Relation

$A = \{a_1, a_2, a_3\}$ $\quad R_1 = \{(a_1, b_1)\ (a_1, b_2)\ (a_1, b_3)\}$.

$B = \{b_1, b_2, b_3\}$ $\quad$ is a relation from A to B.

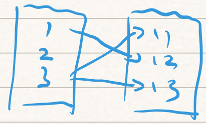
$\emptyset \subseteq A \times B$

$A \times B \subseteq A \times B$.

The domain of the relation $r \subseteq A \times B$ is $\mathrm{Dom}(r) = \{a \in A \mid \exists\, b \in B \text{ such that } (a,b) \in R\}$

The range is $\mathrm{Ran}(r) = \{b \in B \mid \exists\, a \in A \ \text{-----} \}$.

e.g. $A = \{1,2,3\}$ $\quad B = \{11,12,13\}$.



$\mathrm{Dom}(r) = \{1,2\}$

$\mathrm{Ran}(r) = \{11,12,13\}$.

Inverse:

the inverse of R is $R^{-1} = \{(b,a) \in B \times A \mid (a,b) \in R\}$.

Proof: $(R^{-1})^{-1} = R$.

Let $(a,b) \in A \times B$. Then $(a,b) \in (R^{-1})^{-1}$ iff $(b,a) \in R^{-1}$ (definition),

iff $(a,b) \in (R^{-1})^{-1}$.

So $(R^{-1})^{-1} = R$ $\square$.

Proof $\mathrm{Ran}(R^{-1}) = \mathrm{Dom}(R)$.

Let $a \in A$. Then $a \in \mathrm{Ran}(R^{-1})$ iff $\exists\, b \in B\ ((b,a) \in R^{-1})$

iff $\exists\, b \in B\ ((a,b) \in R)$.

iff $a \in \mathrm{Dom}(R)$. $\square$

Def comp.

Given $R \subseteq A \times B$ $\quad S \subseteq B \times C$. the composition of R and S is

$S \circ R = \{(a,c) \in A \times C \mid \exists\, b\ (a,b) \in R \wedge (b,c) \in S\}$.