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Propositional Logic

Definition: Proposition is a statement that is either true of false
absolutely true / false
E.g: there are 400 trees in Western campus
10 is a prime number (p is a prime number is a predicate logic because p is a parameter)
ANY question/request is not a proposition

Connectives in propositional logic:

AND (conjunction, ^) p ^ q: conjunction of p and q
p AND q is true only when both p and q are true

OR (junction, v) p v q: p OR q is false only when both p and q are false

NOT (negation ): given statement p is true, then the negation of p is false

CONDITIONAL (if.... then..../implies): p -> q: p implies q
p implies q is false only when p is true and q is false, all other conditions are true
p (f) -> q(t) is true: contract: A work and get paid. However, A is still paid even he don't work. The fact is that A is paid finally is true.

equivalent ($\equiv$)

Defination: We say two compound propositions $f(p_1, p_2 \cdots p_n)$ $g(q_1, q_2 \cdots q_n)$ are "equivalent" if for any truth values, f and g gives the same truth value

De Morgan's Law:

Given any two properties of p.q, $\neg(p \vee q) \equiv \neg p \wedge \neg q$

$f \equiv g$ $\Rightarrow$ $f \equiv h$.

$g \equiv h$

* XOR (exclusive or, $\oplus$)

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

$p \oplus q$ is true only when one of $p$ or $q$ is true

Either $p$ or $q$ is true.

$p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$

 

* $p \to q \equiv \neg p \vee q$

| $p$ | $q$ | $p \to q$ | $\neg p \vee q$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | F |
| F | T | T | T |
| F | F | T | T |

* Bi-conditional (iff and only iff)

$p \leftrightarrow q \equiv p \text{ iff } q \equiv (p \to q) \wedge (q \to p)$

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| F | T | F |
| T | F | F |
| F | F | T |

$p \to q$ : if $p$ then $q$

converse : $q \to p$ if $q$ then $p$

inverse : $\neg p \to \neg q$ if no $p$ then no $q$

contrapositive : $\neg q \to \neg p$. if no $q$ then no $p$

$$\underbrace{p \wedge (q \vee r)}_{a} \to \underbrace{(\neg p \vee q) \wedge r}_{b}$$

contrapositive of $a \to b$ is $\neg b \to \neg a$

$\neg b \equiv \neg (\neg p \vee q) \wedge r$

$\equiv \neg(\neg p \vee q) \vee \neg r$

$\equiv (p \wedge \neg q) \vee \neg r$

$p \vee (q \wedge r)$

$\equiv (p \wedge r) \vee (q \wedge r)$

e.g.

$p \to (q \wedge r) \equiv \neg p \vee (\neg q \vee \neg r)$