

## Constrained Optimization

- how to optimize a objective function  
subject to certain constraints

e.g. Farmer's Fence.

Farmer get  $F$  feet of fence, and he could choose the width  $W$  and the length  $L$ , so how could she obtain largest area?

choice variables:  $L, w$

constrain:  $F \geq L + w$

objective function:  $L \cdot w$

Formal problem:

max  $L \cdot w$

$L \geq 0$

$w \geq 0$

subject to  $L + w \leq F$

$F \Rightarrow$  parameter



In general,

max (or min)  $f(x)$

$v_1, v_2$

s.t., constrain 1:

constrain 2:

Endogenous : variable determined by the model

Exogenous : variables that is pre set (i.e. parameters)

e.g. marketing: Firm split a million budget on TV and radio

spending	TV	radio
0	0	0
100 k	4750	950
200 k	9000	1800
300 k	12750	2550
⋮	⋮	⋮

800k	24,000	4800
900k	24,750	4950
1 m	25,000	5000

objective: maximum sales  $B(T, R)$

choose variables: spending on TV (T) and radio (R)

constrain:  $1m \geq T+R$

$$\max_{T, R} B(T, R)$$

$$st \quad T+R \leq 1,000,000$$

### Equilibrium Analysis

if the model output doesn't change,  
(when the exogenous variable do not change)  
then it is an equilibrium.

### Example competitive equilibrium:

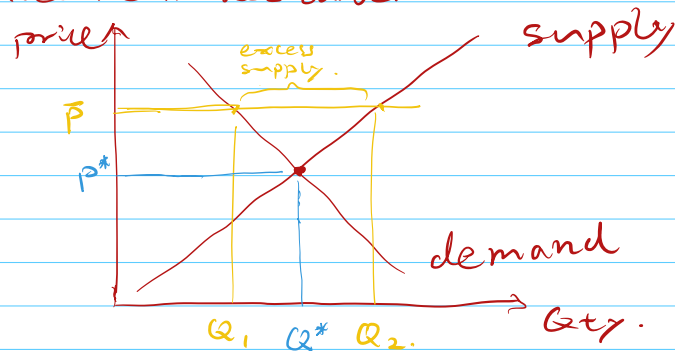
demand curve: quantity the market demand for any given price

supply curve: quantity the market supply for any given price

While there's no influence on supply / demand.

This market is in equilibrium if no seller or buyer want to change their decision

i.e. the market curve.



at the point  $(Q^*, p^*)$ , it reaches equilibrium.

Comparative statics:

- how does the outcome of the model change when we make change in exogenous variable and keep all else constant?

Positive versus normative analysis.

positive: explaining economic phenomena or predicting how systems change in response to other change

normative: how should we achieve social goals, answer questions about what we should be happening