

Q1

Statement 1: False. *integers a, b*

Proof: We assume that  $a-b=3$ , then  $4(a-b)=4 \times 3=12$ . So  $12 \mid 4(a-b)$  holds. However,  $6 \nmid 3$  does not hold. So the statement is false.  $\square$

Statement 2: True.

*n is a nature number*  
Proof: Assume that  $3 \mid n$ , then there exist an integer  $k$  such that  $n=3k$ .  
So  $2n^2+18=2(3k)^2+18=18k^2+18=9(2k^2+2)$ . Since  $k$  is an integer,  $2k^2+2$  is also an integer. Thus,  $9 \mid 2n^2+18$ . So for all  $n \in \mathbb{N}$  that if  $3 \mid n$ , then  $9 \mid 2n^2+18$ .  $\square$

*this should only be roughwork. Only show the y picked.*

Q2.  $x^2y-y-1=0$  could be translate as  $(x^2-1)y=1$ .

Since  $x > 1$ , then  $x^2-1 > 0$ . So  $y = \frac{1}{x^2-1}$ . Because  $x^2-1 > 0$ ,  $\frac{1}{x^2-1} > 0$ . Thus,  $x \in \mathbb{R}$  and  $x > 1$  implies that there exist a  $y \in \mathbb{R}$  such that  $x^2y-y-1=0$  and  $y > 0$ .  $\square$

*this should be "for every  $x \in A$ "*

Q3. Assume there exist  $x \in A$ . Since that every  $A \in \mathcal{F}$  is disjoint from  $B$ ,  $\forall x (x \in A \rightarrow x \notin B)$ . Since  $A \in \mathcal{F}$ ,  $A \subseteq \cup \mathcal{F}$ , so  $\forall x (x \in A \rightarrow x \in \cup \mathcal{F})$ . Since  $A$  is random, every  $x$  in  $A$  is in  $\cup \mathcal{F}$  and is not in  $B$ . Thus, every  $A \in \mathcal{F}$  is disjoint from  $B$  implies that  $\cup \mathcal{F}$  and  $B$  are disjoint.  $\square$ .

Q3. Assume a random  $x \in B$ . Given that every  $A \in \mathcal{F}$  is disjoint from  $B$ ,  $x$  is not in any of  $A$ . Since for every  $y$  in  $A$ ,

$y$  is in  $U^F$  because  $A \in F$ ,  $x$  is not in  $F$ . Since  $x$  is arbitrary, every  $A \in F$  is disjoint from  $B$  implies that  $U^F$  and  $B$  are disjoint  $\square$ .