

Q1

a) The relation is reflexive. Since for all $(a, a) \in \mathbb{Z}^+ \times \mathbb{Z}^+$, (a, a) is in R_1 .

The relation is transitive. Assume that (a, b) and (b, c) are in R_1 , then we can have $\frac{1}{a} \leq \frac{1}{b}$, $\frac{1}{b} \leq \frac{1}{c}$. So $\frac{1}{a} \leq \frac{1}{c}$ and (a, c) is in R_1 .

The relation is not symmetric. We pick $a=2$, $b=1$, $(2, 1)$ is in R_1 since $\frac{1}{2} \leq 1$, but $(1, 2)$ is not in R_1 since $1 > \frac{1}{2}$.

The relation is anti-symmetric. Assume that (a, b) and (b, a) are in R_1 , that is $\frac{1}{a} \leq \frac{1}{b}$ and $\frac{1}{b} \leq \frac{1}{a}$, since $a, b \in \mathbb{Z}^+$, $a \leq b$ and $b \leq a$, so $a=b$.

R_1 is neither partial order nor total order.

b). R_2 is reflexive. Since for all $(a, a) \in \mathbb{Z}$, $|a-a|=0 \leq 2$, so (a, a) is in R_2 .

R_2 is not transitive. Assume that $a, b, c \in \mathbb{Z}$ that $c=a+4$, $b=a+2$. (c, b) , (b, a) are in R_2 since $|c-b|=2 \leq 2$, $|b-a|=2 \leq 2$. However, $|c-a|=4 > 2$, so (c, a) is not in R_2 .

R_2 is symmetric. Assume that $(a, b) \in R_2$. So there exists $k \in \mathbb{Z}$ and $k \in [0, 2]$ that $|a-b|=k$. Since $|b-a|=k$, $(b, a) \in R_2$.

R_2 is neither partial order nor total order.

c)