Economics 701A **Steven A. Matthews**
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Exam

80 points, 80 minutes. Closed books, notes, calculators.
Indicate your reasoning.
Use BOTH clearly written words and math.

1. (30 pts) A firm produces output q from inputs $z = (z_1, \ldots, z_n)$ using a strictly increasing C^2 production function f. Let $c(w, q)$ be the firm's cost function, $z(w, q)$ be its conditional factor demand function, and $D_w z(w, q) = [\partial z_i / \partial w_j]$ be the Jacobian matrix of $z(w, q)$ with respect to w. For each property below, state further assumptions that imply it is true, and prove your answer.

   (a) (10 pts) $c(w, q)$ is convex in q.

   (b) (10 pts) $c(w, q)$ is linear in q.

   (c) (10 pts) $D_w z(w, q) w = 0$.

2. (20 pts) Robinson Crusoe has an endowment $e \in \mathbb{R}_{++}$ of bananas that he can consume or use to make clothing. If he uses $x \in [0, e]$ bananas to make clothing, his utility will be $u(e - x, f(x))$, where $u : \mathbb{R}^2_+ \to \mathbb{R}$ and $f : \mathbb{R}_+ \to \mathbb{R}_+$ are strictly increasing functions. Let

$$x^*(e) := \arg \max_{0 \le x \le e} u(e - x, f(x)).$$

   (a) (10 pts) Show that $x^*(e)$ is convex if u is quasiconcave and f is concave.

   (b) (10 pts) Assume now that $x^*(e)$ is a singleton for any $e > 0$, u and f are C^2 functions, f and u are concave, and $u_{12} \ge 0$. Prove that $x^*(e)$ is a nondecreasing function, stating any further (minor) assumptions you need.

3. (30 pts) Let $\tilde{x} = a + \tilde{\varepsilon}$ be a gamble, where $a \in \mathbb{R}$ and $\tilde{\varepsilon}$ is a random variable with mean zero. A consumer has a C^2 Bernoulli utility function, $u : \mathbb{R} \to \mathbb{R}$, satisfying $u' > 0$ and $u'' \le 0$. Her sale price for the gamble is the minimum amount she would sell the gamble for: it is the number $s(a)$ satisfying

$$u(s(a)) = \mathbb{E} u(a + \tilde{\varepsilon}).$$

   (a) (10 pts) If u exhibits constant absolute risk aversion, what can you say about the derivative $s'(a)$? Prove your answer.

   (b) (20 pts) If u exhibits decreasing absolute risk aversion (DARA), what can you say about the derivative $s'(a)$? Prove your answer.