Mathematics 1229A/B Additional Chapter 1 Exercises

1. If $\mathbf{u} = (4, 1, k)$ and $\mathbf{v} = (5, 1, -3)$ are perpendicular, find k.
2. If $\mathbf{u} = (2, -6, k)$ and $\mathbf{v} = (-1, 3, 2)$ are parallel, find k.
3. Let $\mathbf{u} = (k, 6)$ and $\mathbf{v} = (-5, 1)$. Find k if
   (a) $\mathbf{u}$ and $\mathbf{v}$ are parallel.
   (b) $\mathbf{u}$ and $\mathbf{v}$ are perpendicular.
   (c) $\|\mathbf{u}\| = 10$.
4. Find a nonzero vector which is perpendicular to both $\mathbf{u} = (4, 1, 1)$ and $\mathbf{v} = (-1, 0, 2)$.
5. Find a unit vector which is perpendicular to both $\mathbf{u} = (1, 2, 3)$ and $\mathbf{v} = (1, 1, 2)$.
6. Find the cosine of the angle between the vectors $\mathbf{u} = (2, 1, -2)$ and $\mathbf{v} = (-2, 3, 6)$.
7. Find the area of the parallelogram determined by the vectors $\mathbf{u} = (1, 1, 2)$ and $\mathbf{v} = (2, 0, 1)$.
8. Find the area of the parallelogram determined by the vectors $\mathbf{u} = (-1, 0, 3)$ and $\mathbf{v} = (2, 1, -5)$.
9. Find the area of the triangle with vertices $(0, 0, -1), (1, 0, 1)$ and $(1, 1, 1)$.
10. Find the volume of the parallelepiped determined by the vectors $\mathbf{u} = (2, 1, -2)$, $\mathbf{v} = (-2, 3, 6)$ and $\mathbf{w} = (-1, 1, 1)$.
11. Find the volume of the parallelepiped determined by the vectors $\mathbf{u} = (-1, 0, 3)$, $\mathbf{v} = (2, 1, -5)$ and $\mathbf{w} = (2, -1, 1)$.
12. Line ℓ passes through the points $(1, 2)$ and $(4, -3)$.
    (a) Give a vector parallel to line ℓ.
    (b) Give a normal for line ℓ.
    (c) Give an equation for line ℓ in
        (i) point-parallel form,
        (ii) point-normal form,
        (iii) standard form.
13. Line ℓ passes through the points $(1, 0, -1)$ and $(2, 2, 2)$.
    (a) Give an equation in two-point form for line ℓ.
    (b) Give an equation in point-parallel form for line ℓ.
    (c) Give parametric equations for line ℓ.
14. If the point $(3, 1, k)$ lies on the line $(x, y, z) = (6, 2, 7) + t(3, 1, 5)$, find k.
15. Find a normal to the line passing through the points $(2, -1)$ and $(3, 5)$.
16. Find a vector parallel to the line $4x - 3y = 10$.
17. Find a normal for a line perpendicular to the line $(x, y) = (3, 4) + t(1, -2)$.

18. Give an equation in point-parallel form for the line which passes through the point $(1, -1)$ and is perpendicular to the line $3x + 7y = 11$.

19. Give an equation in standard form for the line $(x, y) = (1, -2) + t(4, -9)$.

20. Consider the plane $4x + 2y - z + 1 = 0$.
    (a) Give a normal to this plane.
    (b) Which, if any, of the points $A(1, -2, 0), B(2, -3, 1), C(-1, 2, 1)$ lie on this plane?

21. Consider the plane through the point $(-1, 1, 3)$ which is parallel to vectors $(1, 4, -1)$ and $(2, -3, 1)$.
    (a) Find a normal for this plane.
    (b) Give an equation in point-normal form for this plane.
    (c) Give an equation in standard form for this plane.

22. Consider the lines

$$\begin{aligned} \ell_1 &: \quad (x, y) = (1, -1) + r(2, 1) \\ \ell_2 &: \quad (x, y) = (0, 3) + s(-2, 4) \\ \ell_3 &: \quad (x, y) = (-2, 3) + t(1, 2) \end{aligned}$$

    (a) Are ℓ_1 and ℓ_2 parallel, perpendicular or neither?
    (b) Find the point of intersection of ℓ_2 and ℓ_3.

23. Find the point of intersection of lines ℓ_1 and ℓ_2 where

$$\ell_1 : \begin{array}{l} x = 9 - t \\ y = 4 + 3t \end{array} \qquad \ell_2 : \begin{array}{l} x = 7 + 2s \\ y = 1 - 3s \end{array}$$

24. Find the point of intersection of lines ℓ_1 and ℓ_2 where

$$\begin{aligned} \ell_1 &: \quad (x, y, z) = (0, -3, -6) + s(1, -1, -5) \\ \ell_2 &: \quad (x, y, z) = (-5, -7, 2) + t(3, 6, 2). \end{aligned}$$

25. Find the point of intersection of the line $(x, y, z) = (-1, 2, 3) + t(2, 3, 4)$ with the plane $2x + y - z = 0$.

26. Find the point of intersection of the line $(x, y, z) = (-3, 1, 2) + t(-1, 2, 3)$ with the plane $x - 3y + 2z = 4$.

27. Find the point of intersection of the line $(x, y, z) = (1, 0, 2) + t(1, 2, 3)$ with the plane $x - y + 3z = 23$.

28. Find the distance from the point $(1, 1, 1)$ to the plane $2x + y - 3z = 1$.

29. Find the distance from the point $(1, 2, 3)$ to the plane $x - 3y + z = 5$.

30. Find the distance from the point $(3, 1)$ to the line $2x - 3y = 5$.