# Regular Languages

COMPSCI 3331

# Outline

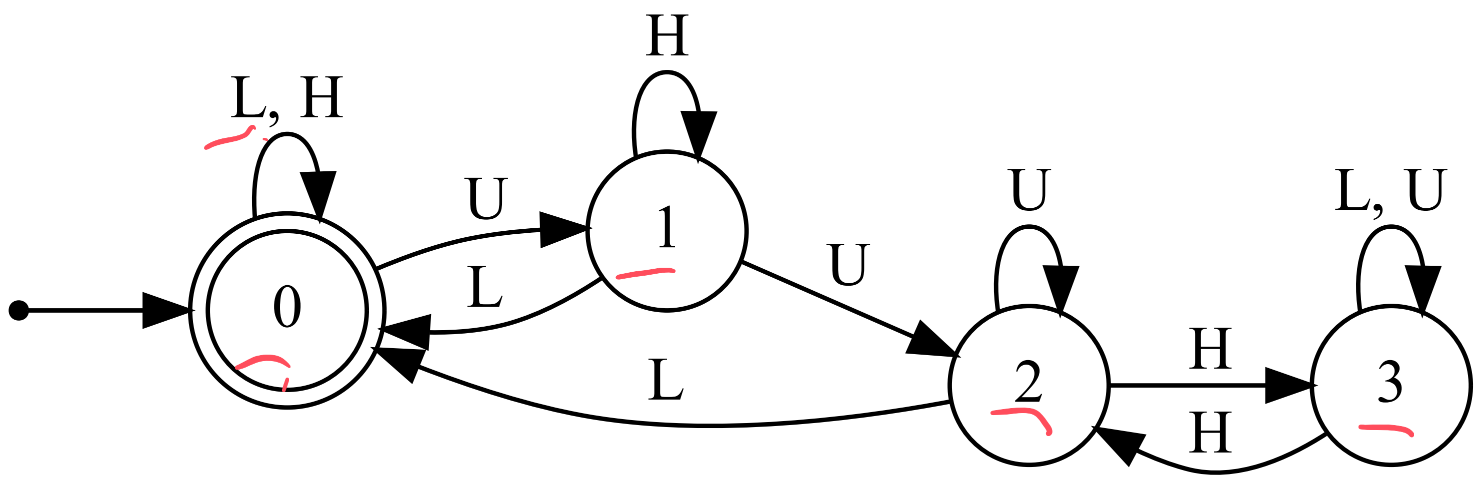
- Motivation for regular languages.
- Regular languages
- Deterministic finite automata.

# Regular Languages - Motivation

- Languages recognized with a fixed amount of memory.
- Developed to model **how circuits work** and early **models of neural behaviour**.
- Used in text matching, regular expressions, compilers, model checking, protocol verification, natural language processing.

# Example: Key Fob

0 - all doors locked
1 - driver's door unlocked




L - lock
U - unlock
H - hatch

2 - all doors unlocked
3 - hatch open

# Deterministic Finite Automaton

A deterministic finite automaton (DFA) consists of:

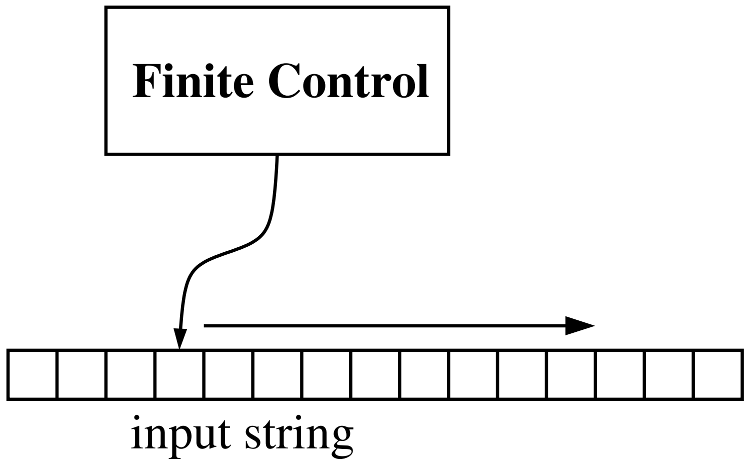
- a finite set of **states** the DFA can be in;
- an **alphabet**, specifying the set of letters the DFA can process;
- a **transition function**, which specifies how to update our state based on the current input letter.
- **start** and **final** states, which specify how to accept or reject words.

# Deterministic Finite Automaton

- The alphabet in the fob example is `L,U,H`.
- Transition Function: what effect do actions from our alphabet have?
- Could give names to states: “hatch open”, “driver’s door unlocked”, etc.

# Deterministic Finite Automaton

How do we visualize our DFA working?




"Finite control": the current state and instructions provided by the transition function.

# Deterministic Finite Automaton

***Formally***, a DFA is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$ where

- $Q$ is a finite set of states,
- $\Sigma$ is a finite alphabet.
- $\delta : \underline{Q \times \Sigma} \to Q$ is the transition function.
- $q_0 \in Q$ is the start state.
- $F \subseteq Q$ is the ~~final state~~ set of final states.

$q, q' \in Q$

$\delta(q, a) = q'$

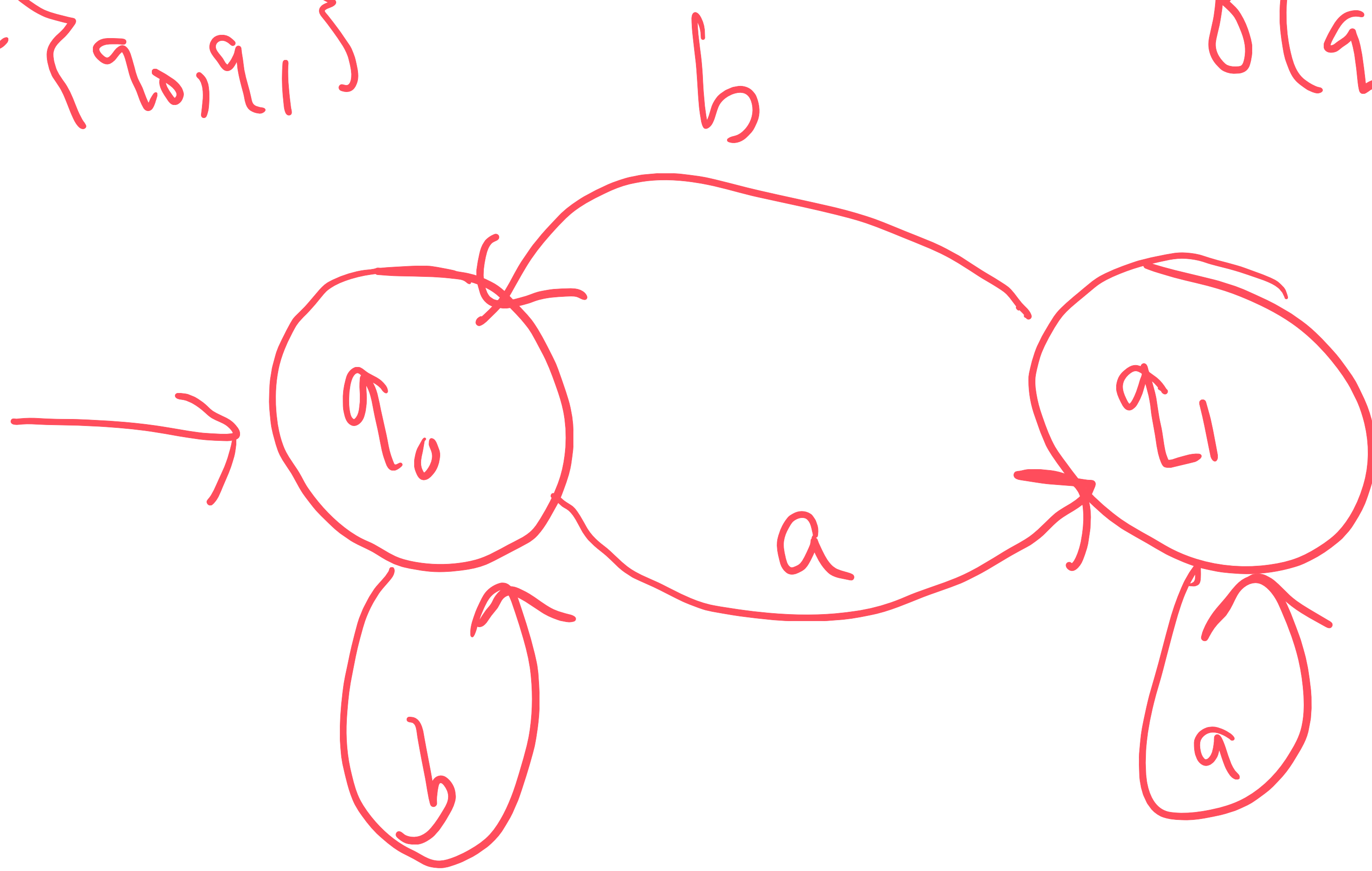
- in state $q$ and we see letter $a$ → go to state $q'$

# Drawing DFAs

- Draw DFA with states, labelled transitions.
- Special indications for start and final states.

$Q = \{q_0, q_1\}$

$\delta(q_0, a) = q_1$



# Accepting words and languages

- Each word $w \in \Sigma^*$ traces a path from the start state to some state in the automaton.
- If the state we reach is a final state ($\in F$) then the word $w$ is **accepted** by $M$. Otherwise, it is rejected.
- The language accepted by a DFA $M$ is the set of all strings accepted by $M$.

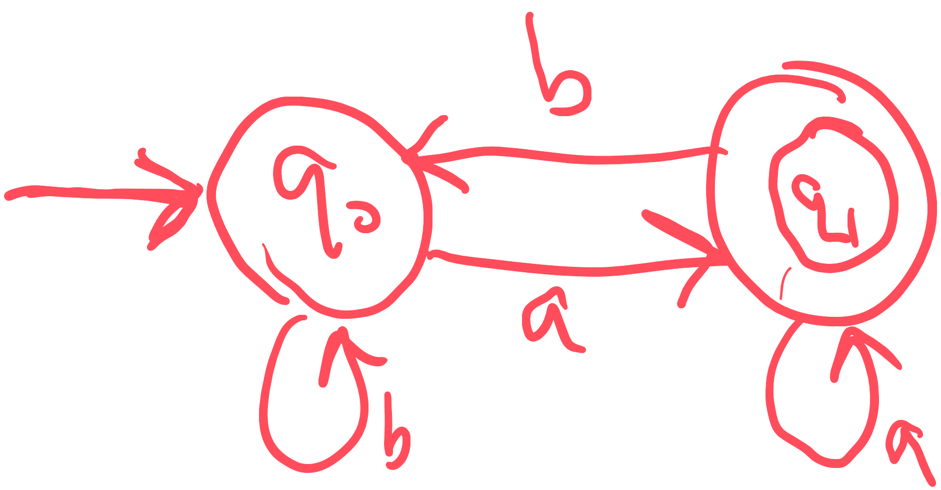
$F = \{q_1\}$

aaba $\rightarrow q_1$ ✓ accepted.

abab $\rightarrow q_0$ X rejected

ab X

aa ✓



# Acceptance: Formal Definition

- The transition function acts on letters from $\Sigma$.
- Extend it to work on **words** from $\Sigma^*$ with a recursive definition:
  - $\delta(q, \varepsilon) = q$ for all $q \in Q$;.
  - $\delta(q, wa) = \delta(\delta(q, w), a)$ for all $q \in Q$, $w \in \Sigma^*$ and $a \in \Sigma$.
- A word $w \in \Sigma^*$ is accepted by a DFA $M = (Q, \Sigma, \delta, q_0, F)$ if $\delta(q_0, w) \in F$. The language accepted by $M$ is

$$\underline{L(M)} = \{w \in \Sigma^* \ : \ \delta(q_0, w) \in F\}.$$

# Language accepted by a DFA

- Can ask "given a DFA $M$, what language does it accept?"
- Establish through proof: define a language $L$ and establish that $L(M) \subseteq L$ and $L \subseteq L(M)$.

$$L := \{ w \in \{a, b\}^* : \text{the last letter of } w \text{ is } a \}$$

# Finding a suitable DFA

- "For this language $L$, find a DFA $M$ such that $L(M) = L$."
- **Assumption**: for the language $L$, **there exists** a DFA $M$ such that $L(M) = L$.
- Tips:
  - Think of the states as "what do we need to remember?"
  - Define set of states, and then the letters that move us from state to state.
  - **Learn by doing!**

# The Regular Languages

A language $L \subseteq \Sigma^*$ is a **regular language** if there exists a DFA $M = (Q, \Sigma, \delta, q_0, F)$ such that $L(M) = L$.

- Not every language is a regular language.

# Are DFAs Unique?

If $M_1, M_2$ are (in some way) distinct DFAs, is it true that $L(M_1) \neq L(M_2)$?

- can always have superfluous unconnected states.
- can have different ways to define the language.

# Return to Motivation

What can DFAs be used to model?

- Finite sets.
- Objects which only require a fixed amount of memory.
- “Easy” jobs in programming language recognition ~~jobs~~.