Short selling

- Short seller borrows shares and sells them
- Proceeds plus additional cash remain with lender as collateral
  - Marked to market daily
  - Earn interest: “rebate” rate
    - often modestly below market interest rate
    - sometimes far below (“specials”), even going negative
- Short-sale cost:
  - market interest rate minus rebate rate
  - like other fees/costs, we will often omit for simplicity
- Short seller pays lender any dividend amounts
- To close short position, buy shares and return them to lender

A Hedge Fund's Balance Sheet

ASSETS	LIABILITIES
Cash posted as collateral for short securities positions	Short security positions
Additional margin requirement for shorts	Equity supporting shorts
Cash in money market account	Equity, additional
Long security positions	Equity supporting longs
	Margin loan for long positions

Total equity

Short-Sale Profit/Loss

- Define

  X_S: minus the current market value of stocks held short (i.e., $X_S < 0$)

  $r_{S,t+1}$: return over the next period on the portfolio of stocks held short

  $r_{f,t+1}$: riskless interest rate (= rebate rate, for simplicity)

  $R_{S,t+1}$: excess return on the portfolio of shorted stocks ($= r_{S,t+1} - r_{f,t+1}$)

- The profit on the short position over the next period:

$$\begin{aligned} \text{short profit} &= X_S r_{S,t+1} - X_S r_{f,t+1} \\ &= X_S R_{S,t+1} \end{aligned}$$

- The short position is a loan of stock—it does not change total invested wealth

- Invested wealth consists of long positions plus the (positive or negative) cash position.

Short-Sales and Portfolio Return

- Define

  X_L: the current market value of stocks held long

  X_C: value of cash position, positive (lending) or negative (borrowing)

  $r_{L,t+1}$: return over the next period on the portfolio of stocks held long

  $R_{L,t+1}$: excess return on the portfolio of stocks held long

- Invested wealth:

$$W = X_L + X_C$$

- Profit on the overall portfolio:

$$\begin{aligned}
\text{total profit} &= \text{(profit on long)} + \text{(profit on cash)} + \text{(profit on short)} \\
&= X_L r_{L,t+1} \quad + \quad X_C r_{f,t+1} \quad + \quad X_S R_{S,t+1} \\
&= X_L r_{L,t+1} \quad + \quad (W - X_L) r_{f,t+1} \quad + \quad X_S R_{S,t+1}
\end{aligned}$$

Short-Sales and Portfolio Return (continued)

- Dividing by W and subtracting $r_{f,t+1}$ gives the long-short portfolio's excess return:

$$\begin{aligned} R_{p,t+1} &= \frac{X_L r_{L,t+1} + (W - X_L) r_{f,t+1} + X_S R_{S,t+1}}{W} - r_{f,t+1} \\ &= \frac{X_L}{W}(r_{L,t+1} - r_{f,t+1}) + \frac{X_S}{W} R_{S,t+1} \\ &= \frac{X_L}{W} R_{L,t+1} + \frac{X_S}{W} R_{S,t+1} \\ &= w_L R_{L,t+1} + w_S R_{S,t+1}, \end{aligned}$$

  defining $w_L = X_L/W$ and $w_S = X_S/W$. (Recall $w_S \leq 0$.)

- Note, in general, $w_L + w_S \neq 1$
- Standard (Reg-T) 50% margin requirement: $w_L + |w_S| \leq 2$

Short-Sales and the Characteristic Line

- The long-short portfolio's excess return given by

$$R_{P,t+1} = w_L R_{L,t+1} + w_S R_{S,t+1},$$

- Substituting characteristic-line regressions for $R_{L,t+1}$ and $R_{S,t+1}$:

$$\begin{aligned} R_{p,t+1} &= w_L \left(\alpha_L + \beta_L R_{M,t+1} + \epsilon_{L,t+1}\right) + w_S \left(\alpha_S + \beta_S R_{M,t+1} + \epsilon_{S,t+1}\right) \\ &= \alpha_P + \beta_P R_{M,t+1} + \epsilon_{P,t+1} \end{aligned}$$

where

$$\begin{aligned} \alpha_P &= w_L \alpha_L + w_S \alpha_S \\ \beta_P &= w_L \beta_L + w_S \beta_S \\ \epsilon_{P,t+1} &= w_L \epsilon_{L,t+1} + w_S \epsilon_{S,t+1} \end{aligned}$$

Simple Case: Symmetric Risks

- Suppose the long and short legs have equal betas and volatilities:

$$\begin{aligned} \beta_L &= \beta_S \\ \sigma\left(\epsilon_L\right) &= \sigma\left(\epsilon_S\right) = \sigma_\epsilon \end{aligned}$$

- Take equal long and short position sizes: $w_L = -w_S = w$
- Then

$$\begin{aligned} \alpha_p &= w\left(\alpha_L - \alpha_S\right) \\ \beta_p &= 0 \\ \epsilon_{P,t+1} &= w\left(\epsilon_{L,t+1} - \epsilon_{S,t+1}\right) \end{aligned}$$

- Variance:

$$\begin{aligned} \sigma^2\left(\epsilon_P\right) &= w^2\left[\sigma_\epsilon^2 + \sigma_\epsilon^2 - 2\mathrm{cov}(\epsilon_L, \epsilon_S)\right] \\ &= 2w^2\sigma_\epsilon^2\left(1-\rho\right), \end{aligned}$$

where $\rho = \mathrm{corr}\left(\epsilon_L, \epsilon_S\right)$.

- Volatility:

$$\sigma\left(\epsilon_P\right) = \sqrt{2}w\sigma_\epsilon\sqrt{1-\rho}$$

Alpha Transport

- Two market segments, e.g.,

  A: domestic U.S. stocks

  B: international stocks

- Manager P trades in segment B

$$R_{P,t} = \alpha_P + \beta_{P,B} R_{B,t} + \epsilon_{P,t}$$

- Also, the manager's portfolio has
  - $\beta_{P,B} = 0$, via long-short or futures/swaps
  - cov $(\epsilon_{P,t}, R_{A,t}) = 0$ (= cov $(R_{P,t}, R_{A,t})$, because $\beta_{P,B} = 0$)

- "Transport" the manager's alpha to market segment A:
  - Invest W dollars in portfolio P
  - Take long position of size W in futures on A

- Resulting return: exposure to A but with the alpha produced in B

$$R_{C,t} = \alpha_P + R_{A,t} + \epsilon_{P,t}$$