Review Session

COMPSCI 3331

Fall 2022

Question 1

1. Let G be the CFG defined by the following set of productions.

$$
\begin{aligned}
S &\rightarrow bbAaA \mid SSaa \mid aa \mid ABC \\
A &\rightarrow Ab \mid Ac \mid CC \\
B &\rightarrow BA \mid bb \mid Dd \\
C &\rightarrow DA \mid \varepsilon \\
D &\rightarrow a
\end{aligned}
$$

Give an equivalent grammar to G that has no ε-productions.

Question 2

2. Let G be the CFG defined by the following set of productions.

$$\begin{array}{lcl} S & \rightarrow & SbaB \mid aa \mid ABC \\ A & \rightarrow & Ba \mid DaDd \\ B & \rightarrow & BA \mid ca \mid Dd \\ C & \rightarrow & DA \mid \varepsilon \\ D & \rightarrow & a \end{array}$$

Convert the grammar to CNF.

Question 3

3. Let $G = (V, \Sigma, P, S)$ be a CFG in CNF. Give an $O(n^3)$ algorithm for the following problem:

- Input: A word w and a nonterminal $A \in V$.
- Output: the value
  $n_A = \max\{|u| \ : \ u \text{ is a suffix of } w \text{ and } A \Rightarrow^* u\}$.

That is, n_A is the length of the longest suffix of w that is generated by A in the grammar.

Question 4

4. Construct PDAs for the following languages:

(a) $L = \{a^n b^m xy \ : \ x, y \in \{0,1,2\}, x \equiv n (\mathrm{mod} 3) \text{ and } y \equiv m (\mathrm{mod} 3)\}$.

(b) $L = \{w\#x \ : \ w, x \in \{a,b\}^*, |w|_a = |x|_a, |w|_b \equiv |x|_b (\mathrm{mod} 3)\}$.

Be sure to indicate what the starting stack symbol is for your PDA and how your PDA accepts words.

Question 5

5. Let C be a fixed integer. Extend the language from Assignment 3 as follows:

$$L_C = \{x\#1^n \ : n \geq 0, x \in \{a,b\}^* \text{ and } \ n - C \leq |x|_a \leq n + C\}$$

Give a context-free grammar for L_C. The productions in your grammar will depend on the value of C. Describe them using a uniform notation (e.g., by using consistently named variables or consistently defined productions, for instance).

Question 6

6. Consider the following modified language from Assignment 3:

$$L = \{u\#v \ : \ u, v \in \{0,1\}^* \text{ and } \mathrm{bin}(v^R) = \mathrm{bin}(u) + 2\}$$

Give a PDA that accepts L.

Question 7

7. A CFG G is in Griebach Normal Form (GNF) if every production has the form

$$A \rightarrow aB_1 B_2 \cdots B_n$$

for some letter a and nonterminals $B_1, B_2, \cdots, B_n$ (where $n \geq 0$). Any grammar (that does not derive ε) can be converted to GNF. Given this fact, show that for any CFG L that does not include ε, you can construct a PDA M that accepts L in the following additional conditions:

- The PDA accepts by empty stack.
- The PDA M does not have any ε-transitions. That is, there are no rules of the form $\delta(q, \varepsilon, \gamma) = \{(q', \beta), \ldots\}$ for any stack symbol γ.

Question 8

8. Prove that the following languages are not context-free:

(a) $L = \{a^p \ : \ p \text{ is a prime number}\}$.

(b) $L = \{a^n b^{n^3} \ : \ n \geq 0\}$.

(c) $L = \{w \in \{a,b\}^* \ : \ |w|_b = 2^{|w|_a}\}$.

Question 9

9. For each of the languages in the previous question, give an informal description of a multi-tape TM that recognizes the language.

(a) $L = \{a^p \ : \ p \text{ is a prime number}\}$.

(b) $L = \{a^n b^{n^3} \ : \ n \geq 0\}$.

(c) $L = \{w \in \{a,b\}^* \ : \ |w|_b = 2^{|w|_a}\}$.

Question 10

10. Show that the following language is r.e.:

$$L = \{e(M_1)\#e(M_2) \ : \ L(M_1) \cap L(M_2) \neq \emptyset\}$$

Question 11

11. Show that the following problem is undecidable by reduction: Given a TM M, is $L(M)$ a finite language?

Question 12

12. Show that the following language is decidable:
$L_{ND} = \{e(M) \;:\; M \text{ is a nondeterministic TM } \}$.

Question 13

13. Show that the following problem is either decidable or undecidable: Given a CFG G is $L(G)$ infinite? (Hint: review the proof of the pumping lemma for CFLs.)