Q1. $x_s = \dfrac{(x+1)\cdot 600}{2} + 200 = \dfrac{x+1}{2} + 500$

$y_s = \dfrac{(y+1)\cdot 100}{2} + 100 = \dfrac{y+1}{2} + 150.$

so the matrix should be $\left(\dfrac{x+1}{2} + 500, \dfrac{y+1}{2} + 150\right)$.

Q2 $M_0 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

$M_1 = TR = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 2 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$M_2 = RT = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{-3+2\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{3\sqrt{3}+2}{2} \\ 0 & 0 & 1 \end{bmatrix}$

$M_0 M_1 = M_0 M_2 = \begin{bmatrix} \frac{-1+2\sqrt{3}}{2} \\ \frac{\sqrt{3}+2}{2} \\ 0 \end{bmatrix}$

transformed by M1: first rotate counterclockwise for 30 degree,
then move up and right.
M2: move upward for 3 and right for 2,
then rotate counterclockwise for 30 degree

Q3: S would be written as $\begin{bmatrix} 1 & m & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

so $S^{-1} = \begin{bmatrix} 1 & -m & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q_4: $\quad v_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \quad w_3 = \begin{bmatrix} \frac{9-\sqrt{3}}{2} \\ \frac{5-\sqrt{3}}{2} \\ 1 \end{bmatrix}$

Assume that $S = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$

$R = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 & \frac{9-\sqrt{3}}{2} \\ 0 & 1 & \frac{9-\sqrt{3}}{2} \\ 0 & 0 & 1 \end{bmatrix}$

$w_1 = \begin{pmatrix} \frac{11-\sqrt{3}}{2} \\ \frac{11-\sqrt{3}}{2} \\ 1 \end{pmatrix} \quad w_2 = \begin{pmatrix} \frac{5-\sqrt{3}}{2} \\ \frac{9-\sqrt{3}}{2} \\ 1 \end{pmatrix}$

$W = TSRV$