MATH 1600 Linear Algebra — Winter 2020

Tutorial 3 - Wednesday

Lines and planes in $\mathbb{R}^3$

1. For the given plane $\mathcal{P}$ and line $\mathcal{L}$ in $\mathbb{R}^3$, calculate the intersection of $\mathcal{P}$ and $\mathcal{L}$.

   (a) $\mathcal{P}$ has the general form $x + 12y + z = 3$;
   $\mathcal{L}$ has the vector form $\langle -8, 5, 13\rangle + t\langle 1, 2, 6\rangle$.

   $x = -8 + t = -8 - \frac{62}{19}$
   $y = 5 + t = 5 - \frac{62}{19}$
   $z = 13 + 6t = 13 - \frac{62}{19} \times 6$

   (b) $\mathcal{P}$ has the vector form $\langle 1, 4, 2\rangle + s\langle 3, 0, -6\rangle + t\langle 4, -1, 3\rangle$;
   $\mathcal{L}$ has the vector form $\langle 1, -10, -29\rangle + t\langle 1, 3, 2\rangle$.

   $(6, 5, -19)$

   $x = 1 + 3s + 4t$ $\quad$ $y = 4 - t$ $\quad$ $z = 2 - 6s + 3t$
   $x + 4y = 17 + 3s$
   $z + 3y = 14 - 6s$

   $z + 2x + 11y = 48$
   $x = 1 + a$ $\quad$ $y = -10 + 3a$
   $z = -29 + 2a$
   $-29 + 2a + 2 + 2a - 110 + 33a = 48$
   $-137 + 37a = 48$
   $a = 5$
   $x = 6$ $\quad$ $y = 5$ $\quad$ $z = -19$

2. Let A, B, C be the points $(1, 3, 0)$, $(2, 4, 1)$ and $(0, 3, 5)$.

   (a) Find a vector form of the line through A and B. $\vec{AB} = (1,1,1)$. $(1,3,0) + t(1,1,1)$.

   (b) Find a vector form of the line through A and C. $\vec{AC} = (-1,0,5)$ $(1,3,0) + s(1,0,5)$.

   (c) Find a normal form of the (unique) plane that passes trough the points A, B and C.
   $\vec{n} = (5, -6, 1)$

Hyperplanes in $\mathbb{R}^4$

3. Let $\mathcal{H}$ be the hyperplane with general form $2x_1 + 15x_2 + 11x_3 + 13x_4 = 19$. Find a normal form of $\mathcal{H}$.
   $(2, 15, 11, 13)$.
   $2(x_1 - 1) + 15(x_2 - 1) + 11(x_3 + 1) + 13(x_4 - 1) = 0$

4. Let $\mathcal{H}$ be the hyperplane with general form $2x_1 + 15x_2 - x_3 + 13x_4 = 19$ and $\mathcal{L}$ be the line through the points $A = (3, 0, 0, 1)$ and $B = (3, 1, 15, 1)$. Show $\mathcal{L}$ is contained in $\mathcal{H}$ (i.e., every point on $\mathcal{L}$ is on $\mathcal{H}$).

   $2 \times 3 + 13 \times 1 = 19$ A is on the plane
   $2 \times 3 + 15 \times 1 + 15 \times (-1) + 13 = 19$ B is on the plane

   $\vec{AB} = (3,0,0,1) + t(0,1,15,0)$.
   $x_1 =$
   $x_2 =$
   $x_3 =$
   $x_4 =$
   $x_1 + x_2 + x_3 + x_4 = 19$.

Parallelism

5. For the following pairs of vectors, determine whether they are parallel.

   (a) $\langle 1, 2\rangle$, $\langle 4, 5\rangle$ in $\mathbb{R}^2$ $\quad$ ×

   (b) $\langle 1, 4\rangle$, $\langle -4, -16\rangle$ in $\mathbb{R}^2$ $\quad$ ✓

   (c) $\langle 3, 7, 1\rangle$, $\langle 1, 2, 3\rangle$ in $\mathbb{R}^3$ $\quad$ ×

6. Which of the following statements are true? (no proof required: a drawing is enough)

   (a) Let $\mathcal{P}$ be a plane in $\mathbb{R}^3$ with a normal vector $\mathbf{n}$. Let $\mathcal{L}$ be a line parallel to $\mathbf{n}$. Does $\mathcal{L}$ intersect $\mathcal{P}$? ✓

   (b) Two lines in $\mathbb{R}^2$ that do not intersect are parallel. ✓

   (c) Two lines in $\mathbb{R}^3$ that do not intersect are parallel. ×

   (d) Two planes in $\mathbb{R}^3$ that do not intersect are parallel. ✓

   (e) For two distinct parallel lines in $\mathbb{R}^3$, there is a unique plane containing both of them. ✓

Vector Product

7. Calculate the vector product of the following two vectors

   (a) $\langle 0,0,0\rangle$ and $\langle 1,1,1\rangle$ $= \left(\det\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix}, -\det\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix}, \det\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix}\right) = (0,0,0).$

   (b) $\langle 1,2,3\rangle$ and $\langle 3,4,5\rangle$ $= \left(\det\begin{bmatrix}2 & 4\\ 3 & 5\end{bmatrix}, -\det\begin{bmatrix}1 & 3\\ 3 & 5\end{bmatrix}, \det\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}\right) = (-2, 4, -2).$

   (c) $\langle 1,0,2\rangle$ and $\langle -3,0,-6\rangle$

   $= \left(0, -\det\begin{bmatrix}1 & -3\\ 2 & -6\end{bmatrix}, 0\right) = (0,0,0).$

8. Let $\mathbf{u}$ and $\mathbf{v}$ be arbitrary vectors in $\mathbb{R}^3$, and show that $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}\times\mathbf{v}$ are orthogonal.

$u = (u_1, u_2, u_3)$

$v = (v_1, v_2, v_3)$. $(u\times v \cdot (u+v)) = 0$.

$u+v = (u_1+v_1, u_2+v_2, u_3+v_3)$.

$u\times v = (u_2v_3 - u_3v_2, -u_1v_3 + u_3v_1, u_1v_2 - u_2v_1)$.

$u\times v \perp u$

$u \times v \perp v$

$u\times v \cdot (u+v) = (u\times v)\cdot u + (u\times v)\cdot v = 0.$

Codes

9. Find the digit d in $\{0,1,2,3,4,5,6,7,8,9,X\}$ for which the ISBN-10 code $103457119d$ is valid.

$(1, 0, 3, 4, 5, 7, 1, 1, 9, d)$.

0 1 2 3 4 5 6 7 8 9

$6+12$

10. Which of the following UPC codes are valid?

   0.

   (a) 725272730706

   (b) 321830130981

   (c) 012345678912

code: abcdefghijk.

$0a+1b+2c+3d+4e+5f+6g+7h+8i+9j+10k=0$.