Review Session

COMPSCI 3331

Fall 2022

Question 1

1. Let G be the CFG defined by the following set of productions.

$$\begin{array}{lcl} S & \rightarrow & bbAaA \mid SSaa \mid aa \mid ABC \\ A & \rightarrow & Ab \mid Ac \mid CC \\ B & \rightarrow & BA \mid bb \mid Dd \\ C & \rightarrow & DA \mid \varepsilon \\ D & \rightarrow & a \end{array}$$

Give an equivalent grammar to G that has no ε-productions.

Question 2

2. Let G be the CFG defined by the following set of productions.

$$
\begin{aligned}
S &\rightarrow SbaB \mid aa \mid ABC \\
A &\rightarrow Ba \mid DaDd \\
B &\rightarrow BA \mid ca \mid Dd \\
C &\rightarrow DA \mid \varepsilon \\
D &\rightarrow a
\end{aligned}
$$

Convert the grammar to CNF.

Question 3

3. Let $G = (V, \Sigma, P, S)$ be a CFG in CNF. Give an $O(n^3)$ algorithm for the following problem:

- Input: A word w and a nonterminal $A \in V$.
- Output: the value
  $n_A = \max\{|u| \ : \ u \text{ is a suffix of } w \text{ and } A \Rightarrow^* u\}$.

That is, n_A is the length of the longest suffix of w that is generated by A in the grammar.

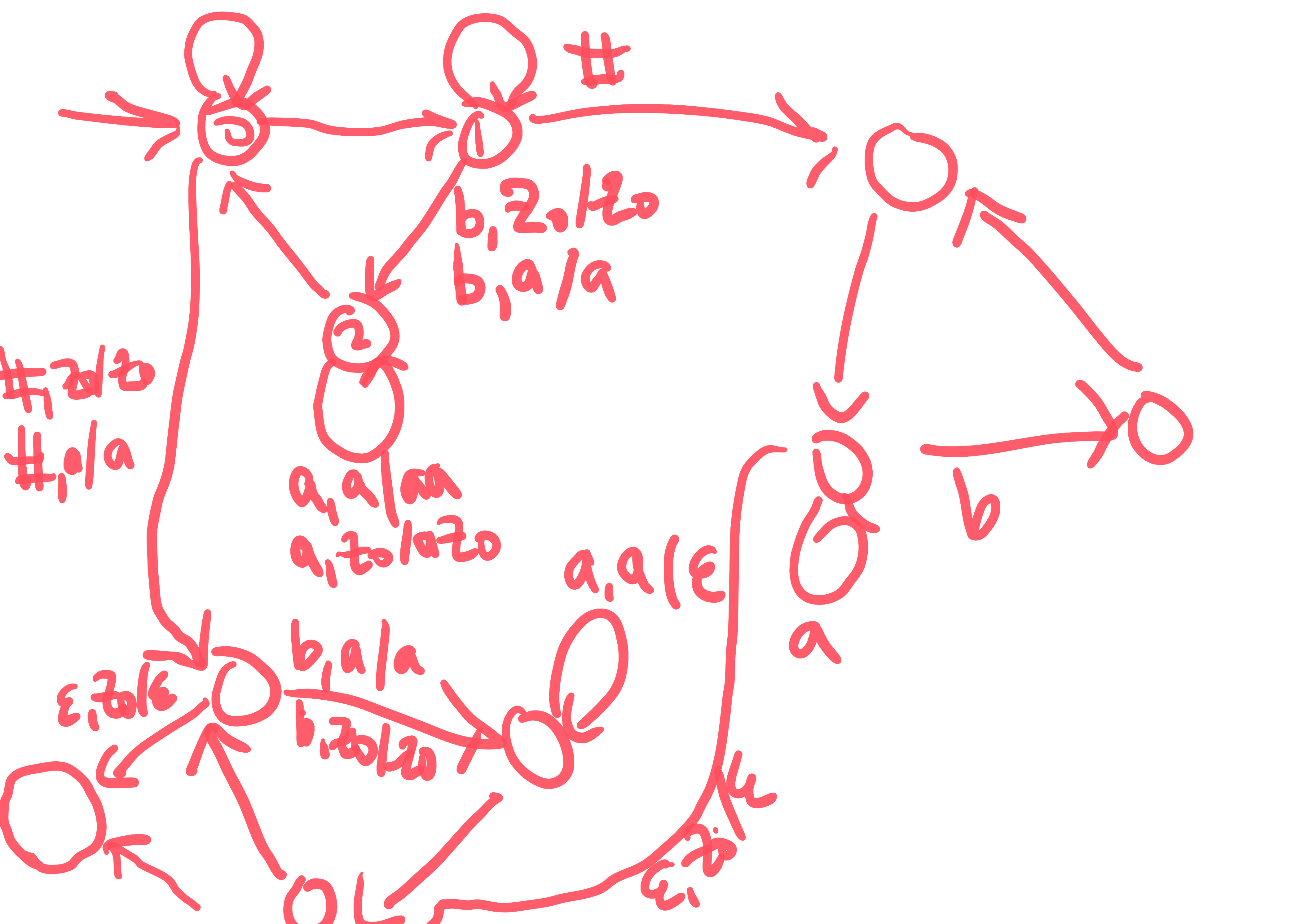
Question 4

4. Construct PDAs for the following languages:

(a) $L = \{a^n b^m xy \ : \ x, y \in \{0,1,2\}, x \equiv n (\mathrm{mod} 3) \text{ and } y \equiv m (\mathrm{mod} 3)\}$.

(b) $L = \{w\#x \ : \ w, x \in \{a,b\}^*, |w|_a = |x|_a, |w|_b \equiv |x|_b (\mathrm{mod} 3)\}$.

Be sure to indicate what the starting stack symbol is for your PDA and how your PDA accepts words.



0
1
2
#
b, Z0/Z0
b, a/a
#, Z0/Z0
#, a/a
a, a/aa
a, Z0/aZ0
a, a/ε
b, a/a
b, Z0/Z0
ε, Z0/ε
a
b

Question 5

5. Let C be a fixed integer. Extend the language from Assignment 3 as follows:

$$L_C = \{x\#1^n \ : n \geq 0, x \in \{a,b\}^* \text{ and } \ n - C \leq |x|_a \leq n + C\}$$

Give a context-free grammar for L_C. The productions in your grammar will depend on the value of C. Describe them using a uniform notation (e.g., by using consistently named variables or consistently defined productions, for instance).

Question 6

6. Consider the following modified language from Assignment 3:

$$L = \{u\#v \ : \ u, v \in \{0,1\}^* \text{ and } \mathrm{bin}(v^R) = \mathrm{bin}(u) + 2\}$$

Give a PDA that accepts L.

Question 7

7. A CFG G is in Griebach Normal Form (GNF) if every production has the form

$$A \rightarrow aB_1 B_2 \cdots B_n$$

for some letter a and nonterminals $B_1, B_2, \cdots, B_n$ (where $n \geq 0$). Any grammar (that does not derive ε) can be converted to GNF. Given this fact, show that for any CFG L that does not include ε, you can construct a PDA M that accepts L in the following additional conditions:

- The PDA accepts by empty stack.
- The PDA M does not have any ε-transitions. That is, there are no rules of the form $\delta(q, \varepsilon, \gamma) = \{(q', \beta), \dots\}$ for any stack symbol γ.

$A \to \alpha$

$\varepsilon, A/\alpha$

$a, a/\varepsilon$

Question 8

8. Prove that the following languages are not context-free:

(a) $L = \{a^p \ : \ p \text{ is a prime number}\}$.

(b) $L = \{a^n b^{n^3} \ : \ n \geq 0\}$.

(c) $L = \{w \in \{a, b\}^* \ : \ |w|_b = 2^{|w|_a}\}$.

$R = a^* b^*$

$L \cap R = \{ a^n b^{2^n} : n \geq 0 \}$. $z = a^n b^{2^n}$

1. v, x cross boundary.
2. v, x all a's
3. v, x all b's
4. v is a's and x is b's.

$z = uvwxy$

$u = a^i$

$v = a^j$

$w = a^{n-i-j} b^k$

$x = b^l$

$y = b^{2^n - l - k}$

EXAMINE TWO SUBCASES.

IF $j = 0 \Rightarrow l \neq 0$.

$uv^2wx^2y = a^n b^{2^n + l}$

NOT IN LANGUAGE.

IF $j \neq 0$.

$uv^2wx^2y = a^{n+j} b^{2^n + l}$.

WANT TO SHOW $2^{\#as} \neq \#bs$

$$2^{(n+j)} \neq 2^n + \ell$$

$$\ell \le n \Rightarrow \ell < 2^n \qquad 2^n + \ell < 2^n + 2^n = 2^{n+1}$$

Question 9

9. For each of the languages in the previous question, give an informal description of a multi-tape TM that recognizes the language.

(a) $L = \{a^p : p \text{ is a prime number}\}$.

(b) $L = \{a^n b^{n^3} : n \geq 0\}$.

(c) $L = \{w \in \{a,b\}^* : |w|_b = 2^{|w|_a}\}$.

Question 10

10. Show that the following language is r.e.:

$$L = \{e(M_1)\#e(M_2) \ : \ L(M_1) \cap L(M_2) \neq \emptyset\}$$

M_U - universal TM.

$w, e(M) \longrightarrow w \in L(M)?$

LET M_0 accept L. NONDETERM.

① NONDETERM WRITE A WORD w ON TWO TAPES.

② RUN M_1 AND M_2 IN PARALLEL

ON SEPERATE COPIES OF w.
(USING M_i)

③ IF BOTH SAY yes → return yes

IF EITHER SAY no → return no

IF EITHER GO INTO INF. LOOP
⇒ ALSO GO INF. LOOP.

Question 11

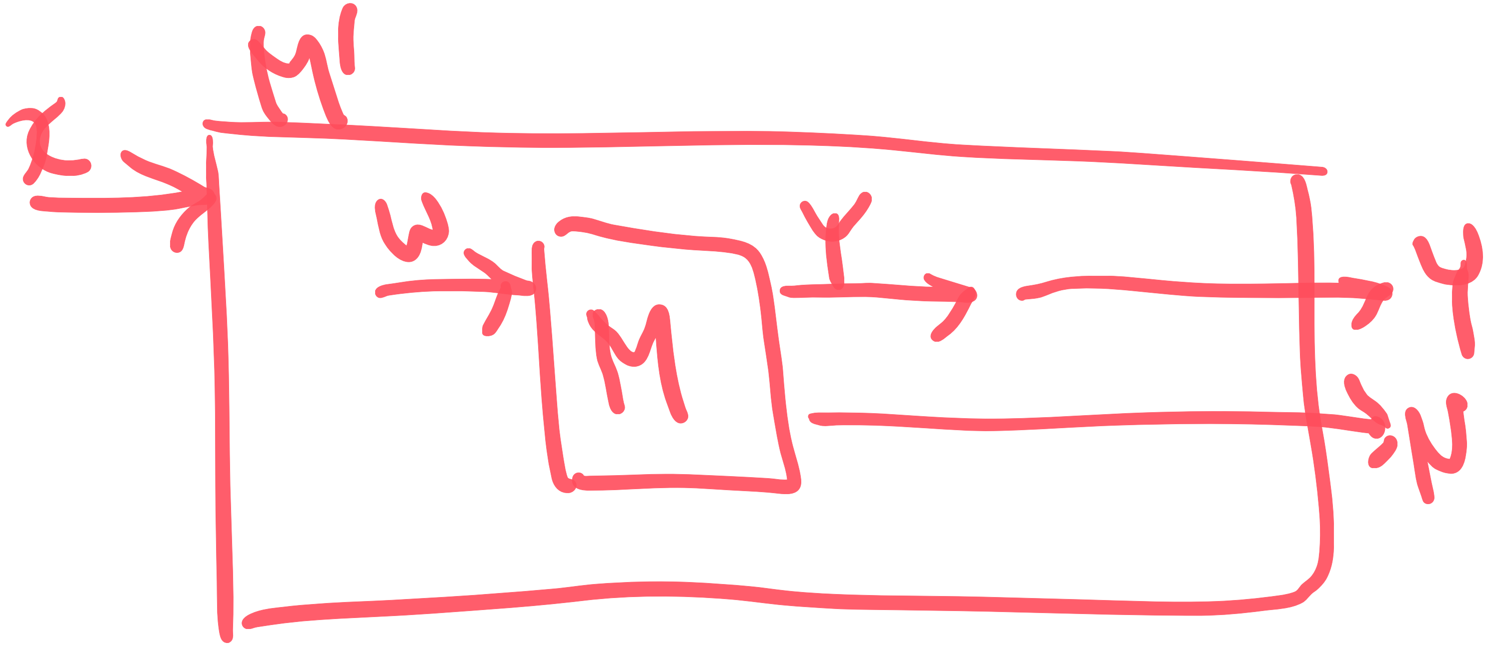
11. Show that the following problem is undecidable by reduction: Given a TM M, is $L(M)$ a finite language?

- CHOOSE HALTING PROBLEM.

$(e(M), \underline{w}) \longrightarrow M'$

$w \in L(M) \Rightarrow L(M')$ finite

$w \notin L(M) \Rightarrow L(M')$ infinite.



$$w \in L(M) \Rightarrow L(M') = \Sigma^*$$

$$w \notin L(M) \Rightarrow L(M') = \phi.$$

Question 12

12. Show that the following language is decidable:
$L_{ND} = \{e(M) \ : \ M \text{ is a nondeterministic TM } \}$.

Question 13

13. Show that the following problem is either decidable or undecidable: Given a CFG G is $L(G)$ infinite? (Hint: review the proof of the pumping lemma for CFLs.)