6.4b

6. 1 $(y)(Fy \supset Lry)$ Valid
2 $\sim(\exists x) Lxr$
3 $Fj$
$\therefore (\exists x)(Lrx \cdot \sim Lxr)$
4 asm: $\sim(\exists x)(Lrx \cdot \sim Lxr)$
5 $\therefore Fj \supset Lrj$ [from 1, 3]
6 $\therefore Lrj$ [from 3, 5].
7 $\therefore (x) \sim(Lrx \cdot \sim Lxr)$ [from 4].
8 $\therefore \sim(Lrj \cdot \sim Ljr)$ [from 3, 7]
9 $\therefore Ljr$ [from 6, 8].
10 $\therefore (x) \sim Lxr$ [from 2]
11 $\therefore \sim Ljr$ [from 3, 10].
12 $\therefore (\exists x)(Lrx \cdot \sim Lxr)$ [from 4; 9 contradicts 11].

9 1. $\sim(x)(\exists y) Lxy$ Valid
$\therefore \sim(x) Lxu$
2 asm: $(x) Lxu$
3 $\therefore (\exists x)(\exists y) \sim Lxy$ [from 1]
4 $\therefore (\exists x) \sim Lxu$ [from 3]
5 $\therefore \sim Lau$ [from 4].
6 $\therefore Lau$ [from 2]
7 $\therefore \sim(x) Lxu$ [from 2; 5 contradicts 6].

11. 1. $(y)(Say \equiv (y) \sim Syy$ Invalid
2. R Saa

16. 1. $(x)(\exists y) Lxy$ Valid.
$\therefore (\exists x) Lxx$
2 asm: $\sim(\exists x) Lxx$
3 $\therefore (x) \sim Lxx$ [from 2]
4 $\therefore (x) Lxx$ [from 1]
5 $\therefore (\exists x) Lxx$ [from 2; 3 contradicts 4]

21. 1 $Gj$ Valid.
2 $Ek$
3 $Cjk$
4 $\sim Pjk$
$\therefore \sim(x)(Gx \supset ((y) Ey \cdot (xy) \supset Pxy))$
5 asm: $(x)(Gx \supset ((y) Ey \cdot (xy) \supset Pxy))$
6 $\therefore (\sim(x)(Gx) \vee ((y) Ey \cdot (xy) \supset Pxy)$ [from 5].
7 $\therefore ((\exists x) \sim Gx) \vee ((y) Ey \cdot (xy) \supset Pxy)$ [from 6].
8 $\therefore (\sim Gj) \vee ((y) Ey \cdot (jy) \supset Pjy)$ [from 7].
9 $\therefore ((y) Ey \cdot (jy) \supset Pjy)$ [from 1, 8].
10 $\therefore \sim((y) Ey \cdot (jy) \vee Pjy$ [from 9]
11 $\therefore (\sim(y) Ey \vee \sim(jy)) \vee Pjy$ [from 10].
12 $\therefore ((\exists y) \sim Ey \vee \sim(jy)) \vee Pjy$ [from 11].
13 $\therefore (\sim Ek \vee \sim(jk)) \vee Pjk$ [from 12]
14 $\therefore Pjk$ [from 2, 3, 13]
15 $\therefore (x)(Gx \supset ((y) Ey \cdot (xy) \supset Pxy))$ [from 5; 4 contradicts 14].