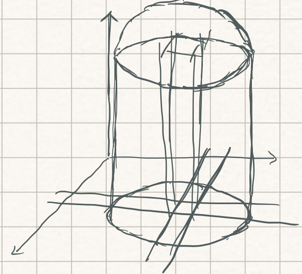
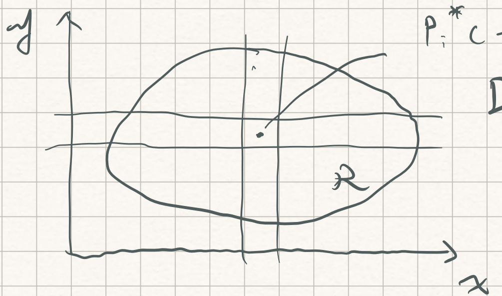
Double Integrals.

Consider a function $z=f(x,y)$, which represent a surface in three-dimension space Assume the surface is above the xy-plane i.e. $f(x,y)\geq 0$ let $f(x,y)$ be defined on a region R whose boundary is a close curve C.



Compute the volum V



Dividing R into n subregions by $x=\text{cost}$ and $y=\text{sint}$. let $P_i^*(x_i^*, y_i^*)$ be an arbitary point in the subregion ΔR_i with ΔA_i

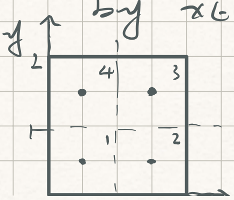
$$\Delta V_i = f(P_i^*)\Delta A_i = f(x_i^*, y_i^*)\Delta A_i$$

$$\iint_R f(x,y)\,dA = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*, y_i^*)\Delta A_i$$

Estimate a double intergral over a rectangular region

- The midpoint rule.

e.g. 1. $\iint_R (16-x^2-y^2)\,dA$ where R is the rectangule defined by $x\in[0,2]$, $y\in[0,2]$. ($R=[0,2]\times[0,2]$.)



$P_1(x_i^*, y_i^*) = (\frac{1}{2}, \frac{1}{2})$ 3 $= (\frac{3}{2}, \frac{3}{2})$

2 $= (\frac{3}{2}, \frac{1}{2})$ 4 $= (\frac{1}{2}, \frac{3}{2})$.

$$\therefore f(x_1^*, y_1^*) = f(\tfrac{1}{2}, \tfrac{1}{2}) = 16 - (\tfrac{1}{2})^2 - 2(\tfrac{1}{2})^2 = \tfrac{61}{4}.$$

$$\quad 2, \quad 2 \quad = f(\tfrac{3}{2}, \tfrac{1}{2}) = \tfrac{53}{4}$$

$$\quad 3 \quad 3 \quad = f(\tfrac{3}{2}, \tfrac{3}{2}) = \tfrac{37}{4}$$

$$\quad 4 \quad 4 \quad = f(\tfrac{1}{2}, \tfrac{3}{2}) = \tfrac{45}{4}.$$

$$\Delta A = \Delta A_1 = \Delta A_2 = \Delta A_3 = \Delta A_4 = (1)(1) = 1.$$

$$V = \iint_R (16 - x^2 - 2y^2)\, dA \approx \sum_{i=1}^{4} f(x_i^*, y_i^*)\, \Delta A \approx 49.$$

very good.

percentage of relative error $= \frac{|48-49|}{48} \times 100\% \approx 2.1\%.$