COMPSCI 3331 - Midterm review questions

Fall 2022

These questions are provided for review purposes and are not guaranteed to be the same or similar to the questions on the midterm. Some questions may be easier than the questions on the midterm and some may be harder.

These questions cover material up to and including Lecture 10. **The final decision on the material on the midterm will be determined later.** The midterm is not guaranteed to cover all material up to Lecture 10.

Solutions to as many questions as possible will be written as I am able. There is no guarantee that all solutions will be available for all questions.

1. Disprove that for words w, x, that if $wx = xw$ then $x = w$. w/ $x = \varepsilon$.

proofs.

2. Prove or disprove that for languages $L_1, L_2 \subseteq \Sigma^*$, where Σ has at least two letters, $\overline{L_1 L_2} = \overline{L_1} L_2 \cup L_1 \overline{L_2}$.

3. Prove or disprove that for languages $L_1, L_2, L_3 \subseteq \Sigma^*$, where Σ has at least two letters, that $L_1(L_2 \cap L_3) = (L_1 L_2) \cap (L_1 L_3)$.

$\rightarrow x \in L_1(L_2 \cap L_3)$. $x = ab$ $a \in L_1, b \in L_2 \cap L_3$

Prove RE :)

4. Show that the following languages are regular. Simply give a regular expression

   (a) $L = \{w \in \{a,b\}^* \ : \ |w|_a = 0 \text{ or } |w|_b \geq 5\}$. =>

   (b) $L = \{w \in \{a,b\}^* \ : \ aa \text{ does not appear in } w\}$. => Constructing DFA/NFA.

   Constructing corresponding DFA

   (c) $L = \{w \in \{0,1\}^* \ : \ w \text{ is divisible by 4 when interpreted as a binary number } \}$ (assume that the numbers are big-endian).

   (d) $L = \{w \in \{a,b,c\}^* \ : \ \exists x \in \{a,b,c\}, u \in \{a,b,c\}^* \text{ such that } w = xux\}$.

For extra practice, try to develop multiple models for each of these languages (DFAs, NFAs, regular expressions). However, some may be difficult to do.

5. For each of the following regular expressions, construct an ε-NFA corresponding to the regular expression.

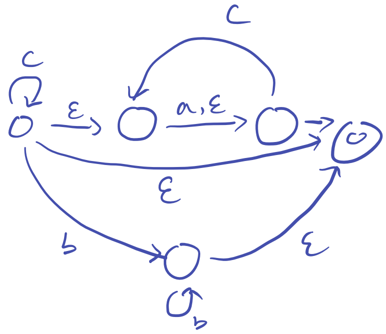
ε-NFA.

(a) $(a+b)(a+b)(a+b)^*$.

(b) $c^*((a+\varepsilon)c)^* + b^*$

(c) $(aa)^* + (bb)^* + cc$

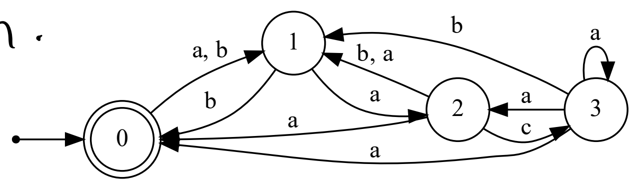
(d) $a(b(cd)^*)^*$



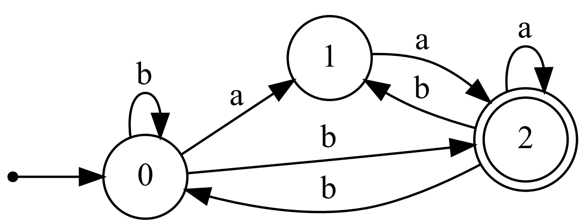
6. For each of the ε-NFAs constructed in the previous question, convert it into an NFA without ε-trasitions.

7. Convert the following NFAs to DFAs.

NFA, DFA conversion.



(a)



(b)

states alpha trans st ∃i

8. Suppose $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA that accepts a language L that is not $\emptyset$, and $M' = (2^Q, \Sigma, \delta', \{q_0\}, F')$ is the DFA constructed from M by the subset construction. Suppose that the state Q is a *reachable* state of M' (by reachable, we mean that there is a word y such that $\delta'(\{q_0\}, y) = Q$).

textured RE.

(a) Is this state a final state? Explain why or why not.

(b) Show that for **some letter** $a \in \Sigma$, the transition on a loops on this state.

(c) Show that the language L is an infinite language (that is, it has infinitely many words in the language).

9. Show that the regular languages are closed under difference. That is, if L_1, L_2 are regular languages, show that $L_1 - L_2$ is regular.

10. Show that the regular languages are closed under the operation of suff(L), which is defined as follows:

$$\text{suff}(L) = \{u \in \Sigma^* \ : \ \exists v \in L \text{ such that } u \text{ is a suffix of } v\}.$$

Pumping Lemma!

11. Show that the following languages are not regular.

pick 2 specific case.

(a) $L_2 = \{w\#y \ : \ w, y \in \{a,b\}^*, |w| < |y|\}$ $(\subseteq \{a,b,\#\}^*)$

(b) $L_3 = \{w\#y \ : \ w, y \in \{a,b\}*, |w|_a = |y|_b\}$.

(c) $L_6 = L^*$ where $L = \{a^n b^n \ : \ n \geq 0\}$

(d) $L_8 = \{w \in \{a,b\}^* \ : \ \exists u \in \{a,b\}^*, w = uu^R\}$.

12. Suppose that L_1 is a regular language and L_2 is a non-regular languages. Which of the following statements are guaranteed to be true? Select all the statements that are true. If they are not true, give a counter example.

- $\overline{L_1}$ is a regular language.
- $\overline{L_2}$ is not a regular language.
- L_1^* is a regular language. ✓
- L_2^* is not a regular language.
- $L_1 \cup L_2$ is not a regular language.
- $L_1 \cap L_2$ is not a regular language.
- $L_1 L_2$ is not a regular language.

CFGs.

13. Let $G = (V, \Sigma, P, S)$ be a context-free grammar. Show that the set of all sentential forms for G is itself a CFL.

14. Let G be the CFG defined by the following set of productions, given in BNF.

$$\begin{aligned}
S &\rightarrow bbAaA \mid SSaa \mid aa \mid ABC \\
A &\rightarrow Ab \mid Ac \mid CC \\
B &\rightarrow BA \mid bb \mid Dd \\
C &\rightarrow DA \mid \varepsilon \\
D &\rightarrow a
\end{aligned}$$

(a) $L_2 = \{w\#y \,:\, w, y \in \{a,b\}^*, |w| < |y|\}$ $(\subseteq \{a,b,\#\}^*)$

(b) $L_3 = \{w\#y \,:\, w, y \in \{a,b\}*, |w|_a = |y|_b\}$.

(c) $L_6 = L^*$ where $L = \{a^n b^n \,:\, n \geq 0\}$

(d) $L_8 = \{w \in \{a,b\}^* \,:\, \exists u \in \{a,b\}^*, w = uu^R\}$.

(a) $z = a^n \# a^{n+1}$ $n < m$.

$= uvw$

$u = a^{n-i}$ $v = a^i$ $w = \# a^{n+1}$

$uv^k w = a^{n+(k-1)i} \# a^{n+1}$

$n + (k-1)i \not\geq i$

$k = 2$ 2

$n + (n+1)i$

(b) $z = a^n \# b^n$

$= uvw$

$u = a^{n-i}$ $v = a^i$ $w = \# b^n$

$uv^k w = a^{n+(k-1)i} \# b^n$

$k = 2$.

$u = a^i$ $v = a^j$ $w = \cdots$

(a) Which nonterminals are nullable?

(b) Remove all ε-productions from the grammar.

15. Let G be the CFG defined by the following set of productions, given in BNF.

$$
\begin{aligned}
S &\rightarrow SbaB \mid aa \mid ABC \\
A &\rightarrow Ba \mid DaDd \\
B &\rightarrow BA \mid ca \mid Dd \\
C &\rightarrow DA \mid \varepsilon \\
D &\rightarrow a
\end{aligned}
$$

Convert the grammar to CNF.

16. Consider the following grammar in CNF.

$$
\begin{aligned}
S &\rightarrow AB \mid BC \\
A &\rightarrow BA \mid a \\
B &\rightarrow CC \mid b \\
C &\rightarrow AC \mid c \mid a
\end{aligned}
$$

Determine if each of these words are generated by the grammar using the CYK algorithm.

(a) cca

(b) $abbcac$

(c) $caabc$

not in mt.

1) let n be the constant described by the pumping lemma.

2) let $z =$ ----, then $z \in L$,

3) Now, by the pumping lemma we can write $z = uvw$ with $|uv| \leq n$ and $v \neq \varepsilon$

4) Thus. $u =$ ---, $v =$ --- and $w =$ ---- for i, j with $i + j \leq n$ and $j \neq 0$

5) Consider $k =$ ---- , then $uv^k w =$ ---- As $k \neq 0$, we must have ---- and so ----. Thus, ---- $\notin L$ and by pumping lemma L is not regular.