Strong Induction:

$P(n)$ be a proposition for $n \in \mathbb{Z}_+$. Then $P(n)$ is true for all n if $\begin{cases} P(1) \text{ is true} \\ P(1), P(2), \cdots P(k) \Rightarrow P(k+1) \text{ is true for all } k \in \mathbb{Z}_+. \end{cases}$

$P(1) \Rightarrow P(2)$

$P(1), P(2) \Rightarrow P(3)$

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Problem: Show that every positive integer as a sum of distinct non-negative powers of 2.

Solution: $1 = 2^0$ $\quad 3 = 2^0 + 2^1$ $\quad 5 = 2^2 + 2^0$ $\quad 7 = 2^2 + 2^1 + 2^0$

$2 = 2^1$ $\quad 4 = 2^2.$ $\quad 6 = 2^2 + 2^1$ $\quad 8 = 2^3.$

Binary Reperest of an Positive integer is unique.

Proof:

$P(n)$: n can be written as sum of distinct non-negative integer powers of 2.

Base Case: $P(1)$ is true as $1 = 2^0$.

Inductive Step: Assume $P(1), P(2) \cdots P(k)$ are true.

Consider the case $P(k+1)$

Let 2^m is the higher power of 2 less than $k+1$.

$k+1 = \text{remainder } r + 2^m$.

$r < 2^m < k+1$ then $P(r)$ is true for

$r = 2^{l_1} + 2^{l_2} \cdots 2^{l_3} \cdots 2^{l_n}$. where $l_n > l_{n-1} \cdots > l_1$.

$\Rightarrow k+1 = 2^m + 2^{l_1} + 2^{l_2} + \cdots + 2^{l_n}$.

where $m > l_n > l_{n-1} \cdots > l_1$.

$\therefore P(k)$ is true for all non-negative integer.

We claim that m is different from any of the l_n

we show this using contradiction:

let $m = l_i \Rightarrow k+1 = 2^{l_1} + 2^{l_2} \cdots\cdots + 2^{m} + 2^{l_i} + 2^{l_{i+1}} + \cdots + 2^{m}$.

$$k+1 > 2^m + 2^m = 2^{m+1}.$$

it is contradiction with the definition of m.

i.e.) the greatest power of 2 is less than m.

$\Rightarrow m, l_1, \cdots\cdots l_n$ are distinct.

therefore, $P_1, P_2 \cdots\cdots \Rightarrow P(k+1)$.

Hence, by Principle of strong induction, $P(n)$ is true for all $n \in \mathbb{Z}_+$.