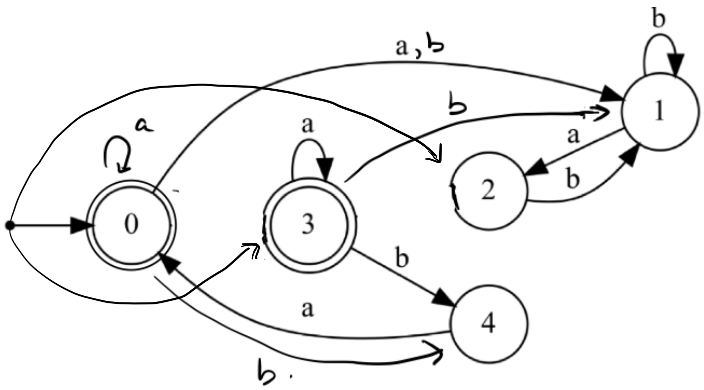
Assignment 2

COMPSCI 3331

Due: October 26, 2022 at 11:59 PM

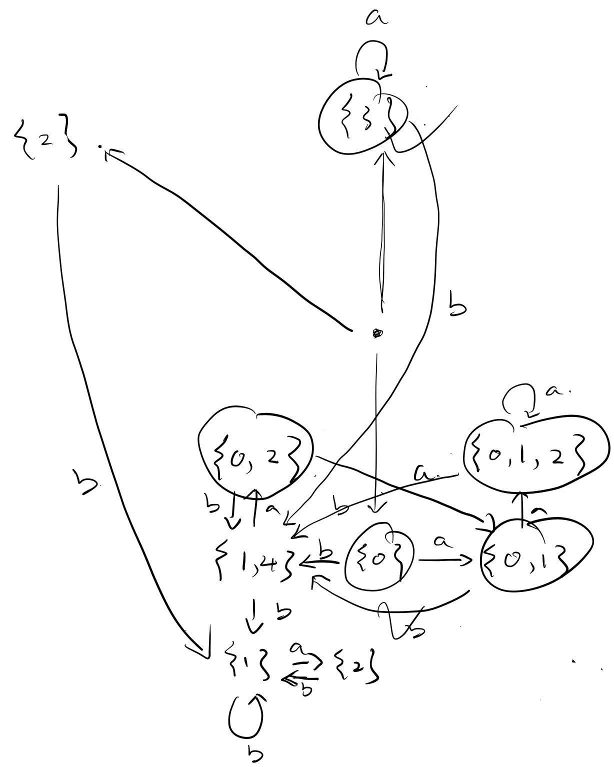
General notes:

- Assignments **must** be submitted on gradescope. You must indicate the locations of all answers for questions using gradescope. A video demonstrating how to do this can be found here.
- Assignments can be hand-written or typeset, as long as they are submitted to gradescope as an electronic file (pdf, png or other accepted format). It is your responsibility to submit a file that can be marked (i.e., images of pages are clear and handwriting, if any, can be read).
- Assignments can be submitted up to 48 hours late. A deduction of 1 % (of the total assignment value) will be applied per hour (rounded up) that the assignment is submitted past the deadline.
- You may also use your **once-per-course** 3-day extension on this assignment. Please submit the form on owl to declare that you want to use this extension. (choose "Individual Extension" from the tool menu on owl.) Recall that extensions do not stack – you may either choose the late submission penalty or the individual extension for an assignment, but not both.

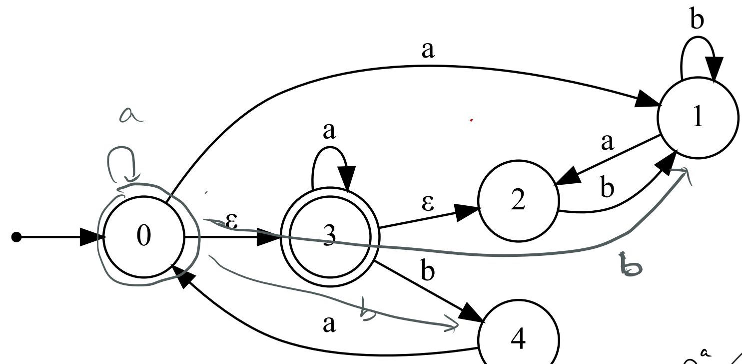


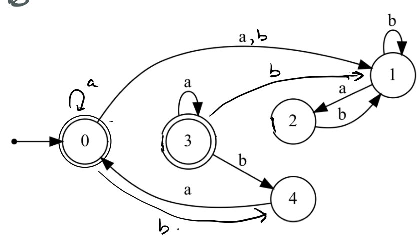
NFA .

DFA .



(5 marks) **1.** Given the ε-NFA below, construct a DFA that accepts the same language. Ensure that all states in the final DFA are appropriately labelled (i.e., with sets of states). Ensure your DFA is complete.





(5 marks) **2.** Let Σ be an arbitrary alphabet with at least one letter and $a \in \Sigma$ be a letter from the alphabet. Let $p_a : 2^{\Sigma^*} \to 2^{\Sigma^*}$ be the following language operation:

$$p_a(L) = \{w \in \Sigma^* \ : \ \exists n \geq 0 \text{ such that } wa^n \in L\}.$$

That is, $p_a(L)$ consists of all words w such that w can be concatenated with some number of copies of the letter a so that the result of this concatenation is in the language L.

Show that the regular languages are closed under p_a. You do not need to give a formal proof of the correctness of your construction, but you must give a explanation for how your construction works.

(6 marks) **3.** (a) Let $\Sigma = \{a\}$ and $L = \{a^p \ : \ p \text{ is a prime integer}\}$. Prove that L is not regular.

(b) Let $\Sigma = \{a\}$ and $L = \{a^n \ : \ n \text{ is a composite integer }\}$. Recall that an integer n is composite if there exists another integer m with $1 < m < n$ such that m divides n. Prove that L is not regular.

(c) Let $\Sigma = \{a, b\}$ and $L \subseteq \Sigma^*$ be the language $L = \{a^i b^j \ : \ j \leq i \leq 2j\}$. Prove that L is not regular.

(4 marks) **4.** Let $L = \{wxw^R \ : \ w, x \in \{a,b\}^+\}$. Recall that $\{a,b\}^+$ is the positive Kleene closure, so that w, x are not ε. Prove or disprove that L is regular.

$c + (k-1)i = 1$

$(k-1)i = 1 - c$

$k = \frac{1-c}{i} + 1.$

$\Sigma = \{a\}$.

Assume L is regular, then L must have a pumping length L and pick string S that $S = xyz$, ~~which could be divided into~~:
in this case, x, y, and z contains a^* only and assume $x = a^{P-L}$, $y = a^L$, $z = \varepsilon$. that is, $xy^iz = a^{P-L}(a^L)^i\varepsilon = a^{P-L+iL}$ In this case, $= P+(i-1)L$
i could be P+1 that $xy^iz = a^{PL}$,

Can i simply pick a specific case (i.e. assume the exact number of P, i, L) to point out in this situation L is not regular?

(c) Let $\Sigma = \{a,b\}$ and $L \subseteq \Sigma^*$ be the language $L = \{a^ib^j : j \le i \le 2j\}$. Prove that L is not regular.

(4 marks) **4.** Let $L = \{wxw^R : w,x \in \{a,b\}^+\}$. Recall that $\{a,b\}^+$ is the positive Kleene closure, so that w,x are not ε. Prove or disprove that L is regular.

c) Assume L is regular, then it has a pumping length P and pick a word w that $w = a^ib^j = xyz$, assume $|y| = y$, $|x| = x$, $|z| = z$
case 1: y contains a only.
2 y contains both a and b.
3 y contains b only.

In case 1, $x = a^x$, $y = a^y$, $z = a^{i-x-y}b^j$,

$$xy^iz = a^x(a^y)^i a^{i-x-y}b^j$$
$$= a^{i-y+yi}b^j$$
$$= a^{i(1+y)-y} \cdot b^j.$$

$i - y + yi > 2j$

$y(1+i) > 2j+i$

$y > \boxed{\frac{2j+i}{i+1}}$

$\boxed{\frac{2j+i}{i+1}}$

$(i+1)(2j+i)$

$A = \{a^n b^n\}$ is not regular.

Assume A is regular, then it need to have a pumping length P. And pick String S that $S = A^P B^P$. $S = xyz$ that

Case 1: y contains only a, that is
Case 2: y contains only b
Case 3: y contains a and b.

1: $xy^1z \Rightarrow xy^2z$

pick a word in the language which has only one variable.

Pick one specific

format as long as it meet the set.

two variables.

(c) Let $\Sigma = \{a,b\}$ and $L \subseteq \Sigma^*$ be the language $L = \{a^i b^j \mid j \leq i \leq 2j\}$. Prove that L is not regular.

let n be a constant defined by pumping lemma,

picking the string $z = a^n b^{2n} \in L$

decompose $z = uvw$, $u = a^x$, $v = a^y$, $w = a^{n-x-y} b^{2n}$.

~~consider~~ $\cancel{uv^{4n}w = a^{3n} b^{2n} \notin L}$.

$uv^k w = a^x a^{ky} a^{n-x-y} b^{2n}$

$x, y > 0$

$x + y \leq 2n$.

$k(y-1)$

$\cancel{x} + ky + \cancel{n} - \cancel{x} - y > \cancel{2}n$

$y \neq 0$.

$k >$

$uv^k w = a^x a^{(n+1)y} a^{n-x-y} b^{2n}$

$\boxed{\text{i.e. } k = n+1.}$

$= a^{2n+y} b^{2n} \notin L$.

$c = xy$

a

$c + (k-1)i$

$x(y-1) + x$

$(k-1)i = 2n$

$a^{x-i} a^{ki} a^{x(y-1)}$

$\Rightarrow a^{\underline{xy + (k-1)i}}$

$k = \frac{2n}{2} + 1$.

$c + (k-1)i = \text{prime}/1/0$.

other prime.

$k = 0$

$\boxed{c = i}$

$a^{n+(k-1)i} b^{2n}$.

$n+(k-1)i > 2n$

$k = n+2$.

$n+(n+1)i > 2n$.

$\boxed{a^{l+(k-1)i}}$.

$l+(k-1)i$.

$n+(k-1)i < n$

$\underline{(k-1)i < 0}$.

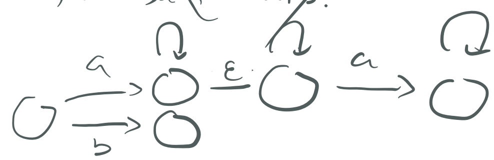
$n \le n+(k-1)i \le 2n$.

$(k-1)i > n$.

(c) Let $\Sigma = \{a,b\}$ and $L \subseteq \Sigma^*$ be the language $L = \{a^i b^j \ : \ j \le i \le 2j\}$. Prove that L is not regular.

(4 marks) **4.** Let $L = \{wxw^R \ : \ w,x \in \{a,b\}^+\}$. Recall that $\{a,b\}^+$ is the positive Kleene closure, so that w,x are not ε. Prove or disprove that L is regular.

$L = \{a,b\}^i ; \{a,b\}^j (\{a,b\}^k)^n$. a.



$wxw^R = xyz$.

P $|xy| < P$.

case 1

cannot keep track of the number of repitition of a/b in w that these two w cannot be the same.

↑

try to make the result diff.

uvw

$u = wx$

$v = w^R$

$w = \varepsilon$

$n \le \cancel{n!} \le 2n$.

$\boxed{|xy| + (k-1)i = 0}$

$u = a^{c-i-j}$

$k =$

$c - i = 0$

$\boxed{i = c}$

$\underline{c + (k-1)i} = 7$.

$\Rightarrow$.

$\frac{(y+1)}{i}$ xy $y\ i$

$k = \boxed{\frac{xy+1}{i}}$

[a, b]

$h+(k-1)i < n$.

$\boxed{k<1}$.

$k<1$

$wx\boxed{w}^R$, $w, x \in \{a,b\}^+$.