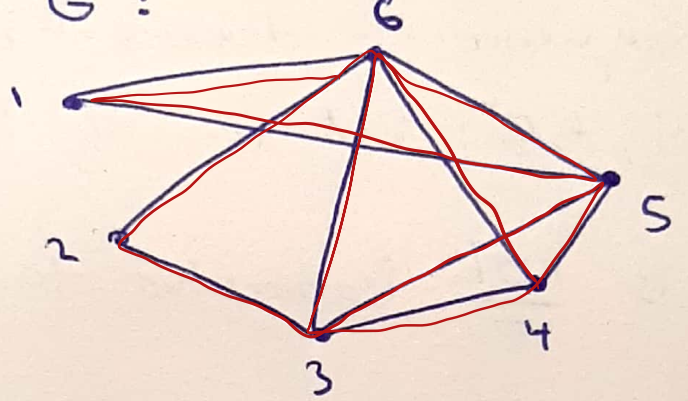
• Consider the following adjacency matrix of graph G:

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

a) reconstruct G :



b) is there any path between nodes 1 & 2 ?

Yes , $1 \rightarrow 6 \rightarrow 2$

c) find degrees of the nodes in this graph:

$d = \{2, 2, 4, 3, 4, 5\}$

d) find #edges in G from the degrees of the vertices :

$$\sum d_i = 2e$$

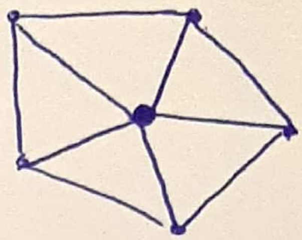
$$\Rightarrow \quad e = (2+2+4+3+4+5)/2 = \frac{20}{2} = \boxed{10}$$

e) is G bipartite? NO,

(Hint : Assume it's bipartite. color of 6 should be different from all other vertices. reach contradiction)

f) is this graph isomorphic to W_6?

→ let's consider W_6:



$|E_G| = 10$ $|V_G| = 6$

$|E_{W_6}| = 10$ $|V_{W_6}| = 6$

However, sequence of degrees for G is:

$\{2, 2, 3, 4, 4, 5\}$ but for W_6 is: $\{3, 3, 3, 3, 3, 5\}$

$\Rightarrow$ G is NOT isomorphic to W_6.

g) Does G have an Euler circuit / path?

Euler path: $4 \to 6 \to 3 \to 4 \to 5 \to 3 \to 2 \to 6 \to 1 \to$ ● $\to 5 \to 6$

* note that the starting & eding vertices of the path have odd degrees.

No Euler circuit. (because there are nodes with odd degree)

:) there's nodes with odd degree, then there's no Euler circuit in this graph.

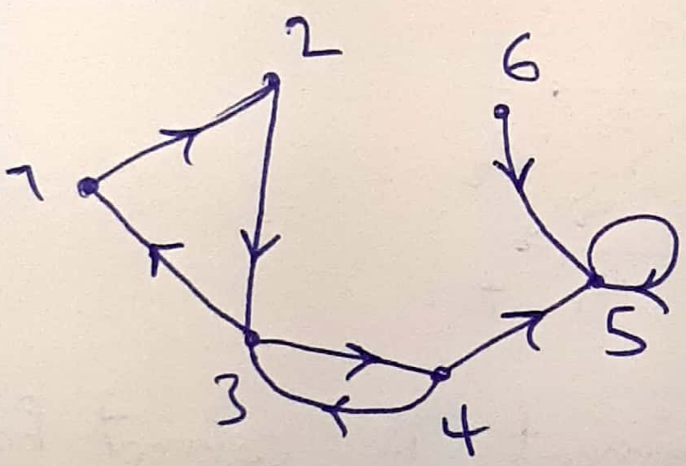
h) is G connected? yes

7) consider the following adjacency list of graph G.

1	2
2	3
3	1 → 4
4	3 → 5
5	5
6	5

a. reconstruct G:

b. tell if it's directed/undirected and simple/multi graph.



G is directed and a multi-graph

c. find the strong/weak components:

strong components : $\begin{cases} C_1 : \{1,2,3,4\} \\ C_2 : \{5\} \\ C_3 : \{6\} \end{cases}$

weak components: {1,2,3,4,5,6}

d. check Euler path/circuit : No Euler path & No Euler circuit

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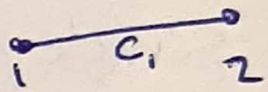
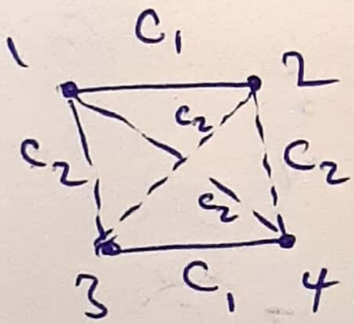
2) consider a complete graph with 2^n vertices. show we can color its edges with n different colors so that the edges of each triangle in the graph has at least 2 different colors.

$\longrightarrow$ base case: $n=1$:



$n=2$:



assume the statement is correct for $n=k$ $\Rightarrow$ we can color the edges of a complete graph with 2^k vertices with k different colors so that each triangle has at least 2 diff colors

$\longrightarrow$ we want to show for $n=k+1$

$\Rightarrow$ put 2 complete graphs with 2^k nodes next to each other & color the edges of each of them with k colors. color the edges between them with color $k+1$. ✓