University of Pennsylvania
The Wharton School

Professor Stambaugh | Investment Management

SAMPLE EXAM 2

Name : ____

Circle one: Fin 720 (10:30) Fin 205 (1:30) Fin 205 (3:00)

Instructions:

1. Do all of your work on this exam paper, and display your answers clearly and legibly in the spaces provided. Do not feel your answer must consume the entire space provided—it is anticipated that it often will not.
2. The time allowed is 1 hour and 20 minutes. Total points: 100.
3. You are allowed one sheet of notes (two-sided) and a calculator.

1. (21 points)

Some investment managers believe that the following behavioral traits are prevalent in financial markets:

- loss aversion
- overconfidence
- recency
- anchoring

Below are three types of long-short strategies we have discussed. For each strategy:

(i) Explain how you would identify which stocks to go long and which to go short

(ii) Pick *one* of the above behavioral traits and explain how its presence could contribute to positive expected profits on the strategy.

[You may pick a different behavioral trait for each strategy. There is not necessarily a unique correct answer to either (i) or (ii).]

a. (7 points) Price momentum

b. (7 points) Earnings momentum

c. (7 points) Value vs. growth

2. (20 points)

a. (10 points) Jim and Joe are both graduating from Wharton this year. Jim is going to work for the SEC. He expects his compensation to grow smoothly over time. Joe is going to work for a long-only equity hedge fund, and he expects his compensation to be much better in years when the stock market does well. Both are considering target-date funds for investing their retirement savings. For whom does a target-date fund make more sense, Jim or Joe? Explain

b. (10 points) Jill and Jane, both in their late 20's, are presented with historical evidence indicating a positive relation between the stock market's earnings-price ratio (E/P) in a given year and returns on the stock market over the subsequent five years. Jill finds the evidence convincing and expects that such a relation will continue to hold in the future. Jane believes this historical correlation is spurious and does not believe that, in the future, E/P will possess any ability to predict stock returns. Which investor, Jill or Jane, should find a target-date fund more appealing? Explain

3. (20 points)

Ed, Hal, and Manny work for a large bank and have been asked to supply an estimate of the expected real rate of return on the small-capitalization segment of the U.S. stock market to use in the firm's asset-allocation models. They have been instructed to view the Russell 2000 index as representative of the small-stock segment, and so far they have assembled the following data:

Current level (share price) of the Russell 2000	\$667
Next year's forecasted earnings per share on the Russell 2000	\$35
Next year's forecasted dividends per share on the Russell 2000	\$11
Long-run forecasted real GDP growth per year	2.5 %
Standard deviation of the annual return on the Russell 2000	30 %

a. (5 points) Ed believes that earnings on the Russell 2000 index will grow in real terms at the same constant rate as GDP and that the dividend-payout rate will remain constant. What expected return on small stocks should Ed recommend? Explain.

b. (5 points) Hal, a veteran at the bank, recalls that he performed Ed's calculation ten years ago, and at that time he got a number higher than what Ed is proposing today. Hal says it's hard to believe that expected small-stock returns are any lower now than they were then, since the average return on small stocks over the past ten years has actually been pretty high—over 11% per year. Ed says that Hal's observation actually makes him feel more confident about his current estimate. Provide a rationale for Ed's statement.

c. (5 points) Manny also argues that Ed's number seems too low. Manny says Ed's number implies that over long horizons there is substantially more than a 50% probability that small stocks will lose money in real terms. Can you make sense of Manny's statement? Explain.

d. (5 points) Manny believes that a reasonable estimate of the expected small-stock return is instead simply the earnings-price ratio, giving an expected real return of $35/667 = 5.25\%$. What can justify Manny's approach?

4. (20 points)

A foundation that is currently invested 100% in an S&P 500 index fund is considering reallocating some of its portfolio to *one* of two market-neutral (zero-beta) hedge funds, A or B. The foundation's objective in selecting the hedge fund, and the amount to allocate to it, is to obtain the highest Sharpe ratio for the foundation's overall portfolio. You have been asked to advise the foundation. Thus far you have estimated the following Sharpe ratios and standard deviations of returns:

Fund (P)	Sharpe ratio	std. dev. of return
A	0.20	0.20
B	0.10	0.10
S&P 500	0.30	0.20

a. (5 points) Of the two hedge funds, which is the better one to receive a portion of the foundation's overall allocation? Explain.

b. (5 points) In the new overall portfolio, for each dollar allocated to the S&P, how much would you allocate to the hedge fund?

c. (5 points) What is the Sharpe ratio of the new overall portfolio?

d. (5 points) If the foundation were instead considering reallocating some of its portfolio to *one or both* of the hedge funds, can you say from the information provided whether the foundation should invest positive amounts in *both* funds? If you cannot say, upon what additional information would the answer depend? Explain.

5. (19 points)

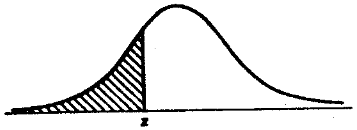
A foundation currently has its \$200 million endowment invested in a portfolio that mimics the S&P 500. The foundation's board is considering placing the entire endowment in a trust. The trust would continue to be managed as an S&P 500 index fund, whose continuously compounded annual return has a standard deviation of 18%. Upon formation, the trust would issue to the foundation two classes of shares, Class A and Class B. The trust would have a 20-year life. At maturity (the end of year 20), the Class A shares would receive the lesser of (i) the trust's total assets or (ii) a "redemption value" equal to the initial trust contribution grown at the continuously compounded riskless rate of 1% per year, ($e^{(20)0.01} \times$ \$200 million). The Class B shares would receive at maturity any trust assets in excess of the "redemption" value. A consultant to the foundation has also provided estimates of what the current prices of the Class A and Class B shares would be, in case the foundation would wish to sell them.

a. (10 points) The consultant has also estimated that there is a 97% probability that the Class A shares will receive the full redemption value. What expected continuously compounded annual rate of return on the S&P must the consultant be assuming? (She is assuming that continuously compounded returns on the S&P are normally distributed, and a table of the standardized normal distribution is attached to the end of the exam.)

b. (9 points) One board member suggests that a purchaser of the Class A shares could also purchase insurance against the possibility that the Class A shares would pay less than the redemption value. Specifically, the insurance would pay the difference, if positive, between the redemption value and the payoff on the Class A shares. The board member argues that the insurance should be very inexpensive, given the consultant's estimated 97% probability of full payoff on the shares. Do you agree with this reasoning? What information already provided to the foundation could be used to estimate the current price of such insurance? Explain. (You are not expected to supply a numerical value here.)

TABLE I Values of the Standard Normal Distribution Function

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}u^2\right) du = P(Z \le z)$$



z	0	1	2	3	4	5	6	7	8	9
−3.	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0126	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0238	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0300	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0570	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
− .9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
− .8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
− .7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
− .6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
− .5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
− .4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
− .3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
− .2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
− .1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
− 0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TABLE 1 Values of the Standard Normal Distribution Function (Continued)

z	0	1	2	3	4	5	6	7	8	9
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9430	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9648	.9656	.9664	.9671	.9678	.9686	.9693	.9700	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9762	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9874	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

Note 1: If a normal variable X is not "standard," its values must be "standardized": $Z = (X - \mu)/\sigma$, i.e., $P(X \le x) = \Phi[(x - \mu)/\sigma]$.

Note 2: For "two-tail" probabilities, see Table Ib.

Note 3: For $z \ge 4$, $\Phi(z) = 1$ to four decimal places; for $z \le -4$, $\Phi(z) = 0$ to four decimal places.

Note 4: Entries opposite 3 and −3 are for 3.0, 3.1, 3.2, etc., and −3.0, −3.1, etc., respectively.

Answers

1. (a) Strategy: go long (short) stocks with the highest (lowest) cumulative returns over the past 6–12 months (generally omitting the most recent month).

    Loss aversion gives rise to what is often called the "disposition" effect: investors resist selling stocks that have gone down, so as to avoid taking realized losses, whereas they do sell stocks that have realized gains (removing the potential for later realized losses). Investors' sales of recent winners temporarily keep the prices of such stocks from going up as much as they should, thereby making them attractive buys. At the same time, investors' reluctance to sell losers temporarily keeps the prices of those stocks from falling as much as they should, thereby making those stocks relatively attractive shorts.

    Overconfidence or *anchoring* can be motivations for a price momentum strategy. Overconfidence leads to investors to attach too much weight to their own opinions and thus too little weight to relevant new information. The latter is then not reflected in prices as quickly as it should be and prices will thus move in a "sticky" persistent fashion, allowing a momentum strategy to profit by catching these short-term trends. Anchoring—giving too much weight to a number that is longer relevant (or even never was)—can also cause similar under-reaction to relevant new information.

    (b) Strategy: go long (short) stocks whose recent earnings announcements are above (below) consensus analyst forecasts or the previous year's same-quarter earnings. Alternatively, go long (short) stocks for which the consensus analyst forecast has moved up (down).

    Overconfidence or *anchoring*—applied to analyst behavior—can also motivate this strategy. Analysts who are overconfident in their own opinions or who become anchored on previous earnings numbers will give too little weight to relevant new information when updating their forecasts, thus allowing earnings surprises and/or past analyst revisions to predict future revisions. If prices reflect analyst forecasts (investors pay attention to analysts), then future price movements become predictable by the same information.

    (c) Strategy: go long (short) stocks whose market values are the lowest (highest) when divided by or otherwise compared to (i) book value, (ii) sales, (iii) fair value from a dividend-discount model, or (iv) an estimate of market value obtained in a cross-sectional regression on a set of accounting variables.

    Overconfidence can lead investors to devote too little effort to uncovering information that is relevant but less easily accessed. The "growth" or "glamour" stocks (with high market values in the above sense) are generally more visible in the media and more heavily followed by analysts than the "value" stocks with relative low market values. As a result, the positive information investors see for the growth/glamour stocks leads them to overvalue those stocks while neglecting and undervaluing the value stocks.

    Recency—placing too much emphasis on recent events—is another potential source of profit. The stocks classified as value have typically experienced several years of poor performance, whereas those classified as growth/glamour have typically experienced strong performance. If investors extrapolate this recent performance well into the future and thereby fail to realize that such performance often reverts to more normal levels, then the value stocks will be underpriced and growth/glamour stocks will be overpriced.

2. (a) Jim. His human capital, represented by the value of his future compensation, can be viewed as asset that is more similar to a bond than to equity, given his anticipated smooth path for that compensation. This bond-like human capital will have its largest value while Jim is young and then decline as Jim ages, as the number of future working years declines. If one views Jim's human capital and his financial capital as together constituting an overall wealth portfolio, the greater value of his bond-like human capital when young justifies investing his financial capital more heavily in equity. As he ages, and his financial capital grows while his bond-like human capital shrinks, reallocating financial capital from equity to bonds preserves a more stable overall mix of bond-like assets and equity in his overall wealth portfolio. This is precisely what a target-date fund does.

   Joe's human capital is less like a bond, given the higher anticipated sensitivity of his compensation to the performance of the equity market, so the above rationale applies less strongly to Joe.

   (b) Jill. The positive correlation means that the expected equity return changes over time in manner that lowers long-run equity volatility, as compared to a setting in which the expected equity return is constant. High values for E/P tend to occur when recent equity returns have been poor (hence making P low and E/P high), and vice versa, so the positive correlation between E/P and future return means that high (low) returns tend to be followed by low (high) returns. This "mean reversion" lowers the volatility of equity from the perspective of a long-run investor, whereas it has no impact on the volatility faced by a short-run investor. As a result, equity becomes relatively more attractive to a long-run investor, which is consistent with target date funds' higher equity allocations for investors with longer horizons.

   The mean-reversion rationale is absent for Jane.

3. (a) Given the constant-growth-rate assumption, Ed can use the simple Gordon model, which implies that the expected simple rate of return $\mathrm{E}(R)$ is the sum of the current dividend yield plus the expected growth rate of dividends (here the same as GDP growth):

$$\mathrm{E}(R) = \frac{D}{P} + g = \frac{11}{667} + 0.025 = 0.0165 + 0.025 = 0.0415\%$$

   (b) A unanticipated drop in expected return during the past ten years is consistent with a high realized return during that period. A drop in the discount rate (expected return) generally raises the value of what is being discounted (expected future dividends), producing an unanticipated capital gain.

   (c) The easiest way to make sense of this is to assume that continuously compounded returns obey a normal distribution. Then, the expected continuously compounded return μ is equal to the expected simple return minus half the return variance:

$$\mu = \mathrm{E}(R) - \frac{1}{2}\sigma^2 = 0.0415 - \frac{1}{2}(.30)^2 = 0.0415 - 0.045 = -0.0035$$

With a negative value for the continuously compounded return, the probability of losing money is greater than 50% and increasing in the length of the horizon.

Note (not required to be included in your answer):

$$\begin{aligned}\text{Prob}\left\{\sum_{t=1}^{T} r_t < 0\right\} &= \text{Prob}\left\{z < \frac{0 - T\mu}{\sqrt{T}\sigma}\right\} \\ &= \text{Prob}\left\{z < -\sqrt{T}\frac{\mu}{\sigma}\right\}.\end{aligned}$$

For $\mu < 0$, note that the z-score on the right-hand side of the inequality is positive and increasing in T. Therefore, the probability exceeds 0.5 and approaches 1 as $T \to \infty$.

(d) The basic assumption required here is that, if the small-cap sector is viewed as a large composite "firm," the rate of return earned on funds retained and invested internally (the ROE) is the same as the expected rate of return $\mathrm{E}(R)$ required by external investors in the firm's stock. The assumption rests on the rationale that capital would flow either into the small-cap sector (if $ROE > \mathrm{E}(R)$) or out of it (if $ROE < \mathrm{E}(R)$) until the rates are equal in equilibrium.

4. (a) The fund with the higher information ratio, $IR = \alpha/\sigma(\epsilon)$. In this case, since the betas are zero, the information ratios are the same as the Sharpe ratios, and thus Fund A is preferred.

(b) Let S_A and S_M denote the Sharpe ratios of Fund A and the S&P, and let σ_A and σ_M denote their respective standard deviations. The fraction of wealth allocated to Fund A is

$$w_A = \frac{w_0}{1 + (1 - \beta_A)w_0} = \frac{w_0}{1 + w_0},$$

where

$$w_0 = \left(\frac{S_A}{S_M}\right)\left(\frac{\sigma_M}{\sigma_A}\right) = \left(\frac{0.2}{0.3}\right)\left(\frac{0.2}{0.2}\right) = 2/3.$$

Therefore,

$$w_A = \frac{2/3}{1 + 2/3} = 2/5,$$

and the amount allocated to Fund A for each dollar allocated to the S&P is $w_A/(1 - w_A) = \$0.667$.

(c) The Sharpe ratio of the new combination, S_Q, is given by the relation

$$S_Q^2 = S_M^2 + IR_A^2 = S_M^2 + S_A^2,$$

so

$$S_Q = \sqrt{0.3^2 + 0.2^2} = 0.3606.$$

(d) The answer depends on the correlation between the returns on the two hedge funds. For a sufficiently high correlation, no positive-weight combination of the two funds will have a higher IR than Fund A's IR. For a sufficiently low correlation, a combination of the two funds will have a higher IR than Fund A's IR.

5. (a) Let μ denote the expected continuously compounded return on the S&P, let V_{20} denote the S&P index fund's value in 20 years for each current dollar invested, let r_t denote the S&P's continuously compounded return in year t, and let σ denote the standard deviation of r_t. The probability that the Class A shares will receive less than the full redemption value is

$$\begin{aligned} 0.03 &= \text{Prob}\left\{V_{20} < e^{(20)0.01}\right\} \\ &= \text{Prob}\left\{\sum_{t=1}^{2} 0 r_t < (20)0.01\right\} \\ &= \text{Prob}\left\{z < \frac{(20)0.01 - (20)\mu}{\sqrt{20}\,\sigma}\right\} \\ &= \text{Prob}\left\{z < \frac{(20)0.01 - (20)\mu}{\sqrt{20}(0.18)}\right\} \end{aligned}$$

We see from the standardized normal distribution that

$$0.03 = \text{Prob}\{z < -1.88\},$$

so

$$\frac{(20)0.01 - (20)\mu}{\sqrt{20}(0.18)} = -1.88,$$

which implies

$$\mu = 0.01 + \frac{0.18(1.88)}{\sqrt{20}} = 0.0857.$$

(b) No—the probability of a full payoff is not what determines the price of the insurance. The insurance cost will be substantially higher than the expected payoff on the insurance. The insurance in this case is equivalent to a 20-year (European) put option on the current $200 million S&P index portfolio, where the strike price is the redemption value. The Class B shares are equivalent to a call option on the same index portfolio, with the same maturity and strike price. The present value of the strike price, discounting at the riskless 1% rate, is simply the current $200 million portfolio value. Thus, by put-call parity, the price of the insuring the Class A shares—the price of the put—is equal to the price of the Class B shares (the call), and the consultant has already provided an estimate of the latter to the foundation.