Nov 29

COMPSCI 3331

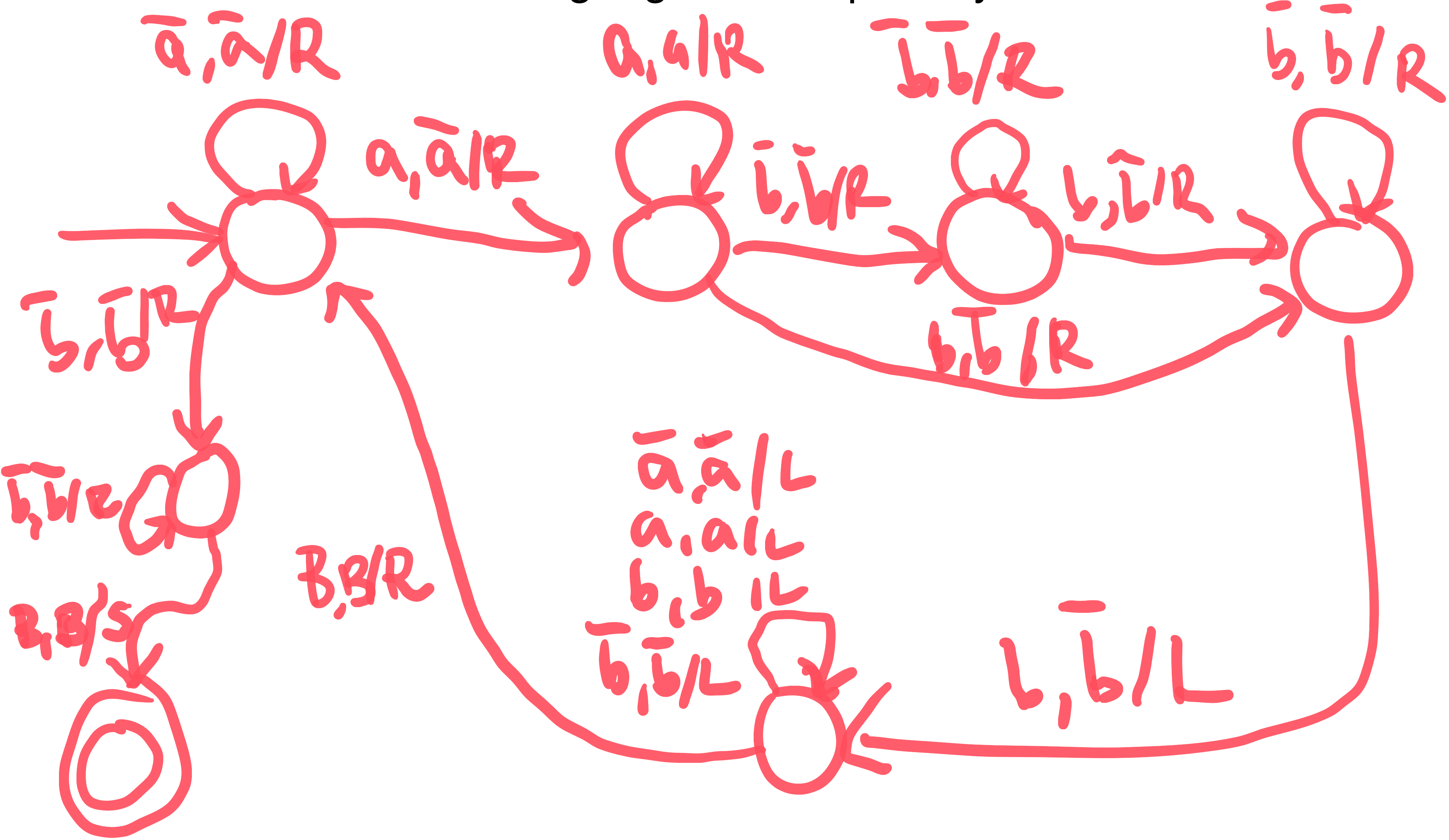
Fall 2022

What's next?

- Please complete feedback for the course: `feedback.uwo.ca`
- Assignment 4: due Dec 7, gradescope available. **Error in Q1 fixed.**
- Last Quiz 8 tomorrow – **Lectures 15 and 16**.
- Quiz 7, Asst 3: being marked.
- Quiz 5,6: grades available.
- Solutions up to Q7, A2 marking guide, MT solutions available.

What language ?

Which context-free language is accepted by this TM?



Recursive / r.e. languages

L is **accepted** by a TM $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ if

$$L = L(M) = \{w \in \Sigma^* \ : \ \exists x_1, x_2 \in \Gamma^*, q_f \in F q_0 w \vdash_M^* x_1 q_f x_2\}.$$

L is **recognized** by a TM M if

(a) $L = L(M)$.

(b) For every word $w \notin L$, M halts and rejects w.

Recursive and Recursively Enumerable

- A language L is **recursive** if there is a TM M such that L is **recognized** by M.
- A language L is **recursively enumerable** (r.e.) if there is a TM M such that L is **accepted** by M.

Recursive / r.e. languages

- Are the recursive languages closed under reversal?
  - i.e., if L is recognized by a TM, is L^R is also recognized by a TM?
- Are the r.e. languages closed under concatenation?
  - i.e., if L_1, L_2 are accepted by TMs, is $L_1 L_2$ is also recognized by a TM?

Reduction

- Let L_0, L_1 be two languages.
- We map between languages using reductions.
- A reduction is a function f which can be computed by a TM and satisfies the following properties:
  - if $x \in L_0$ then $f(x) \in L_1$.
  - if $x \notin L_0$ then $f(x) \notin L_1$.

Reductions

- $L_1 = \{u\#v \ : \ bin(u) + 1 = bin(v^R)\}$
- $L_2 = \{a^n\#a^{n+1} \ : n \geq 0\}$
- Reduction of L_1 to L_2 with a TM.

Encodings of TM

Let $M = (Q, \{0,1\}, \Gamma, \delta, q_1, B, F)$ be a TM.

- $Q = \{q_1, q_2, q_3, \ldots, q_r\}$ for some $r \geq 1$. We can also assume that $F = \{q_r\}$.
- $\Gamma = \{\alpha_1, \alpha_2, \ldots, \alpha_s\}$ for some $s \geq 3$. We assume $\alpha_1 = 0, \alpha_2 = 1$, and $\alpha_3 = B$.

Consider a transition $\delta(q_i, \alpha_j) = (q_k, \alpha_\ell, D)$. We encode this **single** transition as the word

$$0^i 1 0^j 1 0^k 1 0^\ell 1 0^{m(D)}$$

where $m(D)$ is $1, 2, 3$ if D is L, S, R, respectively.
(state from) 1 (symb. read) 1 (state to) 1 (symb. write) 1 (dir.)

Encoding TMs

We now encode the **entire** TM $M = (Q, \Sigma, \Gamma, \delta, q_1, B, F)$.
Let $C_1, C_2, \ldots, C_m$ be the encodings of the m transitions of the TM. Then we encode M as

$$e(M) = C_1 11 C_2 11 C_3 11 \cdots 11 C_m$$