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Assignment 3

Due: Thursday Oct. 22, 2020 before 6:30 PM to be uploaded in Gradescope as a single pdf file.

Please write your name and student number on your submission. Justify each step carefully using previously proven results either in the zyBook or the videos/zoom sessions. When in doubt prove the statement you are going to use. Solutions are graded for correctness as well as clarity.

Exercise 1 (10 point)**.** Let x, y be any non-zero real numbers. Given

$$\frac{x}{y} + \frac{y}{x} = 2$$

Show that $x = y$ by "direct proof".

Exercise 2 (15 point)**.** Prove the following proposition by contraposition:
For every positive integer a, if a is strictly greater than 1 then a does not divide $2a^2 + a + 1$.

$\forall a$, $a > 1$ $2a^2+a+1 \% a != 0$

Exercise 3 (15 points)**.** Prove that $\sqrt[3]{2}$ is not a rational number by contradiction. *Hint:* You may use the fact that if a^3 is an even number then a must also be even, with proper justification.

Exercise 4 (10 point)**.** Solve the following equations for real numbers

$$|x^2 - 1| = |x - 1|$$

using case analysis.

1. $\because x \neq 0, y \neq 0.$

$\frac{x}{y} + \frac{y}{x} = 2$

$x^2 + y^2 - 2xy = 0$

$(x-y)^2 = 0$

$x = y.$

2. directly proof:

For every a greater than 1 and divided $2a^2+a+1$,

there's no positive integer a.

Then, find if there's a meet requirements.

$\frac{2a^2+a+1}{a} = 2a+1+\frac{1}{a}.$

if $a>1$, then $\frac{1}{a}$ cannot be an interger.

Then the proposition is true.

3. assume that $\sqrt[3]{2}$ is a rational number.

then there must be interger. a, b which are relative prime.

$2 = \frac{a^3}{b^3}$ $\therefore$ a is an even number.

then $a^3 = 2b^3 = b^3 + b^3$.

however, a is an even number to meet the requirement.

also, b is an even number.

$\therefore$ the function $2 = \frac{a^3}{b^3}$ is unreachable.

$\therefore \sqrt[3]{2}$ is an irrational number.

4. $|x^2-1| = |x-1|$.

$|(x+1)(x-1)| = |x-1|$

if $x=1$: infinity solutions.

if $x \neq 1$: $x+1 = 1$

$x = 0$.

$\therefore$ $x=1$ or $x=0$.