Ch 11 + Sec. 8.1.

15 MC + 6 L.

e.g. 1 $\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{n!}$ con/div?

ratial test: $|a_n| = \frac{n^n}{n!}$ $\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n}$

$= \frac{(n+1)^{n+1}}{n^n} \cdot \frac{1}{n+1}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = e > 1 \Rightarrow$ divergent. $= (1+\frac{1}{n})^n.$

let $y = (1+\frac{1}{n})^n$

$\ln y = \underbrace{n}_{\infty} \underbrace{\ln(1+\frac{1}{n})}_{0}$

$= \frac{\ln(1+\frac{1}{n})}{\frac{1}{n}}$

$\rightarrow$ apply L'o

$= \frac{1}{1+\frac{1}{n}} \cdot (-\frac{1}{n^2}) / (-\frac{1}{n^2})$

$= \frac{n}{n+1}$ $n \to \infty$ $\ln y \to 1$ $y \to e.$

e.g. 2. Determine whether $\sum_{n=1}^{\infty} (-1)^n \sin(\frac{\pi}{n})$ converges or diverges.

$n \to \infty \Rightarrow \sin(\frac{\pi}{n}) \to \frac{\pi}{n} = b_n$

$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{\pi}{n} = \pi \sum_{n=1}^{\infty} \frac{1}{n} \Leftarrow$ diverges (harmonic series).

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin\frac{\pi}{n}}{\frac{\pi}{n}} = 1 \Rightarrow$ diverses by the limit comparison test.

$\lim_{n\to\infty} a_n = 0$ $\because \frac{\pi}{n} \in (0, \frac{\pi}{2}]$. $\therefore \sin\frac{\pi}{n} > \sin\frac{\pi}{n+1}$

converges by the alternative series test (AST).

e.g. 3. $f(x) = \frac{x-1}{x+2}.$

$f(x) = \frac{(x+2)-2-1}{\quad} = 1 - \frac{3}{\quad} = 1 - 3\frac{1}{1+x}$

$x+2 \qquad x+2 \qquad \frac{}{2} \; 1+\frac{x}{2}.$

$1+x+x^2+x^3+\cdots\cdots = \frac{1}{1-x}.$

$1-x+x^2-x^3-\cdots\cdots = \frac{1}{1+x}.$ $\quad \Rightarrow \quad = 1-\frac{3}{2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{x}{2}\right)^n$

$\left|\frac{x}{2}\right|<1 \quad x\in(-2,2).$

At $x=-2$. Series $= -\frac{1}{2}+3\sum_{n=0}^{\infty}(-1)^n\frac{(-2)^n}{2^n}\cdot\frac{1}{2}.$

$= -\frac{1}{2}+3\sum_{n=0}^{\infty}\frac{1}{2}. = \infty$

$x=2$ Series $= -\frac{1}{2}+3\sum_{n=0}^{\infty}(-1)^n\cdot\frac{1}{2}.$

$=$

$(-2,2).$

e.g. 4. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^{2/3}}$ absolutely / conditionally convergent / divergent.

AST: the series is alternating

1) $a_n=\frac{1}{n^{2/3}}$, $p=2/3<1 \Rightarrow$ diverges because p-series.

2)

$\Rightarrow$ conditional convergent.

e.g. 5. power series of $f(x)=x^2\ln(1-x).$

$\sum_{n=0}^{\infty}\int x^n dx = \int\frac{dx}{1-x}$

$\sum_{n=0}^{\infty}\frac{x^{n+1}}{n+1} = -\ln(1-x).$