$a|b$:?? $b\%a=0$. e.g. $3|6$ ✓ $6|3$ ✗

Distributive laws:

$\forall x(P(x)\wedge Q(x)) = (\forall x P(x))\wedge(\forall x Q(x))$

$\forall x(P(x)\vee Q(x)) \neq \underline{(\forall x Q(x))\vee(\forall x Q(x))}$ ← the x not the same one.

$\exists x(P(x)\wedge Q(x)) \neq (\exists x P(x))\wedge(\exists x Q(x))$

$\exists x(P(x)\vee Q(x)) = (\exists x P(x))\vee(\exists x Q(x))$

list elements: $\{1,2,3\}$.

set builder: $\{x \mid P(x)\}$.

index family: $\{n^2 \mid n\in N\}$. i.e. $A=\{x^2 \mid x\in N\}$.

$P(A)$ has 2^n subsets, n is the number of elements in A.

$\cap\mathcal{F} = \{x \mid \forall A\in\mathcal{F}\,(x\in A)\}$.

$\cup\mathcal{F} = \{x \mid \exists A\in\mathcal{F}\,(x\in A)\}$.

*: Though $\mathcal{F}$ is family of sets and elements in $\mathcal{F}$ are sets,
elements in $\cap\mathcal{F}$, $\cup\mathcal{F}$ are regular elements, NOT sets!

i.e. $\mathcal{F}=\{\{1,2,3\},\{2,3,4\},\{3,4,5\}\}$.

$\cup\mathcal{F}=\{1,2,3,4,5\}$

$\cap\mathcal{F}=\{3\}$

Family of sets, $\mathcal{F}$, $\cap$, $\cup$, $P(A)$.

3.1. Gen Ideas of Proofs: Let [Given], Suppose ----. Since ----, ----
then ----. So, [Given] implies [Goal].

3.2. $\to$: To prove $A \to B$, assume A then try to proof B.
try to prove $\neg B \to \neg A$

$\neg$: Reexpress it in other ways. e.g. $\neg(a \in A \setminus B) = \neg(a \in A \wedge a \notin B) = \dots$

3.3 $\forall$: Let x be arbitrary, then try to proof $P(x)$.
Format: To show ----, we must show for every x that [Giv] [Go[
let x be arbitrary, [proof]. So since x is arbitrary,
we conclude that for every x that [Given], [Goal].

$\forall x P(x)$: pick any of a that $P(a)$. Move $P(a)$ to given.
(used in proof of existence \ not empty sets).

$\exists$: Pick one the one specific answer x. Since x [Given], x [Goal].

3.4 $\wedge$: proof each sides seperately.

$\leftrightarrow$: proof $\to$ and $\leftarrow$ seperately.
same with "iff", "="
you can also try intermedium states that $A \leftrightarrow C \leftrightarrow B$.

3.5 $\vee$ 1) break into two parts. 1) Suppose A is true
2) B is true.

Case 1: Assume A proof
2: Assume B proof the goal.

Since $A \vee B$, these cases are exhaustive, so we have proof our goals.

2) break into seperate cases and try to proof $P \vee Q$ in each cases.

3) translate $P \vee Q$ to $\neg P \to Q$ and add $\neg p$ to the the given

Case 1: P since if P is true the statement is true

Case 2: $\neg P \to Q$

3.6 $\exists!$ existence: pick the specific answer for the goal.

uniqueness: try to prove another z that satisfy the goal, $z = x$.

4.1 $A \times B$. Truth sets: $P = \{(a,b) \in A \times B \mid P(a,b)\}$, $P \subseteq A \times B$ (not every set meets P)

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

$(A \cap D) \times (B \cap C) = (A \times B) \cap (C \times D)$

$(A \cup D) \times (B \cup C) \supseteq (A \times B) \cup (C \times D)$

$A \times \emptyset = \emptyset \times A = \emptyset$

4.2 $\circ, {}^{-1}$ $T \circ (S \circ R) = (T \circ S) \circ R$

$(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

$B \times C \circ A \times B = A \times C$ $A \times B \circ B \times C \neq A \times C$.