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Beyond Equilibrium, the Black-Litterman Approach

Bob Litterman

The Black-Litterman global asset allocation model provides a framework for combining market equilibrium with tactical views about investment opportunities. In order to understand the benefits of the model, it should be recognized that its development was motivated not at all by a belief that equilibrium provides useful short-term forecasts of returns. Rather, it was developed as a solution to a practical problem associated with portfolio optimization. As is well known, the standard mean-variance portfolio optimization discussed in Chapter 4 is not well behaved. Optimal portfolio weights are very sensitive to small changes in expected excess returns. Thus, the historical development of the Black-Litterman model began with a financial engineering question—"How can we make the standard portfolio optimizer better behaved?"—rather than, as developed in this book, as a natural extension of the global CAPM equilibrium.

The problem faced in 1989 in the fixed income research function at Goldman Sachs was a particularly badly behaved optimization exercise. We were advising investors with global bond portfolios, typically with some currency exposures. Many currencies, and most of the yield changes in bonds in the developed fixed income markets, have high correlations to each other. Changes in the forecasts of yields well below the precision with which any forecaster had confidence (for example, on the order of only a few basis points over a period of as much as six months into the future) would create major swings in optimal portfolio allocations. Moreover, it was virtually impossible, without significant constraints on both maximum and minimum holdings, to get portfolios that looked at all reasonable.

At the same time these portfolio optimization issues were being faced, Fischer Black had just finished his "Universal Hedging" paper on the global CAPM equilibrium. It was his suggestion that incorporation of the CAPM equilibrium into the mean-variance optimizer might make it better behaved. In retrospect, the suggestion perhaps seems obvious. It is well known that the properties of many statistical estimators can be improved by some shrinkage toward a neutral point that acts as a

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kind of center of gravity.[1] The more reasonable that point, the better the properties of the estimator. In the Black-Litterman model, the global CAPM equilibrium provides this center of gravity. At the time of Fischer Black's suggestion, though, despite the fact that mean-variance optimization and versions of the CAPM equilibrium had both been well understood for more than 20 years, it was not at all obvious that what the portfolio optimizer needed was the incorporation of such an equilibrium.

In fact, our first naive attempt to use the global equilibrium failed rather miserably. Rather than focus on expected excess returns as unknown quantities to be estimated, we simply tried to take a weighted average of investor-specified expected excess returns with the equilibrium values. We found, as we will show by example, that simply moving away from the equilibrium risk premiums in a naive manner quickly leads to portfolio weights that don't make sense. Further reflection on the nature of the problem led us to think about the uncertainty in the equilibrium risk premiums as well as the nature of information that the investors are trying to incorporate through their views. We also realized that it is essential to take into account the likely correlations among the expected returns of different assets. The estimator that we developed to take these issues into account eliminates the bad behavior of the optimization exercise and provides a robust framework for managing global portfolios.

What we discovered, however, was not simply a better optimizer, but rather a reformulation of the investor's problem. In the context of Black-Litterman, the investor is not asked to specify a vector of expected excess returns, one for each asset. Rather, the investor focuses on one or more views, each of which is an expectation of the return to a portfolio of his or her choosing. We refer to each of these portfolios for which an investor specifies an expected return as a "view portfolio." In the Black-Litterman model, the investor is asked to specify not only a return expectation for each of the view portfolios, but also a degree of confidence, which is a standard deviation around the expectation. This reformulation of the problem can be applied more generally, and among other benefits has greatly facilitated the use of quantitative return forecasting models in asset management.

In an unconstrained optimization context, the Black-Litterman model produces a very simple and intuitive result. The optimal portfolio is a weighted combination of the market capitalization equilibrium portfolio and the view portfolios.[2] The sizes of the tilts toward the view portfolios are a function of both the magnitude and the confidence expressed in the expected returns embedded in the investor-specified views. In fact, the solution is so straightforward one might question whether the model is actually adding value. The answer is that most portfolio optimizations are not so simple. When there are benchmarks, constraints, transactions costs to consider, or other complications, the optimal portfolios are not so obvious

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[1] See, for example, the literature on Bayes-Stein estimation, including C. Stein, "Inadmissability of the Usual Estimator for the Mean of a Multivariate Normal Distribution," Proceedings of the Third Berkeley Symposium on Probability and Statistics (Berkeley, CA: University of California Press, 1955), and Jorion, Philippe, "Bayes-Stein Estimation for Portfolio Analysis," *Journal of Financial and Quantitative Analysis*, September 1986.

[2] The mathematical derivation of these results is included in "The Intuition behind Black-Litterman Model Portfolios," by Guangliang He and Robert Litterman, Goldman Sachs Investment Management Research paper, December 1999.

or easily interpreted. In these contexts the model provides the expected excess returns needed to drive the optimization process.

Let us now illustrate some of the difficulties in using standard portfolio optimizers to create optimal portfolios. One Wall Street prognosticator recently provided us with a nice set of inputs for our example by publishing a set of long-term expected returns for major asset classes. The forecasts and our estimated volatilities are shown in Table 7.1. We suspect our colleague used what he felt was informed judgment to create this outlook, but that he did not try to run the expected returns through an optimizer.

We proceeded to do exactly that, not to criticize our colleague (whose anonymity we shall respect), but rather to illustrate first how an optimizer looks for small inconsistencies in a set of forecasts and forms portfolios based on those inconsistencies, and second how difficult it is to specify a portfolio optimization problem in a way that leads to what might seem to be a reasonable solution. We formed a covariance matrix using historical returns for these various assets classes (and where necessary, as for private equity, used our best proxy). We then created two optimal portfolios, one completely unconstrained except that the weights were normalized to sum to 100 percent, and the other with the addition of no shorting constraints. These optimal portfolios are shown in Table 7.2. What we see in the completely unconstrained portfolio is that indeed the optimizer found some rather interesting opportunities—to create a hugely levered exposure to the global fixed income index while shorting offsetting weights in most of its components. Similarly, the unconstrained optimal portfolio forms a large overweight to the EAFE equity index, while shorting offsetting weights in several of its components. The constrained portfolio cannot take advantage of these long/short opportunities, so it simply chooses to hold large weights in hedge funds and high yield, and a smaller weight in real estate. Notice that the constrained portfolio has a much lower return per unit of volatility. Both portfolios seem quite unreasonable, despite the fact that

TABLE 7.1 A Sample Long-Term Outlook in Early 2002

Asset Class	Return	Volatility
Japanese government bonds	4.7%	4.2%
European government bonds	5.1	3.6
U.S. government bonds	5.2	4.6
U.S. equities	5.4	15.5
Global fixed income	6.0	3.6
European equities	6.1	16.6
U.S. high-grade corporate bonds	6.3	5.4
EAFE	8.0	15.3
Hedge fund portfolio	8.0	5.2
U.S. high yield	8.9	7.3
Private equity	9.0	28.9
Emerging debt	9.0	17.6
REITs	9.0	13.0
Japanese equities	9.5	19.6
Emerging market equities	11.8	23.4

TABLE 7.2 Optimal Portfolio Weights

Asset Class	Unconstrained Portfolio	Portfolio with No Shorting Constraint
Japanese government bonds	–202.7%	0.0%
European government bonds	–321.1	0.0
U.S. government bonds	–484.4	0.0
U.S. equities	–11.3	0.0
Global fixed income	1493.2	0.0
European equities	–258.0	0.0
U.S. high-grade corporate bonds	–385.8	0.0
EAFE	314.3	0.0
Hedge fund portfolio	58.1	55.3
U.S. high yield	–9.9	36.3
Private equity	0.5	0.0
Emerging debt	–28.8	0.0
REITs	4.3	7.7
Japanese equities	–71.7	0.7
Emerging market equities	3.1	0.0
Portfolio volatility	4.9%	5.1%
Portfolio expected return	18.2	8.4

from a mathematical point of view they each optimize the problem that was posed. Given the input forecasts, a large number of relatively tight minimum and maximum holdings would have to be specified (indeed, this is the usual approach) in order to get reasonable-looking answers out of the optimizer. In this situation the optimizer is obviously not adding a lot of value.

In the Black-Litterman approach we don't start with a set of expected returns for all asset classes. Instead, we start with equilibrium expected returns, which lead to the optimal portfolio having market capitalization weights. Though perhaps reasonable looking, this market capitalization portfolio doesn't change very much over time, and the obvious question is how to use an optimizer to tilt away from this portfolio in order to take advantage of perceived opportunities.

We create a simple equity-only example in order to illustrate how sensitive the optimized portfolio is to small changes in expected returns. Equity markets are not as highly correlated as fixed income markets and currencies; if we were to use a more complete set of assets it would only compound the problem. The equity-only equilibrium expected excess returns, shown in Table 7.3 along with market capitalization, differ slightly from those shown for the more complete global market portfolio in Table 6.5. However, since equities dominate the risk of the market portfolio the differences are not that great.

Consider a hypothetical situation in which an investor believes that over the next three months the German economic growth will be slightly weaker than expected and German equity will underperform relative to equilibrium expectations. We suppose that the investor quantifies this view as a 20 basis point lower than equilibrium expected return on the German equity market over the next three

TABLE 7.3 Global Equity Market Portfolio

Country	Market Capitalization	Equilibrium Expected Return	Equilibrium Excess Return
United States	53.98%	8.50%	4.00%
United Kingdom	10.60	7.47	2.97
Japan	9.85	7.07	2.57
France	4.44	8.39	3.89
Switzerland	3.49	7.32	2.82
Germany	3.27	9.11	4.61
Netherlands	2.58	8.19	3.69
Canada	2.28	7.71	3.21
Italy	1.78	8.01	3.51
Australia	1.73	5.99	1.49
Spain	1.37	8.26	3.76
Sweden	0.87	9.59	5.09
Hong Kong	0.83	7.29	2.79
Finland	0.67	11.48	6.98
Belgium	0.48	6.71	2.21
Singapore	0.40	7.05	2.55
Denmark	0.36	6.69	2.19
Ireland	0.30	7.02	2.52
Norway	0.24	6.82	2.32
Portugal	0.19	6.40	1.90
Greece	0.14	6.82	2.32
Austria	0.07	5.20	0.70
New Zealand	0.06	5.35	0.85

months. The investor holds all other expected returns unchanged at their equilibrium values. Given this slight alteration in expected returns, in Table 7.4 we show two new optimal portfolios together with the deviations of these two portfolios from the market capitalization weights. The first portfolio is optimized with no constraints except that weights sum to 100 percent; the second portfolio includes constraints against shorting.

When the portfolio is optimized without constraints the optimizer quickly recognizes a slight inconsistency between the expected return for Germany and the other equity markets and treats this inconsistency as an opportunity. It suggests a 54 percent short position in Germany offset by overweight positions in most of the other equity markets. Notice also, though, the odd short positions in Japan, Finland, Australia, Norway, and New Zealand. When no shorting constraints are imposed the opportunity is significantly reduced. The German equity position is zero and other deviations from market capitalization weights are reduced proportionately.

This unconstrained optimal portfolio has an expected return of 8.1 percent and an annualized volatility of 15.2 percent. These compare to the equilibrium portfolio values of 8.1 percent and 16.2 percent, respectively. The view of a slightly lower expected return on German stocks has provided an opportunity to reduce risk,

TABLE 7.4 Optimal Portfolio Given Bearish View on German Equity

Country	Unconstrained	Change from Market Cap	No Shorting	Change from Market Cap
United States	57.6%	3.6%	54.2%	0.2%
United Kingdom	11.7	1.1	10.6	0.1
Japan	8.4	–1.4	9.8	–0.1
France	18.9	14.4	5.3	0.9
Switzerland	9.2	5.7	3.8	0.3
Germany	–53.7	–57.0	0.0	–3.3
Netherlands	11.5	8.9	3.1	0.5
Canada	2.9	0.6	2.3	0.0
Italy	14.6	12.9	2.5	0.7
Australia	–2.7	–4.4	1.5	–0.2
Spain	3.8	2.4	1.5	0.1
Sweden	8.1	7.3	1.3	0.4
Hong Kong	3.0	2.2	1.0	0.1
Finland	0.1	–0.6	0.6	0.0
Belgium	1.9	1.4	0.6	0.1
Singapore	1.0	0.6	0.4	0.0
Denmark	1.1	0.7	0.4	0.0
Ireland	2.2	1.9	0.4	0.1
Norway	–3.7	–3.9	0.0	–0.2
Portugal	2.8	2.6	0.4	0.2
Greece	0.7	0.5	0.2	0.0
Austria	1.2	1.1	0.1	0.1
New Zealand	–0.4	–0.4	0.0	0.0
Volatility	15.2		16.2	
Expected return	8.1		8.1	

while holding expected return essentially unchanged. In this sense the optimizer is working as it should.

If we compare the portfolio weights in the new unconstrained optimal portfolio with those of the global market capitalization weighted portfolio, however, the changes in country weights are very large, and in some cases inexplicable. This type of behavior is typical of an unconstrained mean-variance optimization. For this reason portfolio optimizations are usually run with many tight constraints on asset weights.

Black-Litterman addresses this excessive sensitivity of portfolio optimizations without adding constraints. The Black-Litterman approach assumes there are two distinct sources of information about future excess returns: investor views and market equilibrium. Both sources of information are assumed to be uncertain and are expressed in terms of probability distributions. The expected excess returns that are used to drive the portfolio optimization are estimates that combine both sources of information.

In the Black-Litterman model a view is a statement about the expected return of any portfolio together with a degree of confidence. Mathematically, a view is expressed as follows:

$$p\mu = q + \varepsilon \tag{7.1}$$

where p = n-vector of weights in the view portfolio, one for each of the n assets
μ = n-vector of expected excess returns on underlying assets
q = Expected excess return of the portfolio
ε = Normally distributed random variable

The confidence in the view is $1/\omega$ where ω is the variance of ε.

As an example, in order to express a bearish view on German equity, let p have weights reflecting a portfolio long 1 percent of German equities, in other words all zeros except a value of .01 for German equity. We let q reflect the 80 basis points less than equilibrium annualized performance suggested above. We specify a degree of confidence of 4 to reflect a one standard deviation uncertainty around q of 50 basis points. The Black-Litterman optimal portfolio, shown in Table 7.5, is simply

TABLE 7.5 Black-Litterman Portfolio Reflecting a Bearish View on German Equity

Country	Unconstrained	Change from Market Cap	Percent Change from Market Cap
United States	58.7%	4.7%	8.8%
United Kingdom	11.5	0.9	8.8
Japan	10.7	0.9	8.8
France	4.8	0.4	8.8
Switzerland	3.8	0.3	8.8
Germany	–5.2	–8.5	–259.6
Netherlands	2.8	0.2	8.8
Canada	2.5	0.2	8.8
Italy	1.9	0.2	8.8
Australia	1.9	0.2	8.8
Spain	1.5	0.1	8.8
Sweden	0.9	0.1	8.8
Hong Kong	0.9	0.1	8.8
Finland	0.7	0.1	8.8
Belgium	0.5	0.0	8.8
Singapore	0.4	0.0	8.8
Denmark	0.4	0.0	8.8
Ireland	0.3	0.0	8.8
Norway	0.3	0.0	8.8
Portugal	0.2	0.0	8.8
Greece	0.2	0.0	8.8
Austria	0.1	0.0	8.8
New Zealand	0.1	0.0	8.8
Volatility	15.9		
Expected return	7.7		

a set of deviations from market capitalization weights in the direction of the view portfolio—that is, a proportional increase in the market portfolio offset by a short position in German equities. The model provides the appropriate weights on the view portfolio, given the stated expected return on the portfolio and the degree of confidence in that view. The model balances the contributions to expected return of the view portfolio and the market portfolio against their contributions to overall portfolio risk. The result is transparent and intuitive.

How does this approach differ from the badly behaved approach of the standard optimizer? In both cases the unconstrained optimal portfolio, w^*, is given by the same matrix equation:

$$w^* = \kappa\Sigma^{-1}\mu^* \tag{7.2}$$

where κ = Risk aversion parameter
Σ = Covariance matrix of excess returns
μ^* = Vector of expected excess returns

The difference between the Black-Litterman approach and the previous approach is that rather than specifying the expected excess returns directly, we define view portfolios, specify expected returns and degrees of confidence in the view portfolios, and apply the following Black-Litterman formula:[3]

$$\mu^* = [(\tau\Sigma)^{-1} + P'\Omega^{-1} P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q] \tag{7.3}$$

This formula creates an expected excess return vector, μ^*, from the information in k views:

$$P\mu = Q + \varepsilon \tag{7.4}$$

and in a prior reflecting equilibrium:

$$\mu = \Pi + \varepsilon^e \tag{7.5}$$

In these formulas P is a $k \times n$ matrix specifying k view portfolios in terms of their weights on the n assets. Q is a k-vector expressing the expected excess returns on the k view portfolios. Ω is the covariance matrix of the random variables representing the uncertainty in the views. Π is the n-vector of equilibrium risk premiums. Finally, τ scales the covariance matrix of returns in order to specify the covariance matrix of the zero-mean distribution for ε^e.

Let us look at the Black-Litterman expected excess returns. These expected excess returns and their deviations from equilibrium are given in Table 7.6. In

[3]This formula was derived in the paper "Global Portfolio Optimization," by Fischer Black and Robert Litterman, *Financial Analysts Journal*, September–October 1992, pages 28–43. In a subsequent paper, "A Demystification of the Black-Litterman Model: Managing Quantitative and Traditional Portfolio Construction," published in the *Journal of Asset Management*, 2000, vol. 1, no. 2, pages 138–150, Stephen Satchell and Alan Scowcroft extend the analysis.

TABLE 7.6 Black-Litterman Expected Excess Returns

Country	Excess Returns	Deviation from Equilibrium
United States	3.64%	−0.36%
United Kingdom	2.61	−0.36
Japan	2.34	−0.23
France	3.38	−0.51
Switzerland	2.46	−0.37
Germany	3.93	−0.68
Netherlands	3.20	−0.49
Canada	2.88	−0.32
Italy	3.02	−0.49
Australia	1.34	−0.15
Spain	3.27	−0.50
Sweden	4.46	−0.63
Hong Kong	2.47	−0.33
Finland	6.17	−0.81
Belgium	1.91	−0.30
Singapore	2.26	−0.30
Denmark	1.93	−0.26
Ireland	2.20	−0.32
Norway	2.04	−0.28
Portugal	1.63	−0.27
Greece	2.03	−0.29
Austria	0.60	−0.10
New Zealand	0.75	−0.10

contrast to the traditional approach, the Black-Litterman model adjusts all of the expected returns away from their starting values in a manner consistent with the views being expressed. Because the view expressed here is bearish on German equities, the expected returns on German equities decline. The total adjustment away from equilibrium is 68 basis points, less than the 80 basis points expressed in the view. This result reflects the assumption that the view has some uncertainty associated with it. The equilibrium is given some weight as well and acts as a center of gravity, pulling the Black-Litterman expected returns away from the view itself, back toward the equilibrium values.

Suppose we add another view. This time let us specify that a portfolio long 100 percent of Japanese equity and short 100 percent of U.K. equity will have a positive expected excess return of 100 basis points. We also give this view a confidence of 4 and assume that its error is uncorrelated with that of the previous view.

The unconstrained Black-Litterman optimal portfolio given these two views is shown in Table 7.7. We can see that the deviations of the optimal portfolio from equilibrium weights are exactly proportional to the sum of the two view portfolios. This result illustrates a very important general property of the Black-Litterman model. In general, the unconstrained optimal portfolio from the Black-Litterman

TABLE 7.7 Optimal Portfolio Given Two Views

Country	Excess Returns	Deviation from Equilibrium	Portfolio Weights	Percent Deviation from Market Cap
United States	3.71%	−0.29%	53.98%	0.00%
United Kingdom	2.59	−0.38	3.96	−6.64
Japan	2.72	0.14	16.49	6.64
France	3.44	−0.46	4.44	0.00
Switzerland	2.48	−0.34	3.49	0.00
Germany	4.04	−0.57	−2.27	−5.54
Netherlands	3.25	−0.45	2.58	0.00
Canada	2.94	−0.26	2.28	0.00
Italy	3.09	−0.42	1.78	0.00
Australia	1.41	−0.09	1.73	0.00
Spain	3.33	−0.43	1.37	0.00
Sweden	4.56	−0.53	0.87	0.00
Hong Kong	2.58	−0.22	0.83	0.00
Finland	6.27	−0.71	0.67	0.00
Belgium	1.93	−0.28	0.48	0.00
Singapore	2.40	−0.15	0.40	0.00
Denmark	1.98	−0.22	0.36	0.00
Ireland	2.25	−0.26	0.30	0.00
Norway	2.08	−0.24	0.24	0.00
Portugal	1.70	−0.21	0.19	0.00
Greece	2.10	−0.23	0.14	0.00
Austria	0.60	−0.10	0.07	0.00
New Zealand	0.79	−0.06	0.06	0.00

model is the market equilibrium portfolio plus a weighted sum of the portfolios about which the investor has views.

We will now investigate how changes in some of the Black-Litterman parameters affect the optimal portfolio tilts. In this simple unconstrained optimization environment,[4] we can characterize the deviations of the optimal portfolios from the market capitalization portfolio by the weights, w_1 and w_2, on the two view portfolios. For example, in Table 7.7, $w_1 = 5.54$ and $w_2 = 6.64$. In Table 7.8 we show how these weights vary with changes in the expected excess returns of the view portfolios (q_1 and q_2), the degrees of confidence ($1/\omega_1$ and $1/\omega_2$), and the correlation between the views. Notice that a view portfolio is given zero weight not when it has zero expected return, but rather when it has a return equal to that implied by a combination of equilibrium and all other views. Thus, adding a view creates a positive tilt toward that view portfolio only when the view is more bullish than the expected return implied by the Black-Litterman model without this particular view.

In an unconstrained optimization environment the Black-Litterman model is, in some respects, a complex tool for solving a relatively straightforward problem.

[4]See He and Litterman (1999).

TABLE 7.8 Effect of Parameter Changes on View Weights

Scenario	Expected Return		Confidence		Correlation	Weights on Views	
	View 1	View 2	View 1	View 2		View 1	View 2
Equilibrium	0.80%	0.40%	0	0	0	0.00%	0.00%
Base case	0.80	0.40	4	4	0	5.54	6.64
Weaker view 1	0.40	0.40	4	4	0	2.28	7.13
Stronger view 1	1.60	0.40	4	4	0	12.07	5.65
More confidence in view 1	0.80	0.40	16	4	0	7.40	6.36
Less confidence in view 1	0.80	0.40	1	4	0	2.77	7.06
No confidence in view 1	0.80	0.40	0	4	0	0.00	7.47
Zero expected return on view 1	0.00	0.40	4	4	0	–0.99	7.62
12 bps expected return on view 1	0.12	0.40	4	4	0	0.00	7.48
Weaker view 2	0.80	0.25	4	4	0	5.73	5.21
Stronger view 2	0.80	1.00	4	4	0	4.80	12.34
More confidence in view 2	0.80	0.40	4	16	0	5.19	9.38
Less confidence in view 2	0.80	0.40	4	1	0	6.01	3.06
No confidence in view 2	0.80	0.40	4	0	0	6.40	0.00
Zero expected return on view 2	0.80	0.00	4	4	0	6.04	2.83
–20 bps expected return on view 2	0.80	–0.20	4	4	0	6.28	0.93
–30 bps expected return on view 2	0.80	–0.30	4	4	0	6.41	–0.02
–40 bps expected return on view 2	0.80	–0.40	4	4	0	6.53	–0.97
Positively correlated views	0.80	0.40	4	4	0.5	6.67	7.74
Negatively correlated views	0.80	0.40	4	4	–0.5	4.68	5.87
Positively colinear views	0.80	0.40	4	4	1	8.26	9.38
Negatively colinear views	0.80	0.40	4	4	–1	3.94	5.37

Once one recognizes that view portfolios provide a flexible format for formulating views, and that the optimal portfolio is simply one that tilts with some set of weights on the view portfolios, it is probably easier to specify weights on those tilt portfolios directly rather than to specify expected returns, degrees of confidence, and correlations between views. There are, however, at least two reasons why the Black-Litterman model is necessary.

First, if one simply specifies weights on view portfolios, one loses the insights that Black-Litterman brings concerning the effects of the different parameters on the optimal weights. Of course that loss has to be balanced against the difficulty of knowing how to set those parameters in the first place. Since the original Black-Litterman paper was written, I have often received the question, "How do you determine the omega matrix?" There is no simple or universal answer. We know what these parameters represent—the expected excess returns on the view portfolios, the degree of uncertainty in the views, and the correlations between views—but the right way to specify such information is certainly context dependent. When the views are the product of quantitative modeling, for example, the expected returns might be a function of historical performance, the degree of confidence might be set proportional to the amount of data supporting the view, and correlations between views might be assumed to be equal to the historical correlations between view portfolio returns.

Other direct approaches to specifying weights on view portfolios can generally be mapped into particular assumptions on the expected excess returns and the omega matrix of Black-Litterman. At least in the context of Black-Litterman, the portfolio manager knows what these parameters represent, and can thus address the issue of whether those specifications make sense.

The second, and perhaps more important, reason that the Black-Litterman framework really is necessary is because in the real world one hardly ever optimizes in an unconstrained environment. The real power of the Black-Litterman model arises when there is a benchmark, a risk or beta target, or other constraints, or when transaction costs are taken into account. In these more complex contexts, the optimal weights are no longer obvious or intuitive. The optimal portfolio is certainly not simply a set of tilts on view portfolios. Nonetheless, the manager can be confident that when the optimizer goes to work using the Black-Litterman expected excess returns, the same trade-off of risk and return—which leads to intuitive results that match the manager's intended views in the unconstrained case—remains operative when there are constraints or other considerations.

Having made this point, it is nonetheless worth noting that, as shown in He and Litterman (1999), in a few special cases the optimal portfolios given constraints retain some intuitive properties. In our paper we consider in turn the case of a risk constraint, a leverage constraint, and a market exposure constraint. In the case of optimizing relative to a specified level of risk, the optimal portfolio is just a linearly scaled version of the solution of the unconstrained optimization problem. However, because of the scaling, the view portfolio deviations no longer tilt away from the market portfolio, but rather from a scaled market portfolio. Otherwise the intuition of the unconstrained portfolio remains.

In the case of a fully invested, no-leverage constraint, a constraint where the portfolio weights sum to 1, another portfolio enters the picture. There exists a "global minimum-variance portfolio" that minimizes the risk of all portfolios that are fully invested in risky assets. When portfolios are optimized subject to being fully invested, the optimal portfolio is a weighted average of the unconstrained optimal portfolio and the global minimum-variance portfolio.

Finally, a common constraint on portfolios is that their market exposure is 1, meaning that the coefficient or beta in a projection on the market portfolio is 1. In this case, the Black-Litterman optimal portfolio is a linear combination of the unconstrained optimal portfolio, the global minimum-variance portfolio, and the equilibrium portfolio.