6.1a Exercise – also LogiCola C (EM & ET)
Translate these English sentences into wffs.

1. Not both A and B.
   ~(A•B)
2. Both A and either B or C.
   A•(B∨C)
3. Either both A and B or C.
   (A•B)∨C
4. If A, then B or C.
   A⊃(B∨C)
5. If A then B, or C.
   (A⊃B)∨C
6. If not A, then not either B or C.
   ~A⊃~(B∨C)
7. If not A, then either not B or C.
   ~A⊃(~B∨C)
8. Either A or B, and C.
   (A∨B)•C
9. Either A, or B and C.
   A∨(B•C)
10. If A then not both not B and not C.
    A⊃(~(~B•~C))
11. If you get an error message, then the disk is bad or it's a Macintosh disk.
    E⊃(B∨M)
12. If I bring my digital camera, then if my batteries don't die then I'll take pictures of my backpack trip and put the pictures on my Web site.
    B⊃(~D⊃(P•W))
13. If you both don't exercise and eat too much, then you'll gain weight.
    (~E•T)⊃W
14. The statue isn't by either Cellini or Michelangelo.
    ~(C∨M)
15. If I don't have either $2 in exact change or a bus pass, I won't ride the bus.
    ~(D∨B)⊃~R
16. If Michigan and Ohio State play each other, then Michigan will win.

(M•O)⊃W

17. Either you went through both Dayton and Cinci, or you went through Louisville.
    (D•C)∨L
18. If she had hamburgers then she ate junk food, and she ate French fries.
    (H⊃J)•F
19. I'm going to Rome or Florence and you're going to London.
    (R∨F)•L
20. Everyone is male or female.
    E ("M∨W" is incorrect, for it doesn't mean "Everyone is male or everyone is female.")

6.2a Exercise – also LogiCola D (TE & FE )

Calculate each truth value.

1. (0 ∨ 1) = 1
2. (0 • 0) = 0
3. (0 ⊃ 0) = 1
4. ~0 = 1
5. (0 ≡ 1) = 0
6. (1 • 0) = 0
7. (1 ⊃ 1) = 1
8. (1 ≡ 1) = 1
9. (0 ∨ 0) = 0
10. (0 ⊃ 1) = 1
11. (0 ≡ 0) = 1
12. (1 ∨ 1) = 1
13. (1 • 1) = 1
14. (1 ⊃ 0) = 0
15. ~1 = 0
16. (1 ∨ 0) = 1
17. (1 ≡ 0) = 0

6.3a Exercise – also LogiCola D (TM & TH)

Assume that A=1 and B=1 (A and B are both true) while X=0 and Y=0 (X and Y are both false). Calculate the truth value of each wff below.

1. ~(A • X) = 1
2. (~A • ~X) = 0
3. ~(~A • ~X) = 1
4. (A ⊃ X) = 0
5. (~X ≡ Y) = 0
6. (~B ⊃ A) = 1
7. ~(A ⊃ X) = 1
8. (B • (X ∨ A)) = 1
9. (~(X • A) ∨ ~B) = 1
10. (~A ∨ ~(X ⊃ Y)) = 0
11. ((A • ~X) ⊃ ~B) = 0
12. ~(A ⊃ (X ∨ ~B)) = 1
13. (~X ∨ ~(~A ≡ B)) = 1
14. (~Y ⊃ (A • X)) = 0
15. ~((A ⊃ B) ⊃ (B ⊃ Y)) = 1

6.4a Exercise – also LogiCola D (UE, UM, & UH)

Assume that T=1 (T is true), F=0 (F is false), and U=? (U is unknown). Calculate the truth value of each wff below.

1. (U • F) = 0
2. (U ⊃ ~T) = ?
3. (U ∨ ~F) = 1
4. (~F • U) = ?
5. (F ⊃ U) = 1
6. (~T ∨ U) = ?
7. (U ⊃ ~T) = ?
8. (~F ∨ U) = 1
9. (T • U) = ?
10. (U ⊃ ~F) = 1
11. (U • ~T) = 0
12. (U ∨ F) = ?

6.6a Exercise – also LogiCola D (AE, AM, & AH)
First appraise intuitively. Then translate into logic (using the letters given) and use the truth-table test to determine validity.

1. If you're a collie, then you're a dog.
You're a dog.
∴ You're a collie. [Use C and D.]
C ⊃ D, D /∴ C INVALID

2. If you're a collie, then you're a dog.
You're not a dog.
∴ You're not a collie. [Use C and D.]
C ⊃ D, ~D /∴ ~C VALID

3. If television is always right, then Anacin is better than Bayer.
If television is always right, then Anacin isn't better than Bayer.
∴ Television isn't always right. [Use T and B.]
T ⊃ B, T ⊃ ~B /∴ ~T VALID

4. If it rains and your tent leaks, then your down sleeping bag will get wet.
Your tent won't leak.
∴ Your down sleeping bag won't get wet. [R, L, W]
(R•L) ⊃ W, ~L /∴ ~W INVALID

5. If I get Grand Canyon reservations and get a group together, then I'll explore canyons during spring break.
I've got a group together.
I can't get Grand Canyon reservations.
∴ I won't explore canyons during spring break. [R, T, E]
(R•T) ⊃ E, T, ~R /∴ ~E INVALID

6. There's an objective moral law.
If there's an objective moral law, then there's a source of the moral law.
If there's a source of the moral law, then there's a God. (Other possible sources, like society or the individual, are claimed not to work.)
∴ There's a God. [Use M, S, and G; this is from C. S. Lewis.]
M, M ⊃ S, S ⊃ G /∴ G VALID

7. If ethics depends on God's will, then something is good because God desires it.
Something isn't good *because* God desires it. (Instead, God desires something because it's already good.)
∴ Ethics doesn't depend on God's will. [Use D and B; this is from Plato's *Euthyphro*.]
D ⊃ B, ~B /∴ ~D VALID

8. It's an empirical fact that the basic physical constants are precisely in the narrow range of what is required for life to be possible. (This "anthropic principle" has considerable evidence behind it.)
The best explanation for this fact is that the basic physical constants were caused by a great mind intending to produce life. (The main alternatives are the "chance coincidence" and "parallel universe" explanations.)
If these two things are true, then it's reasonable to believe that the basic structure of the world was set up by a great mind (God) intending to produce life.
∴ It's reasonable to believe that the basic structure of the world was set up by a great mind (God) intending to produce life. [Use E, B, and R; see Section 5.9.]
E, B, (E•B) ⊃ R /∴ R VALID

9. I'll go to Paris during spring break if and only if I'll win the lottery.
I won't win the lottery.
∴ I won't go to Paris during spring break. [P, W]
P ≡ W, ~W /∴ ~P VALID

10. If we have a simple concept proper to God, then we've directly experienced God and we can't rationally doubt God's existence.
We haven't directly experienced God.
∴ We can rationally doubt God's existence. [S, E, R]
S ⊃ (E•~R), ~E /∴ R INVALID

11. If there is a God, then God created the universe.
If God created the universe, then matter didn't always exist.
Matter always existed.
∴ There is no God. [G, C, M]
G ⊃ C, C ⊃ ~M, M /∴ ~G VALID

12. If this creek is flowing, then either the spring upstream has water or this creek has some other water source.
This creek has no other water source.
This creek isn't flowing.
∴ The spring upstream has no water. [F, S, O]
F ⊃ (S∨O), ~O, ~F / ∴ ~S INVALID

6.7a Exercise – also LogiCola ES
Test for validity using the truth-assignment test.

1. ~(N ≡ H)
N
∴ ~H VALID

2. ((J • ~D) ⊃ Z)
~Z
D
∴ ~J INVALID

3. ((T ∨ M) ⊃ Q)
M
∴ Q VALID

4. P
∴ (P • Q) INVALID

5. ((L • F) ⊃ S)
S
F
∴ L INVALID

6. ((A • U) ⊃ ~B)
B
A
∴ ~U VALID

7. ((W • C) ⊃ Z)
~Z
∴ ~C INVALID

8. Q
∴ (P ⊃ Q) VALID

9. (E ∨ (Y • X))
~E
∴ X VALID

10. (~T ⊃ (P ⊃ J))
P
~J
∴ T VALID

11. ~P
∴ ~(Q ⊃ P) INVALID

12. ((~M • G) ⊃ R)
~R
G
∴ M VALID

13. ~(Q ≡ I)
~Q
∴ I VALID

14. ((Q • R) ≡ S)
Q
∴ S INVALID

15. A
~A
∴ B VALID

6.7b Exercise – also LogiCola EE
First appraise intuitively. Then translate into logic and use the truth-assignment test to determine validity.
1. Some things are caused (brought into existence).
Anything caused is caused by another.
If some things are caused and anything caused is caused by another, then either there's a first cause or there's an infinite series of past causes.
There's no infinite series of past causes.
∴ There's a first cause. [A "first cause" (often identified with God) is a cause that isn't itself caused by another. This is from St Thomas Aquinas.]
S, A, (S•A)⊃(F∨I), ~I / ∴ F VALID

2. If you pass and it's intercepted, then the other side gets the ball.
You pass.
It isn't intercepted.
∴ The other side doesn't get the ball.
(Y•I) ⊃ T, Y, ~I /∴ ~T INVALID

3. If God exists in the understanding and not in reality, then there can be conceived a being greater than God (namely, a similar being that also exists in reality).
"There can be conceived a being greater than God" is false (since "God" is defined as "a being than which no greater can be conceived").
God exists in the understanding.
∴ God exists in reality. [This is St Anselm's famous ontological argument.]
(U•~R) ⊃ G, ~G, U /∴R VALID

4. If existence is a perfection and God by definition has all perfections, then God by definition must exist.
Existence is a perfection.
God by definition has all perfections.
∴ God by definition must exist. [From René Descartes.]
(E•D)⊃G, E, D /∴G VALID

5. If we have sensations of alleged material objects and yet no material objects exist, then God is a deceiver.
God isn't a deceiver.
We have sensations of alleged material objects.
∴ Material objects exist. [From René Descartes, who thus based our knowledge of the external material world on our knowledge of God.]
(W•~M)⊃G, ~G, W /∴M VALID

6. If "good" is definable in experimental terms, then ethical judgments are scientifically provable and ethics has a rational basis.
Ethical judgments aren't scientifically provable.
∴ Ethics doesn't have a rational basis.
G ⊃ (E•R), ~E /∴~R INVALID

7. If it's right for me to lie and not right for you, then there's a relevant difference between our cases.
There's no relevant difference between our cases.
It's not right for you to lie.
∴ It's not right for me to lie.
$(M \bullet {\sim}Y) \supset D, {\sim}D, {\sim}Y \quad /\therefore {\sim}M$ VALID

8. If Newton's gravitational theory is correct and there's no undiscovered planet near Uranus, then the orbit of Uranus would be such-and-such.
Newton's gravitational theory is correct.
The orbit of Uranus isn't such-and-such.
∴ There's an undiscovered planet near Uranus. [This reasoning led to the discovery of the planet Neptune.]
$(N \bullet {\sim}U) \supset O, N, {\sim}O \quad /\therefore U$ VALID

9. If attempts to prove "God exists" fail in the same way as our best arguments for "There are other conscious beings besides myself," then belief in God is reasonable if and only if belief in other conscious beings is reasonable.
Attempts to prove "God exists" fail in the same way as our best arguments for "There are other conscious beings besides myself."
Belief in other conscious beings is reasonable.
∴ Belief in God is reasonable. [From Alvin Plantinga.]
$A \supset (R \equiv C), A, C \quad /\therefore R$ VALID

10. If you pack intelligently, then either this teddy bear will be useful on the hiking trip or you won't pack it.
This teddy bear won't be useful on the hiking trip.
You won't pack it.
∴ You pack intelligently.
$I \supset (T \vee {\sim}P), {\sim}T, {\sim}P \quad /\therefore I$ INVALID

11. If knowledge is sensation, then pigs have knowledge.
Pigs don't have knowledge.
∴ Knowledge isn't sensation. [From Plato.]
$K \supset P, {\sim}P \quad /\therefore {\sim}K$ VALID

12. If capital punishment is justified and justice doesn't demand a vindication for past wrongs, then capital punishment either reforms the offender or effectively deters crime.
Capital punishment doesn't reform the offender.
Capital punishment doesn't effectively deter crime.
∴ Capital punishment isn't justified.
(C•~V) ⊃ (R∨D), ~R, ~D /∴~C INVALID

13. If belief in God were a purely intellectual matter, then either all smart people would be believers or all smart people would be non-believers.
Not all smart people are believers.
Not all smart people are non-believers.
∴ Belief in God isn't a purely intellectual matter.
G ⊃ (A∨B), ~A, ~B /∴~G VALID

14. If you're lost, then you should call for help or head downstream.
You're lost.
∴ You should call for help.
L⊃(C∨H), L /∴C INVALID

15. If maximizing human enjoyment is always good and the sadist's dog-torturing maximizes human enjoyment, then the sadist's act is good.
The sadist's dog-torturing maximizes human enjoyment.
The sadist's act isn't good.
∴ Maximizing human enjoyment isn't always good.
(M•S) ⊃ G, S, ~G /∴~M VALID

16. If there's knowledge, then either some things are known without proof or we can prove every premise by previous arguments infinitely.
We can't prove every premise by previous arguments infinitely.
There's knowledge.
∴ Some things are known without proof. [From Aristotle.]
K ⊃ (S∨I), ~I, K /∴S VALID

17. If you modified your computer or didn't send in the registration card, then the warranty is void.
You didn't modify your computer.
You sent in the registration card.
∴ The warranty isn't void.
$(M \vee \sim S) \supset V, \sim M, S \quad / \therefore \sim V$ INVALID

18. If "X is good" means "Hurrah for X!" and it makes sense to say "If X is good," then it makes sense to say "If hurrah for X!"
It makes sense to say "If X is good."
It doesn't make sense to say "If hurrah for X!"
∴ "X is good" doesn't mean "Hurrah for X!" [From Hector-Neri Castañeda.]
$(H \bullet M) \supset I, M, \sim I \quad / \therefore \sim H$ VALID

19. If we have an idea of substance, then "substance" refers either to a simple sensation or to a complex constructed out of simple sensations.
"Substance" doesn't refer to a simple sensation.
∴ We don't have an idea of substance. [From David Hume.]
$A \supset (S \vee C), \sim S \quad / \therefore \sim A$ INVALID

20. If we have an idea of "substance" and we don't derive the idea of "substance" from sensations, then "substance" is a thought category of pure reason.
We don't derive the idea of "substance" from sensations.
We have an idea of "substance."
∴ "Substance" is a thought category of pure reason. [From Immanuel Kant.]
$(S \bullet \sim D) \supset T, \sim D, S \quad / \therefore T$ VALID

21. If "good" means "socially approved," then what is socially approved is necessarily good.
What is socially approved isn't necessarily good.
∴ "Good" doesn't mean "socially approved."
$G \supset N, \sim N \quad / \therefore \sim G$ VALID

22. [Generalizing the last argument, G. E. Moore argued that we can't define "good" in terms of any empirical term "F" – like "desired" or "socially approved."]
If "good" means "F," then what is F is necessarily good.
What is F isn't necessarily good. (We can consistently say "Some F things may not be good" without thereby violating the meaning of "good.")
∴ "Good" doesn't mean "F."
F ⊃ N, ~N  /∴~F  VALID

23. If moral realism (the belief in objective moral truths) were true, then it could explain the moral diversity in the world.
Moral realism can't explain the moral diversity in the world.
∴ Moral realism isn't true.
M ⊃ E, ~E  /∴~M  VALID

6.8a Exercise – also LogiCola C (HM & HT)
Translate these English sentences into wffs.

1. If she goes, then you'll be alone but I'll be here.
   S⊃(A•H)
2. Your car will start only if you have fuel.
   Y⊃F
3. I will quit unless you give me a raise.
   Q∨G
4. Taking the final is a sufficient condition for passing.
   F⊃P
5. Taking the final is necessary for you to pass.
   P⊃F (or ~F⊃~P)
6. You're a man just if you're a rational animal.
   M≡R
7. Unless you have faith, you'll die.
   F∨D
8. She neither asserted it nor hinted at it.
   ~A•~H (or ~(A∨H))
9. Getting at least 96 is a necessary and sufficient condition for getting an A.
   G≡A
10. Only if you exercise are you fully alive.
    F⊃E (or ~E⊃~F)
11. I'll go, assuming that you go.
    Y⊃I

12. Assuming that your belief is false, you don't know.
    F⊃~K
13. Having a true belief is a necessary condition for having knowledge.
    K⊃B
14. You get mashed potatoes or French fries, but not both.
    (M∨F)•~(M•F)
15. You're wrong if you say that.
    S⊃W

6.9a Exercise – also LogiCola E (F & I)

First appraise intuitively. Then pick out the conclusion, translate into logic, and determine validity using the truth-assignment test. Supply implicit premises if needed.

1. Knowledge can't be sensation. If it were, then we couldn't know something that we aren't presently sensing. [From Plato.]

K ⊃ ~S

S (→implicit premise)

∴~K VALID

2. Presuming that we followed the map, then unless the map is wrong there's a pair of lakes just over the pass. We followed the map. There's no pair of lakes just over the pass. Hence the map is wrong.

F ⊃ (W∨L), F, ~L / ∴ W VALID

3. If they blitz but don't get to our quarterback, then our wide receiver will be open. So our wide receiver won't be open, as shown by the fact that they won't blitz.

(B•~Q) ⊃ O, ~B / ∴ ~O INVALID

4. My true love will marry me only if I buy her a Rolls Royce. It follows that she'll marry me, since I'll buy her a Rolls Royce.

M ⊃ R, R / ∴ M INVALID

5. The basic principles of ethics can't be self-evident truths, since if they were then they'd largely be agreed upon by intelligent people who have studied ethics.

S ⊃ A

~A (→implicit premise)

∴~S VALID

6. That your views are logically consistent is a necessary condition for your views to be sensible. Your views are logically consistent. So <u>your views are sensible</u>.

$S \supset L$, $L$ $/\therefore S$ INVALID

7. If Ohio State wins but Nebraska doesn't, then the Ohio Buckeyes will be national champions. So it looks like <u>the Ohio Buckeyes won't be national champs</u>, since Nebraska clearly is going to win.

$(O \bullet {\sim}N) \supset B$, $N$ $/\therefore {\sim}B$ INVALID

8. <u>The filter capacitor can't be blown</u>. This is indicated by the following facts. You'd hear a hum, presuming that the silicon diodes work but the filter capacitor is blown. But you don't hear a hum. And the silicon diodes work.

$(S \bullet B) \supset H$, ${\sim}H$, $S$ $/\therefore {\sim}B$ VALID

9. There's oxygen present. And so <u>there will be a fire</u>! My reason for saying this is that only if there's oxygen present will there be a fire.

$O$, $F \supset O$ $/\therefore F$ INVALID

10. <u>We have no moral knowledge</u>. This is proved by the fact that if we did have moral knowledge then basic moral principles would be either provable or self-evident.
But they aren't provable. And they aren't self-evident either.

$K \supset (P \vee S)$, ${\sim}P$, ${\sim}S$ $\therefore {\sim}$ VALID

11. <u>It must be a touchdown</u>! We know that it's a touchdown if the ball broke the plane of the end zone.

$B$ ($\rightarrow$implicit premise)

$B \supset T$

$\therefore T$ VALID

12. Assuming that it wasn't an inside job, then the lock was forced unless the thief stole the key. The thief didn't steal the key. We may infer that <u>the robbery was an inside job</u>, inasmuch as the lock wasn't forced.

${\sim}I \supset (L \vee T)$, ${\sim}T$, ${\sim}L$ $/\therefore I$ VALID

13. It must be the case that <u>we don't have any tea bags</u>. After all, we'd have tea bags if your sister Carol drinks tea. Of course, Carol doesn't drink tea.

$D \supset T$, ${\sim}D$ $/\therefore {\sim}T$ INVALID

14. <u>We can't still be on the right trail</u>. We'd see the white Appalachian Trail blazes on the trees if we were still on the right trail.

$R \supset S$

$\sim S$ (→implicit premise)

$\therefore \sim R$ VALID

15. If God is omnipotent, then he could make hatred inherently good – unless there's a contradiction in hatred being inherently good. But there's no contradiction in this. And God is omnipotent. I conclude that <u>God could make hatred inherently good</u>. [From William of Ockham, who saw morality as depending on God's will.]

$O \supset (H \vee C)$, $\sim C$, $O$ $/ \therefore H$ VALID

16. Taking the exam is a sufficient condition for getting an A. You didn't take the exam. This means <u>you don't get an A</u>.

$T \supset A$, $\sim T$ $/ \therefore \sim A$ INVALID

17. If Texas or Arkansas wins, then I win my $10 bet. I guess <u>I win $10</u>. Texas just beat Oklahoma 17-14!

$(T \vee A) \supset W$, $T$ $/ \therefore W$ VALID

18. Unless you give me a raise, I'll quit. Therefore <u>I'm quitting</u>!

$R \vee Q$

$\sim R$ (→implicit premise)

$\therefore Q$ VALID

19. <u>Empirical knowledge must be impossible</u>. My reason for saying this is that there's no independent way to prove that our senses are reliable. Empirical knowledge would be possible, of course, only if there were an independent way to prove that our senses are reliable.

$\sim N$, $E \supset N$ $/ \therefore \sim E$ VALID

20. It's virtuous to try to do what's good. On the other hand, it isn't virtuous to try to do what's socially approved. I conclude that, contrary to cultural relativism, <u>"good" doesn't mean "socially approved</u>." I assume, of course, that if "good" meant "socially approved" and it was virtuous to try to do what's good, then it would be virtuous to try to do what's socially approved.

$G$, $\sim S$, $(M \bullet G) \supset S$ $/ \therefore \sim M$ VALID

21. Moral conclusions can be deduced from non-moral premises only if "good" is definable using non-moral predicates. But "good" isn't so definable. So <u>moral conclusions can't be deduced from non-moral premises</u>.

$M \supset G$, $\sim G$ $/\therefore \sim M$ VALID

22. <u>The world can't need a cause</u>. If the world needed a cause, then so would God.

$W \supset G$

$\sim G$ (→implicit premise)

$\therefore \sim W$

6.10a Exercise – also LogiCola F (SE & SH)

Draw any simple conclusions (a letter or its negation) that follow from these premises. If nothing follows, leave blank.

1. (P • U)
   P and U
2. (L ∨ C)
   --no--
3. (~N ⊃ S)
   --no--
4. ~(F ⊃ M)
   F and ~M
5. ~(R ∨ S)
   ~R and ~S
6. ~(J • ~N)
   --no--
7. ~(I ∨ ~V)
   ~I and V
8. (F ⊃ ~G)
   --no--
9. (~Q • B)
   ~Q and B
10. ~(H ⊃ ~I)
    H and I
11. (~O ∨ ~X)
    --no--
12. (~T ⊃ ~H)
    --no--
13. ~(~N ∨ ~E)
    N and E
14. ~(Q • T)
    --no--
15. (M ∨ ~W)
    --no--
16. (~D • ~Z)
    ~D and ~Z
17. ~(~Y ⊃ G)
    ~Y and ~G
18. ~(~A • ~J)
    --no--
19. ~(~U ⊃ ~L)
    ~U and L
20. (~K ∨ B)
    --no--

6.11a Exercise – also LogiCola F (IE & IH)

Draw any simple conclusions (a letter or its negation) that follow from these premises. If nothing follows, leave blank.

1. ~(W • T)
   W
   ~T
2. (S ∨ L)
   S
   --no--
3. (H ⊃ ~B)
   H
   ~B
4. (X ⊃ E)
   E
   --no--
5. ~(B • S)
   ~S
   --no--
6. (~Y ⊃ K)
   Y
   --no--
7. (K ∨ ~R)
   R
   K
8. ~(~S • W)
   ~W
   --no--
9. (U ⊃ G)
   U
   G
10. (~I ∨ K)
    K
    --no--
11. (C ⊃ ~V)
    ~C
    --no--
12. (~N ∨ ~A)
    A
    ~N
13. ~(V • H)
    ~V
    --no--
14. (~A ⊃ ~E)
    ~E
    --no--
15. ~(~F • ~O)
    ~O
    F
16. (Y ∨ ~C)
    ~C
    --no--
17. (~L ⊃ M)
    ~M
    L
18. (~M ∨ ~B)
    ~M
    --no--
19. ~(~F • ~Q)
    F
    --no--
20. ~(A • ~Y)
    A
    Y

6.12a

1. ~U 2. B and ~C 3. No 4. No 5. P and ~Q 6. I 7. No
8. No 9.~C and ~D 10. ~A 11. ~M and I 12. ~R 13. ~L and S 14. ~T
15. No 16. No

6.13a

1. (A•B) and C 2. No 3. No 4. ~(I∨J) 5. No
6. No 7. ~(A⊃B) and ~C 8. C 9. No

7.1a

1

1. A⊃B /∴~B ⊃ ~A
2. asm: ~(~B ⊃ ~A)
3. ∴ ~B (2. NIF)
4. ∴A (2.NIF, DN)
5. ∴ B (1 and 4, MP)
6. ∴~B ⊃ ~A (from 2, 3 contradicts 4, RAA)

2

1. A /∴A∨B
2. asm: ~(A∨B)
3. ∴ ~A (2. NOR)
4. ∴A∨B (from 2, 1 contradicts 3, RAA)

3

1. A ⊃ B
2. ~A ⊃ B /∴B
3. asm: ~B
4. ∴~A (1 and 3, MT)
5. ∴ B (2 and 4, MP)
6. ∴B (from 3, 3 contradicts 5, RAA)

4

1. (A∨B) ⊃ C /∴~C ⊃ ~B
2. asm: ~(~C ⊃ ~B)
3. ∴ ~C (2, NIF)
4. ∴ B (2, NIF, DN)
5. ∴ ~(A∨B) (1 and 3, MT)
6. ∴~B (5, NOR)
7. ∴ ~C ⊃ ~B (from 2, 4 contradicts 6, RAA)

5

1. A ∨ B
2. A ⊃ C
3. B ⊃ D /∴C ∨ D
4. asm: ~(C ∨ D)
5. ∴ ~C (4, NOR)
4. ∴ ~D (4, NOR)
5. ∴ ~A (2 and 5, MT)
6. ∴~B (3 and 4, MT)
7. ∴ A (1 and 6, DS)
8. ∴ C ∨ D (from 4, 5 contradicts 7, RAA)

6

1. A ⊃ B
2. B ⊃ C /∴A ⊃ C
3. asm: ~(A ⊃ C)
4. ∴ A (3, NIF)
5. ∴ ~C (3, NIF)
4. ∴ ~B (2 and 5, MT)
5. ∴ B (1 and 4, MP)
6. ∴ A ⊃ C (from 3, 4 contradicts 5, RAA)

7

1. A ≡ B /∴ A ⊃ (A•B)
2. asm: ~(A ⊃ (A•B))
3. ∴ A (2, NIF)
4. ∴~(A•B) (2, NIF)
5. ∴ ~B (3 and 4, DS)
6. ∴ A ⊃ B (1, IFF)
7. ∴ ~A (5 and 6, MT)
8. ∴ A ⊃ (A•B) (from 2, 3 contradicts 7, RAA)

8

1. ~(A ∨ B)
2. C ∨ B
3. ~(D•C) /∴ ~D
4. asm: ~~D
5. ∴ ~C (3 and 4, CS)
6. ∴B (2 and 5, DS)
7. ∴ ~B (1, NOR)
8. ∴ ~D (from 4, 6 contradicts 7, RAA)

9

1. A ⊃ B
2. ~B /∴ A ≡ B
3. asm: ~(A ≡ B)
4. ∴~A (1 and 2, MT)
5. ∴ A ∨ B (3, NIFF)
6. ∴ B (4 and 5, DS)
7. ∴ A ≡ B (from 3, 2 contradicts 6, RAA)

10

1. A ⊃ (B ⊃ C) /∴ (A•B) ⊃ C
2. asm: ~((A•B) ⊃ C)
3. ∴ A • B (2, NIF)
4. ∴~C (2, NIF)
5. ∴ A (3, AND)
6. ∴ B ⊃ C (1 and 5, MP)
7. ∴ B (3, AND)
8. ∴ C (6 and 7, MP)
9. ∴(A•B) ⊃ C (from 2, 4 contradicts 8, RAA)

7.1b Exercise – also LogiCola F (TE & TH) and GEV

First appraise intuitively. Then translate into logic (using the letters given) and prove to be valid (all are valid).

1. If Heather saw the butler putting the tablet into the drink and the tablet was poison, then the butler killed the deceased.
Heather saw the butler putting the tablet into the drink.
∴ If the tablet was poison, then the butler killed the deceased. [Use H, T, and B.]

1> (H•T) ⊃ B
2> H /∴ T ⊃ B
3> asm: ~(T ⊃ B)
4> ∴ T (3, NIF)
5> ∴ ~B (3, NIF)
6> ∴ ~(H•T) (1 and 3, MT)
7> ∴ ~H (4 and 6, CS)
8> ∴ T ⊃ B (from 3, 2 contradicts 7, RAA)

2. If we had an absolute proof of God's existence, then our will would be irresistibly attracted to do right.
If our will were irresistibly attracted to do right, then we'd have no free will.
∴ If we have free will, then we have no absolute proof of God's existence. [Use P, I, and F. This is from Immanuel Kant and John Hick, who used it to explain why God doesn't make his existence more evident.]
$P \supset I, I \supset \sim F \,/\therefore\, F \supset \sim P$

3. If racism is clearly wrong, then either it's factually clear that all races have equal abilities or it's morally clear that similar interests of all beings ought to be given equal consideration.
It's not factually clear that all races have equal abilities.
If it's morally clear that similar interests of all beings ought to be given equal consideration, then similar interests of animals and humans ought to be given equal consideration.
∴ If racism is clearly wrong, then similar interests of animals and humans ought to be given equal consideration. [Use W, F, M, and A. This argument is from Peter Singer, who fathered the animal liberation movement.]
$W \supset (F \vee M), \sim F, M \supset A \,/\therefore W \supset A$

4. The universe is orderly (like a watch that follows complex laws).
Most orderly things we've examined have intelligent designers.
We've examined a large and varied group of orderly things.
If most orderly things we've examined have intelligent designers and we've examined a large and varied group of orderly things, then probably most orderly things have intelligent designers.
If the universe is orderly and probably most orderly things have intelligent designers, then the universe probably has an intelligent designer.
∴ The universe probably has an intelligent designer. [Use U, M, W, P, and D. This is a form of the argument from design for the existence of God.]
$U, M, W, (M \bullet W) \supset P, (U \bullet P) \supset D \,/\therefore D$

5. If God doesn't want to prevent evil, then he isn't all good.
If God isn't able to prevent evil, then he isn't all powerful.
Either God doesn't want to prevent evil, or he isn't able.
∴ Either God isn't all powerful, or he isn't all good. [Use W, G, A, and P. This form of the problem-of-evil argument is from the ancient Greek Empiricus.]
$\sim W \supset \sim G, \sim A \supset \sim P, \sim W \vee \sim A \,/\therefore \sim P \vee \sim G$

6. If Genesis gives the literal facts, then birds were created before humans. (Genesis 1:20–26)
If Genesis gives the literal facts, then birds were not created before humans. (Genesis 2:5–20)
∴ Genesis doesn't give the literal facts. [Use L and B. Origen, an early Christian thinker, gave similar textual arguments against taking Genesis literally.]
$L \supset B, L \supset \sim B \ / \therefore \sim L$

7. The world had a beginning in time.
If the world had a beginning in time, there was a cause for the world's beginning.
If there was a cause for the world's beginning, a personal being caused the world.
∴ A personal being caused the world. [Use B, C, and P. This "Kalam argument" for the existence of God is from William Craig and James Moreland; they defend premise 1 by various considerations, including the big-bang theory, the law of entropy, and the impossibility of an actual infinite.]
$B, B \supset C, C \supset P \ / \therefore P$

8. If the world had a beginning in time and it didn't just pop into existence without any cause, then the world was caused by God.
If the world was caused by God, then there is a God.
There is no God.
∴ Either the world had no beginning in time, or it just popped into existence without any cause. [Use B, P, C, and G. This is from J. L. Mackie, who based his "There is no God" premise on the problem-of-evil argument.]
$(B \bullet \sim P) \supset C, C \supset G, \sim G \ / \therefore \sim B \vee P$

9. Closed systems tend toward greater entropy (a more randomly uniform distribution of energy). (This is the second law of thermodynamics.)
If closed systems tend toward greater entropy and the world has existed through endless time, then the world would have achieved almost complete entropy (for example, everything would be about the same temperature).
The world has not achieved almost complete entropy.
If the world hasn't existed through endless time, then the world had a beginning in time.
∴ The world had a beginning in time. [Use G, E, C, and B. This is from William Craig and James Moreland.]
$G, (G \bullet E) \supset C, \sim C, \sim E \supset B \ / \therefore B$

10. If time stretches back infinitely, then today wouldn't have been reached.
If today wouldn't have been reached, then today wouldn't exist.
Today exists.
If time doesn't stretch back infinitely, then there was a first moment of time.
∴ There was a first moment of time. [I, R, T, F]
I ⊃ ~R, ~R ⊃ ~T, T, ~I ⊃ F / ∴ F

11. If there are already laws preventing discrimination against women, then if the Equal Rights Amendment (ERA) would rob women of many current privileges then it is the case both that passage of the ERA would be against women's interests and that women ought to work for its defeat.
The ERA would rob women of many current privileges (like draft exemption).
∴ If there are already laws preventing discrimination against women, then women ought to work for the defeat of the ERA. [L, R, A, W]
L ⊃ (R ⊃ (A•W)), R / ∴ L ⊃ W

12. If women ought never to be discriminated against, then we should work for current laws against discrimination and prevent future generations from imposing discriminatory laws against women.
The only way to prevent future generations from imposing discriminatory laws against women is to pass an Equal Rights Amendment (ERA).
If we should prevent future generations from imposing discriminatory laws against women and the only way to do this is to pass an ERA, then we ought to pass an ERA.
∴ If women ought never to be discriminated against, then we ought to pass an ERA. [N, C, F, O, E]
N ⊃ (C•F), O, (F•O) ⊃ E / ∴ N ⊃ E

13. If the claim that knowledge-is-impossible is true, then we understand the word "know" but there are no cases of knowledge.
If we understand the word "know," then the meaning of "know" comes either from a verbal definition or from experienced examples of knowledge.
If the meaning of "know" comes from a verbal definition, then there's an agreed-upon definition of "know."
There's no agreed-upon definition of "know."
If the meaning of "know" comes from experienced examples of knowledge, then there are cases of knowledge.
∴ The claim that knowledge-is-impossible is false. [Use I, U, C, D, E, and A.]
I ⊃ (U•~C), U ⊃ (D∨E), D ⊃ A, ~A, E ⊃ C / ∴ ~I

14. If *p* is the greatest prime, then *n* (we may stipulate) is one plus the product of all the primes less than *p*.
If *n* is one plus the product of all the primes less than *p*, then either *n* is prime or else *n* isn't prime but has prime factors greater than *p*.
If *n* is prime, then *p* isn't the greatest prime.
If *n* has prime factors greater than *p*, then *p* isn't the greatest prime.
∴ *p* isn't the greatest prime. [Use G, N, P, and F. This proof that there's no greatest prime number is from the ancient Greek mathematician Euclid.]
G ⊃ N, N ⊃ (P∨(~P•F)), P ⊃ ~G, F ⊃ ~G / ∴ ~G

7.2b Exercise – also LogiCola GEC
First appraise intuitively. Then translate into logic (using the letters given) and say whether valid (and give a proof) or invalid (and give a refutation).
1. If the butler shot Jones, then he knew how to use a gun.
If the butler was a former marine, then he knew how to use a gun.
The butler was a former marine.
∴ The butler shot Jones. [Use S, K, and M.]
S ⊃ K, M ⊃ K, M / ∴ S (INVALID)

2. If virtue can be taught, then either there are professional virtue-teachers or there are amateur virtue-teachers.
If there are professional virtue-teachers, then the Sophists can teach their students to be virtuous.
If there are amateur virtue-teachers, then the noblest Athenians can teach their children to be virtuous.
The Sophists can't teach their students to be virtuous and the noblest Athenians (such as the great leader Pericles) can't teach their children to be virtuous.
∴ Virtue can't be taught. [Use V, P, A, S, and N. This is from Plato's *Meno*.]
V ⊃ (P∨A), P ⊃ S, A ⊃ N, ~S•~N / ∴ ~V (VALID)

3. It would be equally wrong for a sadist (through a drug injection that would blind you but not hurt your mother) to have blinded you permanently before or after your birth.
If it would be equally wrong for a sadist (through such a drug injection) to have blinded you permanently before or after your birth, then it's false that one's moral right to equal consideration begins at birth.
If infanticide is wrong and abortion isn't wrong, then one's moral right to equal consideration begins at birth.

Infanticide is wrong.

∴ Abortion is wrong. [Use E, R, I, and A.]

$E, E \supset \sim R, (\sim I \bullet \sim A) \supset R, \sim I \; / \therefore A$ (VALID)

4. If you hold a moral belief and don't act on it, then you're inconsistent.

If you're inconsistent, then you're doing wrong.

∴ If you hold a moral belief and act on it, then you aren't doing wrong. [Use M, A, I, and W. Is the conclusion plausible? What more plausible conclusion follows from these premises?]

$(M \bullet \sim A) \supset I, I \supset W \; / \therefore (M \bullet A) \supset \sim W$ (INVALID)

5. If Socrates escapes from jail, then he's willing to obey the state only when it pleases him.

If he's willing to obey the state only when it pleases him, then he doesn't really believe what he says and he's inconsistent.

∴ If Socrates really believes what he says, then he won't escape from jail. [Use E, W, R, and I. This is from Plato's *Crito*. Socrates had been jailed and sentenced to death for teaching philosophy. He discussed with his friends whether he ought to escape from jail instead of suffering the death penalty.]

$E \supset W, W \supset (\sim R \bullet I) \; / \therefore R \supset \sim E$ (VALID)

6. Either Socrates's death will be perpetual sleep, or if the gods are good then his death will be an entry into a better life.

If Socrates's death will be perpetual sleep, then he shouldn't fear death.

If Socrates's death will be an entry into a better life, then he shouldn't fear death.

∴ Socrates shouldn't fear death. [Use P, G, B, and F. This is from Plato's *Crito* – except for which dropped premise?]

$P \vee (G \supset B), P \supset \sim F, B \supset \sim F \; / \therefore \sim F$ (INVALID)

7. If predestination is true, then God causes us to sin.

If God causes us to sin and yet damns sinners to eternal punishment, then God isn't good.

∴ If God is good, then either predestination isn't true or else God doesn't damn sinners to eternal punishment. [Use P, C, D, and G. This attacks the views of the American colonial thinker Jonathan Edwards.]

$P \supset C, (C \bullet D) \supset \sim G \; / \therefore G \supset (\sim P \vee \sim D)$ (VALID)

8. If determinism is true, then we have no free will.
If Heisenberg's interpretation of quantum physics is correct, some events aren't causally necessitated by prior events.
If some events aren't causally necessitated by prior events, determinism is false.
∴ If Heisenberg's interpretation of quantum physics is correct, then we have free will. [D, F, H, E]
D ⊃ ~F, H ⊃ ~E, ~E ⊃ ~D / ∴ H ⊃ F (INVALID)

9. Government's function is to protect life, liberty, and the pursuit of happiness.
The British colonial government doesn't protect these.
The only way to change it is by revolution.
If government's function is to protect life, liberty, and the pursuit of happiness and the British colonial government doesn't protect these, then the British colonial government ought to be changed.
If the British colonial government ought to be changed and the only way to change it is by revolution, then we ought to have a revolution.
∴ We ought to have a revolution. [Use G, B, O, C, and R. This summarizes the reasoning behind the American Declaration of Independence. Premise 1 was claimed to be self-evident, premises 2 and 3 were backed by historical data, and premises 4 and 5 were implicit conceptual bridge premises.]
G, ~B, O, (G•~B) ⊃ C, (C•O) ⊃ R / ∴ R (VALID)

10. The apostles' teaching either comes from God or is of human origin.
If it comes from God and we kill the apostles, then we will be fighting God.
If it's of human origin, then it'll collapse of its own accord.
If it'll collapse of its own accord and we kill the apostles, then our killings will be unnecessary.
∴ If we kill the apostles, then either our killings will be unnecessary or we will be fighting God. [Use G, H, K, F, C, and U. This argument, from Rabbi Gamaliel in Acts 5:34–9, is perhaps the most complex reasoning in the Bible.]
G∨H, (G•K) ⊃ F, H ⊃ C, (C•K) ⊃ U / ∴ K ⊃ (U∨F) (VALID)

11. If materialism (the view that only matter exists) is true, then idealism is false.
If idealism (the view that only minds exist) is true, then materialism is false.
If mental events exist, then materialism is false.
If materialists *think* their theory is true, then mental events exist.
∴ If materialists *think* their theory is true, then idealism is true. [M, I, E, T]
M ⊃ ~I, I ⊃ ~M, E ⊃ ~M, T ⊃ E / ∴ T ⊃ I (INVALID)

12. If determinism is true and cruelty is wrong, then the universe contains unavoidable wrong actions.
If the universe contains unavoidable wrong actions, then we ought to regret the universe as a whole.
If determinism is true and regretting cruelty is wrong, then the universe contains unavoidable wrong actions.
∴ If determinism is true, then either we ought to regret the universe as a whole (the pessimism option) or else cruelty isn't wrong and regretting cruelty isn't wrong (the "nothing matters" option). [Use D, C, U, O, and R. This sketches the reasoning in William James's "The Dilemma of Determinism." James thought that when we couldn't prove one side or the other to be correct (as on the issue of determinism), it was more rational to pick our beliefs in accord with practical considerations. He argued that these weighed against determinism.]
$(D \bullet C) \supset U, U \supset O, (D \bullet R) \supset U / \therefore D \supset (O \vee (\sim C \bullet \sim R))$ (VALID)

13. If a belief is proved, then it's worthy of acceptance.
If a belief isn't disproved but is of practical value to our lives, then it's worthy of acceptance.
If a belief is proved, then it isn't disproved.
∴ If a belief is proved or is of practical value to our lives, then it's worthy of acceptance. [P, W, D, V]
$P \supset W, (\sim D \bullet V) \supset W, P \supset \sim D / \therefore (P \vee V) \supset W$ (INVALID)

14. If you're consistent and think that stealing is normally permissible, then you'll consent to the idea of others stealing from you in normal circumstances.
You don't consent to the idea of others stealing from you in normal circumstances.
∴ If you're consistent, then you won't think that stealing is normally permissible. [C, N, Y]
$(C \bullet N) \supset Y, \sim Y / C \supset \sim N$ (VALID)

15. If the meaning of a term is always the object it refers to, then the meaning of "Fido" is Fido.
If the meaning of "Fido" is Fido, then if Fido is dead then the meaning of "Fido" is dead.
If the meaning of "Fido" is dead, then "Fido is dead" has no meaning.
"Fido is dead" has meaning.
∴ The meaning of a term isn't always the object it refers to. [Use A, B, F, M, and H.]
$A \supset B, B \supset (F \supset M), M \supset \sim H, H / \therefore \sim A$ (INVALID)

16. God is all powerful.
If God is all powerful, then he could have created the world in any logically possible way and the world has no necessity.
If the world has no necessity, then we can't know the way the world is by abstract speculation apart from experience.
∴ We can't know the way the world is by abstract speculation apart from experience. [Use A, C, N, and K. This is from the medieval William of Ockham.]
$A, A \supset (C \bullet N), N \supset \sim K / \therefore \sim K$ (VALID)

17. If God changes, then he changes for the worse or for the better.
If he's perfect, then he doesn't change for the worse.
If he changes for the better, then he isn't perfect.
∴ If God is perfect, then he doesn't change. [C, W, B, P]
$C \supset (W \vee B), P \supset \sim W, B \supset \sim P / \therefore P \supset \sim C$ (VALID)

18. If belief in God has scientific backing, then it's rational.
No conceivable scientific experiment could decide whether there is a God.
If belief in God has scientific backing, then some conceivable scientific experiment could decide whether there is a God.
∴ Belief in God isn't rational. [B, R, D]
$B \supset R, D, B \supset \sim D / \therefore \sim R$ (INVALID)

19. Every event with finite probability eventually takes place.
If the nations of the world don't get rid of their nuclear weapons, then there's a finite probability that humanity will eventually destroy the world.
If every event with finite probability eventually takes place and there's a finite probability that humanity will eventually destroy the world, then humanity will eventually destroy the world.
∴ Either nations of the world will get rid of their nuclear weapons, or humanity will eventually destroy the world. [E, R, F, H]
$E, \sim R \supset F, (E \bullet F) \supset H / \therefore R \vee H$ (VALID)

20. If the world isn't ultimately absurd, then conscious life will go on forever and the world process will culminate in an eternal personal goal.
If there is no God, then conscious life won't go on forever.
∴ If the world isn't ultimately absurd, then there is a God. [Use A, F, C, and G. This is from the Jesuit scientist, Pierre Teilhard de Chardin.]
$\sim A \supset (F \bullet C), \sim G \supset \sim F / \therefore \sim A \supset G$ (VALID)

21. If it rained here on this date 500 years ago and there's no way to know whether it rained here on this date 500 years ago, then there are objective truths that we cannot know.
If it didn't rain here on this date 500 years ago and there's no way to know whether it rained here on this date 500 years ago, then there are objective truths that we cannot know.
There's no way to know whether it rained here on this date 500 years ago.
∴ There are objective truths that we cannot know. [R, K, O]
(R•~K) ⊃ O, (~R•~K) ⊃ O, ~K / ∴ O (VALID)

22. If you know that you don't exist, then you don't exist.
If you know that you don't exist, then you know some things.
If you know some things, then you exist.
∴ You exist. [K, E, S]
K ⊃ ~E, K ⊃ S, S ⊃ E / ∴ E (INVALID)

23. We have an idea of a perfect being.
If we have an idea of a perfect being, then this idea is either from the world or from a perfect being.
If this idea is from a perfect being, then there is a God.
∴ There is a God. [Use I, W, P, and G. This is from René Descartes, except for which dropped premise?]
I, I ⊃ (W∨P), P ⊃ G / ∴ G (INVALID)

24. The distance from A to B can be divided into an infinity of spatial points.
One can cross only one spatial point at a time.
If one can cross only one spatial point at a time, then one can't cross an infinity of spatial points in a finite time.
If the distance from A to B can be divided into an infinity of spatial points and one can't cross an infinity of spatial points in a finite time, then one can't move from A to B in a finite time.
If motion is real, then one can move from A to B in a finite time.
∴ Motion isn't real. [Use D, O, C, M, and R. This is from the ancient Greek Zeno of Elea, who denied the reality of motion.]
D, O, O ⊃ ~C, (D•~ C) ⊃ ~M, R ⊃ M / ∴ ~R (VALID)

25. If the square root of 2 equals some fraction of positive whole numbers, then (we stipulate) the square root of 2 equals *x*/*y* and *x*/*y* is simplified as far as it can be.
If the square root of 2 equals *x*/*y*, then 2 = *x*2/*y*2.
If 2 = *x*2/*y*2, then 2*y*2 = *x*2.
If 2*y*2 = *x*2, then *x* is even.
If *x* is even and 2*y*2 = *x*2, then *y* is even.
If *x* is even and *y* is even, then *x*/*y* isn't simplified as far as it can be.
∴ The square root of 2 doesn't equal some fraction of positive whole numbers.
[F, E, S, T, T´, X, Y]
F ⊃ (E•S), E ⊃ T, T ⊃ T', T' ⊃ X, (X•T') ⊃ Y, (X•Y) ⊃ ~S /∴~F (VALID)

7.2a (All are invalid)
Omit

7

1. ~A ≡ B /∴~(A ≡ B)
2. asm: A ≡ B
3. ∴ A ⊃ B (2, IFF)
4. ∴ B ⊃ A (2, IFF)
5. ∴ ~A ⊃ B (1, IFF)
6. ∴ B ⊃ ~A (1, IFF)
7. asm: A (break 3)
8. ∴ B (3 and 7, MP)
9. ∴~B (6 and 7, MT)
10. ∴ ~A (from 7, 8 contradicts 9. RAA)
11. ∴ B (5 and 10, MP)
12. ∴ A (4 and 11, MP)
13. ∴~(A ≡ B) (from 2, 10 contradicts 12, RAA)

7 (Prove in other proof theory)

1. ~A ≡ B /∴~(A ≡ B)
2. asm: A ≡ B
3. ∴ A ⊃ B (2, IFF)
4. ∴ B ⊃ A (2, IFF)
5. ∴ ~A ⊃ B (1, IFF)
6. ∴ B ⊃ ~A (1, IFF)
7. ∴ A ⊃ ~A (1 and 6, HS)
8. ∴ ~A ∨ ~A (7, Impl)
9. ∴ ~A (8, Taut)
10. ∴~A ⊃ A (4 and 5, HS)
11. ∴ A ∨ A (10, Impl)
12. ∴ A (11, Taut)
13. ∴~(A ≡ B) (from 2, 9 contradicts 12, RAA)

7.3a (All are valid)

Omit

7

1. ~A ≡ B /∴~(A ≡ B)
2. asm: A ≡ B
3. ∴ A ⊃ B (2, IFF)
4. ∴ B ⊃ A (2, IFF)
5. ∴~A ⊃ B (1, IFF)
6. ∴ B ⊃ ~A (1, IFF)
7. asm: A (break 3)
8. ∴ B (3 and 7, MP)
9. ∴ ~B (6 and 7, MT)
10. ∴ ~A (from 7, 8 contradicts 9, RAA)
11. ∴ B (5 and 10, MP)
12. ∴ ~B (4 and 10, MT)
13. ∴~(A ≡ B) (from 2, 11 contradicts 12, RAA)

7(Prove in other proof theory)

1. ~A ≡ B /∴~(A ≡ B)
2. asm: A ≡ B
3. ∴~A ⊃ B (1, IFF)
4. ∴ B ⊃ ~A (1, IFF)
5. ∴ A ⊃ B (2, IFF)
6. ∴ B ⊃ A (2, IFF)
7. ∴ ~A ⊃ A (3 and 6, HS)
8. ∴ A ∨ A (7, Impl)
9. ∴ A (8, Taut)
10. ∴ A ⊃ ~A (4 and 5, HS)
11. ∴ ~A ∨ ~A (10, Impl)
12. ∴ ~A (11, Taut)
13. ∴ ~(A ≡ B) (from 2, 9 contradicts 12, RAA)

7.3b Exercise – also LogiCola GHV

First appraise intuitively. Then translate into logic (using the letters given) and prove to be valid (all are valid).

1. Either the butler fixed the drink and poisoned the deceased, or the butler added poison later and poisoned the deceased.

If the butler poisoned the deceased, then the butler is guilty.

∴ The butler poisoned the deceased and is guilty. [Use F, P, A, and G.]

(F•P)∨(A•P), P ⊃ G /∴P•G

2. If I'm coming down with a cold and I exercise, then I'll get worse and feel awful.

If I don't exercise, then I'll suffer exercise deprivation and I'll feel awful.

∴ If I'm coming down with a cold, then I'll feel awful. [Use C, E, W, A, and D. This one is easier if you break premise 1 (not 2) to make your assumption.]

(C•E) ⊃ (W•A), ~E ⊃ (D•A) /∴C ⊃ A

3. You'll get an A if and only if you either get a hundred on the final exam or else bribe the teacher.
You won't get a hundred on the final exam.
∴ You'll get an A if and only if you bribe the teacher. [Use A, H, and B.]

A ≡ (H∨B), ~H / ∴ A≡B

1. A ≡ (H∨B)
2. ~H
3. asm: ~(A ≡ B)
4. ∴ A ⊃ (H ∨ B)  (1, IFF)
5. ∴ (H ∨ B) ⊃ A  (1, IFF)
6. ∴ A ∨ B  (3, NIFF)
7. ∴ ~(A • B)  (3, NIFF)
8. asm: A  (break 4)
9. ∴ ~B  (7 and 8, CS)
10. ∴ H ∨ B  (4 and 8, MP)
11. ∴ B  (2 and 10, DS)
12. ∴ ~A  (from 8, 9 contradicts 11, RAA)
13. ∴ B  (6 and 12, DS)
14. ∴ ~(H ∨ B)  (5 and 12, MT)
15. ∴ ~B  (14, NOR)
16. ∴ A≡B  (from 3, 13 contradicts 15, RAA)

Prove in other proof theory

1. A ≡ (H∨B)
2. ~H
3. asm: A (for B, CP)
4. ∴ H ∨ B  (1 and 3, IFF, MP)
5. ∴ B  (2 and 4, DS)
6. ∴ A ⊃ B  (from 3, CP)
7. asm: B  (for A, CP)
8. ∴ B ∨ H  (7, Add)
9. ∴ A  (1 and 8, IFF, MP)
10. ∴ B ⊃ A  (from 7, CP)
11. ∴ (A ⊃ B) • (B ⊃ A) (6 and 10, Conj)
12. ∴ A ≡ B  (11, Equiv)

4. If President Nixon knew about the massive Watergate cover-up, then he lied to the American people on national television and he should resign.
If President Nixon didn't know about the massive Watergate cover-up, then he was incompetently ignorant and he should resign.
∴ Nixon should resign. [K, L, R, I]

K ⊃ (L•R), ~K ⊃ (I•R) / ∴ R

5. If you don't compromise your principles, then you won't get campaign money.
If you won't get campaign money, then you won't be elected.
If you compromise your principles, then you'll appeal to more voters.
If you appeal to more voters, then you'll be elected.
∴ You'll be elected if and only if you compromise your principles. [C, M, E, A]

~C ⊃ ~M, ~M ⊃ ~E, C ⊃ A, A ⊃ E / ∴ E≡C

6. Moral judgments express either truth claims or feelings.
If moral judgments express truth claims, then "ought" expresses either a concept from sense experience or an objective concept that isn't from sense experience.
"Ought" doesn't express a concept from sense experience.
"Ought" doesn't express an objective concept that isn't from sense experience.
∴ Moral judgments express feelings and not truth claims. [T, F, S, O]
$T \vee F$, $T \supset (S \vee O)$, $\sim S$, $\sim O$ / $\therefore F \bullet \sim T$

7. If Michigan either won or tied, then Michigan is going to the Rose Bowl and Gensler is happy.
∴ If Gensler isn't happy, then Michigan didn't tie. [W, T, R, H]
$(W \vee T) \supset (R \bullet H)$ / $\therefore \sim H \supset \sim T$

8. There are moral obligations.
If there are moral obligations and moral obligations are explainable, then either there's an explanation besides God's existence or else God's existence would explain moral obligations.
God's existence wouldn't explain moral obligation.
∴ Either moral obligations aren't explainable, or else there's an explanation besides God's existence. [M, E, B, G]
$M$, $(M \bullet E) \supset (B \vee G)$, $\sim G$ / $\therefore \sim E \vee B$

9. If determinism is true and Dr Freudlov correctly predicts (using deterministic laws) what I'll do, then if she tells me her prediction I'll do something else.
If Dr Freudlov tells me her prediction and yet I'll do something else, then Dr Freudlov doesn't correctly predict (using deterministic laws) what I'll do.
∴ If determinism is true, then Dr Freudlov doesn't correctly predict (using deterministic laws) what I'll do or else she won't tell me her prediction. [D, P, T, E]
$(D \bullet P) \supset (T \supset E)$, $(T \bullet E) \supset \sim P$ / $\therefore D \supset (\sim P \vee \sim T)$

10. If you make this demand on your son [that he leave Suzy or else not have his graduate schooling financed] and he leaves Suzy, then he'll regret being forced to leave her and he'll always resent you.
If you make this demand on your son and he doesn't leave Suzy, then he'll regret not going to graduate school and he'll always resent you.
∴ If you make this demand on your son, then he'll always resent you. [Use D, L, F, A, and G; this one is difficult.]

1. (D•L) ⊃ (F•A)
2. (D•~L) ⊃ (G•A) / ∴ D ⊃ A
3. asm: ~(D ⊃ A)
4. ∴ D (3, NIF)
5. ∴ ~A (3, NIF)
6. asm: F•A (break 1)
7. ∴ A (6, AND)
8. ∴ ~(F•A) (from 6, 5 contradicts 7, RAA)
9. ∴ ~(D•L) (1 and 8, MT)
10. ∴ ~L (4 and 9, CS)
11. asm: ~(D•~L) (break 2)
12. ∴ L (4 and 11, CS)
13. ∴ D•~L (from 11, 10 contradicts 12, RAA)
14. ∴ G•A (2 and 13, MP)
15. ∴ A (14, AND)
16. ∴ D ⊃ A (from 3, 5 contradicts 15, RAA)

7.4a Exercise – also LogiCola GHI
Prove each of these arguments to be invalid (all are invalid).
Omit
1

1. ~(A•B) / ∴ ~A•~B
2. asm: ~(~A•~B)
3. ∴ no contradiction and no rules can be applied—construct a refutation box
   Assume V(~(A•B)) = 1 and V(~A•~B) = 0
   ∴ V(A•B) = 0 and assume V(A) = 1 and V(B) = 0
   Given that, V(~A) = 0 and V(~B) = 1
   ∴ when V(A) = 1 and V(B) = 0, this argument is invalid.

7.4b Exercise – also LogiCola G (HC & MC)

1. If the maid prepared the drink, then the butler didn't prepare it.
The maid didn't prepare the drink.
If the butler prepared the drink, then he poisoned the drink and is guilty.
∴ The butler is guilty. [Use M, B, P, and G.]
$M \supset \sim B, \sim M, B \supset (P \bullet G) \; / \therefore G$ (INVALID)

2. If you tell your teacher that you like logic, then your teacher will think that you're insincere and you'll be in trouble.
If you don't tell your teacher that you like logic, then your teacher will think that you dislike logic and you'll be in trouble.
∴ You'll be in trouble. [Use L, I, T, and D.]
$L \supset (I \bullet T), \sim L \supset (D \bullet T) \; / \therefore T$ (VALID)

3. If we don't get reinforcements, then the enemy will overwhelm us and we won't survive.
∴ If we do get reinforcements, then we'll conquer the enemy and we'll survive.
[Use R, O, S, and C.]
$\sim R \supset (O \bullet \sim S) \; / \therefore R \supset (C \bullet S)$ (INVALID)

4. If Socrates didn't approve of the laws of Athens, then he would have left Athens or would have tried to change the laws.
If Socrates didn't leave Athens and didn't try to change the laws, then he agreed to obey the laws.
Socrates didn't leave Athens.
∴ If Socrates didn't try to change the laws, then he approved of the laws and agreed to obey them. [Use A, L, C, and O. This is from Plato's *Crito*, which argued that Socrates shouldn't disobey the law by escaping from jail.]
$\sim A \supset (L \vee C), (\sim L \bullet \sim C) \supset O, \sim L \; / \therefore \sim C \supset (A \bullet O)$ (VALID)

5. If I hike the Appalachian Trail and go during late spring, then I'll get maximum daylight and maximum mosquitoes.
If I'll get maximum mosquitoes, then I won't be comfortable.
If I go right after school, then I'll go during late spring.
∴ If I hike the Appalachian Trail and don't go right after school, then I'll be comfortable. [A, L, D, M, C, S]
$(A \bullet L) \supset (D \bullet M), M \supset \sim C, S \supset L \; / \therefore (A \bullet \sim S) \supset C$ (INVALID)

6. [Logical positivism says "*Every genuine truth claim is either experimentally testable or true by definition*." This view, while once popular, is self-refuting and hence not very popular today.]
If LP (logical positivism) is true and is a genuine truth claim, then it's either experimentally testable or true by definition.
LP isn't experimentally testable.
LP isn't true by definition.
If LP isn't a genuine truth claim, then it isn't true.
∴ LP isn't true. [T, G, E, D]
$(T \bullet G) \supset (E \vee D)$, $\sim E$, $\sim D$, $\sim G \supset \sim T$ / $\therefore \sim T$ (VALID)

7. If you give a test, then students either do well or do poorly.
If students do well, then you think you made the test too easy and you're frustrated.
If students do poorly, then you think they didn't learn any logic and you're frustrated.
∴ If you give a test, then you're frustrated. [Use T, W, P, E, F, and L. This is from a class who tried to talk me out of giving a test.]
$T \supset (W \vee P)$, $W \supset (E \bullet F)$, $P \supset (\sim L \bullet F)$ / $\therefore T \supset F$ (VALID)

8. If the world contains moral goodness, then the world contains free creatures and the free creatures sometimes do wrong.
If the free creatures sometimes do wrong, then the world is imperfect and the creator is imperfect.
∴ If the world doesn't contain moral goodness, then the creator is imperfect. [M, F, S, W, C]
$M \supset (F \bullet S)$, $S \supset (W \bullet C)$ / $\therefore \sim M \supset C$ (INVALID)

9. We'll find your action's cause, if and only if your action has a cause and we look hard enough.
If all events have causes, then your action has a cause.
All events have causes.
∴ We'll find your action's cause, if and only if we look hard enough. [F, H, L, A]
$F \equiv (H \bullet L)$, $A \supset H$, $A$ / $\therefore F \equiv L$ (VALID)

10. Herman sees that the piece of chalk is white.
The piece of chalk is the smallest thing on the desk.
Herman doesn't see that the smallest thing on the desk is white. (He can't see the whole desk and so can't tell that the piece of chalk is the smallest thing on it.)
If Herman sees a material thing, then if he sees that the piece of chalk is white and the

piece of chalk is the smallest thing on the desk, then he sees that the smallest thing on the desk is white.
If Herman doesn't see a material thing, then he sees a sense datum.
∴ Herman doesn't see a material thing, but he does see a sense datum. [Use H, P, H′, M, and S. This argument attacks direct realism – the view that we directly perceive material things and not just sensations or sense data.]
$H, P, H', M \supset ((H \bullet P) \supset H'), \sim M \supset S \;/\therefore \sim M \bullet S$ (INVALID)

11. If the final capacitor in the transmitter is arcing, then the SWR (standing wave ratio) is too high and the efficiency is lowered.
If you hear a cracking sound, then the final capacitor in the transmitter is arcing.
∴ If you don't hear a cracking sound, then the SWR isn't too high. [A, H, L, C]
$A \supset (H \bullet L), C \supset A \;/\therefore \sim C \supset \sim H$ (INVALID)

12. If we can know that God exists, then we can know God by experience or we can know God by logical inference from experience.
If we can't know God empirically, then we can't know God by experience and we can't know God by logical inference from experience.
If we can know God empirically, then "God exists" is a scientific hypothesis and is empirically falsifiable.
"God exists" isn't empirically falsifiable.
∴ We can't know that God exists. [K, E, L, M, S, F]
$K \supset (E \vee L), \sim M \supset (\sim E \bullet \sim L), M \supset (S \bullet F), \sim F \;/\therefore \sim K$ (VALID)

13. If I perceive, then my perception is either delusive or veridical.
If my perception is delusive, then I don't directly perceive a material object.
If my perception is veridical and I directly perceive a material object, then my experience in veridical perception would always differ qualitatively from my experience in delusive perception.
My experience in veridical perception doesn't always differ qualitatively from my experience in delusive perception.
If I perceive and I don't directly perceive a material object, then I directly perceive a sensation.
∴ If I perceive, then I directly perceive a sensation and I don't directly perceive a material object. [Use P, D, V, M, Q, and S. This argument from illusion attacks direct realism – the view that we directly perceive material objects and not just sensations or sense data.]
$P \supset (D \vee V), D \supset \sim M, (V \bullet M) \supset Q, \sim Q, (P \bullet \sim M) \supset S \;/\therefore P \supset (S \bullet \sim M)$ (INVALID)

14. If you’re romantic and you’re Italian, then Juliet will fall in love with you and will want to marry you.
If you’re Italian, then you’re romantic.
∴ If you’re Italian, then Juliet will want to marry you. [R, I, F, M]
$(R \bullet I) \supset (F \bullet M)$, $I \supset R$ / $\therefore I \supset M$ (VALID)

15. If emotions can rest on factual errors and factual errors can be criticized, then we can criticize emotions.
If we can criticize emotions and moral judgments are based on emotions, then beliefs about morality can be criticized and morality isn’t entirely non-rational.
∴ If morality is entirely non-rational, then emotions can’t rest on factual errors.
[E, F, W, M, B, N]
$(E \bullet F) \supset W$, $(W \bullet M) \supset (B \bullet \sim N)$ / $\therefore N \supset \sim E$ (INVALID)

## Workbook for Applied Logic

### Part 3: Quantificational Logic

8.1a Exercise – also LogiCola H (EM & ET)
Translate these English sentences into wffs.

1. x isn’t a cat.
   $\sim Cx$
2. Something is a cat.
   $(\exists x)Cx$
3. Something isn’t a cat.
   $(\exists x)\sim Cx$
4. It isn’t the case that there is something that isn’t a cat.
   $\sim(\exists x)\sim Cx$
5. Everything is a cat.
   $(x)Cx$
6. If x is a dog, then x is an animal.
   $Dx \supset Ax$
7. All dogs are animals.
   $(x)(Dx \supset Ax)$
8. No one is evil.
   $\sim(\exists x)Ex$
9. Some logicians are evil.

(∃x)(Lx • Ex)

10. No logician is evil.

    ~(∃x)(Lx • Ex)

11. All black cats are unlucky.

    (x)((Bx • Cx) ⊃ Ux)

12. Some dogs are large and hungry.

    (∃x)(Dx • (Lx • Hx))

13. Not all hungry dogs bark.

    ~(x)((Hx • Dx) ⊃ Bx)

14. Some animals aren't barking dogs.

    (∃x)(Ax • ~(Bx • Dx))

15. Some animals are dogs who don't bark.

    (∃x)(Ax • (Dx • ~Bx))

16. All dogs who bark are frightening.

    (x)((Dx • Bx) ⊃ Fx)

17. Not all non-dogs are cats.

    ~(x)(~Dx ⊃ Cx)

18. Some cats who aren't black are unlucky.

    (∃)((Cx • ~Bx) • Ux)

19. Some cats don't purr.

    (∃x)(Cx • ~Px)

20. Not every cat purrs.

    ~(x)(Cx ⊃ Px)

21. Not all animals are dogs or cats.

    ~(x)(Ax ⊃ (Dx ∨ Cx))

22. All who are either dogs or cats are animals.

    (x)((Dx ∨ Cx) ⊃ Ax)

23. All who are both dogs and cats are animals.

    (x)((Dx • Cx) ⊃ Ax)

24. All dogs and cats are animals.

    (x)(Dx ⊃ Ax) • (x)(Cx ⊃ Ax)

25. Everyone is a crazy logician.

    (x)(Cx • Lx)

8.2a Exercise – also LogiCola IEV

Prove each of these arguments to be valid (all are valid).

Omit

8.2b Exercise – also LogiCola IEV

First appraise intuitively. Then translate into logic (using the letters given) and prove to be valid (all are valid).

1. All who deliberate about alternatives believe in free will (at least implicitly).
All deliberate about alternatives.
∴ All believe in free will. [Use Dx and Bx. This is from William James.]
(x)(Dx⊃Bx), (x)Dx /∴(x)Bx

2. Everyone makes mistakes.
∴ Every logic teacher makes mistakes. [Use Mx and Lx.]
(x)Mx /∴(x)(Lx⊃Mx)

3. No feeling of pain is publicly observable.
All chemical processes are publicly observable.
∴ No feeling of pain is a chemical process. [Use Fx, Ox, and Cx. This attacks a form of materialism that identifies mental events with material events. We also could test this argument using syllogistic logic (Chapter 2).]
~(∃x)(Fx•Ox), (x)(Cx⊃Ox) /∴~(∃x)(Fx•Cx)

4. All (in the electoral college) who do their jobs are useless.
All (in the electoral college) who don't do their jobs are dangerous.
∴ All (in the electoral college) are useless or dangerous. [Use Jx for "x does their job," Ux for "x is useless," and Dx for "x is dangerous." Use the universe of discourse of electoral college members: take "(x)" to mean "for every electoral college member x" and don't translate "in the electoral college."]
(x)(Jx⊃Ux), (x)(~Jx⊃Dx) /∴(x)(Ux∨Dx)

1) (x)(Jx⊃Ux)
2) (x)(~Jx⊃Dx) /∴(x)(Ux∨Dx)
3) asm: ~(x)(Ux∨Dx)
4) ∴ (∃x)~(Ux∨Dx) (from 3, RS)
5) ∴ ~(Ua∨Da) (from 4, DE)
6) ∴ ~Ua (from 5, NOR)
7) ∴ ~Da (from 5, NOR)
8) ∴ Ja ⊃ Ua (from 1, DU)
9) ∴ ~Ja ⊃ Da (from 2, DU)
10) ∴ ~Ja (from 6 and 8, MT)
11) ∴ ~~Ja (from 7 and 9, MT)
12) ∴ (x)(Ux∨Dx) (from 3, 10 contradicts 11, RAA)

5. All that's known is experienced through the senses.
Nothing that's experienced through the senses is known.
∴ Nothing is known. [Use Kx and Ex. Empiricism (premise 1) plus skepticism about the senses (premise 2) yields general skepticism.]
(x)(Kx⊃Ex), ~(∃x)(Ex•Kx) / ∴ ~(∃x)Kx

6. No pure water is burnable.
Some Cuyahoga River water is burnable.
∴ Some Cuyahoga River water isn't pure water. [Use Px, Bx, and Cx. The Cuyahoga is a river in Cleveland that used to catch fire.]
~(∃)(Px•Bx), (∃x)(Cx•Bx) / ∴ (∃x)(Cx•~Px)

7. Everyone who isn't with me is against me.
∴ Everyone who isn't against me is with me. [Use Wx and Ax. These claims from the Gospels are sometimes thought to be incompatible.]
(x)(~Wx⊃Ax) / ∴ (x)(~Ax⊃Wx)

8. All basic laws depend on God's will.
∴ All basic laws about morality depend on God's will. [Bx, Dx, Mx]
(x)(Bx⊃Dx) / ∴ (x)((Bx•Mx) ⊃ Dx)

9. Some lies in unusual circumstances aren't wrong.
∴ Not all lies are wrong. [Lx, Ux, Wx]
(∃x)((Lx•Ux)•~Wx) / ∴ ~(x)(Lx⊃Wx)

10. Nothing based on sense experience is certain.
Some logical inferences are certain.
All certain things are truths of reason.
∴ Some truths of reason are certain and aren't based on sense experience. [Bx, Cx, Lx, Rx]
~(∃x)(Bx•Cx), (∃x)(Lx•Cx), (x)(Cx⊃Rx) / ∴ (∃x)(Rx•(Cx•~Bx))

11. No truth by itself motivates us to action.
Every categorical imperative would by itself motivate us to action.
Every categorical imperative would be a truth.
∴ There are no categorical imperatives. [Use Tx, Mx, and Cx. Immanuel Kant claimed that commonsense morality accepts categorical imperatives (objectively true moral judgments that command us to act and that we must follow if we are to be rational); but some thinkers argue against the idea.]
~(∃x)(Tx•Mx), (x)(Cx⊃Mx), (x)(Cx⊃Tx) /∴~(∃x)Cx

12. Every genuine truth claim is either experimentally testable or true by definition.
No moral judgments are experimentally testable.
No moral judgments are true by definition.
∴ No moral judgments are genuine truth claims. [Use Gx, Ex, Dx, and Mx. This is logical positivism's argument against moral truths.]
(x)(Gx⊃(Ex∨Dx)), ~(∃x)(Mx•Ex), ~(∃x)(Mx•Dx) /∴~(∃x)(Mx•Gx)

13. Everyone who can think clearly would do well in logic.
Everyone who would do well in logic ought to study logic.
Everyone who can't think clearly ought to study logic.
∴ Everyone ought to study logic. [Tx, Wx, Ox]
(x)(Tx⊃Wx), (x)(Wx⊃Ox), (x)(~Tx⊃Ox) /∴(x)Ox

8.3a Exercise – also LogiCola IEI
Prove each of these arguments to be invalid (all are invalid).
Omit

8.3b Exercise – also LogiCola IEC
First appraise intuitively. Then translate into logic (using the letters given) and say whether valid (and give a proof) or invalid (and give a refutation).
1. Some butlers are guilty.
∴ All butlers are guilty. [Use Bx and Gx.]
(∃x)(Bx•Gx) /∴(x)(Bx⊃Gx) (INVALID)

2. No material thing is infinite.
Not everything is material.
∴ Something is infinite. [Use Mx and Ix.]
~(∃x)(Mx•Ix), ~(x)Mx /∴(∃x)Ix (INVALID)

3. Some smoke.
Not all have clean lungs.
∴ Some who smoke don't have clean lungs. [Use Sx and Cx.]
(∃x)Sx, ~(x)Cx /∴(∃x)(Sx•~Cx)    (INVALID)

4. Some Marxists plot violent revolution.
Some faculty members are Marxists.
∴ Some faculty members plot violent revolution. [Mx, Px, Fx]
(∃x)(Mx•Px), (∃x)(Fx•Mx) /∴(∃x)(Fx•Px)

5. All valid arguments that have "ought" in the conclusion also have "ought" in the premises.
All arguments that seek to deduce an "ought" from an "is" have "ought" in the conclusion but don't have "ought" in the premises.
∴ No argument that seeks to deduce an "ought" from an "is" is valid. [Use Vx for "x is valid," Cx for "x has 'ought' in the conclusion," Px for "x has 'ought' in the premises," Dx for "x seeks to deduce an 'ought' from an 'is,'" and the universe of discourse of arguments. This one is difficult to translate.]
(x)((Vx•Cx) ⊃ Px), (x)(Dx ⊃ (Cx•~Px)) /∴~(∃x)(Dx•Vx)    (VALID)

6. Every kick returner who is successful is fast.
∴ Every kick returner who is fast is successful. [Kx, Sx, Fx]
(x)((Kx•Sx) ⊃ Fx) /∴(x)((Kx•Fx) ⊃ Sx)    (INVALID)

7. All exceptionless duties are based on the categorical imperative.
All non-exceptionless duties are based on the categorical imperative.
∴ All duties are based on the categorical imperative. [Use Ex, Bx, and the universe of discourse of duties. This is from Kant, who based all duties on his supreme moral principle, called "the categorical imperative."]
(x)(Ex⊃Bx), (x)(~Ex⊃Bx) /∴(x)Bx    (VALID)

8. All who aren't crazy agree with me.
∴ No one who is crazy agrees with me. [Cx, Ax]
(x)(~Cx⊃Ax) /∴~(∃x)(Cx•Ax)    (INVALID)

9. Everything can be conceived.
Everything that can be conceived is mental.
∴ Everything is mental. [Use Cx and Mx. This is from George Berkeley, who attacked materialism by arguing that everything is mental and that matter doesn't exist apart from mental sensations; so a chair is just a collection of experiences.
Bertrand Russell thought premise 2 was confused.]
(x)Cx, (x)(Cx⊃Mx) /∴(x)Mx (VALID)

10. All sound arguments are valid.
∴ All invalid arguments are unsound. [Use Sx and Vx and the universe of discourse of arguments.]
(x)(Sx⊃Vx) /∴(x)(~Vx⊃~Sx) (VALID)

11. All trespassers are eaten.
∴ Some trespassers are eaten. [Use Tx and Ex. The premise is from a sign on the Appalachian Trail in northern Virginia. Traditional logic (Section 2.8) takes "all A is B" to entail "some A is B"; modern logic takes "all A is B" to mean "whatever is A also is B" – which can be true even if there are no A's.]
(x)(Tx⊃Ex) /∴(∃x)(Tx•Ex) (INVALID)

12. Some necessary being exists.
All necessary beings are perfect beings.
∴ Some perfect being exists. [Use Nx and Px. Kant claimed that the cosmological argument for God's existence at most proves premise 1; it doesn't prove the existence of God (a perfect being) unless we add premise 2. But premise 2, by the next argument, presupposes the central claim of the ontological argument – that some perfect being is a necessary being. So, Kant claimed, the cosmological argument presupposes the ontological argument.]
(∃x)Nx, (x)(Nx⊃Px) /∴(∃x)Px (VALID)

13. All necessary beings are perfect beings.
∴ Some perfect being is a necessary being. [Use Nx and Px. Kant followed traditional logic (see problem 11) in taking "all A is B" to entail "some A is B."]
(x)(Nx⊃Px) /∴(∃x)(Nx•Px) (INVALID)

14. No one who isn't a logical positivist holds the verifiability criterion of meaning.
∴ All who hold the verifiability criterion of meaning are logical positivists. [Use Lx and Hx. The verifiability criterion of meaning says that every genuine truth claim is either experimentally testable or true by definition.]
$\sim(\exists x)(\sim Lx \bullet Hx)$ / ∴ $(x)(Hx \supset Lx)$ (VALID)

15. No pure water is burnable.
Some Cuyahoga River water isn't burnable.
∴ Some Cuyahoga River water is pure water. [Use Px, Bx, and Cx.]
$\sim(\exists x)(Px \bullet Bx), (\exists x)(Cx \bullet \sim Bx)$ / ∴ $(\exists x)(Cx \bullet Px)$ (INVALID)

8.4a Exercise – also LogiCola H (HM & HT)
Translate these English sentences into wffs. Recall that our translation rules are rough guides and sometimes don't work; so read your formula carefully to make sure it reflects what the English means.

1. Gensler is either crazy or evil.
   $Cg \vee Eg$
2. If Gensler is a logician, then some logicians are evil.
   $Lg \supset (\exists x)(Lx \bullet Ex)$
3. If everyone is a logician, then everyone is evil.
   $(x)Lx \supset (x)Ex$
4. If all logicians are evil, then some logicians are evil.
   $((x)(Lx \supset Ex)) \supset ((\exists x)(Lx \bullet Ex))$
5. If someone is evil, it will rain.
   $((\exists x)Ex) \supset R$
6. If everyone is evil, it will rain.
   $((x)Ex) \supset R$
7. If anyone is evil, it will rain.
   $((\exists x)Ex) \supset R$
8. If Gensler is a logician, then someone is a logician.
   $Lg \supset (\exists x)Lx$
9. If no one is evil, then no one is an evil logician.
   $(\sim(\exists x)Ex) \supset \sim(\exists x)(Ex \bullet Lx)$
10. If all are evil, then all logicians are evil.
   $((x)Ex) \supset (x)(Lx \supset Ex)$
11. If some are logicians, then some are evil.
   $(\exists x)Lx \supset (\exists x)Ex$
12. All crazy logicians are evil.

(x)((Cx • Lx) ⊃ Ex)

13. Everyone who isn't a logician is evil.

    (x)((~Lx) ⊃ Ex)

14. Not everyone is evil.

    ~(x)Ex

15. Not anyone is evil.

    (x)~Ex

16. If Gensler is a logician, then he's evil.

    Lg ⊃ Eg

17. If anyone is a logician, then Gensler is a logician.

    ((∃x)Lx) ⊃ Lg

18. If someone is a logician, then he or she is evil.

    (∃x)(Lx ⊃ Ex)

19. Everyone is an evil logician.

    (x)(Ex • Lx)

20. Not any logician is evil.

    (x)(Lx ⊃ ~Ex) or ~(∃x)(Lx•Ex)

8.5a Exercise – also LogiCola I (HC & MC)

Say whether each is valid (and give a proof) or invalid (and give a refutation).

| | | | | |
|---|---|---|---|---|
| 1.(VALID) | 2.(VALID) | 3.(INVALID) | 4.(VALID) | 5.(INVALID) |
| 6.(VALID) | 7.(INVALID) | 8.(VALID) | 9.(VALID) | 10.(INVALID) |
| 11.(VALID) | 12.(VALID) | 13.(VALID) | 14.(VALID) | 15.(VALID) |

(2)

1) (x)(Ex ⊃ R)     /∴ (∃x)Ex ⊃ R
2) asm: ~((∃x)Ex ⊃ R)
3) ∴ (∃x)Ex     (from 2, NIF)
4) ∴ ~R     (from 2, NIF)
5) ∴ Ea     (from 3, DE)
6) ∴ Ea ⊃ R     (from 1, DU)
7) ∴ R     (from 5 and 6, MP)
8) ∴ (∃x)Ex ⊃ R     (from 2, 4 contradicts 7, RAA)

(3)

1) (x)Lx ⊃ R /∴(x)(Lx ⊃ R)

2) asn: ~(x)(Lx ⊃ R)

3) ∴ (∃x)~(Lx ⊃ R) (from 2, RS)

4) ∴ ~(La ⊃ R) (from 3, DE)

5) ∴ La (from 4, NIF)

6) ∴ ~R (from 4, NIF)

7) asm: R (break 1)

8) ∴ ~R (from 7, 6 contradicts 7, RAA)

9) ∴ ~(x)Lx (from 1 and 8, MT)

10) ∴ (∃x)~Lx (from 9, RS)

11) ∴ ~Lb (from 10, DE)

No contradiction! Construct a refutation box:

D: {a, b}

V(La) = 1 and V(R) = V(Lb) = 0.

Since V(La) = 1 and V(Lb) = 0, V((x)Lx) = 0, V((x)Lx ⊃ R) = 1.

Since V(La) = 1 and V(R) = 0, V(La ⊃ R) = 0, V((x)(Lx ⊃ R)) = 0.

(4)

1. (∃x)Fx ∨ (∃x)Gx /∴(∃x)(Fx∨Gx)
2. asm: ~(∃x)(Fx∨Gx)
3. ∴ (x)~(Fx∨Gx) (from 2, RS)
4. asm: (∃x)Fx (break 1)
5. ∴ Fa (from 4, DE)
6. ∴ ~(Fa∨Ga) (from 3, DU)
7. ∴ ~Fa (from 6, NOR)
8. ∴ ~(∃x)Fx (from 4, 5 contradicts 7, RAA)
9. ∴ (∃x)Gx (from 1 and 8, DS)
10. ∴ Gb (from 9, DE)
11. ∴ ~(Fb∨Gb) (from 1, DU)
12. ∴ ~Gb (from 11, NOR)
13. ∴ (∃x)(Fx∨Gx) (from 2, 10 contradicts 12, RAA)

(5)

1. (∃x)Fx ⊃ (∃x)Gx  / ∴ (x)(Fx ⊃ Gx)
2. asm: ~(x)(Fx ⊃ Gx)
3. ∴ (∃x)~(Fx ⊃ Gx)   (from 2, RS)
4. ∴ ~(Fa ⊃ Ga)   (from 3, DE)
5. ∴ Fa   (from 4, NIF)
6. ∴ ~Ga   (from 4, NIF)
7. asm: ~(∃x)Fx   (break 1)
8. 错误!未找到引用源。 (x)~Fx   (from 7, RS)
9. ∴ ~Fa   (from 8, DU)
10. ∴ (∃x)Fx   (from 7, 5 contradicts 9, RAA)
11. ∴ (∃x)Gx   (from 1 and 10, MP)
12. ∴ Gb   (from 11, DE)

No contradictions!

D: {a, b}, V(Fa) = V(Gb) = 1 and V(Ga) = V(Fb) = 0.

Since V(Gb) = 1, V((∃x)Gx) = 1, V((∃x)Fx ⊃ (∃x)Gx) = 1.

Since V(Fa ⊃ Ga) = 0, V((Fa ⊃ Ga)• (Fb ⊃ Gb)) = 0, V((x)(Fx ⊃ Gx)) = 0.

(14)

1. (x)Fx ∨ (x)Gx  / ∴ (x)(Fx ∨ Gx)
2. asm: ~(x)(Fx ∨ Gx)
3. ∴ (∃x)~(Fx ∨ Gx)   (from 2, RS)
4. ∴ ~(Fa ∨ Ga)   (from 3, DE)
5. ∴ ~Fa   (from 4, NOR)
6. ∴ ~Ga   (from 4, NOR)
7. asm: (x)Fx   (break 1)
8. ∴ Fa   (from 7, DU)
9. ∴ ~(x)Fx   (from 7, 5 contradicts 8, RAA)
10. ∴ (x)Gx   (from 1 and 9, DS)
11. ∴ Ga   (from 10, DU)
12. ∴ (x)(Fx ∨ Gx)   (from 2, 6 contradicts 11, RAA)

8.5b Exercise – also LogiCola I (HC & MC)

First appraise intuitively. Then translate into logic (using the letters given) and say whether valid (and give a proof) or invalid (and give a refutation).

1. Everything has a cause.
If the world has a cause, then there is a God.
∴ There is a God. [Use Cx for "x has a cause," w for "the world," and G for "There is a God" (which we needn't here break down into "(∃x)Gx" – "For some x, x is a God"). A student of mine suggested this argument; but the next example shows that premise 1 can as easily lead to the opposite conclusion.]
(x)Cx, Cw⊃G / ∴ G (VALID)

2. Everything has a cause.
If there is a God, then something doesn't have a cause (namely, God).
∴ There is no God. [Use Cx and G. The next example qualifies "Everything has a cause" to avoid the problem; some prefer an argument based on "Every *contingent being or set of such beings* has a cause."]
(x)Cx, G ⊃ (∃x)~Cx / ∴ ~G (VALID)

3. Everything that began to exist has a cause.
The world began to exist.
If the world has a cause, then there is a God.
∴ There is a God. [Use Bx, Cx, w, and G. This "Kalam argument" is from William Craig and James Moreland; they defend premise 2 by appealing to the big-bang theory, the law of entropy, and the impossibility of an actual infinite.]
(x)(Bx⊃Cx), Bw, Cw⊃G / ∴ G (VALID)

4. If everyone litters, then the world will be dirty.
∴ If you litter, then the world will be dirty. [Lx, D, u]
(x)Lx ⊃ D / ∴ Lu ⊃ D (INVALID)
Construct a refutation box:
D:{a, u}
V(Lu) = 1, V(D) = 0, V(La) = 0.
Since V(Lu) = 1 and V(D) = 0, V(Lu ⊃ D) = 0.
Since V(Lu) = 1 and V(La) = 0, V((x)Lx) = 0, V((x)L x⊃ D) = 1

5. Anything enjoyable is either immoral or fattening. [Ex, Ix, Fx]

∴ If nothing is immoral, then everything that isn't fattening isn't enjoyable.
(x)(Ex ⊃ (Ix∨Fx)) / ∴ (x)~Ix ⊃ (x)(~Fx ⊃ ~Ex) (VALID)

6. Anything that can be explained either can be explained as caused by scientific laws or can be explained as resulting from a free choice of a rational being.
The totality of basic scientific laws can't be explained as caused by scientific laws (since this would be circular).
∴ Either the totality of basic scientific laws can't be explained or else it can be explained as resulting from a free choice of a rational being (God). [Use Ex for "x can be explained," Sx for "x can be explained as caused by scientific laws," Fx for "x can be explained as resulting from a free choice of a rational being," and t for "the totality of scientific laws." This one is from R. G. Swinburne.]
(x)(Ex⊃(Sx∨Fx)), ~St / ∴ ~Et∨Ft (VALID)

7. If someone knows the future, then no one has free will.
∴ No one who knows the future has free will. [Kx, Fx]
(∃x)Kx ⊃ ~(∃x)Fx / ∴ ~(∃x)(Kx•Fx) (VALID)

1) (∃x)Kx ⊃ ~(∃x)Fx
2) asm: (∃x)(Kx•Fx)
3) ∴ Ka•Fa (from 2, DE)
4) ∴ Ka (from 3, AND)
5) ∴ Fa (from 3, AND)
6) asm: ~(∃x)Kx (break 1)
7) ∴ (x)~Kx (from 6, RS)
8) ∴ ~Ka (from 7, DU)
9) ∴ (∃x)Kx (from 6, 4 contradicts 8, RAA)
10) ∴ ~(∃x)Fx (from 1 and 9, MP)
11) ∴ (x)~Fx (from 10, RS)
12) ∴ ~Fa (from 11, DU)
13) ∴ ~(∃x)(Kx•Fx) (from 2, 5 contradicts 12, RAA)

8. If everyone teaches philosophy, then everyone will starve.

∴ Everyone who teaches philosophy will starve. [Tx, Sx]

(x)Tx ⊃ (x)Sx / ∴ (x)(Tx ⊃ Sx) (INVALID)

1) (x)Tx ⊃ (x)Sx

2) asm: ~(x)(Tx ⊃ Sx)

3) ∴ (∃x)~(Tx ⊃ Sx) (from 2, RS)

4) ∴ ~(Ta ⊃ Sa) (from 3, DE)

5) ∴ Ta (from 4, NIF)

6) ∴ ~Sa (from 4, NIF)

7) asm: (x)Tx (break 1)

8) ∴ Ta (from 7, DU)

No contradiction! Construct a refutation box:

D: {a, b}

V(Ta) = 1 and V(Tb) = V(Sa) = V(Sb) = 0.

Since V(Tb) = 0, V(Ta • Tb) = 0, V((x)Tx) = 0.

Since V((x)Tx) = 0, V((x)Tx ⊃ (x)Sx) = 1.

Since V(Ta) = 1 and V(Sa) = 0, V(Ta ⊃ Sa) = 0, V((Ta ⊃ Sa)•(Tb ⊃ Sb)) = 0, V((x)(Tx ⊃ Sx)) = 0.

9. No proposition based on sense experience is logically necessary.

∴ Either no mathematical proposition is based on sense experience, or no mathematical proposition is logically necessary. [Use Sx, Nx, and Mx, and the universe of propositions. This is from the logical positivist A. J. Ayer.]

~(∃x)(Sx•Nx) / ∴ ~(∃x)(Mx•Sx) ∨ ~(∃x)(Mx•Nx) (INVALID)

10. Any basic social rule that people would agree to if they were free and rational but ignorant of their place in society (whether rich or poor, white or black, male or female) is a principle of justice.

The equal-liberty principle and the difference principle are basic social rules that people would agree to if they were free and rational but ignorant of their place in society.

∴ The equal-liberty principle and the difference principle are principles of justice. [Use Ax, Px, e, and d. This is from John Rawls. The equal-liberty principle says that each person is entitled to the greatest liberty compatible with an equal liberty for all others. The difference principle says that wealth is to be distributed equally, except for inequalities that serve as incentives that ultimately benefit everyone and are equally open to all.]

(x)(Ax ⊃ Px), Ae, Ad / ∴ Pe•Pd (VALID)

11. If there are no necessary beings, then there are no contingent beings.

∴ All contingent beings are necessary beings. [Use Nx and Cx. Aquinas accepted the premise but not the conclusion.]
~(∃x)Nx ⊃ ~(∃x)Cx / ∴ (x)(Cx ⊃ Nx)    (INVALID)

| | | |
|---|---|---|
| 1) | ~(∃x)Nx ⊃ ~(∃x)Cx | |
| 2) | asm: ~(x)(Cx ⊃ Nx) | |
| 3) | ∴ (∃x)~(Cx ⊃ Nx) | (from 2, RS) |
| 4) | ∴ ~(Ca ⊃ Na) | (from 3, DE) |
| 5) | ∴ Ca | (from 4, NIF) |
| 6) | ∴ ~Na | (from 4, NIF) |
| 7) | asm: ~(∃x)Cx | (break 1) |
| 8) | ∴ (x)~Cx | (from 7, RS) |
| 9) | ∴ ~Ca | (from 8, DU) |
| 10) | ∴ (∃x)Cx | (from 7, 5 contradicts 9, RAA) |
| 11) | ∴ (∃x)Nx | (from 1 and 10, MT) |
| 12) | ∴ Nb | (from 11, DE) |

No contradiction!

Construct a refutation box:
D:{a, b}
V(Ca) = V(Nb) = 1,
V(Na) = V(Cb) = 0.
Since V(Nb) = 1, V((∃x)Nx) = 1, V(~(∃x)Nx) = 0.
So V(~(∃x)Nx ⊃ ~(∃x)Cx) = 1.
Since V(Ca ⊃ Na) = 0 and V(Cb ⊃ Nb) = 1, V((x)(Cx ⊃ Nx)) = 0.

12. Anything not disproved that is of practical value to one's life to believe ought to be believed.
Free will isn't disproved.
∴ If free will is of practical value to one's life to believe, then it ought to be believed. [Use Dx, Vx, Ox, f (for "free will"), and the universe of discourse of beliefs. This is from William James.]
(x)((~Dx•Vx) ⊃ Ox), ~Df /Vt ⊃ Ot    (VALID)

13. If the world had no temporal beginning, then some series of moments before the

present moment is a completed infinite series.
There's no completed infinite series.
∴ The world had a temporal beginning. [Use Tx for "x had a temporal beginning," w for "the world," Mx for "x is a series of moments before the present moment," and Ix for "x is a completed infinite series." This one and the next are from Immanuel Kant, who thought our intuitive metaphysical principles lead to conflicting conclusions and thus can't be trusted.]

~Tw ⊃ (∃x)(Mx•Ix), ~(∃x)Ix /∴ Tw (VALID)

1) ~Tw ⊃ (∃x)(Mx•Ix)
2) ~(∃x)Ix
3) asm: ~Tw
4) ∴ (∃x)(Mx•Ix) (from 1 and 3, MP)
5) ∴ (x)~Ix (from 2, RS)
6) ∴ Ma • Ia (from 4, DE)
7) ∴ Ia (from 6, AND)
8) ∴ ~Ia (from 5, DU)
9) ∴ Tw (from 3, 7 contradicts 8, RAA)

14. Everything that had a temporal beginning was caused to exist by something previously in existence.
If the world was caused to exist by something previously in existence, then there was time before the world began.
If the world had a temporal beginning, then there was no time before the world began.
∴ The world didn't have a temporal beginning. [Use Tx for "x had a temporal beginning," Cx for "x was caused to exist by something previously in existence," w for "the world," and B for "There was time before the world began."]
(x)(Tx ⊃ Cx), Cw ⊃ B, Tw ⊃ ~B /∴ ~Tw (VALID)

15. If emotivism is true, then "X is good" means "Hurrah for X!" and all moral judgments are exclamations.
All exclamations are inherently emotional.
"This dishonest income tax exemption is wrong" is a moral judgment.
"This dishonest income tax exemption is wrong" isn't inherently emotional.
∴ Emotivism isn't true. [T, H, Mx, Ex, Ix, t]
T ⊃ (H•(x)(Mx ⊃ Ex)), (x)(Ex ⊃ Ix), Mt, ~It /∴ ~T (VALID)

16. If everything is material, then all prime numbers are composed of physical

particles.
Seven is a prime number.
Seven isn't composed of physical particles.
∴ Not everything is material. [Mx, Px, Cx, s]
$(x)Mx \supset (x)(Px \supset Cx)$, Ps, ~Cs / ∴ ~(x)Mx    (VALID)

17. If everyone lies, the results will be disastrous.
∴ If anyone lies, the results will be disastrous. [Lx, D]
$(x)Lx \supset D$ / ∴ $(\exists x)Lx \supset D$    (INVALID)
1. $(x)Lx \supset D$
2. asm: $\sim((\exists x)Lx \supset D)$
3. ∴ $(\exists x)Lx$    (2, NIF)
4. ∴ ~D    (2, NIF)
5. ∴ ~(x)Lx    (1 and 4, MT)
6. ∴ $(\exists x)\sim Lx$    (5, RS)
7. ∴ La    (3, DE)
8. ∴ ~Lb    (6, DE)
9. No contradiction!
Construct a refutation box:
D: {a, b}
V(D) = 0, V(La) = 1 and V(Lb) = 0.
Since V(La) = 1, $V((\exists x)Lx) = 1$ and V(D) = 0, $V((\exists x)Lx \supset R) = 0$.
Since V(Lb) = 0, V((x)Lx) = 0, $V((x)Lx \supset R) = 1$.

18. Everyone makes moral judgments.
Moral judgments logically presuppose beliefs about God.
If moral judgments logically presuppose beliefs about God, then everyone who makes moral judgments believes (at least implicitly) that there is a God.
∴ Everyone believes (at least implicitly) that there is a God. [Use Mx for "x makes moral judgments," L for "Moral judgments logically presuppose beliefs about God," and Bx for "x believes (at least implicitly) that there is a God." This is from the Jesuit theologian Karl Rahner.]
(x)Mx, L, $L \supset (x)(Mx \supset Bx)$ / ∴ (x)Bx    (VALID)

19. "x=x" is a basic law.

"x=x" is true in itself, and not true because someone made it true.
If "x=x" depends on God's will, then "x=x" is true because someone made it true.
∴ Some basic laws don't depend on God's will. [Use e (for "x=x"), Bx, Tx, Mx, and Dx.]
Be, Te • ~Me, De ⊃ Me / ∴ (∃x)(Bx•~Dx) (VALID)

20. Nothing that isn't caused can be integrated into the unity of our experience.
Everything that we could experientially know can be integrated into the unity of our experience.
∴ Everything that we could experientially know is caused. [Use Cx, Ix, and Ex. This is from Immanuel Kant. The conclusion is limited to objects of possible experience – since it says "Everything *that we could experientially know* is caused"; Kant thought that the unqualified "Everything is caused" leads to contradictions (see problems 1 and 2).]
(x)(Ix ⊃ Cx), (x)(Ex ⊃ Ix) / ∴ (x)(Ex ⊃ Cx) (VALID)

21. If everyone deliberates about alternatives, then everyone believes (at least implicitly) in free will.
∴ Everyone who deliberates about alternatives believes (at least implicitly) in free will. [Dx, Bx]
(x)Dx ⊃ (x)Bx / ∴ (x)(Dx ⊃ Bx) (INVALID)
Construct a refutation box:
D:{a, b}
V(Da) = 1, V(Ba) = V(Db) = 0.
Since V(Db) = 0, V((x)Dx) = 0, V((x)Dx ⊃ (x)Bx) = 1.
Since V(Da) = 1 and V(Ba) = 0, V(Da ⊃ Ba) = 0, V((x)(Dx ⊃ Bx)) = 0

22. All who are consistent and think that abortion is normally permissible will consent to the idea of their having been aborted in normal circumstances.
You don't consent to the idea of your having been aborted in normal circumstances.
∴ If you're consistent, then you won't think that abortion is normally permissible. [Use Cx, Px, Ix, and u. See Gensler's article in January 1986 *Philosophical Studies* or the synthesis chapter of Gensler's *Ethics: A Contemporary Introduction*, 2nd ed. (New York: Routledge, 2011).]
(x)((Cx•Px) ⊃ Ix), ~Iu / ∴ Cu ⊃ ~Pu (VALID)