PART A (18 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE INDICATED ON THE SCANTRON SHEET. YOU SHOULD ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

1 mark

1. Which of the following equations is/are linear in the unknowns x, y and z?

(i) $x - \sqrt{y+2z} = 1$ $\quad$ (ii) $x(y+2z) = 3$ $\quad$ (iii) $\dfrac{1}{x} + 3(y+2z) = 5$

A: (ii) only	B: (iii) only	C: (ii) and (iii) only
D: all of (i), (ii) and (iii)	E: none of (i), (ii) and (iii)	

1 mark

2. Find the augmented matrix for the following system of linear equations:

$$\begin{aligned} x_1 &= 2x_2 - 1 \\ 3x_2 &= 4x_1 \\ 0 &= x_2 + 3 \end{aligned}$$

A: $\left[\begin{array}{cc\|c} 1 & -2 & -1 \\ 4 & 3 & 0 \\ 0 & 1 & 3 \end{array}\right]$	B: $\left[\begin{array}{cc\|c} 1 & 2 & -1 \\ 3 & 0 & 4 \\ 0 & -1 & 3 \end{array}\right]$	C: $\left[\begin{array}{cc\|c} 1 & -2 & -1 \\ 0 & 3 & 4 \\ -1 & 0 & 3 \end{array}\right]$
D: $\left[\begin{array}{cc\|c} 1 & -2 & -1 \\ -4 & 3 & 0 \\ 0 & -1 & 3 \end{array}\right]$	E: $\left[\begin{array}{cc\|c} 1 & 2 & -1 \\ 3 & 4 & 0 \\ 0 & -1 & 3 \end{array}\right]$	

1 mark

3. Which one of the following matrices is in row-reduced echelon form?

A: $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B: $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$	C: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	D: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	E: $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

1 mark

4. Let $\left[\begin{array}{ccccc\|c} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$ be the augmented matrix for a system of linear equations. How many parameters are there in the solution for this system?

A: 1	B: 2	C: 3	D: 4	E: 5

1 mark

5. Find the solution to the system of linear equations whose augmented matrix is

$$\left[\begin{array}{ccc\|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

A: $(4,3,2)$	B: $(4,-1,2)$	C: $(-2,-1,2)$	D: $(0,-1,2)$	E: $(0,1,2)$

1 mark

6. Consider the system of linear equations with augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & k & 1 \end{array}\right]$$

Find the value(s) of k for which the system has infinitely many solutions.

A: all values of k	B: $k = 1$ only	C: all $k \neq 1$	D: $k = 0$ only	E: no value of k

Use the following information for questions 7 and 8.

Consider the system of linear equations with augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & k & k \end{array}\right]$$

1 mark

7. Find the value(s) of k for which the system has no solution.

A: $k = 1$ only	B: all $k \neq 1$	C: $k = 0$ only	D: all $k \neq 0$	E: no value of k

1 mark

8. Find the value(s) of k for which the system has exactly one solution.

A: $k = 1$ only	B: all $k \neq 1$	C: $k = 0$ only	D: all $k \neq 0$	E: no value of k

1 mark

9. Find all solutions to the system of linear equations with augmented matrix

$$\left[\begin{array}{ccccc|c} 1 & 3 & 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 3 \\ 2 & 6 & 0 & 4 & 1 & 4 \end{array}\right]$$

A: $(1 - 3s, s, 3, 0, 2)$	B: $(1 - 3s - 2t, s, 3 + t, t, 2)$	C: $(1 + 3s, s, 3, s, 2)$
D: $(1 + 3s - 2t, s + t, 3, s + t, 2)$	E: none of A, B, C or D	

1 mark

10. Let A be a 3×5 matrix, B be a 5×7 matrix and C be a 7×3 matrix. Which one of the following is not defined?

A: ABC	B: CAB	C: $B^TA^T + C$	D: $AB - C^T$	E: $(AB)^T + AB$

1 mark

11. Solve for x in the equation

$$4\begin{bmatrix} 2 & -1 \\ 3 & x \end{bmatrix} = 2\begin{bmatrix} y & z \\ 1 & x \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -10 & 1 \end{bmatrix}$$

A: -2	B: $-\frac{1}{2}$	C: 0	D: $\frac{1}{2}$	E: 2

1 mark

12. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 & 2 \\ 0 & -1 & -2 \end{bmatrix}$.
Find the $(1,2)$-entry of $2A^TB$.

A: -16	B: 16	C: -4	D: -2	E: 2

1 mark 13. If A is a 2×3 matrix, which of the following describes $(A + A)^{2,2}$?

A: the 2×2 zero matrix	B: the 2×3 zero matrix	C: the 3×2 zero matrix
D: the 3×3 zero matrix	E: the expression is not defined	

1 mark 14. If $A = \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix}$, find A^{43}.

A: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	B: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	C: $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	D: $\begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix}$	E: $\begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}$

1 mark 15. If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, find the first row of A^{-1}.

A: $\begin{bmatrix} -7 & 3 \end{bmatrix}$	B: $\begin{bmatrix} 7 & -3 \end{bmatrix}$	C: $\begin{bmatrix} 7 & 3 \end{bmatrix}$	D: $\begin{bmatrix} -7 & -3 \end{bmatrix}$	E: none of A, B, C or D

1 mark 16. Find the value of k for which the matrix $\begin{bmatrix} 5 & -1 \\ 3 & k \end{bmatrix}$ has no inverse.

A: $\frac{5}{3}$	B: $-\frac{5}{3}$	C: 2	D: $\frac{3}{5}$	E: $-\frac{3}{5}$

1 mark 17. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$ and let I be the 3×3 identity matrix.
Which of the following is/are true?

(i) $A^3 = I$ $\quad$ (ii) $A^{-1} = A^2$

A: (i) only	B: (ii) only	C: both (i) and (ii)
D: neither (i) nor (ii)	E: cannot be determined	

1 mark 18. You are given that $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ has $A^{-1} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$.
Consider the system of linear equations shown here:

$$\begin{aligned} ax + by + cz &= 1 \\ dx + ey + fz &= -1 \\ gx + hy + iz &= 3 \end{aligned}$$

Find the value of y in the unique solution to this system.

A: 2	B: 8	C: 10	D: 5	E: 9

PART B (7 marks)

SHOW YOUR WORK FOR ALL QUESTIONS IN PART B

4 marks 19. In each of the following, find the row-reduced echelon form of the given augmented matrix and then state all solutions to the system of linear equations represented by the given matrix. If there are no solutions, print 'no solutions'.

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ -1 & 1 & 0 & 3 \\ 1 & 0 & 1 & 4 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 0 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 3 \\ 1 & -1 & 1 & 1 \end{array}\right]$

3 marks 20. Use the method of row reduction to find the inverse of $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & -4 \\ 1 & 2 & -1 \end{bmatrix}$.

Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 1229A Test 2

Friday, November 17, 2017 **Code 111** 7:00 p.m. - 8:30 p.m.

INSTRUCTIONS

1. Fill in the tops of this page **and the back of this page** completely. Be sure to print your name **legibly**.
2. Fill in the top of the scantron card completely. **Both print AND code** your Student Number, Section Number (see below) and Exam Code (shown above).
3. CALCULATORS AND NOTES ARE NOT PERMITTED.
4. DO NOT UNSTAPLE THE BOOKLET.
5. There are two parts to this examination: PART A (18 marks) in multiple choice format and PART B (7 marks) in written answer format.
6. In Part A, **circle** the correct answer to each question **on this paper** AND fill in the appropriate box on the **scantron** card with an HB pencil. This question paper will be returned to you.
7. In Part B, write your answer in the space provided.
8. Questions are printed on both sides of the paper. They begin on Page 1 and continue to Page 5. Be sure that your booklet is complete.
9. You must hand in this question paper, your scantron card, and all rough work sheets.
10. Circle your section in the list below.

Instructor	Campus/College	Time	Section
Ghorbanpour	Main	9:30 MWF	001
Moschandreou	Main	12:30 MWF	002
Olds	Main	1:30 MWF	003
Zsamboki	Main	8:30 MWF	004
Florence	Brescia	8:30 MTuTh	530
Florence	Brescia	9:30 MTuW	531
Rastegari	Huron	8:30 MWF	550
Mollahajiaghaei	Huron	11:30 MWF	551
Meredith	King's	9:30 TuTh	570
Meredith	King's	1:30 TuTh	571
Meredith	King's	1:30 MW	572
Kuzmin	King's	7:00 MW	573

11. TOTAL MARKS = 25.

Student Number (**Print**)

Student's Name (**Print**)

FOR GRADING ONLY

PAGE	MARK
1–3	
4	
5	
TOTAL	