WESTERN UNIVERSITY
Department of Applied Mathematics

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Calculus 2402A
ASSIGNMENT 3B
Release: Wednesday September 30, 2020
Due: Thursday October 8, 2020 at 11:59 PM

Show all your work, unjustified answers will receive little or no credit.

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1. **[5 marks]** Given $f(x,y,z) = 4xyz - x^4 - y^4 - z^4$, find the critical points and classify them.

$$\begin{aligned} f_x &= 4yz - 4x^3 = 0 \\ f_y &= 4xz - 4y^3 = 0 \\ f_z &= 4yx - 4z^3 = 0 \end{aligned} \Rightarrow \begin{cases} x=0 \\ y=0 \\ z=0 \end{cases} \text{ or } \begin{cases} x=1 \\ y=1 \\ z=1 \end{cases}$$

the critical points are $(0,0,0)$ and $(1,1,1)$.

$f_{xx} = -12x^2$ $f_{xy} = 4z$ $f_{xz} = 4y$

$f_{yx} = 4z$ $f_{yy} = -12y^2$ $f_{yz} = 4x$

$f_{zx} = 4y$ $f_{zy} = 4x$ $f_{zz} = -12z^2$.

$$H(0,0,0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \det H(0,0,0) = 0.$$

$$H(1,1,1) = \begin{bmatrix} -12 & 4 & 4 \\ 4 & -12 & 4 \\ 4 & 4 & -12 \end{bmatrix} \quad \det H(1,1,1) = -1024 < 0$$

$\therefore (0,0,0)$ needs further dismssion,
$(1,1,1)$ is negative defined and the maximum value is 1

2. [**5 marks**] Given $f(x,y,z) = xy + x^2z - x^2 - y - z^2$, find the critical points and classify them.

$$f_x = y + 2xz - 2x = 0$$
$$f_y = x - 1 = 0 \Rightarrow \begin{cases} x = 1 \\ y = 1 \\ z = \frac{1}{2} \end{cases}$$
$$f_z = x^2 - 2z = 0$$

the only critical point is $(1, 1, \frac{1}{2})$.

$$f_{xx} = 2z - 2 \qquad f_{xy} = 1 \qquad f_{xz} = 2x.$$
$$f_{yx} = 1 \qquad f_{yy} = 0 \qquad f_{yz} = 0$$
$$f_{zx} = 2x \qquad f_{zy} = 0 \qquad f_{zz} = 2.$$

$$H_{(1,1,\frac{1}{2})} = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix}. \qquad \det H_{(1,1,\frac{1}{2})} = -2 < 0.$$

$\therefore$ $(1, 1, \frac{1}{2})$ is negative defined and the maximum value is $-\frac{3}{4}$

3. **[5 marks]** Find the third degree Taylor polynomial of $f(x,y) = \dfrac{1}{2+x-2y}$ near the point $(2,1)$.

$a=2 \quad b=1 \quad h=x \quad k=y.$

$f(x,y) = (2+x-2y)^{-1}$

$= f(2,1) + \frac{1}{1!}\left[(x-2)f_x(2,1) + (y-1)f_y(2,1)\right]^1 + \frac{1}{2!}\left[(x-2)f_{xx}(2,1) + (y-1)f_{yy}(2,1)\right]^2 .$

$+ \frac{1}{3!}\left[(x-2)f_{xxx}(2,1) + (y-1)f_{yyy}(2,1)\right]^3 .$

$f_x = -(2+x-2y)^{-2} \quad f_{xx} = 2(2+x-2y)^{-3} \quad f_{xxx} = -6(2+x-2y)^{-4}$

$= -\frac{1}{4} \quad = \frac{1}{4} . \quad = -\frac{3}{8}$

$f_y = 2(2+x-2y)^{-2} . \quad f_{yy} = 8(2+x-2y)^{-3} \quad f_{yyy} = 48(2+x-2y)^{-4}$

$= -\frac{1}{2} \quad = 1 \quad = 3 .$

$\therefore f(x,y) = \frac{1}{2} - \frac{1}{4}(x-2) - \frac{1}{2}(y-1) + \frac{1}{2}\left[\frac{1}{4}(x-2) + (y-1)\right]^2 + \frac{1}{6}\left[-\frac{3}{8}(x-2) + 3(y-1)\right]^3 .$

4. **[5 marks]** Find the third degree Taylor polynomial of $f(x, y) = \ln(x^2 + y^2)$ near the point $(1, 0)$.

$a=1 \quad b=0 \quad h=x \quad k=y.$

$f(1,0) = 0$

$f(x,y) = \ln(x^2+y^2)$

$= f(1,0) + \frac{1}{1!}\left[(x-1)f_x(1,0) + y f_y(1,0)\right]^1 + \frac{1}{2!}\left[(x-1)f_{xx}(1,0) + y f_{yy}(1,0)\right]^2 .$

$+ \frac{1}{3!}\left[(x-1)f_{xxx}(1,0) + y f_{yyy}(1,0)\right]^3 .$

$f_x = \frac{2x}{(x^2+y^2)} \qquad f_{xx} = \frac{2(x^2+y^2) - 2x^2}{(x^2+y^2)^2} = \frac{2y^2}{(x^2+y^2)^2} \qquad f_{xxx} = -\frac{8xy^2}{(x^2+y^2)^3}$

$= 2 \qquad = 0 \qquad = 0.$

$f_y = \frac{2y}{x^2+y^2} \qquad f_{yy} = \frac{2x^2}{(x^2+y^2)^2} \qquad f_{yyy} = -\frac{8x^2y}{(x^2+y^2)^3}$

$= 0 \qquad = 2 \qquad = 0.$

$f(x,y) = 0 + 2(x-1) + \frac{1}{2} \cdot 2y$

$= 2x + y - 2.$