Express $\neg$ and $\vee$ using $\wedge$, $\to$ and false.

$\neg P$ iff $P \to$ false

$P \vee Q$ iff $\neg P \to Q$.

$\neg(\neg P \wedge \neg Q)$.

§2.2. Cont

Ex: $A = B$

iff $\forall x (x \in A \leftrightarrow x \in B)$.

iff $(\forall x \in A \to x \in B) \wedge (\forall x \in B \to x \in A)$.

iff $(A \subseteq B) \wedge (B \subseteq A)$

Ex: $U = N$

"m divides n": $\exists k \in N$, $n = mk$. notation as $m | n$.

$3 | 12$ T

$12 | 3$ F

$3 | 7$ F

n is composit: $\exists a \exists b (a > 1, b > 1, n = ab)$.

n is prime: $n > 1 \wedge$ (n is not a composit).

Ex: $U \in R$

Every real number have a cube root.

$\forall x \exists y \; y^3 = x$ (T)

Every real number have an unique cube root.

$\forall x \exists y (y^3 = x \wedge \neg \exists z (z^3 = x \wedge z \neq y))$

$\exists! x \; P(x)$: exist an unique x that $P(x)$ is true.

for square root:

$\forall x > 0 \exists y \exists z (y^2 = x \wedge z^2 = x \wedge y \neq z \wedge \neg \exists k (k^2 = x \wedge (k \neq y \vee k \neq z)))$

§ More operations on sets.

Ways to define a set:

list elements: $\{6, 7, 8\}$, $\{\{1\}, \emptyset\}$.

set builder notation: $\{x \mid P(x)\}$.

index family: $\{n^2 \mid n \in N\} \leftarrow \{z_i \mid i \in I\}$

↑ set builder should start with variable.

$\{x^2 \mid x \in R\} = [0, \inf)$.

$\{n^2 \mid n \in \{1, 4, 7, 9\}\} = \{1, 16, 49, 81\}$.

$\{[0,x) \mid x \in \mathbb{R}^+\} = \{x \in \mathbb{R} \mid x \in [0, \inf)\}$.

Power set: given set A, the power set of A is notated as $P(A)$ such that $\forall B \in P(A), B \subseteq A$.

Ex: $P(\{2,3\}) = \{\emptyset, \{2\}, \{3\}, \{2,3\}\}$.

* note that $2 \notin P(\{2,3\})$ while $\{2\} \in P(\{2,3\})$

$2 \nsubseteq \{2,3\}$. $\quad \{2\} \subseteq P(\{2,3\})$ ← definition

$2 \in \{2,3\}$.

$B \in P(A)$ iff $B \subseteq A$. , $\emptyset \subseteq P(A)$

$\emptyset \in P(A)$

$P(\emptyset) = \{\emptyset\}$.

A have n elements, then $P(A)$ has $\underline{2^n}$ elements.

Ex: $x \subseteq P(A)$ $\quad x \in P(A \cup B)$

iff $x \subseteq A$. $\quad$ iff $x \subseteq A \cup B$.

iff $\forall y\,(y \in x \to y \in A)$ $\quad$ iff $\forall y\,(y \in x \to y \in A \vee y \in B)$

$P(A) \cup P(B)$.