Exercise (Logical Equivalence)

1) With truth tables check if $\neg(P \vee (\neg P \wedge q)) \equiv \neg(P \vee q)$ is true or false.

2) Simplify the expression $\neg(P \wedge q) \vee (P \wedge q)$

3) Simplify the expression $(\neg p \wedge (P \vee q))$

$= (\neg p \wedge p) \vee (\neg p \wedge q) = \neg p \wedge q$

4) Without a truth table prove $\neg(P \rightarrow q) \equiv P \wedge \neg q$

5) Without truth table prove $\neg(P \vee (\neg P \wedge q)) \equiv \neg(P \vee q)$

$\neg(P \vee (\neg p \wedge q))$

$= \neg((p \vee \neg p) \wedge (p \vee q))$

$= \neg(T \wedge (p \vee q))$

$= \neg(p \vee q)$.