MATH 1600 Linear Algebra — Winter 2020

Tutorial 2

Part A: Length, Direction, Dot Product

1. Consider the points $A = (3, -1, 4)$, $B = (4, -2, 6)$, $C = (5, 0, 2)$ and vectors $\mathbf{u} = \overrightarrow{AB}$, $\mathbf{v} = \overrightarrow{AC}$ in $\mathbb{R}^3$.

   i) Give the coordinates of the vectors. $u = (1, -1, 2)$ $v = (2, 1, -2)$.

   ii) Compute the dot product $\mathbf{u} \cdot \mathbf{v}$. $u \cdot v = 2 - 1 - 4 = -3$.

   iii) Compute $\text{proj}_{\mathbf{u}}(\mathbf{v})$, the projection of $\mathbf{v}$ onto $\mathbf{u}$. $\text{Proj}_u(v) = \frac{\sqrt{6}}{2}$. $\cos\theta = \frac{\sqrt{6}}{6}$

   iv) Let $\alpha(\mathbf{u}, \mathbf{v})$ denote the angle between $\mathbf{u}$ and $\mathbf{v}$, and compute $\cos(\theta)$ and $\sin(\theta)$ for $\theta = \alpha(\mathbf{u}, \mathbf{v})$. $\sin\theta = \frac{\sqrt{30}}{6}$

   v) If we assume $0 \le \theta \le \pi$, is θ an acute, right or <u>obtuse angle</u>?

2. Let $\mathbf{u}, \mathbf{v}$ be vectors in $\mathbb{R}^n$ satisfying $\mathbf{u} \cdot \mathbf{u} = 3$, $\|\mathbf{v}\| = 2$, and $\cos(\alpha(\mathbf{u}, \mathbf{v})) = 1/2$. Compute:

   i) $\|\mathbf{u}\|$. $|u| = 3$.

   ii) $\|2\mathbf{u} + 3\mathbf{v}\|$. $= 12$.

   $4 + 4 + 9 = 17$.

3. Find all scalars c for which $c(2\mathbf{e}_1 + 2\mathbf{e}_2 + 3\mathbf{e}_3)$ a unit vector in $\mathbb{R}^3$. $\frac{1}{\sqrt{17}}$.

   $2\sqrt{2}\, e_x + 3e_3$.

4. Find the scalar k for which the vectors $k\mathbf{e}_1 + 2\mathbf{e}_2$ and $8\mathbf{e}_1 + 6\mathbf{e}_2$ are orthogonal. $(ke_1 + 2e_2) \cdot (8e_1 + 6e_2) =$

   $8ke_1^2 + 16e_1e_2 + 6ke_1e_2 + 12e_2^2$

   $8k + 12 = 0$ $k = -\frac{3}{2}$.

5. i) Find a vector in $\mathbb{R}^3$ which is orthogonal to itself. $(0, 0, 0)$.

   ii) Show that every *other* vector in $\mathbb{R}^3$ is *not* orthogonal to itself.

   (a, b, c). $(a, b, c) \cdot (a, b, c) = a^2 + b^2 + c^2 \neq 0$ for every $a, b, c \neq 0$.

6. (Bonus) Suppose $n \ge 1$ and $\mathbf{u} \in \mathbb{R}^n$ satisfies $\mathbf{u} \cdot \mathbf{v} = 0$ for <u>every</u> $\mathbf{v} \in \mathbb{R}^n$. Show that $\mathbf{u} = \mathbf{0}$.

   $u = (u_1, u_2, u_3 \cdots u_n)$. $v = (v_1, v_2, \cdots v_n)$.

   $u_1v_1 + u_2v_2 + \cdots + u_nv_n = 0$ for every value of $v_1, v_2 \cdots v_n$.

   $u_1, u_2 \cdots u_n = 0$ $u = 0$.

Part B: Lines, Planes, Distance

7. Write an equation of the line passing through the points $P = (2, -1, 3)$ and $Q = (1, 4, -3)$ in:

   i) Vector form. $\overrightarrow{PQ} = (-1, 5, -6)$.

   ii) Parametric form. $(2 - t, -1 + 5t, 3 - 6t)$.

   iii) Symmetric form.

8. Find the distance between the point $(5, 1)$ and the line $y = 3x + 1$ in the plane. $\frac{3\sqrt{10}}{2}$.

9. Let A, B, C be the points from question 1 in Part A, and let L be the line through B and C.

   i) Find an equation for L. $L: (4, -2, 6) + t(2, 1, -2)$.

   ii) Find the distance between A and L.

$d = |MA \times e|$.

$A = (3, -1, 4)$.

$d = |u \times v|$

$u \times v = \left(\det\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}, -\det\begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}, \det\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\right)$.

$= (0, 6, 3).$

$d = \sqrt{36+9} = 3\sqrt{5}.$