Assignment 3

COMPSCI 3331

Due: November 22, 2022 at 11:59 PM

General notes:

- Assignments **must** be submitted on gradescope. You must indicate the locations of all answers for questions using gradescope. A video demonstrating how to do this can be found here.
- Assignments can be hand-written or typeset, as long as they are submitted to gradescope as an electronic file (pdf, png or other accepted format). It is your responsibility to submit a file that can be marked (i.e., images of pages are clear and handwriting, if any, can be read).
- Assignments can be submitted up to 48 hours late. A deduction of 1 % (of the total assignment value) will be applied per hour (rounded up) that the assignment is submitted past the deadline.
- You may also use your **once-per-course** 3-day extension on this assignment. Please submit the form on owl to declare that you want to use this extension. (choose "Individual Extension" from the tool menu on owl.) Recall that extensions do not stack – you may either choose the late submission penalty or the individual extension for an assignment, but not both.

S -> aF   F -> bF

~~S -> bF~~   F -> #

$S \to F$ $F \to aF1$

$S \to F1$

(4 marks) 1. Construct a context-free grammar for the language

$$L = \{x\#1^n \; : x \in \{a,b\}^* \text{ and } n-1 \le |x|_a \le n+1\}$$

over the alphabet $\Sigma = \{a,b,1,\#\}$. Give some justification for why your construction is correct.

(4 marks) 2. Convert the following grammar to CNF: generating -> reachable -> useful

gen

$S \to aSa \mid aA \mid B$ $S \to aSa \mid a \mid B \mid$

$A \to E$ $A \to E \mid aA$

$B \to bbCaa \mid bb$ $B \to bbaa \mid bb \mid bbCaa$

~~$C \to DC \mid \varepsilon$~~

~~$D \to bDDa$~~

$E \to aA \mid \varepsilon$ $E \to aA$

$S \to aSa \mid aA \mid B$

$A \to aA \cancel{E}$

$B \to bbaa \mid bb$

$S \to aSa \mid a \mid aaA \mid B$

Show, as part of your work, the equivalent grammar that has no ε-productions, no unit productions and no useless symbols (at a minimum).

(3 marks) 3. For a word $z \in \Sigma^*$, define the operation $\text{out}(z)$ as

$$\text{out}(z) = \{uw \; : \; \exists u,v,w \in \Sigma^* \text{ such that } z = uvw \text{ and } uv \neq \varepsilon\}.$$

$u \neq \varepsilon$, $w \neq \varepsilon$ $v = \varepsilon$.

Note that for each word z, $\text{out}(z)$ is a language that consists of non-empty words that can be obtained from z by deleting a subword from the interior of z (including possibly none of the characters of z). For example, $\text{out}(abacca)$ contains words such as $abcca$, aca, $abaa$ and $abacca$. The empty word ε is never in $\text{out}(z)$ by definition.

Let G be a fixed context free grammar in CNF. For an input word z of length n, give an $O(n^3)$ time algorithm to determine if there are any words in $\text{out}(z) \cap L(G)$. Your algorithm should return a Boolean value to answer this question. You must give a justification that the runtime of your algorithm is correct and that the algorithm gives the correct answer.

$L(G) = $ CYK.

(4 marks) 4. For a word $w \in \{0,1\}^*$ let $\text{bin}(w)$ be the value of w when interpreted as a binary number with the most significant digit first. For example, $\text{bin}(1010) = 10$. Construct a PDA for the language

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$$L = \{u\#v \; : \; u,v \in \{0,1\}^* \text{ and } \text{bin}(u) = \text{bin}(v^R) \text{ or } \text{bin}(v^R) = \text{bin}(u)+1\}$$

over the alphabet $\{0,1,\#\}$. For example, $10\#01, 11\#001 \in L$. In this question you may assume that all binary numbers have 1 as their most significant digit. Give a justification for why your PDA is correct.

number of remove

position of remove

```
Boolean b=1;
position=1
for i=0 to |out(z)|-1:   ∀ 讨 —
    Boolean match=0
    for j=position to |z|-1:
        if out(z)[i] == z[j]
```

position = j + 1
match = 1

b ← b && match

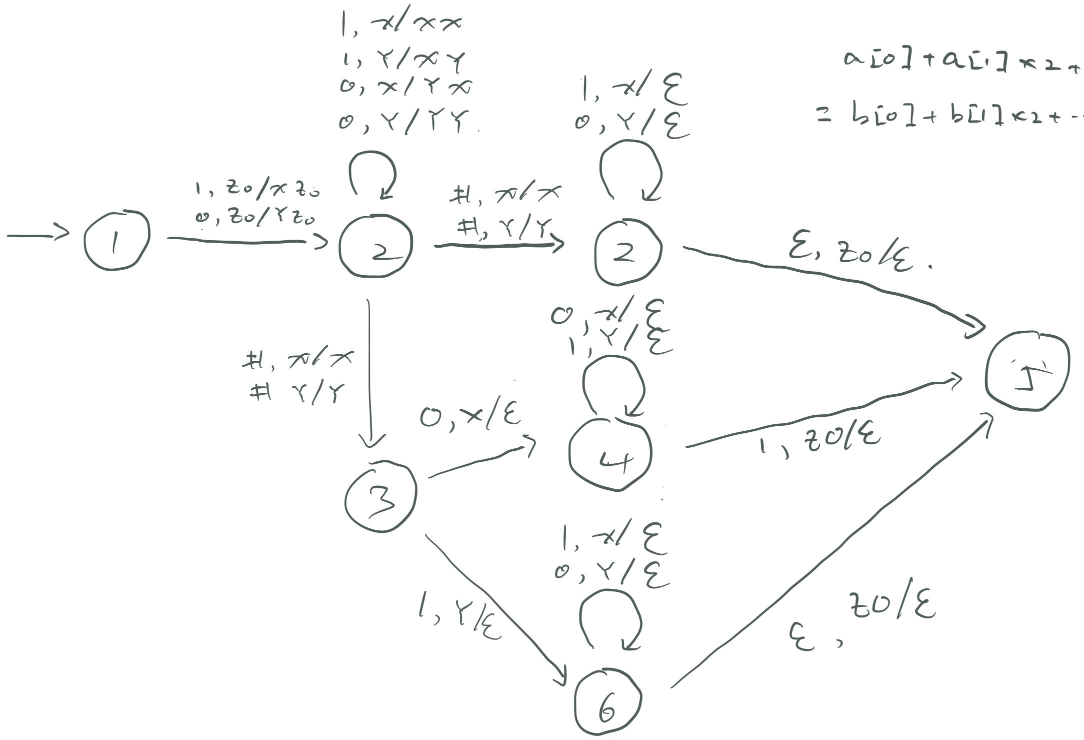
(4 marks) 4. For a word $w \in \{0,1\}^*$ let $\text{bin}(w)$ be the value of w when interpreted as a binary number with the most significant digit first. For example, $\text{bin}(1010) = 10$. Construct a PDA for the language

no leading zero. $2+8=10.$

$$L = \{u\#v \;:\; u, v \in \{0,1\}^* \text{ and } \text{bin}(u) = \text{bin}(v^R) \text{ or } \text{bin}(v^R) = \text{bin}(u) + 1\}$$

$2=2$ $3+1=4.$

over the alphabet $\{0,1,\#\}$. For example, $10\#01, 11\#001 \in L$. In this question you may assume that all binary numbers have 1 as their most significant digit. Give a justification for why your PDA is correct.



a[0]+a[1]×2+.....+1,
= b[0]+b[1]×2+.....

1 for x

0 for Y.

i.e. 10

check last
×
1 : revert all & +1
0 : add 1.

Y.

$\Rightarrow$ Y Y.