ECON3102-005 CHAPTER 5: A CLOSED-ECONOMY ONE-PERIOD MACROECONOMIC MODEL (PART 1)

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- 3. All markets clear (supply=demand for each market).
- 4. The government satisfies its budget constraint:

$$G = T$$

Exogenous and Endogenous Variables

• A model takes exogenous variables, which for the purposes of the problem at hand are determined outside the system we are modelling, and determines values for the endogenous variables.

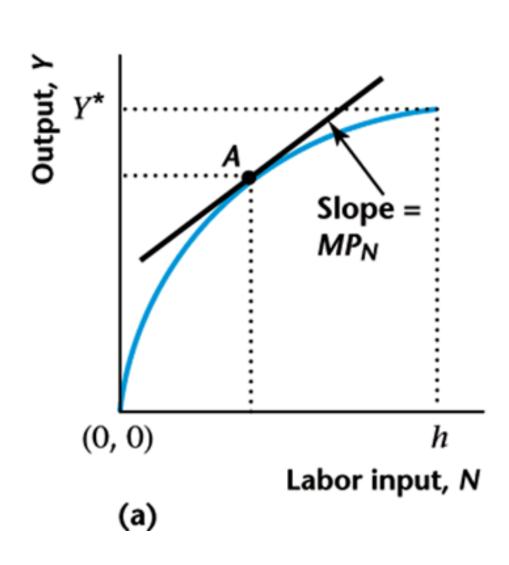
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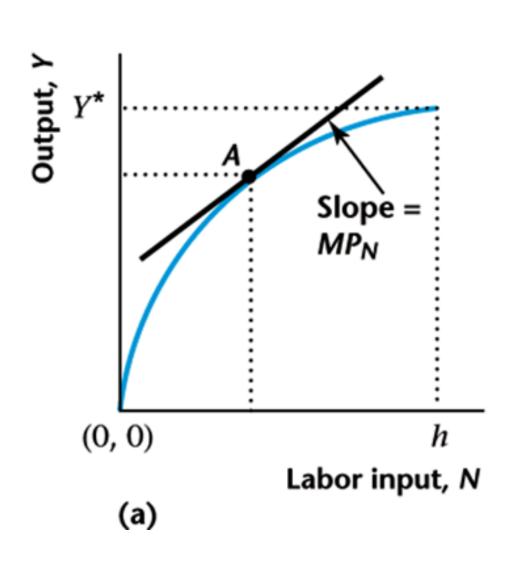
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- In this closed-economy one-period model, the exogenous variables are G, z, K, and the endogenous variables are c, N^d, N^s, T, Y, w .
- Making use of the model is running experiments to see how changes in the exogenous variables change the endogenous variables.

PRODUCTION FUNCTION



- Since in equilibrium labor demand should equal labor supply, then set N^s = N^d = N. Note that this is the market clearing condition for the labor market.
- It follows that we can describe output by Y = zF(K, N).

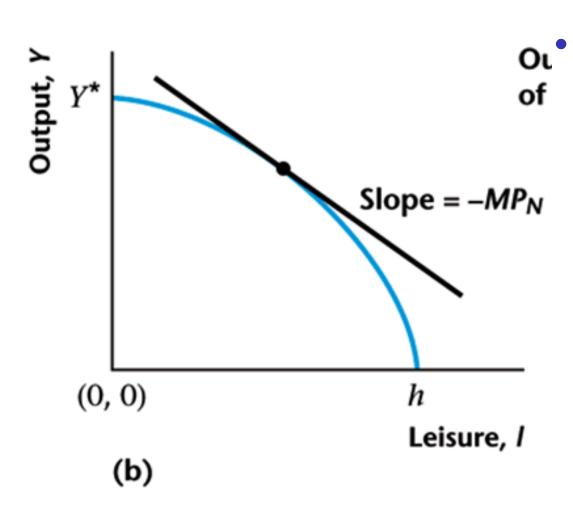
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- Note that the maximum output that can be produced is Y*, where

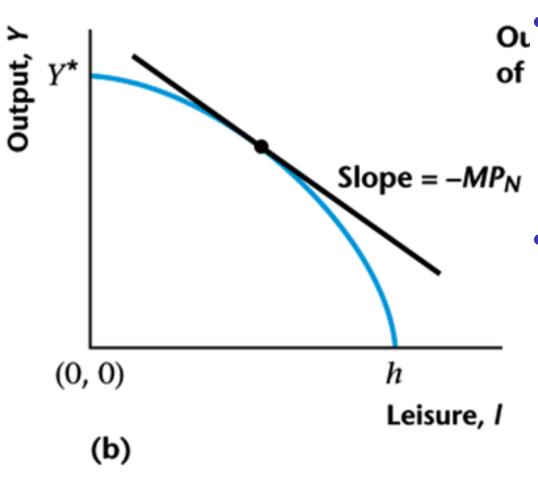
$$Y^* = zF(K, h)$$

Output as a Function of Leisure



Recall that N = h - I. Then it makes sense to put Y = zF(K, h - I), so we can express output as a function of leisure.

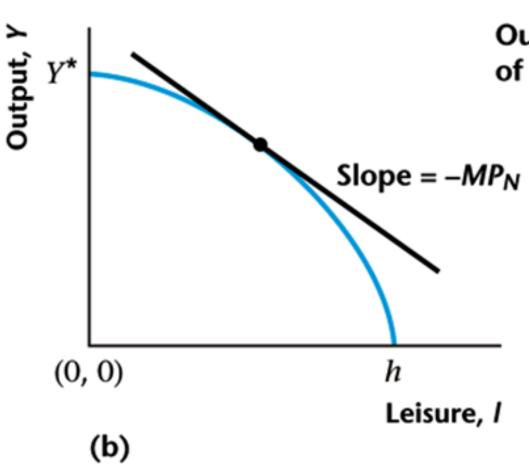
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OUTPUT AS A FUNCTION OF LEISURE



- Recall that N = h I. Then it makes sense to put Y = zF(K, h I), so we can express output as a function of leisure.
- If I = 0, then N = h and Y^* is produced.
- If l = h, then the consumer takes all his time as leisure, and nothing is produced.

PRODUCTION POSSIBILITIES FRONTIER (PPF)

We now want to express the previous graph not as an output-leisure relationship, but rather as a consumption-leisure relationship (the two goods which the consumer cares about).

• Since in equilibrium Y = C + G (because of the income-expenditure identity (aka the market clearing condition for consumption goods), then

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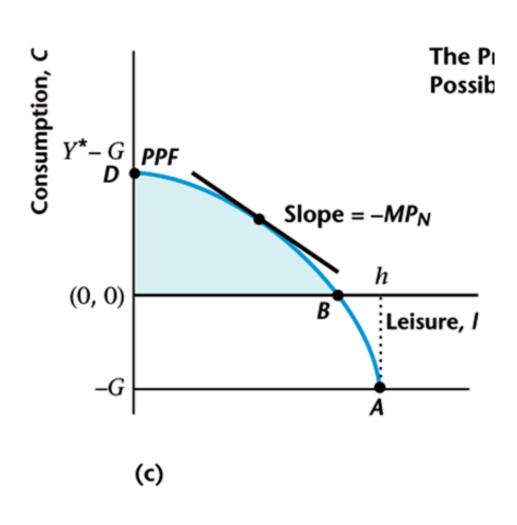
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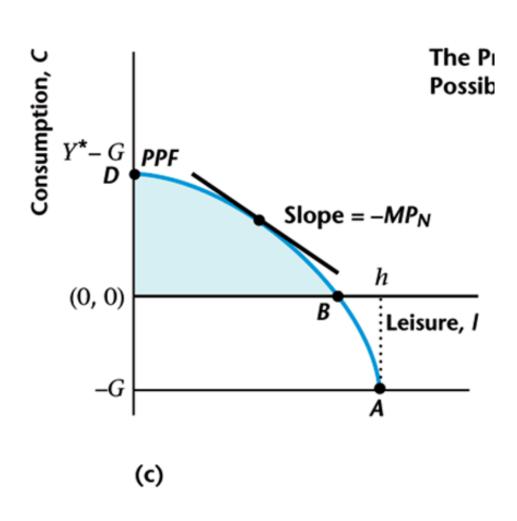
- So, consumption equals output minus the government expenditure.
- This means that we can take the previous graph, shift it down by some amount G, and then get the production possibilities frontier (PPF).

OUTPUT AS A FUNCTION OF LEISURE



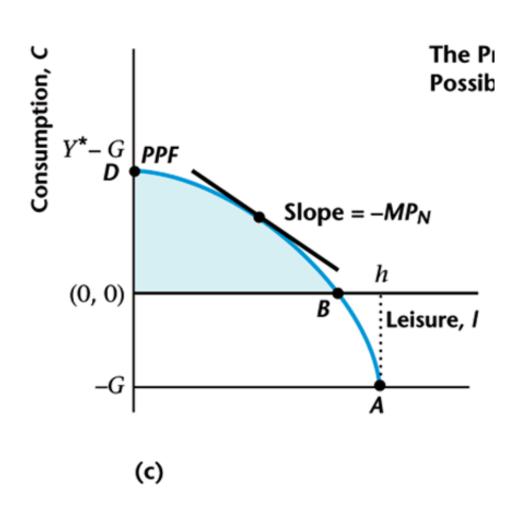
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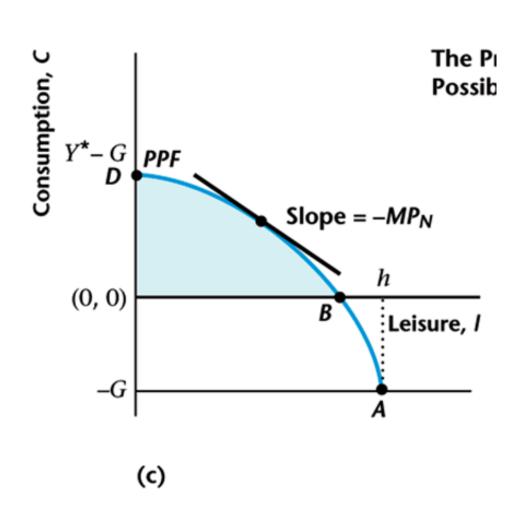
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- Points on segment BD are feasible.

Marginal Rate of Transformation

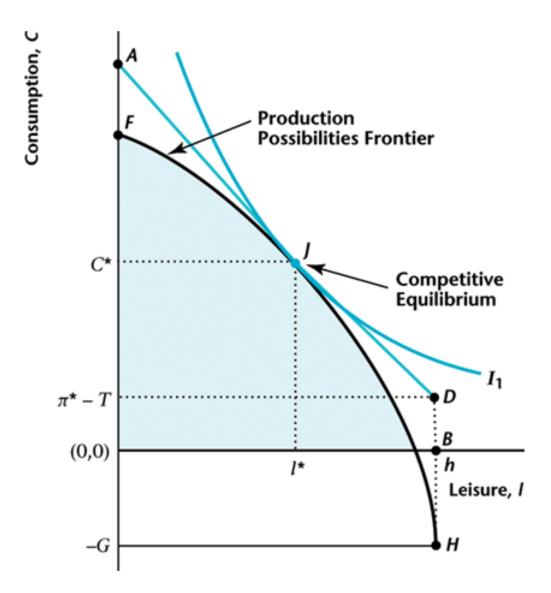
• The negative of the slope of the PPF is also called the **marginal** rate of transformation; this is the rate at which one good can be converted technologically into another.

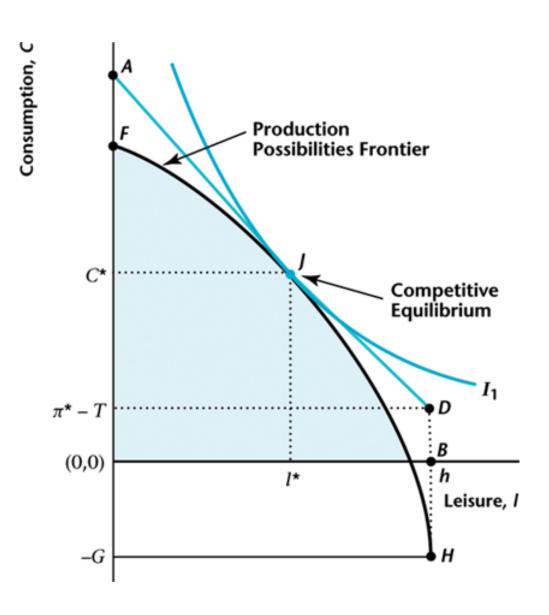
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- Call this rate the $MRT_{I,c}$. In particular, note that

$$MRT_{I,c} = MP_N = -(slope of PPF)$$

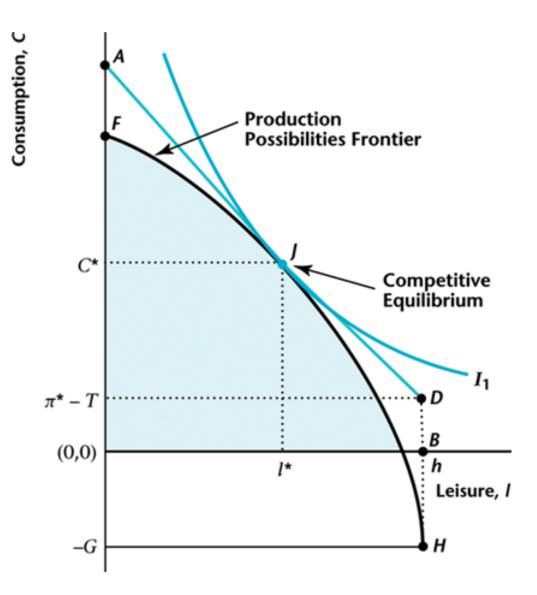
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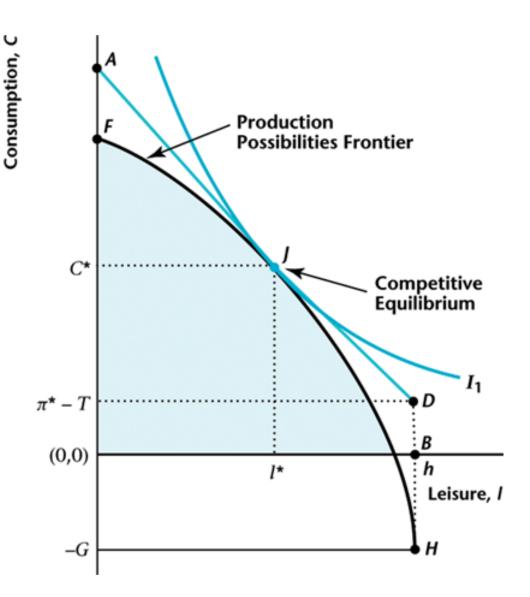
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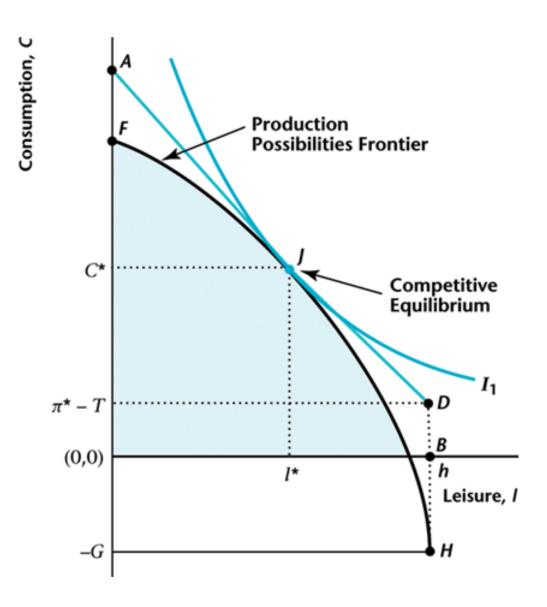
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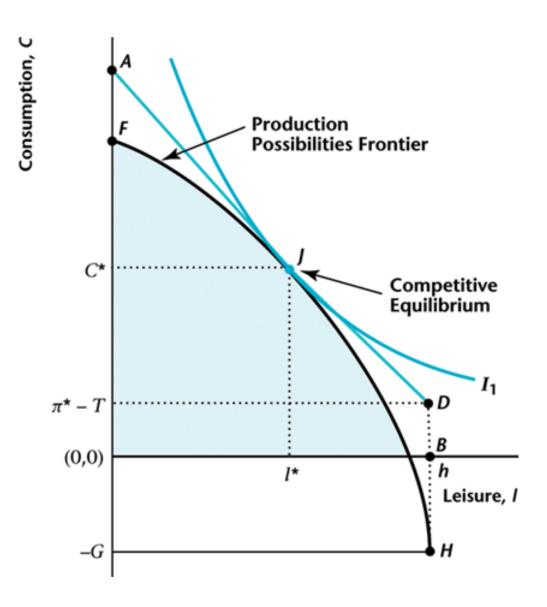
In equilibrium, minus the slope of the PPF equals w; line AD is tangent to the PPF at J; here $MP_N = w$.



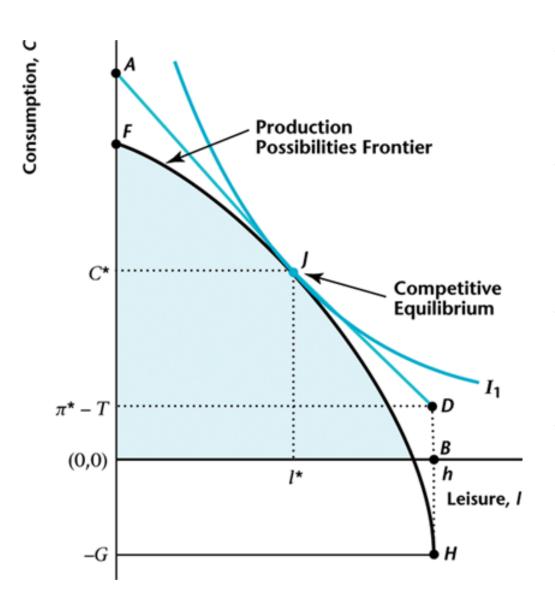
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- Point J is the competitive equilibrium. By consistency, c* is the desired consumption and hl* is the desired labor supply.

A NECESSARY CONDITION FOR COMPETITIVE EQUILIBRIUM

$$MRS_{I,c} = MRT_{I,c} = MP_N$$

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- To answer this, the (almost) universal benchmark is that of Pareto optimality:

Definition A competitive equilibrium is Pareto optimal if there is no way to rearrange production or reallocate goods so that someone is better off without making someone else worse off.

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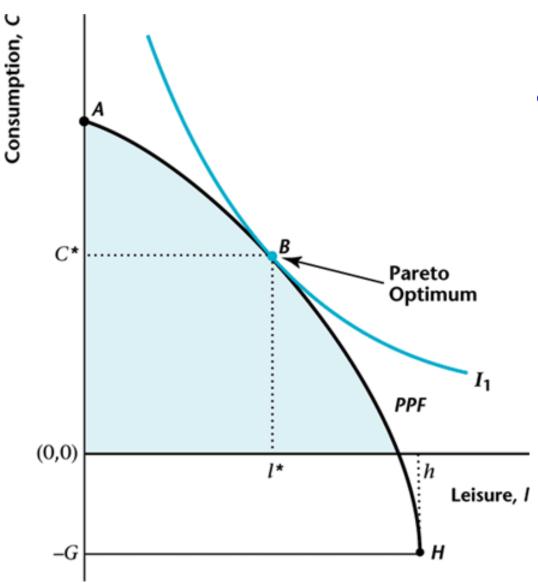
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 - 2. Order the consumer to work $N^s = N$ hours.
 - 3. Take an amount G of output and give the remainder to the consumer.

Hence the planners problem is to choose c and l that, given technological constraints, maximize the utility of the consumer.

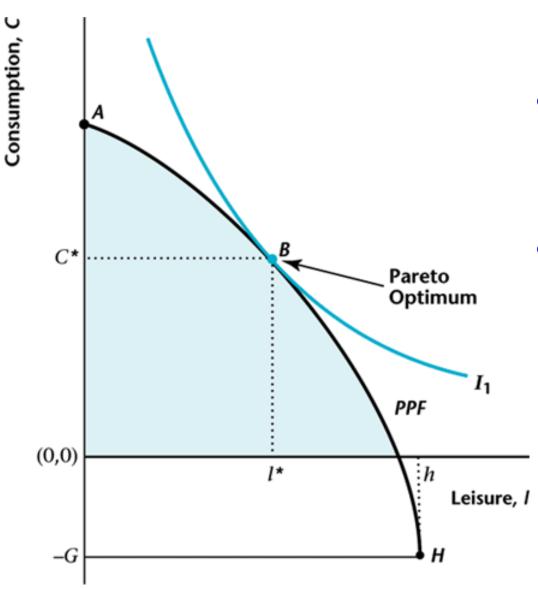
• Formally, he solves:

$$\max_{c,l} U(c,l)$$
 subject to $c = zF(K,h-l) - G$
$$c \ge 0$$

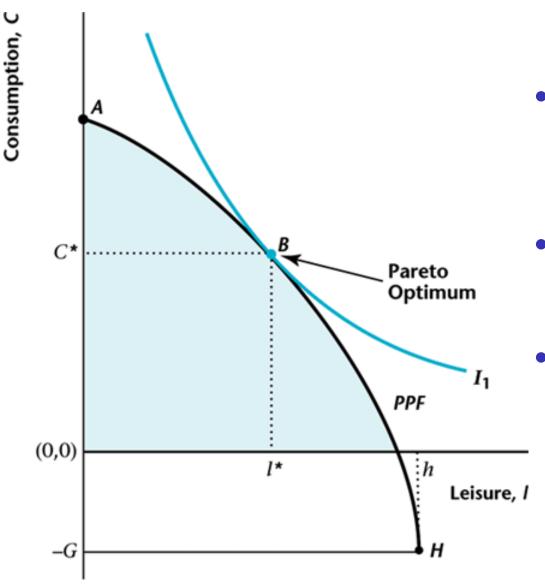
$$0 \le l \le h$$



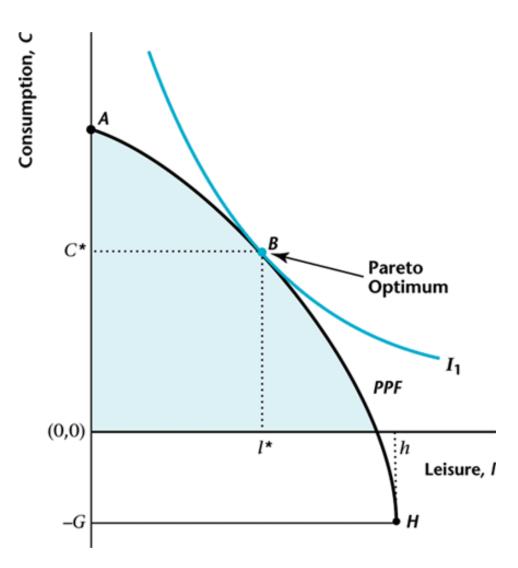
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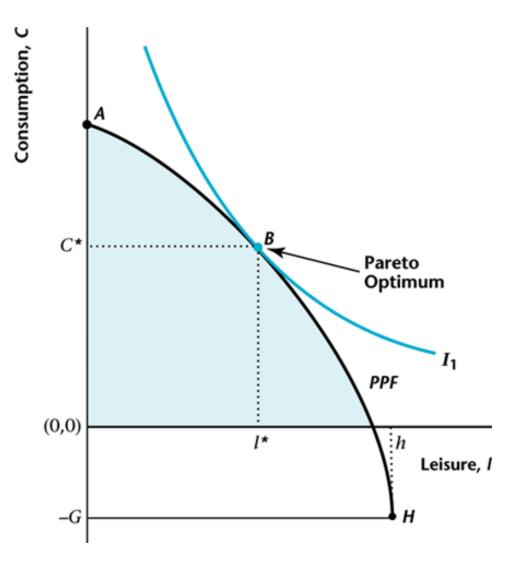


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- This is similar to our previous problem, but we don't get to worry about the budget constraint.



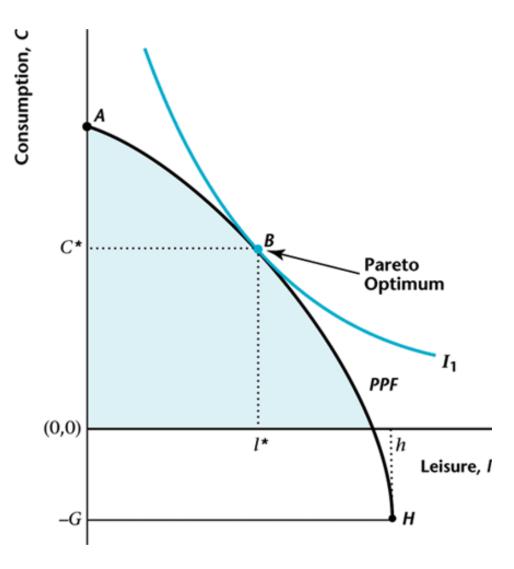
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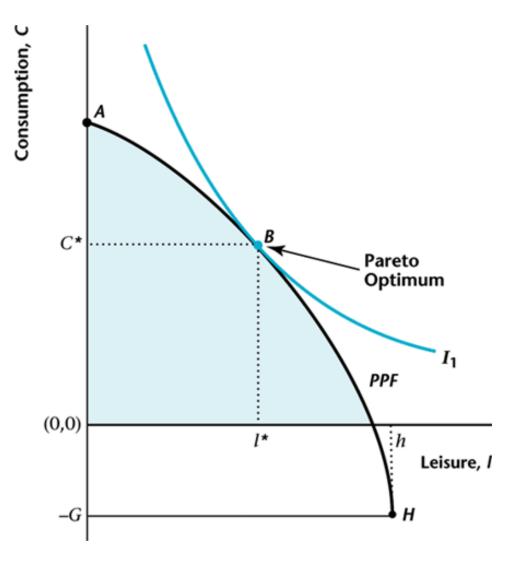
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 - Theorem (The Second Fundamental Theorem of Welfare Economics) Under certain conditions, a Pareto optimal allocation can be established as a competitive equilibrium.
- Free market economies tend to produce socially efficient economic outcomes.



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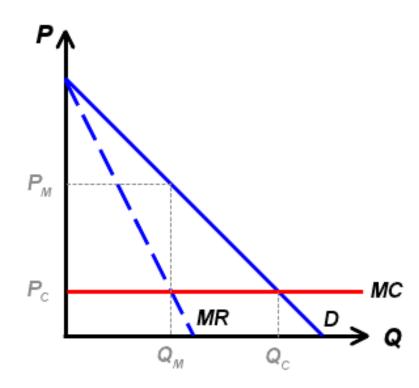
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- Hence, the socially efficient outcome is not reached. A competitive equilibrium is not Pareto optimal.

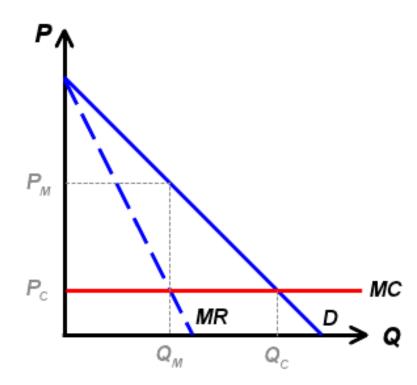
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And the equivalence condition breaks down.



SOLVING THE CE

• To avoid dealing with prices we will use the equivalence between competitive equilibrium and Pareto optimal allocations. Thus, the solution to the planners problem is our competitive equilibrium.