

A set is a collection of elements

e.g.

Obj, Sets, ---

$$S = \{1, 2, \{3, 4\}\}$$

$$\Rightarrow 1 \in S, 2 \in S, 3 \notin S, 4 \notin S, \{3, 4\} \in S.$$

Power Set :

Let  $A$  be a finite set, then the power set of  $A$  is the collection:  $P(A) = \{S : S \subseteq A\}$

put all possible subsets of  $A$  into a set

e.g. Take  $A = \{1, 2, 3\}$ .

$$P(A) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \emptyset\}.$$

Note:  $|A| =$  number of element in  $A$ .

$$|P(A)| = 2^{n^2}$$

Partition of a set:

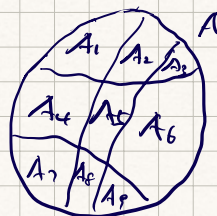
Let  $A$  be a non-empty set, then the partition  $\pi$  of  $A$  is a collection of non-empty subsets of  $A$  that are mutually disjoint and whose union is  $A$ .

$$\pi = \{A_1, \dots, A_n\}$$

Such that the following condition is satisfied.

1.  $A_1, A_2, \dots, A_n \neq \emptyset$
2.  $A_1, A_2, \dots, A_n \subseteq A$ .
3.  $A_i \cap A_j = \emptyset$  where  $i \neq j$ .
4.  $A_1 \cup A_2 \dots A_n = A$ .

$\Rightarrow$



## Sequences and Sums.

A sequence is a function  $f: S \rightarrow \mathbb{R}$  where  $S \subseteq \mathbb{Z}$

$$f: \{1, 2, \dots\} \rightarrow \mathbb{R}.$$

$$f(1), f(2), f(3) \dots \Rightarrow f_1, f_2, f_3 \dots$$

e.g.  $f: \mathbb{Z}_+ \rightarrow \mathbb{R}$ .  $\leftarrow$  a function define as  $f: x^2$ .

定义 1. Constant Sequences.  $\leftarrow$  define by constant number.

$$\text{e.g. } 1, 1, 1, 1, 1$$

定义 2. Geometric Sequences  $\leftarrow$  the ratio of any power consecutive terms

$$\text{e.g. } a, ar, ar^2, ar^3 \dots \text{ is a constant.}$$

定义 3. Arithmetic Sequences  $\leftarrow$  difference of a pair of consecutive term

$$\text{e.g. } a, a+d, a+2d \dots$$

Sum of sequences:

## Induction.

Let  $P(n)$  is a statement parameterized by  $n \in \mathbb{Z}_+$ .

(1)  $P(1)$  is true.

(2)  $P(k) \Rightarrow P(k+1)$  for any positive integer

then  $P(n)$  is true for all positive integer

$$\begin{array}{ccccccc} P(1) & \Rightarrow & P(2) & \Rightarrow & P(3) & \Rightarrow & \dots \\ \text{true} & & \text{true} & & \text{true} & & \\ & \text{true} & & \text{true} & & & \end{array}$$