Solutions (Predicate logic)

For questions 1-5 consider,

$I(x)$ = 'x is an Integer'

$E(x)$ = 'x is even'

$O(x)$ = 'x is odd'

1) $\exists x (I(x) \wedge E(x)) \wedge \exists x (I(x) \wedge O(x))$

2) $\forall x (I(x) \rightarrow E(x))$

3) $\forall x (I(x) \rightarrow (\neg E(x) \rightarrow O(x)))$

OR $\forall x (I(x) \wedge \neg E(x) \rightarrow O(x))$

4) $\exists x (I(x) \wedge O(x))$

5) $\forall x (E(x) \rightarrow I(x))$

6) Let

$S(x)$ = x is a sin

$L(x)$ = x is a form of lying

$\forall x (S(x) \rightarrow L(x))$

7) Let

$H(x)$ = x is happy

Define the constant : Jeff

$H(\text{Jeff})$

8) $D(x)$ = x is a dog

Define constants Tom, Jerry

$D(Tom) \wedge D(Jerry)$

9) $H(x, y)$ = x is happier than y

$S(x, y)$ = x is sadder than y

Define constants Jack, Tim, Bob

$H(Jack, Tim) \wedge S(Jack, Bob)$

10) $T(x)$ = x is trouble maker

$D(x, y)$ = x dislikes y

Define constants Paul, Ben

$D(Ben, Paul) \rightarrow T(Paul)$

12) <u>Proof:</u>

$\neg \forall x\, P(x)$ is true if and only if $\forall x\, P(x)$ is false

Note that $\forall x\, P(x)$ is false if and only if there exists an element in the domain for which $P(x)$ is false

But this holds if and only if there exists an element in the domain for which $\neg P(x)$ is true

The latter holds if and only if $\exists x \neg P(x)$ is true

$$\therefore \neg \forall x\, P(x) \equiv \exists x \neg P(x)$$

13) Proof:

Giving a counter example to the assertion that they have same truth values is enough to prove that they are not equivalent.

Let us assume the domain $D = \mathbb{N}$ (natural numbers)

$P(x)$ = 'x is even'

$Q(x)$ = 'x is odd'

$L.H.S = \forall x (P(x) \vee Q(x))$ (Every natural number is even or odd)

This is True

$R.H.S = \forall x P(x) \vee \forall x Q(x)$ (Every natural number is even or every natural number is odd)

This is False.

$\therefore \forall x (P(x) \vee Q(x)) \not\equiv \forall x P(x) \vee \forall x Q(x)$

14) Proof:

Giving a counter example to the assertion that they have same truth values is enough to prove that they are not equivalent.

Let us assume the domain $D = \mathbb{N}$

$P(x)$ = 'x is even'

$Q(x)$ = 'x is odd'

$R.H.S = (\exists x P(x) \wedge \exists x Q(x))$ (there exists a natural number which is even and there exists a natural number which is odd)

This is True.

$L \cdot H \cdot S = \exists x \, (P(x) \wedge Q(x))$ (there is a natural number which is even and odd)

this is false.

$$\therefore \exists x \, (P(x) \wedge Q(x)) \not\equiv (\exists x \, P(x) \wedge \exists x \, Q(x))$$