Economics 701A **Steven A. Matthews**
Fall 2015 **University of Pennsylvania**

Suggested Solutions to the Quiz

20 points, 40 minutes. Closed books, notes, calculators.
Indicate your reasoning, using clearly written words as well as math.

1. (8 pts) Let $X = \{x, y, z\}$, and consider a choice correspondence C defined on $\mathfrak{B} = 2^X \backslash \varnothing$ (the set of all nonempty subsets of X), with $C(B) \neq \varnothing$ for all $B \in \mathfrak{B}$. Suppose

$$C(\{x, y\}) = \{x\}, \quad C(\{y, z\}) = \{y\}, \quad C(\{x, z\}) = \{z\}.$$

Say which of the following is correct, and prove your answer:

(a) $(\mathfrak{B}, C(\cdot))$ must violate WARP.

(b) $(\mathfrak{B}, C(\cdot))$ must satisfy WARP.

(c) $(\mathfrak{B}, C(\cdot))$ may or may not satisfy WARP.

Soln: The answer is (a), $(\mathfrak{B}, C(\cdot))$ must violate WARP.

Proof. Assume $(\mathfrak{B}, C(\cdot))$ satisfies WARP. Then, by the Revealed Preference Theorem (Lecture Slides 1, page 17), $\succeq^*$ rationalizes $(\mathfrak{B}, C(\cdot))$. Hence,

$$\text{not } y \succeq^* x, \text{ since } y \notin C(\{x, y\}) = C^*(\{x, y\}, \succeq^*).$$

Similarly, we have not $z \succeq^* y$ and not $x \succeq^* z$. Hence,

$$C^*(\{x, y, z\}, \succeq^*) = \varnothing.$$

Since $\succeq^*$ rationalizes $(\mathfrak{B}, C(\cdot))$, this implies the contradiction

$$C(\{x, y, z\}) = \varnothing.$$

We conclude that $(\mathfrak{B}, C(\cdot))$ must violate WARP. ■

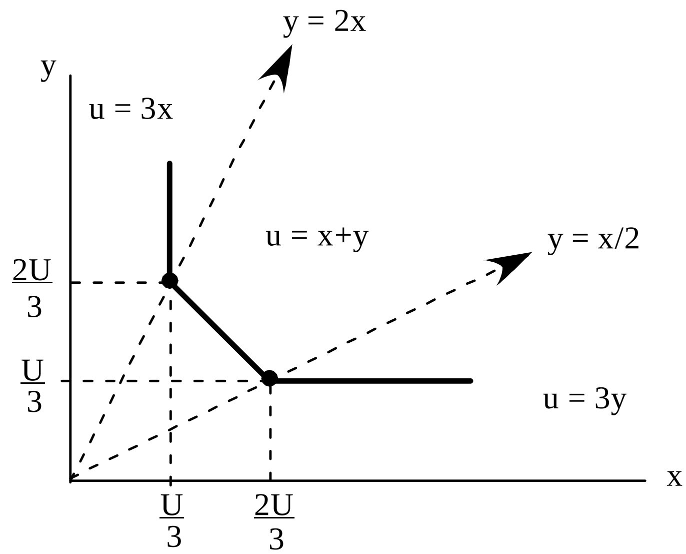
2. (12 pts) (12 pts) A consumer consumes goods x and y, and has the utility function on $\mathbb{R}^2_+$ defined by $u(x,y) = \min(3x, x+y, 3y)$. This utility function can also be written as

$$u(x,y) = \begin{cases} 3y & \text{if } y < \frac{1}{2}x \\ x+y & \text{if } \frac{1}{2}x \le y < 2x \\ 3x & \text{if } 2x \le y \end{cases}$$

The price of good y is fixed at $p_y = 1$.

(a) (4 pts) Graph the indifference curve for a utility level $U \ge 0$, labeling as many points as possible.

Soln:



■

(b) (4 pts) Find the expenditure function $e(p_x, U)$ for all $(p_x, U) \ge (0,0)$.

Soln: We give two alternative solutions.

Solution 1. When the budget lines are steeper than the middle part of the indifference curves, i.e., when $p_x > 1$, we can see that the cost minimizing bundle is on the $y = 2x$ line. Hence, in this case $h(p_x, U) = (\frac{1}{3}U, \frac{2}{3}U)$, and so

$$e(p_x, U) = p_x \tfrac{1}{3}U + \tfrac{2}{3}U = \tfrac{1}{3}(p_x + 2)\,U.$$

Note that $(\frac{1}{3}U, \frac{2}{3}U)$ also minimizes expenditure when $p_x = 1$, so this expression for $e(p_x, U)$ is also valid in this case, with $e(1, U) = U$. Similarly, if $0 < p_x < 1$ then $h(p_x, U) = (\frac{2}{3}U, \frac{1}{3}U)$, and so

$$e(p_x, U) = p_x \tfrac{2}{3}U + \tfrac{1}{3}U = \tfrac{1}{3}(2p_x + 1)\,U.$$

This expression is also valid for $p_x = 0$, since $(\frac{2}{3}U, \frac{1}{3}U)$ is also expenditure minimizing in this case. In summary, for all $(p_x, U) \ge (0,0)$,

$$e(p_x, U) = \begin{cases} \frac{1}{3}(2p_x + 1)\,U & \text{for } p_x \le 1 \\ \frac{1}{3}(p_x + 2)\,U & \text{for } 1 \le p_x \end{cases} \tag{1}$$

Solution 2. Clearly, the expenditure minimizing bundle will lie on one of the two lines, $y = 2x$ or $y = x/2$. That is, either $(\frac{1}{3}U, \frac{2}{3}U)$ or $(\frac{2}{3}U, \frac{1}{3}U)$, or both, solves the EMP. So $e(p_x, U)$ must equal the cost of one of these bundles, whichever has the lower cost:

$$\begin{aligned} e(p_x, U) &= \min\left\{p_x\tfrac{1}{3}U + \tfrac{2}{3}U,\ p_x\tfrac{2}{3}U + \tfrac{1}{3}U\right\} \\ &= \tfrac{1}{3}U \min\left\{p_x + 2, 2p_x + 1\right\} \\ &= \begin{cases} \frac{1}{3}(2p_x + 1)\,U & \text{for } p_x \leq 1 \\ \frac{1}{3}(p_x + 2)\,U & \text{for } 1 \leq p_x \end{cases}. \end{aligned}$$

■

(c) (4 pts) Find the indirect utility function $v(p_x, m)$ for $(p_x, m) \geq (0, 0)$.
Soln: Again, we give two solutions.
Direct Solution. When the budget line is steeper than the middle part of the indifference curves, i.e., when $p_x > 1$, we see that the utility maximizing bundle is on the $y = 2x$ line. We also know it will be on the budget line, so $p_x x + y = m$. Solving these two equations for x and y gives us the demand function for $p_x > 1$:

$$x(p_x, m) = \frac{m}{p_x + 2}, \quad y(p_x, m) = \frac{2m}{p_x + 2}.$$

Since the demand bundle is on the middle line segment of an indifference curve, we know it generates utility $x + y$, and so

$$p_x > 1 \;\Rightarrow\; v(p_x, m) = \frac{m}{p_x + 2} + \frac{2m}{p_x + 2} = \frac{3m}{p_x + 2}.$$

Note that this remains valid for $p_x = 1$, as the above bundle is still a solution of the UMP in this case.
Similarly, if $0 < p_x < 1$, the demand bundle satisfies $y = x/2$ and $p_x x + y = m$. Solving these two equations yields

$$x(p_x, m) = \frac{2m}{2p_x + 1}, \quad y(p_x, m) = \frac{m}{2p_x + 1}.$$

Again, $v = x + y$, and so

$$0 < p_x < 1 \;\Rightarrow\; v(p_x, m) = \frac{2m}{2p_x + 1} + \frac{m}{2p_x + 1} = \frac{3m}{2p_x + 1}.$$

Again, as the above demand bundle is still a solution if $p_x = 0$, this expression is valid for this case too. In summary, for any $(p_x, m) \geq (0, 0)$ we have

$$v(p_x, m) = \begin{cases} \dfrac{3m}{2p_x + 1} & \text{for } p_x \leq 1 \\ \dfrac{3m}{p_x + 2} & \text{for } 1 \leq p_x \end{cases}$$

Indirect Solution. We know that for fixed prices, the expenditure and indirect utility functions are inverses of each other. So we can obtain v by inverting the e we found in (b). From (1) we see that for $p_x \leq 1$,

$$U = \frac{3e(p_x, U)}{2p_x + 1}.$$

Hence, replacing U by $v(p_x, m)$ and $e(p_x, U)$ by m yields

$$v(p_x, m) = \frac{3m}{2p_x + 1}.$$

In the same way, using (1), we find that for $p_x \geq 1$ we have

$$U = \frac{3e(p_x, U)}{p_x + 2} \ \Rightarrow \ v(p_x, m) = \frac{3m}{p_x + 2}.$$

■