

# COMPSCI 3331 - Fall 2022 - Quiz 1

1. (2 marks) Let  $L = \{ab, bba\}$  be a language over  $\Sigma = \{a, b\}$ . List two words of length seven or greater which are in  $L^*$  and two words of length six or greater which are not in  $L^*$ .

*Many solutions. Some examples of word in  $L^*$ :*

- *abababab*
- *bbabbabba*
- *abbbaabbba*

*Some words not in  $L^*$ :*

- *anything that starts with aa, like aaaaaaa*
- *anything that starts with bbb, like bbbbbb*

*Some students noted answers like ccccccc or abce fgh, which are technically correct since the question didn't specify that the words had to be over  $\Sigma$ .*

2. (2 marks) Suppose that  $\Sigma$  is an alphabet with at least two letters. Disprove that

$$L_1 \cap L_2 L_3 = (L_1 \cap L_2)(L_1 \cap L_3)$$

holds for all languages  $L_1, L_2, L_3 \subseteq \Sigma^*$ .

*We disprove this by showing a counter-example. Let  $L_1 = \{ab\}$ ,  $L_2 = \{a\}$  and  $L_3 = \{b\}$ . Then*

$$\begin{aligned} L_1 \cap L_2 L_3 &= \{ab\} \\ (L_1 \cap L_2) &= \emptyset \\ (L_1 \cap L_3) &= \emptyset \\ (L_1 \cap L_2)(L_1 \cap L_3) &= \emptyset \end{aligned}$$

*Thus, the equality does not hold in general.*

*Some common issues in this question:*

- *When two languages have no words in common, then  $L_1 \cap L_2 = \emptyset$ , not  $\{\epsilon\}$*

- $L \cdot \emptyset = \emptyset$ : the concatenation of any language with the empty set is always the emptyset, because there are no words of the form  $xy$  where  $x \in L$  and  $y \in \emptyset$ .

*Note that some people interpreted  $L_1 \cap L_2 L_3$  as  $(L_1 \cap L_2)L_3$ . This was accepted for this solution, but concatenation has higher precedence than intersection and union, so we should always interpret  $L_1 \cap L_2 L_3$  as  $L_1 \cap (L_2 L_3)$ .*