

$$\begin{bmatrix} 1 & 4 & 7 & | & 10 \\ 2 & 5 & 8 & | & 11 \\ 3 & 6 & 9 & | & 12 \end{bmatrix} [A|B]$$

$r(A) \Rightarrow$ 秩子数
 $m \Rightarrow$ 未知数数

$$r(A) = m$$

$$\begin{bmatrix} 1 & 2 & 3 & | & 10 \\ 4 & 5 & 6 & | & 11 \\ 7 & 8 & 9 & | & 12 \end{bmatrix} \Rightarrow \text{unique solution}$$

$$\begin{bmatrix} 1 & 2 & 3 & | & 13 \\ 4 & 5 & 6 & | & 14 \\ 7 & 8 & 9 & | & 15 \end{bmatrix} \Rightarrow \text{no solution} \Rightarrow \begin{matrix} \text{三未知数四个方程} \\ \text{会有冲突} \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & | & 7 \\ 4 & 5 & 6 & | & 8 \end{bmatrix} \Rightarrow \text{infinitely solution} \Rightarrow \begin{matrix} \text{三未知数两个方程} \\ \text{则值无穷多} \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & | & 1 \\ 2 & 1 & | & 3 \\ 3 & 3 & | & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & | & 1 \\ 2 & 0 & | & 1 \\ 0 & 0 & | & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & | & 2 \\ 2 & 0 & | & 1 \\ 0 & 0 & | & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \\ 0 & 0 & | & 1 \end{bmatrix}$$

full rank \Rightarrow 无全零行的矩阵

$$\begin{bmatrix} 1 & 0 & 0 & | & a \\ 0 & 1 & 0 & | & b \\ 0 & 0 & 1 & | & c \end{bmatrix} \Rightarrow \text{unique solution } r(A) = r(B)$$

$$\begin{bmatrix} 1 & 0 & 0 & | & a \\ 0 & 1 & 0 & | & b \\ 0 & 0 & 1 & | & c \\ 1 & 2 & 3 & | & d \end{bmatrix} \Rightarrow \text{no solution}$$

$r(A) = 3$
 $r(A|B) = 4 > r(A)$

$$\begin{bmatrix} 1 & 0 & 0 & | & a \\ 0 & 1 & 0 & | & b \end{bmatrix} \Rightarrow \text{inf solution}$$

$r(A) = 2$
 $r(A|B) = 2$
 $r(A) = 2 < n = 4$

$$\Downarrow$$

$$x + 0y + 0z = a$$

$$0x + y + 0z = b$$

因为 $b \neq 0 \Rightarrow$ 无解

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$$

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array} \right]$$

$$\Downarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

$$\Leftarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 1 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

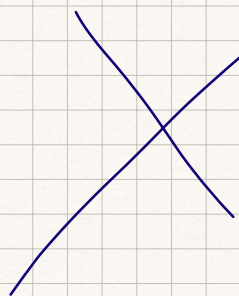
$$\Leftarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & \frac{1}{2} & \frac{1}{2} \end{array} \right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{3}{2} & \frac{1}{2} & \frac{1}{2} \end{array} \right]$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 0 & -1 \\ -\frac{3}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$A^2 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & | & 1 & 0 & 0 \\ 1 & 2 & 3 & | & 0 & 1 & 0 \\ 2 & 3 & 4 & | & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & -1 & 1 & 0 \\ 0 & 1 & 2 & | & -2 & 0 & 1 \end{bmatrix}$$

↓



RRF:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \times \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix} \checkmark \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \checkmark \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 2 \end{bmatrix} \checkmark \quad \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \checkmark \quad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \checkmark$$

[0 0 0]

L⁰

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L

[1 1 2]
[0 0] ✓

1. 递归

找清判定区域很重要!!!

2. 判定区域是1、0

1 上面的只能为0.

非1到0上可为任意数字,

下只能为0 (下行需满足 leading one).

$$a = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$[a|b]$$

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