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Private Equity Performance and Liquidity Risk

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ABSTRACT

Private equity has traditionally been thought to provide diversification benefits. However, these benefits may be lower than anticipated as we find that private equity suffers from significant exposure to the same liquidity risk factor as public equity and other alternative asset classes. The unconditional liquidity risk premium is about 3% annually and, in a four-factor model, the inclusion of this liquidity risk premium reduces alpha to zero. In addition, we provide evidence that the link between private equity returns and overall market liquidity occurs via a funding liquidity channel.

INVESTING IN PRIVATE EQUITY IS among the preferred choices for long-term investors, such as endowments and pension funds, who seek to diversify their portfolios. Such long-term investors are clearly best suited for holding an illiquid asset (i.e., one that cannot be readily traded) such as private equity. The diversification benefits of private equity, however, have not been widely documented. In particular, an issue that has not been addressed so far is whether private equity performance, like that of other asset classes, is affected by liquidity risk (i.e., co-moves with unexpected changes in overall market liquidity).[1] The primary goal of this paper is to quantify liquidity risk in private equity. In

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[1] Note that the focus of this paper is on the systematic risk originating from time-varying liquidity, rather than on the asset-specific liquidity characteristic, the liquidity level. See the Internet Appendix for a more detailed discussion of this distinction. A new strand of literature in asset pricing has established liquidity risk as a priced factor in public equity returns (e.g., Pástor and Stambaugh (2003), Acharya and Pedersen (2005), Sadka (2006)). This evidence has been extended to emerging markets (Bekaert, Harvey, and Lundblad (2007)), bond markets (Beber, Brandt, and Kavajecz (2008), Chordia, Sarkar, and Subrahmanyam (2005), Li et al. (2009), and

Table I
Cash Flows of a Typical Investment

The table shows the cash flows of a representative investment. It lasts for four years, pays a final dividend equal to 1.5 times the original investment, and pays an intermediate dividend in year 2.5 equal to half of the initial investment. We show the computation of the modified IRR (MIRR) with a re-investment rate of 5% per semester. At the bottom of the table we report the present value of the dividends using two different discount rates.

Date (in Years)	Cash Flows	Re-invested Dividend (at 5% per Semester)
0	−100	0
0.5	0	0
1	0	0
1.5	0	0
2	0	0
2.5	50	50
3	0	53
3.5	0	55
4	150	208
MIRR = $(208/100)^{1/4} - 1$ = 20%		
IRR = 21%		
Present value of dividends at 15% discount rate		119
Present value of dividends at 18% discount rate		108

addition, the estimation of a factor model that includes the liquidity risk factor allows us to compute the cost of capital for this asset class and test whether it is efficiently priced.

We use a unique and comprehensive data set containing the exact cash flows generated by a large number of liquidated private equity investments. In order to clarify from the start the peculiar structure of our data, Table I shows a typical cash flow stream. There is an initial negative cash flow (the investment) followed by two positive cash flows (an intermediate distribution and the final dividend corresponding to the divestment). Note that we do not have intermediate valuations for the investment. As a result, there is no time series of returns, which precludes the use of the usual time-series regressions to estimate risk exposures. In such a context, as in Cochrane (2005), Korteweg and Sorensen (2010), and Driessen, Lin, and Phalippou (2012), we exploit variation in returns *across* investments to estimate the risk loadings and abnormal performance of the asset class.

We fit the four-factor model of Pástor and Stambaugh (2003) to the data and find a significant beta on the liquidity risk factor (0.64), on the market factor (1.3), and on the book-to-market factor (1.0), but not on the size factor. The exposure to these factors brings the alpha of this asset class to zero.

Acharya, Amihud, and Bharath (2010)), credit derivative markets (Longstaff, Mithal, and Neis (2005), Bongaerts, de Jong, and Driessen (2010), and Longstaff et al. (2011)), and hedge funds (Sadka (2010) and Boyson, Stahel, and Stulz (2010)).

Importantly, the liquidity risk premium is about 3% annually, which implies a discount of roughly 10% in the valuation of the typical investment (see Table I).[2] We also note that a liquidity risk beta of 0.64 exceeds the corresponding estimate for the large majority (86%) of traded stocks.

These results suggest that private equity is significantly exposed to the same liquidity risk factor as public equity and other asset classes. The diversification gains that can originate from private equity may thus be lower than previously thought given the exposure to liquidity risk.

Prompted by the finding of a significant loading on liquidity risk, we study the economic channel that relates private equity returns to market liquidity. We conjecture that, due to their high leverage, private equity investments are sensitive to the capital constraints faced by the providers of debt to private equity, who are primarily banks and hedge funds. Brunnermeier and Pedersen (2009) develop a theory in which the availability of capital—which they term funding liquidity—is positively related to market liquidity. In our context, their argument suggests that times of low market liquidity are likely to coincide with times when private equity managers may find it difficult to refinance their investments. In these periods, they may be forced to liquidate the investments or to accept higher borrowing costs, which in turn translates into lower returns for this asset class. Accordingly, we conjecture that the link between private equity returns and market liquidity occurs via a funding liquidity channel.

Empirically, we proxy for the evolution in funding liquidity with changes in the credit standards as reported in the Federal Reserve's Senior Loan Officer Survey. This survey asks loan officers at main banks whether they tightened or loosened their lending standards relative to the previous quarter. Axelson et al. (2010) argue that in the private equity context, "this measure captures non-price aspects of credit market conditions, such as debt covenants and quantity constraints." They find this measure to be strongly related to the amount of leverage used to finance private equity investments. In addition, Lown and Morgan (2006) present evidence that this variable strongly correlates with bank loans and is more important than interest rates in explaining loan volume.[3]

Turning to the empirical evidence on this channel, we first document a strong relation between private equity investment returns and the average innovation in market liquidity (as measured by Pástor and Stambaugh (2003)) during the investment's life. The average difference in performance for investments at the extreme deciles of market liquidity innovations is a striking 46% per year. This result is confirmed in a multiple regression setting, in which we control for investment characteristics and macroeconomic variables (credit spreads, M&A cycles, growth in industrial production, change in stock market volatility).

Next, we test our conjecture that funding liquidity is the link between these two variables. We first show that returns are significantly related to the

[2] In addition, adapting Ferson and Harvey's (1999) conditional approach, we find that the expected liquidity risk premium varies over time and is higher than 5% annually in one month out of four. The conditional analysis is reported in the Internet Appendix of the paper, located on the *Journal of Finance* website at http://www.afajof.org/supplements.asp.

[3] Leary (2008) also uses this measure to proxy for loan supply.

tightening in credit standards. A one-standard-deviation increase in this measure of the deterioration in funding liquidity decreases the annual return by 19.5%. Second, when including the measure of funding liquidity as well as that of market liquidity, we observe that funding liquidity absorbs half of the market liquidity effect. In addition, we conduct a time-series test using the aggregate cash flows of all the private equity investments each month. Consistent with the cross-sectional evidence, we find that net cash flows (dividends minus investments) are lower at times of tightening in credit standards and at times of worsening liquidity conditions.

We find these results important for two related reasons. First, they improve our understanding of the economic channel underlying the relationship between private equity returns and market liquidity. Market liquidity is found to be closely related to a measure of funding liquidity, which in turn is a determinant of the ease of refinancing for leveraged deals as shown by Axelson et al. (2010). Second, these results provide empirical support for the theory of Brunnermeier and Pedersen (2009) relating funding liquidity to market liquidity. Our empirical evidence shows that there is indeed a negative relationship between a dry-up in funding liquidity (the tightening in credit standards) and innovations in market liquidity (the Pástor and Stambaugh measure).

The paper continues as follows. Section I describes the data and the liquidity measures. Section II estimates different asset pricing models and computes the cost of capital and alpha for private equity. Section III relates private equity performance to market risk, funding liquidity, macroeconomic variables, and investment characteristics. Section IV discusses the implications of our results and concludes.

I. The Data

In this section, we first detail how the data are collected. We then document the coverage of our data set relative to available commercial data sets, and gauge its representativeness in terms of performance. We next describe how we measure returns in our context, and finally provide some descriptive statistics of returns.

A. Data Source

The data set is provided by the Center for Private Equity Research (CEPRES GmbH), a currently independent advisory firm established in 2001 as a joint venture between VCM Capital Management (part of the Deutsche Bank Group) and the University of Frankfurt. The unique feature of these data is the information on the monthly cash flows generated by private equity investments.

CEPRES obtains data from private equity houses that make use of a service called "The Private Equity Analyzer."[4] Participating private equity houses

[4] "Private equity houses" refers to organizations that run private equity funds that in turn make private equity investments in portfolio companies.

report cash flows (before fees) generated for each investment they have made in the past. In return, the firms receive statistics such as risk-adjusted performance measures. These statistics are then used internally for various purposes such as bonus payments or risk management (e.g., to comply with Solvency II). CEPRES does not benchmark private equity houses to peer groups; this improves data accuracy and representativeness, as it eliminates incentives to manipulate cash flows or cherry-pick past investments.

CEPRES may also be hired as an advisor. In such cases, it may receive data on the past performance of private equity houses. If permitted by the contractual agreement, CEPRES can add these track records to its database. If a track record is already in the Private Equity Analyzer database, then CEPRES systematically cross-checks that the data are accurate.

Earlier versions of this data set have been used in previous studies. A subset covering mainly venture capital investments is used by Cumming, Schmidt, and Walz (2010), Cumming and Walz (2010), and Krohmer, Lauterbach, and Calanog (2009). For this study, CEPRES granted us access to all liquidated buyout investments in its database as of December 2007. Note that we use the wording buyout investments and private equity investments interchangeably. The earliest investment in our sample starts in 1975 and the most recent in 2006. In total, we have 4,403 investments.

We thus have access to a comprehensive and accurate panel of anonymous cash flow streams generated by private equity investments. This enables us to construct precise measures of the investment performance and aggregate liquidity conditions over the investment's life, which is essential for estimating the relation between performance and liquidity risk.[5]

B. Data Coverage and Representativeness

To gauge coverage, we benchmark CEPRES to Standard & Poor's Capital IQ database. Capital IQ is now often perceived as the most comprehensive data set at the investment level (Bernstein et al. (2010) and Strömberg (2007)). Table II shows that CEPRES has a total of 7,198 buyout investments between 1975 and 2006 versus 14,011 in Capital IQ, that is 51%. During the early years, however, the coverage of CEPRES and Capital IQ is remarkably similar. From 1975 to 1994, CEPRES has slightly more investments than Capital IQ. After the mid-1990s, the number of investments in Capital IQ increases exponentially, while the rise is less pronounced for CEPRES. Note that, as mentioned above, we only use data on liquidated buyout investments. Because investments are held for four years on average, the pre-2002 coverage is more informative. Table II shows that, for the period 1975 to 1999, CEPRES coverage is 85% of Capital IQ.

[5] Two proprietary databases similar to the CEPRES data set are used in contemporary research. Ljungqvist, Richardson, and Wolfenzon (2008) have data from a large investor. Our data span a similar time period to theirs, but contain about twice as many investments. Lopez-de-Silanes, Phalippou, and Gottschalg (2009) have a data set containing the performance of private equity investments from hand-collected private placement memoranda, but do not have the detailed cash flows generated for each investment.

Table II

Data Coverage

The table compares the coverage of the CEPRES data set to that of Capital IQ. It shows the number of buyout investments, their total size (equity invested), and their average multiple (size-weighted). Multiple is the sum of all the dividends divided by the sum of all investments. Statistics are shown for both the sample of liquidated investments and the sample of non-liquidated investments. Statistics are also broken down by investment periods.

	Liquidated			Non-liquidated			Total CEPRES	Capital IQ	Fraction
	Number Investments	Size ($ million)	Multiple Mean (VW)	Number Investments	Size ($ million)	Multiple Mean (VW)	Number Investments	Number Investments	CEPRES vs Capital IQ
1975–1989	592	4,581	3.30	10	237	3.56	602	581	1.04
1990–1994	1,320	17,354	3.03	52	1,523	2.20	1,372	1,102	1.25
1995–1999	1,702	38,059	2.38	665	28,569	1.65	2,367	3,416	0.69
2000–2006	789	27,027	2.26	2,068	140,003	1.66	2,857	8,912	0.32
Total	4,403	87,021	2.52	2,795	170,332	1.67	7,198	14,011	0.51

Table III

Data Representativeness

This table compares the success rates of Private Equity (PE) houses included in the CEPRES data set and of PE houses not included in the CEPRES data set. The universe of PE houses and their success ratio comes from Thomson Venture Economics. Successful exit rate is the fraction of investments exited via IPO or M&A over the total number of investments. Only investments made before 2002 and PE houses with more than five investments are considered.

	CEPRES (1)	TVE (ex-CEPRES) (2)	Difference (1) minus (2)
Number of PE houses	117	535	−418
Successful-exit rate			
20th percentile	0.43	0.39	0.04
50th percentile	0.61	0.56	0.05
80th percentile	0.75	0.72	0.03
Mean	0.59	0.55	0.04

Benchmarking performance in private equity is more difficult. The majority of previous studies use Thomson Venture Economics, and measure private equity house performance by a success ratio (the fraction of investments exited via IPO or M&A over the total number of investments).[6]

Results are displayed in Table III. The success ratio is similarly distributed for the two data sets but the CEPRES data set appears to contain firms that are slightly above average performance-wise. We also note that 10% of the investments in our data set are bankrupt, which is similar to the rate reported by Strömberg (2007) for the Capital IQ data set.

C. Performance Measures

As mentioned in the Introduction, our data contain the series of cash flows generated by a given private equity investment. We begin by converting all the cash flows into U.S. dollars. We note that this has little effect on performance. The correlation between performance in original currency and performance in U.S. dollars is 99.8%. This is probably because investments last only four years on average, and hence currency changes do not greatly affect performance. In addition, about half of the cash flows are already in U.S. dollars.

To measure investment performance, we use a Modified Internal Rate of Return (MIRR). MIRR measures the geometric average return for an investor who deposits dividends (D_t) into, and draws her money for intermediate investments (I_t) from, an account that earns an interest rate x_t for each $t = 1, \ldots, T$,

[6] In order to compute a meaningful success ratio, we restrict investments to those made before 2002. It would not be fair to classify as unsuccessful investments made less than five years ago and not exited yet. We also require at least five investments per firm. CEPRES performed the calculations for us in order to preserve anonymity. They counted 117 firms in their data set that satisfy these criteria. In Thomson Venture Economics, they counted 535 firms that satisfy these criteria but were not included in their database.

where T is the number of periods in the investment's life. The investment MIRR is defined as follows:

$$(1+MIRR)^T = \frac{D_1 \prod_{t=1}^{T-1}(1+x_t) + D_2 \prod_{t=2}^{T-1}(1+x_t) + \cdots + D_{T-1}(1+x_{T-1}) + D_T}{I_0 + \frac{I_1}{(1+x_0)} + \frac{I_2}{\prod_{t=0}^{1}(1+x_t)} + \cdots + \frac{I_T}{\prod_{t=0}^{T-1}(1+x_t)}} = \frac{FV(Div, x_t)}{PV(Inv, x_t)}, \tag{1}$$

where $FV(\cdot, x_t)$ and $PV(\cdot, x_t)$, respectively, denote the forward and present value of a stream of cash flows computed using the discount rate x_t. Note that, when no cash is returned to investors (i.e., the dividends are all zero), the MIRR equals -100%.

We now give a numerical example of the construction of MIRR and its sensitivity to the reinvestment assumption. To do so we use the typical cash flow pattern shown in the Introduction (Table I) and assume a constant reinvestment rate of 5% per semester. The final value of the dividends is

$$FV\ (\text{Inv}, 5\%) = 50(1.05^3) + 150 = 208.$$

The annualized MIRR is thus

$$MIRR = \left(\frac{208}{100}\right)^{1/4} - 1 = 20\%.$$

If we use a reinvestment rate of 0%, the MIRR would be 19%. Hence, the sensitivity of MIRR to the reinvestment assumption seems minor in our data. This is due to the relatively short life of the investments and the relatively small size of intermediate cash flows. In the analysis that follows, we use the S&P 500 index as a reinvestment rate. This reinvestment assumption should capture the fact that private equity investors tend to have highly diversified portfolios. We also compute the MIRR of each investment using the risk-free rate as the reinvestment rate and find a correlation coefficient of 99% between the two MIRRs.

D. Descriptive Statistics—Performance

To provide aggregate performance figures, we group investments by their starting year and countries of location. Next, we sum the cash flows of all the investments in the group month by month. Finally, we compute the MIRR of each (pooled) cash flow stream. We thus replicate the rate of return of a buy-and-hold investor who selects all the investments of a certain country/region over a certain time period.

Table IV, Panel A, shows the results. Overall, we find little difference across countries/regions and over time. Returns are highest for Europe (ex-UK) in the second half of the 1990s at 25% annually, but returns in Europe were low

Table IV
Performance by Year and Region

The table reports the modified IRR (MIRR) of a group of investments. Groups are based on the year of investment initiation and the region in which the investment is located. Performance is computed on the pooled cash flows of each group. The re-investment rate is the return on the S&P 500 index.

	1975–1989	1990–1994	1995–1999	2000–2006	1975–2006
Panel A: Modified Internal Rates of Return (S&P as Re-investment Rate)					
US	0.18	0.18	0.19	0.13	0.18
UK	0.17	0.16	0.17	0.20	0.17
Europe (ex-UK)	0.17	0.14	0.25	0.21	0.20
Rest world	0.21	0.15	0.18	0.17	0.17
All countries	0.18	0.17	0.21	0.21	0.19
Panel B: Number of Investments					
US	323	533	534	237	1,627
UK	172	440	526	139	1,277
Europe (ex-UK)	68	269	499	246	1,082
Rest world	17	23	121	152	313
All countries	592	1,320	1,702	789	4,403

in the first half of the 1990s at 14% annually. Returns are stable over time in the U.S., except for more recent years when they drop to 13%. An investor buying all the investments in our sample would have earned 19% annually. The carried interest payable with such a return is about 4% and management fees on invested capital are at least 3%.[7] Hence, after fees, this performance figure is at most 12%, which is similar to the return documented by Kaplan and Schoar (2005) for net-of-fees (fund-level) returns. This further demonstrates that our data are similar performance-wise to those used in previous research.

Table IV, Panel B, shows that our observations are almost evenly distributed across regions: U.S. (37%), UK (29%), and the rest of Europe (25%).

II. The Liquidity Risk Premium in Private Equity Returns

As pointed out by Acharya and Pedersen (2005), among others, liquidity varies over time and displays commonality across securities and asset classes. Recent theoretical and empirical research suggests that this commonality in liquidity is a priced risk factor (liquidity risk). Pástor and Stambaugh (2003) propose a four-factor asset pricing model that includes liquidity risk. If we assume that the public and private equity markets are integrated then this asset pricing model and its corresponding pricing kernel can be applied to private

[7] This is an approximation. Carried interest equals 20% of the returns. The 2% management fee is charged on a mix of capital invested and committed, and is typically equivalent to a 3% fee on capital invested (see Metrick and Yasuda (2010) and Phalippou (2009) for details).

equity to evaluate its cost of capital, which is naturally an important question for such a large asset class. In addition, the finding of a significant loading on the liquidity risk factor would cast a different light on the diversification benefits of private equity.

In this section, we begin by detailing our methodology to estimate a factor model for private equity returns. Next, we provide the estimates of the risk exposures, the alpha, and the resulting cost of capital. In the Internet Appendix,[8] we extend the analysis to time-varying factor loadings and risk premia.

A. Methodology

A.1. The Estimation of Risk Exposures

Because private equity investments are not continuously traded, we cannot compute a time series of returns and use a traditional time-series approach to estimate risk exposures. Instead, we have cash flow streams for a cross-section of investments that we use to compute rates of returns, as described in Section I. This cross-sectional data structure fits into the approach developed by Cochrane (2005), which we adjust to our context.[9]

To start from the simplest case, let us assume that the cash flows of each project i consist of an initial investment, V_0^i, and a final dividend, $V_{T_i}^i$, which is paid at date T_i. Following Cochrane (2005), we assume that one-period returns are log-normal and exhibit a linear factor structure (in logarithm),

$$\ln R_{t+1}^i = \ln \frac{V_{t+1}^i}{V_t^i} = \gamma + \ln R_{t+1}^f + \delta' f_{t+1} + \varepsilon_{t+1}^i, \tag{2}$$

where γ is a constant, R^f is the gross risk-free rate, f_{t+1} is a vector of k risk factors (e.g., the four factors of Pástor and Stambaugh (2003)), δ is a k-vector of risk factor loadings, and ε_{t+1}^i is normal with mean zero and variance σ^2 and is independent of the risk factors.[10]

[8] An Internet Appendix for this article is available online in the "Supplements and Datasets" section at http://www.afajof.org/supplements.asp.

[9] Cochrane (2005) and Korteweg and Sorensen (2010) highlight the role played by sample selection bias when estimating risk models for venture capital investments. In their context, valuations are observed only infrequently, although more often for well-performing investments. Explicit modeling of the selection mechanism is thus required to obtain unbiased estimates of factor loadings and alphas. Because our data do not suffer from such a severe sample selection bias, we can simplify their approach and simply estimate the factor models with OLS regressions.

[10] Given the monthly frequency of the factors, we set the interval length to one month. This choice has no material consequences, except for interpretation of the reported coefficients. Notice also that, unlike Cochrane (2005), we choose to express normally distributed factors in levels rather than logs. The reason is that factors are based on long–short strategies and can take negative values, which does not allow logarithmic transformation. This fact causes minor deviations from Cochrane in the formulas for factor loadings and alphas that we derive in the Internet Appendix.

Given equation (2), the natural logarithm of the (gross) geometric average return on the investment (R_g^i) is given by

$$\ln(R_g^i) = \frac{1}{T_i}\ln\frac{V_T^i}{V_0^i} = \gamma + \frac{1}{T_i}\sum^{T_i}\ln R_{t+1}^f + \delta'\frac{1}{T_i}\sum^{T_i} f_{t+1} + \frac{1}{T_i}\sum^{T_i}\varepsilon_{t+1}^i. \quad (3)$$

The variance of the error term in equation (3) is $\frac{1}{T_i}\sigma^2$. To eliminate this source of heteroskedasticity, we multiply each side of equation (3) by $\sqrt{T_i}$:

$$\sqrt{T_i}\ln(R_g^i) = \gamma\sqrt{T_i} + \frac{1}{\sqrt{T_i}}\sum^{T_i}\ln R_{t+1}^f + \delta'\frac{1}{\sqrt{T_i}}\sum^{T_i} f_{t+1} + \frac{1}{\sqrt{T_i}}\sum^{T_i}\varepsilon_{t+1}^i. \quad (4)$$

This is a GLS transformation that we can perform because we know the form of heteroskedasticity.[11] Equation (4) is the specification that we bring to the data. Notice that the right-hand side is linear in the parameters of interest. We can therefore simply apply a standard OLS regression. The dependent variable is the scaled natural logarithm of gross returns. The explanatory variables are the square root of the investment duration and the time-series averages of the risk factors during the investment's life (which are multiplied by $\sqrt{T_i}$) and the square root of the investment duration. Because this equation does not include a constant, the values of the R^2 from the OLS estimation are not meaningful and we do not report them. In the tables, however, we report the estimate of γ and we label it "Constant" for simplicity.

Because the parameters in equation (2) pertain to the logarithm of returns, we need to derive expressions for alpha and factor loadings for the level of returns. In other words, we need to convert (γ, δ) from equation (2) into the familiar (α, β), which are defined by the equation

$$E\left(R_{t+1}^i\right) = R_{t+1}^f + \alpha + \beta' E\left(f_{t+1}\right). \quad (5)$$

In the Internet Appendix, we show that the formulas for conversion from the parameters in the log of returns to the parameters in the level of returns are

$$\beta = R_f \delta e^{\gamma + \delta'\mu_F + \frac{1}{2}\delta'\sigma_F^2\delta + \frac{1}{2}\sigma^2} \quad (6)$$

$$\alpha = R_f\left(e^{\gamma + \delta'\mu_F + \frac{1}{2}\delta'\sigma_F^2\delta + \frac{1}{2}\sigma^2}(1 - \delta'\mu_F) - 1\right), \quad (7)$$

where μ_F is the k-vector of factor means and σ_F^2 is the $k \times k$ variance-covariance matrix of the factors. Because β turns out to be quite close to δ, we only report the latter in our empirical results.

The approach described above is motivated by the specific structure of the data. As we do not observe periodic valuations of the investment, we cannot construct a time series of investment returns R_{t+1}^i. Instead, we observe the investment cash flows, which allow us to construct a summary measure of

[11] GLS is the most efficient estimation method with non-spherical disturbances.

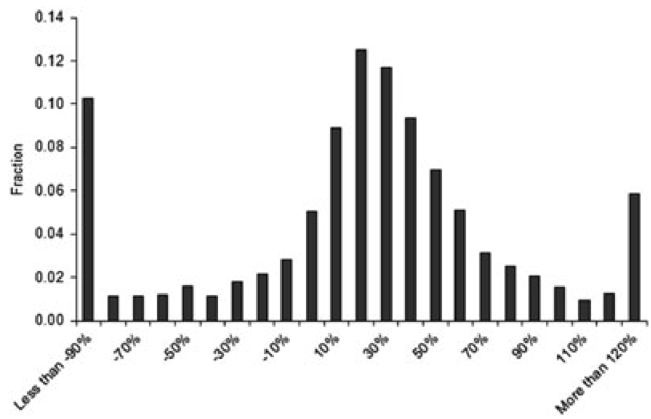

Figure 1. Histogram of MIRR. The figure plots the histogram for the annual MIRR with S&P 500 as the assumed re-investment rate. The first bin contains all the investments with MIRR of −100% annually. The last bin contains all the investments with MIRR of over 125% annually.

performance over the investment's life, that is, the geometric average return R_g^i. This explains why, in equation (4), we relate the average investment return to the average factor realization over the investment's life (with a correction for heteroskedasticity).

This approach amounts to considering each investment as a separate realization of the returns on the asset class, then the variation in returns across investments is used to estimate the risk loadings and the abnormal performance of the asset class. This cross-sectional approach for non-traded assets is essentially the same as in Cochrane (2005), Korteweg and Sorensen (2010), and Driessen, Lin, and Phalippou (2012). Like in these papers, the identification comes from investments that are realized over partly non-overlapping time periods.

A.2. Forming Portfolios

To measure the geometric return in equation (4), we use the investment's MIRR (see the previous section for calculation details) and we need to deal with three issues. First, the logarithm of MIRR is not defined for 10% of the investments (those with a return of −100%). Second, as is well known in the empirical literature on stock returns, the high idiosyncratic risk of individual investments may induce substantial noise in risk estimates. Third, we use the assumption of normality of log-returns to derive alpha and betas, but the distribution of individual investment returns fails a D'Agostino, Belanger, and D'Agostino Jr. (1990) chi-squared normality test. Figure 1 provides the histogram of MIRRs and shows clearly that the probability mass on the tails of the distribution, especially on the left one, explains the rejection of normality.

To get around these three issues, we adopt the typical approach in the asset pricing literature and group individual investments into portfolios.[12] As mentioned above, the statistical identification comes from observing investment returns at different moments in time. Thus, a natural choice is to group together investments that start at the same date (at the monthly frequency). To ensure that portfolios are sufficiently diversified, we require a minimum of 20 investments per portfolio. If the number of investments starting in the same month is below this minimum, we include in the portfolio investments that are started the next month, and continue until the number of investments is at least 20.[13] Portfolio cash flow streams are obtained by summing the cash flows of the individual investments each month. Finally, we compute the MIRR of each portfolio.

By grouping investments based on their starting dates, we reduce idiosyncratic risk and preserve sufficient dispersion in the explanatory variables. In addition, when we form portfolios, we obtain a better behaved distribution of returns and the normality assumption is not rejected (p-value is 0.44). Finally, we do not observe any portfolios with a MIRR of -100%, meaning that the logarithm of the MIRR is always defined. We can thus estimate equation (4) by OLS at the portfolio level.

B. Empirical Estimates of Risk Exposures and Alpha

B.1. The Factor Models

We build up our estimate of the risk premium for private equity moving from the simplest model to a four-factor model that includes liquidity risk. We start with the CAPM, which is the model that Cochrane (2005) estimates for venture capital. Then, recognizing that private equity investments tend to be made predominantly in value companies, we consider the Fama and French (1993) three-factor model. Finally, we augment the three-factor model with the Pástor and Stambaugh (2003) traded liquidity factor. This factor is equal to the return on a portfolio that goes long the tenth liquidity-beta-decile portfolio and short the first liquidity-beta-decile portfolio. Liquidity betas are obtained by regressing individual stock returns on innovations in aggregated liquidity; aggregate liquidity is the sum of stock-level OLS slopes of daily returns on signed daily trading volume within a given month.

Assuming that markets are integrated, or more specifically that there is a unique pricing kernel for private and public equity, allows us to compute the prices of risk for the four factors from their realizations in the public equity market. Table V presents the correlation and distribution of these factors

[12] In a recent paper, Ang, Liu, and Schwarz (2010) compare the cost and benefits of portfolio formation when conducting a two-step Fama and MacBeth (1973) procedure. Forming portfolios improves the estimation of beta but reduces the precision for the estimation of the risk premium. Since we only estimate betas, Ang, Liu, and Schwarz (2010) provide further support for the use of portfolios as opposed to individual investments.

[13] In the Internet Appendix tables, we show robustness to different choices for the minimum number of investments per portfolio.

Table V
Correlations and Distributions of the Traded Factors

This table shows the correlation matrix and summary statistics for the (time series of the) four traded risk factors: the illiquid-minus-liquid factor by Pástor and Stambaugh (2003), the excess market return, HML, and SMB. The time period is from October 1975 to December 2007. The frequency is monthly. Returns are in percentages.

	IML_PS	Rm-Rf	HML	SMB
Correlations:				
IML_PS	1.000			
Rm-Rf	−0.100	1.000		
HML	−0.276	−0.460	1.000	
SMB	0.042	0.236	−0.341	1.000
Mean	0.375	0.630	0.417	0.241
St. deviation	4.138	4.320	3.009	3.166
5th percentile	−5.767	−6.410	−3.960	−4.180
Median	0.608	0.940	0.370	0.120
95th percentile	5.530	7.010	5.330	4.800

during our sample time period (October 1975 to December 2007). In particular, it shows the time-series mean for each factor. We use these means as estimates of the factor risk premia to compute the cost of capital. Multiplying the values in Table V by 12, the liquidity premium is 4.5% annually. The market risk premium is 7.5% annually. The HML and SMB premia are 4.9% and 2.9% annually, respectively. The (unreported) risk-free rate is 5.8% annually.

B.2. Empirical Results

Panel A of Table VI reports the estimates of equation (4) for each of the factor models. In the first specification, the estimate of the CAPM beta is close to one and statistically significant. This number is consistent with the choice of Kaplan and Schoar (2005) to measure private equity performance by a public market equivalent with a beta of one. The second column in Panel A reveals that, after accounting for the other Fama and French (1993) factors, the loading on the market increases. This is due to the fact that private equity investments load positively and significantly on value stock returns and that HML and the market factor are negatively correlated in this sample. The loading on SMB is positive, but not statistically different from zero.

Finally, the last column in Panel A reports the estimates of the model including liquidity risk. The liquidity beta is about 0.64 and statistically significant at the 1% level. We also note that the slope on HML increases, suggesting that in the previous model the importance of HML is mitigated by the negative correlation between HML and the liquidity factor.

For each of the factor models, Panel B of Table VI reports the total risk premium, the cost of capital, and alpha estimates that result from transforming the model in logs to the model in levels according to equations (5) and (6). Each

Table VI

Risk Models and The Cost of Capital

The table reports the results of OLS estimation of three different factor models for private equity returns (Panel A) and the resulting alphas and cost of capital (Panel B). In Panel A, the dependent variable is the logarithm of one plus the monthly MIRR minus the log of one plus the risk-free rate. Each observation corresponds to a portfolio of private equity investments formed by the starting date of the investment. Portfolios must contain at least 20 investments. Each observation is weighted by the square root of the investment duration to correct for unequal variance. Explanatory variables include the Fama and French (1993) three factors (excess market return, HML, SMB) and the illiquid-minus-liquid portfolio (IML_PS) by Pástor and Stambaugh (2003). Each explanatory variable is computed by taking its average value during the portfolio life. All variables are in monthly frequency. All specifications are run between October 1975 and December 2007. The table also reports the estimate of the residual standard deviation (sigma) and the number of observations (N). In Panel B, the risk premium is the sum of the products of the factor loadings times the average factor realizations. The cost of capital is the sum of the average risk-free rate plus the risk premium. Alphas and betas are computed using equations (6) and (7) in the text. The reported values (in %) are annualized by multiplying the monthly estimate by 12. In this panel, we also report the annualized average net risk-free rate. The standard errors for Panel B estimates are computed using the delta method. t-statistics are given in parentheses.

Model:	Market	FF	PS
Panel A: Risk Models			
IML_PS			0.638
			(3.539)
Rm-Rf	0.948	1.395	1.294
	(6.688)	(5.443)	(5.227)
HML		0.719	1.020
		(2.450)	(3.466)
SMB		−0.124	−0.040
		(−0.497)	(−0.167)
Constant	0.006	0.000	−0.002
	(6.003)	(0.035)	(−0.712)
Sigma	0.049	0.048	0.046
Adj. R^2	0.849	0.853	0.865
N	139	139	139
Panel B: Alpha, Risk Premium, and Cost of Capital			
Total risk premium	7.300%	14.048%	17.997%
	(6.723)	(4.485)	(5.635)
Risk premium components:			
$\beta_{liq} \times \mu_{liq}$			2.928%
			(3.599)
$\beta_{mkt} \times \mu_{mkt}$	7.300%	10.745%	9.980%
	(6.723)	(5.505)	(5.312)
$\beta_{hml} \times \mu_{hml}$		3.668%	5.207%
		(2.485)	(3.527)
$\beta_{smb} \times \mu_{smb}$		−0.365%	−0.118%
		(−0.504)	(−0.170)
Risk-free rate (in sample)	5.816%	5.816%	5.816%
Cost of capital (in sample)	13.116%	19.864%	23.813%
	(12.080)	(6.341)	(7.456)
Alpha	9.303%	3.128%	0.413%
	(10.324)	(1.227)	(0.158)

estimated factor risk premium is the product of the estimated factor loading times the average realization of the factor in the sample (described in the previous subsection).

The first line shows the sum of all the risk premia for each factor model. It varies from 7.3% with the CAPM to about 18% with the four-factor model. The cost of capital is the sum of the risk-free rate and the total risk premium. The average risk-free rate in the sample is fairly high (especially compared to current levels). It makes the cost of capital higher by about 6% for all the factor models.

Finally, we present estimates of alpha. Alpha can be interpreted as the portion of expected returns that is not explained by the chosen asset pricing model. The CAPM leaves unexplained a high 9.3% of expected returns. In the second column, once the risk premia on the book-to-market factor and the size factor are taken into account, the alpha drops to about 3.1%, which is still economically (although not statistically) significant. In the model with liquidity risk (third column of Panel B), the premia on the four factors entirely account for average private equity returns. The alpha is virtually zero, both economically and statistically, while the risk premium and the cost of capital are about 18% and 24% per year, respectively.[14] The liquidity risk premium thus appears to be an essential component to fully account for average private equity returns.[15]

B.3. Economic Significance

Implications for company valuation. In the total risk premium, we note that the lion's share belongs to the market risk premium (10%). The book-to-market premium is 5.2%, while the size premium is insignificant. The liquidity risk premium amounts to a statistically significant 2.9% per year.

A simple way to assess the economic significance of a liquidity risk premium for private equity is to use the representative investment cash flow pattern shown in Table I and value this investment using either a 15% discount rate or an 18% discount rate. The present values of this representative investment under the two alternative assumptions are 119 and 108, respectively, or a 10% difference. This simple algebra reveals that even a 3% liquidity premium has important economic implications when valuing investments. In addition, in the

[14] Notice that adding the estimates of alpha and the cost of capital in Panel B of Table VI gives an estimate for the expected return on private equity of about 24%. This estimate is larger than the 19% average return that we report in Panel A of Table IV. The spread is due to the fact that Table IV has the average geometric return (R_g^i), whereas the 24% estimate refers to arithmetic returns (R_{t+1}^i). Whenever the volatility of returns is not zero, the geometric return is smaller than the arithmetic return.

[15] As mentioned above, our choice of a minimum number of 20 investments per portfolio may be considered arbitrary. To address this concern, in the Internet Appendix we report the results with alternative choices for this minimum threshold. The threshold cannot drop below five investments, otherwise we have a portfolio with a −100% return. With a threshold of 50 investments per portfolio, the number of portfolios drops to 68, so we do not raise the threshold further. In general, results seem stable throughout the spectrum of chosen thresholds.

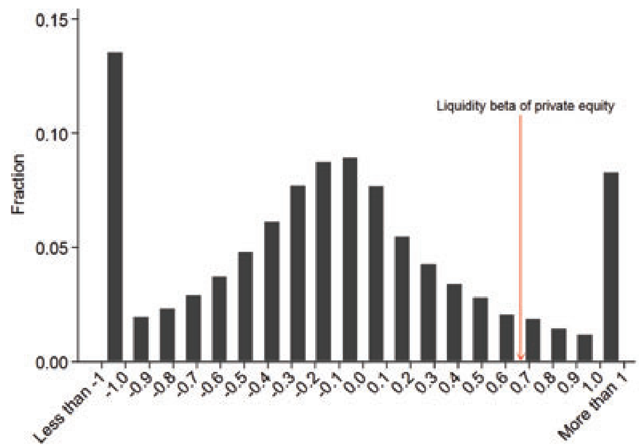

Figure 2. Liquidity betas for listed stocks. The figure plots the histogram for the liquidity betas from the four-factor model devised by Pástor and Stambaugh (2003) for all listed stocks in the CRSP database with at least two years of monthly returns between January 1966 and December 2008 (20,500 stocks).

Internet Appendix, we use a conditional asset pricing framework and find that, in some periods, the liquidity risk premium well exceeds the unconditional value of 3%.

Comparing public equity and private equity liquidity risk exposures. It is also interesting to relate our estimate of liquidity risk for private equity to that for public equity. To do so, we simply compute the liquidity beta of all the stocks in CRSP with a standard time-series regression between January 1966 and December 2008 using the Pástor and Stambaugh (2003) four-factor model. We sort all the stocks by their loading on the liquidity factor and show the resulting histogram in Figure 2. The private equity liquidity beta of 0.638 corresponds to the 86th percentile of the beta distribution for publicly listed equities (as shown by the vertical line in Figure 2). This result indicates that the liquidity risk in private equity is high compared to that of most publicly traded companies.[16]

[16] Note that the 7.5% liquidity risk premium reported by Pástor and Stambaugh (2003) is the beta of a long–short position on the top and bottom decile stocks by liquidity betas and is not the liquidity risk premium paid by the average stock. In fact, by construction the aggregate public equity portfolio will have a zero beta on the liquidity factor, SMB, and HML, and a beta of one on the market factor in a four-factor model. Hence, any asset class with a liquidity beta higher than zero will bear greater liquidity risk than the public equity portfolio. Furthermore, notice that in our sample (October 1987 to December 2007) the Pástor–Stambaugh factor pays a 4.5% premium, which is less than the 7.5% in the 1963 to 1999 sample used in the original Pástor and Stambaugh (2003) paper. This observation gives more economic significance to the 2.9% liquidity premium that we obtain for private equity.

III. The Source of Liquidity Risk in Private Equity

The factor model estimation in Section II shows that a liquidity risk factor is a significant determinant of private equity returns. The premium due to liquidity risk accounts for the abnormal performance of this asset class and is an important component of the cost of capital.

In this section, we set out to identify the channel that links variation in aggregate liquidity to private equity performance. First, we develop our main hypothesis. Second, we test it empirically using the cross-section of investment return data. Third, we provide consistent evidence from a time-series test.

A. Hypothesis Development

The Pástor and Stambaugh (2003) traded liquidity factor that we use in Section II is based on a measure of stock market liquidity. One naturally wonders why private equity returns are related to the liquidity of public equity markets. Our hypothesis is based on two complementary arguments.

The first argument is provided by Brunnermeier and Pedersen (2009). They postulate a relation between market liquidity and funding liquidity, where funding liquidity is the availability of trading capital for investors. For our purposes, the focus is on banks and hedge funds, which are the main providers of finance to private equity companies. Brunnermeier and Pedersen's (2009) theory suggests that times of low funding liquidity are also characterized by poor market liquidity. In their mechanism, labeled a liquidity spiral, a negative shock to investors' trading capital triggers margin calls and forced liquidations, which in turn reduce market liquidity and exacerbate the initial trading losses. Consequently, poor market liquidity conditions, captured by the Pástor and Stambaugh (2003) factor, may reflect a dry-up in funding liquidity.

The second argument is that shocks to funding liquidity may be related to the return of private equity companies. One crucial characteristic of private equity investments, which distinguishes them from investments in public companies, is their higher leverage (e.g., Axelson, Strömberg, and Weisbach (2009)). The fact that these loans need to be refinanced or renegotiated (e.g., following a breach of covenants) makes private equity investments sensitive to the availability of capital of the debt providers (Axelson et al. (2010), Kaplan and Strömberg (2009)). The providers of debt finance to private equity are mainly banks and hedge funds, and they are certainly exposed to variations in funding liquidity of the type described by Brunnermeier and Pedersen (2009). At times of low liquidity, private equity houses may thus find it difficult to refinance their companies and may be forced to liquidate the investments or to accept higher financing costs. This argument relates the returns of private equity companies to funding liquidity.[17]

[17] In a statement that provides support for this refinancing risk channel for private equity, Acharya et al. (2009, p. 9) argue that, as a consequence of the recent liquidity crisis, "[. . .] a large number of leveraged loans that are coming to maturity in 2010 and 2011 [. . .] may go bust once

Taken together these two arguments therefore suggest that private equity returns correlate with market liquidity through the refinancing risk channel. In other words, the observed link between private equity returns and market liquidity results from the dependence of private equity performance on the availability of the capital to debt providers (funding liquidity) and from the link between funding liquidity and market liquidity.

In order to bring this conjecture to the data, we need to find a proxy for funding liquidity that is especially relevant for private equity. Axelson, Strömberg, and Weisbach (2009) propose the Senior Loan Officer Opinion Survey on Bank Lending Practices as an indicator of the credit availability for private equity investments. The survey asks loan officers whether they tightened or loosened their lending standards relative to the previous quarter. Axelson et al. (2010) argue that "this measure captures non-price aspects of credit market conditions, such as debt covenants and quantity constraints." They find this index to be strongly related to the amount of leverage used to finance private equity investments. Also, Lown and Morgan (2006) present evidence that this variable strongly correlates with changes in bank loans and is more important than interest rates in explaining loan volume. Leary (2008) uses this measure to proxy for loan supply. He finds that it helps to explain differences in leverage between firms with and without bond market access. The tightening in credit standards thus has the double advantage of having been used and advocated in a private equity context and having strong empirical support. We therefore expect it to be a potentially accurate instrument to measure funding liquidity.

Given the arguments above, we conjecture that the tightening in credit standards is negatively related to private equity performance as a manifestation of the refinancing risk channel. Furthermore, we postulate that the observed relation between private equity returns and the Pástor and Stambaugh (2003) measure results from the link between funding liquidity and market liquidity. Hence, our chosen measure for the evolution in funding liquidity (the tightening in credit standards) should explain empirically part of the negative relation between private equity returns and market liquidity.

B. Cross-Sectional Evidence

In this subsection, we empirically test the above conjecture using the cross-section of individual investments.

B.1. Main Results

We first need to verify that, consistent with the evidence of liquidity being a priced factor, unexpected market liquidity shocks are correlated with contemporaneous returns. We employ the Pástor and Stambaugh (2003) measure

the refinancing crisis emerges." Also, a recent study by Standard & Poors (2009) corroborates this prediction and shows that, following the last financial crisis, defaults increased substantially due to covenant breaches, especially among large portfolio companies (so-called mega-buyouts).

of unexpected market liquidity shocks. Second, we need to verify that tightening in credit standards explains part of this relation. This analysis is best conducted at the individual investment level and hence we begin from this discussion.

Working with individual investment returns. Different from Section II, where we study portfolio returns, we now cast our analysis at the investment level. Investment characteristics are important control variables when identifying the relation between private equity returns and macroeconomic variables. Forming portfolios would result in a loss of this investment-level information. In addition, the advantages of forming portfolios, which we mentioned in Section II, do not apply to the present analysis.

The investment returns are measured by the annualized MIRR (with S&P 500 reinvestment rate) described in Section I. It measures the performance over the life of the investment. We thus simply compute our explanatory variables as the average realization between the starting date and the ending date of the investment. For instance, we relate the MIRR of an investment to (i) the average realization of market liquidity innovations over the life of that investment (labeled "P&S liquidity conditions"), (ii) the average change in credit standards during the life of that investment (labeled "Tightening of credit standards"), etc.

As we observe outliers on the right tail of returns, we winsorize the MIRR at the 95th percentile.[18] Also, because the Survey of Loan Officers from the Federal Reserve is continuously available only from April 1990, we focus on the investments that are started after this date. This reduces the sample to 3,763 investments out of the original 4,403 observations.[19]

Shocks to market liquidity and investment returns. Figure 3 provides a graphical impression of the relation between investment returns and liquidity conditions by plotting the average MIRR by deciles of P&S liquidity conditions. The relation is almost monotonic and the difference in performance between investments going through bad versus good liquidity conditions is a striking 46% per year.

Of course, this relation does not account for a number of other potentially important determinants of private equity returns. To this purpose, Table VII shows the results from a multiple regression analysis. In all specifications, we include a set of control variables. We have fund size, which is measured as the dollar amount that is committed to the private equity fund that makes the investment (in January 2007 dollars). Fund size is included because, among other things, it may capture the liquidity level of the investment. Larger funds, being more visible and experienced, may be able to exit investments more quickly and with less price impact. Controlling for the liquidity level of the

[18] The 95th percentile is 135% annually and the 99th percentile is 400% annually. Winsorizing at the 99th percentile leads to slightly stronger results.

[19] In the Internet Appendix, we have results for the full sample of 4,403 investments that do not use the credit standards variable. The positive relation between P&S liquidity conditions and private equity returns also holds significantly in the full sample.

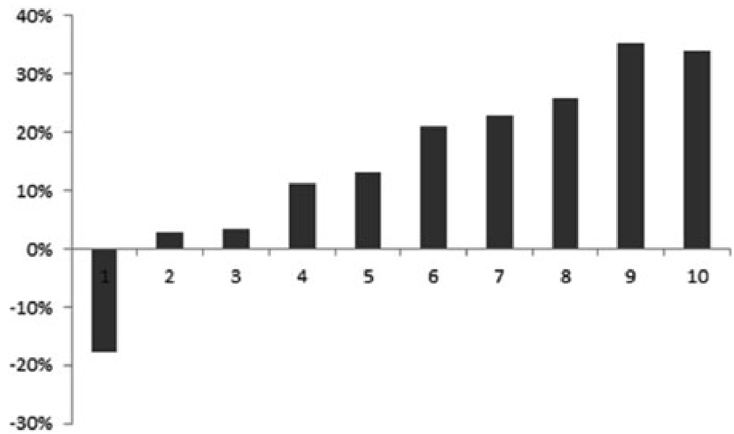

Figure 3. Annual performance by deciles of liquidity conditions. The figure plots the average investment MIRR in each decile of the Pástor and Stambaugh (2003) liquidity condition variable.

investment is important to disentangle the effect of the liquidity level from the effect of liquidity risk on average returns (see the Internet Appendix for a more detailed discussion on this point).

Also, we control for investment-level variables such as the stage (a dummy variable that is one if it is a growth investment), the investment size (equity invested, expressed in January 2007 in U.S. dollars), the country of investment location, the industry of the investment, as well as the return on the stock market (CRSP universe).[20] All the explanatory variables are standardized.[21] In addition, the standard errors are clustered at the investment starting year, because the performance of investments starting at the same time may be driven by unobserved common factors.

In the first specification in Table VII, the effect of liquidity conditions on private equity performance is economically and statistically significant. A one-standard-deviation increase in liquidity conditions raises the annual MIRR by 11.4%. This confirms the results in Section II and Figure 3. A deterioration in market liquidity negatively affects private equity returns.

Shocks to funding liquidity and investment returns. The next step involves the test of our conjecture that the effect of market liquidity on private equity returns originates from the relationship between these two variables and funding

[20] We control for the country and industry of the investment with fixed effects. A growth investment is an investment that is meant to finance the growth of a company. It usually involves less leverage than leveraged buyouts and usually is a minority stake (unlike leveraged buyouts). As investment size increases over time, we subtract the annual mean from each observation. In addition, as there are some outliers in terms of investment size, we winsorize this variable at the 95th percentile.

[21] The coefficients can then be interpreted as the impact of a one-standard-deviation change in the explanatory variable on annual returns. Importantly, inference is not affected by the transformation. The *t*-statistics are exactly the same with and without the standardization.

Table VII

Private Equity Returns and Liquidity Conditions

The table reports the output of OLS regressions. The dependent variable is the investment annualized MIRR. MIRR assumes dividends are re-invested in the S&P 500. A first set of explanatory variables results from taking the average over the investment life of: (i) aggregate liquidity innovations as measured by Pástor and Stambaugh (labeled "P&S liquidity conditions"); (ii) tightening of credit standards, the fraction of survey respondents who report to have tightened their credit standards (source: Fed Senior Officer Survey); (iii) industrial production growth (source: St. Louis Fed); (iv) change in credit spread, difference between Baa and Aaa yield (source: St. Louis Fed); (v) number of M&A deals in a month divided by the number of stocks in CRSP at the beginning of the month; (vi) change in the standard deviation of the next 48 months of CRSP value-weighted stock market index returns (labeled "Delta realized long-term volatility"); and (vii) (monthly) return on the stock market index minus the risk-free rate. A second set of explanatory variables includes investment characteristics: (i) a dummy variable that takes the value one if the investment is classified as growth (zero otherwise), (ii) investment size (i.e., equity invested in January 2007 dollars), and (iii) the size (in January 2007 dollars) of the fund by which the investment is made. A growth investment is one that mainly consists of an equity investment (little or no leverage); the classification is made by CEPRES, our data provider. Investment size is relative to same-year investments and is winsorized (at 5th and 95th percentiles of same-year investments). Country and industry fixed effects are added to the set of explanatory variables. Each explanatory variable is standardized by removing the sample mean and dividing by the sample standard deviation. Standard errors are clustered at the investment year and corresponding t-statistics are reported below each coefficient in parentheses. The sample ranges from April 1990 to December 2007.

P&S liquidity conditions	0.114		0.051	0.107	0.109	0.110	0.114	0.051
	(4.027)		(2.226)	(4.130)	(4.031)	(3.293)	(3.954)	(1.774)
Tightening of credit standards		−0.195	−0.164					−0.172
		(−6.243)	(−6.361)					(−4.898)
Industrial production growth				0.078				−0.003
				(2.180)				(−0.073)
Delta credit spread					−0.029			0.006
					(−1.395)			(0.221)
Relative number of M&A deals						−0.015		0.011
						(−0.471)		(0.241)
Delta realized long term volatility							−0.001	−0.022
							(−0.038)	(−0.633)
Rm-Rf	0.068	−0.013	−0.015	0.011	0.063	0.076	0.068	−0.019
	(2.121)	(−0.424)	(−0.525)	(0.285)	(2.083)	(1.923)	(2.029)	(−0.399)
Growth investment	−0.036	−0.042	−0.041	−0.033	−0.037	−0.037	−0.036	−0.040
	(−2.011)	(−2.425)	(−2.375)	(−1.844)	(−2.027)	(−2.100)	(−2.013)	(−2.422)
Investment size	0.005	−0.001	0.000	0.010	0.003	0.004	0.005	0.001
	(0.361)	(−0.122)	(−0.042)	(0.735)	(0.250)	(0.315)	(0.373)	(0.095)
Fund size	−0.001	−0.003	−0.003	−0.002	−0.001	−0.001	−0.001	−0.002
	(−0.278)	(−0.701)	(−0.565)	(−0.408)	(−0.260)	(−0.166)	(−0.279)	(−0.452)
Country and industry fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adj. R^2	0.093	0.118	0.123	0.100	0.095	0.093	0.093	0.124
N	3,763	3,763	3,763	3,763	3,763	3,763	3,763	3,763

liquidity. Hence, in the second specification in Table VII, we regress the MIRR on the tightening of credit standards (our chosen measure for deteriorating funding liquidity) and the same set of control variables as above. Consistent with our hypothesis, when credit standards get tightened performance is significantly lower.

In the third specification, we test whether the funding liquidity channel explains the impact of market liquidity on private equity performance. Confirming our conjecture, credit conditions absorb half of the Pástor and Stambaugh (P & S) liquidity effect. The coefficient on liquidity conditions decreases from 11.4% to 5.1% when we add the tightening in credit standards and remains significant at the 5% confidence level. This robust and significant effect of market liquidity may be interpreted as the manifestation that the tightening in credit standards is an imperfect measure of funding liquidity. It is also possible that channels other than refinancing risk explain the relation between market liquidity and private equity performance, which opens an area for future research.[22]

The close connection between credit conditions and liquidity conditions is also apparent from the correlation matrix in Table VIII. The variable most tightly linked to P&S liquidity conditions is the tightening in credit standards (−63% correlation), followed by stock market returns (46%). The other variables are only weakly related to credit conditions.

We find these results important for two related reasons. First, they deepen our understanding of the economic channel underlying the relationship between private equity returns and market liquidity. The market liquidity variable is closely related to a measure of funding liquidity, which in turn is a determinant of the ease of refinancing for leveraged deals as shown by Axelson et al. (2010).

Second, as a by-product of our analysis, we find some empirical support for the theory of Brunnermeier and Pedersen (2009) relating funding liquidity to market liquidity. Our evidence shows that there is a negative relationship

[22] Acharya and Pedersen (2005) argue that liquidity risk originates from uncertainty about the transaction costs faced when selling an asset. A simple example illustrates how this story can be relevant in a private equity context. Let us consider the uncertainty about transaction costs that an investor faces when selling two different equity positions, A and B. A is a $10 million position in an S&P 500 company. B is a $10 million investment representing a 100% stake in a privately held business. When selling A, an investor has some flexibility to manage transaction costs; for example, he can limit its price impact by splitting the order (Chan, Jegadeesh, and Lakonishok (1995), Vayanos (2001)). This possibility is not available for asset B. In addition, market depth is probably better for A. The number of potential investors in A certainly varies over time, but there are always enough investors willing to purchase a $10 million stake in an S&P 500 company at a reasonable price. In contrast, the market for full ownership of a privately held business is significantly smaller. To exit a privately held investment, the two main routes are a trade sale or an IPO. Both of these exit channels have proven to be cyclical in the past. When exiting during a trough in the IPO or M&A cycle, transaction costs are probably substantial, but they will be minimal in a buoyant IPO or M&A market. We partly test this story in the robustness analysis by adding the M&A cycles variables. In combination with credit tightening and the other controls, this regressor reduces the significance of P&S liquidity conditions (see the last specification in Table VII).

Table VIII

Correlation of the Main Variables in the Cross-Sectional Analysis.

The table reports the correlation matrix for the variables used in Table VII. The sample ranges from April 1990 to December 2007 and consists of 3,763 investments.

		1	2	3	4	5	6	7	8	9	10	11
1	P&S liquidity conditions	1.00										
2	P&S negative liquidity conditions	0.83	1.00									
3	Tightening of credit standards	−0.63	−0.74	1.00								
4	Industrial production growth	0.43	0.40	−0.74	1.00							
5	Delta credit spread	−0.26	−0.36	0.40	−0.19	1.00						
6	Relative number of M&A deals	−0.05	−0.31	−0.16	0.48	0.21	1.00					
7	Delta realized long term volatility	−0.03	−0.22	−0.12	0.10	0.26	0.61	1.00				
8	Growth investment	−0.03	0.01	0.03	−0.09	0.01	−0.11	0.00	1.00			
9	Investment size	−0.04	−0.01	0.02	−0.11	−0.03	−0.04	0.08	−0.17	1.00		
10	Fund size	−0.01	−0.02	−0.03	0.05	0.00	0.12	0.11	−0.26	0.18	1.00	
11	Rm-Rf	0.46	0.43	−0.70	0.78	−0.23	0.45	0.14	−0.08	−0.06	0.04	1.00

between a dry-up in funding liquidity (the tightening in credit standards) and innovations in market liquidity (the Pástor and Stambaugh measure).

B.2. Robustness

Controlling for the risk of economic conditions. The liquidity effect that we have just shown can be the result of a positive "macroeconomic" environment that fosters both good private equity performance and good liquidity conditions. With the goal of testing alternative explanations for the effect of liquidity conditions, we add controls for the most obvious macroeconomic variables.

Chen, Roll, and Ross (1986) show that two macro factors are priced in the cross-section of expected returns for public equity: the change in credit spreads and the growth in industrial production. According to these authors, the credit spread proxies for the expected risk premium and industrial production growth proxies for changes in future profitability. We thus include these two variables to specifications three and four in Table VII. Neither variable impacts the significance of P&S liquidity conditions. An increase in credit spreads has a negative, although not significant, impact on private equity returns, confirming that credit conditions matter for private equity returns. However, the change in credit spreads appears to be dominated by the tightening of credit standards as a measure of funding liquidity (see also the last specification in Table VII). This is probably due to the fact that the credit spread is a price-based variable that confounds demand and supply effects, unlike the change in credit standards, which is a measure of credit supply. This point is probably best illustrated in the context of the recent financial crisis. In the second semester of 2007, it seems fair to say that refinancing constraints were probably tight. But the credit spreads hardly changed. In fact, they even decreased (0.97% in the first semester of 2007 versus 0.89% in the second semester of 2007). Credit spreads spiked in January 2008 and December 2008 but, in the other months of 2008, they were at similar levels as in 2002.

To capture the cyclicality in exit opportunities we include a measure of merger and acquisition (M&A) waves in column six. The effect of liquidity conditions remains significant with a somewhat lower t-statistic, while the M&A variable is not significant. Spiegel and Wang (2005) and Bandi, Moise, and Russel (2008) argue that the effects on returns of aggregate liquidity and aggregate volatility are closely (negatively) related. So, in the seventh specification, we control for the change in long-term (four-year) realized volatility. This control leaves the significance of the credit standards unaffected and is not significant.

Finally, in the last column of Table VII, all the control variables are included. Liquidity conditions are now significant only at the 10% level. This reduced significance is probably the combined effect of controlling for the tightening of credit standards and for M&A waves. The fact that the latter variable also manages to reduce the significance of P&S liquidity conditions weakly supports the time-varying liquidity of the M&A market as another channel for liquidity risk in private equity (also see footnote 22).

Table IX
Negative Liquidity Conditions

The table reports the output of OLS regressions. The dependent variable is the investment annualized MIRR. MIRR assumes dividends are reinvested in the S&P 500. A first set of explanatory variables results from taking the average over the investment life of: (i) the "P&S negative liquidity conditions" is the average over the investment's life of a variable that is equal to the product of aggregate liquidity innovations as measured by Pástor and Stambaugh (2003) times a dummy variable that is zero if that innovation is positive and one otherwise; (ii) tightening of credit standards, the fraction of survey respondents who report to have tightened their credit standards (source: Fed Senior Officer Survey); (iii) industrial production growth (source: St. Louis Fed); (iv) change in credit spread, difference between Baa and Aaa yield (source: St. Louis Fed); (v) number of M&A deals in a month divided by the number of stocks in CRSP at the beginning of the month; (vi) change in the standard deviation of the next 48 months of CRSP value-weighted stock market index returns (labeled "Delta realized long-term volatility"); and (vii) (monthly) return on the stock market index minus the risk-free rate. A second set of explanatory variables includes investment characteristics: (i) a dummy variable that takes the value one if the investment is classified as growth (zero otherwise), (ii) investment size (i.e., equity invested in January 2007 dollars), and (iii) the size (in January 2007 dollars) of the fund by which the investment is made. A growth investment is one that mainly consists of an equity investment (little or no leverage); the classification is made by CEPRES, our data provider. Investment size is relative to same-year investments and is winsorized (at 5th and 95th percentiles of same-year investments). Country and industry fixed effects are added to the set of explanatory variables. Each explanatory variable is standardized by removing the sample mean and dividing by the sample standard deviation. Standard errors are clustered at the investment year and corresponding t-statistics are reported below each coefficient in parentheses. The sample ranges from April 1990 to December 2007.

P&S negative liquidity conditions	0.138		0.061	0.132	0.135	0.177	0.149	0.085
	(5.785)		(2.244)	(5.984)	(5.527)	(5.502)	(6.173)	(2.294)
Tightening of credit standards		−0.195	−0.144					−0.133
		(−6.243)	(−4.476)					(−3.348)
Industrial production growth				0.075				0.004
				(2.294)				(0.110)
Delta credit spread					−0.010			0.001
					(−0.518)			(0.034)
Relative number of M&A deals						0.065		0.037
						(1.791)		(0.749)
Delta realized long term volatility							0.038	−0.013
							(1.987)	(−0.366)
Rm-Rf	0.061	−0.013	−0.004	0.006	0.060	0.017	0.052	−0.024
	(2.244)	(−0.424)	(−0.123)	(0.176)	(2.249)	(0.442)	(1.776)	(−0.494)
Growth investment	−0.041	−0.042	−0.042	−0.038	−0.041	−0.039	−0.043	−0.040
	(−2.323)	(−2.425)	(−2.450)	(−2.141)	(−2.319)	(−2.306)	(−2.497)	(−2.476)
Investment size	0.001	−0.001	−0.002	0.006	0.000	0.002	−0.002	0.001
	(0.064)	(−0.122)	(−0.128)	(0.444)	(0.035)	(0.181)	(−0.139)	(0.052)
Fund size	−0.001	−0.003	−0.002	−0.002	−0.001	−0.003	−0.002	−0.003
	(−0.155)	(−0.701)	(−0.469)	(−0.281)	(−0.153)	(−0.547)	(−0.401)	(−0.579)
Country and industry fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adj. R^2	0.106	0.118	0.123	0.113	0.107	0.112	0.109	0.124
N	3,763	3,763	3,763	3,763	3,763	3,763	3,763	3,763

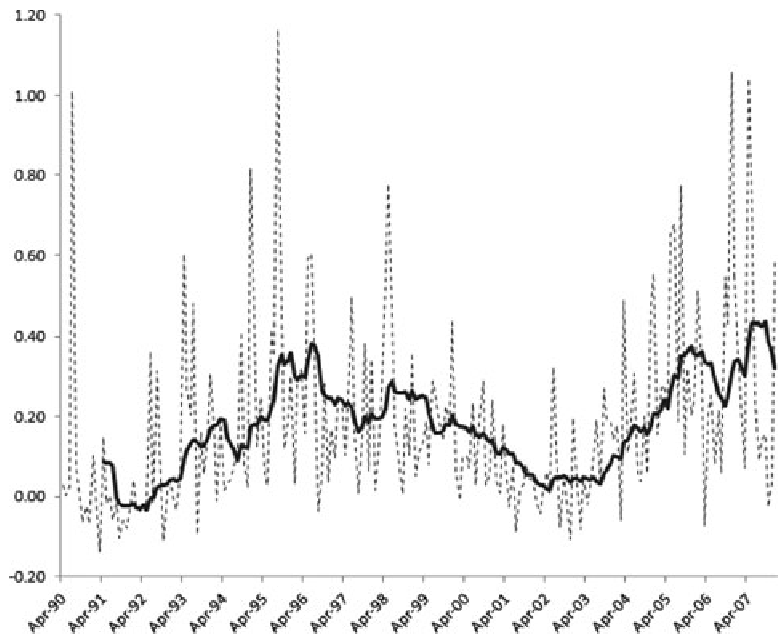

Figure 4. Aggregate net cash flows from private equity investments. The figure shows the aggregate net cash flows from private equity investments from September 1990 to December 2007 (dashed line). Each investment in the sample is scaled so that exactly $1 million is invested in each year. The scaling is pro-rata, based on investment size. Cash flows are dividends minus investments each month after the scaling, in millions of U.S. dollars. The graph also shows the 12-month moving average of the time series of cash flows (solid line).

Asymmetric measure of liquidity conditions. One potential refinement of our measure of liquidity risk is to allow liquidity conditions to affect private equity performance asymmetrically. The intuition stems from the effect of refinancing constraints. If, during the investment's life, liquidity has mild positive and negative shocks, then the refinancing constraint will never be very tight and the investment return should not be much affected. Instead, a large negative shock followed by a large positive shock may lead to the same average liquidity (over time) but will have made the refinancing constraint binding at one point, with a potentially significant impact on performance.

The literature on market timing (e.g., Henriksson and Merton (1981)) captures asymmetric exposures by simply breaking down factor returns into positive and negative realizations. We follow this lead to create both a negative-liquidity-condition variable and a positive-liquidity-condition variable. The negative-liquidity-condition variable results from multiplying each shock by a dummy variable that is zero if a shock is positive and one otherwise. The positive-liquidity-condition variable is defined symmetrically. Then, as usual, we take the average realization of these variables over the investment's life.

Table IX shows the results for the negative-liquidity-condition variable. All the specifications mirror those in the previous table but use this asymmetric measure of liquidity conditions. Consistent with the above intuition, the effect of liquidity conditions is larger than in the original specifications in Table VII. For example, in the first column, a one-standard-deviation decrease in the liquidity variable reduces returns by 13.8% annually. This is a 21% difference in magnitude relative to the slope of the original liquidity variable. Controlling for the tightening in credit standards again cuts the effect of market liquidity in half.[23]

Other robustness tests. In the Internet Appendix, we replicate our analysis using two other market liquidity measures. The Acharya and Pedersen (2005) measure (see also Acharya, Amihud, and Bharath (2010)) is equal to the cross-sectional average of the monthly illiquidity of individual stocks.[24] Stock illiquidity is measured with the Amihud (2002) ratio, which is of the average ratio over the month of absolute daily returns over daily trading volume.

The Sadka (2006) liquidity measure is a market-wide aggregation of estimated price impact at the stock level (also see Sadka (2010)). This price impact consists of a permanent part and a transitory part. These two components are estimated from a micro-structure model and use stock transaction data. The permanent component is the one that, according to Sadka, is priced in public equity and thus is the one that we use here.

The results based on the Acharya and Pedersen measure show that this measure also leads to statistically significant results. We also note that, when using the Acharya and Pedersen measure, the liquidity effect is fully explained by our measure of funding liquidity. The results based on the Sadka measure follow a similar pattern as for the other two liquidity measures.

In the Internet Appendix we also show the same specifications as in Table VII for the subsample of U.S. investments. Although the sample is reduced by half, the economic and statistical significance is remarkably similar.

Averaging shocks over long periods of time naturally leads to a reduction in the dispersion of liquidity conditions across investments and thus a reduction in statistical power. In the Internet Appendix, we re-run our main regressions over subsamples of investments based on investment duration. The results suggest that taking the average realization of the explanatory variables reduces the statistical power with respect to longest-duration investments. Still, there seems to be enough power left to identify a significant effect of liquidity risk and tightening in credit standards for all other investments.

Finally, in the Internet Appendix we extend the set of control variables to additional proxies for the risk of macroeconomic conditions (IPO cycles, VIX, inflation) without impacting the main results and show that the liquidity

[23] The results for the positive-liquidity-condition variable are given in the Internet Appendix. We find that, consistent with the refinancing constraints argument, positive liquidity innovations are not significantly related to returns.

[24] Their original variable captures market illiquidity and so we change its sign for consistency with the other measures.

Table X

Time-Series Analysis

The table reports the output of OLS regressions. The dependent variable is the aggregate net cash flows of all private equity investments in a given month (i.e., total dividends minus total investments, scaled so that the same amount is invested each year, which makes the time-series stationary). The explanatory variables are: (i) aggregate liquidity innovations as measured by Pástor and Stambaugh (labeled "P&S liquidity innovations"); (ii) tightening of credit standards, the fraction of survey respondents who report to have tightened their credit standards (source: Fed Senior Officer Survey); (iii) industrial production growth (source: St. Louis Fed); (iv) change in credit spread, difference between Baa and Aaa yield (source: St. Louis Fed); (v) number of M&A deals in a month divided by the number of stocks in CRSP at the beginning of the month; (vi) change in the standard deviation of the next 48 months of CRSP value-weighted stock market index returns (labeled "Delta realized long-term volatility"); and (vii) (monthly) return on the stock-market index minus the risk-free rate. t-statistics are reported in parentheses. The variable "tightening of credit standards" is available starting in April 1990. Hence, the sample starts in April 1990 and ends in December 2007.

P&S liquidity innovations	0.126		0.101	0.127	0.107	0.124	0.134	0.12	0.112
	(2.061)		(1.836)	(2.013)	(1.722)	(2.228)	(2.137)	(2.022)	(2.128)
Tightening of credit standards		−0.345	−0.337						−0.275
		(−4.601)	(−4.671)						(−3.149)
Industrial production growth				−0.024					−0.164
				(−0.225)					(−1.708)
Delta credit spread					−0.307				−0.113
					(−2.844)				(−1.280)
M&A cycles						0.157			0.086
						(2.635)			(1.533)
Delta realized long term volatility							0.428		0.347
							(2.070)		(1.884)
Rm-Rf								0.03	0.005
								(0.496)	(0.083)
Adj. R^2	0.014	0.091	0.099	0.014	0.046	0.037	0.11	0.014	0.189
N	213	213	213	213	213	213	213	213	213

conditions variable retains significance in the long sample (October 1975 to December 2007) for which the tightening of credit standards is not available.

C. Time-Series Evidence

An alternative approach to the cross-sectional analysis in the previous subsection is to generate an aggregate time series of private equity payoffs and correlate it with the measures of market liquidity, funding liquidity, etc.

We first aggregate the cash flows (positive and negative) every month. Because of the growth of the private equity industry, this series is strongly upward trended. We thus detrend it by scaling the investments so that exactly $100 million is invested each year. The scaling is done pro-rata, based on investment size, such that the cash flows are comparable over time. As shown in Figure 4, the time series of cash flows seems stationary and clearly reveals the private equity cycles (to reduce volatility, in the graph we also show the 12-month moving average of the series). We notice the low-yielding years around the 1991 crisis, after which dividends went back up in the mid-1990s. The downturn in the 2000 to 2003 period is also visible. The large dividends in 2004 to 2005 triggered large fund-raising and large investments, which lowered the net cash flows in 2006. These investments started to yield large payoffs in the first part of 2007. Finally, in late 2007 we see the collapse of private equity payoffs coinciding with the onset of the financial crisis.

Table X shows the results from regressing the time series of aggregate private equity cash flows onto on the same macroeconomic variables (including the liquidity measures) that we used in the cross-sectional analysis. The variable "tightening of credit standards" is available starting in April 1990. Hence, the sample starts in April 1990. The net cash flows are significantly and positively related to the innovations in the Pástor and Stambaugh measure of market liquidity. Also consistent with the preceding analysis, the tightening in credit standards, which we interpret as a deterioration in funding liquidity, is negatively related to private equity net cash flows. Like in the cross-sectional analysis, the tightening in credit standards is the most significant explanatory variable for private equity performance.

These results show that higher net distributions from private equity houses occur during periods of higher liquidity shocks. They bring further empirical support for our hypothesis and above findings. Finally, this exercise generates an aggregate measure of private equity payoffs that can be used in other applications.

IV. Conclusions

Inspired by the recent literature that identifies a liquidity risk factor in the expected returns of stocks and alternative assets, this paper investigates whether private equity returns load on liquidity risk.

Using a novel and comprehensive data set containing the cash flows from liquidated private equity investments, we find a positive and significant beta of private equity returns on the Pástor and Stambaugh (2003) traded liquidity factor in a model that also includes the excess market return, HML, and SMB. The magnitude of the private equity liquidity beta exceeds the corresponding estimate for 86% of publicly listed stocks. The unconditional liquidity risk premium is about 3% annually, the total risk premium is about 18%, and the alpha (gross-of-fees) is not statistically different from zero. In a conditional framework, the premium related to liquidity risk is larger than 5% for a quarter of the sample periods and can represent at times more than half of the total risk premium (these results are in the Internet Appendix).

We explore a potential reason for the observed link between private equity performance and liquidity risk. Private equity investments are sensitive to the liquidity of credit markets because their debts are occasionally refinanced. According to the theory of Brunnermeier and Pedersen (2009), the funding liquidity of private equity lenders (mainly banks and hedge funds) is related to market liquidity, which is the quantity we measure with the Pástor and Stambaugh (2003) factor. Hence, our main conjecture is that the relation between market liquidity and private equity returns is a reflection of the effect of funding liquidity on private equity performance. We test this hypothesis by using the tightening of credit standards from the Federal Reserve's Senior Loan Officer Survey as a measure of the evolution of funding liquidity. Consistent with our conjecture, this variable is strongly related to private equity returns and it accounts for a significant portion of the liquidity effect on returns. This indicates that funding liquidity is an important source of liquidity risk in private equity. Furthermore, as a by-product of our analysis, we find supporting evidence for the link between market and funding liquidity postulated by Brunnermeier and Pedersen (2009).

The results in this paper are relevant for academics, practitioners, and regulators, as we quantify the systematic risks and the pricing efficiency of an asset class that has gained increasing importance in financial markets. Our evidence suggests that the apparently high performance of private equity investments can be largely explained as compensation for the different risk factors to which returns are exposed, and liquidity risk is one important source of this risk premium.

Our results provide practitioners with a hurdle rate to evaluate private equity. Using such a benchmark, they can assess the NPV of their track record. The cost of capital of about 18% in excess of the risk-free rate that we estimate is in sharp contrast to the widely used hurdle rate of 8%. In addition, our results may call current compensation practices into question. Fund managers and, oftentimes, the private equity team within the investor's organization receive performance-based compensation if they achieve returns above 8% per annum, but this hurdle rate seems low in view of our findings. Knowing the risk profile of private equity investments is also an important input for portfolio risk management. At times of liquidity crises, these investments may not offer the risk diversification that investors expect from them.

Regulators may also find some useful insights in our results. Solvency II and Basel II require insurance companies and banks to set aside a provision for the risk on their private equity investments (see Bongaerts and Chalier (2009)).

As the current method of weighting assets by risk does not reflect the large exposure to liquidity risk, this may result in too low a provision.

Finally, for academics, this paper finds that the liquidity risk factor identified in public equity is consistently related to private equity performance. This contributes to the recent literature showing the pervasiveness of liquidity risk across asset classes.

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