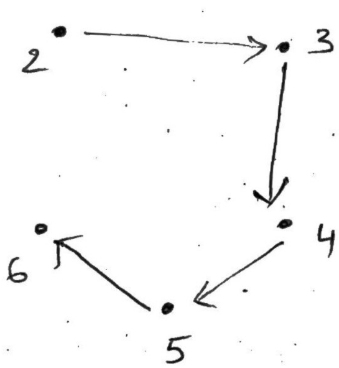
Solutions. (Binary Relations)

(1) a) $R = \{ (a,b) \in A \times A \mid b = a+1 \}$

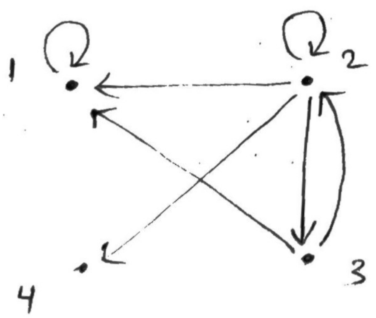
b)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

c)



(2)



Not reflexive, because all vertices not have loops. (Ex:- 3,4)

Not irreflexive, because some vertices have loops (Ex:- 1,2)

Not symmetric, because there are pairs $(x,y) \in R$ for which $(y,x) \notin R$ (Ex:- (2,1))

Not antisymmetric, because (2,3) and (3,2) both belong to R

Not asymmetric, because it is not irreflexive & antisymmetric

Not transitive, because (3,2) and (2,3) belong to R but (3,3) does not

3. Yes, the given relation is a equivalence realion. because it is reflexive, symmetric and transitive

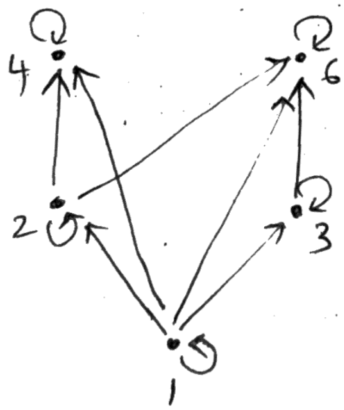
Its 2 equivalence classes are

$$[1] = \{1, 2\} = [2]$$

$$[3] = \{3\}$$

4.

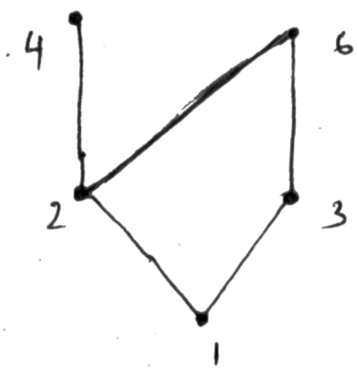
$$R = \{ (1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6) \}$$



It is a partial order since it is clear from the graph that it is reflexive, transitive & antisymmetric

Note that in the graph I already set the position of the vertices in such a way that if aRb then a is lower than b

Now removing self-loops, transitivity & arrowheads



(Hasse Diagram)

(5)

$R_1 = \{(10,11), (11,12), (12,13), (13,14)\}$

$R_2 = \{(10,12), (11,13), (12,14)\}$

a) $\{(10,11), (11,12), (12,13), (13,14), (10,12), (11,13), (12,14)\}$

b) $\{\ \} = \emptyset$

c) $\{(10,11), (11,12), (12,13), (13,14)\}$

d) $\{(11,10), (12,11), (13,12), (14,13)\}$

e) $\{(12,10), (13,11), (14,12)\}$

f) $R_1^2 = R_1 \circ R_1 = \{(a,c) \in A \times A \mid \exists b \in A\ ((a,b) \in R_1 \wedge (b,c) \in R_1)\}$

$= \{(10,12), (11,13), (12,14)\} = R_2$

(BECAUSE $(10,11), (11,12), (12,13), (13,14) \in R_1$)

⑥ $R_2 \cdot R_1 = \{(z,e), (z,d)\}$