Q1.

(a) = (b):

$\to$: Assume an arbitrary a that $a \in (C \setminus A)$. Since $a \in C \setminus A$, $a \in C$ and $a \notin A$. Since $C \subseteq A \cup B$, $a \in A \cup B$. Because $a \notin A$, $a \in B$. So $C \subseteq A \cup B$ implies $(C \setminus A) \subseteq B$. $\square$

$\leftarrow$: Given that $(C \setminus A) \subseteq B$, any random a that $a \in (C \setminus A)$ also in B. So we can have $a \in (C \setminus A) \to a \in B$, which would be rewritten as $\neg(a \in C \wedge a \notin A) \vee a \in B$, and it could be simplified as $a \notin C \vee a \in A \vee a \in B$, so $a \notin C \vee (a \in A \vee a \in B)$, then $a \in C \to (a \in A \vee a \in B)$, which is equal to $a \in C \to a \in (A \cup B)$. Since a is random, $(C \setminus A) \subseteq B$ implies $C \subseteq A \cup B$

From the proof above, we can have (a) = (b) $\square$

(a) = (c) is similar to the proof of (a) = (b).

Thus, we can have (b) = (a) = (c), so they are equivalent. $\square$

Q2.

Statement 1 is true.

Proof: Existence: let $x=3$, the left-hand side of the equation is $(3)^2 - 6 \times 3 = -9$, which is equal to the right-hand side. So $x=3$ can be one solution.

Uniqueness: Assume $z^2 - 6z = -9$. then it could be rewritten as $z^2 - 6z + 9 = 0$, which is $(z-3)^2 = 0$. The only solution in this equation is $z=3$, so $z=x$. Thus, $x=3$ is the only solution. $\square$

Statement 2 is false.

Proof: Existence: Assume there exist x_0 that $x_0^2-6x_0+1<0$,
this would be rewritten as $(x_0-3)^2+8<0$,
since $(x_0-3)^2 \geq 0$, $(x_0-3)^2+8 \geq 8$, so it cannot
be negative and x_0 does not exist.
Thus, statement 2 is false. $\square$.

Q3.

Theorem 1: Existence: let $y=x+1$. The left-hand side would be
$\frac{x+1-1}{x}=1$. Right-hand side would be $x+1-x=1$,
so $y=x+1$ is one solution of the equation. $\square$

Uniqueness: Assume $\frac{z-1}{x}=z-x$, this equation is valid since
$x\neq 0$. Multiply each side by x, we will get:
$z-1=zx-x^2$, and it could be rewritten as:
$z(1-x)=1-x^2$, and it is $z(1-x)=(1+x)(1-x)$.
Since $x\neq 1$, so divide each side by $(1-x)$ and
$z=1+x=y$. Thus, $y=1+x$ is the only solution $\square$

Theorem 2: Existence: Assume $y=3$. The left-hand side would be $(x+3)(x-3)=x^2-9$
and it is equal to the right-hand side. So $y=3$ can
be one solution. $\square$

Uniqueness: Assume z that $(z+x)(x-3)=x^2-9$.
If $x=3$, then the equation would be $0=0$. In this
time z could be any value.
If $x\neq 3$, the equation would be $(z+x)(x-3)=(x+3)(x-3)$
and $z+x=x+3$. so $z=3$. which is the only possible

situation.

Since we've run out all possible cases, there exist an unique y that satisfy all values of x, $(y+x)(x-3) = x^2-9$. □

Q4.

Existence: $m=5$. Since $5|10$, $5|15$ and $5>1$, $m=5$ could be one solution □

Uniqueness: Assume $z>1$, $z|10$ and $z|15$. To satisfy $z|10$, $z \in \{1,2,5,10\}$. and to satisfy $z|15$, $z \in \{1,3,5,15\}$. Since $z>1$, $z|10$ and $z|15$, z could only be 5. □