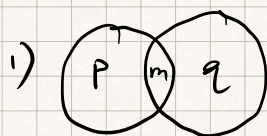Yulun Feng
251113989.

1)

divide A∪B into three parts,

$p = A \cap (\neg B)$ $q = (\neg A) \cap B.$

$m = A \cap B$

$A \triangle B = p \cup q$, $(A \cup B) \triangle (A \cap B)$

$= (p \cup m \cup q) \triangle m = p \cup q.$

$\therefore A \triangle B = (A \cup B) \triangle (A \cap B).$

2) Because there're only 2 sets, so we just need to
   determine there're how many conditions in the first set.

$S_n = C_n^1 + C_n^2 + \cdots + C_n^{n-1}$

$= (C_n^1 + \cdots + C_n^n) - C_n^n$

$= 2^{n-1} - 1.$

3) $a_n = \frac{1}{2} + \frac{1}{2\times3} + \cdots + \frac{1}{n(n+1)}$

$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} \cdots\cdots - \frac{1}{n+1}$

$= \frac{n}{n+1}.$

4) if n is odd, $t_n = C_n^1 \cdot 2^{n-1} + C_n^3 \cdot 2^{n-3} + \cdots + C_n^{\frac{n+1}{2}} \cdot 2^{\frac{n-1}{2}}$

$t_{n-1} = C_{n-1}^1 \cdot 2^{n-2} + C_{n-1}^3 \cdot 2^{n-4} + \cdots + C_{n-1}^{\frac{n-1}{2}} \cdot 2^{\frac{n-1}{2}}$

if n is even, $t_n = C_n^1 \cdot 2^{n-1} + C_n^3 \cdot 2^{n-3} + \cdots + C_n^{\frac{n}{2}} \cdot 2^{\frac{n}{2}}$

$t_{n-1} = C_{n-1}^1 \cdot 2^{n-2} + \cdots\cdots\cdots + C_{n-1}^{\frac{n}{2}} \cdot 2^{\frac{n-2}{2}}.$

$t_n - t_{n-1} = 3^{n-1}$

$t_n = t_{n-1} + 3^{n-1}.$

Bonus :

$$t_1 - t_0 = 1$$
$$t_2 - t_1 = 3$$
$$\vdots \qquad \vdots$$
$$t_n - t_{n-1} = 3^{n-1}$$

$$\therefore \; t_n = 1 + 3 + \cdots + 3^{n-1}$$
$$= 1 \cdot \frac{1 - 3^n}{1 - 3}$$
$$= \frac{3^n - 1}{2}$$