

Assuming that $f(n) = 2^{(2n)} - 1$

Base step: while $n = 1$, $f(1) = 2^{(2 \cdot 1)} - 1 = 3$, which could be divided by 3 obviously.

Inductive step: Assume that $f(n)$ can be divided by 3 while $n > 1$, then we can have

$$f(n) - f(n-1) = 2^{(2n)} - 1 - (2^{(2n-2)} - 1) = 2^{(2n)} - 2^{(2n-2)} = 3 \cdot 2^{(2n-2)}, \text{ which can be divided by 3.}$$

Hence, 3 divides $2^{(2n)} - 1$ for all positive integers n .