

Questions ?

Q. Find Domain of $\ln\left(\frac{5-x^2}{1+x}\right)$.

~~Q. Find domain of $\ln\left(\frac{5-x^2}{1-x}\right)$.~~

Ans: where is \ln defined? when $x > 0$

So, we must have

$$\frac{5-x^2}{1+x} > 0$$

or

Both numerator
and denominator
 > 0

Both numerator
and denominator
 < 0

$$\begin{aligned} 5-x^2 &> 0 \quad \text{--- (1)} \\ \text{and} \\ 1+x &> 0 \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} 5-x^2 &< 0 \\ \text{and} \\ 1+x &< 0 \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad 5-x^2 &> 0 \\ \Rightarrow 5 &> x^2 \\ \Rightarrow \boxed{\sqrt{5} > x > -\sqrt{5}} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad 5-x^2 &< 0 \\ \Rightarrow 5 &< x^2 \\ \Rightarrow \boxed{x < -\sqrt{5} \quad \text{or} \quad x > \sqrt{5}} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 1+x &> 0 \\ \Rightarrow \boxed{x > -1} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad 1+x &< 0 \\ \Rightarrow \boxed{x < -1} \end{aligned}$$

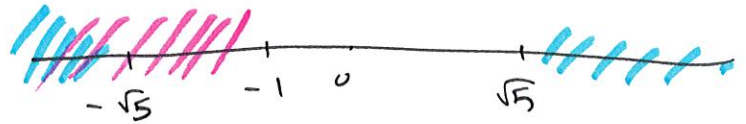
Combining ① and ②

$$\boxed{\sqrt{5} > x > -1}$$

Note: $-1 > -\sqrt{5}$
if we want both
 $x > -1$ and $x > -\sqrt{5}$
we must have
 $x > -1$

③ $x < -\sqrt{5}$ or $x > \sqrt{5}$ and ④ $x < -1$

~~scribbles~~
 $\Rightarrow \boxed{x < -\sqrt{5}}$



Combining everything,

$\boxed{-\sqrt{5} > x > -1}$

or

$x < -\sqrt{5}$



\Rightarrow Domain: $(-\infty, -\sqrt{5}) \cup (-1, \sqrt{5})$

- For domain: ① Start from the outermost function
- ② ~~Inequalities~~ ~~behave~~ be careful when simplifying inequalities.

Q. Find domain of arcsin $(1+x)$.

A: For what x does this make sense?

arcsin has domain: $[-1, 1]$

$\Rightarrow -1 \leq 1+x \leq 1$

$\Rightarrow -1-1 \leq x \leq 1-1$

Subtract 1 from all
3 sides

$$\Rightarrow -2 \leq x \leq 0$$

\Rightarrow Domain of $\arcsin(1+x)$ is $[-2, 0]$.

✎

- For composition of functions, start from the outermost function's domain

Q. Find domain of $\arcsin(\ln x) \cdot \frac{1}{1-x}$.

Ans.

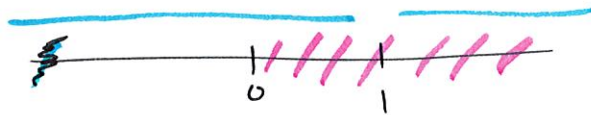
Product \Rightarrow intersect of both domains

Both $\ln x$ and $\frac{1}{1-x}$ should be defined

$$x > 0$$

and

$$x \neq 1$$



$$\Rightarrow \cancel{0 < x < 1} \text{ and } x \in (0, 1) \cup (1, \infty)$$

Domain of $\frac{\ln x}{1-x}$ is $(0, 1) \cup (1, \infty)$

- Inverse of a function \neq reciprocal of the function

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

- Inverse of a function $f(x)$ is defined only if

f is one-to-one.

↓
no two "x-values" give the same "y-value"

$$f(x) = 3$$

$$f(x) = x^2$$

$$f(x) = x^2 + 1$$

} not one-to-one

$f(x) = e^x$ is one-to-one

↓

these functions do not have inverses.

Q. A.10/

2018/5

$$\lim_{x \rightarrow 0^+} \frac{\sin(4x)}{\sqrt{x} \cdot \sin(2\sqrt{x})}$$

① Plug in: $\frac{0}{0}$ \leadsto need to do something

~~②~~

Squeeze

X

identity (*)

we want to use

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \quad (*)$$

$$\lim_{x \rightarrow 0^+} \frac{\sin(4x)}{\sqrt{x} \cdot \sin(2\sqrt{x})}$$

Multiply by $\frac{4x}{4x}$

$$= \lim_{x \rightarrow 0^+} \frac{\sin(4x)}{\sqrt{x} \cdot \sin(2\sqrt{x})} \cdot \frac{4x}{4x}$$

$$= \underbrace{\lim_{x \rightarrow 0^+} \frac{\sin(4x)}{4x}}_{1} \cdot \lim_{x \rightarrow 0^+} \frac{4x}{\sqrt{x} \cdot \sin(2\sqrt{x})}$$

$$= 1 \cdot \lim_{x \rightarrow 0^+} \frac{4\sqrt{x}}{\sin 2\sqrt{x}} \quad \left(\text{as } \frac{x}{\sqrt{x}} = \sqrt{x} \right)$$

$$= 1 \cdot \lim_{x \rightarrow 0^+} \frac{2\sqrt{x}}{\sin(2\sqrt{x})} \cdot 2$$

~~scribbles~~

$$= 1 \cdot \lim_{x \rightarrow 0^+} \frac{1}{\left(\frac{\sin 2\sqrt{x}}{2\sqrt{x}} \right)} \cdot 2$$

$$= 1 \cdot \frac{1}{1} \cdot 2$$

$$= 2$$

□

$$Q. \lim_{x \rightarrow 0} \frac{|5x-1| + |5x+1|}{x}$$

Ans: Plug in

$$\begin{aligned} & \frac{|5 \cdot 0 - 1| + |5 \cdot 0 + 1|}{0} \\ &= \frac{|-1| + |1|}{0} \\ &= \frac{2}{0} \end{aligned}$$

$= \infty$ or $-\infty$ or d.n.e.

• Recall:

① $|x|$ is a piecewise function

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

②

Plug in nearby values and check signs

$$\frac{|5x-1| + |5x+1|}{x}$$

① $x < 0$

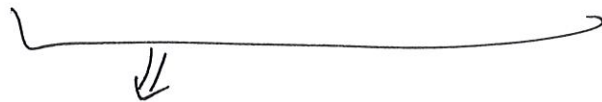
numerator > 0

denominator < 0

② $x > 0$

numerator > 0

denominator > 0



$$\lim_{x \rightarrow 0} \frac{|5x-1| + |5x+1|}{x} \quad \text{d.n.e.}$$

Q. $\lim_{x \rightarrow 0} \frac{|5x-1| - |5x+1|}{x}$

Ans: Plug in $x=0$ $\frac{|-1| - |1|}{0}$

$$= \frac{-1 - 1}{0}$$

$$= \frac{0}{0} \text{ need to do something}$$

(plug in $x=0$)
near $x=0$: $5x-1 < 0$

$$\Rightarrow |5x-1| = -(5x-1)$$

(plug in $x=0$)
near $x=0$: $5x+1 > 0$

$$\Rightarrow |5x+1| = 5x+1$$

Recall:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{|5x-1| - |5x+1|}{x} = \lim_{x \rightarrow 0} \frac{-(5x-1) - (5x+1)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{-5x - 1 - 5x - 1}{x}$$

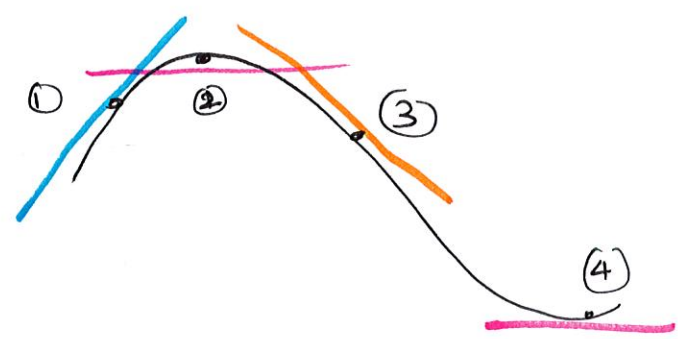
$$= \lim_{x \rightarrow 0} \frac{-10x}{x}$$

$$\boxed{-10}$$

Ans

Ch.4 - applications of derivatives

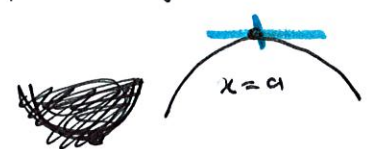
- $f'(a)$ = ~~rest~~ slope of ~~the~~ tangent to graph of $f(x)$ at $x=a$



- (1) $f(x)$ increasing $\Rightarrow f'(x) > 0$
- (4), (2) $f(x)$ is min/max $\Rightarrow f'(x) = 0$
- (3) $f(x)$ is decreasing $\Rightarrow f'(x) < 0$

Def: local maxima / minima:

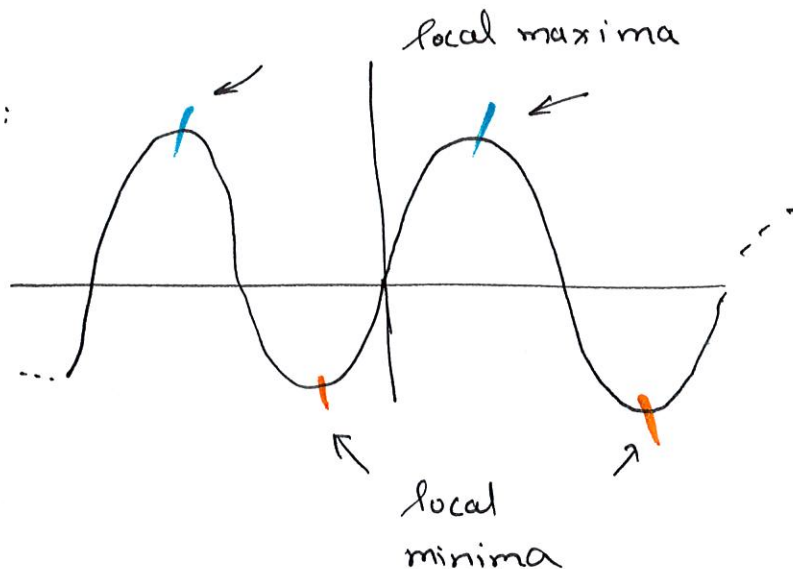
- (1) $x=a$ is a local maxima of $f(x)$ if near $x=a$, $f(x) \leq f(a)$.



- (2) $x=b$ is a local minima of $f(x)$ if near $x=b$, $f(x) \geq f(b)$.



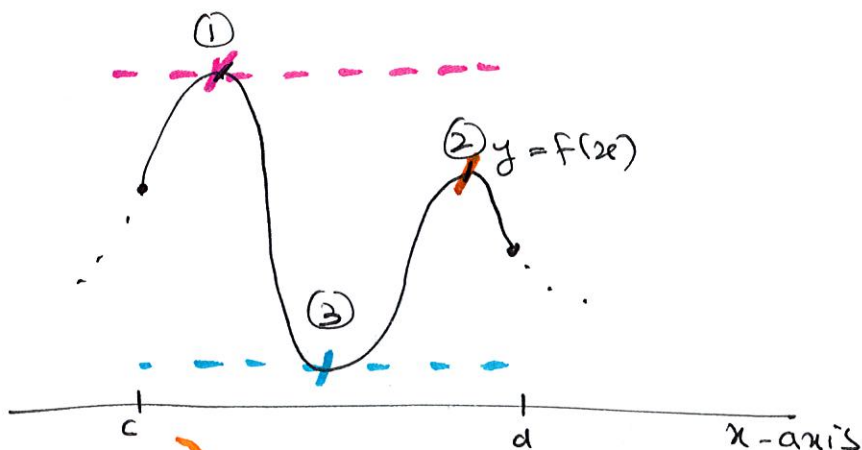
eg:



Def: ~~A function~~ A function $f(x)$ has an absolute maxima at $x=a$, on an interval $[c, d]$ if for all x in $[c, d]$, $f(a) \geq f(x)$.

Similarly; absolute minima.

eg:



①, ②

local maxima

③

local minima

①

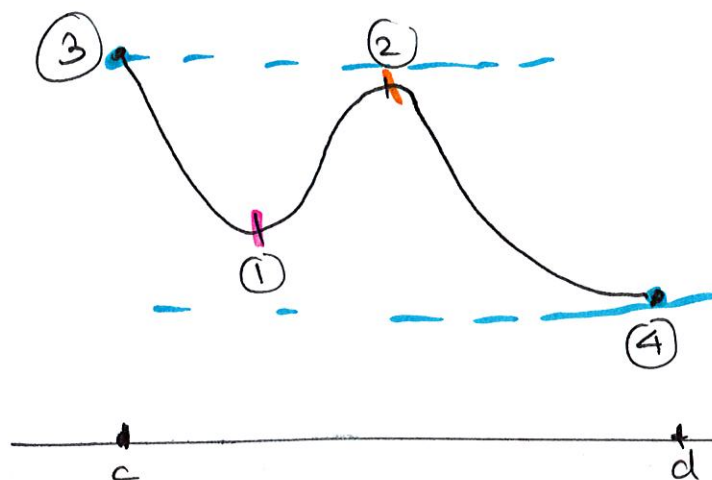
= absolute max

③

absolute minima

(09)

eg:



(1) = local minima

(2) = local max

(3) = absolute max

(4) = absolute min

Q. Find absolute maxima and minima of
 $f(x) = x - \sin x$

on $[-\pi, \pi]$.

Ans:

Find possible candidates for absolute max/min

(1) $f'(x) = 0$

(2) end points

(1) $f'(x) = (x - \sin x)'$

$= 1 - \cos x$

$f'(x) = 0$

$1 - \cos x = 0$

$\Rightarrow \cos x = 1$

$\Rightarrow x = 0$



$x = -\pi$ left end point

$x = \pi$ right end point

Plug in and check:

$f(0) = 0 - \sin 0 = 0$

$f(-\pi) = -\pi - \sin(-\pi) = -\pi$ ← smallest

$f(\pi) = \pi - \sin(\pi) = \pi$ ← largest

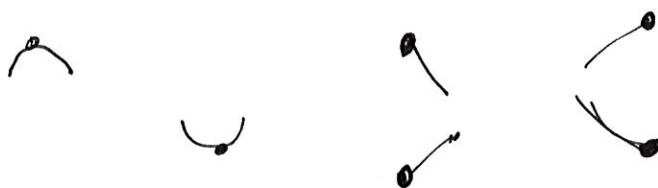
$f(\pi)$ is the largest

$f(-\pi)$ is the smallest

• $x = \pi$ is absolute max.

• $x = -\pi$ is the absolute min.

• for absolute min/max: only need to check at local min/max, endpoints



• Defⁿ: if $f'(a) = 0$ then $x = a$ is called a critical point of $f(x)$.

• local max/min are critical points.



$f'(x) = 0$

For local max/min: ① find x for which $f'(x) = 0$

i.e. find critical points

Second
Derivative
test

② find $f''(x)$ at the critical point

• $f''(x) > 0 \Rightarrow$ local min

• $f''(x) < 0 \Rightarrow$ local max

Local min/max

• (Second derivative test)

- 1) Critical points
- 2) Second derivatives

Q. Find local min/max of

$$f(x) = x - \sin x$$

on $[-\pi, \pi]$.

Ans

① Critical points

$$f'(x) = 1 - \cos x = 0$$

$$\Rightarrow \cos x = 1$$

$$\Rightarrow \boxed{x = 0} \quad (\text{in } [-\pi, \pi])$$

② Second derivative

$$f''(x) = (1 - \cos x)'$$

$$= \sin x$$

$$\text{at } x=0, \quad f''(0) = 0$$

\Rightarrow cannot say anything

\Rightarrow no local min/max

Absolute min/max

11

(~~crit~~)

- 1) critical points
- 2) end points
- 3) Compare their "y-values"

Q. Find absolute min/max

$$f(x) = x - \sin x$$

on $[-\pi, \pi]$

A. we did this

$$\text{abs max } x = \pi$$

$$\text{abs min } x = -\pi$$