Lecture 8.

$\wedge \to$ false.

$\neg P :\Leftrightarrow P \to \text{false}$

$P \vee Q :\Leftrightarrow (P \to \text{false}) \to Q.$

$:\Leftrightarrow \neg(\neg P \wedge \neg Q)$

Like quantifier:

$\forall x \forall y\, P(x,y) :\Leftrightarrow \forall y \forall x\, P(x,y).$

$\exists x \exists y \qquad \exists y \exists x.$

$x \in A\ y \in B \qquad y \in B\ x \in A.$

But it doesn't work for mix case. e.g. $\forall y \exists x$

Free/Bound variable.

$\{x \cdots \mid \cdots\}$, $\forall x$, $\exists x$, $\sum_{x=0}^{\cdots}$ $\Rightarrow$ x bounds.

Any quantifier make a variable bounded.

§ 2.2. Cost.

Distrutive laws: $\forall x\,(P(x) \wedge Q(x)) \equiv \forall x\, P(x) \wedge \forall x\, Q(x).$

$\forall x\,(P(x) \vee Q(x)) \not\equiv \forall x\, P(x) \vee \forall x\, Q(x).$

e.g.

x can be different in these ~~two~~ cases.

$\exists x\,(P(x) \wedge Q(x)) \not\equiv \exists x_1 P(x_1) \wedge \exists x_2 Q(x_2).$

$\exists x\,(P(x) \vee Q(x)) \equiv \exists x\, P(x) \vee \exists x\, Q(x).$

e.g. $A = B$

$:\Leftrightarrow \forall x\,(x \in A \Leftrightarrow x \in B)$

$:\Leftrightarrow \forall x\,[(x \in A \to x \in B) \wedge (x \in B \to x \in A)].$

$\iff [\forall x (x \in A \to x \in B)] \wedge [\forall x (x \in B \to x \in A)]$.

$\iff \overset{\text{subset}}{A \subseteq B} \quad \wedge \quad B \subseteq A$.

e.g. $\mathcal{U} = \mathbb{N}$

m divides n, m/n

$\forall x \in n \quad \exists y \in m \quad y = xk$.

e.g. k is composite $\quad \exists m \exists n \, (k = mn \wedge m > 1 \wedge n > 1)$.

e.g. k is prime. $\quad k > 1 \wedge \neg (\qquad)$

e.g. d is the greatest common divider for m and n.

$(d \mid m/d \wedge n/d) \wedge \neg (c > d \mid m/c \wedge n/c)$.

e.g. $\mathcal{U} = \mathbb{R}$ · Every real number has a cube root.

$\forall x \exists y \quad x = y^3$.

Every real number has a unique cube root.

$(\forall x \exists y \quad x = y^3) \wedge (\neg \exists z \quad z \neq y \wedge x = z^3)$

$\forall x \exists y \, [(x = y^3) \wedge (\forall z \quad x = z^3 \to z = y)]$.

Shorthand: $\exists! y$ exist an unique number y.

$\equiv \exists y \wedge \neg \exists z \, (P(z) \wedge z \neq y)$

Every positive number has two square roots.

$\forall x \quad x > 0 \quad \exists y \exists z \quad x = y^2 = z^2 \wedge y \neq z$

§2.3 More operation done sets.

Power sets: Given a set A, the power sets is the set ~~at~~ of all subsets of A. $P(A) = \{B \mid B \subseteq A\}$.

e.g. $P\{\{2,3\}\} = \{\{2\}, \{3\}, \{2,3\}, \varnothing\}$.

$2 \notin P[(2,3)]$. ← int is not an element.

$\{2\} \in P[(2,3)]$. ← set.

$\{2\} \notin P[(2,3)]$. ← 2 is not an element in any of these sets.

$\{\{2\}\} \subseteq P[(2,3)]$.

$\left.\begin{array}{l} \emptyset \in P[(2,3)] \\ \emptyset \subseteq P[(2,3)]. \end{array}\right]$ special case

$P(\emptyset) = \{\emptyset\}$. the power set of an empty set is a set has only an empty set.

B is the element of P $\Rightarrow$ B is a subset of A.

If A has n element, then the power set of A has 2^n element.