MATH 1600 Linear Algebra — Winter 2020

Tutorial 5 - Wednesday

1. Let $\mathbf{v}_1 = \langle 1, -2, 0\rangle$, $\mathbf{v}_2 = \langle 0, 1, 2\rangle$, $\mathbf{v}_3 = \langle 5, -6, 8\rangle$, $\mathbf{w}_1 = \langle \sqrt{2}, -\sqrt{2}, -2\rangle$, $\mathbf{w}_2 = \langle 2, 1, 6\rangle$, $\mathbf{w}_3 = \langle 2, -1, 6\rangle$ be vectors in $\mathbb{R}^3$.

   (a) Which of the vectors $\mathbf{w}_1$ ✗, $\mathbf{w}_2$ ✗, $\mathbf{w}_3$ ✓ lie in the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

   (b) Write each $\mathbf{w}_i$ that lies in the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$.

   $w_3 = \frac{7}{4} \cdot v_1 + \frac{3}{4} v_3.$

   (c) Are $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ linearly independent? ✓

   (d) Find scalars $a_1, a_2, a_3 \in \mathbb{R}$ such that $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = 0$ and not all the scalars are zero.

   $5 \quad 4. \quad -1$

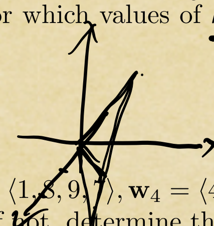
2. Choose four distinct vectors in $\mathbb{R}^3$ and show that they are linearly dependent.

   $(1,0,0) + 2(0,1,0) + 3(0,0,1) = (1,2,3).$

3. Let $\mathbf{v}_1 = \langle 1, -1, 2\rangle$, $\mathbf{v}_2 = \langle 5, -4, -7\rangle$, $\mathbf{v}_3 = \langle -4, 3, 7\rangle$ and $\mathbf{b} = \langle -4, 3, k\rangle$. For which values of k does $\mathbf{b}$ belong to the span of $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$?

   $-4 = 1 + 5t$

   7



4. Show that the vectors $\langle 1, 1, 0\rangle, \langle 1, 2, 3\rangle$ and $\langle 2, 1, -1\rangle$ span $\mathbb{R}^3$.

5. Consider the following vectors in $\mathbb{R}^4$: $\mathbf{w}_1 = \langle 1, 1, 0, -1\rangle, \mathbf{w}_2 = \langle 2, 2, 4, 0\rangle, \mathbf{w}_3 = \langle 1, 8, 9, 7\rangle, \mathbf{w}_4 = \langle 4, 10, 14, 6\rangle$. Determine whether the vectors $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4\}$ are linearly independent. If not, determine the largest subset of these vectors which is linearly independent. ✗.

   $3.$

   $a w_1 + b w_2 + c w_3 = w_4.$

   $$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 1 & 2 & 8 & 10 \\ 0 & 4 & 9 & 14 \\ -1 & 0 & 7 & 6 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 0 & 7 & 6 \\ 0 & 4 & 9 & 14 \\ 0 & 2 & 8 & 10 \end{array}\right]$$

   $$\Rightarrow \begin{matrix} 1 & 2 & 1 & 4 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 7 & 6 \\ 0 & 0 & -7 & -6 \end{matrix} \quad ✗.$$

6. Compute the following.

   (a) $(\sqrt{3} + 2i) + 5i$. $= \sqrt{3} + 7i$

   (b) $(2 + i) - (1 - 6i)$. $= 1 + 7i$

   (c) $(10 + 3i)(i)$. $= -3 + 10i$

   (d) $(2 + i)^2$. $= 4 + 2i - 1 = 3 + 2i$

   (e) $\frac{1+i}{1-i}$. $= \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{2i}{2} = i.$

   (f) Express in polar form the complex numbers $-2\sqrt{3} - 2i$ and $2i$.

   $= -4 \cdot \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i\right) = -4 \cdot \left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right)$

   $= 2 \cdot \left(\cos\frac{\pi}{2} + i \sin\frac{\pi}{2}\right).$

   (g) Solve for x: $(2 + i)x = 1 + 2i$.

   $x = \frac{1+2i}{2+i}.$

   $= \frac{(1+2i)(2-i)}{5}$

   $= \frac{2+4i-i+2}{5}$

   $= \frac{4+3i}{5}.$

   $k(\cos\theta + i \sin\theta).$