Lecture 12.

$n^2 - n + 41$.

$n = 41$ $f(n) = 41^2$ not prime.

$n \in [0, 41) \Rightarrow f(n)$ is a prime number.

HW8:

Q8: $\exists x \forall y (P(x,y) \wedge \neg Q(x,y))$

$\because \exists x \forall y \neg(\neg P(x,y) \vee Q(x,y))$ De Morgan's law.

$\because \exists x \neg \exists y$ ....

$\because \neg \forall x \exists y$ .....

$\because \neg(\ )/\neg(\ )/\neg(\ )\ (P(x,y) \to Q(x,y))$.

three cases above.

Q12: $A = \{1, 2\}$.

$P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$.

only the above four sets are the element of the power set.

$B \subseteq P(A)$ is true for $\underline{2^4}$ numbers of b.

§ 3.3.

To prove $\forall x\, P(x)$.

- Let x be arbitrary - Change goal to. $P(x)$.

Let x be arbitrary

proof of $P(x)$.

Since x is arbitrary, we're shown that for x, $P(x)$ holds.

e.g. three sets A, B and C. $A \setminus B$ and C are disjoint.

Prove that $A \cap C \subseteq B$.

Given	Proof
$(A \setminus B) \cap C = \phi$	$A \cap C \subseteq B$: $\forall x \{x \in A \cap C \to x \in B\}$.
$x \in A \cap C$	$x \in B$.

$x \in A \wedge x \in C$

$\Downarrow$

Given	Proof
$(A \setminus B) \cap C = \phi$	
$x \in A$	$x \in B$.
$x \in C$	

Proof by contradiction: $(A \setminus B) \cap C \neq \phi$

to show $A \cap C \subseteq B$, we have to prove that

$\forall x (x \in A \cap C \to x \in B)$

Let x be arbitory, and assume $x \in A \cap C$,

then $x \in A$ and $x \in C$.

Suppose $x \notin B$. Then $x \in A \setminus B$ and $x \in C$, so

$x \in A \setminus B \cap C$.

This contradict $A \setminus B$ and C are disjoint, so $x \in B$.

Sice x is arbiturty, we have shown that $A \cap C \subseteq B$. $\square$

Shortcut: To prove $\forall x (P(x) \to Q(x))$.

- Let x be arbitrary,
- Add $P(x)$ to the given,
- Change the goal to $Q(x)$.

Let $P(x)$.

Proof of $Q(x)$.

Since x is arbitrary, $\forall x P(x)$, $Q(x)$ holds.

$\forall x (P(x) \to Q(x))$.

e.g. $\forall x \in A \ P(x)$.

$= \forall x (x \in A \to P(x))$

the same as the shortcut above.

if $A \cap B = A$, then $A \subseteq B$.

Given	Goal
(none)	$A \cap B = A \to A \subseteq B$.
$A \cap B = A$	$A \subseteq B$.

Let x is arbitrary and $x \in A$,

$x \in B$.

Since x is arbitrary, $A \cap B = A$ then $A \subseteq B$ holds.