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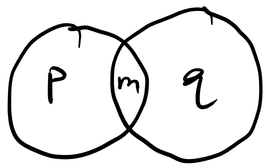
1) 

divide $A \cup B$ into three parts,

$p = A \cap (\neg B)$ $\quad q = (\neg A) \cap B$.

$m = A \cap B$

$A \triangle B = p \cup q$, $\quad (A \cup B) \triangle (A \cap B)$

$= (p \cup m \cup q) \triangle m = p \cup q$.

$\therefore A \triangle B = (A \cup B) \triangle (A \cap B)$.

2) Because there're only 2 sets, so we just need to determine there're how many conditions in the first set.

$S_n = C_n^1 + C_n^2 + \cdots + C_n^{n-1}$

$= (C_n^1 + \cdots + C_n^n) - C_n^n$

$= 2^{n-1} - 1$.

3) $a_n = \frac{1}{2} + \frac{1}{2\times 3} + \cdots + \frac{1}{n(n+1)}$

$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} \cdots\cdots - \frac{1}{n+1}$

$= \frac{n}{n+1}$.

4) if n is odd, $t_n = C_n^1 \cdot 2^{n-1} + C_n^3 \cdot 2^{n-3} + \cdots + C_n^{\frac{n+1}{2}} \cdot 2^{\frac{n-1}{2}}$.

$t_{n-1} = C_{n-1}^1 \cdot 2^{n-2} + C_{n-1}^3 \cdot 2^{n-4} + \cdots + C_{n-1}^{\frac{n-1}{2}} \cdot 2^{\frac{n-1}{2}}$

if n is even, $t_n = C_n^1 \cdot 2^{n-1} + C_n^3 \cdot 2^{n-3} + \cdots + C_n^{\frac{n}{2}} \cdot 2^{\frac{n}{2}}$

$t_{n-1} = C_{n-1}^1 \cdot 2^{n-2} + \cdots\cdots\cdots\cdots + C_{n-1}^{\frac{n}{2}} \cdot 2^{\frac{n-2}{2}}$.

$t_n - t_{n-1} = 3^{n-1}$

$t_n = t_{n-1} + 3^{n-1}$.

Bonus:

$$t_1 - t_0 = 1$$

$$t_2 - t_1 = 3$$

$$\vdots \quad \vdots$$

$$t_n - t_{n-1} = 3^{n-1}.$$

$$\therefore t_n = 1 + 3 + \cdots + 3^{n-1}$$

$$= 1 \cdot \frac{1-3^n}{1-3}$$

$$= \frac{3^n - 1}{2}.$$