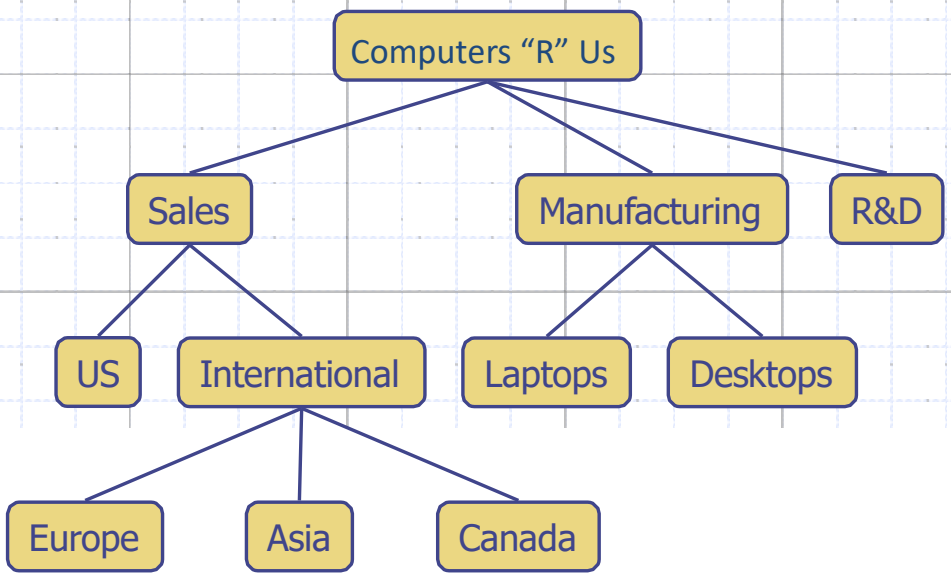
TREES

What is a Tree?

A tree is an abstract model of a hierarchical structure

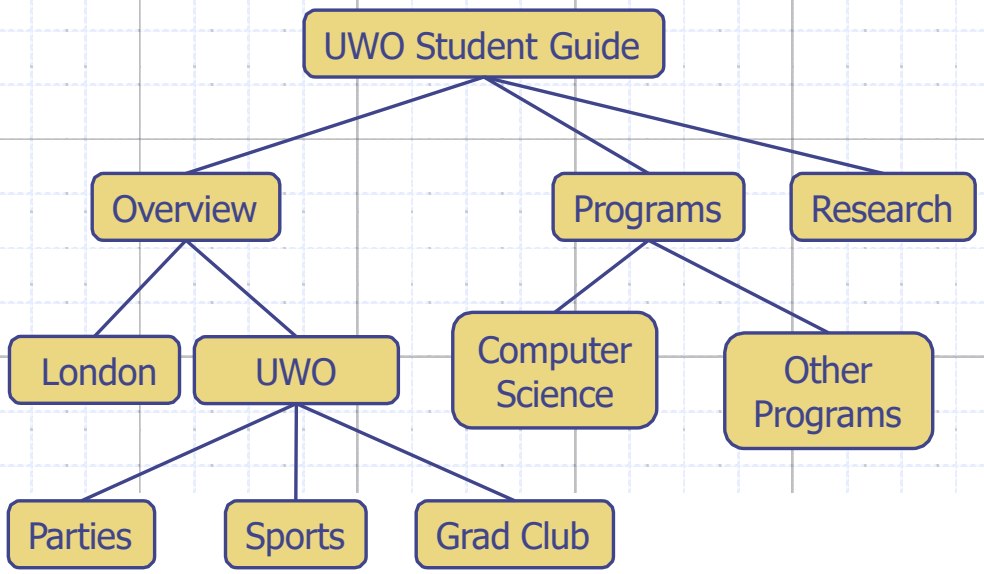
Applications

Organization of a company



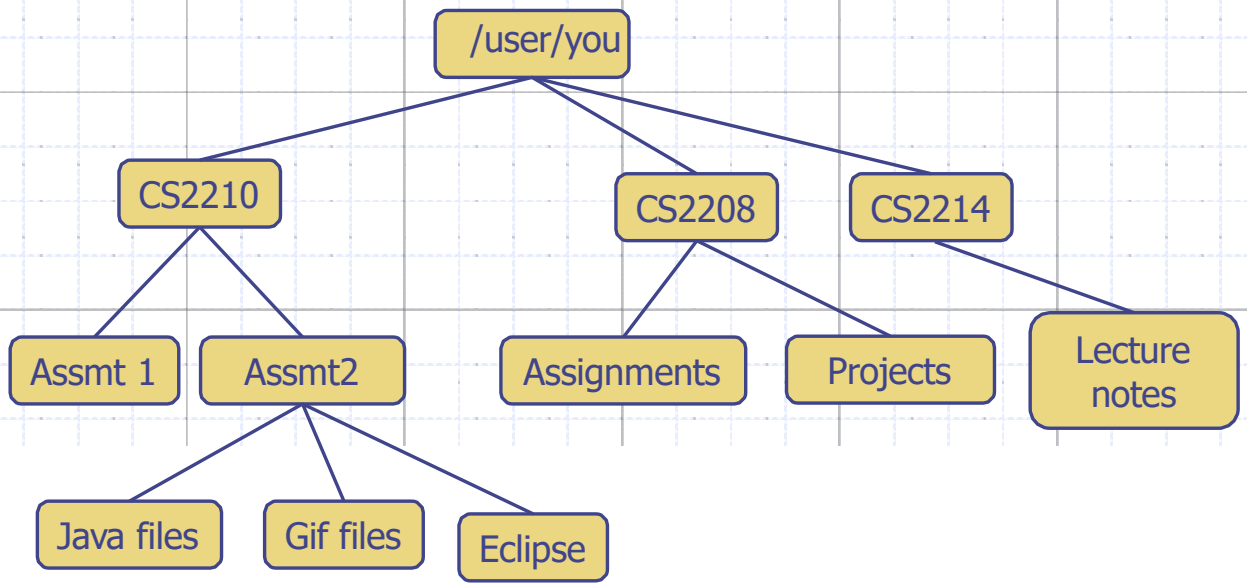
Applications

Table of contents of a book

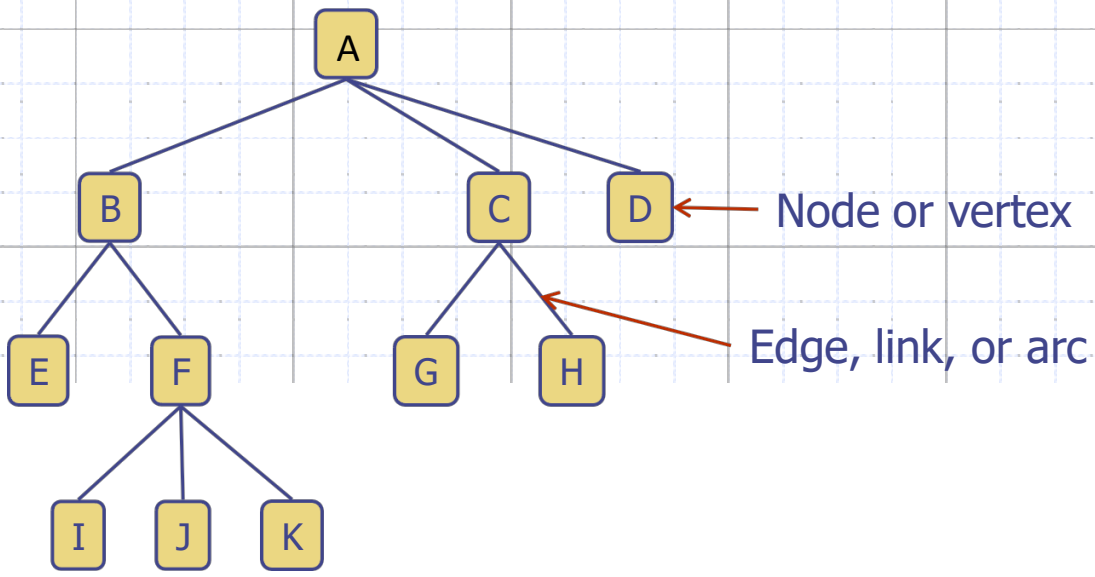


Applications

File system

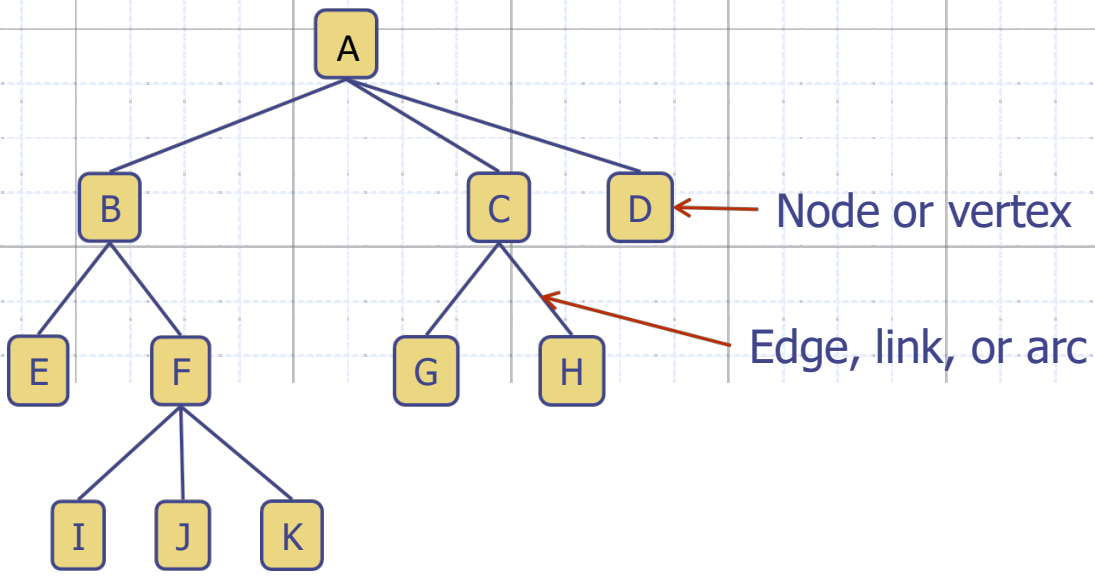


Tree Terminology



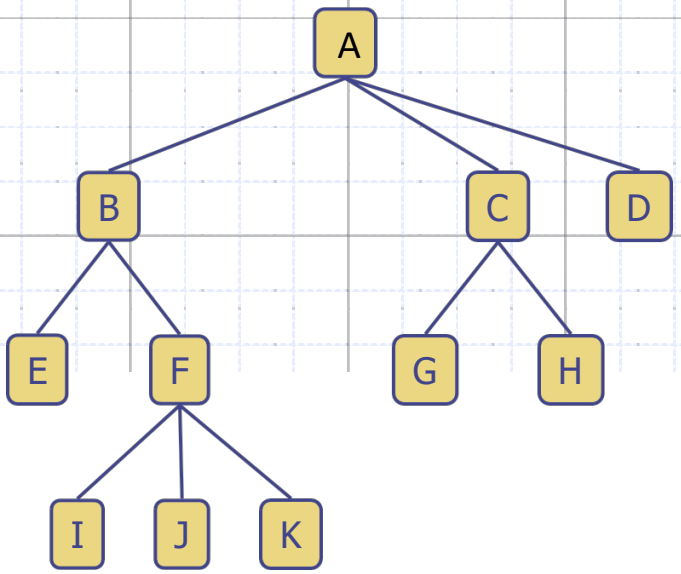
Tree Terminology

A is the root node
B is the parent of E and F
C is the sibling of B and D
B, C, and D are children of A



Tree Terminology

E, F, I, J, K are descendants of B
All nodes, except A, are descendants of A
A, B, F are ancestors of J
A has no ancestors



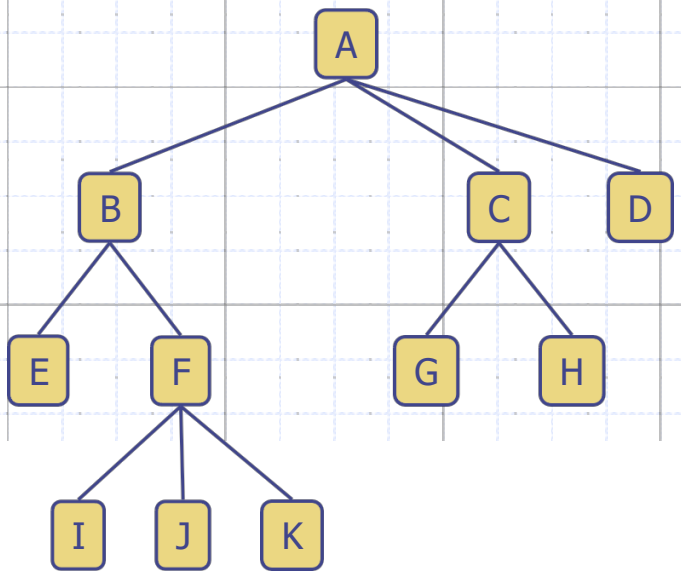
Tree Terminology

Internal node:

node with at least one child (A, B, C, F)

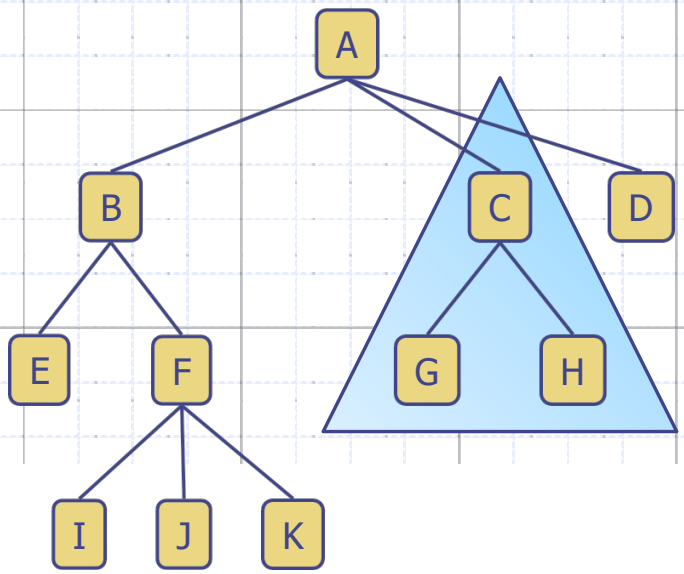
External node or leaf:

node without children (E, I, J, K, G, H, D)



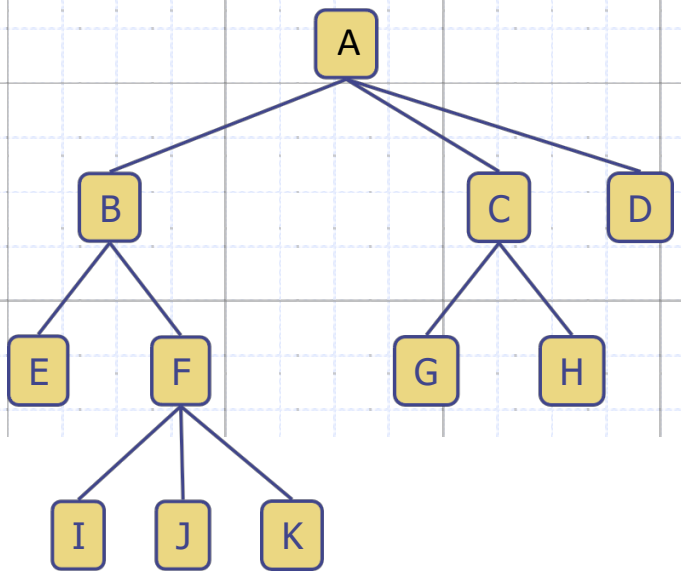
Tree Terminology

Subtree: tree consisting of a node and its descendants



Tree Terminology

Depth or level of a node:
number of ancestors. Depth of E is 2.



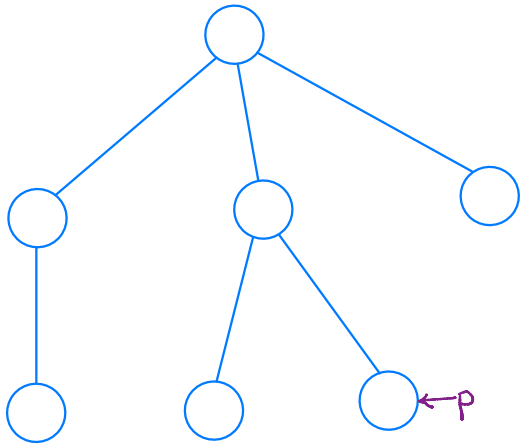
Algorithm level (p)

In: Node p of a tree

Out: Level of p

if p is a root then return 0

else return



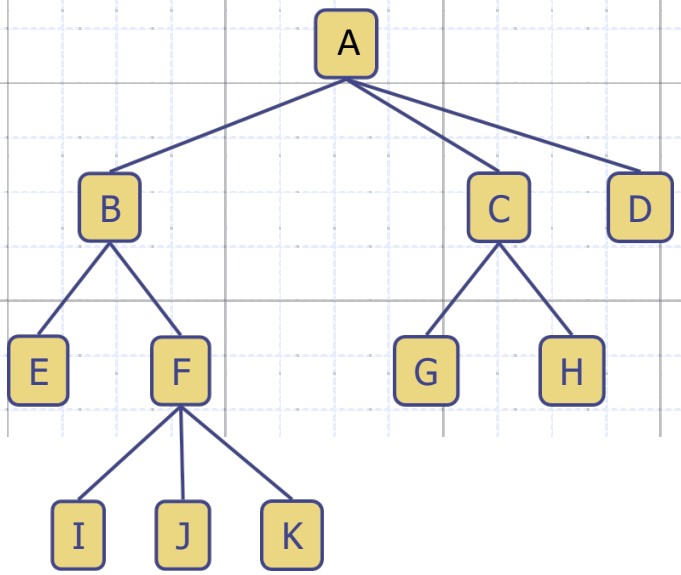
Tree Terminology

Depth or level of a node:

number of ancestors. Depth of E is 2.

Height of tree

maximum depth of any node. Tree has height 3.



Tree Terminology

Depth or level of a node:

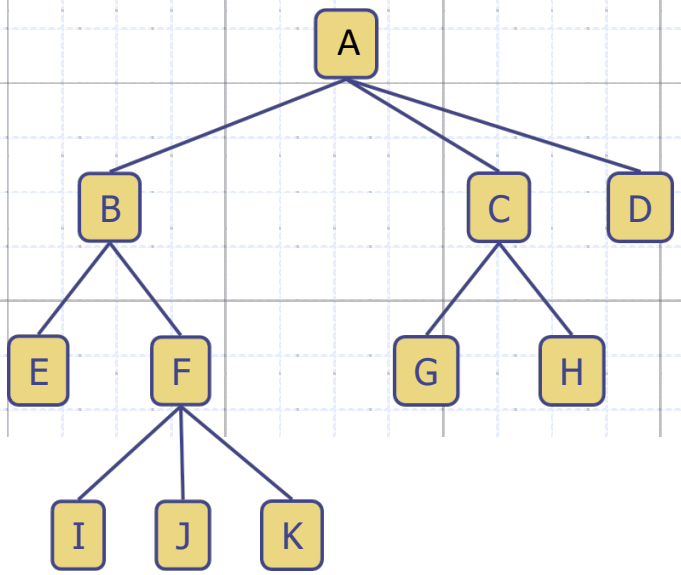
number of ancestors. Depth of E is 2.

Height of tree

maximum depth of any node. Tree has height 3.

Degree of a node:

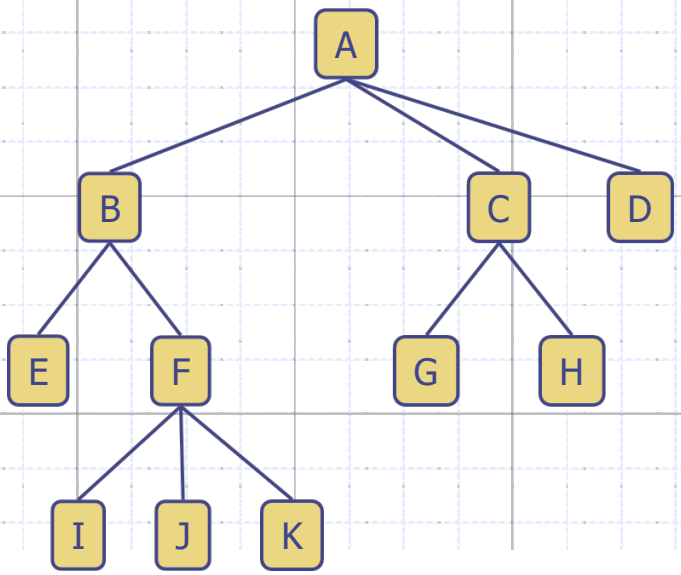
number of children. Degree of F is 3.



$$\sum_{u} \text{degree}(u) = \#\text{edges}.$$

Tree Terminology

Ordered tree: The children of a node are ordered.



Tree Terminology

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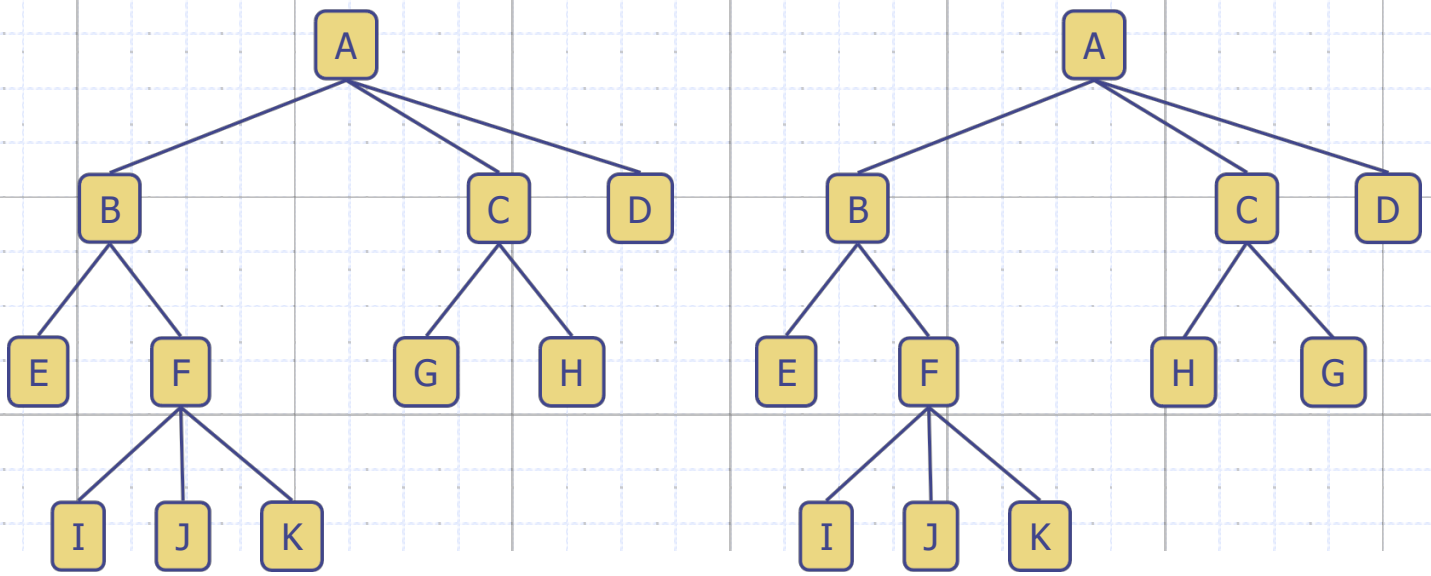
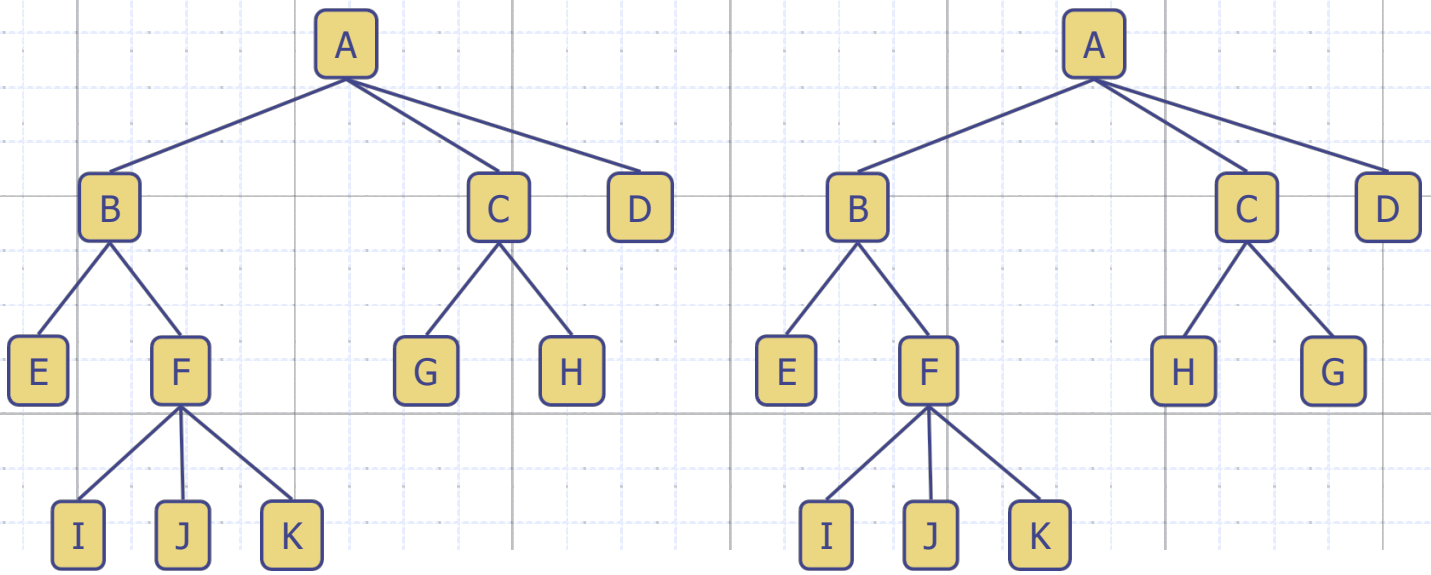


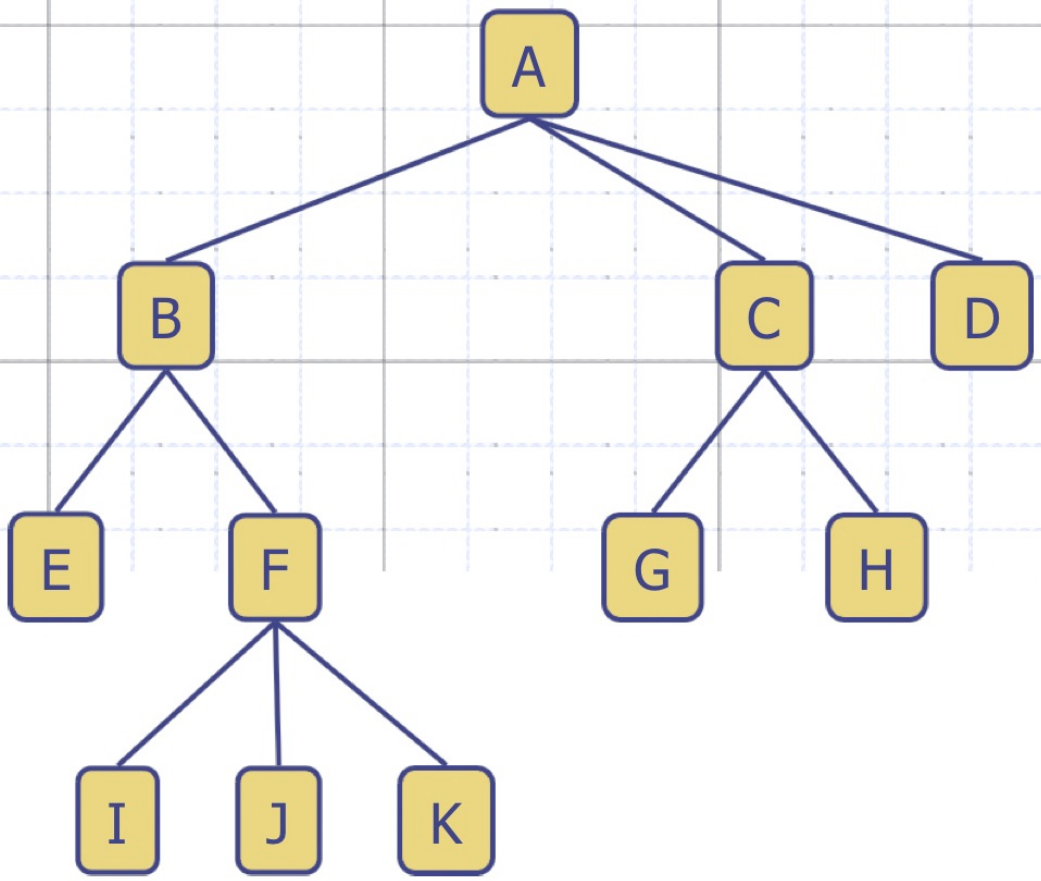
Tree Terminology

Un-Ordered tree: The children of a node are not ordered.



Tree Properties

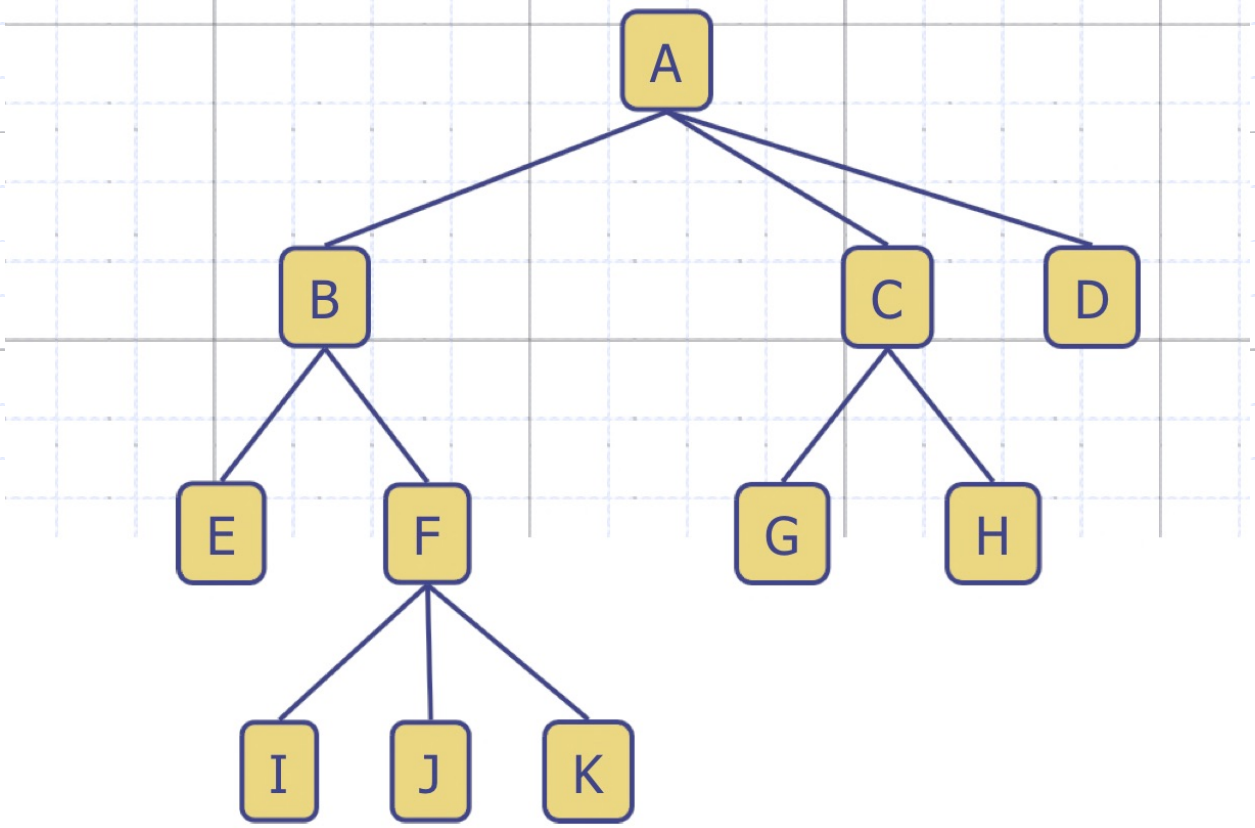
Number of nodes = Number of edges + 1



Tree Properties

Number of nodes = Number of edges + 1

Proof. Glue every node to the edge connecting it to its parent. The root is not glued to any edges.



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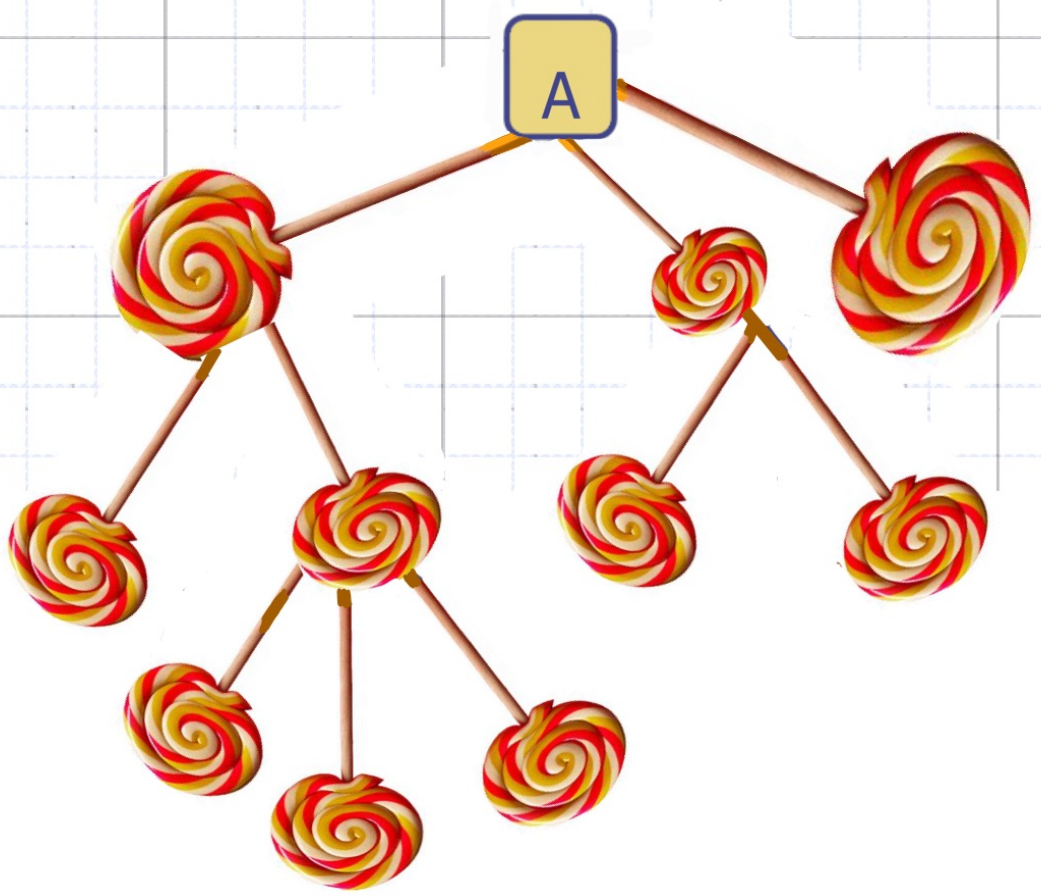
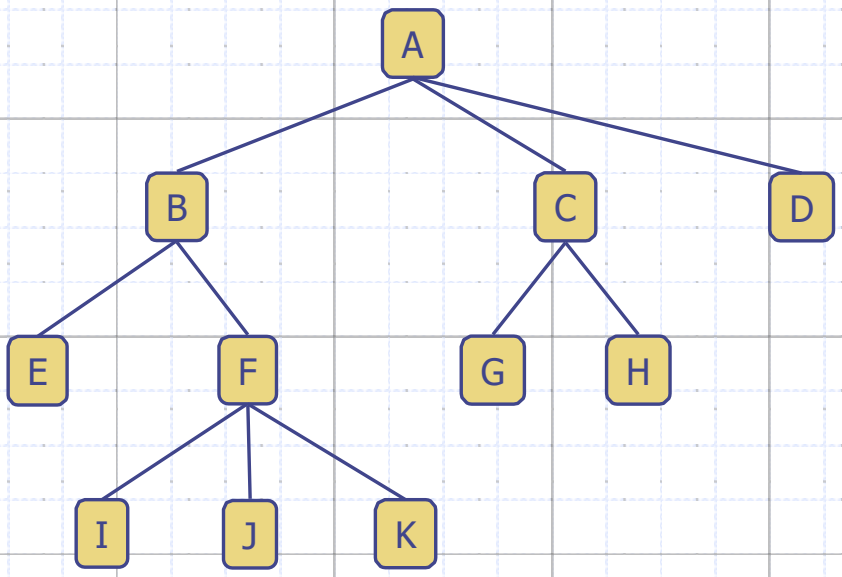


Why is this a Tree?



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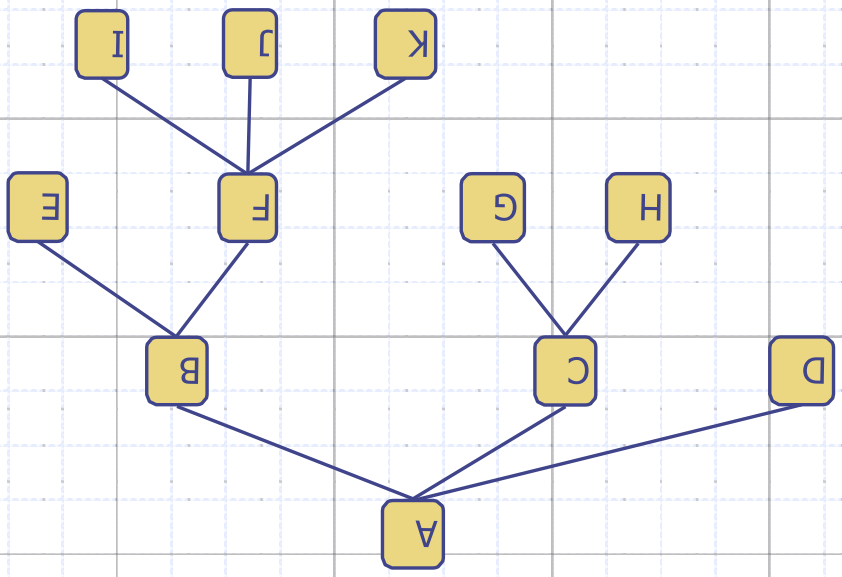

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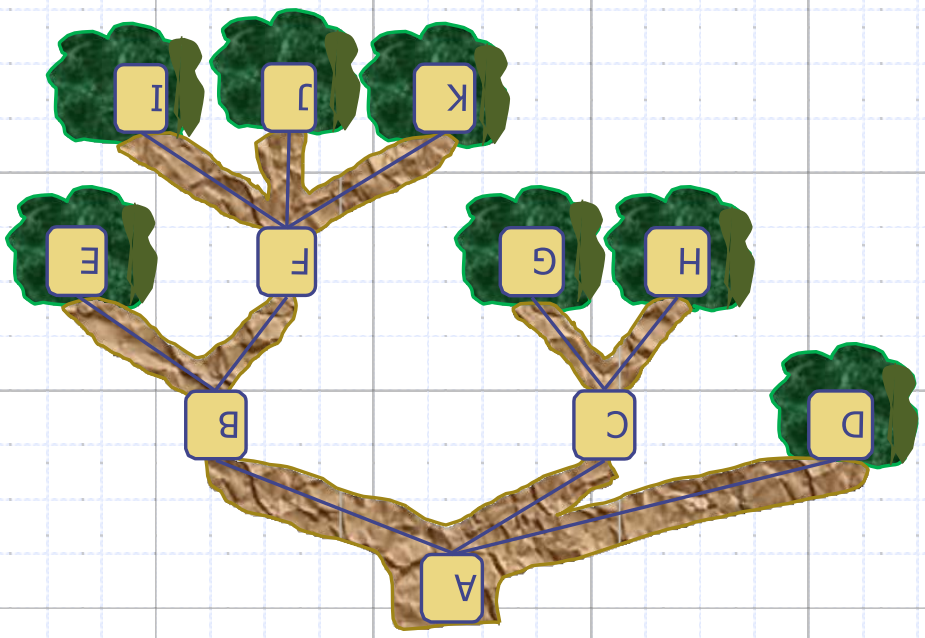
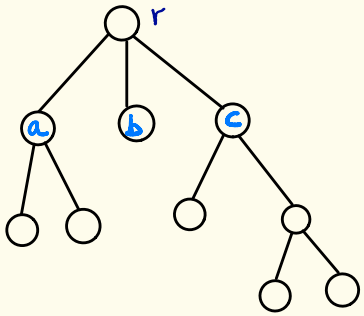
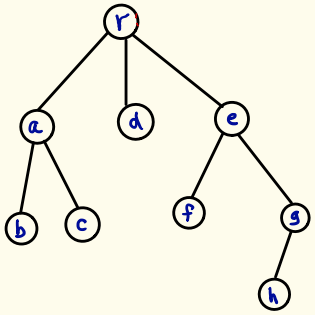


Algorithm to compute the height of a tree





Algorithm height (r)
In: root r of a tree
Out: Height of the tree
if (r.isLeaf())

$7 \to 740 \to 810 \to \Rightarrow$ (B.)