Diversification Returns and Asset Contributions

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For a portfolio with a constant percentage invested in each asset, the compound return is the sum of the contributions of the individual assets in the portfolio. The portfolio compound return is greater than the weighted average of the compound returns on the assets in the portfolio. The incremental return is due to diversification. The contribution of each asset exceeds its compound return by the amount it adds to the portfolio diversification return.

The compound return on an asset is approximately the asset's average return minus one-half the asset's variance. A portfolio's average return is the weighted average of each asset's average return, but a portfolio's variance is the weighted average of each asset's covariance. It follows that the return contribution of an asset can be better approximated by subtracting one-half the covariance than one-half the variance.

The compound return on a portfolio is an important measure of performance because it provides the connection between beginning and ending portfolio values. Knowing the beginning portfolio value, the compound return and the number of investment periods, we can easily compute the ending portfolio value.

The compound return on an asset is, however, a misleading measure of the contribution of the asset to the compound return on a portfolio. We will show that, with a constant percentage invested in each asset, the portfolio compound return is greater than the weighted average of the compound returns on the assets in the portfolio. This means that the contribution of each asset to the portfolio compound return is greater than the asset's compound return. The difference is an incremental return due to diversification. This incremental return is the subject of this article.

Table I presents a simple example using returns on the S&P 500, Treasury bonds and a 50-50 portfolio of the two assets for the 50 years 1941–90. The annualized compound return for the S&P 500 is 11.31%, and the annualized return for Treasury bonds is 4.36%. The average of the two compound returns is 7.84%. But a portfolio that maintains a 50% weight in the S&P 500 and a 50% weight in Treasury bonds has a higher compound return—8.11%. Diversification adds 28 basis points a year to the portfolio return.

Table I also shows the "**return contribution**" of each asset. The return contribution—the central concept of this article—is our estimate of the contribution of each asset to a portfolio's compound return. For the 50-50 portfolio, the return contribution of the S&P 500 is 11.79%, and the return contribution of Treasury bonds is 4.44%. The return contributions are higher than the compound returns (11.31% and 4.36%) because of the gains from diversification. Portfolio diversification adds 48 basis points to the S&P 500 return and 8 basis points to the Treasury bond return. The benefit of diversification for the portfolio (28 basis points) is the average of the benefits for the individual assets.

The increment of the return contribution of an asset over its compound return depends on how much the asset's risk is reduced through diversification. The increment is higher for the S&P 500 because more of its risk is diversified away. Conversely, from a portfolio perspective, judging assets in terms of their compound returns penalizes most those assets that benefit most from diversification.

The Compound Return

The appendix shows that the compound return (continuously compounded) on asset j is well approximated as:

Eq. 1

$$C_j = \ln[1 + E(R_j)] - \frac{s_j^2}{2[1 + E(R_j)]^2}$$

where

$E(R_j)$ = the expected (average) return on asset j,
s_j^2 = the variance (standard deviation squared) of the simple returns on j and

Table I Compound Returns, 1941–1990

	S&P 500 Index	*20-Year Treasury Bonds*	*Portfolio*
Portfolio Weights (%)	50.00	50.00	100.00
Average Monthly Returns (%)	1.03	0.39	0.71
Standard Deviations (%)	4.14	2.39	2.62
Annualized Continuously Compounded Returns (%)			
Compound Return			8.11
Compound Return of Each Asset and the Average Asset Compound Return	11.31	4.36	7.84
Return Due to Diversification	0.48	0.08	0.28
Return Contribution	11.79	4.44	8.11

$\ln[1 + E(R_j)]$ = the average return on j, expressed as a continuously compounded return.

The compound return on portfolio p can be expressed in terms of the mean and **variance** of its simple returns:

Eq. 2

$$C_p = \ln[1 + E(R_p)] - \frac{s_p^2}{2[1 + E(R_p)]^2}.$$

Equations (1) and (2) say that compound returns are lower, the higher the variance of returns. For a portfolio, variance is an appropriate measure of risk. But the risk of an asset in a diversified portfolio is less than the variance of the asset's returns. We show below that the risk reduction due to diversification is what causes an asset's compound return to understate the contribution of the asset to the compound return on the portfolio.

The Risk of an Asset in a Portfolio

Asset j's beta relative to the portfolio is calculated by regressing its returns against the portfolio's returns. It can be expressed as:

Eq. 3

$$b_{jp} = \frac{\text{cov}(R_j, R_p)}{s_p^2}.$$

Here $\text{cov}(R_j, R_p)$ is the covariance of asset j's returns with the portfolio's returns and s_p^2 is the variance of the portfolio's returns.

Rearranging Equation (3), we can express the risk of asset j in terms of its contribution to portfolio p's risk, as measured by its variance:

Eq. 4

$$\text{cov}(R_j, R_p) = b_{jp}s_p^2.$$

The covariance measure of each asset in the portfolio $[\text{cov}(R_i, R_p)]$, weighted by the asset's representation in the portfolio, will sum to the portfolio's variance, s_p^2.

The Return Contribution

One estimate of the contribution of asset j to the compound return on portfolio p is:

Eq. 5

$$D_j = \ln[1 + E(R_j)] - \frac{b_{jp}s_p^2}{2[1 + E(R_j)]^2}.$$

Comparing Equations (1) and (5) shows that the compound return on asset j and its contribution to the compound return on portfolio p have the same first term—$\ln[1 + E(R_j)]$. The difference between the compound return and the return contribution is thus due to the difference between $b_{jp}s_p^2$ and s_j^2. In a nutshell, the return contribution of asset j is greater than its compound return because the contribution of asset j to the variance of the return on the portfolio, $b_{jp}s_p^2$, is less than the variance of the return on j, s_j^2. This risk reduction, a result of diversification, enhances the contribution of asset j to the compound return on the portfolio.

The weighted average of the return contributions estimated from Equation (5) is not exactly the portfolio compound return, because the weighted average of $\ln[1 + E(R_j)]$ is not exactly $\ln[1 + E(R_p)]$. We can arrange for the weighted average of the return contributions to equal the portfolio compound return given by Equation (2) if we change Equation (5) to:

Eq. 6

$$D_j = \frac{E(R_j)\ln[1 + E(R_p)]}{E(R_p)}$$

Glossary

▶***Return Contribution:*** The contribution of an asset to a portfolio's compound return.

▶***Variance:*** The second moment of a distribution around its mean; the average of the squared deviations from the mean.

▶***Asset-Class Investing:*** Investing to capture the returns of an asset class without regard to stock selection. An index fund is one example.

▶***Diversification Returns:*** For a portfolio, the difference between its compound return and the weighted average of each asset's compound return. For an asset in a portfolio, the difference between the return contribution and the compound return.

Table II Portfolio with Small-Cap Stocks, 1941–1990

	S&P 500	*U.S. Small-Cap D6–10*	*20-Year Treasury Bonds*	*One-Month Treasury Bills*	*Portfolio*
Portfolio Weights (%)	50.00	10.00	20.00	20.00	100.00
Average Monthly Returns (%)	1.03	1.24	0.39	0.36	0.79
Standard Deviations (%)	4.14	5.51	2.39	0.28	2.70
Annualized Continuously Compounded Returns (%)					
Compound Return					9.02
Compound Return of Each Asset and the Average Asset Compound Return	11.31	12.96	4.36	4.30	8.68
Return Due to Diversification	0.41	0.99	0.16	−0.00	0.33
Return Contribution	11.72	13.96	4.52	4.29	9.02

$$- \frac{b_{jp}s_p^2}{2[1 + E(R_p)]^2}.$$

The difference between estimating the return contribution using Equation (5) and using Equation (6) is minuscule for most periods. We will use Equation (6) because of its additive property. The weighted average of the first term on the right-hand side is exactly $\ln[1 + E(R_p)]$ and the weighted average of the second term is $s_p^2/2[1 + E(R_p)]^2$. Thus:

Eq. 7

$$D_p = \ln[1 + E(R_p)] - \frac{s_p^2}{2[1 + E(R_p)]^2} = C_p.$$

Examples

We estimate below the return contributions of seven asset classes for different time periods and portfolios.[1] We estimated betas from quarterly data and average returns and standard deviations from monthly data. The continuously compounded returns and return contributions are annualized from the monthly data.

Results

Table II and Table III show results for domestic balanced portfolios for 1941–90. Table II uses deciles 6 through 10 (D6–10) for small-cap stocks and Table III uses deciles 9 through 10 (D9–10). The results for the S&P 500, Treasury bonds and Treasury bills are almost identical in the two tables. The annualized return contribution for the S&P 500 in both is 11.72%, an increase of 0.41% over the index's compound return. The incremental return for Treasury bonds is 0.16%.

The incremental returns due to diversification are greater for small-cap stocks than for the other assets. The incremental returns (the difference between the contribution of small stocks to the compound return on the portfolio and the compound return on small stocks) are 0.99% for D6–10 stocks and 1.36% for D9–10 stocks. Measuring the contributions of these assets to portfolio returns, rather than the compound returns on the assets, substantially increases the premium of small-cap stocks over the S&P 500. The difference between the return contributions of D9–10, 15.42%, and the S&P 500, 11.72%, is 3.70%. The difference between the return contributions of D6–10, 13.96%, and the S&P 500 is 2.24%. The corresponding differences between the compound returns are only 2.75% and 1.65%.

Why are the contributions of small stocks to the compound returns on portfolios so much larger than the compound returns on small stocks? Small stocks have high return variances,

Table III Portfolio with Very Small-Cap Stocks, 1941–1990

	S&P 500	*U.S. Small-Cap D9–10*	*20-Year Treasury Bonds*	*One-Month Treasury Bills*	*Portfolio*
Portfolio Weights (%)	50.00	10.00	20.00	20.00	100.00
Average Monthly Returns (%)	1.03	1.37	0.39	0.36	0.80
Standard Deviations (%)	4.14	6.20	2.39	0.28	2.72
Annualized Continuously Compounded Returns (%)					
Compound Return					9.17
Compound Return of Each Asset and the Average Asset Compound Return	11.31	14.06	4.36	4.30	8.79
Return Due to Diversification	0.41	1.36	0.16	−0.00	0.37
Return Contribution	11.72	15.42	4.52	4.29	9.17

Table IV Domestic Portfolio, 1926–1940

	S&P 500	*U.S. Small-Cap D6–10*	*20-Year Treasury Bonds*	*One-Month Treasury Bills*	*Portfolio*
Portfolio Weights (%)	50.00	10.00	20.00	20.00	100.00
Average Monthly Returns (%)	0.78	0.99	0.42	0.11	0.59
Standard Deviations (%)	9.56	13.62	1.45	0.14	6.09
Annualized Continuously Compounded Returns (%)					
Compound Return					4.94
Compound Return of Each Asset and the Average Asset Compound Return	3.96	1.73	4.87	1.31	3.39
Return Due to Diversification	2.09	4.79	0.10	0.03	1.55
Return Contribution	6.05	6.53	4.96	1.34	4.94

which lower their compound returns (see Equation (1)). Because small stocks are not highly correlated with other assets, however, their risk in a diversified portfolio (the covariance of their returns with portfolio returns) is much less than their return variance. This diversification benefit substantially increases the contribution of small stocks to the compound returns on portfolios. (The story is similar for international stocks, large and small.)

Tables II, IV and V show results for the same domestic portfolio over different time periods. The results for the last 20 years (Table V) are similar to the results for the last 50 years (Table II). The results for the Great Depression (Table IV) are quite different. During the Depression, it was important to be diversified. The return contribution for small-cap stocks in this period is greater than the return contribution for the S&P 500, but the compound return for small-cap stocks is lower than the compound return for the S&P 500.

Tables VI and VII show results for two internationally diversified portfolios. With two additional equity asset classes, the negative bias of the asset compound return is more apparent. Table VI shows that the compound return understates the annualized return contributions for stocks by the following percentages:

S&P 500	0.56
U.S. Small-Cap Stocks (D6–10)	1.31
International Large-Cap Stocks	1.30
International Small-Cap Stocks	1.23

Because the S&P 500 benefits the least from diversification among the four asset classes and it has a large weight in the portfolio, its return increment due to diversification is the lowest of the four asset classes. The U.S. small-cap premium over the S&P is thus understated by 0.75% (1.31–0.56). The international large-cap premium over the S&P is understated by 0.74%.

Table VII shows the results for a portfolio that equally weights all four equity asset classes. Its monthly standard deviation is only slightly greater than that of the portfolio in Table VI—2.99% versus 2.94%. Even though the S&P 500 standard deviation is much lower than the standard deviations of the other equity asset classes, the portfolio standard deviation does not increase significantly because the S&P 500 commitment is reduced. The improvement in diversification about offsets the increase in average standard deviation.

Finally, the difference between the contribution of Treasury bonds to the compound returns on portfolios and the compound return on the bonds has widened in the last 20 years (see Tables I to VII). The reason is that the vari-

Table V Domestic Portfolio, 1971–1990

	S&P 500	*U.S. Small-Cap D6–10*	*20-Year Treasury Bonds*	*One-Month Treasury Bills*	*Portfolio*
Portfolio Weights (%)	50.00	10.00	20.00	20.00	100.00
Average Monthly Returns (%)	0.99	1.11	0.75	0.62	0.88
Standard Deviations (%)	4.65	6.10	3.30	0.22	3.13
Annualized Continuously Compounded Returns (%)					
Compound Return					9.95
Compound Return of Each Asset and the Average Asset Compound Return	10.57	10.97	8.36	7.39	9.53
Return Due to Diversification	0.48	1.19	0.28	−0.00	0.42
Return Contribution	11.06	12.16	8.64	7.39	9.95

Table VI 10% International Portfolio, 1971–1990

	S&P 500	U.S. Small D6–10	Intl. Large	Intl. Small	20-Year T-Bonds	One-Month T-Bills	Portfolio
Portfolio Weights (%)	45.00	5.00	5.00	5.00	20.00	20.00	100.00
Average Monthly Returns (%)	0.99	1.11	1.63	2.02	0.75	0.62	0.96
Standard Deviations (%)	4.65	6.10	5.86	5.50	3.30	0.22	2.94
Annualized Continuously Compounded Returns (%)							
Compound Return							10.95
Compound Return of Each Asset and the Average Asset Compound Return	10.57	10.97	17.46	22.21	8.36	7.39	10.44
Return Due to Diversification	0.56	1.31	1.30	1.23	0.31	−0.00	0.50
Return Contribution	11.13	12.28	18.76	23.45	8.67	7.39	10.95

ance of long-bond returns has increased. In terms of variability, long bonds are now more like stocks. Thus it is now more important that long bonds be held in diversified portfolios that include stocks.

Diversification Returns vs. Investment Uncertainty

Thus far we have shown that diversification improves portfolio compound returns. We will now show that the incremental returns may be lost by engaging in active management.

In order to measure diversification returns, we engaged in **asset-class investing**, investing in asset classes in fixed proportions. This runs counter to active management. The essence of active management is to try to "beat the market" by shifting asset weights.

Active management can either increase or decrease the portfolio compound return, depending on both skill and luck. As we will see, the luck component can be larger than the skill component.

To see how shifting weights introduces uncertainty, suppose we randomly invest in the S&P 500 and Treasury bonds on a monthly basis. One-half of the months we invest 65% in the S&P 500 and 35% in Treasury bonds. The other half of the months we invest 35% in the S&P 500 and 65% in Treasury bonds.

The returns from this strategy will depend on the luck of the draw. To show how uncertain the outcomes can be, we simulated the approach 1000 times for the five years 1986–90. It is as though we had 1000 managers engaged in a market timing contest. The summary of the results are displayed in Table VIII.

We calculated the five-year compound return for each of the simulations, then ranked from highest to lowest. As Table VIII shows, 10% of the annualized returns were 14.33% or higher, while 10% were 10.52% or lower.

The average asset mix is 50% S&P 500 and 50% Treasury bonds. However, the average standard deviation of returns is somewhat higher for the timing portfolios than it is for the 50/50 portfolio, because of the shifting of asset weights. The average standard deviation for the random timing portfolio is equal to that of a constant-mix portfolio invested 53% in the S&P 500 and 47% in Treasury bonds. This 53/47 portfolio serves as a suitable benchmark for comparison with the random timing portfolios. The benchmark outperforms the randomly generated portfolios. Its 12.53% compound return is 14 basis points higher than the average compound return for the randomly generated portfolios, and has a 52-basis-point annual diversification return.

Thus shifting weights lowers expected compound return. But, perhaps more importantly, shift-

Table VII 30% International Portfolio, 1971–1990

	S&P 500	U.S. Small D6–10	Intl. Large	Intl. Small	20-Year T-Bonds	One-Month T-Bills	Portfolio
Portfolio Weights (%)	15.00	15.00	15.00	15.00	20.00	20.00	100.00
Average Monthly Returns (%)	0.99	1.11	1.63	2.02	0.75	0.62	1.14
Standard Deviations (%)	4.64	6.10	5.86	5.50	3.29	0.22	2.99
Annualized Continuously Compounded Returns (%)							
Compound Return							13.04
Compound Return of Each Asset and the Average Asset Compound Return	10.57	10.97	17.46	22.21	8.36	7.39	12.33
Return Due to Diversification	0.66	1.35	1.18	1.08	0.36	−0.01	0.71
Return Contribution	11.23	12.32	18.64	23.30	8.72	7.39	13.04

Table VIII Random Timing Strategy, Annualized Compound Returns (%), 1986–1990

10th Percentile	14.33
50th Percentile	12.39
90th Percentile	10.52
10th–90th	3.81
Benchmark Portfolio	12.53
S&P 500	13.14
Treasury Bills	10.74

ing weights also introduces uncertainty. The median return is the most likely return, but achieving the median return is highly uncertain. About 10% of the random portfolios have returns 200 basis points a year higher than the benchmark return. About 10% of the portfolios have returns 200 basis points a year lower than the benchmark return. Most investors would take 12.53% for certain rather than invest in a strategy that has uncertain returns and an equal chance of a 14.33% return or a 10.52% return.

The uncertainty of achieving benchmark returns extends to real portfolios. Table IX displays the distribution of total plan returns for institutional funds in the SEI Corporation database. The institutional portfolios behave like the randomly generated portfolios. About 10% of the funds have returns 200 basis points a year above the median. About 10% of the funds have returns 200 basis points a year below the median.

The 400-basis-point spread of returns between the 10th and 90th percentiles measures the uncertainty introduced by active management. The implication is clear: Investors should prefer the certain return from a portfolio that maintains fixed asset weights to a strategy whose outcome is uncertain and has an equal chance of underperforming or outperforming the benchmark by 200 basis points per year. Stated differently, investors need a large premium to be willing to incur the additional uncertainty of active management.

Table IX Total Plan Sponsor Annualized Compound Returns (%), 1986–1990*

10th Percentile	12.38
50th Percentile	10.47
90th Percentile	8.35
10th–90th	4.03

Source: SEI Corporation.

Recent studies indicate that investors cannot expect such a premium.[2] The premium return from active management is negative rather than positive. Every year, about half the funds outperform their benchmarks. Those in the upper half in one year have only about a 50% chance of repeating in any other year.

Active management introduces so much uncertainty that we cannot document a premium return over benchmark returns. By contrast, fully diversified portfolios reliably increase portfolio compound returns through the diversification process and eliminate "benchmark risk."

Conclusion

The portfolio compound return is useful for describing portfolio results. But the compound return of an asset in a portfolio understates the contribution of the asset to the portfolio compound return because it ignores the benefit of diversification.

The "return contribution" defined by Equation (6) is an improved measure of the contribution of an asset to the portfolio compound return. The portfolio compound return is the average of the asset return contributions.

The increments of return contribution over compound returns are especially large for small-cap stocks and international stocks, primarily because diversification benefits these assets (reduces their risks) the most. For the S&P 500, the difference is smaller, but important. The increased variance of long-bond returns over the last 20 years has widened the spread between their return contributions and their compound returns.

By shifting asset weights, active management introduces uncertainty about portfolio compound returns. **Diversification returns** are assured only for those portfolios that maintain relatively fixed asset weights.

Appendix

The continuously compounded return on an asset is $\ln[1 + R_j]$, where R_j is the simple return. The expected value of the compound return of an asset, $E[\ln(1 + R_j)]$, can be derived from the Taylor series expansion of the natural logarithm about the mean return of the asset:

Eq. A1

$$C_j = \ln[1 + E(R_j)] - \frac{M_2}{2[1 + E(R_j)]^2} + \frac{M_3}{3[1 + E(R_j)]^3} - \frac{M_4}{4[1 + E(R_j)]^4} + \ldots,$$

where M_k = the kth moment of the asset return about its mean, or:

Eq. A2

$$M_k = E[(R_j - E(R_j))^k].$$

Equation (1) is just Equation (A1) with the higher moments dropped. For the assets and portfolios in Tables I through VII, these higher-order terms are always minuscule, except during the Great Depression. Table AI displays the accuracy of Equation (1) for the portfolios displayed in Tables I through VII.

Table AI Annualized Portfolio Compound Returns Obtained by Linking Monthly Returns Compared with Compound Returns Derived from Equation (1)

Portfolio by Table	*Time Period*	*Actual Compound Return %*	*Derived Compound Return %*	*Difference (Formula Error) %*
I	1941–90	8.11	8.11	0.00
II	1941–90	9.02	9.02	−0.00
III	1941–90	9.17	9.17	−0.00
IV	1926–40	4.97	4.94	0.03
V	1971–90	9.95	9.95	−0.00
VI	1971–90	10.95	10.95	−0.00
VII	1971–90	13.04	13.04	−0.00

Footnotes

1. *S&P 500 and long-term Treasury bond data from R. G. Ibbotson and R. A. Sinquefield,* Stocks, Bonds, Bills and Inflation *(Chicago: Ibbotson Associates). D9–10 is the Dimensional Fund Advisors Inc. (DFA) U.S. 9–10 Small Company Portfolio, net of all fees, from 1982 on and the CRSP 9–10 index (courtesy of the Center for Research in Securities Prices, University of Chicago) for 1926–81. The D6–10 is the DFA U.S. 6–10 Small Company Portfolio, net of administrative fees, from June 1986 on and the CRSP 6–10 index for 1926 through May 1986. The international small-cap portfolio uses the following data. For the U.K.: the Hoare Govett Smaller Co. Index (courtesy London Business School) for 1956 through March 1986 and the DFA United Kingdom Small Company Portfolio, net of all fees, for April 1986 on. For Japan: the smaller half of the first section of the Tokyo Stock Exchange (courtesy of Nomura Securities Investment Trust Management Co., Ltd., Tokyo) for 1970 through March 1986 and the DFA Japanese Small Company Portfolio, net of all fees, for April 1986 on. The international large-cap portfolio uses the following data. For the U.K.: the Financial Times All Shares Index for 1956 on. For Japan: The larger half of the first section of the Tokyo Stock Exchange (courtesy Nomura Securities) for 1970 through June 1986 and the Japan National Index (courtesy of Morgan Stanley Capital International) for July 1986 on. Data on large-cap continental stocks (excluding the U.K.) and on Asia and Australia (excluding Japan) from Morgan Stanley. The international large and small-cap portfolios were weighted as follows. For 1970–June 1988: 50% Japan, 50% U.K. For July 1988–September 1989: 50% Japan, 30% Continent, 20% U.K. For October 1989–March 1990: 40% Japan, 30% Continent, 20% U.K., 10% Asia-Australia. For April 1990 on: 40% Japan, 35% Continent, 15% U.K., 10% Asia-Australia. Returns for various portfolios over various years are available from the authors.*
2. *See G. P. Brinson, L. R. Hood and G. L. Beebower, "Determinants of Portfolio Performance,"* Financial Analysts Journal, *July/August 1986, and G. P. Brinson, B. D. Singer and G. L. Beebower, "Determinants of Portfolio Performance II: An Update,"* Financial Analysts Journal, *May/June 1991.*