Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 1229A Final Examination

Wednesday, December 12, 2018 **Code 111** 9:00 a.m. - 12:00 noon

INSTRUCTIONS

1. Fill in the top of this page, **and the next page**, completely.
2. Fill in the top of the scantron card completely. **You MUST both print AND code** your Student Number, Section Number (below) and Exam Code (above).
3. DO NOT UNSTAPLE THE BOOKLET. The two blank pages at the back of the booklet may be torn off and used for rough work. Do not tear any other pages out of the booklet.
4. CALCULATORS AND NOTES ARE NOT PERMITTED.
5. There are two parts to this examination: PART A (35 marks) in multiple choice format and PART B (15 marks) in show your work format.
6. In Part A, **circle** the correct answer to each question **on this paper** AND fill in the appropriate box on the **scantron** card with an HB pencil.
7. In Part B, show all your work in the space provided.
8. Questions are printed on both sides of the paper, they begin on Page 1 and continue to Page 9. Be sure that your booklet is complete.
9. You must hand in this question paper, your scantron card, and all rough work sheets.
10. Circle your section in the list below.

Instructor	Campus/College	Time	Section
Lindsey	Main	9:30 MWF	001
Pasini	Main	12:30 MWF	002
Olds	Main	1:30 MWF	003
Pasini	Main	8:30 MWF	004
Ghorbanpour	Brescia	8:30 MTuTh	530
O'Hara	Brescia	9:30 MTuW	531
Rastegari	Huron	11:30 MWF	550
Mollahajiaghaei	Huron	8:30 Tu	551
Kuzmin	King's	10:30 Tu, 9:30 Th	570
Turnbull	King's	1:30 Tu, 12:30 Th	571
Turnbull	King's	1:30 M, 12:30 W	572
Kuzmin	King's	7:00 MW	573
Kuzmin	King's	8:30 MW	574

11. TOTAL MARKS = 50.

Student Number (**Print**)

Student's Name (**Print**)

FOR GRADING ONLY

PAGE	MARK
1–5	
6	
7	
8	
9	
TOTAL	

PART A (35 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE INDICATED ON THE SCANTRON SHEET. YOU SHOULD ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

1 mark A

1. Find $\|\mathbf{u}\|$ where $\mathbf{u} = (1, \sqrt{3}, -1, 2, -4)$. 1+3+1+4+16.

A: 5	B: $2\sqrt{3}$	C: $3\sqrt{3}$	D: $\sqrt{31}$	E: 1

1 mark E.

2. For what value(s) of k are the vectors $\mathbf{u} = (1, 0, -1, 2)$ and $\mathbf{v} = (k, 0, 3, k)$ collinear?

A: $k = 1$ only	B: $k = 1$ or $k = 2$	C: $k = -3$ only	D: all values of k	E: no value of k

$k-3+2k=0$ $k=1$.

1 mark A.

3. For what value(s) of k are the vectors $\mathbf{u} = (1, 0, -1, 2)$ and $\mathbf{v} = (k, 0, 3, k)$ orthogonal?

A: $k = 1$ only	B: $k = 0$ only	C: $k = -\frac{3}{5}$ only	D: all values of k	E: no value of k

$\times 2\ (1,-3,1)\cdot(-2,-4,-3)$. $2-12+3=-7$.

1 mark C.

4. Consider the vectors $\mathbf{u} = \mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = -2\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$ in $\Re^3$. Find, if possible, $-2\mathbf{u} \bullet (\mathbf{v} + 3\mathbf{k})$.

$u=(1,-3,1)$ $v=(-2,-4,-6)$.

A: -5	B: 5	C: -14	D: $(4, -24, 6)$	E: the operation is not defined

E number. $-1+6-3-4$.

1 mark D.

5. Let $\mathbf{u} = (1, -3, 1, 1)$, $\mathbf{v} = (-1, -2, -3, -4)$ and $\mathbf{w} = (-2, -1, -2, 0)$. Find, if possible, $(\mathbf{u} \bullet \mathbf{v}) \bullet \mathbf{w}$. -2.

A: 2	B: -2	C: $(2, -6, 6, 0)$	D: $(4, 2, 4, 0)$	E: the operation is not defined

$\cos\theta = \frac{8}{\sqrt{6}\sqrt{18}} = \frac{3}{2\sqrt{3}}$. $\frac{\sqrt{3}}{2}$. $\vec{AB}=(2,1,1)$ $\sqrt{6}$. $\vec{AC}=(3,3,0)$ $\sqrt{18}$.

1 mark A.

6. Find the area of parallelogram $ABCD$ with vertices $A(1,1,1)$, $B(3,2,2)$, $C(4,4,1)$ and $D(2,3,0)$. $S=ab\sin c$. $\frac{1}{2}$. $\vec{AB}=(2,1,2)$ $\vec{DC}=(2,1,2)$.

A: $\sqrt{27}$	B: $2\sqrt{3}$	C: 9	D: 27	E: 3

$\frac{1}{2}\cdot\sqrt{6}\cdot\sqrt{18}$. $\sqrt{27}$.

1 mark D.

7. Find the distance from the point $P(2, 0, -1)$ to the plane $2x - 5y - z = 5$.

A: $\frac{5}{\sqrt{30}}$	B: $\frac{2}{\sqrt{3}}$	C: $\frac{2}{3}$	D: 0	E: 1

$\frac{4+1-5}{\sqrt{4+25+1}}$. $\vec{PQ}=(6,2)$.

1 mark C. AB.

8. Which one of the following is an equation of the line through $P(2, 4)$ and $Q(-4, 2)$?

A: $(6, 2) \bullet ((x, y) - (2, 4)) = 0$	B: $(x, y) = (1 - t)(2, 4) + t(6, 2)$	C: $3y - x = 10$
D: $(x, y) = (2, 4) + t(\cancel{-}4, 2)$	E: $2x - 6\cancel{y} = 0$	

1 mark D.

9. Which one of the following is a point-normal form equation of the plane in $\Re^3$ which contains the lines $(x, y, z) = (1, 1, 2) + t(-2, 1, 4)$ and $(x, y, z) = (1, 1, 2) + r(0, 1, 0)$?

A: $(-4, 0, -2) \bullet ((x, y, z) - (-2, 0, 4)) = 0$	B: $(-2, 0, 4) \bullet ((x, y, z) - (1, 1, 2)) = 0$
C: $(x, y, z) = (1, 1, 2) \bullet (-2, 0, 4)$	D: $(4, 0, 2) \bullet ((x, y, z) - (1, 1, 2)) = 0$
E: there is no such plane	

$(-2,1,4)\cdot(0,1,0)$.

1 mark 10. Which of the matrices shown below are in row-reduced echelon form?

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

B.

A: A and B only	B: C and D only	C: B and C only
D: B, C and D	E: A, B and C	

1 mark 11. Which one of the following is an augmented matrix for a system of linear equations which is *inconsistent*?

C.

A: $\left[\begin{array}{ccc\|c} 1 & 5 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$	B: $\left[\begin{array}{ccc\|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$	C: $\left[\begin{array}{ccc\|c} 1 & 5 & 0 & 1 \\ 0 & 0 & 0 & 5 \\ 0 & 4 & -3 & 4 \end{array}\right]$
D: $\left[\begin{array}{cc\|c} 1 & 5 & 0 \\ 0 & 0 & 0 \\ 0 & 4 & -3 \end{array}\right]$	E: $\left[\begin{array}{ccc\|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$	

1 mark 12. How many solutions does the system of linear equations corresponding to the augmented matrix shown below have?

$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

2 . 5

E.

A: no solutions	B: exactly one solution
C: a 1-parameter family of solutions	D: a 2-parameter family of solutions
E: a 3-parameter family of solutions	

$x_4 = 6$. $x_3 = 6$.

1 mark 13. Consider the following hyperplanes in $\Re^4$: $3x_1 - 2x_2 - x_4 = 4$, $x_3 - x_4 = 0$ and $x_3 = 6$. Which one of the following describes the intersection of these 3 hyperplanes?

D.

A: no points	B: exactly 1 point	C: exactly 5 points	D: a line	E: a plane

Use the following information for questions 14, 15 and 16.

Let $A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & -5 \\ -1 & 4 & 1 \end{bmatrix}$.

1 mark 14. Find $2A + B$. $2A = \begin{bmatrix} -2 & 0 & 2 \\ 4 & 4 & 8 \end{bmatrix} +$ $\begin{matrix} 1 & 0 & -3 \\ 3 & 8 & 9 \end{matrix}$.

A.

A: $\begin{bmatrix} 1 & 0 & -3 \\ 3 & 8 & 9 \end{bmatrix}$	B: $\begin{bmatrix} 2 & 0 & -4 \\ 1 & 6 & 5 \end{bmatrix}$	C: $\begin{bmatrix} 4 & 0 & -8 \\ 2 & 12 & 10 \end{bmatrix}$	D: $\begin{bmatrix} 1 & 0 & -3 \\ 1 & 6 & 5 \end{bmatrix}$	E: not defined

$\begin{matrix} 3 & 0 & -5 \\ -1 & 4 & 1 \end{matrix}$ $\begin{array}{c|c} -1 & 2 \\ 0 & 2 \\ 1 & 4 \end{array}$ $\begin{bmatrix} -8 & -14 \\ 2 & 10 \end{bmatrix}$ $\begin{bmatrix} -8 & 2 \\ -14 & 10 \end{bmatrix}$.

$-2+8+4$.

1 mark 15. Find $(BA^T)^T - AB^T$.

B.

A: $\begin{bmatrix} 0 & -16 \\ -16 & 0 \end{bmatrix}$	B: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	C: $\begin{bmatrix} -3 & -2 \\ 0 & 8 \\ -5 & 4 \end{bmatrix}$	D: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	E: not defined

$\begin{bmatrix} -1 & 0 & 1 \\ 2 & 2 & 4 \end{bmatrix} \left[\begin{array}{c|c} 3 & -1 \\ 0 & 4 \\ -5 & 1 \end{array}\right] - \begin{matrix} -8 & 2 \\ -14 & 10 \end{matrix}$

1 mark 16. Find A^3. *Recall:* $A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 2 & 4 \end{bmatrix}$

E.

A: $\begin{bmatrix} -1 & 0 & 1 \\ 8 & 8 & 64 \end{bmatrix}$	B: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	C: $\begin{bmatrix} 2 & 4 & 10 \\ 46 & 48 & 98 \end{bmatrix}$	D: $\begin{bmatrix} 5 & 4 & 7 \\ 4 & 4 & 8 \\ 7 & 8 & 17 \end{bmatrix}$	E: not defined

$\begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix}$ $A^2 = \begin{bmatrix} -2 & 1 \\ -3 & 1 \end{bmatrix}.$ $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

1 mark 17. If $A = \begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$, find A^2B. $= \begin{bmatrix} 1 & 4 \\ 1 & 5 \end{bmatrix}.$

B.

A: $\begin{bmatrix} -2 & 1 \\ -3 & 1 \end{bmatrix}$	B: $\begin{bmatrix} 1 & 4 \\ 1 & 5 \end{bmatrix}$	C: $\begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$	D: $\begin{bmatrix} 3 & -1 \\ -8 & 3 \end{bmatrix}$	E: $\begin{bmatrix} 2 & -3 \\ -5 & 4 \end{bmatrix}$

1 mark 18. Consider the system of linear equations $A\mathbf{x} = \mathbf{b}$, where A is a 4×4 matrix whose row-reduced echelon form contains exactly 3 leading ones. Which one of the following statements is **true**?

~~A.~~ B

$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

A: A is invertible.
B: A has no inverse.
C: The system cannot be consistent.
D: The system cannot have infinitely many solutions.
E: If the system is homogeneous, it has only the trivial solution.

det A=0 <=> A is not invertible

<=> A has no inverse.

1 mark 19. Find the rank of $\begin{bmatrix} -3 & 6 & 3 & -9 \\ 2 & -4 & -2 & 6 \\ 1 & -2 & -1 & 3 \end{bmatrix}$.

A.

A: 1	B: 2	C: 3	D: 4	E: not defined, since A is not a square matrix

rank. 1 2. 3.

1 mark 20. Let $A\mathbf{x} = \mathbf{b}$ be a system of 2 linear equations in 3 unknowns. Which one of the following statements **MUST** be **false**?

D.

A: The system might have a one-parameter family of solutions.
B: The system might have a two-parameter family of solutions.
C: The system might have no solution.
D: The system might have a unique solution.
E: None of A, B, C or D.

r(A)≠r([A|B]) => no solution.

1 mark 21. Let A be a 5×3 matrix and let $A\mathbf{x} = \mathbf{b}$ be a system of linear equations. If the rank of A is 2 and the rank of the augmented matrix $[A \mid \mathbf{b}]$ is 3, which one of the following describes the solution(s) to this system? 3个式子.

~~B~~ E.

A: a unique solution	B: a 1-parameter family of solutions
C: a 2-parameter family of solutions	D: a 3-parameter family of solutions
E: no solution	

$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}.$ $\left[\begin{array}{cc|c} a & b & m \\ c & d & n \\ e & f & p \end{array}\right]$

1 mark 22. Let $A\mathbf{x} = \mathbf{0}$ be a homogeneous system of n linear equations in n unknowns. If this system has a unique solution, which one of the following statements is **false**?

B

D C

A: The rank of A is n.
B: The row-reduced echelon form of A is an identity matrix.
C: The determinant of A is 0.
D: The unique solution is $\mathbf{x} = \mathbf{0}$.
E: A is invertible.

not.

1 mark 23. Find $\det \begin{bmatrix} -2 & -4 \\ 1 & 3 \end{bmatrix}$.

$-6+4.$

D

A: 2	B: -10	C: 10	D: -2	E: 0

1 mark 24. Find the 3,2-cofactor of the matrix $\begin{bmatrix} 0 & -4 & 1 \\ -3 & 1 & -1 \\ 2 & 2 & 5 \end{bmatrix}$.

$-(0+3).$
$-3.$

A

A: -3	B: 3	C: 8	D: -8	E: 6

1 mark 25. Find $\det \begin{bmatrix} 2 & -1 & -4 & 0 \\ 0 & 2 & 3 & 0 \\ 4 & -3 & 9 & 2 \\ 0 & 1 & -1 & 0 \end{bmatrix}$.

$-2\det\begin{bmatrix} 2 & -1 & -4 \\ 0 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}.$

$= -4\det\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$ $\quad -2-3=-5.$

B

A: -10	B: 20	C: 10	D: -20	E: 0

1 mark 26. If A and B are two 2×2 matrices with $\det A = 2$ and $\det B = -1$, find $\det\left(3A^TB^2\right)$.

A

A: 18	B: -6	C: $\frac{1}{6}$	D: 6	E: $\frac{1}{18}$

$9\det(ABB).$
$9\times 2.$

1 mark 27. Let $A = \begin{bmatrix} 3 & -2 & -4 \\ 0 & 5 & 3 \\ 0 & 0 & -1 \end{bmatrix}$ and let I be the 3×3 identity matrix. Find $\det(A + 2I)$.

C

A: -15	B: 0	C: 35	D: -7	E: -13

1 mark 28. Find $\det \begin{bmatrix} 2 & 4 & 8 & 16 \\ 7 & 5 & 3 & 0 \\ 4 & 8 & 16 & 32 \\ 0 & 2 & -1 & 9 \end{bmatrix}$.

E

A: 16	B: 8	C: 4	D: 2	E: 0

B.

1 mark 29. Find det $\begin{bmatrix} 4 & 5 & 7 & -6 \\ 0 & 0 & 3 & 5 \\ 0 & 1 & 9 & 8 \\ 0 & 0 & 0 & 2 \end{bmatrix}$.

12

A: -12	B: -24	C: 24	D: 12	E: 0

Use the following information for questions 30, 31 and 32.

It is known that det $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} = 6$.

D.

1 mark 30. Find det $\begin{bmatrix} 2a & b & c \\ 2d & e & f \\ -2g & -h & -k \end{bmatrix}$.

×2. ×−1.

A: -2	B: 12	C: 2	D: -12	E: 6

C.

1 mark 31. Find det $\begin{bmatrix} c & b & a \\ f & e & d \\ k-3c & h-3b & g-3a \end{bmatrix}$.

A: 0	B: 6	C: -6	D: 18	E: -18

E.

1 mark 32. Find det $\begin{bmatrix} d & -3a & g \\ e & -3b & h \\ f & -3c & k \end{bmatrix}$.

A: $-\frac{1}{2}$	B: -2	C: 2	D: -18	E: 18

A.

1 mark 33. Find the value of k for which $A = \begin{bmatrix} 0 & k & -1 \\ 1 & 3 & 0 \\ k & 4 & 2 \end{bmatrix}$ has no inverse.

$-1(4-3k) - k \cdot 2 = 0$

$k = 4$.

$-4+k=0$.

A: $k=4$	B: $k=\frac{4}{5}$	C: $k=-4$	D: $k=-\frac{4}{5}$	E: $k=0$

C.

1 mark 34. If A is a 3×3 matrix with det $A = -3$, find det(Adj A).

$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$.

A: -3	B: -27	C: 9	D: $\frac{1}{9}$	E: $-\frac{1}{3}$

B.

1 mark 35. If it is known that det $\begin{bmatrix} a & 2 \\ b & 3 \end{bmatrix} = 1$ and det $\begin{bmatrix} 4 & a \\ 5 & b \end{bmatrix} = 4$, find the unique solution to the system of linear equations

$$\begin{aligned} 4x &+ 2y &= a \\ 5x &+ 3y &= b \end{aligned}$$

$3a-2b=1$. $36-4b=2$.

$4b-5a=4$. $4b-30=4$.

$\boxed{a=6 \quad b=\frac{17}{2}}$

A: $x=4, y=1$	B: $x=\frac{1}{2}, y=2$	C: $x=2, y=\frac{1}{2}$	D: $x=1, y=4$	E: $x=2, y=2$

$6+6y$.

$4x+2y=6$ $12x+6y=18$

$5x + 3y = \frac{17}{2}$. $10x + 6y = 17$.

$x = \frac{1}{2}$.

$y = 2$.

PART B (15 marks)

SHOW YOUR WORK FOR ALL QUESTIONS IN PART B.

NOTE: NO MARKS WILL BE GIVEN if a particular method is specified in a question or question part and you DO NOT use the specified method.

3 marks 36. Consider the lines:

$$\ell_1: \quad (x_1, x_2, x_3, x_4) = (1, 0, 1, 4) + r(1, 3, 0, -2)$$
$$\ell_2: \quad (x_1, x_2, x_3, x_4) = (2, 1, 1, 2) + t(-1, -2, 0, 2)$$

Find the intersection, if any, of ℓ_1 and ℓ_2.

$1 + r = 2 - t$. $2r = 2 - 2t$.

$3r = 1 - 2t$. $3r = 1 - 2t$.

$1 = 1$

$4 - 2r = 2 + 2t$.

$\boxed{r = -1. \quad t = 2.}$

$P \ (0, -3, 1, 6)$.

4 marks 37. Consider the system of linear equations

$$x_3 + 3x_4 + 2 = 0$$
$$3x_2 = x_1 - 2x_4 - 5$$
$$x_1 - 3x_2 - x_3 - 2x_4 = 1$$

(a) Write the augmented matrix for the standard form of this system.

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 3 & -2 \\ -1 & 3 & 0 & 2 & -5 \\ 1 & -3 & -1 & -2 & 1 \end{array}\right].$$

(b) Use your augmented matrix from part (a) to solve the system **using Gauss-Jordan elimination**.

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 3 & -2 \\ -1 & 3 & 0 & 2 & -5 \\ 1 & -3 & -1 & -2 & 1 \end{array}\right] \xrightarrow{r_2 = r_2 + r_3} \left[\begin{array}{cccc|c} 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & -1 & 0 & -4 \\ 1 & -3 & -1 & -2 & 1 \end{array}\right].$$

$$\xrightarrow{r_2 = -r_2} \left[\begin{array}{cccc|c} 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 \\ 1 & -3 & -1 & -2 & 1 \end{array}\right] \xrightarrow[r_3 = r_3 + r_2]{r_1 = \frac{1}{3}(r_1 - r_2)} \left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 4 \\ 1 & -3 & 0 & -2 & 5 \end{array}\right].$$

$$\xrightarrow{r_3 = r_3 + 2r_1} \left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 4 \\ 1 & -3 & 0 & 0 & 1 \end{array}\right].$$

2 marks 38. Let A be a 3×3 matrix with $\det A = 2$ and $\text{Adj } A = \begin{bmatrix} -2 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 4 & -1 \end{bmatrix}$.

(a) Find A^{-1}.

$A^{-1} = \frac{1}{\det A} \text{Adj} A$. $\begin{bmatrix} -1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{1}{2} & 2 & -\frac{1}{2} \end{bmatrix}$.

(b) Find the solution to $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 8 \end{bmatrix}$.

~~$\vec{x} = A^{-1}\vec{b}$.~~

$2x = 0$ $x = 0$.

$2y = 1$ $y = \frac{1}{2}$.

$2z = 8$ $z = 4$.

$A^{-1} = \frac{1}{\det A} \text{Adj} A$.

$A^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{1}{2} & 2 & -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 8 \end{bmatrix}$.

0
1
-2.

$x = 0$
$y = 1$
$z = -2$.

3 marks 39. Let $A = \begin{bmatrix} 4 & -1 & 5 \\ a & b & c \\ d & e & f \end{bmatrix}$ where it is known that

$$\det \begin{bmatrix} a & b \\ d & e \end{bmatrix} = 10, \quad \det \begin{bmatrix} a & c \\ d & f \end{bmatrix} = -2, \quad \det \begin{bmatrix} b & c \\ e & f \end{bmatrix} = 3$$

(a) Find $\det A$.

$\det A = 4 \times 3 - 1 \times 2 + 5 \times 10$.

$= 60$.

(b) Find the (2,1)-entry of Adj A.

$A^T = \begin{bmatrix} 4 & a & d \\ -1 & b & e \\ 5 & c & f \end{bmatrix}$. $\text{Adj} A_{(2,1)} = (-1)^{2+1} \det A_{(2,1)}$.

$= 2$.

3 marks 40. Let $A = \begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & -4 \\ 0 & 1 & 2 \end{bmatrix}$.

(a) Find $\det A$.

$\det A = -(-4-2) + 2(-1+2).$

$= 6 + 2$

$= 8$

(b) **Use Cramer's Rule** to find the value of z in the unique solution to $A\mathbf{x} = \mathbf{b}$ where $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$.

SHOW YOUR WORK. *Note:* You may want to use your answer to part (a).

$\begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & -4 \\ 0 & 1 & 2 \end{bmatrix}.$ $\det A = 8.$

$\begin{bmatrix} 4 & 1 & -1 \\ 0 & -1 & -4 \\ 0 & 1 & 2 \end{bmatrix}.$ $\det A_i = 8.$

$x + y - z = \cancel{4}.$

$-2x - y - 4z = 0.$

$y + 2z = 0.$

$x = 1 \quad y = -2 \quad z = 1.$

$\begin{bmatrix} 1 & 4 & -1 \\ -2 & 0 & -4 \\ 0 & 0 & 2 \end{bmatrix}$ $\det A_j = \cancel{4} -16.$

$\begin{bmatrix} 1 & 1 & 4 \\ -2 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $\det A_k = 8.$