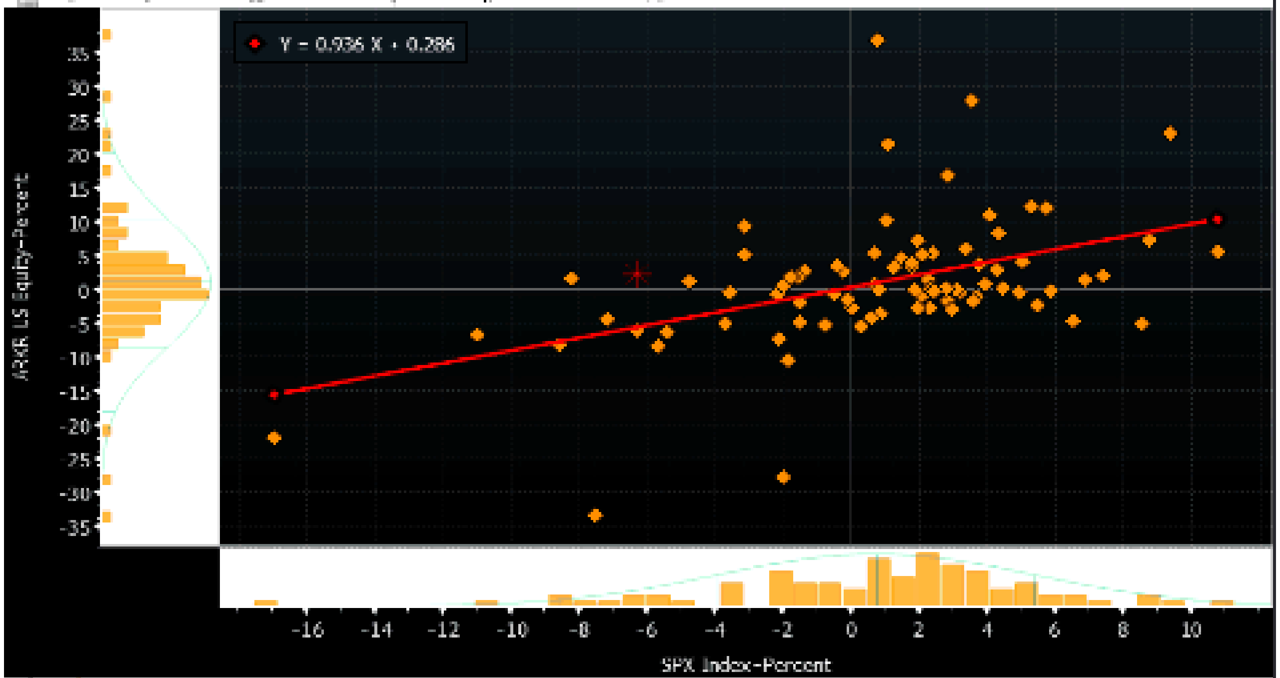
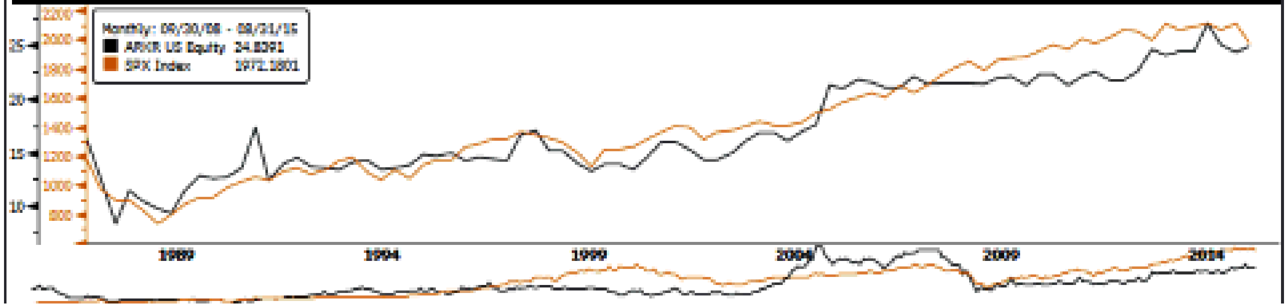
ARKR US Equity | Relative Index | SPX Index | 96) Actions | 97) Edit | Historical Beta

Data Last Price | Data Last Price | Monthly | Linear | Beta +/- | Non-Param | Reg On | Percent

09/01/2008 - 09/01/2015 | 09/01/2012 - 09/01/2014 | Lag 0 | Winsorize | 2 Std Dev | Local CCY

6M YTD 1Y 2Y 5Y Max Monthly | Statistics | Transformations





Y = ARK RESTAURANTS CORP

X = S&P 500 INDEX

Linear Beta	Range 1
Raw BETA	0.936
Adjusted BETA	0.957
ALPHA (Intercept)	0.286
R^2 (Correlation^2)	0.211
R (Correlation)	0.459
Std Dev of Error	8.537
Std Error of ALPHA	0.949
Std Error of BETA	0.201
t-Test	4.649
Significance	0.000
Last T-Value	0.886
Last P-Value	0.811
Number of Points	83
Last Spread	0.00
Last Ratio	0.000

Liquidity and Beta Estimation

- Estimating betas with higher frequency (e.g., daily returns)
  - works well for stocks traded fairly heavily
  - generally under-estimates betas of less liquid stocks
- For a less liquid stock
  - prices don't immediately reflect relevant market-wide information
  - observed day t return, $R_{i,t}$, includes market effects before day t
  - regressing R_{it} on $R_{m,t}$ misses some market sensitivity
- Common fix (Dimson):
  - Run the multiple regression

$$R_{i,t} = a_i + \beta_{i,0} R_{m,t} + \beta_{i,1} R_{m,t-1} + \cdots + \beta_{i,K} R_{m,t-K} + e_{i,t}$$

  - Estimate the stock's beta as the sum of the estimated slopes:

$$\hat{\beta}_i = \hat{\beta}_{i,0} + \hat{\beta}_{i,1} + \cdots + \hat{\beta}_{i,K}$$

Combining Assets

- Consider a portfolio p of two assets, i and j.

$$\begin{aligned} R_{p,t} &= w_i R_{i,t} + w_j R_{j,t} \\ &= w_i (\alpha_i + \beta_i R_{m,t} + \epsilon_{i,t}) + w_j (\alpha_j + \beta_j R_{m,t} + \epsilon_{j,t}) \\ &= \alpha_p + \beta_p R_{m,t} + \epsilon_{p,t}, \end{aligned} \tag{19}$$

where

$$\begin{aligned} \alpha_p &= w_i \alpha_i + w_j \alpha_j \\ \beta_p &= w_i \beta_i + w_j \beta_j \\ \epsilon_{p,t} &= w_i \epsilon_{i,t} + w_j \epsilon_{j,t} \end{aligned}$$

- Same properties as before:

$$\text{cov}\,(\epsilon_{p,t}, R_{m,t}) = 0 \tag{20}$$

$$\beta_p = \frac{\text{Cov}(R_{p,t}, R_{m,t})}{\sigma^2(R_{m,t})} \tag{21}$$

$$\text{Var}(R_{p,t}) = \beta_p^2 \text{Var}(R_{m,t}) + \text{Var}(\epsilon_{p,t}) \tag{22}$$