

Q1.

(a) $L(G) :$

$$A \rightarrow BC$$

$$B \rightarrow AB$$

$$B \rightarrow BC$$

$$B \rightarrow \epsilon.$$

(b).

Q2.

Let n be the pumping lemma length that $n \leq i + j + k$.

$z = uvwxy$ that $|vwx| \leq n$, $|vx| > 0$

1) Assume that one of the v, x contains more than one letter:

$v = a^a b^b$, $a, b \geq 0$, uv^2wx^2y would contain letters that are out of order. If $x = b^a c^b$, $a, b \geq 0$, condition is similar, uv^2wx^2y would be out of order.

2) Assume that both v, x contains 'a' only:

$u = a^a$ $v = a^b$ $w = a^c$ $x = a^d$ $y = a^{i-a-b-c-d} b^j c^k$, then

$uv^2wx^2y = a^{i+b+d} b^j c^k \notin L$ since $|vx| > 0$

3) Assume that both v, x contains 'c' only:

it is very similar with case 2), and finally we'll get

$uv^2wx^2y = a^i b^j c^{k+b+d} \notin L$

4) Assume that both v, x contains 'b' only:

$u = a^i b^a$ $v = b^b$ $w = b^c$ $x = b^d$ $y = b^{j-a-b-c-d} c^k$, then

$uv^2wx^2y = a^i b^{j+b+d} c^k \notin L$ since it is out of order, $j+b+d$ would equal to neither i nor k .

5) Assume that v contains 'a', x contains 'b':

$u = a^i$ $v = a^b$ $w = a^{i-a-b} b^c$ $x = b^d$ $y = b^{i-c-d} c^k$

$uv^2wx^2y = a^{i+b} b^{j+d} c^k \notin L$, since it is out of order.

6) Assume that v contains 'b', x contains 'c':

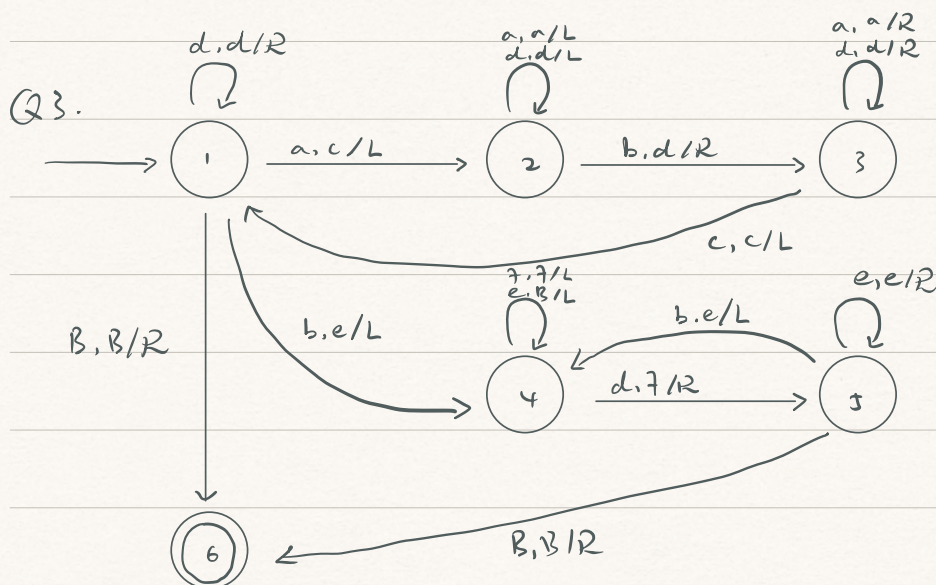
$u = a^i b^a$ $v = b^b$ $w = b^{j-a-b} c^c$ $x = c^d$ $y = c^{k-c-d}$

$uv^2wx^2y = a^i b^{j+b} c^{k+d} \notin L$ since it is also one of order.

7) Assume that v contains 'a', x contains 'c'.

$$u = a^i \quad v = a^b \quad w = a^{i-a-b} b^j c^k \quad x = c^d \quad y = c^{k-d}$$

$uv^2wx^2y = a^{i+b} b^j c^{k+d} \notin L$ since j is equal with neither $i+b$ nor $k+d$.



Q 4.

tape 1 is the tape that used for making up the word.

tape 2 is the tape that used as the counter concerning current loop number.

tape 3 is used for adding letters in current loop to tape 1.

On the start up, the machine loops starting from length $n=1$ till an empty space is detected. When the empty space is detected, the word is accepted only if the counter in tape 3 is at the starting point, else the word is rejected.