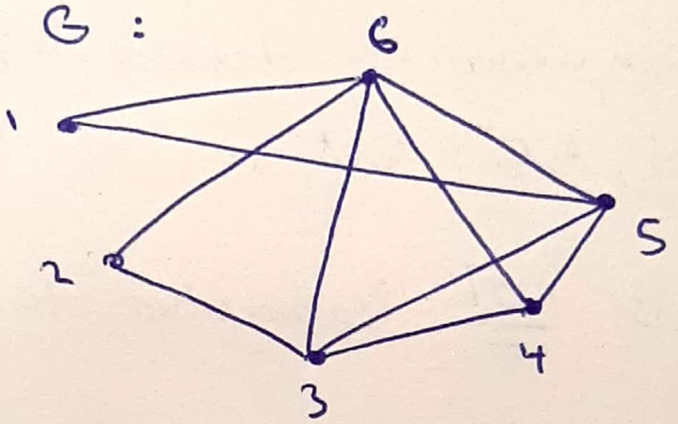
• Consider the following adjacency matrix of graph G:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

a) reconstruct G :



b) is there any path between Nodes 1 & 2 ?

Yes , $1 \rightarrow 6 \rightarrow 2$

c) Find degrees of the nodes in this graph:

$d = \{2, 2, 4, 3, 4, 5\}$

d) Find #edges in G from the degrees of the vertices :

$$\sum d_i = 2e$$

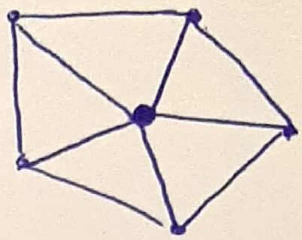
$$\Rightarrow \quad e = (2+2+4+3+4+5)/2 = \frac{20}{2} = \boxed{10}$$

e) is G bipartite ? NO,

(Hint : Assume it's bipartite. color of 6 should be different from all other vertices. reach contradiction)

f) is this graph isomorphic to W_6?

→ let's consider W_6:



$|E_G| = 10 \quad |V_G| = 6$

$|E_{W_6}| = 10 \quad |V_{W_6}| = 6$

However, sequence of degrees for G is:

$\{2, 2, 3, 4, 4, 5\}$ but for W_6 is: $\{3, 3, 3, 3, 3, 5\}$

$\Rightarrow$ G is NOT isomorphic to W_6.

g) Does G have an Euler circuit / path?

Euler path: $4 \to 6 \to 3 \to 4 \to 5 \to 3 \to 2 \to 6 \to 1 \to \bullet \to 5 \to 6$

* note that the starting & eding vertices of the path have odd degrees.

NO Euler circuit. (because there are nodes with odd degrees)

h) is G connected? yes

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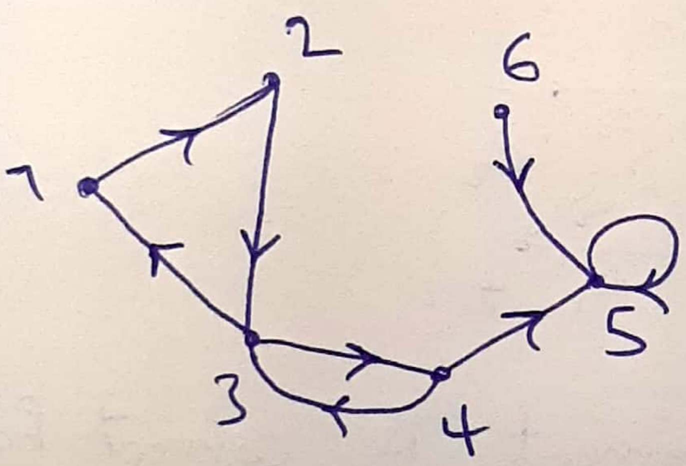
7) consider the following adjacency list of graph G.

1	2
2	3
3	1 → 4
4	3 → 5
5	5
6	5

a. reconstruct G:

b. tell if it's directed/undirected and simple/multi graph.



G is directed and a multi-graph

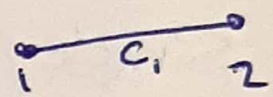
c. find the strong/weak components:

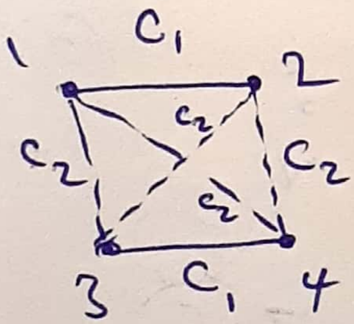
strong components: $\begin{cases} C_1 : \{1,2,3,4\} \\ C_2 : \{5\} \\ C_3 : \{6\} \end{cases}$

weak components: $\{1,2,3,4,5,6\}$

d. check Euler path/circuit: NO Euler path & NO Euler circuit

2) consider a complete graph with 2^n vertices. show we can color its edges with n different colors so that the edges of each triangle in the graph has at least 2 different colors.

→ base case: $n=1$: 

$n=2$: 

assume the statement is correct for $n=k$ ⟹ we can color the edges of a complete graph with 2^k vertices with k different colors so that each triangle has at least 2 diff colors

→ we want to show for $n=k+1$

⟹ put 2 complete graphs with 2^k nodes next to each other & color the edges of each of them with k colors. color the edges between them with color $k+1$. ✓