Math 2155, Fall 2022: Practice questions

The list of practice questions on the course OWL page is the main list. These are just a few supplementary questions for things that are not covered well in the text book.

I'm also giving more on Chapter 7, since this wasn't covered by homework.

Chapter 3:

Q1: Prove that there exists $y \in \mathbb{R}$ such that for every $x \in \mathbb{R}$, $(x-2)(x-3) = x^2 - y^2x + 6$. Is this y unique?

$\cancel{x^2} - 5x + \cancel{6} = \cancel{x^2} - y^2x + \cancel{6}$

$y = \pm\sqrt{5}$

X.

Q2: Prove that for every $x \in \mathbb{R}$, there exists a $y \in \mathbb{R}$ such that $y^2 + xy - 1 = 0$. (Hint: use the quadratic formula.)

$x = \frac{1-y^2}{y}$

Q3: Show that it is not true that for every $x \in \mathbb{R}$, there exists a $y \in \mathbb{R}$ such that $y^2 + xy + 1 = 0$.

pick $x=0$. $x = -\left(\frac{1+y^2}{y}\right)$. $\frac{1}{y} + y$

$x \in (-1,1)$.

Chapter 4:

Q4: For each of the following relations on the specified set, either prove that they are partial orders, or explain why they are not. Also prove that they are total orders, or explain why they are not.

	reflexive	symm	trans	anti-
A_1:	✓	✗	✓	
A_2:	✓	✗		
A_3:	✓			

(a) $A_1 = \{1, 2, 3\}$ and $R_1 = \{(1,1), (1,2), (2,2), (3,2), (3,3)\}$.

(b) $A_2 = \{1, 2, 3\}$ and $R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,3), (1,3)\}$.

(c) $A_3 = \mathbb{N}$ and $R_3 = \{(a, b) \in \mathbb{N} \times \mathbb{N} \mid a - b \leq 1\}$.

Q5: Define a relation S on $\mathbb{R}$ by xSy iff $|x| = |y|$.

(a): Prove that S is an equivalence relation. refl symm tran

(b): For each $x \in \mathbb{R}$, completely describe $[x]_S$, including saying how many elements are in the equivalence class. Then give a set $A \subseteq \mathbb{R}$ such that every equivalence class in $\mathbb{R}/S$ is of the form $[a]_S$ for a unique $a \in A$. Justify.

Q6: Consider the relation S on $\mathbb{R}$ defined by xSy if and only if $\exists a \in \mathbb{R}\,(a > 0 \wedge x = ay)$. Prove that S is an equivalence relation, and completely describe $\mathbb{R}/S$ and all of the equivalence classes.

Chapter 5:

Q7: Write down a function $f : \mathbb{R} \to \mathbb{R}$ which is one-to-one but not onto. Prove both claims.

Q8: Write down a function $f : \mathbb{R} \to \mathbb{R}$ which is onto but not one-to-one. Prove both claims. (Trickier. Hint: think of an odd-degree polynomial.)

Q9: What is the range of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 + 2$?

Q10: Let $A = \mathbb{R} \setminus \{5\}$. What is the range of the function $f : A \to \mathbb{R}$ defined by $f(x) = x/(5 - x)$?

Chapter 6:

The book has lots of great exercises!

Chapter 7:

Q11: Use the Euclidean algorithm to compute $\gcd(490, 35)$. Then factor both numbers into primes, and use the method from Section 7.3 of the text to compute the gcd.

Q12: Using the above, does $[35]_{490}$ have an inverse in $\mathbb{Z}/{\equiv_{490}}$?

Q13: Exactly which integers can be written as $s(35) + t(490)$ for $s, t \in \mathbb{Z}$?

Q14: Use the Euclidean algorithm to compute $\gcd(555, 101)$. Then factor both numbers into primes, and use the method from Section 7.3 of the text to compute the gcd.

Q15: Using the above, does $[101]_{555}$ have an inverse in $\mathbb{Z}/{\equiv_{555}}$?

Q16: Use the Euclidean algorithm to compute $\gcd(777, 180)$. Then factor both numbers into primes, and use the method from Section 7.3 of the text to compute the gcd.

Q17: List all of the divisors of $5^2 \cdot 11^3$. No need to simplify.

Q18: Compute the gcd of $7^3 \cdot 11^2 \cdot 13^3$ and $5^2 \cdot 7 \cdot 13^4$. No need to simplify.

Q19: Write down the addition and multiplication tables for $\mathbb{Z}/{\equiv_4}$.

$+$	$[0]_4$
$[0]_4$	
$[1]_4$	
$[2]_4$	
$[3]_4$	

Q20: In $\mathbb{Z}/{\equiv_6}$, which elements have multiplicative inverses? For each of them, find an inverse.

multiplication table value = 1

Q21: Prove that if $X \in \mathbb{Z}/{\equiv_3}$ and $X \neq [0]_3$, then $X^2 = [1]_3$. Note that there are only two elements to check! But this implies that if $k \in \mathbb{Z}$ and 3 does not divide k, then the remainder when k^2 is divided by 3 is 1.

$x = [1]_3$

$x = [2]_3$

Q22: Prove that there are no rational numbers x and y such that $x^2 + y^2 = 3$. (Tricky. Previous question might help.)

Q11: Use the Euclidean algorithm to compute gcd(490, 35). Then factor both numbers into primes, and use the method from Section 7.3 of the text to compute the gcd.

$490 \div 35 = 14 R 0$ $(35 \times 14 = 490)$.

~~35 = 5×7~~

$35 = 5 \times 7$

$490 = 5 \times 7 \times 2 \times 7$.

$\mathbb{Z}/\equiv_m$.

$a \equiv b \pmod{m}$

$\Rightarrow a \equiv_m b$

e.g. $5 \in \mathbb{Z}/m_3$.

e.g. $[2]_3 = \{b \in \mathbb{Z} \mid b \equiv_3 2\}$

$= \{b \in \mathbb{Z} \mid b \equiv 2 \pmod{3}\}$

$= \{2, 5, 8, 11 \dots\}$.

Q16: Use the Euclidean algorithm to compute gcd(777, 180). Then factor both numbers into primes, and use the method from Section 7.3 of the text to compute the gcd.

$777 \div 180 = 4 R 57$.

$180 \div 57 = 3 R 9$

$57 \div 9 = 6 R 3$

$9 \div 6 = 1 R 3$

$6 \div 3 = 2 R 0$.

$\gcd = 3$.

$777 = 7 \times 3 \times 37$

$180 = 3 \times 3 \times 5 \times 2 \times 2$.

$111 = 3 \times 37$