Instructor's Name (Print)	Student's Name (Print)	

Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO LONDON CANADA

DEPARTMENT OF MATHEMATICS

Mathematics 1229A Final Examination

Wednesday, December 12, 2018

Code 111

9:00 a.m. - 12:00 noon

INSTRUCTIONS

- 1. Fill in the top of this page, and the next page, completely.
- Fill in the top of the scantron card completely. You MUST both print AND code your Student Number, Section Number (below) and Exam Code (above).
- 3. DO NOT UNSTAPLE THE BOOKLET. The two blank pages at the back of the booklet may be torn off and used for rough work. Do not tear any other pages out of the booklet.
- 4. CALCULATORS AND NOTES ARE NOT PERMITTED.
- 5. There are two parts to this examination: PART A (35 marks) in multiple choice format and PART B (15 marks) in show your work format.
- 6. In Part A, circle the correct answer to each question on this paper AND fill in the appropriate box on the **scantron** card with an HB pencil.
- 7. In Part B, show all your work in the space provided.
- 8. Questions are printed on both sides of the paper, they begin on Page 1 and continue to Page 9. Be sure that your booklet is complete.
- You must hand in this question paper, your scantron card, and all rough work sheets.
- 10. Circle your section in the list below.

Instructor Campus/College		Time	Section
Lindsey	Main	9:30 MWF	001
Pasini	Main	12:30 MWF	002
Olds	Main	1:30 MWF	003
Pasini	Main	8:30 MWF	004
Ghorbanpour	Brescia	8:30 MTuTh	530
O'Hara	Brescia	$9:30~\mathrm{MTuW}$	531
Rastegari	Huron	11:30 MWF	550
Mollahajiaghaei	Huron	8:30 Tu	551
Kuzmin	King's	10:30 Tu, 9:30 Th	570
Turnbull	King's	1:30 Tu, 12:30 Th	571
Turnbull	King's	1:30 M, 12:30 W	572
Kuzmin	King's	7:00 MW	573
Kuzmin	King's	8:30 MW	574

11. TOTAL MARKS = 50.

Student Number (Print)	Student's Name (Print)

FOR GRADING ONLY

PAGE	MARK
1–5	
6	
7	
8	
9	
TOTAL	

PART A (35 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE INDICATED ON THE SCANTRON SHEET. YOU SHOULD ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

1. Find $\|\mathbf{u}\|$ where $\mathbf{u}=(1,\sqrt{3},-1,2,-4)$.

A: 5	B: $2\sqrt{3}$	C: $3\sqrt{3}$	D: $\sqrt{31}$	E: 1

mark

2. For what value(s) of k are the vectors $\mathbf{u} = (1, 0, -1, 2)$ and $\mathbf{v} = (k, 0, 3, k)$ collinear?

k-3+2k= 0 k=1.

3. For what value(s) of k are the vectors $\mathbf{u} = (1, 0, -1, 2)$ and $\mathbf{v} = (k, 0, 3, k)$ orthogonal?

A: k = 1 only B: k = 0 only C: $k = -\frac{3}{5}$ only D: all values of k E: no value of k

4. Consider the vectors $\mathbf{u} = \mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = -2\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$ in \Re^3 . Find, if possible,

 \overline{mark} $-2\mathbf{u} \cdot (\mathbf{v} + 3\mathbf{k}).$ u=(1,-3,1) V=(-2,-4,-6).

D: (4, -24, 6)

-1+6-3-4. $-3, 1, 1), \mathbf{v} = (-1, -2, -3, -4) \text{ and } \mathbf{w} = (-2, -1, -2, 0).$ Find, if possible,

A: 2 D: (4, 2, 4, 0)E: the operation is not defined

6. Find the deal of parallelogram ABCD with vertices A(1,1,1), B(3,2,2), C(4,4,1) and

D(2,3,0). S=absine. A: $\sqrt{27}$ C: 9

2.6.18.

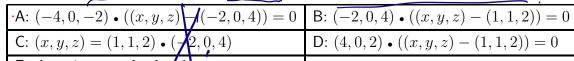
7. Find the distance from the point P(2,0,-1) to the plane 2x-5y-z=5mark

D: 0 $\sqrt{30}$

8. Which one of the following is an equation of the line through P(2,4) and Q(-4,2)

A: $(6,2) \cdot ((x,y) - (2,4)) = 0$ B: (x,y) = (1-t)(2,4) + t(6,2)D: (x,y) = (2,4) + t (4,2)E: 2x - 6y

Which one of the following is a point-normal form equation of the plane in \Re^3 which mark contains the lines (x, y, z) = (1, 1, 2) + t(-2, 1, 4) and (x, y, z) = (1, 1, 2) + r(0, 1, 0)?



E: there is no such plan (-2,1,4). (0,1,0). $\bar{m}ark$

10. Which of the matrices shown below are in row-reduced echelon form?



$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

_	_	- <i>v</i>	_		
2	(, 2)/		1.0		\
l le	1-4-557	A: A and	B nly	$B \colon \ C \text{ and } D \text{ only}$	C: \mathbb{R} and C only
		D: B, C ε	$\operatorname{ing}(D)$	E: $A, B \text{ and } C$	
					_

11. Which one of the following is an augmented matrix for a system of linear equations which $\bar{m}ark$ is inconsistent?



$A: \begin{bmatrix} 1 & 5 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$	$B \colon \left[\begin{array}{ccc c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 &$	C: $\begin{bmatrix} 1 & 5 & 0 & & 1 \\ 0 & 0 & 0 & & 5 \\ 0 & 4 & -3 & & 4 \end{bmatrix}$
$ D: \left[\begin{array}{cc c} 1 & 5 & 0 \\ 0 & 0 & 0 \\ 0 & 4 & -3 \end{array} \right] $	$E \colon \left[\begin{array}{ccc c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$	

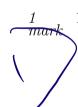
12. How many solutions does the system of linear equations corresponding to the augmented matrix shown below have?



$$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{2} \quad . \quad \mathbf{5} \quad .$$

A: no solutions	B: exactly one solution
C: a 1-parameter family of solutions	D: a 2-parameter family of solutions

E: a 3-parameter family of solutions



13. Consider the following hyperplanes in \Re^4 : $3x_1 - 2x_2 - x_4 = 4$, $x_3 - x_4 = 0$ and $x_3 = 6$.

Which one of the following describes the intersection of these 3 hyperplanes?

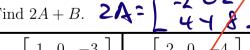
A: no points | R: avastle 1 | | |

A: no points	B: exactly 1 point	C: exactly 5 points	D: a line	E: a plane

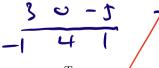
Use the following information for questions 14, 15 and 16.

Let
$$A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 2 & 4 \end{bmatrix}$$
 and $B = \begin{bmatrix} 3 & 0 & -5 \\ -1 & 4 & 1 \end{bmatrix}$.

14. Find 2A + B. **2**



0 1 0 -3E: not defined 2 12 10 1 6

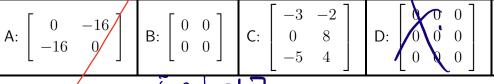


-218+4

E: not defined



15. Find (BA^T)



10.

 $\frac{1}{mark}$ 16. Find A^3 . Recall: $A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 2 & 4 \end{bmatrix}$

A: $ \begin{bmatrix} -1 & 0 & 1 \\ 8 & 8 & 64 \end{bmatrix} $

17. If $A = \begin{bmatrix} 1 & -1 \ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \ 1 & 2 \end{bmatrix}$, find A^2B .

A: $\begin{bmatrix} -2 & 1 \\ -3 & 1 \end{bmatrix}$	$B \colon \left[\begin{array}{cc} 1 & 4 \\ 1 & 5 \end{array} \right]$	$C \colon \left[\begin{array}{cc} 1 & 1 \\ 4 & -1 \end{array} \right]$	D: $\begin{bmatrix} 3 & -1 \\ -8 & 3 \end{bmatrix}$	$ \begin{bmatrix} E: \begin{bmatrix} 2 & -3 \\ -5 & 4 \end{bmatrix} $

Consider the system of linear equations $A\mathbf{x} = \mathbf{b}$, where A is a 4×4 matrix whose row-reduced echelon form contains exactly 3 leading ones. Which one of the following statements is **true**?

0100

0010

3000

A: A is invertible.

B: A has no inverse.

The system cannot be consistent.

The system cannot have infinitely many solutions.

E: If the system is homogeneous, it has only the trivial solution.

det A=0 2=> A is not invertible

<=> A has no inverse.

1 19. Find the rank of $\begin{bmatrix} -3 & 6 & 3 & -9 \\ 2 & -4 & -2 & 6 \\ 1 & -2 & -1 & 3 \end{bmatrix}$.

B: 2 E: not defined, since A is not a square matrix

> vank. 2

20. Let $A\mathbf{x} = \mathbf{b}$ be a system of 2 linear equations in 3 unknowns. Which one of the following markstatements MUST be false?

- The system might have a one-parameter family of solutions.
- The system might have a two-parameter family of solutions.
- The system might have no solution.
- The system might have a unique solution.
- None of A, B, C or D.

21. Let A be a 5×3 matrix and let $A\mathbf{x} = \mathbf{b}$ be a system of linear equations. If the rank of A is 2 and the rank of the augmented matrix $[A \mid \mathbf{b}]$ is 3, which one of the following describes the solution(s) to this system?

A: a unique solution

B: a 1-paramete family of solutions

C: a 2-parameter family of solutions D: a 3-parameter family of solutions

E: no solution

22. Let $A\mathbf{x} = \mathbf{0}$ be a homogeneous system of n linear equations in n knowns. If this system has a unique solution, which one of the following statements is false?



- The rank of A is
- The row-reduced echelon form of A is an identity matrix.
- The determinant of A is Λ .
- The unique solution is $\mathbf{x} = \mathbf{0}$.



23. Find det

-4	_	6+	4
3	•		

A: 2	B: -10	C: 10	D: -2	E: 0
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24. Find the 3,2-cofactor of the matrix
$$\begin{bmatrix} 0 & -4 & 1 \\ -3 & 1 & -1 \\ 2 & 2 & 5 \end{bmatrix}$$
. - $\begin{bmatrix} 6 + 5 \\ -3 & 1 \end{bmatrix}$.

A: -3	B: 3	C: 8	D: -8	E: 6

25. Find det
$$\begin{bmatrix} 2 & -1 & -4 & 0 \\ 0 & 2 & 3 & 0 \\ 4 & -3 & 9 & 2 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$
 - 2 det
$$\begin{bmatrix} 2 & -1 & -4 \\ 0 & 2 & 3 \end{bmatrix}$$
 - 2 - 3 = -5.

A: -10	B: 20	C: 10	D:	-20	E: 0	

26. If A and B are two 2×2 matrices with $\det A = 2$ and $\det B = -1$, find $\det (3A^TB^2)$.

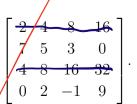
D: 6

and let I be the 3×3 identity matrix. Find $\det(A + 2I)$.

9 det (ABB) 9 ×2.

C: 35

28. Find det



A: 16	B: 8	C: 4	D: 2	E: 0



12

A: −12

D: 12

E: 0

Use the following information for questions 30, 31 and 32.

It is known that $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} = 6.$



 $\begin{array}{cccc} 2d & e & f \\ -2g & -h & -k \end{array}$

X2.

C: 2

D: -12

E: 6

31. Find det k-3c h-3b

B: 6

C: -6

D: 18

E: -18

32. Find det

B: -2

C: 2

D: -18

E: 18

-1 (4-sk)-k.2.=0 0 k-133. Find the value of k for which A =has no inverse.

A: k = 4

B: k =

C: k = -4

C: 9

E: k = 0

34. If A is a 3×3 matrix with det A = -3, find $\det(\operatorname{Adj} A)$

B: -27

D:

= 1 and det $\begin{vmatrix} 4 & a \\ 5 & b \end{vmatrix}$ = 4, find the unique solution to the 35. If it is known that det system of linear equations

A: x = 4, y =

 $\mathsf{B} \colon x =$

C: $x = 2, y = \frac{1}{2}$

D: x = 1, y = 4

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PART B (15 marks)

SHOW YOUR WORK FOR ALL QUESTIONS IN PART B.

NOTE: NO MARKS WILL BE GIVEN if a particular method is specified in a question or question part and you DO NOT use the specified method.

 $\frac{3}{marks}$ 36. Consider the lines:

$$\ell_1: (x_1, x_2, x_3, x_4) = (1, 0, 1, 4) + r(1, 3, 0, -2)$$

 $\ell_2: (x_1, x_2, x_3, x_4) = (2, 1, 1, 2) + t(-1, -2, 0, 2)$

Find the intersection, if any, of ℓ_1 and ℓ_2 .

$$\begin{cases} 3r = 1-2t \cdot 2r = 2-2t \cdot r = -1 \cdot 3r = 1-2t \cdot r = -1 \cdot 1 - 2t \cdot r = -1 \cdot$$

4 37. Consider the system of linear equations

$$x_3 + 3x_4 + 2 = 0$$
$$3x_2 = x_1 - 2x_4 - 5$$
$$x_1 - 3x_2 - x_3 - 2x_4 = 1$$

(a) Write the augmented matrix for the standard form of this system.

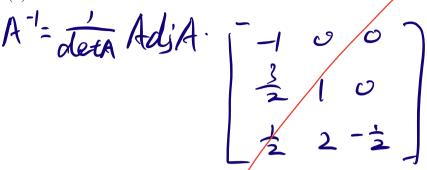
(b) Use your augmented matrix from part (a) to solve the system using Gauss-Jordan elimination.

$$\begin{vmatrix}
-0 & 0 & 1 & 3 & | & -2 \\
-1 & 3 & 0 & 2 & | & -5 \\
1 & -3 & -1 & -2 & | & 1
\end{vmatrix}$$

$$\begin{vmatrix}
r_2 = r_2 + r_3 \\
0 & 0 & -1 & 0 & | & -4 \\
1 & -3 & -1 & -2 & | & 1
\end{vmatrix}$$

$$\frac{V_{2}-V_{2}}{UUU_{1}-3-1-2} = \frac{1}{1} = \frac{1}{3} \frac{(r_{1}-r_{2})}{(r_{3}-r_{3}+r_{2})} = \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} = \frac{1}{3} \frac{(r_{1}-r_{2})}{(r_{3}-r_{3}+r_{2})} = \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} = \frac{1}{3} \frac{(r_{1}-r_{2})}{(r_{3}-r_{3}+r_{2})} = \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} = \frac{1}{3} \frac{(r_{1}-r_{2})}{(r_{3}-r_{3}+r_{2})} = \frac{1}{1} \frac{1}$$

- $\underset{marks}{2} 38. \text{ Let } A \text{ be a } 3 \times 3 \text{ matrix with det } A = 2 \text{ and } Adj A = \begin{bmatrix} -2 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 4 & -1 \end{bmatrix}.$
 - (a) Find A^{-1} .



- (b) Find the solution to $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 8 \end{bmatrix}$.
- 3 39. Let $A = \begin{bmatrix} 4 & -1 & 5 \\ a & b & c \\ d & e & f \end{bmatrix}$ where it is known that

$$\det \begin{bmatrix} a & b \\ d & e \end{bmatrix} = 10, \quad \det \begin{bmatrix} a & c \\ d & f \end{bmatrix} = -2, \quad \det \begin{bmatrix} b & c \\ e & f \end{bmatrix} = 3$$

(a) Find $\det A$.

(b) Find the (2,1)-entry of Adj A.

$$3 \atop marks = 40. \text{ Let } A = \begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & -4 \\ 0 & 1 & 2 \end{bmatrix}.$$

(a) Find $\det A$.

(b) **Use Cramer's Rule** to find the value of z in the unique solution to $A\mathbf{x} = \mathbf{b}$ where $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$.

SHOW YOUR WORK. Note: You may want to use your answer to part (a).