$$\frac{A4 - sol}{(win\ 2023)}$$

① ⓐ Denote:

$$s(x) = smart(x)$$
$$h(x) = hard\_working(x)$$
$$g(x) = graduate\_studies(x)$$

We have:

$$\big((s(x) \wedge h(x)) \rightarrow g(x)\big) \quad \wedge$$
$$\big((s(x) \wedge \neg h(x)) \rightarrow \neg g(x)\big)$$

$$\equiv \big(\neg(s(x) \wedge h(x)) \vee g(x)\big) \wedge$$
$$\big(\neg(s(x) \wedge \neg h(x)) \vee \neg g(x)\big)$$

$$\equiv \big(\neg s(x) \vee \neg h(x) \vee \underline{g(x)}\big) \wedge$$
$$\big(\neg s(x) \vee \underline{h(x)} \vee \neg g(x)\big)$$

one non-negated term

ⓑ

$$\equiv \big((s(x) \wedge h(x)) \rightarrow g(x)\big) \wedge$$
$$\big((s(x) \wedge g(x)) \rightarrow h(x)\big)$$

Prolog:

$$g(x) :- s(x), h(x).$$
$$h(x) :- s(x), g(x).$$

```
1. insert(X, L, [X|L]).
2. insert(X, [H|L], [H|L1]) :- insert(X, L, L1).
3. permute([], []).
4. permute([H|T], P) :- permute(T, P1), insert(H, P1, P).
```

permute $([a, b], Y)$.

④ | $H = a$
   | $T = [b]$

permute $([b], P1)$, insert $(a, P1, Y)$. ⟶ false

④ | $H = b$
   | $T = []$

permute $([], P2)$, insert $(b, P2, P1)$, insert $(a, P1, Y)$.

③ | $P_2 = []$

insert $(b, [], P1)$, insert $(a, P1, Y)$.

① | $X = b$
   | $L = []$
   | $P1 = [b]$

insert $(a, [b], Y)$.

①   ② $X = a$ $Y = [b|L1]$
$X = a$ $H = b$
$L = [b]$ $L = []$

$Y = [a, b]$;

insert $(a, [], L1)$.

① | $X = a$
   | $L = []$
   | $L_1 = [a]$

$Y = [b, a]$;

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```
1. insert(X, L, [X|L]).
2. insert(X, [H|L], [H|L1]) :- insert(X, L, L1).
3. permute([], []).
4. permute([H|T], P) :- permute(T, P1), insert(H, P1, P).
```

permute (Y, [a,b]).

④ | Y = [H|T]
P = [a, b]

permute (T, P1), insert (H, P1, [a, b]).

T = [ ]   ③
P1 = [ ]   ④   T = [H1|T1]

insert (H, [ ], [a, b]).   permute (T1, P2), insert (H1, P2, P1),
                                                        insert (H, P1, [a, b])

backtrack

T1 = [ ]   ③   ④
P2 = [ ]

insert (H1, [ ], P1), insert (H, P1, [a, b]),   •••

① | P1 = [H1]   permute (T1, P2)
                                                 will generate
insert (H, [H1], [a, b]).   infinitely many
                                                 solutions, all of
H = a   ①   ②   H1 = [a|L]   which will fail
H1 = b   , L1 = [b]   insert (..., ..., [a, b]).

Y = [a, b] ;   insert (H, L, [b]).

                              H = b   ①   ②
                              L = [ ]

                              Y = [b, a] ;   insert (H, L2, [ ]).

                                                   backtrack
```

Ⓒ

1. `insert(X, L, [X|L]).`
2. `insert(X, [H|L], [H|L1]) :- insert(X, L, L1).`
3. `permute([], []).`
4. `permute([H|T], P) :- permute(T, P1), insert(H, P1, P).`
5. $not(x) :- x, !, fail.$
6. $not(-).$

$not(permute(Y, [\,]))$.

⑤|     ⑥     ✗      [false]

$permute(Y, [\,]), !, fail.$

③| $Y = [\,]$     ④    ✗

$!, fail$

$fail$

(backtrack)

```prolog
magic_square(X) :- permutation([1,2,3,4,5,6,7,8,9],X),
          X = [X1,X2,X3,X4,X5,X6,X7,X8,X9],
          15 is X1+X2+X3,
          15 is X4+X5+X6,
          15 is X7+X8+X9,
          15 is X1+X4+X7,
          15 is X2+X5+X8,
          15 is X3+X6+X9,
          15 is X1+X5+X9,
          15 is X3+X5+X7.


?- magic_square(X).
X = [2, 7, 6, 9, 5, 1, 4, 3, 8] ;
X = [2, 9, 4, 7, 5, 3, 6, 1, 8] ;
X = [4, 3, 8, 9, 5, 1, 2, 7, 6] ;
X = [4, 9, 2, 3, 5, 7, 8, 1, 6] ;
X = [6, 1, 8, 7, 5, 3, 2, 9, 4] ;
X = [6, 7, 2, 1, 5, 9, 8, 3, 4] ;
X = [8, 1, 6, 3, 5, 7, 4, 9, 2] ;
X = [8, 3, 4, 1, 5, 9, 6, 7, 2] ;
false.
```