

Review Session

COMPSCI 3331

Fall 2022

Question 1

1. Let G be the CFG defined by the following set of productions.

$$S \rightarrow bbAaA \mid SSaa \mid aa \mid ABC$$

$$A \rightarrow Ab \mid Ac \mid CC$$

$$B \rightarrow BA \mid bb \mid Dd$$

$$C \rightarrow DA \mid \varepsilon$$

$$D \rightarrow a$$

Give an equivalent grammar to G that has no ε -productions.

Question 2

2. Let G be the CFG defined by the following set of productions.

$$S \rightarrow SbaB \mid aa \mid ABC$$

$$A \rightarrow Ba \mid DaDd$$

$$B \rightarrow BA \mid ca \mid Dd$$

$$C \rightarrow DA \mid \varepsilon$$

$$D \rightarrow a$$

Convert the grammar to CNF.

Question 3

3. Let $G = (V, \Sigma, P, S)$ be a CFG in CNF. Give an $O(n^3)$ algorithm for the following problem:

- ▶ Input: A word w and a nonterminal $A \in V$.
- ▶ Output: the value
$$n_A = \max\{|u| : u \text{ is a suffix of } w \text{ and } A \Rightarrow^* u\}.$$

That is, n_A is the length of the longest suffix of w that is generated by A in the grammar.

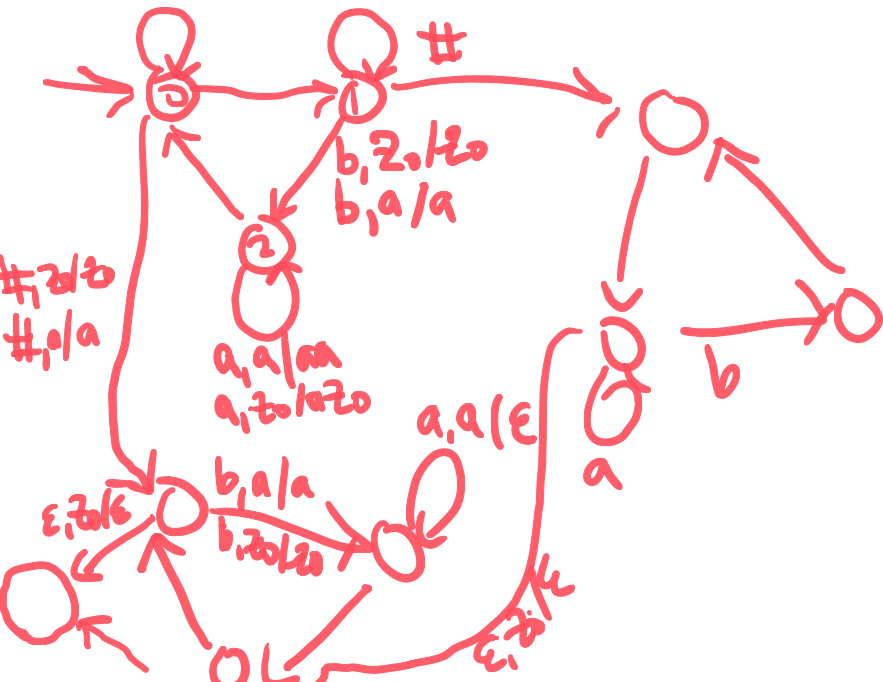
Question 4

4. Construct PDAs for the following languages:

- (a) $L = \{a^n b^m xy : x, y \in \{0, 1, 2\}, x \equiv n(\text{mod } 3) \text{ and } y \equiv m(\text{mod } 3)\}$.
- (b) $L = \{w \# x : w, x \in \{a, b\}^*, |w|_a = |x|_a, |w|_b \equiv |x|_b(\text{mod } 3)\}$.

Be sure to indicate what the starting stack symbol is for your PDA and how your PDA accepts words.





Question 5

5. Let C be a fixed integer. Extend the language from Assignment 3 as follows:

$$L_C = \{x\#1^n : n \geq 0, x \in \{a,b\}^* \text{ and } n - C \leq |x|_a \leq n + C\}$$

Give a context-free grammar for L_C . The productions in your grammar will depend on the value of C . Describe them using a uniform notation (e.g., by using consistently named variables or consistently defined productions, for instance).

Question 6

6. Consider the following modified language from Assignment 3:

$$L = \{u\#v : u, v \in \{0, 1\}^* \text{ and } \text{bin}(v^R) = \text{bin}(u) + 2\}$$

Give a PDA that accepts L .

Question 7

7. A CFG G is in Griebach Normal Form (GNF) if every production has the form

$$A \rightarrow aB_1B_2\cdots B_n$$

for some letter a and nonterminals B_1, B_2, \dots, B_n (where $n \geq 0$). Any grammar (that does not derive ε) can be converted to GNF. Given this fact, show that for any CFG L that does not include ε , you can construct a PDA M that accepts L in the following additional conditions:

- ▶ The PDA accepts by empty stack.
- ▶ The PDA M does not have any ε -transitions. That is, there are no rules of the form $\delta(q, \varepsilon, \gamma) = \{(q', \beta), \dots\}$ for any stack symbol γ .

Question 8

8. Prove that the following languages are not context-free:

(a) $L = \{a^p : p \text{ is a prime number}\}$.

(b) $L = \{a^n b^{n^3} : n \geq 0\}$.

(c) $L = \{w \in \{a, b\}^* : |w|_b = 2^{|w|_a}\}$.

$$R = \frac{a^* b^*}{2^n}$$

$$L \cap R = \{a^n b^{2^n} : n \geq 0\}. \quad z = a^n b^{2^n}$$

① v, x cross boundary.

② v, x all a 's

③ v, x all b 's

④ v is a 's and x is b 's.

$Z = uvwxyz$ EXAMINE TWO SUBCASES.

$$u = a^i$$

$$v = a^j$$

$$w = a^{n-i-j} b^k$$

$$x = b^l$$

$$y = b^{2^n - l - k}$$

IF $j=0 \Rightarrow l \neq 0$.

$$uv^2wx^2y = a^n b^{2^n + l}$$

NOT IN LANGUAGE.

IF $j \neq 0$.

$$uv^2wx^2y = a^{n+j} b^{2^n + l}$$

WANT TO SHOW $2 \nmid \#a's \neq \#b's$

$$2^{(n+j)} \neq 2^n + l.$$

$$0 \leq n \Rightarrow l < 2^n \quad 2^{n+l} < 2^n + 2^n = 2^{n+1}$$



Question 9

9. For each of the languages in the previous question, give an informal description of a multi-tape TM that recognizes the language.

(a) $L = \{a^p : p \text{ is a prime number}\}.$

(b) $L = \{a^n b^{n^3} : n \geq 0\}.$

(c) $L = \{w \in \{a, b\}^* : |w|_b = 2^{|w|_a}\}.$

Question 10

10. Show that the following language is r.e.:

$$L = \{e(M_1)\#e(M_2) : L(M_1) \cap L(M_2) \neq \emptyset\}$$

M_0 - universal TM.

$r \quad w, e(M) \rightarrow w \in L(M)?$

LET M_0 accept L . NONDETERM.

① NONDETERM WRITE A WORD
 w ON TWO TAPES.

② RUN M_1 AND M_2 IN PARALLEL

ON SEPARATE COPIES of w.
(USING Mu)

③ IF BOTH SAM yes \Rightarrow return yes
IF EITHER SAM no \Rightarrow return no
IF EITHER GO INTO INF. LOOP
 \Rightarrow ALSO GO INF. LOOP.

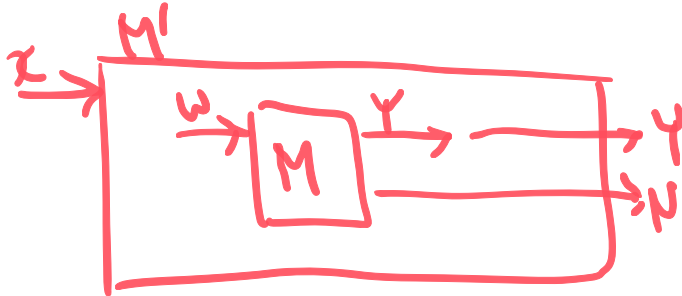
Question 11

11. Show that the following problem is undecidable by reduction: Given a TM M , is $L(M)$ a finite language?

- CHOOSE HAVING PROBLEM.

$(e(M), w) \rightarrow M'$

$w \in L(M') \Rightarrow L(M') \text{ finite}$
 $w \notin L(M) \Rightarrow L(M') \text{ infinite.}$



$$w \in L(M) \Rightarrow L(M') = \Sigma^*$$

$$w \notin L(M) \Rightarrow L(M') = \emptyset.$$

Question 12

12. Show that the following language is decidable:

$L_{ND} = \{ e(M) : M \text{ is a nondeterministic TM} \}$.

Question 13

13. Show that the following problem is either decidable or undecidable: Given a CFG G is $L(G)$ infinite? (Hint: review the proof of the pumping lemma for CFLs.)