Economics 700 **Steven A. Matthews**
Fall 2011 **University of Pennsylvania**

Exam

Closed books, notes, calculators. 1.33 *hours.*
Show your reasoning. Read all before answering any.

Do ONLY four of the following five, equally weighted problems. If you do all five, only 1-4 will be graded.

1. Let $\mathcal{A}$ be a set with at least three elements. Given $u : \mathcal{A} \to \mathbb{R}$, define a binary relation on $\mathcal{A}$ by
$$a \succeq b \;\Leftrightarrow\; u(a) \geq u(b) - 1.$$
Let $\succ$ and $\sim$ be the strict preference and indifference relations derived from $\succeq$ in the usual way. For each of these three relations, is it necessarily (a) complete? Is it necessarily (b) transitive? Prove your answers.

2. A consumer has wealth $w > 0$ and a strictly concave Bernoulli utility function $u : \mathbb{R} \to \mathbb{R}$ for income. She has probability $\pi \in (0, 1)$ of having an accident, in which case she will suffer a monetary loss of $L > 0$. She can purchase insurance: there is a price p such that if she pays px dollars, the insurance company will pay her x dollars if she has an accident. Assume it is feasible for her to buy full insurance: $w > pL$.

   (a) Suppose the price p is equal to the actuarially fair rate. Prove, *with no further assumptions*, that the consumer buys full insurance: $x^* = L$.

   (b) Now suppose p is strictly greater than the actuarially fair rate. Under what (reasonable) additional assumption will she now buy less than full insurance? Give an intuitive explanation of this result.

   (c) Prove the result you stated in (b).

3. Each day a worker consumes leisure, ℓ, and income, x, and has a strictly increasing utility function $u(\ell, x)$. Leisure is measured in hours. She works for the remaining $L = 24 - \ell$ hours, at wage w. The income she consumes must thus satisfy $x \leq wL$. She has differentiable demand functions.

   Prove or disprove: if leisure is a normal good for this consumer, then her labor supply curve must be upward sloping, i.e., $\widehat{L}'(w) \geq 0$.

4. A competitive firm uses skilled and unskilled labor, L and ℓ, to produce one good. It has been observed that when the output price increases, the firm hires more skilled and fewer unskilled workers. Now the wage of the unskilled workers increases (they've unionized), but all other prices stay fixed. The firm has continuously differentiable demand and supply functions.

   State and prove what happens to (a) the firm's demand for unskilled workers, and (b) its supply of output.

5. A competitive firm has a continuous, strictly increasing and strictly concave production function, $f : \mathbb{R}^n_+ \to \mathbb{R}_+$, satisfying $f(\mathbf{0}) = 0$. It gives rise to a C^2 cost function $c(y, \mathbf{w})$. Holding $\mathbf{w} \gg \mathbf{0}$ fixed, derive the necessary properties of the function $c(\cdot, \mathbf{w})$ of output.