

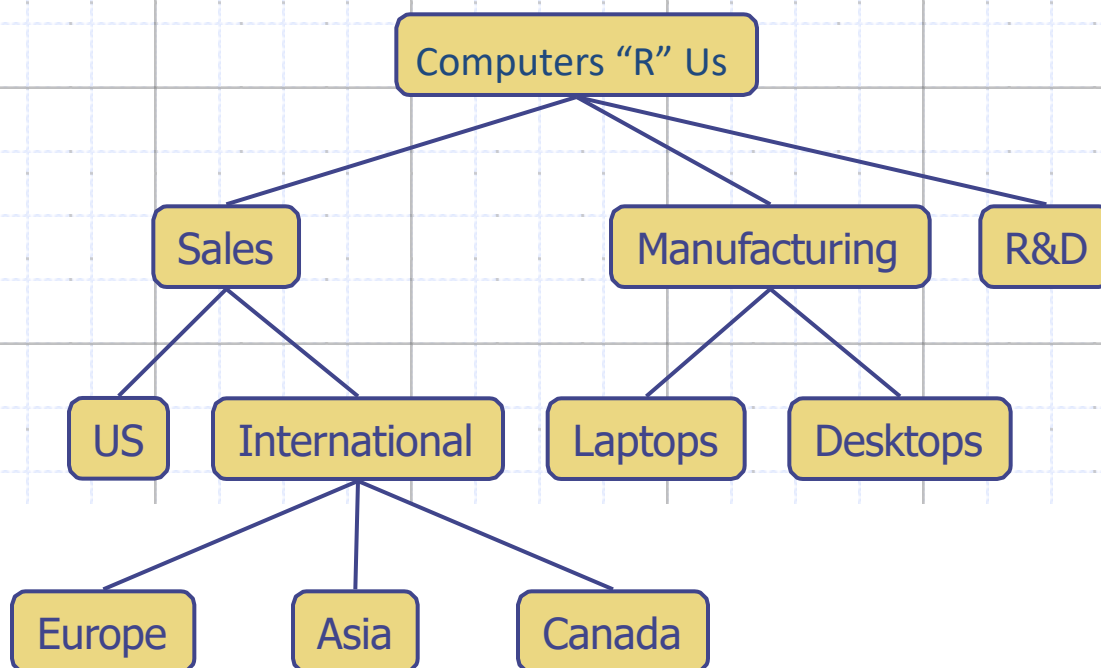
TREES

What is a Tree?

A tree is an abstract model of a hierarchical structure

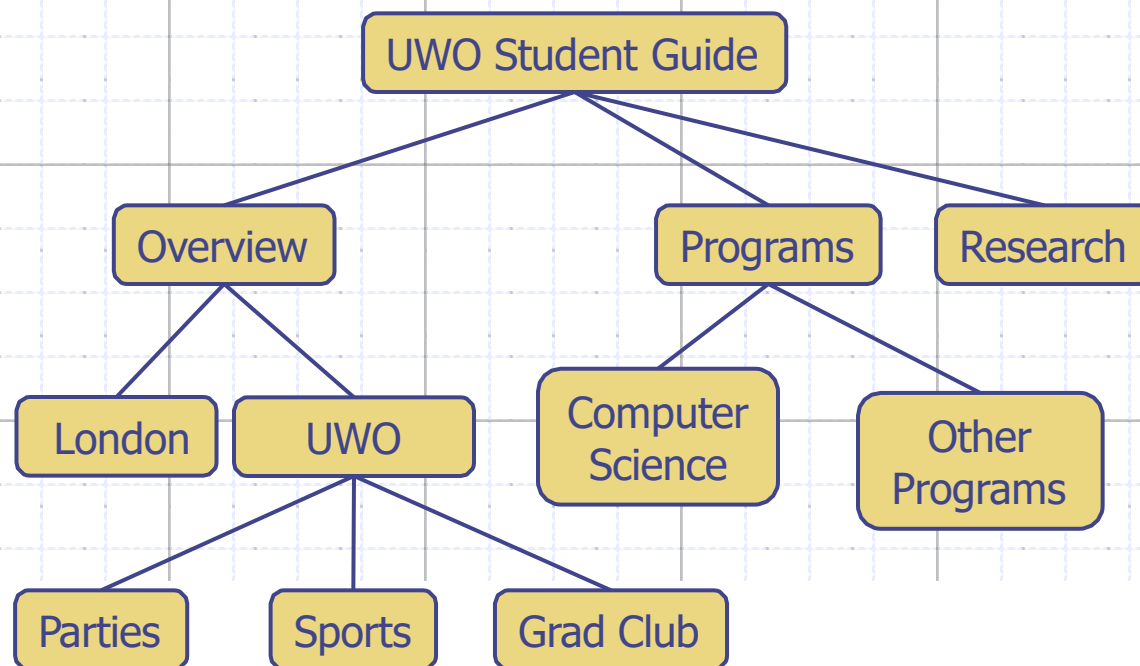
Applications

Organization of a company



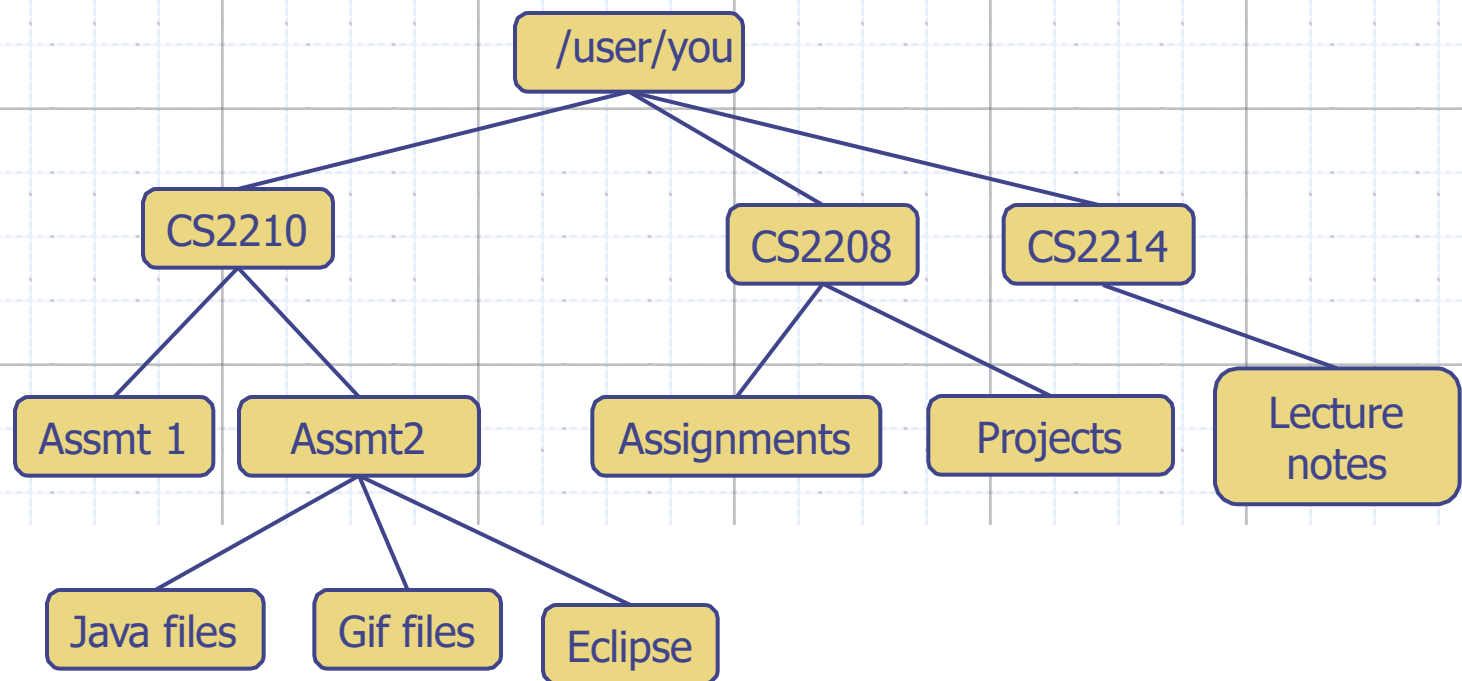
Applications

Table of contents of a book

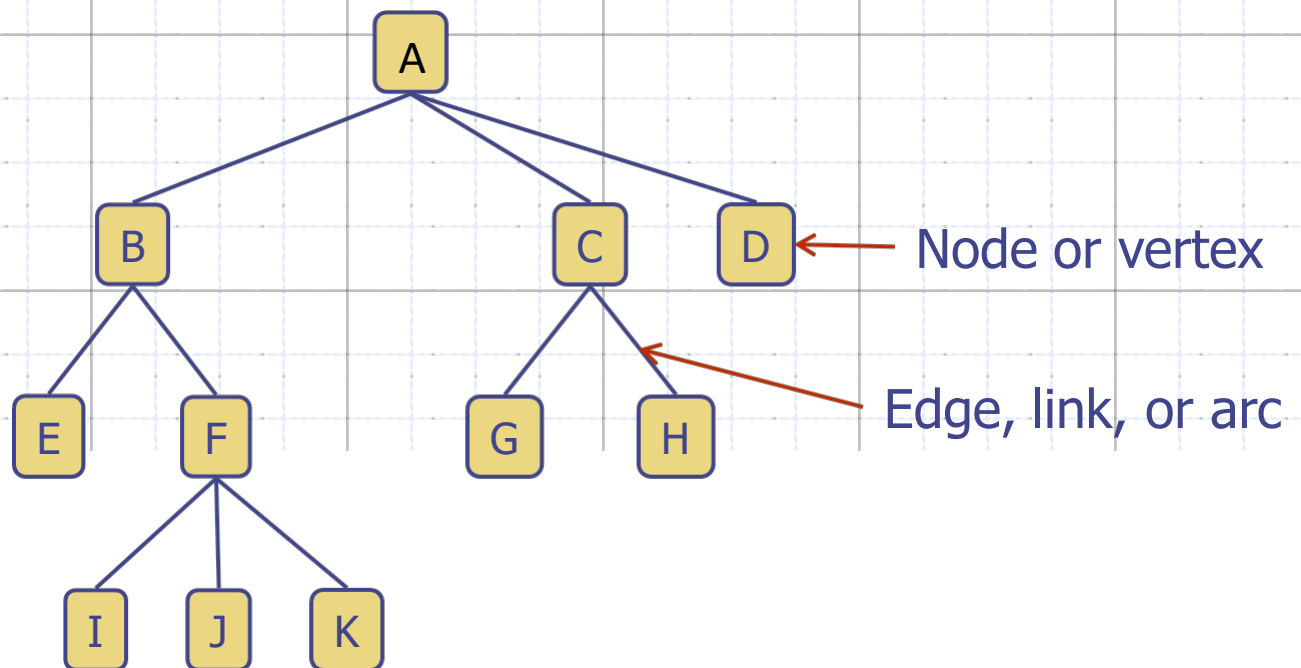


Applications

File system



Tree Terminology



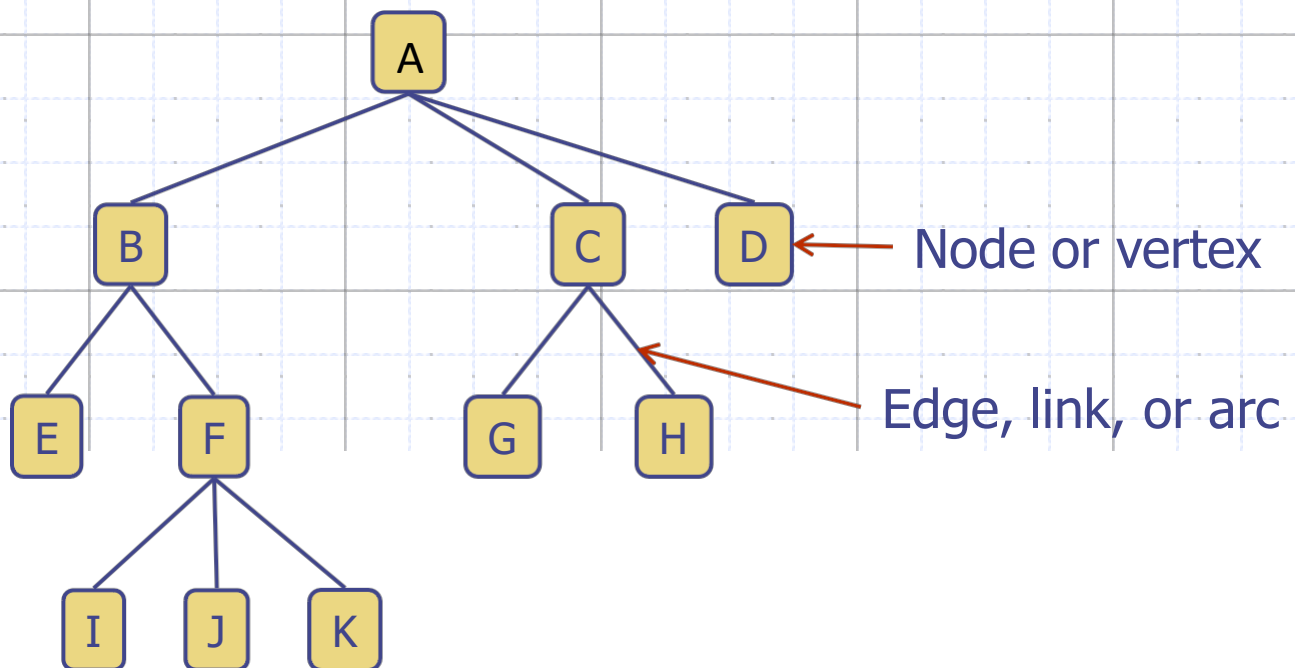
Tree Terminology

A is the root node

B is the parent of E and F

C is the sibling of B and D

B, C, and D are children of A



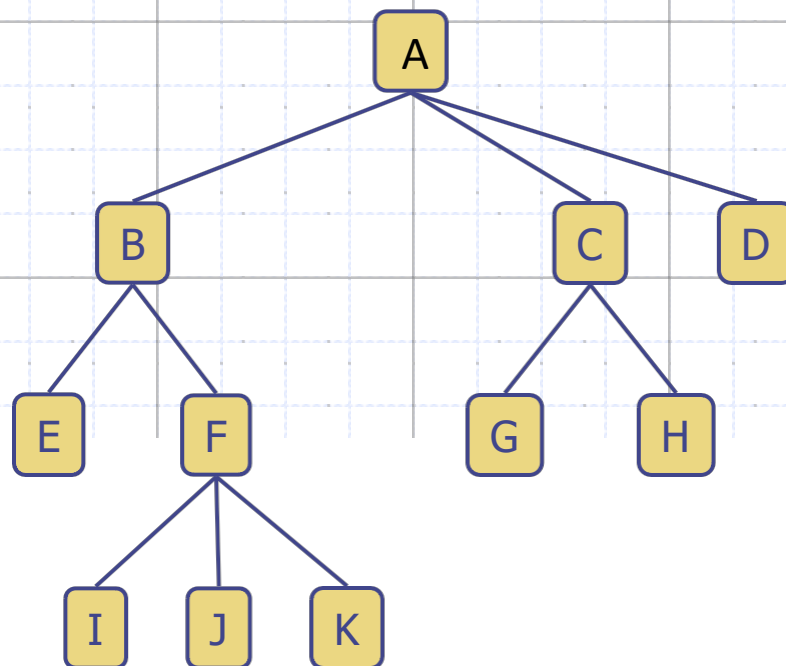
Tree Terminology

E, F, I, J, K are descendants of B

All nodes, except A, are descendants of A

A, B, F are ancestors of J

A has no ancestors



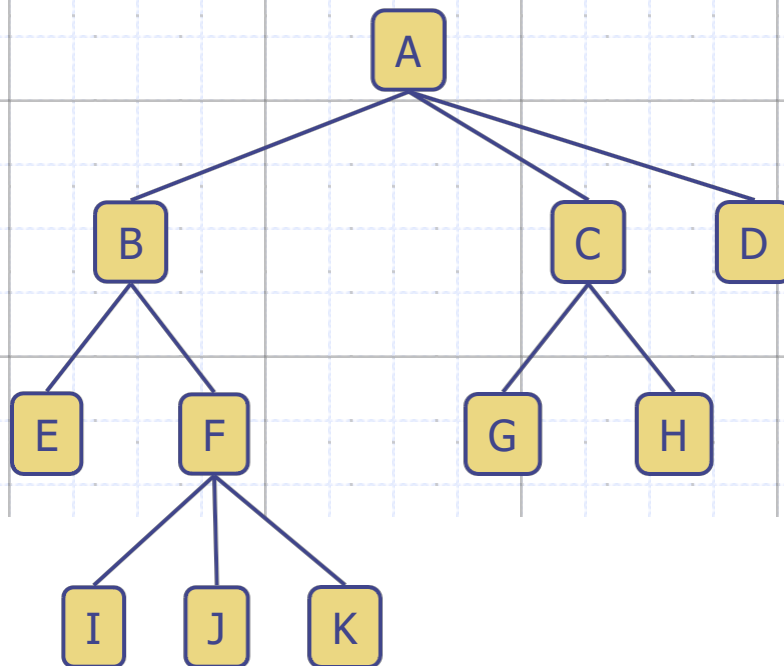
Tree Terminology

Internal node:

node with at least one child (A, B, C, F)

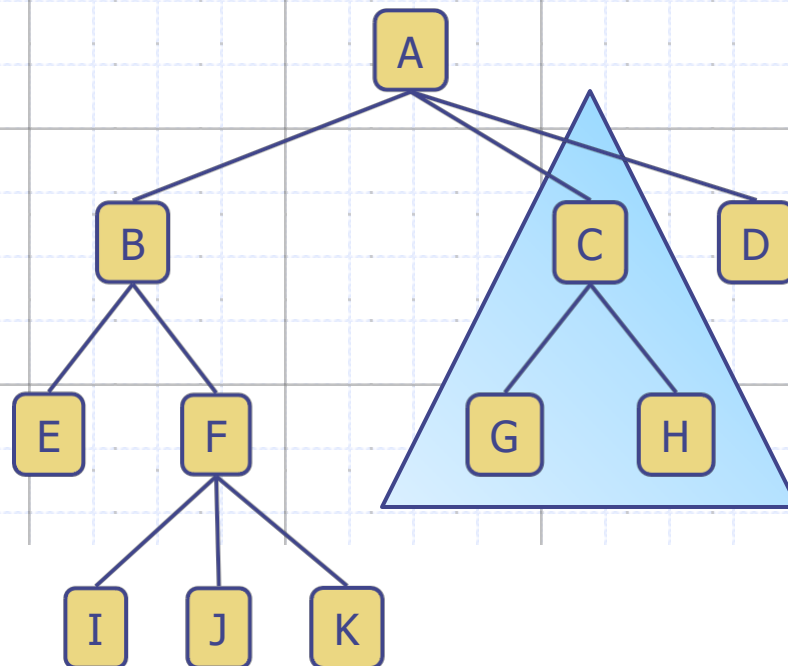
External node or leaf:

node without children (E, I, J, K, G, H, D)



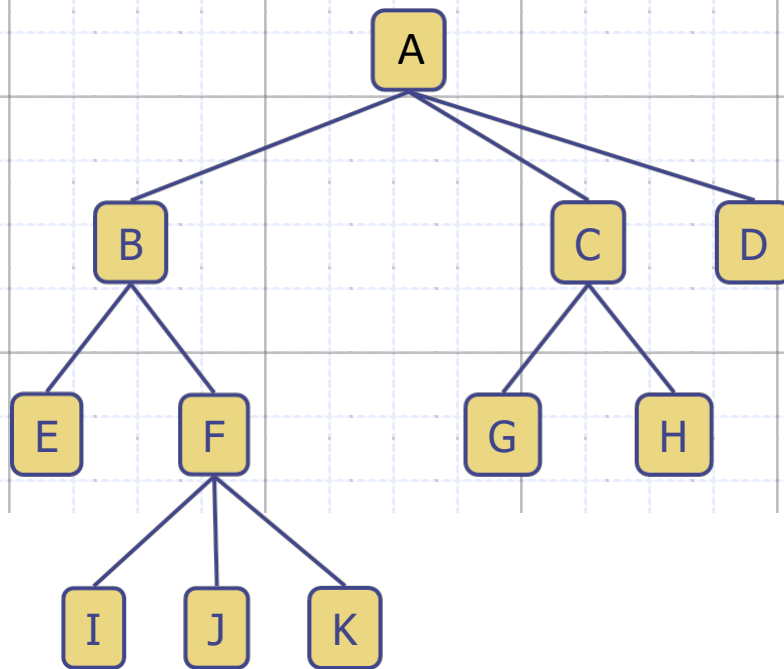
Tree Terminology

Subtree: tree consisting of a node and its descendants

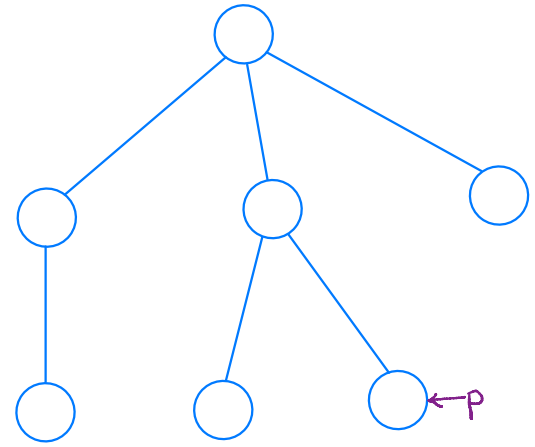


Tree Terminology

Depth or level of a node:
number of ancestors. Depth of E is 2.



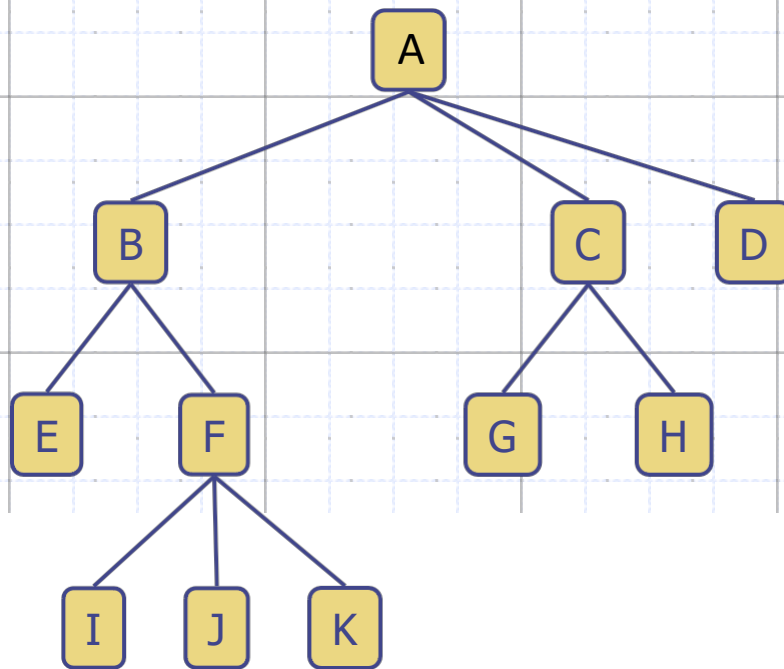
Algorithm level(p)
In: Node p of a tree
Out: Level of p



Tree Terminology

Depth or level of a node:
number of ancestors. Depth of E is 2.

Height of tree
maximum depth of any node. Tree has height 3.



Tree Terminology

Depth or level of a node:

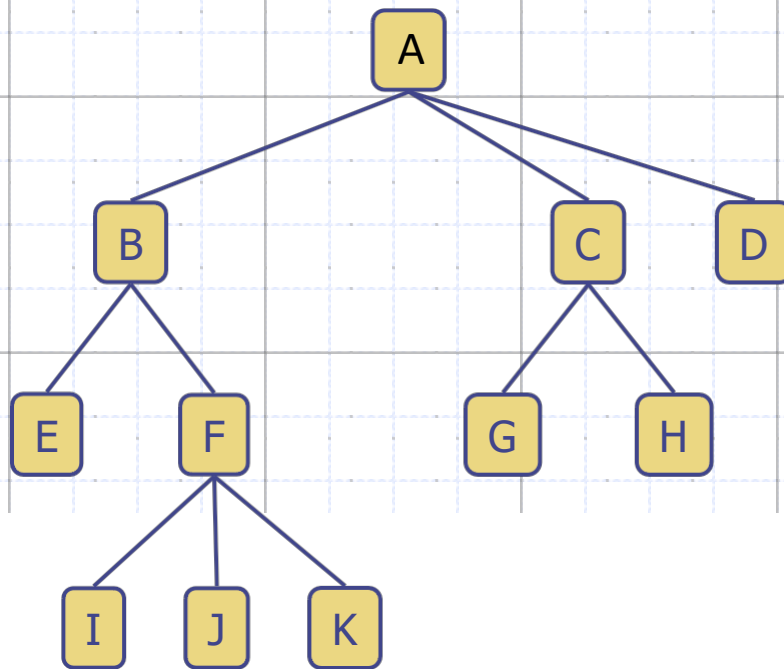
number of ancestors. Depth of E is 2.

Height of tree

maximum depth of any node. Tree has height 3.

Degree of a node:

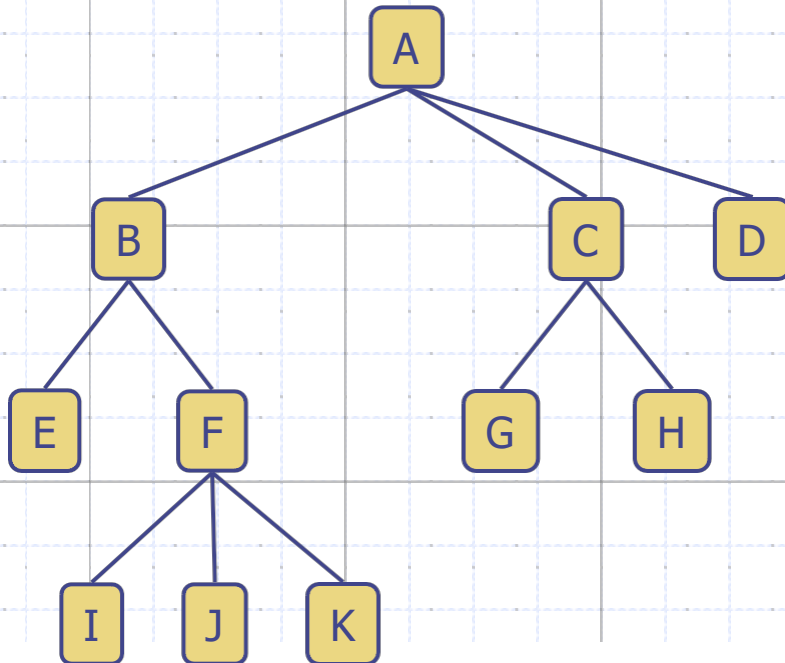
number of children. Degree of F is 3.



$$\sum_u \text{degree}(u) =$$

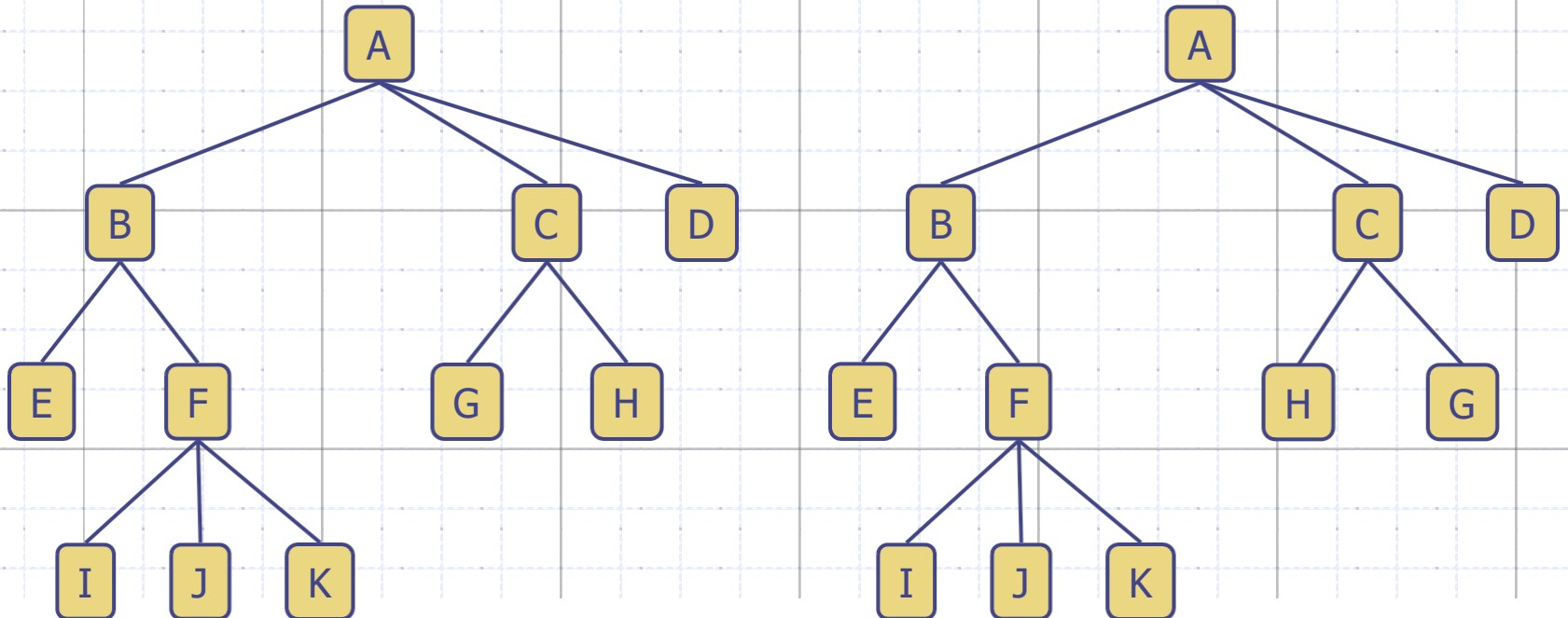
Tree Terminology

Ordered tree: The children of a node are ordered.



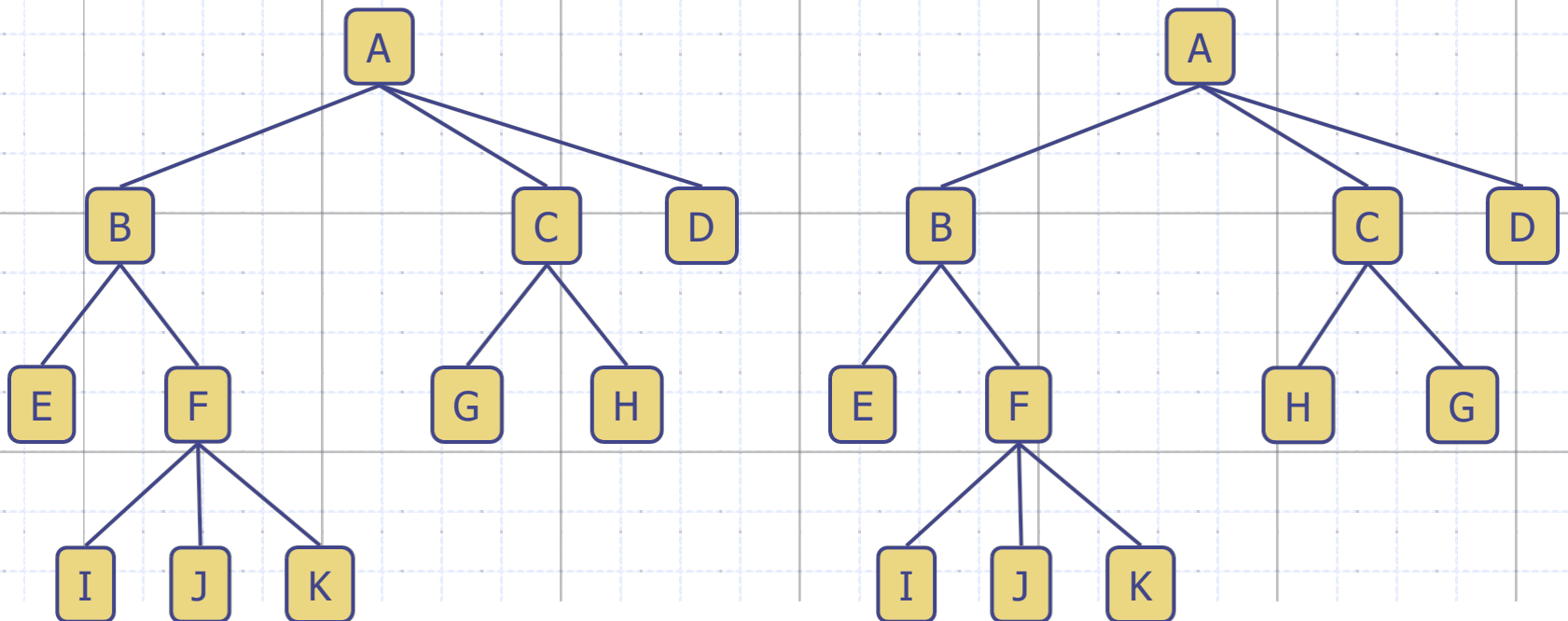
Tree Terminology

Ordered tree: The children of a node are ordered.



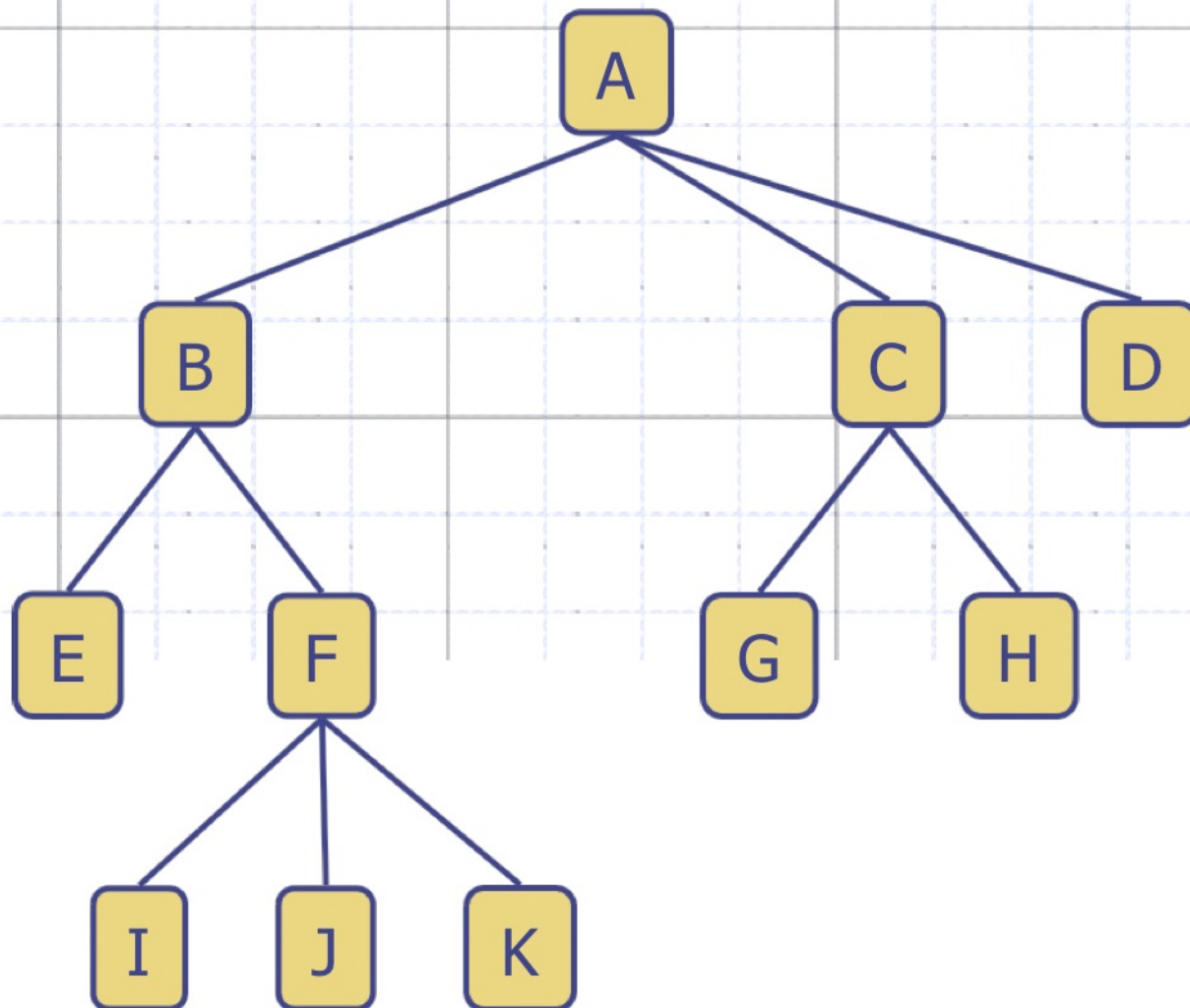
Tree Terminology

Un-Ordered tree: The children of a node are not ordered.



Tree Properties

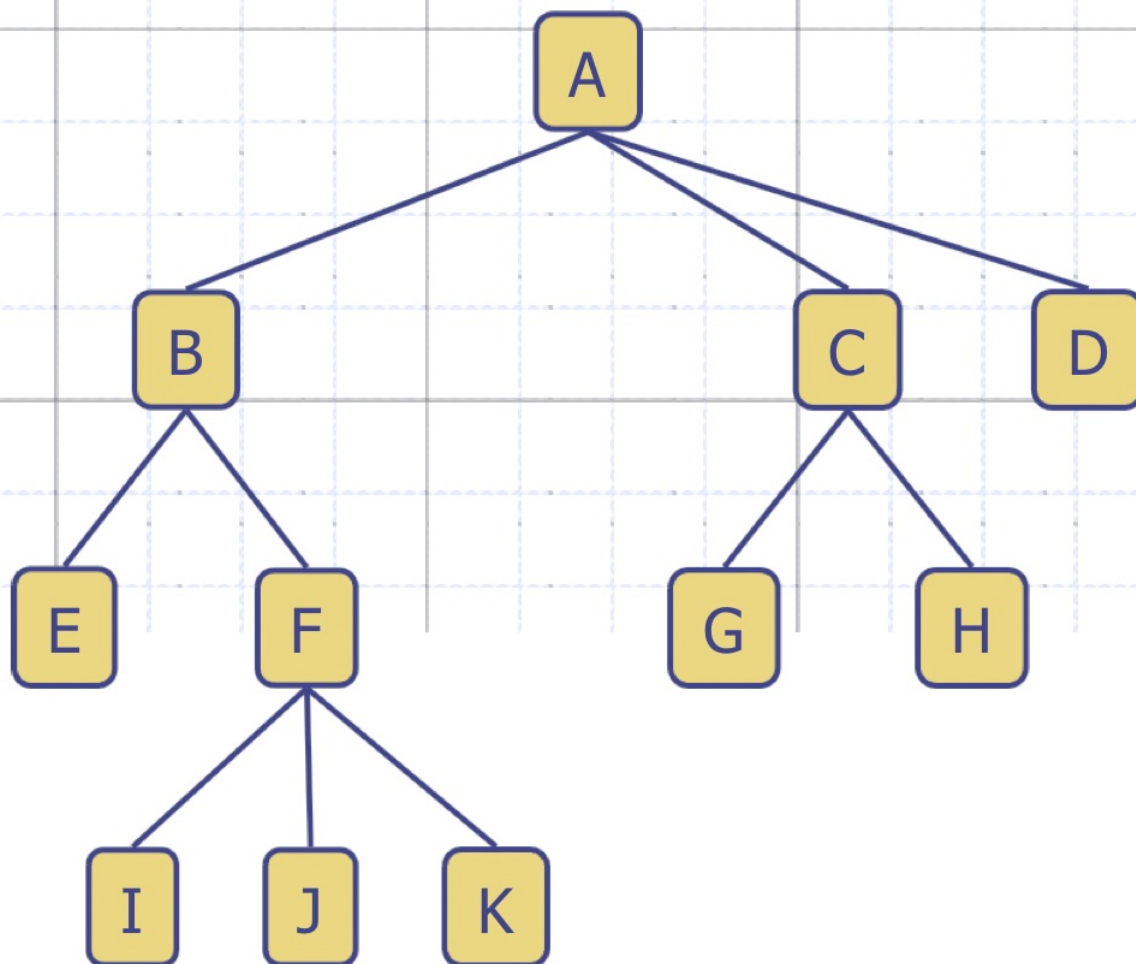
Number of nodes = Number of edges + 1



Tree Properties

Number of nodes = Number of edges + 1

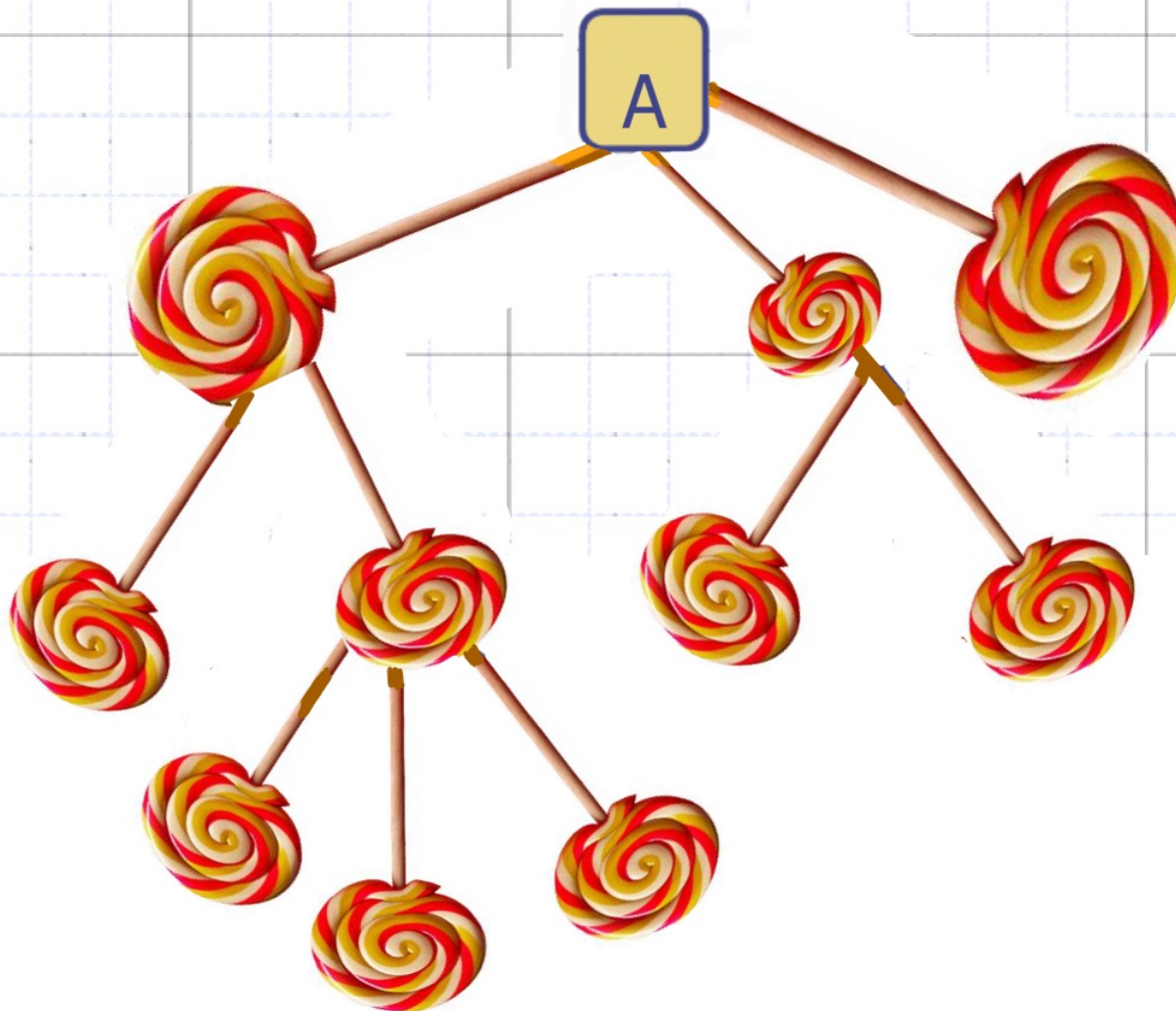
Proof. Glue every node to the edge connecting it to its parent. The root is not glued to any edges.



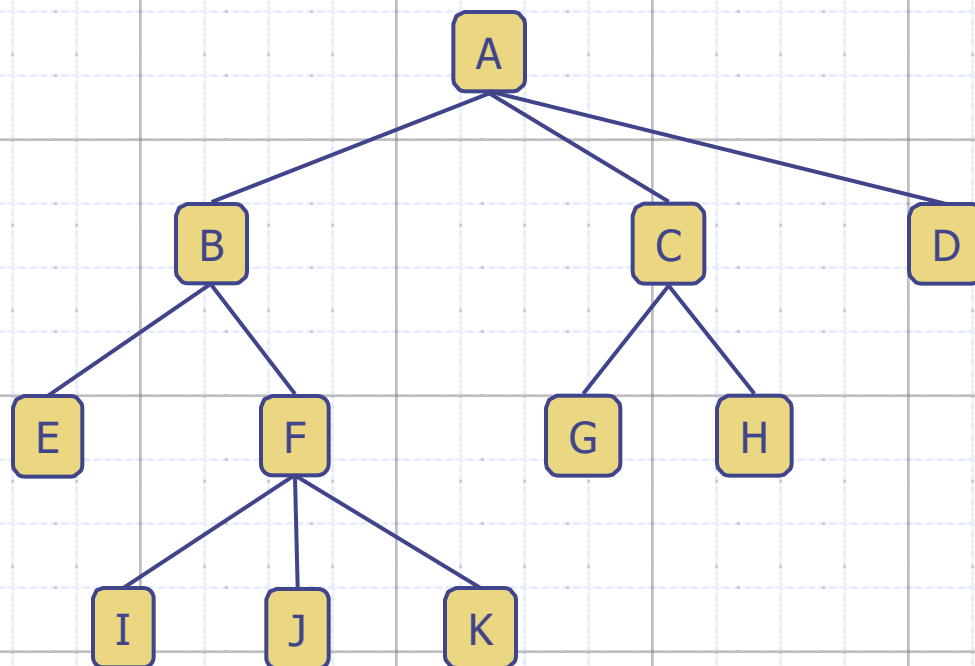
Tree Properties

Number of nodes = Number of edges + 1

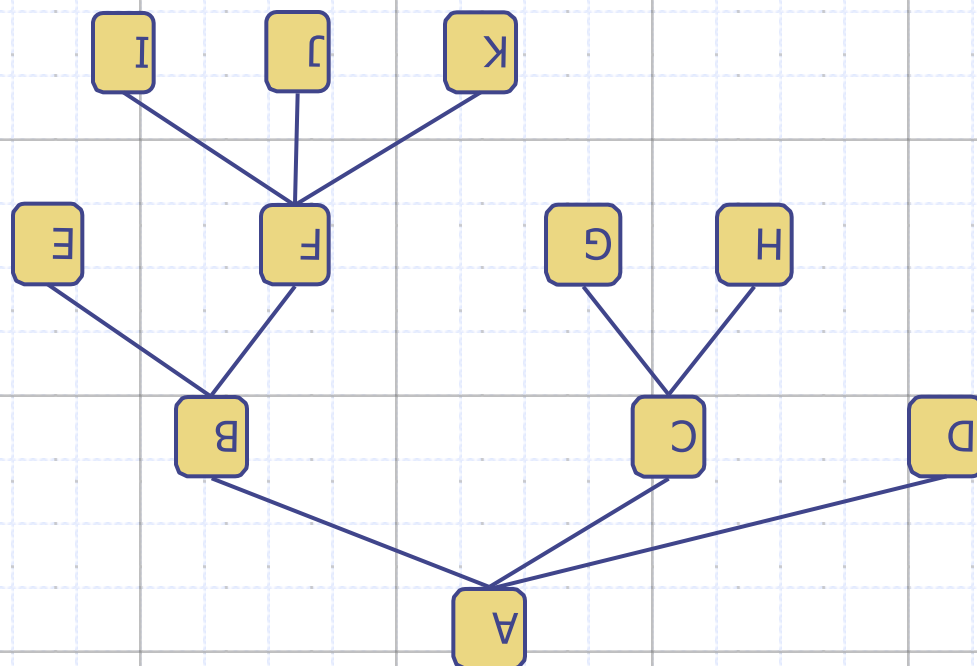
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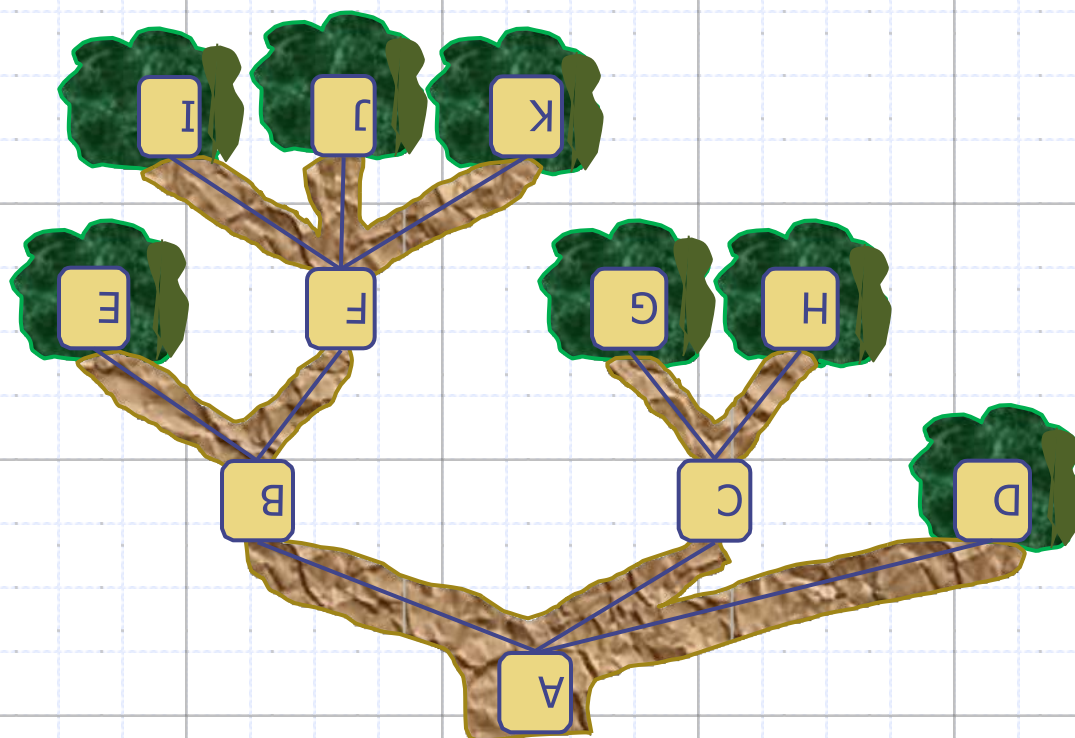
Why is this a Tree?



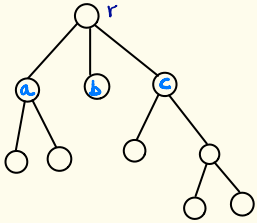
Why is this a Tree?



Why is this a Tree?



Algorithm to compute the height of a tree



Algorithm height(r)
In: root r of a tree
out: Height of the tree

