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# Tutorial 06:

# Combinational Circuits

*Computer Science Department*

*CS2208: Introduction to Computer Organization and Architecture*

*Winter 2021-2022*

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*Office: MC-419*

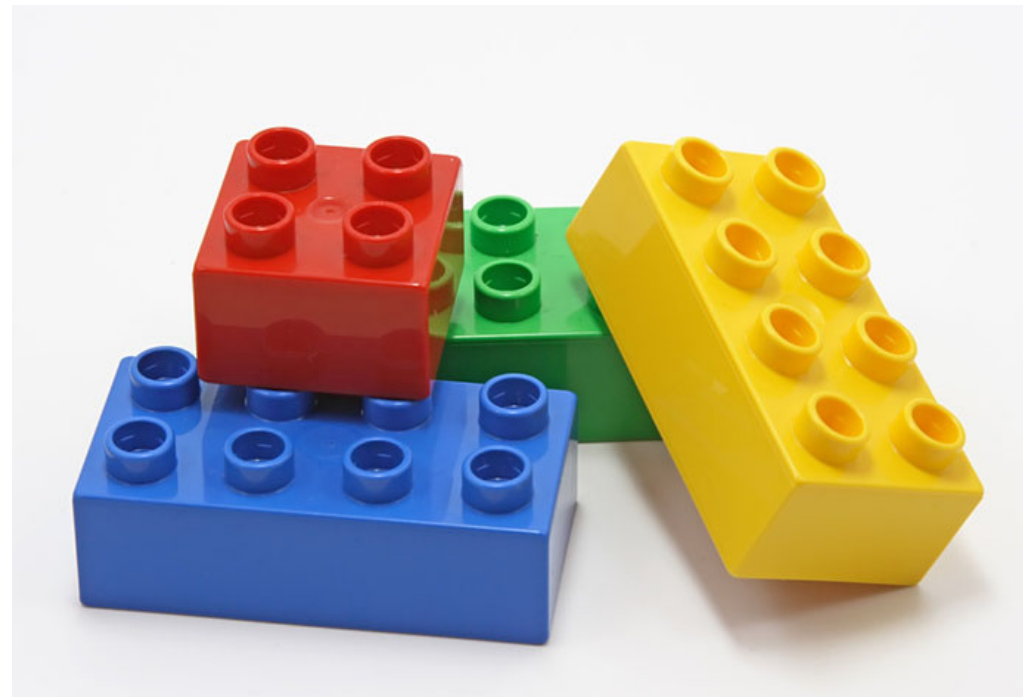
*Email: [elsakka@csd.uwo.ca](mailto:elsakka@csd.uwo.ca)*

*Phone: 519-661-2111 x86996*

# Gates

## ■ Gates

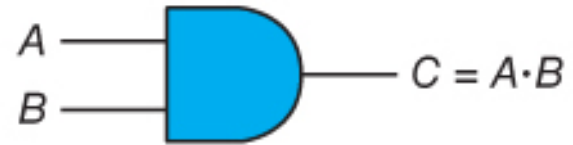
- Are our building blocks when considering hardware realization
- Can be used to build combinational circuits, as well as latches, flip-flops, and sequential circuits
- Come in many flavors, including
  - AND
  - OR
  - NOT
  - NAND (Not AND)
  - NOR (Not OR)
  - XOR



# Gates

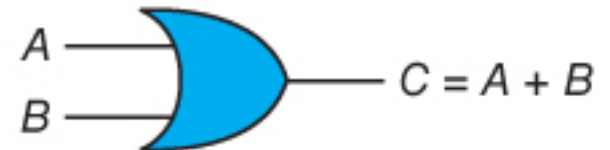
## ■ AND:

- True only when all input are true



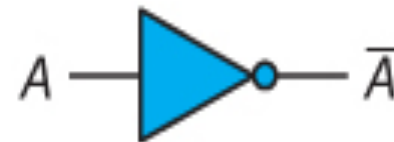
## ■ OR:

- True only when at least one input is true



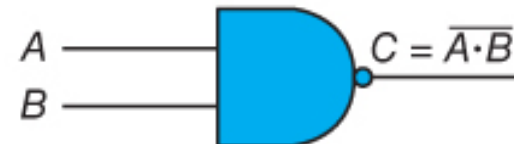
## ■ NOT:

- Complementing the input Boolean value



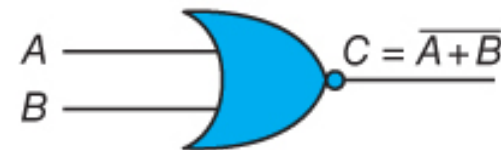
## ■ NAND:

- The complement of an AND result



## ■ NOR:

- The complement of an OR result



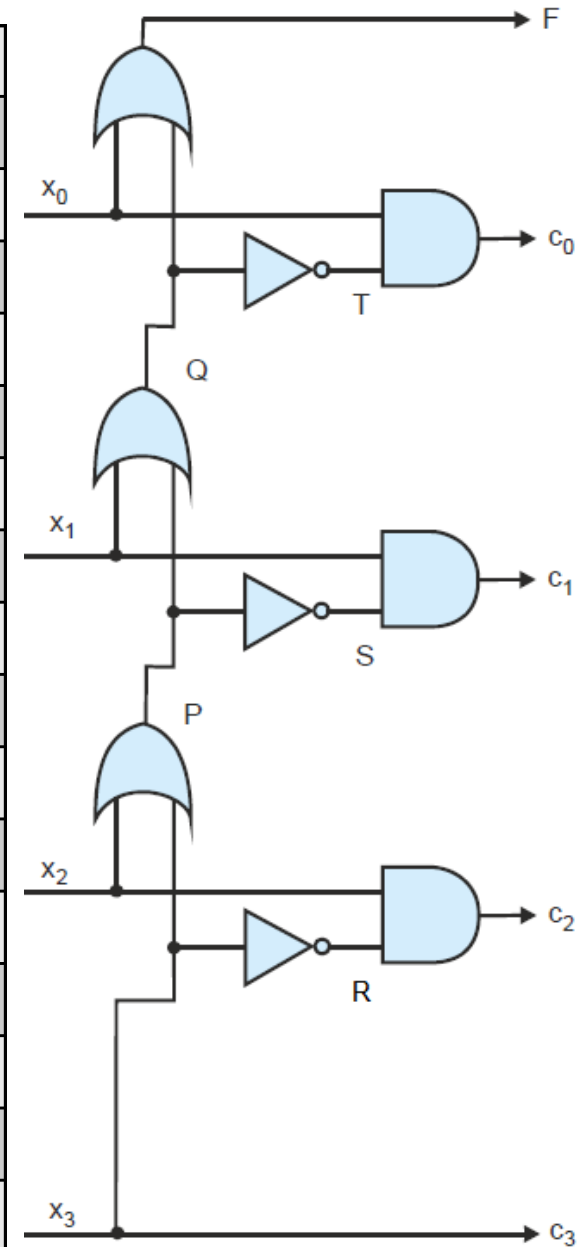
## ■ XOR:

- True only when an odd number of inputs are true



# Example 1

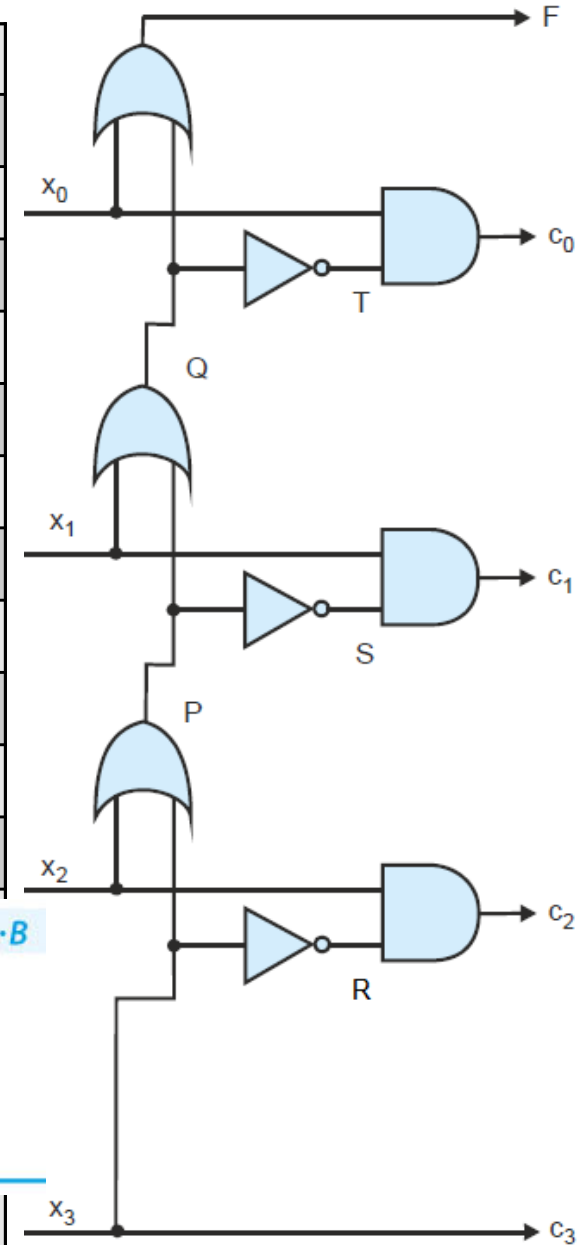
X <sub>0</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	P	S	Q	T	R	C <sub>0</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	F
0	0	0	0	0	1	0	1	1	B      A      C = A + B				
0	0	0	1	1	0	1	0	0	0	0	0		
0	0	1	0	1	0	1	0	1	0	1	1		
0	0	1	1	1	0	1	0	0	1	0	1		
0	1	0	0	0	1	1	0	1					
0	1	0	1	1	0	1	0	0	A	Ā			
0	1	1	0	1	0	1	0	1	1	0			
0	1	1	1	1	0	1	0	0	0	1			
1	0	0	0	0	1	0	1	1					
1	0	0	1	1	0	1	0	0					
1	0	1	0	1	0	1	0	1					
1	0	1	1	1	0	1	0	0					
1	1	0	0	0	1	1	0	1					
1	1	0	1	1	0	1	0	0					
1	1	1	0	1	0	1	0	1					
1	1	1	1	1	0	1	0	0					



# Example 1

$X_0$	$X_1$	$X_2$	$X_3$	P	S	Q	T	R	$C_0$	$C_1$	$C_2$	$C_3$	F
0	0	0	0	0	1	0	1	1	0	0			
0	0	0	1	1	0	1	0	0	0	0			
0	0	1	0	1	0	1	0	1	0	0			
0	0	1	1	1	0	1	0	0	0	0			
0	1	0	0	0	1	1	0	1	0	1			
0	1	0	1	1	0	1	0	0	0	0			
0	1	1	0	1	0	1	0	1	0	0			
0	1	1	1	1	0	1	0	0	0	0			
1	0	0	0	0	1	0	1	1	1	0			
1	0	0	1	1	0	1	0	0	0	0			
1	0	1	0	1	0	1	0	1	0	0			
1	0	1	1	1	0	1	0	0	0	0			
1	1	0	0	0	1	1	0	1	0	1			
1	1	0	1	1	0	1	0	0	0	0			
1	1	1	0	1	0	1	0	1	0	0			
1	1	1	1	1	0	1	0	0	0	0			

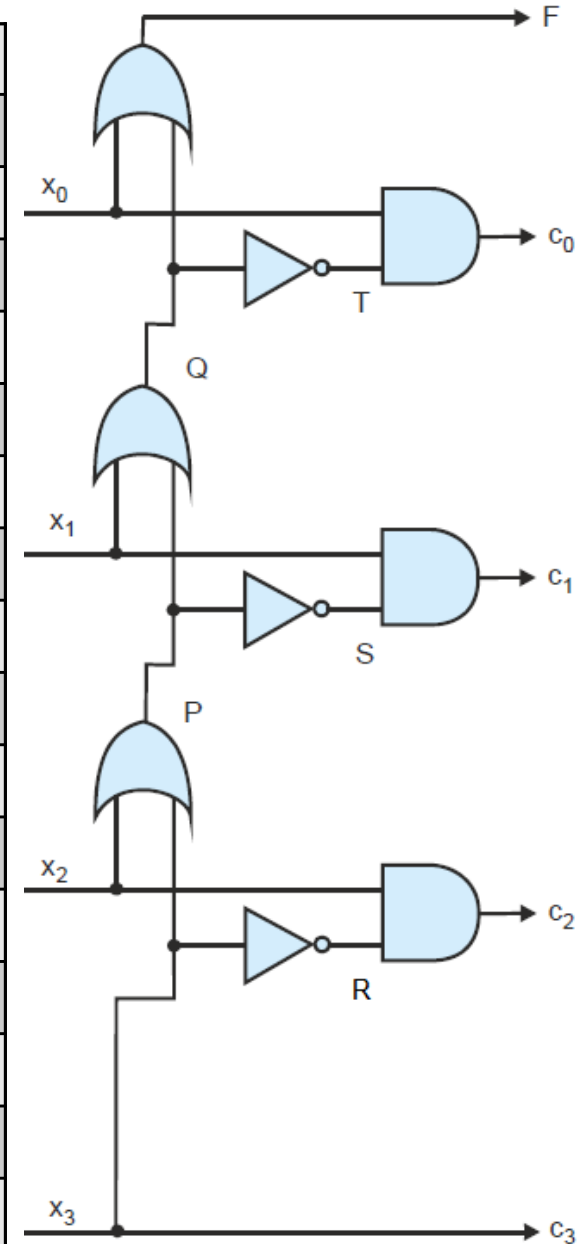
B	A	$C = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1



# Example 1

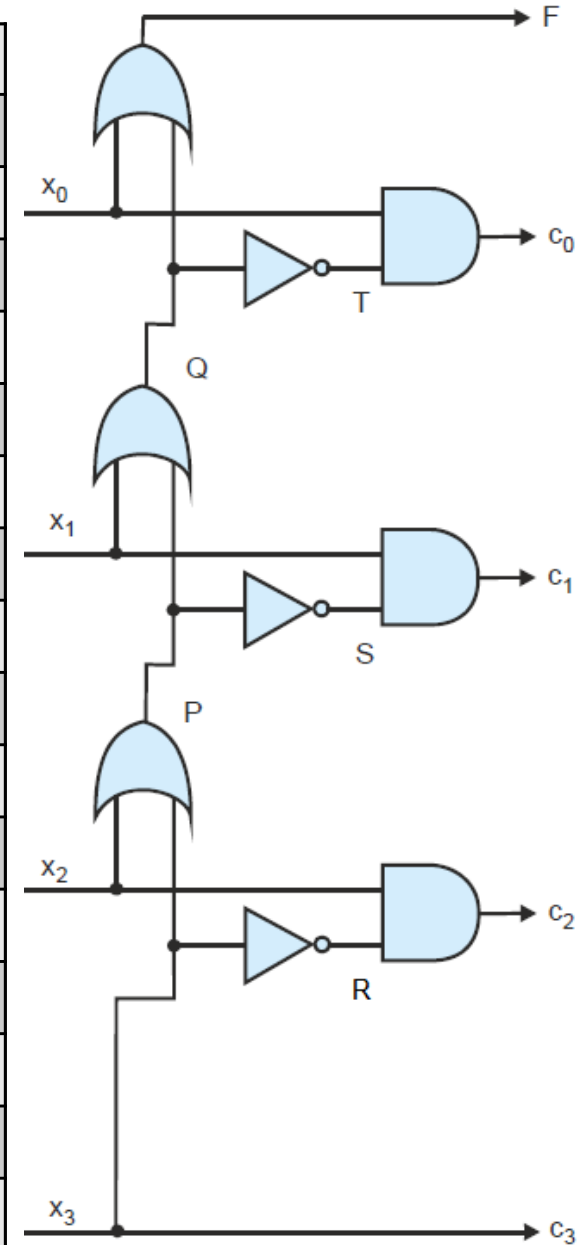
$X_0$	$X_1$	$X_2$	$X_3$	P	S	Q	T	R	$C_0$	$C_1$	$C_2$	$C_3$	F
0	0	0	0	0	1	0	1	1	0	0	0	0	
0	0	0	1	1	0	1	0	0	0	0	0	1	
0	0	1	0	1	0	1	0	1	0	0	1	0	
0	0	1	1	1	0	1	0	0	0	0	0	1	
0	1	0	0										
0	1	0	1										
0	1	1	0										
0	1	1	1										
1	0	0	0	0	1	0	1	1	1	0	0	0	
1	0	0	1	1	0	1	0	0	0	0	0	1	
1	0	1	0	1	0	1	0	1	0	0	1	0	
1	0	1	1	1	0	1	0	0	0	0	0	1	
1	1	0	0	0	1	1	0	1	0	1	0	0	
1	1	0	1	1	0	1	0	0	0	0	0	1	
1	1	1	0	1	0	1	0	1	0	0	1	0	
1	1	1	1	1	0	1	0	0	0	0	0	1	

B	A	$C = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1



# Example 1

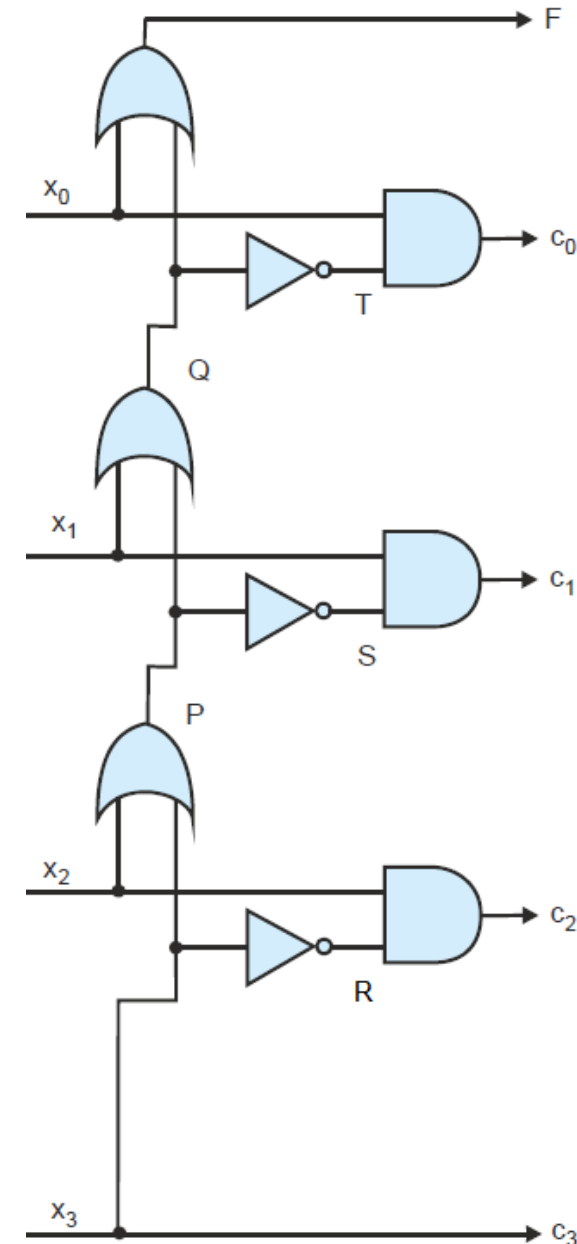
X <sub>0</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	P	S	Q	T	R	C <sub>0</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	F
0	0	0	0	0	1	0	1	1	0	0	0	0	0
0	0	0	1	1	0	1	0	0	0	0	0	1	1
0						1	0	1	0	0	1	0	1
0						1	0	0	0	0	0	1	1
0						1	0	1	0	1	0	0	1
0						1	0	0	0	0	0	1	1
0						1	0	1	0	0	1	0	1
0	1	1	1	1	0	1	0	0	0	0	0	1	1
1	0	0	0	0	1	0	1	1	1	0	0	0	1
1	0	0	1	1	0	1	0	0	0	0	0	1	1
1	0	1	0	1	0	1	0	1	0	0	1	0	1
1	0	1	1	1	0	1	0	0	0	0	0	1	1
1	1	0	0	0	1	1	0	1	0	1	0	0	1
1	1	0	1	1	0	1	0	0	0	0	0	1	1
1	1	1	0	1	0	1	0	1	0	0	1	0	1
1	1	1	1	1	0	1	0	0	0	0	0	1	1



# Example 1

X <sub>0</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	P	S	Q	T	R	C <sub>0</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	F
0	0	0	0	0	1	0	1	1	0	0	0	0	0
0	0	0	1	1	0	1	0	0	0	0	0	1	1
0	0	1	0	1	0	1	0	1	0	0	1	0	1
0	0	1	1	1	0	1	0	0	0	0	0	1	1
0	1	0	0	0	1	1	0	1	0	1	0	0	1
0	1	0	1	1	0	1	0	0	0	0	0	1	1
0	1	1	0	1	0	1	0	1	0	0	1	0	1
0	1	1	1	1	0	1	0	0	0	0	0	1	1
1	0	0	0	0	1	0	1	1	1	0	0	0	1
1	0	0	1	1	0	1	0	0	0	0	0	1	1
1	0	1	0	1	0	1	0	1	0	0	1	0	1
1	0	1	1	1	0	1	0	0	0	0	0	1	1
1	1	0	0	0	1	1	0	1	0	1	0	0	1
1	1	0	1	1	0	1	0	0	0	0	0	1	1
1	1	1	0	1	0	1	0	1	0	0	1	0	1
1	1	1	1	1	0	1	0	0	0	0	0	1	1

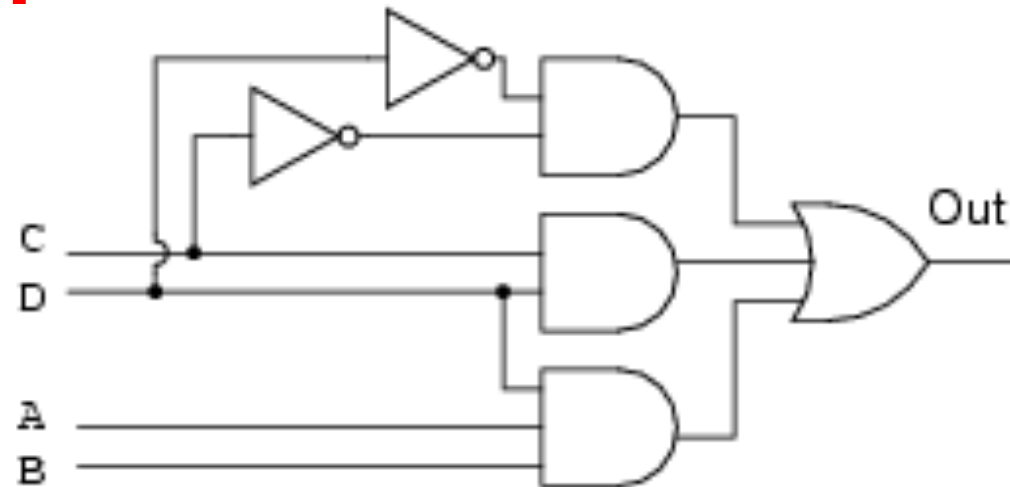
- $P = X_2 + X_3$
- $S = \overline{P} = \overline{(X_2 + X_3)} = \overline{X_2} \cdot \overline{X_3}$  (De Morgan law)
- $Q = X_1 + P = X_1 + X_2 + X_3$
- $T = \overline{Q} = \overline{X_1 + X_2 + X_3} = \overline{X_1} \cdot \overline{X_2} \cdot \overline{X_3}$  (De Morgan law)
- $R = \overline{X_3}$
- $C_0 = X_0 \cdot T = X_0 \cdot \overline{X_1} \cdot \overline{X_2} \cdot \overline{X_3}$
- $C_1 = X_1 \cdot S = X_1 \cdot \overline{X_2} \cdot \overline{X_3}$
- $C_2 = X_2 \cdot R = X_2 \cdot \overline{X_3}$
- $C_3 = X_3$
- $F = X_0 + Q = X_0 + X_1 + X_2 + X_3$



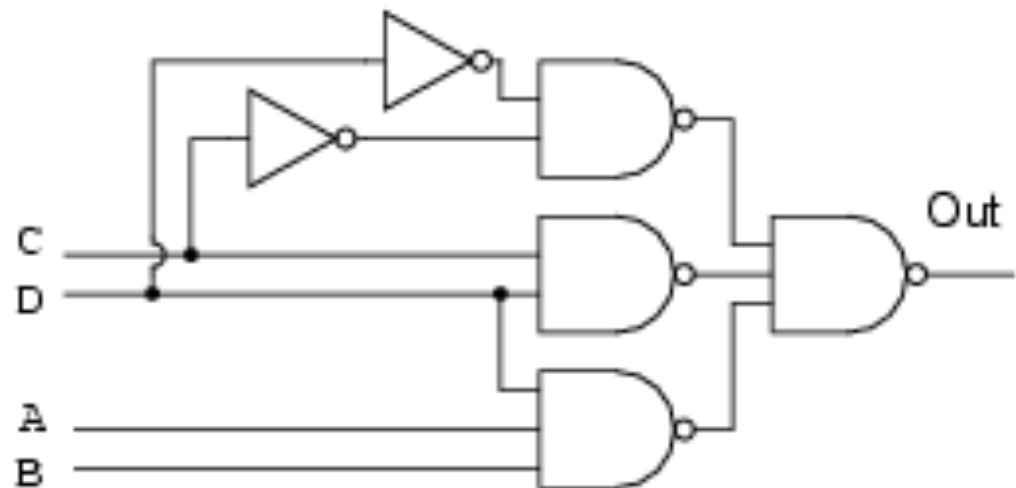


## Example 2

- $out = \overline{C} \overline{D} + CD + ABD$
- $out = \overline{\overline{\overline{C} \overline{D} + CD + ABD}}$
- $out = \overline{(\overline{\overline{C} \overline{D}} \cdot \overline{CD} \cdot \overline{ABD})}$

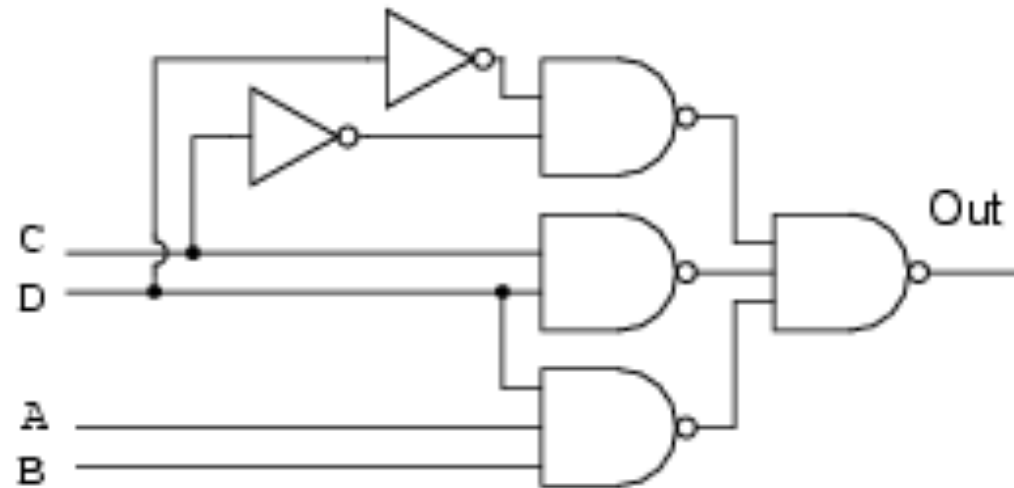
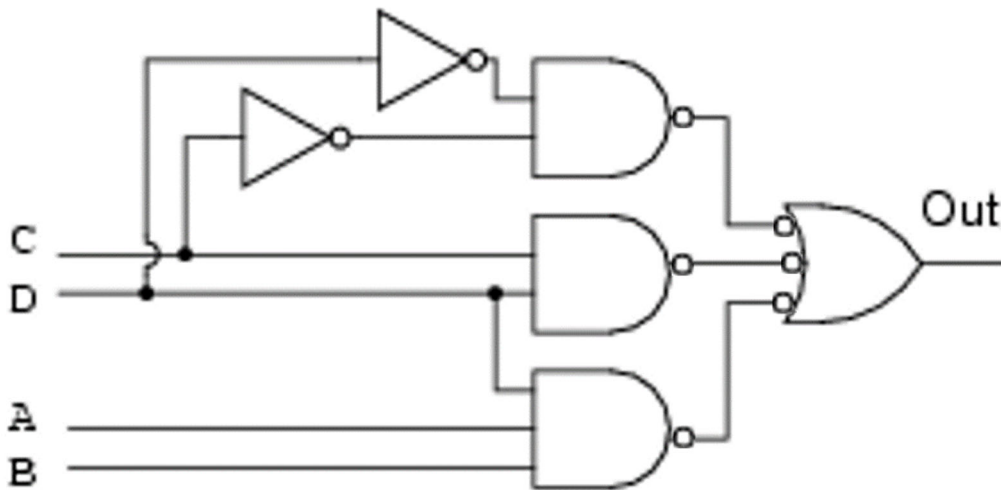
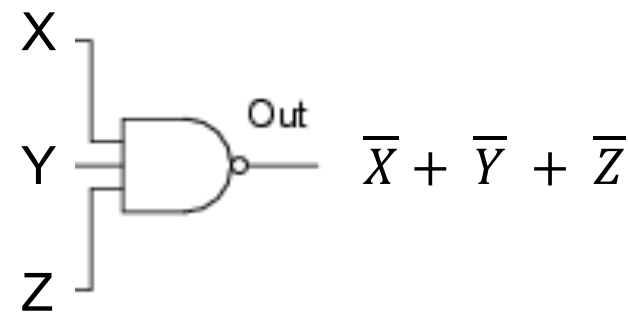
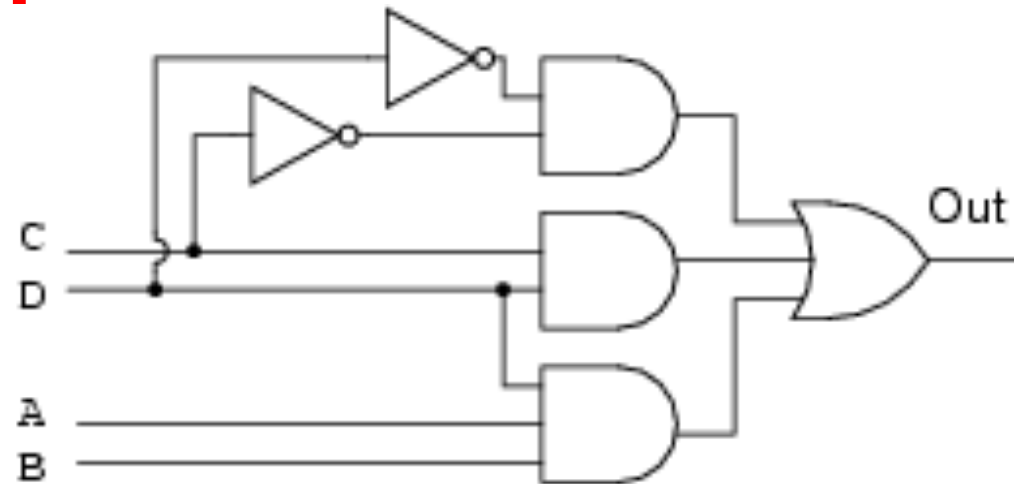
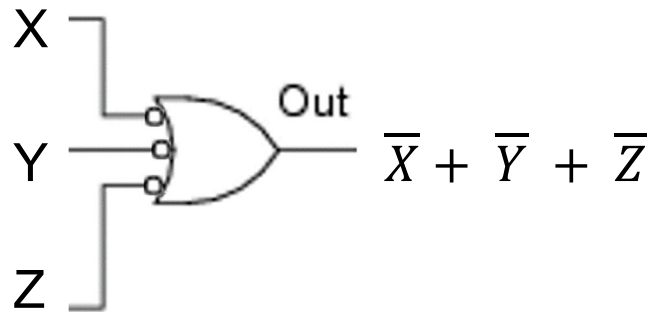


- An inverter can be also replaced by a NAND gate, as  $\overline{X} \cdot \overline{X} = \overline{X}$
- The sum-of-products can be implemented by NAND gates only.



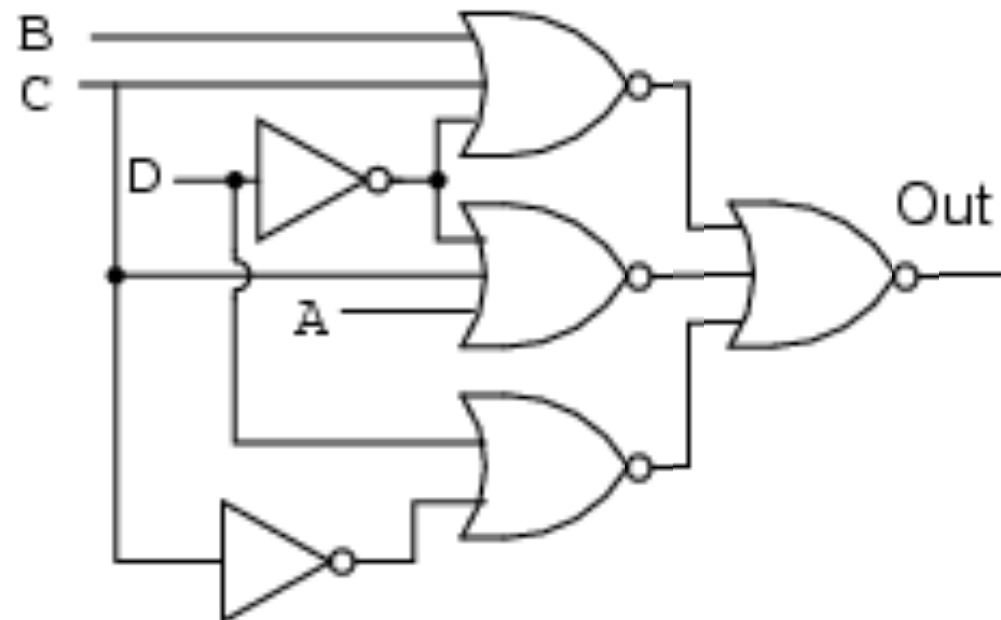
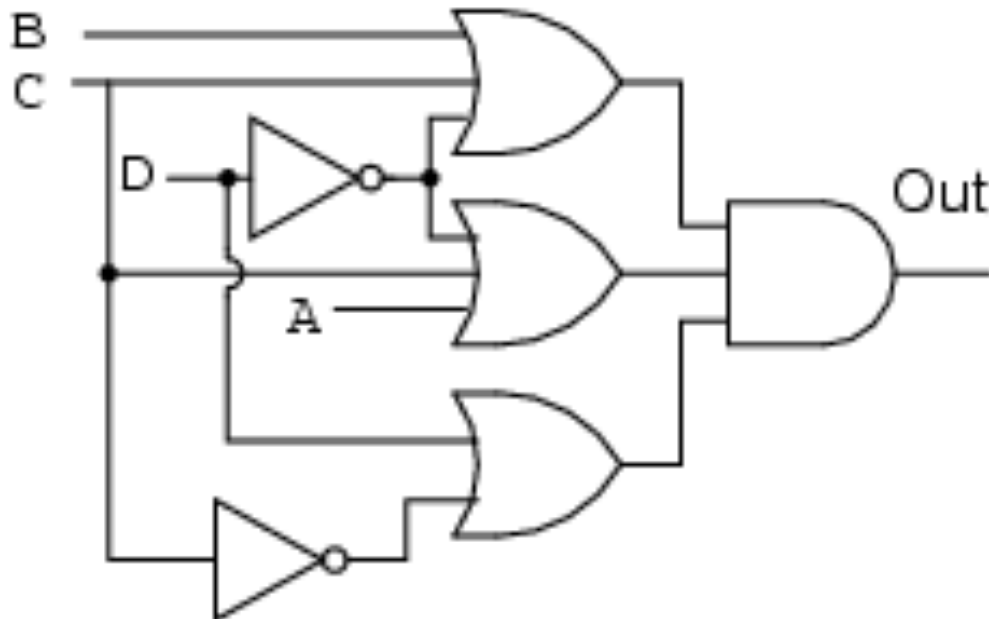
## Example 2

- $out = \overline{C} \overline{D} + CD + ABD$
- $out = \overline{\overline{\overline{C} \overline{D} + CD + ABD}}$
- $out = \overline{(\overline{\overline{C} \overline{D}} \cdot \overline{CD} \cdot \overline{ABD})}$

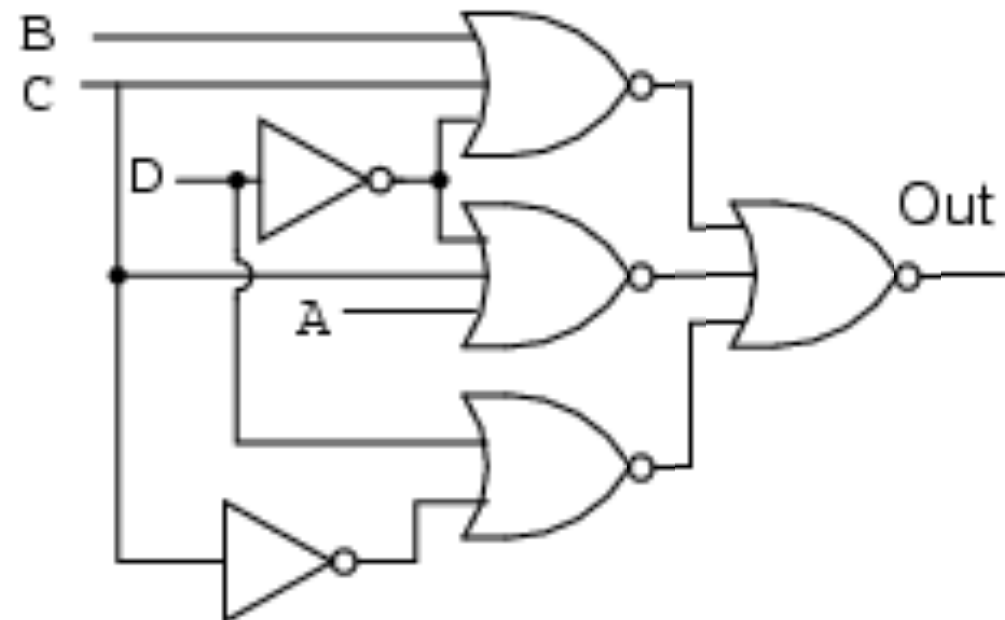
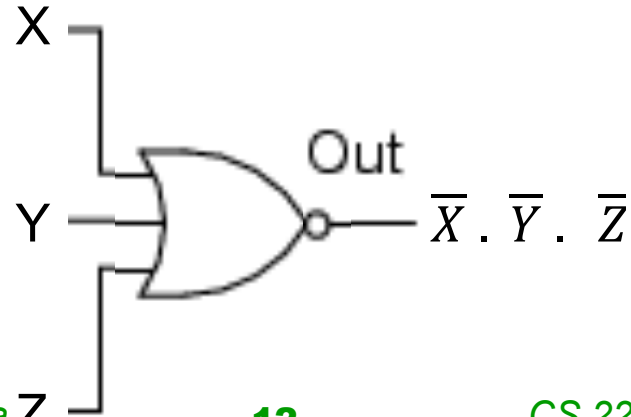
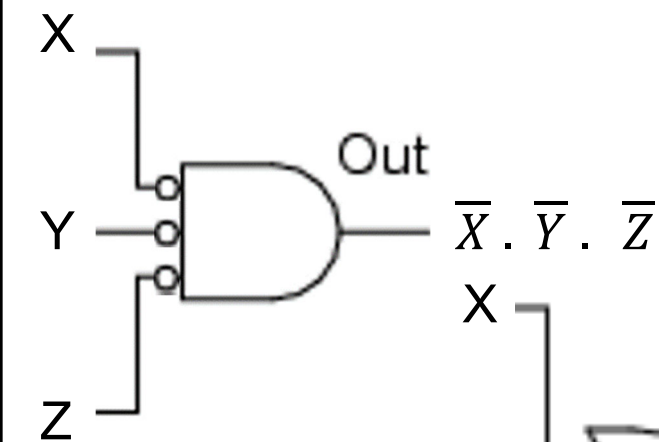
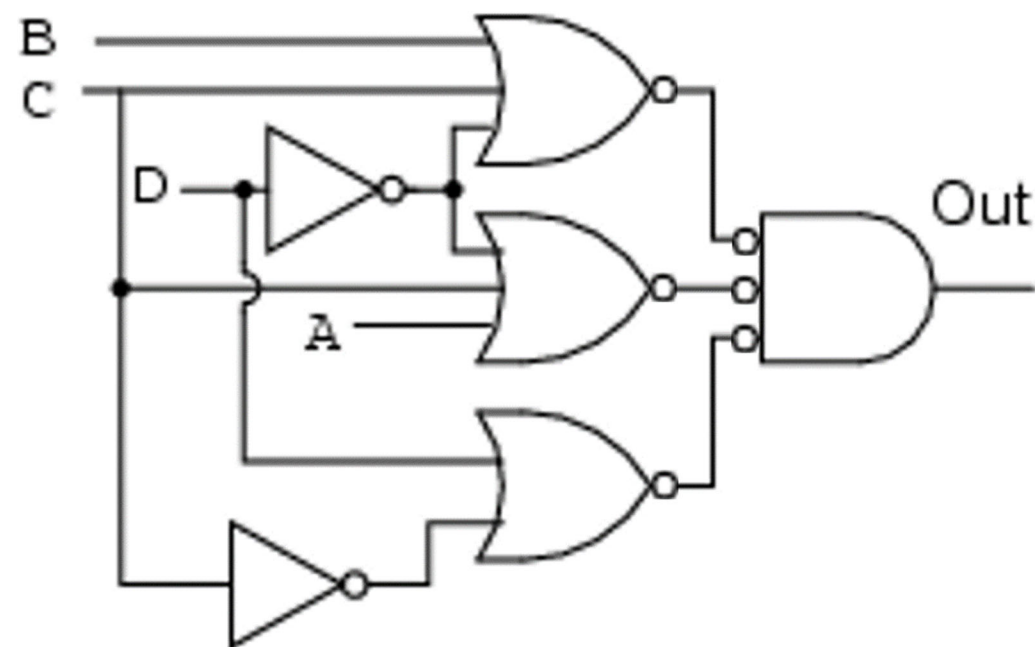
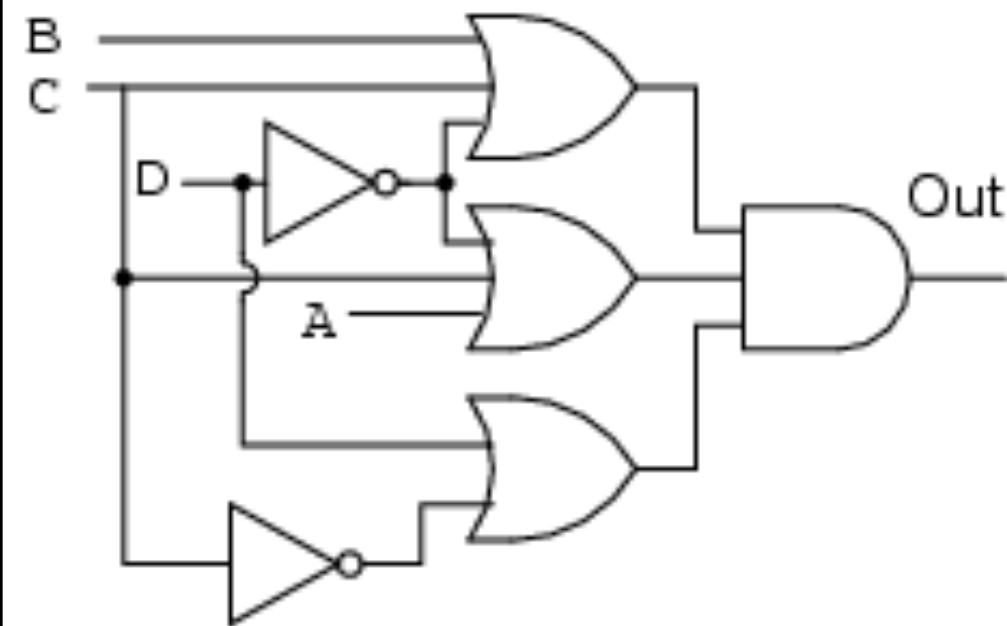


## Example 3

- $out = (B + C + \overline{D}) \cdot (A + C + \overline{D}) \cdot (\overline{C} + D)$
- $out = \overline{\overline{(B + C + \overline{D}) \cdot (A + C + \overline{D}) \cdot (\overline{C} + D)}}$
- $out = \overline{((\overline{B + C + \overline{D}}) + (\overline{A + C + \overline{D}}) + (\overline{\overline{C} + D}))}$
- An inverter can be also replaced by a NOR gate, as  $\overline{X + X} = \overline{X}$
- The product-of-sums can be implemented by NOR gates only.



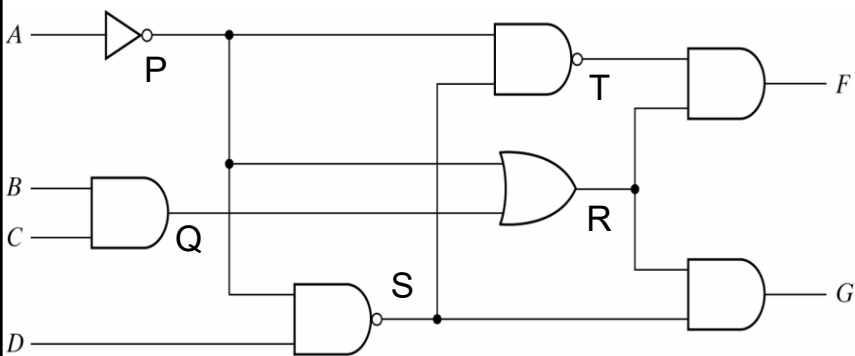
# Example 3



# Example 4

$$P = \bar{A}$$

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1						
0	0	0	1	1						
0	0	1	0	1						
0	0	1	1	1						
0	1	0	0	1						
0	1	0	1	1						
0	1	1	0	1						
0	1	1	1	1						
1	0	0	0	0						
1	0	0	1	0						
1	0	1	0	0						
1	0	1	1	0						
1	1	0	0	0						
1	1	0	1	0						
1	1	1	0	0						
1	1	1	1	0						

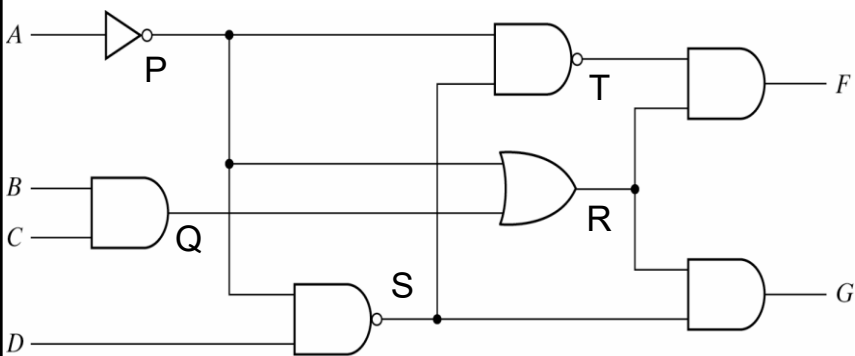


# Example 4

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0					
0	0	0	1	1	0					
0	0	1	0	1	0					
0	0	1	1	1	0					
0	1	0	0	1	0					
0	1	0	1	1	0					
0	1	1	0	1	1					
0	1	1	1	1	1					
1	0	0	0	0	0					
1	0	0	1	0	0					
1	0	1	0	0	0					
1	0	1	1	0	0					
1	1	0	0	0	0					
1	1	0	1	0	0					
1	1	1	0	0	1					
1	1	1	1	0	1					

$$P = \bar{A}$$

$$Q = B \cdot C$$



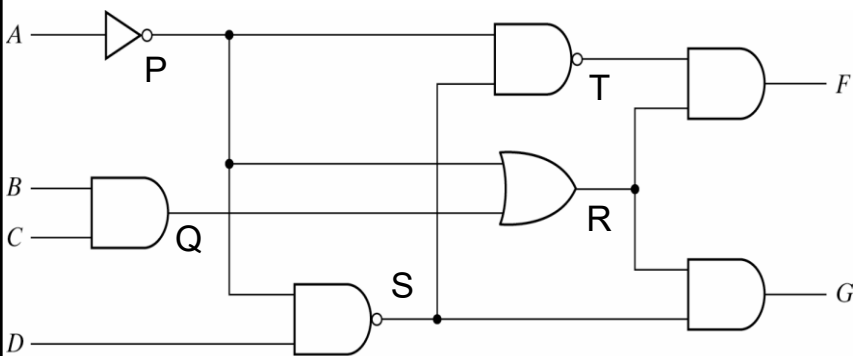
# Example 4

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0	1				
0	0	0	1	1	0	1				
0	0	1	0	1	0	1				
0	0	1	1	1	0	1				
0	1	0	0	1	0	1				
0	1	0	1	1	0	1				
0	1	1	0	1	1	1				
0	1	1	1	1	1	1				
1	0	0	0	0	0	0				
1	0	0	1	0	0	0				
1	0	1	0	0	0	0				
1	0	1	1	0	0	0				
1	1	0	0	0	0	0				
1	1	0	1	0	0	0				
1	1	1	0	0	1	1				
1	1	1	1	0	1	1				

$$P = \bar{A}$$

$$Q = B \cdot C$$

$$R = P + Q = \bar{A} + B \cdot C$$



# Example 4

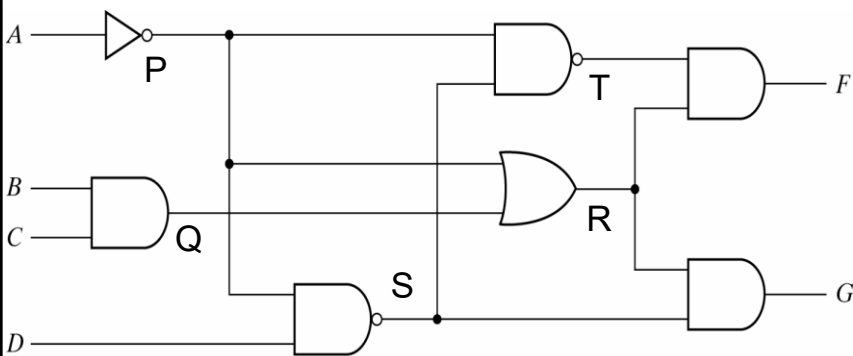
A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0	1	1			
0	0	0	1	1	0	1	0			
0	0	1	0	1	0	1	1			
0	0	1	1	1	0	1	0			
0	1	0	0	1	0	1	1			
0	1	0	1	1	0	1	0			
0	1	1	0	1	1	1	1			
0	1	1	1	1	1	1	0			
1	0	0	0	0	0	0	1			
1	0	0	1	0	0	0	1			
1	0	1	0	0	0	0	1			
1	0	1	1	0	0	0	1			
1	1	0	0	0	0	0	1			
1	1	0	1	0	0	0	1			
1	1	1	0	0	1	1	1			
1	1	1	1	0	1	1	1			

$$P = \bar{A}$$

$$Q = B \cdot C$$

$$R = P + Q = \bar{A} + B \cdot C$$

$$S = \overline{P \cdot D} = \overline{\bar{A} \cdot D} = \bar{\bar{A}} + \bar{D} = A + \bar{D}$$





# Example 4

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0	1	1	0		
0	0	0	1	1	0	1	0	1		
0	0	1	0	1	0	1	1	0		
0	0	1	1	1	0	1	0	1		
0	1	0	0	1	0	1	1	0		
0	1	0	1	1	0	1	0	1		
0	1	1	0	1	1	1	1	0		
0	1	1	1	1	1	1	0	1		
1	0	0	0	0	0	0	1	1		
1	0	0	1	0	0	0	1	1		
1	0	1	1	0	0	0	1	1		
1	1	0	0	0	0	0	1	1		
1	1	0	1	0	0	0	1	1		
1	1	1	0	0	1	1	1	1		
1	1	1	1	0	1	1	1	1		

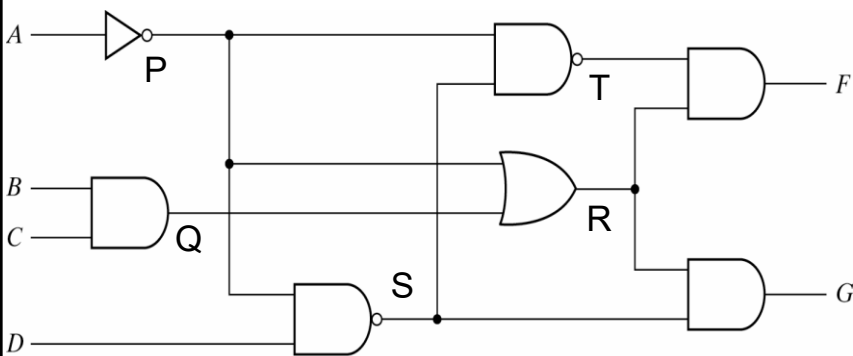
$$P = \bar{A}$$

$$Q = B \cdot C$$

$$R = P + Q = \bar{A} + B \cdot C$$

$$S = \overline{P \cdot D} = \overline{\bar{A} \cdot D} = \bar{\bar{A}} + \bar{D} = A + \bar{D}$$

$$T = \overline{P \cdot S} = \overline{\bar{A} \cdot (A + \bar{D})} = \overline{\bar{A} \cdot A + \bar{A} \cdot \bar{D}} = \overline{\bar{A} \cdot \bar{D}} = A + D$$



# Example 4

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0	1	1	0	0	
0	0	0	1	1	0	1	0	1	1	
0	0	1	0	1	0	1	1	0	0	
0	0	1	1	1	0	1	0	1	1	
0	1	0	0	1	0	1	1	0	0	
0	1	0	1	1	0	1	0	1	1	
0	1	1	0	1	1	1	1	0	0	
0	1	1	1	1	1	1	0	1	1	
1	0	0	0	0	0	0	1	1	0	
1	0	0	1	0	0	0	1	1	0	
1	0	1	0	0	0	0	1	1	0	
1	0	1	1	0	0	0	1	1	0	
1	1	0	0	0	0	0	1	1	0	
1	1	0	1	0	0	0	1	1	0	
1	1	1	0	0	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	

$$P = \bar{A}$$

$$Q = B \cdot C$$

$$R = P + Q = \bar{A} + B \cdot C$$

$$S = \overline{P \cdot D} = \overline{\bar{A} \cdot D} = \bar{\bar{A}} + \bar{D} = A + \bar{D}$$

$$T = \overline{P \cdot S} = \overline{\bar{A} \cdot (A + \bar{D})} = \overline{\bar{A} \cdot A + \bar{A} \cdot \bar{D}} = \overline{\bar{A} \cdot \bar{D}} = A + D$$

$$F = T \cdot R = (A + D) \cdot (\bar{A} + B \cdot C)$$

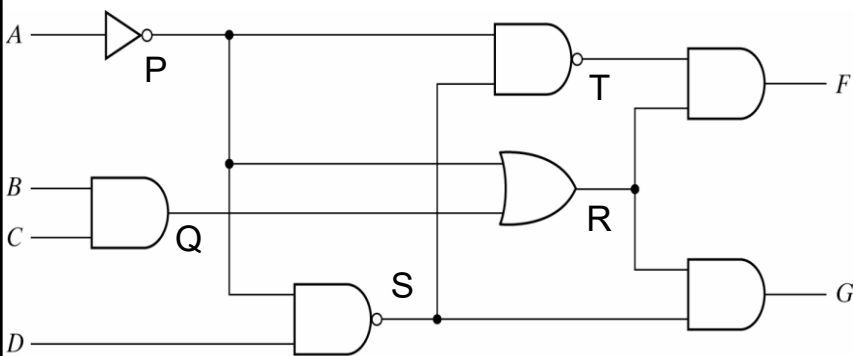
$$= A \cdot \bar{A} + A \cdot B \cdot C + \bar{A} \cdot D + B \cdot C \cdot D$$

$$= 0 + A \cdot B \cdot C + \bar{A} \cdot D + (A \cdot B \cdot C \cdot D + \bar{A} \cdot B \cdot C \cdot D)$$

$$= (A \cdot B \cdot C + A \cdot B \cdot C \cdot D) + (\bar{A} \cdot D + \bar{A} \cdot D \cdot B \cdot C)$$

$$= A \cdot B \cdot C \cdot (1 + D) + \bar{A} \cdot D \cdot (1 + B \cdot C)$$

$$= A \cdot B \cdot C + \bar{A} \cdot D$$



# Example 4

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0	1	1	0	0	1
0	0	0	1	1	0	1	0	1	1	0
0	0	1	0	1	0	1	1	0	0	1
0	0	1	1	1	0	1	0	1	1	0
0	1	0	0	1	0	1	1	0	0	1
0	1	0	1	1	0	1	0	1	1	0
0	1	1	0	1	1	1	1	0	0	1
0	1	1	1	1	1	1	0	1	1	0
1	0	0	0	0	0	0	1	1	0	0
1	0	0	1	0	0	0	1	1	0	0
1	0	1	1	0	0	0	1	1	0	0
1	1	0	0	0	0	0	1	1	0	0
1	1	0	1	0	0	0	1	1	0	0
1	1	1	0	0	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1

$$P = \bar{A}$$

$$Q = B \cdot C$$

$$R = P + Q = \bar{A} + B \cdot C$$

$$S = \overline{P \cdot D} = \overline{\bar{A} \cdot D} = \bar{\bar{A}} + \bar{D} = A + \bar{D}$$

$$T = \overline{P \cdot S} = \overline{\bar{A} \cdot (A + \bar{D})} = \overline{\bar{A} \cdot A + \bar{A} \cdot \bar{D}} = \overline{\bar{A} \cdot \bar{D}} = A + D$$

$$F = T \cdot R = (A + D) \cdot (\bar{A} + B \cdot C)$$

$$= A \cdot \bar{A} + A \cdot B \cdot C + \bar{A} \cdot D + B \cdot C \cdot D$$

$$= 0 + A \cdot B \cdot C + \bar{A} \cdot D + (A \cdot B \cdot C \cdot D + \bar{A} \cdot B \cdot C \cdot D)$$

$$= (A \cdot B \cdot C + A \cdot B \cdot C \cdot D) + (\bar{A} \cdot D + \bar{A} \cdot D \cdot B \cdot C)$$

$$= A \cdot B \cdot C \cdot (1 + D) + \bar{A} \cdot D \cdot (1 + B \cdot C)$$

$$= A \cdot B \cdot C + \bar{A} \cdot D$$

$$G = S \cdot R = (A + \bar{D}) \cdot (\bar{A} + B \cdot C)$$

$$= A \cdot \bar{A} + A \cdot B \cdot C + \bar{A} \cdot \bar{D} + B \cdot C \cdot \bar{D}$$

$$= 0 + A \cdot B \cdot C + \bar{A} \cdot \bar{D} + (A \cdot B \cdot C \cdot \bar{D} + \bar{A} \cdot B \cdot C \cdot \bar{D})$$

$$= (A \cdot B \cdot C + A \cdot B \cdot C \cdot \bar{D}) + (\bar{A} \cdot \bar{D} + \bar{A} \cdot \bar{D} \cdot B \cdot C)$$

$$= A \cdot B \cdot C \cdot (1 + \bar{D}) + \bar{A} \cdot \bar{D} \cdot (1 + B \cdot C)$$

$$= A \cdot B \cdot C + \bar{A} \cdot \bar{D}$$

