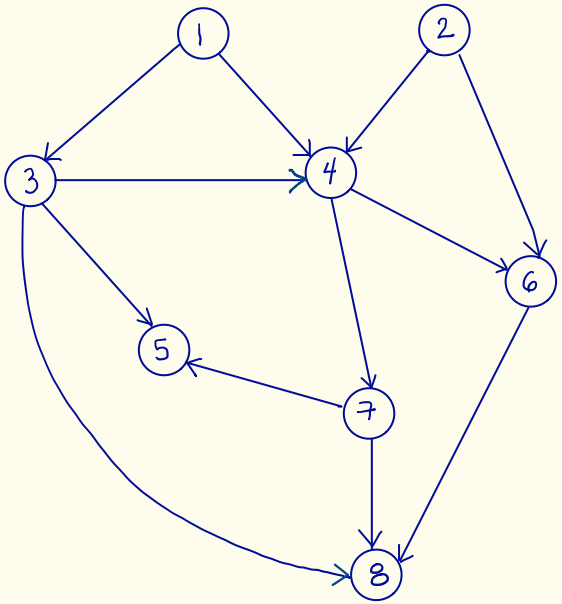
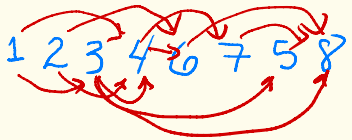
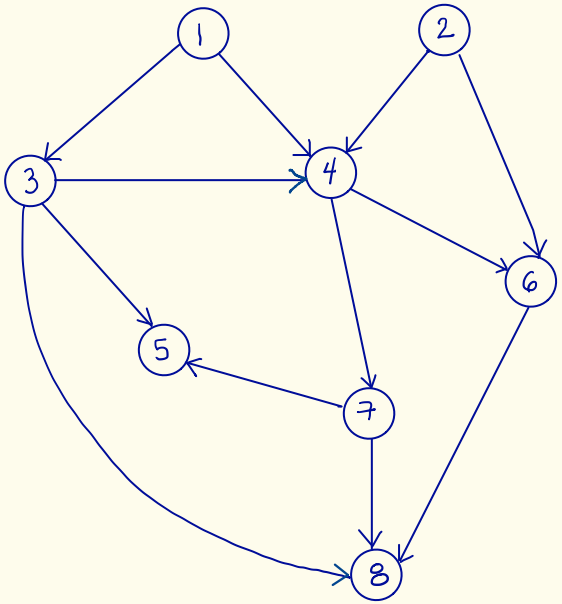


Digraph

Topological ordering

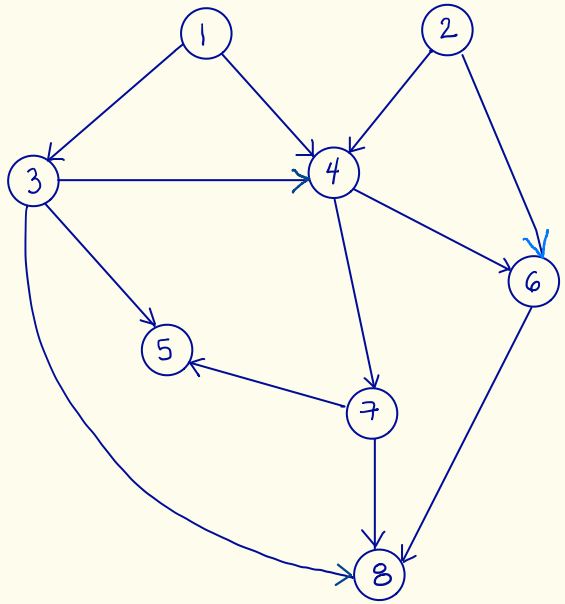




Digraph

Topological ordering?

1 2 3 4 5 6 7 8



Digraph

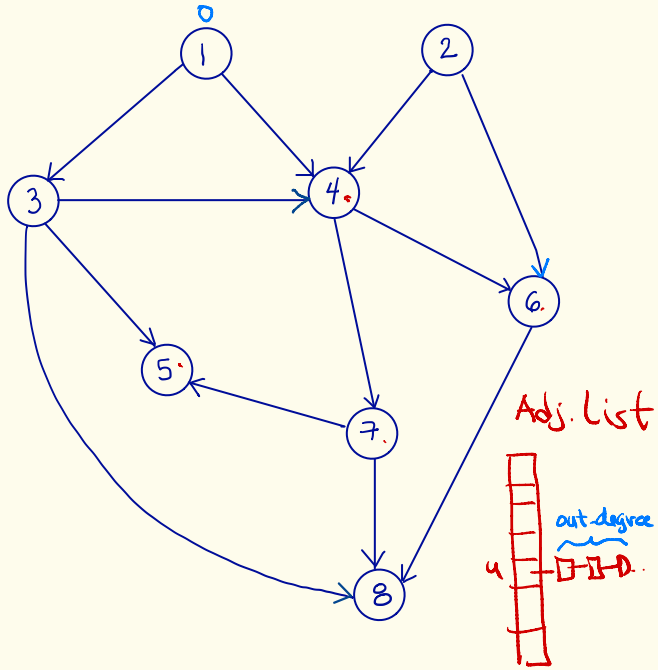
Topological orderings

1 2 3 4 6 7 5 8

2 1 3 4 6 7 5 8

1 2 3 4 7 5 6 8

DAG
Directed
Acyclic
Graph



Adj. List

Algorithm TopologicalOrdering(G)
In: Directed graph G=(V,E)
Out: Topological ordering for the vertices of G

Q ← empty queue } c_2
For each node u of G do
  if u.in-degree = 0 then } c_1 } $c_1 n$
    Q.enqueue(u)

#iter = n
While Q is not empty do {
  c_4 { u ← Q.dequeue
  print(u) } $c_4 + c_3 \times$ out-deg(u)
  $c_3 \times$ out-deg(u) { For each edge (u,v) do {
    c_3 { v.in-degree ← v.in-degree − 1
    if v.in-degree = 0 then
      Q.enqueue(v) }
  }
}

$$c_2 + c_1 n + \sum_{u \in V} (c_4 + c_3 \times \text{out-degree}(u))$$

$$= c_2 + c_1 n + \sum_{u \in V} c_4 + c_3 \sum_{u \in V} \text{out-degree}(u)$$

$$= c_2 + c_1 n + c_4 n + c_3 m \text{ is } O(n+m)$$