


19. Given that f is a differentiable function with $f(2, 5) = 6$, $f_x(2, 5) = 1$, and $f_y(2, 5) = -1$, use a linear approximation to estimate $f(2.2, 4.9)$.

-  20. Find the linear approximation of the function $f(x, y) = 1 - xy \cos \pi y$ at $(1, 1)$ and use it to approximate $f(1.02, 0.97)$. Illustrate by graphing f and the tangent plane.

21. Find the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(3, 2, 6)$ and use it to approximate the number $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$.

22. The wave heights h in the open sea depend on the speed v of the wind and the length of time t that the wind has been blowing at that speed. Values of the function $h = f(v, t)$ are recorded in feet in the following table. Use the table to find a linear approximation to the wave height function when v is near 40 knots and t is near 20 hours. Then estimate the wave heights when the wind has been blowing for 24 hours at 43 knots.

		Duration (hours)						
Wind speed (knots)	t	5	10	15	20	30	40	50
	20	5	7	8	8	9	9	9
	30	9	13	16	17	18	19	19
	40	14	21	25	28	31	33	33
	50	19	29	36	40	45	48	50
	60	24	37	47	54	62	67	69

23. Use the table in Example 3 to find a linear approximation to the heat index function when the temperature is near 94°F and the relative humidity is near 80%. Then estimate the heat index when the temperature is 95°F and the relative humidity is 78%.

24. The wind-chill index W is the perceived temperature when the actual temperature is T and the wind speed is v , so we can write $W = f(T, v)$. The following table of values is an excerpt from Table 1 in Section 14.1. Use the table to find a linear approximation to the wind-chill index function when T is near -15°C and v is near 50 km/h. Then estimate the wind-chill index when the temperature is -17°C and the wind speed is 55 km/h.

		Wind speed (km/h)					
Actual temperature (°C)	T v	20	30	40	50	60	70
	−10	−18	−20	−21	−22	−23	−23
	−15	−24	−26	−27	−29	−30	−30
	−20	−30	−33	−34	−35	−36	−37
	−25	−37	−39	−41	−42	−43	−44

- 25–30 Find the differential of the function.

25. $z = e^{-2x} \cos 2\pi t$

26. $u = \sqrt{x^2 + 3y^2}$

27. $m = p^5 q^3$

28. $T = \frac{v}{1 + uvw}$

29. $R = \alpha\beta^2 \cos \gamma$

30. $L = xze^{-y^2 - z^2}$

31. If $z = 5x^2 + y^2$ and (x, y) changes from $(1, 2)$ to $(1.05, 2.1)$, compare the values of Δz and dz .

32. If $z = x^2 - xy + 3y^2$ and (x, y) changes from $(3, -1)$ to $(2.96, -0.95)$, compare the values of Δz and dz .

33. The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

34. Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.

35. Use differentials to estimate the amount of tin in a closed tin can with diameter 8 cm and height 12 cm if the tin is 0.04 cm thick.

36. The wind-chill index is modeled by the function

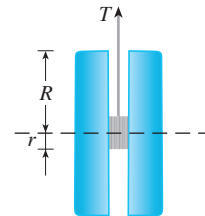
$$W = 13.12 + 0.6215T - 11.37v^{0.16} + 0.3965Tv^{0.16}$$

where T is the temperature (in $^\circ\text{C}$) and v is the wind speed (in km/h). The wind speed is measured as 26 km/h, with a possible error of ± 2 km/h, and the temperature is measured as -11°C , with a possible error of $\pm 1^\circ\text{C}$. Use differentials to estimate the maximum error in the calculated value of W due to the measurement errors in T and v .

37. The tension T in the string of the yo-yo in the figure is

$$T = \frac{mgR}{2r^2 + R^2}$$

where m is the mass of the yo-yo and g is acceleration due to gravity. Use differentials to estimate the change in the tension if R is increased from 3 cm to 3.1 cm and r is increased from 0.7 cm to 0.8 cm. Does the tension increase or decrease?



38. The pressure, volume, and temperature of a mole of an ideal gas are related by the equation $PV = 8.31T$, where P is measured in kilopascals, V in liters, and T in kelvins. Use differentials to find the approximate change in the pressure if the volume increases from 12 L to 12.3 L and the temperature decreases from 310 K to 305 K.

39. If R is the total resistance of three resistors, connected in parallel, with resistances R_1, R_2, R_3 , then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If the resistances are measured in ohms as $R_1 = 25 \Omega$, $R_2 = 40 \Omega$, and $R_3 = 50 \Omega$, with a possible error of 0.5% in each case, estimate the maximum error in the calculated value of R .

40. A model for the surface area of a human body is given by $S = 0.1091w^{0.425}h^{0.725}$, where w is the weight (in pounds), h is the height (in inches), and S is measured in square feet. If the errors in measurement of w and h are at most 2%, use differentials to estimate the maximum percentage error in the calculated surface area.
41. In Exercise 14.1.39 and Example 14.3.3, the body mass index of a person was defined as $B(m, h) = m/h^2$, where m is the mass in kilograms and h is the height in meters.
- (a) What is the linear approximation of $B(m, h)$ for a child with mass 23 kg and height 1.10 m?
- (b) If the child's mass increases by 1 kg and height by 3 cm, use the linear approximation to estimate the new BMI. Compare with the actual new BMI.
42. Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2, 1, 3)$. You don't have an equation

for S but you know that the curves

$$\mathbf{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle$$

$$\mathbf{r}_2(u) = \langle 1 + u^2, 2u^3 - 1, 2u + 1 \rangle$$

both lie on S . Find an equation of the tangent plane at P .

43–44 Show that the function is differentiable by finding values of ε_1 and ε_2 that satisfy Definition 7.

43. $f(x, y) = x^2 + y^2$ 44. $f(x, y) = xy - 5y^2$

45. Prove that if f is a function of two variables that is differentiable at (a, b) , then f is continuous at (a, b) .

Hint: Show that

$$\lim_{(\Delta x, \Delta y) \rightarrow (0, 0)} f(a + \Delta x, b + \Delta y) = f(a, b)$$

46. (a) The function

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

was graphed in Figure 4. Show that $f_x(0, 0)$ and $f_y(0, 0)$ both exist but f is not differentiable at $(0, 0)$. [Hint: Use the result of Exercise 45.]

- (b) Explain why f_x and f_y are not continuous at $(0, 0)$.

APPLIED PROJECT

THE SPEEDO LZR RACER

Many technological advances have occurred in sports that have contributed to increased athletic performance. One of the best known is the introduction, in 2008, of the Speedo LZR racer. It was claimed that this full-body swimsuit reduced a swimmer's drag in the water. Figure 1 shows the number of world records broken in men's and women's long-course freestyle swimming events from 1990 to 2011.¹ The dramatic increase in 2008 when the suit was introduced led people to claim that such suits are a form of technological doping. As a result all full-body suits were banned from competition starting in 2010.

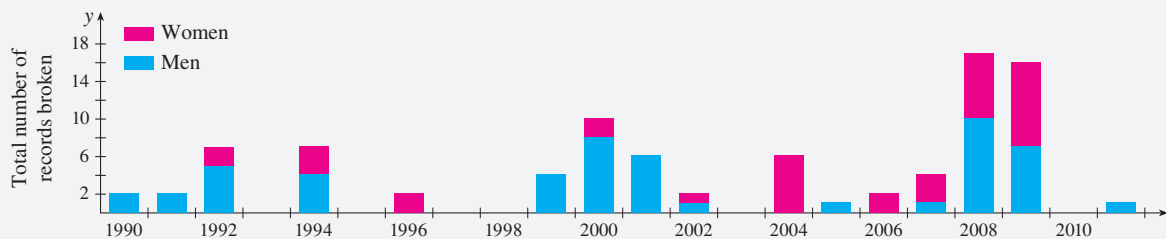


FIGURE 1 Number of world records set in long-course men's and women's freestyle swimming event 1990–2011

It might be surprising that a simple reduction in drag could have such a big effect on performance. We can gain some insight into this using a simple mathematical model.²

1. L. Foster et al., "Influence of Full Body Swimsuits on Competitive Performance," *Procedia Engineering* 34 (2012): 712–17.

2. Adapted from <http://plus.maths.org/content/swimming>.