

Solutions (Logical Equivalence)

1)

P	Q	$\neg P$	$(\neg P \wedge Q)$	$P \vee (\neg P \wedge Q)$	$P \vee Q$	$\neg(P \vee (\neg P \wedge Q))$	$\neg(P \vee Q)$
0	0	1	0	0	0	1	1
0	1	1	1	1	1	0	0
1	0	0	0	1	1	0	0
1	1	0	0	1	1	0	0

True

$$2) \quad \neg(P \wedge Q) \vee (P \wedge Q)$$

$$= \neg A \vee A$$

[consider $(P \wedge Q) = A$]

$$= \top$$

[Law of excluded middle]

$$3) \quad (\neg P \wedge (P \vee Q))$$

[expanding the expression
by distributivity law]

$$= (\neg P \wedge P) \vee (\neg P \wedge Q)$$

[Law of non-contradiction]

$$= \perp \vee (\neg P \wedge Q)$$

[Identity law]

$$= \neg P \wedge Q$$

$$4) \quad \neg(P \rightarrow q)$$

$$= \neg(\neg P \vee q) \quad [\text{we know that } A \rightarrow B \equiv \neg A \vee B]$$

$$= \neg(\neg P) \wedge \neg(q) \quad [\text{De Morgan's Law}]$$

$$= P \wedge \neg(q) \quad [\text{Double negation law}]$$

$$= P \wedge \neg q$$

$$\therefore \underline{\neg(P \rightarrow q) \equiv P \wedge \neg q}$$

$$5) \quad \neg(P \vee (\neg P \wedge q))$$

$$= \neg((P \vee \neg P) \wedge (P \vee q)) \quad [\text{Distributive law}]$$

$$= \neg(T \wedge (P \vee q)) \quad [\text{Law of excluded middle}]$$

$$= \neg(P \vee q) \quad [\text{Identity Law}]$$

$$\therefore \underline{\neg(P \vee (\neg P \wedge q)) \equiv \neg(P \vee q)}$$