UWO CS2214

Tutorial #1

Problem 1 First, express each of the compound propositions below using basic (atomic) propositions $p, q, r, \ldots$ (define them) and logical connectives such as implications ($\rightarrow$). Then, **determine if the following set of compound propositions is consistent**, i.e. if there is an assignment of truth values to each of the atomic propositions that makes all compound propositions true.

1. The campus server does not work if the internet is off.
2. Students can skype during the test when the prof is distracted.
3. If the classroom phone does not ring then the prof is not distracted.
4. Students cannot skype during the test unless the internet is on.
5. If the classroom phone rings then the campus server works.

Solution 1

1. $\neg I \rightarrow \neg C$, where I and C stand respectively for "internet is on" and "the campus server works".
2. $D \rightarrow S$, where D and S stand respectively for "the prof is distracted" and "Students can skype during the test".
3. $\neg R \rightarrow \neg D$, where R and D stand respectively for "the classroom phone rings" and "the prof is distracted".
4. $S \rightarrow I$, where I and S stand respectively for "internet is on" and "Students can skype during the test".
5. $R \rightarrow C$, where R and C stand respectively for "'the classroom phone rings" and "the campus server works".

For an assignment of truth values of the atomic propositions that makes all compound propositions true, we must have

$$(\neg I \rightarrow \neg C) \wedge (D \rightarrow S) \wedge (\neg R \rightarrow \neg D) \wedge (S \rightarrow I) \wedge (R \rightarrow C)$$

which is satisfiable with

$$I = \text{true}, S = \text{true}, R = \text{true}, D = \text{true}, C = \text{true}.$$

Problem 2 Show that

$$p \leftrightarrow q \;\equiv\; (p \wedge q) \vee (\neg p \wedge \neg q)$$

is a tautology. You should do it in two different ways using

(a) truth tables,

(b) logical equivalences.

Solution 2 Using a truth table poses no difficulty. For the solution based on logical equivalence, replace $p \leftrightarrow q$ with $(p \Rightarrow q) \wedge (q \Rightarrow p)$, then with $(\neg p \vee q) \wedge (\neg q \vee p)$, then apply distributivity of $\wedge$ over $\neg q$, then cancel trivial terms like $\neg p \wedge p$, finally apply commutativity.

Problem 3 Determine which of the propositional formulas below are satisfiable. In case a propositional formula, determine an assignment of the propositional variables for which the formula is satisfied.

1. $p \wedge (q \vee \neg p) \wedge (\neg q \vee \neg r)$
2. $p \wedge (q \vee \neg r) \wedge (\neg q \vee \neg r)$
3. $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee \neg r)$

Solution 3

1. satisfiable with $(p, q, r) = (\text{true}, \text{true}, \text{false})$,
2. satisfiable with $(p, q, r) = (\text{true}, \text{true}, \text{false})$,
3. satisfiable with $(p, q, r) = (\text{true}, \text{false}, \text{false})$.