

Q1

Statement 1: False.

Proof: We assume that $a-b=3$, then $4(a-b)=4 \times 3=12$. So $12|4(a-b)$ holds. However, $6|3$ does not hold. So the statement is false. \square

Statement 2: True.

Proof: Assume that $3|n$, then there exist an integer k such that $n=3k$. So $2n^2+18=2(3k)^2+18=18k^2+18=9(2k^2+2)$. Since k is an integer, $2k^2+2$ is also an integer. Thus, $9|2n^2+18$. So for all $n \in \mathbb{N}$ that if $3|n$, then $9|2n^2+18$. \square

Q2. $x^2y-y-1=0$ could be translate as $(x^2-1)y=1$.

Since $x>1$, then $x^2-1>0$. So $y=\frac{1}{x^2-1}$. Because $x^2-1>0$, $\frac{1}{x^2-1}>0$. Thus, $x \in \mathbb{R}$ and $x>1$ implies that there exist a $y \in \mathbb{R}$ such that $x^2y-y-1=0$ and $y>0$. \square

Q3 Assume there exist $x \in A$. Since that every $A \cap F$ is disjoint from B , $\forall x (x \in A \rightarrow x \notin B)$. Since $A \subseteq F$, $A \subseteq U \cap F$, so $\forall x (x \in A \rightarrow x \in U \cap F)$. Since A is random, every x in A is in $U \cap F$ and is not in B . Thus, every $A \cap F$ is disjoint from B implies that $U \cap F$ and B are disjoint. \square .