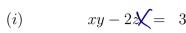
PART A (18 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE INDICATED ON THE SCANTRON SHEET. YOU SHOULD ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.



1. Which of the following equations is/are linear in the unknowns x, y and z?



(i)
$$xy - 2x = 3$$
 (ii) $3x - \sqrt{2}y + \frac{1}{2}z = 0$ (iii) $5\sqrt{x} - 2y + 2x = 1$ (iv) $4x - y - 4x = 0$

(iii)
$$5\sqrt{x} - 2y + 2\chi =$$

$$(iv) 4x - y - x = 0$$

A: (ii) only	B: (ii) and (iv) only	C: (i) and (iii) only
D: (iv) only	E: (i) , (iii) and (iv) only	

 $\frac{1}{mark}$

2. Which of the following is an augmented matrix corresponding to the system of linear equations shown below?

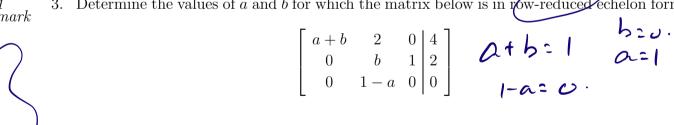
$$x_1 = 5 + x_2 - x_3$$

 $x_2 = 3x_4 - x_1$
 $x_3 = 2x_1 - x_4 + 6$

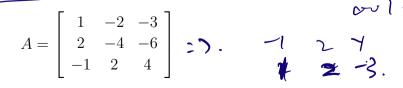
																	_	
A:	3	$1 \\ -1 \\ -1$	0	1		B:	$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$	1 1 0	-1 0 1	0 3 -1	$\begin{bmatrix} 5 \\ 0 \\ 6 \end{bmatrix}$	C:	$\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$	$ \begin{array}{c} -1 \\ 1 \\ 0 \end{array} $	$ \begin{array}{c} 1 \\ 0 \\ 1 \end{array} $	0/ /-3 1	5 0 6	
	Γ 1	1	_	$2 \mid 5$	5]		Г1 -	-5	-1	1	0]							

D:
$$\begin{bmatrix} 1 & 1 & -2 & 5 \\ -1 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \end{bmatrix}$$
 E:
$$\begin{bmatrix} 1 & -5 & -1 & 1 & 0 \\ 1 & -3 & 1 & 0 & 0 \\ 1 & -2 & 1 & -6 & 0 \end{bmatrix}$$

3. Determine the values of a and b for which the matrix below is in row-reduced echelon form. $\frac{1}{mark}$



4. Let B be the row-reduced echelon form of the matrix A below. Find the first row of B.



B: A: 1 0 0 1 -2 0 $\frac{1}{mark}$

5. Find the solution(s) to the system of linear equations whose augmented matrix is shown here:

$$\left[
\begin{array}{ccc|ccc|c}
1 & 0 & 3 & 0 & 7 \\
0 & 1 & -2 & 0 & 0 \\
0 & 0 & 0 & 1 & -6 \\
0 & 0 & 0 & 0 & 0
\end{array}
\right]$$

A: $(7-3t, 2t, t, -6)$	B: $(4, 2, -6, 0)$	C: $(-3t, 2t, -6, 0)$
D: $(3-7t, -2, t, -6)$	E: The system has no solution.	

mark

6. Find the solution(s) to the system of linear equations whose augmented matrix is shown here:

$$\left[\begin{array}{ccc|c}
1 & -1 & 0 & 0 \\
0 & 1 & 1 & 1 \\
2 & -1 & 1 & 2
\end{array}\right]$$

A: $(0,1,2)$ B: $(1-t,1-t,t)$	C: (1, 1, 1)	$D \colon (t,t,t)$	E: The system has no solution.
-------------------------------	--------------	--------------------	--------------------------------

Use the following information for questions 7, 8 and 9.

Consider the system of linear equations whose augmented matrix is:

$$\begin{bmatrix}
1 & 0 & 0 & 5 \\
0 & 1 & -5 & 10 \\
0 & 0 & k+3 & 3k+k^2
\end{bmatrix}$$

 $\frac{1}{mark}$

7. Find the value(s) of k for which the system has no solution.

1 8. Find the value(s) of k for which the system has a unique solution.

man					
	A: $k = -3$ only	B: all $k \neq -3$	C: $k = 0$ only	D: all $k \neq 0$	E: no value of k

9. Find the value(s) of k for which the system has infinitely many solutions.

$mar \kappa$					
	A: $k = -3$ only	B: all $k \neq -3$	C: $k = 0$ only	D: all $k \neq 0$	E: no value of k

1 10. Let A be a 3×4 matrix, B be a 2×4 matrix and C be a 3×5 matrix. Which one of the following describes the matrix BA^TC ?

A: a 3×4 matrix	B: a 5×3 matrix	C: a 2×3 matrix
D: a 4×2 matrix	E: a 2×5 matrix	

11. If A is a 4×4 matrix and B is a 3×4 matrix, which one of the following expressions is **not defined**?

A: BB^T B: BA^T	$C \colon B + B^T A$	D: BA	$E \colon B + BA^T$
---------------------	----------------------	-------	---------------------

A: -1	B: 21	C: -9	D: 6	E: 0
-------	-------	-------	------	------

13. If $AB = C = [c_{ij}]$, where A and B are the matrices shown below, find c_{23} .

$$A = \begin{bmatrix} 2 & 1 & 7 & 0 \\ -1 & 0 & 2 & 3 \\ 3 & 1 & -4 & 7 \end{bmatrix}$$
 and
$$B = \begin{bmatrix} 6 & 2 & 4 & 0 \\ 3 & -5 & -5 & 8 \\ -4 & -1 & 1 & -3 \\ 1 & 0 & 2 & 5 \end{bmatrix}$$

A: 1	B: 2	C: 3	D: 4	E: 5

 $1 \atop mark$ 14. Let $A = \begin{bmatrix} 3 & 7 \\ 1 & 4 \end{bmatrix}$. What is the (2,2)-entry of A^{-1} ?

3		1		1
Δ. Ξ	B. 3	$C \cdot -\frac{1}{2}$	D· _7	F. <u>†</u>
1 7. <u>E</u>	D. 3	$C\frac{1}{5}$	D. – 1	L. <u>E</u>
J		J		J

1 mark 15. Find the value of k for which the matrix $\begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -1 \\ 3 & -10 & k \end{bmatrix}$ has no inverse.

A: 3	B: -7	C: 10	D: -3	E: 7

1 16. Let A be an $n \times n$ matrix and I be the $n \times n$ identity matrix. Which one of the following statements is **false**?

A: If $A^2 = I$ then $A = A^{-1}$.

B: If A has a row of (only) zeros, it is not invertible.

C: If all main diagonal entries of A are ones, it must be invertible.

D: If $A^T A = I$ then $A^T = A^{-1}$.

 $\begin{array}{ccc}
1 & 17. & \text{If } \begin{bmatrix} 3 & c \\ -1 & d \end{bmatrix} \text{ is the inverse of } \begin{bmatrix} a & 2 \\ b & -3 \end{bmatrix}, \text{ find the value of } c.$

A: -1	B: 1	C: -2	D: 0	E: 2

 $\frac{1}{mark}$ 18. Let A be the coefficient matrix of the system of linear equations shown here:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = -3$$

 $a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 5$
 $a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 7$

Find the value of x_2 in the unique solution to the system if $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$.

A: 7 B: -10	C: 49	D: -8	E: 3
-------------	-------	-------	------

PART B (7 marks)

SHOW YOUR WORK FOR ALL QUESTIONS IN PART B

4 19. Consider the following system of linear equations:

(a) Write the augmented matrix corresponding to this system of equations.

(b) Find the row-reduced echelon form of your augmented matrix from part (a).

(c) Use your row-reduced echelon form from part (b) to find all solutions to the system.

 $\frac{3}{marks}$ 20. Use the method of row reduction to find the inverse of $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 4 \\ -3 & -3 & -2 \end{bmatrix}$.