

Assuming that  $a = (A \setminus B) \setminus C$ ,  $b = (B \setminus A) \setminus C$ ,  $c = (C \setminus A) \setminus B$ ,  $d = (A \cap B) \setminus C$ ,  $e = (B \cap C) \setminus A$ ,  $f = (A \cap C) \setminus B$ ,  $g = A \cap B \cap C$

So,  $A = a \cup d \cup g \cup f$ ,  $B = b \cup d \cup g \cup e$ ,  $C = c \cup e \cup g \cup f$

According to the definition of symmetric difference,  $A \Delta B = a \cup f \cup b \cup e$ ,  $B \Delta C = b \cup d \cup c \cup f$ ,  $A \Delta C = a \cup d \cup c \cup e$ .

$$1. \quad A \Delta (B \Delta C) = A \Delta (b \cup d \cup c \cup f) = (a \cup d \cup f \cup g) \Delta (b \cup d \cup c \cup f) = a \cup b \cup c \cup g$$

$$(A \Delta B) \Delta C = (a \cup f \cup b \cup e) \Delta C = (a \cup f \cup b \cup e) \Delta (c \cup e \cup g \cup f) = a \cup b \cup c \cup g$$

Thus,  $A \Delta (B \Delta C) = (A \Delta B) \Delta C$ , the operation  $\Delta$  is associative

$$2. \quad A^- = b \cup e \cup c, B^- = a \cup f \cup c, C^- = a \cup d \cup b$$

$$A \cap B \cap C = g, A \cap B^- \cap C^- = a, B \cap C^- \cap A^- = b, C \cap B^- \cap A^- = c, A \cap B \cap C^- = d, B \cap C \cap A^- = e, C \cap A \cap B^- = f$$

$$(A \cap B \cap C) \cup (A \cap B^- \cap C^-) \cup (B \cap C^- \cap A^-) \cup (C \cap B^- \cap A^-) \cup (A \cap B \cap C^-) \cup (B \cap C \cap A^-) \cup (C \cap A \cap B^-) = g + a + b + c + d + e + f$$

$$A \cup B \cup C = a + b + c + d + e + f + g$$

$$\text{Thus, } A \cup B \cup C = (A \cap B \cap C) \cup (A \cap B^- \cap C^-) \cup (B \cap C^- \cap A^-) \cup (C \cap B^- \cap A^-) \cup (A \cap B \cap C^-) \cup (B \cap C \cap A^-) \cup (C \cap A \cap B^-)$$

$$3. \quad \text{Convert the logic into English: If } A \Delta (B \Delta C) = \emptyset, \text{ then } A \cup B \cup C = ((A \cap B) \setminus C) \cup ((A \cap C) \setminus B) \cup ((B \cap C) \setminus A)$$

Given that  $A \Delta (B \Delta C) = \emptyset$ , it can be inferred that  $a \cup b \cup c \cup g = \emptyset$ ,  $a = b = c = g = \emptyset$

$$A \cup B \cup C = a \cup b \cup c \cup d \cup e \cup f \cup g = d \cup f \cup e$$

$$((A \cap B) \setminus C) \cup ((A \cap C) \setminus B) \cup ((B \cap C) \setminus A) = ((d \cup g) \setminus (c \cup f \cup g \cup e)) \cup ((g \cup f) \setminus (b \cup d \cup g \cup e)) \cup ((g \cup e) \setminus (a \cup d \cup g \cup f)) \\ = d \cup f \cup e$$

Thus,  $A \cup B \cup C = ((A \cap B) \setminus C) \cup ((A \cap C) \setminus B) \cup ((B \cap C) \setminus A)$  when  $A \Delta (B \Delta C) = \emptyset$