Full CNF example

COMPSCI 3331

Fall 2022

This is the full result of the example done in class.

Original Grammar

$$
\begin{aligned}
S &\rightarrow ABDE \\
A &\rightarrow BDa \mid D \mid Ba \cdot \\
B &\rightarrow aaE \\
D &\rightarrow \cancel{\varepsilon} \; G \\
E &\rightarrow DD \mid aab \mid D \\
F &\rightarrow DE \; \cancel{} \\
G &\rightarrow aaa
\end{aligned}
$$

Remove ε productions

Which variables are nullable?

- F, D, A, E

Updated grammar: Add all productions that you can get by striking one or more nullable variables from the right-hand side of a production. Also remove all ε productions.

$$
\begin{aligned}
S &\rightarrow ABDE \mid AB \mid BD \mid BE \mid B \mid ABD \mid BDE \mid ABE \\
A &\rightarrow BDa \mid D \mid Ba \\
B &\rightarrow aaE \mid aa \\
D &\rightarrow G \\
E &\rightarrow DD \mid D \mid aab \\
F &\rightarrow DE \mid D \mid E \\
G &\rightarrow aaa
\end{aligned}
$$

Remove Unit Productions

What are the unit pairs?

- $(A,D), (S,B), (D,G), (E,D), (F,D), (F,E), (A,G), (E,G), (F,G)$

Updated grammar. Add productions that simulate using a unit-pair derivation. Also remove all unit productions.

$$
\begin{aligned}
S &\rightarrow ABDE \mid AB \mid BD \mid BE \mid ABD \mid BDE \mid ABE \mid aa \mid aaE \\
A &\rightarrow BDa \mid Ba \mid aaa \\
B &\rightarrow aaE \mid aa \\
D &\rightarrow aaa \\
E &\rightarrow DD \mid aab \mid aaa \\
F &\rightarrow DE \mid aaa \mid DD \mid aab \\
G &\rightarrow aaa
\end{aligned}
$$

Remove useless symbols

What are reachable / generating symbols?

- Reachable: S, A, B, D, E

- Generating S, A, B, D, E, F, G

- Useless F, G.

Updated grammar. Remove all useless variables.

$$
\begin{aligned}
S &\rightarrow ABDE \mid AB \mid BD \mid BE \mid ABD \mid BDE \mid ABE \mid aa \mid aaE \\
A &\rightarrow BDa \mid Ba \mid aaa \\
B &\rightarrow aaE \mid aa \\
D &\rightarrow aaa \\
E &\rightarrow DD \mid aab \mid aaa
\end{aligned}
$$

Introduce letter productions

Updated grammar: Introduce X_a, X_b, X_c and replace all letters in productions.

$$
\begin{aligned}
S &\rightarrow ABDE \mid AB \mid BD \mid BE \mid ABD \mid BDE \mid ABE \mid X_a X_a \mid X_a X_a E \\
A &\rightarrow BD X_a \mid B X_a \mid X_a X_a X_a \\
B &\rightarrow X_a X_a E \mid X_a X_a \\
D &\rightarrow X_a X_a X_a \\
E &\rightarrow DD \mid X_a X_a X_b \mid X_a X_a X_a \\
X_a &\rightarrow a \\
X_b &\rightarrow b
\end{aligned}
$$

CNF

Break long productions. Note that new variables like Y_{ABD} are still just regular nonterminals. **Final grammar** in CNF:

$$
S \rightarrow Y_{ABD}E \mid AB \mid BD \mid BE \mid Y_{AB}D \mid Y_{BD}E \mid Y_{AB}E \mid X_a X_a \mid Y_{aa}E
$$

$$
\begin{aligned}
A &\rightarrow Y_{BD}X_a \mid BX_a \mid Y_{aa}X_a \\
B &\rightarrow Y_{aa}E \mid X_aX_a \\
D &\rightarrow Y_{aa}X_a \\
E &\rightarrow DD \mid Y_{aa}X_b \mid Y_{aa}X_a \\
Y_{ABD} &\rightarrow Y_{AB}D \\
Y_{BD} &\rightarrow BD \\
Y_{AB} &\rightarrow AB \\
Y_{aa} &\rightarrow aa \\
X_a &\rightarrow a \\
X_b &\rightarrow b
\end{aligned}
$$