

## Chapter 3 & 4.1-4.3.

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|----------------------------|---------------------------------|
| 3.1: proof strategy.       | 4.1 x                           |
| 3.2: $\neg, \rightarrow$ . | *4.2 relations                  |
| 3.3: $\forall, \exists$    | *4.3 $R \subseteq A \times A$ . |
| 3.4: $\wedge, \exists!$    |                                 |
| 3.5: $\vee$ ,              |                                 |
| *3.6: $\exists!$           |                                 |
| 3.7: more examples.        |                                 |

4.1  $A \times B = \{(a, b) \mid a \in A, b \in B\}$ .  
 $(a_1, b_1) = (a_2, b_2) \iff a_1 = a_2, b_1 = b_2$ .

4.2 A relation from  $A$  to  $B$  is  $R \subseteq A \times B$ .  
 $aRb$  to mean  $(a, b) \in R$ .

The domain of  $R$  is  $\text{Dom}(R) = \{a \in A \mid \exists b \in B, aRb\}$ .  
the range of  $R$  is  $\text{Ran}(R) = \{b \in B \mid \exists a \in A, aRb\}$ .  
the inverse of  $R$  is  $R^{-1} = \{(b, a) \mid aRb\}$ .  
the composition of  $R \subseteq A \times B$  and  $S \subseteq B \times C$  is  
 $S \circ R = \{(a, c) \in A \times C \mid \exists b \in B, aRb \wedge bSc\}$ .

4.3: A relation on  $A$  is  $R \subseteq A \times A$ .

$R$  is reflexive if  $\forall a \in A, aRa \iff (i_A \subseteq R)$ .

$R$  is symmetric if  $\forall x, y \in A, xRy \rightarrow yRx \iff (R = R^{-1})$ .

$R$  is transitive if  $\forall x, y, z \in A, xRy \wedge yRz \rightarrow xRz$   
 $\iff R \circ R \subseteq R$ .

3.6  $\exists! x \in A, P(x)$ .

$\exists x \in A (P(x) \wedge \forall y \in A (P(y) \rightarrow x=y))$ .

$\exists x \in A, P(x)$ . at  $x = \dots$