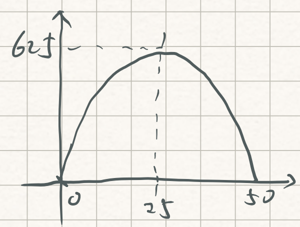
$x + y = 50.$

$A = xy.$

$= x(50 - x).$

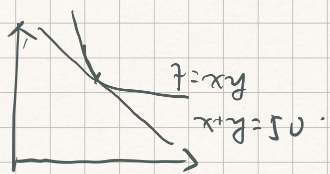


$A_{max} = 625$ at $x = 25.$

$\max f(x, y) = xy$ along the curve $x + y = 50.$

the level curve of $f(x, y)$ is in the figure below:

Let $P(x_0, y_0)$ is the point where $f(x, y)$ has its extreme value, ∇f and ∇g are parell.



then $\nabla f = \lambda \nabla g.$

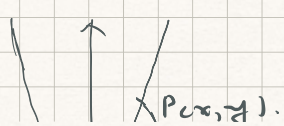
$\nabla f = y\hat{i} + x\hat{j} \quad \nabla g = \hat{i} + \hat{j}$

$\therefore (y, x) = \lambda(1, 1).$

$\Rightarrow \begin{array}{l} y = \lambda, \\ x = \lambda. \end{array} \quad x = y = 25.$

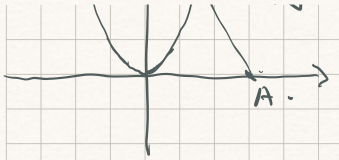
Method of Lagrange Multiplier: To find the maximum and minimum values of the function $f(x, y, z)$. subject to $g(x, y, z) = k$. Let (x_0, y_0, z_0) be the point where f has its extreme value then $\nabla f = \lambda \nabla g$. λ is called a Lagrange Multiplier.

e.g. 1. Find the shortest distance from $(3, 0)$ to the parabola $y = x^2$.



Sol: $d = AP = \sqrt{(x-3)^2 + y^2}.$

$f(x, y) = d^2 = (x-3)^2 + y^2.$



$f(x,y) = x^2 - y = 0$.

At the critical point, $\nabla F = \lambda \nabla f$.

$2(x-3)\hat{i} + 2y\hat{j} = \lambda(2x\hat{i} - \hat{j})$.

$$\Rightarrow \begin{cases} 2(x-3) = 2x\lambda \\ 2y = -\lambda \\ y = x^2. \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$$

$d_{min} = \sqrt{5}$ // Ans.

Problem that has more than one constant:

Suppose $F(x,y,z)$ $f(x,y,z) = 0$, $h(x,y,z) = 0$.

Sol: Let C be interception curve of $f(x,y,z) = 0$ and $h(x,y,z) = 0$

assume F has extreme value at $P(x_0, y_0, z_0)$.

let $\vec{T}$ be the tangent curve of C at P.

$\underline{\vec{T} = \nabla f \times \nabla h.}$

$\Downarrow$

$\vec{T} \perp \nabla f \wedge \vec{T} \perp \nabla h$.