Strategy: To use the given of the form $\forall x\, P(x)$:

- For any a you can choose, add $P(a)$ to the given (more than once if you want)

Ex: Suppose F and G are families of sets, their intersection is not empty. Then, proof $\bigcap F \subseteq \bigcup G$.

	Given	Goal
	$F \cap G \neq \emptyset$	$\bigcap F \subseteq \bigcup G$.
	$\exists A (A \in F \wedge A \in G)$	$\cancel{\forall x (x \in \bigcap F \rightarrow x \in \bigcup G)}$
one use of A	$A_0 \in F \wedge A_0 \in G$	$x \in \bigcup G$
	$x \in \bigcap F$	$\exists B \in G (x \in B)$
	$\forall A \in F (x \in A)$	$B = A_0$
	$x \in A_0$	$x \in B$

Proof: Let $x \in \bigcap F$. Since $F \cap G \neq \emptyset$, let A_0 be a set with $A_0 \in F$ and $A_0 \in G$. Since $x \in \bigcap F$, $x \in A_0$ But $A_0 \in G$ so $x \in \bigcup G$. Since x is arbitrary, $\bigcap F \subseteq \bigcup G$ $\square$

Exercise 3.3.16: if $F \subseteq P(B)$, then $\bigcup F \subseteq B$

Proof: Suppose $F \subseteq P(B)$, let $x \in \bigcup F$. So let $A \in F$ such that $x \in A$, Since $A \in F$ and $F \subseteq P(B)$, so we have $A \in P(B)$. So $A \subseteq P(B)$. Since $x \in A$, we get that $x \in B$. Since x is arbitrary, $\bigcup F \subseteq P(B)$. Therefore, $F \subseteq P(B)$ implies $\bigcup F \subseteq B$ $\square$

Exercise 6(a) Let $x \in \mathbb{R}$. Prove that if $x \neq 1$ then there exist a y that $\frac{y+1}{y-2} = x$.

Given $\qquad\qquad\qquad$ Goal

$x \in \mathbb{R}$ $\quad x \neq 1$ $\qquad\qquad$ $\exists y\ \frac{y+1}{y-2} = x \longrightarrow y = \frac{-2x-1}{1-x}$

Proof: Let x be a real number and assume $x \neq 1$. Let $y = \frac{-2x-1}{1-x}$, which is well-defined since $x \neq 1$. Note that $y \neq 2$ since if $y = 2$, $2 - 2x = -2x - 1$, which is a contradiction.

Now: $\frac{y+1}{y-2} = \frac{\frac{-2x-1}{1-x} + 1}{\frac{-2x-1}{1-x} - 2} = x$ as required. $\square$

the transition should write in a chain form.