PROVE THAT $\forall n \in \mathbb{N}$

$$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$$

- BASE CASE: FOR $\boxed{n=0}$ // $\sum_{i=0}^{0} x^i = x^0 = 1$ ✓

RHS: $\frac{x^{0+1}-1}{x-1} = \frac{x^1-1}{x-1} = 1$

- INDUCTION STEP: ASSUME THAT, FOR $k \in \mathbb{N}$,

$$\boxed{\sum_{i=0}^{k} x^i} = \boxed{\frac{x^{k+1}-1}{x-1}} \quad (I.H)$$

WANT: $\sum_{i=0}^{k+1} x^i = \frac{x^{k+1+1}-1}{x-1} = \frac{x^{k+2}-1}{x-1}$

$$\sum_{i=0}^{k+1} x^i = \boxed{\sum_{i=0}^{k} x^i} + x^{k+1} \overset{I.H}{=} \boxed{\frac{x^{k+1}-1}{x-1}} + x^{k+1} =$$

$$= \frac{x^{k+1}-1+(x-1)\cdot x^{k+1}}{x-1} = \frac{x^{k+1}-1+x^{k+2}-x^{k+1}}{x-1}$$

✓

- CONCLUSION: BY ∎ INDUCTION, $\forall n \in \mathbb{N}$

$$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$$

LET $F(0)=0$, $F(1)=1$,

$F(n+2) = F(n) + F(n+1) \quad \forall n \in \mathbb{N}$

PROVE: $\forall n \in \mathbb{N}\setminus\{0\}\ F(n-1)\cdot F(n+1) - F(n)^2 = (-1)^n$

- BASE CASE: $\boxed{n=1}$: $F(0)F(2) - F(1)^2 =$

$= 0\cdot(0+1) - 1^2 = 0 - 1 = (-1)^1$ ✓

- IND. STEP: ASSUME, FOR A $k \in \mathbb{N}\setminus\{0\}$, $\boxed{k\geq 1}$

$\rightarrow \boxed{F(k-1)F(k+1) - F(k)^2} = (-1)^k$ (I.H.)

WANT: $F(k+1-1)F(k+1+1) - F(k+1)^2 = (-1)^{k+1}$

$F(k)\cdot F(k+2) - F(k+1)^2 = (-1)^{k+1}$

$F(k)\left(F(k)+F(k+1)\right) - F(k+1)^2 =$

$F(k)^2 + F(k)F(k+1) - F(k+1)^2 =$

$F(k)^2 + F(k+1)\underbrace{\left(F(k) - F(k+1)\right)}_{-F(k-1)} =$

$F(k+1) = F(k) + F(k-1)$

$-F(k-1) = F(k) - F(k+1)$

$F(k)^2 - F(k+1)F(k-1) = -\left(F(k+1)F(k-1) - F(k)^2\right)$

$\overset{\text{I.H.}}{=} -\boxed{(-1)^k} = (-1)^{k+1}$ ✓

- CONCLUSION: BY INDUCTION, $\forall n \in \mathbb{N}\setminus\{0\}$

$F(n-1)F(n+1) - F(n)^2 = (-1)^n$

EQUIV.

$\forall n \in \mathbb{N},\ n \geq 1$