Q1.

(a) = (b): Suppose $C \subseteq A \cup B$, and

$\rightarrow$: Assume an arbitrary a that $a \in (C \setminus A)$. Since $a \in C \setminus A$, $a \in C$ and $a \notin A$. Since $C \subseteq A \cup B$, $a \in A \cup B$. Because $a \notin A$, $a \in B$. So $C \subseteq A \cup B$ implies $(C \setminus A) \subseteq B$. $\square$

avoid the symbols, use "implies" instead.

$\leftarrow$: Given that $(C \setminus A) \subseteq B$, any random a that $a \in (C \setminus A)$ also in B. So we can have $a \in (C \setminus A) \rightarrow a \in B$, which would be rewriten as $\neg(a \in C \wedge a \notin A) \vee a \in B$, and it could be simplified as $a \notin C \vee a \in A \vee a \in B$, so $a \notin C \vee (a \in A \vee a \in B)$, then $a \in C \rightarrow (a \in A \vee a \in B)$, which is equal to $a \in C \rightarrow a \in (A \cup B)$. Since a is random, $(C \setminus A) \subseteq B$ implies $C \subseteq A \cup B$

From the proof above, we can have (a) = (b) $\square$

(a) = (c) is similar to the proof of (a) = (b) $\Leftarrow$ Must include every proof!

Thus, we can have (b) = (a) = (c), so they are equivalent. $\square$

* To show a=b=c, we have to proof that a->b, b->c, c->a.

Q_2.

Statement 1 is true.

Proof: Existence: let $x=3$, the left-hand side of the equation is $(3)^2 - 6 \times 3 = -9$, which is equal to the right-hand side. So $x=3$ can be one solution.

Uniqueness: Assume $z^2 - 6z = -9$. then it could be rewritten as $z^2 - 6z + 9 = 0$, which is $(z-3)^2 = 0$. The only solution in this equation is $z=3$, so $z=x$. Thus, $x=3$ is the only solution. $\square$

Statement 2 is false.

Proof: Existence: Assume there exist x_0 that $x_0^2-6x_0+1<0$,

this would be rewritten as $(x_0-3)^2+8<0$,

it would be better if a specific result is picked. i.e. $x=1$

Since $(x_0-3)^2 \geq 0$, $(x_0-3)^2+8 \geq 8$, so it cannot be negative and x_0 does not exist.

Thus, statement 2 is false. □

Q3.

Still need to show the uniqueness part. i.e. pick two specific value and says the result is not unique.

Theorem 1: Existence: Let $y=x+1$. The left-hand side would be $\frac{x+1-1}{x}=1$. Right-hand side would be $x+1-x=1$,

Then, we have $\frac{y-1}{x}=\frac{x+1-1}{x}=\frac{x}{x}=1$

Also, $y-x=x+1-x=1$

incorrect opening sentence

Start with "Let $x\in\mathbb{R}$ be arbitrary with $x\neq 0$, $x\neq 1$".

so $y=x+1$ is one solution of the equation. □

Uniqueness: Assume $\frac{z-1}{x}=z-x$, this equation is valid since $x\neq 0$. Multiply each side by x, we will get:

"Proof of theorem 1".

$z-1=zx-x^2$, and it could be rewritten as:

$z(1-x)=1-x^2$, and it is $z(1-x)=(1+x)(1-x)$.

Since $x\neq 1$, so divide each side by $(1-x)$ and

"Proof of theorem 2"

$z=1+x=y$. Thus, $y=1+x$ is the only solution. □

Theorem 2: Existence: Assume $y=3$. Let x be arbitrary real number. The left-hand side would be, for any real number x, $(x+3)(x-3)=x^2-9$ and it is equal to the right-hand side. So $y=3$ ~~can be~~ is one solution. □

Uniqueness: Assume z that $(z+x)(x-3)=x^2-9$. For all $x\in\mathbb{R}$.

If $x=3$, then the equation would be $0=0$. In this time z would be any value.

If $x\neq 3$, the equation would be $(z+x)(x-3)=(x+3)(x-3)$

and $z+x=x+3$, so $z=3$, which is the only possible situation.

Since we've run out all possible cases, there exist an unique y that satisfy all values of x, $(y+x)(x-3) = x^2-9$. $\square$

Q4.

Existence: $m=5$. Since $5|10$, $5|15$ and $5>1$, $m=5$ could be one solution $\square$

Uniqueness: Assume $z>1$, $z|10$ and $z|15$. To satisfy $z|10$, $z\in\{1,2,5,10\}$. and to satisfy $z|15$, $z\in\{1,3,5,15\}$. Since $z>1$, $z|10$ and $z|15$. z could only be 5. $\square$