Problem 1:

I set sigma = 0.1, initial value p_0 = 100, simulation time n = 10000.

Here is the output:

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Expected mean of Classical Brownian Motion: 100
Expected std of Classical Brownian Motion: 0.1
mean of Classical Brownian Motion: 100.00065659055961
std of Classical Brownian Motion: 0.09998735079801287
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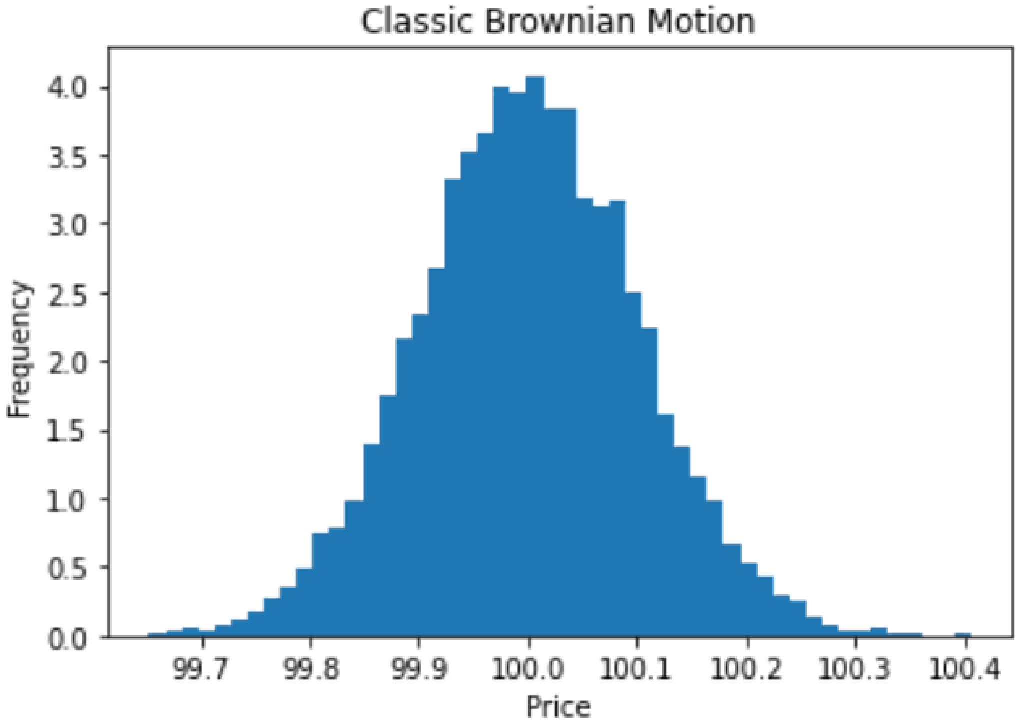
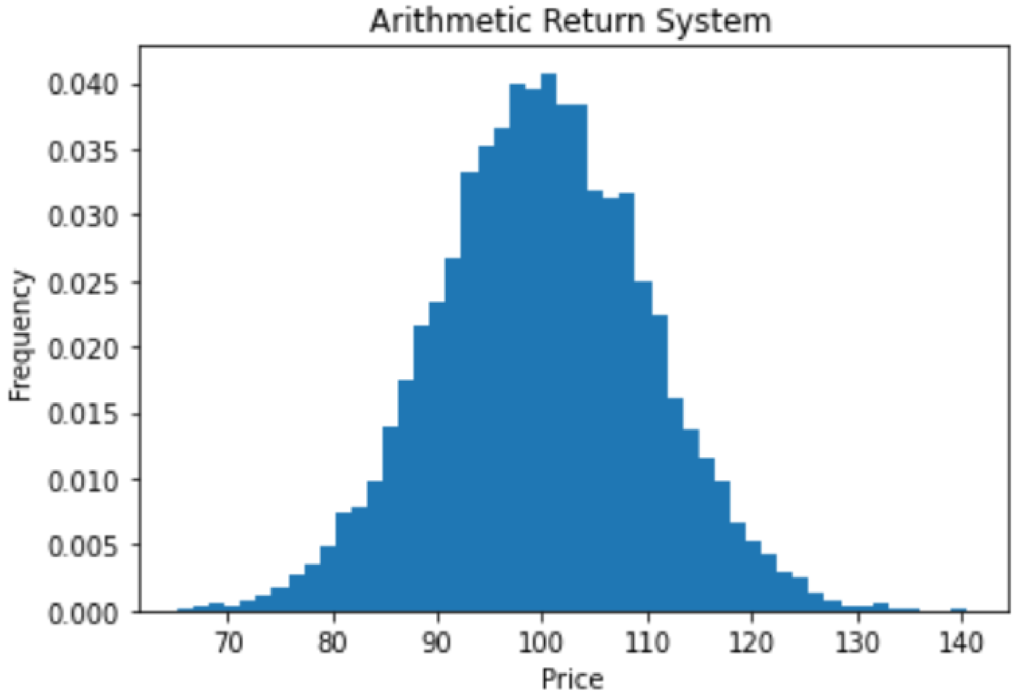
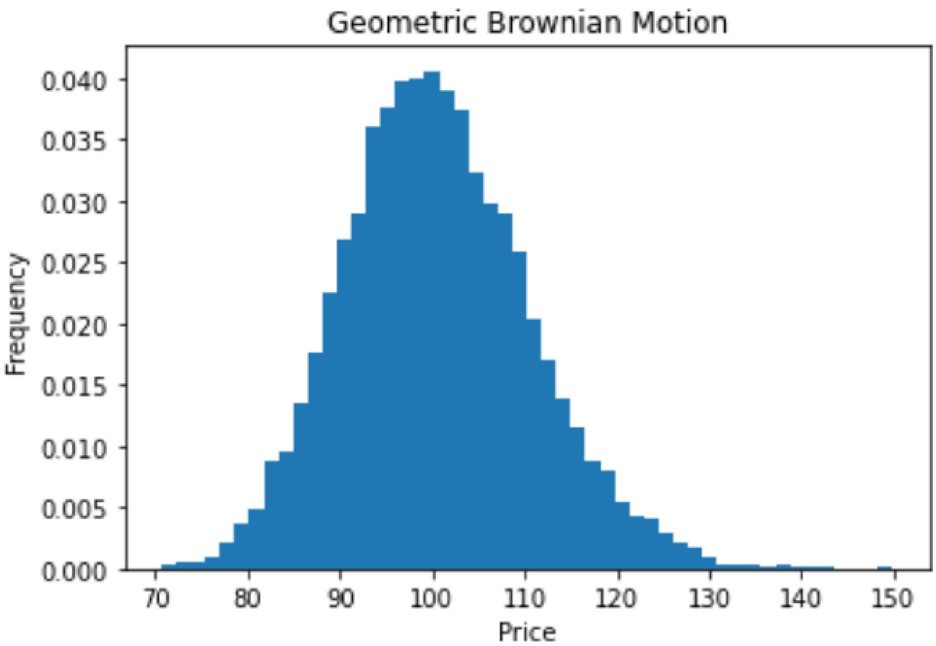


Expected mean of Arithmetic Return System: 100
Expected std of Arithmetic Return System: 10.0
mean of Arithmetic Return System: 100.06565905596139
std of Arithmetic Return System: 9.998735079801287



Expected mean of Geometric Brownian Motion: 100.5012520859401
Expected std of Geometric Brownian Motion: 10.075302944620395
mean of Geometric Brownian Motion: 100.56762050142633
std of Geometric Brownian Motion: 10.095780508016846



We can see that the results match our expectations. For Classic Geometric Motion, the expected mean is equal to p_0, the expected standard deviation is equal to sigma. The simulated mean and standard deviation are very close to the expected value. For Arithmetic Return System, the expected mean is equal to p_0, the expected standard deviation is equal to p_0 * sigma. The simulated mean and standard deviation are also very close to the expected value. For Geometric Brownian Motion, the expected mean is equal to p_0 * e ^ ((sigma^2)/2), the expected standard deviation is equal to p_0 * sqrt(e^(sigma^2-1) * e^(sigma^2)). The simulated mean and standard deviation are also very similar to the expected value. The difference between the expected value and the simulated value is smallest for Classic Geometric Motion. It's bigger for Arithmetic Return System and biggest for Geometric Brownian Motion. But still, with the given parameters, the difference can be considered very small.

Problem 2:

Here are the results:

VaR for normal distribution is 0.06573897817672866
VaR for normal distribution with exponentially weighted variance is: 0.09675016393120545
VaR for MLE fitted T Distribution is: 0.061473796278451426
VAR for fitted AR(1) is: 0.0667485628054164
VAR for Historic Simulation is: 0.06088703100509518
The results suggest that the VaR estimated using the MLE fitted T Distribution method has the lowest value, indicating a lower expected maximum loss as compared to the other methods. On the other hand, the VaR obtained through the normal distribution with exponentially weighted variance has the highest value, indicating a higher expected maximum loss. Except the VaR obtained through the normal distribution with exponentially weighted variance, the results of the other four methods are very close.

Problem 3:

I used two methods to calculate VaR: delta normal and historic simulation. The delta normal method assumes that the portfolio return is linearly related to the return of the underlying assets and that the covariance matrix of the assets returns is constant over the VaR horizon. I choose historic simulation because it is useful when there is sufficient historical data available and when the assumption of normality is not appropriate. It also considers the distribution of the returns in the past of the portfolio, which can be useful in this situation.
Here are the results:
The current value for A is: 299950.06
VaR for A using Delta Normal is: 5670.20
VaR for A using Historic Simulation is: 7109.30

The current value for B is: 294385.59
VaR for B using Delta Normal is: 4494.60
VaR for B using Historic Simulation is: 7273.7

The current value for C is: 270042.83
VaR for C using Delta Normal is: 3786.59
VaR for C using Historic Simulation is: 5310.07

The results show that the VaR estimates for all three portfolios are higher with historic simulation than with the delta normal method. This is expected, as the delta normal method assumes that the returns are normally distributed, which may not be an accurate representation of the true distribution of the returns. In contrast, the historic simulation method uses actual historical returns to estimate the VaR, which can capture the non-normality and other characteristics of the true distribution of returns.