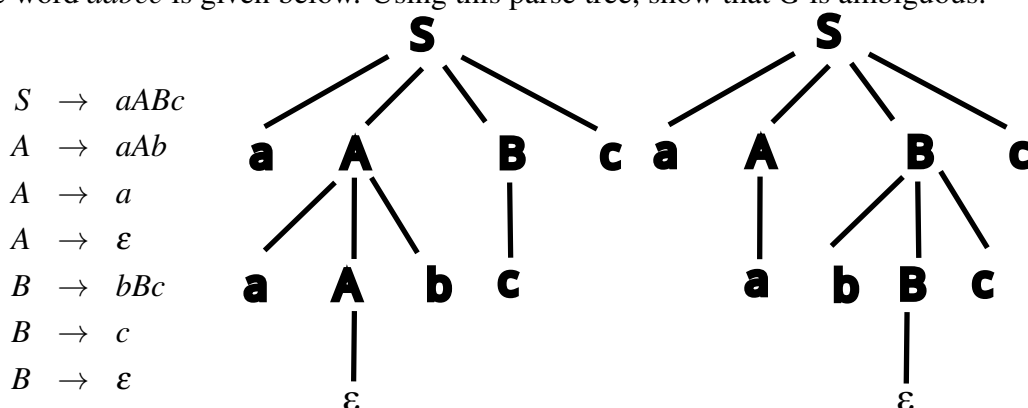# COMPSCI 3331 - Fall 2022 - Quiz 5

(1 mark) 1. Consider the CFG $G = (\{S,A,B\}, \{a,b,c\}, P, S)$ with P given below. A parse tree for the word *aabcc* is given below. Using this parse tree, show that G is ambiguous.

$$
\begin{aligned}
S &\rightarrow aABc \\
A &\rightarrow aAb \\
A &\rightarrow a \\
A &\rightarrow \varepsilon \\
B &\rightarrow bBc \\
B &\rightarrow c \\
B &\rightarrow \varepsilon
\end{aligned}
$$

Solution: the parse tree on the right gives an alternate parse tree for aabcc. As this word has two parse trees, the grammar is ambiguous.

(1 mark) 2. Suppose that a CFG G over the alphabet $\Sigma = \{a,b,c\}$ has nullable nonterminals A, B, C, D. Suppose that G also has productions $A \rightarrow aEFc$, $A \rightarrow \varepsilon$ and $C \rightarrow CbbD$ (there may be others, but you are only asked about these). Based on these productions only, list the **new** productions that are **added** to G when removing ε productions.

Solution: the only production from the list that is affected is $C \rightarrow CbbD$, as none of the others have nullable nonterminals on the right-hand side. So the added productions are

$$
\begin{aligned}
C &\rightarrow bbD \\
C &\rightarrow Cbb \\
C &\rightarrow bb
\end{aligned}
$$

(1 mark) 3. Suppose that a CFG G over the alphabet $\Sigma = \{a,b\}$ has a unit pair (A, Z) and the productions $A \rightarrow Z$, $A \rightarrow ZZ$, $Z \rightarrow aBaa$ and $Z \rightarrow BaaAa$ (there may be others, but you are only asked about these). Based on these productions only, list the **new** productions that are **added** to G when removing unit productions.

Solution: The added productions are

$$A \rightarrow aBaa$$
$$A \rightarrow BaaAa$$

This reflects adding the two productions for Z to now be derivable from A directly, instead of using the unit production $A \rightarrow Z$.

*Note that you should **not** include, for example, $A \rightarrow aBaaaBaa$. That's because $A \rightarrow ZZ$ remains as a production, which allows us to obtain $A \Rightarrow^* aBaaaBaaa$ through three steps of the remaining productions. Adding $A \rightarrow aBaaaBaa$ is not a part of removing unit productions.*