

Structural Induction:

$$x \neq \emptyset$$

Let X be a well-ordered set. Let $P(x)$ be a proposition

for $x \in X$. If:

1) $P(x_0)$ holds where x_0 is the least element of X

2) $\left\{ \begin{array}{l} \text{for all } y < x \\ \text{if } P(y) \text{ is true} \end{array} \right\} \Rightarrow P(x) \text{ is true.}$

then $P(x)$ is true for all $x \in X$.

e.g. $\mathbb{N} \times \mathbb{N} = \{(a, b)\}$. $(a, b) < (x, y)$ if $a < x$
or $a = x$ $b < y$.

e.g. Let T be a binary tree with height h and n nodes

Show that $n \leq 2^{h+1} - 1$