A Fast and Robust Failure Analysis of Memory Circuits Using Adaptive Importance Sampling Method

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ABSTRACT

Performance failure has become a growing concern for the robustness and reliability of memory circuits. It is challenging to accurately estimate the extremely small failure probability when failed samples are distributed in multiple disjoint failure regions. In this paper, we develop an adaptive importance sampling (AIS) method. AIS has several iterations of sampling region adjustments, while existing methods pre-decide a static sampling distribution. By iteratively searching for failure regions, AIS may lead to better efficiency and accuracy. This is validated by our experiments. For SRAM cell with single failure region, AIS uses 5-10X fewer samples and reaches better accuracy when compared to several recent methods. For sense amplifier circuit with multiple failure regions, AIS is 4369X faster than MC without compromising accuracy, while other methods fail to cover all failure regions in our experiment.

CCS CONCEPTS

• **Hardware → Failure prediction;**

KEYWORDS

Process Variation, Failure Probability, Adaptive Importance Sampling, Failure Regions

1 INTRODUCTION

As microelectronic devices shrink to nano-meter scale, process variation has become a growing concern for efficient circuit sizing and design. For memory circuits, such as SRAM cells and their peripheral circuits, which are replicated for millions of times, the tolerable failure probability is extremely small. In order to achieve robust design, we need to establish very accurate estimation of failure probability for each circuit component. Traditional circuit simulation approaches perform worst case analysis (WCA) or build analytical model to deterministically estimate the failure probability. It is, however, infeasible in "rare-event" scenario.

In general, the probability estimation of "rare-event" is achieved by sampling methods. Among those methods, Monte Carlo (MC) approach remains the gold standard, which repeatedly generates samples and evaluates circuit performance with transistor-level SPICE simulation. However, MC is extremely time-consuming under the "rare-event" scenario because millions of simulations are required to capture one single failure event.

To address this issue, Boundary Searching (BS) method has been developed to speed up the failure probability estimation. For example, Statistical Blocade (SB) [1] applies a classifier to filter the more likely-to-fail Monte Carlo samples and only runs simulation on these samples. More recently, Recursive SB [2] and REscope [3] construct conditional classifier and SVM-based nonlinear classifier, respectively. However, training such efficient classifiers is expensive and the complexity grows exponentially as failure probability decreases.

In addition, Importance Sampling (IS) has been proposed based on the insight that drawing samples in the likely-to-fail regions can improve the estimation accuracy and meanwhile accelerate the estimation convergence. For example, Mixture Importance Sampling (MixIS) [4] utilizes a mixture of a uniform distribution, the original distribution and a shifted distribution centered at the failure region as the optimal sampling distribution. The approaches in [5, 6] simply shift the center of sampling distribution toward the minimum L_2-norm point of a set of failure samples, while the method in [7] shifts the sample mean to the centroid of failure samples. In order to tackle multi-failure-region problems, approaches in [8, 9] attempt to construct multiple mean shift vectors to cover the failure events distributed in different failure regions. However, all these methods construct a static likely-to-fail region to sample from, which makes them vulnerable to poor initializations.

In this paper, we present an efficient and accurate algorithm based on Adaptive Importance Sampling (AIS) method for the failure probability estimation of memory circuits. At each iteration, AIS generates samples from current proposed distribution. Next, AIS carefully assigns weight to each sample based on its tilted occurrence probability between failure region and current proposed distribution. Then we resample with replication among these samples to update the proposed distribution. Unlike the traditional

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static importance sampling approaches, this procedure can proceed iteratively to search for target failure region.

Although the idea of AIS was initially developed in the statistics field [10, 11], it was previously unknown how to be adapted to the failure probability estimation of memory circuits. In general, our major contribution is presented in three aspects: first, a primary difficulty with AIS algorithm is to initialize the parameterized sampling distritbution. To solve this issue, spherical presampling is performed in order to collect a number of failure events and sketch initial proposed distribution. Second, we select circuit failure event distribution as the AIS target distribution, which connects AIS algorithm to circuit failure prediction problem. Moreover, we implement multinomial resampling scheme to prevent weight degeneracy, which discards low-weight samples and regenerates high-weight samples. To the best of our knowledge, this is the first work on successfully developing adaptive importance sampling method for circuit failure probability estimation.

2 BACKGROUND

2.1 Rare Event Analysis

In general, it is of great interest to estimate the probability of Y belongs to a subset S of the entire parametric space. For example, under circuit failure rate estimation scenario, a tiny subset S can denote the "failure region" which includes all the failed samples. Thereby, we introduce indicator function $I(X)$ to identify pass/fail of Y:

$$I(X) = \begin{cases} 0, & if \quad Y \notin S \\ 1, & if \quad Y \in S \end{cases} \tag{1}$$

Therefore, the probability P_{fail} can be calculated as

$$P_{fail} = P(Y \in S) = \int I(X) \cdot f(X) dX \tag{2}$$

2.2 Importance Sampling

When S denotes a rare event, such as circuit failure, standard MC becomes infeasible because it requires millions times of simulations to capture one single failure event. To avoid massive MC simulations, importance sampling (IS) has been introduced to sample from a "distorted" proposed sampling distribution $g(X)$ that tilts towards the failure region S.

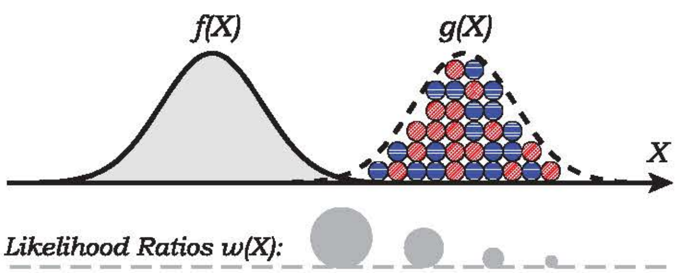

Figure 1: Mean-shift importance sampling

As shown in one-dimensional illustrative Figure 1, IS samples from a proposed distribution $g(X)$ which contains more statistically likely-to-fail samples. Here failure probability can be expressed as:

$$P_{fail} = P(Y \in S) = \int I(X) \cdot \frac{f(X)}{g(X)} \cdot g(X) dX \tag{3}$$

$$= \int I(X) \cdot w(X) \cdot g(X) dX \tag{4}$$

where $w(X)$ denotes the likelihood ratio between original PDF $f(X)$ and shifted proposed PDF $g(X)$. $w(X)$ compensates for the discrepancy between $f(X)$ and $g(X)$. Then unbiased IS estimator $\hat{P}_{IS,fail}$ can straightforwardly collect samples $\{x_j\}_{j=1}^{M}$ from $g(X)$:

$$\hat{P}_{IS,fail} = \hat{P}_{IS}(Y \in S) = \frac{1}{M} \sum_{j=1}^{M} w(x_j) I(x_j) \xrightarrow{M \to +\infty} P(Y \in S) \tag{5}$$

We notice that IS sample set $\{x_j\}_{j=1}^{M}$ in (6) has much smaller size than MC sample set $\{x_i\}_{i=1}^{N}$ in (3) because failure in $g(X)$ is not a "rare event".

Theoretically, it is obvious that the optimal sampling distribution $g^{opt}(X)$ is the failure event distribution $\pi(X)$:

$$g^{opt}(X) = \frac{I(X) \cdot f(X)}{P_{fail}} \tag{6}$$

However, $g^{opt}(X)$ cannot be evaluated with (7) directly because the analytical expression of $I(X)$ is unknown and P_{fail} is indeed the desired failure rate. In practice, most existing approaches attempt to shift the sample mean toward either accept/fail boundary [5], centroid of a cluster of failure samples [7], or centroids of multiple clusters [8]. However, current mean-shift IS implementations suffer from two major drawbacks:

First, existing IS approaches consist of two steps: a presampling step and an importance sampling step. Generally speaking, presampling is to construct proposed distribution and importance sampling is to evaluate P_{fail}. However, it is extremely inefficient because the shifted samples in importance sampling step are pre-fixed. Their algorithms are therefore vulnerable to poor initial conditions.

Moreover, the likelihood ratio $w(X)$ is substantially biased because the huge discrepancy between $f(X)$ and $g(X)$. This could lead to disastrous consequences when only a few samples contribute to the failure probability calculation. It is the major challenge to obtain consistent estimation.

3 ADAPTIVE IMPORTANCE SAMPLING ALGORITHM

3.1 Algorithm Description

The pseudo-code of Adaptive Importance Sampling (AIS) algorithm is summarized in Algorithm 1. The objective is to approximate the target failure event distribution, $\pi(x)$, by a set of samples and weights. Since the analytical expression of $\pi(x)$ is unavailable, we cannot directly draw samples from it. Our novel approach is to "learn" from the past simulations and construct the sequence of adaptivei probability densities. First of all, AIS start with a mixture density $g^{(0)}(x)$ built from a class of N proposed distributions $q_i^{(0)}(x|\mu_i^{(0)}, \Sigma_i^{(0)})$ with location parameters $\mu_i^{(0)}$ and covariance matrices $\Sigma_i^{(0)}$. At each iteration t, we independently generate N samples $x_i^{(t)}$ from the N previous proposed distributions

Algorithm 1: AIS Algorithm

Initialization: Set iteration index $t = 0$, construct N Guassian distributions,

$$q_i^{(0)}(x) = q_i^{(0)}(x|\mu_i^{(0)}, \Sigma_i^{(0)}) \qquad i = 1, \ldots, N,$$

with $\mu_i^{(0)} = x_i^{(0)} \qquad i = 1, \ldots, N,$

and $\Sigma_i^{(0)} = \Sigma_i \qquad i = 1, \ldots, N,$ where Σ_i is the predefined covariance matrices.

Repeat

Update iteration index $t = t + 1$.

1. **Sample propagation:**
   Generate a new set of N samples $\{x_i^{(t)}\}_{i=1}^N$ in the parametric space:
   $$x_i^{(t)} \sim q_i^{(t-1)}(x|\mu_i^{(t-1)}, \Sigma_i^{(t-1)}) \qquad i = 1, \ldots, N.$$
2. **Update:**
   (a) Compute incremental importance weights
   $$w_{i,t} = \frac{\pi(x)}{g^{(t-1)}(x)} = \frac{f(x)I(x)}{\frac{1}{N}\sum_{i=1}^N q_i^{(t-1)}(x|\mu_i^{(t-1)}, \Sigma_i^{(t-1)})} \qquad i = 1, \ldots, N.$$
   (b) Update the unbiased estimator
   $$\hat{P}_{fail,t} = \frac{1}{tN}\sum_{j=1}^{t}\sum_{i=1}^{N} w_{i,t} \qquad i = 1, \ldots, N.$$
   (c) Normalize importance weights for resampling
   $$\bar{w}_{i,t} = \frac{w_{i,t}}{\sum_{i=1}^N w_{i,t}} \qquad i = 1, \ldots, N.$$
3. **Resampling:**
   Resample with replication from N samples in current iteration $\{x_i^{(t)}\}_{i=1}^N$ based on the normalized importance weights $\{\bar{w}_{i,t}\}_{i=1}^N$. We obtain a set of new location parameters $\{\mu_i^{(t)}\}_{i=1}^N$ for next iteration.

Until

Relative standard deviation (FOM): $\rho = \frac{\sqrt{\sigma^2_{\hat{P}_{fail}}}}{\hat{P}_{fail}} \leq 0.1$

$q_i^{(t-1)}(x|\mu_i^{(t-1)}, \Sigma_i^{(t-1)})$. Then we update the location parameter $\mu_i^{(t)}$ through a carefully designed resampling procedure which keeps estimator unbiased and reduces variance of sample weights. This sampling and resampling procedure can proceed iteratively until the discrepancy between the target distribution and proposed distribution is not reduced anymore.

3.2 Initializing and Resampling Scheme

3.2.1 ***Initialization.*** One major challenge in implementing our AIS algorithm is that the selection of initial location parameters $\{\mu_i^{(0)}\}_{i=1}^N$ can significantly affect the efficiency of our iterative search procedure. An appropriate initialization can achieve high sample diversity, quick convergence, and better capability to explore parametric space.

In order to develop a good initialization, as shown in Figure 2, we start with circuit nominal distribution, which denotes a unit hypersphere in parametric space. The radius of hypersphere indicates the variance of circuit parameters. We notice that most likely no failure event is captured under nominal distribution. To cover multiple failure regions, we gradually increase the radius of hypersphere until N failure samples are captured. Here variable N, which denotes the number of proposed distributions in Algorithm 1 can be arbitrarily user-defined. The trade-off between estimation accuracy and algorithm complexity is obvious: the larger N is, the higher estimation accuracy we can achieve while sacrificing specific simulation runtime. In our experiments, we choose N to be 128 and collect 1000 samples on each hypersphere.

An alternative view of our spherical presampling method is to project high dimensional samples onto surface of multiple hyperespheres, which is a dimension reduction process. This process is capable of accelerating the exploration in entire open space while maintaining the sample diversity.

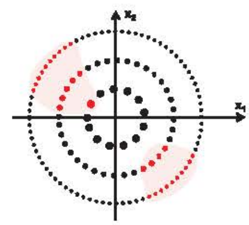

Figure 2: Spherical Presampling

3.2.2 ***Resampling.*** As explained in Section 2, standard importance sampling suffers from weight bias in the estimates. That is, one or a few samples occupy most of the weights while others are trivial. Our solution is to replicate samples with larger weights and eliminate samples with smaller weights. Multinomial resampling technique is applied in our AIS algorithm to iteratively improve the sample set $\{x_i^{(t)}, w_{i,t}\}_{i=1}^N$, as shown in Figure 3. To be specific, we resample with replication from a random measure $\{x_i^{(t)}\}_{i=1}^N$ according to $\{w_{i,t}\}_{i=1}^N$, generating a new set of location parameters $\{\mu_i^{(t)}\}_{i=1}^N$. Then, a set of updated random measure $\{x_i^{(t+1)}, w_{i,t+1}\}_{i=1}^N$ are generated from $q_i^{(t)}(x|\mu_i^{(t)}, \Sigma_i^{(t)})$. This resampling approach effectively mitigates weight discrimination, and thus guarantees each sample uniformly contribute to the estimator.

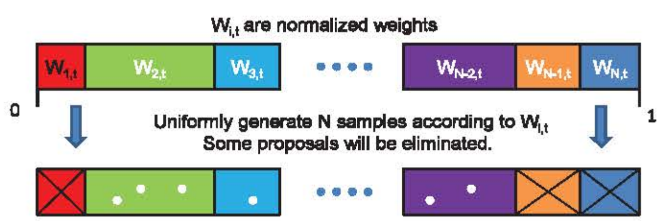

Figure 3: A description of multinomial resampling procedure.

3.3 AIS Estimator Analysis

3.3.1 ***Unbiasedness of AIS Estimator.*** First of all, from equation (8) to (10), we will derive that our AIS estimator $\hat{P}_{fail}$ keeps the unbiasedness of traditional static importance sampling estimator. It guarantees the estimation built from final random measure $\{x_i^{(t)}, w_{i,t}\}_{i=1}^N$ converge to the failure probability P_{fail}.

$$E[\hat{P}_{fail}] = E[\frac{1}{tN}\sum_{k=1}^{t}\sum_{i=1}^{N} w_{i,j}] \tag{7}$$

$$= \frac{1}{N}\sum_{i=1}^{N} E[\frac{f(x)I(x)}{\frac{1}{N}\sum_{j=1}^{N} q_j^{(t)}(x|\mu_j^{(t)}, \Sigma_j^{(t)})}] \tag{8}$$

$$= \frac{1}{N}\sum_{i=1}^{N}\int \frac{f(x)I(x)}{\frac{1}{N}\sum_{j=1}^{N} q_j^{(t)}(x)} q_i^{(t)}(x) = P_{fail} \tag{9}$$

3.3.2 ***Weight balancing.*** In this section, we investigate how our AIS estimator outperforms other importance sampling based estimator in terms of weight balancing. As we explained in Section 2, one major challenge of traditional IS technique occurs when the probability discrepancy between original distribution $f(X)$ and proposed distribution $g(X)$ is large. These overwhelming fluctuations in likelihood ratio $w(X)$ can be prevented by our adaptive steps. To quantitatively analyze this feature, we introduce Effective Sample Size (ESS), which reflects the degree of weight degeneracy. ESS is extensively defined as [12]:

$$ESS = \frac{1}{\sum_{i=1}^{N}(\bar{w}_{i,t})^2} \tag{10}$$

which involves only the normalized weights $\{\bar{w}_{i,t}\}_{i=1}^N$. As we have illustrated in Section 3.2.2, our resampling scheme generates samples from "parent" samples with larger weights and eliminates samples with smaller weights. Thus $\{\bar{w}_{i,t}\}_{i=1}^N$ is naturally balanced and converge to optimal $w^* = \frac{1}{N}$. An alternative view is that our entire AIS samples set $\{x_i^{(t)}, w_{i,t}\}_{i=1}^{NT}$ have equivalent influence on the estimator $\hat{P}_{fail} = \frac{1}{TN}\sum_{t=1}^{T}\sum_{i=1}^{N} w_{i,t}$. To this end, the resampling method can be adapted to maintain the robustness of AIS estimator.

4 EXPERIMENT RESULT

The proposed AIS method is first evaluated on a typical 6T-SRAM circuit with 54 variables, then on a sense amplifier circuit with 63 variables. In our experiment settings, the sense amplifier circuit is known to have multiple failure regions. We also implemented different methods, including Monte Carlo (MC), High Dimensional Importance Sampling (HDIS) [7], and Hyperspherical Clustering and Sampling (HSCS) [8] for comparison purpose. We run HSPICE with SMIC 40nm model.

4.1 Experiments on 6T SRAM Bit Cell

The schematic of typical 6-transistors SRAM bit cell is shown in Figure 4: four transistors $MN1$, $MP2$, $MN3$ and $MP4$ form two cross-coupled inverters and utilize two steady states to store either '0' or '1'; transistor $MN5$ and $MN6$ control accessing to the storage cell during read, write or standby operations. The SRAM reading failure occurs when the voltage difference between BL and $\bar{BL}$ is too small to be captured by sense amplifier at the end of reading operation. We will compare in terms of accuracy and efficiency.

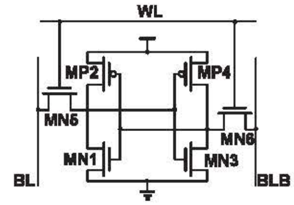

Figure 4: The schematic of typical 6T SRAM cell

4.1.1 ***Accuracy Comparison.*** In order to evaluate the accuracy, Figure of Merit, ρ, is employed to characterize the accuracy and confidence of our estimation[5]. It is defined as:

$$\rho = \frac{\sqrt{\sigma^2_{\hat{P}_{fail}}}}{\hat{P}_{fail}} \tag{11}$$

where $\hat{P}_{fail}$ denotes the estimation of failure probability and $\sigma_{\hat{P}_{fail}}$ denotes the standard deviation of $\hat{P}_{fail}$. In fact, we can declare one estimate is $(1-\epsilon)100\%$ accurate with $(1-\delta)100\%$ confidence when $\rho < \epsilon\sqrt{log(1/\delta)}$. Here we use $\rho < 0.1$ to indicate the estimation reach a steady state with 90% confidence interval. ρ has been extensively used in the literature [7, 8].

The evolutions of failure probability estimation and calculation of FOM are plotted in Figure 5. From Figure 5(a), we notice that HDIS, HSCS and proposed method all converge. The stable estimation value is reached when the FOM is smaller than 0.1, which is denoted by the dashed line in Figure 5(b). As illustrated in Table 1, the ground truth Monte Carlo result is 1.66e-5 (4.26σ). Our proposed method obtains only 0.6% relative error, while the estimations from HSCS and HDIS have 6.6% and 22.1% relative error, respectively.

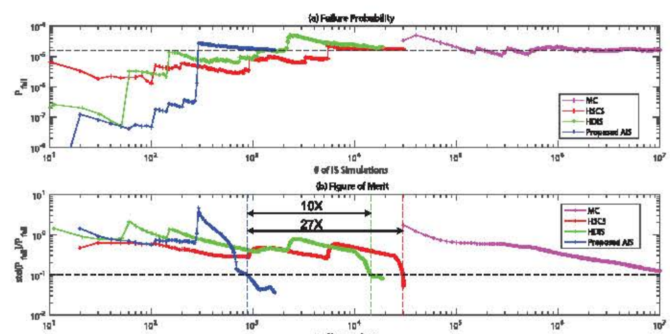

Figure 5: Evolution comparison of failure prob. and FOM on SRAM cell

4.1.2 ***Efficiency Comparison.*** Figure 5(b) illustrates the efficiency of our proposed algorithm, which has the fastest convergence speed among the different methods. It is attributed to our

Table 1: Accuracy and efficiency evolution on 54-dimensional SRAM circuit

	Monte Carlo	HSCS	HDIS	Proposed AIS
Failure prob.(error)	1.66e-5(0%)	1.77e-5(6.6%)	2.01e-5(22.1%)	1.67e-5(0.6%)
Presampling # sim.	0	4500	8000	3000
Importance Sampling # sim.	1e7	38750	14480	1430
Total # sim. (speedup)	1e7(1X)	43250(231X)	22480(445X)	4430(2257X)

dynamic search process, which is far more efficient than sampling from static distribution.

In fact, in the importance sampling (IS) step, our proposed method can reach steady state with 90% confidence interval within only 1430 samples. On the contrary, HSCS and HDIS need 38750 and 14480 times IS simulations to reach the same FOM value, respectively. Thus we can obtain 27X and 10X speed up w.r.t HSCS and HDIS method in the IS step.

Apart from the IS samples, we note that all importance sampling based approaches need extra presampling simulations to initialize the proposed distribution. To be specific, as indicated in Table 1, our proposed method need 3000 presampling simulations, while HSCS and HDIS need 4500 and 8000 times. Then Table 1 calculates the total number of simulations and reveals that our AIS method can achieve 2257X speedup over Monte Carlo, 10X speedup over HSCS and 5X over HDIS.

4.2 Experiments on Sense Amplfier circuit

We also evaluate the proposed AIS method using a sense amplifier (SA) circuit, which is one of the most critical memory peripheral circuits. A simplified schematic of the sense amplifier consisting of two cross-coupled inverters is presented in Figure 6. When SAE is high, arbitrary small ΔV between BL and BLB can be amplified to expected value by positive feedback.

Ideally, the structure of SA is symmetrical, that is, $MN2/MN3$, $MP4/MP5$, $MP6/MP7$ have the same sizing. However, in practice, it is difficult to realize because of process variation. In the following experiments, we consider a failure if the SA cannot recognize a specified voltage difference $\Delta V = \pm 150mV$. It could be induced by the discharge current mismatch between left and right branch. Therefore, the SA circuit has two failure modes, which results in at least two failure regions.

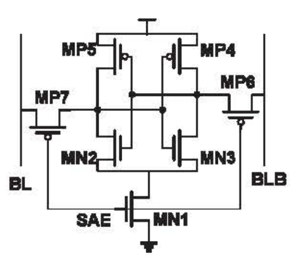

Figure 6: The schematic of sense amplifier

4.2.1 *Accuracy and Efficiency Comparison.*

In order to validate the accuracy of AIS algorithm, we plot the evolution of failure probability estimation and FOM vs. number of IS simulations in Figure 7. To generate ground truth, MC takes 33 millions simulations to obtain confident estimation of P_{fail} at 2.07e-6 (4.75σ). We notice that only AIS method can effectively estimate failure probability, while HDIS fails to discover any failure samples and HSCS converges to a wrong failure probability. The FOM of our proposed AIS method descends to 0.1 within 7530 simulations (including 5000 presampling simulations), which represents a 4369X speed up over Monte Carlo method.

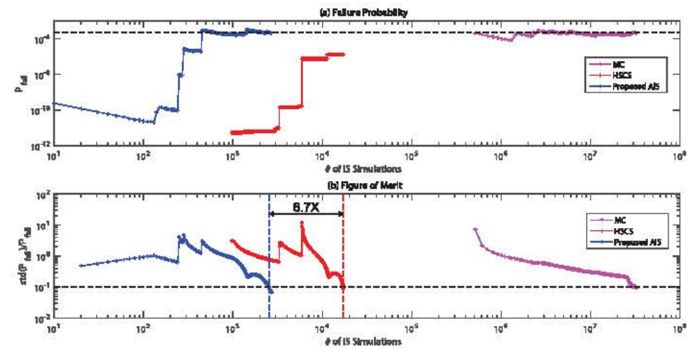

Figure 7: Evolution comparison of failure prob. and FOM on SA circuit

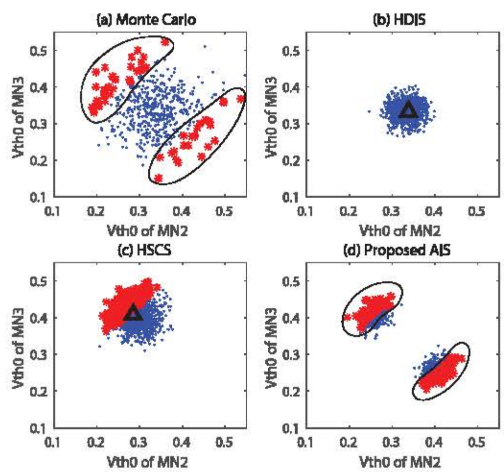

Figure 8: Multiple failure region coverage test (failure samples are colored and failure regions are circled, triangular denote the shifted mean for IS)

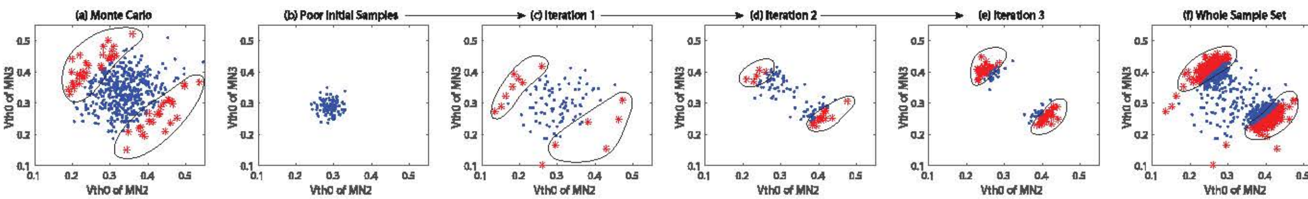

Figure 9: 2D visualized plot of proposed AIS iterative step (failure samples are colored and failure regions are circled)

To further investigate the difference among aforementioned methods, we project the samples toward two most significant dimensions to plot a visualized 2D sample space in Figure 8. Figure 8(a) shows some of the Monte Carlo samples in 2-dimension. Notice that we keep more failure points to make the failure region visualizable. As illustrated in Figure 8(b), HDIS cannot capture any failure samples because it attempts to sample around the centroid of failure samples, which is close to the origin (i.e. the nominal value). In Figure 8(c), HSCS converges to wrong P_{fail} value because only one failure region is notified. Proposed AIS method quickly moves the proposed distribution to failure regions and preserves excellent failure region coverage after several iterations. The AIS samples in final iterations are ploted in Figure 8(d), and the iterative steps are described in next section.

4.2.2 ***Robustness***. In this section, we demonstrate the robustness of our proposed method from two aspects.

First, AIS method is managed to recover from different initialization conditions. Figure 9(b) illustrates a set of poor initial samples. At iteration 1, AIS automatically reaches out to search for failure samples during the sample propagation step. Next, AIS heuristically optimizes the proposed mixture distribution by approximating target distribution and balancing sample weights. This explains how samples are accumulating to the boundary of failure regions, as shown in Figure 9(d,e). In this circuit example, our proposed distribution is fundamentally stabilized after Iteration 3. Figure 9(f) depicts our whole sample set in 2-dimension.

Furthermore, our AIS method can consistently generate accurate estimations. Figure 10 demonstrates the evolution curve of failure probability after 10 replications, which is shown to converge to the ground truth within 7.5e+3 simulations.

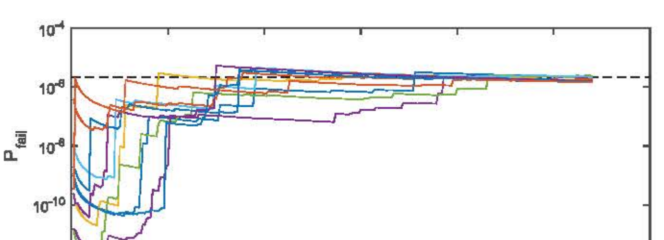

Figure 10: Robustness test of proposed AIS with 10 replications

5 CONCLUSION

In this paper, we presented an Adaptive Importance Sampling method to efficiently estimate the rare event failure probability of memory circuits. This method first applies spherical presampling to construct initial sampling distribution. Next, an iterative sampling and resampling scheme is performed to search for failure regions and balance sample weights. The experiments demonstrate that proposed algorithm can provide extremely high accuracy and efficiency. Experiments on SRAM cell circuit indicate that AIS achieves 2257X speedup over Monte Carlo and 5-10X over other state-of-the-art methods. On another sense amplifier circuit, AIS is 4369X faster than Monte Carlo method, while other IS based approaches fail to provide reasonable accuracy.

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