Assignment 4

Due: Saturday Nov. 21, 2020 before 6:30 PM to be uploaded in Gradescope as a single pdf file.

Please write your name and student number on your submission. Justify each step carefully. When in doubt prove the statement you are going to use. Solutions are graded for correctness as well as clarity.

Exercise 1 (10 point)**.** Prove that for sets A, B and C, we have

$$A\Delta B = (A \cup B)\Delta(A \cap B) \tag{1}$$

where Δ denotes the symmetric difference of sets (sometimes also denoted by $\oplus$).

Exercise 2 (10 point)**.** Let S_n denote the number of set partitions of the set $\{1, 2, \dots, n\}$ into two sets (sometimes called *parts*). For example, $S_3 = 3$, because $\{\{1, 2\}, \{3\}\}$, $\{\{1, 3\}, \{2\}\}$ and $\{\{2, 3\}, \{1\}\}$ are the only set partitions of $\{1, 2, 3\}$ into two parts. Show that

$$S_n = 2^{n-1} - 1 \tag{2}$$

Exercise 3 (15 points)**.** Write the following sum in Σ notation

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n \cdot (n+1)} \tag{3}$$

and show using induction that for $n \in \mathbb{Z}_+$, the above sum equals $\frac{n}{n+1}$.

Exercise 4 (15 point)**.** There are 3^n ternary sequences (sequences whose alphabet set is $\{0, 1, 2\}$) of length n. Let t_n denotes the number of ternary sequences of length n having odd number of zeros. Show that

$$t_n = t_{n-1} + 3^{n-1} \tag{4}$$

[**Bonus:**(10pts) Find a closed formula for t_n.]