BOOLEAN ALGEBRA. A CORRECTION

BY

EDWARD V. HUNTINGTON

In my paper in these Transactions for January, 1933, the Example 4.5 on page 286 is erroneous, and Postulate 4.5 on page 280 is in fact redundant. Hence the "fourth set" of postulates for Boolean algebra, on the base $(K, +, ')$, should read as follows (the class K being understood to contain at least two distinct elements):

POSTULATE 4.1. *If a and b are in K, then $a+b$ is in K.*

POSTULATE 4.2. *If a is in K, then a' is in K.*

POSTULATE 4.3. $a+b=b+a$.

POSTULATE 4.4. $(a+b)+c=a+(b+c)$.

POSTULATE 4.6. $(a'+b')'+(a'+b)'=a$ [or, $ab+ab'=a$, where, by definition, $ab=(a'+b')'$].

The steps by which the proposition 4.5 $(a+a=a)$ is deduced as a theorem from Postulates 4.1, 4.2, 4.3, 4.4, and 4.6, are as follows.*

4.10. $a''=a$. (Proof as on page 281.)

4.11. $a+a'=b+b'$.

Proof (without using 4.5). By 4.6, with 4.3 and 4.4,

$$a + a' = [(a' + b'')' + (a' + b')'] + [(a'' + b'')' + (a'' + b')']$$
$$= [(b' + a'')' + (b' + a')'] + [(b'' + a'')' + (b'' + a')'] = b + b'.$$

4.12. Definition. $U=a+a'=$ the "universe element" of the system.

In particular, $U=U+U'$.

4.15. $a+U'=a$.

By 4.6, 4.10, 4.12,

(a) $$U' = (U + U)' + (U + U')' = (U + U)' + U'.$$

By 4.12, (a), 4.4, 4.12,

$$U = U + U' = U + [(U + U)' + U']'$$
$$= (U + U') + (U + U)' = U + (U + U)'.$$

Hence by 4.4, 4.12,

(b) $$U + U = U + [U + (U + U)'] = (U + U) + (U + U)' = U.$$

From (a), (b), $U'=U'+U'$, whence by 4.12,

---

* For an essential step in this proof I am indebted to Mr. B. Notcutt, a Commonwealth Fellow from Oxford University, at present a graduate student in Harvard University. The following article appeared after my brief bibliography was completed: W. V. Quine, *A note on Nicod's postulate*, Mind, vol. 41 (1932), pp. 345–350.

License or copyright restrictions may apply to redistribution; see https://www.ams.org/journal-terms-of-use

(c) $$(a' + a)' = (a' + a)' + (a' + a)'.$$

By 4.12, 4.6, 4.4,

$$a + U' = a + (a' + a)' = [(a' + a')' + (a' + a)'] + (a' + a)'$$
$$= (a' + a')' + [(a' + a)' + (a' + a)'],$$

whence by (c), 4.6,

$$a + U' = (a' + a')' + (a' + a)' = a.$$

4.5. $a+a=a$.

By 4.15, 4.3, 4.12, 4.6, 4.10,

$$(a + a)' = U' + (a + a)' = (a + a')' + (a + a)' = a'.$$

Hence, by 4.10, $a+a=a$.

The number of postulates in the "fourth set," as thus corrected, is no larger than the number in the "fifth set."

HARVARD UNIVERSITY,
    CAMBRIDGE, MASS.