Solutions (Logical Equivalence)

1.)

P	q	$\neg P$	$(\neg P \wedge q)$	$P \vee (\neg P \wedge q)$	$P \vee q$	$\neg(P \vee (\neg P \wedge q))$	$\neg(P \vee q)$
0	0	1	0	0	0	1	1
0	1	1	1	1	1	0	0
1	0	0	0	1	1	0	0
1	1	0	0	1	1	0	0

True

2) $\neg(P \wedge q) \vee (P \wedge q)$

$= \neg A \vee A$ [consider $(P \wedge q) = A$]

$= \top$ [Law of excluded middle]

3) $(\neg P \wedge (P \vee q))$

[expanding the expression by distributivity law]

$= (\neg P \wedge P) \vee (\neg P \vee q)$

[Law of non-contradiction]

$= 1 \vee (\neg P \vee q)$

[Identity Law]

$= \neg P \vee q$

4) $\neg(P \rightarrow q)$

$\equiv \neg(\neg P \vee q)$ [we know that $A \rightarrow B \equiv \neg A \vee B$]

$= \neg(\neg P) \wedge \neg(q)$ [De Morgan's Law]

$= P \wedge \neg(q)$ [Double negation law]

$= P \wedge \neg q$

$\therefore \neg(P \rightarrow q) \equiv P \wedge \neg q$

5) $\neg(P \vee (\neg P \wedge q))$

$= \neg((P \vee \neg P) \wedge (P \vee q))$ [Distributive law]

$= \neg(T \wedge (P \vee q))$ [Law of excluded middle]

$= \neg(P \vee q)$ [~~Identity~~ Identity Law]

$\therefore \neg(P \vee (\neg P \wedge q)) \equiv \neg(P \vee q)$