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1. (1 point)

**Midterm Practice Introduction**

This problem set is for practice and is **not** worth anything.

You can attempt each problem as many times as you like and you can view the answers to a question by selecting the CorrectAnswers checkbox.

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2. (1 point)

Consider the statements:

$P$ : I will go to the library today.

$Q$ : I will go to the gym today.

Which of the following statements is equivalent to the statement “I will go to either the library or the gym today, but I won’t go to both?”

- $(P \wedge Q) \vee \neg(P \wedge Q)$
- $(P \wedge Q) \vee (\neg P \wedge \neg Q)$
- $(P \vee Q) \wedge (\neg P \wedge \neg Q)$
- ✓  $(P \vee Q) \wedge \neg(P \wedge Q)$
- None of the above

Answer(s) submitted:

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(incorrect)

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3. (1 point)

Consider the statements:

$P$ : Billy gets the job.

$Q$ : Bob gets the job.

Which of the following statements is the **negation** of the statement “Either Billy or Bob will get the job, but not both”? (The “negation” of a statement  $R$  is  $\neg R$ .)

- $(P \wedge \neg Q) \vee (\neg P \wedge Q)$
- $(\neg P \wedge Q) \wedge (P \wedge \neg Q)$
- $(\neg P \vee Q) \wedge (P \vee \neg Q)$
- $(P \vee \neg Q) \vee (\neg P \vee Q)$

- None of the above

Answer(s) submitted:

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(incorrect)

4. (1 point)

Consider the statements:

$S$ : Steve will go to the concert.

$K$ : Kate will go to the concert.

Match the following propositions with the English sentences that they represent.

\_\_\_ 1.  $(S \wedge K) \vee (\neg S \wedge \neg K)$

\_\_\_ 2.  $(S \vee K) \wedge \neg S$

\_\_\_ 3.  $S \vee (\neg S \wedge \neg K)$

A. Either Steve or Kate will go to the concert, and Steve won't go.

B. Either both Steve and Kate will go to the concert, or both of them won't go.

C. Either Steve will go to the concert, or neither Steve nor Kate will go.

Answer(s) submitted:

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(incorrect)

5. (1 point)

Select all the sentences that are logically equivalent to

$$x \in S \rightarrow x \in T.$$

• A.  $x \in S \vee x \notin T$

• B.  $x \in S \wedge x \in T$

• C.  $x \notin T \rightarrow x \notin S$

• D.  $x \in T \vee x \notin S$

• E.  $x \in T \rightarrow x \in S$

Partial credit is given, except when no boxes are marked.

Answer(s) submitted:

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(incorrect)

Complete the following truth table by filling in the blanks with T or F as appropriate.

$\neg P$	$Q$	$(P \rightarrow Q) \leftrightarrow (Q \rightarrow P)$
$\neg T$	$T$	—
$\neg T$	$F$	—
$\neg F$	$T$	—
$\neg F$	$F$	—

Answer(s) submitted:

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(incorrect)

Complete the following truth table by filling in the blanks with T or F as appropriate.

$P$	$Q$	$P \rightarrow Q$	$Q \rightarrow P$	$\neg P$	$\neg Q$	$\neg P \rightarrow \neg Q$	$\neg Q \rightarrow \neg P$
T	T	—	—	—	—	—	—
T	F	—	—	—	—	—	—
F	T	—	—	—	—	—	—
F	F	—	—	—	—	—	—

Answer(s) submitted:

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(incorrect)

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8. (1 point)

Which of the following statements are equivalent to the statement  $P \wedge \neg Q$ ? Select all correct answers.

- A.  $\neg(\neg P \vee Q)$
- B.  $\neg Q \wedge (P \vee Q)$
- C.  $\neg(P \vee Q)$
- D.  $(P \vee \neg Q) \wedge (Q \vee \neg P)$

Partial credit is given, except when no boxes are marked.

Answer(s) submitted:

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(incorrect)

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9. (1 point)

Which of the statements below has the following truth table?

$\neg P$	$Q$	???
$\neg T$	T	T
$\neg T$	F	F
$\neg F$	T	T
$\neg F$	F	T

- $P \vee \neg Q$
- $Q \vee \neg(P \wedge Q)$
- $P \vee (\neg P \wedge \neg Q)$
- $(P \wedge Q) \vee \neg P$
- None of the above

Answer(s) submitted:

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(incorrect)

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10. (1 point)

Determine if each of the following statements is a tautology, a contradiction or neither:

$(P \wedge \neg R) \vee \neg(P \vee Q)$

- ?
- Tautology
- Contradiction
- Neither

$$(P \vee Q) \vee (\neg P \vee \neg Q)$$

- ?
- Tautology
- Contradiction
- Neither

Answer(s) submitted:

- 
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(incorrect)

### 11. (1 point)

Let  $C(x)$  be the statement "x has a cat", let  $D(x)$  be the statement "x has a dog" and let  $F(x)$  be the statement "x has a ferret". Let the universe of discourse consist of all students in your class. Match each logical expression to its equivalent English statement by putting the appropriate letter beside the logical expression.

- \_\_\_1.  $\exists x(C(x) \wedge D(x) \wedge F(x))$
- \_\_\_2.  $\exists x(C(x)) \wedge (\exists x D(x)) \wedge (\exists x F(x))$
- \_\_\_3.  $\forall x(C(x) \vee D(x) \vee F(x))$
- \_\_\_4.  $\neg \exists x(C(x) \wedge D(x) \wedge F(x))$
- \_\_\_5.  $\exists x(C(x) \wedge F(x) \wedge \neg D(x))$

- A. A student in your class has a cat, a dog, and a ferret.
- B. All students in your class have a cat, a dog, or a ferret.
- C. Some student in your class has a cat and a ferret but not a dog.
- D. No student in this class has a cat, a dog, and a ferret.
- E. For each of the three kinds of animals (cats, dogs, and ferrets), there is a student in your class who has one of these animals.

Answer(s) submitted:

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(incorrect)

### 12. (1 point)

Let  $F(x)$  denote the statement  $x$  is friendly and  $H(x)$  denote the statement  $x$  is happy.

Analyze the logical form of the following statement and choose the correct answer.

*Everyone is happy only if everyone is friendly.*

- ✓  $\forall x H(x) \rightarrow \forall x F(x)$   
 1)  $H \rightarrow F$   
 2) "Everyone"  $\rightarrow x$  is random, not the specific one,  
 NOT  
 •  $\forall x (F(x) \rightarrow H(x))$   
 •  $\forall x (H(x) \rightarrow F(x))$   
 •  $\forall x F(x) \rightarrow \forall x H(x)$   
 • None of the above

Answer(s) submitted:

(incorrect)

Select all the correct answers. There might be more than one right answer.

- A.  $\forall x\forall y(P(x,y) \wedge Q(x,y))$
- B.  $\exists x\exists y(\neg P(x,y) \vee \neg Q(x,y))$
- C.  $\exists x\exists y(Q(x,y) \rightarrow \neg P(x,y))$
- D.  $\neg\forall x\forall y\neg(P(x,y) \rightarrow \neg Q(x,y))$

Answer(s) submitted:

(incorrect)

Let  $P(x,y)$  stand for the statement “ $x$  is the parent of  $y$ ”. Here  $x$  and  $y$  each denote a human being.

- There exists a human who has no human parent.
- Every human is the parent of at least one human.
- There exists a human who is the parent of every human.
- It is not the case that every human is the parent of every human.
- None of the above

Answer(s) submitted:

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(incorrect)

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**15.** (1 point)

Select all the correct answers. There might be more than one right answer.

In which of these four universes is the following statement true.

$$\exists x (x \text{ is odd} \rightarrow x > 8)$$

- A. {19}
- B. {10, 20}
- C. {1, 3, 7}
- D. {8}
- E. None of the above

Answer(s) submitted:

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(incorrect)

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**16.** (1 point)

Let  $A$ ,  $B$ , and  $C$  be sets. Which of the following sets is always equal to  $A \setminus (B \setminus C)$  ?

- $(A \cap C) \setminus (B \cap C)$
- $(A \setminus B) \cap (A \setminus C)$
- $(A \setminus C) \cup (B \setminus C)$
- $(A \setminus B) \cup (A \cap C)$
- None of the above

Hint: For this problem it might be useful to draw Venn diagrams.

Answer(s) submitted:

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(incorrect)

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17. (1 point)

Evaluate the following sets:

$$\bigcup \{[-n, n] \mid n \in \mathbb{N}\} = \underline{\hspace{2cm}}$$

$$\{x^2 + 4 \mid x \in \mathbb{R}\} = \underline{\hspace{2cm}}$$

$$\{3n + 1 \mid n \in \{0, 1, 2, 3\}\} = \underline{\hspace{2cm}}$$

Enter your answers using **set notation** or **interval notation**.

Answer(s) submitted:

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(incorrect)

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Consider the family of sets  $\mathcal{F} = \{\{1, 2, 3, 4, 5\}, \{2, 3, 4, 5, 6\}, \{3, 4, 5, 6, 7\}, \{4, 5, 6, 7, 8\}\}$ . Evaluate the following sets.

$$\bigcup \mathcal{F} = \underline{\hspace{2cm}}$$

$$\bigcap \mathcal{F} = \underline{\hspace{2cm}}$$

Enter your answers using **set notation**.

Answer(s) submitted:

- 
- 

(incorrect)

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The power set of  $\{3, 7\}$  is  $\underline{\hspace{2cm}}$

Enter your answer using **set notation**.

Answer(s) submitted:

- 

(incorrect)

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20. (1 point) Which of the following statements are true? Select all correct answers.

- A.  $\emptyset \in \{\emptyset\}$
- B.  $\{0\} \subseteq \emptyset$
- C.  $\emptyset \subseteq \{\emptyset\}$
- D.  $0 \in \emptyset$



- E.  $\emptyset \subseteq \{0\}$

Answer(s) submitted:

•

(incorrect)

21. (1 point) Let  $A = \{1, 2\}$  and  $\mathcal{P}(A)$  be the power set of  $A$ .

Which of the following statements are true? Select all correct answers.

✓ A.  $\emptyset \in \mathcal{P}(A)$   $\mathcal{P}(A) = \{ \emptyset, \{1\}, \{2\}, \{1, 2\} \}$

• B.  $\emptyset \in A$   $\emptyset \in \mathcal{P}(A)$   $\emptyset \subseteq \mathcal{P}(A)$   
 $\emptyset \subseteq A$

• C.  $\{\emptyset\} \in \mathcal{P}(A)$

✓ D.  $\emptyset \subseteq A$

• E.  $A \subseteq \mathcal{P}(A)$   $A \in \mathcal{P}(A)$

Partial credit is given, except when no boxes are marked.

Answer(s) submitted:

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(incorrect)

22. (1 point) Which of the following sets are subsets of {integers divisible by 2}? Select all correct answers.

✓ A.  $\{-82, 104, 16\}$  integers:  $\mathbb{Z}, \mathbb{Z}^+, \mathbb{Z}^-, 0 \leq \mathbb{Z}$

• B. {prime numbers}

✓ C. {even natural numbers} natural numbers:  $\mathbb{N}$

• D.  $\{44, 221, 90\}$

• E.  $\{1\}$

Answer(s) submitted:

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(incorrect)

23. (1 point)

Let  $W = \{1, 5, 8, 9, 11\}$  and  $X = \{0, 3, 5, 8, 9\}$ .

Determine the following sets. Express your answers using set notation.

$W \setminus X =$  \_\_\_\_\_

$X \setminus W =$  \_\_\_\_\_

Answer(s) submitted:

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(incorrect)

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**24.** (1 point)

Let  $A = \{1, 2, 3, 5, 10\}$  and  $B = \{1, 5, 8, 10\}$ .

Determine the following sets. Express your answers using **set notation**.

$A \cap B =$  \_\_\_\_\_

$A \cup B =$  \_\_\_\_\_

Answer(s) submitted:

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- 

(incorrect)

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**25.** (1 point)

Let  $A = \{7, 10, 4, 8, 11\}$  and  $B = \{x \mid x \text{ is an even positive integer less than } 10\}$ .

Determine the following sets. Express your answers using **set notation**.

$A \cap B =$  \_\_\_\_\_

$A \cup B =$  \_\_\_\_\_

Answer(s) submitted:

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(incorrect)

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**26.** (1 point) Consider the following statement:

$$a - b \notin \{c \in \mathbb{R} \mid c \text{ is odd}\}.$$

Determine whether each variable is free or bound.

$a$  is a:

- A. Free variable
- B. Bound variable

$b$  is a:

- A. Free variable
- B. Bound variable

$c$  is a:

- A. Free variable
- B. Bound variable

Answer(s) submitted:

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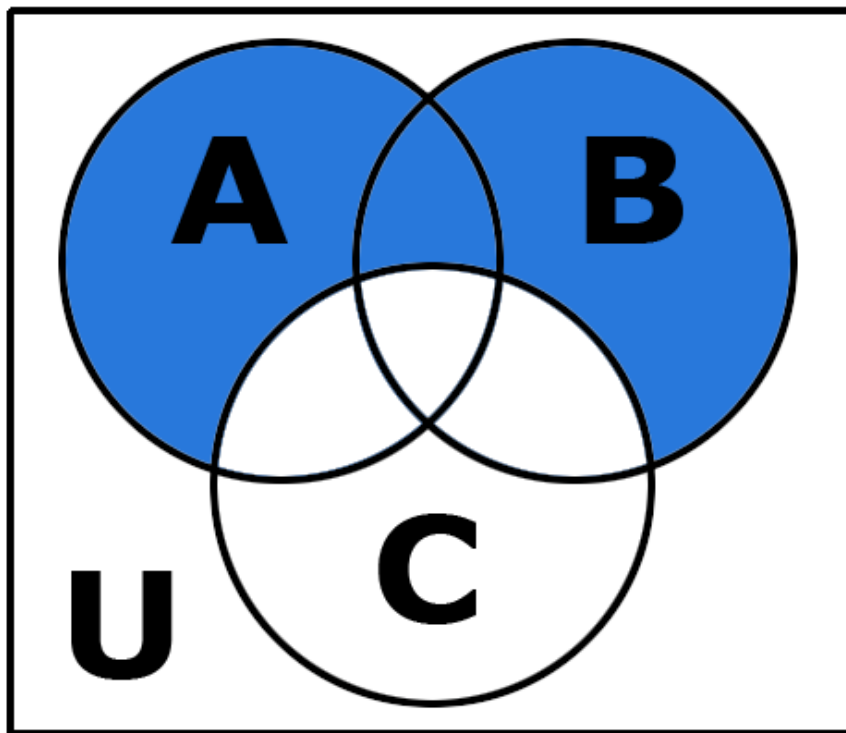
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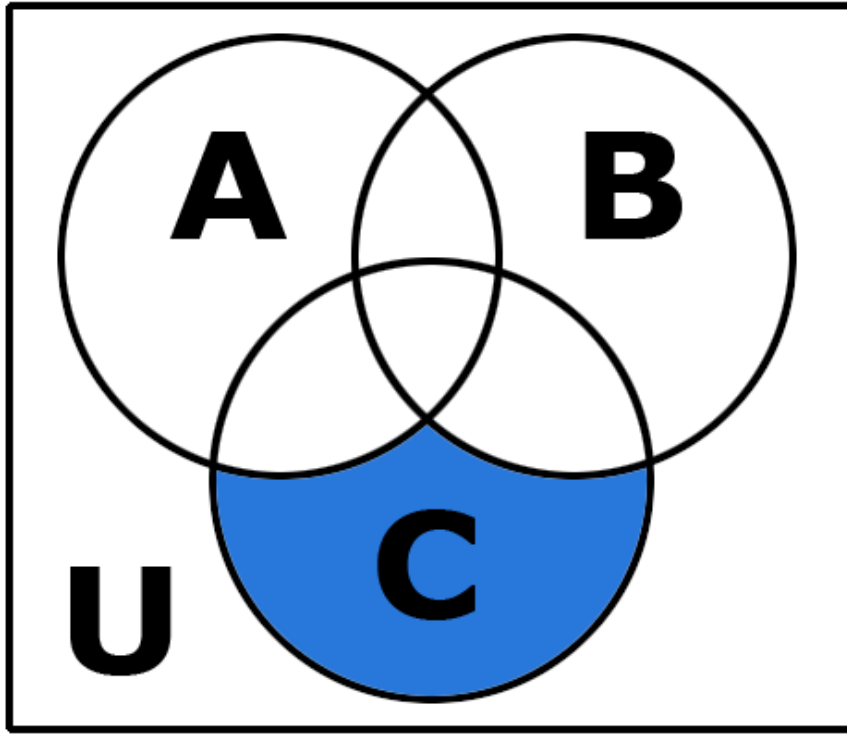
27. (1 point)

Let  $A$ ,  $B$ , and  $C$  be sets in a universe  $U$ .

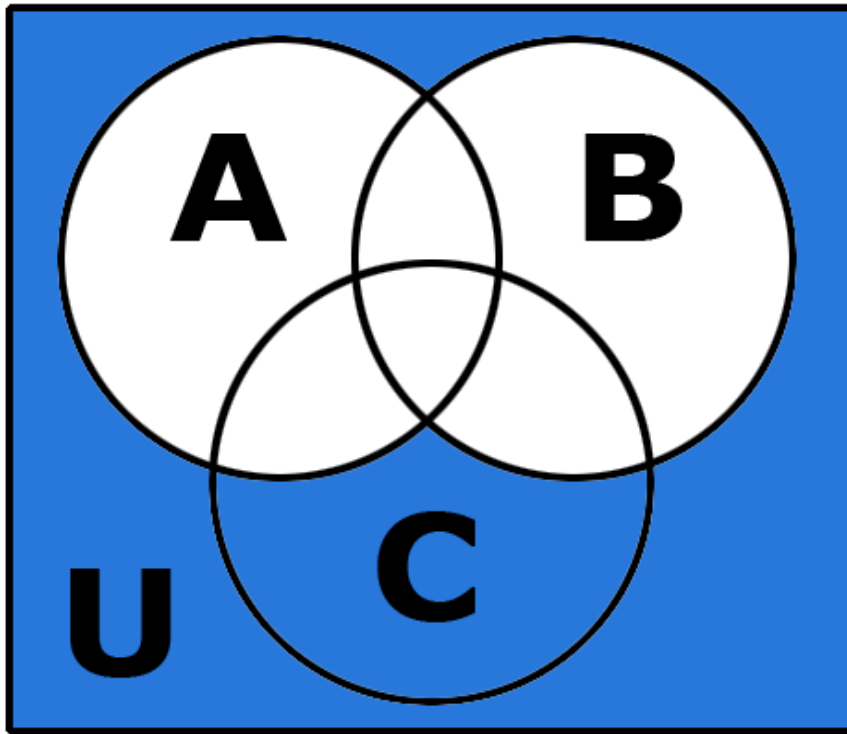
Which of the following Venn diagrams corresponds to  $U \setminus (A \cup B)$  ?



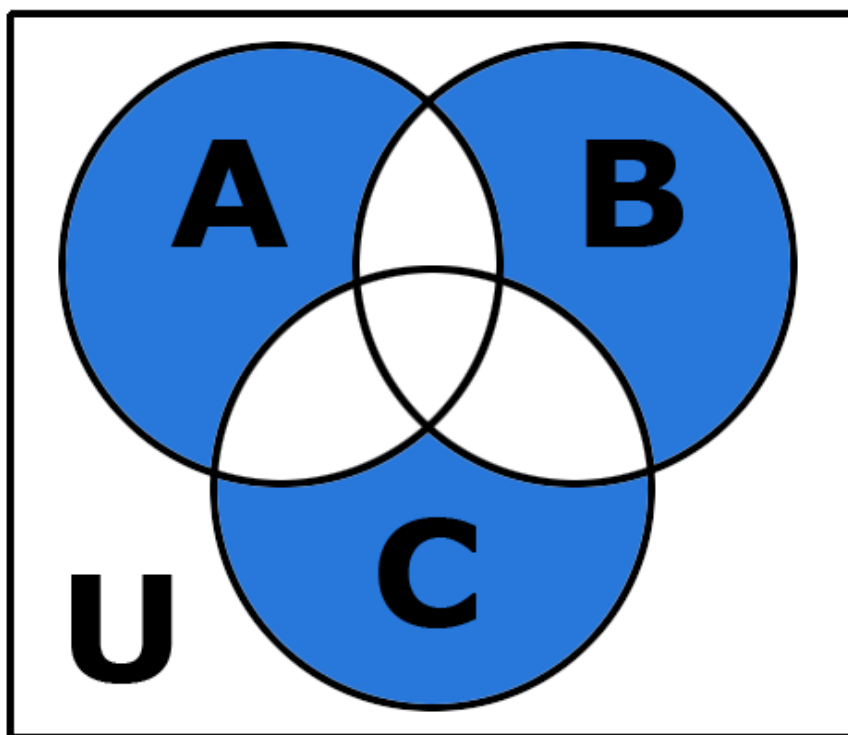
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*Answer(s) submitted:*

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(incorrect)

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