(20pt) Show that the following problem is decidable:

Given $\Sigma = \{a, b\}$ and α a regular expression, is it true that $L(\alpha)$ contains only non-empty even-length strings in Σ^* and no string consisting only of b's?

3.

We'll show that the problem is decidable by defining a decision procedure to answer it:

1. From α, use **Thompson's Construction Algorithm** to build FSM M_α such that $L(M_\alpha) = L(\alpha)$
2. Let regular expression β = (bb)*(ba+a(b+a))((b+a)(b+a))*). β is such that L(β) exclusively accepts all non-empty, even-length strings in Σ^* and no string consisting only of b's.
3. From β, use **Thompson's Construction Algorithm** to build FSM M_β such that $L(M_\beta) = L(\beta)$
4. Construct M_A to accept $L(M_\alpha) - L(M_\beta)$ **(Difference Closure)**
5. Return **emptyFSM(M_A)**

Algorithms and closure properties used:

- Thompson' Construction to create FSM from a regular expression.
- Difference Closure to create a set which is Set α without all the non-empty, even-length strings in Σ^* that have at least one a
- emptyFSM algorithm. This will return true if the set is empty, and false otherwise. This makes sense because the set would only have elements in it that are not in M_β which means it has elements that don't make the requirements.