



Using call-by-value reduction:

$$\begin{aligned} & \text{XOR } p \text{ (NOT } p) \\ &= (\lambda p q. p \text{ (NOT } q) \text{) } p \text{ (NOT } p) \text{ (applying XOR definition)} \\ &= (\lambda q. p \text{ (NOT } q) \text{) } p \text{ (NOT } p) \text{ (applying } p \text{ for } q) \\ &= p \text{ (NOT (NOT } p)) \text{ (applying NOT for } p \text{ and (NOT } p)) \\ &= p \text{ ((}\lambda p q r. p \text{ } r \text{ } q) \text{) } p \text{ (NOT } p) \text{ (applying NOT definition for (NOT } p)) \\ &= p \text{ ((}\lambda q r. p \text{ (NOT } p) \text{) } q) \text{) } p \text{ (NOT } p) \text{ (applying } p \text{ for } p \text{ and (NOT } p)) \\ &= p \text{ (} p \text{ (NOT } p) \text{ (NOT } p)) \text{ (applying } (\lambda q r. p \text{ (NOT } p) \text{) } q) \text{ for } p \text{ and (NOT } p)) \\ &= p \text{ (} p \text{ (}\lambda p q r. p \text{ } r \text{ } q) \text{ (NOT } p) \text{ (NOT } p)) \text{ (applying (NOT } p) \text{ for } q) \\ &= p \text{ (} p \text{ (}\lambda r. \text{(NOT } p) \text{) } r \text{ (NOT (NOT } p))) \text{ (NOT } p) \text{ (applying } \lambda p q r. p \text{ } r \text{ } q \text{ for } p, \text{(NOT } p) \text{ and (NOT } p)) \\ &= p \text{ (} p \text{ (NOT } p) \text{ (NOT (NOT } p))) \text{ (applying } \lambda r. \text{(NOT } p) \text{) } r \text{ (NOT (NOT } p)) \text{ for } r) \\ &= p \text{ (NOT } p) \end{aligned}$$

Using call-by-name reduction:

$$\begin{aligned} & \text{XOR } p \text{ (NOT } p) \\ &= (\lambda p q. p \text{ (NOT } q) \text{) } p \text{ (NOT } p) \text{ (applying XOR definition)} \\ &= p \text{ (NOT (NOT } p)) \text{ } q \text{ (applying } p \text{ for } p \text{ and (NOT } p)) \\ &= p \text{ ((}\lambda p q r. p \text{ } r \text{ } q) \text{) } p \text{ (NOT } p)) \text{ } q \text{ (applying NOT definition for (NOT } p)) \\ &= p \text{ ((}\lambda q r. p \text{ (NOT } p) \text{) } q) \text{) } p \text{ (NOT } p)) \text{ } q \text{ (applying } p \text{ for } p \text{ and (NOT } p)) \\ &= p \text{ (} p \text{ (NOT } p) \text{ (NOT } p)) \text{ } q \text{ (applying } (\lambda q r. p \text{ (NOT } p) \text{) } q) \text{ for } p \text{ and (NOT } p)) \\ &= p \text{ (} p \text{ (}\lambda p q r. p \text{ } r \text{ } q) \text{ (NOT } p) \text{ (NOT } p)) \text{ } q \text{ (applying (NOT } p) \text{ for } q) \\ &= p \text{ (} p \text{ (}\lambda r. \text{(NOT } p) \text{) } r \text{ (NOT (NOT } p))) \text{ (NOT } p)) \text{ } q \text{ (applying } \lambda p q r. p \text{ } r \text{ } q \text{ for } p, \text{(NOT } p) \text{ and (NOT } p)) \\ &= p \text{ (NOT } p) \text{ } q \text{ (applying } \lambda r. \text{(NOT } p) \text{) } r \text{ (NOT (NOT } p)) \text{ for } r) \\ &= p \text{ (NOT } p) \end{aligned}$$

In both call-by-value and call-by-name reductions, the final result is $p \text{ (NOT } p)$, which represents the logical operator for the boolean equality.

