Q1. $\hat{k} = (0,0,1)$, $\hat{i} = (1,0,0)$, $\hat{j} = (0,1,0)$

$\hat{k}$ is orthogonal to $\hat{i}$: $\hat{k} \cdot \hat{i} = 0\times1 + 0\times0 + 1\times0 = 0$, so $\hat{k}$ is orthogonal to $\hat{i}$

$\hat{k}$ is orthogonal to $\hat{j}$: $\hat{k} \cdot \hat{j} = 0\times0 + 0\times1 + 1\times0 = 0$, so $\hat{k}$ is orthogonal to $\hat{j}$.

Q2.

$$A = \begin{bmatrix} 1 & -4 & 8 \\ 11 & 2 & 24 \\ 12 & 4 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -9 & 8 & 6 \\ 0 & 15 & 2 \\ 3 & 14 & 0 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 1\times(-9) - 4\times0 + 8\times3 & 1\times8 - 4\times15 + 8\times14 & 1\times6 - 4\times2 + 8\times0 \\ 11\times(-9) + 2\times0 + 24\times3 & 11\times8 + 2\times15 + 24\times14 & 11\times6 + 2\times2 + 24\times0 \\ 12\times(-9) + 4\times0 + 1\times3 & 12\times8 + 4\times15 + 1\times14 & 12\times6 + 4\times2 + 1\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 60 & -2 \\ -27 & 404 & 70 \\ -105 & 170 & 80 \end{bmatrix}$$

Q3. Assume $\vec{v'}=(x,y)$, $|\vec{v'}|=1$. Since the angle between $\vec{n}$ and $\vec{v}$ is same as the angle between $\vec{v'}$ and $\vec{n}$. Thus we could have:

$$\frac{\vec{v}\cdot\vec{n}}{|\vec{v}|\cdot|\vec{n}|} = \frac{\vec{n}\cdot\vec{v'}}{|\vec{v'}|\cdot|\vec{n}|}$$

$$\frac{-2+6}{\sqrt{13}\cdot\sqrt{5}} = \frac{-x+2y}{\sqrt{x^2+y^2}\cdot\sqrt{5}}$$

$$\frac{4\sqrt{13}}{13} = -x+2y$$

giving that $|\vec{v'}|=1$, we could have $x^2+y^2=1$, then

$$\begin{cases} x^2+y^2=1 \\ -x+2y=\frac{4\sqrt{13}}{13} \end{cases}, \quad \text{so} \quad \begin{cases} x_1=-\frac{22\sqrt{13}}{65} \\ y_1=\frac{\sqrt{13}}{65} \end{cases} \quad \begin{cases} x_2=\frac{2\sqrt{13}}{13} \\ y_2=\frac{3\sqrt{13}}{13} \end{cases}$$

Since $\vec{v}$ is the reflection of $\vec{v}$ across $\vec{n}$, so

$$\vec{v'}=\left(-\frac{22}{65}\sqrt{13}, \frac{1}{65}\sqrt{13}\right)$$

$$\cos\langle\vec{v},\vec{n}\rangle=\left|\frac{-2+6}{\sqrt{13}\cdot\sqrt{5}}\right|=\frac{4}{65}\sqrt{65}$$

$$\cos\langle\vec{v'},\vec{n}\rangle=\left|\frac{-\frac{22}{65}\sqrt{13}+\frac{2}{65}\sqrt{13}}{1\cdot\sqrt{5}}\right|=\left|\frac{-\frac{4}{\sqrt{13}}}{\sqrt{5}}\right|=\frac{4}{65}\sqrt{65}$$

Q4 $p=(3,2,4)$ $q=(1,-3,4)$, $r=(1,3,-1)$

$\vec{qp}=(2,5,0)$, $\vec{rp}=(2,-1,3)$

To find the normalized normal vector to the plane containing these three points, we could have $\vec{v}=k(\vec{qp}\times\vec{rp})=k(15,-6,-12)$

Since $\vec{v}$ is normalized, $255k^2+36k^2+144k^2=1$, $k=\frac{1}{405}\sqrt{405}$

So one of $\vec{v}$ is $\left(\frac{15}{405}\sqrt{405}, -\frac{6}{405}\sqrt{405}, -\frac{12}{405}\sqrt{405}\right)$.