Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 1229A Test 2

Friday, November 16, 2018 **Code 111** 7:00 p.m. - 8:30 p.m.

INSTRUCTIONS

1. Fill in the tops of this page **and the back of this page** completely. Be sure to print your name **legibly**.

2. Fill in the top of the scantron card completely. **Both print AND code** your Student Number, Section Number (see below) and Exam Code (shown above).

3. CALCULATORS AND NOTES ARE NOT PERMITTED.

4. DO NOT UNSTAPLE THE BOOKLET.

5. There are two parts to this examination: PART A (18 marks) in multiple choice format and PART B (7 marks) in written answer format.

6. In Part A, **circle** the correct answer to each question **on this paper** AND fill in the appropriate box on the **scantron** card with an HB pencil. This question paper will be returned to you.

7. In Part B, write your answer in the space provided.

8. Questions are printed on both sides of the paper. They begin on Page 1 and continue to Page 5. Be sure that your booklet is complete.

9. You must hand in this question paper, your scantron card, and all rough work sheets.

10. Circle your section in the list below.

Instructor	Campus/College	Time	Section
Lindsey	Main	9:30 MWF	001
Pasini	Main	12:30 MWF	002
Olds	Main	1:30 MWF	003
Pasini	Main	8:30 MWF	004
Ghorbanpour	Brescia	8:30 MTuTh	530
O'Hara	Brescia	9:30 MTuW	531
Rastegari	Huron	11:30 MWF	550
Mollahajiaghaei	Huron	8:30 Tu	551
Kuzmin	King's	10:30 Tu, 9:30 Th	570
Turnbull	King's	1:30 Tu, 12:30 Th	571
Turnbull	King's	1:30 M, 12:30 W	572
Kuzmin	King's	7:00 MW	573
Kuzmin	King's	8:30 MW	574

11. TOTAL MARKS = 25.

Student Number (**Print**)

Student's Name (**Print**)

FOR GRADING ONLY

PAGE	MARK
1–3	
4	
5	
TOTAL	

PART A (18 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE INDICATED ON THE SCANTRON SHEET. YOU SHOULD ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

1 mark 1. Consider the systems of equations shown below. Which of the systems is/are linear in the unknowns x, y and z?

B.

(i) $\begin{aligned} x - y + 2^2 z &= 3 \\ x - y + 2z^2 &= \tfrac{1}{2} \end{aligned}$ (ii) $\begin{aligned} x + 2y + 3z &= 2 \\ 2x + 2(2y + 3z) &= 8 \end{aligned}$ (iii) $\begin{aligned} 2^2 x - y(2 - z) &= 0 \\ 2x + \tfrac{1}{y} + z &= 3 \end{aligned}$

A: (i) only	B: (ii) only	C: (iii) only
D: none of (i), (ii) and (iii)	E: all of (i), (ii) and (iii)	

1 mark 2. Find the augmented matrix for the following system of linear equations:

C.

$$\begin{aligned} x_1 - x_3 - 1 &= 3x_2 \\ 2x_1 + x_2 - x_4 &= 5 \end{aligned}$$

1 -3 -1 0 = 1.
2 1 0 -1 = 5

A: $\left[\begin{array}{ccc\|c} 1 & -1 & -1 & 3 \\ 2 & 1 & -1 & 5 \end{array}\right]$	B: $\left[\begin{array}{ccc\|c} 1 & -3 & -1 & 1 \\ 2 & 1 & -1 & 5 \end{array}\right]$	C: $\left[\begin{array}{cccc\|c} 1 & -3 & -1 & 0 & 1 \\ 2 & 1 & 0 & -1 & 5 \end{array}\right]$
D: $\left[\begin{array}{cccc\|c} 1 & -3 & -1 & -1 & 0 \\ 2 & 1 & 0 & -1 & 5 \end{array}\right]$	E: $\left[\begin{array}{cccc\|c} 1 & -3 & -1 & 0 & 1 \\ 2 & 1 & -1 & 0 & 5 \end{array}\right]$	

1 mark 3. Which one of the following matrices is **not** in row-reduced echelon form?

D.

A: $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	B: $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$	C: $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	D: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$	E: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

~~1, 0~~ (0, 1.) (1, 0). 000 x
1, 1. 00

1 mark 4. Find all values (a, b) for which $\begin{bmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & 0 & b \end{bmatrix}$ is in row-reduced echelon form.

B.

A: $(1,1)$ only	B: $(1,0)$ and $(0,0)$ only	C: $(1,1)$, $(0,0)$, $(1,0)$ and $(0,1)$
D: $(0,0)$ only	E: $(0,1)$ and $(1,0)$ only	

1 0 0
0 1 0
0 0 1

(1, 0) ✓ (0, 0)

1 mark 5. The row-reduced echelon form of the augmented matrix corresponding to a system of linear equations is shown below. How many parameters are needed to express all solutions for the system?

2

D

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \\ \end{array}$$
$$\left[\begin{array}{cccccc|c} 1 & 1 & 0 & 0 & 0 & 8 & 7 \\ 0 & 0 & 1 & 1 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 3 \end{array}\right]$$

$x_1 + x_2 + 8x_6 = 7$
$x_3 + x_4 + 2x_6 = 2$

A: 0	B: 1	C: 2	D: 3	E: 4

$x_1 + x_2 + 8x_6 = 7$
$x_3 + x_4 + 2x_6 = 2.$

仍需写3个式子 - ?

1 mark 6. Let B be the row-reduced echelon form of the matrix A shown here:

$$A = \begin{bmatrix} 2 & 4 & 8 \\ -1 & 2 & 8 \\ 2 & 0 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 4 \\ -1 & 2 & 8 \\ 1 & 0 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 4 \\ 0 & 4 & 12 \\ 0 & 2 & 6 \end{bmatrix}$$

Find the first row of matrix B.

A: $\begin{bmatrix} 1 & 0 & 2 \end{bmatrix}$	B: $\begin{bmatrix} 1 & 0 & -2 \end{bmatrix}$	C: $\begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$	D: $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$	E: $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

B.

1 mark 7. Find the value(s) of k for which the system of linear equations whose augmented matrix is shown below has exactly one solution. x_1+2x_2

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & k & 3 \end{array}\right]$$

A: $k = 1$ only	B: all $k \neq 0$	C: $k = 3$ only	D: all values of k	E: no value of k

E.

1 mark 8. The system of linear equations with augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 4 & 3 \\ 0 & 0 & k & k+1 & k+2 \end{array}\right]$$

$k=0.$

化简得到一个 $y=kx+b$ —?

represents the intersection of 3 hyperplanes in $\Re^4$. Find the value(s) of k for which the intersection is a line.

A: all values of k	B: $k = -1$ only	C: $k = 0$ only	D: all $k \neq -2$	E: no value of k

E. A

1 mark 9. The matrix shown below is the augmented matrix for a system of 3 linear equations in the 3 unknowns x, y and z.

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & k & -1 \\ 0 & k & 16 & 4 \end{array}\right]$$

$k=-4$ $k=4.$ $k=4.$

Find the value(s) of k for which the system of linear equations has no solution.

A: all values of k	B: no value of k	C: all $k \neq \pm 4$	D: $k = -4$ only	E: $k = 4$ only

1 mark 10. A is a 3×4 matrix, B is a 4×3 matrix and C is a 3×3 matrix. Which one of the following operations is not defined?

A: $A^T + B$	B: BA	C: $CB^T A^T$	D: BAC	E: $AB + C$

$[3\times4]\times[4\times3]$ $[4\times4]$ $[4\times3]\cdot[3\times4]$ $[3\times3]$

C.

1 mark 11. Consider the matrices A and B shown below. If $C = [c_{ij}] = 2B^T + 3A$, find c_{23}.

$$A = \begin{bmatrix} 2 & 1 & -3 \\ -4 & 5 & -2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 5 & 2 \\ -1 & 4 \\ 2 & 3 \end{bmatrix}$$

$\begin{bmatrix} 10 & -2 & 4 \\ 4 & 8 & 6 \end{bmatrix}$

$3A = \begin{bmatrix} 6 & 3 & -9 \\ -12 & 15 & -6 \end{bmatrix}$

A: 4	B: -3	C: 0	D: 1	E: not defined

C

1 mark 12. For the matrices given below, what is the (3, 2)-entry of AB?

$$A = \begin{bmatrix} 5 & 2 \\ -1 & 4 \\ 2 & -3 \end{bmatrix} \quad \text{and } B^T = \begin{bmatrix} -3 & 2 \\ -2 & -1 \\ 5 & -4 \end{bmatrix}$$

$B = \begin{bmatrix} -3 & -2 & 5 \\ 2 & -1 & -4 \end{bmatrix}$

A: -1	B: -3	C: 4	D: 0	E: not defined

1 mark 13. For what value(s) of k does the matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 1-k \end{bmatrix}$ have an inverse?

A: all $k \neq -5$	B: $k = -5$ only	C: all $k \neq 1$	D: $k = 1$ only	E: no value of k

1 mark 14. For what value(s) of k does the matrix $\begin{bmatrix} k & -2 & 3 \\ -2 & 4 & -6 \\ 0 & 0 & k \end{bmatrix}$ have **no transpose**?

A: $k = 3$ only	B: all $k \neq 3$	C: $k = 0$ only	D: $k = 1$ only	E: no value of k

1 mark 15. Let $A = \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$. Find A^{2018}.

$A^2 = \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$

A: $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$	B: $\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$	C: $\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$	D: $\begin{bmatrix} -1 & -2 \\ 0 & 0 \end{bmatrix}$	E: $\begin{bmatrix} (-1)^{2018} & 2^{2018} \\ 0 & 0 \end{bmatrix}$

1 mark 16. If $\begin{bmatrix} 3 & 0 & g \\ 1 & -1 & h \\ 0 & 2 & i \end{bmatrix}$ is the inverse of the matrix $\begin{bmatrix} a & d & 2 \\ b & e & 2 \\ c & f & -3 \end{bmatrix}$, find the value of i.

$6 - 3g = 0$, $h = 0$, $4 - 3i = 1$, $i = 1$.

A: 2	B: 0	C: -1	D: 1	E: cannot be determined

1 mark 17. If A is an $n \times n$ square matrix such that $A - A^T = \text{O}$, where O is the $n \times n$ zero matrix, which one of the following **must always** be true?

A: $A = \text{O}$	B: $A = A^T$	C: A is a diagonal matrix
D: A is an identity matrix	E: A is invertible	

1 mark 18. You are given that $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ has $A^{-1} = \begin{bmatrix} 4 & 0 & 0 \\ 7 & -1 & 2 \\ 9 & 5 & -8 \end{bmatrix}$.

Consider the system of linear equations shown here:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$

If it is known that $(4, 11, -2)$ is the unique solution to the system of equations, find the value of b_1.

A: 1	B: 2	C: -1	D: 0	E: 4

PART B (7 marks)

SHOW YOUR WORK FOR ALL QUESTIONS IN PART B

3 marks 19. Consider the following system of linear equations in the unknowns x_1, x_2, x_3 and x_4.

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & & & + & x_4 & = & 2 \\ 2x_1 & - & 4x_2 & - & 3x_3 & - & 4x_4 & = & 1 \\ x_1 & - & 2x_2 & - & 3x_3 & - & 5x_4 & = & -1 \end{array}$$

(a) Write the augmented matrix corresponding to this system of equations.

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 2 \\ 2 & -4 & -3 & -4 & 1 \\ 1 & -2 & -3 & -5 & -1 \end{array}\right].$$

(b) Find the row-reduced echelon form of the matrix you wrote in part (a).

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 2 \\ 2 & -4 & -3 & -4 & 1 \\ 1 & -2 & -3 & -5 & -1 \end{array}\right] \xrightarrow[R_2 = R_2 - 2R_1]{R_3 = R_3 - R_1} \left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 2 \\ 0 & 2 & 3 & 6 & 3 \\ 0 & 1 & 3 & 6 & 3 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 2 \\ 0 & 1 & \frac{3}{2} & 3 & \frac{3}{2} \\ 0 & 1 & 3 & 6 & 3 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 \end{array}\right]. \Leftarrow \left[\begin{array}{cccc|c} 1 & 0 & \frac{3}{2} & 4 & \frac{7}{2} \\ 0 & 1 & \frac{3}{2} & 3 & \frac{3}{2} \\ 0 & 0 & \frac{3}{2} & 3 & \frac{3}{2} \end{array}\right]$$

(c) Use your row-reduced echelon form augmented matrix from part (b) to find **all** solutions to the system of equations.

$$\begin{cases} x_1 + x_4 = 2 \\ x_2 = 0 \\ x_3 + x_4 = 0. \end{cases}$$

$x_1 - x_3 = 2.$

2 marks 20. Using the method of row reduction, find A^{-1} where $A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 5 & -5 \\ 3 & -7 & 6 \end{bmatrix}$.

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 1 & 0 & 0 \\ -2 & 5 & -5 & 0 & 1 & 0 \\ 3 & -7 & 6 & 0 & 0 & 1 \end{array}\right] \Rightarrow$$

2 marks 21. Let $B = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}$. Find $\left(2B^{-1}B - B^T\right)^2$.