Calculus 2402 A Lecture 8

For a function of three variables f(x,y,z). Let (x_0,y_0,z_0) be a CP of f, ie, $\nabla f(x_0,y_0,z_0) = \vec{O}$. Also, suppose that all 2nd partial derivatives of f are continuous on a neighborhood of (x_0,y_0,z_0) . The Hessian of f at (x_0,y_0,z_0) is defined as

$$H(x_{0},y_{0},z_{0}) = \begin{cases} f_{xx}(x_{0},y_{0},z_{0}) & f_{xy}(x_{0},y_{0},z_{0}) & f_{xz}(x_{0},y_{0},z_{0}) \\ f_{yx}(x_{0},y_{0},z_{0}) & f_{yy}(x_{0},y_{0},z_{0}) & f_{zz}(x_{0},y_{0},z_{0}) \end{cases}$$

$$f_{zx}(x_{0},y_{0},z_{0}) & f_{zy}(x_{0},y_{0},z_{0}) & f_{zz}(x_{0},y_{0},z_{0}) \end{cases}$$

then

- a) If $H(x_0, y_0, \overline{z}_0)$ is the definite, then f has a local min $(x_0, y_0, \overline{z}_0)$
- L) If H (xo, yo, 70) is ve definite, then f has a local max (a) (x, yo, 2)
- c) If H(xo,yo, 20) is indefinite, then t has (xo,yo, 20) as a saddle point
- d) If det (H(xo, yo, zo)) = O, the test is inconclusive.

In order to apply the 2nd derivative test, the following theorem is very useful. It helps us to determine whether a given symmetric matrix is either the definite or -ve definite or indefinite.

Theorem: If A is a symmetric matrix then the KxK matrix A_k obtained by deleting all except the first k nows and the 1st K Wolumns of A is called the KxK principal mina of A. By convention $A_n = A$.

- (1) A is +ve definite iff det (Ak)>0 for all k.
- (ii) A is -ve definite ; if $(-1)^k$ det $(A_k) > 0$ for all k. In other words,

if k is odd then det (Ax) <0

if it is even then det (Ax) >0

(iii) If det (Azn) <0 for some K, then A is indefinite.

Ex: let f(1, y, 7) = 2 + y + 2 - xy + x - 27. Find the CP's of f

and c Solution	lassiby them.
fy	$= 2z - y + 1 = 0 \Rightarrow 2(2y) - y + 1 = 0 \Rightarrow 3y + 1 = 0 \Rightarrow y = -\frac{1}{3}$ $= 2y - 2 = 0 \Rightarrow x = 2y \Rightarrow z = -\frac{2}{3}$
52	$= 27 - 2 = 0 \implies 7 = 1$ $\therefore \left(-\frac{2}{3}, -\frac{1}{3}, 1\right) \text{ is } \Leftrightarrow CP.$
	$f_{xx} = 2$ $f_{xy} = -1$, $f_{xz} = 0$ $f_{yy} = 2$, $f_{yx} = -1$, $f_{yz} = 0$
<i>i</i> .	$f_{22} = 2 , f_{xz} = 0 , f_{xy} = 0$ $H(-\frac{2}{3}, -\frac{1}{3}, 1) = \begin{bmatrix} 2 & -1 & 1 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ -1 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ -1 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} + $
	$A_1 = \begin{bmatrix} 2 \end{bmatrix} \Rightarrow det(A_1) = 2 > 0$
	$A_{2} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, Let $(A_{2}) = (2)(2) - (-1)(-1) = 9 - 1 = 3 > 0$ $A_{3} = A \Rightarrow det(A_{3}) = Let(A) = 2 \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 2(3) = 6 > 0$
F	$\left(-\frac{2}{3}, -\frac{1}{3}, 1\right)$ is the definite. Hence, f has a local minimum the $\left(-\frac{2}{3}, -\frac{1}{3}, 1\right)$ and
	$ \frac{1}{3}\left(-\frac{2}{3}, -\frac{1}{3}, 1\right) = \left(-\frac{2}{3}\right)^{2} + \left(-\frac{1}{3}\right)^{2} + \left(1\right)^{2} - \left(-\frac{2}{3}\right)\left(-\frac{1}{3}\right) + \left(-\frac{2}{3}\right) - 2\left(1\right) $ $ = \frac{4}{9} + \frac{1}{9} + 1 - \frac{2}{9} - \frac{2}{3} - 2 = -\frac{4}{3} \text{ // Ams}. $
	Mean Value Theorem (See Stewart, p. 287) f be a function that Satisfies the following conditions
fh	1. f is continuous on a closed interval [a1b] 2. f is differentiable on the open interval (a,b) en there is a number c ∈ (a,b) Such that
	$f'(c) = \underbrace{f(b) - f(a)}_{b-a} \tag{1}$
	Let $\theta = \frac{c-a}{b-a}$ then $0 < \theta < 1$ Let $h = b-a \implies c = a + \theta h$
_	(1) becomes $f(a+h) = f(a) + h f'(a+th)$

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The MVT can be generalize to Taylor's theorem
      f(a+l) = f(a) + l f'(a) + \frac{l^2}{2!} f''(a) + \cdots + \frac{l^n}{n!} f^{(n)}(a)
                                                       + \frac{\int_{1}^{2\pi i} \int_{1}^{2\pi i} \left(a + \theta L\right)}{\left(n+i\right)!} \int_{1}^{2\pi i} \left(a + \theta L\right)
0 < \theta < 1 \qquad (2)
    Taylor sens of a function of two variables
              Let F(t) = f(a+ht, b+kt)
     Apply Taylor's theorem to F(t) with t = 1 and t = 0
          F(1) = F(0) + \frac{F'(0)}{1!} + \frac{F''(0)}{2!} + \cdots + \frac{F^{(n)}(0)}{n!} + \frac{1}{(n+1)!} F^{(n+1)}(\theta)
                                                                                              0 < <del>0</del> < 1
                                                                                                     (3)
       We also note that
                       F(1) = f(a+h,b+k)
    F'(+) = h fx (a + ht, b + kt) + k fy (a + ht, b + kt)
    F"(+) = h [ h fxx (a+ht, b+kt) + K fxy (a+ht, b+kt)]
                + K ] h fyx (a+ Lt, b+ kt) + K fyy (a+ht, b+ k+)
                = h2 fxx (a+ht, b+kt) + 2hk fxy (a+ht, b+kt) +
                                                                                  12 fyy (a+ht, b+kt)
       Set t=0,
          F'(0) = h fx (a,b) + K fy (a,b)
                      = \left( \frac{1}{2} + \frac{3}{24} \right) + \left( \frac{3}{24} \right)
         F''(0) = h^2 f_{xx}(a,b) + 2hk f_{xy}(a,b) + k^2 f_{yy}(a,b)
                    = \left(\frac{h}{2} + \frac{3}{2}\right)^{2} + \left(\frac{3}{2}\right)^{2}
           F^{(n)}(0) = \left(h \frac{3}{2} + k \frac{3}{2n}\right)^n f(a,b)
      Then (3) becomes
    f (a+h, b+k) = f(a,h) + hfx(a,b) + kfy(a,h)+
                               1 h fxx (a,b) + 2hk fxg (a,b) + K2 fyy (a,b) +
                               \frac{1}{3!} \left( \left\{ \frac{\partial x}{\partial} + \left\{ \frac{\partial y}{\partial} \right\} \right\} \left\{ \left( a'P \right) + \cdots + \frac{1}{1!} \left( \left\{ \frac{\partial x}{\partial} + \left\{ \frac{\partial x}{\partial} \right\} \right\} \right\} \left\{ \left( a'P \right) \right\}
                             + \frac{1}{(n+1)!} \left( k \frac{3}{2x} + k \frac{3}{2y} \right)^{n+1} f \left( a + \theta k, b + \theta k \right) \tag{4}
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This is called Taylor's formula of f(x,y). Let n > 00, (4)
  becomes a Taylor Series.
 N.B
   (i) If n=1, (4) becomes
    f(a+h, b+k) = f(a,b) + h fx(a,b) + k fy (a,b) +
                                                            \frac{1}{2!} \left[ h^2 f_{xx} \left( a + \theta h, b + \theta k \right) + 2 h k f_{xy} \left( a + \theta h, b + \theta k \right) \right]
                                                                                 + 12 fyy (a+th, b+tk)
                                                                                                                            0<0<1
    We note that when h, K are very small
               f(a+b,b+k) \approx f(a,b) + k f_{x}(a,b) + k f_{y}(a,b)
      we have a linear approximation of f ( Sec 14, 4).
    (ii) If n=7, (4) becomes
   f (a+h, b+1c) = f(a,h) + h fx (a,h) + 1c fy (a,b) +
                                                 1 21 [ h2 fxx (a,h) + 2hk fxy (a,h) + k2 fyy (a,h)] +
                                                     \frac{3!}{3!}\left(\frac{1}{2}\frac{3}{3}+\frac{1}{2}\frac{3}{3}\right)^{3}f(a+\theta h,b+\theta lc), \quad (x,\theta < 1)
    When h, k are very small,
         f(a+h,b+1c) ~ f(a,b) + lfx(a,b) + lc fy(a,b) +
                                                           we have a quadratic approximation.
       We also note that if (a,h) is a C.P (fx (a,h)=0, fy (a,h)=0)
               f(a+b,b+k) - f(a,b) \simeq \frac{1}{2!} \left[ h^2 f_{xx}(a,b) + 2kk f_{xy}(a,b) + f_{yy}(a,b) \right]
                                                                                                                               fyy (a, b)
                                                                             \simeq \frac{1}{2} \left( f_{1}(k) \left[ f_{xx}(a_{1}b) f_{xy}(a_{1}b) \right] \left[ f_{xx}(a_{1}b) f_{yy}(a_{1}b) f_{yy}(a_{1}b) f_{yy}(a_{1}b) \right] \left[ f_{xx}(a_{1}b) f_{yy}(a_{1}b) f_{yy}(a_
                                                                                                                                 H(a,b)
            f(a+h,b+l()-f(a,h) \ge 0 if H(a,b) is the definite
            f(a+h, b+k) - f(a,b) < 0 if +1 (a,b) is -ve definite
  If sometimes f(a+h,b+1)-f(a,b) > 0 and sometimes
            f(a+h, b+k) - f(a,b) <0 when H(a,b) is indefinite.
 We rediscover the 2nd Deivative Test of firs of two variables.
   Ex: Obtain Taylor's polynomial of degre 2 in the
        Taylor expansion of f(x,y) = exts by
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b) expensive $e^{2\pi i 3}$ in a large of $e^{2\pi i 3}$. e) multiplicity the series expression of $e^{2\pi i 3}$. Subtraction of $f(y_1,y_2) = e^{2\pi i 3} = f(x_1,x_2) + 2\pi f_{2}(x_1,x_2) + 3\pi f_{2}(x_1,x_2) +$	a) hsi	Taylor expansion (4),
c) multiply the series expension of e^{x} and e^{y} . Shitm a) Here $a \ge 0$, $b \ge 0$, $b \ge 2$, $b = y$ from (4) $f(y_{12}) = e^{y+2} = f(0,0) + x f_{x}(0,0) + y f_{y}(0,0) + y^{2} f_{yy}(0,0) = 1$ $f_{y}(x_{12}) = e^{x+2}$ $f_{y}(x_{13}) = e^{x+2}$ $f_{y}(x_{13}) = e^{x+2}$ $f_{y}(x_{13}) = e^{x+2}$ $f_{y}(x_{13}) = f_{y}(x_{13}) = 1$ $f_{y}(x_{13}) = f_{y}(x_{$		
Shtm. a) Flux $z = 0$, $l = 0$, $l = x$, $k = y$ $f(x_{1},y_{1}) = e^{x+y_{2}} = f(x_{1},0) + x + f_{2}(x_{1},0) + y + f_{3}(x_{1},0) + y + f_{3}(x_{1},0$		
from (4) $f(x_{1-2}) = e^{x+3} = f(o_{1}o) + x f_{x}(o_{1}o) + y f_{y}(o_{1}o) + \frac{1}{2!} \left[\frac{x^{2}}{2!} f_{xx}(o_{1}o) + 2 \frac{x}{2} f_{xy}(o_{1}o) + y^{2} f_{yy}(o_{1}o) \right] + \frac{1}{2!} \left[\frac{x^{2}}{2!} f_{xx}(o_{1}o) + 2 \frac{x}{2} f_{xy}(o_{1}o) + y^{2} f_{yy}(o_{1}o) \right] \right]$ $f_{x}(x_{1}y) = e^{x+3} \qquad f_{y}(o_{1}o) = 1$ $f_{y}(x_{1}y) = e^{x+3} \qquad f_{y}(o_{1}o) = 1$ $f_{y}(o_{1}o) = 1$ $f_{$		
from (4) $f(x_{1-2}) = e^{x+3} = f(o_{1}o) + x f_{x}(o_{1}o) + y f_{y}(o_{1}o) + \frac{1}{2!} \left[\frac{x^{2}}{2!} f_{xx}(o_{1}o) + 2 \frac{x}{2} f_{xy}(o_{1}o) + y^{2} f_{yy}(o_{1}o) \right] + \frac{1}{2!} \left[\frac{x^{2}}{2!} f_{xx}(o_{1}o) + 2 \frac{x}{2} f_{xy}(o_{1}o) + y^{2} f_{yy}(o_{1}o) \right] \right]$ $f_{x}(x_{1}y) = e^{x+3} \qquad f_{y}(o_{1}o) = 1$ $f_{y}(x_{1}y) = e^{x+3} \qquad f_{y}(o_{1}o) = 1$ $f_{y}(o_{1}o) = 1$ $f_{$	a) Here	a = 0, $k = 0$, $k = y$
$f_{x}(\lambda_{1}): e^{x+y} \implies f_{y}(e, 0) = 1$ $f_{y}(\lambda_{1}) = e^{x+y} \qquad f_{y}(e, 0) = 1$ $f_{xx}(x_{1}) = e^{x+y} \qquad f_{xy}(e, 0) = 1$ $f_{xy}(x_{1}) = e^{x+y} \qquad f_{xy}(e, 0) = 1$ $f_{xy}(e, 0) = $	f(x	5) = ex +5 = f(0,0) + x fx (0,0) + 5 fy (0,0)
$f_{Y}(\lambda_{1}) : e^{x+y} \implies f_{Y}(0,0) = 1$ $f_{Y}(\lambda_{1}) = e^{x+y} \qquad f_{Y}(0,0) = 1$ $f_{YY}(x_{1}) = e^{x+y} \qquad f_{YY}(0,0) = 1$ $f_{YY}(x_{1}) : e^{x+y} \qquad f_{YY}(0,0) = 1$ $f_{YY}(x_{1}) : e^{x+y} \qquad f_{YY}(0,0) = 1$ $Subst $		
$f_{\gamma}(x,y) = e^{x+y} \qquad f_{\gamma}(x,0) = 1$ $f_{\gamma\gamma}(x,y) = e^{x+y} \qquad f_{\gamma\gamma}(x,0) = 1$ $f_{\gamma\gamma}(x,y) = e^{x+y} \qquad f_{\gamma\gamma}(x,y) = 1$ $f_{\gamma\gamma}(x,y) = e^{x+y} \qquad f_{\gamma\gamma}$	f _{>} (
$\int_{7x} (x_{1}y) = e^{x+y} \qquad f_{xy}(0,0) = 1$ $\int_{7y} (x_{1}y) = e^{x+y} \qquad f_{yy}(0,0) = 1$ $\int_{8x} (x_{1}y) = e^$		
$f_{xy}(x_{1},y) = e^{x+3} \qquad f_{xy}(x_{1},y) = 1$ $Subst \text{ then int } (**),$ $e^{x+y} = 1 + x + y + \frac{1}{2} \left[x^{2} + 2xy + y^{2} \right]$ $= 1 + x + y + \frac{1}{2} x^{2} + xy + \frac{1}{2} y^{2} \qquad Am_{1}.$ $e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{2}}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^{n}}{n!}$ $\therefore e^{x+y} = \sum_{n=0}^{\infty} \frac{(x+y)^{n}}{n!}$ $= 1 + (x+y) + \frac{1}{2!} (x+y)^{2}$ $= 1 + 2 + y + \frac{1}{2!} x^{2} + xy + \frac{1}{2!} y^{2} + \dots$ $e^{x} = 1 + x + y + \frac{y^{2}}{2!} + xy + \frac{x^{2}}{2!} + \dots$ $= 1 + x + y + \frac{y^{2}}{2!} + xy + \frac{x^{2}}{2!} \qquad Am_{1}.$		
$f_{xy}(x_{1},y) = e^{x+3} \qquad f_{xy}(x_{1},y) = 1$ $Subst \text{ then int } (**),$ $e^{x+y} = 1 + x + y + \frac{1}{2} \left[x^{2} + 2xy + y^{2} \right]$ $= 1 + x + y + \frac{1}{2} x^{2} + xy + \frac{1}{2} y^{2} \qquad Am_{1}.$ $e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{2}}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^{n}}{n!}$ $\therefore e^{x+y} = \sum_{n=0}^{\infty} \frac{(x+y)^{n}}{n!}$ $= 1 + (x+y) + \frac{1}{2!} (x+y)^{2}$ $= 1 + 2 + y + \frac{1}{2!} x^{2} + xy + \frac{1}{2!} y^{2} + \dots$ $e^{x} = 1 + x + y + \frac{y^{2}}{2!} + xy + \frac{x^{2}}{2!} + \dots$ $= 1 + x + y + \frac{y^{2}}{2!} + xy + \frac{x^{2}}{2!} \qquad Am_{1}.$	f_{γ} .	$f_{\gamma\gamma}(0,0) = e^{\lambda + 2}$ $f_{\gamma\gamma}(0,0) = 1$
$e^{x+y} = 1 + x + y + \frac{1}{2} \left[x^{2} + 2xy + y^{2} \right]$ $= 1 + x + y + \frac{1}{2} x^{2} + xy + \frac{1}{2} y^{2} $	f	(2,7) = ex+5 fxy (0,0) = 1
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Sulst -	hose into (**),
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	e ^{x.}	+9 = 1 + 2 + 1 + 1 + 2 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 3 + 3
$e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^{n}}{n!}$ $\therefore e^{x+y} = \sum_{n=0}^{\infty} \frac{(x+y)^{n}}{n!}$ $\approx \sum_{n=0}^{\infty} \frac{(x+y)^{n}}{n!}$ $\approx 1 + (x+y) + \frac{1}{2!} (x+y)^{2}$ $\approx 1 + x + y + \frac{1}{2!} + x + y + \frac{1}{2!} + \dots$ $\approx 1 + x + y + \frac{y^{2}}{2!} + x + y + \frac{x^{2}}{2!} + \dots$ $\approx 1 + x + y + \frac{y^{2}}{2!} + x + y + \frac{x^{2}}{2!} + \dots$		
$e^{x+y} = \sum_{n=0}^{\infty} \frac{(x+y)^n}{n!}$ $\approx \sum_{n=0}^{\infty} \frac{(x+y)^n}{n!}$ $\approx 1 + (x+y) + \frac{1}{2!} (x+y)^2$ $\approx 1 + 2 + 3 + \frac{1}{2} x^2 + 2 + 3 + \frac{1}{2} y^2 \text{ (Ass.)}$ $e^{x} e^{y} = (1+x+y+y+\frac{y^2}{2!}+x) + \frac{x^2}{2!}$ $\approx 1 + x + y + \frac{y^2}{2!} + x + \frac{x^2}{2!} \text{ (Ass.)}$	L) Le	e Stewart, sec 11.10
$e^{x+y} = \sum_{n=0}^{\infty} \frac{(x+y)^n}{n!}$ $\approx \sum_{n=0}^{\infty} \frac{(x+y)^n}{n!}$ $\approx 1 + (x+y) + \frac{1}{2!} (x+y)^2$ $\approx 1 + 2 + 3 + \frac{1}{2} x^1 + 2 + 3 + \frac{1}{2} y^2 $ $\approx 1 + 2 + 3 + \frac{3}{2!} + \cdots) \left(\frac{1}{2} + 3 + \frac{3}{2!} + \cdots \right)$ $\approx 1 + 2 + 3 + \frac{3}{2!} + 2 + 3 + \frac{2}{2!}$ $\approx 1 + 2 + 3 + \frac{3}{2!} + 2 + 3 + \frac{2}{2!}$ $\approx 1 + 2 + 3 + \frac{3}{2!} + 2 + 3 + \frac{2}{2!}$ $\approx 1 + 2 + 3 + \frac{3}{2!} + 2 + 3 + \frac{2}{2!}$ $\approx 1 + 2 + 3 + \frac{3}{2!} + 2 + 3 + \frac{2}{2!}$ $\approx 1 + 2 + 3 + \frac{3}{2!} + 2 + 3 + \frac{2}{2!}$ $\approx 1 + 2 + 3 + \frac{3}{2!} + 2 + \frac{3}{2!} + 2 + \frac{3}{2!}$ $\approx 1 + 2 + 3 + \frac{3}{2!} + 2 + \frac{3}{2!} + 2 + \frac{3}{2!}$ $\approx 1 + 2 + 3 + \frac{3}{2!} + 2 + \frac{3}{2!} + 2 + \frac{3}{2!}$ $\approx 1 + 2 + 3 + \frac{3}{2!} + 2 + \frac{3}$		$e^{x} = 1 + x + \frac{x^{2}}{x^{2}} + \frac{x^{3}}{x^{3}} + \dots = \frac{x^{n}}{x^{n}}$
$= \frac{1}{2!} + \frac{1}{2!$		$\frac{h}{2}$ $\frac{h}{2}$ $\frac{h}{2}$ $\frac{h}{2}$ $\frac{h}{2}$ $\frac{h}{2}$
$= \frac{1}{2!} + \frac{1}{2!$:. e	$\frac{2}{2} \left(\frac{2}{2} + \frac{1}{3} \right)$
$= \frac{1}{2!} + \frac{1}{2!$		$\frac{n_{-0}}{2} \left(\frac{1}{2c + 4} \right)^{\frac{n}{2}}$
$= \frac{1}{2!} + \frac{1}{2!$		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		$=$ $1 + (x+y) + \frac{1}{2!}(x+y)^2$
c) $e^{x} e^{y} = (1 + x + \frac{2^{2}}{2!} + \cdots) (\frac{1}{2} + \frac{3}{2!} + \cdots)$ $\frac{2}{2} + x + y + \frac{3^{2}}{2!} + \frac{2^{2}}{2!} + \cdots)$ $\frac{2}{2} + x + y + \frac{3^{2}}{2!} + \frac{2^{2}}{2!} + \cdots$ $\frac{2}{2} + x + y + \frac{3^{2}}{2!} + \frac{2^{2}}{2!} + \cdots$ $\frac{2}{2} + \frac{2}{2} + \frac{2}{2!} + \cdots$ $\frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \cdots$ $\frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \cdots$		= 1 + 2 + 5 + \frac{1}{2} \tau^2 + \tau_3 + \frac{1}{2} \gamma^2 \pi Ans.
$\frac{2}{2} + x + y + \frac{y^{2}}{2!} + xy + \frac{z^{2}}{2!}$ $\frac{2}{2} + x + y + \frac{y^{2}}{2} + xy + \frac{z^{2}}{2!} $ Ans.		
$\frac{2}{2} + x + y + \frac{y^{2}}{2!} + xy + \frac{z^{2}}{2!}$ $\frac{2}{2} + x + y + \frac{y^{2}}{2} + xy + \frac{z^{2}}{2!} $ Ans.	c) e ^x	$e^{y} = \left(1 + 2 + \frac{2^{2}}{2!} + \cdots\right) \left(\frac{1}{2} + 3 + \frac{3^{2}}{2!} + \cdots\right)$
$\simeq 1 + x + 5 + \frac{y^2}{2} + 35 + \frac{x^2}{2}$ // Ans.		
		$\frac{2}{2!} + \frac{1}{2!} + \frac{1}{2!} + \frac{1}{2!}$
		$\simeq 1 + x + y^2 + 3y + \frac{x^2}{2} \parallel Ans.$
See yn on Monday!		
		Lee yn on Monday!