University of Pennsylvania
The Wharton School

Professor Stambaugh | Investment Management

SAMPLE EXAM 1

Name : ______________________________

Circle one: Fin 720 (10:30) Fin 205(1:30) Fin 205 (3:00)

Instructions:

1. Do all of your work on this exam paper, and display your answers clearly and legibly in the spaces provided. Do not feel your answer must consume the entire space provided—it is anticipated that it often will not.
2. The time allowed is 1 hour and 20 minutes. Total points: 100.
3. You are allowed one sheet of notes (two-sided) and a calculator.

1. (20 points)

You are advising clients about investment allocations in their employer-sponsored retirement accounts. You believe each client's risk/reward attitudes can be well approximated by an objective of the form,

$$\mathrm{E}(r_p) - \frac{A}{2}\mathrm{var}(r_p),$$

where r_p is the return over the next period on the client's portfolio, and A is the client's coefficient of risk aversion. (Each client's value of A is generally different from that of other clients.) Currently you are deciding what to advise two clients, Carol and Ted, both of whom have taken new jobs.

Carol's previous employer offered each employee the ability to allocate his/her retirement account among three funds, all from the ABC mutual fund company: the Market Fund, the Growth Fund, and the Money Fund. Carol's allocation to these funds has been as follows:

Market Fund	35%
Growth Fund	35%
Money Fund	30%

You had recommended this allocation to her, after consulting with her and assessing her risk tolerance. She has recently changed jobs. Her new employer's retirement plan also offers ABC funds, but just the Market Fund and the Money Fund. You must now recommend a new allocation to Carol that includes just those two funds.

Ted is in the opposite situation. His previous employer offered just two ABC funds, the Market Fund and the Money Fund, but his new employer offers those two funds plus the Growth Fund. Ted's previous allocation, which you had also recommended to him, was as follows:

Market Fund	50%
Money Fund	50%

You must now recommend a new allocation to Ted that includes all three funds.

In advising both clients, you have been assuming that the Money Fund returns the riskless interest rate, and you believe that the properties of the annual returns (in excess of the interest rate) on their *current* portfolios above—*before* any re-allocations—are as follows:

	Expected Excess Return	Standard Deviation of Excess Return
Carol	0.10	0.20
Ted	0.06	0.14

These values reflect underlying assumptions about properties of the mutual funds (expected returns, standard deviations, correlations), and you have not changed those fund-level assumptions.

a. (10 points) What new allocation would you now recommend to Carol?

Market Fund	________
Money Fund	________
Total	100%

b. (10 points) What new allocation would you now recommend to Ted?

Market Fund	______
Growth Fund	______
Money Fund	______
Total	100%

2. (20 points)

Consider the following four alternative allocations to a stock portfolio, a bond portfolio, and cash.

	A	B	C	D
Stocks	.3	.4	.4	.6
Bonds	.3	.4	.6	.4
Cash	.4	.2	0	0
Standard Deviation of Overall Return	0.075	0.100	0.114	0.137

Assume:

- the compositions within the stock and bond portfolios are identical across the allocations
- borrowing (a negative cash position) is prohibited
- stocks have a higher expected return than bonds
- stocks have a higher standard deviation of return than bonds
- bonds have an expected return greater than the riskless return on cash

a. (10 points) Three of the four allocations above *are* mean-variance efficient. Which allocation is then *not* mean-variance efficient? Explain briefly.

b. (10 points) Give the allocations for the mean-variance efficient portfolio whose standard deviation of return is equal to 0.11.

Stocks	____________
Bonds	____________
Cash	____________
Total	100%

3. (20 points)

You have a client, Paul, who is currently holding a portfolio invested 70% in an S&P 500 index fund and 30% in cash. Paul has been considering adding one additional fund to his portfolio, hoping to improve his portfolio's expected return without increasing the standard deviation of his return. He has gathered historical track records for the index fund and three additional mutual funds he is considering (A, B, and C), and he has computed the following statistics for each fund's annual excess returns. ("Excess" return is the rate of return minus the riskless interest rate.)

	S&P 500 index fund	Fund A	Fund B	Fund C
Arithmetic average excess return	0.08	0.10	0.11	0.12
Beta (using the S&P 500 as the market)	1.00	1.35	1.40	1.45
Standard deviation of excess return	0.20	0.30	0.35	0.40

Paul is willing to assume that the above historical statistics provide reasonable estimates of the true underlying parameter values that he is facing over the upcoming year.

a. (10 points) Which fund should Paul add to his portfolio? You do *not* need to recommend allocation percentages.

Please circle one: Fund A Fund B Fund C

b. (10 points) Explain your recommendation.

4. (20 points)

Jane and Lynne have started a small fund, investing \$2 million of their own capital, with hopes of establishing a track record that will later help them to attract money from outside investors. Their quantitative strategy has identified 16 stocks to buy and hold over the next month, and they will buy equal initial dollar amounts of each stock. They would like to hedge their long position in the 16 stocks with a position in S&P 500 futures, so as to make their overall portfolio market-neutral (zero-beta).

For each stock, the two partners have estimated characteristic line regressions of the form,

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + \epsilon_{i,t}$$

where $R_{i,t}$ is a stock's return in month t, and $R_{M,t}$ is the return on the S&P 500. (Both $R_{i,t}$ and $R_{M,t}$ denote returns in excess of the riskless interest rate.) The partners estimate that, across the 16 stocks, the average α_i equals 0.03, and the average β_i equals 1.25. Based on historical estimates, they are also assuming that

- the standard deviation of $\epsilon_{i,t}$ equals 0.12 for each individual stock from the list of 16
- the correlation between $\epsilon_{i,t}$ and $\epsilon_{j,t}$ equals zero for every possible pair of stocks from the list of 16
- the expected excess monthly return on the market equals 0.005
- the standard deviation of the monthly market return is 0.06

a. (5 points) Give the dollar amount of the futures position they should establish, and indicate whether that futures position should be long or short.

b. (7 points) What dollar profit on the overall hedged position should the partners expect over the next month?

c. (8 points) What is the standard deviation of the dollar profit on the overall hedged position over the next month?

5. (20 points)

The "policy" portfolio of an endowment fund is given below, along with some characteristics of the asset classes.

Asset class	Weight in Policy Portfolio	Standard Deviation	Correlations	
			Equity	Fixed Income
Equity	0.60	0.20	1.00	0.30
Fixed Income	0.40	0.10	0.30	1.00

The fund believes that the Sharpe ratio of the policy portfolio is equal to 0.25.

b. (12 points) If the policy portfolio is mean-variance efficient, what must be the expected excess return on equity? (Excess return is the rate of return minus the riskless interest rate.)

c. (8 points) If the policy portfolio is mean-variance efficient, what must be the expected excess return on fixed income?

Answers

1. (a) Carol's current portfolio, which is mean-variance efficient, combines Money (cash) with what must be the tangent portfolio of Market and Growth—a 50-50 combination of those two funds. Given that she allocates 70% to the latter, the mean and standard deviation of the tangent portfolio's excess return are given by

$$E_T = \frac{0.10}{0.70} = 0.1429 \ , \qquad \sigma_T = \frac{0.20}{0.70} = 0.2857.$$

Her risk-aversion is given by

$$A_{\text{Carol}} = \frac{0.1429}{0.70(.2857)^2} \left(= \frac{.10}{.20^2} \right) = 2.50.$$

Given that Ted currently allocates 50% to Market, the mean and standard deviation of that fund's excess return are given by

$$E_M = \frac{0.06}{0.50} = 0.12 \ , \qquad \sigma_M = \frac{0.14}{0.50} = 0.28.$$

Carol's new allocation to Market is therefore

$$y_M = \frac{E_M}{A_{\text{Carol}}\sigma_M^2} = \frac{0.12}{2.50(0.28)^2} = 0.6122,$$

with $1 - 0.6122 = 0.3878$ in Money.

(b) Ted's risk-aversion is given by

$$A_{\text{Ted}} = \frac{0.12}{0.50(.28)^2} \left(= \frac{.06}{.14^2} \right) = 3.06.$$

Ted's new allocation to the tangent portfolio of Market and Growth is therefore

$$y_T = \frac{E_T}{A_{\text{Ted}}\sigma_T^2} = \frac{0.1429}{3.06(0.2857)^2} = 0.5717,$$

or 0.2858 in each of Market and Growth, with $1 - 0.5717 = 0.4283$ in Money.

2. (a) Since portfolios A and B contain the same stock-bond ratio, either both are efficient or both are inefficient, and the latter would violate the statement that three of the four are efficient. The tangent portfolio must therefore be a 50-50 combination of stocks and bonds. With no borrowing, any efficient portfolio for a risk level above that of the tangent portfolio will contain no cash, and it must have higher expected return than the tangent portfolio. To accomplish the latter, the weight in stocks must exceed the stock weight in the tangent portfolio, because the expected return on stocks exceeds that of bonds. Portfolio D satisfies that requirement, whereas Portfolio C does not. Thus, the inefficient portfolio must be C.

(b) Since portfolio A allocates 60% to the tangent portfolio and has a standard deviation of 0.075, the standard deviation of the tangent portfolio is

$$\sigma_T = \frac{0.075}{0.60} = 0.125$$

(Using portfolio B instead gives the same value: $\sigma_T = 0.10/0.80 = 0.125$.) To achieve a standard deviation of 0.11, the weight in the tangent portfolio must therefore be

$$y_T = \frac{0.11}{0.125} = 0.88$$

or 0.44 in each of stocks and bonds, and $1 - 0.88 = 0.12$ in cash.

3. (a) Fund C—it's the only fund with a positive alpha:

$$\begin{aligned}
\alpha_A &= E_A - \beta_A E_{\text{S\&P}} = 0.10 - 1.35(0.08) = -0.008 \\
\alpha_B &= E_B - \beta_B E_{\text{S\&P}} = 0.11 - 1.40(0.08) = -0.002 \\
\alpha_C &= E_C - \beta_C E_{\text{S\&P}} = 0.12 - 1.45(0.08) = 0.004
\end{aligned}$$

(b) An asset's alpha is what indicates whether it is possible to obtain a higher Sharpe ratio than the benchmark by combining the benchmark with the asset. If the alpha is positive, then some positive allocation to the asset will produce a higher Sharpe ratio than holding just the benchmark. In this case, only one of the funds has that property. Thus, if Paul is considering adding one fund, it should be that fund.

4. (a) Since the average β is 1.25, that value is also the beta of the equally weighted portfolio. Hedging the long stock position therefore requires a *short* futures position of size 1.25($2 million) = $ 2.5 million.

(b) The expected dollar profit on the hedged position, in excess of the interest rate, is determined by the portfolio's alpha. Since the average alpha is 0.03, that value is also the alpha of the equally weighted portfolio. Therefore, the expected profit is 0.03($2 million) = $ 60,000, plus interest on the $2 million. (Full credit is also given if the interest portion is omitted.)

(c) With market risk hedged, the standard deviation of the rate of return is simply the standard deviation of the portfolio's non-market return—the standard deviation of the portfolio's epsilon. With an equally weighted portfolio,

$$\epsilon_{P,t} = \sum_{i=1}^{16} \frac{1}{16} \epsilon_{i,t}.$$

Since the $\epsilon_{i,t}$'s are mutually uncorrelated, the variance of $\epsilon_{P,t}$ is given by

$$\sigma^2(\epsilon_P) = \sum_{i=1}^{16} \left(\frac{1}{16}\right)^2 \sigma^2(\epsilon_i) = 16 \left(\frac{1}{16}\right)^2 (0.12)^2 = \frac{0.12^2}{16},$$

and thus the standard deviation of the return is

$$\sigma(\epsilon_P) = \sqrt{\frac{0.12^2}{16}} = \frac{0.12}{4} = 0.03.$$

The standard deviation of the dollar profit is therefore 0.03(\$ 2 million) = \$ 60,000.

5. (a) Mean variance efficiency of the policy portfolio implies that it is the tangent portfolio T for combinations of equity and fixed income (since the portfolio contains no cash). For each asset i in the tangent portfolio, the asset's expected excess return E_i is proportional to the covariance of its return with that of the tangent portfolio, σ_{iT}. Specifically,

$$E_i = \frac{\sigma_{iT}}{\sigma_T^2} E_T = \frac{\sigma_{iT}}{\sigma_T} S_T, \tag{1}$$

where E_T and σ_T^2 denote the mean and variance of the tangent portfolio's excess return, and S_T denotes the Sharpe ratio of the tangent portfolio—equal to 0.25 in this case. For σ_{ET}, the covariance of the equity return with the tangent return, we have

$$\begin{aligned} \sigma_{ET} &= \mathrm{Cov}\,(R_E, R_T) \\ &= \mathrm{Cov}\,(R_E, 0.6R_E + 0.4R_{FI}) \\ &= 0.6\mathrm{Cov}\,(R_E, R_E) + 0.4\mathrm{Cov}\,(R_E, R_{FI}) \\ &= 0.6(0.2)^2 + 0.4(0.2)(0.1)(0.3) \\ &= 0.0264, \end{aligned}$$

where the second term in the next-to-last line uses the fact that $\mathrm{Cov}\,(R_E, R_{FI})$ is the product of each asset's standard deviation and the correlation between the assets. For the variance of the tangent portfolio we have

$$\begin{aligned} \sigma_T^2 &= \mathrm{Var}\,(0.6R_E + 0.4R_{FI}) \\ &= 0.6^2\mathrm{Var}\,(R_E) + 0.4^2\mathrm{Var}\,(R_{FI}) + 2(0.6)(0.4)\mathrm{Cov}\,(R_E, R_{FI}) \\ &= 0.6^2(0.20)^2 + 0.4^2(0.10)^2 + 2(0.6)(0.4)(0.2)(0.1)(0.3) \\ &= 0.0189, \end{aligned}$$

so $\sigma_T = 0.1374$. Therefore, substituting into (1) gives the expected excess return on equity as

$$E_E = \frac{\sigma_{ET}}{\sigma_T} S_T, = \frac{0.0264}{0.1374} 0.25 = 0.0480.$$

(b) Since $S_T = E_T/\sigma_T$, we have

$$E_T = \sigma_T S_T = (0.1374)(0.25) = 0.0344.$$

Then

$$\begin{aligned} E_T &= 0.6E_E + 0.4E_{FI} \\ 0.0344 &= 0.6(0.0480) + 0.4E_{FI}, \end{aligned}$$

and the expected excess return on fixed income is

$$\begin{aligned} E_{FI} &= \frac{0.0344 - 0.6(0.0480)}{0.4} \\ &= 0.0138. \end{aligned} \tag{2}$$