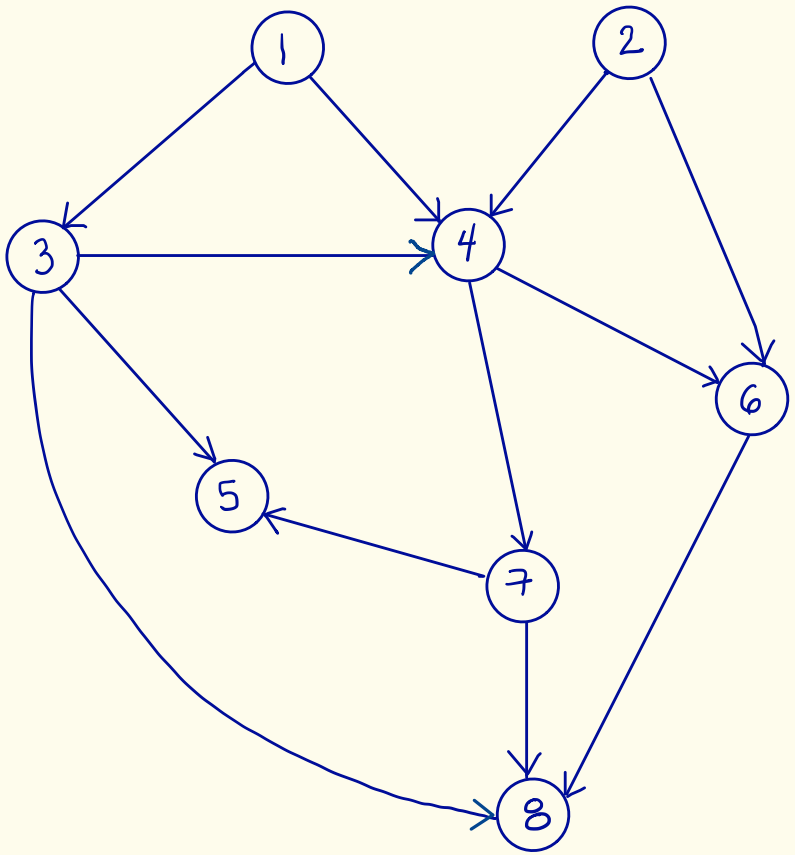
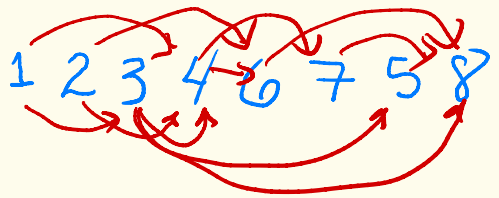
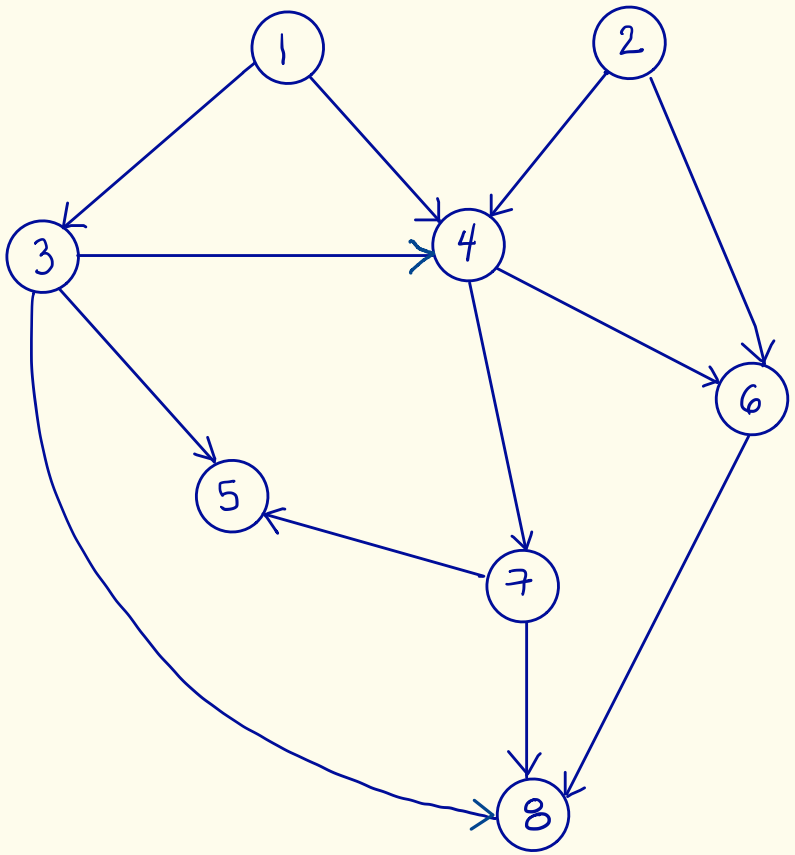


Digraph

Topological ordering

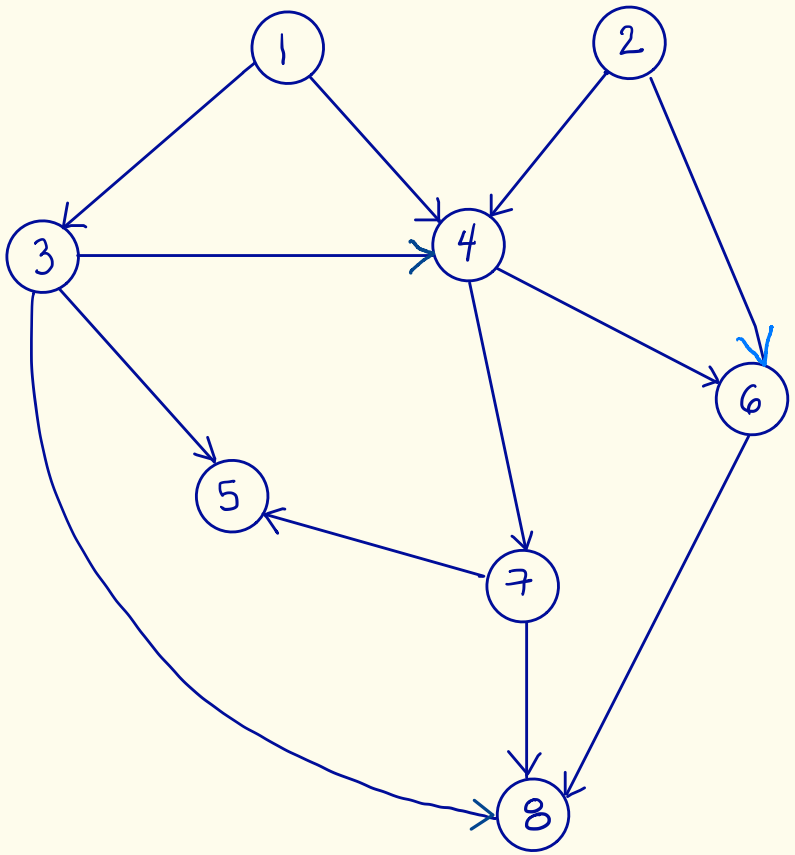




Digraph

Topological ordering?

1 2 3 4 5 6 7 8



Digraph

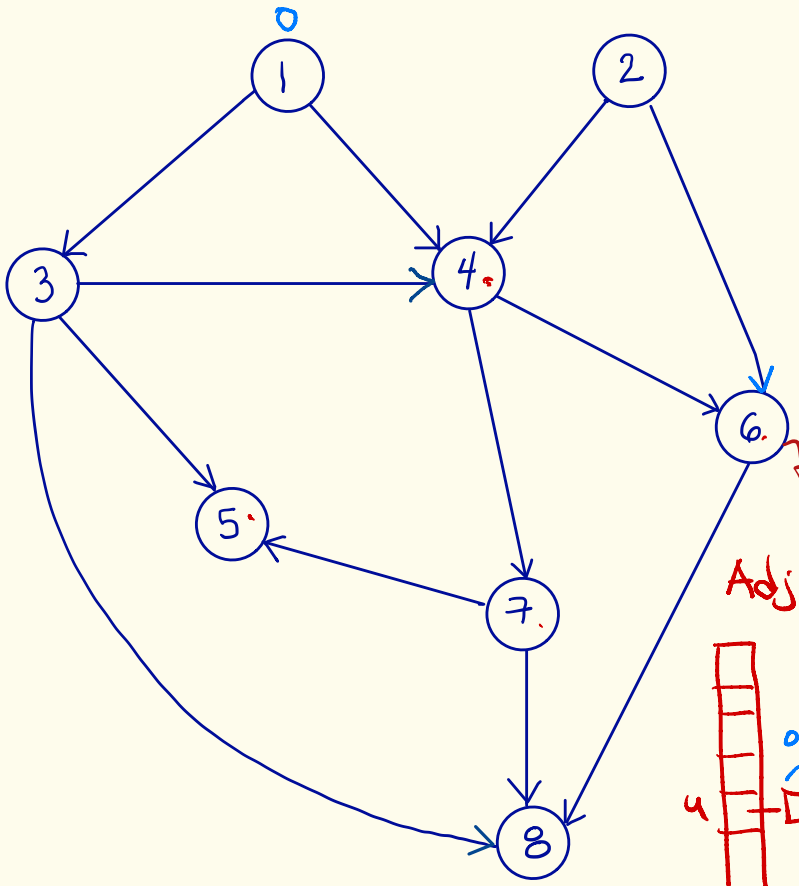
Topological orderings

1 2 3 4 6 7 5 8

2 1 3 4 6 7 5 8

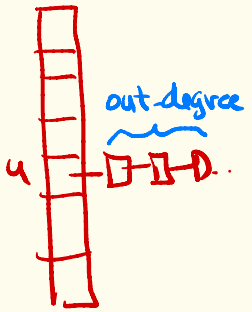
1 2 3 4 7 5 6 8

DAG
Directed
Acyclic
Graph



enqueue for the 1st time.

Adj. List



Algorithm TopologicalOrdering(G)
In: Directed graph G=(V,E)
Out: Topological ordering for the vertices of G

```
Q ← empty queue                                   } c2
For each node u of G do
    if u.in-degree = 0 then                       } c1   } c1 n
        Q.enqueue(u)
#iter = n
While Q is not empty do {
    u ← Q.dequeue                                 } c4   } c4 + c3 x out-deg(u)
    print(u)
    For each edge (u,v) do {                      } c3 x out-deg(u)
        v.in-degree ← v.in-degree - 1             } c3
        if v.in-degree = 0 then
            Q.enqueue(v)
}
```

$$c_2 + c_1 n + \sum_{u \in V} (c_4 + c_3 \times \text{out-degree}(u))$$

$$= c_2 + c_1 n + \sum_{u \in V} c_4 + c_3 \sum_{u \in V} \text{out-degree}(u)$$

$$= c_2 + c_1 n + c_4 n + c_3 m \text{ is } O(n+m)$$