Review Session

COMPSCI 3331

Fall 2022

Question 1

1. Let G be the CFG defined by the following set of productions.

$$
\begin{array}{lcl}
S & \rightarrow & bbAaA \mid SSaa \mid aa \mid ABC \mid AB \mid bbAa \mid bbaA \mid BC \mid B \mid bba \\
A & \rightarrow & Ab \mid Ac \mid CC \mid C \mid b \mid b \\
B & \rightarrow & BA \mid bb \mid Dd \mid B \\
C & \rightarrow & DA \mid \cancel{\varepsilon} \mid D \\
D & \rightarrow & a
\end{array}
$$

Give an equivalent grammar to G that has no ε-productions.

Question 2

2. Let G be the CFG defined by the following set of productions.

$$
\begin{aligned}
S &\rightarrow SbaB \mid aa \mid ABC \mid AB\,. \\
A &\rightarrow Ba \mid DaDd \\
B &\rightarrow BA \mid ca \mid Dd \\
C &\rightarrow DA \mid \cancel{\varepsilon} \\
D &\rightarrow a \\
E &\rightarrow b \\
F &\rightarrow c \\
G &\rightarrow d\,.
\end{aligned}
$$

Convert the grammar to CNF.

$S \to Y_{SE} Y_{DB} \mid DD \mid Y_{AB} C \mid AB$

$A \to BD \mid Y_{DD} Y_{DG}$

$B \to BA \mid FD \mid Y_{DG}$.

$C \to DA$

$D \to a$

$E \to b$

$F \to c$

$G \to d$.

$Y_{AB} \to AB$

$Y_{DD} \to DD$

$Y_{DB} \to DB$

$Y_{SE} \to SE$

$Y_{DG} \to DG$

Question 3

3. Let $G = (V, \Sigma, P, S)$ be a CFG in CNF. Give an $O(n^3)$ algorithm for the following problem:

- Input: A word w and a nonterminal $A \in V$.
- Output: the value
  $n_A = \max\{|u| \ : \ u \text{ is a suffix of } w \text{ and } A \Rightarrow^* u\}$.

That is, n_A is the length of the longest suffix of w that is generated by A in the grammar.

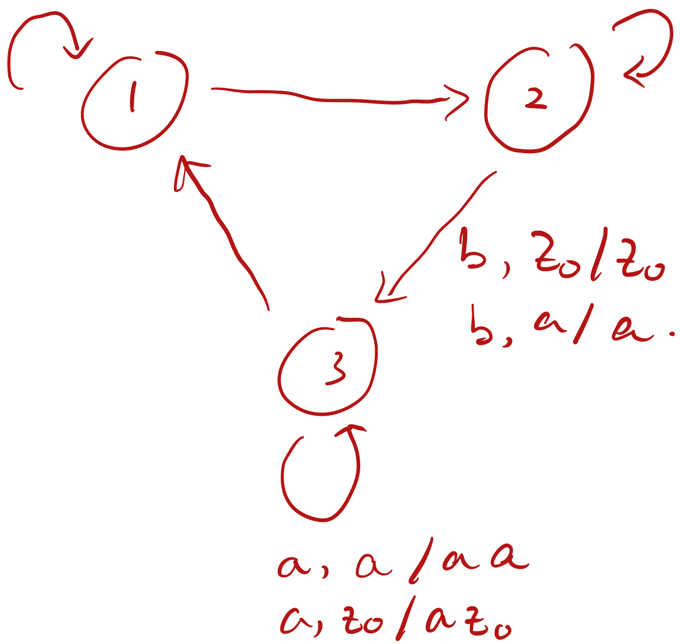
Question 4

4. Construct PDAs for the following languages:

(a) $L = \{a^n b^m xy \ : \ x, y \in \{0, 1, 2\}, x \equiv n(\mathrm{mod} 3) \text{ and } y \equiv m(\mathrm{mod} 3)\}$.

(b) $L = \{w\#x \ : \ w, x \in \{a, b\}^*, |w|_a = |x|_a, |w|_b \equiv |x|_b(\mathrm{mod} 3)\}$.

Be sure to indicate what the starting stack symbol is for your PDA and how your PDA accepts words.



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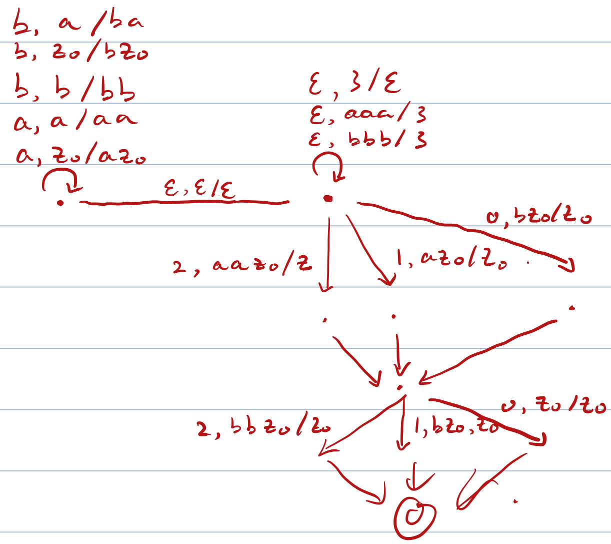

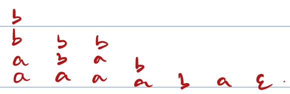

Question 5

5. Let C be a fixed integer. Extend the language from Assignment 3 as follows:

$$L_C = \{x\#1^n \ : n \geq 0, x \in \{a,b\}^* \text{ and } \ n - C \leq |x|_a \leq n + C\}$$

LFCs

Give a context-free grammar for L_C. The productions in your grammar will depend on the value of C. Describe them using a uniform notation (e.g., by using consistently named variables or consistently defined productions, for instance).

$S \to bS$

$S \to aS1$

$S \to$

$\{a,b\}^* S\, 1^{|x|_a}$

$\{a,b\}^* \# T$

$M \to 1^c$

Question 6

6. Consider the following modified language from Assignment 3:

$$L = \{u\#v \;:\; u, v \in \{0,1\}^* \text{ and } \mathrm{bin}(v^R) = \mathrm{bin}(u) + 2\}$$

Give a PDA that accepts L.

Question 7

7. A CFG G is in Griebach Normal Form (GNF) if every production has the form

$$A \to aB_1 B_2 \cdots B_n$$

for some letter a and nonterminals $B_1, B_2, \cdots, B_n$ (where $n \geq 0$). Any grammar (that does not derive ε) can be converted to GNF. Given this fact, show that for any CFG L that does not include ε, you can construct a PDA M that accepts L in the following additional conditions:

- The PDA accepts by empty stack.
- The PDA M does not have any ε-transitions. That is, there are no rules of the form $\delta(q, \varepsilon, \gamma) = \{(q', \beta), \dots\}$ for any stack symbol γ.

Question 8

8. Prove that the following languages are not context-free:
   (a) $L = \{a^p \;:\; p \text{ is a prime number}\}$.
   (b) $L = \{a^n b^{n^3} \;:\; n \geq 0\}$.
   (c) $L = \{w \in \{a, b\}^* \;:\; |w|_b = 2^{|w|_a}\}$. $R = a^* b^*$.

$L \cap R = \{a^n b^{2^n}\}$.

convert it to a regular language.

1) v, x cross boundary:
2) vx all a's
3) vx all b's
4) v in a's x in b's

$z = uvwxy$.

4) $u = a^i$ $v = a^j$ $w = a^{n-i-j} b^k$ $x = b^l$ $y = b^{2^n - k - l}$

if $j = 0 \Rightarrow l \neq 0$ $uv^2wx^2y = a^n b^{2^n + l}$

if $j \neq 0 \Rightarrow uv^2wx^2y = a^{n+j} b^{2^n + l}$

to show $\# 2^{as} \neq bs \Rightarrow 2^{n+j} \neq 2^n + l$

$2^n (2^j - 1) \neq l$ $j + l \leq n$

$\Rightarrow l \leq n - 1$ since $j \neq 0$.

$l \leq 2^n - 1$

Question 9

9. For each of the languages in the previous question, give an informal description of a multi-tape TM that recognizes the language.

(a) $L = \{a^p : p \text{ is a prime number}\}$.

(b) $L = \{a^n b^{n^3} : n \geq 0\}$.

(c) $L = \{w \in \{a,b\}^* : |w|_b = 2^{|w|_a}\}$.

Question 10

10. Show that the following language is r.e.:

=> accepted by turning machine.

$$L = \{e(M_1)\#e(M_2) \;:\; L(M_1) \cap L(M_2) \neq \emptyset\}$$

M_u - universial TM

Question 11

11. Show that the following problem is undecidable by reduction: Given a TM M, is $L(M)$ a finite language?

Question 12

12. Show that the following language is decidable:
$L_{ND} = \{e(M) \ : \ M \text{ is a nondeterministic TM } \}$.

Question 13

13. Show that the following problem is either decidable or undecidable: Given a CFG G is $L(G)$ infinite? (Hint: review the proof of the pumping lemma for CFLs.)