14.

6. $F_{(x,y,z)} = 8x^2 + 8y$

$F_{(x,y,z)} = 8x^2 + 8y^2 - 4z^2 - 2x(y+z) = 0$. $8x$

$F_x = 16x - 2(y+z)$ $F_z = -8z - 2x$ $\frac{dz}{dx} = -\frac{F_x}{F_z}$

$= \frac{y+z-8x}{x+4z}$

7. $F = xe^y \cos z - z - 2$. $(4,0,0)$.

$F_x = e^y \cos z$ $F_y = xe^y \cos z$ $F_z = -xe^y \sin z - 1$.

$\therefore \vec{n} = (e^y \cos z,\ xe^y \cos z,\ -xe^y \sin z - 1)$.

$\vec{n}|_{(4,0,0)} = (1,4,-1)$ $\Rightarrow$ $\frac{x-4}{1} = \frac{y}{4} = \frac{z}{-1}$.

$\therefore$ ~~$1(x-4) + 4(y-0) - (z-0) = 0$.~~

~~$x + 4y - z - 4 = 0$.~~

8. $f(x,y) = x^2 - xy + y^2 - 9x + 6y + 9$

$f_x = 2x - y - 9 = 0$

$f_y = -x + 2y + 6 = 0$. $\Rightarrow \begin{cases} x = 4 \\ y = -1 \end{cases}$

$f_{xx} = 2$ $f_{yy} = 2$ $f_{xy} = f_{yx} = -1$.

$f_{xx} f_{yy} - (f_{xy})^2 = 4 - 1 = 3 > 0$.

$\therefore$ there's a local maximum at $(4,-1)$

9. $x + y + z = 400$ $f(x,y,z) = xyz$.

$F = xyz + \lambda(x + y + z - 400)$.

$F_x = yz + \lambda x = 0$ $F_y = xz + \lambda y = 0$ $F_z = xy + \lambda z = 0$.

$F_\lambda = x + y + z - 400 = 0$

$\Rightarrow$ $x = y = z = \frac{400}{3}$, $\lambda = -\frac{400}{3}$.

$\therefore$ the maximum value is $\frac{64000000}{27}$.

10. $f(x,y,z) = 5x - y - 5z$, $x + 2y - z = 0$ $x^2 + 4y^2 = 1$.

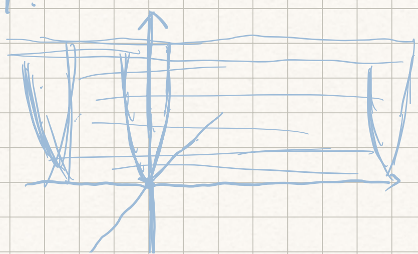
$L = 5x - y - 5z + \lambda_1 (x + 2y - z) + \lambda_2 (x^2 + 4y^2 - 1)$.

$$\left.\begin{aligned} L_x &= 5 + \lambda_1 + 2x\lambda_2 = 0. \\ L_y &= -1 + 2\lambda_1 + 8y\lambda_2 = 0 \\ L_z &= -5 - \lambda_1 = 0 \\ L_{\lambda_1} &= x + 2y - z = 0 \\ L_{\lambda_2} &= x^2 + 4y^2 - 1 = 0. \end{aligned}\right\} \begin{aligned} x &= 0. \\ y &= \tfrac{1}{2} \ / -\tfrac{1}{2}. \\ z &= 1 \ / -1 \\ \lambda_1 &= -5. \\ \lambda_2 &= 0 \end{aligned}$$

$\therefore \max = 0 + \frac{1}{2} + 5 = \frac{11}{2}$

$\min = 0 - \frac{1}{2} - 5 = -\frac{11}{2}$.

11.



16. $\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{16x + \frac{9}{2}\sqrt{\frac{y}{x}}}{\frac{9}{2}\sqrt{\frac{y}{x}} - 7}$.

19. $f(x, y, z) = x^4 e^{yz}$ $P(-2, 0, -9)$.

$f_x = 4x^3 e^{yz} = -32$

$f_y = z x^4 e^{yz} = -144$. $\nabla f = (-32, -144, 0)$.

$f_z = y x^4 e^{yz} = 0$. $\nabla f = -32i - 144j$

20. $x + y + z = 368$ $f(x, y, z) = xyz$

$f_{max} = (\frac{368}{3})^3$.