Sept 20

COMPSCI 3331

Fall 2022

What’s next?

- Current location: up to Lecture 3, Part 1
- Tomorrow: more on Lecture 3.
- Assignment 1: out by Sept 27 (at the latest), due Oct 11.
- Quiz 1: Sept 28 (including Lecture 3 material).
- Updates - this file available before class, “unanswered questions” file.

Software tools

- For assignments, you can use a software tool to draw automata.
- Two that I know of:
  - JFLAP: `https://www.jflap.org/` - self-contained software.
  - FAdo: `https://fado.dcc.fc.up.pt/` - py library
- Not necessary for the course.
- Fine to use for assignments.

Reversal

- Inductive definition.
- Proof of $(xy)^R = y^R x^R$

$\varepsilon^R = \varepsilon$ ← empty string

$(xa)^R = a x^R \ \forall x \in \varepsilon^R$
$\forall a \in \varepsilon$

$(abcd)^R = dcba$

induction on length of y:

base case: $|y| = 0 \Rightarrow y = \varepsilon$

$(xy)^R = (x\varepsilon)^R = x^R = \varepsilon \cdot x^R = y^R x^R$

inductive case:

assume the statement holds for all $y \in \Sigma^*$ with $|y| = n$.

let y be a word with $|y| = n+1$

$\Rightarrow y = wa$ for $w \in \Sigma^*$, $a \in \Sigma$

$(xy^R) = (x(wa))^R = ((xw)a)^R = a(xw)^R$ — definition

$= a w^R x^R = y^R x^R$ — definition

letter, words. Since a is a single letter, no need of reverse itself

w — subword; a — the last letter

Σ^*: a set of languages.

Language Examples

- $L_1 = \{w \in \{a, b\}^* \ : \ |w|_a > |w|_b\}$. 1. w is made up from a and b. 2. w has more a than b.
- $L_2 = \{x\#y \ : \ x, y \in \{0, 1, 2, \ldots, 9\}^* \text{ and } x^2 = y\}$

just a symbol, which has no meaning. aka. placeholder.

e.g. 0#0, 1#1, 2#4 since the order matter, so 4#2 is not in the language.

Language Identities

- $L_1(L_2 \cup L_3) = L_1L_2 \cup L_1L_3$? ✓
- $(L_1L_2)^R = L_2^R L_1^R$? ✓
- $\overline{L^*} = (\overline{L})^*$? ✓

$L_1(L_2 \cup L_3) \subseteq L_1L_2 \cup L_1L_3$.

let $x \in L_1(L_2 \cup L_3)$

$\exists y \in L_1, z \in L_2 \cup L_3$ so

$x = yz$

$L = \phi$ $L^* = \{\varepsilon\}$ $(\overline{L^*}) = \Sigma^*$

$(\overline{L})^* = \Sigma^*$

$\overline{L}$: complement of L.

$\overline{L^*}$: words not in L

$\Sigma = \{a, b\}$ $L = \{a^i ; i \geq 0\}$.

$L^* = \bigcup_{i=0}^{\infty} L^i = L$ (in this case).

$\overline{L^*} = \{w \in \{a,b\}^*, |w|_b > 0\} = (\overline{L})^*$.

a set of words of a, b which has at least one b in each word.

DFAs

- $L_1 = \{x \in \{a,b,c\}^* \ : \ |x|_b \equiv 0 \pmod 3\}$.
- $L_2 = \{(aabc^i)^j \ : \ i \geq 0, j \geq 0\}$
- $L_3 = \{(abbd^i)^j \ : \ i \geq 0, j \geq 0\}$
- $L_4 = L_2 \cup L_3$