3) if $a = b \pmod N$, $c = d \pmod N$ $\Rightarrow a+c = b+d \pmod N$

proof: $\left.\begin{matrix} a-b = k_1 N \\ c-d = k_2 N \end{matrix}\right\} \downarrow + \quad (a+c)-(b+d) = N(k_1 - k_2)$

$\Rightarrow a+c = b+d \pmod N$

e.g: $50 + 148 \overset{24}{\equiv} 2 + 4 \overset{24}{\equiv} 6$

if $\left.\begin{matrix} a \overset{N}{\equiv} b \\ c \overset{N}{\equiv} d \end{matrix}\right\} \Rightarrow ac \overset{N}{\equiv} bd$

proof: $a-b = k_1 N$, $c-d = k_2 N$, $ac - bd = ac - bc + bc - bd$

$= c(a-b) + b(c-d)$

$= c k_1 N + b k_2 N$

$= N(ck_1 + bk_2)$

$\Rightarrow ac - bd = N(ck_1 + bk_2)$

$\Rightarrow ac = bd \pmod N$

e.g: $8 \times 16 \times 43 \times 71 \overset{7}{\equiv} 1 \times 2 \times 1 \times 1 = 2$

★ if $a \overset{N}{\equiv} b \Rightarrow a^k \overset{N}{\equiv} b^k$

e.g: $3^{20} \overset{4}{\equiv} (-1)^{20} \overset{4}{\equiv} 1$

$3^{22} \overset{7}{\equiv} 3 \times 3^{21} \overset{7}{\equiv} 3 \times (3^3)^7 \overset{7}{\equiv} 3 \times (27)^7 \overset{7}{\equiv}$

$3 \times (-1)^7 \overset{7}{\equiv} -3 \overset{7}{\equiv} \boxed{4}$

4) Find the Bézout coefficients of 533 and 195 using Euclidean algorithm:

→ step 1: finding gcd(533, 195)

$$195\,\overline{)533} \quad 2$$
$$-390$$
$$143\,\overline{)195} \quad 1$$
$$-143$$
$$52\,\overline{)143} \quad 2$$
$$-104$$
$$39\,\overline{)52} \quad 1$$
$$-39$$
$$13\,\overline{)39} \quad 3$$
$$-39$$
$$0$$

→ $533 = 195 \times 2 + 143$ IV

→ $195 = 143 \times 1 + 52$ III

→ $143 = 52 \times 2 + 39$ II

→ $52 = 39 \times 1 + 13$ I

→ $39 = 13 \times 3 + \underline{\underline{0}}$

$\Longrightarrow \gcd(533, 195) = \boxed{13}$

→ step 2: using substitutions and writing 13 as a linear combination of 195 and 533.

I → $13 = 52 - 39$

II ↓ $13 = 52 - (143 - 2 \times 52) = -1 \times 143 + 3 \times 52$

III ↓ $13 = -1 \times 143 + 3 \times (195 - 143) = 3 \times 195 - 4 \times 143$

IV ↓ $13 = 3 \times 195 - 4 \times (533 - 2 \times 195) = -4 \times 533 + 11 \times 195$

→ $13 = \boxed{-4} \times 533 + \boxed{11} \times 195$ ✓