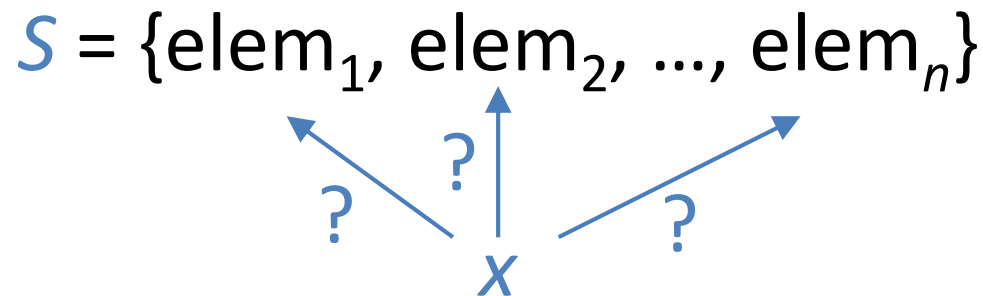


CS 2210 Data Structures and Algorithms

The Search Problem

A Fundamental Problem

Given a set S of n elements and a particular element x the **search problem** is to decide whether x is in S .



A Fundamental Problem

Given a set S of n elements and a particular element x the **search problem** is to decide whether x is in S .

- If x is in S we want to know where in S it appears

$$S = \{\text{elem}_1, \text{elem}_2, \dots, x, \dots, \text{elem}_n\}$$

- If x is not in S we want an indication that this is the case

$$x \text{ is not in } S = \{\text{elem}_1, \text{elem}_2, \dots, \text{elem}_n\}$$

The Search Problem

This problem has a large number of applications:

- S = Names in a phone book
 - x = name of a person

Application: Find
phone number of
person



The Search Problem

This problem has a large number of applications:

- S = Student records
 x = student ID

Application:
Print a
transcript

A screenshot of a web application titled "UWO Database" with a sub-header "Japan Drug Master Files". It features a search bar, a table of drug records, and navigation controls. The table has columns for Registration Number, Registration Date, Registration Name, Alternative Name, Registration Address, Registration Status, Registration Agency, Registration Classification, UWO ID Number, Status, and Remarks. The first row shows a registration for "Daiichi Sankyo Co., Ltd." with a status of "Canceled". The second row shows a registration for "Daiichi Sankyo Co., Ltd." with a status of "Active". The third row shows a registration for "Daiichi Sankyo Co., Ltd." with a status of "Active". The fourth row shows a registration for "Daiichi Sankyo Co., Ltd." with a status of "Active". The fifth row shows a registration for "Daiichi Sankyo Co., Ltd." with a status of "Active". The sixth row shows a registration for "Daiichi Sankyo Co., Ltd." with a status of "Active". The seventh row shows a registration for "Daiichi Sankyo Co., Ltd." with a status of "Active". The eighth row shows a registration for "Daiichi Sankyo Co., Ltd." with a status of "Active". The ninth row shows a registration for "Daiichi Sankyo Co., Ltd." with a status of "Active". The tenth row shows a registration for "Daiichi Sankyo Co., Ltd." with a status of "Active". The table is paginated, showing "Page 1 of 100" and "Records Per Page: 10".

The Search Problem

This problem has a large number of applications:

- S = Variables in a program
 x = name of a variable

Application:
Find compilation
errors

```
/* When the user has selected a play, this  
   process the selected play */  
public void actionPerformed(ActionEvent ev)  
    if(event.getSource() instanceof JButton) { /* Some position of the  
                                                board was selected */  
  
        int row = -1, col = -1;  
        PosPlay pos;  
  
        if (game_ended) System.exit(0);  
        /* Find out which position was selected by the player */  
        for (int i = 0; i < board_size; i++) {  
            for (int j = 0; j < board_size; j++)  
                if(event.getSource() == board[i][j]) {  
                    row = i;  
                    col = j;  
                    break;  
                }  
            if (row != -1) break;  
        }
```

The Search Problem

This problem has a large number of applications:

- S = Variables in a program

S = Symbol Table

x = name of a variable

```
/* When the user has selected a play, this method is invoked to
process the selected play */
public void actionPerformed(ActionEvent event) {
    if(event.getSource() instanceof JButton) { /* Some position of the
board was selected */
        int row = -1, col = -1;
        PosPlay pos;

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            for (int j = 0; j < board_size; j++)
                if(event.getSource() == board[i][j]) {
                    row = i;
                    col = j;
                    break;
                }
            if (row != -1) break;
        }
    }
}
```

The Search Problem

This problem has a large number of applications:

- S = Web host names

x = URL

Application:
Display a
Web page



Solving a Problem

The solution of a problem has 2 parts:

- How to organize data

Data structure:

➤ a systematic way of organizing and accessing data

- How to solve the problem

Algorithm:

➤ a step-by-step procedure for performing some task in finite time

Data Structure for the Search Problem

For simplicity, let us assume that S is a set of n different integers stored in non-decreasing order in an array L .

L	3	9	11	17	18	26	29	43	48	55
	0	1	2	3	4	5	6	7	8	9

x

Algorithms

- How to solve the problem

An algorithm **must** have two properties:

1. It must be correct: Always produces the correct answer/outcome
2. It must be efficient.

Algorithms

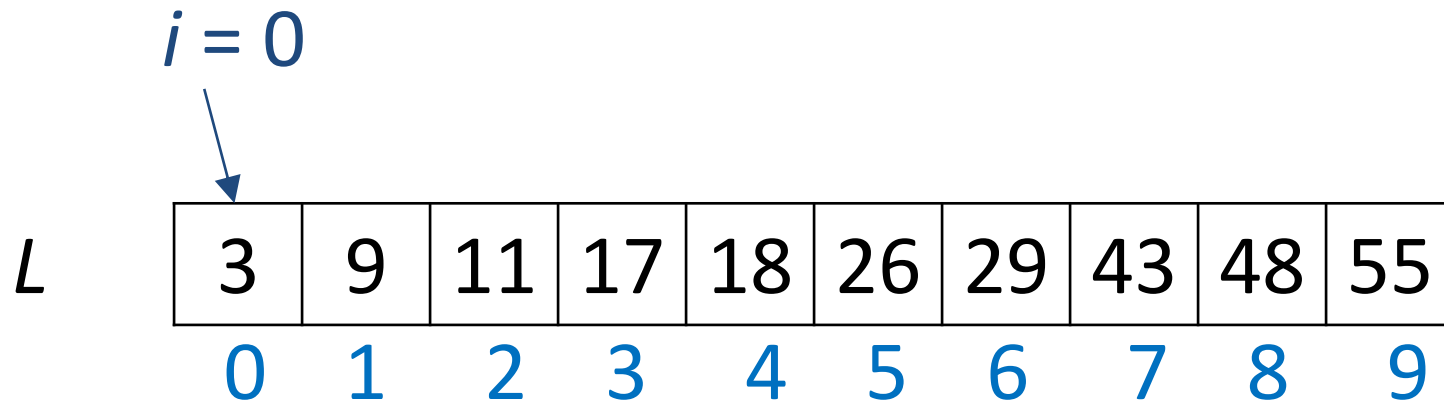
Given a problem that can be solved using a computer there is an **infinite** number of different algorithms to solve it.

The job of a programmer is to select/design the most appropriate algorithm for a particular situation.

Software Development Life Cycle

- Specification
- Design
- Implementation
- Testing and Debugging
- Deployment

Linear Search




x

Compare x with $L[0]$

Linear Search

$i = 1$



L


3	9	11	17	18	26	29	43	48	55
0	1	2	3	4	5	6	7	8	9

x

Compare x with $L[1]$

Linear Search

$i = 2$




L	3	9	11	17	18	26	29	43	48	55
	0	1	2	3	4	5	6	7	8	9

x

Compare x with $L[2]$...

Linear Search

$i = 5$



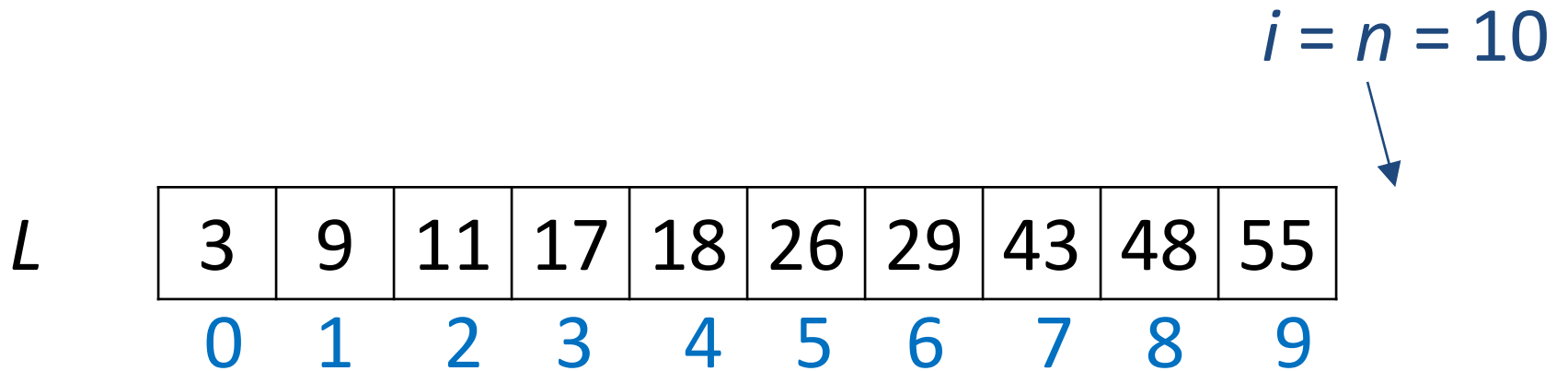
L

3	9	11	17	18	26	29	43	48	55
0	1	2	3	4	5	6	7	8	9

$x = 26$

Compare x with $L[5]$. Value x found!

Linear Search



$x = 40$

If x is not in L then eventually i will have value n and the algorithm then terminates.

Writing an Algorithm

It is not a good practice to write algorithms directly on the computer in some programming language.

Instead, we should write the algorithms in **pseudocode**.

Pseudocode

Pseudocode is a combination of English statements and programming-like control statements that

- allows us to express in detail an algorithm
- without having to deal with syntactic rules of a programming language
(;, variable declarations, public, private, protected, static, casting, generics, ...)

Pseudocode

Writing an algorithm in pseudocode makes it easier to design an algorithm because

- we only need to think about how to solve a problem and
- we do not have to think about how to express that algorithm in a particular programming language.

Algorithm LinearSearch (L,n,x)

Input: Array L of size n and value x

Output: Position i, $0 \leq i < n$, such that $L[i] = x$
if x in L, or -1 if x not in L

Specification

Algorithm LinearSearch (L,n,x)

Input: Array L of size n and value x

Output: Position i, $0 \leq i < n$, such that $L[i] = x$, if x in L, or -1, if x not in L

$i \leftarrow 0$

while ($i < n$) **and** ($L[i] \neq x$) **do**

$i \leftarrow i+1$

if $i=n$ **then return** -1

else return i

Proving the Correctness of an Algorithm

To prove that an algorithm is correct we need to show 2 things:

- The algorithm terminates
- The algorithm produces the correct output

Correctness of Linear Search

Termination

- i takes values 0, 1, 2, 3, ...
- The while loop cannot perform more than n iterations because of the condition ($i < n$)

Correctness of Linear Search

Correct Output

- The algorithm compares x with $L[0]$, $L[1]$, $L[2]$, ...
- Hence, if x is in L then $x = L[i]$ in some iteration of the **while** loop; this ends the loop and then the algorithm correctly returns the value i
- If x is not in L then in some iteration $i = n$; this ends the loop and the algorithm returns -1.

Another Algorithm for the Search Problem

mid = 4
↓


<i>L</i>	3	9	11	17	18	26	29	43	48	55
	0	1	2	3	4	5	6	7	8	9

x

Compare *x* with the value stored in the middle of array *L*

Another Algorithm for the Search Problem

mid = 4



L	3	9	11	17	18	26	29	43	48	55
	0	1	2	3	4	5	6	7	8	9

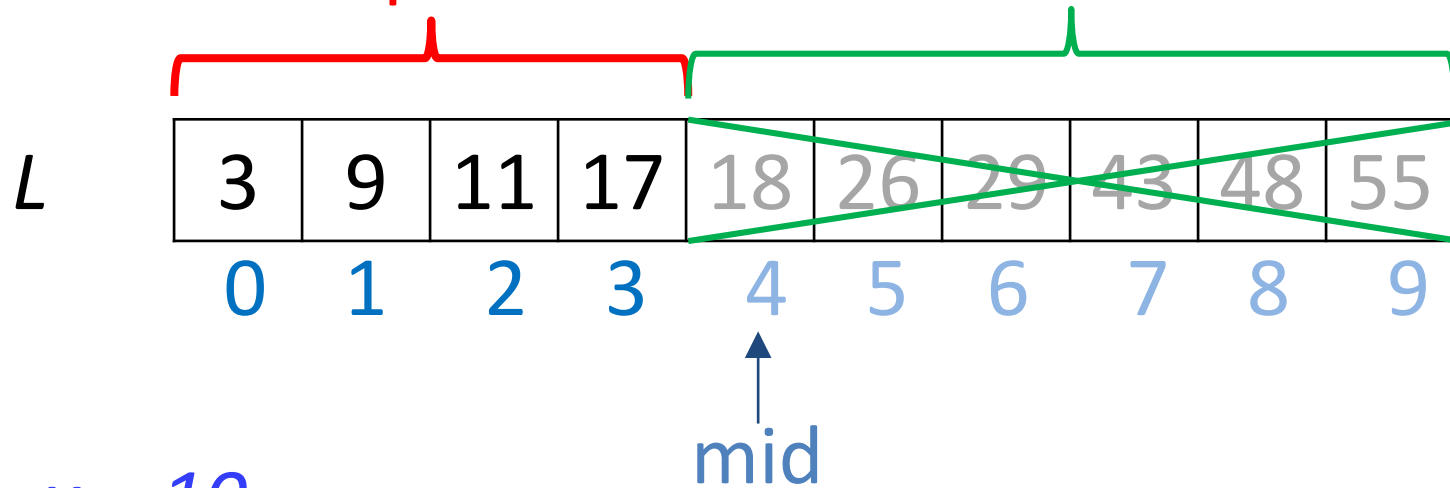
$x = 18$

If $x = L[\text{mid}]$ the algorithm ends

Another Algorithm for the Search Problem

Search continues
in this part of L

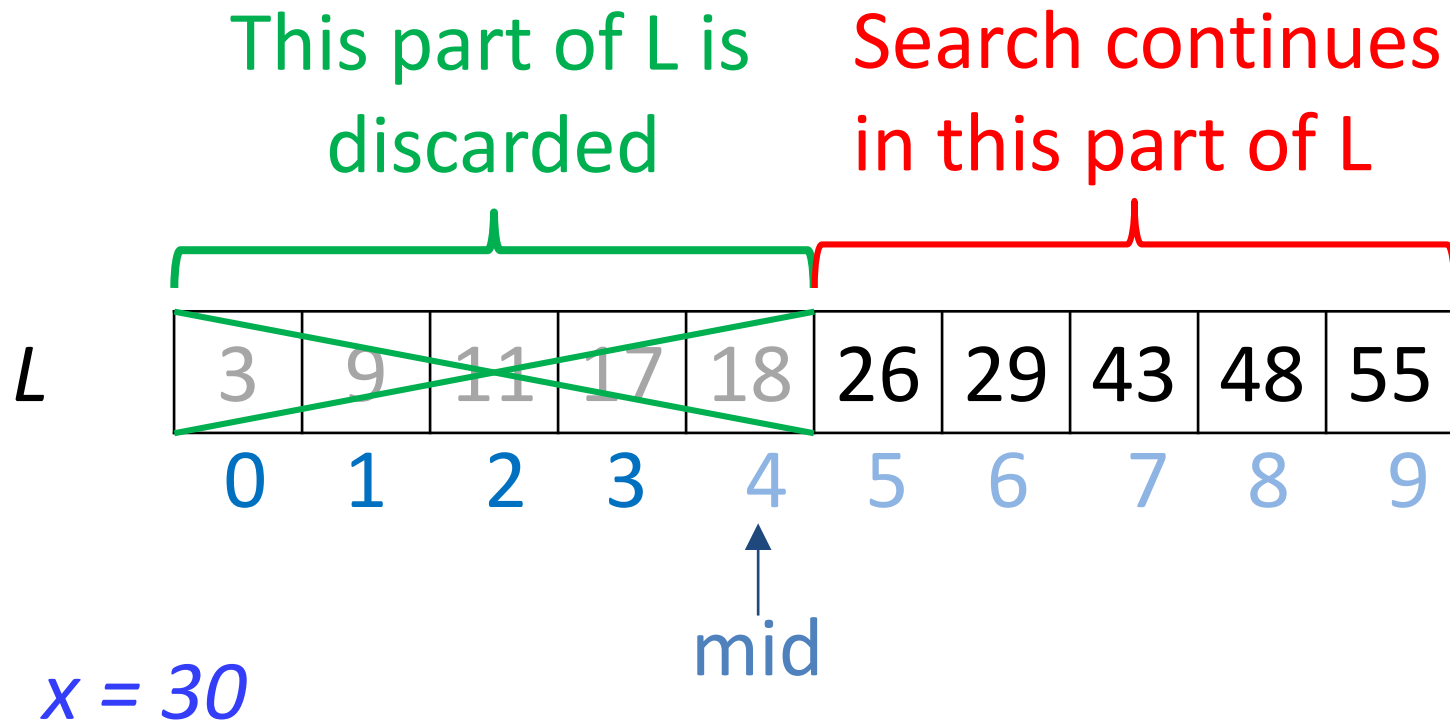
This part of L is
discarded



$x = 10$

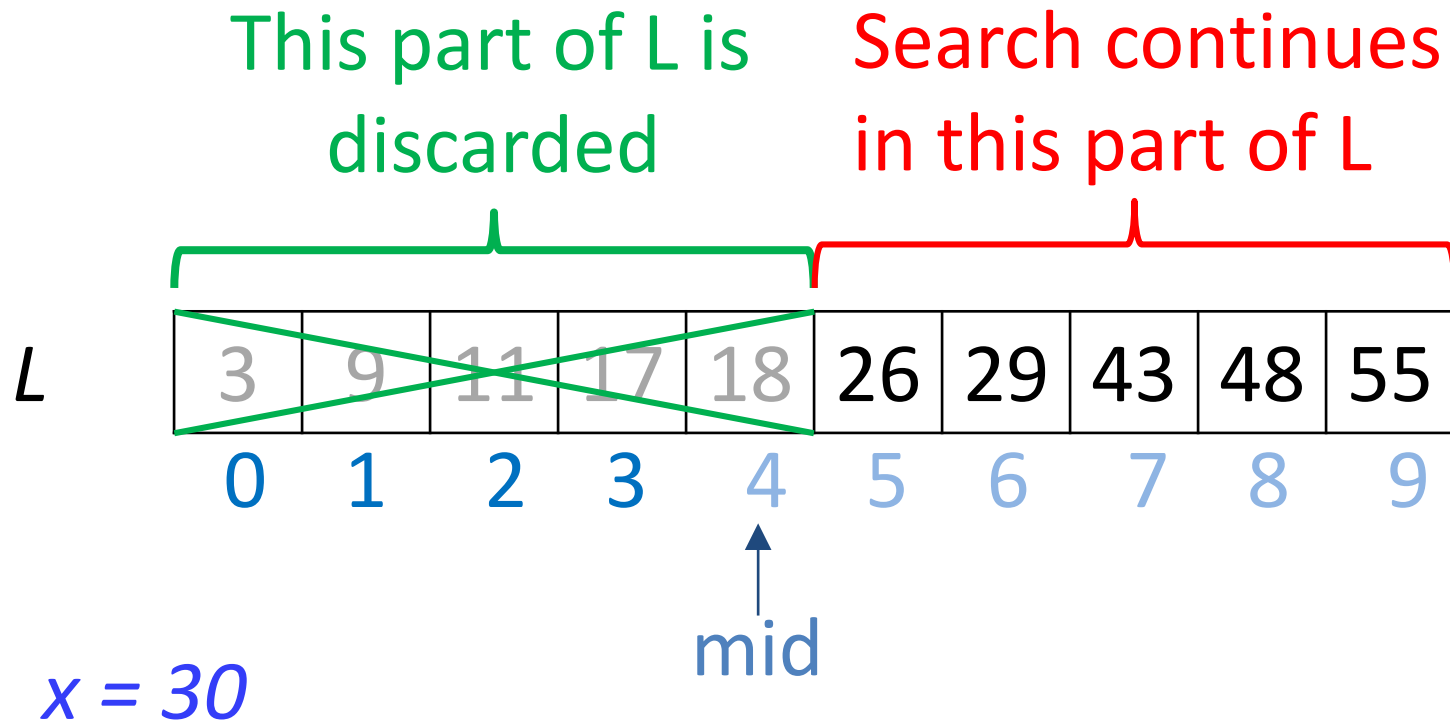
If $x < L[\text{mid}]$ the search continues on the first half of the array

Another Algorithm for the Search Problem



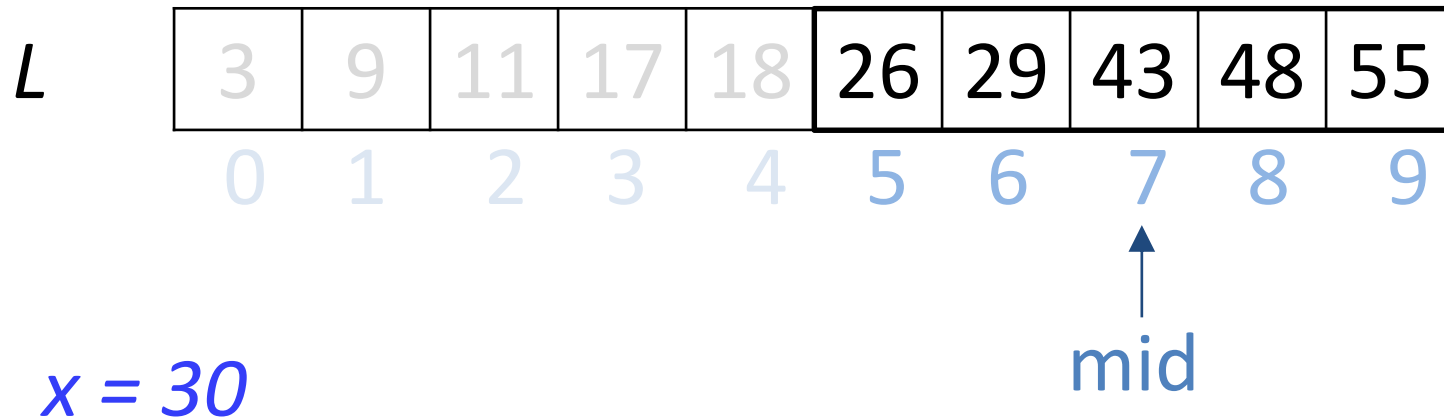
If $x > L[\text{mid}]$ the search continues on the second half of the array

Another Algorithm for the Search Problem



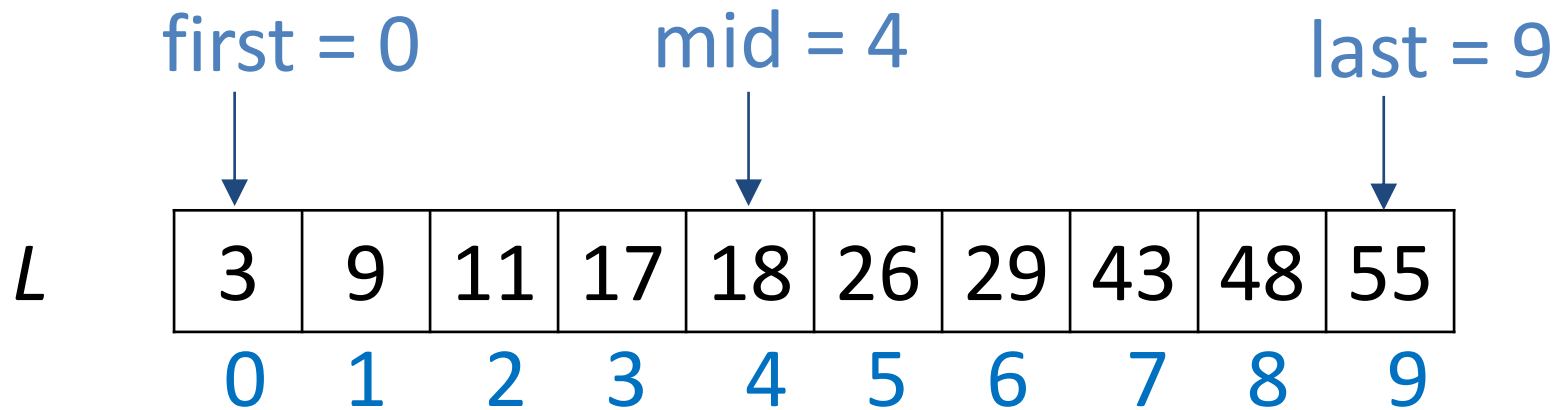
The search proceeds in the same manner in the remaining part of the array

Another Algorithm for the Search Problem



The search proceeds in the same manner in the remaining part of the array. This algorithm is called **binary search**.

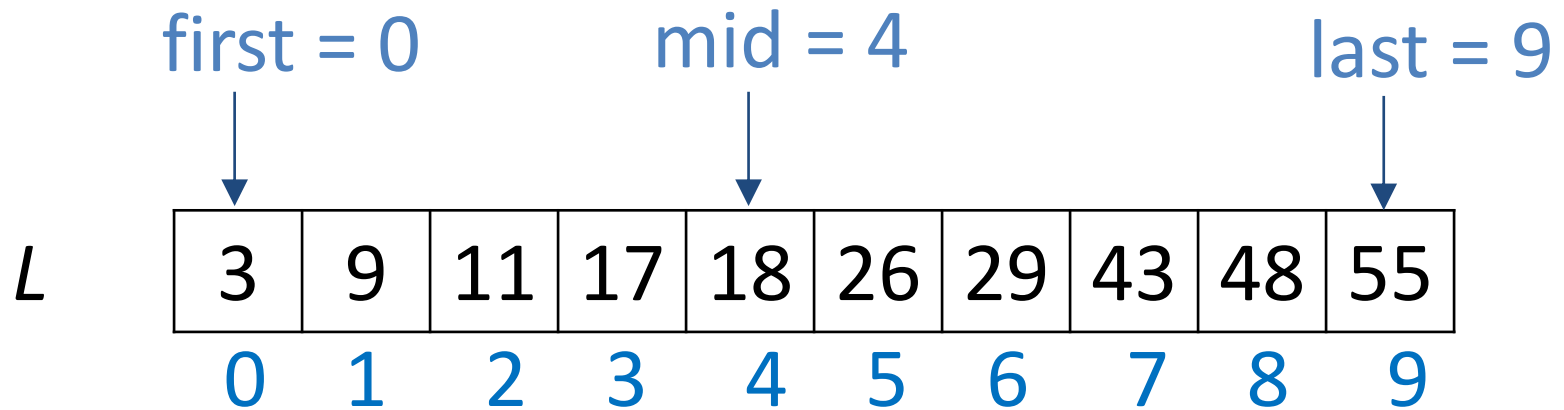
Binary Search



x

We use two indices: *first* and *last* to bound the part of the array where the search is performed

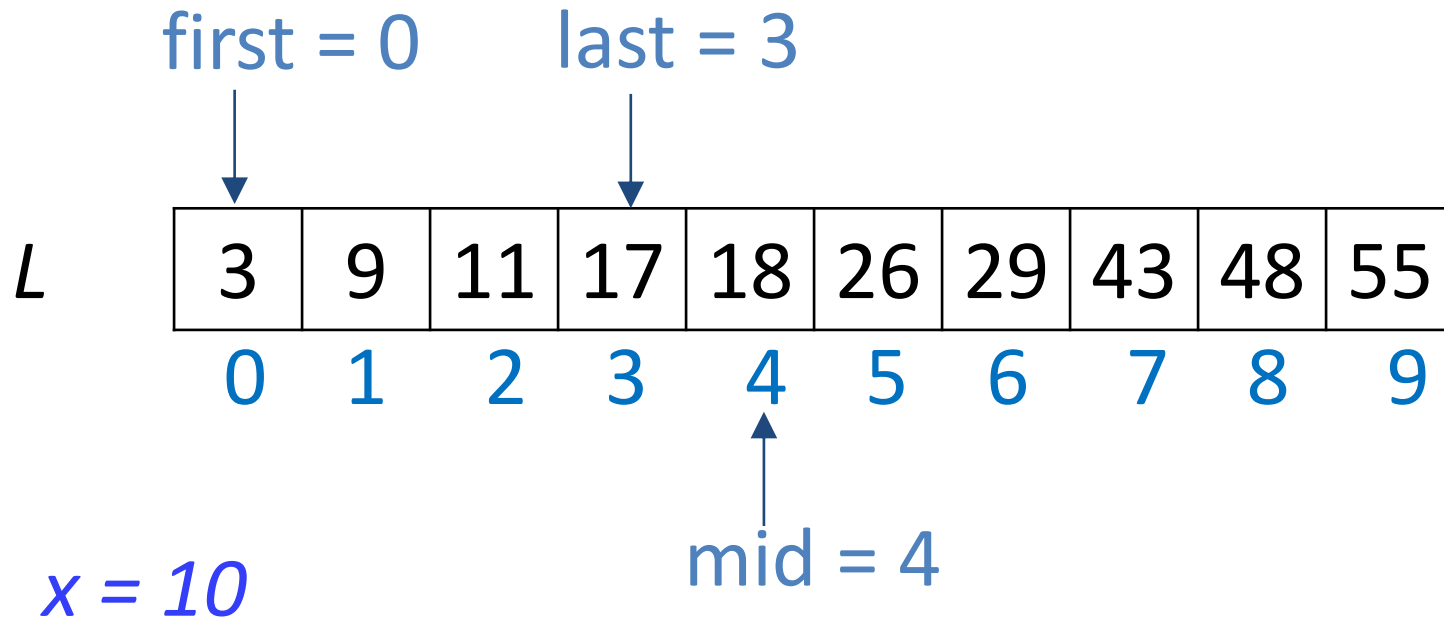
Binary Search



$x = 18$

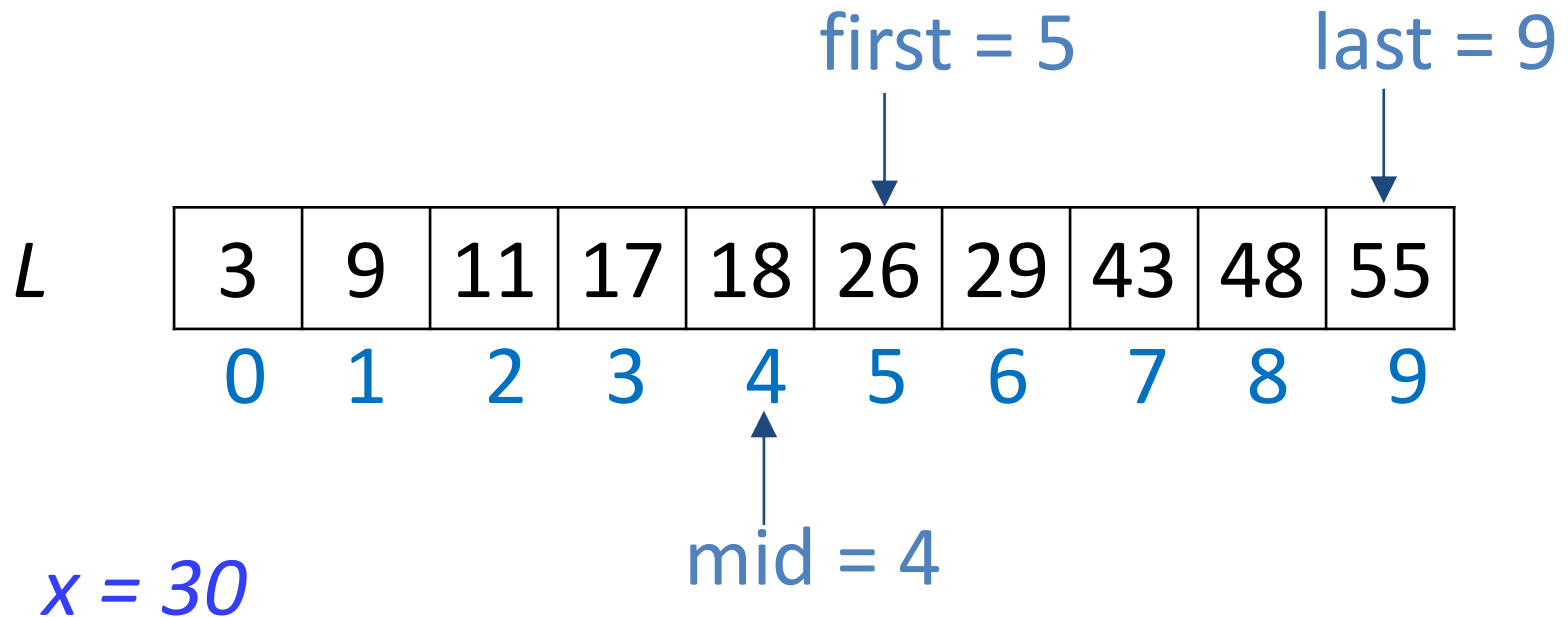
If $x = L[\text{mid}]$ the algorithm ends

Binary Search



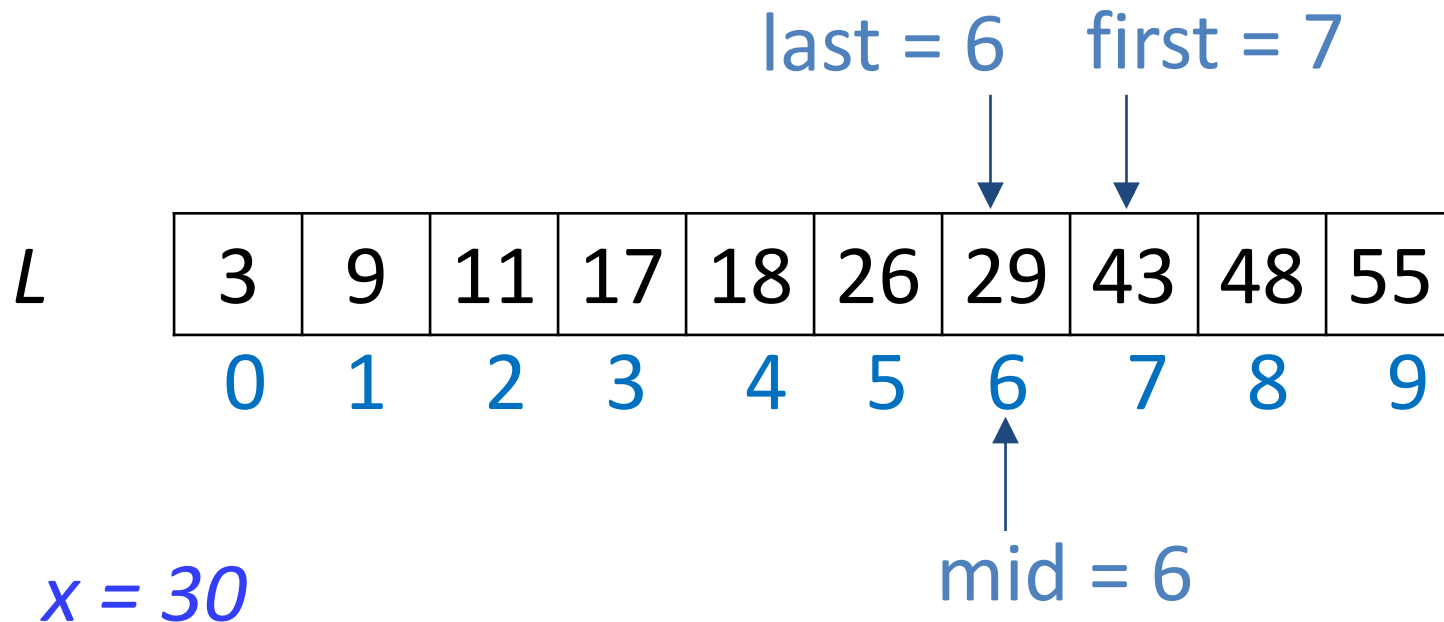
If $x < L[\text{mid}]$ the value of last changes to $\text{mid} - 1$, so the search continues in the first half of L .

Binary Search



If $x > L[\text{mid}]$ the value of **first** changes to **mid + 1**, so the search continues in the second half of L

Binary Search



If $first > last$ the algorithm terminates as x is not in L

Algorithm BinarySearch (L,x, first, last)

Input: Array L of size n and value x

Output: Position i, $0 \leq i < n$, such that $L[i] = x$, if x
in L, or -1, if x not in L

if first > last **then return** -1

else mid $\leftarrow \lfloor (first + last) / 2 \rfloor$

This symbol means
"round down"

if x = L[mid] **then return** mid

else if x < L[mid] **then**

return BinarySearch (L,x,first,mid -1)

else return BinarySearch (L,x,mid +1,last)

Correctness of Binary Search

Termination

- If $x = L[mid]$ the algorithm terminates
- If $x < L[mid]$ or $x > L[mid]$, the value $L[mid]$ is discarded from the next recursive call. Hence, in each recursive call the size of L decreases by at least 1.

L	3	9	11	17	18	26	29	43	48	55
-----	---	---	----	----	----	----	----	----	----	----

↑
mid

Correctness of Binary Search

Termination

- If $x = L[mid]$ the algorithm terminates
- If $x < L[mid]$ or $x > L[mid]$, the value $L[mid]$ is discarded from the next recursive call.
Hence, in each recursive call the size of L decreases by at least 1.
- After a finite number of recursive calls the size of L must be zero and the algorithm ends

Correctness of Binary Search

Correct Output. Case 1: x is not in L

In each recursive call the algorithm only discards values **different** from x

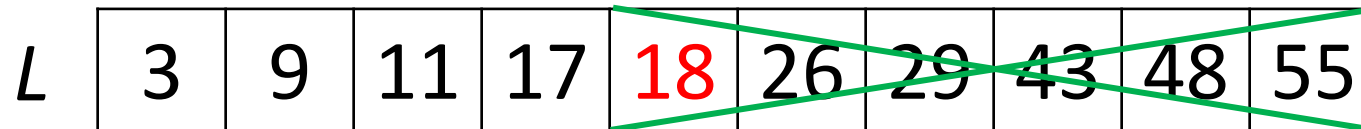
Correctness of Binary Search

Correct Output. Case 1: x is not in L

In each recursive call the algorithm only discards values **different** from x

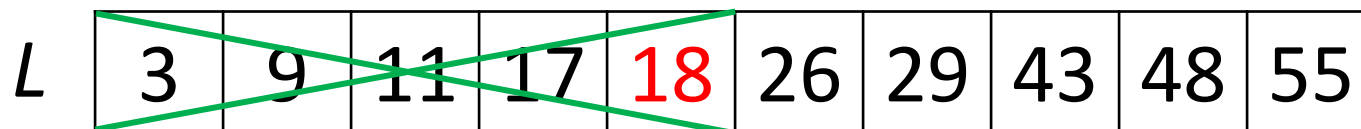
if $x < L[\text{mid}]$ then

return BinarySearch ($L, x, \text{first}, \text{mid}-1$)



$x < L[\text{mid}]$

else return BinarySearch ($L, x, \text{mid}+1, \text{last}$)



$x > L[\text{mid}]$

Correctness of Binary Search

Correct Output. Case 1: x is not in L

Since the algorithm only discards values different from x , all values of L will be discarded (so L is empty) and the algorithm correctly returns -1:

if first > last then return -1

Correctness of Binary Search

Correct Output. Case 2: x is in L

Since the algorithm terminates then either

- return -1, or
- return mid

must be executed.

Since return -1 is executed only if x is not in L then

if $x = L[mid]$ then return mid

is executed when x is in L and so the algorithm correctly returns the position of x in L .