HW3 due tonight 11:55
OH Today 3-4 pm MC130
7-8 pm Zoom.
Read 3.1 3.2.

Puzzle:
$72 = 2\times 2\times 2\times 3\times 3$
$2+6+6=14$ ⇐ these two have same sum, so the woman
√ $3+3+\underline{8}=14$ could not get the answer
↑ oldest child.

§ 3.1
Proofs involving conditional

Suppose $n \in \mathbb{N}$ Proof: if $n>1$, $n^2>n$.
Rough work:

Given	Goal
$n\in\mathbb{N}$	$(n>1) \to (n^2>n)$

Technique: To prove $P \to Q$. Assume P, and prove Q.
$n\in\mathbb{N}$ $n>1$ $n^2>n$
Idea: Since $n>0$, mutiply both sides by n.

Proof: Let $n \in \mathbb{N}$, suppose $n>1$. Then, since n is a positive, we can multiply both sides of the inequality by n to get $n^2>n$. Therefore, $n>1$ implies $n^2>n$. $\square$

Strategy: To prove $P \to Q$:
1. Add P to the given
2. Change goal to Q.

Suppose P
[Proving of Q]
Therefore P implies Q.

Counterexamples:
__Wrong__: Let $n\in\mathbb{N}$ if $n>0$, then $n^2>n$
Give a specific counterexample.

the hypothesis is true ↓ the conclusion is not hold ↓

when $n=1$, $n>0$ is true, $n^2=1$ and it is not greater then 1, so it is false

Technique 2: To prove $P \to Q$,

1. Assume $\neg Q \to \neg P$.
2. Add $\neg Q$ to the given
3. Switch the goal to $\neg P$.

Suppose $\neg Q$

[proof]

Therefore P implies Q.

Ex: Let $x \in \mathbb{R}$, prove that if $\sqrt{x^2+7} = x+2$, then $x \neq 3$.

Rough work

Given	Goal
$x \in \mathbb{R}$	$\sqrt{x^2+7} = x+2 \to x \neq 3$
$x \in \mathbb{R}$	$x = 3 \to \sqrt{x^2+7} \neq x+2$.
$x \in \mathbb{R}, x=3$	$\sqrt{x^2+7} \neq x+2$

$\sqrt{x^2+7} = 4$, $x+2=5$.

Proof: Let $x \in \mathbb{R}$. Assume $x=3$, then $\sqrt{x^2+7} = \sqrt{3^2+7} = 4$ and $x+2 = 5 \neq 4$ so $\sqrt{x^2+7} \neq x+2$.

Therefore $\sqrt{x^2+7} = x+2$ implies $x \neq 3$ $\square$

§ 3.2. Proofs involve $\neg P$.

Strategy 1: To prove a goal in form of $\neg P$, reexpress it in another way.

Ex 3.2.1: Suppose $A \cap C \subseteq B$ and $a \notin C$. Prove that $a \in A \setminus B$.

Given	Goal
$A \cap C \subseteq B$, $a \notin C$.	$a \notin A \setminus B$

$$\boxed{\begin{aligned} a \notin A \setminus B \ &\text{iff}\ \neg(a \in A \setminus B) \\ &\text{iff}\ \neg(a \in A \wedge a \notin B) \\ &\text{iff}\ a \notin A \vee a \in B \\ &\text{iff}\ a \in A \to a \in B \end{aligned}}$$

Given	Goal
$A \cap C \subseteq B$, $a \notin C$	$a \in A \to A \in B$
$A \cap C \subseteq B$, $a \in A$, $a \notin C$	$A \in B$

Proof: Suppose $a \in A$. Then, since $a \notin C$, but $a \in A \cap C$.
Since $A \cap C \subseteq B$, we get $a \in B$. It can't be the case that $a \in A$ and $a \notin B$. Therefore, $a \notin A \setminus B$ □.

Strategy 2: To prove the goal at from $\neg P$:

1. Add P to the given
2. Change goal to "contradiction"

Suppose P is true

[proof]

Thus, P is false.

Ex: Let $x, y \in \mathbb{R}$ Suppose $y + x = 2y - x$ and y and x are not both 0. Prove $y \neq 0$

Given	Goal
$x, y \in \mathbb{R}$, $y + x = 2y - x$	~~$y \neq 0$~~. contradiction.
y and x are not both 0	
$y = 0$	

$0 + x = 0 - x$

$x = 0$

however, $\neg(x = 0 \land y = 0)$