Q1

$B = \mathbb{R} \setminus \{5\}$, $f^{-1} = \frac{y}{y-5}$

Since $B = \mathbb{R} \setminus \{5\}$, $x = \frac{y}{y-5}$, $xy - 5x = y$, so $y = \frac{5x}{x-1}$ for $x \neq 1$. So for every $y \in B$, there exist an unique x that $f(x) = y$ $\square$.

Q2.

Assume that $f: A \to B$ is onto and $h: B \to C$ is not one-to-one. Since h is not one to one, there exist $b_1, b_2 \in B, b_1 \neq b_2$ $C_0 \in C$ that $h(b_1) = h(b_2) = c_0$. Since f is onto, there exist $a_1, a_2 \in A$. $a_1 \neq a_2$ that $f(a_1) = b_1$, $f(a_2) = b_2$. Thus, $h(f(a_1)) = h(f(a_2)) = c_0$, that is $h \circ f(a_1) = h \circ f(a_2)$. Thus, $h \circ f$ is not one-to-one. $\square$

Q3

We prove by induction. Assume $\sum_{i=0}^{n} n(n+3) = \frac{n(n+1)(n+5)}{3}$

Base case: if $n=0$, $0 \times (0+3) = \frac{0 \times (0+1) \times (0+5)}{3} = 0$

Inductive case: if $m = n-1$, $\frac{m(m+1)(m+5)}{3} + n(n+3) = \frac{(n-1)n(n+4)}{3} + n(n+3))$

$$= \frac{n^3 + 6n^2 + 5n}{3}$$

$$\frac{n(n+1)(n+5)}{3} = \frac{n^3 + 6n^2 + 5n}{3}$$

Since $\frac{n(n+1)(n+5)}{3} = \frac{(n-1)[(n-1)+1][(n-1)+5]}{3} + n(n+3)$, by induction we can conclude that for $n \in \mathbb{N}$, $0 \cdot 3 + 1 \cdot 4 + 2 \cdot 5 + \cdots + n(n+3) = \frac{n(n+1)(n+5)}{3}$ $\square$.

Q4.

$n = 4$.

Prove: $4! = 24$, $4^2+2=18$, $4! > 4^2+2$.

For smaller natural numbers,

$3! = 6$, $3^2+2=11$, $4! < 3^2+2$.

$2! = 2$, $2^2+2=6$, $2! < 2^2+2$

$1! = 1$, $1^2+2=3$, $1! < 1^2+2$.