Semantic (Truth / Meaning)

T/F →, ←, ∧, ∨

$P_1, P_2 \cdots P_n \models q$

if $P_1, \cdots P_n$ are all true, then q is true.

P_1
⋮
P_n
———
q is valid

$P_1 \wedge P_2 \cdots \wedge P_n \to q$ is a tautology.

$f(p,q) \equiv g(p,q)$

$\Rightarrow (f(p,q) \to g(p,q)) \wedge (g(p,q) \to f(p,q))$ is a tautology.

$f(p,q) \models g(p,q)$

Syntax don't have truth value.

$\frac{p \wedge q}{\therefore p}$, $\frac{p \wedge q}{\therefore q}$ ← Rules of Natural deduction.

and elimination.

$\frac{p, q}{\therefore p \wedge q}$ $\wedge i$ and introduction.

$\frac{p}{p \vee q}$, $\frac{q}{p \vee q}$ $\vee i_1$, $\vee i_2$.

$P_1, P_2 \cdots P_n \vdash q$

need to prove.

1) $\phi \to \psi$ is a premis
2) $\neg\psi$ ↓ ↓ ↓
3) ϕ assumption
4) ψ (1, 3 → e).

$\frac{p \to q}{\neg q \to \neg p}$.

1) $p \to q$
2) $\neg q$

5) ⊥ (2,47e).

$\frac{}{\neg\phi}$ $\neg i$ 3-5.

3) $\frac{\neg p \quad (1,2, MT)}{\neg q \to \neg p}$