Exam Questions

COMPSCI 3331 - Final review questions

Fall 2022

These questions are provided for review purposes and are not guaranteed to be the same or similar to the questions on the final. Some questions may be easier than the questions on the final and some may be harder.

Solutions to as many questions as possible will be written as I am able. There is no guarantee that all solutions will be available for all questions.

1. Recall that u is a prefix of v if $v = ux$ for some word x. Let Σ be an alphabet with at least two letters. Let pref(L) the language operation defined by

$$\text{pref}(L) = \{w \in \Sigma^* \ : \ \exists x \in L \text{ such that } w \text{ is a prefix of } x\}.$$

Give a counter-example to the equation $(\text{pref}(L))^* = \text{pref}(L^*)$.

2. Let G be the CFG defined by the following set of productions.

$$\begin{aligned} S &\rightarrow bbAaA \mid SSaa \mid aa \mid ABC \\ A &\rightarrow Ab \mid Ac \mid CC \\ B &\rightarrow BA \mid bb \mid Dd \\ C &\rightarrow DA \mid \varepsilon \\ D &\rightarrow a \end{aligned}$$

Give an equivalent grammar to G that has no ε-productions.

3. Let G be the CFG defined by the following set of productions.

$$\begin{aligned} S &\rightarrow SbaB \mid aa \mid ABC \\ A &\rightarrow Ba \mid DaDd \\ B &\rightarrow BA \mid ca \mid Dd \\ C &\rightarrow DA \mid \varepsilon \\ D &\rightarrow a \end{aligned}$$

Convert the grammar to CNF.

4. Let $G = (V, \Sigma, P, S)$ be a CFG in CNF. Give an $O(n^3)$ algorithm for the following problem:

1. Recall that u is a prefix of v if $v = ux$ for some word x. Let Σ be an alphabet with at least two letters. Let $\mathrm{pref}(L)$ the language operation defined by

$$\mathrm{pref}(L) = \{w \in \Sigma^* \ : \ \exists x \in L \text{ such that } w \text{ is a prefix of } x\}.$$

Give a counter-example to the equation $(\mathrm{pref}(L))^* = \mathrm{pref}(L^*)$.

Assume that $L = \{aac\}$, then $L^* = (aac)^*$, $\mathrm{pref}(L) = \{a, aa\}$.

$(\mathrm{pref}(L))^* = a^*$

$\mathrm{pref}(L^*) = \{a, aa, aac, aaca, aacaa \dots\}$.

$aaa \in (\mathrm{pref}(L))^*$, $aaa \notin \mathrm{pref}(L)^*$

2. Let G be the CFG defined by the following set of productions.

$S \rightarrow bbAaA \mid SSaa \mid aa \mid ABC \mid \cancel{\varepsilon} \mid bba \mid BC \mid B \mid AB$.

$A \rightarrow Ab \mid Ac \mid \cancel{CC}\ \varepsilon \mid b \mid c$

$B \rightarrow BA \mid bb \mid Dd \mid B$

$C \rightarrow DA \mid \cancel{\varepsilon} \mid D$

$D \rightarrow a$

3. Let G be the CFG defined by the following set of productions.

remove ε production.

remove unit production.

check generating

check reachable.

$S_0 \rightarrow SX_bDB \mid DD \mid ABC \mid AB$.

$S \rightarrow S\cancel{ba}XDB \mid \cancel{aa}\,DD \mid ABC \mid AB$

$A \rightarrow B\cancel{a}D \mid D\cancel{a}DDZ$

$B \rightarrow BA \mid \cancel{ca}\,YD \mid DZ$

$C \rightarrow DA \mid \cancel{\varepsilon}$

$D \rightarrow a$

$X \rightarrow b$

$Y \rightarrow c$

$Z \rightarrow d$

$S_0 \rightarrow SXDB \mid DD \mid ABC \mid AB$

$S \rightarrow SXDB \mid DD \mid ABC \mid AB$

$A \rightarrow BD \mid DDDZ$

$B \rightarrow BA \mid YD \mid DZ$

$C \rightarrow DA$

$D \rightarrow a$

$X \rightarrow b$

$Y \rightarrow c$

$Z \rightarrow d$.

$S_0 \rightarrow SX_bDB \mid DD \mid ABX_dA$

$S \rightarrow SX_bDB \mid \cancel{aa}\,DD \mid ABX_dA$

$A \rightarrow BD \mid DDDX_d$.

$B \rightarrow BA \mid X_cD \mid DX_d$

~~$C \rightarrow X_dA \mid \varepsilon$~~

$D \rightarrow a$

$X_b \rightarrow b$

$X_c \rightarrow c$

$X_d \rightarrow d$

Y_D

$S \rightarrow aSa$

$S \rightarrow B$

$B \rightarrow bb \cancel{/} aa$.

~~$C \rightarrow cC$~~

~~$C \rightarrow \varepsilon$~~

$B \rightarrow bb$

$S \rightarrow aSa$

~~$S \rightarrow bbaa$~~ BX_a

$B \rightarrow \cancel{bb}\ X_bX_b$

$X_b \rightarrow b$

$X_a \rightarrow a$

$S \rightarrow X_aBX_aX_a$

- Input: A word w and a nonterminal A.
- Output: the value $n_A = \max\{|u| \,:\, u \text{ is a suffix of } w \text{ and } A \Rightarrow^* u\}$.

That is, n_A is the length of the longest suffix of w that is generated by A in the grammar.

5. Construct PDAs for the following languages:

(a) $L = \{a^n b^m xy \,:\, x, y \in \{0,1,2\}, x \equiv n(\text{mod}3) \text{ and } y \equiv m(\text{mod}3)\}$.

(b) $L = \{w\#x \,:\, w, x \in \{a,b\}^*, |w|_a = |x|_a, |w|_b \equiv |x|_b(\text{mod}3)\}$.

6. Let C be a fixed integer. Extend the language from Assignment 3 as follows:

$$L_C = \{x\#1^n \;: n \geq 0, x \in \{a,b\}^* \text{ and } \; n - C \leq |x|_a \leq n + C\}$$

Give a context-free grammar for L_C. The productions in your grammar will depend on the value of C. Describe them using a uniform notation (e.g., by using consistently named variables or consistently defined productions, for instance).

7. Consider the following modified language from Assignment 3:

$$L = \{u\#v \,:\, u, v \in \{0,1\}^* \text{ and } \text{bin}(v^R) = \text{bin}(u) + 2\}$$

Give a PDA that accepts L.

8. A CFG G is in Griebach Normal Form (GNF) if every production has the form

$$A \to aB_1B_2\cdots B_n$$

for some letter a and nonterminals $B_1, B_2, \cdots, B_n$ (where $n \geq 0$).

Any grammar (that does not derive ε) can be converted to GNF. Given this fact, show that for any CFG L , you can construct a PDA M that accepts L in the following additional conditions:

- The PDA accepts by empty stack.
- The PDA M does not have any ε-transitions. That is, there are no rules of the form $\delta(q, \varepsilon, \gamma) = \{(q', \beta), \ldots\}$ for any stack symbol γ.

9. Prove that the following languages are not context-free:

- $L = \{a^p \,:\, p \text{ is a prime number}\}$.
- $L = \{a^n b^{n^3} \,:\, n \geq 0\}$.
- $L = \{w \in \{a,b\}^* \,:\, |w|_b = 2^{|w|_a}\}$.

10. For each of the languages in the previous question, give an informal description of a multi-tape TM that recognizes the language.

11. Show that the following language is r.e.:

$$L = \{e(M_1)\#e(M_2) \ : \ L(M_1) \cap L(M_2) \neq \emptyset\}$$

12. Show that the following problem is undecidable by reduction: Given a TM M, is $L(M)$ a finite language?

13. Show that the following language is decidable: $L_{ND} = \{e(M) \ : \ M \text{ is a nondeterministic TM }\}$.

14. Show that the following problem is decidable: Given a CFG G is $L(G)$ infinite? (Hint: review the proof of the pumping lemma for CFLs.)