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1. (i) $p \vee q$
$p \to r$
$\neg q$
___
$\therefore r$

$\neg q \equiv T$

$q \equiv F$

$p \vee F \equiv T$

$p \equiv T$ (domination laws)

$p \to r \equiv \neg p \vee r$ (conditional identity)

$\equiv F \vee r$

$F \vee r \equiv T$

$r \equiv T$ (domination laws)

$\therefore$ the proof is valid.

(ii) $\forall x\,(P(x)) \to (Q(x)) \wedge (R(x))$
$\forall x\,(S(x) \vee P(x))$
$\neg \forall x\, Q(x)$
___
$\exists x\, S(x)$.

$\neg \forall x\, Q(x). \equiv \exists x\, \neg Q(x)$.

$\therefore \forall x\, Q(x) \equiv F$.

(condition identity)

$\forall x\,(P(x)) \to (Q(x)) \wedge (R(x)) \equiv \neg(\forall x\,(P(x))) \vee ((Q(x)) \wedge (R(x)))$

$\equiv \neg(\forall x\,(P(x))) \vee (F \wedge (R(x)))$

$\equiv \neg(\forall x\,(P(x))) \vee F$ (domination laws).

$\neg(\forall x\,(P(x))) \vee F \equiv T$

$\neg(\forall x\,(P(x))) \equiv T$ (domination laws).

$\forall x\,(P(x)) \equiv F$.

$\forall x\,(S(x) \vee P(x)) \equiv (\forall x\, S(x)) \vee (\forall x\, P(x))$

$\equiv (\forall x\, S(x)) \vee F$.

$(\forall x\, S(x)) \vee F \equiv T$.

$\forall x\, S(x) \equiv T$ (domination laws).

$\therefore$ $\exists x\, S(x) \equiv T$

$\therefore$ the proof is valid.

2. (i) Some student don't have an official email address.

(ii) Exist a real number x that there is no real number y satisfies $x = y^2$

3. Theorem: $P_1 \cdots P_n$ is a collection of propositions.

$$\neg(P_1 \vee \cdots \vee P_n) = (\neg P_1 \wedge \cdots \wedge \neg P_n).$$

Proof: $(P_1 \vee \cdots \vee P_n)$ is false iff all propositions from P_1 to P_n are false, so $\neg(P_1 \vee \cdots \vee P_n)$ is true iff all propositions are false.

Assume a new collection $q_1 \cdots q_n$ where $q_n = \neg P_n$.

$q_1 \wedge \cdots \wedge q_n$ is true iff all propositions from q_1 to q_n are true, so $\neg P_1 \wedge \cdots \wedge \neg P_n$ is true iff all propositions are false.

the only ture condition is the sam from both sides, so $\neg(P_1 \vee \cdots \vee P_n) = (\neg P_1 \wedge \cdots \wedge \neg P_n)$ ■

4. Theorem: P a predicate depends on variables $x_1 \cdots x_n$

Q denote $\forall x_n \forall \cdots \forall x_n\, P$ for all $n \geq 1$.

$$\neg Q = \exists x_n \exists x_{n-1} \exists \cdots \exists x_1 \neg P$$

Proof: P is false iff $\exists x_n \exists x_{n-1} \cdots \exists x_1 P$ is false, so $\neg P$ is true iff $\exists x_n \exists x_{n-1} \cdots \exists x_1 \neg P$ is true.

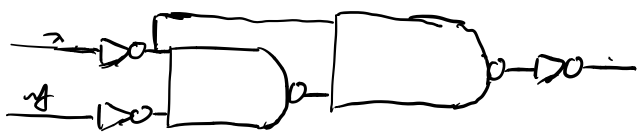
Q is false iff $\exists x_n \exists x_{n-1} \cdots \exists x_1$ P is false. so

$\neg Q$ is true iff $\exists x_n \exists x_{n-1} \cdots \exists x_1 \neg P$ is true. ■

5. (i) $f(x,y) = xy + x\bar{y} + \bar{x}\bar{y}$.

(ii) $x \uparrow y = \overline{x \cdot y} = \bar{x} + \bar{y}$.

$f(x,y) = x + \bar{x}\bar{y} = x + \overline{\overline{\bar{x}\bar{y}}} = \overline{\bar{x} \cdot \overline{\bar{x}\bar{y}}}$.



(iii). $g(x,y) = x(\bar{x}+\bar{y})$

$= x\bar{x} + x\bar{y}$.

$= F + x\bar{y}$

$= x\bar{y}$.

Not the same.

6 (1) One which can be used to express all possible turth tables by combining members of the set into a Boolean expression.

(2) $xy = (x+y) \cdot \overline{x \cdot y}$

$= \overline{\bar{x}\bar{y}} \cdot \overline{x \cdot y}$ $\bar{a} + \bar{b} = \overline{ab}$.

$= \overline{\overline{\overline{\bar{x}\bar{y}} \cdot \overline{xy}}}$.

7. To prove $f(x,y) \cdot g(x,y)$ is satisfiable when both $f(x,y)$, $g(x,y)$ are satisfiable, is equal to prove its contrapositive:

If either $f(x,y)$, $g(x,y)$ is not satisfiable or both $f(x,y)$ and $g(x,y)$ are unsatisfiable, then $f(x,y) \cdot g(x,y)$ is not satisfiable

when either $f(x,y)$, $g(x,y)$ is not satisfiable, the turth value of $f(x,y) \cdot g(x,y)$ is 0

when both $f(x,y), g(x,y)$ are not satisfiable, the turth value of $f(x,y) \cdot g(x,y)$ is 0.

Thus, the contrapositive is true. So $f(x,y) \cdot g(x,y)$ is satisfiabe when both $f(x,y), g(x,y)$ are satisfiable.

8. Assume $F(n) = 5^n - 1$.

Base case: $n=1$ $5^n-1=4$ can be divided by 4.

when $n \geq 2$., $F(n) - F(n-1) = 5^n - 1 - 5^{n-1} + 1 = 4 \cdot 5^{n-1}$.

so k can be 5^{n-1} that 5^n-1 can be divided by 4 for any $n \geq 2$.

So 5^n-1 can be divided by 4.