Recall: $F \subseteq A \times B$ is a function if $\forall a \in A\ \exists! b\ (a,b) \in F$
while $(a,b) \in F$, write $F(a) = b$.

Some functions are defined by "subs".
Ex: $A = \mathbb{Z}$, $B = \{1,2,3\}$.
Rule: For $x \in \mathbb{Z}$,

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is even} \\ 2 & \text{if } x = 3 \\ 3 & \text{if } x \text{ is odd and } x \neq 3. \end{cases}$$

this define $f \subseteq \mathbb{Z} \times B$.

$f: \mathbb{R} \to \mathbb{R}$ $f(x) = x^2 + 3$.
$h: \mathbb{Z} \to \mathbb{R}$ $h(x) = x^2 + 3$.

Theorm 5.1.4. For f, g, $A \to B$ if $\forall a \in A$ $f(a) = g(a)$ $f = g$.
pf sketch: Let $(a,b) \in A \times B$.
$(a,b) \in f$ iff $f(a) = b$
iff $g(a) = b$
iff $(a,b) \in g$ $\square$.

Ex: $f, g : \{0,1,2\} \to \mathbb{R}$
$f(x) = x^2 + x + 2$
$g(x) = 2^{x+1}$

$f(0) = 2$ $g(0) = 2$
$f(1) = 4$ $g(1) = 4$
$f(2) = 8$ $g(2) = 8$

so $f = g$.

Composition

Thm 5.1.5: if $A \to B$ and $g: B \to C$ then $g \circ f$ is a function
proof: