Econ 681
Public Goods and Externalities

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December 5, 2017

Introduction

- An economy with externalities is one in which the actions of some agents *directly* affect the well-being of others.
- Examples:
  - air and water pollution (negative externality)
  - loud car stereos (negative externality)
  - the smell of baking bread (positive externality)
  - the neighbor's well-kept front yard (positive externality)
  - a public park, roads (positive externality, public good, excludable)
  - national defense (positive externality, pure public good, non-excludable)
- It is customary to define externalities so that they lead to a failure of the FWT. An external effect that occurs solely through markets, i.e., that affects prices, does not lead to a failure of the FWT, and is called a *pecuniary externality*. Reconsider the first two positive externality examples above.
- **Game theory** is the modern tool of choice for studying externalities.

Game Theory in a Nutshell

Definition. A **normal (strategic) form game** consists of $n \geq 1$ players and, for each player i, a set of strategies S_i and a payoff (Bernoulli utility) function,

$$u_i : S_1 \times \cdots \times S_n \to \mathbb{R}$$

Definition. A **Nash equilibrium** of such a game is a *strategy profile*,

$$s^* = (s_1^*, \ldots s_n^*) \in S_1 \times \cdots \times S_n$$

such that for each i,

$$s_i^* \in \max_{s_i \in S_i} u_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*)$$

Thus, s^* is a NE iff it specifies strategies that are mutual "best replies".

- If the players believe s^* will be played, no player has a reason to play otherwise.
- Any stable, commonly known way of playing the game must be a NE.

Example: The Driving Game

- Each of $n > 2$ people must decide to drive on the left or right side of roads:

$$S_i = \{L, R\}.$$

- After these decisions are made, a pair of players are chosen randomly to drive on a mountain road, one to drive up and the other to drive down.
- There will be a crash iff the two drivers are on the same side of the road. A crash gives each of them a payoff of -1. Their payoffs are 0 if they do not crash. The payoff function of player i is thus

$$u_i(s) = -\Pr(\text{crash} \mid s) = -\frac{x_i(s)}{n-1},$$

where $x_i(s) := \#\{j \neq i : s_j \neq s_i\}$.
- **Efficient Nash equilibria:** $(L, \ldots, L)$ and $(R, \ldots, R)$.
- **An Inefficient Nash equilibrium:** each player i flips a fair coin to decide whether $s_i = L$ or R. (This equilibrium is in *mixed strategies*.)

A Quasilinear Public Goods Environment

$g =$ public good (e.g., nat'l defense, park, firefighting,. . . .)

x_i, $e_i =$ consumer i's private good consumption and endowment

$u^i(g, x_i) = v_i(g) + x_i$, where $v_i' > 0$ and $v_i'' < 0$

$C(g) = cg$ is the private good cost of producing public good amount g

$(g, x_1, \ldots, x_n) \geq 0$ is an allocation, and it is **feasible** iff

$$C(g) + \sum_i x_i \leq \sum_i e_i$$

Efficiency in our Public Goods Environment

Proposition. Suppose each $u^i(g, x_i)$ is differentiable, with $u^i_x > 0$, and that C is differentiable. Then a strictly positive $(g^*, x_1^*, \ldots, x_n^*)$ is Pareto efficient only if

1. (**Budget Balance**) $C(g^*) + \sum_i x_i^* = \sum_i e_i$, and
2. (**Samulsonion Condition**) The sum of the MRS's equals MC:

$$\sum_i \frac{u^i_g(g^*, x_i^*)}{u^i_x(g^*, x_i^*)} = C'(g^*).$$

These conditions are sufficient for $(g^*, x_1^*, \ldots, x_n^*)$ to be Pareto efficient if each u^i is also quasiconcave and C is convex.

Remark. For quasilinear utility and linear cost, the Samuelsonian condition is

$$\sum_i v_i'(g^*) = c.$$

Thus, in a quasilinear setting efficiency requires only the maximization of the "surplus," $\sum_i v_i(g) - C(g)$. How the private good is distributed is irrelevant; it does not affect "the efficient level" of public good (no income effects).

Proof of the Proposition

- $(g^*, x_1^*, \ldots, x_n^*)$ is Pareto efficient iff it solves this Pareto problem:

$$\max_{g, x_1, \ldots, x_n \geq 0} u^1(g, x_1) \text{ such that}$$

$$u^i(g, x_i) \geq u^i(g^*, x_i^*) \ \forall i > 1, \text{ and}$$

$$C(g) + \sum_i x_i \leq \sum_i e_i.$$

- Let $\lambda_1 = 1$. Then the FOC are the constraints and (recall the allocation is strictly positive)

$$\sum_i \lambda_i u_g^i(g^*, x_i^*) - \mu C'(g^*) = 0,$$

$$\lambda_i u_x^i(g^*, x_i^*) - \mu = 0 \ \forall i,$$

and the multipliers $\lambda_2, \ldots, \lambda_n, \mu$ are nonnegative.
- Note that $\mu = \lambda_1 u_x^1 > 0$. This implies Budget Balance.
- We get the Samuelsonian condition from the FOC:

$$\sum_i \frac{u_g^i(g^*, x_i^*)}{u_x^i(g^*, x_i^*)} = \sum_i \frac{\lambda_i u_g^i(g^*, x_i^*)}{\lambda_i u_x^i(g^*, x_i^*)} = \sum_i \frac{\lambda_i u_g^i(g^*, x_i^*)}{\mu} = C'(g^*). \blacksquare$$

Competitive Market for Public Goods

- Consider the quasilinear linear model, with just one consumer who can buy public good at price p.
- Her inverse demand function for g would be $p(g) = v'(g)$. Her demand function would be $d(p) = v'^{-1}(p)$.
- If a competitive firm produces the public good, a competitive equilibrium is determined by the intersection of the demand curve and the marginal cost curve, and so $v'(g^*) = c$. Efficiency achieved.
- With more than one consumer, the left side of the Samuelsonian condition, $\sum_i v_i'(g^*)$, is the social marginal benefit of increasing the public good, and can be interpreted as the **vertical** sum of the individual demand curves for the public good, society's demand for the public good. The efficient outcome is determined by its intersection with the marginal cost curve.
- Does this efficient allocation arise from a "competive equilibrium"?

The Voluntary Contribution Game

- Suppose each consumer can purchase as much public good as she wants, for a price equal to its marginal cost c.
- This defines a game in which each consumer's strategy is an amount of public good to purchase. Letting $n = 2$, a strategy profile is a pair (g_1, g_2). The game's payoff functions are

$$\hat{u}_i(g_1, g_2) = v_i(g_1 + g_2) + e_i - cg_i.$$

- Assume $v_i' > 0$, $v_i'' < 0$, and $v_i'(0) > c > \lim_{g \to e_i/c} v_i'(g)$.
- If only consumer i, existed, she would purchase the $\bar{g}_i$ defined by $v_i'(\bar{g}_i) = c$.
- The efficient amount with two consumers is the g^* defined by $v_1'(g^*) + v_2'(g^*) = c$. Note that $g^* > \bar{g}_i$.

Proposition. If $v_1'(\cdot) < v_2'(\cdot)$, the unique NE is $(g_1, g_2) = (0, \bar{g}_2)$.

Discussion. The amount of public good provided in the NE is

$$g_1 + g_2 = \bar{g}_2 < g^*.$$

Neither player takes into account the marginal benefit to the other player of purchasing one more unit of the public good. The equilibrium is inefficient because of "free riding"; in this extreme case, player 1 is 100% free riding on player 2.

Personalized Prices (Lindahl Equilibrium)

- If there is a separate market for each consumer for the public good, there is an efficient competitive equilibrium.
- Let p_i be the personal (Lindahl) price of consumer i. Suppose she thinks she can unilaterally purchase the level of public good at price p_i :

$$g_i(p_i) = \arg\max_{g_i} v_i(g) - p_i g$$

- Thus, if $p_i = v_i'(g^*)$, each consumer will want to purchase the efficient amount g^* ("unanimity"). And the total amount of private good collected will just pay for the public good:

$$p_1 g^* + p_2 g^* = \left(v_1'(g^*) + v_2'(g^*)\right) g^* = cg^* = C(g^*).$$

- But each of these markets only has one consumer – the price-taking assumption seems far fetched!
- Also, how can a rational consumer believe she will not benefit from the other consumers' purchases of public good?

A Simple Two-Agent Externality Model

- Agent 1 takes an action $h \in \mathbb{R}$ that has an external effect on agent 2
- Utility: $v_i(x^i, h) + y_i$, where $x \in \mathbb{R}_+^{n-1}$ is a bundle of marketed goods
- Agent 1 first chooses h, then both agents buy goods
- Solve each agent's problem "backwards": given h, the consumer problems are

$$\phi_i(p, h) := \max_{x \in \mathbb{R}_+^{n-1}} v_i(x^i, h) - p \cdot x = v_i(x^i(p, h), h) - p \cdot x$$

- Suppress p, as will be constant: $\phi_i(h)$
- Assumptions:
  - $\phi_i'' < 0$
  - $h_1 > 0$ exists such that $\phi_1'(h_1) = 0$ (so h_1 is the unique maximizer of ϕ_1)
  - either $\phi_2' > 0$ (h is a **positive externality),**
    or $\phi_2' < 0$ (h is a **negative externality**)

Efficiency in the Externality Model

- Given a choice of h and amounts y_1, y_2, utilities are $\phi_i(h) + y_i$
- Quasilinear utility $\Rightarrow$ (h°, y_1, y_2) is Pareto efficient only if h^* maximizes the surplus:

$$h^* \in \arg\max_h \phi_1(h) + \phi_2(h)$$

- The efficient level of the externality is determined by

$$\phi_1'(h^*) = -\phi_2'(h^*)$$

- "Marginal social benefit = marginal social cost" picture

Competitive Equilibrium in the Externality Model

- The competitive equilibruim level of the externality is

$$h_1 \in \arg\max_h \phi_1(h),$$

  determined by $\phi_1'(h_1) = 0$.
- Inefficiency:
  - Negative externality $\Rightarrow h_1 > h^*$, since $\phi_1'' < 0$ (picture)
  - Positive externality $\Rightarrow h_1 < h^*$, since $\phi_1'' < 0$ (picture)
  - Agent 1 does not take into account the external marginal benefit/cost of her action

Remedy 1: Government-Imposed Quota

- Henceforth, restrict attention to the case of a negative externality
- The government tells agent 1 that she can only choose h satisfying

$$h \leq h^*$$

  Then agent 1's optimal action is indeed h^*.
- Drawbacks:
  - The government needs to know the functions ϕ_i to be able to calculate h^*
  - The agents have no incentive to correctly reveal their ϕ_i functions
  - h must be measureable so the gov't can impose a penality on agent 1 if $h > h^*$
  - administrative costs

Remedy 2: Pigouvian Taxes

- The government imposes a tax of t dollars on agent 1 for each unit of h he chooses
- Agent 1 now chooses h to maximize

$$\phi_1(h) - th,$$

and so chooses the h satisfying $\phi_1'(h) = t$.

- So she chooses the efficient amount h^* if the tax is set equal to the social marginal cost,

$$t = -\phi_2'(h^*)$$

- Same drawbacks as quotas:
  - The government needs to know the functions ϕ_i to be able to calculate $\phi_2'(h^*)$
  - The agents have no incentive to correctly reveal their ϕ_i functions
  - h must be measureable so the gov't can collect the correct tax, th
  - administrative costs
- Additional drawback: must tax just the externality, not an associated good (smoke and CO_2 emissions, not jet fuel)

Remedy 3: Restore the Missing Market

- Create a market in "pollution permits"
  - If agent 1 wants to pollute in the amount h, she must buy h permits from agent 2
  - Denote the price of one permit as p
- Competitive equilibrium:
  - demand: $h_d(p) \in \arg\max_h \phi_1(h) - ph$
  - supply: $h_s(p) \in \arg\max_h \phi_2(h) + ph$
  - market clearing: $h_d(p) = h_s(p)$
  - Walrasian equilibrium outcome:

$$h = h^*, \quad p = \phi_1'(h^*) = -\phi_2'(h^*)$$

- Advantage relative to remedies $1-2$: gov't does not need to know ϕ_i or h^*
- New drawbacks:
  - price-taking is a bad assumption in small markets
  - convexity of preferences is required to prove existence of WE, and this convexity is actually problematic in an externalities model (see, if interested, MWG, ch 11, Appendix A)

Remedy 4: Property Rights + Bargaining (Coase Theorem)

- Give one agent the "property right" to have h set to whatever level she wants. For concreteness, let's give the property right to agent 2, so that agent 1 requires her permission to choose any $h > 0$
- Then let them bargain over the level of h to set, and the amount of money T that agent 1 will pay agent 2
- There are a variety of bargaining games we could use to model the bargaining. For concreteness, consider one that gives all the bargaining power to agent 1:
  - Agent 1 makes a take-it-or-leave-it (tioli) offer (h, T) to agent 2. Agent 2's "sequentially rational" strategy is to accept any such offer satisfying

    $$\phi_2(h) + T \geq \phi_2(0) = \text{ her disagreement payoff}$$

  - Agent 1's best reply to this acceptance strategy is an offer (h, T) that solves the program

    $$\max_{h,T} \phi_1(h) - T \text{ such that } \phi_2(h) + T \geq \phi_2(0)$$

  - Thus, the equilibrium strategy of agent 1 is $(h, T) = (h^*, \phi_2(0) - \phi_2(h^*))$.
- Advantage: gov't needs only to establish and enforce property rights
- Drawback: bargaining may be costly, e.g., if there are many agents who must bargain with each other, or if their preferences are private information.