Q1. $x_s = \frac{(x+1)\cdot 600}{2} + 200 = \frac{x+1}{2} + 500$

$y_s = \frac{(y+1)\cdot 100}{2} + 100 = \frac{y+1}{2} + 150$.

so the matrix should be $(\frac{x+1}{2} + 500, \frac{y+1}{2} + 150)$.

Q2 $M_0 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

$M_1 = TR = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 2 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$M_2 = RT = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{-3+2\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{3\sqrt{3}+2}{2} \\ 0 & 0 & 1 \end{bmatrix}$

$M_0 M_1 = M_0 M_2 = \begin{bmatrix} \frac{-1+2\sqrt{3}}{2} \\ \frac{\sqrt{3}+2}{2} \\ 0 \end{bmatrix}$

transformed by M_1: first rotate counterclockwise for 30 degree,
then move up and right.

M_2: move upward for 3 and right for 2,
then rotate counterclockwise for 30 degree

Q3: S would be written as $\begin{bmatrix} 1 & m & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

so $S^{-1} = \begin{bmatrix} 1 & -m & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q_4: $\quad V_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$, $\quad W_3 = \begin{bmatrix} \frac{9-\sqrt{3}}{2} \\ \frac{5-\sqrt{3}}{2} \\ 1 \end{bmatrix}$

Assume that $S = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $T = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix}$, $R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

$R = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\quad S = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad T = \begin{bmatrix} 1 & 0 & \frac{9-\sqrt{3}}{2} \\ 0 & 1 & \frac{9-\sqrt{3}}{2} \\ 0 & 0 & 1 \end{bmatrix}$

$W_1 = \begin{pmatrix} \frac{11-\sqrt{3}}{2} \\ \frac{11-\sqrt{3}}{2} \\ 1 \end{pmatrix}$ $\quad W_2 = \begin{pmatrix} \frac{5-\sqrt{3}}{2} \\ \frac{9-\sqrt{3}}{2} \\ 1 \end{pmatrix}$

$W = TSRV$