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1) if x_1 is true.

The turth table for $x_1 \cdot g_1(x_2, x_3)$:

x_1	x_2	x_3	$x_1 \cdot g_1(x_2, x_3)$
1	1	1	*
1	1	0	*
1	0	1	*
1	0	0	*.

The turth table for $\bar{x}_1 \cdot g_2(x_2, x_3)$:

$\bar{x}_1$	x_2	x_3	$x_1 \cdot g_1(x_2, x_3)$
0	1	1	*
0	1	0	*
0	0	1	*
0	0	0	*.

The turth table for $x_1 \cdot g_1(x_2, x_3) + \bar{x}_1 g_2(x_2, x_3)$ is:

x_1	x_2	x_3	$x_1 \cdot g_1(x_2, x_3)$
1	1	1	*
1	1	0	*
1	0	1	*
1	0	0	*.
0	1	1	*
0	1	0	*
0	0	1	*
0	0	0	*.

, which is the same as the turth table of $f(x_1, x_2, x_3)$.

if x is false, the condition is similar, the turth

table for $x_1 \cdot f_1(x_2, x_3) + \bar{x_1} \cdot f_2(x_2, x_3)$ the same as the turth table above.

2) For Boolean function that has n variables, it can have 2^n possible minterms at most. Assume this boolean function has m cases of true value, then it must could be written as the sum of m minterms of these 2^n minterms. because these 2^n minterms cover all cases that could happen.

For example, if $n=2$. then all possible minterms are:
$x \cdot y, \bar{x} \cdot y, x \cdot \bar{y}, \bar{x} \cdot \bar{y}$.
all these minterms cover all possible conditions when there are two variables.

3)