Q1

Assume that $n \in \mathbb{Z}$, $n \equiv 0 \pmod 3$. So there exist $x \in \mathbb{Z}$ that $n = 3x$

Case 1: Assume that n is an odd number. Since $n = 3x$, x must be odd and there exist $k \in \mathbb{Z}$ that $x = 2k+1$. Thus, $n^2 = (3x)^2 = [3(2k+1)^2]$, which is $36k^2+36k+9$, and it could be rewritten as $6(6k^2+6k+1)+3$, Since $k \in \mathbb{Z}$, $n^2 \equiv 3 \pmod 6$.

Case 2: Assume that n is a even number. Since $n = 3x$, x must be even and there exist $k \in \mathbb{Z}$ that $x = 2k$. Thus, $n^2 = (3x)^2 = [3(2k)]^2$, which is $36k^2$, and it could be rewritten as $6(6k)$. Since $k \in \mathbb{Z}$, $n^2 \equiv 0 \pmod 6$.

Since these cases above are exhaustive, so for all $n \in \mathbb{Z}$, $n \equiv 0 \pmod 3$ implies that $n^2 \equiv 0 \pmod 6$ or $n^2 \equiv 3 \pmod 6$ $\square$.

Q2

$E = \{(1,1), (0,0), (2,2), (1,-1), (-1,1), (-2,2), (2,-2)\}$.

$\{2\} = \{2, -2\}$

$\{1\} = \{1, -1\}$

$\{0\} = \{0\}$.

Q3.

(a) it is not a function since they have (2,7), (2,10), one element in A has more than one relation with element in B.

(b). R^{-1} is a function and it is onto since every element in A has a corresponding relation.
It is not one-to-one since (7,2) and (10,2) are in R^{-1}

(c) It is a function. It is one-to-one and onto since every element in E has only one relation with an element in D.

Q4

(a) $f \cap g = \{(2,2), (-1,1)\}$.