Questions ?

Q. Find Domain of $\ln\left(\frac{5-x^2}{1+x}\right)$.

Ans: where is ln defined? when $x > 0$

So, we must have

$$\frac{5-x^2}{1+x} > 0$$

or

Both numerator and denominator > 0	Both numerator and denominator < 0
$5-x^2 > 0$ — ① and $1+x > 0$ — ②	$5-x^2 < 0$ and $1+x < 0$
① $5-x^2 > 0$ $\Rightarrow 5 > x^2$ $\Rightarrow \boxed{\sqrt{5} > x > -\sqrt{5}}$	③ $5-x^2 < 0$ $\Rightarrow 5 < x^2$ $\Rightarrow \boxed{x < -\sqrt{5} \text{ or } x > \sqrt{5}}$
② $1+x > 0$ $\Rightarrow \boxed{x > -1}$	④ $1+x < 0$ $\Rightarrow \boxed{x < -1}$
Combining ① and ② $\boxed{\sqrt{5} > x > -1}$	

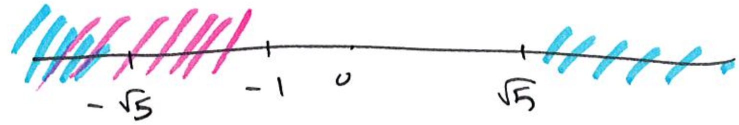
~~①. find domain of $\ln\left(\frac{5-x^2}{1-x}\right)$.~~

Note: $-1 > -\sqrt{5}$

if we want both $x > -1$ and $x > -\sqrt{5}$ we must have $x > -1$

③ $x < -\sqrt{5}$ or $x > \sqrt{5}$ and ④ $x < -1$

$\Rightarrow \boxed{x < -\sqrt{5}}$



Combining everything,

$\boxed{\sqrt{5} > x > -1}$ or $x < -\sqrt{5}$

$\Rightarrow$ Domain : $(-\infty, -\sqrt{5}) \cup (-1, \sqrt{5})$

- For domain: ① Start from the outermost function

  ② ~~Inequalities~~ ~~behav~~ be careful when simplifying inequalities.

Q. Find domain of arcsin $(1+x)$.

A: For what x does this make sense?

arcsin has domain : $[-1, 1]$

$$\Rightarrow \quad -1 \leq 1+x \leq 1$$

$$\Rightarrow \quad -1-1 \leq x \leq 1-1$$

[Subtract 1 from all 3 sides

$\Rightarrow \quad -2 \leq x \leq 0$

$\Rightarrow$ Domain of $\arcsin(1+x)$ is $[-2, 0]$.

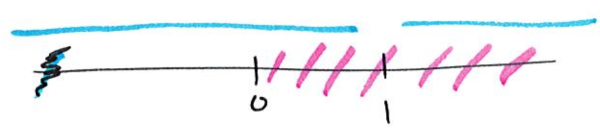
- $\boxed{\text{For composition of functions, start from the outermost function's domain}}$

Q. Find domain of $\cancel{ar}(\ln x) \cdot \dfrac{1}{1-x}$.

Ans. Product $\Rightarrow$ intersect of both domains

. Both $\underbrace{\ln x}$ and $\underbrace{\dfrac{1}{1-x}}$ should be defined

$x > 0$ and $x \neq 1$



$\Rightarrow$ ~~0 < x < 1 and~~ $x \in (0,1) \cup (1, \infty)$

Domain of $\dfrac{\ln x}{1-x}$ is $(0,1) \cup (1,\infty)$

- Inverse of a function $\neq$ reciprocal of the function

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

- Inverse of a function $f(x)$ is defined only if f is $\underbrace{\text{one-to-one}}$.

↓

no two "x-values" give the same "y-value"

- $\left.\begin{array}{l} f(x)=3 \\ f(x)=x^2 \\ f(x)=x^2+1 \end{array}\right\}$ not one-to-one

↓

these functions donot have inverses.

$f(x)=e^x$ is one-to-one

Q. A.10

2015

$$\lim_{x \to 0^+} \frac{\sin(4x)}{\sqrt{x} \cdot \sin(2\sqrt{x})}$$

we want to use

$$\lim_{h \to 0} \frac{\sin h}{h} = 1 \quad -(*)$$

① Plug in : $\frac{0}{0}$ ⇝ need to do something

Squeeze ✗   identity (*)

$$\lim_{x \to 0^+} \frac{\sin(4x)}{\sqrt{x} \cdot \sin(2\sqrt{x})}$$

Multiply by $\frac{4x}{4x}$

$$= \lim_{x \to 0^+} \frac{\sin(4x)}{\sqrt{x} \cdot \sin(2\sqrt{x})} \cdot \frac{4x}{4x}$$

$$= \underbrace{\lim_{x \to 0^+} \frac{\sin(4x)}{4x}} \cdot \lim_{x \to 0^+} \frac{4x}{\sqrt{x} \cdot \sin(2\sqrt{x})}$$

$$= 1 \cdot \lim_{x \to 0^+} \frac{4\sqrt{x}}{\sin 2\sqrt{x}}$$

$\left(\text{as } \frac{x}{\sqrt{x}} = \sqrt{x} \right)$

$$= 1 \cdot \underbrace{\lim_{x \to 0^+} \frac{2\sqrt{x}}{\sin(2\sqrt{x})}} \cdot 2$$

$$= 1 \cdot \lim_{x \to 0^+} \frac{1}{\left(\frac{\sin 2\sqrt{x}}{2\sqrt{x}} \right)} \cdot 2$$

$$= 1 \cdot \frac{1}{1} \cdot 2$$

$$= 2$$

Q. $\lim_{x \to 0} \dfrac{|5x-1| + |5x+1|}{x}$

• Recall :

(1) $|x|$ is a piecewise function

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

(2)

Ans: Plug in

$$\frac{|5.0-1| + |5.0+1|}{0}$$

$$= \frac{|-1| + |1|}{0}$$

$$= \frac{2}{0}$$

$$= \infty \text{ or } -\infty \text{ or d.n.e.}$$

Plug in nearby values and check signs

$$\frac{|5x-1| + |5x+1|}{x}$$

(1) $x < 0$ numerator > 0

denominator < 0

(2) $x > 0$ numerator > 0

denominator > 0

$\Downarrow$

$\lim_{x \to 0} \dfrac{|5x-1| + |5x+1|}{x}$ d.n.e.

Q. $\lim_{x \to 0} \frac{|5x-1| - |5x+1|}{x}$

Ans: Plug in $x=0$ $\frac{|-1| - |1|}{0}$

$= \frac{1-1}{0}$

$= \frac{0}{0}$ ⇝ need to do something

(plug in $x=0$)

near $x=0$: $5x-1 < 0$

$\Rightarrow |5x-1| = -(5x-1)$

(plug in $x=0$)

near $x=0$: $5x+1 > 0$

$\Rightarrow |5x+1| = 5x+1$

Recall:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\Rightarrow \lim_{x \to 0} \frac{|5x-1| - |5x+1|}{x} = \lim_{x \to 0} \frac{-(5x-1) - (5x+1)}{x}$$

$$= \lim_{x \to 0} \frac{-5x + \cancel{1} - 5x - \cancel{1}}{x}$$

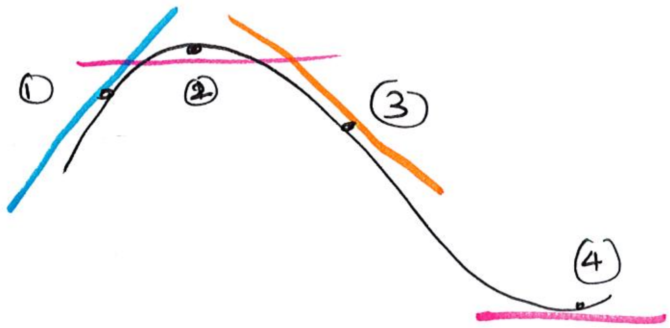
$$= \lim_{x \to 0} \frac{-10\cancel{x}}{\cancel{x}}$$

$$\boxed{= -10}$$

Ans

Ch.4 - applications of derivatives

- $f'(a)$ = ~~rat~~ slope of ~~gra~~ tangent to graph of $f(x)$ at $x = a$



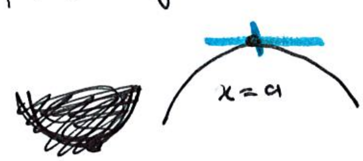
① $f(x)$ increasing $\Rightarrow$ $f'(x) > 0$

④, ② $f(x)$ is min/max $\Rightarrow$ $f'(x) = 0$

③ $f(x)$ is decreasing $\Rightarrow$ $f'(x) < 0$

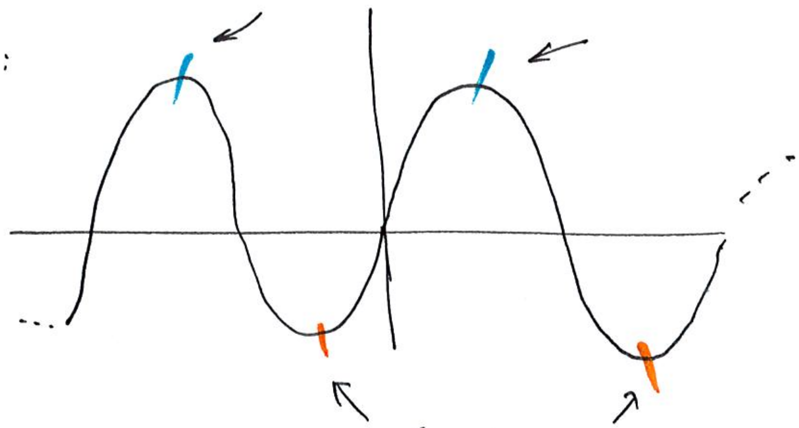
Def: Local maxima / minima:

① $x = a$ is a local maxima of $f(x)$ if near $x = a$, $f(x) \leq f(a)$.



② $x = b$ is a local minima of $f(x)$ if near $x = b$, $f(x) \geq f(b)$.

eg:

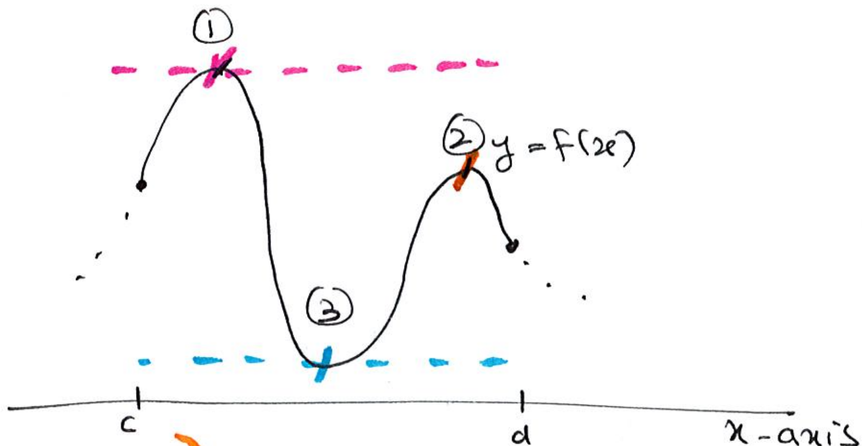


Def: ~~A fun~~ A function $f(x)$ has an <u>absolute maxima</u> at $x = a$, on an <u>interval</u> $[c, d]$ if for all x in $[c, d]$,

$$f(a) \geq f(x).$$

Similarly; absolute minima.

eg:

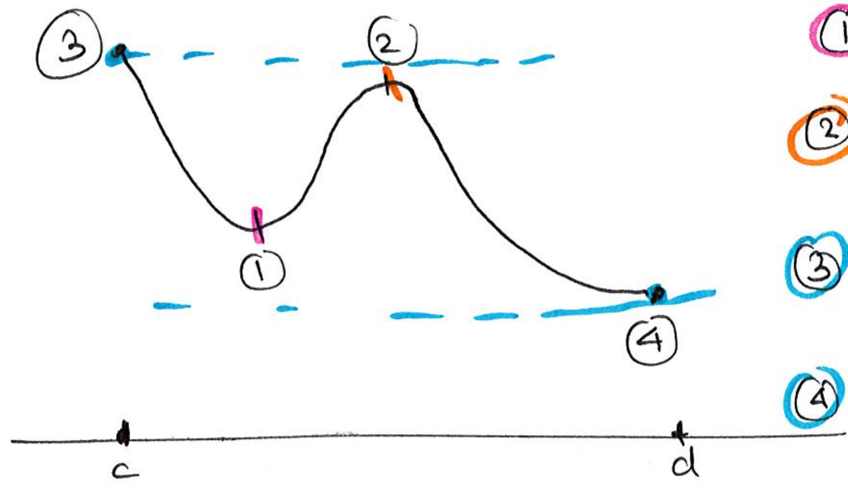


①, ② local maxima

③ local minima

① = absolute max ③ absolute minima

eg:



1 = local minima

2 = local max

3 = absolute max

4 = absolute min

Q. Find absolute maxima and minima of

$$f(x) = x - \sin x$$

on $[-\pi, \pi]$.

Ans: Find possible candidates for absolute max/min

1. $f'(x) = 0$
2. end points

1. $f'(x) = (x - \sin x)'$

$= 1 - \cos x$

$f'(x) = 0$

$1 - \cos x = 0$

$\Rightarrow \cos x = 1$

$\Rightarrow \boxed{x = 0}$

- $\boxed{x = -\pi}$ left end point
- $\boxed{x = \pi}$ right end point

Plug in and check:

$\boxed{f(0) = 0 - \sin 0 = 0}$

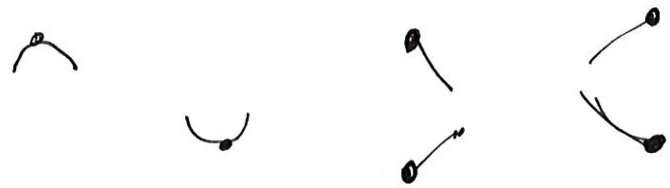
$\boxed{f(-\pi) = -\pi - \sin(-\pi) = -\pi}$ ← smallest

$\boxed{f(\pi) = \pi - \sin(\pi) = \pi}$ ← largest

$f(\pi)$ is the largest $\qquad$ $f(-\pi)$ is the smallest

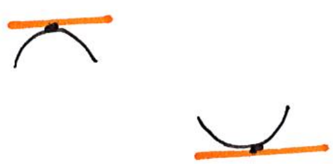
- $x = \pi$ is absolute max. $\qquad$ • $x = -\pi$ is the absolute min.

- for absolute min/max: only need to check at local min/max, endpoints



- Def^n: • if $f'(a) = 0$ then $x = a$ is called a critical point of $f(x)$.

  • local max / min are critical points.



$f'(x) = 0$

For local max/min: $\boxed{\text{Second Derivative test}}$

① Find x for which $f'(x) = 0$

i.e. find critical points

② Find $f''(x)$ at the critical point

- $f''(x) > 0 \Rightarrow$ local min
- $f''(x) < 0 \Rightarrow$ local max

Local min/max

- (Second~~er~~ derivative test)

1) Critical points

2) Second derivaties

Absolute min/max

(~~crit~~

1) critical points

2) end points

3) Compare their "y-values"

Q. Find local min/max of

$$f(x) = x - \sin x$$

on $[-\pi, \pi)$.

Ans ① Critical points

$$f'(x) = 1 - \cos x = 0$$

$$\Rightarrow \cos x = 1$$

$$\Rightarrow \boxed{x = 0} \quad (\text{in } [-\pi, \pi])$$

② Second derivative

$$f''(x) = (1 - \cos x)'$$

$$= \sin x$$

at $x = 0$, $f''(0) = 0$

$\Rightarrow$ cannot say anything

$\Rightarrow$ no local min/max

Q Find absolute min/max

$$f(x) = x - \sin x$$

on $[-\pi, \pi]$

A. we did this

abs max $x = \pi$

abs min $x = -\pi$