Algorithms and Decision Procedures for Regular Languages

Chapter 9

Decision Procedures

A **decision procedure** is an algorithm whose result is a Boolean value. It must:

- Halt
- Be correct

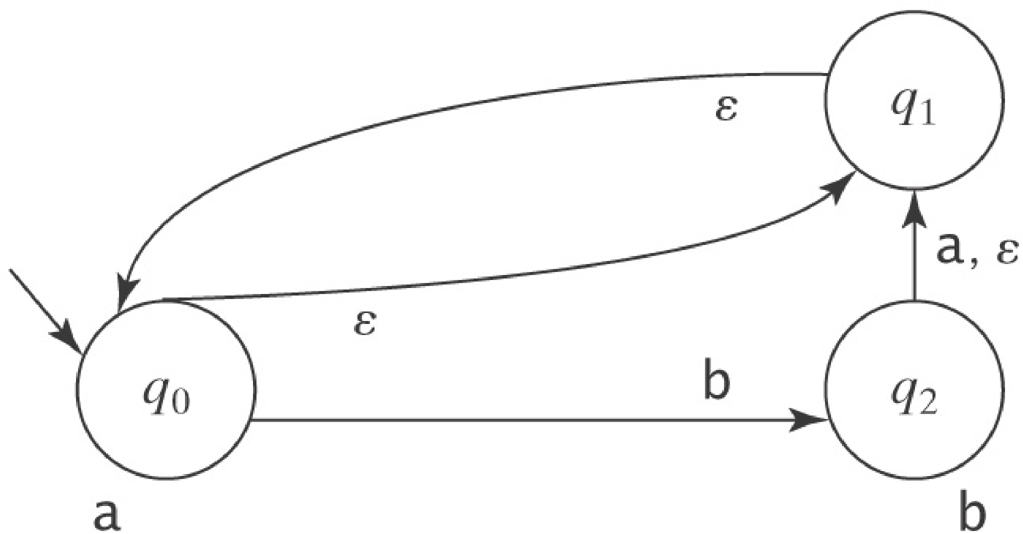
Important decision procedures exist for regular languages:

- Given an FSM M and a string s, does M accept s?
- Given a regular expression α and a string w, does α generate w?

Membership

We can answer the membership question by running an FSM.

But we must be careful:



Membership

decideFSM(*M*: FSM, *w*: string) =

If *ndfsmsimulate*(*M*, *w*) accepts then return *True*

else return *False*.

decideregex(α: regular expression, *w*: string) =

From α, use *regextofsm* to construct an FSM *M*

such that $L(\alpha) = L(M)$.

Return *decideFSM*(*M*, *w*).

Emptiness, Finiteness, Equivalence

- Given an FSM M, is $L(M)$ empty?
- Given an FSM M, is $L(M) = \Sigma_M^*$?
- Given an FSM M, is $L(M)$ finite?
- Given an FSM M, is $L(M)$ infinite?
- Given two FSMs M_1 and M_2, are they equivalent?

Emptiness

- Given an FSM *M*, is *L*(*M*) empty?

- The graph analysis approach:

  1. Mark all states that are reachable via some path from the start state of *M*.
  2. If at least one marked state is an accepting state, return *False*. Else return *True*.

- The simulation approach:

  1. Let $M' = ndfsmtodfsm(M)$.
  2. For each string *w* in Σ^* such that $|w| < |K_{M'}|$ do:
     Run $decideFSM(M', w)$.
  3. If M' accepts at least one such string, return *False*. Else return *True*.

Totality

- Given an FSM M, is $L(M) = \Sigma_M{}^*$?

1. Construct M' to accept $\neg L(M)$.
2. Return *emptyFSM*(M').

Finiteness

- Given an FSM M, is $L(M)$ finite?
  - The graph analysis approach:

Finiteness

- Given an FSM M, is $L(M)$ finite?
  - The graph analysis approach:

    The mere presence of a loop does not guarantee that $L(M)$ is infinite. The loop might be:

    - labeled only with ε,
    - unreachable from the start state, or
    - not on a path to an accepting state.

Finiteness

- Given an FSM *M*, is *L*(*M*) finite?
  - The graph analysis approach:

1. *M*´ = *ndfsmtodfsm*(*M*).
2. *M*´´ = *minDFSM*(*M*´).
3. Mark all states in *M*´´ that are on a path to an accepting state.
4. Considering only marked states, determine whether there are any cycles in *M*´´.
5. If there are cycles, return *False*. Else return *True*.

Finiteness

- Given an FSM M, is $L(M)$ finite?
  - The simulation approach:

1. $M' = ndfsmtodfsm(M)$.
2. For each string w in Σ^* such that $|K_{M'}| \leq |w| \leq 2 \cdot |K_{M'}| - 1$ do:
3. Run $decideFSM(M', w)$.
4. If M' accepts at least one such string, return *False*. Else return *True*.

Equivalence

- Given two FSMs M_1 and M_2, are they equivalent? In other words, is $L(M_1) = L(M_2)$?

Two solutions.

Equivalence

- Given two FSMs M_1 and M_2, are they equivalent? In other words, is $L(M_1) = L(M_2)$?

equalFSMs$_1$(M_1: FSM, M_2: FSM) =

1. $M_1{}'$ = *buildFSMcanonicalform*(M_1).
2. $M_2{}'$ = *buildFSMcanonicalform*(M_2).
3. If $M_1{}'$ and $M_2{}'$ are equal, return *True*, else return *False*.

Equivalence

- Given two FSMs M_1 and M_2, are they equivalent? In other words, is $L(M_1) = L(M_2)$?

  Observe that M_1 and M_2 are equivalent iff:

$$(L(M_1) - L(M_2)) \cup (L(M_2) - L(M_1)) = \varnothing.$$

equalFSMs$_2$(M_1: FSM, M_2: FSM) =

1. Construct M_A to accept $L(M_1) - L(M_2)$.
2. Construct M_B to accept $L(M_2) - L(M_1)$.
3. Construct M_C to accept $L(M_A) \cup L(M_B)$.
4. Return *emptyFSM*(M_C).

Minimality

- Given DFSM *M*, is *M* minimal?

  1. $M' = minDFSM(M)$.
  2. If $|K_M| = |K_{M'}|$ return *True*; else return *False*.

Answering Specific Questions

Given two regular expressions α_1 and α_2, is:

$$(L(\alpha_1) \cap L(\alpha_2)) - \{\varepsilon\} \neq \varnothing?$$

1. From α_1, construct an FSM M_1 such that $L(\alpha_1) = L(M_1)$.
2. From α_2, construct an FSM M_2 such that $L(\alpha_2) = L(M_2)$.
3. Construct M' such that $L(M') = L(M_1) \cap L(M_2)$.
4. Construct M_ε such that $L(M_\varepsilon) = \{\varepsilon\}$.
5. Construct M'' such that $L(M'') = L(M') - L(M_\varepsilon)$.
6. If $L(M'')$ is empty return *False*; else return *True*.

Answering Specific Questions

Given two regular expressions α_1 and α_2, are there at least 3 strings that are generated by both of them?

Summary of Closure Properties

- Compute functions of languages defined as FSMs:
  - Given FSMs M_1 and M_2, construct a FSM M_3 such that
    $$L(M_3) = L(M_1) \cup L(M_2).$$
  - Given FSMs M_1 and M_2, construct a new FSM M_3 such that
    $$L(M_3) = L(M_1)\, L(M_2).$$
  - Given FSM M, construct an FSM M_3 such that
    $$L(M_3) = (L(M))^*.$$
  - Given a DFSM M, construct an FSM M_3 such that
    $$L(M_3) = \neg L(M).$$
  - Given two FSMs M_1 and M_2, construct an FSM M_3 such that
    $$L(M_3) = L(M_1) \cap L(M_2).$$
  - Given two FSMs M_1 and M_2, construct an FSM M_3 such that
    $$L(M_3) = L(M_1) - L(M_2).$$
  - Given an FSM M, construct an FSM M_3 such that
    $$L(M_3) = (L(M))^R.$$

Summary of Decision Procedures

- Decision procedures that answer questions about languages defined by FSMs:
  - Given an FSM M and a string s, decide whether s is accepted by M.
  - Given an FSM M, decide whether $L(M)$ is empty.
  - Given an FSM M, decide whether $L(M)$ is finite.
  - Given two FSMs, M_1 and M_2, decide whether $L(M_1) = L(M_2)$.
  - Given an FSM M, is M minimal?