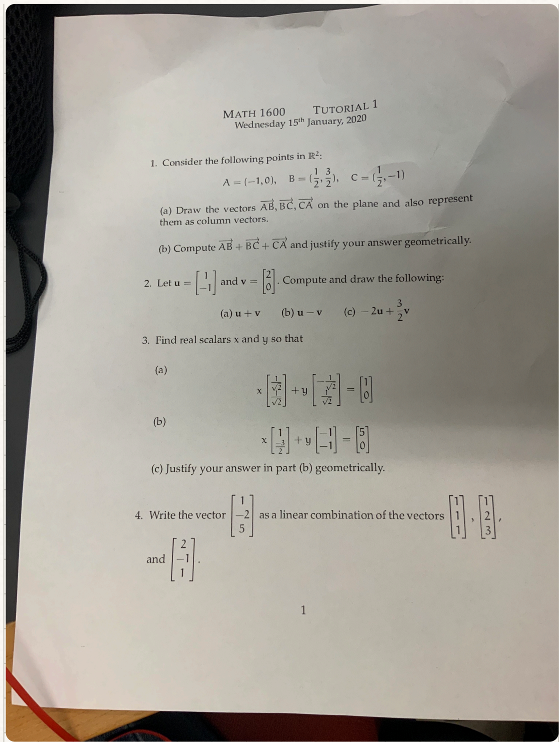

MATH 1600 TUTORIAL 1
Wednesday 15th January, 2020

1. Consider the following points in $\mathbb{R}^2$:
$$A = (-1, 0), \quad B = (\tfrac{1}{2}, \tfrac{3}{2}), \quad C = (\tfrac{1}{2}, -1)$$
(a) Draw the vectors $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$ on the plane and also represent them as column vectors.
(b) Compute $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$ and justify your answer geometrically.

2. Let $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$. Compute and draw the following:
(a) $\mathbf{u} + \mathbf{v}$ (b) $\mathbf{u} - \mathbf{v}$ (c) $-2\mathbf{u} + \frac{3}{2}\mathbf{v}$

3. Find real scalars x and y so that
(a)
$$x \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} + y \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
(b)
$$x \begin{bmatrix} 1 \\ -\frac{3}{2} \end{bmatrix} + y \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$
(c) Justify your answer in part (b) geometrically.

4. Write the vector $\begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$.

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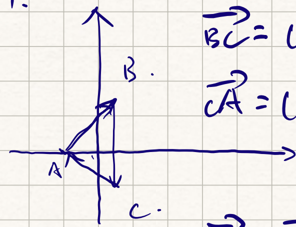
1. $\vec{AB} = (\frac{3}{2}, \frac{3}{2})$



$\vec{BC} = (0, -1)$.

$\vec{CA} = (-\frac{3}{2}, 1)$.

$\vec{AB} + \vec{BC} + \vec{CA} = 0$.

2. $u = (1, -1)$ $v = (2, 0)$

$u + v = (3, -1)$ $u - v = (-1, -1)$

$-2u + \frac{3}{2}v = (-2, 2) + (3, 0) = (1, 2)$.

3. $\frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}y = 1$. $x = \frac{1}{\sqrt{2}}$ $y = -\frac{1}{\sqrt{2}}$.

$\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y = 0$.

$x - y = 5$ $x = 2$

$-\frac{3}{2}x - y = 0$. $y = -3$.

4. $(1, -2, 5) = x(1, 1, 1) + y(1, 2, 3)$
$+ z(2, -1, 1)$.

$1 = x + y + 2z$ $-3 = 3x + 5y$.
$-2 = x + 2y - z$ $x = -6$.
$5 = x + 3y + z$. $3 = 2x + 5y$ $y = 3$
$z = 2$.

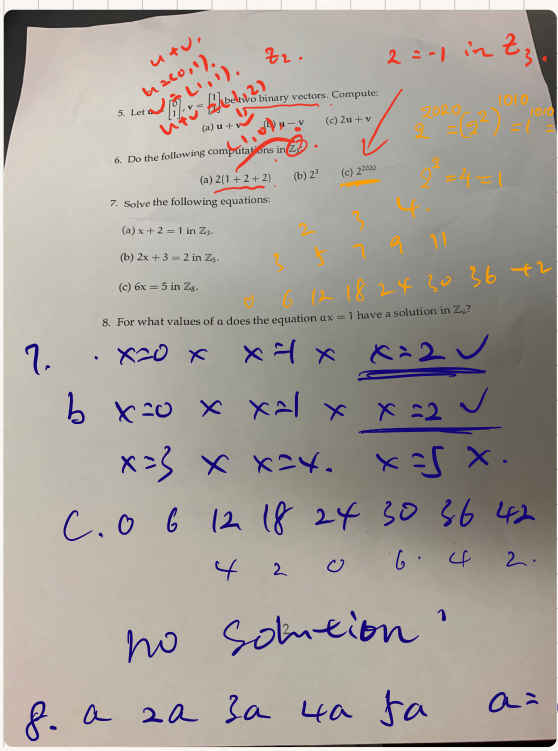
$\therefore (1, -2, 5) = -6(1, 1, 1) + 3(1, 2, 3)$
$+ 2(2, -1, 1)$.

5. $u = (0, 1)$ $v = (1, 1)$. in $\mathbb{Z}_2$.

$u + v = (1, 0)$. $u - v = (1, 0)$

$2u + v = (1, 1)$.

6. $a = 1$ $b = 2$ $c = 1$.



5. Let $\mathbf{u} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ be two binary vectors. Compute:
(a) $\mathbf{u} + \mathbf{v}$ (b) $\mathbf{u} - \mathbf{v}$ (c) $2\mathbf{u} + \mathbf{v}$

6. Do the following computations in $\mathbb{Z}_3$:
(a) $2(1 + 2 + 2)$ (b) 2^3 (c) 2^{2020}

7. Solve the following equations:
(a) $x + 2 = 1$ in $\mathbb{Z}_3$.
(b) $2x + 3 = 2$ in $\mathbb{Z}_5$.
(c) $6x = 5$ in $\mathbb{Z}_8$.

8. For what values of a does the equation $ax = 1$ have a solution in $\mathbb{Z}_6$?

7. $x = 0$ × $x = 1$ × $\underline{x = 2}$ ✓

b $x = 0$ × $x = 1$ × $\underline{x = 2}$ ✓

$x = 3$ × $x = 4$. $x = 5$ ×.

c. 0 6 12 18 24 30 36 42

4 2 0 6. 4 2.

no solution.

8. a $2a$ $3a$ $4a$ $5a$ $a = 1 / \frac{1}{2} / \frac{1}{3} / \frac{1}{4} / \frac{1}{5}$.