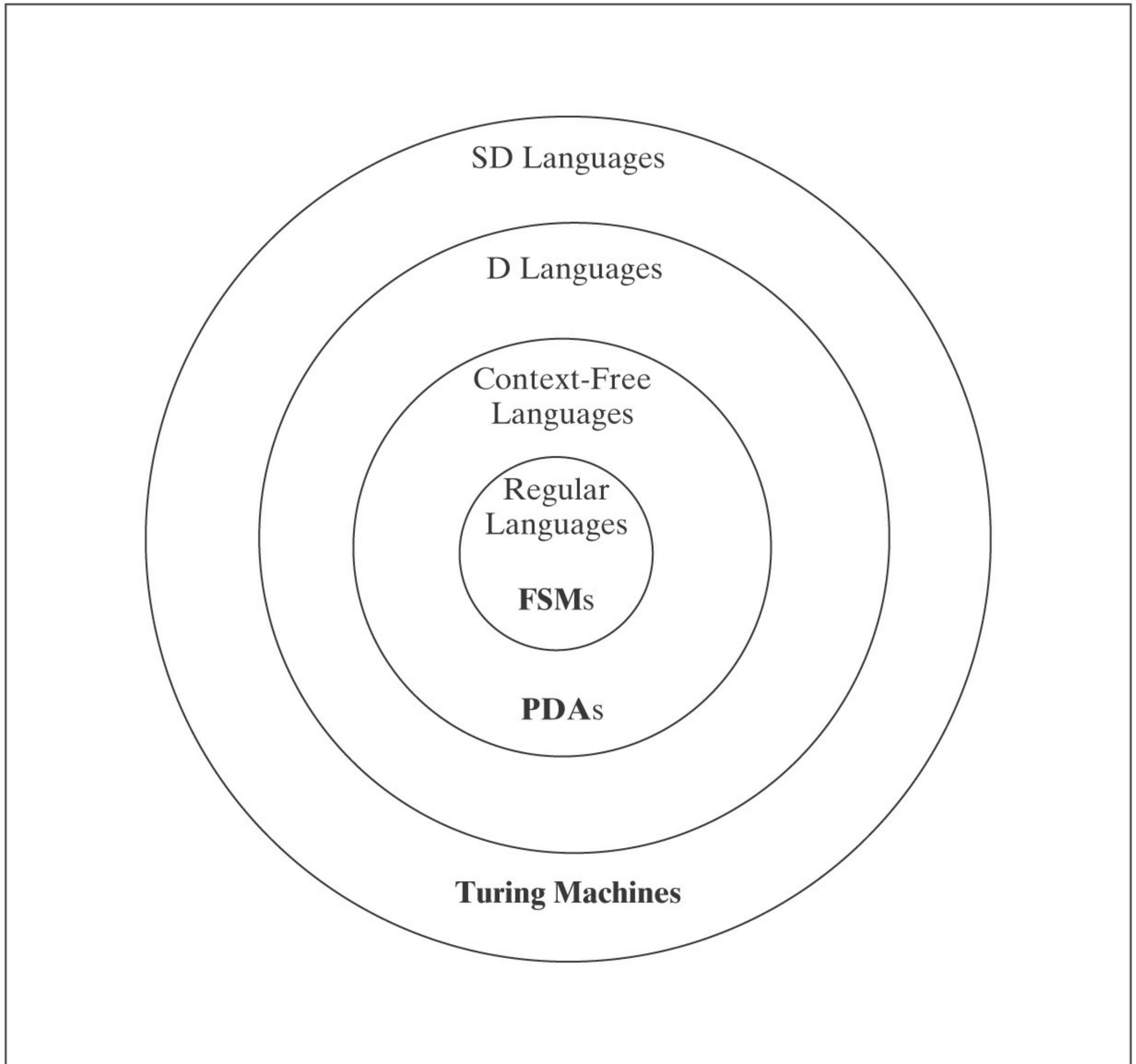
Context-Free Languages: Review

Chapter 16

Languages and Machines



Regular and CF Languages

Regular Languages

- regular exprs.
  or
- regular grammars
- = DFSMs
- recognize
- minimize FSMs

- closed under:
  - ♦ concatenation
  - ♦ union
  - ♦ Kleene star
  - ♦ complement
  - ♦ intersection
- pumping theorem
- D = ND

Context-Free Languages

- context-free grammars

- = NDPDAs
- parse
- find unambiguous grammars

- closed under:
  - ♦ concatenation
  - ♦ union
  - ♦ Kleene star
  - ♦ intersection w/ reg. langs
- pumping theorem
- D $\neq$ ND

Example

$\{\mathtt{b}\mathtt{a}^{m1}\mathtt{b}\mathtt{a}^{m2}\mathtt{b}\mathtt{a}^{m3}\ldots\ \mathtt{b}\mathtt{a}^{mn} : n \geq 2, m_1, m_2, \ldots, m_n \geq 0,$

and

$m_i \neq m_j$ for some $i, j\}$

A PDA:

A CFG:

Is L regular?

$$L = \{\mathtt{a}^i\mathtt{b}^j : j = 4i + 2\}$$

Is L Regular, Context Free, or Neither?

$L = \{x\, x_{neg} : x \in \{\texttt{0}, \texttt{1}\}^*\}$.

x_{neg} = x with all `0`'s replaced by `1`'s and `1`'s replaced by `0`'s.

Is *L* Regular, Context Free, or Neither?

$L = \{w \in \{\mathtt{0}, \mathtt{1}\}^* : \exists k$ (w is a binary encoding, leading zeros allowed, of $2k+1$)$\}$

Is L Regular, Context Free, or Neither?

$L = \{w \in \{\texttt{a}, \texttt{b}, \texttt{c}\}^*$: every $\texttt{a}$ has a matching $\texttt{b}$ and a matching $\texttt{c}$ somewhere in w (and no $\texttt{b}$ or $\texttt{c}$ is considered to match more than one $\texttt{a}$)$\}$

An Example

$L = \{\mathrm{a}^i\mathrm{b}^j\mathrm{c}^k : k \leq i \text{ or } k \leq j\}$

Construct a context-free grammar for L.

Functions on Languages

Again, let $L = \{\mathrm{a}^i\mathrm{b}^j\mathrm{c}^k : k \leq i \text{ or } k \leq j\}$.

Describe *precisely* the language $L' = maxstring(L)$, where:

$maxstring(L) =$

$$\{x: x \in L \text{ and } (\forall y \in \Sigma^* \ (y \neq \varepsilon) \rightarrow (xy \notin L))\}$$

Is $L' = maxstring(L)$ context free?

Are the context-free languages closed under *maxstring*?

Regular, Context Free or Neither?

$L_1 = \{\mathtt{a}^n\mathtt{b}^m\mathtt{c}^k : m \leq min(n, k)\}$.

$L_2 = \{\mathtt{a}^n\mathtt{b}^m\mathtt{c}^k : n = m + k \text{ or } m = n + k\}$.