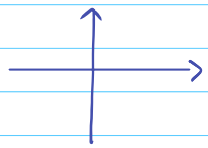
1.1 Coordinate System.

1 dim: number line : 

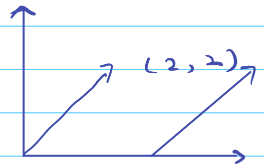
2 dim: cartesian plane: 

1) origin
2) ortho vectors

=> tuples indicate position.

n-dim:

1.2 Vectors: the direction and a line.



parallel vectors are equivalent.

we construct vectors using displacements.

$\vec{v} = (2, 3)$

$p + \vec{v} = (x+2, y+3)$

scalar multiplication: $\vec{v} = (v_1, v_2, v_3)$ $\quad c \in \mathbb{R}$

$c\vec{v} = (cv_1, cv_2, cv_3)$

simply change the length of a vector.

<u>no change in direction</u>

Euclear norm: $|\vec{v}| = \sqrt{(v_1)^2 + (v_2^2) + \dots + (v_n)^2}$

$|c\vec{v}| = c|\vec{v}|$

vector addition: $\vec{V_a} = (V_{ax}, V_{ay})$

$\vec{V_b} = (V_{bx}, V_{by})$

$\vec{V_a} + \vec{V_b} = (V_{ax} + V_{bx}, V_{ay} + V_{b}$

simply connect one to the other.

e.g. $\begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

vector addition is commutive $\vec{v} + \vec{w} = \vec{w} + \vec{v}$

subtraction: $\vec{v} - \vec{w} = \vec{v} + (-\vec{w})$ with a -1 scaler multi

Dot product (inner product)

$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ $\vec{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$ $\vec{v} \circ \vec{w} = \begin{pmatrix} v_1 w_1 \\ v_2 w_2 \end{pmatrix}$

$\cos\theta = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|}$

normalization: transform a vector to the length of 1:

$\frac{\vec{v}}{|\vec{v}|}$

orthonormal vectors: vectors that are normalized and perpendicular to each other.

i.e. $\vec{v} \cdot \vec{w} = 0$

parallel: $\frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} = \cos\theta = \pm 1$

$\theta < 90°$ iff $\vec{v} \cdot \vec{w} > 0$

$> 90°$ $<$

Direction is a normalized vector

Orthonormal bases of demension of n is a set of n normalized vectors that are perpendictular to each other.

e.g. $\hat{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\hat{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\hat{i} \cdot \hat{j} = 0$

3-dm: $\hat{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\hat{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\hat{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Cross product: $\vec{u} \times \vec{v}$ gives a third vector that is perpendicular to these two.

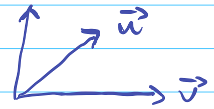
assume $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

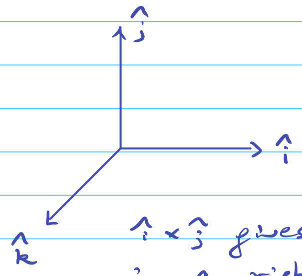
$$\vec{u} \times \vec{v} = \begin{pmatrix} u_2 v_3 - u_3 v_2 \\ u_1 v_3 - u_3 v_1 \\ u_1 v_2 - u_2 v_1 \end{pmatrix} \begin{matrix} i \\ j \\ k \end{matrix}$$

geometry of cross product:

any two non-parallel vectors spanes a plane, then a third vector perpendicults to this plane.

handness of coordinate system:





right-hand system: $\vec{v}\times\vec{u}$

left-hand : $\vec{u}\times\vec{v}$

$\hat{i}\times\hat{j}$ gives an outward $\hat{k}$ in a right-hand system, the openGL adapt this one.

$$\frac{|\vec{u}\times\vec{v}|}{|\vec{u}|\cdot|\vec{v}|} = \sin\theta$$

Matrcies

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

if A, B, C are square matrices, then

$ABC = A(BC) = A(BC)$

* but it is not communtive. $A(BC) \neq A(CB)$

$$A_{n\times m}\cdot B_{m\times n} = C_{n\times n} = (C_{ij}) = \sum_{k=1}^{m} a_{ik}b_{kj}$$