CS3388B, Winter 2023

Problem Set 1

Due: January 13, 2023

Exercise 1. Show that the basis vector $\hat{k}$ is orthogonal to both $\hat{\imath}$ and $\hat{\jmath}$.

$\hat{k} \cdot \hat{\imath} = 0$, $\hat{k} \cdot \hat{\jmath} = 0$

Exercise 2. Compute the matrix product of A and B given :

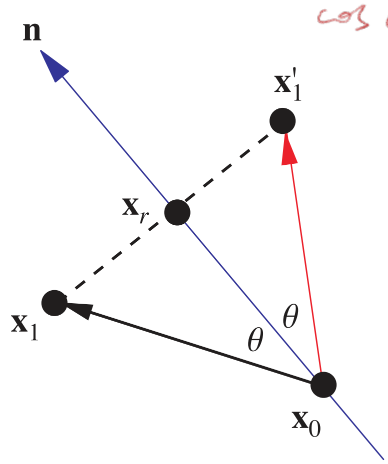
$$A = \begin{bmatrix} 1 & -4 & 8 \\ 11 & 2 & 24 \\ 12 & 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -9 & 8 & 6 \\ 0 & 15 & 2 \\ 3 & 14 & 0 \end{bmatrix}$$

$15 = 1 \times (-9) + (-4) \times 0 + 3 \times 8$

$$A \cdot B = \begin{bmatrix} 15 & 60 & -2 \\ -27 & 454 & 70 \\ -105 & 170 & 80 \end{bmatrix}$$

Exercise 3. Consider the vector $\vec{v} = (2, 3)$ and the vector $\vec{n} = (-1, 2)$. Find a vector that is in the same direction as the *reflection* of $\vec{v}$ across $\vec{n}$? You don't have to exactly find the reflection, just one in the same direction. Consider the below image.

$\cos\theta = \frac{\vec{v} \cdot \vec{n}}{|\vec{v}| \cdot |\vec{n}|}$



Then, find the angle between $\vec{v}$ and $\vec{n}$ and show that it is the same angle as between $\vec{n}$ and your computed reflection.

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = (\frac{-\sqrt{5}}{5}, \frac{2\sqrt{5}}{5})$$
$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = (\frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13})$$
$$\hat{n} \cdot \hat{v} = \cos(\theta) = \frac{4\sqrt{5}\sqrt{13}}{65} \approx 0.4961389$$

Let $\hat{r} = (x, y)$ be the direction of the reflection of $\vec{v}$ across $\vec{n}$. It must be that $\hat{r} \cdot \hat{n} = \hat{v} \cdot \hat{n}$. Therefore:

assuming these two have same length

$$\begin{cases} \frac{-\sqrt{5}}{5}x + \frac{2\sqrt{5}}{5}y = 0.4961389 \\ x^2 + y^2 = 1 \end{cases}$$

This second equation ensures that $|\hat{r}| = 1$. Otherwise, the first equation could include a denominator on the left hand side to divide by the length of $\vec{r}$. This gives:

$$\frac{-\sqrt{5}}{5}x + \frac{2\sqrt{5}}{5}y = (0.4961389)\sqrt{x^2 + y^2}$$

Exc ME?

Solving a system of equations is hard by hand, so let's use the more complicated equation. Since we only care about direction, let $y = 1$ (geometrically we know that $\hat{r}$ should have a negative x and a positive y). Now solve for x:

$$\frac{-\sqrt{5}x + 2\sqrt{5}}{5} = (0.4961389)\sqrt{x^2 + 1}$$

$$(-\sqrt{5}x + 2\sqrt{5}) = 2.4806945(\sqrt{x^2 + 1}) \quad \text{then square both sides}$$

$$5x^2 - 20x + 20 = 2.480695(x^2 + 1)$$

$$x = \frac{2}{3}, -18$$

We take the negative solution, otherwise we get the direction of $\vec{v}$. After normalizing for length, we get:

$$\vec{r} = (-18, 1)$$
$$\hat{r} = (-0.99846, 0.05547)$$

It is then easy to verify that $\hat{n} \cdot \hat{v} = \hat{n} \cdot \hat{r}$.

Exercise 4. Let $p = (3, 2, 4), q = (1, -3, 4), r = (1, 3, -1)$ be three points. Find the normalized normal vector to the plane containing these three points. Remember there are two such normals. Find one.

Find two displacement vectors. Say $p - q$ and $q - r$:

$$\vec{pq} = (\overset{x_1}{-2}, \overset{y_1}{-5}, \overset{z_1}{0}), \quad \vec{qr} = (\overset{x_2}{0}, \overset{y_2}{6}, \overset{z_2}{-5})$$

Get cross product, normalize: $\hat{n} = \frac{1}{\sqrt{869}}(25, -10, -12)$

$x_3 = y_1 z_2 - y_2 z_1$

$y_3 = x_1 z_2 - x_2 z_1$

$z_3 = x_1 y_2 - x_2 y_1$.