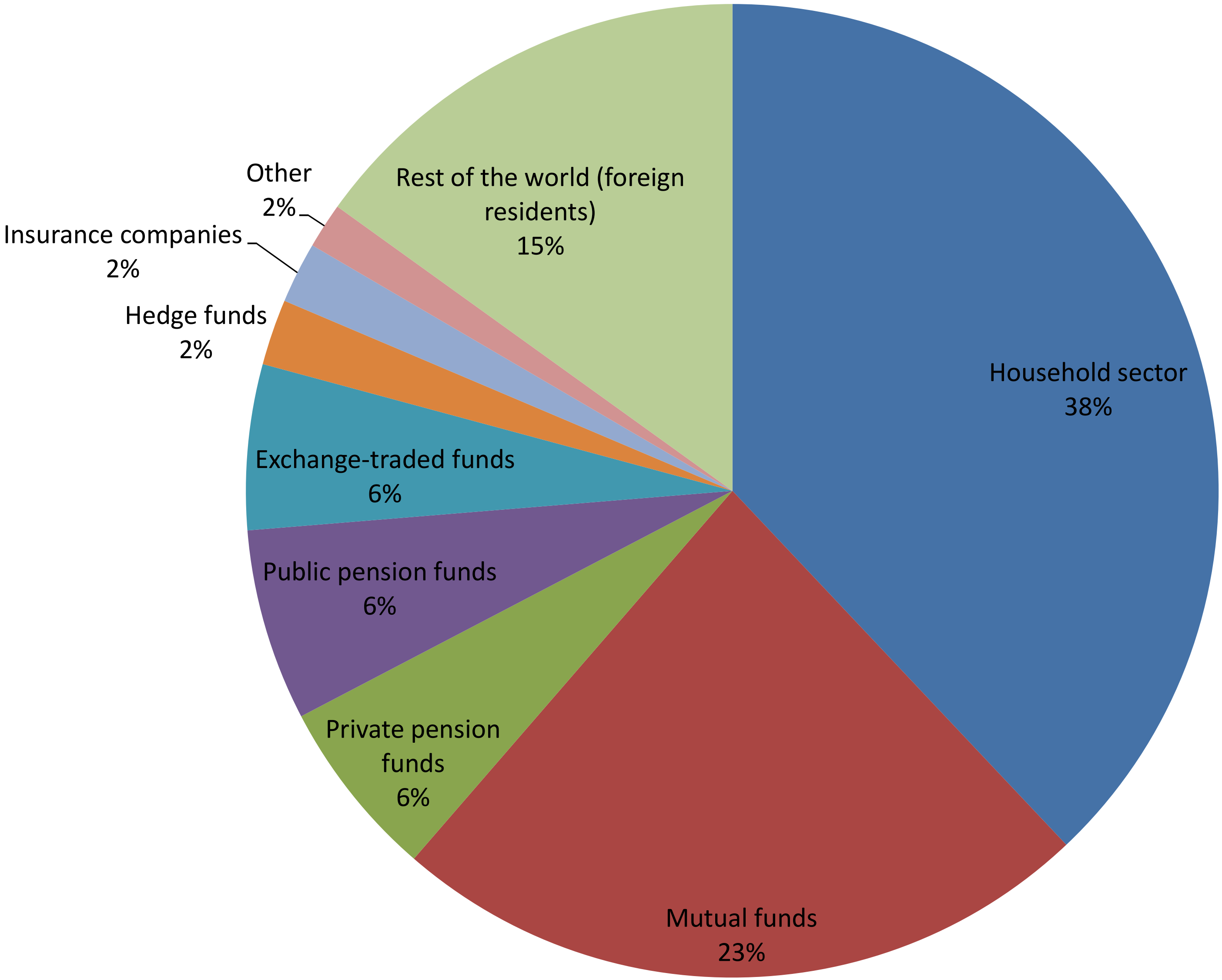
Outline

- Ownership of U.S. equities
- Historical distributions of returns on U.S. equities
- Sharpe ratios
- Shortfall probability
- Role of return horizon
- Four-century perspective on stock markets

Holdings of US Corporate Equity



U.S. Equity Excess Returns

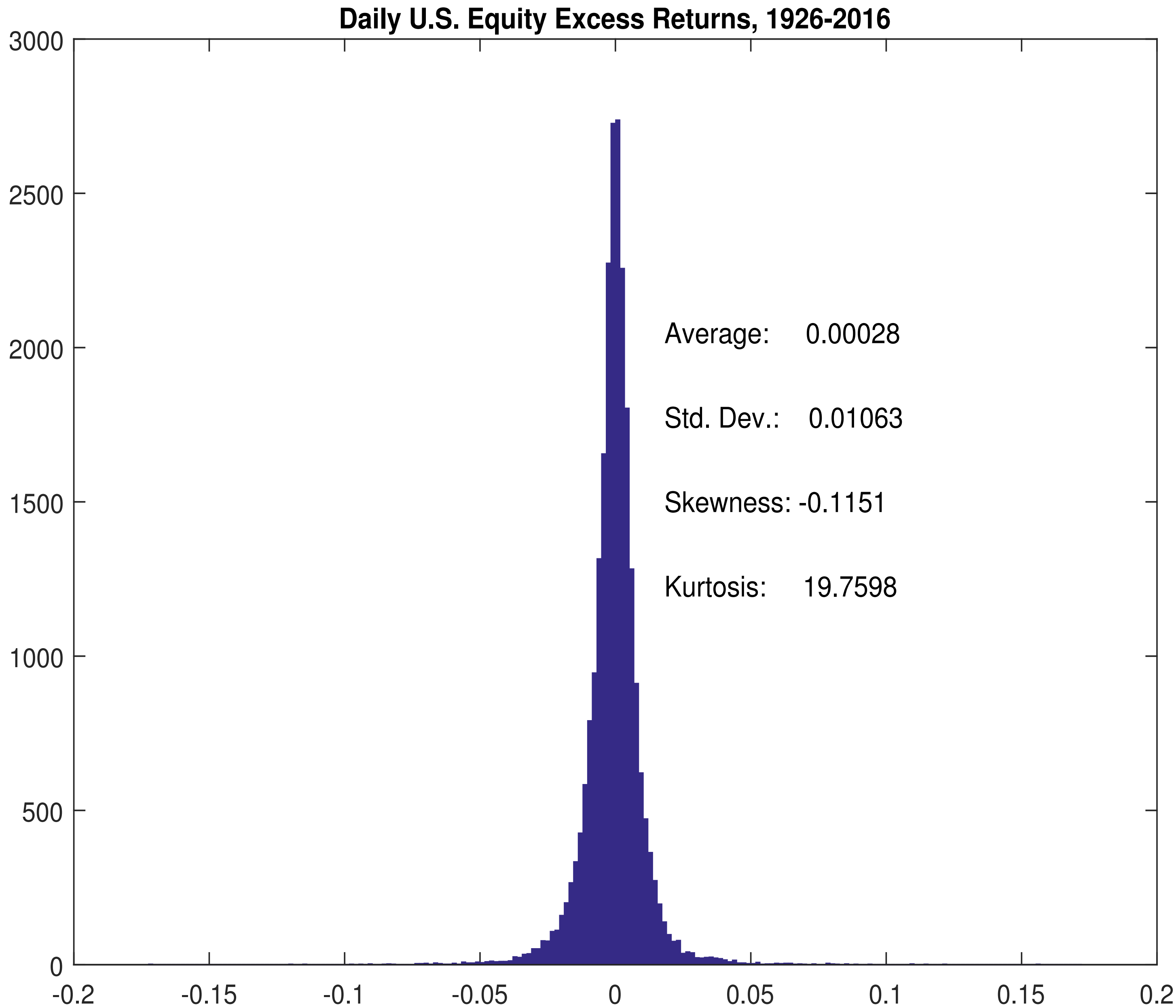
- Return on value-weighted portfolio of U.S Stocks
  minus
  Return on U.S. Treasury ("riskless" or "cash" rate)
- Daily excess returns
  - 1/2/1926–12/30/2016
  - 24,038 trading days
  - subperiods: 1/2/1926–5/13/1969, 5/14/1969–12/30/2016
- Monthly excess returns
  - 1/1926–12/2016
  - 1092 months
  - subperiods: 1/1926–6/1971, 7/1971–12/2016
- Annual excess returns
  - 1926–2016
  - 91 years
  - subperiods: 1926–1971, 1971–2016

Lowest Daily Excess Returns (%), 1926–2016

October 19, 1987	-17.2
October 29, 1929	-12.0
October 28, 1929	-11.3
November 06, 1929	-9.7
July 21, 1933	-9.3
October 15, 2008	-9.0
December 01, 2008	-8.9
July 20, 1933	-8.5
October 26, 1987	-8.3
September 29, 2008	-8.3
October 18, 1937	-8.2
October 05, 1932	-7.3
October 09, 2008	-7.3
May 14, 1940	-7.2
July 26, 1934	-7.2
May 21, 1940	-7.0
November 20, 2008	-7.0
May 28, 1962	-7.0
September 03, 1946	-6.9
August 08, 2011	-6.9

Highest Daily Excess Returns (%), 1926–2016

March 15, 1933	15.7
October 30, 1929	12.2
October 13, 2008	11.5
October 06, 1931	11.1
September 21, 1932	11.0
October 28, 2008	9.5
September 05, 1939	8.9
October 21, 1987	8.6
October 20, 1937	8.6
April 20, 1933	8.3
February 13, 1932	8.2
July 24, 1933	8.0
December 18, 1931	7.9
October 08, 1931	7.9
November 14, 1929	7.9
August 03, 1932	7.8
February 11, 1932	7.4
June 19, 1933	7.3
April 19, 1933	7.1
March 23, 2009	6.9


Daily U.S. Equity Excess Returns, 1926-2016
3000
2500
2000
1500
1000
500
0
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Average: 0.00028
Std. Dev.: 0.01063
Skewness: -0.1151
Kurtosis: 19.7598

Threshold Probabilities: Daily Returns

R_t: excess return on any given day t

$E(R_t)$: estimated with arithmetic sample mean (not the geometric mean)

$\sigma(R_t)$: estimated with sample standard deviation

$$\text{Prob}\{R_t < E(R_t) + x \cdot \sigma(R_t)\}$$

x	Data Frequency	Normal Probability
-3.0	0.0092	0.0013
-2.0	0.0264	0.0228
-1.5	0.0488	0.0668
-1.0	0.0988	0.1587
0.0	0.4770	0.5000
1.0	0.9120	0.8413
1.5	0.9601	0.9332
2.0	0.9791	0.9772
3.0	0.9913	0.9987

Lowest Daily Excess Returns (%), 1926–1969

October 29, 1929	-12.0
October 28, 1929	-11.3
November 06, 1929	-9.7
July 21, 1933	-9.3
July 20, 1933	-8.5
October 18, 1937	-8.2
October 05, 1932	-7.3
May 14, 1940	-7.2
July 26, 1934	-7.2
May 21, 1940	-7.0
May 28, 1962	-7.0
September 03, 1946	-6.9
May 31, 1932	-6.8
September 24, 1931	-6.8
September 26, 1955	-6.5
September 12, 1932	-6.5
June 15, 1933	-6.3
August 12, 1932	-6.3
September 21, 1933	-6.2
October 16, 1933	-6.0

Lowest Daily Excess Returns (%), 1969–2016

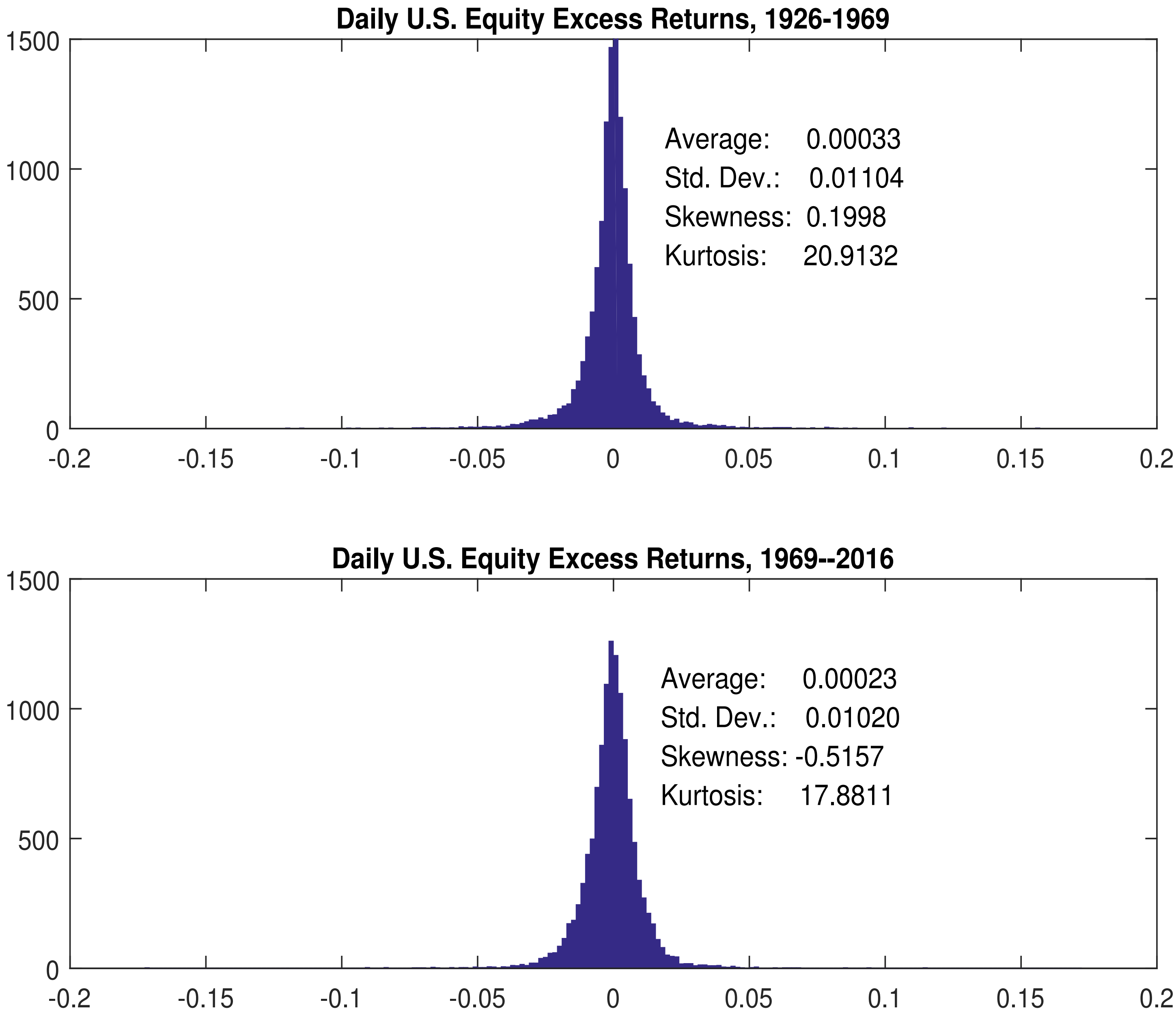
October 19, 1987	-17.2
October 15, 2008	-9.0
December 01, 2008	-8.9
October 26, 1987	-8.3
September 29, 2008	-8.3
October 09, 2008	-7.3
November 20, 2008	-7.0
August 08, 2011	-6.9
April 14, 2000	-6.6
August 31, 1998	-6.6
October 27, 1997	-6.5
November 19, 2008	-6.4
October 22, 2008	-5.9
October 07, 2008	-5.8
January 08, 1988	-5.5
January 20, 2009	-5.5
October 13, 1989	-5.4
November 12, 2008	-5.4
September 17, 2001	-5.1
November 05, 2008	-5.1

Highest Daily Excess Returns (%), 1926–1969

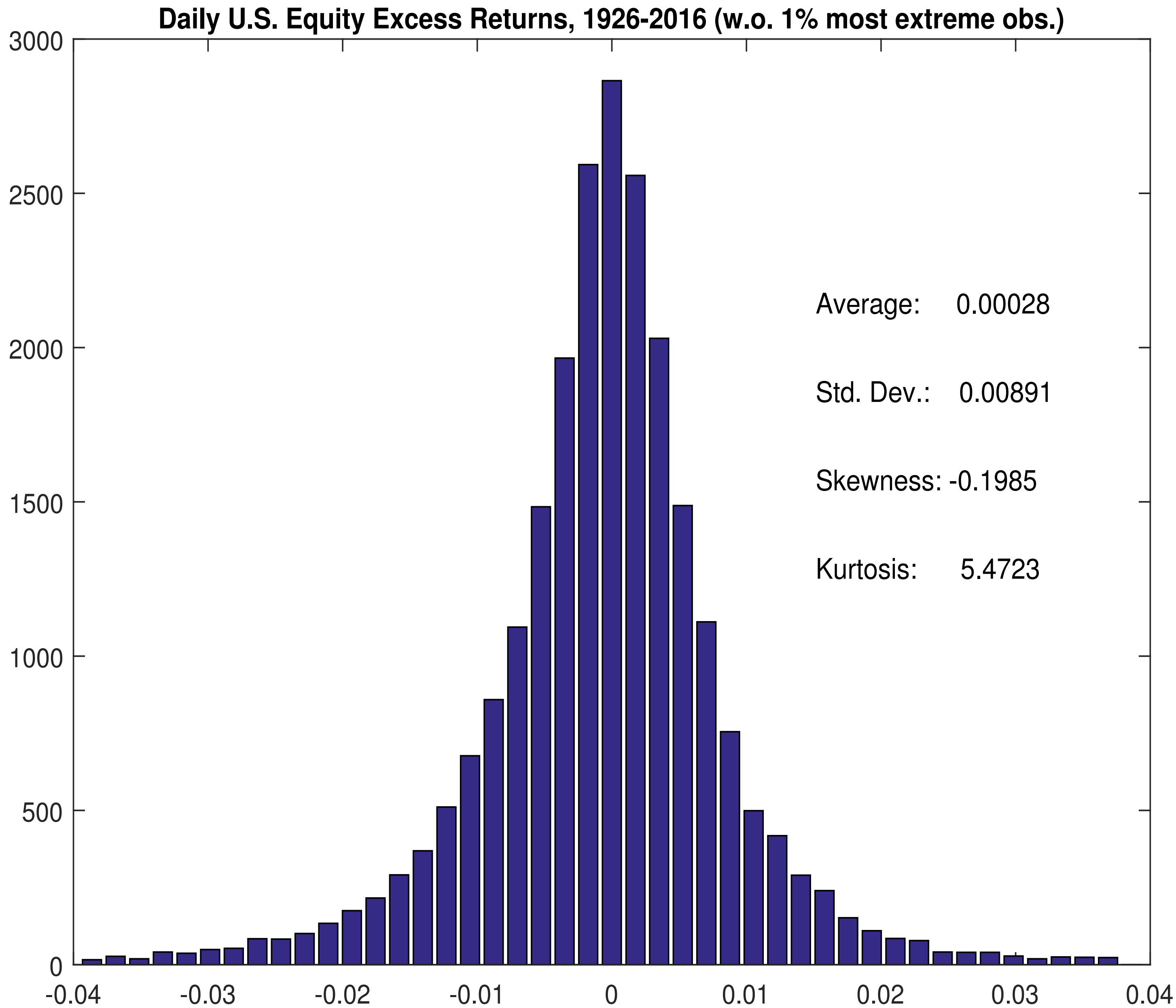
March 15, 1933	15.7
October 30, 1929	12.2
October 06, 1931	11.1
September 21, 1932	11.0
September 05, 1939	8.9
October 20, 1937	8.6
April 20, 1933	8.3
February 13, 1932	8.2
July 24, 1933	8.0
December 18, 1931	7.9
October 08, 1931	7.9
November 14, 1929	7.9
August 03, 1932	7.8
February 11, 1932	7.4
June 19, 1933	7.3
April 19, 1933	7.1
October 11, 1932	6.9
September 23, 1931	6.6
June 03, 1931	6.5
January 06, 1932	6.5

Highest Daily Excess Returns (%), 1969–2016

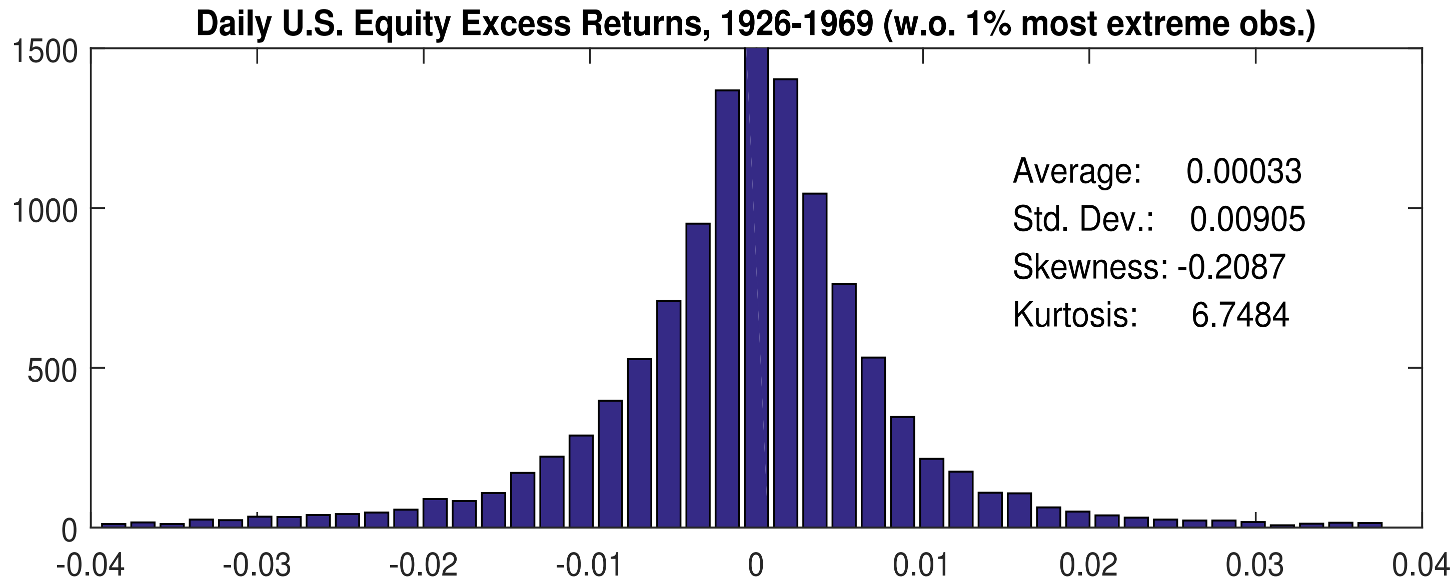
October 13, 2008	11.5
October 28, 2008	9.5
October 21, 1987	8.6
March 23, 2009	6.9
November 13, 2008	6.8
November 24, 2008	6.6
March 10, 2009	6.3
November 21, 2008	6.1
July 29, 2002	5.3
January 03, 2001	5.3
December 16, 2008	5.3
May 27, 1970	5.3
July 24, 2002	5.2
August 09, 2011	5.1
September 08, 1998	4.8
September 30, 2008	4.7
October 20, 2008	4.6
September 19, 2008	4.6
April 05, 2001	4.6
October 29, 1987	4.6

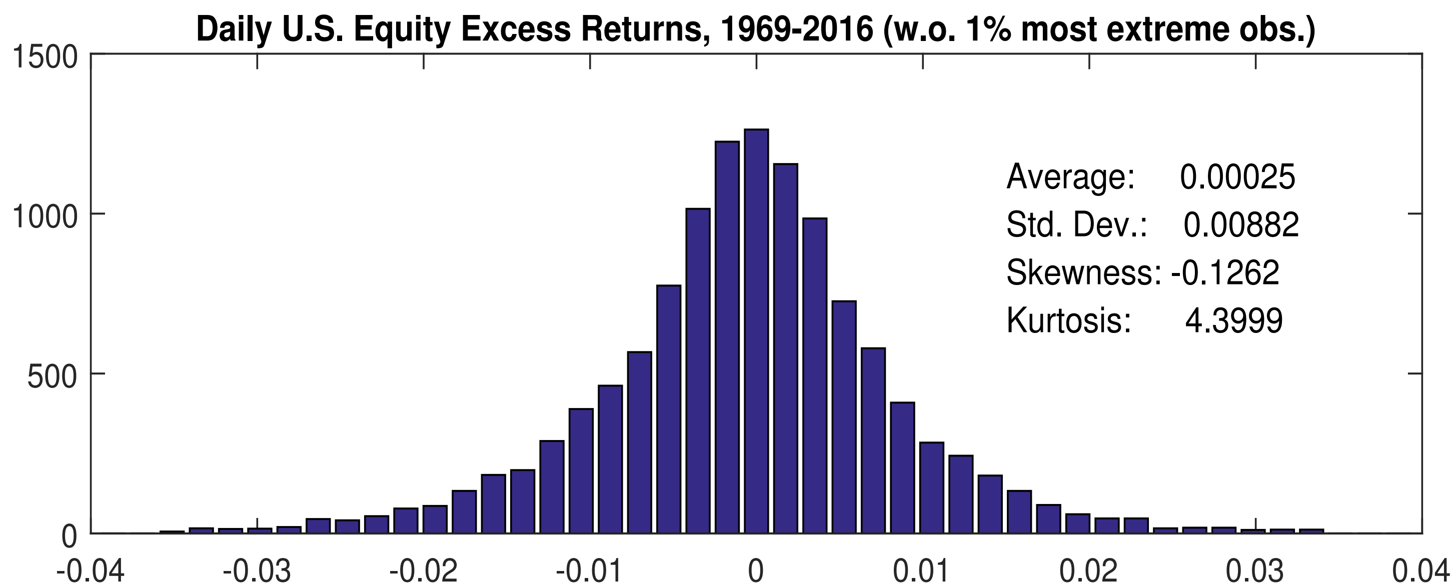


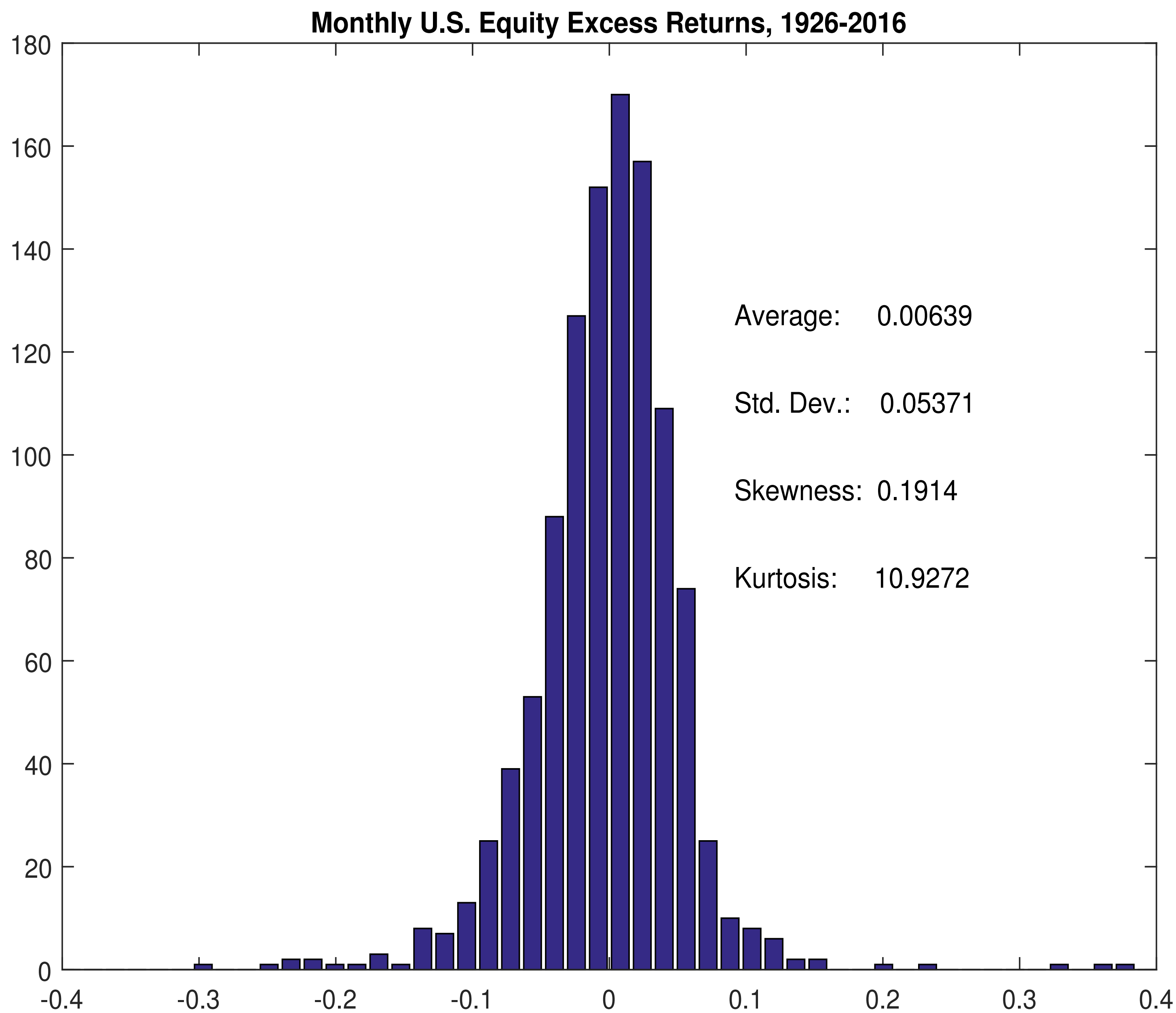
Daily U.S. Equity Excess Returns, 1926-1969
Average: 0.00033
Std. Dev.: 0.01104
Skewness: 0.1998
Kurtosis: 20.9132
1500
1000
500
0
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Daily U.S. Equity Excess Returns, 1969--2016
Average: 0.00023
Std. Dev.: 0.01020
Skewness: -0.5157
Kurtosis: 17.8811
1500
1000
500
0
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2



Daily U.S. Equity Excess Returns, 1926-2016 (w.o. 1% most extreme obs.)
3000
2500
2000
1500
1000
500
0
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
Average: 0.00028
Std. Dev.: 0.00891
Skewness: -0.1985
Kurtosis: 5.4723


Daily U.S. Equity Excess Returns, 1926-1969 (w.o. 1% most extreme obs.)
1500
1000
500
0
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
Average: 0.00033
Std. Dev.: 0.00905
Skewness: -0.2087
Kurtosis: 6.7484


Daily U.S. Equity Excess Returns, 1969-2016 (w.o. 1% most extreme obs.)
1500
1000
500
0
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
Average: 0.00025
Std. Dev.: 0.00882
Skewness: -0.1262
Kurtosis: 4.3999


Monthly U.S. Equity Excess Returns, 1926-2016
Average: 0.00639
Std. Dev.: 0.05371
Skewness: 0.1914
Kurtosis: 10.9272
180
160
140
120
100
80
60
40
20
0
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4

Threshold Probabilities: Monthly Returns

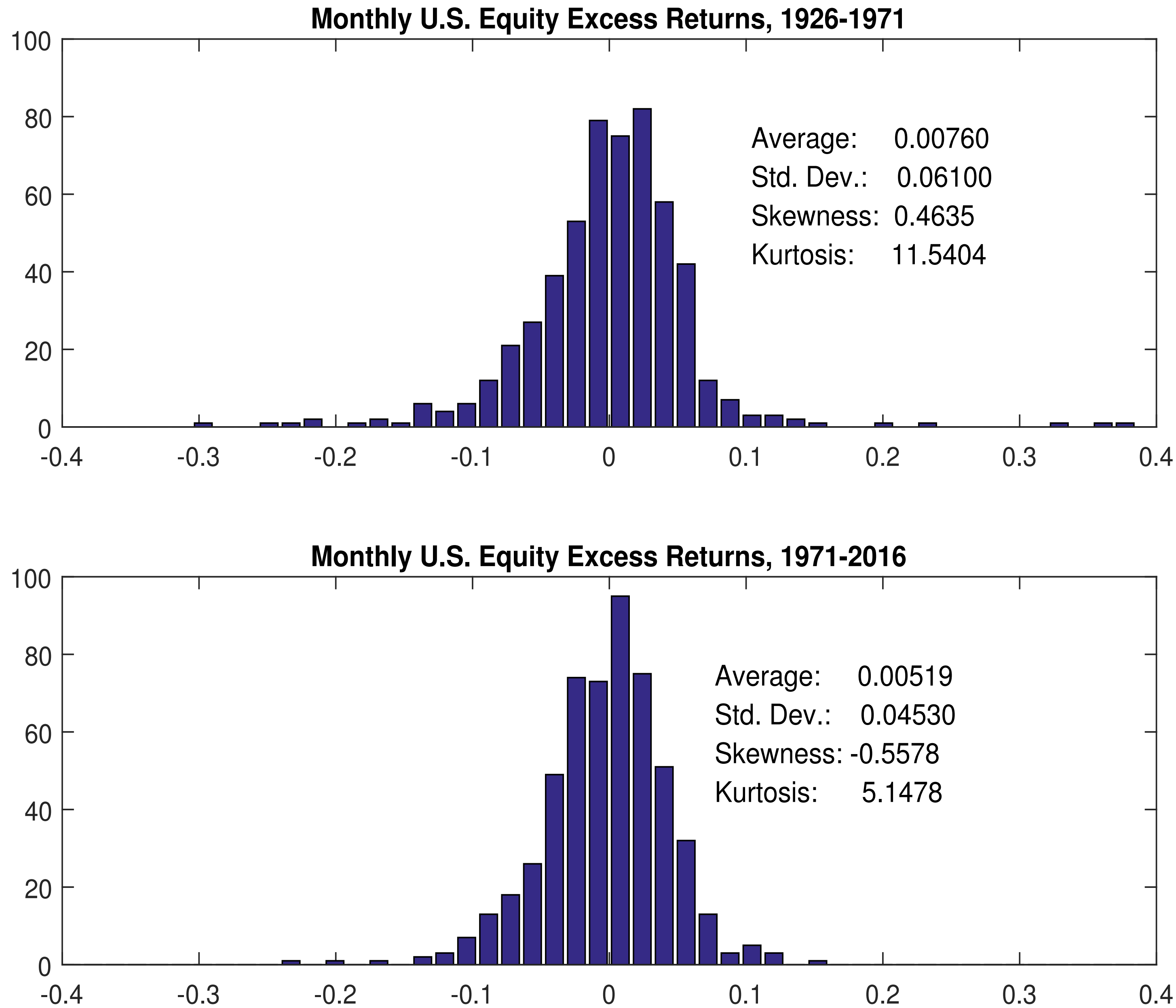
R_t: excess return in any given month t

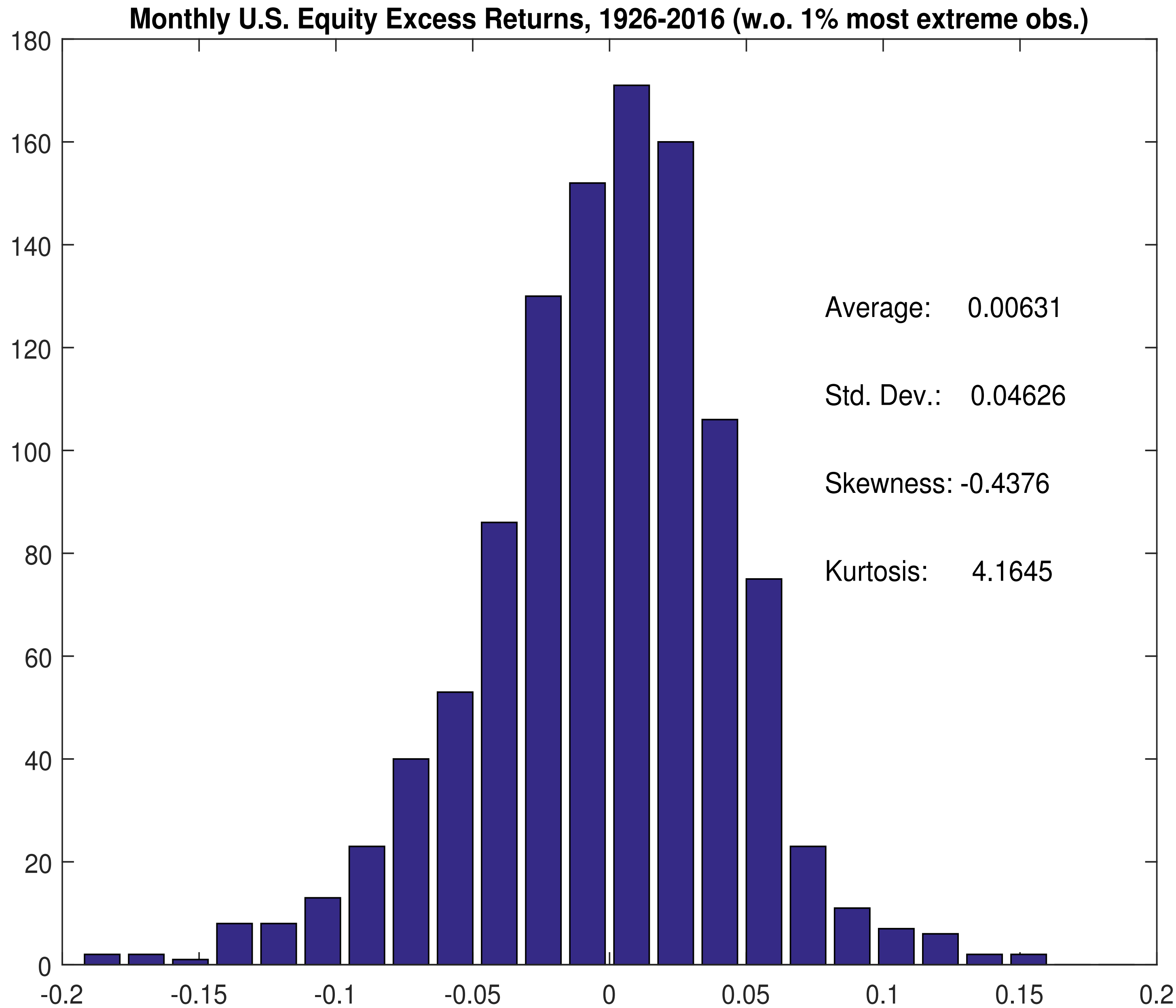
$E(R_t)$: estimated with arithmetic sample mean (not the geometric mean)

$\sigma(R_t)$: estimated with sample standard deviation

$$\text{Prob}\{R_t < E(R_t) + x \cdot \sigma(R_t)\}$$

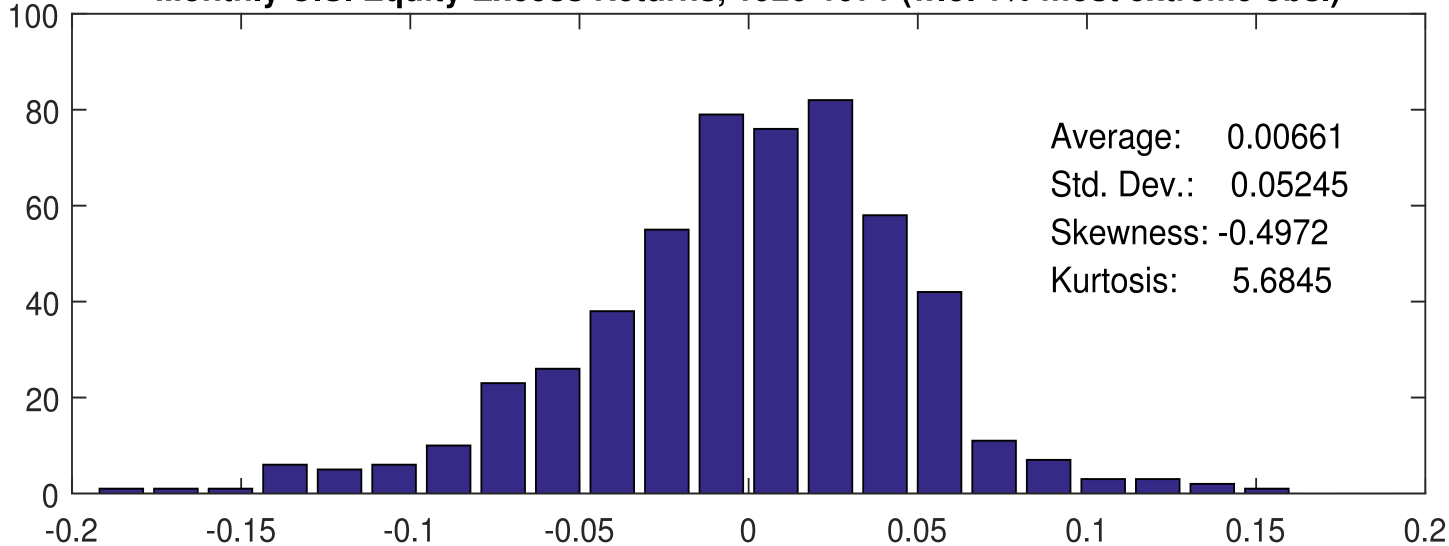
x	Data Frequency	Normal Probability
-3.0	0.0092	0.0013
-2.0	0.0284	0.0228
-1.5	0.0577	0.0668
-1.0	0.1209	0.1587
0.0	0.4634	0.5000
1.0	0.9048	0.8413
1.5	0.9679	0.9332
2.0	0.9844	0.9772
3.0	0.9954	0.9987


Monthly U.S. Equity Excess Returns, 1926-1971
Average: 0.00760
Std. Dev.: 0.06100
Skewness: 0.4635
Kurtosis: 11.5404
100
80
60
40
20
0
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
Monthly U.S. Equity Excess Returns, 1971-2016
Average: 0.00519
Std. Dev.: 0.04530
Skewness: -0.5578
Kurtosis: 5.1478
100
80
60
40
20
0
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4

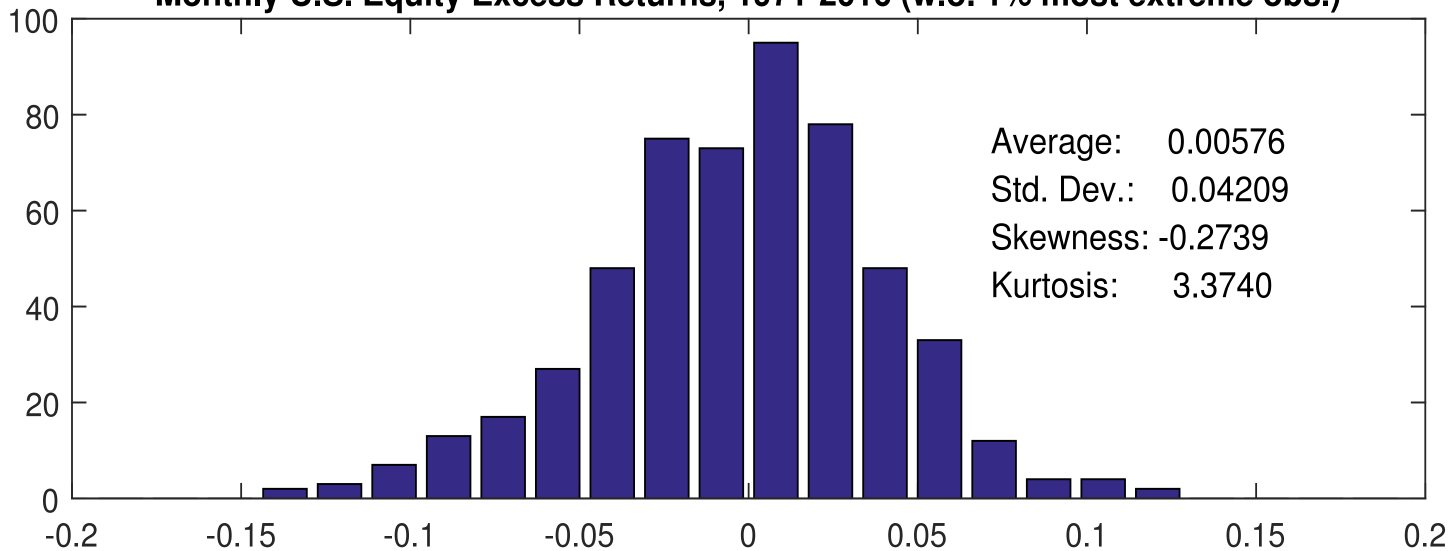


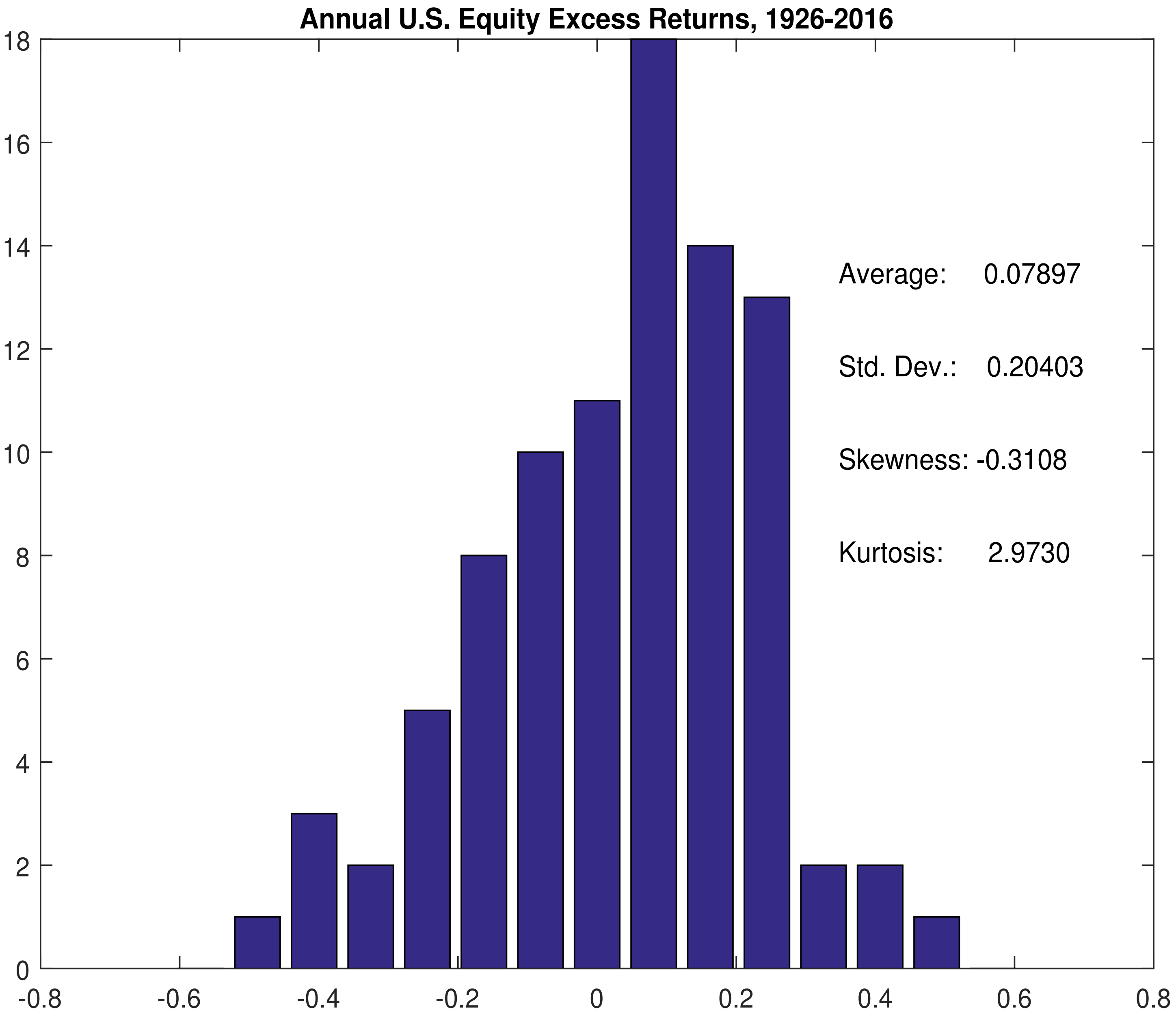
Monthly U.S. Equity Excess Returns, 1926-2016 (w.o. 1% most extreme obs.)
Average: 0.00631
Std. Dev.: 0.04626
Skewness: -0.4376
Kurtosis: 4.1645
180
160
140
120
100
80
60
40
20
0
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2

Monthly U.S. Equity Excess Returns, 1926-1971 (w.o. 1% most extreme obs.)



Monthly U.S. Equity Excess Returns, 1971-2016 (w.o. 1% most extreme obs.)





Annual U.S. Equity Excess Returns, 1926-2016
Average: 0.07897
Std. Dev.: 0.20403
Skewness: -0.3108
Kurtosis: 2.9730
0
2
4
6
8
10
12
14
16
18
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8

Threshold Probabilities: Annual Returns

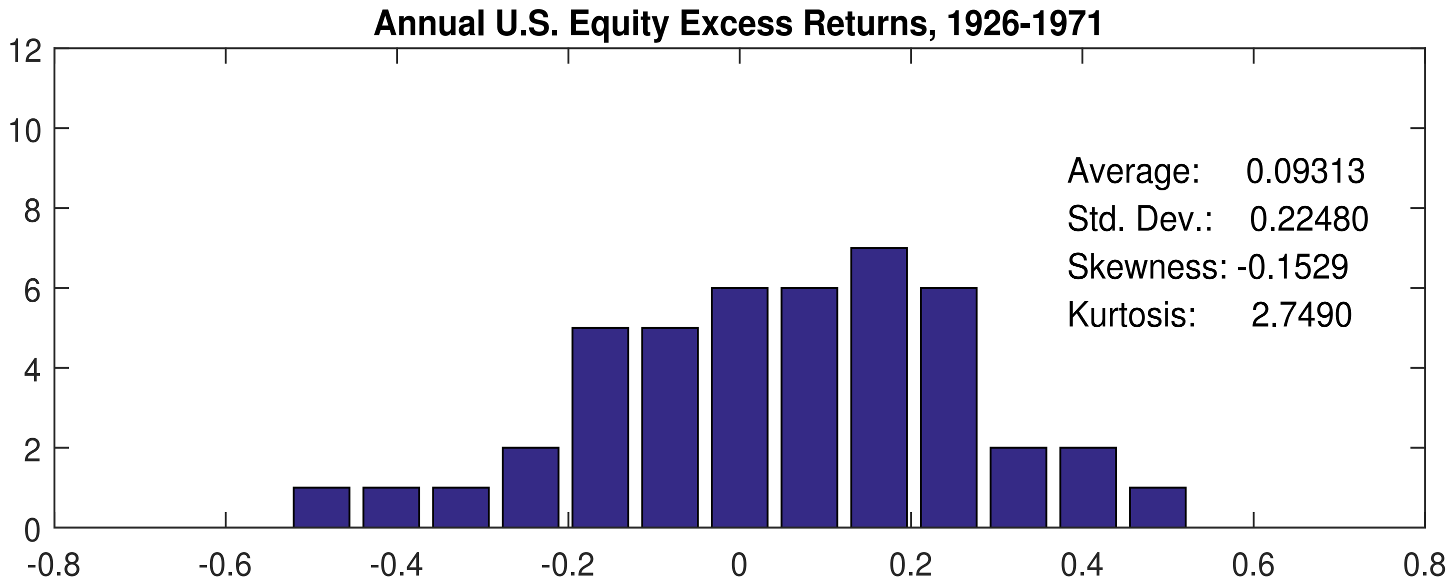
R_t: excess return in any given year t

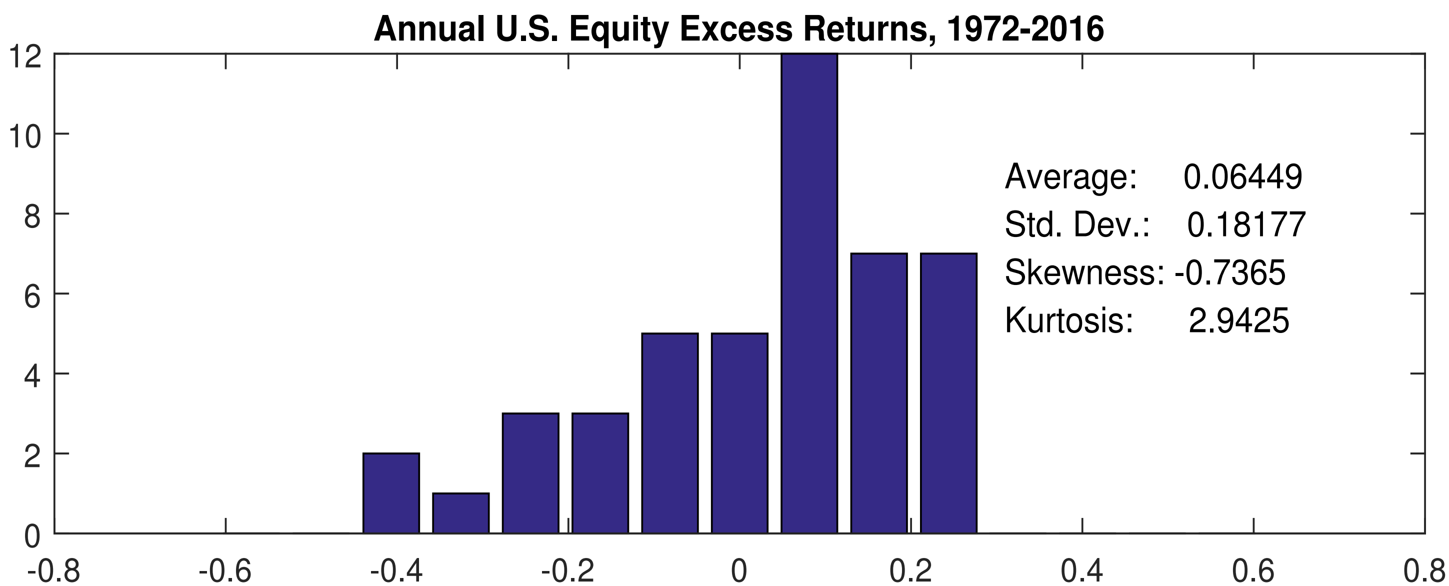
$E(R_t)$: estimated with arithmetic sample mean (not the geometric mean)

$\sigma(R_t)$: estimated with sample standard deviation

$$\text{Prob}\,\{R_t < E(R_t) + x \cdot \sigma(R_t)\}$$

x	Data Frequency	Normal Probability
-3.0	0.0000	0.0013
-2.0	0.0440	0.0228
-1.5	0.0659	0.0668
-1.0	0.1758	0.1587
0.0	0.4396	0.5000
1.0	0.8352	0.8413
1.5	0.9451	0.9332
2.0	0.9780	0.9772
3.0	1.0000	0.9987


Annual U.S. Equity Excess Returns, 1926-1971
Average: 0.09313
Std. Dev.: 0.22480
Skewness: -0.1529
Kurtosis: 2.7490


Annual U.S. Equity Excess Returns, 1972-2016
Average: 0.06449
Std. Dev.: 0.18177
Skewness: -0.7365
Kurtosis: 2.9425

Sharpe Ratios and Probabilities

- An investment's ***Sharpe ratio***:

$$S = \frac{E(R_t)}{\sigma(R_t)}$$

- If R_t is normally distributed,

$$z = \frac{R_t - E(R_t)}{\sigma(R_t)}$$

is a *standardized* normal variable with $E(z) = 0$ and $\sigma(z) = 1$

- Probability of beating cash, approximated using normal distribution:

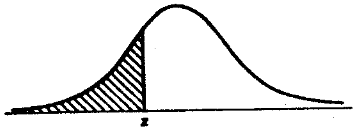
$$\text{Prob}\{R_t > 0\} = \text{Prob}\left\{\frac{R_t - E(R_t)}{\sigma(R_t)} > \frac{-E(R_t)}{\sigma(R_t)}\right\} = \text{Prob}\{z > -S\},$$

- Examples:

S	Prob $\{R_t > 0\}$
0.10	0.540
0.38	0.648
0.76	0.776
1.00	0.841
2.00	0.977

TABLE I Values of the Standard Normal Distribution Function

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}u^2\right) du = P(Z \le z)$$



z	0	1	2	3	4	5	6	7	8	9
−3.	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0126	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0238	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0300	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0570	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
− .9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
− .8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
− .7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
− .6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
− .5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
− .4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
− .3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
− .2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
− .1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
− 0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TABLE 1 Values of the Standard Normal Distribution Function (Continued)

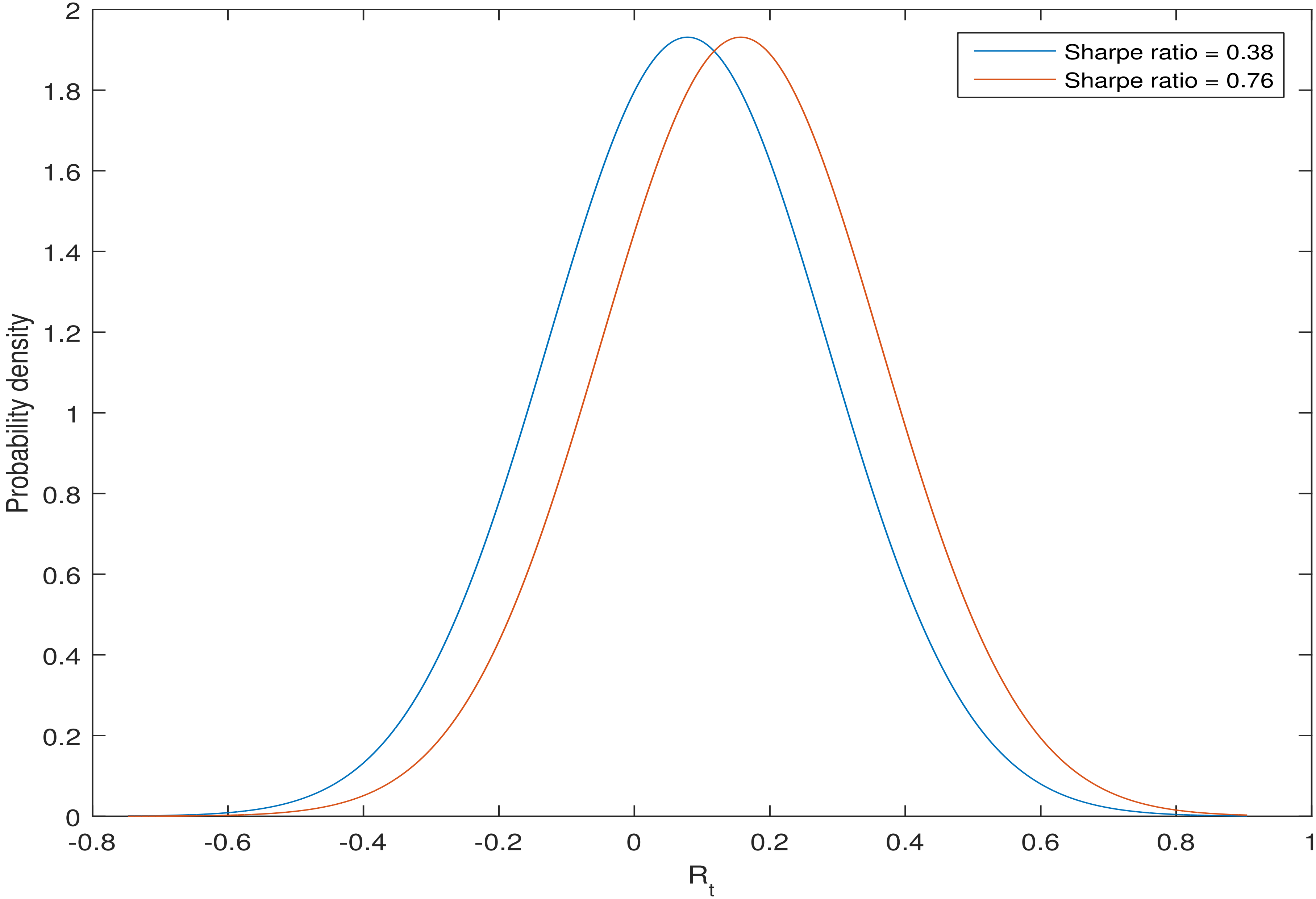
z	0	1	2	3	4	5	6	7	8	9
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9430	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9648	.9656	.9664	.9671	.9678	.9686	.9693	.9700	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9762	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9874	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

Note 1: If a normal variable X is not "standard," its values must be "standardized": $Z = (X - \mu)/\sigma$, i.e., $P(X \le x) = \Phi[(x - \mu)/\sigma]$.

Note 2: For "two-tail" probabilities, see Table Ib.

Note 3: For $z \ge 4$, $\Phi(z) = 1$ to four decimal places; for $z \le -4$, $\Phi(z) = 0$ to four decimal places.

Note 4: Entries opposite 3 and −3 are for 3.0, 3.1, 3.2, etc., and −3.0, −3.1, etc., respectively.



Sharpe ratio = 0.38
Sharpe ratio = 0.76
Probability density
R_t
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Sharpe Ratios and Investment Horizons

- S is (approximately) proportional to the square root of the investment horizon
- An investment
  - with a monthly S of $\mathbf{0.10}$
  - has an annual S of about $\mathbf{0.10 \times \sqrt{12} = 0.346}$
  - and a daily S of about $\mathbf{0.10 \div \sqrt{21} = 0.0218}$
- For the U.S. equity market,
  - daily: $S = \mathbf{0.000277/.01065 = 0.0260}$
  - monthly: $S = \mathbf{0.00635/.0539 = 0.1178}$
  - annual: $S = \mathbf{0.0785/.2051 = 0.3825}$
- Note

$$
\begin{aligned}
\sqrt{21} \times 0.0260 &= 0.1191 \quad \text{(versus } 0.1178\text{)} \\
\sqrt{12} \times 0.1178 &= 0.4081 \quad \text{(versus } 0.3825\text{)} \\
\sqrt{250} \times 0.0260 &= 0.4111 \quad \text{(versus } 0.3825\text{)}
\end{aligned}
$$

Why the Square Root?

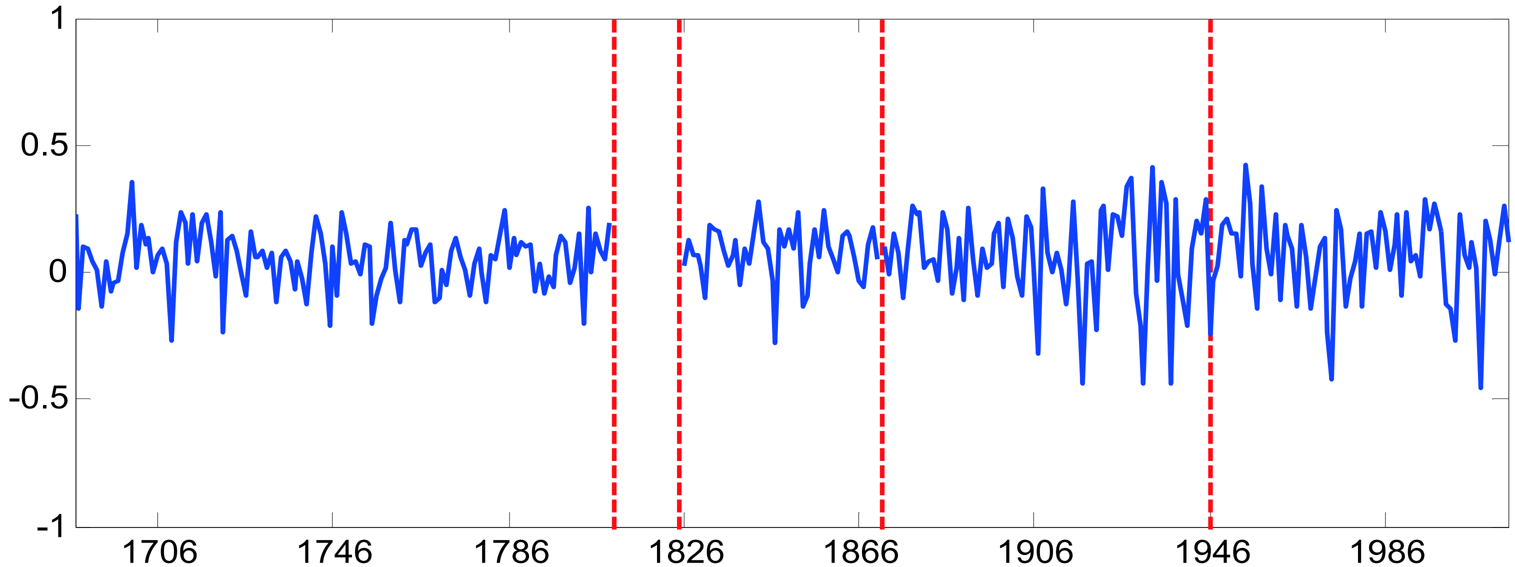
- In each of the next N periods invest one dollar (i.e., don't compound returns)
- Profit (in excess of cash return) : $P = R_{t+1} + R_{t+2} + \cdots + R_{t+N}$
- Assume R_t is distributed identically and independently across periods
- $E(P) = N \cdot E(R_t)$ [expected value of sum = sum of expected values]
- $\sigma^2(P) = N \cdot \sigma^2(R_t)$ [variance of sum = sum of variances, ***if*** all of the terms are uncorrelated with each other]
- The Sharpe ratio of the profit per dollar invested (i.e., return):

$$S = \frac{E(P)}{\sigma(P)} = \frac{N \cdot E(R_t)}{\sqrt{N} \cdot \sigma(R_t)} = \sqrt{N}\left(\frac{E(R_t)}{\sigma(R_t)}\right)$$

- S is $\sqrt{N}$ times the one-period Sharpe ratio

A Four-Century Perspective on Stock Markets

	Neth./U.K. 1686–1809	U.K. 1825–1870	U.S 1871–1945	U.S. 1945–2014	Full period
Mean (total) return	0.055	0.069	0.069	0.103	0.071
Std. Dev.	0.093	0.069	0.190	0.161	0.136
Cash rate	0.032	0.034	0.025	0.041	0.032
Sharpe ratio	0.249	0.508	0.232	0.382	0.283



Source: “Four Centuries of Return Predictability,” by Benjamin Golez and Peter Koudijs, working paper, 2016, University of Notre Dame and Stanford University