CS3331 - Assignment 3 - 2018

Decidable and Semi-Decidable Languages I

Due: Tuesday, Nov 27, 2018 (Latest to submit: Friday, Nov 30)

1. (20pt) Construct a deterministic Turing machine M that decides the language

$$L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ has } \mathtt{ab} \text{ as a substring and ends with } \mathtt{ba}\}.$$

   M starts with the initial configuration $(s, \sqcup w)$ and halts with the configuration $(q, \sqcup w)$, for the appropriate $q \in \{y, n\}$. Describe M in detail using a directed graph whose edges are labelled by transitions (such as the one in Example 17.2, p. 368 of textbook).

2. (20pt) Construct a deterministic Turing machine M that subtracts one from its binary input if it is positive and sets it to zero if its input is zero. This machine computes the function

$$f(n) = \begin{cases} 0 & \text{if } n \text{ is } 0 \\ n-1 & \text{otherwise} \end{cases}$$

   M starts with the initial configuration $(s, \sqcup w)$, where $w \in \{\mathtt{0}, \mathtt{1}\}^*$; the binary input w is interpreted as an integer number. The machine must remove all leading zeros from the input and halt in the appropriate configuration $(h, \sqcup(w - \mathtt{1})_{(2)})$ or $(h, \sqcup\mathtt{0})$, where $w_{(2)}$ is the binary representation of w. If $w = \varepsilon$, treat it as a representation of $\mathtt{0}$. Here are some examples of M s behaviour:

   $(s, \sqcup) \mid\!-^* (h, \sqcup\mathtt{0})$

   $(s, \sqcup\mathtt{000}) \mid\!-^* (h, \sqcup\mathtt{0})$

   $(s, \sqcup\mathtt{01}) \mid\!-^* (h, \sqcup\mathtt{0})$

   $(s, \sqcup\mathtt{111}) \mid\!-^* (h, \sqcup\mathtt{110})$

   $(s, \sqcup\mathtt{001100}) \mid\!-^* (h, \sqcup\mathtt{1011})$

   Describe M using the macro language (such as the one in Example 17.8, p. 377 of textbook).

3. (20pt) Construct a deterministic Turing machine M that multiplies two unary numbers. Specifically, given the input string $< x >; < y >$, where $< x >$ is the unary encoding of a natural number x and $< y >$ is the unary encoding of a natural number y, M should output $< z >$, the unary encoding of $z = xy$. For example, on input $\mathtt{111;1111}$, M should output $\mathtt{111111111111}$. Describe M using the macro language

4. (20pt) Consider the language $L = \{< M > \mid M \text{ accepts at least two strings}\}$.

   (a) Describe in clear English a Turing machine M that semidecides L.

   (b) Suppose we changed the definition of L just a bit. We now consider:

$$L' = \{< M > \mid M \text{ accepts } \textit{exactly } 2 \text{ strings}\}.$$

   Can you tweak the Turing machine you described in part (a) to semidecide L'?

5. (20pt) Describe in clear English a Turing machine that semidecides the language

$$L = \{< M > \mid M \text{ accepts the binary encodings of the first four Fibonacci numbers}\}.$$

Note well: You may submit your assignment in one of two ways:

- Ideally, submit your solution as a pdf file on OWL (scanned written assignments are fine). Assignments submitted this way will not be accepted after 11:59 pm on November 30.
- Otherwise **staple your assignment** and hand in solutions in class or to the 3331 dropbox (locker #306, across from the elevator on the 3rd floor of Middlesex College). Assignments submitted this way will not be accepted after 5:00 pm on November 30.