ECON3102-005
Chapter 6: Economic growth: The Solow growth model (Part 1)

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Motivations

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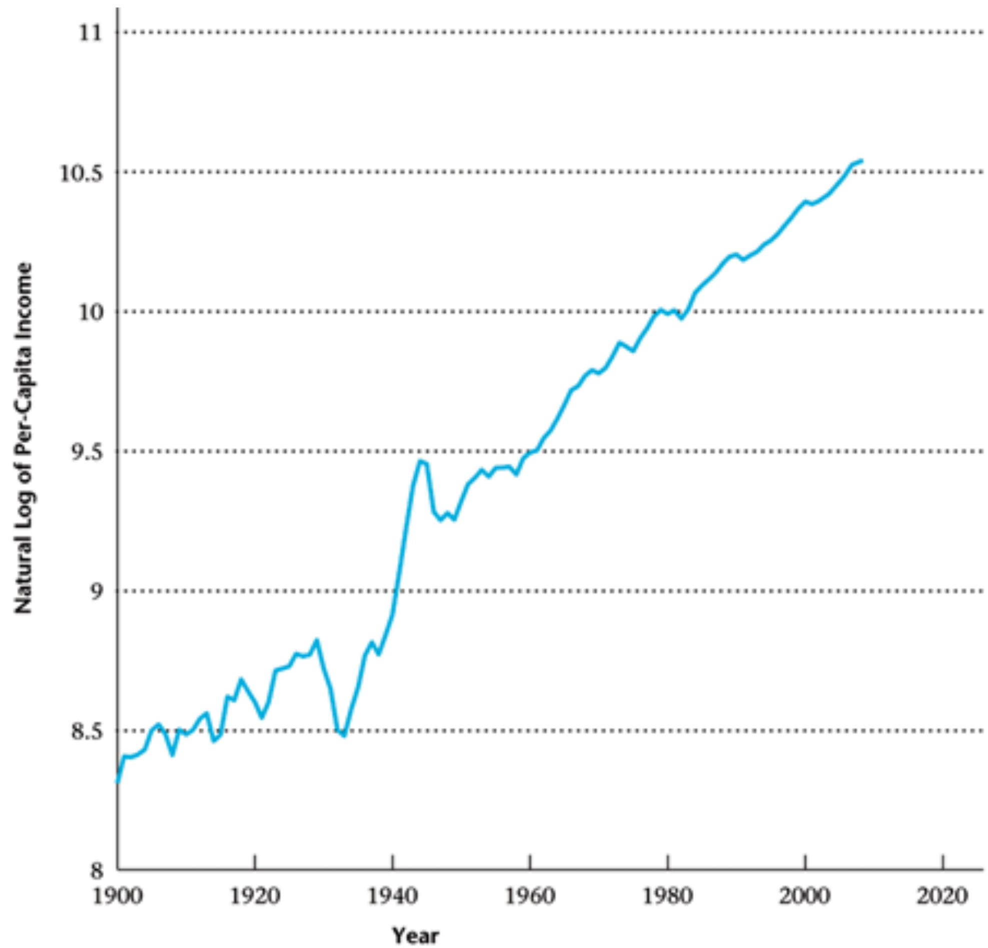
- As Robert Lucas put it, "Once you start thinking about growth, its hard to think about anything else."
- We'll use the framework we have learned and try to get some answers to the questions above. Now, there are some empirical facts that could help to motivate the discussion.

Observations



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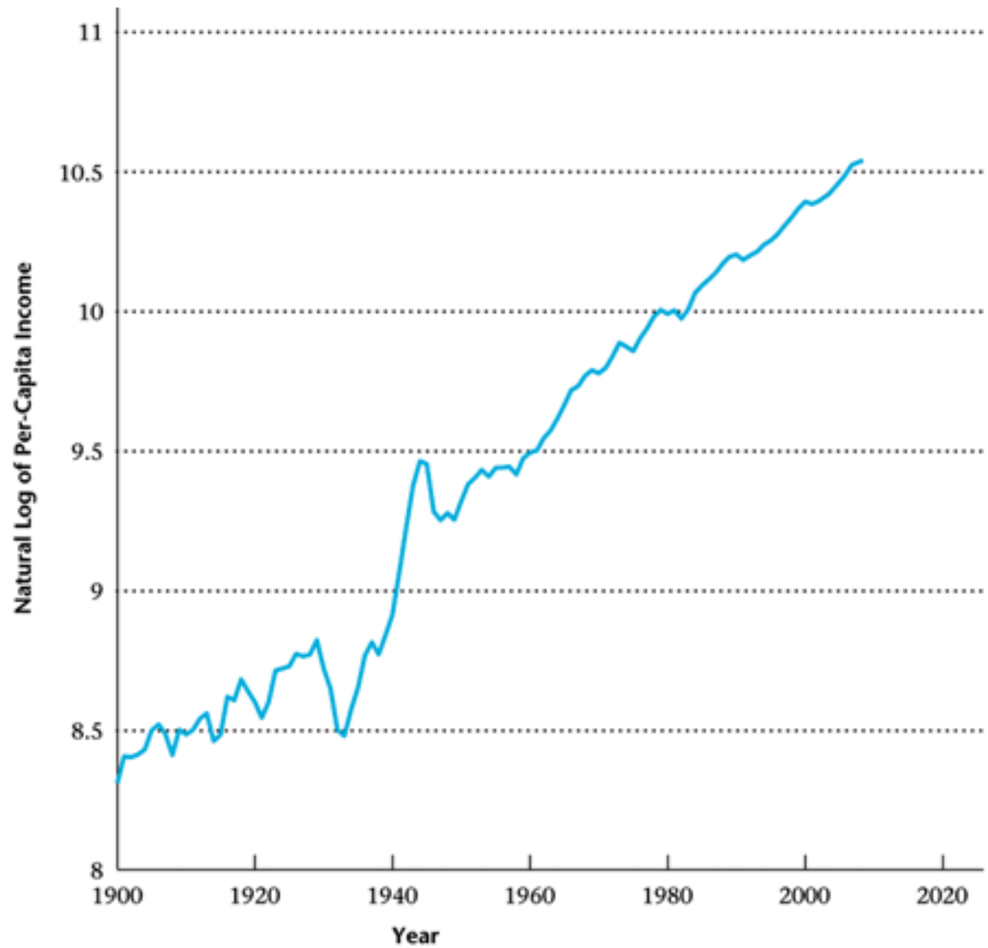

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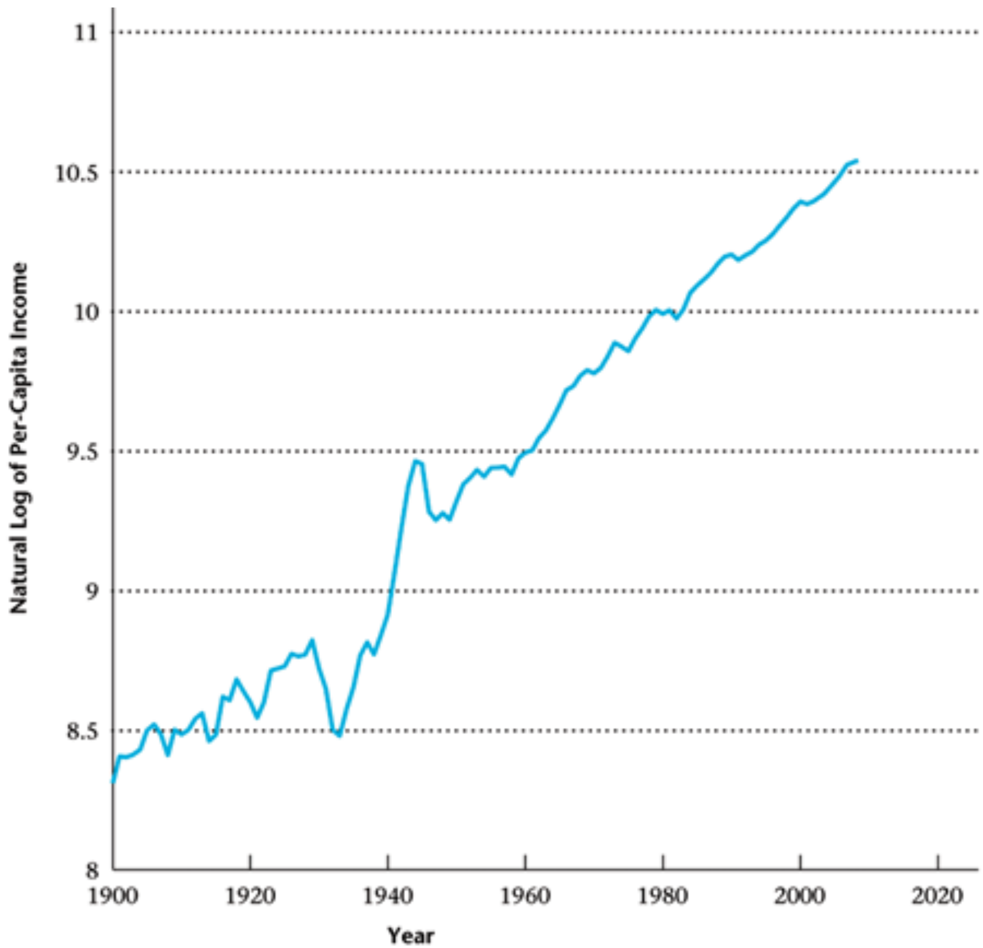
- Since the industrial revolution, per capita income growth has been sustained in the richest countries. In the US, average annual growth in per capita income has been about 2% since 1869.



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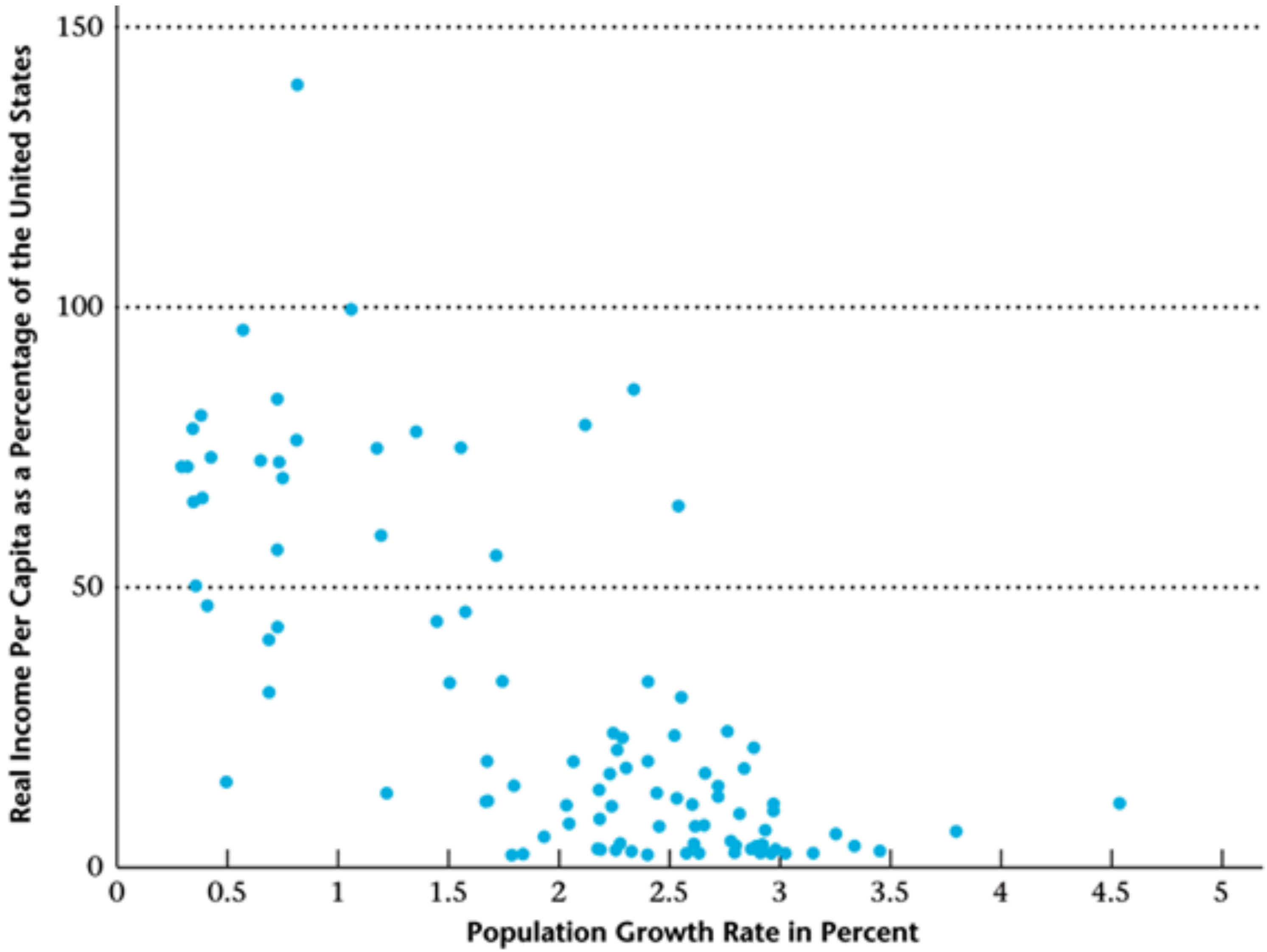
OBSERVATIONS



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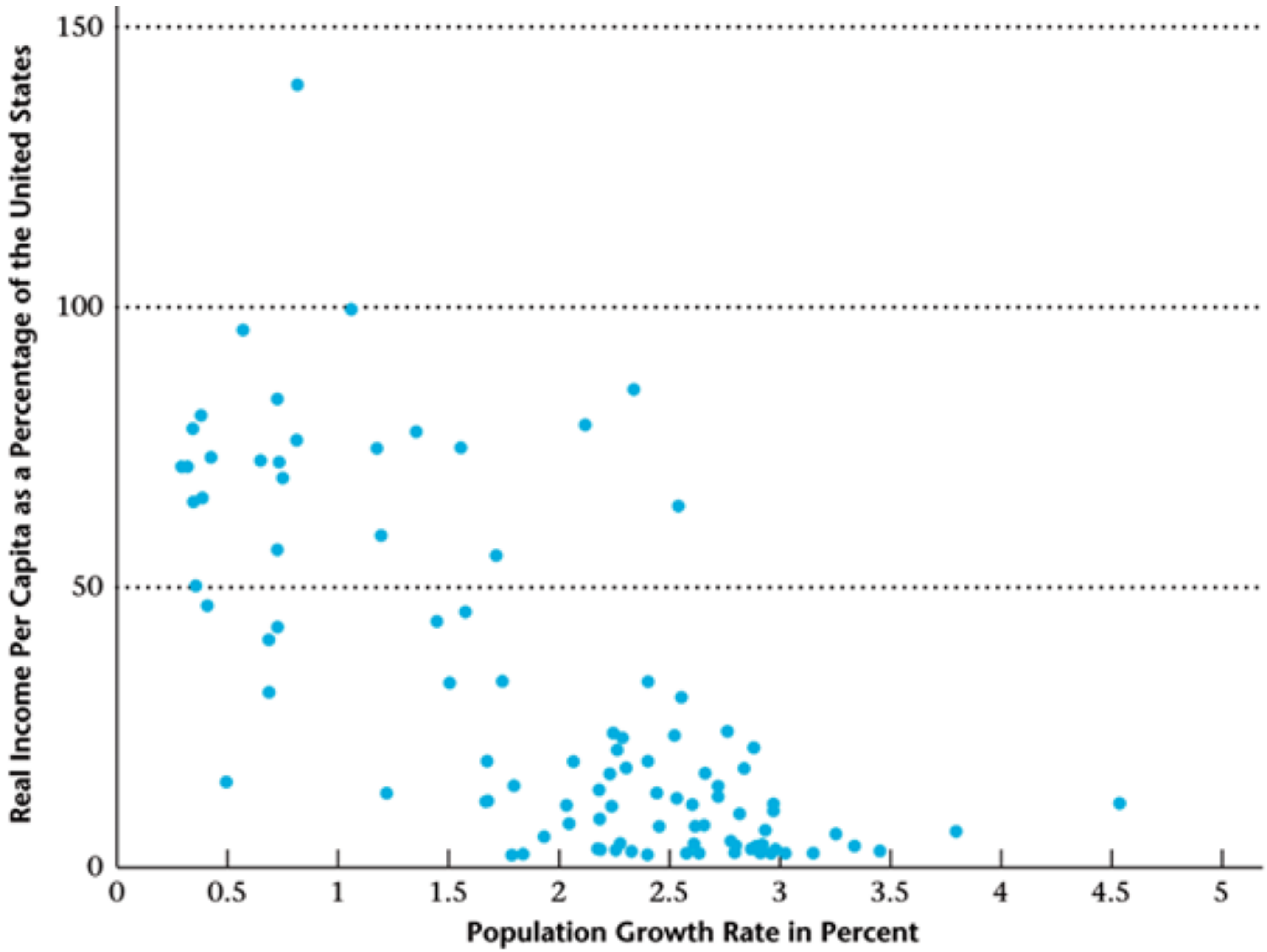
- There is a negative correlation between the population growth rate and output per worker across countries.



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- There is no government; consequently, no taxes.

The Solow Growth Model (cont'd)

- Consumers receive Y, current real output, as income. They face the decision of how much of current income to save and how much to consume. We assume they consume a constant fraction of income:

$$C = (1 - s)Y, \quad s < 1,$$

where C is current consumption, s the savings rate, and current savings are $S = sY$.

The Solow Growth Model (cont'd)

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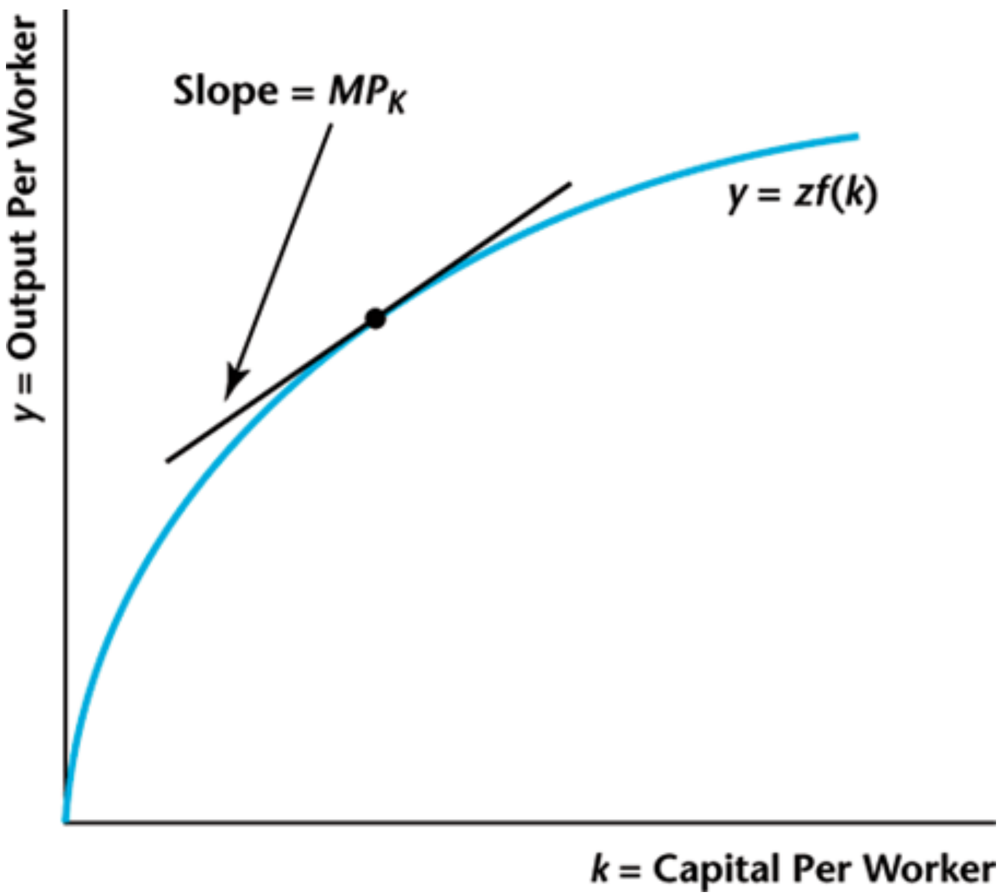
- Here we let $\frac{Y}{N}$ be output per worker, and $\frac{K}{N}$ capital per worker. Then (2) tells that the output per worker depends on the capital per worker.

The Solow Growth Model (cont'd)



- Rewrite (2) as

$$y = zf(k), \tag{3}$$

where $y = Y/N$, $k = K/N$, $f(k) = F(k, 1)$.

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- Here, depreciation is denoted by d, where $0 < d < 1$.
- Now we can talk about dynamics. Given the depreciation rate, the capital stock changes over time according to

  $$K' = (1 - d)K + I,$$

  where I denotes investment.

The Solow Growth Model (cont'd)

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- In the labor market, current consumption goods are traded for current labor.
- In the assets market, current consumption goods are traded for capital.
- Capital is the asset in this economy, and consumers save by accumulating it.

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- Substituting for I and C from equations, $K' = (1 - d)K + I$ and $C = (1 - s)Y$, gives

$$Y = (1 - s)Y + K' - (1 - d)K$$

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- Equation (5) says that future capital equals the amount of savings plus capital left over from the current period that has not depreciated.

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- Using equations, $Y = zF(K, N)$ and $K' = sY + (1 - d)K$, we have

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- We can divide by N to express in per worker terms

$$\frac{K'}{N} = szF(\frac{K}{N}, 1) + (1 - d)\frac{K}{N},$$

multiplying the first term by $1 = N'/N'$

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- Dividing across by $(1+n)$ gives the key equation of the model:

$$k' = \frac{szf(k)}{1+n} + \frac{(1-d)k}{1+n} \qquad (*)$$

Steady States

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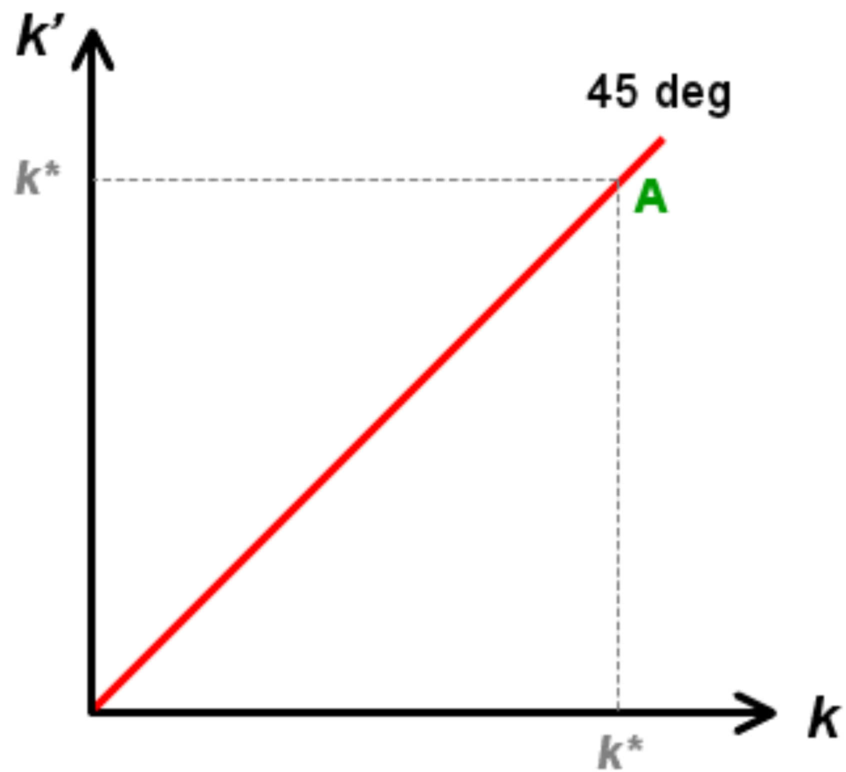
- Note that when we graph in $k'k$ space, any point that crosses the 45 degree line satisfies $k' = k$.



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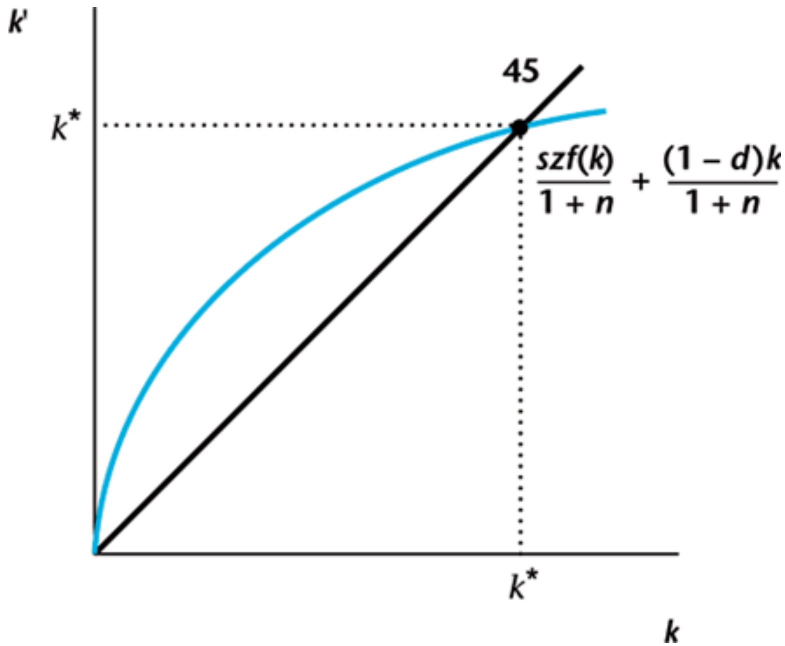

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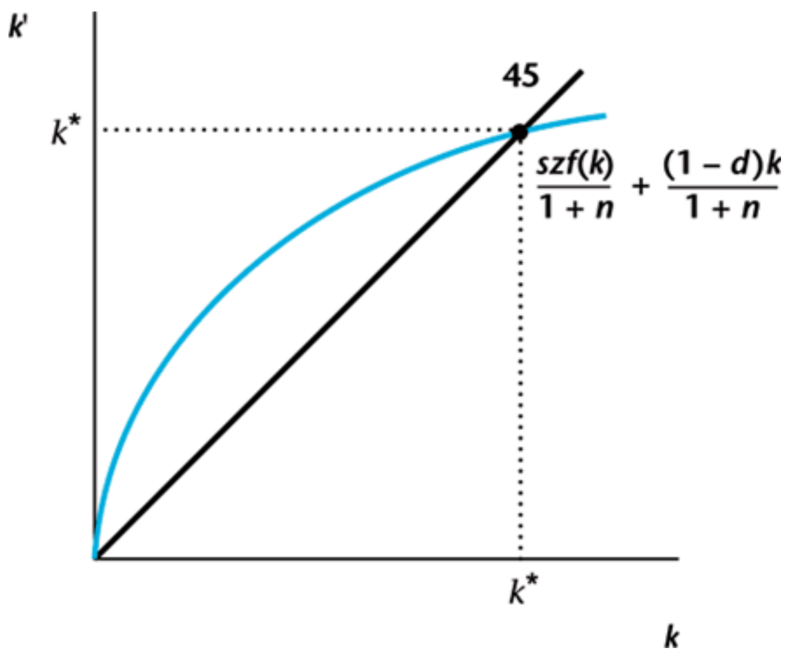

- At the steady state, $k = k^*$ and $k' = k^*$; k^* is the equilibrium level of capital in the economy.

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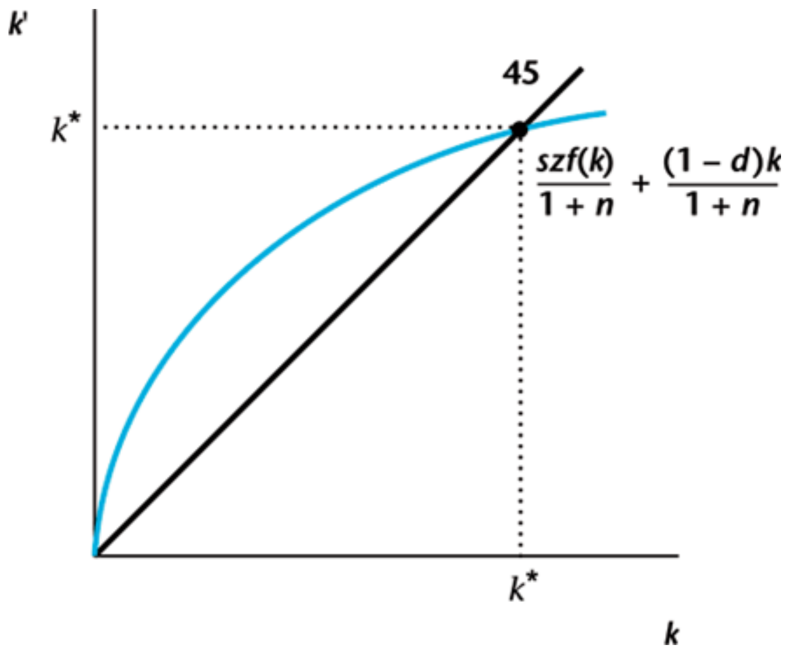

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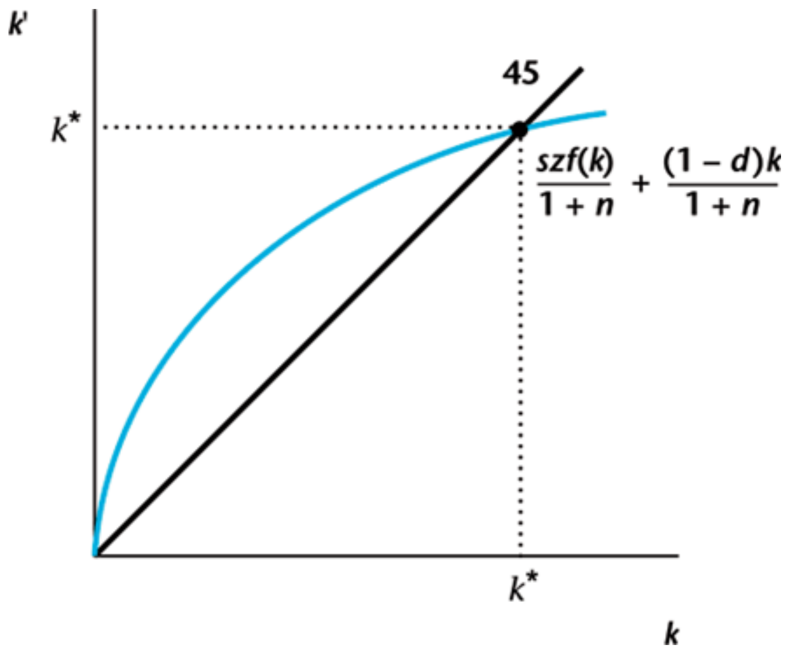

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- Here, current investment is relatively large with respect to depreciation and labor force growth.

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- So, theres no growth in here? Are we forgetting something?

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- In this way, the Solow growth model is an exogenous growth model.