Question:	1	2	3	4	5	6	7	Total
Marks:	8	2	4	6	8	5	7	40
Score:								

Name (print): ______________________________

Signature: ______________________________

UWO ID number: ______________________________

CIRCLE THE NUMBERS OF YOUR LECTURE SECTION AND LAB SECTION IN THE TABLES BELOW:

001	MWF 8:30	Hugo Bacard
002	MWF 10:30	Dan Christensen

003	Wed 9:30	Youlong Yan	006	Wed 3:30	Javad Rastegari Koopaei
004	Thu 2:30	Allen O'Hara	007	Thu 12:30	Gaohong Wang
005	Thu 11:30	Jason Haradyn	008	Wed 11:30	Jason Haradyn

THE UNIVERSITY OF WESTERN ONTARIO
DEPARTMENT OF MATHEMATICS

MATHEMATICS 1600A MIDTERM EXAMINATION
3 October 2013 7:00–8:30 PM

INSTRUCTIONS:

1. This exam is 9 pages long. There are 7 questions. **Check that your exam is complete.**
2. All questions must be answered in the space provided. Indicate your answer clearly. Should you need extra space, a blank page is provided at the end of the booklet.
3. Show all your of your work and explain your answers fully. Unjustified, irrelevant or illegible answers will receive little or no credit.
4. Do not unstaple the exam booklet.
5. **No aids are permitted. In particular, calculators, cell phones, ipods, etc. are not allowed and may be confiscated.**
6. If not stated otherwise, all vectors and equations involve real numbers.
7. In your final answers you must give all numbers in $\mathbb{Z}_m$ as a number between 0 and $m-1$.

1. For each of the following statements, circle **T** if the statement is always true and **F** if it can be false. Give a one-sentence justification for your answer.

[2] (a) Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be non-zero vectors in $\mathbb{R}^3$. If $\vec{u}$ and $\vec{v}$ are both orthogonal to $\vec{w}$, then $\vec{u}$ is parallel to $\vec{v}$.

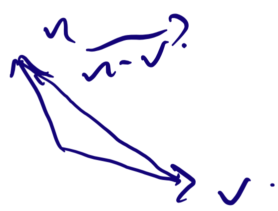
T **F**

[2] (b) Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^n$. Then $\|\vec{u}-\vec{v}\| \le \|\vec{u}\| + \|\vec{v}\|$.

T **F**

$u^2+v^2-2uv\cos\theta$

u^2+v^2+2uv



[2] (c) The planes $2x-3y+z=4$ and $-4x+6y-2z=1$ in $\mathbb{R}^3$ are parallel.

T **F**

[2] (d) Let A denote the coefficient matrix of a system of 4 linear equations in 4 unknowns. If the rank of A is 3, then this system has infinitely many solutions.

T **F**

The system may not be consistent.

可能无解.

[2] 2. Given that $\vec{u}\cdot\vec{v}=0$, $\vec{u}\cdot\vec{w}=1$, $\vec{v}\cdot\vec{w}=2$ and $\|v\|=1$, compute $(2\vec{u}+\vec{v})\cdot(2\vec{v}+3\vec{w})$.

$(2\vec{u}+\vec{v})\cdot(2\vec{v}+3\vec{w})$.

$= 4\vec{u}\cdot\vec{v} + 2\vec{v}\cdot\vec{v} + 6\vec{u}\vec{w} + 3\vec{v}\vec{w}$.

$= 0 + 2 + 6 + 6$.

$= 14$.

3. Let $\vec{u}=[1,\sqrt{2},1]$ and $\vec{v}=[0,0,1]$ be vectors in $\mathbb{R}^3$.

[2] (a) Find the unit vector in the same direction as $\vec{u}$.

assume the unit vector $\vec{n} = (a, \sqrt{2}a, a)$.

$1 = a^2+2a^2+a^2$.

$a = \frac{1}{2}$.

$\therefore \vec{n} = (\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2})$.

[2] (b) Compute the angle between $\vec{u}$ and $\vec{v}$.

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}$$

$$= \cancel{\frac{1}{4}} \quad \frac{1}{2}.$$

$\theta = 60°$

4. Let ℓ be the line through the points $P = (1, 2)$ and $Q = (5, 5)$.

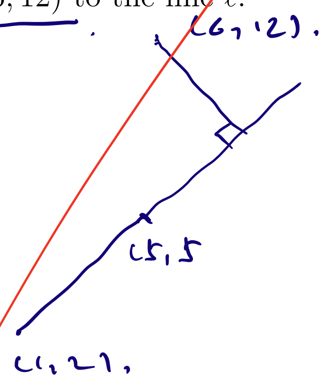
[2] (a) Find a direction vector for the line ℓ and write parametric equations of the line ℓ.

$\vec{PQ} = (4, 3)$.

$x = 1 + 4\lambda$

$y = 2 + 3\lambda$.

[4] (b) Find the distance from the point $R = (6, 12)$ to the line ℓ.



assume that $l' \perp l$

$l': y - 12 = -\frac{4}{3}(x - 6)$.

$l: y - 2 = \frac{3}{4}(x - 1)$.

$\begin{cases} x = 1 + 4\lambda \\ y = 2 + 3\lambda \end{cases}$ $\begin{cases} x = 6 + 3n \\ y = 12 - 4n \end{cases}$.

$1 + 4\lambda = 6 + 3n$

$2 + 3\lambda = 12 - 4n$.

$3 + 12\lambda = 18 + 9n$

$8 + 12\lambda = 48 - 16n$.

$5 = 30 - 25n$

$n = 1$

$\lambda = 2$.

$\therefore (9, 8)$.

$d = \sqrt{(9-6)^2 + (8-12)^2}$

$= 5$

5. Let $\mathcal{P}$ be the plane in $\mathbb{R}^3$ given by the parametric equations

$$\begin{aligned} x &= -5 + \boxed{1s} \ \boxed{0} \\ y &= \quad \boxed{-2s} \boxed{+1t} \\ z &= 1 \boxed{+6s} \boxed{-3t} \end{aligned}$$

$-5 + s - 30s + 15t$

$+5 + 30s - 15t = 0$

[3] (a) Find a normal vector to the plane $\mathcal{P}$.

$P: (-5+s, -2s+t, 1+6s-3t)$.

direction vectors $: (1, -2, 6)\ (0, 1, -3)$.

$n: (a, b, c)$. $\Rightarrow \vec{n} = (0, 3, 1)$.

$(-5+s)a + (-2s+t)b + (1+6s-3t)c = 0$.

$-5a + sa - 2sb + tb + c + 6sc - 3tc = 0$.

$(-5a+c) + (a-2b+6c)s + (b-3c)t = 0$.

$\begin{cases} -5a + c = 0 \\ a - 2b + 6c = 0 \\ b - 3c = 0 \end{cases}$ $b = 3c = 15a$. $c = 5a$. $n: (a, 15a, 5a)$.

$\vec{n} = (1, 15, 5)$

[2] (b) Find a general equation for the plane $\mathcal{P}$.

$3y + z = 1$.

还要下的是这个……

[3] (c) Give the general equation for a plane $\mathcal{P}'$ that intersects $\mathcal{P}$ in a line, and explain how you know that the intersection is exactly a line.

any plane whose normal vector is not parallel to $\vec{n}$, $x = 0$. / $x + y + z = 1$?.

if normal vectors are not parallel, then planes are not parallel, so they must intersect at a line.

6. Recall that the Universal Product Code (UPC) uses code words in $\mathbb{Z}_{10}^{12}$ and has check vector $\vec{c} = [3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1]$.

[3] (a) Find the missing digit y in the UPC $[0, 4, 3, 7, 0, 6, 5, y, 9, 1, 2, 1]$.

3 1 3 1 3 1 3 1 3 1 3 1.

$4 + 9 + 7 + 6 + 15 + y + 27 + 1 + 6 + 1$

$= y + 6 = 10,$

$y = 4.$

[2] (b) Find a valid UPC code with only one non-zero digit, or explain why this is not possible.

It is impossible.

For $\forall$ $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $b \in \{1, 3\}$.

$3a \neq 10b$.

注意表述方式

7. Consider the system of linear equations

$$\begin{aligned} 2x + 4y - 2z &= 2 \\ 2x + \ \ y + \ \ z &= 5 \\ \cancel{x + 4y - 3z} &= \cancel{-1} \end{aligned}$$

$2x + 8y - 6z = -2.$

$4y - 4z = -4.$

$7y - 7z = -7.$

[1] (a) Write down the augmented matrix of this linear system.

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 2 & 1 & 1 & 5 \\ 1 & 4 & -3 & -1 \end{array}\right].$$

$x + z = 3$

$y - z = -1.$

[3] (b) Compute the reduced row-echelon form of the augmented matrix above. Indicate all elementary row operations that you are performing.

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 2 & 1 & 1 & 5 \\ 1 & 4 & -3 & -1 \end{array}\right] \xrightarrow[R_1 = \frac{1}{2}R_1]{R_2 = R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 3 & 3 \\ 1 & 4 & -3 & -1 \end{array}\right].$$

$$\xrightarrow{R_3 = R_3 - R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 3 & 3 \\ 0 & 2 & -2 & -2 \end{array}\right] \xrightarrow[R_2 = \frac{1}{2}R_2]{R_3 = -\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & -1 & -1 \end{array}\right].$$

$$\xrightarrow[R_3 = R_3 - R_2]{R_1 = R_1 - 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

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[2] (c) Use the result of the previous part to find all solutions of the linear system.

for all $x+z=3$
$y-z=-1$.
the linear system has infinitely many solutions.

无穷多解

$x=3-t$
$y=-1+t$
$z=t$

⇒ 用参数表示.

for any value of t.

[1] (d) What is the rank of the augmented matrix you found in part (a)?

2.

Use this page if you need extra space for your work.

Did you write your name and student ID on the first page?
Did you give full explanations and show all of your work?