Ex: $x \in P(A) \cup P(B)$

iff $x \in P(A) \vee x \in P(B)$

iff $x \subseteq P(A) \vee x \subseteq P(B)$

iff $\forall y (y \in x \to y \in A) \vee \forall z (z \in x \to z \in B)$.

$P(A) \cup P(B) \neq P(A \cup B)$

e.g. $A = \{2\}$ $B = \{3\}$.

$P(A) \cup P(B) = \{\{2\}, \{3\}, \emptyset\}$.

$P(A \cup B) = \{\{2\}, \{3\}, \{2,3\}, \emptyset\}$.

$P(A) \cap P(B) = P(A \cap B)$.

Intersections + Unions of family of sets.

Family of sets: a set of sets.

$\mathcal{F}$ be a family of sets: $\cap \mathcal{F} = \{x \mid \forall A \in \mathcal{F} (x \in A)\}$.

$\cup \mathcal{F} = \{x \mid \exists A \in \mathcal{F} (x \in A)\}$

e.g. $\mathcal{F} = \{\{1,2,3\}, \{2\}, \{2,4,6\}\}$.

$\cap \mathcal{F} = \{1,2,3\} \cap \{2\} \cap \{2,4,6\} = \{2\}$

$\cup \mathcal{F} = \{1,2,3\} \cup \{2\} \cup \{2,4,6\} = \{1,2,3,4,6\}$.

Ex: $\cap \{[0,x) \mid x \in \mathbb{R}\} = \{0\}$

$\cup \{[0,x) \mid x \in \mathbb{R}\} = [0,\infty)$.

Ex: $x \in \cup \mathcal{F}$ iff $\exists A \in \mathcal{F} (x \in A)$

$x \in P(\cup \mathcal{F})$ iff $x \subseteq \cup \mathcal{F}$

iff $\forall y (y \in x \to y \in \cup \mathcal{F})$

iff $\forall y (y \in x \to \exists A \in \mathcal{F} (y \in A))$

If $\mathcal{F} = \{A_i \mid i \in I\}$.

$\cup \mathcal{F} = \bigcup_{i \in I} A_i$

$\cap \mathcal{F} = \bigcap_{i \in I} A_i$

Ex: $\bigcap_{x \in \mathbb{R}^+} [0,x) = \{0\}$.

$\bigcup_{x \in \mathbb{R}^+} [0,x) = [0,\infty)$.

Ex: For $n \in \mathbb{Z}^+$, let $D(n) = \{d \in \mathbb{Z}^+ \mid d \mid n\}$.

$D(6) = \{1, 2, 3, 6\}$.

$D(0) = \mathbb{Z}^+$.

$\bigcap_{n \in \mathbb{N}} D(n) = \{1\}$.

$\bigcup_{n \in \mathbb{N}} D(n) = \mathbb{Z}^+$.

$\bigcup_{n \geq 30} D(n) = \mathbb{Z}^+$

$\bigcap_{n \geq 30} D(n) = \{1\}$.