

Exam

80 points, 75 minutes. Closed books, notes, calculators.

Indicate your reasoning, using clearly written words as well as math.

1. (40 pts) A consumer with a strictly increasing utility function $u : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ has the expenditure function

$$e(p, U) = \left(p_1^a + p_2^b + 2p_1^c p_2^c \right) U^2, \text{ where } a, b, c > 0.$$

- (a) (10 pts) What can you say about the exponents a , b , and c ? Prove your answers.
- (b) (10 pts) Find the Hicksian demand functions, $h_1(p, U)$ and $h_2(p, U)$.
- (c) (10 pts) Find the indirect utility function, $v(p, m)$.
- (d) (10 pts) Find the utility function u .
2. (40 pts) Urn R contains 49 green balls and 51 blue balls. Urn U contains 100 balls, each of which is green or blue. One ball will be randomly drawn from one urn. The decision maker, DM, will be allowed to choose the urn.
- In scenario G , DM will be paid \$1000 if the ball that is drawn is green, and nothing if it is blue. In this scenario DM chooses urn R . That is, referring to the gambles as G_R when she chooses urn R and G_U when she chooses urn U , her preference is $G_R \succ G_U$.
- In scenario B , DM will be paid \$1000 if the ball she chooses is blue, and nothing if it is green. In this scenario DM also chooses urn R . That is, referring to these gambles as B_R when she chooses urn R and B_U when she chooses urn U , her preference is $B_R \succ B_U$.
- (a) (20 pts) Are DM's choices consistent with maximizing expected utility, with subjective probabilities for urn U and objective ones for urn R ? Prove your answer.
- (b) (10 pts) Describe a set of states such that each of the four gambles is a Savage "act," i.e., a function from the set of states to a set of consequences. Depict them in a table as we did in lecture.
- (c) (10 pts) Are DM's choices consistent with the Sure-Thing Principle? Prove your answer directly from the definition of the principle.