

## Translation

$$v = \begin{pmatrix} x \\ y \end{pmatrix} \quad \vec{d} = \begin{pmatrix} dx \\ dy \end{pmatrix} \quad v + \vec{d} = \begin{pmatrix} x+dx \\ y+dy \end{pmatrix}$$

homogeneous coordinates  $v = v' = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$  for a point  
 $(x, y, 0)$  for a direction

translation matrix:  $\begin{pmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x+dx \\ y+dy \\ 1 \end{pmatrix} = \begin{pmatrix} x+dx \\ y+dy \end{pmatrix}$

direction does not affect the translation  $\Rightarrow \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} x+dx \\ y+dy \\ 0 \end{pmatrix}$

## 1-homogeneous 2D Affine.

$$\left( \begin{array}{cc|c} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{array} \right) \left( \begin{array}{cc|c} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{array} \right) \left( \begin{array}{cc|c} 1 & m & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right)$$

composition of transformation

$$\begin{pmatrix} S_x & & \\ & S_y & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & dx \\ & 1 & dy \\ & & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \leftarrow \text{a combination of transformation by the associative rule of matrices,}$$

Though these matrices could be combined, we still could not change the order of translation applied.

we could put matrices together.  $M_1 M_2 M_3 \dots M_n = (M_n) M_o$   
 all these are applied in order from right to left. e.g.  $S_4 R_3 T_2 R_1 v$

$$\begin{pmatrix} S_x & & \\ & S_y & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & dx \\ & 1 & dy \\ & & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} S_x & & \\ & S_y & \\ & & 1 \end{pmatrix} \begin{pmatrix} x+dx \\ y+dy \\ 1 \end{pmatrix} = \begin{pmatrix} S_x(x+dx) \\ S_y(y+dy) \\ 1 \end{pmatrix}$$

$\Uparrow$  This order MATTER!!!

For a left-hand system, the transition is the same.

if we're applying a rotation on the system, we could ---

1) directly apply a rotation on the point itself

2) apply a rotation to every later movements.

Basic vertices  $\hat{i}$   $\hat{j}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} i_1 & j_1 \\ i_2 & j_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} i_1 x + i_2 y \\ j_1 x + j_2 y \end{pmatrix}, \text{ where } i_1^2 + i_2^2 = j_1^2 + j_2^2 = 1$$

$\uparrow$   
modified coordinate system.

then, we can let  $\sin\theta = i_1$ , so  $\cos\theta = i_2$ ,

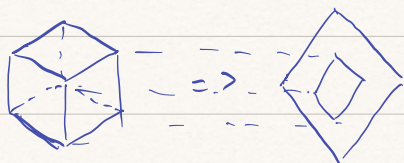
$\sin\alpha = j_1$ ,  $\cos\alpha = j_2$ , and  $\alpha = \theta + 90^\circ$  by def

moving the shape is the moving a collection of vertices.

so for rotation

## Projection

orthogonal projection have: 1) parallel projection lines



2) projection lines that are perpendicular to the image plane.

perspective projection: closer is bigger, further away is smaller



OpenGL does not have a 2D square bound, so we use a 3D cube space instead.

$NDC = P \cdot V$   
|  
| projection matrices | world vector  
|  
normalized

$[-1, 1] = P \cdot (1200, 500)$

$glOrtho()$  is a function compute projected matrix.

we have to provide the viewing volume of all six faces.

$x$   $y$   $z$   
(left, right, bottom, top, near, far)

OpenGL is a global state machine, so we just have to call  $glOrtho()$  once.  $glMatrixMode$  let us modify the projection matrix:  $glMatrixMode(GL_PROJECTION)$

given current matrix  $\rightarrow glLoadIdentity()$

load with identity  $glOrtho(L, R, B, T, N, F)$

$\uparrow$   
multiply the current matrix with ortho one. It is a transform