HW2. Due Fri 11:59 pm.

OH: Thurs Fri 7-8 pm

Fri 3-4 pm

§1.4 Operations & Sets.

Recall:

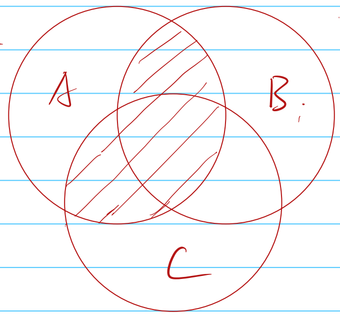
$A \cap B = \{x \mid x \in A \wedge x \in B\}$

$A \cup B = \{x \mid x \in A \vee x \in B\}$.

$A \setminus B = \{x \mid x \in A \wedge x \notin B\}$.

Ex: proof: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Venn diagram:



equal: two sets has same elements.

$x \in A \cap (B \cup C)$

iff $x \in A \wedge x \in (B \cup C)$.

iff $x \in A \wedge (x \in B \vee x \in C)$

iff $(x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$

iff $x \in (A \cap B) \vee x \in (A \cap C)$

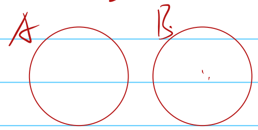
iff $x \in (A \cap B) \cup (A \cap C)$.

So $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$ are equal.

Def: A is a subset of B iff $\forall x \in A$, $x \in B$.
written as $A \subseteq B$.

Ex: $\emptyset \subseteq \mathbb{N} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{R}$.

Def: A and B are disjoint iff $A \cap B = \emptyset$. i.e. $\forall x \in A, x \notin B$.

Venn diagram: 

1.4.7. Theorm:

$(A \cup B) \setminus B \subseteq A$.

Proof: suppose $x \in (A \cup B) \setminus B$.

These proofs should be in one paragh.

$x \in (A \cup B) \setminus B$

iff $x \in (A \cup B) \wedge x \notin B$.

iff $(x \in A \vee x \in B) \wedge \neg x \in B$ <= $(p \vee q) \wedge \neg q = p$

Since we cannot have both $x \in B$ and $x \notin B$.

So $x \in A$.

So every element of $(A \cup B) \setminus B$ is an element of A.

Ex: For any sets A, B, $A \cap B$ and $A \setminus B$ are disjoint.

Proof: suppose $x \in A \cap B$.

$x \in A \cap B$

iff $x \in A \wedge x \in B$.

§1.5 Conditional & Biconditional

Conditional

$P \to Q$: if P then Q.

P	Q	$P \to Q$	$\neg P \vee Q$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

true statement.

Ex: if x is an integer, then $2x$ is an integer.

$x = 1$: 1, 2 . $T \to T$ T

$x = 1/2$: 1/2, 1 $F \to T$ T

$x=1/4$ $1/4, 1/2$ $F \to F$ T

Ex: If it rains, then I won't attend the class

$R \to \neg A$

to attain classes, you must be vaccinated.

$A \to V$ (each person attend is vaccinated)

Equivelent forms:

P	q	$\neg p \vee q$	$\neg(p \wedge \neg Q)$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

Thus, $P \to Q$ could be replaced with $\neg p \vee q$.

Ex: $A \to V$

iff $\neg A \vee V$. either you get vaccinated or you don't come to class.

iff $\neg(A \wedge \neg V)$. you can't come to class and no-vaccinated.

Contrapositive. $P \to Q$ iff $\neg Q \to \neg p$

P	Q	$P \to Q$	$\neg Q$	$\to$	$\neg p$
F	F	T	T	$\boxed{T}$	T
F	T	T	F	$\boxed{T}$	T
T	F	F	T	$\boxed{F}$	F
T	T	T	F	$\boxed{T}$	T

$\neg V \to \neg A$: the one who not vaccinated can't attend the class.

Converse of $P \to Q$ is $Q \to P$.

the converse has no relation with the origin statement.

Equivelent things: $P \to Q$ $\quad \neg Q \to \neg p$

$\neg p \vee q$ $\quad \neg(p \wedge \neg q)$

if P then Q

P implies Q

Q if P

P only if Q.

P is a sufficient condition for Q.

Q is a necessary condition for P