Last name (please print)	
First name (please print)	
Student Number	

WESTERN UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE

CS3331 - Foundations of Computer Science – Fall 2015 – Final Exam
Instructor: Dr. Lucian Ilie
Friday, Dec. 11, 2015, 7:00pm - 10:00pm
P&AB106 (Absi - Meht), P&AB148 (Meye - Zhu)

This exam consists of 6 questions worth a total of 100 marks. **No other materials are allowed, such as cheat-sheets (or any other sheets), books, or electronic devices.** All answers are to be written in this booklet. For each question, if use the back of the page or the scrap work sheets at the end to write your answers, please indicate this clearly on the page that contains the question. The exam is 120 minutes long and comprises 30% of your final mark.

(1) 10pt	
(2) 10pt	
(3) 30pt	
(4) 10pt	
(5) 30pt	
(6) 10pt	
Grade	

1. (10pt) Construct a deterministic Turing machine M that decides the language

$$L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ contains at least three } \mathtt{a}\text{'s}\} \ .$$

M starts with the initial configuration $(s, \underline{\square}w)$ and halts with the configuration $(q, \underline{\square}w)$, in the appropriate state $q \in \{y, n\}$. Describe M using the macro language (e.g.: $\mathtt{a}R_{\square}\mathtt{b}L_{\#}$)

2. (10pt) The set SD is closed under intersection. Is the following a correct proof of this fact? Explain your answer.

   Assume $L_1, L_2 \in$ SD, semidecided by two Turing machines M_1 and M_2, respectively. We can build a machine M to decide $L_1 \cap L_2$ as follows:
   1. on input w
   2. run M_1 on w
   3. if M_1 accepts, then
   4.   run M_2 on w
   5.   if M_2 accepts, then accept

3. (30pt) Can you give an example of a language L such that:

   (a) $L \in \text{D}$ and $\neg L \in \text{SD} - \text{D}$?

   (b) $L, \neg L \in \text{SD} - \text{D}$?

   (c) $L \in \text{SD}$, $\neg L \notin \text{SD}$?

   (d) $L, \neg L \notin \text{SD}$?

   Prove your answers. ($\neg L$ is the complement of L.)

4. (10pt) Assume a language L and a Turing machine M that lexicographically enumerates L. Construct a Turing machine M' that decides $\neg L^R$ (the complement of the reversal of L). (Only clear English description is required.)

5. (30pt) For each of the following languages, prove whether it is in D, SD − D, or ¬SD. Explain first intuitively why you think it is in D, SD − D, or ¬SD, then prove your assertion rigurously.

   (a) $L_1 = \{< M >\mid L(M) \text{ is finite}\}$.

   (b) $L_2 = \{< M >\mid L(M) \neq \emptyset\}$.

   (c) $L_3 = \{< M_1, M_2 >\mid L(M_1) - L(M_2) \text{ is infinite}\}$.

   (d) $L_4 = \{< M, w >\mid M \text{ accepts } w \text{ and rejects } w^R\}$.

6. (10pt)

(a) Does the following instance of PCP have any solutions? Prove your answer.

a	bbb	aab	b
bab	bb	ab	a

(b) If you can determine, as you did at (a), whether a PCP instance has solutions or not, how is it possible that the Post Correspondence Problem is undecidable?

(c) Can you give an example of an instance of PCP that has only one solution? Prove your answer.

(scrap work)

(scrap work)

(scrap work)

(scrap work)

(scrap work)

(scrap work)