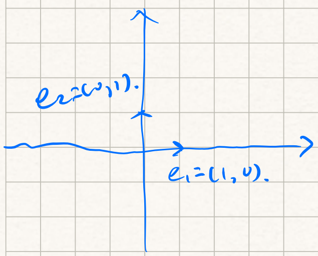
$c_1, c_2, \cdots c_k$ is called the coordinate of B.

e.g. $S = \mathbb{R}^2$.



$v = xe_1 + ye_2$.

the sum of scaler mutipled by e_1, e_2.

$[x_1, x_2] \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = v$.

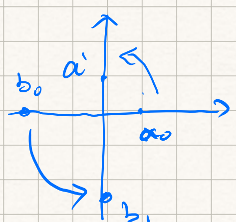
Linear transformers

Def: $T: \mathbb{R}^n \to \mathbb{R}^m$.

1) $T(x+y) = T(x) + T(y)$. for any T in $\mathbb{R}^n$.

2) $T(\lambda \bar{x}) = \lambda T(\bar{x})$, $\forall \lambda, \bar{x}$.

e.g. $T: \mathbb{R}^2 \to \mathbb{R}^2$: turn $90°$



$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}$.

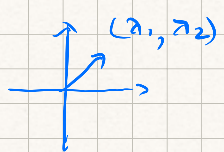
$T(\lambda_1 v_1 + \lambda_2 v_2 + \cdots \lambda_n v_n) = \lambda_1 T v_1 + \cdots + \lambda_n T v_n$.

$T(0) = 0$

$T_A(x) = Ax$.

$n = 2$.

$A = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$.



$AV = \lambda V$

$(A-\lambda)V=0$

$(A-\lambda I)V=0$.

Find possible eigen values of A.

If λ is an eigen value A if and only if $A-\lambda I$
is a nontrivial null space.

$\begin{matrix} 3-\lambda & 1 \\ 0 & 2-\lambda \end{matrix} = A-\lambda I.$

rank (B) => pivot num
+
nullility (B) => non-pivot column num
||
n

$\begin{bmatrix} 3-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix}$ is in REF

when the matrix has a non-pivot column

$\lambda=3 \Rightarrow V$

$\lambda=2 \Rightarrow V \Rightarrow (A-2I)V=0$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad x+y=0 \Rightarrow$ line.