

A low-angle, upward-looking photograph of several large, light-colored stone columns of a classical building. The columns are fluted and have ornate capitals. The sky is a clear, bright blue. The perspective creates a sense of height and grandeur.

# **COST-BENEFIT ANALYSIS**

## **Chapter 3**

Presentation prepared by Sevren Williams to teach Economics  
2160A: Public Finance – Revenue at Western University in  
Summer 2023. These slides are for that purpose only.  
swill43@uwo.ca  
©2016 by McGraw-Hill Education Limited

# Learning Objectives

- Distinguish between a Pareto improving allocation of resources and a potential Pareto improvement.
- Calculate the present value of future cash flows.
- State the present value criteria for project evaluation.
- Explain how to use changes in consumer surplus to estimate project benefits.

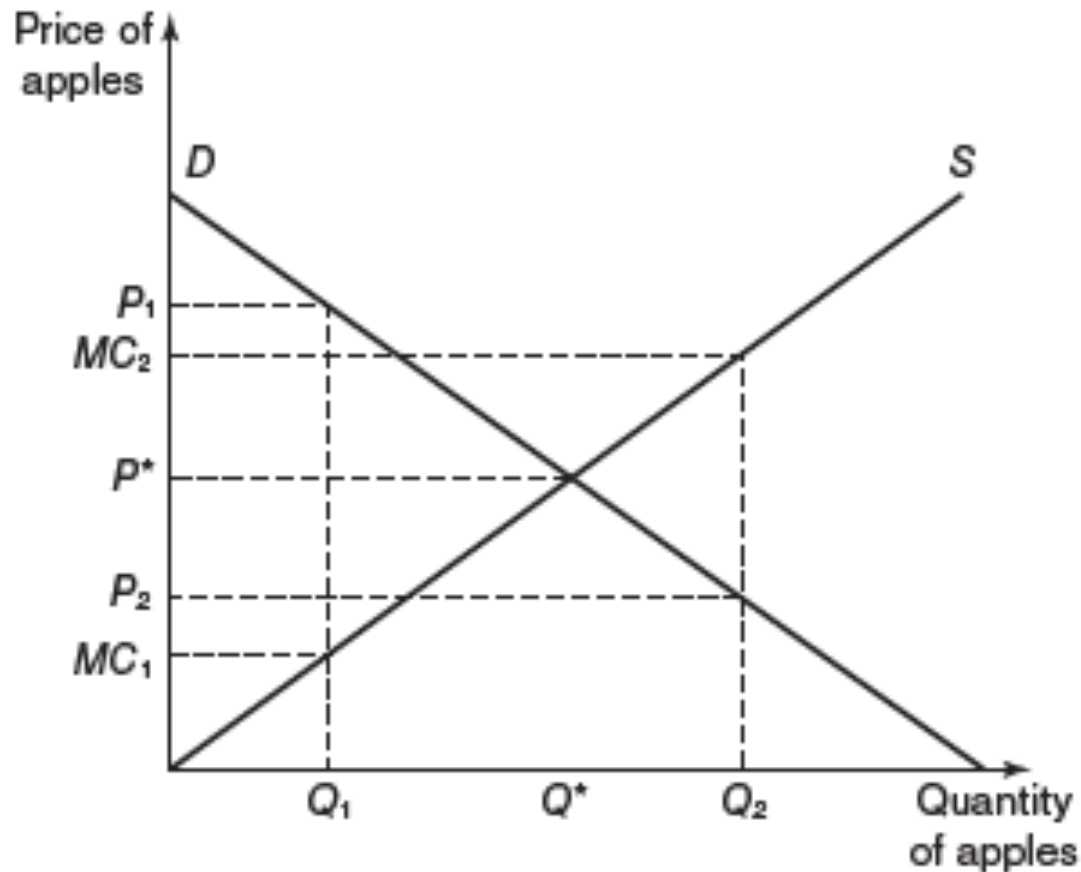
# Governments Implement Policies to Improve Market Outcomes

- For example, the federal government proposed a project to build a new airport in Pickering to open in 2027
- The price is approximately \$2 billion
- The federal government argues the airport will be necessary to meet future travel demands
- Critics are concerned about financial and environmental costs
- How should policymakers evaluate whether the benefits are worth the costs?

# Economic Efficiency

- When resources are allocated inefficiently, then there is a Pareto improvement available: someone can gain and no one need lose
- Suppose the benefits of a project exceed its costs, thereby increasing the “size of the economic pie”
  - Beneficiaries can fully compensate the losers and still have some benefits left over, generating a Pareto improvement
- The difference between benefits (B) and costs (C) can be regarded as the contribution of a project to economic efficiency
  - $\text{Net Return} = B - C$

# Efficiency of the Competitive Equilibrium



**Figure 3.1**

# The First Fundamental Theorem of Welfare Economics

## 1. Given:

- All producers and consumers act as perfect competitors; no one has any market power
- A market exists for each and every commodity

## 2. The First Fundamental Theorem of Welfare Economics states that a Pareto Efficient allocation of resources emerges

- A competitive economy “automatically” allocates resources efficiently without any need for centralized direction

# The Second Fundamental Theorem of Welfare Economics

1. Addresses equity concerns in allocations of goods
2. Second Fundamental Theory of Welfare Economics states that society can attain any Pareto efficient allocation of resources – one that is more equitable – by making a suitable assignment of initial endowments and then letting people freely trade with each other
3. Equity can be achieved without inhibiting efficiency

# Consumer Surplus

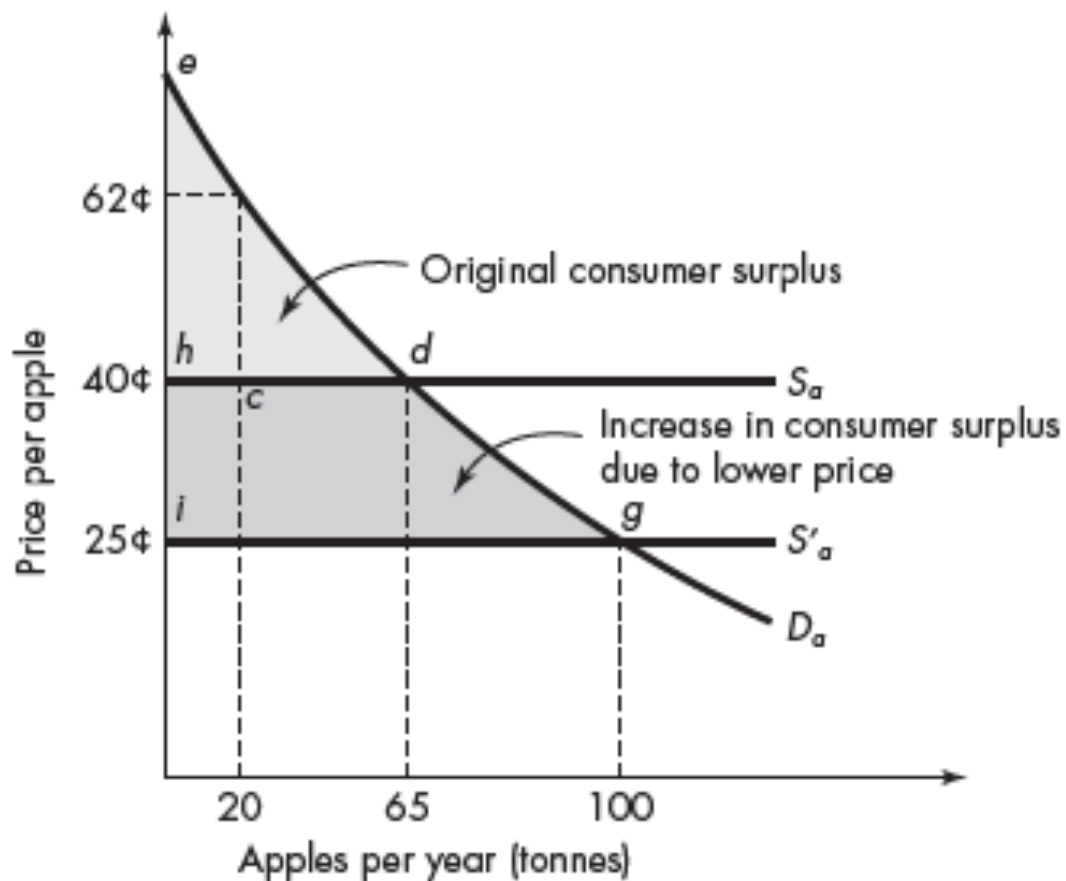


Figure 3A.1



# Producer Surplus

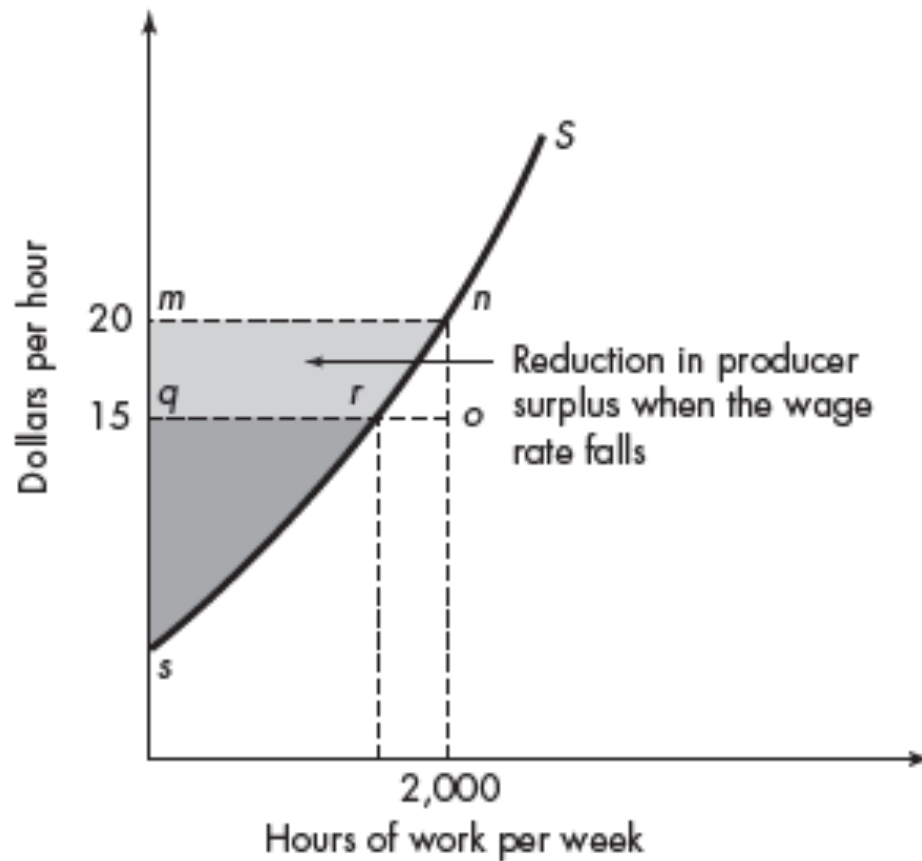


Figure 3A.2

# Distributional Considerations

- Should who gains and who loses be taken into account? Should benefits and costs be weighted?
- NO: Hicks-Kaldor Criterion – a project should be undertaken if it has positive net present value, regardless of distributional consequences
- NO: Let the government costlessly correct any undesirable distributional aspects
- NO: Relies too much on value judgments and politics

# Present Value

- Project evaluation usually requires comparing costs and benefits from different time periods.
- The present value of a future amount of money is the maximum amount you would be willing to pay today for the right to receive the money in the future.

# Projecting Present Dollars Into The Future

R=\$ T=years r=interest rate

How much will \$100 earn in 2 years at an interest rate of 5%?

$$R_0 = \$100$$

$$R_1 = \$100 * (1 + .05) = \$105$$

$$R_2 = \$105 * (1 + .05) = \$110.25$$

$$R_2 = \$100 * (1 + .05)^2 = \$110.25$$

$$R_T = R_0 * (1 + r)^T$$

# Discounting Future Dollars Into The Present

How much will \$100 earned in 2 years at an interest rate of 5% be worth today?

$$\text{Since } R_T = R_0 * (1+r)^T$$

Present Value

$$R_0 = R_T / \underbrace{(1+r)^T}_{\text{discount factor}}$$

discount rate

Low  $r$  – more future-oriented and benefits projects in which returns are concentrated further into the future

High  $r$  – more present-oriented and benefits projects in which returns are concentrated closer into the future

## Present Value of a Stream of Money

$$PV = R_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \dots + \frac{R_T}{(1+r)^T}$$

# Inflation

How to incorporate inflation – price level increases – into the procedure?

Given:  $\pi$  = inflation rate

$$PV = R_0 + \frac{(1 + \pi)R_1}{(1 + \pi)(1 + r)} + \frac{(1 + \pi)^2 R_2}{(1 + \pi)^2 (1 + r)^2} + \dots + \frac{(1 + \pi)^T R_T}{(1 + \pi)^T (1 + r)^T}$$

**However,  $(1 + \pi)$  terms cancel out, leaving the PV equation from previous slide!**

**CAUTION:** \$ values and  $r$  values must be measured consistently – if real values are used for  $R$ , the  $r$  must be measured in real terms

# Private Sector Project Evaluation

**A useful starting point:** a private firm's point of view.

- A project is admissible only if its net return is positive; that is, the benefits exceed the costs.
- If both projects are admissible, the firm should choose the project with the higher net return.
- In reality, most projects involve a stream of real benefits and returns that occur over time rather than instantaneously.

The **Present-Value Criterion** is the most reliable evaluation guide.

- Both IRR and Benefit-Cost Ratio Criteria can lead to incorrect inferences.



# Private Sector Project Evaluation – Present Value Criteria

$$PV = B_0 - C_0 + \frac{B_1 - C_1}{1 + r} + \frac{B_2 - C_2}{(1 + r)^2} + \dots + \frac{B_T - C_T}{(1 + r)^T}$$

Annual Net Return			Present Value		
Year	R&D	Oil Well	$r =$	R&D	Oil Well
0	\$1,000	-\$1,000	0	\$150	\$200
1	600	0	0.01	128	165
2	0	0	0.05	46	37
3	550	1,200	0.07	10	-21

**Table 3.2**

**Note** choice of  $r$  is critical:

- Low  $r$  benefits Oil Well; High  $r$  benefits R&D.

# Private Sector Project Evaluation – Internal Rate of Return

**IRR:** Discount rate that would make a project's NPV zero

$$PV = B_0 - C_0 + \frac{B_1 - C_1}{1 + \rho} + \frac{B_2 - C_2}{(1 + \rho)^2} + \dots + \frac{B_T - C_T}{(1 + \rho)^T} = 0$$

Project	Year 0	Year 1	$\rho$	Profit	PV
X	-\$100	\$110	10%	\$4	3.77
Y	-\$1,000	\$1,080	8%	\$20	18.87

This criterion is flawed when comparing projects of much differing sizes. Although X has the higher IRR, Y yields the higher profit.

**Note** that the PV criteria, using  $r=6\%$ , would prefer Y.

# Private Sector Project Evaluation Benefit-Cost Ratio

$$B = B_0 + \frac{B_1}{1+r} + \frac{B_2}{(1+r)^2} + \dots + \frac{B_T}{(1+r)^T}$$

$$C = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_T}{(1+r)^T}$$

Benefit-cost ratio =  $B/C$

- Admissibility requires that a project's benefit–cost ratio exceed one.
- Application of this rule always gives correct guidance.
- To see why, note simply that  $B/C > 1$  implies that  $B - C > 0$ , which is equivalent to the present value criterion for admissibility.

# Problems with the Benefit-Cost Ratio: “Costs or Negative Benefits?”

Method	B	C	B/C
I	\$250M	\$100M	2.5
II	\$200M	\$100M	2.0
Suppose that \$40 of costs need to be added to method I			
I: Subtract \$40M from B?	\$210M	\$100M	2.1
OR			
I: Add \$40M to C?	\$250M	\$140M	1.79

Benefit-cost criterion can lead to incorrect inferences

## Questions For Discussion (1 of 5)

Suppose the interest rate is  $r = 5\%$ . The government creates a project with an up-front cost of \$100 in year 1, and a return of \$100 in years 2,3 and 4.

- What is the present value of the project?
- Is the project admissible?
- Suppose the inflation rate is 2% and the inflation is anticipated. What is the present value of the project now?
- What if the inflation is not anticipated in the payments? What is the present value of the project?

## Questions For Discussion (2 of 5)

Suppose there are two competing projects. The interest rate is 4%. The cost and benefit schedule for each is listed below:

Project 1			Project 2		
Year	Cost	Benefit	Year	Cost	Benefit
0	\$0	\$100	0	\$1000	\$0
1	0	200	1	800	1,200
2	0	300	2	600	1,600
3	0	400	3	400	2,000

- What is the present value of each project?
- Which projects, if any, are admissible?
- Which project is optimal?

## Questions For Discussion (3 of 5)

A project yields an annual benefit of \$25 a year, starting next year and continuing forever. What is the present value of the benefits if the interest rate is 10 percent? (*Hint:* The infinite sum  $x + x^2 + x^3 + \dots$  is equal to  $x/(1 - x)$ , where  $x$  is a number less than 1.) Generalize your answer to show that if the perpetual annual benefit is  $B$  and the interest rate is  $r$ , then the present value is  $B/r$ .

## Questions For Discussion (4 of 5)

A proposed irrigation project would lower the constant marginal cost of producing rice from \$1,000 to \$600 per unit. The local market for rice is competitive and has a demand curve given by  $Q^D = 7,500 - 0.5P$  where  $P$  is the price. Draw a graph of the supply and demand for rice to show the change in consumer surplus due to the irrigation project. Calculate the increase in the net benefits to consumers. If the irrigation project will cost taxpayers \$28.1 million, is it worth undertaking?



## Questions For Discussion (5 of 5)

Bill rides the subway at a cost of 75 cents per trip, but would switch if the price were any higher. His only alternative is a bus that takes five minutes longer, but costs only 50 cents. He makes ten trips per year. The city is considering renovations of the subway system that would reduce the trip by ten minutes, but fares would rise by 40 cents per trip to cover the costs. The fare increase and reduced travel time both take effect in one year and last forever. The interest rate is 25 percent.

- a. As far as Bill is concerned, what are the present values of the project's benefits and costs?
- b. The city's population consists of 55,000 middle-class people, all of whom are identical to Bill, and 5,000 poor people. Poor people are either unemployed or have jobs close to their homes, so they do not use any form of public transportation. What are the total benefits and costs of the project for the city as a whole? What is the net present value of the project?
- c. Some members of the city council propose an alternative project that consists of an immediate tax of \$1.25 per middle-class person to provide "free" legal services for the poor in both of the following two years. The legal services are valued by the poor at a total of \$62,500 per year. (Assume this amount is received at the end of each of the two years.) What is the present value of the project?
- d. If the city must choose between the subway project and the legal services project, which should it select?
- e. \* What is the "distributional weight" of each dollar received by a poor person that would make the present values of the two projects just equal? That is, how much must each dollar of income to a poor person be weighted relative to that of a middle-class person? Interpret your answer.