

EQUITY SWAPS AND EQUITY INVESTING

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In an equity swap, two parties make a series of payments to each other with at least one set of payments determined by a stock or index return. The other set of payments can be a fixed or floating rate or the return on another stock or index. Equity swaps are used to substitute for a direct transaction in stock. This article explores equity swaps, describing variations, applications, and advantages and disadvantages. It also presents and illustrates formulas for pricing and valuation and provides empirical evidence comparing the performance of equity swaps against comparable strategies involving direct investment in equity.

EQUITY SWAPS AND EQUITY INVESTING

With the proliferation of exchange-traded funds, the possibilities for direct investing in the equity market have greatly increased in recent years. In contrast to direct investing in the equity market, however, are a number of indirect methods. Stock index futures contracts are well-known and widely used as a means of investing in a stock index, and the early but developing market in futures on individual stocks offers even more possibilities. This article focuses on another method for investing in the stock market, one that is little known but increasingly used: equity swaps.

A swap is a financial transaction in which one party agrees to make a series of payments to another at regularly scheduled dates. The opposite party, in turn, agrees to make a series of payments to the first party. At least one set of payments must be determined by the course of an unknown future factor such as an interest rate, exchange rate, commodity price, or stock price. The other set of payments can be determined by another factor or it can be fixed. When at least one set of payments is determined by a stock or stock index, the transaction is referred to as an equity swap, or sometimes equity-index(-ed) swap, or equity-linked swap.¹ The equity payments on an equity swap are determined as the rate of return of the underlying equity applied to a pre-determined notional principal. Because the return typically includes dividends, equity swaps are often referred to as total return swaps, though it is possible (and in fact often the case) that the underlying asset in a total return swap is a bond. Given, however, that equity swaps result in payment of the return on a stock or stock index, they provide cash flows that mimic equity returns. We emphasize the word “mimic” here, because some types of equity swaps precisely replicate the return on a stock or index; others come very close but are not identical. In any case, however, equity swaps can, as with other equity derivatives, be used as substitutes for direct transactions in equity.

It is not clear when equity swaps first started, but *Investment Dealers' Digest* in 1990 reports on an equity swap used by the \$2.5 billion Amoco pension fund to earn a return on a Japanese stock index.² The article indicates that this was not the first equity swap. Anecdotal evidence suggests that equity swaps may have started in the United States in the late 1980's in response to a tax issue associated with investments in foreign equities.³ Investors holding foreign stocks are often subject to withheld dividends. Even though these dividends can often be eventually recovered, the interest on them is still lost. A derivatives dealer can, through a foreign

¹Technically, the other series of payments could be an interest rate, exchange rate or commodity price. A series of payments linked to any random factor can be swapped for a series of payments linked to any other random factor or for a series of fixed payments.

²See Schwimmer [1990].

subsidiary in the particular country, invest in the foreign securities without the withholding tax and enter into a swap with the parent dealer company, which can then enter a swap with the American investor, effectively passing on the dividends without the withholding tax.⁴

Equity swaps are offered by derivatives dealers in much the same manner as their offerings of interest rate, currency, and commodity swaps. Dealers typically charge a bid-ask spread and hedge any risk they have assumed by transacting in the underlying stock or index or another derivative on the stock or index.

Equity swaps have since been used for a variety of purposes, which are explored in this paper. Though they are not as widely used as interest rate and currency swaps, the market is still quite large. A summary of results from the last four years of surveys taken by the Bank for International Settlements in Basel, Switzerland is provided in Exhibit 1. As of December 2002, equity swaps worldwide have an estimated total notional principal of over \$300 billion and a market value of over \$60 billion⁵.

This article provides an examination of equity swaps as a form of indirect equity investing. The next section contains a more thorough description of the instrument and its major variations. The remaining sections illustrate common applications of equity swaps, discuss their advantages and disadvantages, explore the pricing and valuation of equity swaps, and provide some empirical results on the returns on equity swaps in comparison to the returns on direct equity investing. The final section contains the conclusions.

I. EQUITY SWAPS: FORMAL DEFINITION AND VARIATIONS

As previously described, an equity swap is a transaction between two parties in which each party agrees to make a series of payments to the other, with at least one set of payments determined by the return on a stock or stock index. The return is calculated based on a given notional principal and may or may not include dividends. The payments occur on regularly scheduled dates over a specified period of time. In the following subsections we illustrate specific examples of three main types of equity swaps.

An Equity Swap with the Equity Return Paid Against a Fixed Rate

The first type of equity swap we examine is one in which one party pays the return on a stock or stock index and the other pays a fixed rate. Consider the following example:

On December 15 of a given year, Dynamic Money Management enters into a swap to pay a fixed rate of 5% with payment terms

³See Gastineau [1993].

⁴At about that time, Merton [1990] also suggested another reason for the use of equity swaps, which is to achieve cross-border diversification.

⁵In contrast, the total notional principal and market value of interest rate swaps is estimated at \$79 trillion and \$3.9 trillion, respectively, as of December 2002.

*of 30/360 and receive the return on the S&P 500 with payments to occur on March 15, June 15, September 15, and December 15 for one year. Payments will be calculated on a notional principal of \$20 million. The counterparty is the swaps dealer Total Swaps, Inc. The S&P 500 is at 1105.15 on the day the swap is initiated.*⁶

The time line of this swap is illustrated in Exhibit 2. Note that, as in all swaps, no money changes hands up front. This is because the fixed rate of 5% has been calculated so that the present value of the stream of equity payments is the same as the present value of the stream of fixed payments. This is the process of pricing the swap, which we shall cover in a later section. The 5% rate is an interest rate and is therefore adjusted by an appropriate factor, in this case, 90/360. Other adjustments, such as using the actual day count divided by 360 or 365, are possible if the parties agree.

Note that the equity payments are based on the index rate of return and, therefore, are not determined until the settlement date on which the payment is due. This feature contrasts with that of an interest rate swap, in which the variable (floating) payment is set at the beginning of the settlement period and the payment is made at the end of the period. In that case, the two parties always know the upcoming variable payment, although they would not know any variable payments beyond the upcoming one.

Obviously no one knows what the equity payments will be on an equity swap. Exhibit 3 illustrates a hypothetical set of payments with illustrations showing how the payments are calculated. On each settlement date, the return on the S&P 500 is calculated and applied to the \$20 million notional principal to determine the payment to be received. The fixed payment is based on 5% and an adjustment factor of 90/360 applied to \$20 million. As is customary in swaps, the difference in the two payments is determined and only the net, as indicated in the last column, is paid.

Note here an important distinction between equity swaps and interest rate or currency swaps. For the period of March 15 to June 15 and also from June 15 to September 15, the S&P 500 decreased. Thus, the equity payment that this party is due to receive is actually negative. Hence, this party must pay the equity return as well as the fixed return. In interest rate and currency swaps, all payments are based on interest rates, which are always positive so all such payments are positive. But this feature is not a defect of any sort in equity swaps. It simply reflects the fact that equity returns can be negative. A party committing to receive the equity

⁶ It is possible and quite common that the S&P 500 total return index, which includes the reinvestment of dividends on a daily basis, would be used. For illustrative purposes, we use the traditional S&P 500 index.

return must be prepared to accept negative returns and, therefore, make payments based on the equity performance.

An Equity Swap with the Equity Return paid against a Floating Rate

In the previous swap, the party received the equity return and paid a fixed rate. Now let us change the fixed rate to a floating rate. We shall alter the previous example just slightly, as follows:

On December 15 of a given year, Dynamic Money Management enters into a swap to pay a floating rate of 90-day LIBOR with payment terms of 30/360 and receive the return on the S&P 500 with payments to occur on March 15, June 15, September 15, and December 15 for one year. Payments will be calculated on a notional principal of \$20 million. The counterparty is the swaps dealer Total Swaps, Inc. The S&P 500 is at 1105.15 and 90-day LIBOR is 4.75% on the day the swap is initiated.

LIBOR, the London Interbank Offer Rate, is the Eurodollar rate and varies as banks borrow and lend with each other. LIBOR is the most common interest rate used in dollar-based derivative transactions. Exhibit 4 illustrates this equity swap, which differs only slightly in structure from that of the previous example. Note that we would know the current 90-day LIBOR on December 15, the date the swap is initiated, and this rate of 4.75% determines the first floating payment.

Exhibit 5 shows a set of hypothetical payments on this swap for the same set of values of the S&P 500 on the dates key dates. We have added hypothetical values for LIBOR on the settlement dates as well. Note that the LIBOR payment at the beginning of the settlement period determines the interest payment at the end of the period. This is, of course, the way that floating interest payments are determined in plain vanilla interest rate swaps as well as standard floating rate loans.

An Equity Swap with the Equity Return Paid Against Another Equity Return

Another common variation of the equity swap involves both sides paying the return on a stock or stock index. In the example below, we change the interest payment to the return on another stock index.

On December 15 of a given year Dynamic Money Management enters into a swap to pay the return on the NASDAQ Composite index and receive the return on the S&P 500 with payments to occur on March 15, June 15, September 15, and December 15 for one year. Payments will be calculated on a notional principal of \$20 million. The counterparty is the swaps dealer Total Swaps, Inc. The S&P 500 is at 1105.15 and NASDAQ is at 1705.51.

Exhibit 6 shows the structure of this swap and Exhibit 7 shows a set of hypothetical payments based on an assumed set of values for the S&P 500 and NASDAQ. Note in this case that since Dynamic Money Management is paying the NASDAQ return, it can actually end up receiving the NASDAQ return. This occurs on June 15. The NASDAQ index lost 3.5144% from March 15 to June 15, resulting in a negative payment of \$702,880. Since Dynamic is obligated pay the NASDAQ return, which is negative, Dynamic would receive the NASDAQ payment. Dynamic receives the S&P 500 payment, which is a negative \$800,020, so the net effect is that Dynamic pays \$97,140. On September 15, however, the NASDAQ payment is more negative than the S&P 500 payment, so Dynamic ends up receiving a net of \$407,100.

Other Variations of Equity Swaps

Another familiar variation of the third swap presented above is when a set of payments is based on a foreign stock index. In that case, there are even two further varieties. One is for the payment based on the foreign stock index to be made in the foreign currency. Hence, there would be two notional principals determined at the start, which would differ according to the exchange rate at the start of the transaction. This type of transaction combines elements of both equity and currency swaps and is similar to investing directly in foreign stock. The other variation is for the foreign stock index return to be made without regard to any exchange rate adjustment. In other words, the foreign stock return would be paid based on the domestic notional principal. This type of swap would be similar to a currency-hedged position in foreign stock.

Another common variation of an equity swap involves a variable notional principal. Consider the following example. Let the underlying stock index be at 100 when the swap is initiated. Let the swap life be two years and involve annual payments. The notional principal will be \$10 million. Suppose the stock index goes up to 115 at the end of the first year and then is at 138 a year later. Look at this problem from the perspective of the party receiving the equity payments.⁷ The returns are obviously

$$\left(\frac{115}{100}\right) - 1 = 0.15$$

$$\left(\frac{138}{115}\right) - 1 = 0.20$$

For a fixed notional principal, the equity payments are

$$0.15(\$10,000,000) = \$1,500,000$$

$$0.20(\$10,000,000) = \$2,000,000.$$

⁷Obviously there is a payment on the other side of the swap, but we can ignore this to illustrate the essential point.

Suppose that instead of doing the swap, the party had invested \$10 million directly in the stock index. At the end of one year, the \$10 million would clearly be worth 15 % more or \$11,500,000, an unrealized capital gain of \$1,500,000, which is the same as the swap payment. Assume the investor does not liquidate the stock. Then one year later, it will be worth $\$11,500,000(1.20) = \$13,800,000$, an unrealized capital gain of \$2,300,000. Note that the unrealized capital gain the first period matches the first swap payment, but the unrealized capital gain the second period is higher than the swap payment. This result occurs because of the assumption regarding reinvestment of the funds. In the swap, there is no reinvestment assumption. The swap payments would match the actual stock gains if the stock gains were realized. In other words, assume that at the end of the first year, the stock investor withdrew the gain of \$1,500,000 and left \$10 million invested. At the end of the second year, the gain of \$2 million is withdrawn.

In some equity swaps, the parties attempt to make the transaction more like a reinvested position in stock by specifying that the notional principal adjusts by the stock return. If that is done in this case, the notional principal the second year would be adjusted upward to $\$10,000,000(1.15) = \$11,500,000$. Then the swap payment at the end of the second year would be

$$(0.20)\$11,500,000 = \$2,300,000.$$

This appears to match the assumption that the stock investor reinvested the capital gains, but this adjustment actually overcompensates. The swap party receives cash of \$1.5 million in one year and \$2.3 million one year later. The buy-and-hold stock investor does not receive any cash until liquidating the entire position in two years at which time he sells the stock for \$3.8 million, a capital gain of \$3.8 million. The swap party also earned capital gains of \$3.8 million but earned them in the form of \$1.5 million in one year and \$2.3 million one year later. At the very least, the swap party could have invested the \$1.5 million earned after the first year in risk-free bonds and ended up with more than \$3.8 million a year later. Hence, the notional principal adjustment on the swap overcompensates for the reinvestment of capital gains. In short, when the notional principal is indexed, the swap party receives the best of both worlds: realized capital gains, which can be withdrawn and spent, that are then assumed to be reinvested. This is not the same as realizing capital gains and then reinvesting them, in which case the party cannot spend them.

This example also illustrates another aspect of equity swaps that makes them difficult to compare to direct equity investing. Swaps generate only the returns on stock. They require no initial investment. The party receiving the equity return simply promises to make a set of payments. Thus, equity swaps can be viewed as fully leveraged equity. We shall be forced to

address this point in a later section when we compare the performance of equity swaps with that of direct equity investing.

For further discussion of variations of equity swaps, see Allen and Showers [1991]. For discussion of the tax treatment of equity swaps, see Laatsch [2000].

II. APPLICATIONS OF EQUITY SWAPS

Equity swaps can be used in almost any scenario in which a party might wish to buy or sell stock. In this section we illustrate five examples.

Diversifying a Concentrated Portfolio

Consider the following hypothetical example. Atlantic Coast Technical University is a small technology-based school that has recently received a donation of 100,000 shares of VAMA Technology, a company founded by a graduate of the school. This donation is worth \$6 million, which doubles the school's endowment. The manager of the endowment is concerned now that the portfolio is poorly diversified. Of course, the manager can always sell the stock but the donor might be offended. An equity swap will solve the problem. Suppose the manager enters into a swap to pay the return on VAMA to a swap dealer, which pays the endowment the return on a diversified stock portfolio. We illustrate this situation in Exhibit 8, where the diversified portfolio return is the Russell 3000.

Achieving International Diversification

Consider a hypothetical money management firm called American Assets, which has always eschewed international investing. It has finally become convinced that it should invest internationally, but it is hesitant to do so directly. A swap dealer has convinced it that an equity swap will be the simplest and lowest cost way to invest internationally and will not entail any direct holdings of foreign stocks. American Assets decides to put 25% or \$200 million of its \$800 million portfolio in foreign stock through an equity swap.⁸ Specifically, it enters into a swap to pay the dealer the return on the S&P 500 on \$200 million and receive the return on the Morgan Stanley EAFE Index, which it deems an acceptable proxy for the return on international stocks. The structure of the swap is illustrated in Exhibit 9. American Assets has achieved international diversification without selling any of its U. S. stock.⁹

⁸Perhaps the first thing American Assets should do after entering into this swap is to change its name.

⁹Bodie and Merton [2002] mention another related advantage of using an equity swap to achieve foreign diversification, which is that it avoids the problem where some countries prohibit foreign investors from owning more than a limited percentage of a company.

Executing an Asset Allocation Decision

Quest Assets is a money management firm with a \$150 million portfolio that is currently invested 75% in domestic and international stock and 25% in U. S. government and corporate bonds. Within each asset class, the asset allocation is as follows:

Stock (75%):

60% domestic large-cap stock

40% foreign large-cap stock

Bonds (25%):

80% U. S. government long-term bonds

20% domestic corporate intermediate-term bonds

Quest would like to change the allocation to 90% stock and 10% bonds, while leaving the allocations within the various asset classes unchanged. Assume that it accepts the S&P 500 as a good proxy for the domestic stock, the EAFE Index as a good proxy for the foreign stock, the Lehman Brothers Long-Term Government Bond Index as a good proxy for the government bonds, and the Merrill Lynch 1-10 Year Corporate Bond Index as a good proxy for the corporate bonds.

Since the \$150 million portfolio is allocated 75% to stock and 25% to bonds, the value of the stock component is \$112.5 million and the value of the bond component is \$37.5 million. If the allocation were at the desired levels of 90% stock and 10% bonds, the stock value would be \$135 million and the bond value would be \$15 million. Thus, the portfolio needs an additional investment of $\$135 - \$112.50 = \$22.5$ million in stock and a reduction of the investment in bonds of $\$37.5 - \$15 = \$22.5$ million. Quest can enter into a swap to earn the return on \$22.5 million of stock and pay the return on \$22.5 million in bonds, but there are several technical issues that must be addressed.

As noted, it must enter into an equity swap to receive the return on \$22.5 million of stock, but the swap must be structured carefully. Quest wants to maintain its stock allocation at 60% domestic large-cap stock and 40% foreign large-cap stock. Thus, the equity swap to receive \$22.5 million must actually be split into two components, one to receive $(0.60)\$22.5 \text{ million} = \13.5 million on domestic large-cap stock. This payment will be tied to the S&P 500. The other part of the equity swap payment should be the return on $(0.40)\$22.5 = \9 million on foreign large-cap stock. This payment will be tied to the EAFE.

Quest must also pay the return on certain fixed income indices. Technically we have not addressed this type of swap. It is neither an interest rate swap nor an equity swap, but it is more like the latter. It is, in fact, a fixed income swap, which has payments based on a bond or bond

index. Quest must pay a fixed income return on \$22.5 million and this return must be divided as 80% U. S. government long-term bonds and 20% domestic corporate intermediate term bonds. Thus, Quest will pay the return on $(0.80)\$22.5 = \18 million based on the Lehman Brothers government bond index and the return on $(0.20)\$22.5$ million = \$4.5 million based on the Merrill Lynch corporate bond index. Thus, Quest enters into a swap with a dealer on notional principal of \$22.5 million to

- Receive the S&P 500 return on \$13.5 million
- Receive the EAFE return on \$9 million
- Pay the Lehman Brothers return on \$18 million
- Pay the Merrill Lynch return on \$4.5 million

This transaction is illustrated in Exhibit 10.

If Quest wanted to change the allocations within each asset class, it could easily do so with this swap. Consider, for example, the allocation to equity. We have already determined that Quest wants to increase the allocation from 75% stock to 90% stock. This adjustment will be implemented with a swap to receive payments of the return on equity indices on \$22.5 million notional principal and to make payments of the return on fixed income indices on \$22.5 million notional principal. The original allocation of 60% domestic stock and 40% foreign stock means that the market value of the domestic stock was \$90 million and the market value of the foreign stock was \$60 million. After the swap is executed, the portfolio will be equivalent to \$150 million allocated 90% to stock and 10% to bonds. Thus, the portfolio is a synthetic version of one with $(0.9)\$150$ million = \$135 million allocated to stock and $(0.1)\$150$ million = \$15 million to bonds. The stock allocation is equivalent to $(0.60)\$135$ million = \$81 million in domestic stock and $(0.40)\$135$ million = \$54 million to foreign stock.

Now suppose Quest wants to change the allocation to 75% domestic stock and 25% foreign stock. Thus, it wants its \$135 million allocation to stock to be equivalent to $(0.75)\$135$ million = \$101.25 million to domestic stock and $(0.25)\$150 = \33.75 to foreign stock. Therefore, it needs to increase its domestic stock allocation from \$81 million to \$101.25 million, an increase of \$20.25 million, and reduce its foreign stock allocation from \$54 million to \$33.75 million, a decrease of \$20.25 million. It can accomplish this by entering into another swap to

- Receive the S&P 500 return on \$20.25 million
- Pay the EAFE return on \$20.5 million.

Now let us look at all of the transactions:

- Receive the S&P 500 return on \$13.5 million
- Receive the EAFE return on \$9 million

Pay the Lehman Brothers return on \$18 million

Pay the Merrill Lynch return on \$4.5 million

Receive the S&P 500 return on \$20.25 million

Pay the EAFE return on \$20.25 million

These transactions net out to the following:

Receive the S&P 500 return on \$33.75 million

Pay the EAFE return on \$11.25 million

Pay the Lehman Brothers return on \$18 million

Pay the Merrill Lynch return on \$4.5 million

Thus, the overall swap would have a notional principal of \$33.75 million and would involve a receipt of the S&P 500 return and payments based on the EAFE, Lehman, and Merrill Lynch indices. If Quest wanted to change the bond allocation, it could do so in a similar manner and this might further change the notional principal.

Creating an Index Fund

Total Indexers is a hypothetical new asset management company that creates and manages index funds. It would like to organize a new \$500 million fund based on the Amex Composite Index. It can do this by entering into a swap to receive the return on the AMEX Index, but it must also take into account the fact that the swap return is actually a leveraged position in the index. As we saw above, a swap will obligate a party to pay either a fixed return, a floating return, on the return on another index. In the first case, the swap is like borrowing at a fixed rate and investing in stock. In the second case, the swap is like borrowing at a floating rate and investing in stock. In the third case, the swap is like shorting one stock index and using the proceeds to invest in the other. To create an index fund using a swap would involve raising some money, entering into the swap and then using the money to undo or offset the payments owed on the swap. In this case, we shall assume that Total Indexers takes its \$500 million of investment capital, purchases fixed rate bonds and enters into a swap to pay a fixed rate. The fixed rate bonds should mature at the termination date of the swap to avoid capital gains and losses from the sale of the bonds. The structure of this arrangement is shown in Exhibit 11.

Hedging an Equity Position by a Corporate Executive

Corporate executives typically have a significant investment in the stock of their employers. Given that so much of their compensation and portfolio wealth is tied to the performance of a single company, their investments are poorly diversified. A few executives have used swaps to effectively sell some of the exposure in their stock, while maintaining their position in the stock. Consider the following hypothetical example.

David R. Keller is CEO of Keller Technology, a small but publicly traded software company that he founded. Keller's annual cash compensation is modest at only \$300,000 per year, but he holds four million shares of stock, currently worth \$9 a share. These holdings are just sufficient for Keller to maintain voting control of the firm so he would be reluctant to sell his shares. The highly concentrated nature of his wealth, however, has him uneasy. Keller would like to synthetically reduce his exposure to the stock by entering into an equity swap. He decides to enter into such a swap on one million of his four million shares. Thus, the notional principal would be \$9 million. He agrees to pay the return on the one million shares and receive the return on the Wilshire 5000. The structure of the arrangement is shown in Exhibit 12.

There are a number of important and somewhat controversial implications of this transaction. For one, it is considered an insider sale and, thus, may send a signal to the market that is not as obvious as direct selling of stock. When examining the executive's position in an annual report, 10-K, or proxy statement, it can be difficult to learn whether the executive has sold off some exposure through swaps. Other issues and concerns are discussed in Norris [1994] and Bolster, Chance, and Rich [1998].

We have examined applications of equity swaps. Let us now consider the advantages and disadvantages of equity swaps.

III. ADVANTAGES AND DISADVANTAGES OF EQUITY SWAPS

Equity swaps have several significant advantages. For one, being an over-the-counter derivatives transaction, they have the attractive feature of being customizable for a particular user's situation. Investors may have specific time horizons, portfolio compositions, or other terms and conditions that are not matched by exchange-listed derivatives.

Equity swaps are private transactions that are not directly reportable to any regulatory authority. Thus, in contrast to exchange-listed derivatives, they are not created in a public forum that can send signals to investors of the intention and position of a particular investor. As noted in previous sections, however, when insiders use equity swaps to substitute for the sale of shares, the transactions must be reported as insider sales. Also, if an executive has sold shares synthetically through an equity swap, this information must be reported to the shareholders in some form. These rules regarding insider transactions always apply and equity swaps are not exempted. In general, specific equity swaps need not be reported, but new accounting and disclosure requirements mean that equity swaps would be indirectly reported when derivatives positions are marked to market and recorded on financial statements and disclosures about risk management are made. In general, however, there are no requirements that specific equity swaps have to be publicly reported.

Like other over-the-counter derivatives, equity swaps are essentially unregulated. Thus, there are no governmental restrictions on whether an entity can engage in an equity swap. Of course, there may be firm-specific prohibitions.

As with nearly all derivatives, equity swaps provide an excellent means of capturing the performance of the underlying asset at low transaction costs. In fact, low transaction costs are one of the primary advantages of derivatives in general. These low transaction costs are reflective of the relative ease of establishing and maintaining a position in the transaction, as well as the low cost to the dealer to hedge the position. In addition, equity swaps incur no custodial costs that would ordinarily be associated with the holding of stock and no withholding taxes on positions related to foreign stock indices. There are, however, some additional transaction costs in the form of legal and documentation costs, but these are relatively small.

Equity swaps can be customized to any specific stock, portfolio, or index and are an excellent means of investing in markets that would otherwise not be very accessible. Foreign markets, in particular, can be more easily accessed by a derivatives dealer, which can then offer its advantage to investors through equity swaps.¹⁰ In this regard, Merton [1990] notes that equity swaps can enable one to invest freely in foreign markets without worrying about restrictions that might prohibit foreigners from owning more than a certain percentage of a given stock.¹¹

Merton [1990] also notes that equity swaps have the advantage of not being particularly susceptible to manipulation. While this would tend to be true for any index-based derivative, equity swaps have an additional advantage that would apply even if the equity swap were based on an individual stock. As Merton notes, because equity swap payments are based on the rate of return, any attempt to manipulate the stock price at the end of a period would have the opposite effect that would carry over into the next period.

Equity swaps are not without disadvantages as well. In particular, the cash flows from equity swaps can be significant. Hence, there can be large cash outflows that must be funded. When such outflows are not offset by gains on stock positions, the gains on the stock may have to be liquidated to fund cash outflows. This can offset one of the primary attractions of equity swaps, which is to avoid engaging in transactions in the actual securities.

Being derivative contracts, equity swaps must also have a termination date. Therefore, while they can be used to replicate a position in equity, they are not open-ended. They must be

¹⁰See the aforementioned papers by Merton [1990] and Bodie and Merton [2002].

¹¹For example, suppose the laws of a country prohibit a foreign investor from owning more than a given percentage of the shares of any firm domiciled in that country. Using an equity swap, however, the investor can get around that restriction. This can be done if the dealer firm has a foreign subsidiary that owns the foreign stock that would then be used to hedge the dealer's position as it pays out returns on the stock to the investor.

re-established from time to time. This imposes an additional cost and, for certain equity swaps, can lead to less favorable terms than were already in place.

As noted above, equity swaps can be tailored to a specific portfolio, but the more customized the swap, the more costly in terms of the bid-ask spread that a dealer would impose. A user of an equity swap might find it too costly to engage in a transaction customized to a specific, non-standard portfolio. Thus, the swap might be better structured to a commonly used stock index. In that case, however, tracking error will occur. For example, we saw in the previous section cases of portfolios that are synthetically sold using equity swaps tied to an index. If the portfolio does not match the index, the returns on the equity swap will not be matched by the returns on the portfolio being held.

Equity swaps are also subject to credit risk that would not otherwise exist when investing in equity directly. Specifically, the user of an equity swap assumes the risk that the dealer will default.

We now turn to a determination of how the financial terms of an equity swap are determined and how the market value is assessed.

IV. PRICING AND VALUATION OF AN EQUITY SWAP

In a previous section, we illustrated common variations of equity swaps. In the first case, the party received the return on an equity index and paid a fixed interest rate. In the second case, the party received the return on an equity index and paid a floating interest rate. In the third case, the party received the return on one equity index and paid the return on another. When these transactions are initiated, there is no exchange of cash up front. Hence, the transactions have a market value of zero at the start. In the first example, the fixed rate is determined so that the present value of the equity payments equals the present value of the fixed payments, thereby giving the transaction zero market value. In the second transaction, there is no fixed rate or set of terms. Hence, the transaction must automatically have a zero market value at the start. In the third transaction, there is also no fixed rate or set of terms. Hence, that transaction must also have zero market value at the start.

For the first transaction we need to determine the appropriate fixed rate. This process is called *pricing the swap*, although the term is a bit misleading. No “price” is determined. The fixed rate is set such that the market value is zero. During the life of the swap, market conditions change and the swap’s market value will deviate from zero. The determination of the value of the swap is called, quite naturally, *valuing the swap*. In this section, we illustrate how pricing and valuation of an equity swap is done. For the second and third swaps, there is no initial pricing,

but we must verify that the present values of both sets of payments are equal at the start and we must identify how to value the swaps during their lives.

We shall start by setting up the conditions and assumptions. We assume that the swap is initiated at time 0 and has a set of $n+1$ payments that occur at times $t, t+1, t+2, \dots, t+n$. We choose an arbitrary time point j , where $0 \leq j \leq t$. Let $S(j)$ be the price of the stock or level of the index at time j . The fixed rate that we shall solve for is R , which will be a periodic rate. Thus, it is not an annualized interest rate that must be multiplied by a factor such as $90/360$ to determine the actual payment. In practice, this periodic rate would be annualized by multiplying by $360/90$ or some other appropriate time factor. We shall do this in the examples shown in the appendix. Let $B(j,k)$ be the price of a zero coupon bond at time j that pays \$1 at its maturity at time $k, k \geq j$. The value of the swap at time j is denoted as $V(j;0,t+n)$, which is interpreted as the value at time j given that the swap was initiated at time 0 and has a final payment at time $t+n$. We shall assume a notional principal of \$1.00, which we can do because all results are linearly related to the notional principal. The actual payments and market value can be obtained by multiplying by the actual notional principal.

The procedure will be that we determine the value of the swap at an arbitrary time j for a given fixed rate R . To price the swap, we simply let $j = 0$, set the swap value to zero, and solve for R . To determine the value of the swap, we follow the general procedure applicable to nearly all derivative contracts. We find a trading strategy that replicates the cash flows from the derivative. The market value of that trading strategy at time 0 will be the value of the derivative. Hence, we need to find a trading strategy that generates the cash flows on these equity swaps.

Pricing and Valuation of the Equity Swap to Pay a Fixed Rate and Receive the Equity Return

The cash flows on this swap to the party paying a fixed rate of R and receiving the equity return are as follows:

<u>Time t</u>	<u>Cash Flows</u>
t	$S(t)/S(0) - (1 + R)$
t+1	$S(t+1)/S(t) - (1 + R)$
t+2	$S(t+2)/S(t+1) - (1 + R)$
...	
t+n	$S(t+n)/S(t+n-1) - (1 + R)$

To replicate the payments on this swap, we shall need to engage in a series of transactions at time $j, 0 \leq j \leq t$. Consider the first payment, $S(t)/S(0) - (1 + R)$. It can be replicated by doing the following:

At j:

Invest the amount $S(j)/S(0)$ in the stock

Borrow $(1 + R)B(j,t)$

At time t, the following occurs:

The stock will be worth

$$\left(\frac{S(j)}{S(0)} \right) \left(\frac{S(t)}{S(j)} \right) = \frac{S(t)}{S(0)}.$$

Pay off bonds in the amount $1 + R$.

The total cash flow will be

$$\frac{S(t)}{S(0)} - (1 + R).$$

This amount is equivalent to the first swap payment.

To replicate the second swap payment, do the following:

At time j:

Invest $B(j,t)$ in a risk-free bond

Borrow $(1+R)B(j,t+1)$

At time t:

The bond matures and is worth \$1.

Invest the \$1 in the stock

At time t+1:

The stock is worth

$$\frac{S(t+1)}{S(t)}.$$

Pay off bonds in the amount $1 + R$.

The total cash flow will be

$$\frac{S(t+1)}{S(t)} - (1 + R).$$

This amount is equivalent to the second swap payment.

The other swap payments can be replicated in a similar manner. Stock is purchased at time j, and funds are invested in the risk-free asset to mature to a value of \$1 one period prior to the date of the cash flow we are replicating. Then that \$1 is invested in stock. Also, at time j funds are borrowed to be paid back in the amount of $1 + R$ at the time of the swap payment.

The total value of these transactions at j is

$$\begin{aligned}
& \frac{S(j)}{S(0)} - (1+R)B(j,t) + B(j,t) - (1+R)B(j,t+1) + \\
& B(j,t+1) - (1+R)B(j,t+2) + \dots + B(j,t+n-1) - (1+R)B(j,t+n) \\
& = \frac{S(j)}{S(0)} - B(j,t) - RB(j,t) + B(j,t) - B(j,t+1) - RB(j,t+1) + B(j,t+1) \\
& - B(j,t+2) - RB(j,t+2) + \dots + B(j,t+n-1) - B(j,t+n) - RB(j,t+n)
\end{aligned}$$

This result can be simplified and gives the market value of the swap at time j :

$$V(j;0,t+n) = \frac{S(j)}{S(0)} - B(j,t+n) - R \sum_{i=0}^n B(j,t+i).$$

In this example, the swap was created at time 0 and the upcoming payment is the first one. This formula is sufficiently general, however, that it is not necessary that the upcoming payment be the first. In other words, we can, without loss of generality, let time 0 simply be the last date on which a payment was made or the swap initiation date, whichever is more recent, and $n+1$ be the number of upcoming payments.

Now that we have the market value of the swap, we can easily price the swap. As noted above, we let $j = 0$. Noting that we now have $S(0)/S(0) = 1$ for the first term, we set the market value to zero:

$$1 - B(0,t+n) - R \sum_{i=0}^n B(0,t+i) = 0.$$

Solving for R , we obtain

$$R = \frac{1 - B(0,t+n)}{\sum_{i=0}^n B(0,t+i)}.$$

This extremely simple formula is familiar to users of interest rate swaps. It is, in fact, the same formula as that of the fixed rate on a plain vanilla interest rate swap. Perhaps somewhat surprisingly, the level of the stock is irrelevant in pricing the pay-fixed equity swap. Of course, the level of the stock is not irrelevant in determining the value of the swap during its life.

Once the fixed rate is obtained, a dealer would add a small premium if the dealer is receiving the fixed payment and subtract a premium if the dealer is making the fixed payment. Thus, the dealer would effectively be quoting a bid-ask spread, which would reflect the cost of hedging the transaction and other costs associated with providing the service. The hedging of the swap as done by the dealer is somewhat complex. It is easy for the dealer to determine the sensitivity of the swap to the underlying stock or index. From the valuation formula above, we

see that the dealer should hold $1/S(0)$ units of the index if paying the equity return.¹² Hedging the fixed rate component of the swap, however, requires the use of a model that incorporates movements in the term structure. Though there are a variety of such models, the choice of model is not obvious and a number of parameters must also be selected and carefully estimated. Nonetheless, most equity swap dealers make markets in interest rate swaps and do this type of hedging on a regular basis.

Pricing and Valuation of the Equity Swap to Pay a Floating Rate and Receive the Equity Return

Now consider the swap to pay a floating rate and receive the equity return. First observe that this swap can be replicated as follows:

- Pay floating, receive equity \equiv (a) a swap to pay fixed, receive equity, plus
 (b) a swap to pay floating, receive fixed

We know the value of swap (a) from the swap we did in the previous section. Note that swap (b) is a plain vanilla interest rate swap, the value of which is well-known to be

$$R \sum_{i=0}^n B(j, t+i) + B(j, t+n) - (1+r(0, t))B(j, t),$$

where $r(0, t)$ is the one-period floating rate that was observed at time 0 and determines the next payment. Adding the values of swaps (a) and (b), we obtain

$$\frac{S(j)}{S(0)} - B(j, t+n) - R \sum_{i=0}^n B(j, t+i) + R \sum_{i=0}^n B(j, t+i) + B(j, t+n) - (1+r(0, t))B(j, t).$$

Thus, the value of the pay floating, receive-equity swap is

$$V(j; 0, t+n) = \frac{S(j)}{S(0)} - (1+r(0, t))B(j, t).$$

As noted, there is no pricing of this swap, because it has no fixed rate. We need to verify, however, that this value is zero at time $j = 0$. Otherwise, an investor could earn an arbitrage profit. Setting j to 0 in the above equation, we obtain

$$V(0; 0, t+n) = \frac{S(0)}{S(0)} - (1+r(0, t))B(0, t) = 1 - (1+r(0, t))B(0, t).$$

By definition, $B(0, t) = 1/(1+r(0, t))$. Hence, the value of the swap is zero at the start.

Since the dealer would not be quoting a fixed rate, it would typically incorporate the bid-ask spread into its floating rate. Alternatively, it could build the spread into the equity payment.

¹²An alternative hedging strategy would entail engaging in the replicating strategy that was used to price the swap, taking the opposite positions to the ones described here.

Hedging the position would require the holding of $1/S(0)$ units of the stock or index if paying the equity return. Hedging the interest component of the model would require the use of a term structure model, as noted above in the previous section. The floating rate component, however, has relatively low sensitivity to changes in interest rates.¹³

Pricing and Valuation of the Equity Swap to Pay the Return on One Equity Index and Receive the Return on Another Equity Index

The cash flows on this swap can be easily replicated by going short one index and long the other. Let $S_1(j)$ be the value of one index at time j and $S_2(j)$ be the corresponding value of the other. We shall assume we are paying the return on index 2 and receiving the return on index 1.

The first cash flow will be

$$\left(\frac{S_1(t)}{S_1(0)} \right) - 1 - \left(\left(\frac{S_2(t)}{S_2(0)} \right) - 1 \right) = \left(\frac{S_1(t)}{S_1(0)} \right) - \left(\frac{S_2(t)}{S_2(0)} \right).$$

The second cash flow will be

$$\left(\frac{S_1(t+1)}{S_1(t)} \right) - 1 - \left(\left(\frac{S_2(t+1)}{S_2(t)} \right) - 1 \right) = \left(\frac{S_1(t+1)}{S_1(t)} \right) - \left(\frac{S_2(t+1)}{S_2(t)} \right).$$

This pattern continues and the final cash flow will be

$$\left(\frac{S_1(t+n)}{S_1(t+n-1)} \right) - 1 - \left(\left(\frac{S_2(t+n)}{S_2(t+n-1)} \right) - 1 \right) = \left(\frac{S_1(t+n)}{S_1(t+n-1)} \right) - \left(\frac{S_2(t+n)}{S_2(t+n-1)} \right).$$

This payment stream can be replicated by investing $S_1(j)/S_1(0)$ dollars in index 1 and selling short $S_2(j)/S_2(0)$ dollars of index 2. At time t , the position will have a market value of

$$\left(\frac{S_1(j)}{S_1(0)} \right) \left(\frac{S_1(t)}{S_1(j)} \right) - \left(\frac{S_2(j)}{S_2(0)} \right) \left(\frac{S_2(t)}{S_2(j)} \right) = \left(\frac{S_1(t)}{S_1(0)} \right) - \left(\frac{S_2(t)}{S_2(0)} \right)$$

Liquidate this position, which will produce the above value in cash and match the first cash flow from the swap. At time t , to reproduce the second cash flow, we need only sell short \$1 of index 2 and use the proceeds to buy \$1 of index 1. This will require no funds and have the following market value at time $t+1$:

$$\left(\frac{S_1(t+1)}{S_1(t)} \right) - \left(\frac{S_2(t+1)}{S_2(t)} \right).$$

We liquidate this position, which will produce the same cash flow as the swap. We then continue in this manner, shorting \$1 of stock 2 and buying \$1 of stock 1 on each swap settlement date, and then liquidate the position on the next swap settlement date.

¹³The floating rate component of the swap is like a floating rate bond, which has relatively low sensitivity to

At time j , the value of the position is the value of the stock purchased minus the value of the stock sold short:

$$V(j;0,t+n) = \frac{S_1(j)}{S_1(0)} - \frac{S_2(j)}{S_2(0)}.$$

Thus, this formulation gives us the market value at any arbitrary time j . Of course, there is no initial pricing of the stock, but we must be sure that this value is zero at time $j = 0$. Otherwise, a party could earn an arbitrage profit. Setting j to 0, the formula becomes

$$V(0;0,t+n) = \frac{S_1(0)}{S_1(0)} - \frac{S_2(0)}{S_2(0)} = 1 - 1 = 0.$$

As in the examples in the previous sections, the dealer would incorporate a bid-ask spread into the transaction by either paying less than the full return on one stock or index or receiving more than the full return on the other stock or index. Hedging the swap would involve $1/S_1(0)$ units of equity 1 and $1/S_2(0)$ of equity 2, being short the one the dealer is receiving payment on and long the one the dealer is making payment on.

Numerical examples of each of these swap pricing and valuation problems are presented in the appendix. Chance and Rich [1998] have derived the above formulas, and an alternative but more complex approach to pricing equity swaps using stochastic discount factors is in Jarrow and Turnbull [2000]. Rich [1995] and Marshall, Sorensen, and Tucker [1992] also examine the pricing of swaps without employing the arbitrage-free approach. For pricing swaps with a variable notional principal, see Kijima and Muromachi [2001].

V. EVIDENCE ON THE PERFORMANCE OF EQUITY SWAPS

As we have noted above, the equity swap involving the payment of one equity return and receipt of another is identical to shorting the first equity and going long the second. In the other two cases – equity vs. a fixed interest rate and equity vs. a floating interest rate, an equity swap is not identical to a buy and hold position in the equity. It was apparent in our derivation of the pricing and valuation formulas for the first type of swap that an equity swap is equivalent to a dynamic trading strategy that does not involve being invested in equity at all times. Since equity swaps are widely used as a proxy for investing in equity, it is of considerable interest to know how close equity swaps are to a buy-and-hold equity strategy. We explore this issue in this section.

First, we must recognize that we cannot directly compare the performance of an equity swap with a buy-and-hold position in equity. An equity swap is leveraged. There is no initial outlay; hence, it is not comparable to a strategy involving the commitment of capital to an equity

movements in interest rates.

position. By paying an interest rate, whether fixed or floating, an equity swap appears to be similar to a leveraged position in equity. Thus, we should compare an equity swap with a position in equity financed by borrowing at either a fixed or floating rate. We will do this and calculate the difference in returns from the equity and debt portions of the positions. Though we shall call this difference a return, it is technically based on a zero net investment of capital.

The most commonly used interest rate in dollar-denominated swaps is LIBOR. Data on one-, three- and six-month LIBOR are available on the Federal Reserve's web site <http://www.federalreserve.gov/releases/h15/data.htm> for every business day starting in 1971. This data, along with returns on an appropriate stock index, are sufficient for constructing a hypothetical time series of the performance of six-month equity swaps with three-month settlements for every day since 1971. It is tempting to use all of the data, but that would entail the use of mostly overlapping observations. Thus, we elect to use end-of-quarter data. Starting with the end of March in 1971, we construct a hypothetical six-month equity swap that would have its first payment one quarter later and its second and last payment one quarter after that. On the day the swap is constructed we solve for the fixed rate. Then three months later, we calculate the return on the swap for the first three months. Three months after that, we calculate the return on the swap for the second and final three months. Then on that date, we construct a new equity swap. This process is repeated until the final observation, which was on June 30, 2000, and gives a total of 117 observations. We construct two types of equity swaps and two types of leveraged equity transactions. The first type of each involves a fixed interest rate and the second involves a floating interest rate. We explain these transactions here.¹⁴

We assume that the equity pays no dividends. First, let the swap be constructed at time 0. Three months later is time t and three months after that is time $t+1$. We examine four strategies. The first two do not involve equity swaps but are comparable strategies and serve as a benchmark.

Strategy I: Fixed-Rate Leveraged Equity

We assume that the investor borrows funds at a fixed rate of six-month LIBOR and uses the proceeds to invest in the equity. At the end of the first three months, the return is as follows:

$$r_I(0,3) = \frac{S(3)}{S(0)} - \frac{B(3,6)}{B(0,6)}.$$

The first term on the right-hand side is the value at month three of a \$1 initial investment in the stock. The second term is the return on the debt issued at time 0. Recall that we issued a six-

¹⁴In these strategies, we calculate interest based on 360 days in a year and use the exact day count rather than assume 30 days per month.

month zero coupon bond in the amount of \$1 to generate the funds to buy the stock. To determine the value of this debt in three months, we discount the maturity value, $1/B(0,6)$, using the three-month discount factor, $B(3,6)$. Thus, $B(3,6)/B(0,6)$ is the value of the debt in three months.¹⁵

The return over the second three months will be

$$r_I(3,6) = \frac{S(6)}{S(3)} - \frac{1}{B(3,6)}.$$

The first term on the right-hand side is the value of the stock if \$1 were invested in it at month three. The second term is the return on the bond over months three to six. Note how this value arises. The face value of the debt due at month 6 is $1/B(0,6)$. The market value of this debt at month three, as described above, was $B(3,6)/B(0,6)$. The return on this bond from month three to month 6 is, therefore, $(1/B(0,6))/(B(3,6)/B(0,6)) = 1/B(3,6)$.

Strategy II: Floating-Rate Leveraged Equity

In this strategy we assume that at month 0, we issue a one-period floating rate bond. This bond will have a present value of \$1 and a face value of $1/B(0,3)$. At month three, the value of the position will be

$$r_{II}(0,3) = \frac{S(3)}{S(0)} - \frac{1}{B(0,3)}.$$

The first term is obviously the value of the \$1 invested in stock. The second term is the face value of the maturing floating-rate debt. We liquidate the position and reestablish it by borrowing \$1 of debt, which will have a face value of $1/B(3,6)$, and invest the money in stock. The value of the position three periods later is

$$r_{II}(3,6) = \frac{S(6)}{S(3)} - \frac{1}{B(3,6)}.$$

The first term is the value of the \$1 invested in the stock. The second term is the return on \$1 of debt over months three to six.

Strategies I and II provide benchmarks for examining the performance of equity swaps. We now turn to two equity swap strategies.

Strategy III: Fixed-Rate Equity Swap

Here we enter into a two-period equity swap, promising to pay a fixed rate of R . To determine the performance of the equity swap, we use the formula for the market value of the swap at the first settlement period. We assume the swap payment has not been made, so it is

¹⁵It is important to note that we are not refinancing the debt at the new one-period rate. We are simply holding on to

included in the market value calculation, which also includes the value of the remaining payment. Using the formula we derived for the market value of the fixed-rate equity swap, the market value of the swap is

$$r_{III}(0,3) = \frac{S(3)}{S(0)} - B(3,6) - R(1 + B(3,6)).$$

The return on the swap over the second three periods is

$$r_{III}(3,6) = \frac{S(6)}{S(3)} - 1 - R.$$

This formula is easily obtained from the valuation formula we derived for this swap. The term -1 is $-B(6,6)$ and the term $-R$ is multiplied by an implicit discount factor of 1.¹⁶

Strategy IV: Floating-Rate Equity Swap

Here we make floating-rate payments on the equity swap. The value of the swap over the first period is

$$r_{IV}(0,3) = \frac{S(3)}{S(0)} - \frac{1}{B(0,3)}.$$

This result is obtained from the valuation formula we derived for this type of swap. We would normally have $(1 + r(0,3))$ times the discount factor $B(6,6)$, which is 1.0. Thus, we have $1 + r(0,3)$, which is $1/B(0,3)$.

For the second three months, the return is

$$r_{IV}(3,6) = \frac{S(6)}{S(3)} - \frac{1}{B(3,6)}.$$

This formula is also obtained directly from the valuation formula in the same manner as the return over the first three months.

Comparisons of the Returns

Before examining the empirical evidence, it is useful to look at how these returns should compare. Simple algebra reveals a number of interesting findings.

For the first three months, the return from leveraged equity at a fixed rate exceeds the return from leveraged equity at a floating rate if

$$\frac{B(0,6)}{B(0,3)} > B(3,6).$$

This statement is equivalent to the simple statement that the forward price exceeds the realized future spot price. If this is the case, then it is better to have borrowed at a fixed rate, but of

the same position.

¹⁶Technically, this interpretation is very simple: the swap will pay $S(6)/S(3) - 1 - R$ in an instant.

course, the best choice ex post would not be known ex ante. For the second three months, the return from leveraged equity at a fixed rate is the same as the return from leveraged equity at a floating rate.

For the first three months, the return from an equity swap at a fixed rate exceeds the return from an equity swap at a floating rate if

$$\frac{B(0,6)}{B(0,3)} > B(3,6).$$

This is the same criterion we obtained with leveraged equity. For the second three months, the return from the equity swap with a fixed rate exceeds the return from the equity swap with a floating rate if

$$R < \frac{1 - B(3,6)}{B(3,6)}.$$

The left-hand side is obviously the swap fixed rate. The right-hand side is the realized rate on a one-period bond in one period. Thus, if the swap fixed rate is less than the realized rate on a one-period bond the second period, the equity swap with a fixed rate is better.

For the first three months, the return on leveraged equity with a fixed rate exceeds the return on an equity swap with a fixed rate if

$$\frac{B(0,6)}{B(0,3)} > B(3,6).$$

This is the same criterion we have seen twice already. The left-hand side is the forward price. The right-hand side is the realized spot price in three months. For the second three months, leveraged equity with a fixed rate exceeds the return on the fixed-rate swap if

$$R > \frac{1 - B(3,6)}{B(3,6)}.$$

This is the same criterion we obtained when comparing a fixed-rate equity swap with a floating-rate equity swap, but the sign is reversed. So, leveraged equity is preferred if the swap fixed rate exceeds the return from a one-period bond.

For leveraged equity with a floating rate, the performance is identical to that of the floating-rate equity swap for both periods.

Empirical Results

Using the above formulas, the returns for the first and second three months of a series of six-month swaps and leveraged equity transactions were computed over the period of March 1971 to June 2000. As a first glance, Exhibit 13 illustrates the time series of returns from the leveraged equity transactions. Panel (a) is the case in which equity is financed with a fixed rate loan. Panel

(b) is the case in which equity is financed with a floating rate loan. Panel (c) is the difference between the two cases. Exhibit 14 shows the corresponding cases for equity swaps. Exhibit 15 shows the difference between the fixed-rate leveraged equity and a fixed-rate equity swap. Of course, as noted, floating-rate leveraged equity and a floating-rate equity swap give the same result, so the difference is not pictured.

Summary statistics for these results are presented in Exhibit 16. First observe the mean quarterly returns. For the fixed-rate leveraged equity, the average is 0.0058. For floating-rate leveraged equity (and the floating-rate equity swap), the average is about 0.0061. For a fixed-rate equity swap, the average is 0.0056. Thus, the average returns are within five basis points of each other. The standard deviations are, not surprisingly, very close at around 0.079 to 0.081. The floating-rate leveraged equity and swap are the riskiest strategies and the fixed-rate equity swap is the least risky, but again, the differences are extremely small. The maximum and minimum returns are also very similar, ranging from around 0.195 to -0.296 . The number of positive returns is at least 59% of the total. A z-test for the hypothesis that the mean is different from zero reveals that none of the means are statistically different from zero.

Of more interest, however, are the results for the differences. In comparing fixed-rate leveraged equity with floating-rate leveraged equity (I – II), we see that the average difference is negative but not statistically different from zero. Nonetheless, a negative difference was observed in 89 of 117 cases or 76% of the time. The difference between fixed-rate leveraged equity and the fixed-rate equity swap is positive (I – III), meaning that the former is greater, but the difference is not statistically significant. Finally, the average difference between a fixed-rate equity swap and a floating-rate equity swap (III – IV) is negative, but not statistically significant. In both of these cases (fixed-rate leveraged equity versus fixed-rate equity swap and fixed-rate equity swap versus floating-rate equity swap), the number of positive differences and the number of negative differences are nearly equal.

Thus, while a fixed-rate equity swap does not perfectly replicate a fixed-rate leveraged equity position, it comes extremely close. On average the difference is about two basis points per three-month period, though this difference is not statistically significant. In about half the cases, the swap does better. In about half the cases, leveraged equity does better. The difference between a fixed-rate equity swap and a floating-rate equity swap is about 4.5 basis points per three month period, with the floating-rate swap doing better on average. This difference, however, is quite variable and not statistically significant. About half the time, the difference is positive and about half the time the difference is negative. Of course, floating-rate leveraged equity is the same as a floating-rate swap.

Since the formulas for the first and second quarter returns differ, it may be the case that our conclusions would be altered by focusing on the first and second returns separately. We conducted a similar analysis in this manner and our conclusions are essentially the same. We do find, however, that the second return tends to be negative for all strategies, while the first tends to be positive. Also, we find that for the first period, the fixed-rate equity swap tends to have a slightly higher return and higher risk than fixed-rate leveraged equity, but the difference is extremely small, with the average difference less than one basis point.

VI. CONCLUSIONS

Equity swaps are derivative transactions that can substitute for direct transactions in equity. They offer a number of advantages over the direct purchase and sale of stock. This article has explored strategies for using equity swaps, identified advantages and disadvantages, provided pricing and valuation formulas for different types of equity swaps, and explored the differences among equity swaps and between equity swaps relative to comparable direct equity investments. It shows that while equity swaps are not in all cases perfect substitutes for direct equity investing, they are, nonetheless, extremely close substitutes. For fixed-rate borrowing, equity swaps offer slightly lower risk, albeit with slightly lower returns. For floating-rate borrowing, however, equity swaps offer the same performance as direct equity investing.

APPENDIX

Pricing and Valuation Examples

This appendix provides examples that illustrate the pricing and valuation of equity swaps. Consider transactions initiated on January 15 of a non-leap year. A one-year swap will be established on that day and will have payments on April 15, July 15, October 15, and the following January 15. Interest rate information is given by the following term structure of LIBOR.

<u>Maturity date</u>	<u>Days to Maturity</u>	<u>Rate</u>
April 15	90	5.00 %
July 15	181	5.50 %
October 15	273	5.75%
January 15	365	5.875

We must first determine the discount factors for 90, 180, 273, and 365 days. These are as follows:

$$B(0,90) = \frac{1}{1 + .05\left(\frac{90}{360}\right)} = 0.987654$$

$$B(0,181) = \frac{1}{1 + .055\left(\frac{181}{360}\right)} = 0.973091$$

$$B(0,273) = \frac{1}{1 + .0575\left(\frac{273}{360}\right)} = 0.958218$$

$$B(0,365) = \frac{1}{1 + .05875\left(\frac{365}{360}\right)} = 0.943783$$

We shall set up fixed and floating-rate equity swaps for a specific stock index currently at 1435.52. A second stock index, currently at 4721.63, will be used for the swap involving payment of one index and receipt of another. We shall first establish the swap on January 15, identifying the fixed rate where applicable. Then on June 30, we shall determine the market value of the swap. At that point the first stock index is at 1525.81 and the second stock index is at 4985.57. The new term structure will be

<u>Maturity date</u>	<u>Days to Maturity</u>	<u>Rate</u>	<u>Discount Factor</u>
July 15	15	6.25 %	0.997403
October 15	107	6.5 %	0.981047
January 15	199	6.625 %	0.964672

Calculations for the discount factors are done in the same manner as shown above for January 15 with a different rate and days to maturity.

Pricing and Valuation of the Fixed-Rate Equity Swap

Using the formula in the text, we obtain the fixed rate on this swap as

$$R = \frac{1 - B(0, t + n)}{\sum_{i=0}^n B(0, t + i)}$$

$$= \frac{1 - 0.943783}{0.987654 + 0.973091 + 0.958218 + 0.943783} = 0.014554.$$

Thus, the fixed payment per period would be \$0.014554 per \$1 notional principal. If this rate is quoted as an annual rate, it would be $0.014554(360/90) = .058216$, with an understanding that the payment would be calculated as the annual rate times days/360.

Now, let us move forward to June 30 and determine the market value of the swap. We take the perspective of the party paying a fixed rate and receiving the equity return. The formula is

$$V = \frac{S(j)}{S(0)} - B(j, t + n) - R \sum_{i=0}^n B(j, t + i)$$

$$= \frac{1525.81}{1435.52} - 0.964672 - 0.014554(0.997403 + 0.981047 + 0.964672)$$

$$= 0.055391$$

Thus, the swap is worth \$0.055391 per \$1 notional principal.

Pricing and Valuation of the Floating-Rate Equity Swap

Since this swap does not have a fixed rate, there is no pricing required at the start. To determine the market value on June 30, we must know the floating rate that was in effect on April 15. This rate determines the payment on July 15. Assume that this rate was 5.9375, which means that the upcoming interest payment would be $.059375(91/360) = 0.015009$, based on 91 days between April 15 and July 15. Using the formula we previously obtained, we obtain:

$$V = \frac{1525.81}{1435.52} - 1.015009(0.997403)$$

$$= 0.050524.$$

Thus, the value of this swap is \$0.050524 per \$1 notional principal.

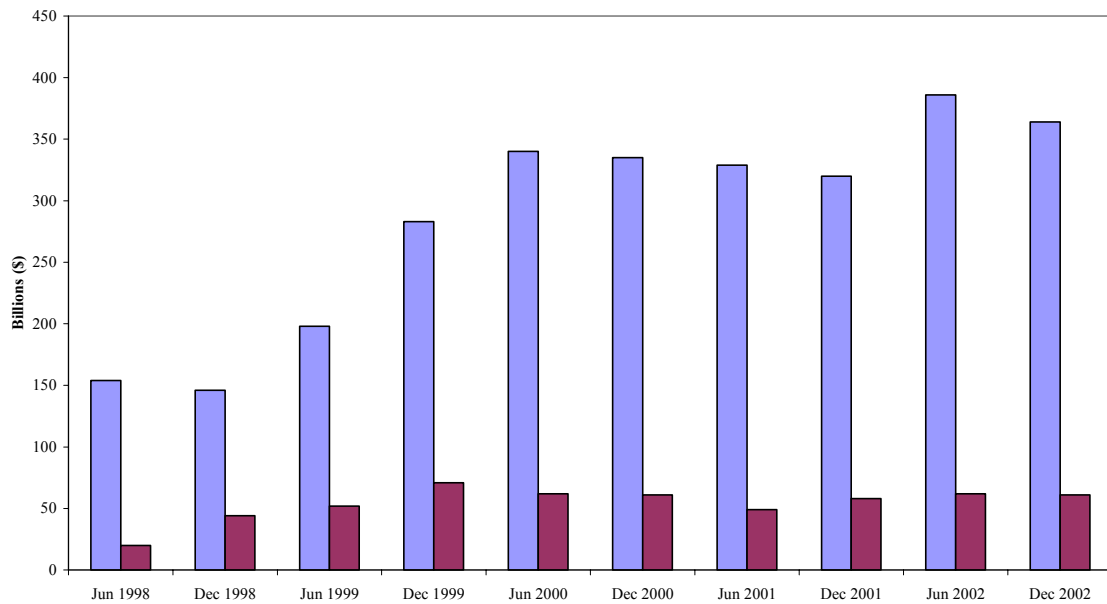
Pricing and Valuation of the Equity Swap to Pay the Return on One Stock or Index and Receive the Return on Another Stock or Index

Now let us assume that we enter into a swap to pay the return on the second stock index and receive the return on the first. There is no initial pricing on this swap. We determine the market value using the formula in the text:

$$V = \frac{1525.81}{1435.52} - \frac{4985.57}{4721.63} \\ = 0.006997.$$

Thus, the market value is \$0.006997 per \$1 notional principal.

Exhibit 1. Notional Principal and Market Value of Equity Swaps



Source: Bank for International Settlements,
<http://www.bis.org/publ/regpubl.htm>
 Note: includes swaps and a small amount of forwards

■ Notional Principal ■ Market Value

Exhibit 2. One-Year Equity Swap with Quarterly Settlements to Pay Fixed Rate of 5% and Receive the Return on the S&P 500 on Notional Principal of \$20 million.

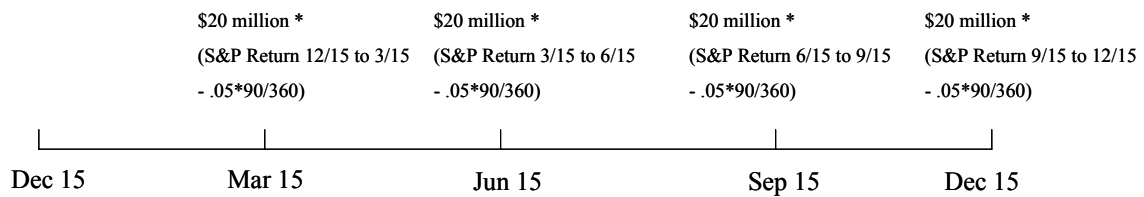


Exhibit 3. Hypothetical Payments on One-Year Equity Swap with Quarterly Settlement to Pay a Fixed Rate of 5% and Receive the Return on the S&P 500 on Notional Principal of \$20 Million

Date	S&P 500 Index	Periodic Return on S&P 500 ¹	S&P 500 Cash Flow ²	Fixed Interest Cash Flow ³	Net Cash Flow ⁴
December 15	1105.15				
March 15	1129.48	2.2015%	\$440,300	-\$250,000	\$190,300
June 15	1084.30	-4.0001%	-800,020	-250,000	-1,050,020
September 15	1055.29	-2.6755%	-535,100	-250,000	-785,100
December 15	1099.52	4.1913%	838,260	-250,000	588,260

¹This return is calculated over the three month period ending with the date in the first column on the same line. For example,

$$\left(\left(\frac{1129.48}{1105.15} \right) - 1 \right) * 100 = 2.2015\%$$

²For example,

$$\$20,000,000(.022015) = \$440,300$$

³For example,

$$\$20,000,000(.05) \left(\frac{90}{360} \right) = \$250,000$$

⁴For example,

$$\$440,300 - \$250,000 = \$190,300$$

Exhibit 4. One-Year Equity Swap with Quarterly Settlements to Pay LIBOR and Receive the Return on the S&P 500 on Notional Principal of \$20 million

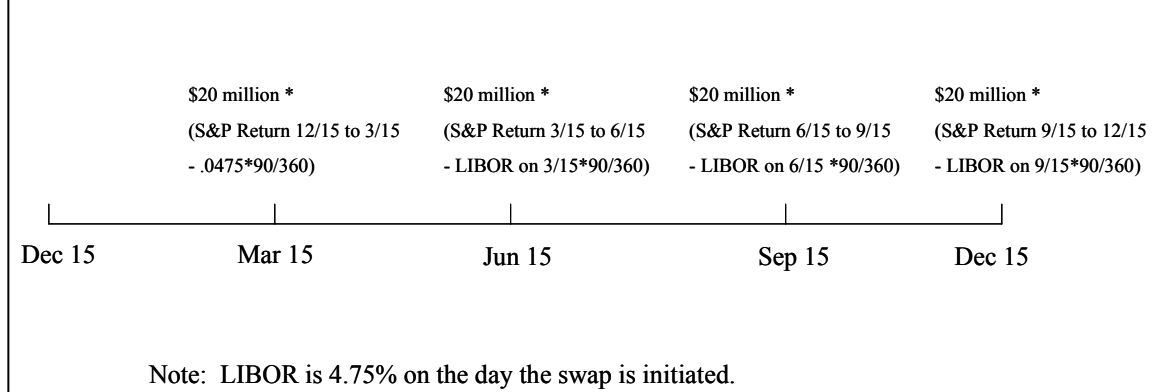


Exhibit 5. Hypothetical Payments on One-Year Equity Swap with Quarterly Settlement to Pay LIBOR and Receive the Return on the S&P 500 on Notional Principal of \$20 Million

Date	S&P 500 Index	Periodic Return on S&P 500	S&P 500 Cash Flow	LIBOR	LIBOR Cash Flow	Net Cash Flow
December 15	1105.15			4.75%		
March 15	1129.48	2.2015%	\$440,300	4.875%	-\$237,500	\$202,800
June 15	1084.30	-4.0001%	-800,020	5.125%	-243,750	-1,043,770
September 15	1055.29	-2.6755%	-535,100	4.9375%	-256,250	-791,350
December 15	1099.52	4.1913%	838,260	not applicable	-246,875	591,385

Note: See Exhibit 3 for examples of how the calculations are done. The LIBOR payment is determined as $\text{rate} \times 90/360 \times \$20 \text{ million notional principal}$ but based on LIBOR at the beginning of the settlement period. Thus, 4.75% on December 15 determines the payment on March 15. 4.875% on March 15 determines the payment on June 15, etc.

Exhibit 6. One-Year Equity Swap with Quarterly Settlements to Pay the Return on NASDAQ and Receive the Return on the S&P 500 on Notional Principal of \$20 million

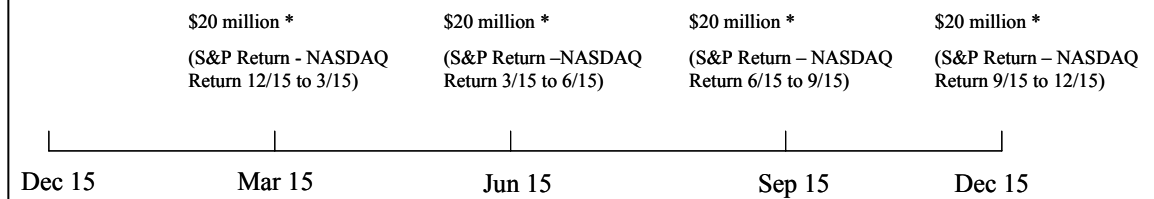


Exhibit 7. Hypothetical Payments on One-Year Equity Swap with Quarterly Settlement to Pay the NASDAQ Return and Receive the Return on the S&P 500 on Notional Principal of \$20 Million

Date	S&P 500 Index	Periodic Return on S&P 500	S&P 500 Cash Flow	NASDAQ Index	Periodic Return on NASDAQ	NASDAQ Cash Flow	Net Cash Flow
December 15	1105.15			1705.51			
March 15	1129.48	2.2015%	\$440,300	1750.78	2.6543%	-\$530,860	-\$90,560
June 15	1084.30	-4.0001%	-800,020	1689.25	-3.5144%	+702,880	-97,140
September 15	1055.29	-2.6755%	-535,100	1609.67	-4.7110%	+942,200	407,100
December 15	1099.52	4.1913%	838,260	1678.51	4.2767%	-855,340	-17,080

Note: See Exhibit 3 for examples of how the calculations are done. The NASDAQ payment is done in the same manner as the S&P 500 payment.

Exhibit 8. The Structure of an Equity Swap for an Endowment to Pay the Return on VAMA Technology and Receive the Return on the Russell 3000

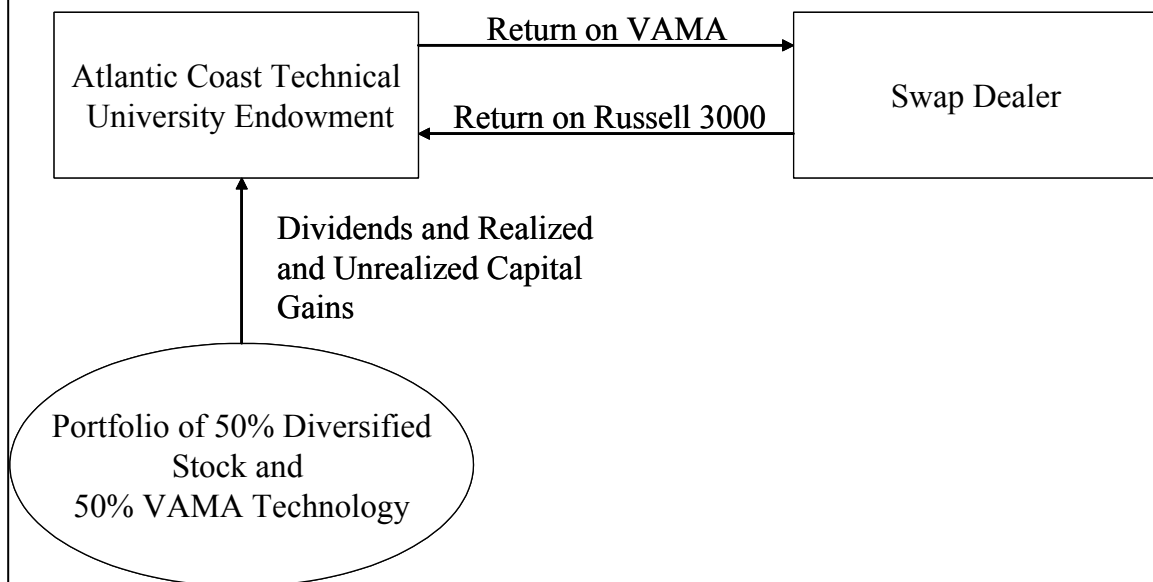


Exhibit 9. The Structure of an Equity Swap for An Asset Management Company to Pay the Return on the S&P 500 and Receive the Return on the Morgan Stanley Capital International EAFE Index

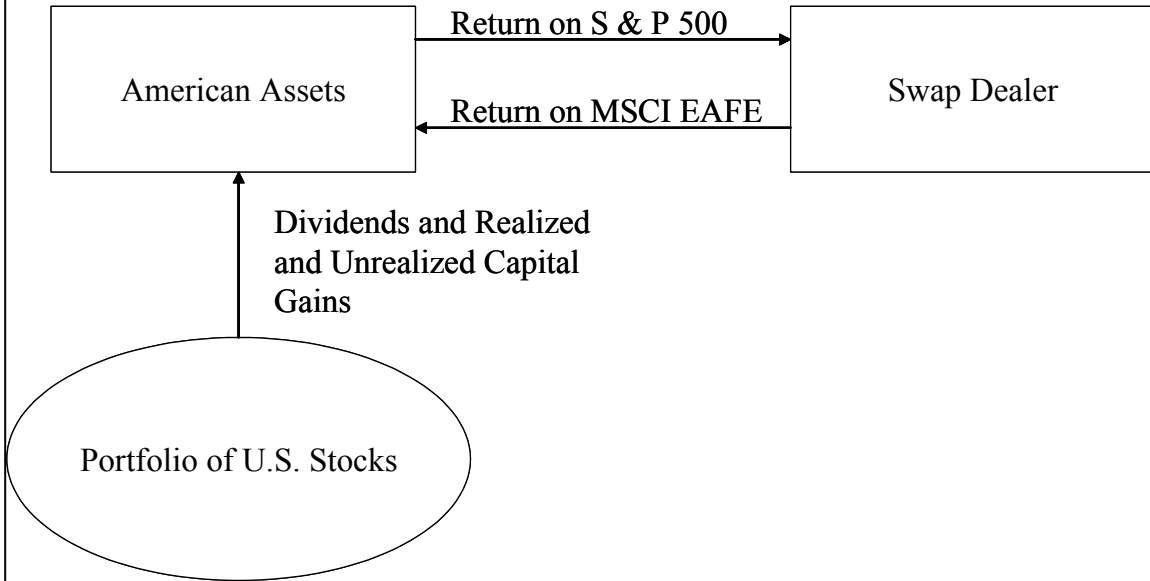


Exhibit 10. The Structure of an Equity Swap for An Asset Management Company to Pay the Return on Two Bond Indices and Receive the Return on Two Stock Indices

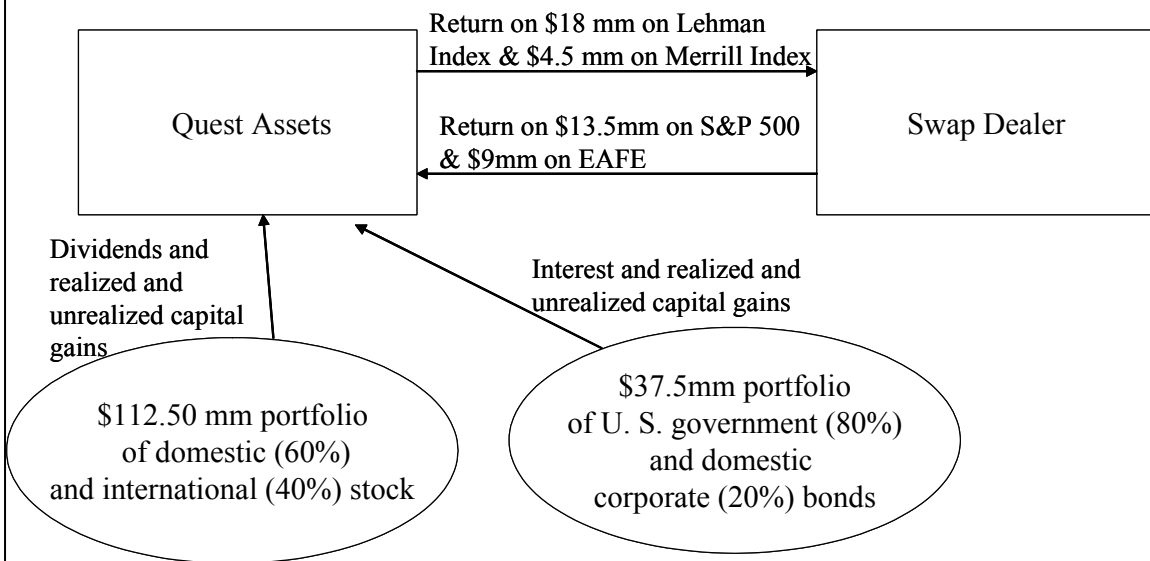


Exhibit 11. Constructing an Index Fund Based on the AMEX Composite Index Using an Equity Swap to Pay a Fixed Rate

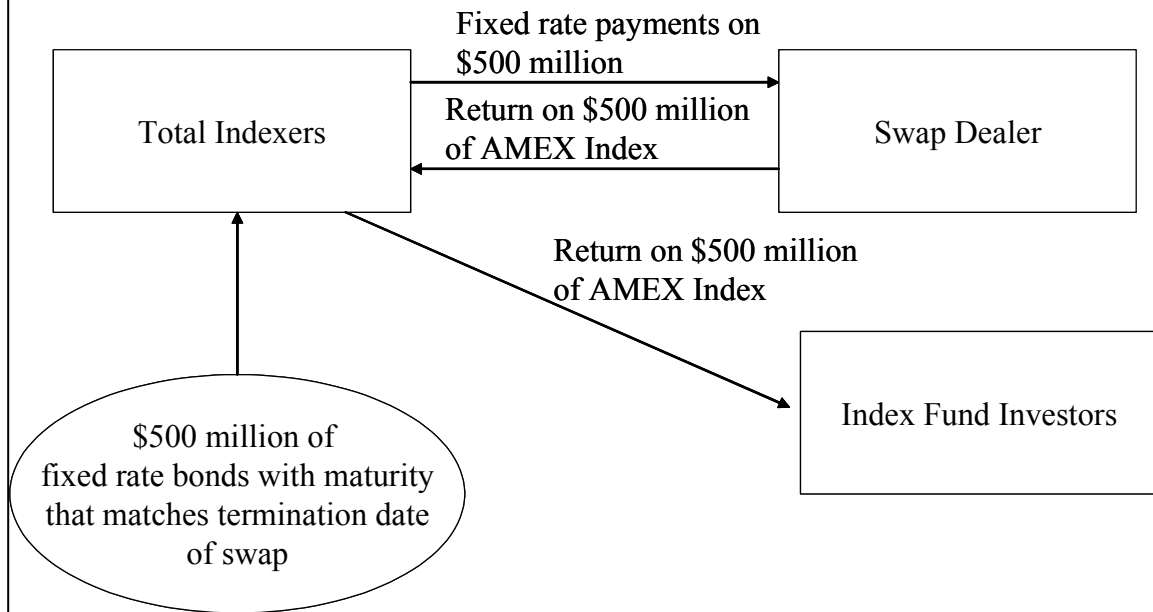


Exhibit 12. The Structure of an Equity Swap for an Executive to Pay the Return on One Million of His Four Million Shares and Receive the Return on the Wilshire 5000

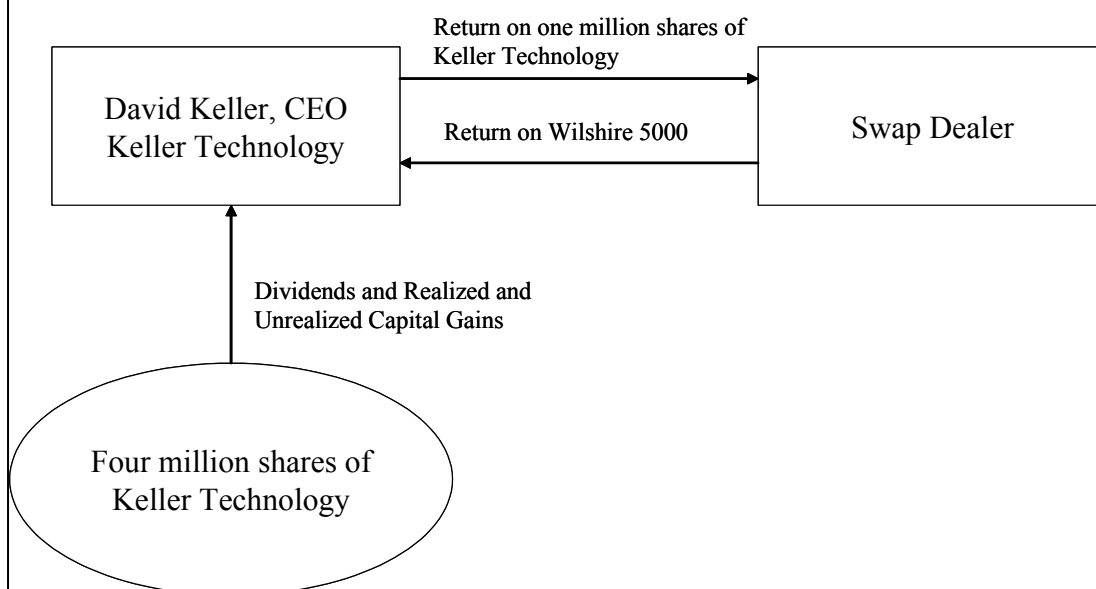


Exhibit 13(a). Return on Fixed-Rate Leveraged Equity

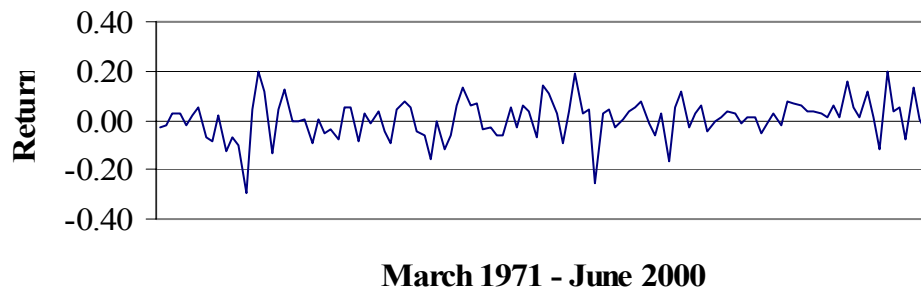


Exhibit 13(b). Return on Floating-Rate Leveraged Equity

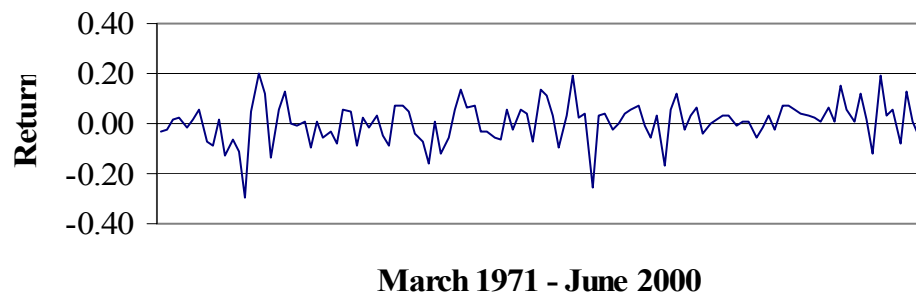
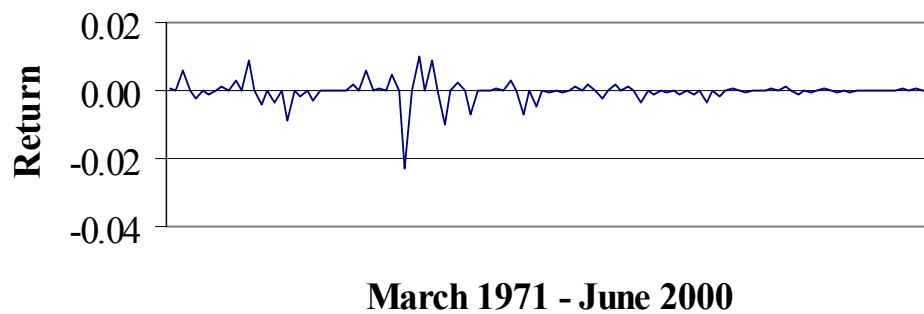
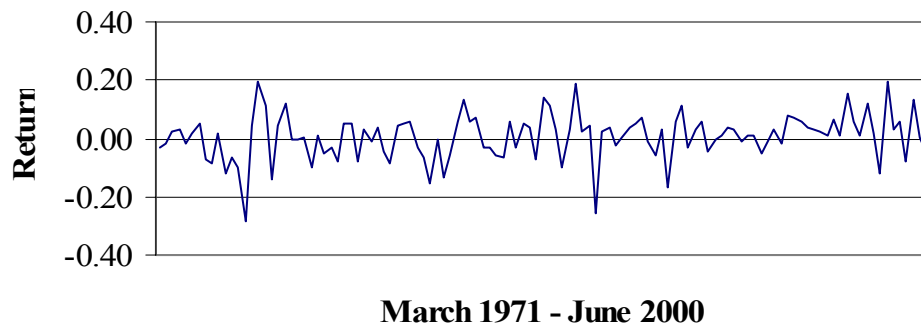


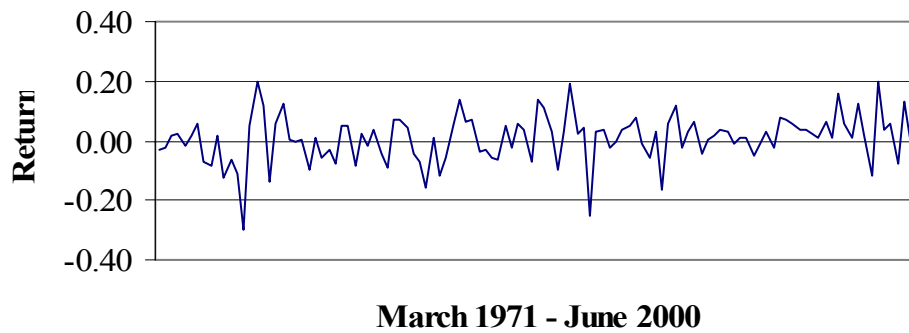
Exhibit 13(c). Difference Between Return on Fixed- and Floating-Rate Leveraged Equity



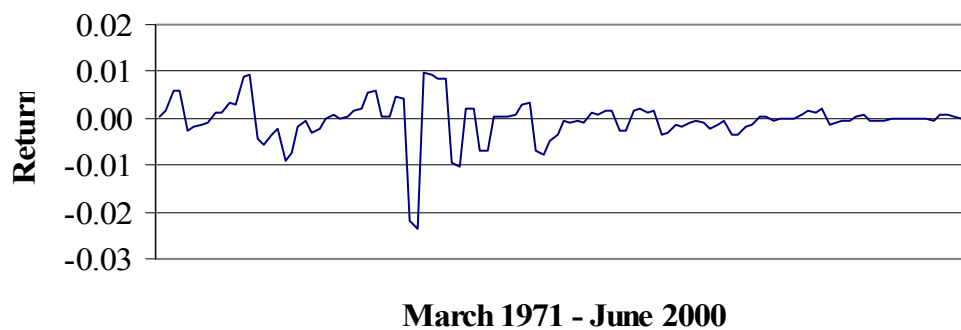
**Exhibit 14(a). Return on Fixed-Rate
Equity Swap**



**Exhibit 14(b). Return on Floating-Rate
Equity Swap**



**Exhibit 14(c). Difference Between Return on
Fixed- and Floating-Rate Equity Swap**



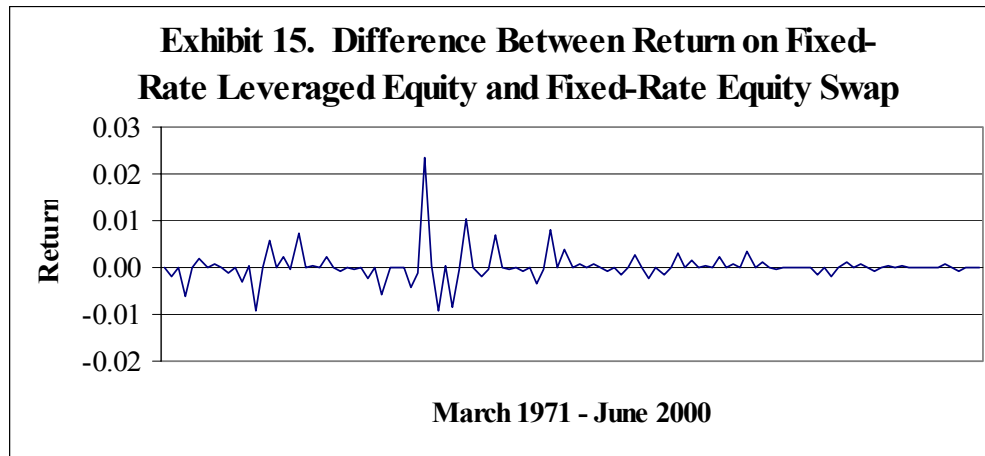


Exhibit 16. Summary Statistics from the Four Strategies for Equity Investing

	Strategy			Differences between Strategies		
	I	II (& IV)	III	I – II	I – III	III - IV
Mean	0.005800	0.006067	0.005612	-0.000267	0.000188	-0.000454
Standard Deviation	0.080169	0.080762	0.079494	0.003367	0.003376	0.004671
Maximum	0.198984	0.198984	0.195103	0.010202	0.023653	0.009859
Minimum	-0.296099	-0.296099	-0.286683	-0.022778	-0.009417	-0.023653
# > 0	69	71	69	28	57	57
# < 0	48	46	48	89	60	60
Z-statistic for Mean = 0	0.78	0.81	0.76	-0.86	0.60	-1.05

Note: Strategy I is equity leveraged using fixed-rate debt. Strategy II is equity leveraged using floating-rate debt. Strategy III is an equity swap with a fixed interest rate. Strategy IV is an equity swap leveraged with a floating interest rate. As shown in the paper, Strategies II and IV produce identical results.

References

- Allen, Julie A., and Janet L. Showers. "Equity-Index-Linked Derivatives: An Investor's Guide." Salomon Brothers (1991).
- Bodie, Zvi, and Robert C. Merton. "International Pension Swaps." *Journal of Pension Economics and Finance*, 1 (2002), pp. 77-83.
- Bolster, Paul, Don M. Chance, and Don Rich. "Executive Equity Swaps and Corporate Insider Holdings." *Financial Management*, 25 (Summer, 1996), pp. 14-25.
- Chance, Don M., and Don Rich. "The Pricing of Equity Swaps and Swaptions." *The Journal of Derivatives*, 5 (Summer, 1998), pp. 19-31.
- Gastineau, Gary L. "Adding Value with Equity Derivatives: Part I." *Derivative Strategies for Managing Portfolio Risk*. Charlottesville, Virginia: Association for Investment Management and Research, (1993), pp. 54-61.
- Jarrow, Robert, and Stuart Turnbull. *Derivative Securities*, 2nd ed. Cincinnati: South-Western College Publishing, 2000.
- Kijima, Masaaki, and Yukio Muromachi. "Pricing Equity Swaps in a Stochastic Interest Rate Economy." *The Journal of Derivatives*, 8 (Summer, 2001), pp. 19-35.
- Laatsch, Francis E. "Tax Clienteles, Arbitrage, and the Pricing of Total Return Equity Swaps." *The Journal of Derivatives*, 8 (Winter, 2000), pp. 37-46.
- Marshall, John E., Eric Sorensen, and Alan Tucker. 1992. "Equity Derivatives: The Plain Vanilla Equity Swap and Its Variants." *Journal of Financial Engineering*, 1 (1992), pp. 219-241.
- Merton, Robert C. "The Financial System and Economic Performance." *Journal of Financial Services Research*, 4 (1990), pp. 263-300.
- Norris, Floyd. "Keeping Votes as Stock is Sold." *The New York Times*, (March 29, 1994), pp. D1, D16.
- Rich, Don. "A Note on the Valuation and Hedging of Equity Swaps." *Journal of Financial Engineering*, 4 (1995), pp. 323-334.
- Schwimmer, Anne. "Total-Return Swaps Could Go Mainstream with Amoco Deal." *Investment Dealers Digest*, 59 (August 30, 1990), pp. 16.