

CS 2211

Systems Programming

Recursion

Recursive Definitions

- **Recursion:**
defining something *in terms of itself*
- **Recursive definition**
 - Uses the word or concept being defined ***in the definition itself***
 - Includes a **base case** that is defined directly, ***without*** self-reference

Recursive Definitions

- A recursive definition consists of two parts:
 - The *base case*: this defines the “*simplest*” case or starting point
 - The *recursive part*: this is the “*general case*”, that describes all the other cases in terms of “*smaller*” versions of itself
- Why is a base case needed?
 - A definition without a non-recursive part causes *infinite recursion*

Recursive Definitions

- Mathematical formulas are often expressed recursively
- **Example**: the formula for **factorial**
*for any positive integer n , $n!$ (n factorial) is defined to be the product of all integers between **1** and n inclusive*
- Express this definition recursively
$$\begin{aligned} 1! &= 1 \text{ (the base case)} \\ n! &= n * (n-1)! \quad \text{for } n \geq 2 \end{aligned}$$
- Now determine the value of **4!**

Discussion

- *Recursion* is an alternative to *iteration*, and can be a very powerful problem-solving technique
- What is *iteration*? repetition, as in a loop
- What is *recursion*? defining something in terms of a *smaller* or *simpler* version of itself (why smaller/simpler?)

Recursive Programming

- ***Recursion*** is a programming technique in which a method can ***call itself*** to solve a problem
- A function in C that invokes itself is called a ***recursive method*** and must contain code for
 - The ***base case***
 - The ***recursive part***

Example of Recursive Programming

- Consider the problem of computing the sum of all the numbers between **1** and **n** inclusive

e.g. if **n** is **5**, the sum is (in an iterative processes)

$$1 + 2 + 3 + 4 + 5$$

- How can this problem be expressed recursively?

Hint: the above sum is the same as

$$5 + 4 + 3 + 2 + 1$$

Recursion in C

END OF Part 1


```
#include <stdio.h>

int SumIter(int );

int main()
{
    int i = 3 , x;

    x = SumIter(i);
    printf("\nIteration x: %d\n", x);

    return 0;
}

int SumIter(int ir)
{
    if (ir == 1)
        return (1);
    else
    {
        int totalSum = 0;
        for (int k = 1; k <= ir; k++)
        {
            totalSum = totalSum + k;
        }
        return (totalSum);
    }
}
```

[illegible]

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```
int totalSum = 0;
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```

[illegible]

```
totalSum = totalSum + k;
```

i	400 - 403	3
x	404 - 407	

```
return (totalSum);
```

	i	400 - 403	3
	x	404 - 407	


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        }
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}

```

Label	Address	Value
	399	
i	400 - 403	3
x	404 - 407	6
	...	
	...	
	...	
	...	
	...	
	...	
	...	
	...	
	...	
	...	
	...	
	...	

OUTPUT:
Iteration x: 6


```
int i = 3 , x;
```

[illegible]

```
#include <stdio.h>

int SumRec(int );

int main()
{
    int i = 3 , x;
    x = SumRec(i);
    printf("\nFactorial x: %d\n", x);

    return 0;
}

int SumRec(int ir)
{
    int result;
    if (ir == 1)
    {
        result = 1;
    }
    else
    {
        result = (ir + SumRec(ir - 1));
    }
    return (result);
}
```

[illegible]


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int result;
```

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```
int SumRec(int );
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```
int main()
```

 $\{$

```
int i = 3 , x;
```

```
x = SumRec(i);
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```
printf("\nFactorial x: %d\n", x);
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return 0;
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}

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int result;
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result = 1;
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}

else

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result = (ir + SumRec(ir - 1));
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return (result);
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[illegible]


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    }
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}

```

Label	Address	Value
	399	
i	400 - 403	3
x	404 - 407	
ir	440 - 443	3
result	444 - 447	
	...	
	...	
	...	
	...	
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3 + 3

result = (ir + SumRec(ir - 1));

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OUTPUT:
Factorial x: 6

Recursion in C

END OF Part 2

How Recursion Works

- What happens when **any** function is called?
 - A **call frame** is set up
 - That call frame is pushed onto the **runtime stack**
- What happens when a recursive method “***calls itself***”?
It’s actually just like calling any other method!
 - A **call frame** is set up
 - That call frame is pushed onto the **runtime stack**

How Recursion

- What happens when *any* function is called
 - A **call frame** is set up
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onto the *runtime stack*

How Recursion

- What happens when ~~any~~ **function** is called
 - A **call frame** is set up
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Label	Address	Value
	399	
i	400 - 403	3
x	404 - 407	
ir	440 - 443	3
result	444 - 447	
	...	
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onto the *runtime stack*

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- What happens when a recursive method “*calls itself*”?
It’s actually just like calling any other method!
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Label	Address	Value
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i	400 - 403	3
x	404 - 407	
ir	440 - 443	3
result	444 - 447	
ir	444 - 447	2
result	448 - 451	
	...	
	...	
	...	
	...	
	...	
	...	
	...	
	...	
	...	

- What happens when a recursive function is called? It's actually just like calling any other function.
 - A **call frame** is set up
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x	404 - 407	
ir	440 - 443	3
result	444 - 447	
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result	448 - 451	
ir	468 - 471	1
result	472 - 475	1
	...	
	...	
	...	
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- What happens when a recursive function is called?
 - It's actually just like calling any other function
 - A **call frame** is set up
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How Recursion Works

- Note: For a recursive **function** , how many copies of the code are there?
 - Just one! (like any other **function**)
- When does the recursive function stop calling itself?
 - When the base case is reached
- What happens then?
 - *That invocation* of the function completes, its call frame is popped off the runtime stack, and control returns to *the function that invoked it*

How Recursion Works

- But which function invoked it?
the *previous invocation* of the recursive function
 - This function now completes,
its call frame is popped off the runtime stack,
and
control returns to *the function that invoked it*
- And so on until we get back to the first invocation of the recursive function

How Recursion Works

- But which function invoked it?
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How Recursion Works

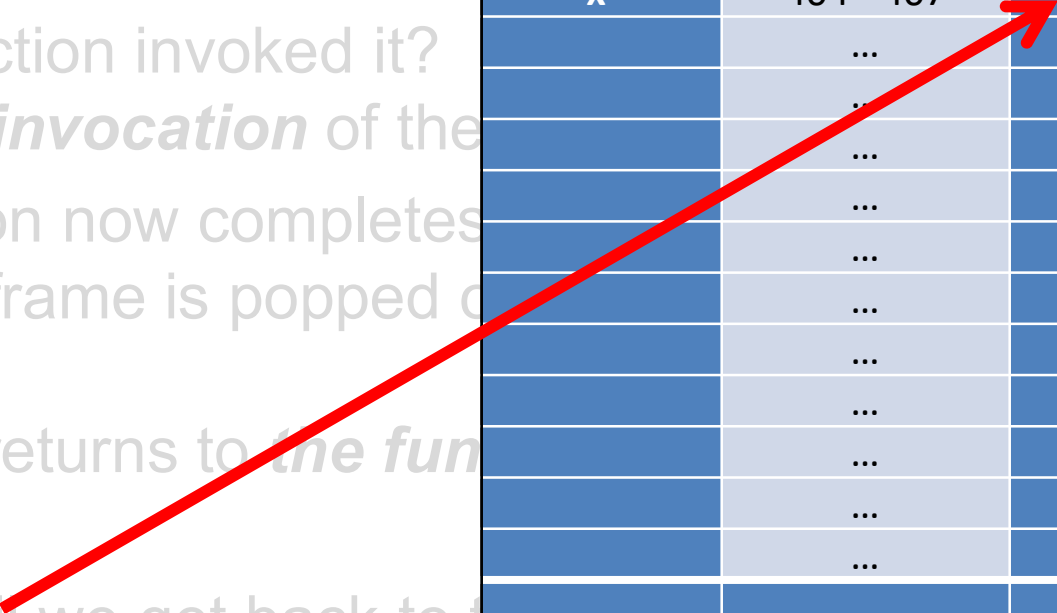
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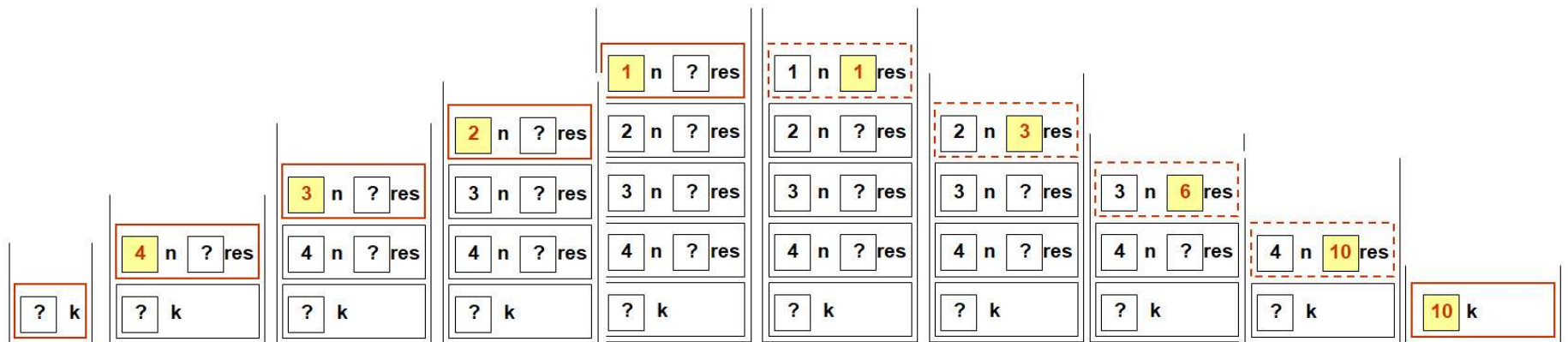
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    int i = 4 , k;

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    printf("\nFactorial k: %d\n", x);

    return 0;
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int SumRec(int n)
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    int res;
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Recursion in C

END OF Recursion

