Proof: using strong induction on n

let $n \in \mathbb{N}$. Assume that for every $0 \le k < n$, there exist $Q, R \in \mathbb{N}$ that $k = qm + r$ and $r < m$

Case 1: $n < m$: let $q = 0$ and $r = n$, then $qm + r = r = n$, and $r = n < m$.

Case 2: $n \geq m$: let $k = n - m$. Since $m \in \mathbb{Z}^+$, $k < n$. Since $n \geq m$, $k \geq 0$. So by inductive hypothesis, there exist $q, r \in \mathbb{N}$ that $k = qm + r$ and $r < m$. Let $q' = q + 1$, $r' = r$, then $q'm + r = (q+1)m + r = k + m = (n - m) + m = n$, $r' = r < m$.

Recall: $n > 1$ is prime if $\neg \exists a \in \mathbb{N}\ \exists b \in \mathbb{N}\ (n = ab \wedge a < n \wedge b < n)$

Thm 6.4.2: every nature number is either prime or a product of two primes.

Rough work: $P(n) = \begin{cases} \text{prime} \\ \text{product of primes} \end{cases}$

Strong induction:

Given	Goal
$n \in \mathbb{N}$	n is prime / n is product of primes.
$\forall k < n$. k is either a prime or a product of primes.	

Proof: We use strong induction. Assume $n \in \mathbb{N}$, $n > 1$, Suppose that for all k that $1 < k < n$, k is either prime or a product of primes.

if n is prime, we're done.

Assume n is not prime, then there exist $a, b \in \mathbb{N}$ that $n = ab$, $a < n$ and $b < n$. Since $a < n$, $b > 1$. Similarly, since $b < n$, $a > 1$. So by inductive hypo, a and b are prime or product of prime. Therefore $n = ab$ is a product or primes. $\square$.

Ex: define a sequence T_n by $T_1 = T_2 = T_3 = 1$
and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$
Proof that $T_n < 2^n$ for all $n \geq 1$.

Proof: We use strong induction Let $n \geq 1$ Assume that for all
k with $1 \leq k < n$, $T_k < 2^n$.
Base cases: $n = 1, 2, 3$. $T_n = 1$ and $2^n > T_n$.
Inductive cases: $n \geq 4$. Then $T_n = T_{n-1} + T_{n-2} + T_{n-3}$.

$$< 2^{n-1} + 2^{n-2} + 2^{n-3}$$
$$= \frac{7}{8} 2^n < 2^n.$$

Thm 6.4.4. well ordering principle.
Every non-empty subset of nature number have
smallest element.
Roughwork:
$\forall s \subseteq N \ (s \neq \phi \rightarrow s$ has a smallest number$)$.
Proof:
Let $S \subseteq N$ We'll prove the contrapositive. Assume s
does not have a smallest element. we'll show $\forall n \in N$
$n \notin s$.
We use strong induction. let $n \in N$. Assume that for
every $k < n$, $k \notin S$.
Then if $n \in S$, it is a smallest element.
But S does not have smallest, so $n \notin S$ $\triangleleft$