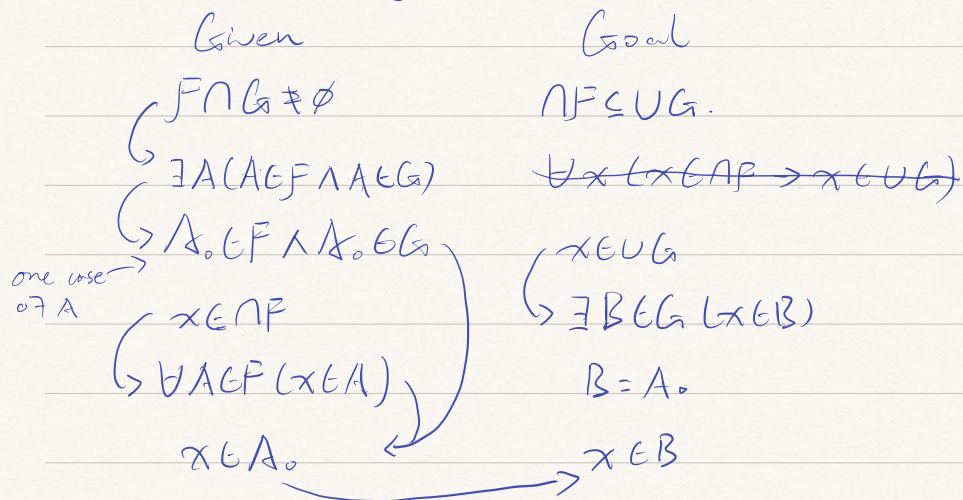


Strategy: To use the given of the form $\forall x P(x)$:

- For any a you can choose, add $P(a)$ to the given
(more than once if you want)

Ex: Suppose F and G are families of sets, their intersection is not empty. Then, prove $\cap F \subseteq \cup G$.



Proof: Let $x \in \cap F$. Since $F \cap G \neq \emptyset$, let A_0 be a set with $A_0 \in F$ and $A_0 \in G$. Since $x \in \cap F$, $x \in A_0$. But $A_0 \in G$ so $x \in \cup G$. Since x is arbitrary, $\cap F \subseteq \cup G$ \square

Exercise 3.3.16: If $F \subseteq P(B)$, then $\cup F \subseteq B$

Proof: Suppose $F \subseteq P(B)$, let $x \in \cup F$. So let $A \in F$ such that $x \in A$. Since $A \in F$ and $F \subseteq P(B)$, so we have $A \subseteq B$. So $x \in B$. Since x is arbitrary, $\cup F \subseteq B$. Therefore, $F \subseteq P(B)$ implies $\cup F \subseteq B$ \square

Exercise 6(a) Let $x \in \mathbb{R}$. Prove that if $x \neq 1$ then there exist a y that $\frac{y+1}{y-1} = x$.

$$y \neq 2$$

Given

Goal

$$x \in \mathbb{R} \quad x \neq 1 \quad \exists y \frac{y+1}{y-2} = x \rightarrow y = \frac{-2x-1}{1-x}$$

Proof: Let x be a real number and assume $x \neq 1$. Let

$y = \frac{-2x-1}{1-x}$, which is well-defined since $x \neq 1$. Note that

$y \neq 2$ since if $y=2$, $2-2x = -2x-1$, which is a contradiction

$$\text{Now: } \frac{y+1}{y-2} = \frac{\frac{-2x-1}{1-x} + 1}{\frac{-2x-1}{1-x} - 2} = x \quad \text{as required. } \square$$

\rightarrow

the transition should write in a chain form.