

PART A (18 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE INDICATED ON THE SCANTRON SHEET. YOU SHOULD ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

1. Which of the following equations is/are linear in the unknowns  $x$ ,  $y$  and  $z$ ?

(i)  $xy - 2z = 3$  (ii)  $3x - \sqrt{2}y + \frac{1}{4}z = 0$   
(iii)  $5\sqrt{x} - 2y + z = 1$  (iv)  $4x - y - 5z = 0$

A: (i) only	B: (ii) and (iv) only	C: (i) and (iii) only
D: (iv) only	E: (i), (iii) and (iv) only	

2. Which of the following is an augmented matrix corresponding to the system of linear equations shown below?

$x_1 = 5 + x_2 - x_3$   
 $x_2 = 3x_4 - x_1$   
 $x_3 = 2x_1 - x_4 + 6$

A: $\left[ \begin{array}{cccc c} 5 & 1 & -1 & 1 & 1 \\ 3 & -1 & 0 & 1 & 1 \\ 2 & -1 & 6 & 1 & 1 \end{array} \right]$	B: $\left[ \begin{array}{cccc c} 1 & 1 & -1 & 0 & 5 \\ -1 & 1 & 0 & 3 & 0 \\ 2 & 0 & 1 & -1 & 6 \end{array} \right]$	C: $\left[ \begin{array}{cccc c} 1 & -1 & 1 & 0 & 5 \\ 1 & 1 & 0 & -3 & 0 \\ -2 & 0 & 1 & 1 & 6 \end{array} \right]$
D: $\left[ \begin{array}{cccc c} 1 & 1 & -2 & 5 & 5 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -3 & 1 & 0 & 0 \end{array} \right]$	E: $\left[ \begin{array}{cccc c} 1 & -5 & -1 & 1 & 0 \\ 1 & -3 & 1 & 0 & 0 \\ 1 & -2 & 1 & -6 & 0 \end{array} \right]$	

3. Determine the values of  $a$  and  $b$  for which the matrix below is in row-reduced echelon form.

$\left[ \begin{array}{ccc|c} a+b & 2 & 0 & 4 \\ 0 & b & 1 & 2 \\ 0 & 1-a & 0 & 0 \end{array} \right]$

$a+b=1$   
 $b=0$   
 $a=1$   
 $1-a=0$

A: $\begin{matrix} a = 1 \\ b = 1 \end{matrix}$	B: $\begin{matrix} a = 1 \\ b = 0 \end{matrix}$	C: $\begin{matrix} a = 0 \\ b = 1 \end{matrix}$	D: $\begin{matrix} a = -1 \\ b = 1 \end{matrix}$	E: $\begin{matrix} a = 0 \\ b = 0 \end{matrix}$
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4. Let  $B$  be the row-reduced echelon form of the matrix  $A$  below. Find the first row of  $B$ .

$A = \left[ \begin{array}{ccc} 1 & -2 & -3 \\ 2 & -4 & -6 \\ -1 & 2 & 4 \end{array} \right]$

$\Rightarrow$   
 $\begin{matrix} 1 & 2 & 7 \\ 2 & -3 & -3 \end{matrix}$

A: $\left[ \begin{array}{ccc} 1 & -2 & -3 \end{array} \right]$	B: $\left[ \begin{array}{ccc} 1 & 0 & 0 \end{array} \right]$	C: $\left[ \begin{array}{ccc} 1 & 0 & 1 \end{array} \right]$	D: $\left[ \begin{array}{ccc} 1 & -2 & 0 \end{array} \right]$	E: $\left[ \begin{array}{ccc} 1 & 0 & -3 \end{array} \right]$
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- 1  
mark

5. Find the solution(s) to the system of linear equations whose augmented matrix is shown here:

$$\left[ \begin{array}{cccc|c} 1 & 0 & 3 & 0 & 7 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

A: $(7 - 3t, 2t, t, -6)$	B: $(4, 2, -6, 0)$	C: $(-3t, 2t, -6, 0)$
D: $(3 - 7t, -2, t, -6)$	E: The system has no solution.	

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6. Find the solution(s) to the system of linear equations whose augmented matrix is shown here:

$$\left[ \begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 2 & -1 & 1 & 2 \end{array} \right]$$

A: $(0, 1, 2)$	B: $(1 - t, 1 - t, t)$	C: $(1, 1, 1)$	D: $(t, t, t)$	E: The system has no solution.
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Use the following information for questions 7, 8 and 9.

Consider the system of linear equations whose augmented matrix is:

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & -5 & 10 \\ 0 & 0 & k + 3 & 3k + k^2 \end{array} \right]$$

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7. Find the value(s) of  $k$  for which the system has no solution.

A: $k = -3$ only	B: all $k \neq -3$	C: $k = 0$ only	D: all $k \neq 0$	E: no value of $k$
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8. Find the value(s) of  $k$  for which the system has a unique solution.

A: $k = -3$ only	B: all $k \neq -3$	C: $k = 0$ only	D: all $k \neq 0$	E: no value of $k$
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9. Find the value(s) of  $k$  for which the system has infinitely many solutions.

A: $k = -3$ only	B: all $k \neq -3$	C: $k = 0$ only	D: all $k \neq 0$	E: no value of $k$
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10. Let  $A$  be a  $3 \times 4$  matrix,  $B$  be a  $2 \times 4$  matrix and  $C$  be a  $3 \times 5$  matrix. Which one of the following describes the matrix  $BA^TC$ ?

A: a $3 \times 4$ matrix	B: a $5 \times 3$ matrix	C: a $2 \times 3$ matrix
D: a $4 \times 2$ matrix	E: a $2 \times 5$ matrix	

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11. If  $A$  is a  $4 \times 4$  matrix and  $B$  is a  $3 \times 4$  matrix, which one of the following expressions is **not defined**?

A: $BB^T$	B: $BA^T$	C: $B + B^TA$	D: $BA$	E: $B + BA^T$
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12. If  $7 \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} - 2 \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , find the value of  $a$ .

A: $-1$	B: $21$	C: $-9$	D: $6$	E: $0$
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13. If  $AB = C = [c_{ij}]$ , where  $A$  and  $B$  are the matrices shown below, find  $c_{23}$ .

$$A = \begin{bmatrix} 2 & 1 & 7 & 0 \\ -1 & 0 & 2 & 3 \\ 3 & 1 & -4 & 7 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 6 & 2 & 4 & 0 \\ 3 & -5 & -5 & 8 \\ -4 & -1 & 1 & -3 \\ 1 & 0 & 2 & 5 \end{bmatrix}$$

A: 1	B: 2	C: 3	D: 4	E: 5
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14. Let  $A = \begin{bmatrix} 3 & 7 \\ 1 & 4 \end{bmatrix}$ . What is the  $(2, 2)$ -entry of  $A^{-1}$ ?

A: $\frac{3}{5}$	B: 3	C: $-\frac{1}{5}$	D: $-7$	E: $\frac{1}{5}$
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15. Find the value of  $k$  for which the matrix  $\begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -1 \\ 3 & -10 & k \end{bmatrix}$  has no inverse.

A: 3	B: $-7$	C: 10	D: $-3$	E: 7
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16. Let  $A$  be an  $n \times n$  matrix and  $I$  be the  $n \times n$  identity matrix. Which one of the following statements is **false**?

A: If $A^2 = I$ then $A = A^{-1}$ .
B: If $A$ has a row of (only) zeros, it is not invertible.
C: If all main diagonal entries of $A$ are ones, it must be invertible.
D: If $A^T A = I$ then $A^T = A^{-1}$ .

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17. If  $\begin{bmatrix} 3 & c \\ -1 & d \end{bmatrix}$  is the inverse of  $\begin{bmatrix} a & 2 \\ b & -3 \end{bmatrix}$ , find the value of  $c$ .

A: $-1$	B: 1	C: $-2$	D: 0	E: 2
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mark

18. Let  $A$  be the coefficient matrix of the system of linear equations shown here:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = -3$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 5$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 7$$

Find the value of  $x_2$  in the unique solution to the system if  $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$ .

A: 7	B: $-10$	C: 49	D: $-8$	E: 3
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**PART B (7 marks)**

**SHOW YOUR WORK FOR ALL QUESTIONS IN PART B**

- 4* marks 19. Consider the following system of linear equations:

$$\begin{array}{rrcrcl} x & + & 2y & + & 3z & = & -5 \\ -x & + & y & - & 3z & = & 2 \\ -2x & - & 3y & - & 6z & = & 9 \end{array}$$

- (a) Write the augmented matrix corresponding to this system of equations.
- (b) Find the row-reduced echelon form of your augmented matrix from part (a).
- (c) Use your row-reduced echelon form from part (b) to find all solutions to the system.

*3 marks*

20. Use the method of row reduction to find the inverse of  $A =$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 4 \\ -3 & -3 & -2 \end{bmatrix}.$$