

Tutorial on resolution in predicate logic

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Example 1

- ① All hounds howl at night.
 - ② Anyone who has any cats will not have any mice.
 - ③ Light sleepers do not have anything which howls at night.
 - ④ John has a cat or a hound.
- ∴ If John is a light sleeper, then John does not have any mice.

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① $\forall x(Hound(x) \rightarrow Howl(x))$

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- ② $\forall x\forall y((Have(x, y) \cdot Cat(y))$

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④ $\exists x (Have(John, x) \cdot (Cat(x) \vee Hound(x)))$

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 $\{Have(John, a)\}, \{Cat(a), House(a)\}$

Example 1

$$④ \exists x (Have(John, x) \cdot (Cat(x) \vee Hound(x)))$$

$$Have(John, a) \cdot (Cat(a) \vee House(a))$$

$$\{Have(John, a)\}, \{Cat(a), House(a)\}$$

$$\therefore \neg(Ls(John) \rightarrow \neg \exists z (Have(John, z) \cdot Mouse(z)))$$

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$$④ \exists x (Have(John, x) \cdot (Cat(x) \vee Hound(x)))$$

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$$Ls(John) \cdot \exists z (Have(John, z) \cdot Mouse(z))$$

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$$④ \quad \exists x (Have(John, x) \cdot (Cat(x) \vee Hound(x)))$$

$$Have(John, a) \cdot (Cat(a) \vee House(a))$$

$$\{Have(John, a)\}, \{Cat(a), House(a)\}$$

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$$\neg (\neg Ls(John) \vee \neg \exists z (Have(John, z) \cdot Mouse(z)))$$

$$Ls(John) \cdot \exists z (Have(John, z) \cdot Mouse(z))$$

$$Ls(John) \cdot Have(John, b) \cdot Mouse(b)$$

Example 1

$$4 \quad \exists x (Have(John, x) \cdot (Cat(x) \vee Hound(x)))$$

$$Have(John, a) \cdot (Cat(a) \vee House(a))$$

$$\{Have(John, a)\}, \{Cat(a), House(a)\}$$

$$\therefore \neg(Ls(John) \rightarrow \neg \exists z (Have(John, z) \cdot Mouse(z)))$$

$$\neg(\neg Ls(John) \vee \neg \exists z (Have(John, z) \cdot Mouse(z)))$$

$$Ls(John) \cdot \exists z (Have(John, z) \cdot Mouse(z))$$

$$Ls(John) \cdot Have(John, b) \cdot Mouse(b)$$

$$\{Ls(John)\}, \{Have(John, b)\}, \{Mouse(b)\}$$

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$$4 \quad \exists x (Have(John, x) \cdot (Cat(x) \vee Hound(x)))$$

$$Have(John, a) \cdot (Cat(a) \vee House(a))$$

$$\{Have(John, a)\}, \{Cat(a), House(a)\}$$

$$\therefore \neg(Ls(John) \rightarrow \neg \exists z (Have(John, z) \cdot Mouse(z)))$$

$$\neg(\neg Ls(John) \vee \neg \exists z (Have(John, z) \cdot Mouse(z)))$$

$$Ls(John) \cdot \exists z (Have(John, z) \cdot Mouse(z))$$

$$Ls(John) \cdot Have(John, b) \cdot Mouse(b)$$

$$\{Ls(John)\}, \{Have(John, b)\}, \{Mouse(b)\}$$

Example 1

- ① $\{\neg Hound(x), Howl(x)\}$
- ② $\{\neg Have(x, y), \neg Cat(y), \neg Have(x, z), \neg Mouse(z)\}$
- ③ $\{\neg Ls(x), \neg Have(x, y), \neg Howl(y)\}$
- ④ $\{Have(John, a)\}$
- ⑤ $\{Cat(a), Hound(a)\}$
- ⑥ $\{Ls(John)\}$
- ⑦ $\{Have(John, b)\}$
- ⑧ $\{Mouse(b)\}$

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- ⑥ $\{Ls(John)\}$
- ⑦ $\{Have(John, b)\}$
- ⑧ $\{Mouse(b)\}$
- ⑨ $\{Cat(a), Howl(a)\} [1,5]$

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- ④ $\{Have(John, a)\}$
- ⑤ $\{Cat(a), Hound(a)\}$
- ⑥ $\{Ls(John)\}$
- ⑦ $\{Have(John, b)\}$
- ⑧ $\{Mouse(b)\}$
- ⑨ $\{Cat(a), Howl(a)\} [1,5]$
- ⑩ $\{\neg Have(x, y), \neg Cat(y), \neg Have(x, b)\} [2,8]$

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- ⑨ $\{Cat(a), Howl(a)\} [1,5]$
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- ⑪ $\{\neg Have(John, y), \neg Cat(y)\} [10,7]$

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- ⑩ $\{\neg Have(x, y), \neg Cat(y), \neg Have(x, b)\} [2,8]$
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- ④ $\{Have(John, a)\}$
- ⑤ $\{Cat(a), Hound(a)\}$
- ⑥ $\{Ls(John)\}$
- ⑦ $\{Have(John, b)\}$
- ⑧ $\{Mouse(b)\}$
- ⑨ $\{Cat(a), Howl(a)\} [1,5]$
- ⑩ $\{\neg Have(x, y), \neg Cat(y), \neg Have(x, b)\} [2,8]$
- ⑪ $\{\neg Have(John, y), \neg Cat(y)\} [10,7]$
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- ⑬ $\{Howl(a)\} [4,12]$

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- ② $\{\neg Have(x, y), \neg Cat(y), \neg Have(x, z), \neg Mouse(z)\}$
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- ④ $\{Have(John, a)\}$
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- ⑥ $\{Ls(John)\}$
- ⑦ $\{Have(John, b)\}$
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- ⑨ $\{Cat(a), Howl(a)\} [1,5]$
- ⑩ $\{\neg Have(x, y), \neg Cat(y), \neg Have(x, b)\} [2,8]$
- ⑪ $\{\neg Have(John, y), \neg Cat(y)\} [10,7]$
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- ⑫ $\{\neg Have(John, a), Howl(a)\} [9,11]$
- ⑬ $\{Howl(a)\} [4,12]$
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- ⑮ $\{\neg Ls(John)\} [4,14]$

Example 1

- ① $\{\neg Hound(x), Howl(x)\}$
- ② $\{\neg Have(x, y), \neg Cat(y), \neg Have(x, z), \neg Mouse(z)\}$
- ③ $\{\neg Ls(x), \neg Have(x, y), \neg Howl(y)\}$
- ④ $\{Have(John, a)\}$
- ⑤ $\{Cat(a), Hound(a)\}$
- ⑥ $\{Ls(John)\}$
- ⑦ $\{Have(John, b)\}$
- ⑧ $\{Mouse(b)\}$
- ⑨ $\{Cat(a), Howl(a)\} [1,5]$
- ⑩ $\{\neg Have(x, y), \neg Cat(y), \neg Have(x, b)\} [2,8]$
- ⑪ $\{\neg Have(John, y), \neg Cat(y)\} [10,7]$
- ⑫ $\{\neg Have(John, a), Howl(a)\} [9,11]$
- ⑬ $\{Howl(a)\} [4,12]$
- ⑭ $\{\neg Ls(x), \neg Have(x, a)\} [3,13]$
- ⑮ $\{\neg Ls(John)\} [4,14]$
- ⑯ $\{\} [6,15]$

Example 1

- ① $\{\neg Hound(x), Howl(x)\}$
- ② $\{\neg Have(x, y), \neg Cat(y), \neg Have(x, z), \neg Mouse(z)\}$
- ③ $\{\neg Ls(x), \neg Have(x, y), \neg Howl(y)\}$
- ④ $\{Have(John, a)\}$
- ⑤ $\{Cat(a), Hound(a)\}$
- ⑥ $\{Ls(John)\}$
- ⑦ $\{Have(John, b)\}$
- ⑧ $\{Mouse(b)\}$
- ⑨ $\{Cat(a), Howl(a)\} [1,5]$
- ⑩ $\{\neg Have(x, y), \neg Cat(y), \neg Have(x, b)\} [2,8]$
- ⑪ $\{\neg Have(John, y), \neg Cat(y)\} [10,7]$
- ⑫ $\{\neg Have(John, a), Howl(a)\} [9,11]$
- ⑬ $\{Howl(a)\} [4,12]$
- ⑭ $\{\neg Ls(x), \neg Have(x, a)\} [3,13]$
- ⑮ $\{\neg Ls(John)\} [4,14]$
- ⑯ $\{\} [6,15]$

Valid

Resolution in predicate

Example 2

- ① Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
 - ② Every dog chases some rabbit.
 - ③ Mary buys carrots by the bushel.
 - ④ Anyone who owns a rabbit hates anything that chases any rabbit.
 - ⑤ John owns a dog.
 - ⑥ Someone who hates something owned by another person will not date that person.
- ∴ If Mary does not own a grocery store, she will not date John.

Resolution in predicate

Example 2

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 - ⑥ Someone who hates something owned by another person will not date that person.
- ∴ If Mary does not own a grocery store, she will not date John.

① $\forall x (Buy(x) \rightarrow \exists y (Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$

Resolution in predicate

Example 2

- ① Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
 - ② Every dog chases some rabbit.
 - ③ Mary buys carrots by the bushel.
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 - ⑥ Someone who hates something owned by another person will not date that person.
- ∴ If Mary does not own a grocery store, she will not date John.

- ① $\forall x (Buy(x) \rightarrow \exists y (Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
- ② $\forall x (Dog(x) \rightarrow \exists y (Rabbit(y) \cdot Chase(x, y)))$

Resolution in predicate

Example 2

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 - ② Every dog chases some rabbit.
 - ③ Mary buys carrots by the bushel.
 - ④ Anyone who owns a rabbit hates anything that chases any rabbit.
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 - ⑥ Someone who hates something owned by another person will not date that person.
- ∴ If Mary does not own a grocery store, she will not date John.

- ① $\forall x (Buy(x) \rightarrow \exists y (Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
- ② $\forall x (Dog(x) \rightarrow \exists y (Rabbit(y) \cdot Chase(x, y)))$
- ③ $Buy(mary)$

Resolution in predicate

Example 2

- ① Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
 - ② Every dog chases some rabbit.
 - ③ Mary buys carrots by the bushel.
 - ④ Anyone who owns a rabbit hates anything that chases any rabbit.
 - ⑤ John owns a dog.
 - ⑥ Someone who hates something owned by another person will not date that person.
- ∴ If Mary does not own a grocery store, she will not date John.

- ① $\forall x (Buy(x) \rightarrow \exists y (Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
- ② $\forall x (Dog(x) \rightarrow \exists y (Rabbit(y) \cdot Chase(x, y)))$
- ③ $Buy(mary)$
- ④ $\forall x \forall y (Owns(x, y) \cdot Rabbit(y) \rightarrow \forall z \forall w (Rabbit(w) \cdot Chase(z, w) \rightarrow Hates(x, z)))$

Resolution in predicate

Example 2

- ① Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
 - ② Every dog chases some rabbit.
 - ③ Mary buys carrots by the bushel.
 - ④ Anyone who owns a rabbit hates anything that chases any rabbit.
 - ⑤ John owns a dog.
 - ⑥ Someone who hates something owned by another person will not date that person.
- ∴ If Mary does not own a grocery store, she will not date John.

- ① $\forall x (Buy(x) \rightarrow \exists y (Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
- ② $\forall x (Dog(x) \rightarrow \exists y (Rabbit(y) \cdot Chase(x, y)))$
- ③ $Buy(mary)$
- ④ $\forall x \forall y (Owns(x, y) \cdot Rabbit(y) \rightarrow \forall z \forall w (Rabbit(w) \cdot Chase(z, w) \rightarrow Hates(x, z)))$
- ⑤ $\exists x (Dog(x) \cdot Owns(john, x))$

Resolution in predicate

Example 2

- ① Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
 - ② Every dog chases some rabbit.
 - ③ Mary buys carrots by the bushel.
 - ④ Anyone who owns a rabbit hates anything that chases any rabbit.
 - ⑤ John owns a dog.
 - ⑥ Someone who hates something owned by another person will not date that person.
- ∴ If Mary does not own a grocery store, she will not date John.

- ① $\forall x (Buy(x) \rightarrow \exists y (Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
- ② $\forall x (Dog(x) \rightarrow \exists y (Rabbit(y) \cdot Chase(x, y)))$
- ③ $Buy(mary)$
- ④ $\forall x \forall y (Owns(x, y) \cdot Rabbit(y) \rightarrow \forall z \forall w (Rabbit(w) \cdot Chase(z, w) \rightarrow Hates(x, z)))$
- ⑤ $\exists x (Dog(x) \cdot Owns(john, x))$
- ⑥ $\forall x \forall y \forall z (Owns(y, z) \cdot Hates(x, z) \rightarrow \neg Date(x, y))$

Resolution in predicate

Example 2

- ① Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
 - ② Every dog chases some rabbit.
 - ③ Mary buys carrots by the bushel.
 - ④ Anyone who owns a rabbit hates anything that chases any rabbit.
 - ⑤ John owns a dog.
 - ⑥ Someone who hates something owned by another person will not date that person.
- ∴ If Mary does not own a grocery store, she will not date John.

- ① $\forall x (Buy(x) \rightarrow \exists y (Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 - ② $\forall x (Dog(x) \rightarrow \exists y (Rabbit(y) \cdot Chase(x, y)))$
 - ③ $Buy(mary)$
 - ④ $\forall x \forall y (Owns(x, y) \cdot Rabbit(y) \rightarrow \forall z \forall w (Rabbit(w) \cdot Chase(z, w) \rightarrow Hates(x, z)))$
 - ⑤ $\exists x (Dog(x) \cdot Owns(john, x))$
 - ⑥ $\forall x \forall y \forall z (Owns(y, z) \cdot Hates(x, z) \rightarrow \neg Date(x, y))$
- ∴ $((\neg \exists x (Grocery(x) \cdot Owns(mary, x))) \rightarrow \neg Date(mary, john))$

Example 2

① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$

Example 2

- 1 $\forall x (Buy(x) \rightarrow \exists y (Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x(Dog(x) \rightarrow \exists y(Rabbit(y) \cdot Chase(x, y)))$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x(Dog(x) \rightarrow \exists y(Rabbit(y) \cdot Chase(x, y)))$
 $Dog(x) \rightarrow (Rabbit(b) \cdot Chase(x, b))$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x(Dog(x) \rightarrow \exists y(Rabbit(y) \cdot Chase(x, y)))$
 $Dog(x) \rightarrow (Rabbit(b) \cdot Chase(x, b))$
 $\neg Dog(x) \vee (Rabbit(b) \cdot Chase(x, b))$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x(Dog(x) \rightarrow \exists y(Rabbit(y) \cdot Chase(x, y)))$
 $Dog(x) \rightarrow (Rabbit(b) \cdot Chase(x, b))$
 $\neg Dog(x) \vee (Rabbit(b) \cdot Chase(x, b))$
 $(\neg Dog(x) \vee Rabbit(b)) \cdot (\neg Dog(x) \vee Chase(x, b))$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x(Dog(x) \rightarrow \exists y(Rabbit(y) \cdot Chase(x, y)))$
 $Dog(x) \rightarrow (Rabbit(b) \cdot Chase(x, b))$
 $\neg Dog(x) \vee (Rabbit(b) \cdot Chase(x, b))$
 $(\neg Dog(x) \vee Rabbit(b)) \cdot (\neg Dog(x) \vee Chase(x, b))$
 $\{\neg Dog(x), Rabbit(b)\}, \{\neg Dog(x), Chase(x, b)\}$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x(Dog(x) \rightarrow \exists y(Rabbit(y) \cdot Chase(x, y)))$
 $Dog(x) \rightarrow (Rabbit(b) \cdot Chase(x, b))$
 $\neg Dog(x) \vee (Rabbit(b) \cdot Chase(x, b))$
 $(\neg Dog(x) \vee Rabbit(b)) \cdot (\neg Dog(x) \vee Chase(x, b))$
 $\{\neg Dog(x), Rabbit(b)\}, \{\neg Dog(x), Chase(x, b)\}$
- ③ $Buy(mary)$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x(Dog(x) \rightarrow \exists y(Rabbit(y) \cdot Chase(x, y)))$
 $Dog(x) \rightarrow (Rabbit(b) \cdot Chase(x, b))$
 $\neg Dog(x) \vee (Rabbit(b) \cdot Chase(x, b))$
 $(\neg Dog(x) \vee Rabbit(b)) \cdot (\neg Dog(x) \vee Chase(x, b))$
 $\{\neg Dog(x), Rabbit(b)\}, \{\neg Dog(x), Chase(x, b)\}$
- ③ $Buy(mary)$
 $\{Buy(mary)\}$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x(Dog(x) \rightarrow \exists y(Rabbit(y) \cdot Chase(x, y)))$
 $Dog(x) \rightarrow (Rabbit(b) \cdot Chase(x, b))$
 $\neg Dog(x) \vee (Rabbit(b) \cdot Chase(x, b))$
 $(\neg Dog(x) \vee Rabbit(b)) \cdot (\neg Dog(x) \vee Chase(x, b))$
 $\{\neg Dog(x), Rabbit(b)\}, \{\neg Dog(x), Chase(x, b)\}$
- ③ $Buy(mary)$
 $\{Buy(mary)\}$
- ④ $\forall x \forall y (Owns(x, y) \cdot Rabbit(y) \rightarrow \forall z \forall w (Rabbit(w) \cdot Chase(z, w) \rightarrow Hates(x, z)))$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x(Dog(x) \rightarrow \exists y(Rabbit(y) \cdot Chase(x, y)))$
 $Dog(x) \rightarrow (Rabbit(b) \cdot Chase(x, b))$
 $\neg Dog(x) \vee (Rabbit(b) \cdot Chase(x, b))$
 $(\neg Dog(x) \vee Rabbit(b)) \cdot (\neg Dog(x) \vee Chase(x, b))$
 $\{\neg Dog(x), Rabbit(b)\}, \{\neg Dog(x), Chase(x, b)\}$
- ③ $Buy(mary)$
 $\{Buy(mary)\}$
- ④ $\forall x \forall y (Owns(x, y) \cdot Rabbit(y) \rightarrow \forall z \forall w (Rabbit(w) \cdot Chase(z, w) \rightarrow Hates(x, z)))$
 $\neg (Owns(x, y) \cdot Rabbit(y)) \vee (\neg (Rabbit(w) \cdot Chase(z, w)) \vee Hates(x, z))$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x(Dog(x) \rightarrow \exists y(Rabbit(y) \cdot Chase(x, y)))$
 $Dog(x) \rightarrow (Rabbit(b) \cdot Chase(x, b))$
 $\neg Dog(x) \vee (Rabbit(b) \cdot Chase(x, b))$
 $(\neg Dog(x) \vee Rabbit(b)) \cdot (\neg Dog(x) \vee Chase(x, b))$
 $\{\neg Dog(x), Rabbit(b)\}, \{\neg Dog(x), Chase(x, b)\}$
- ③ $Buy(mary)$
 $\{Buy(mary)\}$
- ④ $\forall x \forall y (Owns(x, y) \cdot Rabbit(y) \rightarrow \forall z \forall w (Rabbit(w) \cdot Chase(z, w) \rightarrow Hates(x, z)))$
 $\neg (Owns(x, y) \cdot Rabbit(y)) \vee (\neg (Rabbit(w) \cdot Chase(z, w)) \vee Hates(x, z))$
 $\neg Owns(x, y) \vee \neg Rabbit(y) \vee \neg Rabbit(w) \vee \neg Chase(z, w) \vee Hates(x, z)$

Example 2

- ① $\forall x (Buy(x) \rightarrow \exists y (Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x (Dog(x) \rightarrow \exists y (Rabbit(y) \cdot Chase(x, y)))$
 $Dog(x) \rightarrow (Rabbit(b) \cdot Chase(x, b))$
 $\neg Dog(x) \vee (Rabbit(b) \cdot Chase(x, b))$
 $(\neg Dog(x) \vee Rabbit(b)) \cdot (\neg Dog(x) \vee Chase(x, b))$
 $\{\neg Dog(x), Rabbit(b)\}, \{\neg Dog(x), Chase(x, b)\}$
- ③ $Buy(mary)$
 $\{Buy(mary)\}$
- ④ $\forall x \forall y (Owns(x, y) \cdot Rabbit(y) \rightarrow \forall z \forall w (Rabbit(w) \cdot Chase(z, w) \rightarrow Hates(x, z)))$
 $\neg (Owns(x, y) \cdot Rabbit(y)) \vee (\neg (Rabbit(w) \cdot Chase(z, w)) \vee Hates(x, z))$
 $\neg Owns(x, y) \vee \neg Rabbit(y) \vee \neg Rabbit(w) \vee \neg Chase(z, w) \vee Hates(x, z)$
 $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$

Example 2

- ① $\forall x(Buy(x) \rightarrow \exists y(Owns(x, y) \cdot (Rabbit(y) \vee Grocery(y))))$
 $\neg Buy(x) \vee (Owns(x, a) \cdot (Rabbit(a) \vee Grocery(a)))$
 $(\neg Buy(x) \vee Owns(x, a)) \cdot (\neg Buy(x) \vee Rabbit(a) \vee Grocery(a))$
 $\{\neg Buy(x), Owns(x, a)\}, \{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ② $\forall x(Dog(x) \rightarrow \exists y(Rabbit(y) \cdot Chase(x, y)))$
 $Dog(x) \rightarrow (Rabbit(b) \cdot Chase(x, b))$
 $\neg Dog(x) \vee (Rabbit(b) \cdot Chase(x, b))$
 $(\neg Dog(x) \vee Rabbit(b)) \cdot (\neg Dog(x) \vee Chase(x, b))$
 $\{\neg Dog(x), Rabbit(b)\}, \{\neg Dog(x), Chase(x, b)\}$
- ③ $Buy(mary)$
 $\{Buy(mary)\}$
- ④ $\forall x \forall y (Owns(x, y) \cdot Rabbit(y) \rightarrow \forall z \forall w (Rabbit(w) \cdot Chase(z, w) \rightarrow Hates(x, z)))$
 $\neg (Owns(x, y) \cdot Rabbit(y)) \vee (\neg (Rabbit(w) \cdot Chase(z, w)) \vee Hates(x, z))$
 $\neg Owns(x, y) \vee \neg Rabbit(y) \vee \neg Rabbit(w) \vee \neg Chase(z, w) \vee Hates(x, z)$
 $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$

Example 2

⑤ $\exists x(Dog(x) \cdot Owns(John, x))$

Example 2

- ⑤ $\exists x(Dog(x) \cdot Owns(John, x))$
 $Dog(c) \cdot Owns(john, c)$

Example 2

- ⑤ $\exists x(Dog(x) \cdot Owns(John, x))$
 $Dog(c) \cdot Owns(john, c)$
 $\{Dog(c)\}, \{Owns(john, c)\}$

Example 2

- ⑤ $\exists x(Dog(x) \cdot Owns(John, x))$
 $Dog(c) \cdot Owns(john, c)$
 $\{Dog(c)\}, \{Owns(john, c)\}$
- ⑥ $\forall x\forall y\forall z(Owns(y, z) \cdot Hates(x, z) \rightarrow \neg Date(x, y))$

Example 2

- ⑤ $\exists x(Dog(x) \cdot Owns(John, x))$
 $Dog(c) \cdot Owns(john, c)$
 $\{Dog(c)\}, \{Owns(john, c)\}$
- ⑥ $\forall x\forall y\forall z(Owns(y, z) \cdot Hates(x, z) \rightarrow \neg Date(x, y))$
 $\neg(Owns(y, z) \cdot Hates(x, z)) \vee \neg Date(x, y)$

Example 2

- ⑤ $\exists x(Dog(x) \cdot Owns(John, x))$
 $Dog(c) \cdot Owns(john, c)$
 $\{Dog(c)\}, \{Owns(john, c)\}$
- ⑥ $\forall x\forall y\forall z(Owns(y, z) \cdot Hates(x, z) \rightarrow \neg Date(x, y))$
 $\neg(Owns(y, z) \cdot Hates(x, z)) \vee \neg Date(x, y)$
 $\neg Owns(y, z) \vee \neg Hates(x, z) \vee \neg Date(x, y)$

Example 2

- ⑤ $\exists x(Dog(x) \cdot Owns(John, x))$
 $Dog(c) \cdot Owns(john, c)$
 $\{Dog(c)\}, \{Owns(john, c)\}$
- ⑥ $\forall x\forall y\forall z(Owns(y, z) \cdot Hates(x, z) \rightarrow \neg Date(x, y))$
 $\neg(Owns(y, z) \cdot Hates(x, z)) \vee \neg Date(x, y)$
 $\neg Owns(y, z) \vee \neg Hates(x, z) \vee \neg Date(x, y)$
 $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$

Example 2

⑤ $\exists x(Dog(x) \cdot Owns(John, x))$

$Dog(c) \cdot Owns(john, c)$

$\{Dog(c)\}, \{Owns(john, c)\}$

⑥ $\forall x\forall y\forall z(Owns(y, z) \cdot Hates(x, z) \rightarrow \neg Date(x, y))$

$\neg(Owns(y, z) \cdot Hates(x, z)) \vee \neg Date(x, y)$

$\neg Owns(y, z) \vee \neg Hates(x, z) \vee \neg Date(x, y)$

$\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$

$\therefore \neg((\neg \exists x(Grocery(x) \cdot Owns(Mary, x))) \rightarrow \neg Date(Mary, John))$

$\neg(\forall x\neg(Grocery(x) \cdot Owns(mary, x)) \rightarrow \neg Date(mary, john))$

Example 2

$$\textcircled{5} \exists x(Dog(x) \cdot Owns(John, x))$$

$$Dog(c) \cdot Owns(john, c)$$

$$\{Dog(c)\}, \{Owns(john, c)\}$$

$$\textcircled{6} \forall x \forall y \forall z (Owns(y, z) \cdot Hates(x, z) \rightarrow \neg Date(x, y))$$

$$\neg(Owns(y, z) \cdot Hates(x, z)) \vee \neg Date(x, y)$$

$$\neg Owns(y, z) \vee \neg Hates(x, z) \vee \neg Date(x, y)$$

$$\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$$

$$\neg \therefore \neg((\neg \exists x(Grocery(x) \cdot Owns(Mary, x))) \rightarrow \neg Date(Mary, John))$$

$$\neg(\forall x \neg(Grocery(x) \cdot Owns(mary, x)) \rightarrow \neg Date(mary, john))$$

$$\neg((Grocery(x) \cdot Owns(mary, x)) \vee \neg Date(mary, john))$$

Example 2

⑤ $\exists x(Dog(x) \cdot Owns(John, x))$

$Dog(c) \cdot Owns(john, c)$

$\{Dog(c)\}, \{Owns(john, c)\}$

⑥ $\forall x\forall y\forall z(Owns(y, z) \cdot Hates(x, z) \rightarrow \neg Date(x, y))$

$\neg(Owns(y, z) \cdot Hates(x, z)) \vee \neg Date(x, y)$

$\neg Owns(y, z) \vee \neg Hates(x, z) \vee \neg Date(x, y)$

$\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$

$\neg \therefore \neg((\neg \exists x(Grocery(x) \cdot Owns(Mary, x))) \rightarrow \neg Date(Mary, John))$

$\neg(\forall x\neg(Grocery(x) \cdot Owns(mary, x)) \rightarrow \neg Date(mary, john))$

$\neg((Grocery(x) \cdot Owns(mary, x)) \vee \neg Date(mary, john))$

$\neg(Grocery(x) \cdot Owns(mary, x)) \cdot Date(mary, john))$

Example 2

⑤ $\exists x(Dog(x) \cdot Owns(John, x))$

$$Dog(c) \cdot Owns(john, c)$$

$$\{Dog(c)\}, \{Owns(john, c)\}$$

⑥ $\forall x\forall y\forall z(Owns(y, z) \cdot Hates(x, z) \rightarrow \neg Date(x, y))$

$$\neg(Owns(y, z) \cdot Hates(x, z)) \vee \neg Date(x, y)$$

$$\neg Owns(y, z) \vee \neg Hates(x, z) \vee \neg Date(x, y)$$

$$\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$$

$\neg \therefore \neg((\neg \exists x(Grocery(x) \cdot Owns(Mary, x))) \rightarrow \neg Date(Mary, John))$

$$\neg(\forall x\neg(Grocery(x) \cdot Owns(mary, x)) \rightarrow \neg Date(mary, john))$$

$$\neg((Grocery(x) \cdot Owns(mary, x)) \vee \neg Date(mary, john))$$

$$\neg(Grocery(x) \cdot Owns(mary, x)) \cdot Date(mary, john))$$

$$(\neg Grocery(x) \vee \neg Owns(mary, x)) \cdot Date(mary, john))$$

Example 2

⑤ $\exists x(Dog(x) \cdot Owns(John, x))$

$Dog(c) \cdot Owns(john, c)$

$\{Dog(c)\}, \{Owns(john, c)\}$

⑥ $\forall x\forall y\forall z(Owns(y, z) \cdot Hates(x, z) \rightarrow \neg Date(x, y))$

$\neg(Owns(y, z) \cdot Hates(x, z)) \vee \neg Date(x, y)$

$\neg Owns(y, z) \vee \neg Hates(x, z) \vee \neg Date(x, y)$

$\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$

$\neg \therefore \neg((\neg \exists x(Grocery(x) \cdot Owns(Mary, x))) \rightarrow \neg Date(Mary, John))$

$\neg(\forall x\neg(Grocery(x) \cdot Owns(mary, x)) \rightarrow \neg Date(mary, john))$

$\neg((Grocery(x) \cdot Owns(mary, x)) \vee \neg Date(mary, john))$

$\neg(Grocery(x) \cdot Owns(mary, x)) \cdot Date(mary, john))$

$(\neg Grocery(x) \vee \neg Owns(mary, x)) \cdot Date(mary, john))$

$\{\neg Grocery(x), \neg Owns(mary, x)\}, \{Date(mary, john)\}$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

12 $\{Owns(mary, a)\} [1,5]$

- 1 $\{\neg Buy(x), Owns(x, a)\}$
- 2 $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- 3 $\{\neg Dog(x), Rabbit(b)\}$
- 4 $\{\neg Dog(x), Chase(x, b)\}$
- 5 $\{Buy(mary)\}$
- 6 $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- 7 $\{Dog(c)\}$
- 8 $\{Owns(john, c)\}$
- 9 $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- 10 $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- 11 $\{Date(mary, john)\}$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

$$\textcircled{12} \{Owns(mary, a)\} [1,5]$$

$$\textcircled{13} \{Rabbit(a), Grocery(a)\} [2,5]$$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$
- ⑮ $\{Rabbit(a)\} [13,14]$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$
- ⑮ $\{Rabbit(a)\} [13,14]$
- ⑯ $\{\neg Owns(x, a), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\} [6,15]$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$
- ⑮ $\{Rabbit(a)\} [13,14]$
- ⑯ $\{\neg Owns(x, a), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\} [6,15]$
- ⑰ $\{\neg Rabbit(w), \neg Chase(z, w), Hates(mary, z)\} [12,16]$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$
- ⑮ $\{Rabbit(a)\} [13,14]$
- ⑯ $\{\neg Owns(x, a), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\} [6,15]$
- ⑰ $\{\neg Rabbit(w), \neg Chase(z, w), Hates(mary, z)\} [12,16]$
- ⑱ $\{Rabbit(b)\} [3,7]$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$
- ⑮ $\{Rabbit(a)\} [13,14]$
- ⑯ $\{\neg Owns(x, a), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\} [6,15]$
- ⑰ $\{\neg Rabbit(w), \neg Chase(z, w), Hates(mary, z)\} [12,16]$
- ⑱ $\{Rabbit(b)\} [3,7]$
- ⑲ $\{Chase(c, b)\} [4,7]$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$
- ⑮ $\{Rabbit(a)\} [13,14]$
- ⑯ $\{\neg Owns(x, a), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\} [6,15]$
- ⑰ $\{\neg Rabbit(w), \neg Chase(z, w), Hates(mary, z)\} [12,16]$
- ⑱ $\{Rabbit(b)\} [3,7]$
- ⑲ $\{Chase(c, b)\} [4,7]$
- ⑳ $\{\neg Chase(z, b), Hates(mary, z)\} [17,18]$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$
- ⑮ $\{Rabbit(a)\} [13,14]$
- ⑯ $\{\neg Owns(x, a), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\} [6,15]$
- ⑰ $\{\neg Rabbit(w), \neg Chase(z, w), Hates(mary, z)\} [12,16]$
- ⑱ $\{Rabbit(b)\} [3,7]$
- ⑲ $\{Chase(c, b)\} [4,7]$
- ⑳ $\{\neg Chase(z, b), Hates(mary, z)\} [17,18]$
- ㉑ $\{Hates(mary, c)\} [19,20]$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$
- ⑮ $\{Rabbit(a)\} [13,14]$
- ⑯ $\{\neg Owns(x, a), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\} [6,15]$
- ⑰ $\{\neg Rabbit(w), \neg Chase(z, w), Hates(mary, z)\} [12,16]$
- ⑱ $\{Rabbit(b)\} [3,7]$
- ⑲ $\{Chase(c, b)\} [4,7]$
- ⑳ $\{\neg Chase(z, b), Hates(mary, z)\} [17,18]$
- ㉑ $\{Hates(mary, c)\} [19,20]$
- ㉒ $\{\neg Hates(x, c), \neg Date(x, john)\} [8,9]$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$
- ⑮ $\{Rabbit(a)\} [13,14]$
- ⑯ $\{\neg Owns(x, a), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\} [6,15]$
- ⑰ $\{\neg Rabbit(w), \neg Chase(z, w), Hates(mary, z)\} [12,16]$
- ⑱ $\{Rabbit(b)\} [3,7]$
- ⑲ $\{Chase(c, b)\} [4,7]$
- ⑳ $\{\neg Chase(z, b), Hates(mary, z)\} [17,18]$
- ㉑ $\{Hates(mary, c)\} [19,20]$
- ㉒ $\{\neg Hates(x, c), \neg Date(x, john)\} [8,9]$
- ㉓ $\{\neg Date(mary, john)\} [21,22]$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$
- ⑮ $\{Rabbit(a)\} [13,14]$
- ⑯ $\{\neg Owns(x, a), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\} [6,15]$
- ⑰ $\{\neg Rabbit(w), \neg Chase(z, w), Hates(mary, z)\} [12,16]$
- ⑱ $\{Rabbit(b)\} [3,7]$
- ⑲ $\{Chase(c, b)\} [4,7]$
- ⑳ $\{\neg Chase(z, b), Hates(mary, z)\} [17,18]$
- ㉑ $\{Hates(mary, c)\} [19,20]$
- ㉒ $\{\neg Hates(x, c), \neg Date(x, john)\} [8,9]$
- ㉓ $\{\neg Date(mary, john)\} [21,22]$
- ㉔ $\{\} [11,23]$

Resolution in predicate

- ① $\{\neg Buy(x), Owns(x, a)\}$
- ② $\{\neg Buy(x), Rabbit(a), Grocery(a)\}$
- ③ $\{\neg Dog(x), Rabbit(b)\}$
- ④ $\{\neg Dog(x), Chase(x, b)\}$
- ⑤ $\{Buy(mary)\}$
- ⑥ $\{\neg Owns(x, y), \neg Rabbit(y), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\}$
- ⑦ $\{Dog(c)\}$
- ⑧ $\{Owns(john, c)\}$
- ⑨ $\{\neg Owns(y, z), \neg Hates(x, z), \neg Date(x, y)\}$
- ⑩ $\{\neg Grocery(x), \neg Owns(mary, x)\}$
- ⑪ $\{Date(mary, john)\}$

- ⑫ $\{Owns(mary, a)\} [1,5]$
- ⑬ $\{Rabbit(a), Grocery(a)\} [2,5]$
- ⑭ $\{\neg Grocery(a)\} [10,12]$
- ⑮ $\{Rabbit(a)\} [13,14]$
- ⑯ $\{\neg Owns(x, a), \neg Rabbit(w), \neg Chase(z, w), Hates(x, z)\} [6,15]$
- ⑰ $\{\neg Rabbit(w), \neg Chase(z, w), Hates(mary, z)\} [12,16]$
- ⑱ $\{Rabbit(b)\} [3,7]$
- ⑲ $\{Chase(c, b)\} [4,7]$
- ⑳ $\{\neg Chase(z, b), Hates(mary, z)\} [17,18]$
- ㉑ $\{Hates(mary, c)\} [19,20]$
- ㉒ $\{\neg Hates(x, c), \neg Date(x, john)\} [8,9]$
- ㉓ $\{\neg Date(mary, john)\} [21,22]$
- ㉔ $\{\} [11,23]$

Valid

Example 3 (invalid)

- ① Every child loves Santa.
 - ② Everyone who loves Santa loves any reindeer.
 - ③ Rudolph is a reindeer, and Rudolph has a red nose.
 - ④ Anything which has a red nose is weird or is a clown.
 - ⑤ No reindeer is a clown.
 - ⑥ Scrooge does not love anything which is weird.
- ∴ Scrooge is a child.

Example 3 (invalid)

- ① Every child loves Santa.
 - ② Everyone who loves Santa loves any reindeer.
 - ③ Rudolph is a reindeer, and Rudolph has a red nose.
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- ∴ Scrooge is a child.

① $\forall x(Child(x) \rightarrow Loves(x, Santa))$

Example 3 (invalid)

- ① Every child loves Santa.
 - ② Everyone who loves Santa loves any reindeer.
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 - ⑤ No reindeer is a clown.
 - ⑥ Scrooge does not love anything which is weird.
- ∴ Scrooge is a child.

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$

Example 3 (invalid)

- ① Every child loves Santa.
 - ② Everyone who loves Santa loves any reindeer.
 - ③ Rudolph is a reindeer, and Rudolph has a red nose.
 - ④ Anything which has a red nose is weird or is a clown.
 - ⑤ No reindeer is a clown.
 - ⑥ Scrooge does not love anything which is weird.
- ∴ Scrooge is a child.

- ① $\forall x (Child(x) \rightarrow Loves(x, Santa))$
- ② $\forall x (Loves(x, Santa) \rightarrow \forall y (Reindeer(y) \rightarrow Loves(x, y)))$
- ③ $Reindeer(Rudolph) \cdot Rednose(Rudolph)$

Example 3 (invalid)

- ① Every child loves Santa.
 - ② Everyone who loves Santa loves any reindeer.
 - ③ Rudolph is a reindeer, and Rudolph has a red nose.
 - ④ Anything which has a red nose is weird or is a clown.
 - ⑤ No reindeer is a clown.
 - ⑥ Scrooge does not love anything which is weird.
- ∴ Scrooge is a child.

- ① $\forall x (Child(x) \rightarrow Loves(x, Santa))$
- ② $\forall x (Loves(x, Santa) \rightarrow \forall y (Reindeer(y) \rightarrow Loves(x, y)))$
- ③ $Reindeer(Rudolph) \cdot Rednose(Rudolph)$
- ④ $\forall x (Rednose(x) \rightarrow Weird(x) \vee Clown(x))$

Example 3 (invalid)

- ① Every child loves Santa.
- ② Everyone who loves Santa loves any reindeer.
- ③ Rudolph is a reindeer, and Rudolph has a red nose.
- ④ Anything which has a red nose is weird or is a clown.
- ⑤ No reindeer is a clown.
- ⑥ Scrooge does not love anything which is weird.
- ∴ Scrooge is a child.

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$
- ③ $Reindeer(Rudolph) \cdot Rednose(Rudolph)$
- ④ $\forall x(Rednose(x) \rightarrow Weird(x) \vee Clown(x))$
- ⑤ $\neg \exists x(Reindeer(x) \cdot Clown(x))$

Example 3 (invalid)

- ① Every child loves Santa.
 - ② Everyone who loves Santa loves any reindeer.
 - ③ Rudolph is a reindeer, and Rudolph has a red nose.
 - ④ Anything which has a red nose is weird or is a clown.
 - ⑤ No reindeer is a clown.
 - ⑥ Scrooge does not love anything which is weird.
- ∴ Scrooge is a child.

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$
- ③ $Reindeer(Rudolph) \cdot Rednose(Rudolph)$
- ④ $\forall x(Rednose(x) \rightarrow Weird(x) \vee Clown(x))$
- ⑤ $\neg \exists x(Reindeer(x) \cdot Clown(x))$
- ⑥ $\forall x(Weird(x) \rightarrow \neg Loves(Scrooge, x))$

Resolution in predicate

Example 3 (invalid)

- ① Every child loves Santa.
- ② Everyone who loves Santa loves any reindeer.
- ③ Rudolph is a reindeer, and Rudolph has a red nose.
- ④ Anything which has a red nose is weird or is a clown.
- ⑤ No reindeer is a clown.
- ⑥ Scrooge does not love anything which is weird.
- ∴ Scrooge is a child.

- ① $\forall x (Child(x) \rightarrow Loves(x, Santa))$
- ② $\forall x (Loves(x, Santa) \rightarrow \forall y (Reindeer(y) \rightarrow Loves(x, y)))$
- ③ $Reindeer(Rudolph) \cdot Rednose(Rudolph)$
- ④ $\forall x (Rednose(x) \rightarrow Weird(x) \vee Clown(x))$
- ⑤ $\neg \exists x (Reindeer(x) \cdot Clown(x))$
- ⑥ $\forall x (Weird(x) \rightarrow \neg Loves(Scrooge, x))$
- ∴ $Child(Scrooge)$

Example 3 (invalid)

① $\forall x (Child(x) \rightarrow Loves(x, Santa))$

Example 3 (invalid)

- 1 $\forall x (Child(x) \rightarrow Loves(x, Santa))$
 $\neg Child(x) \vee Loves(x, Santa)$

Example 3 (invalid)

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
 $\neg Child(x) \vee Loves(x, Santa)$
 $\{\neg Child(x), Loves(x, Santa)\}$

Example 3 (invalid)

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
 $\neg Child(x) \vee Loves(x, Santa)$
 $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$

Example 3 (invalid)

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
 $\neg Child(x) \vee Loves(x, Santa)$
 $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$
 $\forall x\forall y(\neg Loves(x, Santa) \vee (\neg Reindeer(y) \vee Loves(x, y)))$

Example 3 (invalid)

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
 $\neg Child(x) \vee Loves(x, Santa)$
 $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$
 $\forall x\forall y(\neg Loves(x, Santa) \vee (\neg Reindeer(y) \vee Loves(x, y)))$
 $\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)$

Example 3 (invalid)

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
 $\neg Child(x) \vee Loves(x, Santa)$
 $\{ \neg Child(x), Loves(x, Santa) \}$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$
 $\forall x \forall y(\neg Loves(x, Santa) \vee (\neg Reindeer(y) \vee Loves(x, y)))$
 $\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)$
 $\{ \neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y) \}$

Example 3 (invalid)

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
 $\neg Child(x) \vee Loves(x, Santa)$
 $\{ \neg Child(x), Loves(x, Santa) \}$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$
 $\forall x \forall y(\neg Loves(x, Santa) \vee (\neg Reindeer(y) \vee Loves(x, y)))$
 $\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)$
 $\{ \neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y) \}$
- ③ $Reindeer(Rudolph) \cdot Rednose(Rudolph)$

Example 3 (invalid)

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
 $\neg Child(x) \vee Loves(x, Santa)$
 $\{ \neg Child(x), Loves(x, Santa) \}$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$
 $\forall x \forall y(\neg Loves(x, Santa) \vee (\neg Reindeer(y) \vee Loves(x, y)))$
 $\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)$
 $\{ \neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y) \}$
- ③ $Reindeer(Rudolph) \cdot Rednose(Rudolph)$
 $\{ Reindeer(Rudolph) \}, \{ Rednose(Rudolph) \}$

Example 3 (invalid)

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
 $\neg Child(x) \vee Loves(x, Santa)$
 $\{ \neg Child(x), Loves(x, Santa) \}$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$
 $\forall x \forall y(\neg Loves(x, Santa) \vee (\neg Reindeer(y) \vee Loves(x, y)))$
 $\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)$
 $\{ \neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y) \}$
- ③ $Reindeer(Rudolph) \cdot Rednose(Rudolph)$
 $\{ Reindeer(Rudolph) \}, \{ Rednose(Rudolph) \}$
- ④ $\forall x(Rednose(x) \rightarrow Weird(x) \vee Clown(x))$

Example 3 (invalid)

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
 $\neg Child(x) \vee Loves(x, Santa)$
 $\{ \neg Child(x), Loves(x, Santa) \}$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$
 $\forall x \forall y(\neg Loves(x, Santa) \vee (\neg Reindeer(y) \vee Loves(x, y)))$
 $\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)$
 $\{ \neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y) \}$
- ③ $Reindeer(Rudolph) \cdot Rednose(Rudolph)$
 $\{ Reindeer(Rudolph) \}, \{ Rednose(Rudolph) \}$
- ④ $\forall x(Rednose(x) \rightarrow Weird(x) \vee Clown(x))$
 $\neg Rednose(x) \vee Weird(x) \vee Clown(x)$

Example 3 (invalid)

- ① $\forall x(Child(x) \rightarrow Loves(x, Santa))$
 $\neg Child(x) \vee Loves(x, Santa)$
 $\{ \neg Child(x), Loves(x, Santa) \}$
- ② $\forall x(Loves(x, Santa) \rightarrow \forall y(Reindeer(y) \rightarrow Loves(x, y)))$
 $\forall x \forall y(\neg Loves(x, Santa) \vee (\neg Reindeer(y) \vee Loves(x, y)))$
 $\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)$
 $\{ \neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y) \}$
- ③ $Reindeer(Rudolph) \cdot Rednose(Rudolph)$
 $\{ Reindeer(Rudolph) \}, \{ Rednose(Rudolph) \}$
- ④ $\forall x(Rednose(x) \rightarrow Weird(x) \vee Clown(x))$
 $\neg Rednose(x) \vee Weird(x) \vee Clown(x)$
 $\{ \neg Rednose(x) \vee Weird(x) \vee Clown(x) \}$

Example 3 (invalid)

④ $\neg \exists x (Reindeer(x) \cdot Clown(x))$

Example 3 (invalid)

- ④ $\neg \exists x (Reindeer(x) \cdot Clown(x))$
 $\forall x \neg (Reindeer(x) \cdot Clown(x))$

Example 3 (invalid)

- ④ $\neg \exists x (Reindeer(x) \cdot Clown(x))$
 $\forall x \neg (Reindeer(x) \cdot Clown(x))$
 $\neg Reindeer(x) \vee \neg Clown(x)$

Example 3 (invalid)

- ④ $\neg \exists x (Reindeer(x) \cdot Clown(x))$
 $\forall x \neg (Reindeer(x) \cdot Clown(x))$
 $\neg Reindeer(x) \vee \neg Clown(x)$
 $\{\neg Reindeer(x), \neg Clown(x)\}$

Example 3 (invalid)

- ④ $\neg \exists x (Reindeer(x) \cdot Clown(x))$
 $\forall x \neg (Reindeer(x) \cdot Clown(x))$
 $\neg Reindeer(x) \vee \neg Clown(x)$
 $\{\neg Reindeer(x), \neg Clown(x)\}$
- ④ $\forall x (Weird(x) \rightarrow \neg Loves(Scrooge, x))$

Example 3 (invalid)

- ④ $\neg \exists x (Reindeer(x) \cdot Clown(x))$
 $\forall x \neg (Reindeer(x) \cdot Clown(x))$
 $\neg Reindeer(x) \vee \neg Clown(x)$
 $\{\neg Reindeer(x), \neg Clown(x)\}$
- ④ $\forall x (Weird(x) \rightarrow \neg Loves(Scrooge, x))$
 $\neg Weird(x) \vee \neg Loves(Scrooge, x)$

Example 3 (invalid)

- ④ $\neg \exists x (Reindeer(x) \cdot Clown(x))$
 $\forall x \neg (Reindeer(x) \cdot Clown(x))$
 $\neg Reindeer(x) \vee \neg Clown(x)$
 $\{\neg Reindeer(x), \neg Clown(x)\}$
- ④ $\forall x (Weird(x) \rightarrow \neg Loves(Scrooge, x))$
 $\neg Weird(x) \vee \neg Loves(Scrooge, x)$
 $\{\neg Weird(x), \neg Loves(Scrooge, x)\}$

Example 3 (invalid)

$$\begin{aligned} 4 \quad & \neg \exists x (Reindeer(x) \cdot Clown(x)) \\ & \forall x \neg (Reindeer(x) \cdot Clown(x)) \\ & \neg Reindeer(x) \vee \neg Clown(x) \\ & \{\neg Reindeer(x), \neg Clown(x)\} \end{aligned}$$

$$\begin{aligned} 4 \quad & \forall x (Weird(x) \rightarrow \neg Loves(Scrooge, x)) \\ & \neg Weird(x) \vee \neg Loves(Scrooge, x) \\ & \{\neg Weird(x), \neg Loves(Scrooge, x)\} \end{aligned}$$

$$\neg \therefore \neg Child(Scrooge)$$

Example 3 (invalid)

$$\begin{aligned} 4 \quad & \neg \exists x (Reindeer(x) \cdot Clown(x)) \\ & \forall x \neg (Reindeer(x) \cdot Clown(x)) \\ & \neg Reindeer(x) \vee \neg Clown(x) \\ & \{\neg Reindeer(x), \neg Clown(x)\} \end{aligned}$$

$$\begin{aligned} 4 \quad & \forall x (Weird(x) \rightarrow \neg Loves(Scrooge, x)) \\ & \neg Weird(x) \vee \neg Loves(Scrooge, x) \\ & \{\neg Weird(x), \neg Loves(Scrooge, x)\} \end{aligned}$$

$$\begin{aligned} \neg \therefore & \neg Child(Scrooge) \\ & \{\neg Child(Scrooge)\} \end{aligned}$$

Example 3 (invalid)

$$\begin{aligned} 4 \quad & \neg \exists x (Reindeer(x) \cdot Clown(x)) \\ & \forall x \neg (Reindeer(x) \cdot Clown(x)) \\ & \neg Reindeer(x) \vee \neg Clown(x) \\ & \{\neg Reindeer(x), \neg Clown(x)\} \end{aligned}$$

$$\begin{aligned} 4 \quad & \forall x (Weird(x) \rightarrow \neg Loves(Scrooge, x)) \\ & \neg Weird(x) \vee \neg Loves(Scrooge, x) \\ & \{\neg Weird(x), \neg Loves(Scrooge, x)\} \end{aligned}$$

$$\begin{aligned} \neg \therefore & \neg Child(Scrooge) \\ & \{\neg Child(Scrooge)\} \end{aligned}$$

Example 3 (invalid)

- ① $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\{\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)\}$
- ③ $\{Reindeer(Rudolph)\}$
- ④ $\{Rednose(Rudolph)\}$
- ⑤ $\{\neg Rednose(x) \vee Weird(x) \vee Clown(x)\}$
- ⑥ $\{\neg Reindeer(x), \neg Clown(x)\}$
- ⑦ $\{\neg Weird(x), \neg Loves(Scrooge, x)\}$
- ⑧ $\{\neg Child(Scrooge)\}$

Resolution in predicate

Example 3 (invalid)

$$\textcircled{9} \{Weird(Rudolph), Clown(Rudolph)\} \\ [4,5]$$

- $\textcircled{1} \{\neg Child(x), Loves(x, Santa)\}$
- $\textcircled{2} \{\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)\}$
- $\textcircled{3} \{Reindeer(Rudolph)\}$
- $\textcircled{4} \{Rednose(Rudolph)\}$
- $\textcircled{5} \{\neg Rednose(x) \vee Weird(x) \vee Clown(x)\}$
- $\textcircled{6} \{\neg Reindeer(x), \neg Clown(x)\}$
- $\textcircled{7} \{\neg Weird(x), \neg Loves(Scrooge, x)\}$
- $\textcircled{8} \{\neg Child(Scrooge)\}$

Resolution in predicate

Example 3 (invalid)

- ① $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\{\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)\}$
- ③ $\{Reindeer(Rudolph)\}$
- ④ $\{Rednose(Rudolph)\}$
- ⑤ $\{\neg Rednose(x) \vee Weird(x) \vee Clown(x)\}$
- ⑥ $\{\neg Reindeer(x), \neg Clown(x)\}$
- ⑦ $\{\neg Weird(x), \neg Loves(Scrooge, x)\}$
- ⑧ $\{\neg Child(Scrooge)\}$
- ⑨ $\{Weird(Rudolph), Clown(Rudolph)\}$
[4,5]
- ⑩ $\{\neg Clown(Rudolph)\}$ [3,6]

Resolution in predicate

Example 3 (invalid)

- ① $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\{\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)\}$
- ③ $\{Reindeer(Rudolph)\}$
- ④ $\{Rednose(Rudolph)\}$
- ⑤ $\{\neg Rednose(x) \vee Weird(x) \vee Clown(x)\}$
- ⑥ $\{\neg Reindeer(x), \neg Clown(x)\}$
- ⑦ $\{\neg Weird(x), \neg Loves(Scrooge, x)\}$
- ⑧ $\{\neg Child(Scrooge)\}$
- ⑨ $\{Weird(Rudolph), Clown(Rudolph)\}$
[4,5]
- ⑩ $\{\neg Clown(Rudolph)\}$ [3,6]
- ⑪ $\{Weird(Rudolph)\}$ [9,10]

Resolution in predicate

Example 3 (invalid)

- ① $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\{\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)\}$
- ③ $\{Reindeer(Rudolph)\}$
- ④ $\{Rednose(Rudolph)\}$
- ⑤ $\{\neg Rednose(x) \vee Weird(x) \vee Clown(x)\}$
- ⑥ $\{\neg Reindeer(x), \neg Clown(x)\}$
- ⑦ $\{\neg Weird(x), \neg Loves(Scrooge, x)\}$
- ⑧ $\{\neg Child(Scrooge)\}$
- ⑨ $\{Weird(Rudolph), Clown(Rudolph)\}$
[4,5]
- ⑩ $\{\neg Clown(Rudolph)\}$ [3,6]
- ⑪ $\{Weird(Rudolph)\}$ [9,10]
- ⑫ $\{\neg Loves(Scrooge, Rudolph)\}$ [7,12]

Resolution in predicate

Example 3 (invalid)

- ① $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\{\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)\}$
- ③ $\{Reindeer(Rudolph)\}$
- ④ $\{Rednose(Rudolph)\}$
- ⑤ $\{\neg Rednose(x) \vee Weird(x) \vee Clown(x)\}$
- ⑥ $\{\neg Reindeer(x), \neg Clown(x)\}$
- ⑦ $\{\neg Weird(x), \neg Loves(Scrooge, x)\}$
- ⑧ $\{\neg Child(Scrooge)\}$
- ⑨ $\{Weird(Rudolph), Clown(Rudolph)\}$
[4,5]
- ⑩ $\{\neg Clown(Rudolph)\}$ [3,6]
- ⑪ $\{Weird(Rudolph)\}$ [9,10]
- ⑫ $\{\neg Loves(Scrooge, Rudolph)\}$ [7,12]
- ⑬ $\{\neg Loves(x, Santa), Loves(x, Rudolph)\}$ [2,3]

Resolution in predicate

Example 3 (invalid)

- ① $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\{\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)\}$
- ③ $\{Reindeer(Rudolph)\}$
- ④ $\{Rednose(Rudolph)\}$
- ⑤ $\{\neg Rednose(x) \vee Weird(x) \vee Clown(x)\}$
- ⑥ $\{\neg Reindeer(x), \neg Clown(x)\}$
- ⑦ $\{\neg Weird(x), \neg Loves(Scrooge, x)\}$
- ⑧ $\{\neg Child(Scrooge)\}$
- ⑨ $\{Weird(Rudolph), Clown(Rudolph)\}$ [4,5]
- ⑩ $\{\neg Clown(Rudolph)\}$ [3,6]
- ⑪ $\{Weird(Rudolph)\}$ [9,10]
- ⑫ $\{\neg Loves(Scrooge, Rudolph)\}$ [7,12]
- ⑬ $\{\neg Loves(x, Santa), Loves(x, Rudolph)\}$ [2,3]
- ⑭ $\{\neg Loves(Scrooge, Santa)\}$ [12,13]

Resolution in predicate

Example 3 (invalid)

- ① $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\{\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)\}$
- ③ $\{Reindeer(Rudolph)\}$
- ④ $\{Rednose(Rudolph)\}$
- ⑤ $\{\neg Rednose(x) \vee Weird(x) \vee Clown(x)\}$
- ⑥ $\{\neg Reindeer(x), \neg Clown(x)\}$
- ⑦ $\{\neg Weird(x), \neg Loves(Scrooge, x)\}$
- ⑧ $\{\neg Child(Scrooge)\}$
- ⑨ $\{Weird(Rudolph), Clown(Rudolph)\}$
[4,5]
- ⑩ $\{\neg Clown(Rudolph)\}$ [3,6]
- ⑪ $\{Weird(Rudolph)\}$ [9,10]
- ⑫ $\{\neg Loves(Scrooge, Rudolph)\}$ [7,12]
- ⑬ $\{\neg Loves(x, Santa), Loves(x, Rudolph)\}$ [2,3]
- ⑭ $\{\neg Loves(Scrooge, Santa)\}$ [12,13]
- ⑮ $\{\neg Child(Scrooge)\}$ [1,14]

Resolution in predicate

Example 3 (invalid)

- ① $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\{\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)\}$
- ③ $\{Reindeer(Rudolph)\}$
- ④ $\{Rednose(Rudolph)\}$
- ⑤ $\{\neg Rednose(x) \vee Weird(x) \vee Clown(x)\}$
- ⑥ $\{\neg Reindeer(x), \neg Clown(x)\}$
- ⑦ $\{\neg Weird(x), \neg Loves(Scrooge, x)\}$
- ⑧ $\{\neg Child(Scrooge)\}$
- ⑨ $\{Weird(Rudolph), Clown(Rudolph)\}$
[4,5]
- ⑩ $\{\neg Clown(Rudolph)\}$ [3,6]
- ⑪ $\{Weird(Rudolph)\}$ [9,10]
- ⑫ $\{\neg Loves(Scrooge, Rudolph)\}$ [7,12]
- ⑬ $\{\neg Loves(x, Santa),$
 $Loves(x, Rudolph)\}$ [2,3]
- ⑭ $\{\neg Loves(Scrooge, Santa)\}$ [12,13]
- ⑮ $\{\neg Child(Scrooge)\}$ [1,14]
- ⑯ $\{\neg Loves(Scrooge, Santa),$
 $\neg Reindeer(Santa)\}$ [2,15]

Resolution in predicate

Example 3 (invalid)

- ① $\{\neg Child(x), Loves(x, Santa)\}$
- ② $\{\neg Loves(x, Santa) \vee \neg Reindeer(y) \vee Loves(x, y)\}$
- ③ $\{Reindeer(Rudolph)\}$
- ④ $\{Rednose(Rudolph)\}$
- ⑤ $\{\neg Rednose(x) \vee Weird(x) \vee Clown(x)\}$
- ⑥ $\{\neg Reindeer(x), \neg Clown(x)\}$
- ⑦ $\{\neg Weird(x), \neg Loves(Scrooge, x)\}$
- ⑧ $\{\neg Child(Scrooge)\}$

- ⑨ $\{Weird(Rudolph), Clown(Rudolph)\}$ [4,5]
- ⑩ $\{\neg Clown(Rudolph)\}$ [3,6]
- ⑪ $\{Weird(Rudolph)\}$ [9,10]
- ⑫ $\{\neg Loves(Scrooge, Rudolph)\}$ [7,12]
- ⑬ $\{\neg Loves(x, Santa), Loves(x, Rudolph)\}$ [2,3]
- ⑭ $\{\neg Loves(Scrooge, Santa)\}$ [12,13]
- ⑮ $\{\neg Child(Scrooge)\}$ [1,14]
- ⑯ $\{\neg Loves(Scrooge, Santa), \neg Reindeer(Santa)\}$ [2,15]
- ⑰ $[\neg Clown(Rudolph), Weird(Rudolph), \neg Loves(x, Santa), \neg Loves(Scrooge, Rudolph), \neg Child(Scrooge)]$

Invalid