Puzzle: $\exists x \in \mathbb{R}\ \sqrt{x+2}+2=0$.

proof: $\sqrt{x+2}=-2$

$(\sqrt{x+2})^2=(-2)^2$

$x+2=4$ ⇐ This step is incorrect.

$x=2$

Ex 3.6.3. if $x\in\mathbb{R}\wedge x\neq 2$, then there exist an unique $y\in\mathbb{R}$ such that $\frac{2y}{y+1}=x$

Proof: Let $x\in\mathbb{R}$ and assume $x\neq 2$. Let $y=\frac{x}{2-x}$, which is well-defined since $x\neq 2$. Then:

$$\frac{2y}{y+1}=\frac{2\frac{x}{2-x}}{\frac{x}{2-x}+1}=\frac{2x}{x+2-x}=x.$$ (exist).

To see that the result is unique. suppose $\frac{2z}{z+1}=x$. Then $2z=x(z+1)$ and $z=\frac{x}{2-x}=y$. ← (unique).

Exercise 8: Let U be a set. $\forall A\subseteq U$, there is a unique $B\subseteq U$ such that $\forall C\subseteq U,\ C\setminus A=C\cap B$.

Given

$A\subseteq U$

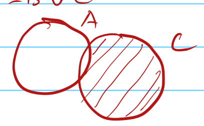
$B\subseteq U$

Goal

$\forall A\ \exists! B\ \forall C.\ C\setminus A=C\cap B$.

$\exists! B\ \forall C\ C\setminus A=C\cap B$.

$\exists$ $\exists B\ \forall C$ ......



$B=U\setminus A$.

!

$\forall C\ (C\setminus A=C\cap(U\setminus A))$

$C\subseteq U$

$C\setminus A=C\cap(U\setminus A)$

$C\setminus A\subseteq C\cap(U\setminus A)$

$C\cap(U\setminus A)\subseteq C\setminus A$

these are easy to proof.

$A\subseteq U$

$\forall D\ (\forall C\ C\setminus A=C\cap D)\rightarrow D=(U\setminus A)$

$D=U\setminus A$

$\forall D\ (\forall C\ C\setminus A=C\cap D)$

$C=U$ $\forall D\ (U\setminus A=U\cap D)$

$U\cap D=D$

Proof: let U be a set and let $A \subseteq U$.

Existence: Take $B = U \setminus A$. let arbitrary $C \subseteq U$. let $x = C \setminus A$.
so $x \in C \wedge x \notin A$. Since $C \subseteq U$ $x \in U$. so $x \in U \wedge x \notin A$.
so $x \in U \setminus A$. Since $x \in C$. so $x \in C \cap (U/A)$. let $x \in C \cap (U \setminus A)$
then $x \in C \cap (U \setminus A)$, $x \in C$, $x \in U$, $x \notin A$, so $x \in C \setminus A$,
Therefore $C \setminus A = C \cap (U \setminus A)$ $\square$

Uniqueness, Suppose $D \subseteq U$, as the property, $C \setminus A = C \cap D$ for.
every $C \subseteq U$. Taking $C = U$, we get that $U \setminus A = U \cap D$..
Since $D \subseteq U$, we have $U \cap D = D$, so $D = U \setminus A = B$. $\square$

Ex 3.6.4. Suppose A, B, C are sets. $A \cap B \neq \emptyset$, $A \cap C \neq \emptyset$, A has exactly one elenent, prove $B \cap C \neq \emptyset$

Given	Goal
$A \cap B \neq \emptyset$	$\exists y \; y \in B \cap C$
$A \cap C \neq \emptyset$	
$\exists! x \; x \in A$.	
$\exists x \; x \in A \cap B$	
$\exists x \; x \in A \cap C$	
$b \in B \cap A$	
$c \in C \cap A$	
$b = x$	
$c = x$	
$b = c = x$.	

Proof: Given A, B are not disjoint, there exist b that $b \in A$ and $b \in B$. Similarly, given A, C are not disjoint, there exist c such that $c \in A$ and $c \in C$. Since A has only one element, $b = c$. Therefore, $b \in C$ and $c \in B$. Since $b \in C$ and $b \in B$, B and C are not disjoint $\square$.