4.5.4. R an equivelence relation

1. $\bigcup_{x \in A} [x] = A$.

2. $\forall x, y \in A, ([x]_R \neq [y]_R \to [x]_R \cap [y]_R = \phi)$

Proof:

1. Clearly $\bigcup_{x \in A} [x] \subseteq A$.
   Let $z \in A$, then $z \in [z]$.
   So $z \in \bigcup_{x \in A} [x]$, so $\bigcup_{x \in A} [x] = A$.

2. Let x, y be two elements of A.
   Suppose $[x] \cap [y] \neq \phi$. so let $z \in [x] \cap [y]$.
   so $[x] = [z] = [y]$

   done. by contrapositive.

Def 4.5.2. $\mathcal{F} \subseteq P(A)$. is called a partition of A if:

1. $\bigcup_{x \in \mathcal{F}} x = A$.
2. $\forall\, a \neq b \in \mathcal{F}$, $a \neq b \to a \cap b = \phi$.
3. $\forall x \in \mathcal{F} \to x \neq \phi$

For every equivlence relation R on a set A.
A/R is a partition of R.

$A = \{1, 2, 3, 4\}$ $\mathcal{F} = \{\{3\}, \{1, 2\}, \{4\}\}$
$\{\{3\}\ \{1, 2\}\ \{4\}\ \{4\}\}$ is still a partition (but confusing).

Def 5.1.1 $F \subseteq A \times B$ is a function if $\forall a \in A\ \exists!\, b \in B\ x F y$
if $(a, b) \in F$, $b = F(a)$.