A3 sol

(win 2022

Q1

(a) s(n) and p() always use the global a:

s(0) – $a_g = 0$

f() : s(1) : $a_g = 1$

p() : print 1

p(): print 1

g() : s(2) : $a_g = 2$

p() : print 2

p() : print 2

output: 1, 1, 2, 2

(b) most recent a : a_g

s(0) : $a_g = 0$

f() : s(1) : $a_g = 1$

p() : print 1

p() : print 1

g() : most recent a: a_e

s(2) : $a_e = 2$

p() : print 2

most recent a : a_g

p() : print 1

output : 1, 1, 2, 1

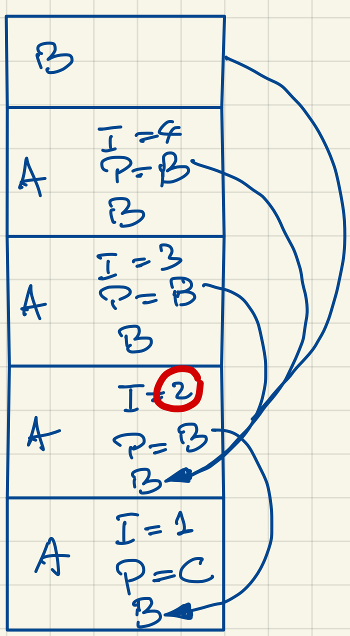
Q2 We add the print statements below:

```python
def A(I, P):
    def B():
        print(I)
    print("A call:  I = " + str(I) + "  B = " + str(B) + "  P = " + str(P))
    if I > 3:
        P()
    elif I > 2:
        A(4, P)
    elif I > 1:
        A(3, B)
    else:
        A(2, B)
def C():
    print(0)
print("C = ", end="")
print(C)
A(1, C)
```

The output is:

```
C = <function C at 0x7fd978159048>
A call:  I = 1  B = <function A.<locals>.B at 0x7fd978159268>  P = <function C at 0x7fd978159048>
A call:  I = 2  B = <function A.<locals>.B at 0x7fd9781596a8>  P = <function A.<locals>.B at 0x7fd978159268>
A call:  I = 3  B = <function A.<locals>.B at 0x7fd978159730>  P = <function A.<locals>.B at 0x7fd9781596a8>
A call:  I = 4  B = <function A.<locals>.B at 0x7fd9781597b8>  P = <function A.<locals>.B at 0x7fd9781596a8>
2
```

The stack:



Python uses deep binding, which is why the deeper I = 2 is printed.

Q3 (a) - call by value

$1+3 \equiv +\ 1\ 3 \equiv$

$(\lambda mnab.\ ma\ (n\ a\ b))(\lambda fc.\ fc)(\lambda fc.\ f(f(f\ c))) \Rightarrow_\beta$

$(\lambda nab.\ (\lambda fc.fc)\ a\ (n\,a\,b))\ (\lambda fc.\ f(f(f\ c))) \Rightarrow_\beta$

$(\lambda nab.\ (\lambda c.\ ac)\ (n\,a\,b))(\lambda fc.\ f(f(fc))) \Rightarrow_\beta$

$(\lambda nab.\ a\ (n\,a\,b))\ (\lambda fc.\ f(f(fc))) \Rightarrow_\beta$

$\lambda ab.\ a\ ((\lambda fc.\ f(f(fc)))\ a\ b) \Rightarrow_\beta$

$\lambda ab.\ a((\lambda c.\ a(a(ac)))\ b) \Rightarrow_\beta$

$\lambda ab.\ a\ (a(a(a\,b))) \equiv 4$

- call by name

$1+3 \equiv +\ 1\ 3 \equiv$

$(\lambda mnab.\ ma\ (n\ a\ b))(\lambda fc.\ fc)(\lambda fc.\ f(f(f\ c))) \Rightarrow_\beta$

$(\lambda nab.\ (\lambda fc.fc)\ a\ (n\,a\,b))\ (\lambda fc.\ f(f(f\ c))) \Rightarrow_\beta$

$\lambda ab.\ (\lambda fc.\ fc)\ a\ ((\lambda fc.\ f(f(fc)))\ a\ b) \Rightarrow_\beta$

$\lambda ab.\ (\lambda c.\ ac)\ ((\lambda fc.\ f(f(fc)))\ a\ b) \Rightarrow_\beta$

$\lambda ab.\ a\ ((\lambda fc.\ f(f(fc)))\ a\ b) \Rightarrow_\beta$

$\lambda ab.\ a\ ((\lambda c.\ a(a(ac)))\ b) \Rightarrow_\beta$

$\lambda ab.\ a\ (a(a(a\,b))) \equiv 4$

(b) - call by value

$$0 * 2 \equiv * \; 0 \; 2 \equiv$$

$$(\lambda mna.\, m(na))(\lambda fc.c)(\lambda fc.\, f(fc)) \Rightarrow_\beta$$

$$(\lambda na.\, (\lambda fc.c)(na))(\lambda fc.\, f(fc)) \Rightarrow_\beta$$

$$(\lambda na.\, (\lambda c.c))(\lambda fc.\, f(fc)) \Rightarrow_\beta$$

$$\lambda a.\, \lambda c.c \equiv \lambda ac.c \equiv 0$$

- call by name

$$0 * 2 \equiv * \; 0 \; 2 \equiv$$

$$(\lambda mna.\, m(na))(\lambda fc.c)(\lambda fc.\, f(fc)) \Rightarrow_\beta$$

$$(\lambda na.\, (\lambda fc.c)(na))(\lambda fc.\, f(fc)) \Rightarrow_\beta$$

$$\lambda a.\, (\lambda fc.c)((\lambda fc.\, f(fc))a) \Rightarrow_\beta$$

$$\lambda a.\, \lambda c.c \equiv \lambda ac.c \equiv 0$$

Q4

- call by value

$$XOR\ T\ T \equiv$$
$$(\lambda xy.x(yFT)y)\ T\ T \Rightarrow_\beta$$
$$(\lambda y.T(yFT)y)\ T \equiv$$
$$(\lambda y.(\lambda xy.x)(yFT)y)\ T \Rightarrow_\alpha$$
$$(\lambda y.(\lambda xz.x)(yFT)y)\ T \Rightarrow_\beta$$
$$(\lambda y.(\lambda z.yFT)y)\ T \Rightarrow_\beta$$
$$(\lambda y.yFT)\ T \Rightarrow_\beta$$
$$TFT \equiv$$
$$(\lambda xy.x)(\lambda xy.y)\ T \Rightarrow_\beta$$
$$(\lambda y.(\lambda xy.y))\ T \Rightarrow_\beta$$
$$\lambda xy.y \equiv F$$

- call by name

$$XOR\ T\ T \equiv$$
$$(\lambda xy.x(yFT)y)\ T\ T \Rightarrow_\beta$$
$$(\lambda y.T(yFT)y)\ T \Rightarrow_\beta$$
$$T(TFT)T \equiv$$
$$(\lambda xy.x)(TFT)\ T \Rightarrow_\beta$$
$$(\lambda y.TFT)\ T \Rightarrow_\beta$$
$$TFT \equiv$$
$$(\lambda xy.x)(\lambda xy.y)\ T \Rightarrow_\beta$$
$$(\lambda y.(\lambda xy.y))\ T \Rightarrow_\beta$$
$$\lambda xy.y \equiv F$$