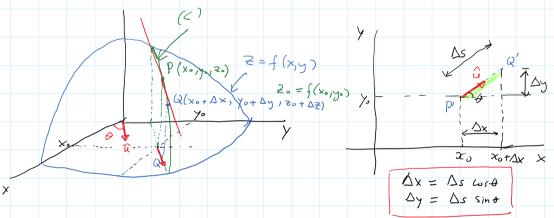


Directional derivatives and the gradient (sec 14.6)

Imagine that we are in a norm which is very cold in the winter. Let's put a fireplace at the centre of the room. Then the temperature T(x,y) is a function of x and y (a location on the floon). If you feel old you likely move forward to the fireplace to obtain the fastest rate of change in temperature. If you feel quite but, you likely move away from the fireplace to obtain the fastest rate of change in the fireplace to obtain the fastest deceasing rate in temperature. Thus, the rate of change of temperature with distance depends on the direction you move. This kind of reate of change is called a directional derivative.



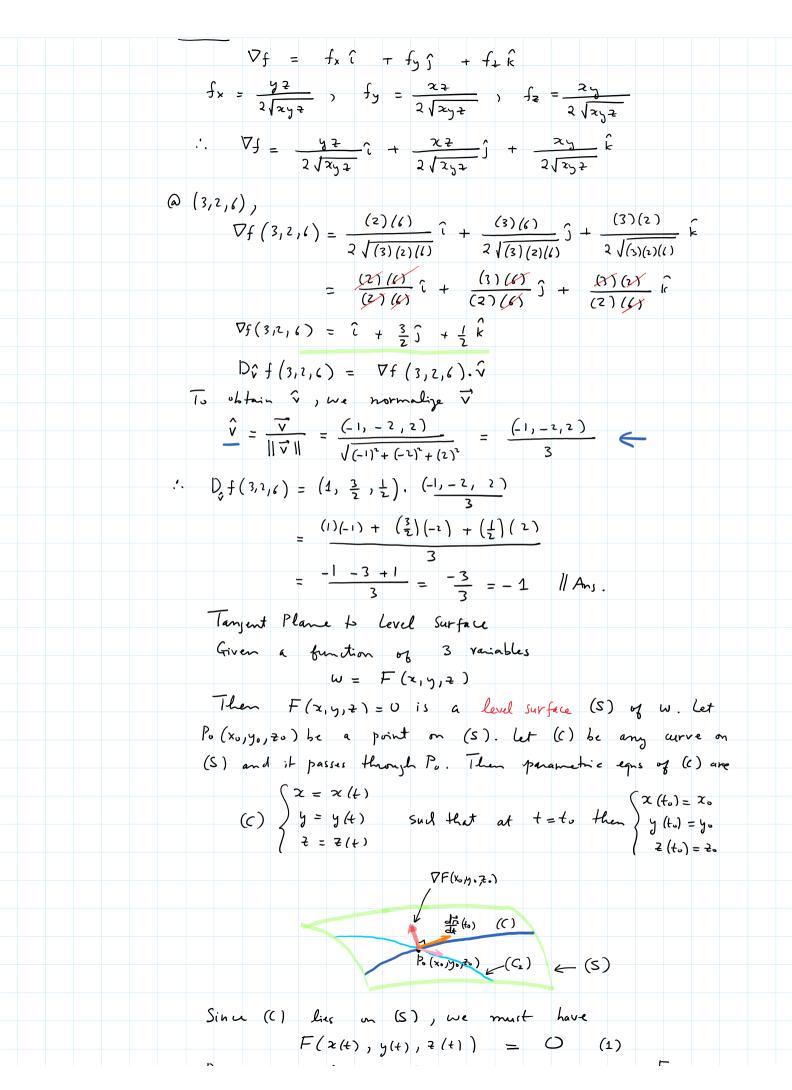
Suppose that we want to find the rate of change of Z = f(x,y) at  $(x_0, y_0)$  in the direction of a unit vector  $\hat{u} = \omega s \theta \hat{i} + \delta n \theta \hat{j}$ . The vertical plane passing throu  $P(x_0, y_0, z_0)$  where  $Z = f(x_0, y_0)$  in the  $\hat{u}$  direction cuts the surface Z = f(x,y) along a curve (C). The shope of Z = f(x,y) in the  $\hat{u}$  direction. We have

$$\overrightarrow{P'Q'} | | \widehat{u} \quad \text{and if we let } \overrightarrow{P'Q'} = \Delta s \text{ then } \overrightarrow{P'Q'} = \Delta s \widehat{u} = \Delta s (\omega s \theta \widehat{i} + \sin \theta \widehat{j})$$

$$= (\Delta s \cos \theta \widehat{i} + \Delta s \sin \theta \widehat{j})$$

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= ( \Ds Cust i + \Ds Fint ) ]
Than the directional derivative of z = f(z, y) in the direction of in
            D_{\hat{u}} f(x_0, y_0) = \lim_{\Delta s \to 0} \frac{\Delta f}{\Delta s}
                                   =\lim_{\Delta c \to 0} \frac{f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0)}{\Delta c}
                                   = \lim_{\Delta s \to 0} f_x(x_0, y_0) \Delta z + f_y(x_0, y_0) \Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y
                             = \lim_{\Delta s \to 0} f_{x}(x_{0}, y_{0}) \Delta s \cos \theta + f_{y}(x_{0}, y_{0}) \Delta s \sin \theta + \xi_{1} \Delta s \cos \theta + \xi_{2} \Delta s \sin \theta
                             = lim As [fx (xo,yo) cost + fy (xo,yo) sint + (2) cost + (2) sint]
As + 0
                 Dûf(x,y,)= fx (x,y,) ws + fy (x,y,) sn+ (I)
           The above expression can be written as a dit product
                   D\hat{x} f(x_0, y_0) = \left[ f_{x}(x_0, y_0) \hat{i} + f_{y}(x_0, y_0) \hat{j} \right] - \left[ c_{x_0} + c_{y_0} + c_{y_0} + c_{y_0} + c_{y_0} \right]
            \nabla f(x_0, y_0) = \nabla f(x_0, y_0) \cdot \hat{u} \qquad (\overline{\underline{u}})
             where \nabla f(x,y) = f(x,y)\hat{i} + f_y(x,y)\hat{j}
is called the gradient of f at the point (x,y).
      N.B:
                   If \hat{u} = \hat{i} then \theta = 0 \implies \omega_s \theta = 1 and \delta n \theta = 0
                            D_{\hat{i}} f(x_0, y_0) = f_x(x_0, y_0)
                  If \hat{u} = \hat{j} then \theta = \frac{\pi}{2} \implies \omega_{S} \theta = 0 and \delta u \theta = 1
                                \therefore \quad D_{\hat{J}} f(x_{\epsilon}, y_{\epsilon}) = f_{\hat{J}} (x_{\epsilon}, y_{\epsilon})
          (II) can be expressed as
                                                                                              U.V= || U || || V || COSX
                    D_{\hat{n}} f(x_0, y_0) = \| \nabla f(x_0, y_0) \| \| \hat{u} \| \cos \alpha
                     DC_{\delta}f(x_{0},y_{0}) = \| \nabla f(x_{0},y_{0}) \| \omega_{\delta} \alpha \qquad (\overline{\mathbf{m}})
             Dûf is maximum when d = U, ie, û has the same direction
                  as Pf. In this case,
                                      pv. t | − | | Δτ ||
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$Dif \mid_{\text{max}} = \ \nabla f\ $ $Dif \mid_{\text{max}} = \ \nabla f $ $Dif \mid_{\text{max}} = \ \nabla$	as Pf. In this case,	
When $\alpha = \pi$ , $\alpha_{1}\pi = -4$ , then $D^{\alpha}_{1}f = -1 \nabla f \Pi$ In this case, if deceases must repidly in the treating $-\nabla f$ .  Est: Given $f(x_{1},y_{1}) = \frac{x}{2}$ , find the graduat of $f$ at $P(2_{1})$ and find the rate of change of $f$ at $f$ in the direction of $f$ is $f$ in $f$	V-J	
In this case , $f$ deceases must regidly in the director $g$ $-\nabla f$ .  Est: Given $f(x_{1,y_1}) = \frac{\chi}{y}$ , find the graduat of $f$ of $P(2,1)$ and brind the rate of change of $f$ at $P$ in the director $g$ $\chi = 3\hat{i} + 4\hat{j}$ .  Solution $D_{ij}^{ij} \{(x_{1,y_1}) = \nabla f(x_{1,y_2}), \hat{i}_{ij}$ $\nabla f = f_{ij} \hat{i} + f_{ij} \hat{j}$ $= \frac{1}{3} \hat{i} + (-\frac{1}{3}) \hat{j}$ $\therefore \nabla f(2,1) = \frac{1}{(1)} \hat{i} - \frac{(2)}{(1)^3} \hat{j}$ $= \hat{i} - 2\hat{j} \text{ // Ans}.$ Since $\vec{u} = 3\hat{i} + 4\hat{j} \text{ is not a unit vector, we must rainable sit as  \hat{u} = \frac{\vec{u}}{\ \vec{u}\ } = \frac{3\hat{i} + 4\hat{j}}{\sqrt{(3\hat{i}^3 + (1)^3)}} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{2\hat{i}}} = \frac{5\hat{i} + 4\hat{j}}{\sqrt{2\hat{i}}} \therefore D_{ij}^{ij} \{(2,1)\} = (1,-2) - (\frac{1}{3},\frac{1}{3}) = \frac{(1)(x_1) + (1-2)(x_2)}{5} = \frac{3-8}{5} = \frac{5}{5} = -1 \text{ // Ans}.  For functions of three variables, we can show that the directoral derivative of f(x_1,y_1,x_2) at (x_2,y_3,x_4) in a director of x_1 where x_2 where x_3 is x_4 and x_4 and$	(xo, yo)	
In this case , $f$ deceases must regidly in the director $g$ $-\nabla f$ .  Est: Given $f(x_{1,y_1}) = \frac{\chi}{y}$ , find the graduat of $f$ of $P(2,1)$ and brind the rate of change of $f$ at $P$ in the director $g$ $\chi = 3\hat{i} + 4\hat{j}$ .  Solution $D_{ij}^{ij} \{(x_{1,y_1}) = \nabla f(x_{1,y_2}), \hat{i}_{ij}$ $\nabla f = f_{ij} \hat{i} + f_{ij} \hat{j}$ $= \frac{1}{3} \hat{i} + (-\frac{1}{3}) \hat{j}$ $\therefore \nabla f(2,1) = \frac{1}{(1)} \hat{i} - \frac{(2)}{(1)^3} \hat{j}$ $= \hat{i} - 2\hat{j} \text{ // Ans}.$ Since $\vec{u} = 3\hat{i} + 4\hat{j} \text{ is not a unit vector, we must rainable sit as  \hat{u} = \frac{\vec{u}}{\ \vec{u}\ } = \frac{3\hat{i} + 4\hat{j}}{\sqrt{(3\hat{i}^3 + (1)^3)}} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{2\hat{i}}} = \frac{5\hat{i} + 4\hat{j}}{\sqrt{2\hat{i}}} \therefore D_{ij}^{ij} \{(2,1)\} = (1,-2) - (\frac{1}{3},\frac{1}{3}) = \frac{(1)(x_1) + (1-2)(x_2)}{5} = \frac{3-8}{5} = \frac{5}{5} = -1 \text{ // Ans}.  For functions of three variables, we can show that the directoral derivative of f(x_1,y_1,x_2) at (x_2,y_3,x_4) in a director of x_1 where x_2 where x_3 is x_4 and x_4 and$	When $\alpha = \pi$ , $\omega_s \pi = -1$ , then	
$\frac{-\nabla f}{SN}: \text{ Given } f(z_1, y_1) = \frac{\lambda}{3}, \text{ find the gradual of } f \text{ of } P(z_1) \text{ and find the rate of charge of } f \text{ of } P \text{ in the deciden } f \text{ is } = 3\hat{i}, + 4\hat{j}.$ Solution $D_0^2 f(x_1, y_2) = \nabla f(x_1, y_2).\hat{u}.$ $\nabla f = f_1 \hat{i} + f_2 \hat{j}.$ $= \frac{1}{3} \hat{i} + f_3 \hat{j}.$ Since $\vec{u} = 3\hat{i} + 4\hat{j}.$ is not a unit vector, we must remarkly it as $\hat{u} = \frac{\vec{u}}{\ \vec{u}\ } = \frac{3\hat{i} + 4\hat{j}}{\sqrt{3\hat{i}^3 + f(\hat{i})^2}} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{2\pi}} = \frac{5\hat{i} + 4\hat{j}}{5}.$ $\therefore D_0^2 f(z_1) = (1, -2) \cdot (3, 4)$ $= \frac{(1)(\hat{z}) + (-2)(\hat{z})}{5} = \frac{3 - 8}{5} = \frac{5}{5} = -1  \text{   Ans. }$ For functions of these variables, we can show that the distributed derivative of $f(x_1, y_1, z_1)$ at $(x_2, y_1, z_2).\hat{u}$ where $\nabla f(x_1, y_2) = f_3\hat{i} + f_3\hat{j} + f_3\hat{k}$ $\frac{S_{AZ}}{S}: \text{Find the directional derivative of } f(x_3, y_2, z_1) = \sqrt{2}(y_3z_1) = \sqrt{2}(y_3z_2) = \sqrt{2}(y_3z_1) = \sqrt{2}(y_3z_2).$ Solution	$D_{u}^{\wedge}f = - \  \nabla f \ $	
$\frac{-\nabla f}{SN}: \text{ Given } f(z_1, y_1) = \frac{\lambda}{3}, \text{ find the gradual of } f \text{ of } P(z_1) \text{ and find the rate of charge of } f \text{ of } P \text{ in the deciden } f \text{ is } = 3\hat{i}, + 4\hat{j}.$ Solution $D_0^2 f(x_1, y_2) = \nabla f(x_1, y_2).\hat{u}.$ $\nabla f = f_1 \hat{i} + f_2 \hat{j}.$ $= \frac{1}{3} \hat{i} + f_3 \hat{j}.$ Since $\vec{u} = 3\hat{i} + 4\hat{j}.$ is not a unit vector, we must remarkly it as $\hat{u} = \frac{\vec{u}}{\ \vec{u}\ } = \frac{3\hat{i} + 4\hat{j}}{\sqrt{3\hat{i}^3 + f(\hat{i})^2}} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{2\pi}} = \frac{5\hat{i} + 4\hat{j}}{5}.$ $\therefore D_0^2 f(z_1) = (1, -2) \cdot (3, 4)$ $= \frac{(1)(\hat{z}) + (-2)(\hat{z})}{5} = \frac{3 - 8}{5} = \frac{5}{5} = -1  \text{   Ans. }$ For functions of these variables, we can show that the distributed derivative of $f(x_1, y_1, z_1)$ at $(x_2, y_1, z_2).\hat{u}$ where $\nabla f(x_1, y_2) = f_3\hat{i} + f_3\hat{j} + f_3\hat{k}$ $\frac{S_{AZ}}{S}: \text{Find the directional derivative of } f(x_3, y_2, z_1) = \sqrt{2}(y_3z_1) = \sqrt{2}(y_3z_2) = \sqrt{2}(y_3z_1) = \sqrt{2}(y_3z_2).$ Solution	In this case, of decreases most rapidly in the director of	
P(2,1) and find the sist of change of $f$ at $f$ in the director $f$		
P(2,1) and find the sist of change of $f$ at $f$ in the director $f$	$\frac{\mathcal{E}_{x}}{ }$ : Given $f(z,y) = \frac{x}{y}$ , find the gradient of fat	
Solution $ \begin{array}{llll}                                  $	P(2,1) and bind the rate of change of f at P in the	
Dû $f(x_1, y_1) = \nabla f(x_1, y_2) \cdot \hat{u}$ $\nabla f = f_1 \hat{i} + f_2 \hat{j}$ $= \frac{1}{3} \hat{i} + \left(-\frac{x_1}{3^2}\right) \hat{j}$ $\therefore \nabla f(z_1) = \frac{1}{(1)} \hat{i} - \frac{(2)}{(1)^3} \hat{j}$ $= \hat{i} - 2\hat{j}  ///////////////////////////////////$	direction of $\overline{u} = 3\hat{i} + 4\hat{j}$ .	
$ \nabla f = f_{x} \hat{i} + f_{y} \hat{j} \\ = \frac{1}{y} \hat{i} + \left(-\frac{x}{y^{2}}\right) \hat{j} \\ \therefore \nabla f(z,1) = \frac{1}{(1)} \hat{i} - \frac{(27)}{(1)^{3}} \hat{j} \\ = \hat{i} - 2\hat{j} \text{ // Ans}. $ Since $\vec{u} = 3\hat{i} + 4\hat{j}$ is not a unit vector, we must ramalize it as $\hat{u} = \frac{\vec{u}}{\ \vec{u}\ } = \frac{3\hat{i} + 4\hat{j}}{\sqrt{3}\hat{j}^{2} + (\hat{j})^{3}} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{27}} = \frac{5\hat{i} + 4\hat{j}}{5}$ $\therefore D\hat{u} f(z,1) = (1,-2) - (3,4)$ $= \frac{(1)(z) + (-2)(z)}{5} = \frac{3 - 8}{5} = -\frac{7}{5} = -1 \text{ // Ans}. $ For functions of three varieties, we can show that the directional derivative of $f(x_{1},y_{1},z)$ at $(x_{0},y_{0},z_{0})$ in a direction of $\hat{u}$ where $\nabla f(x_{1},y_{1},z) = f_{x}\hat{i} + f_{y}\hat{j} + f_{z}\hat{k}$ $\frac{Exz}{5} : \text{ Find the directional derivative of } \frac{1}{5}$ $\frac{Exz}{5} : \text{ Find the directional derivative of } \frac{1}{5}$ $\frac{Exz}{5} : \text{ Find the directional derivative of } \frac{1}{5}$ $\frac{Exz}{5} : \text{ Find the directional derivative of } \frac{1}{5}$ $\frac{Exz}{5} : \text{ Find the directional derivative of } \frac{1}{5}$ $\frac{1}{5} : \frac{1}{5} : \frac{1}{5$		
$ \begin{array}{c} =\frac{1}{3}\hat{\imath}+\left(\frac{2}{3^2}\right)\hat{3} \\ \\ \therefore $		
$ \begin{array}{lll} \overrightarrow{\nabla}f\left(2,1\right) &=& \frac{1}{\left(1\right)}\widehat{i} - \frac{(27)}{\left(1\right)^{2}}\widehat{j} \\ &=& \widehat{i} - 2\widehat{j}  \text{// Ans.} \\ Since  \overrightarrow{u} &=& 3\widehat{i} + 4\widehat{j}  \text{is not a unit vector, , we must} \\ &\text{nonmalize it as} \\ \widehat{u} &=& \frac{\overrightarrow{u}}{\ \overrightarrow{u}\ } &=& \frac{3\widehat{i} + 4\widehat{j}}{\sqrt{33^{2} + (4)^{3}}} &=& \frac{3\widehat{i} + 4\widehat{j}}{\sqrt{27}} &=& \frac{5\widehat{i} + 4\widehat{j}}{5} \\ &\vdots  D_{\widehat{u}} f(z_{1}) = (1, -2) - (\frac{3}{3}, 4) \\ &=& \frac{(1)(z_{1}) + (-2)(4)}{5} &=& \frac{3-8}{5} &=& -5}{5} &=& -1  \text{// Ans.} \\ &\text{For functions of three variables, , we can show that the directional derivative of } f(x_{1}, y_{1}, z_{1}) &=& \frac{3-8}{5} &=& -5 \\ &=& \frac{1}{5}  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& \frac{3-8}{5} &=& -5 \\ &=& -1  \text{// Ans.} \\ &=& $		
Since $\vec{u} = 3\hat{i} + 4\hat{j}$ is not a unit vector, we must namely it as $\hat{u} = \frac{\vec{u}}{\ \vec{u}\ } = \frac{3\hat{i} + 4\hat{j}}{\sqrt{(3)^2 + (4)^3}} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{2\tau}} = \frac{5\hat{i} + 4\hat{j}}{5}$ $\therefore D_{\hat{u}} f(z,1) = (1,-2) \cdot (\frac{3}{3},\frac{4}{3})$ $= \frac{(1)(z) + (-2)(+)}{5} = \frac{3-8}{5} = \frac{-5}{5} = -1 \text{ $  } A_{\text{MS}}.$ For functions of three variables, we can show that the directional derivative of $f(x,y,z)$ at $(x_0,y_0,z_0)$ in a direction of $a$ unit vactor $a$ is $D_{\hat{u}} f(x_0,y_0,z_0) = \nabla f(x_0,y_0,z_0).\hat{u}$ where $\nabla f(x_0,z_0) = f_{\hat{u}}\hat{i} + f_{\hat{u}}\hat{j} + f_{\hat{u}}\hat{k}$ $\frac{2}{5}x^2 : \text{ Find the directional derivative of } \frac{1}{5}\hat{k}$ $\frac{2}{5}x^2 : \text{ Find the directional derivative of } \frac{1}{5}\hat{k}$ $\frac{2}{5}x^2 : \frac{1}{5}\hat{i} + \frac{1}{5}\hat$	$= \frac{1}{3} \hat{i} + \left(-\frac{2}{3^2}\right) \hat{j}$	
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Since $\vec{u} = 3\hat{i} + 4\hat{j}$ is not a unit vector, we must namely a it as $\hat{u} = \frac{\vec{u}}{\ \vec{u}\ } = \frac{3\hat{i} + 4\hat{j}}{\sqrt{(3)^2 + (4)^4}} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{27}} = \frac{5\hat{i} + 4\hat{j}}{5}$ $\therefore \hat{D}\hat{u} f(z,1) = (1,-2) \cdot (\frac{3}{3},\frac{4}{3})$ $= \frac{(1)(z) + (-2)(4)}{5} = \frac{3-8}{5} = -\frac{5}{5} = -1 \text{ If Ans.}$ For functions of three varieties, we can show that the directional derivative of $f(x,y,z)$ at $(x_0,y_0,z_0)$ in a direction of $a$ where $\hat{u} \text{ is}$ $\hat{D}\hat{u} f(x_0,y_0,z_0) = \nabla f(x_0,y_0,z_0).\hat{u}$ where $\nabla f(x_0,y_0,z_0) = f_{x_0}\hat{i} + f_{y_0}\hat{j} + f_{z_0}\hat{k}$ $\frac{\xi_{x_0}}{V}: \text{ Find the directional derivative of } \frac{1}{2}\hat{k}$ $\frac{\xi_{x_0}}{V}: \text{ Find the directional derivative of } \frac{1}{2}\hat{k}$ Solution		
namelize it as $\hat{u} = \frac{\vec{u}}{\ \vec{u}\ } = \frac{3\hat{i} + 4\hat{j}}{\sqrt{(3)^2 + (4)^3}} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{25}} = \frac{5\hat{i} + 4\hat{j}}{5}$ $\therefore  D\hat{u} f(z,1) = (1,-2) \cdot (\frac{3}{3},\frac{4}{3})$ $= \frac{(1)(z) + (-2)(4)}{5} = \frac{3-8}{5} = \frac{-5}{5} = -1 \text{ II Ans.}$ For functions of three variebles, we can show that the directional derivative of $f(x,y,z)$ at $(x_0,y_0,z_0)$ in a direction of $a$ unit vachous $\hat{u}$ is $D\hat{u} f(x_0,y_0,z_0) = \nabla f(x_0,y_0,z_0).\hat{u}$ where $\nabla f(x_0,z_0) = f_{x,\hat{i}} + f_{y,\hat{j}} + f_{z,\hat{k}}$ $\frac{\mathcal{E}xz}{f(x_0,y_0)} = \sqrt{xy_0} \text{ at the print } (3,2,6) \text{ in the direction of } \hat{v} = (-1,-2,2).$ Solution	$= \hat{c} - 2\hat{j}$ // Ans.	
namelize it as $\hat{u} = \frac{\vec{u}}{\ \vec{u}\ } = \frac{3\hat{i} + 4\hat{j}}{\sqrt{(3)^2 + (4)^3}} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{25}} = \frac{5\hat{i} + 4\hat{j}}{5}$ $\therefore  D\hat{u} f(z,1) = (1,-2) \cdot (\frac{3}{3},\frac{4}{3})$ $= \frac{(1)(z) + (-2)(4)}{5} = \frac{3-8}{5} = \frac{-5}{5} = -1 \text{ II Ans.}$ For functions of three variebles, we can show that the directional derivative of $f(x,y,z)$ at $(x_0,y_0,z_0)$ in a direction of $a$ unit vachous $\hat{u}$ is $D\hat{u} f(x_0,y_0,z_0) = \nabla f(x_0,y_0,z_0).\hat{u}$ where $\nabla f(x_0,z_0) = f_{x,\hat{i}} + f_{y,\hat{j}} + f_{z,\hat{k}}$ $\frac{\mathcal{E}xz}{f(x_0,y_0)} = \sqrt{xy_0} \text{ at the print } (3,2,6) \text{ in the direction of } \hat{v} = (-1,-2,2).$ Solution	Since $\vec{u} = 3\hat{i} + 4\hat{j}$ is not a unit vector, we must	
$\hat{u} = \frac{\vec{u}}{\ \vec{u}\ } = \frac{3\hat{i} + 4\hat{j}}{\sqrt{(3)^2 + (4)^2}} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{25}} = \frac{5\hat{i} + 4\hat{j}}{\sqrt{5}}$ $\therefore D\hat{u} f(z,1) = (1,-2) \cdot (\frac{3}{3},\frac{4}{3})$ $= \frac{(1)(z) + (-2)(4)}{5} = \frac{3-8}{5} = \frac{5}{5} = -1 \text{ $I$ Ans.}$ For functions of three variables, we can show that the directional derivative of $f(x,y,z)$ at $(x_0,y_0,z_0)$ in a direction of $u$ is $D\hat{u} f(x_0,y_0,z_0) = \nabla f(x_0,y_0,z_0).\hat{u}$ where $\nabla f(x,y,z) = fx\hat{i} + fy\hat{j} + fz\hat{k}$ $\frac{\mathcal{E}_{XZ}}{f(x_0,y_0,z_0)} = \sqrt{2}y_0z_0$ $\mathcal{E$	namalize it as	
Dû $f(z,1) = (1,-2) \cdot (3,4)$ $= \frac{(1)(z) + (-z)(+)}{5} = \frac{3-8}{5} = -1 \text{ MAns.}$ For functions of three variables, we can show that the directional derivative of $f(x,y,\pm)$ at $(x_0,y_0,\pm)$ in a direction of a unit vector $\hat{u}$ is $D\hat{u} f(x_0,y_0,z_0) = \nabla f(x_0,y_0,\pm).\hat{u}$ where $\nabla f(x_0,y_0,z_0) = f(x_0,y_0,\pm).\hat{u}$ $\nabla f(x_0,y_0,z_0,\pm).\hat{u}$ $\nabla f(x_0,y_0,z_0,\pm).\hat{u}$ $\nabla f(x_0,y_0,z_0,\pm).\hat{u}$ $\nabla f$	$\hat{\lambda} = \frac{\vec{u}}{\vec{u}} = \frac{3\hat{i} + 4\hat{j}}{3\hat{i} + 4\hat{j}} = \frac{3\hat{i} + 4\hat{j}}{3\hat{i}} = \frac{3\hat{i}}{3\hat{i}} = 3$	
For functions of three variables, we can show that the directional derivative of $f(x, y, z)$ at $(x_0, y_0, z_0)$ in a direction of a unit vector $\hat{u}$ is		
For functions of three variables, we can show that the directional derivative of $f(x, y, z)$ at $(x_0, y_0, z_0)$ in a direction of a unit vector $\hat{u}$ is	$\therefore D_{\hat{u}} \neq (z, 1) = (1, -2) \cdot (3, 4)$	
For functions of three variables, we can show that the directional derivative of $f(x,y,z)$ at $(x_0,y_0,z_0)$ in a direction of a unit vector $\hat{u}$ is $D\hat{u}f(x_0,y_0,z_0) = \nabla f(x_0,y_0,z_0).\hat{u}$ where $\nabla f(x,y,z) = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$ $\hat{z}$		
derivative of $f(x,y,\pm)$ at $(x_0,y_0,\pm)$ in a direction of a unit vector $\hat{u}$ is $D_{u}^{2}f(x_0,y_0,\pm) = \nabla f(x_0,y_0,\pm).\hat{u}$ where $\nabla f(x,y,\pm) = f_{x}\hat{i} + f_{y}\hat{j} + f_{\pm}\hat{k}$ $\frac{\mathcal{E}_{x2}}{f(x_0,y,\pm)} = \sqrt{x_0} + \frac{1}{2} \int_{0}^{\infty} dx_0 dx_0 dx_0 dx_0 dx_0 dx_0 dx_0 dx_0$	$\frac{1}{5} = \frac{1}{5} = -1     Ans.$	
$\hat{u} \text{ is}$ $D_{u}^{2} f(x_{0}, y_{0}, z_{0}) = \nabla f(x_{0}, y_{0}, z_{0}). \hat{u}$ $\nabla f(x_{0}, y_{0}, z_{0}) = f_{x}\hat{i} + f_{y}\hat{j} + f_{z}\hat{k}$ $\frac{\mathcal{E}_{xz}}{\hat{j}} : \text{ Find the directional derivative of } z$ $f(x_{0}, y_{0}, z_{0}) = \sqrt{xyz} \text{ at the point } (3, z_{0}, k_{0}) \text{ in the direction of } z$ $\vec{v} = (-1, -2, z_{0}).$ Solution	For functions of three variables, we can show that the directional	
$D_{x}^{2} f(x_{0}, y_{0}, z_{0}) = \nabla f(x_{0}, y_{0}, z_{0}).\hat{u}$ where $\nabla f(x_{0}, y_{0}, z_{0}) = f_{x}\hat{i} + f_{y}\hat{j} + f_{z}\hat{k}$ $\frac{\mathcal{E}_{x2}}{\hat{i}}: \text{ Find the directoral derivative of } z$ $f(x_{0}, y_{0}, z_{0}) = \sqrt{xyz} \text{ at the point } (3, 2, 6) \text{ in the direction of } z$ $\frac{1}{\sqrt{x_{0}}} = \sqrt{xyz} \text{ at the point } (3, 2, 6) \text{ in the direction of } z$ $\frac{1}{\sqrt{x_{0}}} = \sqrt{xyz} \text{ at the point } (3, 2, 6) \text{ in the direction of } z$ $\frac{1}{\sqrt{x_{0}}} = \sqrt{xyz} \text{ at the point } (3, 2, 6) \text{ in the direction of } z$		
where $\nabla f(x,y,z) = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$ $\frac{\mathcal{E}xz}{\hat{j}} : \text{ Find the directional derivative of } z$ $f(x,y,z) = \sqrt{xyz} \text{ at the point } (3,2,6) \text{ in the direction of } z$ $\vec{V} = (-1,-2,2).$ Solution		
$\nabla f(x,y,z) = fx\hat{i} + fy\hat{j} + fz\hat{k}$ $\frac{\mathcal{E}x2}{\hat{i}} : \text{ Find the directional derivative of } z$ $f(x,y,z) = \sqrt{xyz} \text{ at the point } (3,2,6) \text{ in the direction of } z$ $\vec{V} = (-1,-2,2).$ Solution		
$\frac{\mathcal{E}x2}{\mathcal{E}}$ : Find the directional derivative of $\frac{\mathcal{E}x2}{\mathcal{E}}$ : $\frac{\mathcal{E}x2}{\mathcal{E}}$ : Find the directional derivative of $\frac{\mathcal{E}x2}{\mathcal{E}}$ :		
$f(x,y,z) = \sqrt{xyz}$ at the point $(3,2,6)$ in the direction of $V = (-1,-2,2)$ .  Solution	V + (x,y,z) = tx + tyj + tz K	
$f(x,y,z) = \sqrt{xyz}$ at the point $(3,2,6)$ in the direction of $V = (-1,-2,2)$ .  Solution	Ext. Find H. Jinstand Janutra	
$\overrightarrow{V} = (-1, -2, 2)$ . Solution		
Selution		
$\nabla f = f_x \hat{i} + f_y \hat{k}$		
	$\nabla f = f_x \hat{i} + f_y \hat{k}$	



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F(x(t), y(t), z(t)) = O(1)
 Differentiating (1) wat t,
  )ifferentiating (1) what t,
\frac{\partial F}{\partial x} \frac{dx}{dt} + \frac{\partial F}{\partial y} \frac{dy}{dt} + \frac{\partial F}{\partial z} \frac{dz}{dt} = 0
(2)
\begin{cases}
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{cases}
 (2) can be expressed in a dit product
    \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right) \cdot \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = 0
                                                           (3)
  At t = t., (3) becomes
       \nabla F(x_0, y_0, z_0) \cdot \frac{d\vec{n}}{dt}(t_0) = 0
   This means \nabla F(x_0, y_0, z_0) to the tangent \frac{d\vec{n}}{dt}|_{t_0} to (c) at Po.
 Since (C) is an arbitrary curve passing through Po, we conclude that
   VF(P.) is the normal rechn to the level surface (5) at Po.
 .. The equation of the tangent plane to the level surface
  (5) defined by F(x,y, =) = C at Po (20, y0, 20) is
 F_{x}(x_{0},y_{0},z_{0})(x-x_{0})+F_{y}(x_{0},y_{0},z_{0})(y_{y_{0}}+F_{z}(x_{0},y_{0},z_{0})(z_{z_{0}})=0
Ex3: Find equations of the tangent plane and the normal line
 to the surface (3) y = x2 - 22 at the point (4,7,3).
 Solutin
   Rewriting the equation of (S) in the form of F(x,y,z) = 0 to
   obtuin
                     F(x,y,z) = x^2 - y - z^2 = 0
     then \nabla F = 2\hat{i} - \hat{j} - 2 + k
    \therefore \quad \nabla F(4,7,3) = 2(4)\hat{i} - \hat{j} - 2(7)\hat{k}
                       = 8î -î - 6 k
  :. An equation of the tangent plane to (S) @ (4,7,3) is
     8 (x-4) + (-1) (y-7) + (-6) (z-3) = 0
         8x - 9 - 62 - 32 + 7 + 18 = 0
                 8x - 3 - 6x - 7 = 0 // Ans.
        and the egns of the normal line are
                 \frac{x-4}{8} = \frac{y-7}{-1} = \frac{z-3}{-6} | Ans.
         or in parametric from
                  \begin{cases} x = 4 + 8t \\ y = 7 - t \end{cases}
                                                      / Ans.
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\
} y = 7 - + // Ams.
2 = 3 - 64
Ex5: The temperature at a point (x,y,2) is given by
$\frac{1}{(x_{1}y_{1}+1)} = 200 e^{-x^{2}-3y^{2}-9z^{2}}$
where T is measured in °C and 2,4,7 in meters.
a) Find the rate of change of temperature at the print P(2,-1,2)
in the direction toward the point (3,-3,3).
b) In which direction does the temperature increase fastest at P?
c) Find the maximum nate of increase at P.
Solution
$\sum_{n=0}^{\infty} \frac{3^{n}}{2^{n}} \int_{0}^{\infty} 3^$
$= -400xe^{-x^{2}-31^{2}-9+2}\hat{i} - 1200ye^{-x^{2}-3y^{2}-9+2}\hat{j} - 3600+e^{-x^{2}-3y^{2}-9+2}\hat{k}$
$\nabla T(2,-1,2) = [-400(2)\hat{i}-1200(-1)\hat{j}-3600(2)\hat{k}] e^{-(2)^2-3(-1)^2-9(2)^2}$
= (-800î +1200ĵ - 7200k) = 43
$\vec{u} = \vec{P}\vec{Q} = (3, -3, 3) - (2, -1, 2)$
=(3-2,-3-(-1),3-2)
= (1) -2,1)
$\hat{u} = \frac{\vec{u}}{\ \vec{u}\ } = \frac{(1,-2,1)}{\sqrt{(1)^2 + (-2)^2 + (1)^2}} = \frac{(1,-2,1)}{\sqrt{6}}$
$D_{\hat{u}} T(2)_{-1,2} = \nabla T(2)_{-1,2} \cdot \hat{u}$
$=(-800\bar{e}^{43}, 1200\bar{e}^{43}, -7200\bar{e}^{43}).(1,-2,1)$
$= (-\frac{900 - 2400 - 7200}{\sqrt{6}})e^{-43}$
$= -\frac{5200\sqrt{6}}{3}e^{43} \circ C/m$ //Ams.
3
b) In the direction of $\nabla T(2,-2,2)$ which is the direction of
(-2,3,-18) the temperature increases fastest.
$D_{\hat{u}}^{2} f(z,-z,z) \Big _{m_{x_{x}}} = \  \nabla T(z,-z,z) \ $
max = 11 V 1 (2, 2, 2)11
= \( \left( -800)^2 + \left( 1200 \right)^2 + \left( -7200 \right)^2 \end{array}
~ 7343 E43 °C/m // Am.
See you on Monday!