Ch6. Mathmetic Induction

§6.1 Proof by mathmatical induction

Strategy: To proof a goal of the form $\forall n \in N\ P(n)$.

$P(0)$ (base case).

$\forall n \in N\ P(n) \to P(n+1)$.

Exercise 3: $\forall n \in R,\ 0^3 + 1^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$.

RW: $P(n) : 0^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$.

(not shown)

Base step: $P(0) = 0 = \left[\frac{0(0+1)}{2}\right]^2$

Inductive step: $\forall n \in R,\ P(n) \to P(n+1)$

Given	Goal
$\sum_{i=0}^{n} n^3 = \left[\frac{n(n+1)}{2}\right]^2$.	$\sum_{i=0}^{n+1} n^3 = \left[\frac{(n+1)(n+2)}{2}\right]^2$
$= (n+1)^2 \cdot \frac{n^2}{4}$	$= (n+1)^2 \cdot \frac{n^2+4n+4}{4}$

Proof: We use induction on variable n (tell reader the method and the variable)

Base case: $n=0$ $\quad 0^3 = 0 = \frac{0(0+1)}{2}$

Inductive step: Let $n \in N$, and assume:

$$0^3 + 1^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Then
$$0^3 + 1^3 + \cdots + n^3 + (n+1)^3 = \left[\frac{n(n+1)}{2}\right]^2 + (n+1)^3$$
$$= (n+1)^2 \left[\frac{n^2}{4} + n + 1\right]$$
$$= \frac{(n+1)^2}{4}\left[n^2 + 4n + 4\right].$$
$$= \left[\frac{(n+1)(n+2)}{2}\right]^2 \qquad \square.$$

* the inductive step should be write in detail, it is not allowed to be something like foo = foo

Ex 6.1.2. $\forall n \in N,\ (3 \mid n^3 - n)$.

Base case: $3 \mid 0$

Inductive step: Let $n \in N$, assume $3 \mid n^3 - n \in N$. So there is an integer k such that $(n^3 - n) = 3k$.

Notice that $(n+1)^3 - (n+1) = n^3 + 3n^2 + 2n$

$= (n^3-n) + 3n^2+3n$
$= 3k + 3n^2 + 3n$

Since k, n are integers, $3 \mid (n+1)^3-(n+1)$ $\square$.

Ex 6.1.3 $\forall n \geq 5$, $2^n > n^2$

Rw: $n=5$: $2^5 > 5^2$

$n>5$: Given   Goal

$2^n > n^2$ $2^{n+1} > (n+1)^2$.

$= 2^n \cdot 2^n$, $(n+1)^2 = n^2+2n+1$.

Proof: Base case: $n=5$, $2^n = 32$, $n^2=25$, $2^n \geq n^2$ holds.

Inductive step. Let $n \in N$. $n \geq 5$. Assume $2^n \geq n^2$.

$2^{n+1} = \underline{2^n \cdot 2} \geq 2n^2$ (inductive hyp)
$= n^2+n^2$
$\geq n^2+5n$ ($n \geq 5$)
$= n^2+2n+3n$
$\geq n^2+2n+1$ ($3n \geq 15$)
$= (n+1)^2$ $\square$.