CHAPTER 3
Basic Propositional Logic

Propositional logic studies arguments whose validity depends on "if-then," "and," "or," "not," and similar notions. This chapter covers the basics, and the next covers proofs. Our later systems will build on what we learn here.

3.1 Easier translations

We'll now create a little "propositional language," with precise rules for constructing arguments and testing validity. Our language will help us to test English arguments.

Our language uses capital letters for true-or-false statements and parentheses for grouping. And it uses five special symbols: "~" (squiggle), "·" (dot), "∨" (vee), "⊃" (horseshoe), and "≡" (threebar):

~P	=	Not-P
(P · Q)	=	Both P and Q
(P ∨ Q)	=	Either P or Q
(P ⊃ Q)	=	If P then Q
(P ≡ Q)	=	P if and only if Q

A grammatically correct formula of our language is called a **wff,** or **well-formed formula**. Wffs are sequences that we can construct using these rules:[1]

1. Any capital letter is a wff.
2. The result of prefixing any wff with "~" is a wff.
3. The result of joining any two wffs by "·" or "∨" or "⊃" or "≡" and enclosing the result in parentheses is a wff.

These rules let us build wffs like the following:

P	=	I went to Paris.
~Q	=	I didn't go to Quebec.
(P · ~Q)	=	I went to Paris and I didn't go to Quebec.
(N ⊃ (P · ~Q))	=	If I'm Napoleon, then I went to Paris and I didn't go to Quebec.

[1] Pronounce "wff" as "wŏŏf" as—as in "wood" or "woofer." This book will consider letters with primes (like **A′** and **A″**) to be distinct additional letters.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

Parentheses are tricky. Note how these formulas differ:

Wrong:	(~Q)	~P • Q	P • Q ⊃ R
Right:	~Q	(~P • Q) ~(P • Q)	(P • (Q ⊃ R)) ((P • Q) ⊃ R)

"~Q" doesn't need parentheses. A wff requires a pair of parentheses (to avoid ambiguity) for each "·," "∨," "⊃," or "≡." These two differ:

(~P • Q) = Both not-P and Q
~(P • Q) = Not both P and Q

The first is very definite and says that P is false and Q is true. The second just says that not both are true (at least one is false). Don't read both the same way, as "not P and Q." Read "both" for the left-hand parenthesis, or use pauses:

(~P • Q) = Not-P (pause) and (pause) Q
~(P • Q) = Not (pause) P and Q

Logic is easier if you read the formulas correctly. These two also differ:

(P • (Q ⊃ R)) = P, and if Q then R
((P • Q) ⊃ R) = If P-and-Q, then R

The first says P is definitely true, but the second leaves us in doubt about this.

Here's a useful rule for translating from English into logic:

> Put "(" wherever you see "both," "either," or "if."

Here are examples:

Either not A or B = (~A ∨ B)
Not either A or B = ~(A ∨ B)

If both A and B, then C = ((A • B) ⊃ C)
Not both not A and B = ~(~A • B)

Here's another useful rule, with examples:

> Group together parts on either side of a comma.

If A, then B and C = (A ⊃ (B • C))
If A then B, and C = ((A ⊃ B) • C)

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a
onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

If you're confused on where to divide a sentence that lacks a comma, ask yourself where a comma would naturally go—and then translate accordingly:

If it snows then I'll go outside and I'll ski	=	If it snows, then I'll go outside and I'll ski	=	$(S \supset (G \cdot K))$

Be sure to have your capital letters stand for whole statements:

Wrong:	Gensler is happy	=	$(G \cdot H)$
Right:	Gensler is happy	=	G

Here's a more subtle example:

Wrong:	Bob and Lauren got married to each other	=	$(B \cdot L)$
Right:	Bob and Lauren got married to each other	=	M

The first is wrong because the English sentence doesn't mean "Bob got married and Lauren got married" (which omits "to each other"). So "and" in our example doesn't connect whole sentences—as it does here:

Bob and Lauren were sick = $(B \cdot L)$

This means "Bob was sick and Lauren was sick."

Long sentences, like this one, can be confusing to translate:

> If attempts to prove "God exists" fail in the same way as our best proofs for "There are other conscious beings besides myself," then belief in God is reasonable if and only if belief in other conscious beings is reasonable.

Focus on logical terms, like "if-then" and "not," and translate part by part. Our long sentence has this form:

If …, then … if and only if …. = $(F \supset (G \equiv O))$

Don't let complex wording intimidate you. Instead, divide and conquer.

It doesn't matter what letters you use, as long as you're consistent. Use the same letter for the same idea, and different letters for different ideas. If you use "P" for "I went to Paris," then use "~P" for "I didn't go to Paris."

Our translation rules have exceptions and need to be applied with common sense. So don't translate "I saw them both" as "S("—which isn't even a wff.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

3.1a Exercise—LogiCola C (EM & ET)[1]

Translate these English sentences into wffs.

Both not A and B.	(~A·B)

1. Not both A and B.
2. Both A and either B or C.
3. Either both A and B or C.
4. If A, then B or C.
5. If A then B, or C.
6. If not A, then not either B or C.
7. If not A, then either not B or C.
8. Either A or B, and C.
9. Either A, or B and C.
10. If A then not both not B and not C.
11. If you get an error message, then the disk is bad or it's a Macintosh disk.
12. If I bring my digital camera, then if my batteries don't die then I'll take pictures of my backpack trip and put the pictures on my Web site.
13. If you both don't exercise and eat too much, then you'll gain weight.
14. The statue isn't by either Cellini or Michelangelo.
15. If I don't have either $2 in exact change or a bus pass, I won't ride the bus.
16. If Michigan and Ohio State play each other, then Michigan will win.
17. Either you went through both Dayton and Cinci, or you went through Louisville.
18. If she had hamburgers then she ate junk food, and she ate French fries.
19. I'm going to Rome or Florence and you're going to London.
20. Everyone is male or female.

3.2 Simple truth tables

Let "P" stand for "I went to Paris" and "Q" for "I went to Quebec." Each could be *true* or *false* (the two **truth values**)—represented by "1" and "0." There are four possible truth-value combinations:

PQ	
0 0	Both are false
0 1	Just Q is true
1 0	Just P is true
1 1	Both are true

[1] Exercise sections have a boxed sample problem worked out and refer to any corresponding LogiCola computer exercises. Some further problems are worked out at the back of the book.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

In the first case, I went to neither Paris nor Quebec. In the second, I went to Quebec but not Paris. And so on.

A **truth table** gives a logical diagram for a wff. It lists all possible truth-value combinations for the letters and says whether the wff is true or false in each case. The truth table for "·" ("and") is very simple:

P Q	(P·Q)
0 0	0
0 1	0
1 0	0
1 1	1

"I went to Paris and I went to Quebec."

"(P·Q)" is a **conjunction;**
P and Q are its **conjuncts**.

"(P·Q)" claims that *both* parts are true. So "I went to Paris *and* I went to Quebec" is false in the first three cases (where one or both parts are false)—and true only in the last case. These truth equivalences give the same information:

(0·0)=0	(false·false)=false
(0·1)=0	(false·true)=false
(1·0)=0	(true·false)=false
(1·1)=1	(true·true)=true

Here "(0·0)=0" says that an AND statement is false if both parts are false. The next two say that an AND is false if one part is false and the other part is true. And "(1·1)=1" says that an AND is true if both parts are true.[1]

Here are the truth table and equivalences for "∨" ("or"):

P Q	(P∨Q)	
0 0	0	(0∨0)=0
0 1	1	(0∨1)=1
1 0	1	(1∨0)=1
1 1	1	(1∨1)=1

"I went to Paris or
I went to Quebec."

"(P∨Q)" is a **disjunction;**
P and Q are its **disjuncts**.

"(P∨Q)" claims that *at least one* part is true. So "I went to Paris *or* I went to Quebec" is true if I went to one or both places; it's false if I went to neither place. Our "∨" thus symbolizes the *inclusive* sense of "or." English also can use "or" in an *exclusive* sense, which claims that at least one part is true *but not both.* Here is how both senses of "or" translate into our symbolism:

- Inclusive "or": A or B or both=(A∨B)
- Exclusive "or": A or B but not both=((A∨B)·~(A·B))

[1] Our "·" is simpler than the English "and," which can mean things like "and then" (as it does in "Suzy got married and had a baby"—which differs from "Suzy had a baby and got married").

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

The exclusive sense requires a longer symbolization.[2]

Here are the truth table and equivalences for "⊃" ("if-then"):

P Q	P⊃Q		
0 0	1	(0⊃0)=1	"If I went to Paris, then I went to Quebec."
0 1	1	(0⊃1)=1	"(P⊃Q)" is a **conditional;**
1 0	0	(1⊃0)=0	P is the **antecedent**
1 1	1	(1⊃1)=1	and Q the **consequent**.

"(P⊃Q)" claims that what we *don't* have is the first part true and the second false. Suppose you say this:

"*If* I went to Paris, *then* I went to Quebec."

By our table, you speak truly if you went to neither place, or to both places, or to Quebec but not Paris. You speak falsely if you went to Paris but not Quebec. Does that seem right to you? Most people think so, but some have doubts.

Our truth table can produce strange results. Take this example:

If I had eggs for breakfast, then the world will end at noon. (E⊃W)

Suppose that I didn't have eggs, and so E is false. By our table, the conditional is then *true*—since if E is false then "(E⊃W)" is true. This is strange. We'd normally tend to take the conditional to be *false*—since we'd take it to claim that my having eggs would *cause* the world to end. So translating "if-then" as "⊃" doesn't seem satisfactory. There's something fishy going on here.

Our "⊃" symbolizes a simplified "if-then" that ignores elements like causal connections and temporal sequence. "(P⊃Q)" has a very simple meaning; it just *denies* that we have P-true-and-Q-false:

(P⊃Q)	=	~(P·~Q)
If P is true, then Q is true.		We don't have P true and Q false.

Translating "if-then" this way is a useful simplification, since it captures the part of "if-then" that normally determines validity. The simplification usually works; in the few cases where it doesn't, we can use a more complex translation (as we'll sometimes do in the chapters on modal logic).

The truth conditions for "⊃" are hard to remember. These slogans may help:

[2] People sometimes use *"Either* A *or* B" to indicate the exclusive "or." We won't do this; instead, we'll use "either" to indicate grouping and we'll translate it as a left-hand parenthesis.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

Falsity implies anything.	(0⊃)=1
Anything implies truth.	(⊃1)=1
Truth doesn't imply falsity.	(1⊃0)=0

The "Falsity implies anything" slogan, for example, means that the whole if-then is true if the first part is false; so "If I'm a billionaire, then…" is true, regardless of what replaces "…," since I'm *not* a billionaire.

Here are the table and equivalences for "≡" ("if-and-only-if"):

P Q	(P≡Q)	
0 0	1	(0≡0)=1
0 1	0	(0≡1)=0
1 0	0	(1≡0)=0
1 1	1	(1≡1)=1

"I went to Paris if and only if I went to Quebec."

"(P≡Q)" is a **biconditional**.

"(P≡Q)" claims that both parts have the *same* truth value: both are true or both are false. So "≡" is much like "equals."

Here are the table and equivalences for "~" ("not"):

P	~P	
0	1	~0=1
1	0	~1=0

"I didn't go to Paris."

"~P" is a **negation**.

"~P" has the *opposite* value of "P." If "P" is true then "~P" is false, and if "P" is false then "~P" is true.

Most of the rest of this book presupposes that you know these truth equivalences; try to master them right away.

3.2a Exercise—LogiCola D (TE & FE)

Calculate each truth value.

$\boxed{(0 \cdot 1)}$ $\boxed{(0 \cdot 1) = 0}$

1. (0 ∨ 1)
2. (0 • 0)
3. (0 ⊃ 0)
4. ~0
5. (0 ≡ 1)
6. (1 • 0)
7. (1 ⊃ 1)
8. (1 ≡ 1)
9. (0 ∨ 0)
10. (0 ⊃ 1)
11. (0 ≡ 0)
12. (1 ∨ 1)
13. (1 • 1)
14. (1 ⊃ 0)
15. ~1
16. (1 ∨ 0)
17. (1 ≡ 0)

3.3 Truth evaluations

We can calculate the truth value of a wff if we know the truth value of its letters. Consider this problem:

Suppose that P=1, Q=0, and R=0.
What is the truth value of "((P⊃Q)≡~R)"?

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

To figure this out, we write "1" for "P," "0" for "Q," and "0" for "R"; then we simplify from the inside out, using our equivalences, until we get "1" or "0":

$((P \supset Q) \equiv \sim R)$	←	original formula
$((1 \supset 0) \equiv \sim 0)$	←	substitute "1" and "0" for the letters
$(0 \equiv 1)$	←	put "0" for "$(1 \supset 0)$," and "1" for "~ 0"
0	←	put "0" for "$(0 \equiv 1)$"

Here the formula is false.

Some people work out truth values vertically, as above. Others work them out horizontally:

$$((1 \supset 0) \equiv \sim 0) \;=\; (0 \equiv 1) \;=\; 0$$

Still others work out the values in their heads.

Simplify parts inside parentheses first. With a wff of the form "~(…)," first work out the part inside parentheses to get 1 or 0; then apply "~" to the result. Study these two examples:

Right: $\sim(1 \vee 0) \;=\; \sim 1 \;=\; 0$
Wrong: $\sim(1 \vee 0) \;=\; (\sim 0 \vee \sim 1) \;=\; (1 \vee 0) \;=\; 1$

Don't distribute "~" as the wrong example does it. Instead, first evaluate whatever is inside the parentheses.

3.3a Exercise—LogiCola D (TM & TH)

Assume that A=1 and B=1 (A and B are both true) while X=0 and Y=0 (X and Y are both false). Calculate the truth value of each wff below.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a
onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

3.4 Unknown evaluations

We can sometimes figure out a formula's truth value even if we don't know the truth value of some letters. Take this example:

Suppose that P=1 and Q=? (unknown).
What is the truth value of "(P∨Q)"?

We first substitute "1" for "P" and "?" for "Q"

(1∨?)

We might just see that this is true, since an OR is true if at least one part is true. Or we might try it both ways. Since "?" could be "1" or "0," we write "1" above the "?" and "0" below it. Then we evaluate the formula for each case:

$$\begin{array}{l} \quad\;\; 1 = 1 \\ (1 \vee ?) \\ \quad\;\; 0 = 1 \end{array}$$

The formula is true, since it's true either way.

Here's another example:

Suppose that P=1 and Q=?
What is the truth value of "(P•Q)"?

We first substitute "1" for "P" and "?" for "Q":

(1·?)

We might see that this is unknown, since the truth value of the whole depends on the unknown letter. Or we might try it both ways; then we write "1" above the "?" and "0" below it—and we evaluate the formula for each case:

$$\begin{array}{l} \quad\;\; 1 = 1 \\ (1 \cdot ?) \\ \quad\;\; 0 = 0 \end{array}$$

The formula is unknown, since it could be either true or false.

3.4a Exercise—LogiCola D (UE, UM, & UH)

Assume that T=1 (T is true), F=0 (F is false), and U=? (U is unknown). Calculate the truth value of each wff below.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a
onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

$\boxed{(\sim T \cdot U)}$ $\boxed{(\sim 1 \cdot ?) = (0 \cdot ?) = 0}$

1.	$(U \cdot F)$	4.	$(\sim F \cdot U)$	7.	$(U \supset \sim T)$	10.	$(U \supset \sim F)$
2.	$(U \supset \sim T)$	5.	$(F \supset U)$	8.	$(\sim F \vee U)$	11.	$(U \cdot \sim T)$
3.	$(U \vee \sim F)$	6.	$(\sim T \vee U)$	9.	$(T \cdot U)$	12.	$(U \vee F)$

3.5 Complex truth tables

A truth table for a wff is a diagram listing all possible truth-value combinations for the wff s letters and saying whether the wff would be true or false in each case. We've done simple tables already; now we'll do complex ones.

A formula with n distinct letters has 2^n possible truth-value combinations:

One letter gives 2 (2^1) combinations.
Two letters give 4 (2^2) combinations.
Three letters give 8 (2^3) combinations.

n letters give 2^n combinations.

A	A B	A B C
0	0 0	0 0 0
1	0 1	0 0 1
	1 0	0 1 0
	1 1	0 1 1
		1 0 0
		1 0 1
		1 1 0
		1 1 1

To get every combination, alternate 0's and 1's below the last letter the required number of times. Then alternate 0's and 1's below each earlier letter at half the previous rate: by twos, fours, and so on. This numbers each row in ascending order in base 2.

A truth table for "~(A∨~B)" begins like this:

A B	~(A∨~B)
0 0	
0 1	
1 0	
1 1	

The right side has the wff. The left side has each letter used in the wff; we write each letter just once, regardless of how often it occurs. Below the letters, we write all possible truth-value combinations. Finally we figure out the wff's truth value for each line. The first line has A and B both false—which makes the whole wff false:

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

~(A∨~B)	←	original formula
~(0∨~0)	←	substitute “0” for each letter
~(0∨1)	←	put “1” for “~0”
~1	←	put “1” for “(0∨1)”
0	←	put “0” for “~1”

The wff comes out “1,” “0,” and “0” for the next three lines—which gives us this truth table:

A B	~(A∨~B)
0 0	0
0 1	1
1 0	0
1 1	0

So “~(A∨~B)” is true if and only if A is false and B is true. The simpler wff “(~A·B)” is equivalent, in that it's true in the same cases. Both wffs are true in some cases and false in others—making them **contingent statements**.

The truth table for “(P∨~P)” is true in all cases—which makes the formula a **tautology:**

P	(P∨~P)
0	1
1	1

“I went to Paris or I didn't go to Paris.”

This formula, called the **law of the excluded middle,** says that every statement is true or false. This law holds in propositional logic, since we stipulated that capital letters stand for true-or-false statements. The law doesn't always hold in English, since English allows statements that are too vague to be true or false. Is “It's raining” true if there's a slight drizzle? Is “My shirt is white” true if it's a light cream color? Such claims can be too vague to be true or false. So the law is an idealization when applied to English.

The truth table for “(P·~P)” is false in all cases—which makes the formula a **self-contradiction:**

P	(P·~P)
0	0
1	0

“I went to Paris and I didn't go to Paris.”

“P and not-P” is always false in propositional logic, which presupposes that “P” stands for the same statement throughout. English is looser and lets us shift the meaning of a phrase in the middle of a sentence. “I went to Paris and I didn't go to Paris” may express a truth if it means this:

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

> "I went to Paris (in that I landed once at the Paris airport)—but I didn't really go there (in that I saw almost nothing of the city)."

Because of the shift in meaning, this would better translate as "(P·~Q)."

3.5a Exercise—LogiCola D (FM & FH)

Do a truth table for each formula.

46 INTRODUCTION TO LOGIC

((P ∨ Q) ⊃ R)

P Q R	((P ∨ Q) ⊃ R)
0 0 0	1
0 0 1	1
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

3.6 The truth-table test

> Take a propositional argument. Construct a truth table showing the truth value of the premises and conclusion for all possible cases. The argument is **VALID** if and only if no possible case has the premises all true and conclusion false.

Suppose we want to test this invalid argument:

If you're a dog, then you're an animal.	(D⊃A)
You're not a dog.	~D
∴ You're not an animal.	∴ ~A

First we do a truth table for the premises and conclusion. We start as follows:

D A	(D⊃A),	~D	∴	~A
0 0				
0 1				
1 0				
1 1				

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

Then we evaluate the three wffs on each truth combination. The first combination has D=0 and A=0, which makes all three wffs true:

(D⊃A)	~D	~A
(0⊃0)	~0	~0
1	1	1

So the first line of our truth table looks like this:

DA	(D⊃A),	~D	∴	~A
0 0	1	1		1

We work out the other three lines:

D A	(D⊃A),	~D	∴	~A		
0 0	1	1		1		
0 1	1	1		0	←	**Invalid**
1 0	0	0		1		
1 1	1	0		0		

The argument is invalid, since some possible case has the premises all true and conclusion false. The case in question is one where you aren't a dog but you are an animal (perhaps a cat).

Consider this valid argument:

If you're a dog, then you're an animal.	(D⊃A)
You're a dog.	D
∴ You're an animal.	∴ A

Again we do a truth table for the premises and conclusion:

D A	(D⊃L),	D	∴	A	**Valid**
0 0	1	0		0	
0 1	1	0		1	
1 0	0	1		0	
1 1	1	1		1	

Here the argument is valid, since no possible case has the premises all true and conclusion false.

There's a short-cut form of the truth-table test. Recall that all we're looking for is 110 (premises all true and conclusion false). The argument is invalid if 110 sometimes occurs; otherwise, it's valid. To save time, we can first evaluate an easy wff and cross out any lines that couldn't be 110. In our previous example, we might work out "D" first:

D A	(D⊃A),	D	∴	A
0 0	– – – – – – –	0	– – – – –	

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

0 1	– – – – – – –	0	– – – – –
1 0		1	
1 1		1	

The first two lines can't be 110 (since the second digit is 0); so we cross them out and ignore the remaining values. The last two lines could be 110, so we need to work them out further. Next we might evaluate "A":

D A	(D⊃A),	D	∴	A
0 0	– – – – – – –	0	– – – – –	
0 1	– – – – – – –	0	– – – – –	
1 0		1		0
1 1	– – – – – – –	1	– – –	1 –

The last line can't be 110 (since the last digit is 1); so we cross it out. Then we have to evaluate "(D⊃A)" for only one case—for which it comes out false. Since we never get 110, the argument is valid:

D A	(D⊃A),	D	∴	A	**Valid**
0 0	– – – – – – –	0	– – – – –		
0 1	– – – – – – –	0	– – – – –		
1 0	– – – 0 – – –	1	– – –	0 –	
1 1	– – – – – – –	1	– – –	1 –	

The short-cut method can save much time if otherwise we'd have to evaluate a long formula for eight or more cases.

With a two-premise argument, we look for 110. With three premises, we look for 1110. Whatever the number of premises, we look for a case where the premises are all true and conclusion false. The argument is valid if and only if this case never occurs.

The truth-table test can get tedious for long arguments. Arguments with 6 letters need 64 lines—and ones with 10 letters need 1024 lines. So we'll use the truth-table test only on fairly simple arguments.[1]

3.6a Exercise—LogiCola D (AE, AM, & AH)

First appraise intuitively. Then translate into logic (using the letters given) and use the truth-table test to determine validity.

[1] An argument that tests out "invalid" is either invalid or else valid on grounds that go beyond the system in question. Let me give an example to illustrate the second possibility. "This is green, therefore something is green" translates into propositional logic as "T ∴ S" and tests out invalid. But the argument is in fact valid—as we later could show using quantificational logic.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

It's in my left hand or my right hand.
It's not in my left hand.
∴ It's in my right hand.

L R	(L ∨ R),	~L	∴	R	Valid
0 0	0	1		0	
0 1	1	1		1	we never get 110
1 0	1	0		0	
1 1	1	0		1	

1. If you're a collie, then you're a dog.
   You're a dog.
   ∴ You're a collie. [Use C and D.]

2. If you're a collie, then you're a dog.
   You're not a dog.
   ∴ You're not a collie. [Use C and D.]

3. If television is always right, then Anacin is better than Bayer.
   If television is always right, then Anacin isn't better than Bayer.
   ∴ Television isn't always right. [Use T and B.]

4. If it rains and your tent leaks, then your down sleeping bag will get wet.
   Your tent won't leak.
   ∴ Your down sleeping bag won't get wet. [Use R, L, and W.]

5. If I get Grand Canyon reservations and get a group together, then I'll explore canyons during spring break.
   I've got a group together.
   I can't get Grand Canyon reservations.
   ∴ I won't explore canyons during spring break. [Use R, T, and E.]

6. There is an objective moral law.
   If there is an objective moral law, then there is a source of the moral law.
   If there is a source of the moral law, then there is a God. (Other possible sources, like society or the individual, are claimed not to work.)
   ∴ There is a God. [Use M, S, and G; this argument is from C.S.Lewis.]

7. If ethics depends on God's will, then something is good because God desires it.
   Something isn't good because God desires it. (Instead, God desires something because it's already good.)
   ∴ Ethics doesn't depend on God's will. [Use D and B; this argument is from Plato's *Euthyphro*.]

8. It's an empirical fact that the basic physical constants are precisely in the narrow range of what is required for life to be possible. (This "anthropic principle" has considerable evidence behind it.)
   The best explanation for this fact is that the basic physical constants were caused by an intelligent being intending to produce life. (The main alternatives are the "chance coincidence" and "parallel universe" explanations.)
   If these two things are true, then it's reasonable to believe that the basic structure of the world was set up by an intelligent being (God) intending to produce life.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

∴ It's reasonable to believe that the basic structure of the world was set up by an intelligent being (God) intending to produce life. [Use E, B, and R; this argument is from Peter Glynn.]

9. I'll go to Paris during spring break if and only if I'll win the lottery.
   I won't win the lottery.
   ∴ I won't go to Paris during spring break. [Use P and W.]

10. If we have a simple concept proper to God, then we've directly experienced God and we can't rationally doubt God's existence.
    We haven't directly experienced God.
    ∴ We can rationally doubt God's existence. [Use S, E, and R.]

11. If there is a God, then God created the universe.
    If God created the universe, then matter didn't always exist.
    Matter always existed.
    ∴ There is no God. [Use G, C, and M.]

12. If this creek is flowing, then either the spring upstream has water or this creek has some other water source.
    This creek has no other water source.
    This creek isn't flowing.
    ∴ The spring upstream has no water. [Use F, S, and O.]

3.7 The truth-assignment test

> Take a propositional argument. Set each premise to 1 and the conclusion to 0. The argument is **VALID** if and only if no consistent way of assigning 1 and 0 to the letters will make this work—so we can't make the premises all true and conclusion false.

Suppose we want to test this valid argument:

It's in my left hand or my right hand.	$(L \vee R)$
It's not in my left hand.	$\sim L$
∴ It's in my right hand.	$\therefore R$

First we set each premise to 1 and the conclusion to 0 (just to see if this works):

$$
\begin{aligned}
&(L \vee R) = 1\\
&\sim L = 1\\
\therefore\ &R = 0
\end{aligned}
$$

Then we figure out the values of as many letters as we can. The conclusion has R false. We show this by writing a 0-superscript after each R:

$$
\begin{aligned}
&(L \vee R^0) = 1\\
&\sim L = 1\\
\therefore\ &R^0 = 0
\end{aligned}
$$

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

The second premise has ~L true—and so L is false. So we write a 0-superscript after each L (showing that L is false):

$$\begin{aligned} (L^0 \vee R^0) &= 1 \\ {\sim}L^0 &= 1 \\ \therefore R^0 &= 0 \end{aligned}$$

Then the first premise is false, since an OR is false if both parts are false:

$$\begin{aligned} \overset{0}{\overline{(L^0 \vee R^0)}} &\neq 1 \quad \textbf{Valid} \\ {\sim}L^0 &= 1 \\ \therefore R^0 &= 0 \end{aligned}$$

Since 0 isn't 1, we slash the "=." It's impossible to make the premises all true and conclusion false, since this would make the first premise both 1 and 0. So the argument is valid.

In doing the test, first assign 1 to the premises and 0 to the conclusion. Then figure out the truth values for the letters—and then the truth values for the longer formulas. If you have to cross something out, then the assignment isn't possible, and so the argument is valid.

Consider this invalid argument:

It's in my left hand or my right hand.	(L∨R)
It's not in my left hand.	~L
∴ It's not in my right hand.	∴ ~R

Again, we first set each premise to 1 and the conclusion to 0:

$$\begin{aligned} (L \vee R) &= 1 \\ {\sim}L &= 1 \\ \therefore {\sim}R &= 0 \end{aligned}$$

Then L is false (since ~L is true)—and R is true (since ~R is false). But then the first premise comes out true:

$$\begin{aligned} \overset{1}{\overline{(L^0 \vee R^1)}} &= 1 \quad \textbf{Invalid} \\ {\sim}L^0 &= 1 \\ \therefore {\sim}R^1 &= 0 \end{aligned}$$

Since we *can* make the premises all true and conclusion false, the argument is invalid. A truth-table gives the same result when L=0 and R=1:

L R	(L ⊃ R),	~L	∴ ~R	
0 1	1	1	0	← **Invalid**

The truth-assignment test gives this result more quickly.[1]

[1] Some find lines like "$\sim L^0=1$" confusing. Here the *larger complex "~L"* is true but the *letter "L"* is false. When I write an 0-superscript above the letter, I mean that the letter is false.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

Here's another invalid argument:

It's in my left hand or my right hand.	(L∨R)
∴ It's in my right hand.	∴ R

If we work this out, we get R false—but we get no value for L:

$$
\begin{array}{l}
(L \vee R^0) = 1 \\
\therefore R^0 = 0
\end{array}
$$

Since the value for L matters, we could try both values (first *true* and then *false*); the argument is invalid if *either* value makes the premises all true and conclusion false. Here making L *true* gives this result:

$$
\begin{array}{ll}
\underline{\quad 1 \quad} & \\
(L^1 \vee R^0) = 1 & \textbf{Invalid} \\
\therefore R^0 = 0 &
\end{array}
$$

In working out the truth values for the letters, try to make the premises all true and conclusion false. The argument is invalid if there's some way to do this.

3.7a Exercise—LogiCola ES

Test for validity using the truth-assignment test.

(K ⊃ (I ∨ S)) ~I K ∴ S	$(K^1 \supset (I^0 \vee S^0)) \neq 1$ **Valid** $\sim I^0 = 1$ $K^1 = 1$ (we can't ∴ $S^0 = 0$ have 1110)

1. ~(N ≡ H)
N
∴ ~H
2. ((J · ~D) ⊃ Z)
~Z
D
∴ ~J
3. ((T ∨ M) ⊃ Q)
M
∴ Q
4. P
∴ (P · Q)
5. ((L · F) ⊃ S)
S
F
∴ L
6. ((A · U) ⊃ ~B)
B
A
∴ ~U
7. ((W · C) ⊃ Z)
~Z
∴ ~C
8. Q
∴ (P ⊃ Q)
9. (E ∨ (Y · X))
~E
∴ X
10. (~T ⊃ (P ⊃ J))
P
~J
∴ T
11. ~P
∴ ~(Q ⊃ P)
12. ((~M · G) ⊃ R)
~R
G
∴ M
13. ~(Q ≡ I)
~Q
∴ I
14. ((Q · R) ≡ S)
Q
∴ S
15. A
~A
∴ B

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

3.7b Exercise—LogiCola EE

First appraise intuitively. Then translate into logic and use the truth-assignment test to determine validity.

If our country will be weak, then there will be war. Our country will not be weak. ∴ There will not be war.	$(K^0 \supset R^1)=1$ **Invalid** $\sim K^0=1$ (we can have 110) ∴ $\sim R^{1}=0$

1. Some things are caused (brought into existence).
   Anything caused is caused by another.
   If some things are caused and anything caused is caused by another, then either there's a first cause or there's an infinite series of past causes.
   There's no infinite series of past causes.
   ∴ There's a first cause. [A "first cause" (often identified with God) is a cause that isn't itself caused by another. This argument is from St Thomas Aquinas.]

2. If you pass and it's intercepted, then the other side gets the ball.
   You pass.
   It isn't intercepted.
   ∴ The other side doesn't get the ball.

3. If God exists in the understanding and not in reality, then there can be conceived a being greater than God (namely, a similar being that also exists in reality).
   "There can be conceived a being greater than God" is false (since "God" is defined as "a being than which no greater can be conceived").
   God exists in the understanding.
   ∴ God exists in reality. [This is St Anselm's ontological argument—one of the most widely discussed arguments of all time.]

4. If existence is a perfection and God by definition has all perfections, then God by definition must exist.
   God by definition has all perfections.
   Existence is a perfection.
   ∴ God by definition must exist. [From René Descartes.]

5. If we have sensations of alleged material objects and yet no material objects exist, then God is a deceiver.
   God isn't a deceiver.
   We have sensations of alleged material objects.
   ∴ Material objects exist. [From René Descartes—who thus based our knowledge of the external material world on our knowledge of God.]

6. If "good" is definable in experimental terms, then ethical judgments are scientifically provable and ethics has a rational basis.
   Ethical judgments aren't scientifically provable.
   ∴ Ethics doesn't have a rational basis.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a
onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

7. If it's right for me to lie and not right for you, then there's a relevant difference between our cases.
   There's no relevant difference between our cases.
   It's not right for you to lie.
   ∴ It's not right for me to lie.

8. If Newton's gravitational theory is correct and there's no undiscovered planet near Uranus, then the orbit of Uranus would be such-and-such.
   Newton's gravitational theory is correct.
   The orbit of Uranus isn't such-and-such.
   ∴ There's an undiscovered planet near Uranus. [This reasoning led to the discovery of the planet Neptune.]

9. If attempts to prove "God exists" fail in the same way as our best arguments for "There are other conscious beings besides myself," then belief in God is reasonable if and only if belief in other conscious beings is reasonable.
   Attempts to prove "God exists" fail in the same way as our best arguments for "There are other conscious beings besides myself."
   Belief in other conscious beings is reasonable.
   ∴ Belief in God is reasonable. [From Alvin Plantinga.]

10. If you pack intelligently, then either this teddy bear will be useful on the hiking trip or you won't pack it.
    This teddy bear won't be useful on the hiking trip.
    You won't pack it.
    ∴ You pack intelligently.

11. If knowledge is sensation, then pigs have knowledge.
    Pigs don't have knowledge.
    ∴ Knowledge isn't sensation. [From Plato.]

12. If capital punishment is justified and justice doesn't demand a vindication for past injustices, then capital punishment either reforms the offender or effectively deters crime.
    Capital punishment doesn't reform the offender.
    Capital punishment doesn't effectively deter crime.
    ∴ Capital punishment isn't justified.

13. If belief in God were a purely intellectual matter, then either all smart people would be believers or all smart people would be non-believers.
    Not all smart people are believers.
    Not all smart people are non-believers.
    ∴ Belief in God isn't a purely intellectual matter. [From the Jesuit theologian John Powell.]

14. If you're lost, then you should call for help or head downstream.
    You're lost.
    ∴ You should call for help.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

15. If maximizing human enjoyment is always good and the sadist's dog-torturing maximizes human enjoyment, then the sadist's act is good.
    The sadist's dog-torturing maximizes human enjoyment.
    The sadist's act isn't good.
    ∴ Maximizing human enjoyment isn't always good.

16. If there's knowledge, then either some things are known without proof or we can prove every premise by previous arguments infinitely.
    We can't prove every premise by previous arguments infinitely.
    There's knowledge.
    ∴ Some things are known without proof. [From Aristotle.]

17. If you modified your computer or didn't send in the registration card, then the warranty is void.
    You didn't modify your computer.
    You sent in the registration card.
    ∴ The warranty isn't void.

18. If "X is good" means "Hurrah for X!" and it makes sense to say "If X is good," then it makes sense to say "If hurrah for X!"
    It makes sense to say "If X is good."
    It doesn't make sense to say "If hurrah for X!"
    ∴ "X is good" doesn't mean "Hurrah for X!" [From Hector-Neri Castañeda.]

19. If we have an idea of substance, then "substance" refers either to a simple sensation or to a complex constructed out of simple sensations.
    "Substance" doesn't refer to a simple sensation.
    ∴ We don't have an idea of substance. [From David Hume.]

20. If we have an idea of "substance" and our idea of "substance" doesn't derive from sensations, then "substance" is a thought category of pure reason.
    Our idea of "substance" doesn't derive from sensations.
    We have an idea of "substance."
    ∴ "Substance" is a thought category of pure reason. [From Immanuel Kant.]

21. If "good" means "socially approved," then what is socially approved is necessarily good.
    What is socially approved isn't necessarily good.
    ∴ "Good" doesn't mean "socially approved."

22. [Generalizing the last argument, G.E.Moore argued that we can't define "good" in terms of *any* empirical term "F"—like "desired" or "socially approved."]
    If "good" means "F," then what is F is necessarily good.
    What is F isn't necessarily good. (We can consistently say "Some F things may not be good" without thereby violating the meaning of "good.")
    ∴ "Good" doesn't mean "F."

23. If moral realism (the belief in objective moral truths) were true, then it could explain the moral diversity in the world.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

Moral realism can't explain the moral diversity in the world.
∴ Moral realism isn't true.

3.8 Harder translations

Now we'll learn how to symbolize idiomatic English. We'll still sometimes use these earlier rules:

- Put "(" wherever you see "both," "either," or "if."
- Group together parts on either side of a comma.

Here are three additional rules, with examples:

> Translate "but" ("yet," "however," "although," and so on) as "and."

Michigan played *but* it lost = $(P \cdot L)$

The translation loses the contrast (or surprise), but this doesn't affect validity.

> Translate "unless" as "or."

You'll die *unless* you breathe	=	$(D \vee B)$	=	$(B \vee D)$
Unless you breathe you'll die	=	$(D \vee B)$	=	$(B \vee D)$

"Unless" also is equivalent to "if not"; so we could use "$(\sim B \supset D)$"—"If you don't breathe, then you'll die."

> Translate "just if" and "iff" (a logician word) as "if and only if."

I'll agree *just if* you pay me $1,000	=	$(A \equiv P)$
I'll agree *iff* you pay me $1,000	=	$(A \equiv P)$

The order of the letters doesn't matter with "·" or "∨" or "≡."

Our next two rules are tricky. The first governs most conditional words:

> The part after "if" ("provided that," "assuming that," and so on) is the antecedent (the "if"-part, the part before the horseshoe).

If you're a dog, then you're an animal = $(D \supset A)$

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

Provided that you're a dog, you're an animal = $(D \supset A)$
You're an animal, ***if*** you're a dog = $(D \supset A)$
You're an animal, ***provided that*** you're a dog = $(D \supset A)$

"Only if" is different and follows its own rule:

> The part after "only if" is the consequent (the "then"-part, the part after the horseshoe).
>
> Equivalently: Write "⊃" for "only if."

You're alive *only if* you have oxygen = $(A \supset 0)$
Only if you have oxygen are you alive = $(A \supset 0)$

Using the second form of the rule, "Only if O, A" would become "⊃ O, A"—which we'd put as "$(A \supset 0)$."

Sometimes the **contrapositive** form gives a more intuitive translation. The contrapositive of "$(A \supset B)$" is "$(\sim B \supset \sim A)$"; both are equivalent (true in the same cases). Consider these translations:

You pass *only if* you take the exam=
$(P \supset E)$ (If you pass then you take the exam.)
$(\sim E \supset \sim P)$ (If you don't take the exam then you don't pass.)

The second translation sounds better in English, since we tend to read a temporal sequence into an if-then (even though "⊃" abstracts from this and claims only that we don't have the first part true and the second part false).

Here's the rule for translating "sufficient" and "necessary":

> "A is *sufficient* for B" means "If A then B."
> "A is *necessary* for B" means "If not A then not B."
> "A is *necessary and sufficient* for B" means "A if and only if B."

Oxygen is *sufficient for* life = $(O \supset L)$
Oxygen is *necessary for* life = $(\sim O \supset \sim L)$
Oxygen is *necessary and sufficient for* life = $(0 \equiv L)$

The order of the letters matters only with "⊃."

These translation rules are rough and don't always work. Sometimes you have to puzzle out the meaning on your own.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a
onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

3.8a Exercise—LogiCola C (HM & HT)

Translate these English sentences into wffs.

A, assuming that B.	(B⊃A)

1. If she goes, then you'll be alone but I'll be here.
2. Your car will start only if you have gas.
3. I will quit unless you give me a raise.
4. Taking the final is a sufficient condition for passing.
5. Taking the final is necessary for you to pass.
6. You're a man just if you're a rational animal.
7. Unless you have faith, you'll die.
8. She neither asserted it nor hinted at it.
9. Getting at least 96 is a necessary and sufficient condition for getting an A.
10. Only if you exercise are you fully alive.
11. I'll go, assuming that you go.
12. Assuming that your belief is false, you don't know.
13. Having a true belief is a necessary condition for having knowledge.
14. You get mashed potatoes or French fries, but not both.
15. You're wrong if you say that.

3.9 Idiomatic arguments

Our arguments so far have been phrased in a clear premise-conclusion format. Unfortunately, real-life arguments are seldom so neat and clean. Instead we may find convoluted wording or extraneous material. Important parts of the argument may be omitted or only hinted at. And it may be hard to pick out the premises and conclusion. It often takes hard work to reconstruct a clearly stated argument from a passage.

Logicians like to put the conclusion last:

Socrates is human. If he's	H
human, then he's mortal. So	(H⊃M)
Socrates is mortal.	∴ M

But people sometimes put the conclusion first, or in the middle:

		H
Socrates must be mortal. After all, he's human. And if he's human, he's mortal.	Socrates is human. So *he must be mortal*— since if he's human, he's mortal.	(H⊃M)
		∴ M

In these examples, "must" and "so" indicate the conclusion (which always goes *last* when we translate the argument into logic). Here are some typical words that help us pick out the premises and conclusion:

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

These often indicate premises:		*These often indicate conclusions:*
Because, for, since, after all… I assume that, as we know… For these reasons…	$\therefore$	Hence, thus, so, therefore… It must be, it can't be… This proves (or shows) that…

When you don't have this help, ask yourself what is argued *from* (these are the premises) and what is argued *to* (this is the conclusion).

In reconstructing an argument, first pick out the conclusion. Then translate the premises and conclusion into logic; this step may involve untangling idioms like "A unless B" (which translates into "A or B"). If you don't get a valid argument, try adding unstated but implicit premises; using the "principle of charity," interpret unclear reasoning in the way that gives the best argument. Finally, test for validity.

Here's an easy example:

The gun must have been shot recently! It's still hot.

First we pick out the premises and conclusion:

The gun is still hot.	H
∴ The gun was shot recently.	∴ S

Since this seems to presume an implicit premise, we add the most plausible one that we can think of that makes the argument valid. Then we translate into logic and test for validity:

If the gun is still hot, then it was shot recently, (implicit)	$(H \supset S)$	**Valid**
The gun is still hot.	H	
∴ The gun was shot recently.	∴ S	

3.9a Exercise—LogiCola E (F I)

First appraise intuitively; then translate into logic (making sure to pick out the conclusion correctly) and determine validity using the truth-assignment test. Supply implicit premises where needed.

Knowledge is good in itself only if it's desired for its own sake. So knowledge is good in itself, since it's desired for its own sake.	$(G^0 \supset D^1)=1$ **Invalid** $D^1=1$ $\therefore G^0=0$ (The conclusion is "So *knowledge is good in itself*"—"*G*.")

1. Knowledge can't be sensation. If it were, then we couldn't know something that we aren't presently sensing. [From Plato.]
2. Presuming that we followed the map, then unless the map is wrong there's a pair of lakes just over the pass. We followed the map. There's no pair of lakes just over the pass. Hence the map is wrong.
3. If they blitz but don't get to our quarterback, then our wide receiver will be open. So our wide receiver won't be open, as shown by the fact that they won't blitz.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

4. My true love will marry me only if I buy her a Rolls Royce. It follows that she'll marry me, since I'll buy her a Rolls Royce.
5. The basic principles of ethics can't be self-evident truths, since if they were then they'd largely be agreed upon by intelligent people who have studied ethics.
6. That your views are logically consistent is a necessary condition for your views to be sensible. Your views are logically consistent. So your views are sensible.
7. If Ohio State wins but Nebraska doesn't, then the Ohio Buckeyes will be national champions. So it looks like the Ohio Buckeyes won't be national champs, since Nebraska clearly is going to win.
8. The filter capacitor can't be blown. This is indicated by the following facts. You'd hear a hum, presuming that the silicon diodes work but the filter capacitor is blown. But you don't hear a hum. And the silicon diodes work.
9. There will be a fire! My reason for saying this is that only if there's oxygen present will there be a fire. Of course there's oxygen present.
10. We have no moral knowledge. This is proved by the fact that if we did have moral knowledge then basic moral principles would be either provable or self-evident. But they aren't provable. And they aren't self-evident either.
11. It must be a touchdown! We know that it's a touchdown if the ball broke the plane of the end zone.
12. Assuming that it wasn't an inside job, then the lock was forced unless the thief stole the key. The thief didn't steal the key. We may infer that the robbery was an inside job, inasmuch as the lock wasn't forced.
13. It must be the case that we don't have any tea bags. After all, we'd have tea bags if your sister Carol drinks tea. Of course, Carol doesn't drink tea.
14. We can't still be on the right trail. We'd see the white Appalachian Trail blazes on the trees if we were still on the right trail.
15. If God is omnipotent, then he could make hatred inherently good—unless there's a contradiction in hatred being inherently good. But there's no contradiction in this. And God is omnipotent. I conclude that God could make hatred inherently good. [From William of Ockham, who saw morality as depending on God's will.]
16. Taking the exam is a sufficient condition for getting an A. You didn't take the exam. This means you don't get an A.
17. If Texas or Arkansas wins, then I win my $10 bet. I guess I win $10. Texas just beat Oklahoma 17–14!
18. Unless you give me a raise, I'll quit. Therefore I'm quitting!
19. There's no independent way to prove that our senses are reliable. So empirical knowledge is impossible—since, of course, empirical knowledge would be possible only if there were an independent way to prove that our senses are reliable.
20. It's virtuous to try to do what's good. On the other hand, it isn't virtuous to try to do what's socially approved. I conclude that, contrary to cultural relativism, "good" doesn't mean "socially approved." I assume, of course, that if "good" meant "socially approved" and it was virtuous to try to do what's good, then it would be virtuous to try to do what's socially approved.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

21. Moral conclusions can be deduced from non-moral premises only if "good" is definable using non-moral predicates. But "good" isn't so definable. So moral conclusions can't be deduced from non-moral premises.
22. The world can't need a cause. If the world needed a cause, then so would God.

3.10 S-rules

We'll now learn some **inference rules,** which state that certain formulas can be derived from certain other formulas. These rules are important in their own right, since they reflect common forms of reasoning. The same rules will be building blocks for formal proofs, which we start in the next chapter; formal proofs reduce a complex argument to a series of small steps, each based on an inference rule.

The **S-rules,** which we'll study in this section, are used to *simplify* statements. Our first rule deals with "and"; here it is in English and in symbols:

This and that. ∴ This. ∴ That.	$\dfrac{(P \cdot Q)}{P, Q}$	AND statement, so both parts are true.

From an AND statement, we can infer each part: "It's cold and windy; therefore it's cold, therefore it's windy." Negative parts work the same way:

It isn't cold and it isn't windy.
∴ It isn't cold.
∴ It isn't windy.

$$\frac{(\sim C \cdot \sim W)}{\sim C, \sim W}$$

But from a *negative* AND statement (where "~" is outside the parentheses), we can infer nothing about the truth or falsity of each part:

You're *not both* in Paris and in Quebec.
∴ No conclusion.

$$\frac{\sim(P \cdot Q)}{\text{nil}}$$

You can't be in both cities at the same time. But you might be in Paris (and not Quebec), or in Quebec (and not Paris), or in some third place. From "~(P·Q)" we can't tell the truth value for P or for Q; we only know that *not both* are true (at least one is false).

Our second S-rule deals with "or":

Not either this or that. ∴ Not this. ∴ Not that.	$\dfrac{\sim(P \vee Q)}{\sim P, \sim Q}$	NOT-EITHER is true, so both are false.

From a NOT-EITHER statement, we can infer the opposite of each part: "It isn't either cold or windy, therefore it isn't cold, therefore it isn't windy." Negative parts work the same way: we infer the opposite of each part (the opposite of "~A" being "A"):

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

	Not either not-A or not-B.	$\dfrac{\sim(\sim A \lor \sim B)}{A,\ B}$
∴	A	
∴	B	

But from a *positive* OR statement we can infer nothing about the truth or falsity of each part:

	You're in either Paris or Quebec.	$\dfrac{(P \lor Q)}{\text{nil}}$
∴	No conclusion.	

You might be in Paris (and not Quebec), or in Quebec (and not Paris). From "(P∨Q)" we can't tell the truth value for P or for Q; we only know that *at least one* is true.

Our third S-rule deals with "if-then":

	False if-then.	$\dfrac{\sim(P \supset Q)}{P,\ \sim Q}$	FALSE IF-THEN,
∴	First part true.		so first part true,
∴	Second part false.		second part false.

Recall that our truth tables make an if-then false just in case the first part is true and the second false. Recall also that "(P⊃Q)" means "We *don't* have P-true-and-Q-false"; so "~(P⊃Q)" means "We *do* have P-true-and-Q-false."

This FALSE IF-THEN rule isn't very intuitive; I suggest memorizing it instead of appealing to logical intuitions or concrete examples. You'll use this rule so much in doing proofs that it'll soon become second nature.

If a FALSE IF-THEN has negative parts, we again infer the first part and the opposite of the second part:

$$\frac{\sim(\sim A \supset B)}{\sim A,\ \sim B} \qquad \frac{\sim(A \supset \sim B)}{A,\ B} \qquad \frac{\sim(\sim A \supset \sim B)}{\sim A,\ B}$$

This diagram might help you follow what is going on here:

$$\frac{\sim(\text{first part} \supset \text{second part})}{\begin{array}{cc}\text{write the} & \text{write the opposite}\\ \text{first part} & \text{of the second part}\end{array}}$$

But if the if-then is itself *positive* (there's no "~" outside the parentheses), then we can infer nothing about the truth or falsity of each part. So from "(A⊃B)" we can infer nothing about A or about B.

Let me sum up our three S-rules:

If we have		*we can infer*
AND	→	each part
NOT-EITHER	→	opposite of each part
FALSE IF-THEN	→	part-1 & opposite of part-2

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

So we can simplify these:

$$\frac{(P \cdot Q)}{P,\ Q} \qquad \frac{\sim(P \vee Q)}{\sim P,\ \sim Q} \qquad \frac{\sim(P \supset Q)}{P,\ \sim Q}$$

But we can't simplify these:

$$\frac{\sim(P \cdot Q)}{\text{nil}} \qquad \frac{(P \vee Q)}{\text{nil}} \qquad \frac{(P \supset Q)}{\text{nil}}$$

3.10a Exercise—LogiCola F (SE & SH)

Draw any simple conclusions (a letter or its negation) that follow from these premises. If nothing follows, leave blank.

$$\boxed{\frac{(C \cdot \sim R)}{}} \qquad \boxed{\frac{(C \cdot \sim R)}{C,\ \sim R}}$$

1. $\underline{\sim(I \vee \sim V)}$
2. $\underline{\sim(J \cdot \sim N)}$
3. $\underline{(\sim O \vee \sim X)}$
4. $\underline{\sim(H \supset \sim I)}$
5. $\underline{(F \supset \sim G)}$
6. $\underline{(\sim Q \cdot B)}$
7. $\underline{\sim(\sim A \cdot \sim J)}$
8. $\underline{(\sim T \supset \sim H)}$
9. $\underline{\sim(\sim N \vee \sim E)}$
10. $\underline{\sim(F \supset M)}$
11. $\underline{(M \vee \sim W)}$
12. $\underline{(\sim N \supset S)}$
13. $\underline{(P \cdot U)}$
14. $\underline{\sim(R \vee S)}$
15. $\underline{(\sim D \cdot \sim Z)}$
16. $\underline{(L \vee C)}$
17. $\underline{\sim(\sim U \supset \sim L)}$
18. $\underline{\sim(Q \cdot T)}$
19. $\underline{\sim(\sim Y \supset G)}$
20. $\underline{(\sim K \vee B)}$

3.11 I-rules

The **I-rules** are used to *infer* a conclusion from two premises. There are six I-rules—two each for "·," "∨," and "⊃."

Our first two I-rules deal with "and":

Not both are true. This one is true. ∴ The other isn't.	$\dfrac{\sim(P \cdot Q) \quad P}{\sim Q} \qquad \dfrac{\sim(P \cdot Q) \quad Q}{\sim P}$	*Deny* AND. *Affirm* one part. ∴ *Deny* other part.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

With a NOT-BOTH, we must affirm one part. Here are examples:

You're *not both* in Paris and also in Quebec.
You're in Paris.
∴ You're not in Quebec.

You're *not both* in Paris and also in Quebec.
You're in Quebec.
∴ You're not in Paris.

Negative parts work the same way; if we affirm one, we can deny the other:

$$\begin{array}{c} \sim(\sim A \cdot \sim B) \\ \sim A \\ \hline B \end{array} \qquad \begin{array}{c} \sim(A \cdot \sim B) \\ A \\ \hline B \end{array} \qquad \begin{array}{c} \sim(A \cdot \sim B) \\ \sim B \\ \hline \sim A \end{array}$$

In each case, the second premise affirms (says the same as) one part. And the conclusion denies (says the opposite of) the other part.

If we deny one part, we can't draw a conclusion about the other part:

$$\begin{array}{c} \text{Not both are true.} \\ \text{The first is false.} \\ \hline \text{No conclusion.} \end{array} \qquad \begin{array}{c} \sim(P \cdot Q) \\ \sim P \\ \hline \text{nil} \end{array}$$

You may want to conclude Q; but maybe Q is false too (maybe both parts are false). Here's an example:

You're *not both* in Paris and also in Quebec.
You're not in Paris.
∴ No conclusion.

You needn't be in Quebec; maybe you're in Chicago. To get a conclusion from a NOT-BOTH, we must *affirm* one part.

Our second pair of I-rules deals with "or":

At least one is true.
This one isn't.
∴ The other is.

$$\begin{array}{c} (P \vee Q) \\ \sim P \\ \hline Q \end{array} \qquad \begin{array}{c} (P \vee Q) \\ \sim Q \\ \hline P \end{array}$$

Affirm OR.
Deny one part.
∴ *Affirm* other part.

With an OR, we must deny one part. Here are examples:

At least one hand (left or right) has candy.
The left hand doesn't.
∴ The right hand does.

At least one hand (left or right) has candy.
The right hand doesn't.
∴ The left hand does.

Negative parts work the same; if we deny one part, we can affirm the other:

$$\begin{array}{c} (\sim A \vee \sim B) \\ A \\ \hline \sim B \end{array} \qquad \begin{array}{c} (A \vee \sim B) \\ \sim A \\ \hline \sim B \end{array} \qquad \begin{array}{c} (A \vee \sim B) \\ B \\ \hline A \end{array}$$

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

In each case, the second premise denies (says the opposite of) one part. And the conclusion affirms (says the same as) the other part.

If we affirm one part, we can't draw a conclusion about the other part:

$$\begin{array}{c} \text{At least one is true.} \\ \text{The first is true.} \\ \hline \text{No conclusion.} \end{array} \qquad \begin{array}{c} (L \vee R) \\ L \\ \hline \text{nil} \end{array}$$

You may want to conclude ~R; but maybe R is true too (maybe both parts are true). Here's an example:

At least one hand (left or right) has candy.
The left hand has candy.
∴ No conclusion.

We can't conclude "The right hand doesn't have candy," since maybe both hands have it. To get a conclusion from an OR, we must *deny* one part.

Our final I-rules are ***modus ponens*** (MP—affirming mode) and ***modus tollens*** (MT—denying mode). Both deal with "if-then":

IF-THEN. Affirm first. ∴ Affirm second.	$\begin{array}{c} (P \supset Q) \\ P \\ \hline Q \end{array}$	IF-THEN. Deny second. ∴ Deny first.	$\begin{array}{c} (P \supset Q) \\ {\sim}Q \\ \hline {\sim}P \end{array}$

With an if-then, we must affirm the first part or deny the second part:

If you're a dog, then you're an animal. You're a dog. ∴ You're an animal.	$\begin{array}{c} (D \supset A) \\ D \\ \hline A \end{array}$	If you're a dog, then you're an animal. You're not an animal. ∴ You're not a dog.	$\begin{array}{c} (D \supset A) \\ {\sim}A \\ \hline {\sim}D \end{array}$

Negative parts work the same. If we affirm the first, we can affirm the second:

$$\begin{array}{c} ({\sim}A \supset {\sim}B) \\ {\sim}A \\ \hline {\sim}B \end{array} \qquad \begin{array}{c} (A \supset {\sim}B) \\ A \\ \hline {\sim}B \end{array} \qquad \begin{array}{c} ({\sim}A \supset B) \\ {\sim}A \\ \hline B \end{array}$$

And if we deny the second, we can deny the first:

$$\begin{array}{c} ({\sim}A \supset {\sim}B) \\ B \\ \hline A \end{array} \qquad \begin{array}{c} (A \supset {\sim}B) \\ B \\ \hline {\sim}A \end{array} \qquad \begin{array}{c} ({\sim}A \supset B) \\ {\sim}B \\ \hline A \end{array}$$

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

If we deny the first part or affirm the second, we can't conclude anything about the other part:

If you're a dog, then you're an animal. You're not a dog. ∴ No conclusion.	$\dfrac{\begin{array}{c}(D \supset A)\\ \sim D\end{array}}{\text{nil}}$	If you're a dog, then you're an animal. You're an animal. ∴ No conclusion.	$\dfrac{\begin{array}{c}(D \supset A)\\ A\end{array}}{\text{nil}}$

In the first case, you may want to conclude "You're not an animal"; but you might be a cat. In the second, you may want to conclude "You're a dog"; but again, you might be a cat. To get a conclusion from an if-then, we must *affirm the first part or deny the second part:* "(+⊃−)."

Let me sum up our I-rules:

If we have		*we can infer*
NOT-BOTH+one part	→	opposite of other part
OR+opposite of one part	→	other part
IF-THEN+part-1	→	part-2
IF-THEN+opposite of part-2	→	opposite of part-1

So we can infer with these:

$$\dfrac{\begin{array}{c}\sim(P \cdot Q)\\ P\end{array}}{\sim Q} \qquad \dfrac{\begin{array}{c}(P \vee Q)\\ \sim P\end{array}}{Q} \qquad \dfrac{\begin{array}{c}(P \supset Q)\\ P\end{array}}{Q} \qquad \dfrac{\begin{array}{c}(P \supset Q)\\ \sim Q\end{array}}{\sim P}$$

But we can't infer with these:

$$\dfrac{\begin{array}{c}\sim(P \cdot Q)\\ \sim P\end{array}}{\text{nil}} \qquad \dfrac{\begin{array}{c}(P \vee Q)\\ P\end{array}}{\text{nil}} \qquad \dfrac{\begin{array}{c}(P \supset Q)\\ \sim P\end{array}}{\text{nil}} \qquad \dfrac{\begin{array}{c}(P \supset Q)\\ Q\end{array}}{\text{nil}}$$

Since formal proofs depend so much on the S- and I-rules, it's important to master these rules before starting the next chapter.

3.11a Exercise—LogiCola F (IE & IH)

Draw any simple conclusions (a letter or its negation) that follow from these premises. If nothing follows, leave blank.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

(~Q ∨ ~M)
Q
____

(~Q ∨ ~M)
Q
____
~M

1. ~(W · T)
   W
   ____

2. (~Y ⊃ K)
   Y
   ____

3. (S ∨ ~L)
   ~S
   ____

4. (X ⊃ E)
   E
   ____

5. (~M ∨ ~B)
   ~M
   ____

6. ~(B · S)
   ~S
   ____

7. (U ⊃ G)
   U
   ____

8. ~(~F · ~Q)
   ~F
   ____

9. (C ⊃ ~V)
   ~C
   ____

10. (H ⊃ ~B)
    H
    ____

11. (~N ∨ ~A)
    A
    ____

12. ~(V · H)
    ~V
    ____

13. (~A ⊃ ~E)
    ~E
    ____

14. (K ∨ ~R)
    R
    ____

15. (Y ∨ ~C)
    ~C
    ____

16. (~L ⊃ M)
    ~M
    ____

17. ~(~F · ~O)
    ~O
    ____

18. ~(~S · W)
    ~W
    ____

19. (~I ∨ K)
    K
    ____

20. ~(A · ~Y)
    A
    ____

3.12 Combining S- and I-rules

Our next exercise mixes S- and I-rule inferences. This should cause you little trouble, so long as you remember to use S-rules to simplify *one* premise and I-rules to infer from *two* premises. Here's a quick review:

S-rules (Simplifying): S-1 to S-3	*I-rules (Inferring): I-1 to I-6*
(P·Q) → P, Q	~(P·Q), P → ~Q ~(P·Q), Q → ~P
~(P∨Q) → ~P, ~Q	(P∨Q), ~P → Q (P∨Q), ~Q → P
~(P⊃Q) → P, ~Q	(P⊃Q), P → Q (P⊃Q), ~Q → ~P

3.12a Exercise—LogiCola F (CE & CH)

Draw any simple conclusions (a letter or its negation) that follow from these premises. If nothing follows, leave blank.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

$(A \supset \sim B)$ $\sim A$ ________	(no conclusion)

1. $\sim(W \cdot \sim X)$, $\sim W$ ________
2. $(\sim I \supset \sim N)$, N ________
3. $\sim(\sim B \vee C)$ ________
4. $\sim(L \cdot M)$ ________
5. $(D \vee \sim J)$, D ________
6. $\sim(\sim C \supset D)$ ________
7. $(X \supset F)$, $\sim X$ ________
8. $\sim(M \vee \sim I)$ ________
9. $(P \cdot \sim Q)$ ________
10. $\sim(\sim R \cdot A)$, $\sim R$ ________
11. $(\sim S \vee T)$ ________
12. $\sim(R \cdot \sim G)$, $\sim G$ ________
13. $(\sim L \cdot S)$ ________
14. $(\sim L \vee \sim T)$, L ________
15. $(A \supset \sim B)$ ________
16. $\sim(U \cdot T)$, T ________

3.13 Extended inferences

From an AND statement, we can conclude that both parts are true; so from "(P·Q)," we can get "P" and also "Q." The rule also works on larger formulas:

$$\frac{((C \equiv D) \cdot (E \supset F))}{(C \equiv D),\ (E \supset F)}$$

AND statement, so both parts are true.

Visualize the premise as a big AND with two parts—blurring out the details: "($$$$$· #####)." We can infer each part, even if these parts are complex.

Here's another inference using an S-rule:

$$\frac{\sim((C \cdot D) \supset (E \supset F))}{(C \cdot D),\ \sim(E \supset F)}$$

FALSE IF-THEN, so first part true, second part false.

Again, blur the details; read the long formula as just "FALSE IF-THEN." From such a formula, we can conclude that the first part is true and the second false; so we write the first part and the opposite of the second.

Consider this formula (which I suggest you read to yourself as "IF-THEN"):

$$((C \cdot D) \supset (E \supset F))$$

Since this is an if-then, we can't break it down using an S-rule. But we can conclude something from it if we have the first part true:

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

((C · D) ⊃ (E ⊃ F))	IF-THEN.
(C · D)	Affirm first.
(E ⊃ F)	Affirm second.

And we can infer if we have the second part false:

((C · D) ⊃ (E ⊃ F))	IF-THEN.
~(E ⊃ F)	Deny second.
~(C · D)	Deny first.

These are the only legitimate I-rule inferences. We get no conclusion if we deny the first part or affirm the second:

((C · D) ⊃ (E ⊃ F))	((C · D) ⊃ (E ⊃ F))
~(C · D)	(E ⊃ F)
nil	nil

Also, we get no conclusion if we just have "E" as an additional premise:

((C · D) ⊃ (E ⊃ F))
E
―――――――――
nil

Here we don't know that "(E⊃F)" is true—but just that it *would* be true *if* "(C·D)" were true.

3.13aExercise—No LogiCola exercise

Draw any conclusions that follow from these premises by a single application of the S-or I-rules. If nothing follows in this way, then leave blank.

~(~A ∨ (B · C))	~(~A ∨ (B · C))
	A, ~(B · C)

1. ~((A · B) ⊃ ~C)

2. ((A · B) ⊃ ~C)
~(A · B)

3. ~((G ∨ H) · (I ∨ J))

4. ~((G ∨ H) · (I ∨ J))
(G ∨ H)

5. ((A · B) ∨ (C ⊃ D))

6. ((A · B) ∨ (C ⊃ D))
C

7. ~((A ⊃ B) ∨ C)

8. ((A ⊃ B) ⊃ C)
(A ⊃ B)

9. ((G ≡ H) ⊃ ~(I · J))
~(I · J)

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.

3.14 Logic gates and computers

Digital computers were developed using ideas from propositional logic. The key insight is that electrical devices can simulate logic formulas.

Computers represent "1" and "0" by different physical states; "1" might be a positive voltage and "0" a zero voltage. An **and-gate** would then be a physical device with two inputs and one output, where the output has a positive voltage if and only if *both* inputs have positive voltages:

An **or-gate** would be similar, except that the output has a positive voltage if and only if *at least one* input has a positive voltage. For any formula, we can construct an input-output device (a **logic gate**) that mimics that formula.

A computer basically converts input information into 1's and 0's, manipulates these 1's and 0's by logic gates and memory devices, and converts the resulting 1's and 0's back into a useful output. So propositional logic is central to the operation of computers. One of my logic teachers at the University of Michigan, Art Burks, was part of the team in the 1940s that produced the ENIAC—the first large-scale electronic computer. So logic had an important role in moving us into the computer age.

Copyright © 2001. Taylor & Francis Group. All rights reserved.

Gensler, H. (2001). Introduction to logic. ProQuest Ebook Central <a
onclick=window.open('http://ebookcentral.proquest.com','_blank') href='http://ebookcentral.proquest.com' target='_blank' style='cursor: pointer;'>http://ebookce
Created from west on 2021-09-08 13:46:07.