From p-values to decision making

Suppose you are testing the hypotheses

$$H_0 : p = 0.6$$

$$H_a : p > 0.6$$

based on data that have produced

specific observed value $\rightarrow \hat{p} = 0.75$

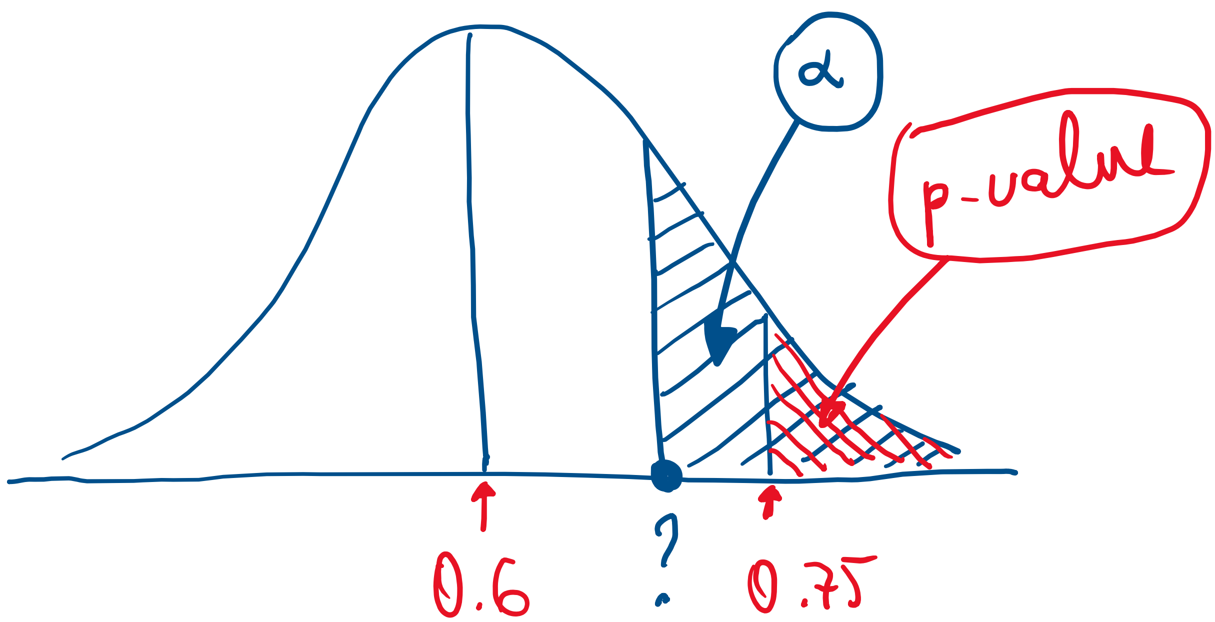
For this, you calculate the p-value

specific value

$$\text{p-value} = P(\hat{p} > 0.75)$$

random variable

and compare it with the significance level α.



What is the value of "?" that gives the area under the normal curve equal to α?

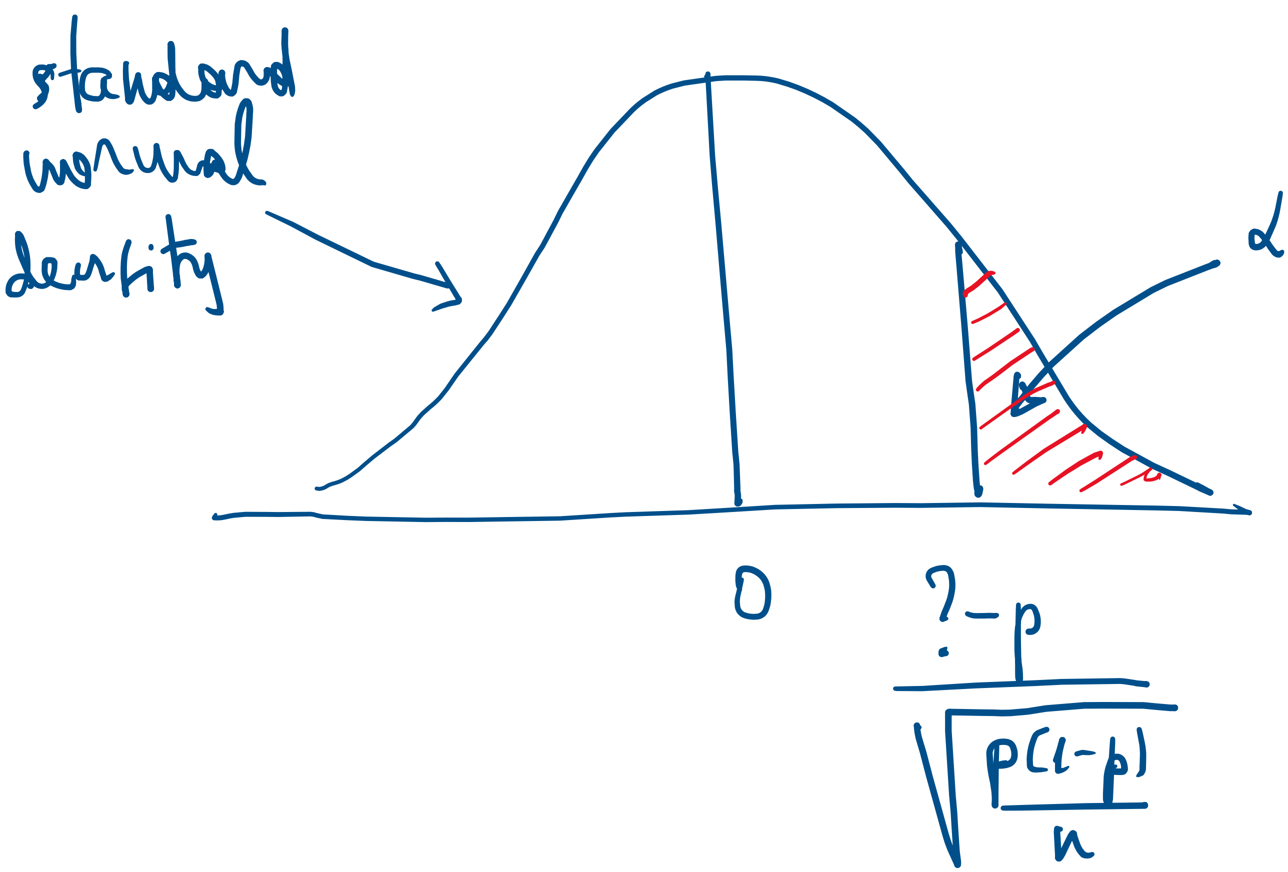
This value of "?" is very important: if the observed $\hat{p}$ is to the right of "?", then we reject the null hypothesis, and we retain it otherwise.

Let's find "?"

$$P(\hat{p} > ?) = \alpha$$

same as

$$P\left(\underbrace{\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}}_{Z} > \frac{?-p}{\sqrt{\frac{p(1-p)}{n}}}\right) = \alpha$$



Suppose that $\alpha = 0.05$

Then $$\frac{? - p}{\sqrt{\frac{p(1-p)}{n}}} = 1.64 \quad \text{(see Table 21.1)}$$

and so

$$? = p + 1.64\sqrt{\frac{p(1-p)}{n}}$$

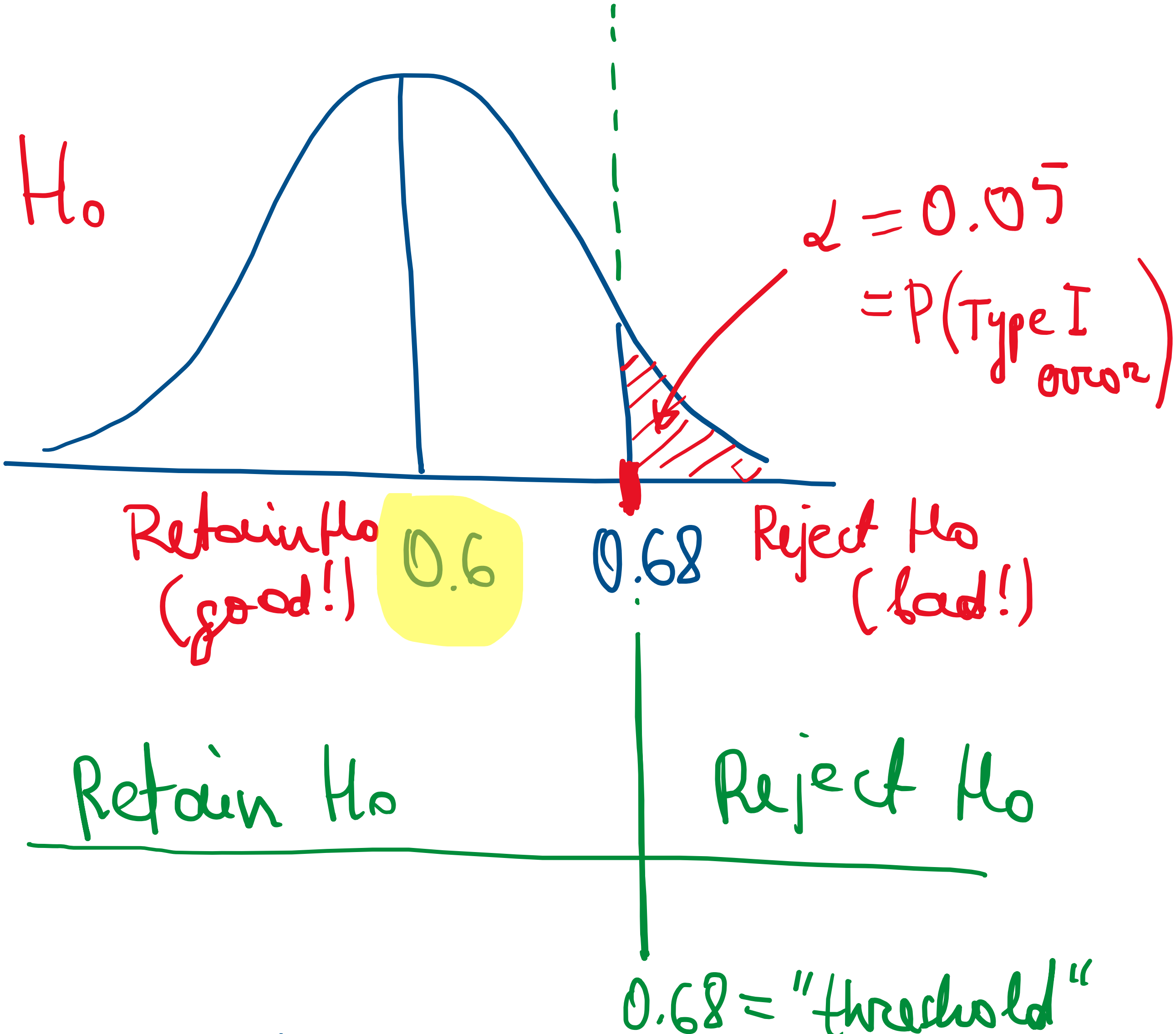
$$= 0.6 + 1.64\sqrt{\frac{0.6 \times 0.4}{100}}$$

$$= 0.6803433\ldots$$

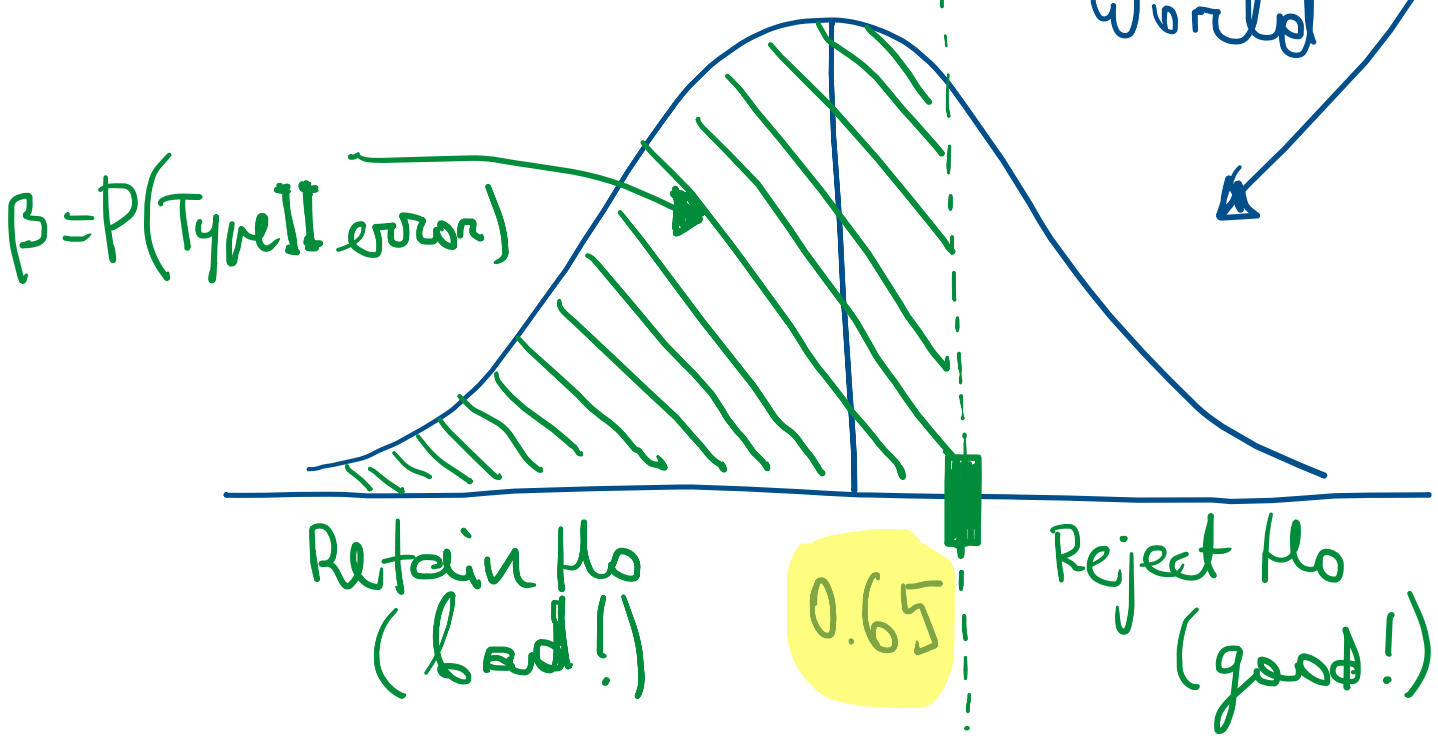
Consequently, we have the decision rule:

If $\hat{p} > 0.68$, then reject H_0

If $\hat{p} < 0.68$, then retain H_0



But what if the truth is this alternative world



Note what happens when we move the threshold to the right or left:

If we move it to the right, α decreases but β increases

If we move it to the left, α increases but β decreases

As we see, we can't decrease the two probabilities at the same time. Hence, we have to make a choice: either α is small or β is small.

See the file entitled "Decisions (credit card) type I and II errors" for a visualization. Run the slides quickly to see how the probabilities of type I and type II errors interact.