1a)

Let A represent the set of alphanumeric characters.

G = {V, Σ , R, S}

V = {A, $\varnothing, (,), \cup, \cdot, *, S, X, Y, Z\}$

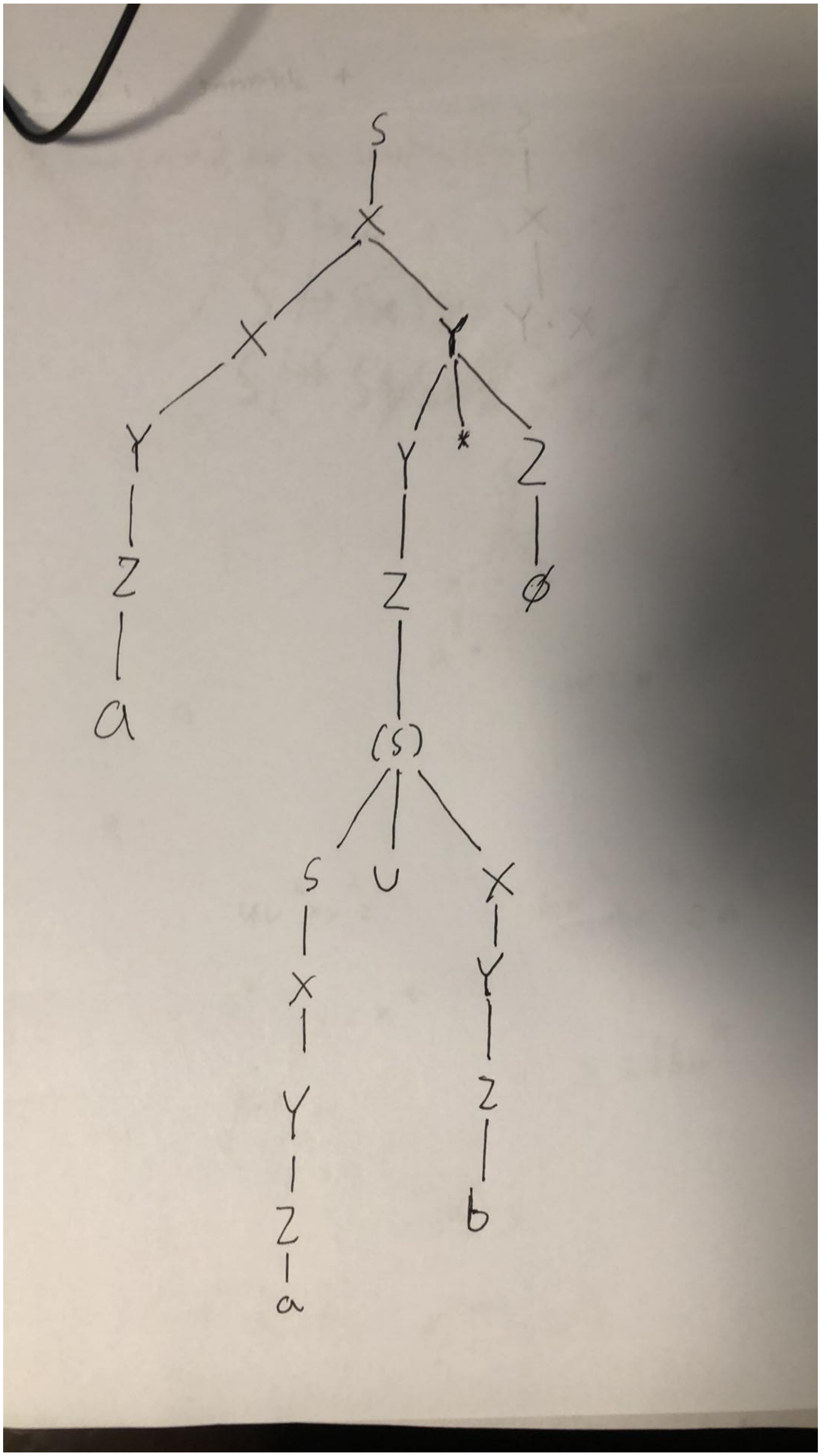
$A, \varnothing, (,), \cup, \cdot, *¿$
$\Sigma = ¿$

R = {

$S \rightarrow S \cup X \vee X \quad X \rightarrow X \cdot Y | XY | Y \quad Y \rightarrow Y * Z \vee Z \quad Z \rightarrow (S) | A | \varnothing$

}

1b)



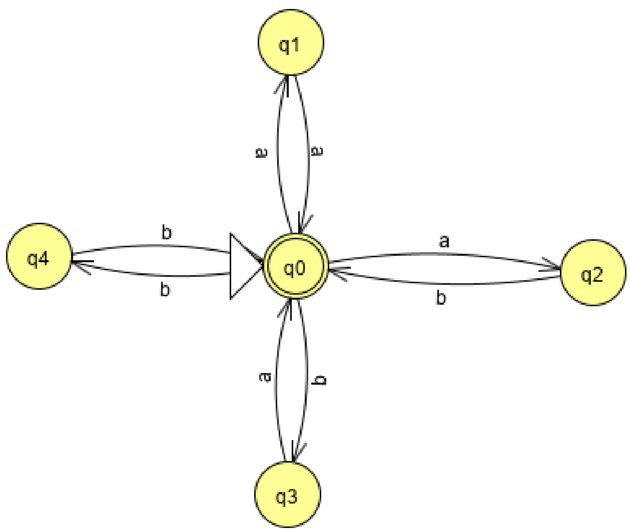
2a) **Context-free and regular**

Language can be reworded as all **even length words that only contain a's and b's**, since the length is |x| + |y| which is equal to |x|+|x| if |x| = |y|. Therefore length is 2|x|. Since |x| is an integer, the length must be even.

Prove context-free by creating CFG that generates this language:

S → $\epsilon|SS|aa|bb|ab \vee ba$

Prove regular by creating FSM:



2b) **Context-free and not regular**

Prove context-free by creating CFG that will accept:

$$S \to \epsilon | SS \vee aX | Xa \vee YaY \qquad X \to bb \vee Xb \qquad Y \to b \vee X$$

Prove not regular using pumping lemma:

- let w = $a^{2k} b^k .\, Note\, that: w \in L .\, |w| \geq k\, for\, all\, k \geq 1.$
- let xy = $a^v\, where\, v \leq k$.
  $Note\, that\, this\, general\, form\, captures\, all\, possibilites\, of\ xy\, , as\, |xy| \leq k$
- let x = $a^i \wedge y = a^j .\, i + j \leq k\, , j = v - i\, , j > 0$
- let z = $a^{k-v} b^k$
- Therefore, we can write w as w = xyz
- w = $a^i a^j a^{2k-i-j} b^k$
- Assume language is regular. Therefore, for all q >= 0, $x\, y^q z \in L$
- Pick q = 3
- a
  $(¿¿ 3j) a^{2k-i-j} b^k \quad ¿ a^{2j} a^{2k} b^k$
  $x\, y^q z = a^i ¿$

  Since j > 0, therefore $a^{2j} a^{2k} b^k \notin L$

  Therefore we have proved by contradiction that L is not regular.

2c) **Not context-free**

Prove not context-free using pumping lemma:

Let w = $ab c^k c^k baab c^k \quad \in L$

Rewrite w as w = uvxyz such that:

- u = ab
- v = c^i j+i <= k, i + j > 0
- x = ϵ
- y = c^j j+i <= k, i + j > 0
- z = $c^{k-i-j} c^k baab c^k$

Note that $u, v, x, y, z \in \Sigma^{¿}, |vxy| \leq k, |w| \geq k, \wedge vy\ is\ n'tempty$

- Assume language is context-free. Then, for all q $\geq 0, y v^q x y^q z \in L$
- $w = u v^q x y^q z$
- Let q = 2
- w = $ab c^{2i} c^{2j} c^{k-i-j} baab c^k$
- w = $ab c^{k+i+j} baab c^k$
- Since i + j > 0, then i+k+j > k.
- Therefore w $\notin L$
- Therefore, we have proven by contradiction that L is not context-free.

Not regular because all regular languages are context-free.

3) L is context-free. This proof is correct because it shows a valid context-free grammar. Since any language that can be generated with a context-free grammar is itself context-free, this proof has sufficiently showed that L is context-free. We can further confirm by checking if all the rules R follow the rules for context-free grammars. Since all the rules are in the form (V - Σ) X V*,

The second proof is incorrect, because it only proves that $u v^q x y^q z \notin L$ when v = a^p and y = a^q .

The pumping lemma theorem states there is SOME u,v,x,y,z and SOME k that satisfies the criteria. If the pumping lemma theorem said ALL u,v,x,y,z and ALL k have to satisfy the criteria, then the proof would be correct.

By not writing the exponents in terms of k, the proof has not proved the general case. Also, because exponents of v and y are not put in terms of k, the proof did not necessarily satisfy the requirement that |vxy| <= k.

Ultimately, the proof is incorrect as proving that because there exists some element(s) that don't satisfy the criteria does not prove that there is **no possibility** that satisfy the criteria.

4) Show decidable through decision procedure.

- Let G be a context-free grammar
- Let L be the language that G generates.
- Let O be the regular language that forms the set of all odd length strings.
- Generate CFG G1 = L ∩ R (Intersection of context-free language and regular language is context-free)
- Let L1 be the language generated from G1
- If decideCFLempty(L1) is true, return No
- Otherwise, return Yes.