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1. i) the left hand side:

$(p \wedge q) \to r \equiv \neg(p \wedge q) \vee r \equiv \neg p \vee \neg q \vee r.$

the right hand side:

$$\begin{aligned}(p \wedge \neg r) \to \neg q &\equiv \neg(p \wedge \neg r) \vee \neg q \\ &\equiv \neg p \vee r \vee \neg q \\ &\equiv \neg p \vee \neg q \vee r \equiv \text{left hand side.}\end{aligned}$$

ii) $$\begin{aligned}(p \to q) \vee (q \to p) &\equiv (\neg p \vee q) \vee (\neg q \vee p) \\ &\equiv \neg p \vee q \vee \neg q \vee p \\ &\equiv (\neg p \vee p) \vee (\neg q \vee q) \\ &\equiv T \vee T \equiv T.\end{aligned}$$

2. 1) Valid.

$\neg p \equiv T$ $\qquad \dfrac{p \vee q \quad \neg p}{q} \qquad \dfrac{p \vee r \quad \neg p}{r}.$

$p \equiv F$

Disjunctive Syllgism

$\therefore q \wedge r \equiv T \wedge T \equiv T.$

2) Invalid.

$\neg q \equiv T. \quad p \equiv T \quad p \to q \equiv T.$

$q \equiv F. \qquad \neg p \vee q \equiv T.$ (conditional identity)

$F \vee F \equiv T$

it is invalid.

3) Invalid.

$p \to q \equiv T.$

p	q	$p \to q$
T	T	T

$\neg p \vee q \equiv T.$

(conditional identities).

the proposition is invalid while both p and q are false.

T	F	F
F	T	T
F	F	T

3 i) x, z are free variables

y is bound variable.

ii) There's a prime number P there exist no integer k satisfy $p=2k+1$.

iii) Yes.

It is true because when $P=2$, there's no integer satisfy $p=2k+1$.

4. A: There are some trees always shed their leaves.

B: There are some trees always change color.

C: There are some trees shorter than 25 m.

$$\begin{array}{c} \neg A \\ A \to B \\ \neg A \to C \\ \hline \neg B \to C. \end{array}$$

$\neg A \equiv T$

$A \equiv F$

$A \to B \equiv T.$

$\neg A \vee B \equiv T$ (Conditional identity)

$T \vee B \equiv T.$

$$\begin{array}{c} \neg A \to C \\ \neg A \\ \hline C \end{array}$$ (disjunctive syllogism)

$\neg B \to C \equiv B \vee C$

$\equiv B \vee T$

$\equiv T.$ (Domination Laws).

5 i) direct proof:

$$n = \frac{x+y+z}{3}.$$

assume that z is the least one of the number.

$$n - z = \frac{x+y-2z}{3} = \frac{(x-z)+(y-z)}{3}.$$

$\because z<x,\ z<y \quad \therefore x-z>0,\ y+z>0.$

$\therefore \frac{(x-z)+(y-z)}{3} > 0 \quad \therefore n-z>0$

$n>z.$

ii) proof by contradiction.

assume $a+b\sqrt{2}$ is rational.

given that a is rational number, $b\sqrt{2}$ should also be a rational number.

however, b is a rational number $\sqrt{2}$ is an irrational number. so $b\sqrt{2}$ can't be a rational number.

$\therefore$ $a+b\sqrt{2}$ is irrational.

6. the contraposition:

x is odd if x^2+3x+5 is odd.

if x is odd, x^2 is odd,

$3x$ is odd.

5 is an odd number.

x^2+3x+5 is an odd number.