Ch. 13 exercises

13.24 Let A be the event that the student is enrolled in a four-year college and B the event that the student is female. The proportion of females enrolled in a four-year college is expressed in probability notation as

a. $P(A \text{ and } B)$.

b. $P(A|B)$.

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Correct is c.

13.29 **Universal Blood Donors.** People with type O-negative blood are referred to as universal donors, although if you give type O-negative blood to any patient, you run the risk of a transfusion reaction due to certain antibodies present in the blood. However, any patient can receive a transfusion of O-negative *red blood cells*. Only 7.2% of the American population have O-negative blood. If 10 people appear at random to give blood, what is the probability that at least one of them is a universal donor?

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By rule #4 of probability

P (at least one is O-negative) = $1 - P$ (none are O-negative)

P (none are O-negative) = $(1 - 0.072)^{10} = 0.4737$ → *By multiplication rule under independence*

→ P (at least one is O-negative) = $1 - P$ (none are O-negative) = 1 - 0.4737 = 0.5263

13.35 Student Debt. At the end of 2016, the average outstanding student debt for bachelor's degree recipients was \$28,500. Here is the distribution of outstanding education debt (in thousands of dollars):[17]

Debt	< 10	10 to < 20	20 to < 30	30 to < 40	40 to < 50	≥ 50
Probability	0.40	0.13	0.17	0.12	0.08	0.10

a. What is the probability that a randomly chosen student has an outstanding debt of \$20,000 or more?
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(a) P($ 20,000 or more) = 0.17 + 0.12 + 0.08 + 0.10 =0.47

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That's the intersection of the two events

P(>= 50 and >=20) = P($\boxed{\text{>=50}}$)

(b) $P(\text{at least \$50,000} \mid \text{at least \$20,000}) = \dfrac{\boxed{P(\text{at least \$50,000})}}{P(\text{at least \$20,000})} = \dfrac{0.10}{0.47} = 0.2128.$

13.37 College Degrees. A striking trend in higher education is that more women than men reach each level of attainment. The National Center for Education Statistics (which classifies all subjects as male or female) provides projections for the number of degrees earned, classified by level and by the sex of the degree recipient. Here are the projected number of earned degrees (in thousands) in the United States for the 2027–28 academic year:[19]

	Associate's	Bachelor's	Master's	Doctorate	Total
Female	653	1106	473	101	2333
Male	411	816	340	90	1657
Total	1064	1922	813	191	3990

a. If you choose a degree recipient at random, what is the probability that the person you choose is a man?
b. What is the conditional probability that you choose a man, given that the person chosen received a master's?
c. Are the events "choose a man" and "choose a master's degree recipient" independent? How do you know?

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Let *M* be the event " the person is a man" and *S* be the event "the person earned a Master's degree."
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(b) $P(\text{M} \mid S) = 340/813 = 0.4182$.

$$= P(\text{M and S}) / P(\text{S}) = \frac{\frac{340}{3990}}{\frac{813}{3990}}$$

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$$= P(M \text{ and } S) / P(S) = \frac{\frac{340}{3990}}{\frac{813}{3990}}$$

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(c) The events " choose a man" and "choose a Master's degree recipient" are not independent. If they were, the two probabilities in part (a) and part (b) would be equal.

13.49 **Winning at Tennis.** A player serving in tennis has two chances to get a serve into play. If the first serve is out, the player serves again. If the second serve is also out, the player loses the point. Here are probabilities based on four years of the Wimbledon Championship:[22]

$$\begin{aligned} P\,(\text{1st serve in}) &= 0.59 \\ P\,(\text{win point} \mid \text{1st serve in}) &= 0.73 \\ P\,(\text{2st serve in} \mid \text{1st serve out}) &= 0.86 \\ P\,(\text{win point } \mid \text{1st serve out and 2nd serve in}) &= 0.59 \end{aligned}$$

Make a tree diagram for the results of the two serves and the outcome (win or lose) of the point. (The branches in your tree have different numbers of stages, depending on the outcome of the first serve.) What is the probability that the serving player wins the point?

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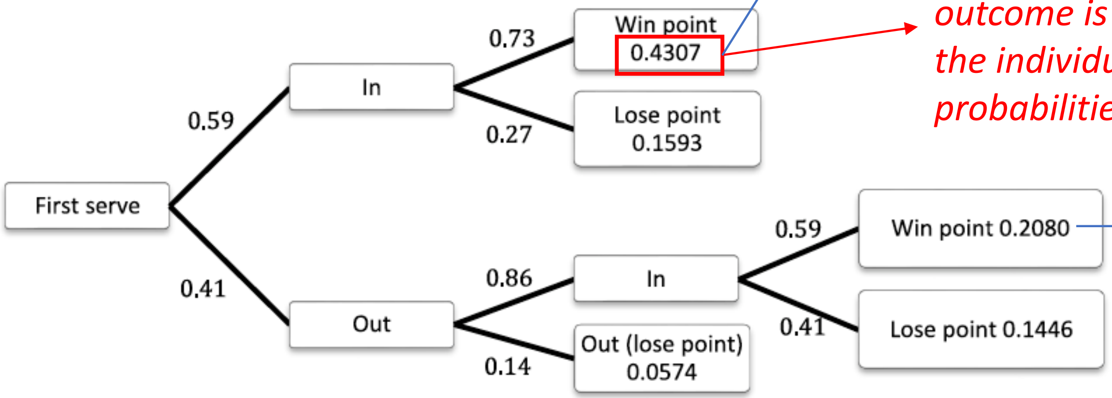
P(1st serve in and win point)
= 0.59 x 0.73 = 0.4307

the probability of each outcome is the product of the individual branch probabilities leading to it



P(1st serve out AND 2nd serve in AND win point)
= 0.41 x 0.86 x 0.59 = 0.2080

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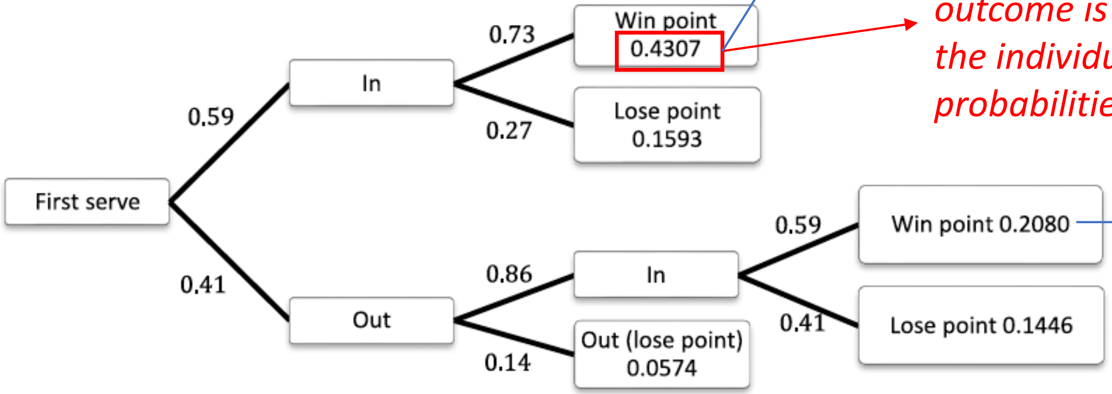

P(1st serve out AND 2nd serve in AND win point)
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→ **Probability that the serving players wins the point is 0.4307 + 0.2080 = 0.6387**