§4.2 cont

Recall: $E = \{(s,c) \in S \times C \mid s \text{ is enrolled in } c\}$.

Ex: $E^{-1} \circ E = \{(s,s) \in S \times S \mid \exists c, (s_1, c) \in S \wedge (s_2, c) \in S^{-1}$
$= \{(s,s) \in S \times S \mid \exists c\ (s_1, c) \in E \wedge (s_2, c) \in E$

$\Rightarrow$ two students are related if they have a course together.

Ex2: $E \circ E^{-1} = \{(c,c) \in C \times C \mid \exists s, (s, c_1) \in C \wedge (s, c_2) \in C^{-1}$

Thm 4.2.5: For $R \subseteq A \times B$, $S \subseteq B \times C$, $T \subseteq C \times D$

$T \circ (S \circ R) = (T \circ S) \circ R$

$(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

Proof: both of these are relations from C to A. Let $(c,a) \in C \times A$

Then:

$(c,a) \in R^{-1} \circ S^{-1}$

iff $\exists b$ where $(c,b) \in S^{-1} \wedge (b,a) \in R^{-1}$

iff $(b,c) \in S \wedge (a,b) \in R$

iff $(a,c) \in S \circ R$

iff $(c,a) \in (S \circ R)^{-1}$

So, ... $\square$

*Midterm Tue Nov 8, 8-10 pm, WSC-55

OH Thur + Fri, Nov 3+4 in class (zoom)

Mon, Nov 7 4:30-5:30 pm

Tue Nov 8 11:30-12:30

Cover up to 4.2.

Questions similar to HW, two more sets on webwork.

style + correctness

Focus chapter 3+4 more.

3.1 general ideas for proof

3.2 $\neg$, $\rightarrow$

3.3, $\forall$, $\exists$

3.4 $\leftrightarrow$, $\wedge$

3.5 $\vee$

3.6 $\exists!$

3.7 Ex

4.1 $A \times B$

4.2 $\circ$, $^{-1}$, Dom

Summary

4.1: $A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$

$(a,b) = (a',b')$ iff $a = a' \wedge b = b'$

4.2 A relation from A to B is a subset $R \subseteq A \times B$

The domain $\mathrm{Dom}(R) = \{a \in A \mid \exists b\ (a,b) \in R\}$

The range $\mathrm{Ran}(R) = \{b \in B \mid \exists a\ (a,b) \in R\}$.

The inverse $R^{-1} = \{(b,a) \in B \times A \mid (a,b) \in R\}$.

The composite $R \subseteq A \times B$, $S \subseteq B \times C$,

$S \circ R = \{(a,c) \in A \times C \mid \exists b \in B\ (a,b) \in R \wedge (b,c) \in S\}$.

§3.6. exercise 3.

$\forall x$, if $x \neq 0 \wedge x \neq 1$, then $\exists! y \in \mathbb{R}$ $y/x = y - x$

Rough work: $y/x = y - x$

$y = xy - x^2$

$y = \frac{-x^2}{1-x}$ move the "if" part to assume.

Proof: Let $x \in \mathbb{R}$, assume $x \neq 0 \wedge x \neq 1$

Existence: let $y = \frac{-x^2}{1-x}$.

then left-hand side: $y/x = \frac{\frac{-x^2}{1-x}}{x} = -\frac{x}{1-x}$

right-hand side: $y - x = -\frac{x^2}{1-x} - x = -\frac{x}{1-x}$

so ....

Uniqueness: Assume $z \in \mathbb{R}$ that $z/x = z - x$

$z/x = z - x$

$z = xz - x^2$

$z = -\frac{x^2}{1-x}$

so ....

3.6.2 $\exists! x\ (\forall y\ \ xy + x - 4 = 4y)$

in this case, x would only be a number, not an equation

Roughwork: Take $y=0$: $x=4$

Take $y=1$: $x=4$

Proof: Existence: let $x=4$: $4y+4-4=4y$ holds.

Uniqueness: Assume $z \in \mathbb{R}$ and $zy + z - 4 = 4y$ for all y:0.

take $y=0$, then $z=4$ □.