Principle of strong induction:

PMI:

Given a proposition $P(n)$ parameterized by $n \in \mathbb{Z}_+$

1) If $P(1)$ is is true

2) If $P(n) \to P(n+1)$ is true for all $P(n)$

then $P(n)$ is true for all $n \in \mathbb{Z}_+$

Variant I:

If $P(1)$, $P(2)$ is true

$P(k), P(k+1) \Rightarrow P(k+2)$ is true

Then $P(n)$ is true for all $n \in \mathbb{Z}_+$.

e.g. Let $\{a_i\}_{i=0}^{\infty} = (a_0, a_1, a_2, a_3 \cdots\cdots)$ be a recursive sequence.

defined by $a_0 = 1$ $a_1 = 3$ $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2$.

Show that $a_n = 2n+1$.

$a_n - a_{n-1} = a_{n-1} - a_{n-2} = \cdots\cdots = a_1 - a_0 = 2$

$a_n - a_{n-1} = 2 \Rightarrow a_n = 2n+1$.

Proof: Using Principle of Mathenetical Induction

P_0 holds. $\quad 2n+1$

Base Case: $a_0 = 1 \quad 1$

$a_1 = 3 \quad 2(1)+1 = 3$

$P(1)$ holds.

Inductive hypotlesis: $P(k)$ $P(k+1)$.

$P(k)$: $a(k) = 2k+1 \wedge P(k+1)$. $a_{(k+1)} = 2k+3$.

$a_n = 2a_{n-1} - a_{n-2}$.

when $n = k+2$: $a_{k+2} = 2a_{k+1} - a_k$.

$= 2(2k+3) - (2k+1)$

$= 2(k+2)+1$

$= P(k+2)$.

therefore, $P(k) \wedge P(k+1) \Rightarrow P(k+2)$.

Hence, by PMI(II) $a_n = 2n+1$ for all $n \in N$