

ECON3102-005

# Chapter 9: A Real Intertemporal Model of Investment

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Spring 2014

# What do we study in this chapter?

- Construct a real intertemporal model that will serve as a basis for studying money and business cycles in Chapters 11-13.
- Understand the investment decision of the firm.
- Show how macroeconomic shocks affect the economy.

# What's new in this chapter?

- The key things added to chapter 8 are a work-leisure decision and a representative firm.
- We are moving from an endowment economy to a production economy.
- The representative firm and consumer interact on the labor and consumption markets.

# The Representative Consumer

- has  $h$  units of time each period, for work and leisure ( $\ell$ ).
- today, earns real wage  $w(h - \ell)$  receives dividend income  $\pi$ , pays taxes  $T$ , and saves  $S^P$ .
- hence the current period BC:

$$C + S^P = w(h - \ell) + \pi - T$$

- tomorrow, earns real wage  $w'(h - \ell')$  receives dividend income  $\pi'$ , pays taxes  $T'$ , and receives  $S^P(1 + r)$ .
- hence, the future period BC:

$$C' = w'(h - \ell') + \pi' - T' + S^P(1 + r)$$

# The Representative Consumer

- This gives the consumer's PVBC:

$$C + \frac{C'}{1+r} = w(h - \ell) + \frac{w'(h - \ell')}{1+r} + \pi + \frac{\pi'}{1+r} - T - \frac{T'}{1+r}$$

- The representative consumer chooses  $C, C', \ell, \ell'$  to be as well off as possible, given this PVBC.

# The Representative Consumers Optimization Conditions

- The *MRS* of leisure for consumption is the price of leisure (wage):  
 $MRS_{\ell, C} = w$  and  $MRS_{\ell', C'} = w'$ .
- The *MRS* of current for future consumption is the price of current consumption in terms of future consumption ( $1 + r$ ):  
 $MRS_{C, C'} = 1 + r$

# The Representative Consumers Labor Supply

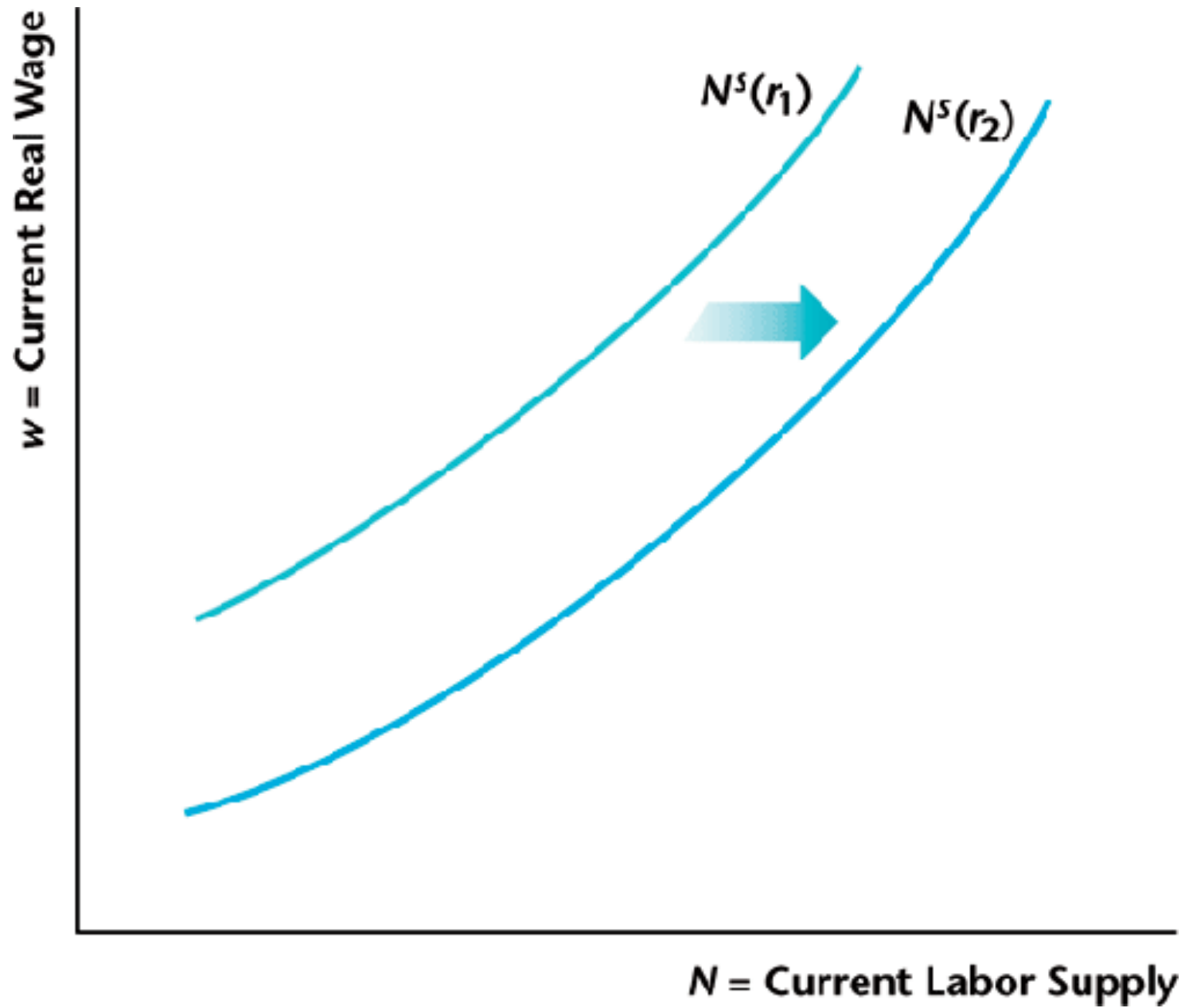
- Recall from Chapter 4 that as the real wage increases, the substitution effect  $\Rightarrow$  labor supply  $\uparrow$ .
- However, the income effect  $\Rightarrow$  leisure  $\uparrow$ , i.e. labor supply  $\downarrow$ . (assuming leisure is normal.)
- Here, we assume substitution  $>$  income effect, so that  $w \uparrow \Rightarrow$  labor supply  $\uparrow$ .

# Current Labor Supply and Real Interest Rate

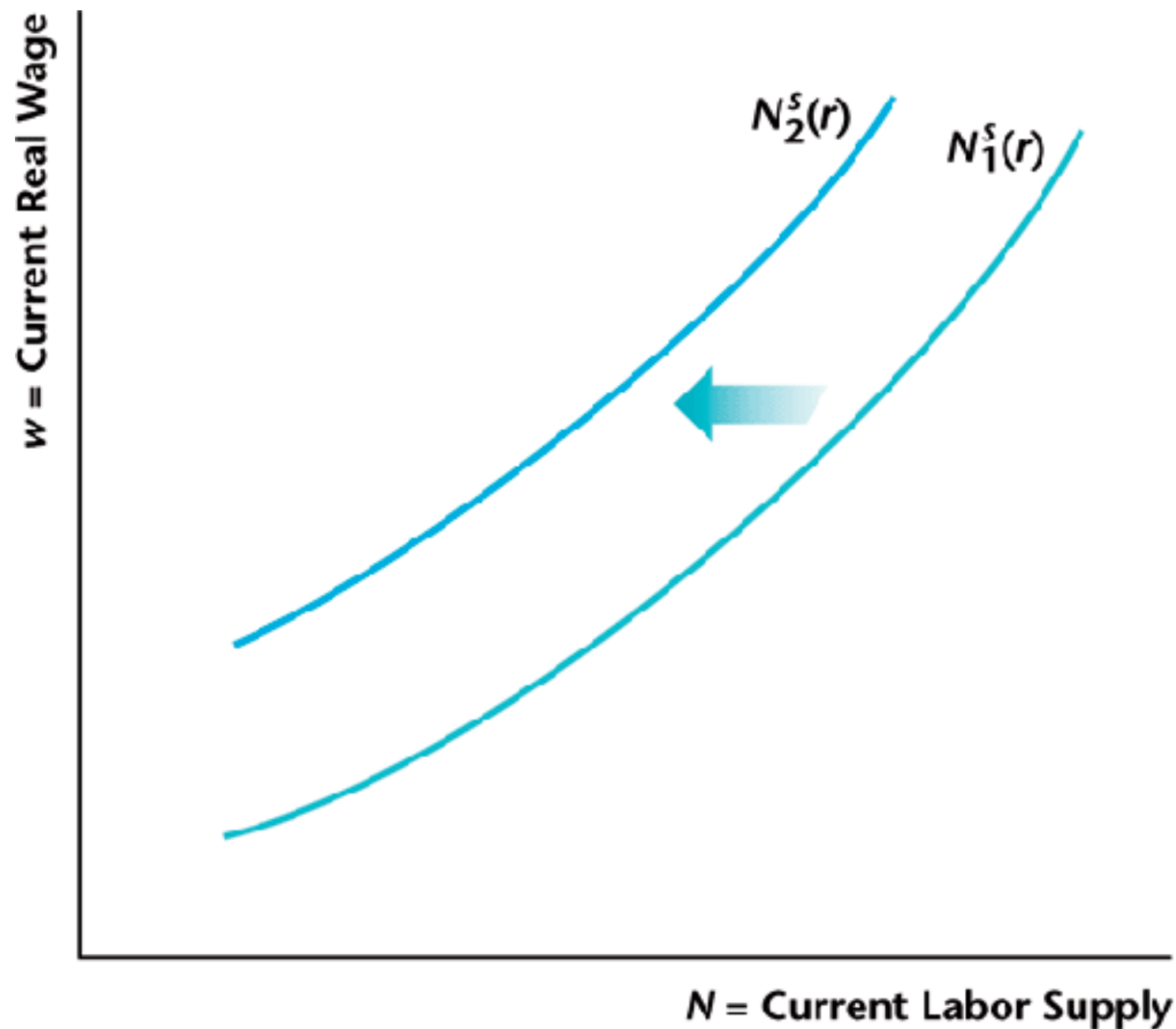
- The price of current relative to future leisure is:  $\frac{w(1+r)}{w'}$
- As  $r \uparrow$ , the consumer substitutes current for future consumption and current for future leisure.
- Hence, assuming substitution > income effect,  $r \uparrow \Rightarrow$  labor supply  $\uparrow$ .
- Labor supply  $\downarrow$  as non-wage disposable income  $\uparrow$  (income effect) (e.g:  $\downarrow$  in lifetime taxes.)



# An Increase in the Real Interest Rate Shifts the Current Labor Supply Curve to the Right



## Effects of an Increase in Lifetime Wealth



# The Representative Firm

- Let  $z$  and  $z'$  denote present and future TFP. Current Period Production function is:

$$Y = zF(K, N)$$

- Future Period Production function:

$$Y' = z'F(K', N')$$

# The Representative Firms Investment

- We assume it requires one unit of current consumption may be transformed into one unit of capital.
- The representative firm foregoes current profits, invests in current capital to increase capacity tomorrow:

$$K' = (1 - d)K + I$$

- We assume that in the last period, the remaining capital  $(1 - d)K$  is converted back into the consumption good and sold to consumers.

# The Representative Firm's Profit

- The objective of the firm is to max the PV of prots.

- Current Prots are:

$$\pi = Y - wN - I$$

- Future Prots are:

$$\pi' = Y' - w'N' + (1 - d)K$$

- The present value of prots is

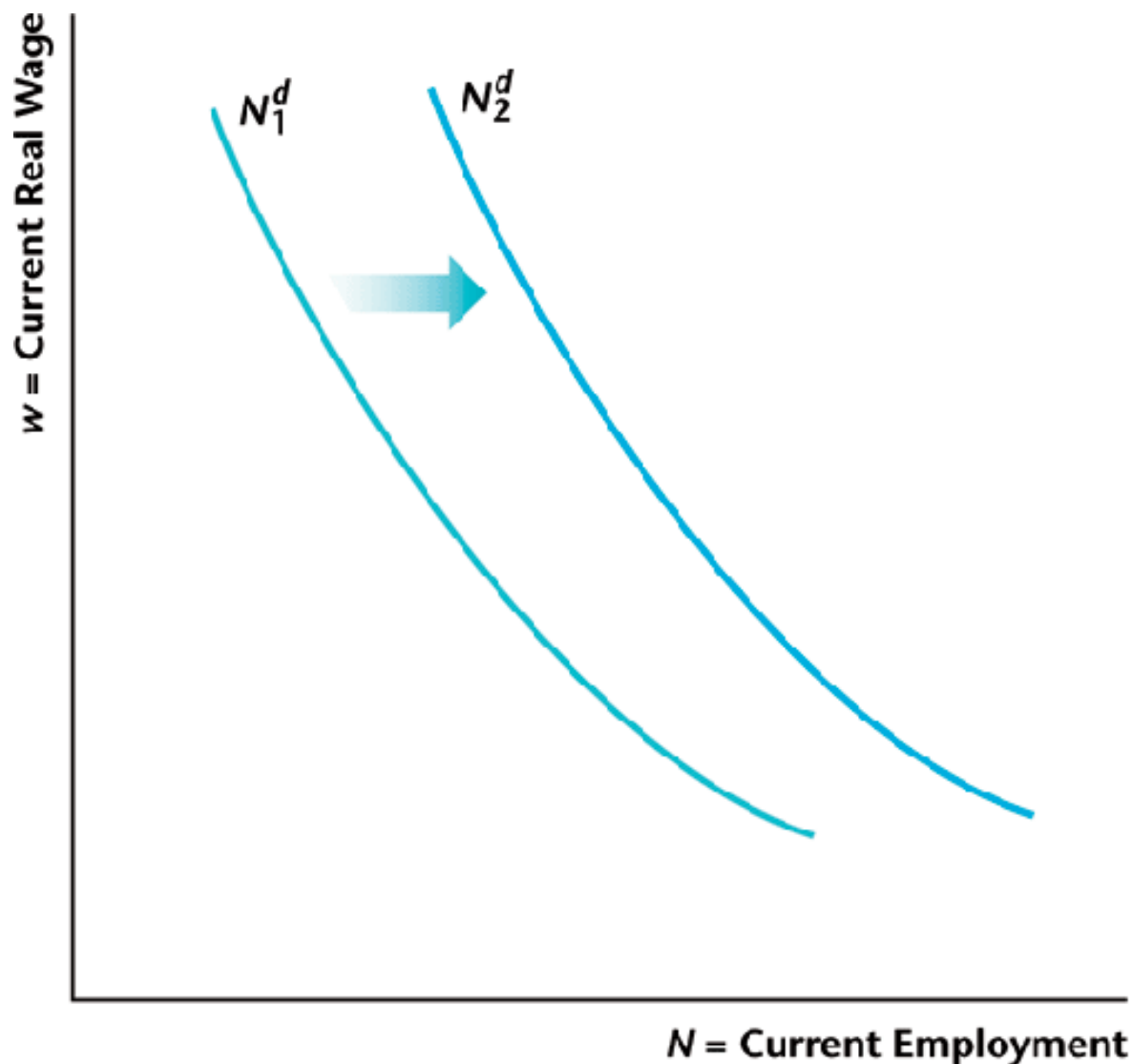
$$\pi + \frac{\pi'}{1 + r}$$

# The Representative Firms Current Labor Demand

The optimization rule for labor demand from the firm is:

- The objective of the firm is to max the PV of prots.  $MP_N = w$ .
- Recall from Ch 4 that because  $MP_N$  is  $\downarrow$  in  $N$ , the firms labor demand is also  $\downarrow$ .

# The Current Demand Curve for Labor Shifts Due to Changes in Current Total Factor Productivity $z$ and in the Current Capital Stock $K$



# The Representative Firms Investment Decision

- The marginal cost of investment is what is given up to make an additional unit of investment. It equals one unit of the consumption good:

$$MC(I) \equiv 1$$

.

- An additional unit of investment transforms into one unit of capital that tomorrow is used for production and sold after depreciation.
- Hence, the marginal benefit of investment in terms of units of current consumption is:

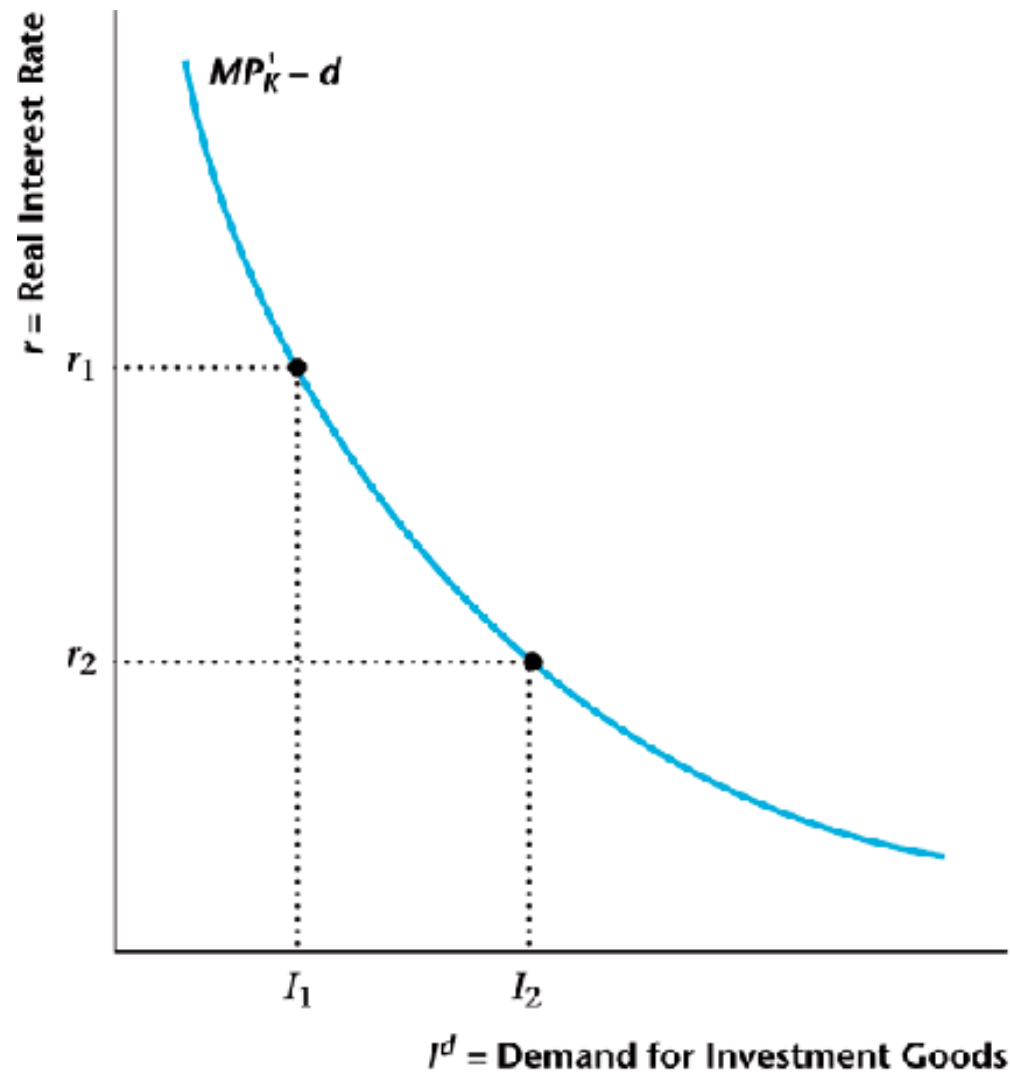
$$MB(I) \equiv \frac{MP_{k'} + 1 - d}{1 + r}$$



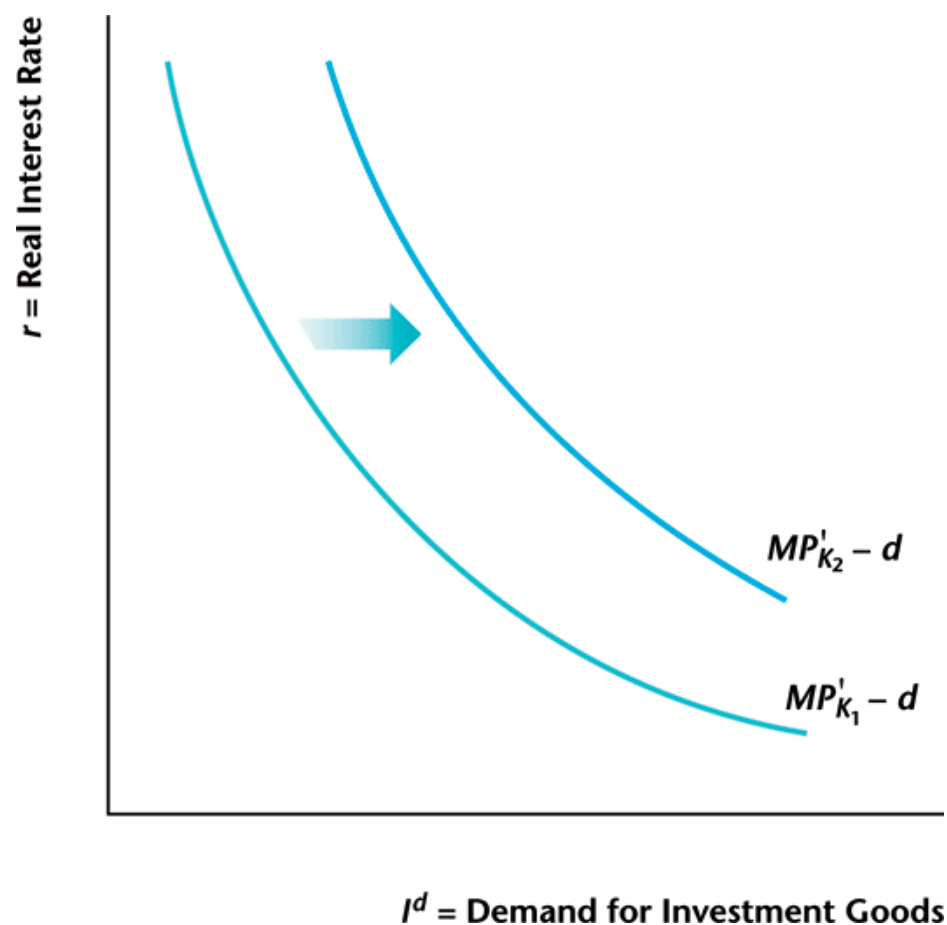
# The Representative Firms Investment Decision

- The firm invests until:  $MC(I) = MB(I)$ ,
- $\Rightarrow MB(I) \equiv \frac{MP_{k'} + 1 - d}{1 + r} = 1 \Rightarrow \underbrace{MP_{k'} - d}_{\text{net marginal product of capital}} = r.$
- Hence, as  $r \uparrow \Rightarrow MP_{k'} \uparrow \Rightarrow K' \downarrow \Rightarrow I = (K' - (1 - d)K) \downarrow.$
- The net marginal product of capital is the marginal product of capital, net of its depreciation.
- Intuition: The bond and capital are the two assets in the economy and should yield identical returns (the firm often borrows to finance investment).

# Optimal Investment Schedule for the Representative Firm



The Optimal Investment Schedule Shifts to the Right if  
Current Capital Decreases or Future Total Factor  
Productivity Is Expected to Increase



# The Government

- Governments behavior is the same as in Chapter 8:

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

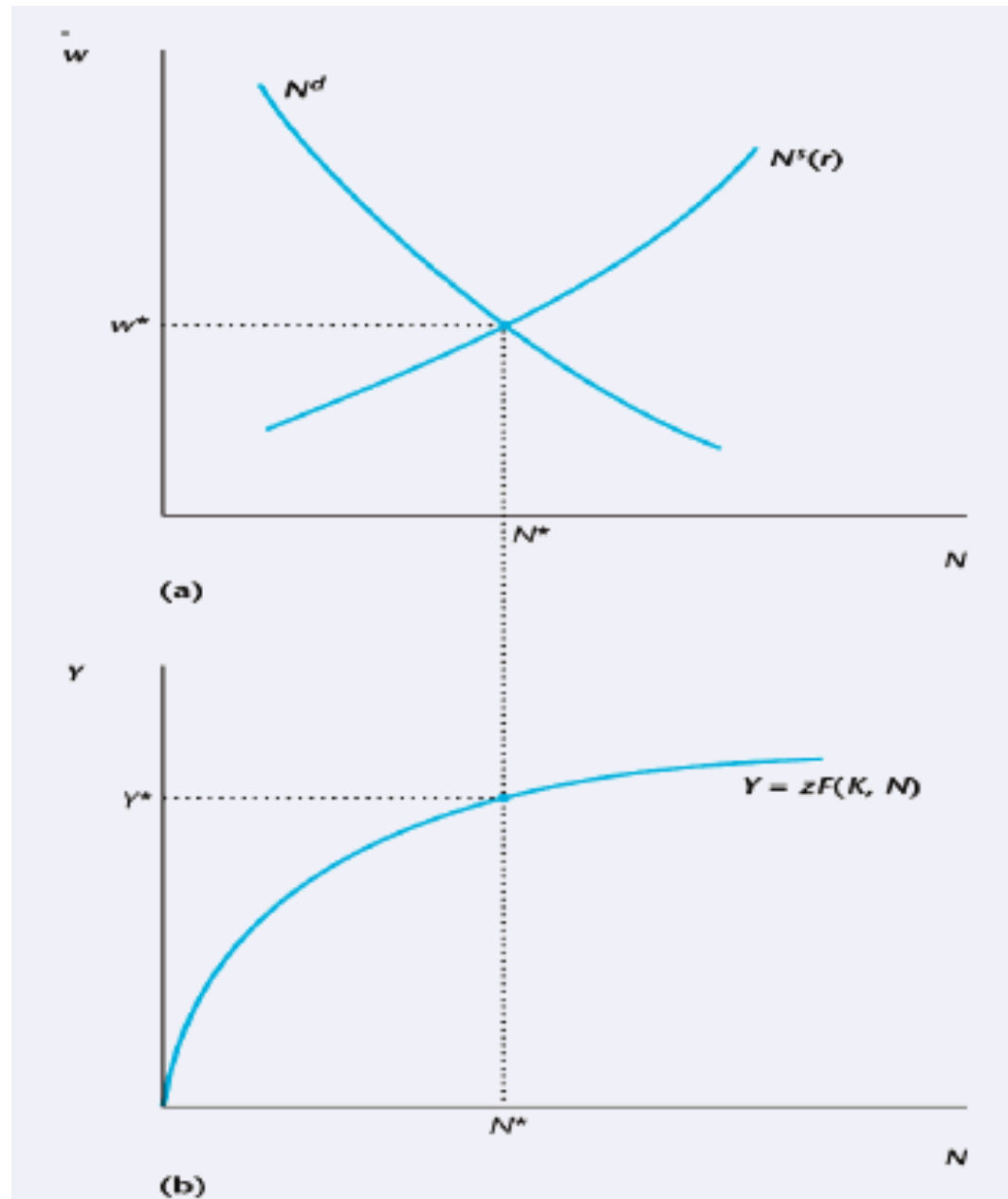
# Competitive Equilibrium

- A CE in this economy is a situation in which consumers and firms optimize, taking prices and taxes as given, GBC holds and markets clear.
- We will focus on current period markets only.
- Eventually, in this model, we will show that this is WLOG, as the clearing of current markets implies that of future markets.

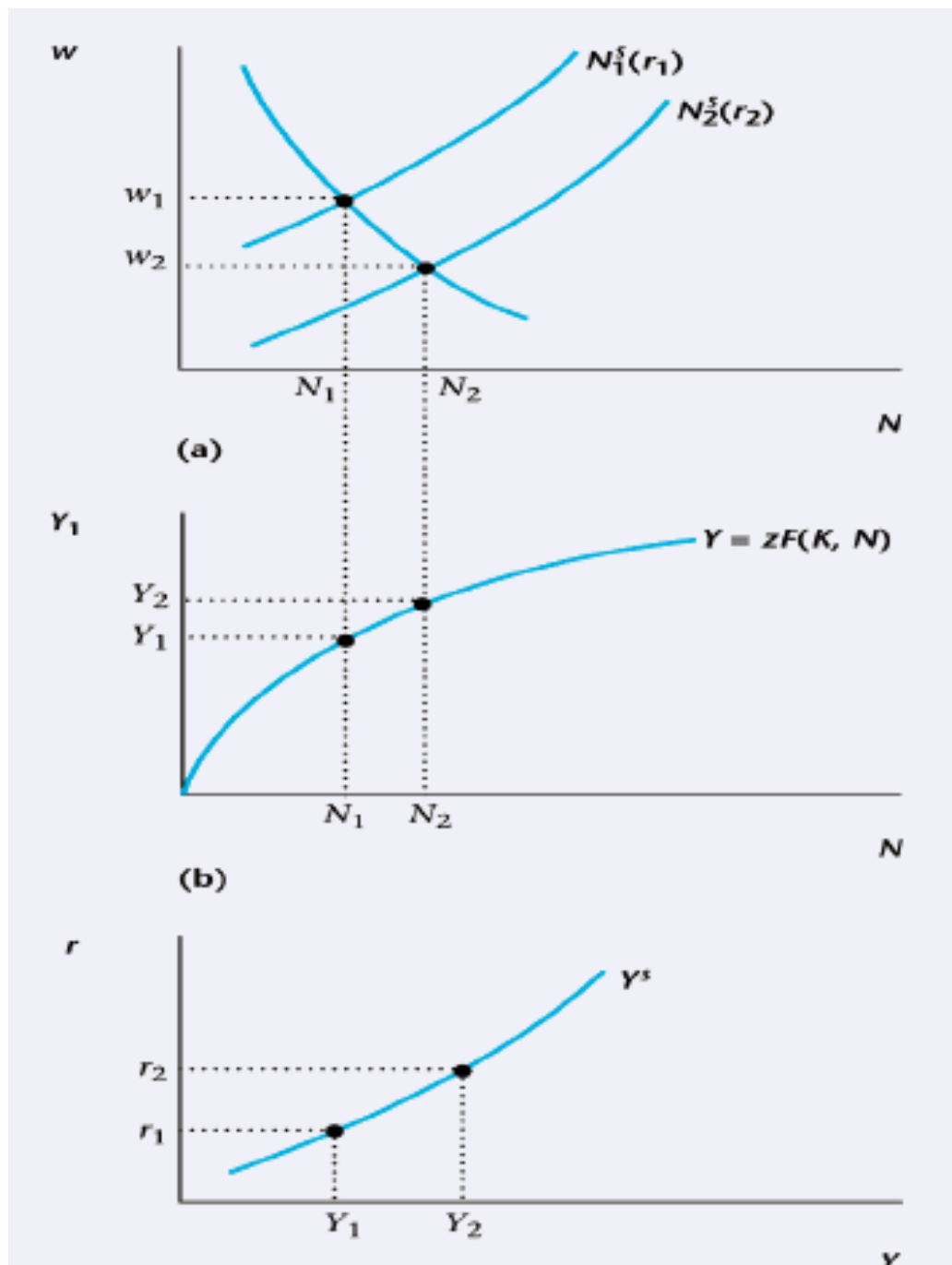
# The Output Supply Curve

- The next 2 slides explain the determination of the output supply curve by the firm, as a function of  $r$ .
- We will find that because of the intertemporal substitution effect effect on labor supply, the the output supply curve is  $\uparrow$  in  $r$ .

# Determination of Equilibrium in the Labor Market Given the Real Interest Rate $r$



# Construction of the Output Supply Curve





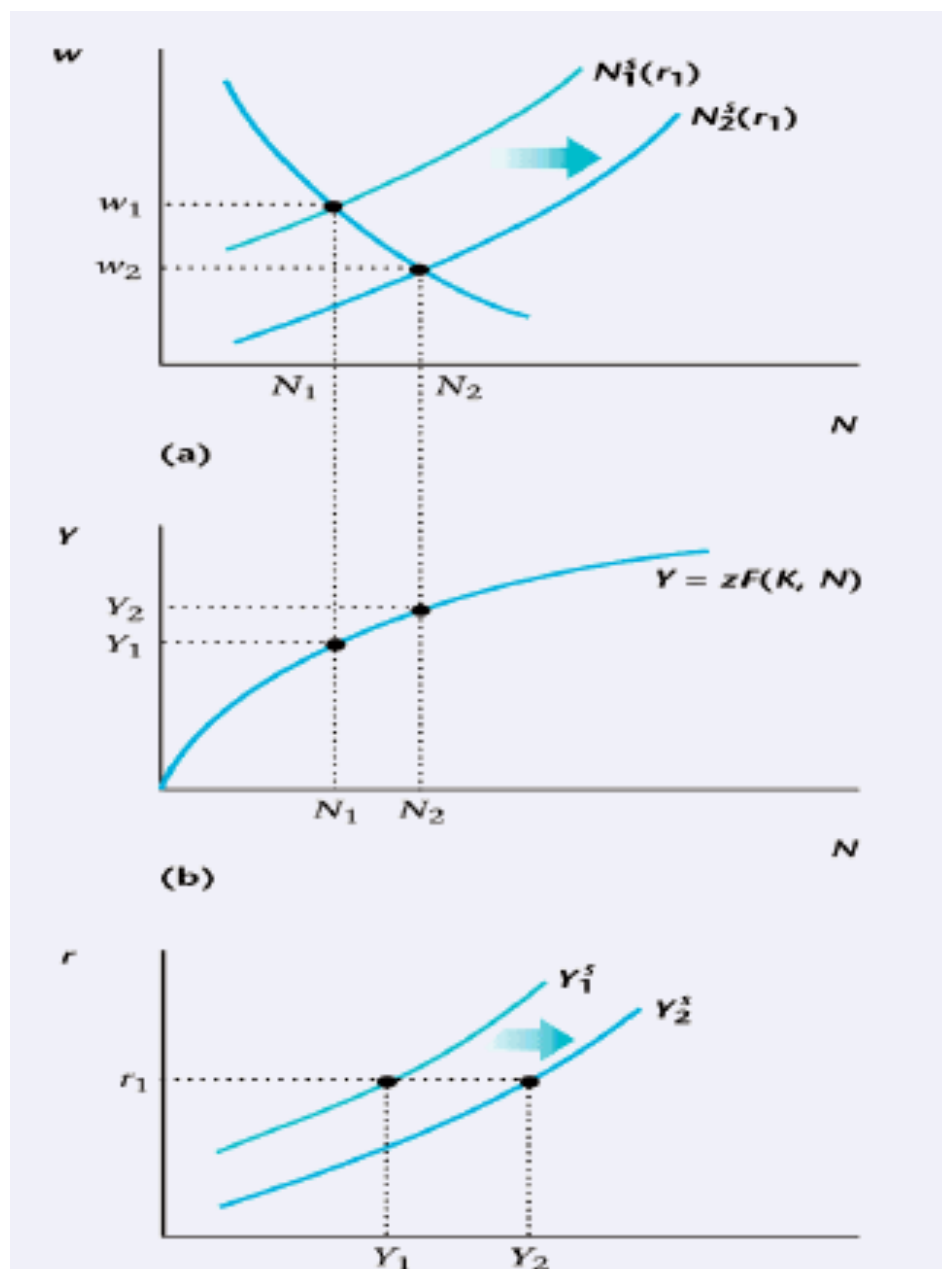
# Shifts of the Output Supply Curve

- The output supply curve (function of  $r$ ) shifts because of:
  - A shift in the current labor supply curve (e.g. from a change in lifetime wealth) (by the income effect)
  - A shift in the current labor demand curve (e.g from changes in  $z$ ,  $K$ ).

# Shifts of the Output Supply Curve in response to a change in $G, G'$

- Hence an  $\uparrow$  in  $G$  or  $G' \Rightarrow \uparrow$  in lifetime taxes  $\Rightarrow \downarrow$  in lifetime wealth  $\Rightarrow$  leisure  $\downarrow \Rightarrow$  labor supply  $\uparrow$ .

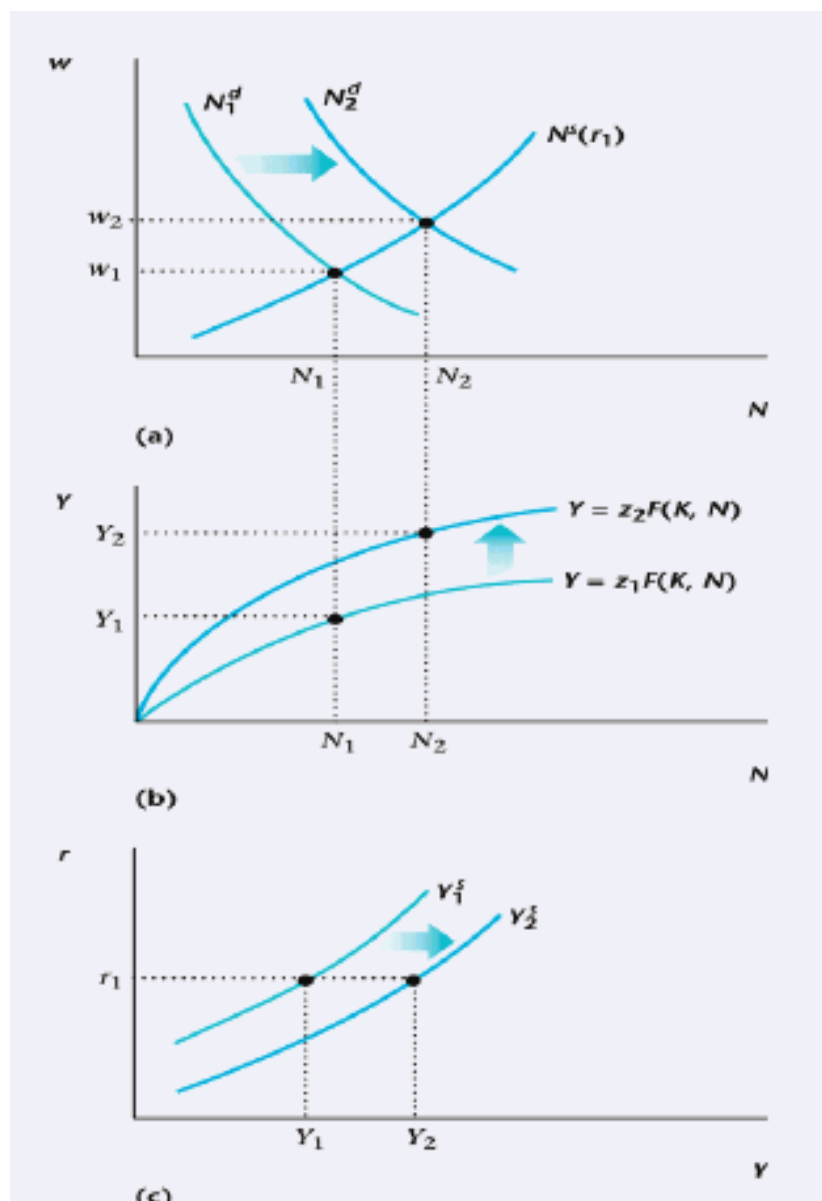
# Shifts of the Output Supply Curve in response to an increase in $G$ or $G'$



# Shifts of the Output Supply Curve in Response to a Change in $z$ (same for an increase in $K$ )

- $z \uparrow \Rightarrow$  the production function shifts to the right (ch 4)
- Also, an  $\uparrow$  in  $z \Rightarrow$  labor demand  $\uparrow$  as  $MP_N$  increases.

# Shifts of the Output Supply Curve in response to an increase in $z$ (same for an increase in $K$ )



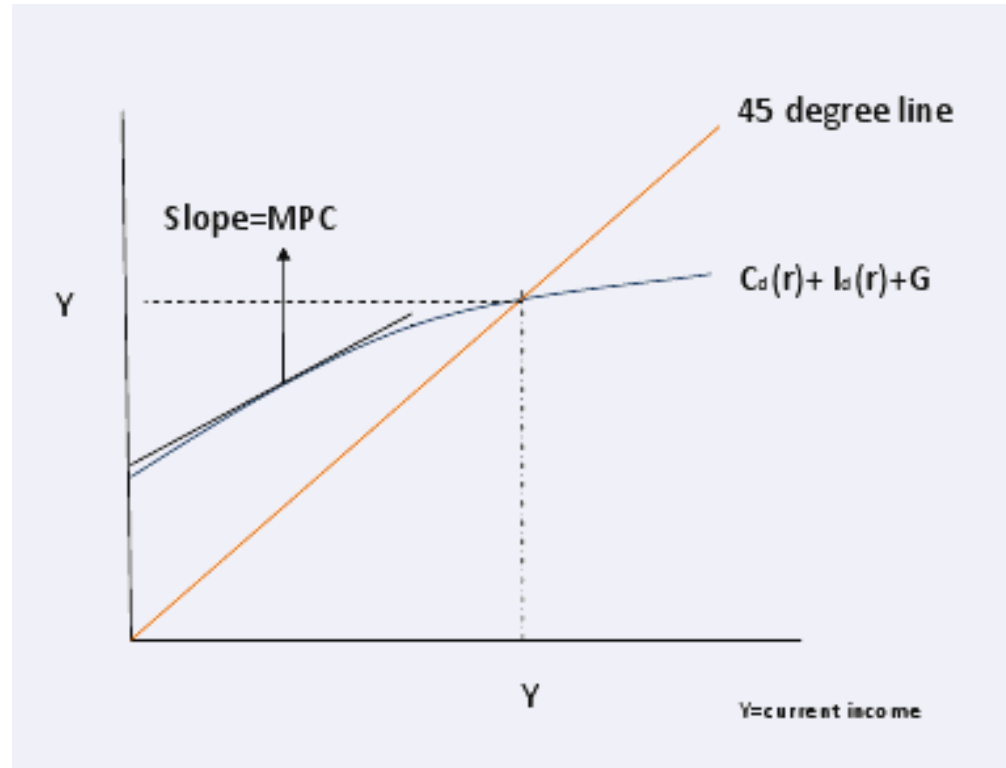
# The Output Demand Curve

- Let  $Y$  represents the representative consumers current income.
- Let  $C^d(Y, r)$  denote the representative consumers current demand for the consumption good ( a  $\downarrow$  function of  $r$  assuming the substitution effect dominates).

$$Y^d = C^d(Y^d, r) + I^d(r) + G$$

# The Total Demand for current goods

- Since  $I^d$  and  $G$  do not depend on current income  $Y$ , only  $C^d$  influences  $C^d(r) + I^d(r) + G$  as a function of  $Y$ .
- Hence, the slope of  $Y^d = MPC$ .



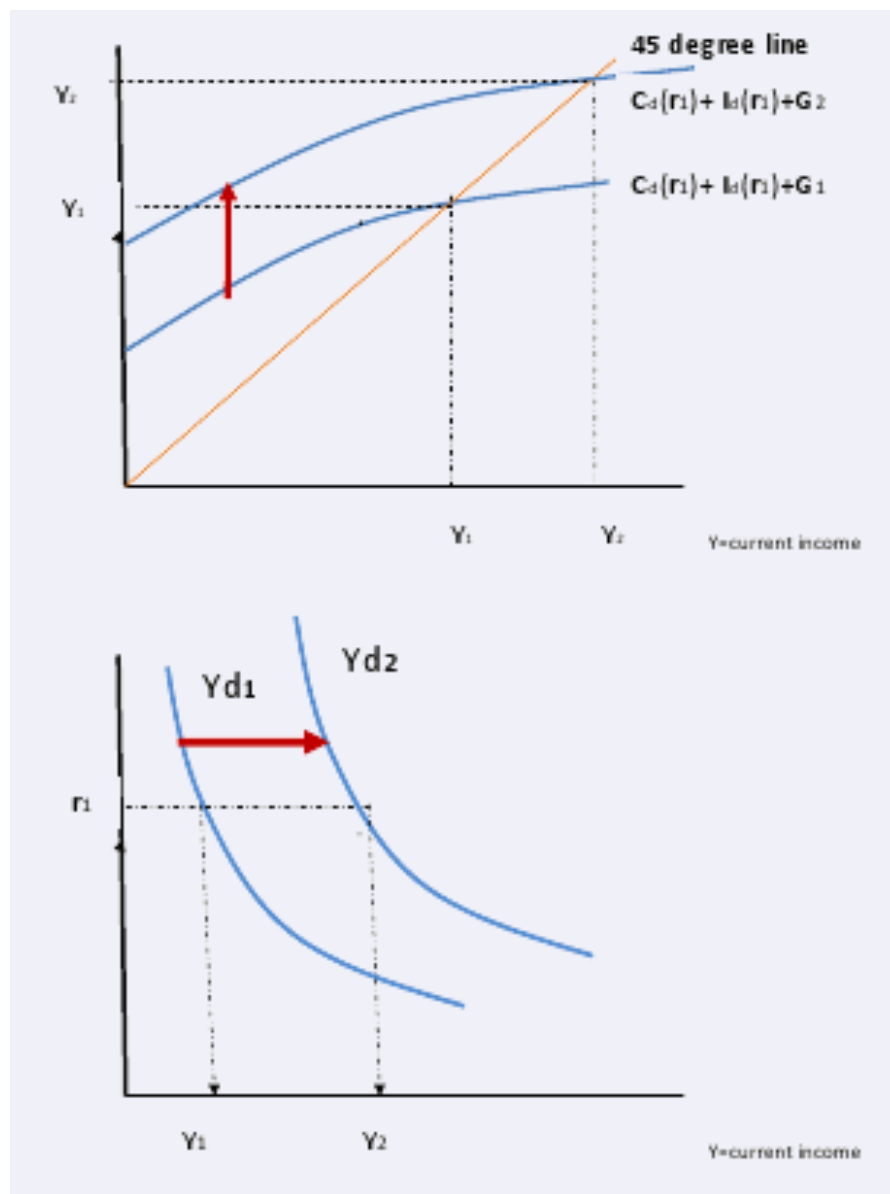
- We are not done yet. We need to get  $Y^d$  as a function of  $r$ .

# The Total Demand for current goods

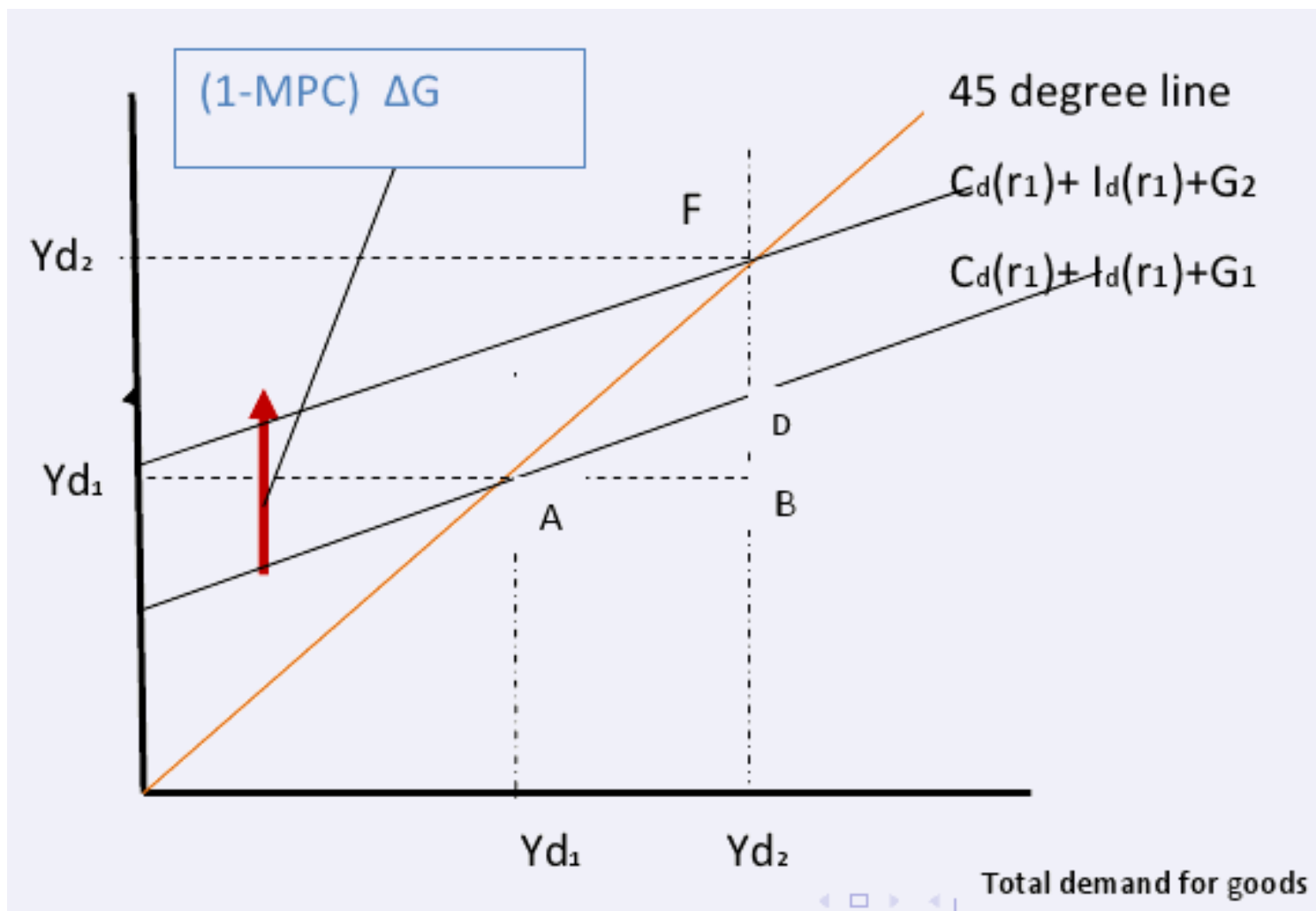
- Recall  $C^d \downarrow$  as  $r \uparrow$  (assuming substitution  $>$  income effect).
- Recall  $I^d \downarrow$  as  $r \uparrow$
- Hence,  $Y^d = C^d(Y^d, r) + I^d(r) + G \Rightarrow Y^d \downarrow$  as  $r \uparrow$ .



# An Increase in Current or Future Government Spending Shifts the $Y^d$ Curve



But the magnitude of  $\frac{\Delta Y}{\Delta G}$  is different from what Keynesian theory says!

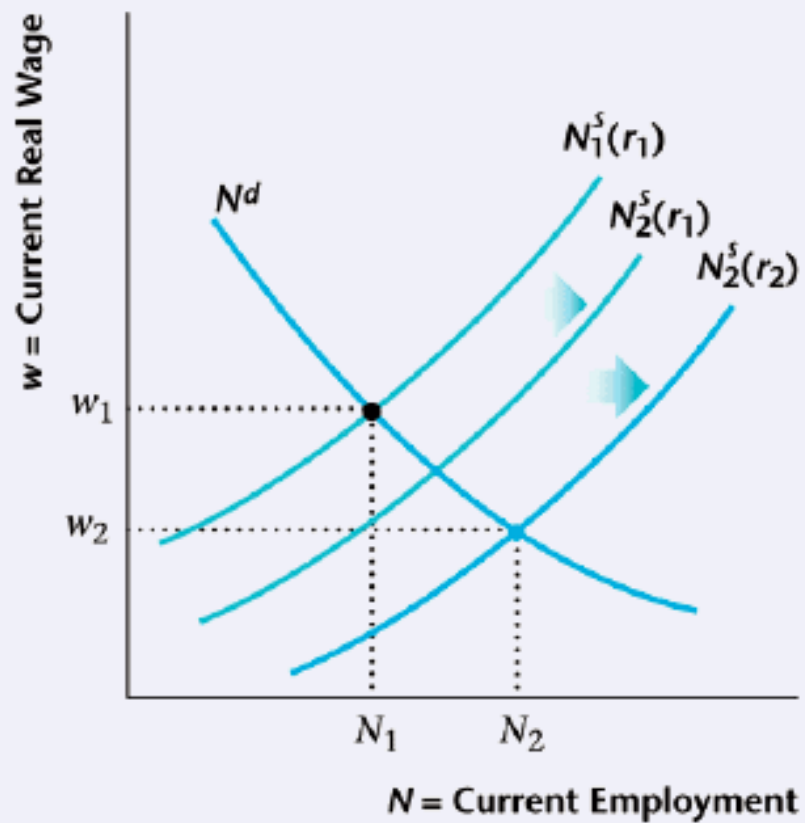


Here we have:

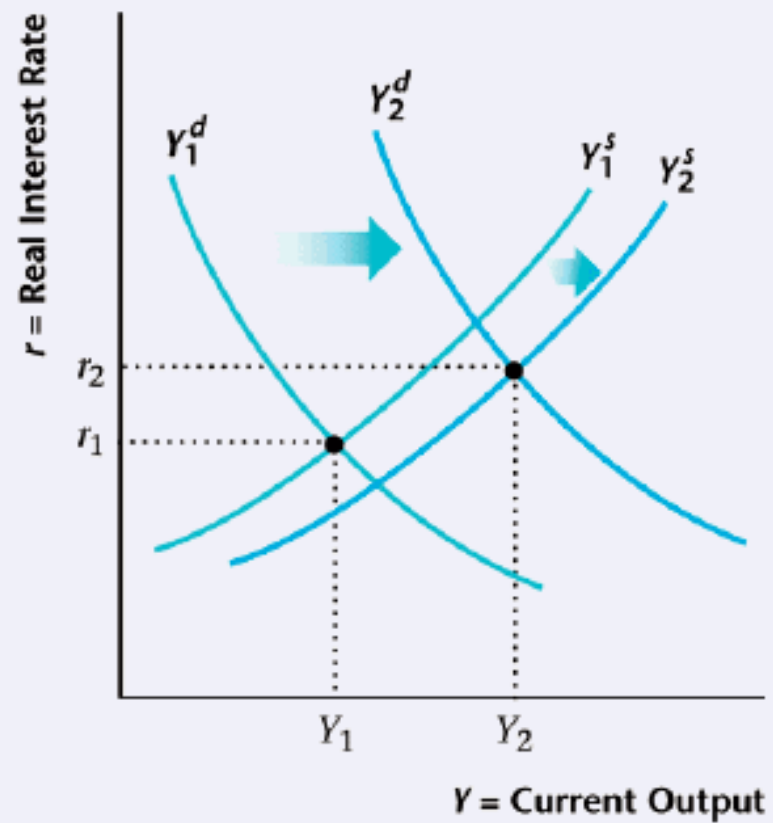
- $MPC \equiv \frac{DB}{AB} = \frac{FB-FD}{Y_2^d - Y_1^d} = \frac{[Y_2^d - Y_1^d] - FD}{Y_2^d - Y_1^d} = \frac{[Y_2^d - Y_1^d] - [(1-MPC)\Delta G]}{Y_2^d - Y_1^d}$
- $\Rightarrow Y_2^d - Y_1^d = G_2 - G_1 \Rightarrow \frac{\Delta Y}{\Delta G} = 1$
- Not what you know from your Chapter on Keynes in Econ 1102!

# What went wrong in the Keynesian model?

- In Econ 1102, we had  $\Delta Y = \frac{1}{1-MPC} \Delta G$
- This is because the Keynesian theory forgets the impact of the change in taxes on consumption.
- The new classical model is not perfect either, as it assumes perfect foresight.
- So far, we have only looked at the effect of  $\Delta G$  on  $Y^d$ . What about the effect on  $Y^s$ ? Taxes  $\uparrow \Rightarrow$  labor supply  $\uparrow$  (income effect  $\Rightarrow$  leisure  $\downarrow$ ). This  $\Rightarrow Y^s$  shifts to the right.



(a)



(b)

# General Equilibrium effects of an increase in $G$

- The horizontal shift of  $Y^d$  is of length  $= \Delta G$ .
- But because  $Y^s$  does not shift to the right by the same magnitude, the change in  $Y$  is less than the original change in  $\Delta G$ .
- The Keynesian model does not account for substitution effects and cannot capture the impact on  $y^s$ .

# $G$ crowds out private consumption and private investment

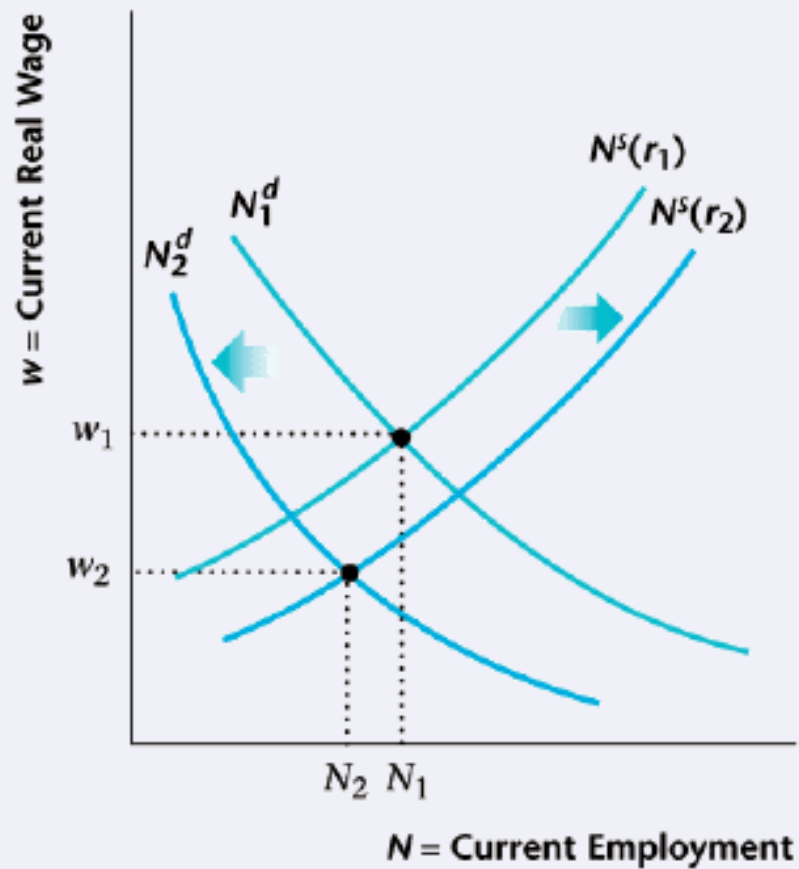
- Note from the previous graph, that  $r \uparrow$  if the income effect( that causes leisure to  $\downarrow$  and shifts  $Y^s$  to the right) is not too high. Otherwise,  $r \downarrow$ .
- Lets assume conditions are met for an  $\uparrow$  in  $r$ .
- Then, because of intertemporal substitution, current consumption will  $\downarrow$ .
- The  $\uparrow$  in  $r$  also  $\Rightarrow \downarrow$  in private investment. This crowding out effect is ignored in the Keynesian theory.
- Note further that the  $\uparrow$  in  $r$  causes the consumer to substitute current for future leisure, shifting  $N^s$  and  $Y^s$  further to the right.

## General Equilibrium Effects of $\downarrow$ in $K$

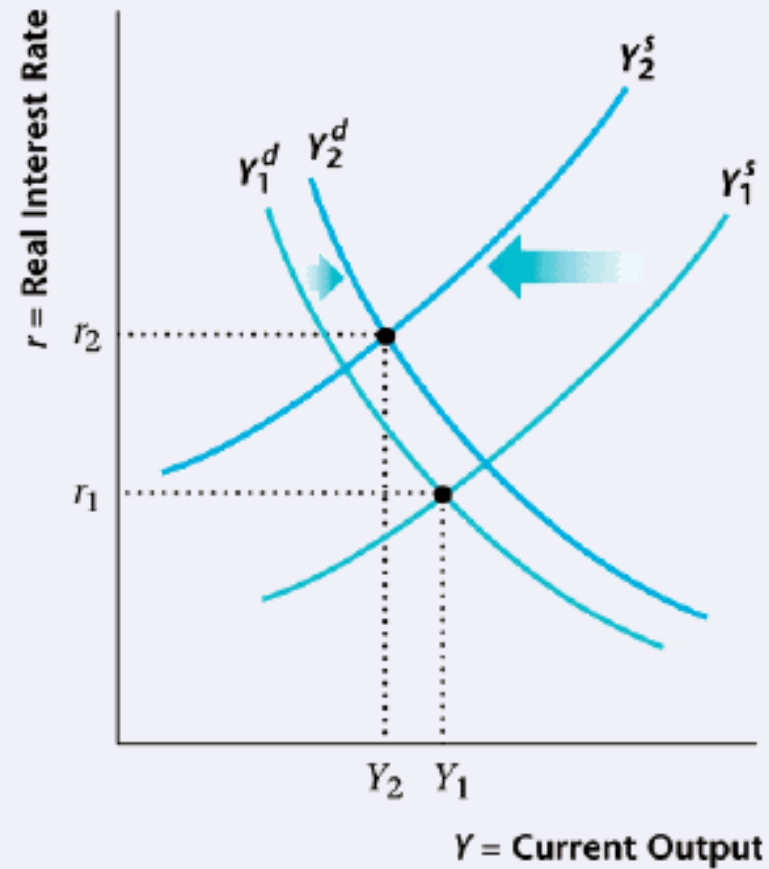
- $\Rightarrow I^d \uparrow \Rightarrow Y^d$  shifts to the right.
- Also, the  $\downarrow$  in  $K \Rightarrow$  a  $\downarrow$  in  $MP_N \Rightarrow N^d$  and  $Y^s$  shifts to the left. In total,  $Y^*$  may  $\uparrow$  or  $\downarrow$ .
- Hence, in equilibrium,  $r$  must necessarily  $\uparrow$ . So, current consumption must  $\downarrow$ .
- $I^d$  may  $\uparrow$  (as  $K$  has  $\downarrow$ ) or  $\downarrow$  as  $r$  has  $\uparrow$ .
- As  $r \uparrow$ , the consumer substitutes current for future leisure, which  $\Rightarrow N^s$  shifts to the right, depressing wages further. Overall,  $w$  may  $\downarrow$  or  $\uparrow$ .



# General Equilibrium Effects of $\downarrow$ in $K$



(a)

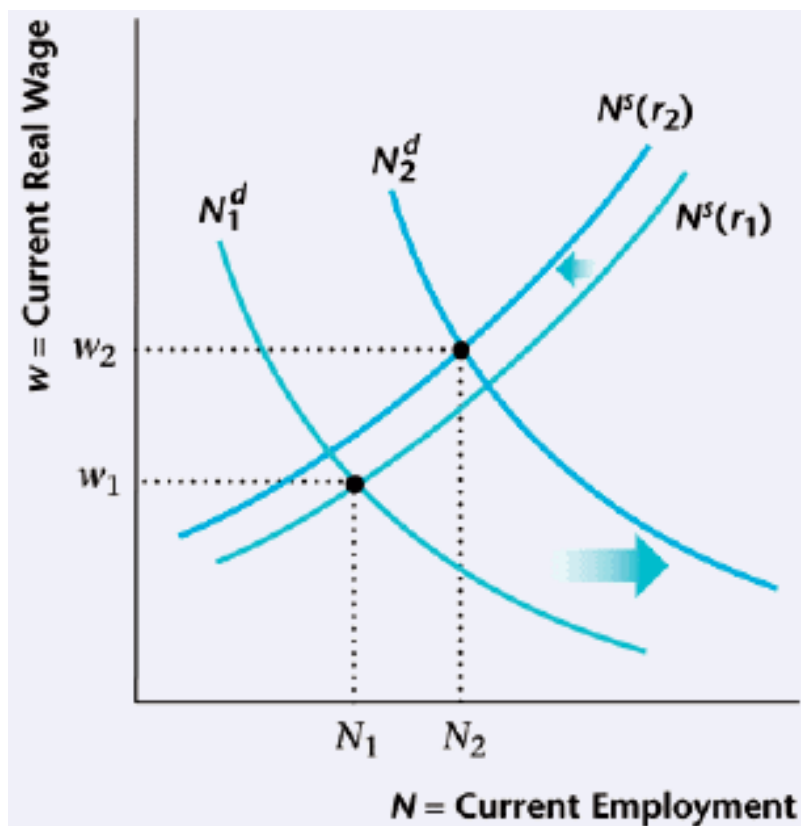


(b)

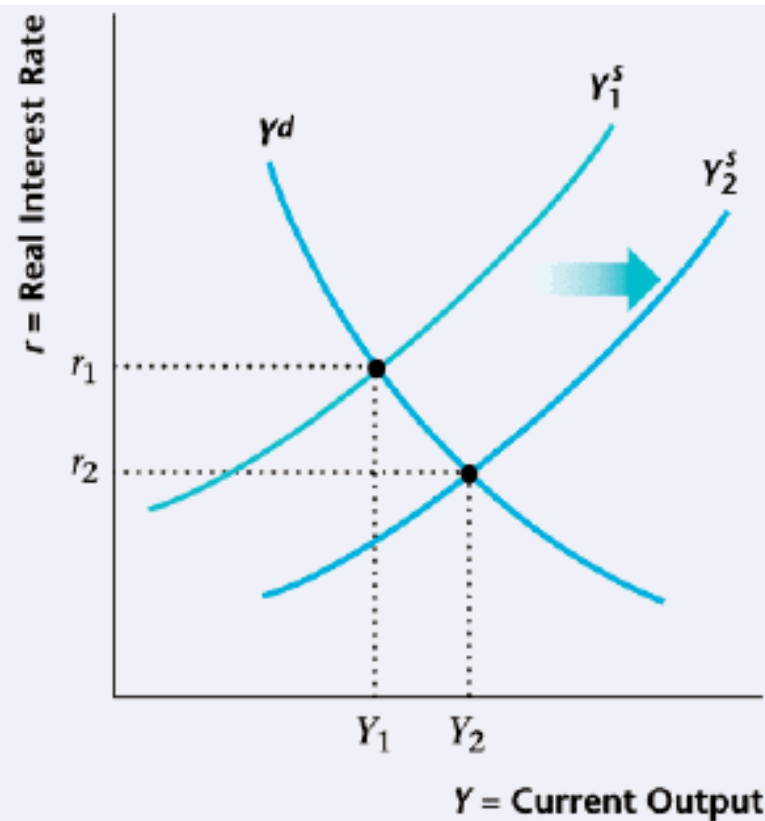
# General Equilibrium effects of an increase in TFP $z$

- As  $z \uparrow$ ,  $MP_N \uparrow$ , and so,  $N^d$  and  $Y^s$  shifts to the right.
- $Y^s$  shifts to the right  $\Rightarrow r \downarrow \Rightarrow N^s$  shifts to the left as the consumer substitutes future for current leisure.

# General Equilibrium effects of an increase in TFP $z$



(a)



(b)