UNIVERSITY OF PENNSYLVANIA
The Wharton School

Investments | Project 2: Optimization
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Goal: Use historical data and portfolio optimization to explore the extent to which investment opportunities for a mean-variance investor are enhanced by (1) adding portfolios segmented by value/growth and market capitalization and (2) allowing long-short strategies.

Setting and Analysis: An endowment is currently invested 80% in a market-index fund and 20% in cash. The endowment's investment committee is considering the potential benefits of adding portfolios that provide exposure to segments of the market stratified by firms' market capitalizations and book-to-market ratios. (Stocks with high book-to-market ratios are known as "value" stocks, and those with low book-to-market ratios are known as "growth" stocks.) Specifically, the committee is considering the potential addition of four portfolios to the fund's investment universe:

1. Small-cap growth
2. Small-cap value
3. Large-cap growth
4. Large-cap value

The committee is also considering whether to add the above portfolios in a long-only capacity or whether allocations involving short positions are sufficiently more attractive to justify going in that direction. Finally, some committee members note that small-cap stocks are substantially riskier, and they wonder whether excluding the two small-cap portfolios above would significantly limit the endowment's investment opportunities.

The committee is interested in learning the potential attractiveness of the following four scenarios from a mean-variance perspective:

	Eligible risky assets	**Constraints on risky-asset weights**	**Constraints on borrowing/lending**
1	Market index, large-cap growth, large-cap value	Long only	No borrowing allowed
2	Market index, large-cap growth, large-cap value	Weight on each asset between -0.5 and 1.5	No constraint on borrowing/lending
3	Market index, large-cap growth, large-cap value, small-cap growth, small-cap value	Long only	No borrowing allowed
4	Market index, large-cap growth, large-cap value, small-cap growth, small-cap value	Weight on each asset between -0.5 and 1.5	No constraint on borrowing/lending

For each of the above four scenarios, report:

1. the mean-variance-efficient (optimal) allocation
2. the monthly Sharpe ratio of that allocation
3. the added benefit over the current portfolio in terms of a monthly certainty-equivalent rate of return
4. the 5% monthly Value-at-Risk (VaR) of the recommended allocation under normally distributed returns

In an overall summary of the results you obtained across the four scenarios, comment on whether the additional portfolios appear to offer the endowment substantial benefits over the existing allocation. If there are substantial benefits, comment on which added dimensions (i.e., market-cap tilts, value/growth tilts, shorting) appear most responsible for those potential benefits.

Guidelines:

1. Assume a riskless interest rate (for lending or borrowing) equal to 0.003 (0.3%) per month.
2. Use a mean-variance objective function of the form

$$U = E(R_p) - (1/2)\, A\, Var(R_p).$$

   The value of U is also known as the investment's "certainty-equivalent" rate of return.
3. The endowment's degree of risk aversion (A) is such that the current allocation between the market index and cash is optimal for the endowment when the investment universe is restricted to only those two assets.
4. For inputs pertaining to the four additional portfolios, use values produced by the Wharton Backtester with the following specifications:
   a. Returns from the end of December 1981 through the end of December 2016
   b. Market-capitalization portfolio weights, annual resorting, using all available stocks
   c. Create four portfolios by sorting (independently) into two partitions for market cap and two partitions for book-to-market
5. Backtester output also includes a market benchmark (the capitalization-weighted NYSE/AMEX/NASDAQ), which you should treat as the market-index fund in this analysis.
6. The "Optimize Backtest" button in Backtester transfers values to a web-based optimization program and includes the option to add one or two additional assets. Adding one asset, for example, is useful for including a single riskless interest rate for borrowing and/or lending.[1]
7. The 5% monthly VaR is the monthly rate of return for which there is only a 5% probability of a worse outcome. For this calculation, assume returns are normally distributed.

[1] The optimizer can also be accessed directly at http://finance.wharton.upenn.edu/~stambaug/portopt.html, where all input values must be entered manually.