CS3342 – Assignment 4
due Apr. 7, 2022
2-day no-penalty extension until: Apr. 9, 11:55pm
(SRA's cannot be used to extend the due date further)

1. (15pt) Consider the following Prolog rule:

   ```
   a(X, Y) :- b(Z), c(X, Z), d(X, Y, Z), e(Z, W).
   ```

   inside quantifier: ✓
   outside ---- : 1

   Rewrite this rule in predicate calculus, using:

   (a) four universal quantifiers;
   (b) two universal quantifiers and two existential quantifiers;
   (c) four existential quantifiers.

2. (25pt) Implement a sorting algorithm (of your choice) in Prolog as a predicate `my_sort(X, Sorted_X)`, working as shown below:

   ```
   ?- my_sort([1], X).
   X = [1] .
   ?- my_sort([2,1,3], X).
   X = [1, 2, 3] .
   ```

   Submit your code as a file `my_sort.pl`.

3. (60pt) Implement the following Scheme functions:

   ```
   (define my-permutations
     (lambda (L) ...
   (define list-lex-less?
     (lambda (L1 L2) ...
   (define my-sort
     (lambda (L comp-pred?) ...
   ```

   `my-permutations` generates a list of all permutations of a given list (in any order); `list-lex-less?` compares two lists and outputs true iff the first is smaller than the second in lexicographic order; `my-sort` sorts a given list (with an algorithm of your choice) using the given `comp-pred?` predicate to compare elements. The three functions are then put together into `sorted-permutations` to produce all permutations of a given list in lexicographical order.

   ```
   (define sorted-permutations
     (lambda (L) ...
   ```

   Here are some examples:

   ```
   (my-permutations '(1 3 2))  => '((1 3 2) (3 1 2) (3 2 1) (1 2 3) (2 1 3) (2 3 1))
   (list-lex-less? '(1 2) '(1 2)) => #t
   (list-lex-less? '(1 2) '(1 2 3)) => #t
   (list-lex-less? '(2) '(1 3)) => #f
   (list-lex-less? '(1 2 3) '(1 2)) => #f
   (my-sort '(2 3 1) <) => '(1 2 3)
   (my-sort '((3) (1 2 3) (1 3)) list-lex-less?) => '((1 2 3) (1 3) (3))
   (sorted-permutations '(1 3 2)) => '((1 2 3) (1 3 2) (2 1 3) (2 3 1) (3 1 2) (3 2 1))
   ```

1. (15pt) Consider the following Prolog rule:

```
a(X, Y) :- b(Z), c(X, Z), d(X, Y, Z), e(Z, W).
```

Rewrite this rule in predicate calculus, using:

(a) four universal quantifiers;
(b) two universal quantifiers and two existential quantifiers;
(c) four existential quantifiers.

(a) $\forall x \forall y \forall z \forall w \; (b(z) \wedge c(x,z) \wedge d(x,y,z) \wedge e(z,w)) \rightarrow a$

$\equiv (\neg b(z)) \vee (\neg c) \vee (\neg d) \vee (\neg e)$

(b) $\forall x \forall y \; (a(x,y) \leftarrow (\exists z (b(z) \wedge c(x,z) \wedge d(x,y,z)) \wedge \exists w \, e(z,w)).$

$\equiv \forall x \forall y$

You are required to provide pure functional implementations from scratch, that do not employ advanced functions or imperative features. Therefore, you are allowed to use *only* the following basic Scheme functional constructs:
– function creation: `lambda`
– binding: `define`, `let`, `let*`, `letrec`
– booleans: `not`, `and`, `or`
– conditionals: `if`, `cond`
– basic list operations: `car`, `cdr`, `cons`, `list`, `append`, `null?`
– mapping: `map`, `apply`

Submit your code as a file `sorted-permutations.rkt`.

READ ME! Submit your answers as a *single pdf file* in OWL. Solutions should be typed but readable (by others!) hand-written solutions are acceptable. Source code, if required, is submitted as separate files.

LaTeX: For those interested, the best program for scientific writing is LaTeX. It is far superior to all the other programs, it is free, and you can start using it in minutes; here is an introduction: `https://tobi.oetiker.ch/lshort/lshort.pdf`