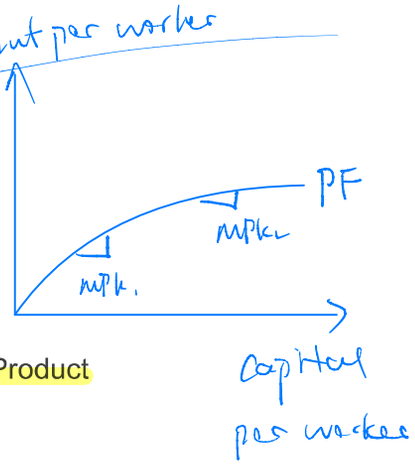
知识点 1. SOLOW GROWTH MODEL

- **Solow Growth Model ：**是由伟大的 Robert Solow 所提出并创建，所以是由他的名字所命名。这个模型是用来解释经济体在生存中，Capital 对于生产的重要性，以及对于未来经济发展的一种预测。

Production Function ：$Y = K^a L^{1-a}$



- ✓ Y：代表 GDP，National Income
- ✓ K：Capital，L：Labour
- ✓ a：Capital Share，Investor keep a 比例的 GDP
- ✓ $1-a$：Labour Share，Labour keep $1-a$ 比例的 GDP
- ✓ **Properties ：**Constant returns to scale， Diminishing Marginal Product

- **Capital Accumulation:**

$$K' = (1-\delta)K + I$$

- ✓ 明天的机器数量 = 今天的机器数量 – 报废的机器数量 + 新投资的机器

- **Saving Function ：**

$$S = \sigma Y$$

- ✓ 在借贷市场中，Equilibrium 发生在 $S = I$

- **Intensive Form ：（Variable Per Worker）：**

$$y = \frac{Y}{L} = \frac{K^a L^{1-a}}{L} = \frac{K^a L^{1-a}}{L^a L^{1-a}} = \left(\frac{K}{L}\right)^a = k^a$$

$$k' = (1-\delta)k + \sigma y$$

$$k' = k - \delta k + \sigma k^a$$

- **Steady State: when all variables are constant or growing at a constant pace**

$$\sigma k^a = \delta k$$

新投资的机器 = 报废掉的机器，所以 Capital Per Worker 不变

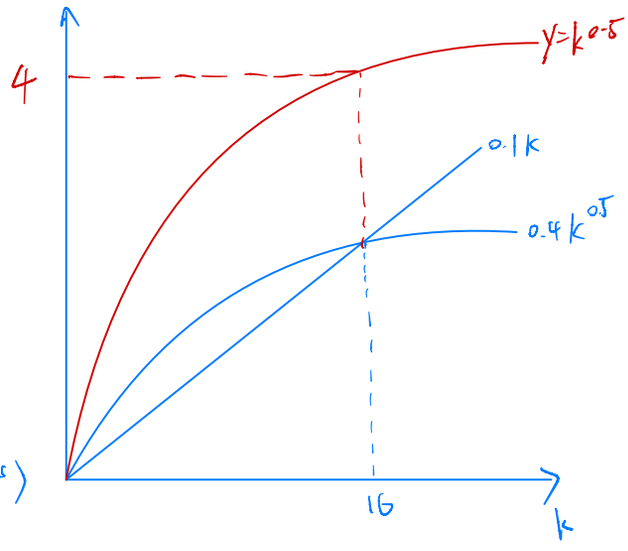
EXAMPLE：

1. A Solow Growth Model has production function $Y = K^{1/2}L^{1/2}, a = 0.5, \delta = 0.1$ and $\sigma = 0.4$. Solve for the steady state Equilibrium and Provide a labelled diagram.

$0.4k^{0.5} = 0.1k$

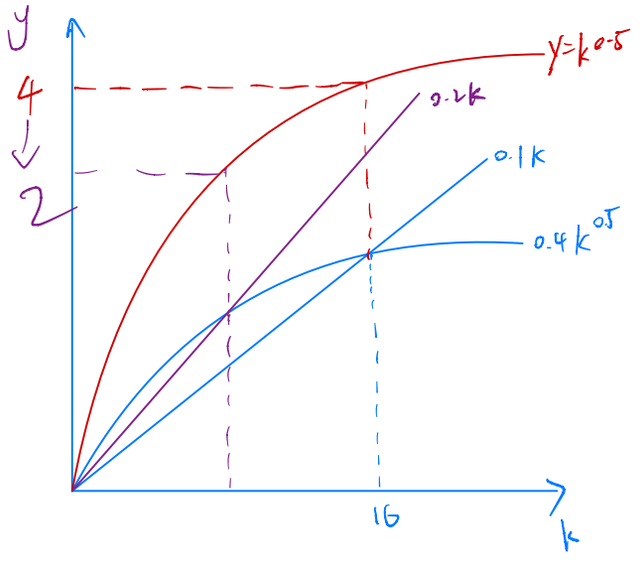
$k = 16$ (capital per worker)

$y = k^{\alpha} = 16^{0.5} = 4$ (output per worker)



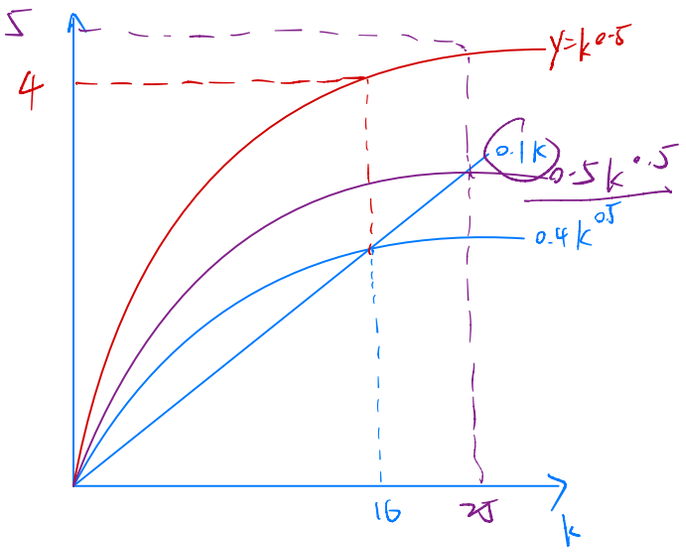
- **Prediction One: Catch Up Effect, 随着国家的发展，经济增长的速度会越来越慢，也就是说时间线足够长的话，发达国家一定会被发展中国家所追赶上**

2. Quantify and Illustrate the impact of a higher rate of depreciation rate $\delta = 0.2$. Prediction Two：**更高的 Depreciation rate 会导致更低的 GDP per worker**



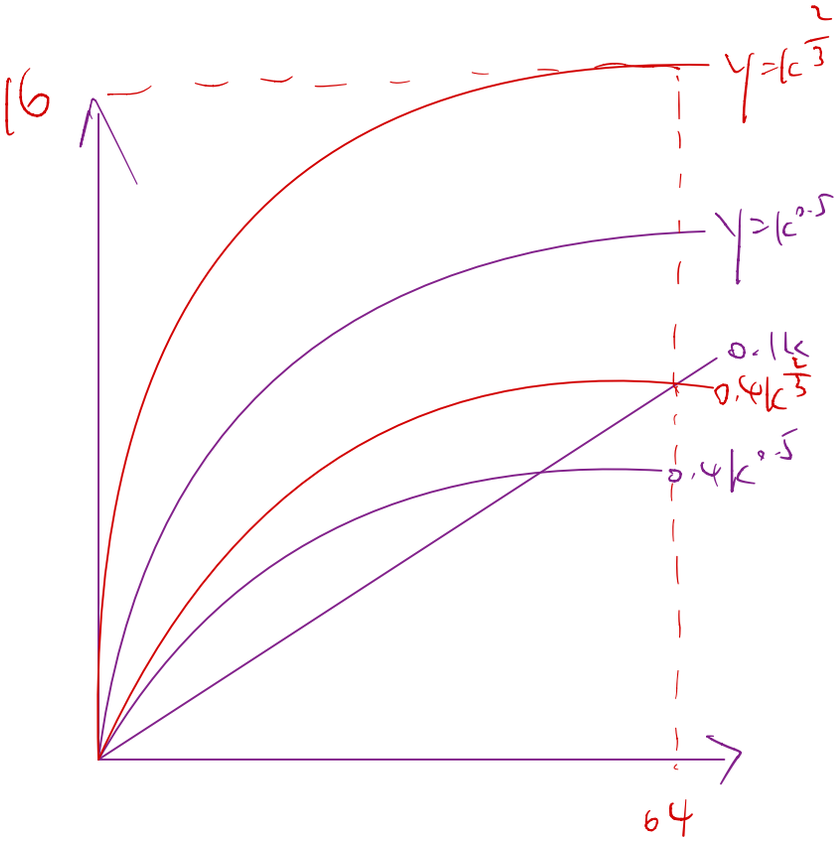
3. Quantify and Illustrate the impact of a higher rate of saving $\sigma = 0.5$ when $\delta = 0.1$. Prediction Three：**Higher Saving 会导致更高的 GDP per worker**

$$\begin{cases} k = 25 \\ y = 5 \end{cases}$$



4. Quantify and Illustrate the impact of a higher rate of Capital Share $a = 2/3$ when $\sigma = 0.4$, $\delta = 0.1$. Prediction Four：**Higher Capital Share 会导致 higher GDP per worker**

$$\begin{cases} k = 64 \\ y = 16 \end{cases}$$

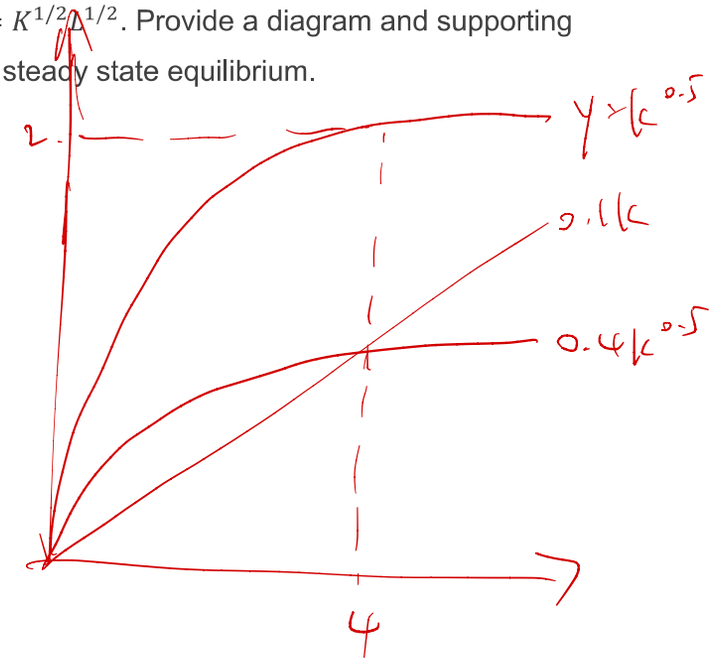


PAST TEST 大题：

1. A Solow Growth model economy has a savings rate of 20 percent, a 10 percent rate of depreciation and a production function $Y = K^{1/2}L^{1/2}$. Provide a diagram and supporting calculations that illustrate and quantify the steady state equilibrium.

$$K^{0.5} = 2$$

$$\begin{cases} K = 4 \\ Y = 2 \end{cases}$$



HW

2. A Solow Growth model economy has a savings rate of 20 percent, a 10 percent rate of depreciation and a production function $Y = K^{1/2}L^{1/2}$.
   a. Provide a diagram and supporting calculations that illustrate and quantify the steady state equilibrium.
   b. Quantify the impact on steady state equilibrium if the depreciation rate was only 5 percent. Modify your diagram from part a. to illustrate this equilibrium

- **在现实生活中，人口是有增长的，人口肯定不是不变的：**

$$L' = L + \eta L = (1+\eta)L$$

$$\frac{K'}{L} = \frac{K}{L} + \frac{\delta K}{L} + \frac{\sigma K^a}{L}$$

$$\frac{K'}{L'}\frac{L'}{L} = \frac{K}{L} + \frac{\delta K}{L} + \frac{\sigma K^a}{L}$$

$$k'(1+\eta) = k + \delta k + \sigma k^a$$

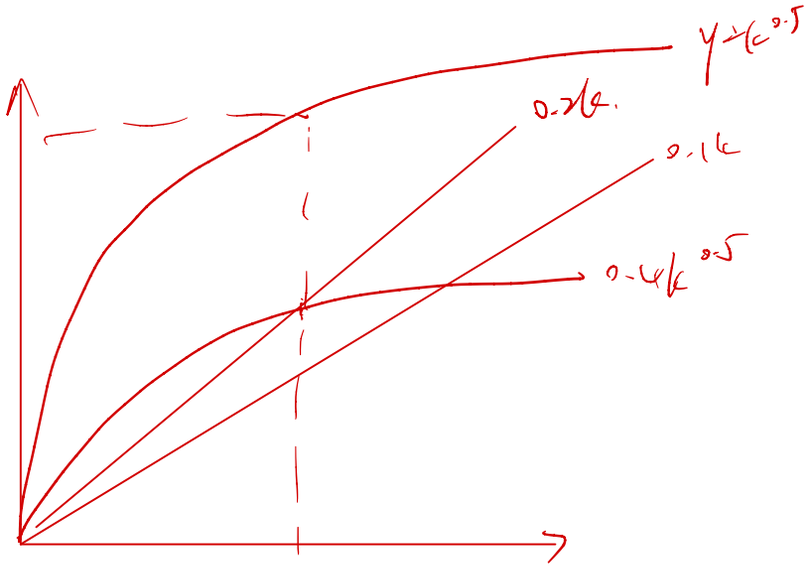
- **In steady State:**

$$\sigma k^a = (\delta + \eta)k$$

EXAMPLE:

Solve for steady state equilibrium: $\eta = 0.1, a = 0.5, \delta = 0.1$ and $\sigma = 0.4$. Provide a labelled diagram. How does GDP per worker in this country compare to an economy where $\eta = 0$

Prediction Five: higher 人口增长率，会导致更低的 GDP per worker

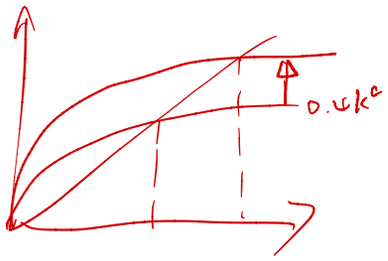


- $y = Y/L$，**所以** $Y = yL$，**如果我们可以 ln 在每一项中，**$lnY = lny + lnL$，**所以 Prediction Six：**$\%\Delta Y = \%\Delta y + \%\Delta L$，**GDP 的增长率 = GDP per worker 的增长率 + 人口的增长率**

TUT PRACTICE :

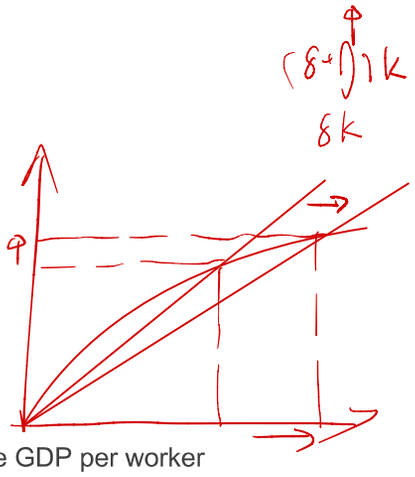
1. An increase in the population, with everything else held constant, will cause.
   a. An increase in per capita output.
   b. A decrease in per capita output.
   c. A decrease in the capital to output ratio.
   d. An inward shift in the production possibilities boundary.

2. Rising levels of output per capita could be caused by
   a. Higher depreciation rates.
   b. Lower savings rates.
   c. Lower investment.
   d. Higher savings rates.



3. Anincreaseinthelabourforce_____totaloutputand_____outputperworker.
   a. Increases; Increases
   b. Increases; Decreases
   c. Decreases; Increases
   d. Decreases; Decreases

4. Sustained rising material living standards can only be explained by
   a. Growth in human capital
   b. Growth in physical capital
   c. Growth in the labour force
   d. Exogenous technological change



5. The Solow Growth Model predicts that a lower depreciation rate will
   a. increase steady state savings per worker and increase steady state GDP per worker
   b. decrease steady state savings per worker and decrease steady state GDP per worker
   c. decrease steady state savings per worker and increase steady state GDP per worker
   d. increase steady state savings per worker and decrease steady state GDP per worker

6. The Solow Growth Model predicts that slower growth in the labour force will
   a. increase steady state savings per worker and increase steady state GDP per worker
   b. decrease steady state savings per worker and decrease steady state GDP per worker
   c. decrease steady state savings per worker and increase steady state GDP per worker
   d. increase steady state savings per worker and decrease steady state GDP per worker

HW

PAST TEST 大題：

1. A Solow Growth model economy with production function $Y = K^{1/2}(AL)^{1/2}$ has a savings rate of 48 percent, a 6 percent labour force growth rate and a 10 percent rate of depreciation. There is no technological progress.
   a. Provide a labelled diagram and supporting calculations to illustrate and quantify the steady state equilibrium for this economy.
   b. Use the same diagram to show how the steady state would be affected if immigration policies are relaxed and the labour force growth rate increased from 6 percent to 14 percent.

- **Technological Progress：**

$$(AL)' = (1 + \gamma + \eta)AL$$

  - ✓ A: Human Capital 知识水平
  - ✓ Human Capital 也是可以积累，和增长的，增长率是 γ
  - ✓ AL: Effective Worker

- In Steady State Equilibrium：

$$\sigma k^{a} = (\delta + \gamma + \eta)k$$

- 因此：GDP per Effective Worker：$y = \frac{Y}{AN}$

EXAMPLE:

Solve for Steady State Equilibrium $\gamma = 0.2, \eta = 0.1, a = 0.5, \delta = 0.1, and\ \sigma = 0.4$ and provide a labelled diagrams **Prediction Seven: Higher human capital growth rate, will cause lower GDP per effective worker.**

- **$y = Y/AL$，所以 $Y = yAL$，如果我 take ln 在每一项中，$lnY = lny + lnA + lnL$，所以 Prediction Six：$\%\Delta Y = \%\Delta y + \%\Delta A + \%\Delta L$，GDP 的增长率 = GDP per worker 的增长率 + 人口的增长率 + 知识水平的增长率**

TUT PRACTICE : Hw

1. A Solow Growth economy with $Y = K^{1/2}(AL)^{1/2}$, depreciation of 2 percent, labour force growth of 2 percent and technological progress of 1 percent will have equilibrium GDP per effective worker equal to _____

Hw.

2. A Solow Growth economy with $Y = K^{1/2}(AL)^{1/2}$, depreciation of 2 percent, labour force growth of 1 percent and technological progress of 2 percent will have Consumption per Worker that grows at a rate of _____ percent per year.

PAST TEST 大题：

1. A Solow Growth model economy with production function $Y = K^{1/2}(AL)^{1/2}$ has a savings rate of 48 percent, a 3 percent labour force growth rate and a 5 percent rate of depreciation. Labour augmenting technological progress occurs at a rate of 4 percent per year.
   a. Provide a labelled diagram and supporting calculations to illustrate and quantify the steady state equilibrium for this economy.
   b. Steady state output per worker grows at a rate of ____0____ percent per year and output grows at a rate of ____7____ percent per year.
   c. Update your diagram to show what would happen if the depreciation rate fell to 1 percent per year.

HW.

a) $0.48k^{0.5} = (0.03+0.05+0.04)k$

$k = 16$

$y = 4$

b) In S.S. k^* constant, y^* constant

$\%\Delta Y = \%\Delta y + \%\Delta A + \%\Delta L$

$0.07 = 0.00 \quad 0.04 + 0.03$