

P	f1(P)	f2(P)	f3(P)= $\sim$ P	f4(P)					
0	1	0	1	0					
1	1	0	0	1					
P	Q	f1(P,Q)	f2(P,Q)	.....					
0	0	0	0	0	.....				
0	1	0	0	0	.....				
1	0	0	0	0	.....				
1	1	0	1	1	.....				

Don't need to worry about proving all of these as you only need to show that your set can generate  $\{\sim, *, \vee, \Rightarrow, \Leftrightarrow\}$  because as per the slides this set is already adequate

So to prove  $\cdot$  is not adequate you can show it doesn't generate  $f3(P)=\sim P$

P	P*P	P*P*P	P*P*P*P	.....	$\sim P$				
0	0	0	0	0	0	1			
1	1	1	1	1	1	0			

no way to generate  $\sim$  with  $\cdot$  so  $\{*\}$  is not adequate (also can't generate  $f1(P)$ ,  $f2(P)$  above)

$\sim$  and  $*$  are given as they are in  $\{\sim, *\}$

Use truth table to show other ones (maybe you can do it without truth tables but it is good to check)

Show we can generate $\vee$					Not unique				
P	Q	$P \vee Q$	$\sim P * \sim Q$	$\sim(\sim P * \sim Q)$	$\sim((\sim P * \sim Q).(\sim P * \sim Q))$				
0	0	0	1	0	0				
0	1	1	0	1	1				

1	0	1	0	1	1			
1	1	1	0	1	1			
Show we can generate $P \Rightarrow Q$								
From textbook								
$P \Rightarrow Q$	=	$\sim P \vee Q$						
Substitute the formula we already have for $\vee$								
$\sim P \vee Q$	=	$\sim(\sim P) * \sim Q$	=	$\sim(P * \sim Q)$				
Don't need to check with truth table but I will do it so you can see								
P	Q	$P \Rightarrow Q$	$P * \sim Q$	$\sim(P * \sim Q)$				
0	0	1	0	1				
0	1	1	0	1				
1	0	0	1	0				
1	1	1	0	1				
It works the other way too. If you have $\Rightarrow$ you can convert to $\vee$ using $\sim \sim P = P$								
Similarly from the text								
$P \Leftrightarrow Q$	=	$(P \Rightarrow Q) \cdot (Q \Rightarrow P)$						
Again substitute the formula we have for $P \Rightarrow Q$ from above								
$(P \Rightarrow Q) * (Q \Rightarrow P)$	=	$\sim(P * \sim Q) * \sim(\sim P * Q)$						

Then we are done as we have show we can write all standard connectives with $\{\sim, *\}$									
(I will not do the truth table for this one)									
So once you have $\sim, *, \vee$ you can use the substitution rules to get $\Rightarrow$ and $\Leftrightarrow$									