Schedule

Today: intersection points and distances

Next Monday: Extra problem sheet for Chapter 1.

Next Tuesday: Start Chapter 2.

Reminder: Term test 1 is at 1:30 - 3:00 p.m on 5th Oct

Test 1 covers to Section 2.1

Distances and intersection points

Recap: Lines in $\mathbb{R}^2$ or $\mathbb{R}^3$

	lines in $\mathbb{R}^2$	lines in $\mathbb{R}^3$
Point-parallel form	$\vec{x}(t) = (p_1, p_2) + t(v_1, v_2)$	$\vec{x}(t) = (p_1, p_2, p_3) + t(v_1, v_2, v_3)$
Parametric form	$x = p_1 + tv_1$ and $y = p_2 + tv_2$	$x = p_1 + tv_1, y = p_2 + tv_2$ and $z = p_3 + tv_3$
Two-point form	$\vec{x}(t) = (1-t)(p_1, p_2) + t(q_1, q_2)$	$\vec{x}(t) = (1-t)(p_1, p_2, p_3) + t(q_1, q_2, q_3)$
Point-normal form	$(n_1, n_2) \cdot (\vec{x} - (p_1, p_2)) = 0$	?
Standard form	$ax + by = c$	?

Point-normal form for a plane in $\mathbb{R}^3$

$$(n_1, n_2, n_3) \cdot (\vec{x} - (p_1, p_2, p_3)) = 0.$$

Standard form for a plane in $\mathbb{R}^3$

$$ax + by + cz = d.$$

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Let $P(p_1, p_2, p_3)$ be a point in $\mathbb{R}^3$ and let $ax + by + cz = d$ be a plane in $\mathbb{R}^3$. We want to find a point $Q(q_1, q_2, q_3)$ on Π such that $\overrightarrow{PQ}$ is normal to Π.

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Let $P(p_1, p_2, p_3)$ be a point in $\mathbb{R}^3$ and let $ax + by + cz = d$ be a plane in $\mathbb{R}^3$. We want to find a point $Q(q_1, q_2, q_3)$ on Π such that $\overrightarrow{PQ}$ is normal to Π.

- $aq_1 + bq_2 + cq_3 = d$ since Q is on Π
- (a, b, c) is parallel to $\vec{q} - \vec{p}$

where $\vec{q} = (q_1, q_2, q_3)$ and $\vec{p} = (p_1, p_2, p_3)$.

Theorem Consider any plane Π. Let $\vec{n}$ be any normal vector for the plane Π and let Q be any point on plane Π. Consider any other point P which is not on the plane Π. Then the distance between point P and plane Π is given by

$$distance = \frac{|\vec{n} \cdot (\vec{q} - \vec{p})|}{\|\vec{n}\|}.$$

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Example 1. Find the distance between the point $P(1,2,3)$ and the plane with point-normal form equation $(1,2,1) \cdot (\vec{x} - (3,-1,0)) = 0$.

2. Find the distance from the origin to the plane $x + 2y + 3z = 2$.

Distance between a point to a line

Let P be a point and let L be a line. Then the distance between P and L is the shortest distance between P and point Q on the line.

Idea: find Q on the line L such that $\overrightarrow{PQ}$ is perpendicular to L

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Example Find the distance between the point $P(1, 2)$ and the line L described by $2x + y = 1$.

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Use appropriate forms of lines or planes and find solutions of equations.

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a) use parametric forms of lines
or
b) use one parametric form and one standard form of lines

Find intersection points

Find the intersection point(s) of two lines or find the intersection point(s) of a line and a plane.

Use appropriate forms of lines or planes and find solutions of equations.

- To find the intersection point of two lines

a) use parametric forms of lines
or
b) use one parametric form and one standard form of lines

- To find the intersection point of a line and a plane

use the parametric form of a line and the standard form of a plane.

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1) Two lines are the same;

2) Two different lines but they are parallel;

3) Not the cases above.

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You may also come across different situations in order to find the intersection point(s) of a line and a plane:

1) the line is on the plane;

2) the line is not on the plane but is parallel to the plane;

3) Not the cases above.

You may also come across different situations in order to find the intersection point(s) of two lines:

1) two lines are the same; infinitely many intersection points

2) two different but they are parallel; They do not have an intersection point;

3) Not the cases above.
If they are lines in $\mathbb{R}^2$, then these two lines have a single intersection point. If they are lines in $\mathbb{R}^3$, it might have one intersection point but it might also not intersect.

You may also come across different situations in order to find the intersection point(s) of a line and a plane:

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You may also come across different situations in order to find the intersection point(s) of a line and a plane:

1) the line is on the plane;
infinitely many intersection points, i.e., all the points of the line are on the plane;

2) the line is not on the plane but is parallel to the plane;
They do not have an intersection point;

3) Not the cases above. only one intersection point.

Examples

1. Find the point of intersection of the line L_1: $\vec{x}(t) = (1, 1, 2) + t(2, 1, -1)$ with the line L_2: $\vec{x}(s) = (0, 1, 2) + s(1, -1, 1)$.

2. Find the point of intersection of the line $\vec{x}(t) = (2, 1, 3) + t(2, -2, 1)$ with the plane $x + 2y - z = 7$.

3. Find the point of intersection of the lines L_1: $\vec{x}(t) = (1, 0) + t(3, -1)$ and L_2: $2x - y = 6$.