Answers to Assigned Homework Problems

Section 1.1

1. $(-4, -8, 12)$
2. $(-6, 0, -3)$
3. $(1, -1)$
4. $(5, -2)$
5. $(5, 1, 1)$
6. $(4, -3)$
7. $(-3, -2, 2)$
8. $(-2, 5)$
9. $(-9, -3, 1)$
10. $\sqrt{13}$
11. $\sqrt{14}$
12. $\sqrt{5}$
13. $\sqrt{17}$
14. $\sqrt{17}$
15. $\sqrt{14} + \sqrt{5}$
16. $5\sqrt{17}$
17. $\left(\frac{3}{5}, -\frac{4}{5}\right)$
18. $\left(\frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}\right)$
19. $\left(-\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$
20. $\left(\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right)$
21. $(4, 2)$
22. $(-3, 2, 4)$
23. $(1, -4, -7)$
24. $(-3, 5)$
25. $2\mathbf{i} - 3\mathbf{k}$
26. $(3, 2, -1)$
27. $\sqrt{14}$
37. Yes

Answers to Assigned Homework Problems

Section 1.2

1. 4
2. 12
3. 4
4. -24
5. $(11, -7, -10)$
6. -38
7. $(-22, 14, 20)$
8. $(-57, -57, 0)$
9. -38
13. $-\dfrac{8}{\sqrt{5}\sqrt{29}}$
14. $\dfrac{5}{\sqrt{14}\sqrt{5}}$
15. $-\dfrac{1}{2}$
16. $\dfrac{1}{\sqrt{2}}$
17. any multiple of $(-4, 7, -1)$
18. any multiple of $(4, 5, -2)$
19. $\dfrac{\sqrt{14}}{2}$
20. $3\sqrt{3}$

22. The cross product of a vector and a scalar is not defined.

23. The dot product of a vector and a scalar is not defined.

34. **Note:** The formula given in question 33 is **wrong**. There should be absolute value signs rather than magnitude signs around the scalar quantity $(\mathbf{u} \times \mathbf{v}) \bullet \mathbf{w}$.

Answer: $V = |\ (\mathbf{u} \times \mathbf{v}) \bullet \mathbf{w}\ | = 1$.

Answers to Assigned Homework Problems

Section 1.3

1. $(3,1,2) \bullet (\mathbf{x} - (2,-1,-4)) = 0$; $3x + y + 2z = -3$
2. $(-3,0,1) \bullet (\mathbf{x} - (1,2,3)) = 0$; $-3x + z = 0$
3. $(2,1,3) \bullet (\mathbf{x} - (0,0,0)) = 0$; $2x + y + 3z = 0$
4. $(1,2,-2) \bullet (\mathbf{x} - (-1,2,3)) = 0$; $x + 2y - 2z = -3$
5. $(3,6,1) \bullet (\mathbf{x} - (-1,2,3)) = 0$; $3x + 6y + z = 12$
6. $(2,-1,1) \bullet (\mathbf{x} - (0,0,5)) = 0$
7. $(2,3,0) \bullet \left(\mathbf{x} - \left(\frac{1}{2},0,0\right)\right) = 0$
8. $(0,0,1) \bullet (\mathbf{x} - (0,0,0)) = 0$
9. $(2,1) \bullet (\mathbf{x} - (-1,2)) = 0$
10. $(0,2) \bullet (\mathbf{x} - (2,-1)) = 0$
11. $(2,-1) \bullet (\mathbf{x} - (-2,5)) = 0$
12. $(4,3) \bullet (\mathbf{x} - (1,-3)) = 0$
13. $\mathbf{x}(t) = (2,1,-3) + t(1,2,2)$; $x = 2 + t, y = 1 + 2t, z = -3 + 2t$
14. $\mathbf{x}(t) = (3,-1) + t(2,3)$; $x = 3 + 2t, y = -1 + 3t$
15. $\mathbf{x}(t) = (2,-3,1) + t(1,0,0)$; $x = 2 + t, y = -3, z = 1$
16. $\mathbf{x}(t) = (1-t)(1,2,-1) + t(2,-1,3)$; $x = 1 + t, y = 2 - 3t, z = -1 + 4t$
17. $\mathbf{x}(t) = (1-t)(2,0,-2) + t(1,4,2)$; $x = 2 - t, y = 4t, z = -2 + 4t$
18. $\mathbf{x}(t) = (1,2,3) + t(2,-1,-2)$; $x = 1 + 2t, y = 2 - t, z = 3 - 2t$
19. $\mathbf{x}(t) = (2,4,5) + t(5,-5,-10)$; $x = 2 + 5t, y = 4 - 5t, z = 5 - 10t$
20. $\mathbf{x}(t) = (3,-2) + t(3,-5)$; $x = 3 + 3t, y = -2 - 5t$
21. $\mathbf{x}(t) = (1,-1) + t(1,-3)$; $x = 1 + t, y = -1 - 3t$

25. $\dfrac{3}{\sqrt{6}}$ 27. $\dfrac{9}{\sqrt{5}}$ 29. $\left(\dfrac{13}{5}, 1, -\dfrac{7}{5}\right)$

Answers to Assigned Homework Problems

Section 2.1

1. $(1, 2, 5, 1)$
2. $(11, -2, 7, 11)$
3. $(12, -6, 14, 40)$
4. $\sqrt{47}$
5. 2
6. $2\sqrt{11}$
7. $6\sqrt{11}$
8. $\left(\frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, 0, \frac{3}{\sqrt{11}}\right)$
9. $\sqrt{31}$
10. $(-10, 10, 0, -30)$
11. $2\sqrt{47}$
12. 30

13. $\mathbf{u}_1$ and $\mathbf{u}_3$; $\mathbf{u}_2$ and $\mathbf{u}_3$; $\mathbf{u}_2$ and $\mathbf{u}_4$; $\mathbf{u}_3$ and $\mathbf{u}_4$.

16. $\left(\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, 0, \frac{5}{6}\right)$
17. $\left(\frac{2}{\sqrt{31}}, \frac{1}{\sqrt{31}}, -\frac{1}{\sqrt{31}}, 0, \frac{3}{\sqrt{31}}, \frac{4}{\sqrt{31}}\right)$
18. $x_1 - x_3 + x_5 = c$ for any real c
19. $-2x_1 - x_2 + x_4 = c$ for any real c

20. $\mathbf{x}(t) = (1-t)(2, 1, 0, 3, 1) + t(1, -1, 3, 0, 5)$
and $\mathbf{x}(t) = (2, 1, 0, 3, 1) + t(-1, -2, 3, -3, 4)$
and $x_1 = 2 - t, x_2 = 1 - 2t, x_3 = 3t, x_4 = 3 - 3t, x_5 = 1 + 4t$

21. $\mathbf{x}(t) = (1-t)(-1, 0, 3, 2) + t(-1, 0, 4, 5)$ and $\mathbf{x}(t) = (-1, 0, 3, 2) + t(0, 0, 1, 3)$
and $x_1 = -1, x_2 = 0, x_3 = 3 + t, x_4 = 2 + 3t$

22. $(1, 2, -1, 3) \bullet (\mathbf{x} - (-2, 1, 4, 0)) = 0$ and $x_1 + 2x_2 - x_3 + 3x_4 = -4$.

23. $(1, -1, 1, -1, 1) \bullet (\mathbf{x} - (3, 4, 5, 6, 7)) = 0$ and $x_1 - x_2 + x_3 - x_4 + x_5 = 5$.

24. $(2, -3, 0, 1, -1) \bullet (\mathbf{x} - (1, 0, 0, 0, 0)) = 0$

25. $(2, -3, 0, 1, -1, 0) \bullet (\mathbf{x} - (1, 0, 0, 0, 0, 0)) = 0$

Answers to Assigned Homework Problems

Section 2.2

Problems assigned as 2.2 homework:

1. Linear

2. Not linear

3. Not linear

4. Linear

5. Not linear

6. Not linear

7. $$\begin{array}{rcrcrcr} & & y & + & z & = & 6 \\ x & + & y & - & z & = & 0 \\ x & - & y & - & z & = & -3 \end{array}$$

8. $$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & + & x_3 & - & x_4 & = & 3 \\ & & x_2 & & & & & = & 0 \\ x_1 & + & x_2 & + & x_3 & & & = & 1 \end{array}$$

9. $$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & + & x_3 & - & x_4 & = & 0 \\ x_1 & & & & & & & = & 1 \\ & & -x_2 & & & + & x_4 & = & 3 \end{array}$$

10. $$\begin{array}{rcrcrcr} x & + & y & & & = & 0 \\ x & & & - & z & = & 1 \\ x & - & y & & & = & 1 \end{array}$$

15. $x = 1 + 4t, y = t,$ for any real t

See Section 2.3 Answers for answers to 2.2 problems assigned as 2.3 homework

Answers to Assigned Homework Problems

Section 2.3

Problems assigned from 2.3:

1. $\left[\begin{array}{ccc|c} 2 & -3 & 1 & 0 \\ 1 & 0 & -2 & 1 \\ 0 & -4 & 1 & -1 \end{array}\right]$

2. $\left[\begin{array}{cccc|c} 1 & 1 & -1 & 0 & 3 \\ 0 & 1 & 1 & -1 & 0 \\ -1 & 0 & 1 & -1 & 1 \end{array}\right]$

3. $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$

4. $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{array}\right]$

5. No

6. Yes

7. Yes

8. No

9. No

10. No

11. $x_1 = 0$, $x_2 = 2$, $x_3 = -1$

12. $x_1 = -s$, $x_2 = s$, $x_3 = s$ for any real s

13. No solution

14. $x_1 = 0$, $x_2 = 0$, $x_3 = s$ for any real s

15. $x_1 = 4 - 2s - 3t$, $x_2 = s$, $x_3 = t$ for any real s and t

16. $x_1 = 0$, $x_2 = 1$, $x_3 = t$ for any real t

17. $x_1 = 1 + 2s$, $x_2 = s$ for any real s

19. $x_1 = -\frac{3}{4}$, $x_2 = -\frac{5}{4}$, $x_3 = \frac{13}{4}$

21. $x_1 = 2s$, $x_2 = \frac{5s-1}{3}$, $x_3 = s$ for any real s

23. No solution

24. $x_1 = \frac{5s}{7}$, $x_2 = \frac{s}{7}$, $x_3 = -\frac{10s}{7}$, $x_4 = s$ for any real s

25. $u = \frac{1}{2} + s$, $v = 1 + 2s - t$, $w = s$, $z = t$ for any real s and t

26. $x_1 = -1 + s$, $x_2 = -\frac{1}{3}$, $x_3 = \frac{4}{3} - s$, $x_4 = s$ for any real s

37. infinite: no value of k
no solution: $k = 2$

Problems assigned from 2.2:

19. point 20. no intersection 21. line 22. plane

23. When $c = -1$ the solutions are $x = t, y = 2t, z = t$ for any real t.

24. No solution when $k \neq 1$.

Answers to Assigned Homework Problems

Section 3.1

1. $\begin{bmatrix} -1 & 4 & -1 \\ 5 & 0 & 5 \\ -1 & 1 & -1 \end{bmatrix}$

2. $\begin{bmatrix} -1 & -4 & 3 \\ -1 & -2 & 1 \\ 1 & 1 & -3 \end{bmatrix}$

3. $\begin{bmatrix} -2 & -12 & 8 \\ -5 & -5 & 0 \\ 3 & 2 & -7 \end{bmatrix}$

4. $\begin{bmatrix} 0 & 0 & 1 \\ 2 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix}$

5. $\begin{bmatrix} -1-\lambda & 0 & 1 \\ 2 & -1-\lambda & 3 \\ 0 & 1 & -2-\lambda \end{bmatrix}$

6. $\begin{bmatrix} -1 & 4\lambda & 1-2\lambda \\ 2+3\lambda & -1+\lambda & 3+2\lambda \\ -\lambda & 1 & -2+\lambda \end{bmatrix}$

7. $\begin{bmatrix} -1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 3 & -2 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

9. $\begin{bmatrix} -1 & -4 & 3 \\ -6 & 7 & -3 \\ 5 & 1 & 0 \end{bmatrix}$

10. $\begin{bmatrix} -1 & -6 & 5 \\ -4 & 7 & 1 \\ 3 & -3 & 0 \end{bmatrix}$

11. $\begin{bmatrix} -1 & -6 & 5 \\ -4 & 7 & 1 \\ 3 & -3 & 0 \end{bmatrix}$

12. $\begin{bmatrix} 8 & -1 & 1 \\ -6 & 1 & 1 \\ 16 & 2 & -3 \end{bmatrix}$

13. $\begin{bmatrix} 2 & 1 \\ -1 & 0 \\ \frac{1}{2} & -2 \end{bmatrix}$

14. $(A^T)^T = A$

17. $AB = \begin{bmatrix} 7 & -1 & 1 \\ 0 & -4 & -2 \\ 6 & -6 & 0 \end{bmatrix}$ and $BA = \begin{bmatrix} -1 & 2 & 4 \\ 5 & 6 & -1 \\ 6 & 4 & -2 \end{bmatrix}$

18. $AB = \begin{bmatrix} -3 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $BA = [-3]$

19. AB and BA are not defined

20. $AB = \begin{bmatrix} 3 & 2 & 1 \\ 14 & 36 & -22 \end{bmatrix}$ but BA is not defined

21. $AB = A$ but BA is not defined

22. AB is not defined and BA is the 4×3 zero matrix

23. $\begin{bmatrix} -9 & 1 & 8 \\ -2 & -3 & -10 \end{bmatrix}$

24. $\begin{bmatrix} \frac{15}{2} & 6 \\ 15 & \frac{27}{4} \end{bmatrix}$

25. $A^2 = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 5 & -8 \\ -8 & 13 \end{bmatrix}$

26. $A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 37 & 54 \\ 81 & 118 \end{bmatrix}$

27. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 10 & 4 & 1 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 21 & 6 & 1 \end{bmatrix}$

35. $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

36. $A^n = 2^{n-1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

37. Any 2×2 matrix A, which is not the 2×2 identity matrix I, and which does have the property that $A^2 = I$, is a correct answer. The easiest one to find is $A = -I$. All of the others have the main diagonal entries summing to 0. Here are some other possible answers:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{or } A = \begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix} \quad \text{or } A = \begin{bmatrix} -5 & -8 \\ 3 & 5 \end{bmatrix}$$

38. Any 2×2 matrix A, which is not the 2×2 identity matrix I and also not the 2×2 zero matrix, and which does have the property that $A^2 = A$, is a correct answer. All of the possibilities have the main diagonal entries summing to 1. Here are a few of the possibilities:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{or } A = \begin{bmatrix} 2/5 & 1/5 \\ 6/5 & 3/5 \end{bmatrix} \quad \text{or } A = \begin{bmatrix} 5 & 40 \\ -1/2 & -4 \end{bmatrix}$$

Answers to Assigned Homework Problems

Section 3.2

1. $\begin{bmatrix} 1 & -1 & 3 \\ 1 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

2. $\begin{bmatrix} 2 & -3 & 0 & 1 \\ 0 & 1 & -1 & 3 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$

3. $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$

5 through 7: In each case, $AB = I$, so A and B are inverses of one another.

9. $\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$

10. $\begin{bmatrix} 2 & 1 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$

11. $\begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

12. Not invertible

13. $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{5}{2} & \frac{1}{2} & 1 \end{bmatrix}$

15. Not invertible

16. $\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$

17. $c = -6$

18. $c = -\frac{3}{2}$

19. $\begin{bmatrix} -3 \\ -8 \end{bmatrix}$

21. $\frac{1}{2}\begin{bmatrix} 7 \\ -3 \\ -3 \end{bmatrix}$

23. $\begin{bmatrix} 12 \\ 7 \end{bmatrix}$

25. $\begin{bmatrix} 4 \\ -4 \\ -3 \end{bmatrix}$

30. $\begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ -\frac{1}{14} & \frac{3}{14} \end{bmatrix}$

31. $\begin{bmatrix} 1 & \frac{1}{4} \\ 0 & \frac{1}{4} \end{bmatrix}$

Answers to Assigned Homework Problems

Section 3.3

1. Homogeneous

2. Nonhomogeneous

3. 2 4. 2 5. 3 6. 3

9. False 10. False 11. True

12. True 13. False 14. True

15. Infinitely many solutions.

16. No solutions, or infinitely many solutions.

17. One solution, or infinitely many solutions.

18. No solutions, one solution or infinitely many solutions.

21. (*a*) No solutions when $x = 0$ and $y \neq 0$.
(*b*) If $x \neq 0$, there is a unique solution for *any* real value of y.
(*c*) If $x = y = 0$, there are infinitely many solutions.

Answers to Assigned Homework Problems

Section 4.1

1. 3,1-minor is -7; 3,1-cofactor is -7; 3,2-minor is -1; 3,2-cofactor is 1.

2. 2,2-minor is -8; 2,2-cofactor is -8; 2,3-minor is -6; 2,3-cofactor is 6.

3. 3	4. -5	5. -5	6. 0
7. 15	8. 10	9. 0	10. 51
11. 120	12. 0	13. a^3	14. $k(ad-bc)$
15. $k^2(ad-bc)$	16. $\lambda^2-3\lambda-10$	17. t^2-3t+3	18. 24
19. -8	20. -15	21. 0	22. 0
27. $x=-1$ or 3	28. $x=0$ or 2 or 3		

29. $a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}-a_{13}a_{22}a_{31}-a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}$

34. 24

Answers to Assigned Homework Problems

Section 4.2

1. 2
2. 6
3. -24
4. 0
5. $-\dfrac{1}{720}$
6. $(.02)(.03)(.01)[(1)(7)(27)] = .001134$
7. $-x(x+1)(2x-1)$
8. $(t-3)(t^2-t-8)$
9. $-(\lambda-1)(\lambda-2)(\lambda+1)(\lambda+2)$

10. Only the matrix in question 4 is not invertible; the matrices in 1, 2, 3, 5 and 6 are all invertible.

11. The matrix in 7 is invertible for any $x \neq -1, 0, \frac{1}{2}$.
    The matrix in 8 is invertible for any $t \neq 3$ and $t \neq \dfrac{1 \pm \sqrt{33}}{2}$.
    The matrix in 9 is invertible for any $\lambda \neq \pm 1, \pm 2$.

13. Has a unique solution
14. Does not have a unique solution
15. Does not have a unique solution
16. Has a unique solution
17. $\det(A^T) = 4; \det(2A) = 32$

19. Appropriate row-reduction produces a matrix with a row of only 0's.

Answers to Assigned Homework Problems

Section 4.3

1. $x = 1, y = -1$

3. Cramer's Rule cannot be used.

5. $x = \dfrac{3}{5}, y = \dfrac{3}{2}, z = \dfrac{17}{10}$

7. $w = 0, x = 2, y = 1, z = -1$

9. Adj $A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$; $A^{-1} = \begin{bmatrix} \dfrac{2}{5} & -\dfrac{1}{5} \\ \dfrac{3}{10} & \dfrac{1}{10} \end{bmatrix}$

11. Adj $A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$; $A^{-1} = \begin{bmatrix} 1 & \dfrac{1}{2} & 1 \\ 0 & \dfrac{1}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$

13. Adj $A = \begin{bmatrix} -27 & 9 & 9 \\ 18 & -6 & -6 \\ 9 & -3 & -3 \end{bmatrix}$; A has no inverse