

Nov 15

COMPSCI 3331

Fall 2022

What's next?

- ▶ Assignment 3: due Nov 22.
- ▶ Quiz 6 tomorrow - **Lectures 9 and 10.**
- ▶ Midterm marks: now available. Remark requests due Nov 24.
- ▶ Assignment 2, Quiz 5: being marked.

Assignment 3

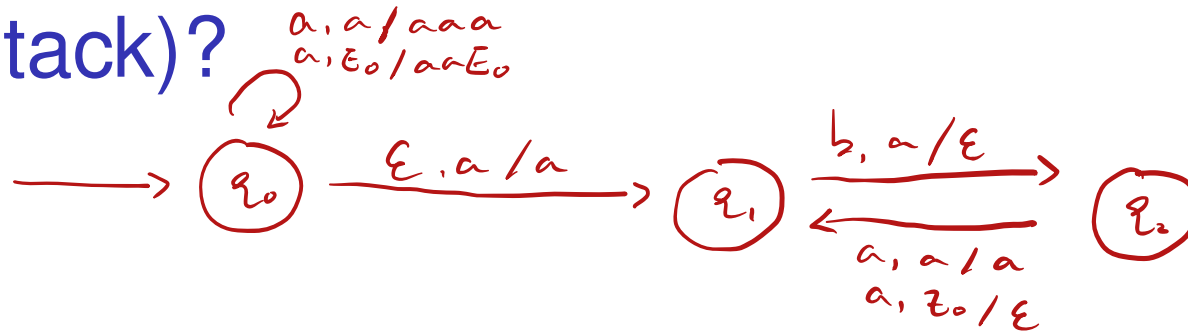
- ▶ Question 3: Algorithm to determine if $\exists x \in \text{out}(z) \cap L(G)$.
- ▶ G is in CNF.
- ▶ What doesn't work?

out(A) gives you $O(n^2)$ words where $|z| > n$.

e.g. what does not work: $\text{Out}(z) = \{x_1, x_2 \dots x_m\}$.

test $x_i \in L(G)$? $O(n^5)$

What language is accepted by this PDA (empty stack)?



$a^n b^{2n}$

$a^n (ba)^n$

$a^n (bca)^{2n}$

$a^{2n} b^n$

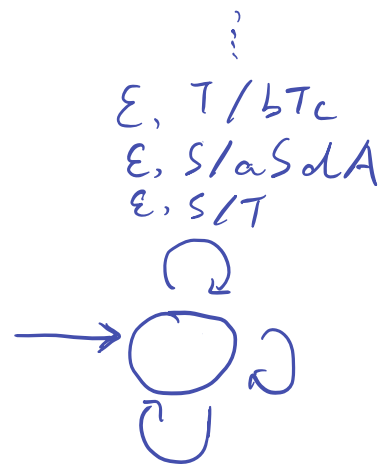
$a^{2n} (ba)^n$

Converting CFG to PDA

$$S \rightarrow aSdA \mid T$$

$$T \rightarrow bTc \mid \varepsilon$$

$$A \rightarrow eA \mid e$$



Simply replacing
left-hand side with
right-hand sides.

$$\begin{aligned}
 S &\Rightarrow aSdA \\
 &\Rightarrow aTdA \\
 &\Rightarrow abTcdA \\
 &\Rightarrow abcde.
 \end{aligned}$$

e.g. ~~abcde~~

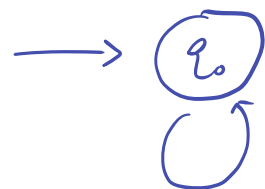
~~a~~
~~b~~
~~T~~
~~c~~
~~d~~
~~e~~

$S \Rightarrow T \Rightarrow C \Rightarrow \dots \Rightarrow$

$S \Rightarrow A$

an empty stack.

Converting PDAs to CFGs



$a, a/aa$

$a, z_0/az_0$

$b, a/\epsilon$

$\epsilon, z_0/\epsilon$

⇒ this is an example
of open-close, like
" () "

$S \rightarrow [\bar{q}_0, z_0, q_0]$

$A: [\bar{q}_0, a, q_0] \rightarrow a[\bar{q}_0, a, q_0]^2$

$B: [\bar{q}_0, z_0, q_0] \rightarrow a[\bar{q}_0, a, q_0][\bar{q}_0, z_0, q_0]$

$[\bar{q}_0, a, q_0] \rightarrow b$

$[\bar{q}_0, z_0, q_0] \rightarrow \epsilon$

$S \rightarrow B$

$A \rightarrow aAA$

$B \rightarrow aAB$

$A \rightarrow b$

$B \rightarrow \epsilon$

Non-CFLs

recursive language

neither regular nor context-free.

not context free
 \Rightarrow not regular.

$L = \{a^n b^n c^n : n \geq 0\}$. \therefore we cannot use the stack twice.

let n be the pumping lemma constant

$z = a^n b^n c^n$. so $|z| = 3n \geq n$ and z is in L

Then we can write $z = uvwx$ where $|vwx| \leq n$, $|v| > 0$

	a	b	c
1	v	x	
2	v x		
3		v x	
4			v x
5		v	x
6	v		x
7)	v	x	

either v or x cross the boundary

(find all decomposition of $uvwx$)

1) Assume v is in a 's and x is in b 's.

$u = a^i, v = a^j, w = a^{n-i-j} b^k, x = b^l, y = b^{n-k-l} c^n$

pick $r=2$, $uv^2wx^2y = a^{n+j} b^{n+l} c^n$

it is easy to see it is not in L . is the same.

x, y here is used to prove it is not only regular but also not context-free

it is required to show all cases. the number we pump

2) Assume v is in a 's, x is in a 's.

$u = a^i, v = a^j, w = a^k, x = a^l, y = a^{n-i-j-k-l} b^n c^n, j+l > 0$

let $r=2$ $uv^2wx^2y = a^{n+j+l} b^n c^n \notin L$ since $j+l > 0$

3) v, x are both in b 's. 4) v, x are both in c 's.

Similar with case 2), but we just pump out b 's and c 's.

5) v in b 's, x in c 's. similar with case 1). instead pumping b 's and c 's.

6) v in a 's x in c 's.

this case is invalid since $|vwx| > n$ so we don't need to consider.

7) v or x contains more than one letter:

$$v = a^i b^j \quad i, j > 0$$

uv^2wx^2y will contain letters that are out of order, certainly $uv^2wx^2y \notin L$
since there exist some cases does not satisfy the language.