Lecture 10.

$a \le b \le c$.

$a+b+c = h$.

$abc = 72. = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$.

"my oldest child likes strawberry" => only one oldest child

=> the only case could be 3, 3, 8.

Chapter 03: proofs.

§3.1. Proof Strategies.

$P \to Q$ if P then Q.

e.g. Suppose $n \in \mathbb{N}$

Prove that if $n > 1$, then $n^2 > n$.

Given	Goal
$n \in \mathbb{N}$	$n > 1 \to n^2 > n$.

Strategy: To prove $P \to Q$. Assume P then try to prove Q. => Add P to the given, change the goal to Q.

Given	Goal.
$n \in \mathbb{N} \wedge n > 1$	$n^2 > n$

Since $n > 1$, then $n > 0$.

Multiply each sides by n, to get $n^2 > n$

Proof: Let $n \in \mathbb{N}$ and suppose $n > 1$. Since $n > 0$, we can multiply both sides by n to get

$n^2 > n$, therefore, $n > 1$ implies $n^2 > n$ $\square$

Q.E.D box.

Proof: Assume P

proof of Q.

Therefore, P implies Q $\square$.

Counter:

Proof: Let $n \in \mathbb{N}$, if $n > 0$, then $n^2 > n$.

Counterexample: $n = 1$ then $n > 0$, $n^2 = n$. and not $n^2 > n$.

Stratgy 2: To prove $P \to Q$,

Assume Q false, prove P is false.

$P \to Q$ iff $\neg Q \to \neg P$.

e.g. Proof: $x \in \mathbb{R}$, prove that $\sqrt{x^2+7} = x+2$. then $x \neq 3$.

given that $x = 3$, then $\sqrt{x^2+7} = 4$ $x+2 = 5$, $4 \neq 5$.

Therefore, $\sqrt{x^2+7} = x+2$, then $x \neq 3$ is true.

Suppose $\neg Q$,

prove $\neg P$

Therefore, $P \to Q$.

§ 3.2. Prove revolve $\neg P$ and $P \to Q$.

P_1 P_2

Suppose $A \cap C \subseteq B$, and $a \in C$,

prove $a \notin A \setminus B$. (Q).

Given	Goal
$A \cap C \subseteq B$	$a \notin A \setminus B$.
$a \in C$	$\Updownarrow$
	$a \in A \to a \in B$.

$a \notin A \setminus B$ iff $\neg(a \in A \wedge a \notin B)$

$\equiv (a \notin A \vee a \in B)$ not P or Q form.

$\equiv a \in A \to a \in B$

Given

$A \cap C \subseteq B$

$a \in C$

$a \in A$

Goal

$a \in B$.

$\Rightarrow$ Easy to see.