Midterm 1 : Feb 8, Sat 7:00 pm - 9:00 pm.
section 005 FMB 1240
15 multiple-choice questions. 2% each 6 long questions, 36%.
(7.6, 7.7) X

# Section 11.6

## Absolute Convergence, Conditional Converges

Given any series $\sum_{n=1}^{\infty} a_n$ (a could be +ve/-ve),
we can construct a new series which has a same terms but write the absolute value $|a_n|$.
i.e. the new series in the form of $S_n = |a_1| + |a_2| \cdots + |a_n|$.
$= \sum_{n=1}^{\infty} |a_n|$.

Defination: a series $\sum a_n$ is called absolute convergent if the series of $|a_n|$ is convergent

e.g. 1. $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{n^2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - \cdots\cdots$
is absolute convergent.
proof: $\sum_{n=1}^{\infty} |a_n| = 1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2}$. which is convergent
because it is a p series where $p = 2 > 1$.

Recall: $\int_a^{\infty} \frac{1}{x^p}$ $p > 1$ converges. $\int_0^a \frac{1}{x^p}$ $p > 1$ diverges
$p < 1$ diverges $p < 1$ converges.

Defination $\sum_{n=1}^{\infty} a_n$ is called conditionally convergent
if it is convergent but not absolutely convergent.

e.g. $\sum \frac{(-1)^{n+1}}{n}$ is convergent by the — series test.

but not absolutely converges.

Recall: $\frac{1}{n}$ is a harmonic series and it is divergent

absolute convergent $\underset{\not\Leftarrow}{\overset{\checkmark}{\Rightarrow}}$ conditional convergent.
(convergent).

proof: $0 \le a_n + |a_n| \le 2|a_n|$.

If $a_n$ is absolutely convergent:

By defination, $2\sum |a_n|$ is convergent.

the difference between two convergent series is also convergent

e.g. 3. $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$

$$\sum_{n=1}^{\infty} \left| \frac{\cos n}{n^2} \right| = \sum_{n=1}^{\infty} \frac{|\cos n|}{n^2}$$

convergent.

$\because |\cos n| \in [0,1]$. $\therefore$ $0 \le \sum_{n=1}^{\infty} \frac{|\cos n|}{n^2} \le \sum_{n=1}^{\infty} \frac{1}{n^2} \leftarrow$ p series

$\Rightarrow$ the original series is absolute convergent

$\Rightarrow$ the original series is convergent.

Similarily, $\frac{\sin}{n^2}$ is convergent.

Consider a geometric series.

$a + an + an^2 + an^3 + \cdots = \sum_{x=0}^{\infty} an^{x-1}$.

if $|n| < 1 \Rightarrow$ convergent

$\quad > 1 \quad$ divergent.

Consider the ratial $\left|\frac{a_{x+1}}{a_x}\right|$ when $a_x = a n^{x-1}$.

If the $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1 \Rightarrow$ converges.

$>$ diverges.

Ratial test:

(i) If $\left|\frac{a_{n+1}}{a_n}\right|$ $<$ / $>$ $1$, the series is absolute converges / diverges.

(ii) If $\left|\frac{a_{n+1}}{a_n}\right| = \infty$, the series is diverses.

e.g. 4. determie the series is absolutly or conditionally convergent.

(i) $\sum_{n=0}^{\infty} \frac{(-10)^n}{n!}$ (ii) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n^4}$

(i) $a_n = \frac{(-10)^n}{n!}$.

$$\frac{a_{n+1}}{a_n} = \frac{n!\,(-10)^{n+1}}{(n+1)! \cdot (-10)^n} = -\frac{10}{n+1}.$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{10}{n+1} = \frac{\frac{10}{n}}{1+\frac{1}{n}} \underset{n\to\infty}{=} 0 < 1$$

$\Rightarrow$ absolutly convergent $\Rightarrow$ convergent.

(ii) $a_n = (-1)^{n+1}\frac{2^n}{n^4}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{2n^4}{(n+1)^4}$$

$$\lim_{n\to\infty}\frac{n^4}{(n+1)^4} = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^4 = \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)^4.$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{2n^4}{(n+1)^4} = 2 > 1.$$

$\Rightarrow$ divergent.

$\left|\frac{a_{n+1}}{a_n}\right| < n$.

$\left|\frac{a_{n+1}}{n}\right|$ $>1$ / $<1$ $=1$ ?

The root test:

If $\lim_{n\to\infty} |a_n|^{\frac{1}{n}} = L$ $<1$ $\Rightarrow$ the series absolute convergent

$>1$ $\Rightarrow$ diverges.

$=1$ $\Rightarrow$ the test fail, change test.