

知识点 1. SOLOW GROWTH MODEL

- **Solow Growth Model** : 是由伟大的 Robert Solow 所提出并创建，所以是由他的名字所命名。这个模型是用来解释经济体在生存中，Capital 对于生产的重要性，以及对于未来经济发展的一种预测。

$$\text{Production Function : } Y = K^a L^{1-a}$$

- ✓ Y : 代表 GDP , National Income
 - ✓ K : Capital , L : Labour
 - ✓ a : Capital Share , Investor keep a 比例的 GDP
 - ✓ $1 - a$: Labour Share , Labour keep $1 - a$ 比例的 GDP
 - ✓ **Properties** : Constant returns to scale , Diminishing Marginal Product
- **Capital Accumulation:**

$$K' = (1 - \delta)K + I$$

- ✓ 明天的机器数量 = 今天的机器数量 - 报废的机器数量 + 新投资的机器

- **Saving Function :**

$$S = \sigma Y$$

- ✓ 在借贷市场中，Equilibrium 发生在 $S = I$

- **Intensive Form : (Variable Per Worker) :**

$$y = \frac{Y}{L} = \frac{K^a L^{1-a}}{L} = \frac{K^a L^{1-a}}{L^a L^{1-a}} = \left(\frac{K}{L}\right)^a = k^a$$

$$k' = (1 - \delta)k + \sigma y$$

$$k' = k - \delta k + \sigma k^a$$

- **Steady State : when all variables are constant or growing at a constant pace**

$$\sigma k^a = \delta k$$

新投资的机器 = 报废掉的机器，所以 Capital Per Worker 不变

EXAMPLE :

1. A Solow Growth Model has production function $Y = K^{1/2}L^{1/2}$, $\alpha = 0.5$, $\delta = 0.1$ and $\sigma = 0.4$. Solve for the steady state Equilibrium and Provide a labelled diagram.

- **Prediction One: Catch Up Effect**, 随着国家的发展，经济增长的速度会越来越慢，也就是说时间线足够长的话，发达国家一定会被发展中国家所追赶上
2. Quantify and Illustrate the impact of a higher rate of depreciation rate $\delta = 0.2$. Prediction Two : **更高的 Depreciation rate 会导致更低的 GDP per worker**

3. Quantify and Illustrate the impact of a higher rate of saving $\sigma = 0.5$ when $\delta = 0.1$. Prediction

Three : **Higher Saving 会导致更高的 GDP per worker**

$$0.5k^{0.5} = 0.1k$$

$$k^{0.5} = \frac{1}{5}k$$

$$2 = k^{0.5}$$

$$y = \frac{1}{2}k$$

$$y = k^{0.5}$$

$$\frac{1}{2}k = k^{0.5}$$

$$k = 4$$

$$y = 2$$

$$0.5k^{0.5} = 0.1k$$

$$k = 25$$

$$y = 5$$

4. Quantify and Illustrate the impact of a higher rate of Capital Share $\alpha = 2/3$ when $\sigma = 0.4$,

$\delta = 0.1$. Prediction Four : **Higher Capital Share 会导致 higher GDP per worker**

$$0.4k^{\frac{2}{3}} = 0.1k$$

$$4 = k^{\frac{1}{3}}$$

$$\boxed{k = 64}$$

$$y = 16$$

$$4 \times 4 \times 4$$

PAST TEST 大题：

1. A Solow Growth model economy has a savings rate of 20 percent, a 10 percent rate of depreciation and a production function $Y = K^{1/2}L^{1/2}$. Provide a diagram and supporting calculations that illustrate and quantify the steady state equilibrium.



$$\alpha = \frac{1}{2}$$

$$s = 0.2$$

$$\delta = 0.1$$

$$Y =$$

$$s k^{\alpha} = \delta k$$

$$0.2 k^{\frac{1}{2}} = 0.1 k$$

$$k = 4$$

$$Y = k^{\alpha} = 2$$

2. A Solow Growth model economy has a savings rate of 20 percent, a 10 percent rate of depreciation and a production function $Y = K^{1/2}L^{1/2}$. s = 0.2.
- Provide a diagram and supporting calculations that illustrate and quantify the steady state equilibrium.
 - Quantify the impact on steady state equilibrium if the depreciation rate was only 5 percent. Modify your diagram from part a. to illustrate this equilibrium

- 在现实生活中，人口是有增长的，人口肯定不是不变的：

$$L' = L + \eta L = (1 + \eta)L$$

$$\frac{K'}{L} = \frac{K}{L} + \frac{\delta K}{L} + \frac{\sigma K^a}{L}$$

$$\frac{K'}{L'} \frac{L'}{L} = \frac{K}{L} + \frac{\delta K}{L} + \frac{\sigma K^a}{L}$$

$$k'(1 + \eta) = k + \delta k + \sigma k^a$$

- In steady State:

$$\sigma k^a = (\delta + \eta)k$$

EXAMPLE:

Solve for steady state equilibrium: $\eta = 0.1$, $a = 0.5$, $\delta = 0.1$ and $\sigma = 0.4$. Provide a labelled diagram. How does GDP per worker in this country compare to an economy where $\eta = 0$

Prediction Five: higher 人口增长率，会导致更低的 GDP per worker

$$0.4 k^{0.5} = (0.1 + 0.1) k$$

$$k = 4$$

$$y = 2$$

- $y = Y/L$ ，所以 $Y = yL$ ，如果我们可以在每一项中取对数， $\ln Y = \ln y + \ln L$ ，所以 **Prediction**

Six : $\% \Delta Y = \% \Delta y + \% \Delta L$ ，GDP 的增长率 = GDP per worker 的增长率 + 人口的增长率

TUT PRACTICE :

1. An increase in the population, with everything else held constant, will cause.
 - a. An increase in per capita output.
 - b. A decrease in per capita output.
 - c. A decrease in the capital to output ratio.
 - d. An inward shift in the production possibilities boundary.
2. Rising levels of output per capita could be caused by
 - a. Higher depreciation rates.
 - b. Lower savings rates.
 - c. Lower investment.
 - d. Higher savings rates.

$y = k^{\alpha} L$
3. An increase in the labour force _____ total output and _____ output per worker.
 - a. Increases; Increases
 - b. Increases; Decreases
 - c. Decreases; Increases
 - d. Decreases; Decreases
4. Sustained rising material living standards can only be explained by
 - a. Growth in human capital
 - b. Growth in physical capital
 - c. Growth in the labour force
 - d. Exogenous technological change
5. The Solow Growth Model predicts that a lower depreciation rate will
 - a. increase steady state savings per worker and increase steady state GDP per worker
 - b. decrease steady state savings per worker and decrease steady state GDP per worker
 - c. decrease steady state savings per worker and increase steady state GDP per worker
 - d. increase steady state savings per worker and decrease steady state GDP per worker
6. The Solow Growth Model predicts that slower growth in the labour force will
 - a. increase steady state savings per worker and increase steady state GDP per worker
 - b. decrease steady state savings per worker and decrease steady state GDP per worker
 - c. decrease steady state savings per worker and increase steady state GDP per worker
 - d. increase steady state savings per worker and decrease steady state GDP per worker



PAST TEST 大题：

1. A Solow Growth model economy with production function $Y = K^{1/2}(AL)^{1/2}$ has a savings rate of 48 percent, a 6 percent labour force growth rate and a 10 percent rate of depreciation. There is no technological progress.
 - a. Provide a labelled diagram and supporting calculations to illustrate and quantify the steady state equilibrium for this economy.
 - b. Use the same diagram to show how the steady state would be affected if immigration policies are relaxed and the labour force growth rate increased from 6 percent to 14 percent.

➤ **Technological Progress :**

$$(AL)' = (1 + \gamma + \eta)AL$$

- ✓ A : Human Capital 知识水平
- ✓ Human Capital 也是可以积累, 和增长的, 增长率是 γ
- ✓ AL : Effective Worker

➤ **In Steady State Equilibrium :**

$$\sigma k^a = (\delta + \gamma + \eta)k$$

➤ 因此 : GDP per Effective Worker : $y = \frac{Y}{AN}$

EXAMPLE:

Solve for Steady State Equilibrium $\gamma = 0.2, \eta = 0.1, a = 0.5, \delta = 0.1$, and $\sigma = 0.4$ and provide a labelled diagrams **Prediction Seven: Higher human capital growth rate, will cause lower GDP per effective worker.**

➤ $y = Y/AL$, 所以 $Y = yAL$, 如果我 take ln 在每一项中 , $\ln Y = \ln y + \ln A + \ln L$, 所以

Prediction Six : $\% \Delta Y = \% \Delta y + \% \Delta A + \% \Delta L$, GDP 的增长率 = GDP per worker 的增长率 + 人口的增长率 + 知识水平的增长率

TUT PRACTICE :

1. A Solow Growth economy with $Y = K^{1/2}(AL)^{1/2}$, depreciation of 2 percent, labour force growth of 2 percent and technological progress of 1 percent will have equilibrium GDP per effective worker equal to _____
2. A Solow Growth economy with $Y = K^{1/2}(AL)^{1/2}$, depreciation of 2 percent, labour force growth of 1 percent and technological progress of 2 percent will have Consumption per Worker that grows at a rate of _____ percent per year.

PAST TEST 大题：

1. A Solow Growth model economy with production function $Y = K^{1/2}(AL)^{1/2}$ has a savings rate of 48 percent, a 3 percent labour force growth rate and a 5 percent rate of depreciation.

Labour augmenting technological progress occurs at a rate of 4 percent per year.

- a. Provide a labelled diagram and supporting calculations to illustrate and quantify the steady state equilibrium for this economy.
- b. Steady state output per worker grows at a rate of _____ percent per year and output grows at a rate of _____ percent per year.
- c. Update your diagram to show what would happen if the depreciation rate fell to 1 percent per year.

$$48 - 0.48k^{0.5} = 0.12k$$

$$k = 16$$

$$\gamma = 4$$

$$\sigma k^a = \delta k$$