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## D and SD

TM  $M$  decides a language  $L \subseteq \Sigma^*$  iff for any string  $w \in \Sigma^*$ :

- $M$  accepts  $w$  if  $w \in L$
- $M$  rejects  $w$  if  $w \notin L$

A language is decidable (in D) iff there is a Turing machine that decides it.

TM  $M$  semidecides a language  $L \subseteq \Sigma^*$  iff for any string  $w \in \Sigma^*$ :

- $M$  accepts  $w$  if  $w \in L$
- $M$  rejects  $w$  OR loops (not halt) if  $w \notin L$

A language is semidecidable (in D) iff there is a Turing machine that decides it.

D is closed under complement - If  $L$  is in D, then  $\neg L$  is in D.

SD is not closed under complement.

A language is in D iff both the language and its complement are in SD

A language is in SD iff it is Turing-enumerable

A language is in D iff it is lexicographically Turing-enumerable

To show in D:

- Make deciding TM
- Lexic. Enumeration
- $L$  and  $\neg L$  are in SD, then  $L$  is in D

To show not in D:

- Diagonalize Reduction

To show in SD

- Make semideciding TM
- Enumerable
- Unrestricted grammar

To show not in SD:

- Reduction

Using reduction to show undecidability:

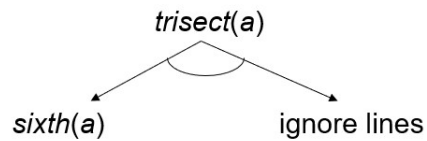
**Theorem:** There exists no general procedure to solve the following problem:

Given an angle  $A$ , divide  $A$  into sixths using only a straightedge and a compass.

**Proof:** Suppose that there were such a procedure, which we'll call *sixth*. Then we could trisect an arbitrary angle:

*trisect*( $a$ : angle) =

1. Divide  $a$  into six equal parts by invoking *sixth*( $a$ ).
2. Ignore every other line, thus dividing  $a$  into thirds.



*sixth* exists  $\rightarrow$  *trisect* exists.

Since we know trisect doesn't exist, we know that sixth doesn't exist