# How to calculate p-values

## Chapter 22

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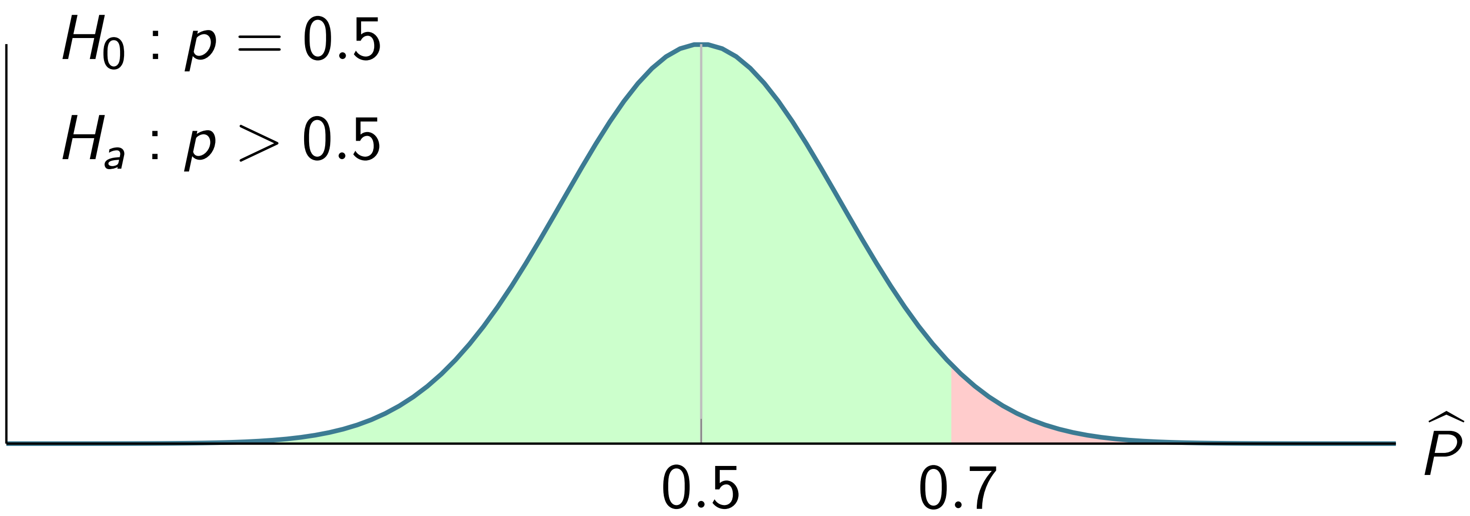
1. Proportion: right-hand side
2. Proportion: left-hand side
3. Proportion: two sided
4. Mean: right-hand side
5. Mean: left-hand side
6. Mean: two sided




A data set of size $n = 16$ has resulted in $\widehat{p} = 0.7$

Does this evidence reject or retain $H_0$?

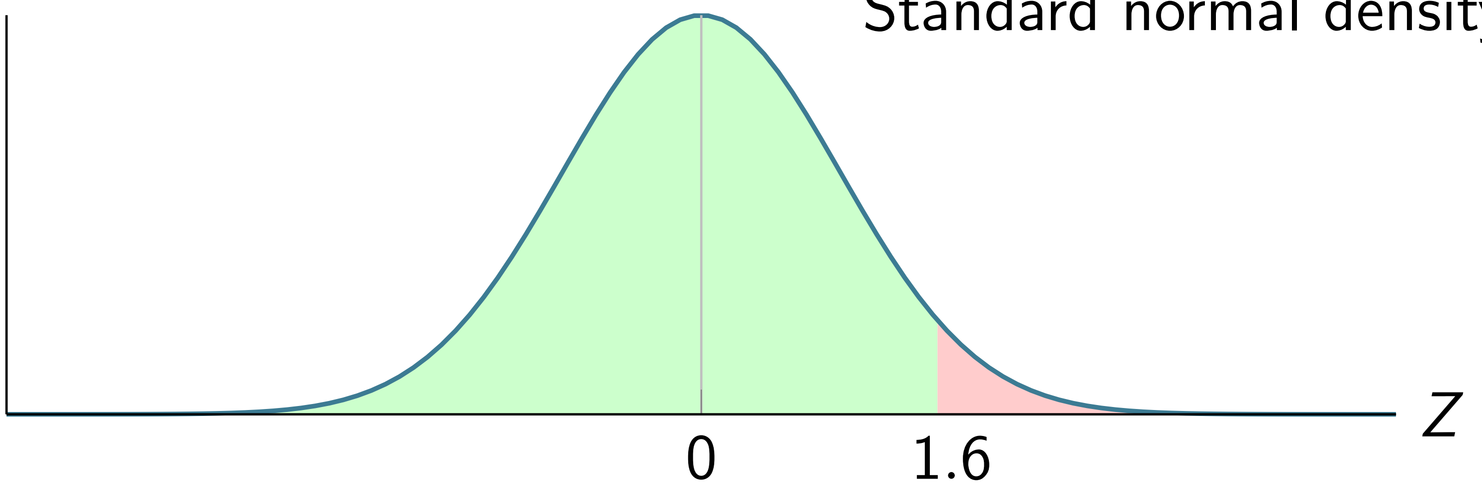
We need to calculate the p-value

$$\begin{aligned} \text{p-value} &= \text{red area} \\ &= \Pr\left(\widehat{P} > 0.7\right) \end{aligned}$$

# Reducing to the standard normal

$$\begin{aligned}
\text{p-value} &= \Pr\left(\widehat{P} > 0.7\right) \\
&= \Pr\left(\frac{\widehat{P} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} > \frac{0.7 - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}\right) \quad \text{with} \quad p_0 = 0.5 \;\&\; n = 16 \\
&\approx \Pr\left(Z > \frac{0.7 - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}\right) \qquad \text{(approximately normal)} \\
&= \Pr\left(Z > \frac{0.2}{0.5/4}\right) \\
&= \Pr\left(Z > 1.6\right)
\end{aligned}$$

Standard normal density (Table B)




$$
\begin{aligned}
\text{p-value} &\approx \Pr(Z > 1.6) \qquad \text{(approximately normal)}\\
&= 1 - \Pr(Z \leq 1.6)\\
&= 1 - 0.9452\\
&= 0.0548
\end{aligned}
$$

Note a little trick: due to symmetry around 0, the probability $\Pr(Z > 1.6)$ is equal to $\Pr(Z < -1.6)$, which is convenient when using Table B.

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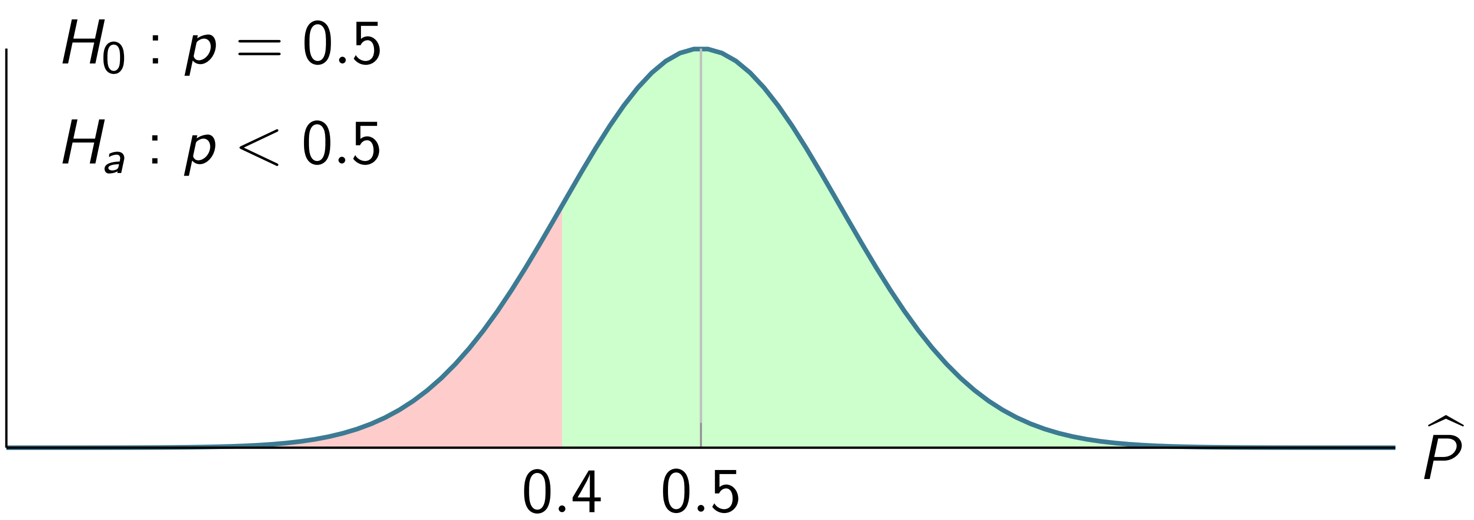



A data set of size $n = 100$ has resulted in $\widehat{p} = 0.4$

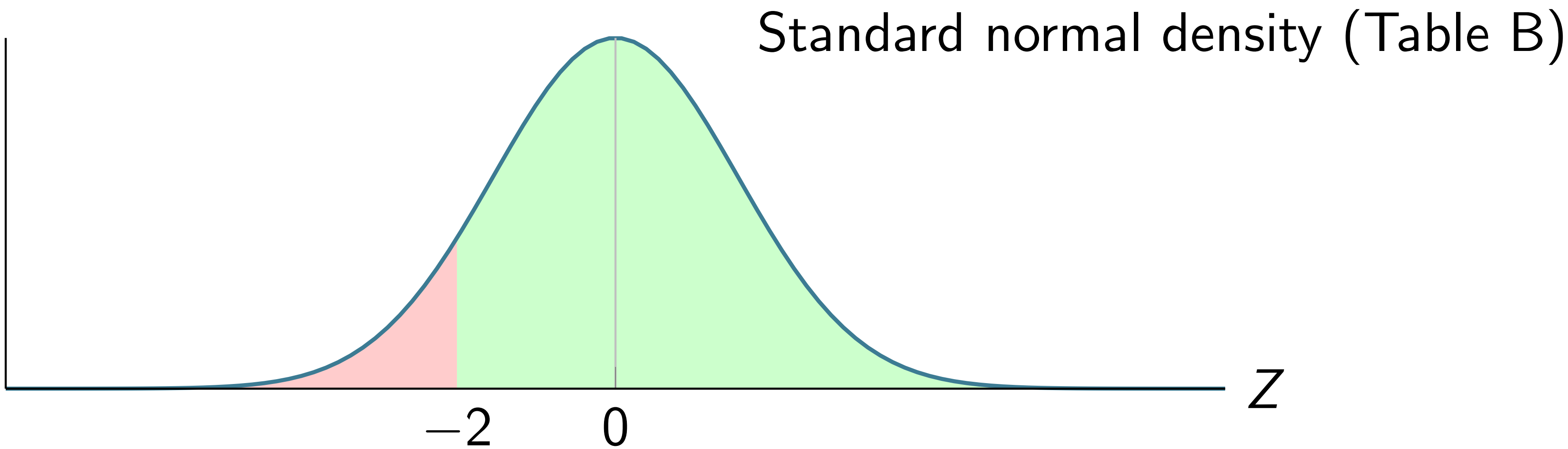
Does this evidence reject or retain $H_0$?

We need to calculate the p-value

$$\begin{aligned} \text{p-value} &= \text{red area} \\ &= \Pr\left(\widehat{P} < 0.4\right) \end{aligned}$$

## Reducing to the standard normal

$$
\begin{aligned}
\text{p-value} &= \Pr\left(\widehat{P} < 0.4\right) \\
&= \Pr\left(\frac{\widehat{P} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} < \frac{0.4 - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}\right) \quad \text{with} \quad p_0 = 0.5 \;\&\; n = 100 \\
&\approx \Pr\left(Z < \frac{0.4 - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}\right) \qquad \text{(approximately normal)} \\
&= \Pr\left(Z < \frac{-0.1}{0.5/10}\right) \\
&= \Pr(Z < -2)
\end{aligned}
$$




$$\text{p-value} \approx \Pr\left(Z < -2\right) \qquad \text{(approximately normal)}$$
$$= 0.0227$$

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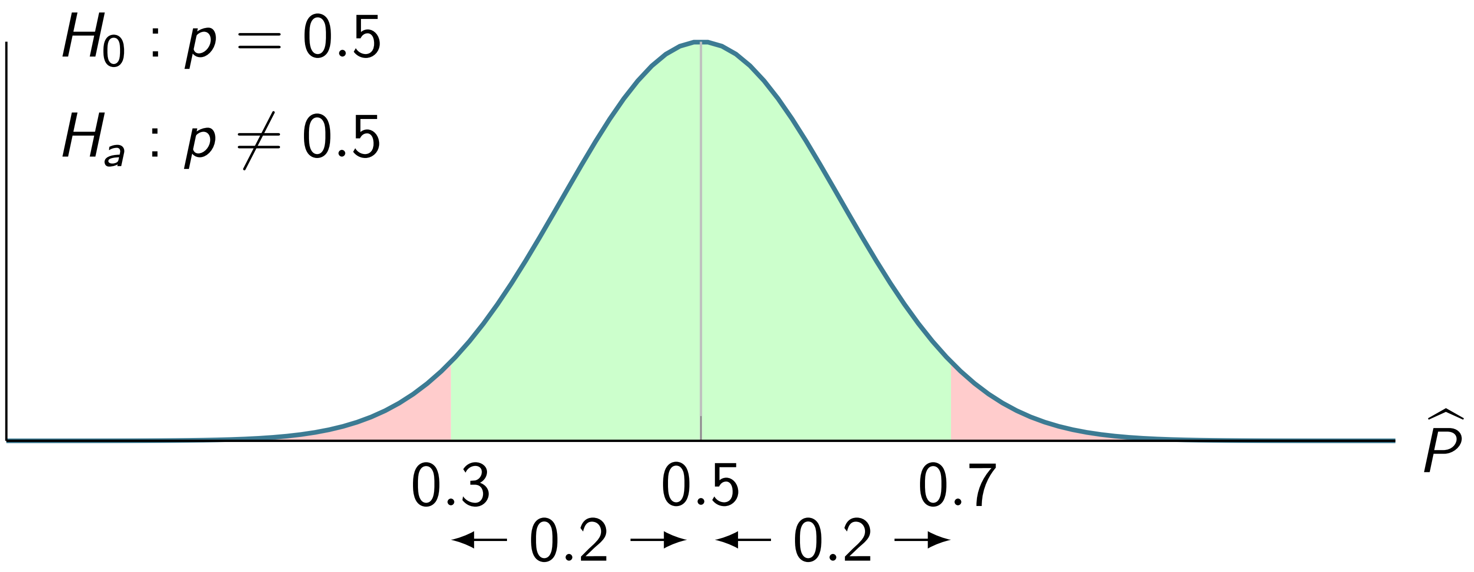


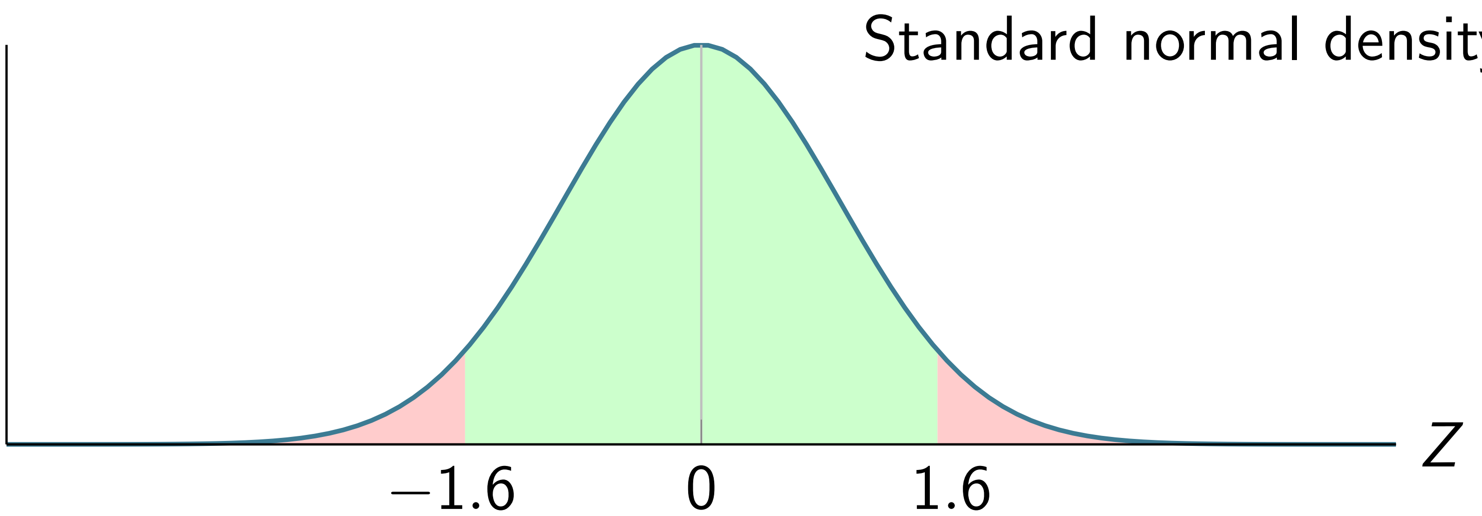

A data set of size $n = 16$ has resulted in $\widehat{p} = 0.7$

Does this evidence reject or retain $H_0$?

We need to calculate the p-value

$$\begin{aligned} \text{p-value} &= \text{red area} + \text{red area} \\ &= \Pr\left(\widehat{P} < 0.3\right) + \Pr\left(\widehat{P} > 0.7\right) \\ &= 2\Pr\left(\widehat{P} > 0.7\right) \end{aligned}$$

# Reducing to the standard normal




$$\text{p-value} = 2\,\Pr\left(\widehat{P} > 0.7\right) \quad \text{with} \quad p_0 = 0.5 \;\&\; n = 16$$

$$\approx 2\,\Pr\left(Z > 1.6\right) \qquad \text{(approximately normal)}$$

$$= 2 \times 0.0548$$

$$= 0.1096$$

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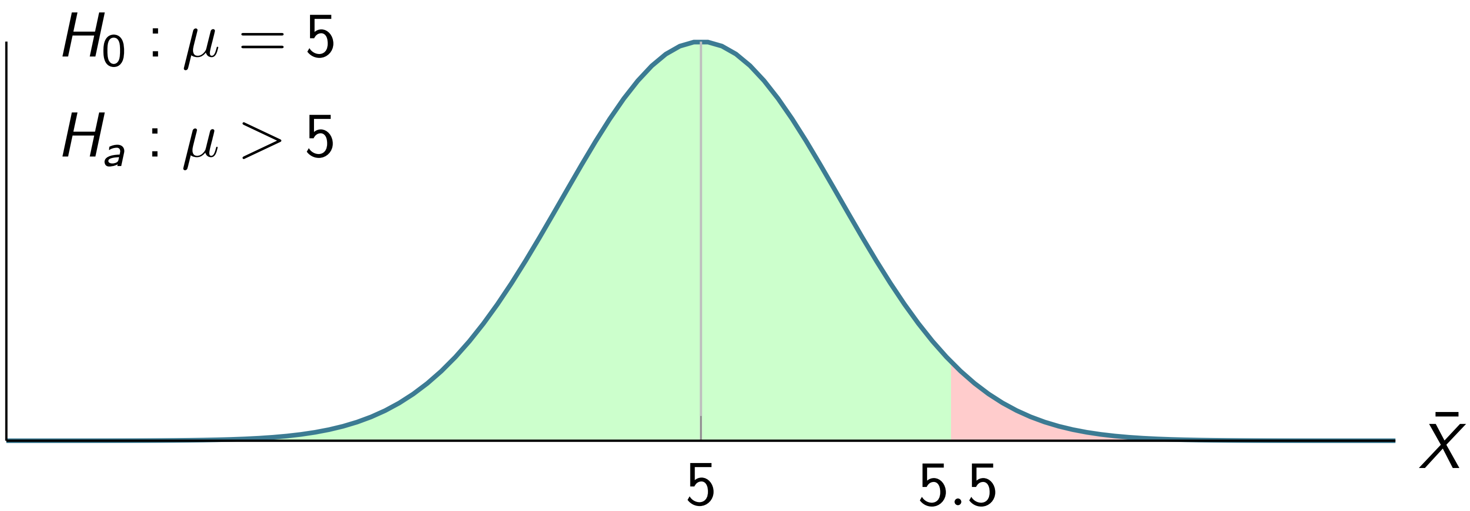



A data set of size $n = 36$ has resulted in $\bar{x} = 5.5$ and $s = 2$

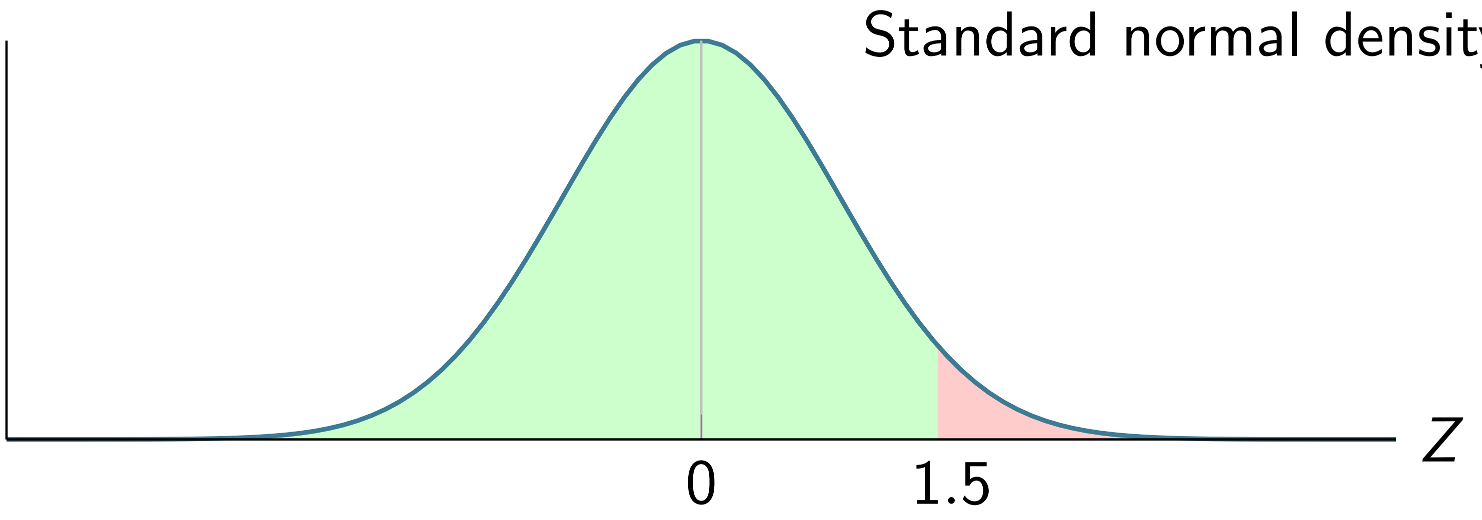
Does this evidence reject or retain $H_0$?

We need to calculate the p-value

$$\begin{aligned} \text{p-value} &= \text{red area} \\ &= \Pr\left(\bar{X} > 5.5\right) \end{aligned}$$

## Reducing to the standard normal

$$
\begin{aligned}
\text{p-value} &= \Pr\left(\bar{X} > 5.5\right) \\
&= \Pr\left(\frac{\bar{X}-\mu_0}{\frac{\sigma}{\sqrt{n}}} > \frac{5.5-\mu_0}{\frac{\sigma}{\sqrt{n}}}\right) \quad \text{with} \quad \mu_0 = 5 \\
&\approx \Pr\left(Z > \frac{5.5-\mu_0}{\frac{\sigma}{\sqrt{n}}}\right) \qquad \text{(approximately normal)} \\
&= \Pr\left(Z > \frac{0.5}{2/6}\right) \\
&= \Pr\left(Z > 1.5\right)
\end{aligned}
$$




$$\begin{aligned} \text{p-value} &\approx \Pr(Z > 1.5) \qquad \text{(approximately normal)} \\ &= 1 - \Pr(Z \leq 1.5) \\ &= 1 - 0.9332 \\ &= 0.0668 \end{aligned}$$

Note a little trick: due to symmetry around 0, the probability $\Pr(Z > 1.5)$ is equal to $\Pr(Z < -1.5)$, which is convenient when using Table B.

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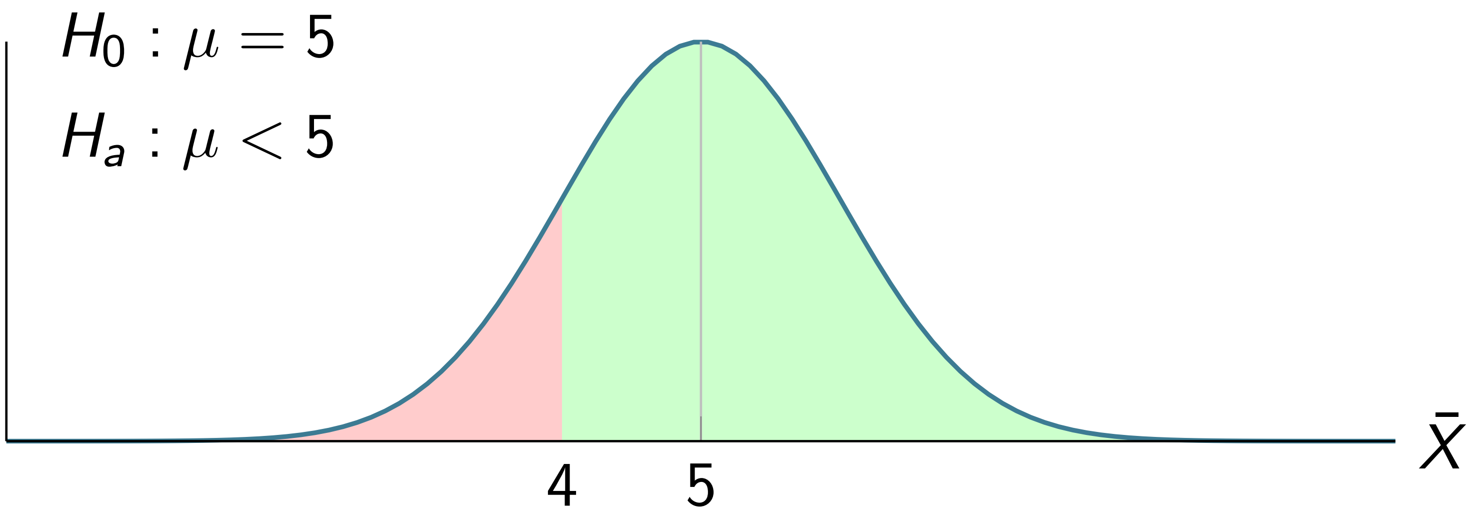



A data set of size $n = 36$ has resulted in $\bar{x} = 4$ and $s = 2$

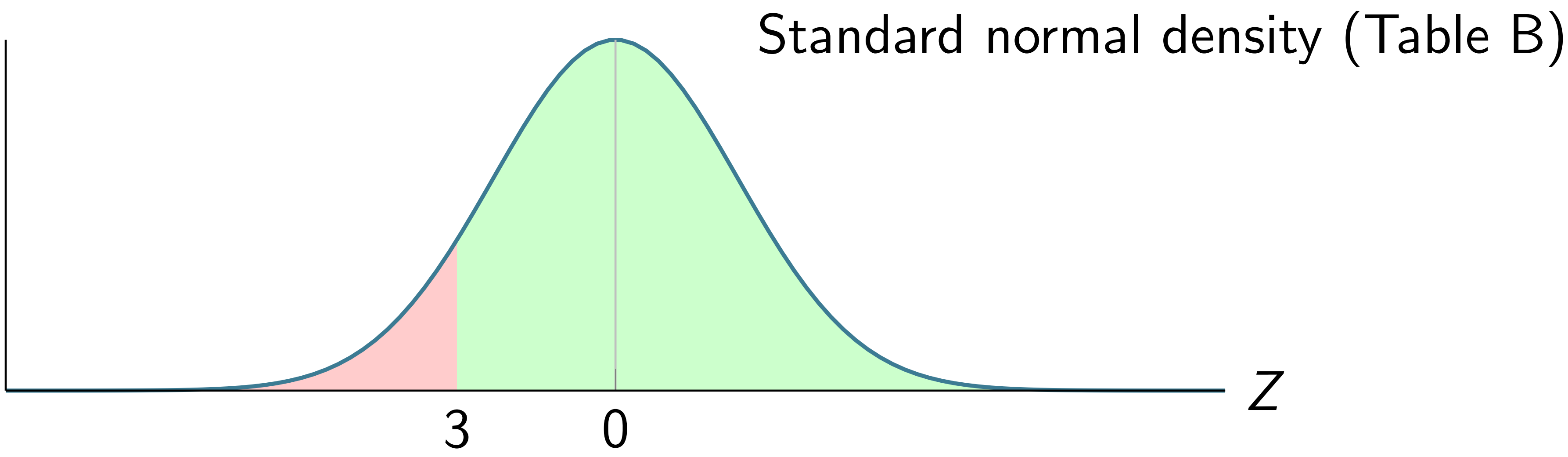
Does this evidence reject or retain $H_0$?

We need to calculate the p-value

$$\begin{aligned} \text{p-value} &= \text{red area} \\ &= \Pr\left(\bar{X} < 4\right) \end{aligned}$$

# Reducing to the standard normal

$$
\begin{aligned}
\text{p-value} &= \Pr\left(\bar{X} < 4\right) \\
&= \Pr\left(\frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} < \frac{4 - \mu_0}{\frac{\sigma}{\sqrt{n}}}\right) \quad \text{with} \quad \mu_0 = 5 \\
&\approx \Pr\left(Z < \frac{4 - \mu_0}{\frac{\sigma}{\sqrt{n}}}\right) \qquad \text{(approximately normal)} \\
&= \Pr\left(Z < \frac{-1}{2/6}\right) \\
&= \Pr\left(Z < -3\right)
\end{aligned}
$$




$$\text{p-value} \approx \Pr(Z < -3) \qquad \text{(approximately normal)}$$

$$= 0.0013$$

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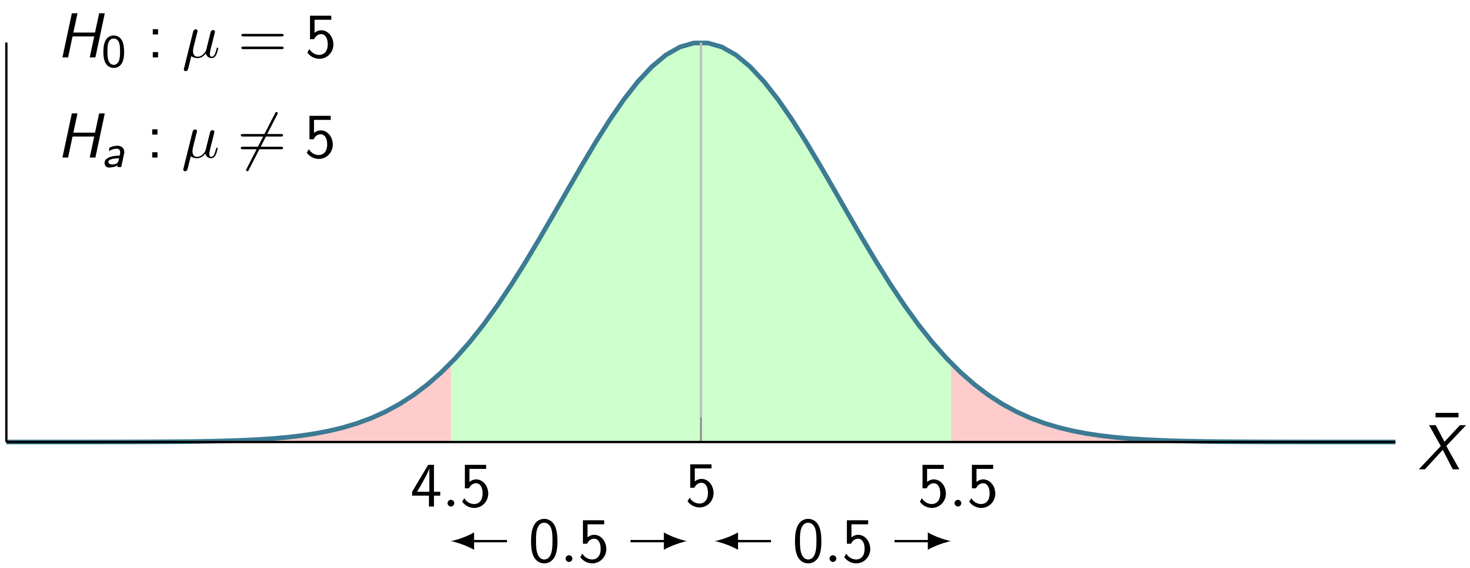


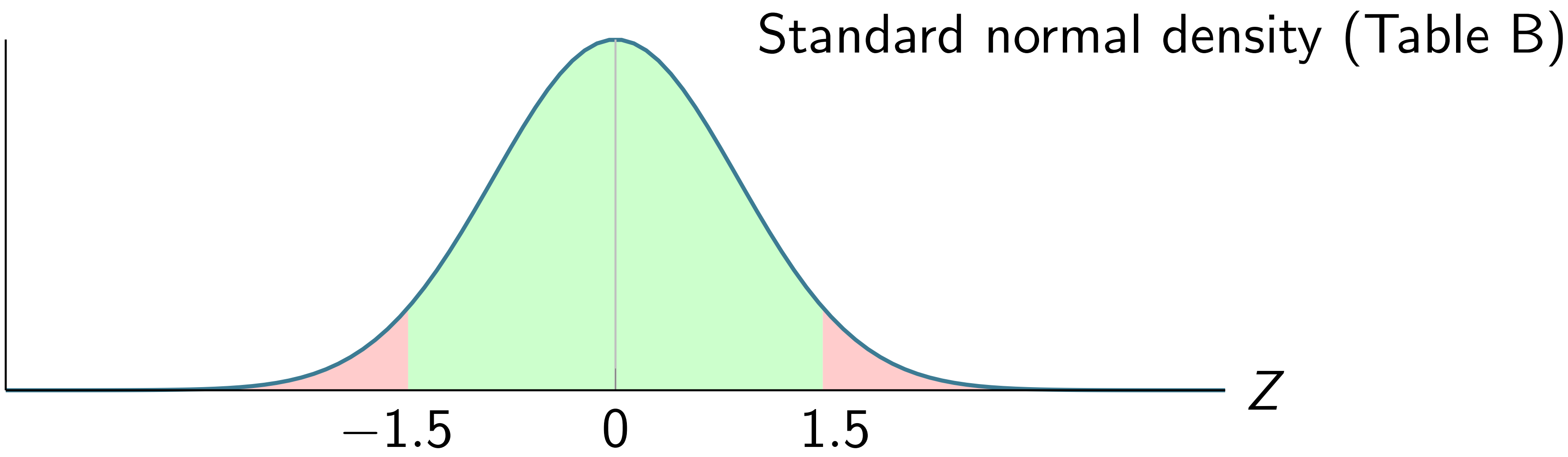

A data set of size $n = 36$ has resulted in $\bar{x} = 5.5$ and $s = 2$

Does this evidence reject or retain $H_0$?

We need to calculate the p-value

$$\begin{aligned} \text{p-value} &= \text{red area} + \text{red area} \\ &= \Pr\left(\bar{X} < 4.5\right) + \Pr\left(\bar{X} > 5.5\right) \\ &= 2\Pr\left(\bar{X} > 5.5\right) \end{aligned}$$

# Reducing to the standard normal




$$\begin{aligned} \text{p-value} &\approx 2\,\Pr\left(Z > 1.5\right) \qquad \text{(approximately normal)} \\ &= 2 \times 0.0668 \\ &= 0.1336 \end{aligned}$$