Strategy of intersection.

in this section, we combined all the techniques we have learnt in previous section to evaluate a given intergrate.

e.g. 1. $\int \sec x \, dx.$

$= \int \frac{1}{\cos x} dx.$

$= \int \frac{1}{\cos x} \cdot \frac{\cos x}{\cos x} dx.$

$= \int \frac{\cos x}{\cos^2 x} dx.$

$u = \sin x \quad u' dx = \cos x dx.$

$= \int \frac{1}{1-u^2} du.$

$= \int \frac{1}{(1+u)(1-u)}$

$A(1-u) + B(1+u) = 1.$

$A = B = \frac{1}{2}.$

$= \frac{1}{2} \int \frac{1}{1+u} - \frac{1}{2} \int \frac{1}{u-1}.$

$= \frac{1}{2} \ln|1+u| - \frac{1}{2} \ln|u-1| + C.$

$\Rightarrow \int \sec x \, dx = \frac{1}{2} \ln|1+\sin x| - \frac{1}{2} \ln|\sin x - 1| + C.$

$= \frac{1}{2} \ln\left|\frac{\sin x + 1}{\sin x - 1}\right| + C.$

$= \frac{1}{2} \ln\left|\frac{\sin x + 1}{\sin x - 1} \cdot \frac{\sin x + 1}{\sin x + 1}\right| + C.$

$= \ln\left|\frac{\sin x + 1}{\cos x}\right| + C.$

$= \ln|\sec x + \tan x| + C.$

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e.g. 2. $\int \frac{1}{x^2 + x\sqrt{x}} dx.$

$u = x^{\frac{1}{2}}. \quad u' dx = \frac{1}{2} x^{-\frac{1}{2}}.$

$= \int \frac{\frac{1}{2} x^{-\frac{1}{2}} dx.}{\frac{1}{2} x^{-\frac{1}{2}} (x^2 + x^{\frac{3}{2}})}$

$= 2 \int \frac{1}{u^3 + u^2} du.$

$= 2 \int \frac{1}{u^2(u+1)} du.$

$= 2 \int \left[\frac{A}{u^2} + \frac{B}{u} + \frac{C}{u+1}\right] du.$

$A(u+1) + Bu(u+1) + Cu^2 = 1.$

$u = -1 \quad C = 1$

$u = 0 \quad A = 1.$

$u = 1 \quad 2 + 2B + 1 = 1 \quad B = -1.$

$$= 2\left[\int \frac{1}{u^2}du - \int \frac{1}{u}du + \int \frac{1}{u+1}du\right].$$

$$= 2(-u^{-1} - \ln|u| + \ln|u+1|)$$

$$= 2\left(-\frac{1}{\sqrt{x}} - \frac{1}{2}\ln x + \ln(\sqrt{x}+1)\right)$$

$$= -\frac{2}{\sqrt{x}} - \ln x + 2\ln(\sqrt{x}+1) + C.$$

e.g. 3 $\int_0^{\frac{\pi}{4}} \sec^4 x\,dx$

$$\int \sec^4 x\,dx = \int (1+\tan^2 x)\sec^2 x\,dx.$$

$$= \int \sec^2 x\,dx + \int \tan^2 x \sec^2 x \cdot dx.$$

$$= \tan x + \int \tan^2 x \sec^2 x\,dx$$

$u = \tan x.\quad du = \sec^2 x\,dx$

$$= \tan x + \int u^2\,du.$$

$$= \tan x + \frac{1}{3}\tan^3 x + C.$$

e.g. 4 $\int x^2 \cos x\,dx$

$u = x^2 \qquad dv = \cos x\,dx$

$du = 2x\,dx \qquad v = \sin x.$

$$= x^2 \sin x - \int \sin x \cdot 2x\,dx.$$

$$\int \sin x \cdot x\,dx.$$

$$= x\cdot(-\cos x) + \int \cos x\,dx$$

$$= -x\cos x + \sin x.$$

$$\Rightarrow \quad = x^2 \sin x + 2x\cos x - 2\sin x.$$

x^2	$\cos x$
$2x$	$\sin x$
2	$-\cos x$
0	$-\sin x$

$$= x^2\sin x + 2x\cos x - 2\sin x.$$

$$\int_0^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_0^b f(x)\,dx.$$

$$\int_{-\infty}^{0} f(x)\,dx = \lim_{b\to-\infty} \int_b^0 f(x)\,dx.$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{b\to-\infty} \int_b^a f(x)\,dx + \lim_{c\to\infty} \int_a^c f(x)\,dx.$$

undefine at $x=a$: e.g. $\int_0^2 \frac{1}{x}$.

$$\int_a^b f(x)\,dx = \lim_{\alpha\to a} \int_\alpha^b f(x)\,dx$$

e.g. $\int_0^1 \frac{1}{x^2}\,dx. = \lim_{\alpha\to 0} \int_\alpha^1 \frac{1}{x^2}\,dx$

$= -1 + \alpha^{-3}.\quad \alpha \to 0.$

$= -1.$

$$\int_0^{\infty} e^{-x} \cos x\,dx = \lim_{b\to\infty} \int_0^b e^{-x}\cos x\,dx.$$

$$\int e^{-x}\cos x\,dx. = \frac{1}{2} e^{-x} (\sin x - \cos x)\,dx.$$

$\Rightarrow \quad 0 + \frac{1}{2} = \frac{1}{2}.$

$$\int_0^1 \frac{1}{(x-1)^2}\,dx.$$

$u = x-1.\quad u'dx = du.$

$\Rightarrow \int_{-1}^{0} \frac{1}{u^2}\,du.$

$= -\frac{1}{b} + 1 \quad b \to 0.$

$\Rightarrow -\infty.$