Computer Science 2209A
Midterm
October 25, 2020. 11:00 AM - 1:00 PM
(submission by 1:30PM)

Instructions

1. No calculators allowed.
2. There is a cheat sheet for laws of propositional logic and rules of inference at the end of the question paper.
3. Every proof must be justified carefully. Clarity in arguments is weighed heavily. You can lose marks (especially in Question 5 and 6) if you write a poor proof even with the correct idea.

1. (20 points, 10 each) Prove the following equivalences using laws of propositional logic. Clearly state which laws you are using in each step.

   (i.) $(p \wedge q) \to r \equiv (p \wedge \neg q) \to \neg q$.

   (ii.) $(p \to q) \vee (q \to p) \equiv \mathrm{T}$

2. (25 points) Assume p, q, r are propositions. Prove the validity or invalidity of the following arguments. If the argument is valid give a proof using rules of inference/ laws of propositional logic in the format discussed in the class/textbook. If the argument is invalid prove it using truth values.

   (i) (10 points)

$$\begin{array}{r} p \vee q \\ p \vee r \\ \neg p \\ \hline \therefore q \wedge r \end{array}$$

   (ii.) (10 points)

$$\begin{array}{r} p \to q \\ \neg q \\ p \\ \hline \therefore r \end{array}$$

   (iii.) (5 points)

$$\begin{array}{r} p \to q \\ \hline \therefore p \end{array}$$

3. (9 points, 3 each)

   (i.) Identify the free and bound variables in the predicate $\forall x \exists y\ P(x, y, z)$.

   (ii.) Write the negation of the following predicate using De Morgan's law. Show your work.

   For every prime number p there exists an integer k such that $p = 2k + 1$.

   (iii.) Is the statement in 3(ii.) a proposition? If yes, is it True or False?

4. (16 points) Prove the validity of the following argument by converting it into the language of predicate logic and using appropriate laws/ rules of inference. State the domain and predicates clearly.

$$\begin{array}{r} \text{There are some trees which do not shed their leaves} \\ \text{The trees which change color always shed their leaves} \\ \text{All trees which do not change color are shorter than 25m.} \\ \hline \therefore \text{There is some tree which does not change color and shorter than 25m.} \end{array}$$

5. Prove one of the following theorems. Mention the method of proof you are using (direct proof, proof by contradiction etc.). Justify each step carefully.

   (i.) (15 points) Prove that the average of three real numbers x, y, z is greater than or equal to at least one of the numbers.
   Hint: What is the negation of this statement?

   OR

   (ii.) (15 points) Let a and b be arbitrary rational numbers, assume that $b \neq 0$. Prove that $a + b\sqrt{2}$ is not rational.
   Hint Use the fact that $\sqrt{2}$ is not rational.

6. (15 points) Let x be a positive integer. Prove that

$$\text{if } x^2 + 3x + 5 \text{ is even, then } x \text{ is also even.}$$

using 'proof by contraposition'.

Rules of inference:

Modus ponens	p $p \to q$ ——— q	Modus tollens	$\neg q$ $p \to q$ ——— $\neg p$
Addition	p ——— $p \lor q$	Simplification	$p \land q$ ——— p
Conjunction	p q ——— $p \land q$	Hypothetical Syllogism	$p \to q$ $q \to r$ ——— $p \to r$
Disjunctive Syllogism	$p \lor q$ $\neg p$ ——— q	Resolution	$p \lor q$ $\neg p \lor r$ ——— $q \lor r$

Laws of propositional logic

Idempotent laws:	$p \lor p \equiv p$	$p \land p \equiv p$
Associative laws:	$(p \lor q) \lor r \equiv p \lor (q \lor r)$	$(p \land q) \land r \equiv p \land (q \land r)$
Commutative laws:	$p \lor q \equiv q \lor p$	$p \land q \equiv q \land p$
Distributive laws:	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
Identity laws:	$p \lor F \equiv p$	$p \land T \equiv p$
Domination laws:	$p \land F \equiv F$	$p \lor T \equiv T$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \land \neg p \equiv F, \neg T \equiv F$	p $\lor \neg p \equiv T, \neg F \equiv T$
De Morgan's laws:	$\neg(p \lor q) \equiv \neg p \land \neg q$	$\neg(p \land q) \equiv \neg p \lor \neg q$
Absorption laws:	$p \lor (p \land q) \equiv p$	$p \land (p \lor q) \equiv p$
Conditional identities:	$p \to q \equiv \neg p \lor q$	$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$