If we compute the length $L$ for $f$ defined at $[0,\sqrt{2}]$.

$$L = \int_0^{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2-x^2}}\,dx = \sqrt{2}\,\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\Big|_0^{\sqrt{2}}$$

$$= \sqrt{2}\left[\sin^{-1}1 - 0\right]$$

$$= \sqrt{2}\cdot\frac{\pi}{2}$$

$$= \frac{\sqrt{2}\pi}{2} = 1/4 \text{ of a circle}$$

e.g. Find the arc length of a curve $y = \frac{x^3}{3} + \frac{1}{4x}$ when $x \in [1,2]$.

$$y' = x^2 - \frac{1}{4x^2}$$

$$ds = \sqrt{1+y'^2}\,dx = \sqrt{1+\left(x^2-\frac{1}{4x^2}\right)^2}\,dx$$

$$L = \int_1^2 \sqrt{1+\left(x^2-\frac{1}{4x^2}\right)^2}\,dx.$$

$$= \int_1^2 \sqrt{1+\left(x^4-\frac{1}{2}+\frac{1}{16x^4}\right)}\,dx$$

$$= \int_1^2 \left(x^2+\frac{1}{4x^2}\right)dx$$

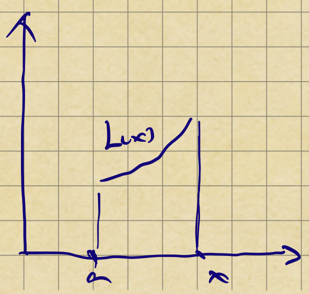
$$= \left(\frac{1}{3}x^3 - \frac{1}{4x}\right)\Big|_1^2$$

$$= \frac{1}{3}\times 8 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4}$$

$$= \frac{59}{24}.$$

Arc length function.

$$L(x) = \int \sqrt{1+(f'(t))^2}\,dt \quad t \in [a, x]$$



eg 3. $f(x)=\ln(\sin x)$ $x\in(0,\pi)$.

$$L(x)=\int_0^{\pi}\sqrt{1+[\ln(\sin x)]^2}$$

$x\to 0^+$ : $L(x)\to -\infty$
$x\to\pi$ : $L(x)\to -\infty$
} vertical asymp

$f'(x)=\frac{1}{\sin x}\frac{d}{dx}(\sin x)=\frac{\cos x}{\sin x}$. $L(x)=\int_{\frac{\pi}{2}}^{x}\sqrt{1+[f'(t)]^2}\,dt$.

$$L(x)=\int_{\frac{\pi}{2}}^{x}\sqrt{1+\cot^2 t}\;dt.$$

$$=\int_{\frac{\pi}{2}}^{x}\csc t\,dt.$$

$$=\ln|\csc t-\cot t|\,\Big|_{\frac{\pi}{2}}^{x}$$

OR $=-\ln|\csc t+\cot t|\,.\Big|_{\frac{\pi}{2}}^{x}$

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