HW3

Q3: Things equivelent to $\exists x \forall y (P(x,y) \wedge \neg Q(x,y))$

$\exists x \forall y (P(x,y) \wedge \neg Q(x,y))$

iff $\exists x \forall y \neg(\neg P(x,y) \vee Q(x,y))$.

iff $\exists x \neg \exists y (P(x,y) \rightarrow Q(x,y))$

iff $\neg \forall x \exists y (P(x,y) \rightarrow Q(x,y))$.

Q12: $A = \{1,2\}$.

$P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$.

$x \in P(A)$ $\quad x = \phi, \{1\}, \{2\}, \{1,2\}$.

$B \subseteq P(A)$ $\quad B = 2^4 = 16$ possible solutions.

$\phi, \{\phi\}, \{\{1\}\}, \{\phi, \{1\}\}$.

Ex: Let $x, y \in \mathbb{R}$, Suppose $y + x = 2y - x$, and not both y and x are zero, Prove $y \neq 0$.

Proof:

Assume $y = 0$, Then $x \neq 0$ But the equation $x = -x$ is a contradiction. Thus $y \neq 0$ $\square$.

Strategy: To use given of form $\neg P$. when the goal is a contradiction, try making P your goal

Ex: Suppose $A \setminus B$ is disjoint from C and $x \in A$. prove that if $x \in C$, then $x \in B$.

Given	Goal
$(A \setminus B) \cap C = \phi$	~~$x \in C \rightarrow x \in B$~~
$x \in A$	~~$x \in B$~~.
$x \in C$	~~Contradiction~~.
$x \notin B$	$\exists y \in (A \setminus B) \cap C$

Proof: Suppose x in C. Assume $x \notin B$.

then since $x \in A$, $x \notin B$. we have $x \in A \setminus B$.

Since $x \in C$, $x \in (A \setminus B) \cap C$. This contradict our assumption. So $x \in B$. Therefore $x \in C \rightarrow x \in B$ $\square$

Strategy: To use given of form $\neg P$, reexpress it.
To use a given of form $P \to Q$:
Modus ponens: if P is provable, add Q as given.
Modus tollens: contrapositive

Ex 3.2.5: Suppose $A \subseteq B$, $a \in A$ and $a \notin B \setminus C$, Prove $a \in C$.

Given	Goal
$A \subseteq B$	$a \in C$.
$a \in A$,
~~$a \notin B \setminus C$~~
~~$\neg(a \in B \wedge a \notin C)$~~
~~$a \notin B \vee a \in C$~~
$a \in B \to a \in C$.
$A \subseteq B$
iff $\forall x \in A \to x \in B$
$a \in A \to a \in B$

Proof: Since $a \in A$ and $A \subseteq B$, $a \in B$. Since $a \notin B \setminus C$, it follows that $a \in C$. $\square$.

See Ex 3.2.4 in text.