10/26 $S = \{\text{uwo Students}\}$ $C = \{\text{uwo course}\}$

$E = \{(s, c) \in S \times C \mid s \text{ is enrolled in } c\}$

ex: $(\text{Chris}, \text{M2155}) \in E$, $(\text{Chris}, \text{Psych1001}) \notin E$

E is truth set of $P(s, c)$: "s is enrolled in c"

Def 4.2.1: Let A and B be sets. A relation from A to B is a subset $R \subseteq A \times B$.

Ex: • E above is a relation from S to C

• $P = \{\text{uwo prof}\}$. $T = \{(c, p) \in C \times P \mid p \text{ is teaching } c\}$

$(\text{Math2155}, \text{me}) \in T$. T is a relation from C to P

• $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ $R_1 = \{(1,3), (1,5), (3,3)\}$

R_1 is a relation from A to B

• $G = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x > y\}$ is a relation from $\mathbb{R}$ to $\mathbb{R}$.

• $D = \{1, 2\}$, $R_2 = \{(x, y) \in D \times P(D) \mid x \in y\}$

$= \{(1, \{1\}), (1, \{1,2\}), (2, \{2\}), (2, \{1,2\})\}$

is a relation from D to $P(D)$. The "$\in$" relation

• For any A, B: $A \times B$ is a relation from $A \times B$. So is $\emptyset$.

Def 4.2.3: The domain of $R \subseteq A \times B$ is

$$\text{Dom}(R) := \{a \in A \mid \exists b \in B, (a, b) \in R\}.$$

and the range of R is

$$\text{Ran}(R) := \{b \in B \mid \exists a \in A, (a, b) \in R\}$$

Ex: $\text{Dom}(E) = \{\text{all students taking at least one course}\}$

$\text{Ran}(E) = \{\text{courses with at least one student}\}$

$Dom(R_1) = \{1,3\}$, $Ran(R_1) = \{3,5\}$

$Dom(R_2) = \{1,2\} = D$, $Ran(R_2) = \{\{1\},\{2\},\{1,2\}\} \subseteq P(D)$

Def 4.2.3 (cont): The inverse of $R \subseteq A \times B$ is the relation R^{-1} from B to A defined by $R^{-1} = \{(b,a) \in B \times A \mid (a,b) \in R\}$.

Ex: $R_1^{-1} = \{(3,1), (5,1), (3,3)\}$

$$G^{-1} = \{(x,y) \in \mathbb{R}\times\mathbb{R} \mid (y,x) \in G\}$$
$$= \{(x,y) \in \mathbb{R}\times\mathbb{R} \mid y > x\}$$
$$= \{(x,y) \in \mathbb{R}\times\mathbb{R} \mid x < y\}$$

E^{-1}: reverse arrows.

Thm 4.2.5 (part): Let $R \subseteq A \times B$. Then

1. $(R^{-1})^{-1} = R$
2. $Dom(R^{-1}) = Ran(R)$
3. $Ran(R^{-1}) = Dom(R)$

Proof of 1: Let $(a,b) \in A \times B$:

Then $(a,b) \in (R^{-1})^{-1}$

iff $(b,a) \in R^{-1}$

iff $(a,b) \in R$

so $(R^{-1})^{-1} = R$

2. see text.

3. let $a \in A$, then $a \in Ran(R^{-1})$

iff $\exists b \in B, (b,a) \in R^{-1}$

iff $\exists b \in B, (a,b) \in R$

iff $a \in Dom(R)$

So $Ran(R^{-1}) = Dom(R)$ $\square$

Def 4.2.3 (cont.) Given $R \subseteq A \times B$ and $S \subseteq B \times C$, then composition (or composite) of R and S is

$$S \circ R = \{(a,c) \in A \times C \mid \exists b \in B \ (a,b) \in R \text{ and } (b,c) \in S\}$$

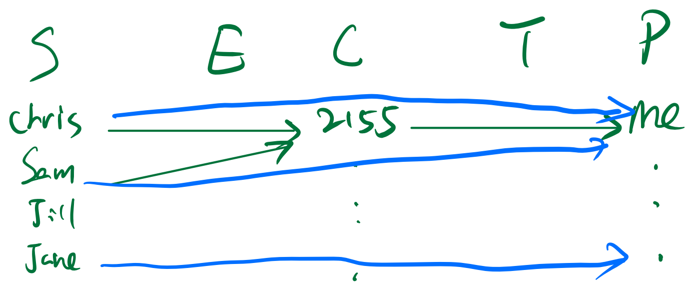
a relation from A to C

Ex: $E = \{(s,c) \in S \times C \mid s \text{ is enrolled in } c\}$

$T = \{(c,p) \in C \times P \mid p \text{ is teaching } c\}$

Then $T \circ E = \{(s,p) \in S \times P \mid \exists c \in C \ (s,c) \in E \text{ and } (c,p) \in T\}$

$\underset{\underset{S \times P}{\shortparallel}}{C \times P \ \ S \times C} = \{(s,p) \in S \times P \mid s \text{ is taking at least one course taught by } p\}$



10/28. Recall: $E = \{(s,c) \in S \times C \mid s \text{ is enrolled in } c\}$

Ex: $E^{-1} \circ E = \{(s_1,s_2) \in S \times S \mid \exists c \in C \ (s_1,c) \in E \wedge (c,s_2) \in E^{-1}\}$

$= \{(s_1,s_2) \in S \times S \mid \exists c \ (s_1,c) \in E \wedge (s_2,c) \in E\}$

$= \{(s_1,s_2) \in S \times S \mid s_1 \ \& \ s_2 \text{ have a course together}\}$

Ex: $E \circ E^{-1} = \{(c_1,c_2) \in C \times C \mid \text{there is a student taking both}\}$

Thm 4.2.5 (cont.): For $R \subseteq A \times B$, $S \subseteq B \times C$, $T \subseteq C \times D$

4. $T \circ (S \circ R) = (T \circ S) \circ R$ see text

5. $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$