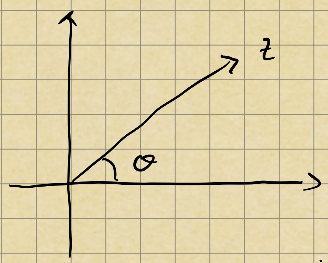
Complex Numbers.



$z = a + \theta i$.

multiplication.

properties: $|z \cdot w| = |z| \cdot |w|$.

$|z + w| \le |z| + |w|$ triangle equality.

If z has argument θ and w has argument φ.

then zw has argument $\theta + \varphi$

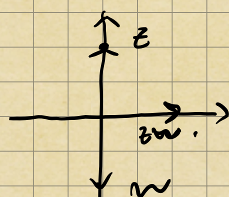
e.g. $z = i$ $\quad w = i$.

$\theta = \frac{\pi}{2}$. $\quad \varphi = \frac{\pi}{2}$.

$zw = i^2 = -1$

$z = i$ $\quad w = -i$.

$\theta = \frac{\pi}{2}$ $\quad \varphi = -\frac{\pi}{2}$.

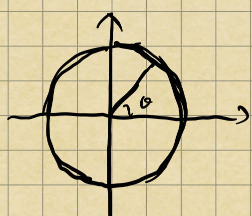


$zw = 1$

$\theta + \varphi = 0$

define: $e^{bi} = \cos(b) + i\sin(b)$.

$e^{a+bi} = e^a \cdot e^{bi} = e^a(\cos b + i\sin b)$



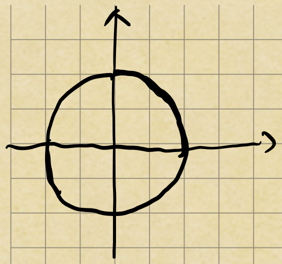
If z is a complex number $a+bi$,

then $e^z = e^a(\cos b + i\sin b)$.

Therom: If $z_1 = r_1 e^{i\theta_1}$

$z_2 = r_2 e^{i\theta_2}$.

then $z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$.



Root of 1.

$n=2$: $1, -1$.

$n=3$: $1, e^{\frac{2\pi i}{3}}, e^{\frac{4\pi i}{3}}$.

$x_2^3 = e^{2\pi i} = \cos 2\pi + i \sin 2\pi = 1$.

$x_3^3 = e^{4\pi i} = \cos 4\pi + i \sin 4\pi = 1$.