ECON3102-005
Chapter 9: A Real Intertemporal Model of Investment

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What do we study in this chapter?

- Construct a real intertemporal model that will serve as a basis for studying money and business cycles in Chapters 11-13.

- Understand the investment decision of the firm.

- Show how macroeconomic shocks affect the economy.

What's new in this chapter?

- The key things added to chapter 8 are a work-leisure decision and a representative firm.

- We are moving from an endowment economy to a production economy.

- The representative firm and consumer interact on the labor and consumption markets.

The Representative Consumer

- has h units of time each period, for work and leisure (ℓ).

- today, earns real wage $w(h - \ell)$ receives dividend income π, pays taxes T, and saves S^p.

- hence the current period BC:

$$C + S^p = w(h - \ell) + \pi - T$$

- tomorrow, earns real wage $w'(h - \ell')$ receives dividend income π', pays taxes T', and receives $S^p(1 + r)$.

- hence, the future period BC:

$$C' = w'(h - \ell') + \pi' - T' + S^p(1 + r)$$

The Representative Consumer

- This gives the consumer's PVBC:

$$C + \frac{C'}{1+r} = w(h - \ell) + \frac{w'(h - \ell')}{1+r} + \pi + \frac{\pi'}{1+r} - T - \frac{T'}{1+r}$$

- The representative consumer chooses C, C', ℓ, ℓ' to be as well off as possible, given this PVBC.

The Representative Consumers Optimization Conditions

- The *MRS* of leisure for consumption is the price of leisure (wage): $MRS_{\ell,C} = w$ and $MRS_{\ell',C'} = w'$.

- The *MRS* of current for future consumption is the price of current consumption in terms of future consumption $(1 + r)$: $MRS_{C,C'} = 1 + r$

The Representative Consumers Labor Supply

- Recall from Chapter 4 that as the real wage increases, the substitution effect $\Rightarrow$ labor supply $\uparrow$.

- However, the income effect $\Rightarrow$ leisure $\uparrow$, i.e. labor supply $\downarrow$. (assuming leisure is normal.)

- Here,we assume substitution$>$income effect, so that $w \uparrow \Rightarrow$ labor supply $\uparrow$.

Current Labor Supply and Real Interest Rate

- The price of current relative to future leisure is: $\frac{w(1+r)}{w'}$

- As $r \uparrow$, the consumer substitutes current for future consumption and current for future leisure.

- Hence, assuming substitution>income effect, $r \uparrow \Rightarrow$ labor supply $\uparrow$.

- Labor supply $\downarrow$ as non-wage disposable income $\uparrow$ (income effect) (e.g: $\downarrow$ in lifetime taxes.)

An Increase in the Real Interest Rate Shifts the Current Labor Supply Curve to the Right
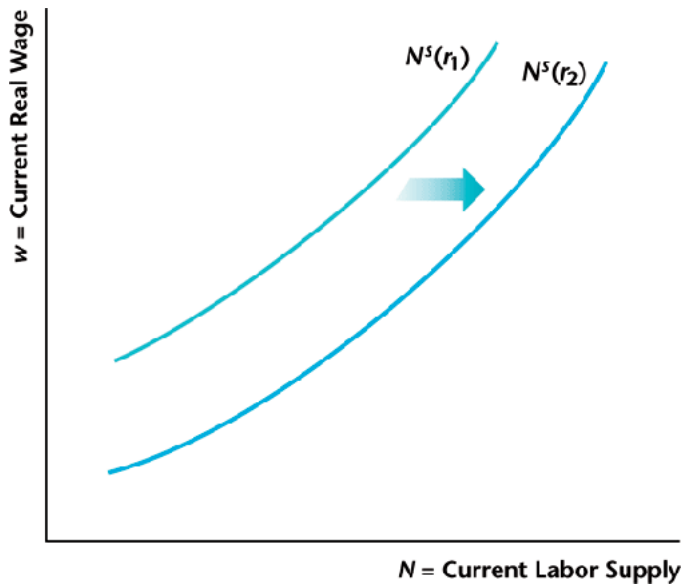
Effects of an Increase in Lifetime Wealth
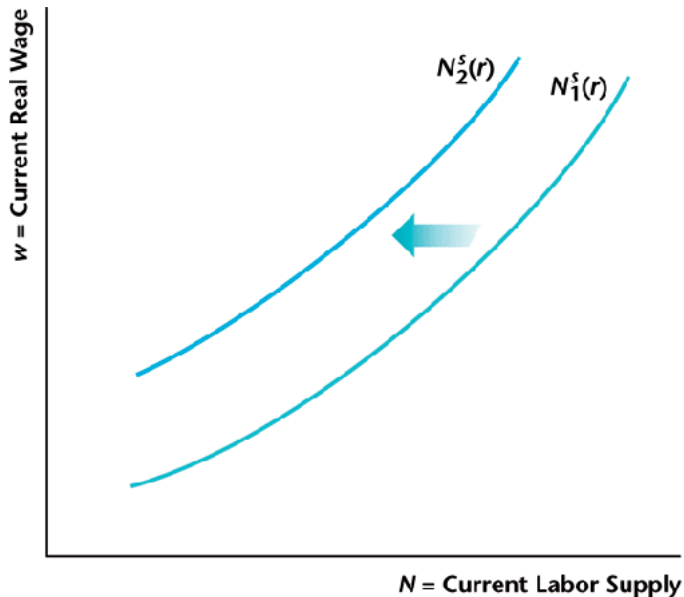
The Representative Firm

- Let z and z' denote present and future TFP. Current Period Production function is:

$$Y = zF(K, N)$$

- Future Period Production function:

$$Y' = z'F(K', N')$$

The Representative Firms Investment

- We assume it requires one unit of current consumption may be transformed into one unit of capital.

- The representative rm foregoes current profits, invests in current capital to increase capacity tomorrow:

$$K' = (1 - d)K + I$$

- We assume that in the last period, the remaining capital $(1 - d)K$ is converted back into the consumption good and sold to consumers.

The Representative Firm's Profit

- The objective of the firm is to max the PV of prots.

- Current Prots are:
$$\pi = Y - wN - I$$

- Future Prots are:
$$\pi' = Y' - w'N' + (1 - d)K$$

- The present value of prots is
$$\pi + \frac{\pi'}{1 + r}$$

The Representative Firms Current Labor Demand

The optimization rule for labor demand from the firm is:

- The objective of the firm is to max the PV of prots. $MP_N = w$.

- Recall from Ch 4 that because MP_N is $\downarrow$ in N, the firms labor demand is also $\downarrow$.

The Current Demand Curve for Labor Shifts Due to Changes in Current Total Factor Productivity z and in the Current Capital Stock K
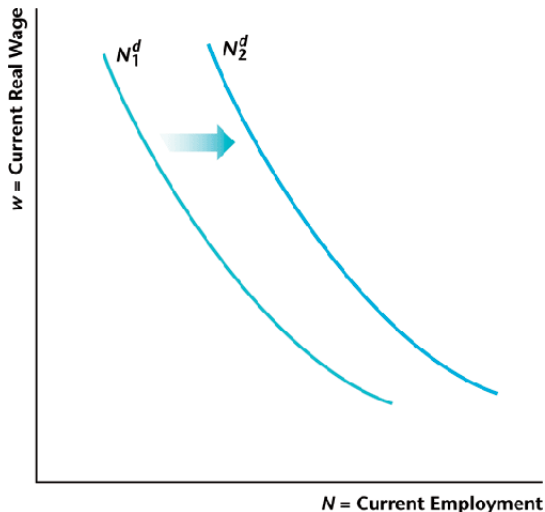
The Representative Firms Investment Decision

- The marginal cost of investment is what is given up to make an additional unit of investment. It equals one unit of the consumption good:

$$MC(I) \equiv 1$$

.

- An additional unit of investment transforms into one unit of capital that tomorrow is used for production and sold after depreciation.

- Hence, the marginal benet of investment in terms of units of current consumption is:

$$MB(I) \equiv \frac{MP_{k'} + 1 - d}{1 + r}$$

The Representative Firms Investment Decision

- The firm invests until: $MC(I) = MB(I)$,

- $\Rightarrow MB(I) \equiv \frac{MP_{k'} + 1 - d}{1 + r} = 1 \Rightarrow \underbrace{MP_{k'} - d}_{\text{net marginal product of capital}} = r.$

- Hence, as $r \uparrow \Rightarrow MP_{k'} \uparrow \Rightarrow K' \downarrow \Rightarrow I = (K' - (1-d)K) \downarrow$.

- The net marginal product of capital is the marginal product of capital, net of its depreciation.

- Intuition: The bond and capital are the two assets in the economy and should yield identical returns (the rm often borrows to nance investment).

Optimal Investment Schedule for the Representative Firm
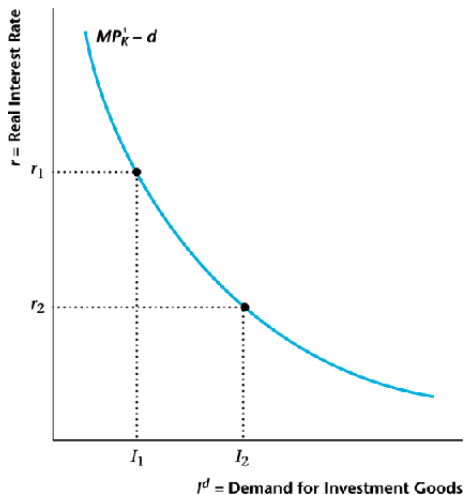
The Optimal Investment Schedule Shifts to the Right if Current Capital Decreases or Future Total Factor Productivity Is Expected to Increase

I^d = Demand for Investment Goods
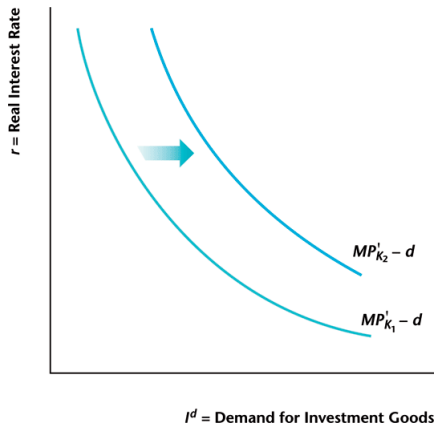
The Government

- Governments behavior is the same as in Chapter 8:

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

Competitive Equilibrium

- A CE in this economy is a situation in which consumers and rms optimize, taking prices and taxes as given, GBC holds and markets clear.

- We will focus on current period markets only.

- Eventually, in this model, we will show that this is WLOG, as the clearing of current markets implies that of future markets.
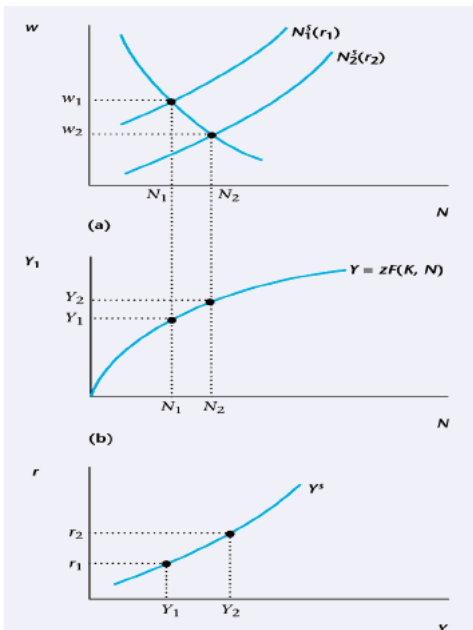
The Output Supply Curve

- The next 2 slides explain the determination of the output supply curve by the firm, as a function of r.

- We will find that because of the intertemporal substitution effect effect on labor supply, the the output supply curve is $\uparrow$ in r.

Determination of Equilibrium in the Labor Market Given the Real Interest Rate r
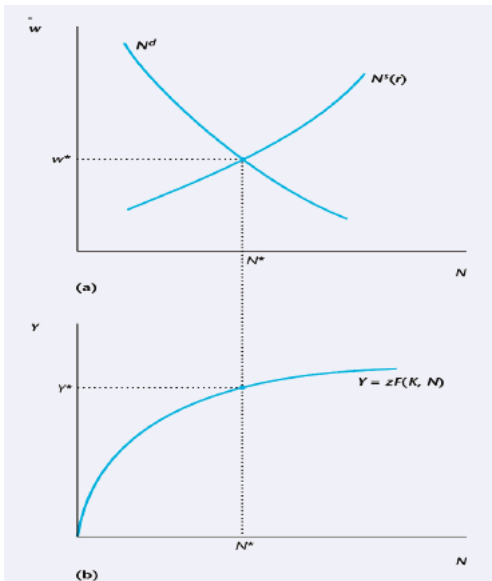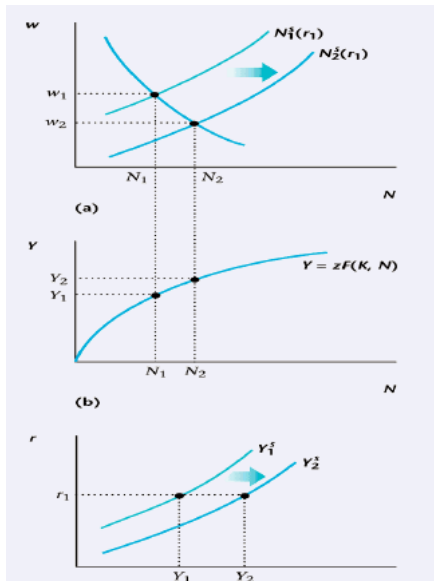
Construction of the Output Supply Curve

Shifts of the Output Supply Curve

- The output supply curve (function of r) shifts because of:

  - A shift in the current labor supply curve (e.g. from a change in lifetime wealth) (by the income effect)

  - A shift in the current labor demand curve (e.g from changes in z, K).

Shifts of the Output Supply Curve in response to a change in G, G'

- Hence an $\uparrow$ in G or $G' \Rightarrow \uparrow$ in lifetime taxes $\Rightarrow \downarrow$ in lifetime wealth $\Rightarrow$ leisure $\downarrow \Rightarrow$ labor supply $\uparrow$.
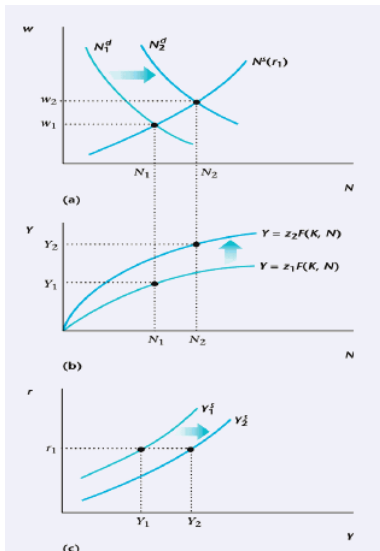
Shifts of the Output Supply Curve in response to an increase in G or G'

Shifts of the Output Supply Curve in Response to a Change in z (same for an increase in K)

- $z \uparrow \Rightarrow$ the production function shifts to the right (ch 4)

- Also, an $\uparrow$ in $z \Rightarrow$ labor demand $\uparrow$ as MP_N increases.

Shifts of the Output Supply Curve in response to an increase in z (same for an increase in K)

The Output Demand Curve

- Let Y represents the representative consumers current income.

- Let $C^d(Y, r)$ denote the representative consumers current demand for the consumption good (a $\downarrow$ function of r assuming the substitution effect dominates).

$$Y^d = C^d(Y^d, r) + I^d(r) + G$$

The Total Demand for current goods

- Since I^d and G do not depend on current income Y, only C^d inuences $C^d(r) + I^d(r) + G$ as a function of Y.

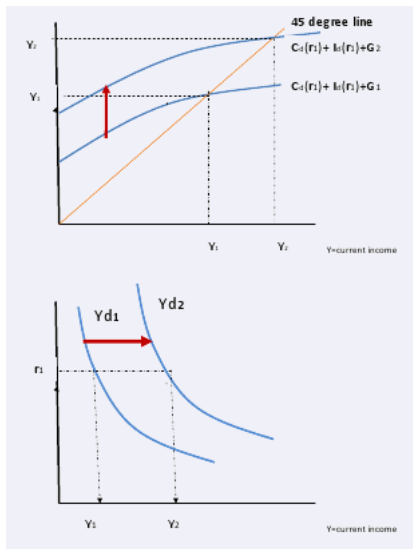- Hence, the slope of $Y^d = MPC$.
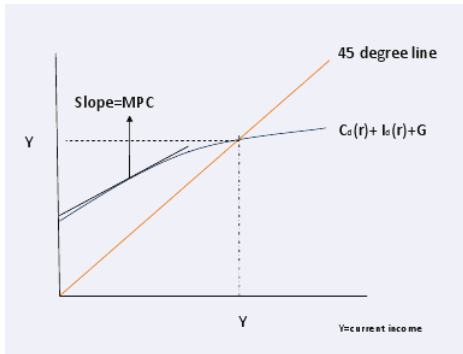
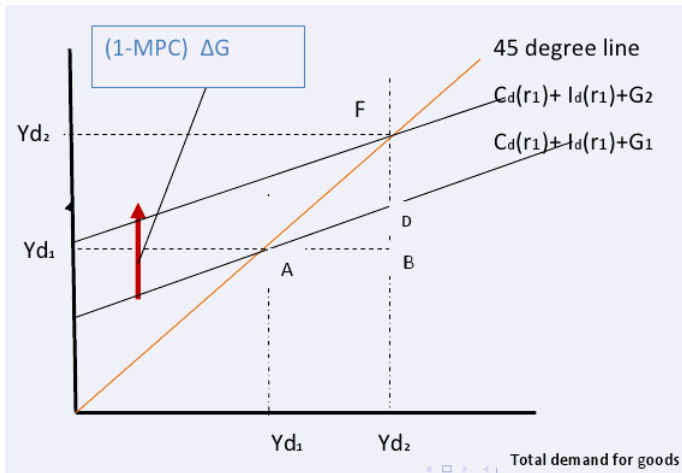- We are not done yet. We need to get Y^d as a function of r.

The Total Demand for current goods

- Recall $C^d \downarrow$ as $r \uparrow$ (assuming substitution $>$ income effect).

- Recall $I^d \downarrow$ as $r \uparrow$

- Hence, $Y^d = C^d(Y^d, r) + I^d(r) + G \Rightarrow Y^d \downarrow$ as $r \uparrow$.

An Increase in Current or Future Government Spending Shifts the Y^d Curve

But the magnitude of $\frac{\Delta Y}{\Delta G}$ is different from what Keynesian theory says!

Here we have:

- $MPC \equiv \frac{DB}{AB} = \frac{FB-FD}{Y_2^d-Y_1^d} = \frac{[Y_2^d-Y_1^d]-FD}{Y_2^d-Y_1^d} = \frac{[Y_2^d-Y_1^d]-[(1-MPC)\Delta G]}{Y_2^d-Y_1^d}$

- $\Rightarrow Y_2^d - Y_1^d = G_2 - G_1 \Rightarrow \frac{\Delta Y}{\Delta G} = 1$

- Not what you know from your Chapter on Keynes in Econ 1102!

What went wrong in the Keynesian model?

- In Econ 1102, we had $\Delta Y = \frac{1}{1-MPC}\Delta G$

- This is because the Keynesian theory forgets the impact of the change in taxes on consumption.

- The new classical model is not perfect either, as it assumes perfect foresight.

- So far, we have only looked at the effect of ΔG on Y^d. What about the effect on Y^s? Taxes $\uparrow \Rightarrow$ labor supply $\uparrow$ (income effect $\Rightarrow$ leisure $\downarrow$). This $\Rightarrow Y^s$ shifts to the right.
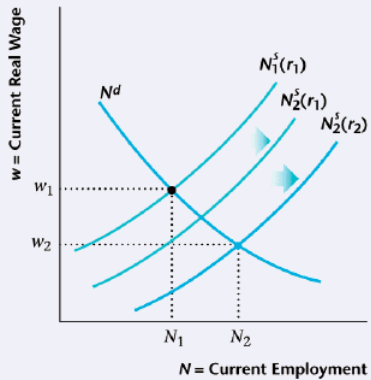
(a)
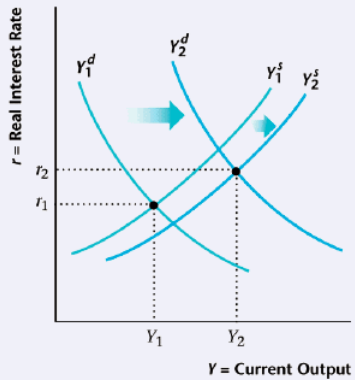
(b)

General Equilibrium effects of an increase in G

- The horizontal shift of Y^d is of length $= \Delta G$.

- But because Y^s does not shift to the right by the same magnitude, the change in Y is less than the original change in ΔG.

- The Keynesian model does not account for substitution effects and cannot capture the impact on y^s.
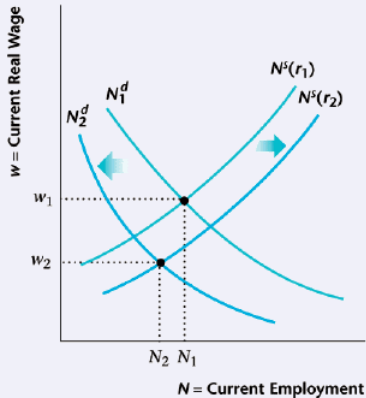
G crowds out private consumption and private investment

- Note from the previous graph, that $r \uparrow$ if the income effect(that causes leisure to $\downarrow$ and shifts Y^s to the right) is not too high. Otherwise, $r \downarrow$.

- Lets assume conditions are met for an $\uparrow$ in r.

- Then, because of intertemporal substitution, current consumption will $\downarrow$.

- The $\uparrow$ in r also $\Rightarrow \downarrow$ in private investment. This crowding out effect is ignored in the Keynesian theory.

- Note further that the $\uparrow$ in r causes the consumer to substitute current for future leisure, shifting N^s and Y^s further to the right.
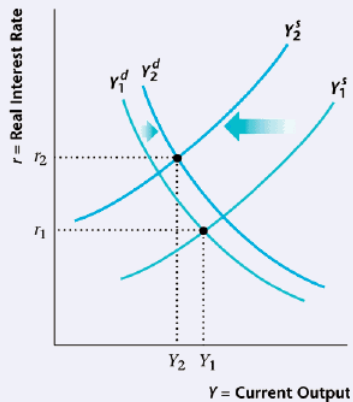
General Equilibrium Effects of $\downarrow$ in K

- $\Rightarrow I^d \uparrow \Rightarrow Y^d$ shifts to the right.

- Also, the $\downarrow$ in $K \Rightarrow$ a $\downarrow$ in $MP_N \Rightarrow N^d$ and Y^s shifts to the left. In total, Y^* may $\uparrow$ or $\downarrow$.

- Hence, in equilibrium, r must necessarily $\uparrow$. So, current consumption must $\downarrow$.

- I^d may $\uparrow$ (as K has $\downarrow$) or $\downarrow$ as r has $\uparrow$.

- As $r \uparrow$, the consumer substitutes current for future leisure, which $\Rightarrow N^s$ shifts to the right, depressing wages further. Overall, w may $\downarrow$ or $\uparrow$.

General Equilibrium Effects of ↓ in K

(a) w = Current Real Wage, N = Current Employment

(b) r = Real Interest Rate, Y = Current Output

General Equilibrium effects of an increase in TFP z

- As $z \uparrow$, $MP_N \uparrow$, and so, N^d and Y^s shifts to the right.

- Y^s shifts to the right $\Rightarrow r \downarrow \Rightarrow N^s$ shifts to the left as the consumer substitutes future for current leisure.

General Equilibrium effects of an increase in TFP z

(a)
w = Current Real Wage
N = Current Employment
N_1^d, N_2^d, $N^s(r_2)$, $N^s(r_1)$
w_2, w_1
N_1, N_2

(b)
r = Real Interest Rate
Y = Current Output
Y^d, Y_1^s, Y_2^s
r_1, r_2
Y_1, Y_2