1.

BASE STEP:

When height is 0, then $l(Q) = 0$, $i(Q) = 1$, $n(Q) = 1 = l(Q)+i(Q)$

INDUCTIVE STEP:

Assuming the inductive hypothesis that we can reach h, currently we can have

$i(Q) = 4^0+4^1+4^2+\cdots\cdots+4^{(h-1)} = (4^{(h-1)}-1)/3$, $l(Q) = 4^{(h-1)}$, $n(Q) = (4^h-1)/3$

then when the height is h, all leave nodes in h full quadtree are changed to internal nodes, then we have $i(Q) = (4^h-1)/3$, and new leave nodes $l(Q) = 4^{(h-1)}$, add up two parts, we can get $i(Q)+l(Q)=(4^h)/3$. Also, the current nodes number equals to $4^0+4^1+4^2+\cdots\cdots+4^h = (4^h)/3$, which is the same.

Hence, $p(h) \to p(h+1)$ is true for all positive integers h. For any full quadtree Q, we have: $n(Q)=l(Q)+i(Q)$.

2.

BASE STEP:

when $h = 0$, $l(Q) = 0$, $4^{(h(Q))} = 1$, Hence, $l(Q) = 0<4^{(h(Q))} = 1$

RECURSIVE STEP:

Assuming the inductive hypothesis that we can reach h, then $l(Q) = 4^{(h+1)}$, in this case, $l(Q)-4^{(h(Q))} = 4^{(h)}-4^{(h+1)}$, which is easy to see it is a negative result.

Hence, $p(h) \to p(h+1)$ is true for all positive integers h. For any full quadtree Q, we have: $n(Q)=l(Q)+i(Q)$.

3.

BASIS STEP: The result holds for a full quadtree consisting only of a root, $n(Q) = 1$ and $h(Q) = 0$: Hence, $n(Q) = n(Q) \leqslant \Sigma(j=0,\ 1)\,4\hat{}j = 4$

RECURSIVE STEP: $l(Q1)<4^{(h(Q1))}$, $l(Q2)<4^{(h(Q2))}$, $l(Q3)<4^{(h(Q3))}$, $l(Q4)<4^{(h(Q4))}$ whenever Q1, Q2, Q3 and Q4 are full quadtrees.