§1.3. Free & Bound variables.

$y \in \{x \in \mathbb{R} \mid x^2 \leq 4\}$.

free variable bound variable.

Free variable: - the truth value depends on which.
- you can substitute a value for which.

$\Rightarrow P(y)$

bound variable: you cannot choose the value.
(dummy variable) the variable could be replaced by the other one without changing the meaning.

A variable is free unless it is bound by:

1) $\{x \in U \mid \cdots\}$ 2) $\sum_{x=a}^{b} \cdots$

e.g. $a+b \in \{x \in \mathbb{N} \mid x+y \text{ is even}\}$.

* in this case, the only bound variable is x.
  a, b, y, are free variables.

$c \in \{x \in \mathbb{N} \mid 3x+y \in \{z \mid z \text{ is even}\}\}$.

bound variables: x, z

free variables: c, y.

$\sum_{i=a}^{b} i^k < l$

bound variable: i

free variable: a, b, k, l.

§1.4 Operation on Sets.

two sets A, B.

Their intersection $A \cap B = \{x \mid x \in A \wedge x \in B\}$.

union $A \cup B = \{x \mid x \in A \vee x \in B\}$.

the difference $A \setminus B = \{x \mid x \in A \wedge \neg(x \in B)\}$.

if $A = \{x \mid P(x)\}$ $B = \{x \mid Q(x)\}$.

then $A \cap B = \{x \mid P(x) \wedge Q(x)\}$.

$A \cup B = \{x \mid P(x) \vee Q(x)\}$.

$A \setminus B = \{x \mid P(x) \wedge \neg Q(x)\}$.

if u is a universe

d is a discourse

$u \setminus B = \{x \in u \mid x \notin B\}$.

$= \{x \mid x \notin B\}$.

e.g. $A = \{1, 2, 3\}$. $B = \{2, 4, 6\}$.

$A \cap B = \{2\}$.

$A \cup B = \{1, 2, 3, 4, 6\}$.

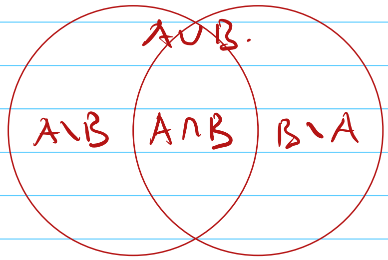
$A \setminus B = \{1, 3\}$.

$(A \cup B) \setminus (A \cap B) = \{1, 3, 4, 6\}$.

$(A \setminus B) \cup (B \setminus A) = \{1, 3, 4, 6\}$ <= symmetric difference.

$A \triangle B$.

in Venn diagram:



to prove $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$.

we will show they have same elements.

Assume that $x \in (A \cup B) \setminus (A \cap B)$

iff $(x \in (A \cup B)) \wedge \neg (x \in (A \cap B))$

iff $((x \in A) \vee (x \in B)) \wedge \neg((x \in A) \wedge (x \in B))$

iff $((x \in A) \vee (x \in B)) \wedge (\neg (x \in A) \vee \neg (x \in B))$

iff $(x \in A \wedge x \notin A) \vee (x \in B \wedge x \notin A) \vee (x \in B \wedge x \notin B) \vee (x \in A \wedge x \notin B)$

iff $(x \in B \wedge x \notin A) \vee (x \notin B \wedge x \in A)$

iff $x \in (A \setminus B) \vee x \in (B \setminus A)$