Assignment 5

Due: Saturday Dec 5th, 2020 before 11:55 PM to be uploaded in Gradescope as a single pdf file.

Please write your name and student number on your submission. Justify each step carefully. When in doubt prove the statement you are going to use. Solutions are graded for correctness as well as clarity.

Exercise 1 (10 point)**.** Let $f(x_1, x_2, x_3)$ be a non-zero Boolean function in three variables x_1, x_2, x_3. Prove that $f(x_1, x_2, x_3) = x_1.g_1(x_2, x_3) + \overline{x_1}.g_2(x_2, x_3)$, where g_1, g_2 are Boolean functions obtained as follows:

Cut the truth table for $f(x_1, x_2, x_3)$ in half and delete the column for x_1. Now you have two truth tables in the variables x_2 and x_3, each representing the functions $g_1(x_2, x_3)$ and $g_2(x_2, x_3)$.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
1	1	1	*
1	1	0	*
1	0	1	*
1	0	0	*
0	1	1	*
0	1	0	*
0	0	1	*
0	0	0	*

$g_2(x_2, x_3)$

$g_3(x_2, x_3)$.

Exercise 2 (20 points)**.** Prove that any non-zero Boolean function can be written as a sum of minterms.

Hint: See section 6.2 in the zyBook for definitions. Draw a 'truth table' with n variables and identify the 'truth table' with $n-1$ variables. Use the idea from exercise-1 as explained in the zoom class and induction.

Exercise 3 (10 points, 5 each)**.** Construct a Boolean circuit for the Boolean function $f(x, y, z) = (x + \bar{y})(y + \bar{z})$ なはし.

i. ONLY using NAND gates.

ii. ONLY using NOR gates.