Assignment 3

Due: Thursday Oct. 22, 2020 before 6:30 PM to be uploaded in Gradescope as a single pdf file.

Please write your name and student number on your submission. Justify each step carefully using previously proven results either in the zyBook or the videos/zoom sessions. When in doubt prove the statement you are going to use. Solutions are graded for correctness as well as clarity.

Exercise 1 (10 point)**.** Let x, y be any non-zero real numbers. Given

$$\frac{x}{y} + \frac{y}{x} = 2$$

Show that $x = y$ by "direct proof".

Exercise 2 (15 point)**.** Prove the following proposition by contraposition:
For every positive integer a, if a is strictly greater than 1 then a does not divide $2a^2 + a + 1$.

Exercise 3 (15 points)**.** Prove that $\sqrt[3]{2}$ is not a rational number by contradiction.
Hint: You may use the fact that if a^3 is an even number then a must also be even, with proper justification.

Exercise 4 (10 point)**.** Solve the following equations for real numbers

$$|x^2 - 1| = |x - 1|$$

using case analysis.