$\det A$: ↘ − ↗ A_{ij} => i 行 j 列.

i,j minor of A => $\det A_{ij}$ 行在前!!!

★ i,j cofactor of A => $(-1)^{i+j} \det A_{ij}$.

含全零行矩阵 => $\det A = 0$ (自己推导看?).

$A = \begin{bmatrix} a & e & f & g \\ 0 & b & h & i \\ 0 & 0 & c & j \\ 0 & 0 & 0 & d \end{bmatrix}$ => $\det A = abcd$.

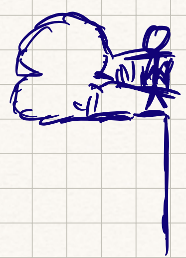
$B = \begin{bmatrix} 2a & 2c \\ 2b & 2d \end{bmatrix}$ $\det B = 2\det\begin{bmatrix} a & c \\ 2b & 2d \end{bmatrix} = 2\det\begin{bmatrix} a & 2c \\ b & 2d \end{bmatrix} = 4\det\begin{bmatrix} a & c \\ b & d \end{bmatrix}$.

↑

前提是 $n \times n$ matrix.

Square matrix A has two identical rows/columns => $\det A = 0$.

e.g. $\det\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 12 & 18 \end{bmatrix} = 0$. 矩阵塞端.



今时已到?

哭哭.

矩阵交换两行/列 => 乘 −1

(i,j)-entry of $\mathrm{Adj}\,A = (-1)^{i+j} \det A_{ij}^{T}$.

=> $\mathrm{Adj}\,A$ 是一个矩阵. 其中每一 i,j 都要计算.

$A^{-1} = \frac{1}{\det A} \mathrm{Adj}\,A$.

$A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}$, find a_{12}.

$-3 = \frac{1}{\det A} \mathrm{Adj}\,A$.

$\det(AB) = \det A \cdot \det B$.

$\det(2A) = 2^n \det A$ n = row/column.

$A\vec{x} = \vec{b}$.

$x_{(j)} = \dfrac{\det A(j)}{\det A}$. $x(i) \Rightarrow$ 位于i位的未知数.

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\begin{bmatrix} e \\ f \end{bmatrix}$. only if $\det \neq 0$

$[a \quad b \quad c \mid d]$.

$\Rightarrow ax + by + cz = d$.

$A_{(1)} = \begin{bmatrix} e & b \\ f & d \end{bmatrix}$

Cramer's rule. $\Rightarrow$ 求解用.

$A_{(2)} = \begin{bmatrix} a & e \\ c & f \end{bmatrix}$.

2	-3	5
-1	2.	-4.

$\det A = 7$. $\det A(j) = -3$.

$\det A_{(1)} = 22$

$x = \dfrac{\det A_{(1)}}{\det A} = \dfrac{22}{7}$ $y = \dfrac{-3}{7}$.

$2x + 3y = 5$

$-x + 2y = -4$.

$7y = -3$.

$y = \frac{-3}{7}$.

$\cancel{-x} - \frac{6}{7} = -4$.

$x = \frac{22}{7}$.

求 det $\Rightarrow$ 先令其化为RREF再乘对角线.

no inverse $\Rightarrow \det A = 0$.

$\det A = \dfrac{1}{\det A^{-1}}$.

Adj $A \Rightarrow \det A$ · (不用乘系数, 不是cofactor).

$A^{-1} = \dfrac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.