Tutorial 03: Addition/Subtraction using 2's Complement

Computer Science Department
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Binary Arithmetic

- These tables cover the fundamental arithmetic operations.

Addition

0 + 0 = 0 (carry 0)
0 + 1 = 1 (carry 0)
1 + 0 = 1 (carry 0)
1 + 1 = 0 (carry 1)

Subtraction

0 – 0 = 0 (borrow 0)
0 – 1 = 1 (borrow 1)
1 – 0 = 1 (borrow 0)
1 – 1 = 0 (borrow 0)

Multiplication

0 × 0 = 0
0 × 1 = 0
1 × 0 = 0
1 × 1 = 1

Addition (three bits)

0 + 0 + 0 = 0 (carry 0)
0 + 0 + 1 = 1 (carry 0)
0 + 1 + 0 = 1 (carry 0)
0 + 1 + 1 = 0 (carry 1)
1 + 0 + 0 = 1 (carry 0)
1 + 0 + 1 = 0 (carry 1)
1 + 1 + 0 = 0 (carry 1)
1 + 1 + 1 = 1 (carry 1)

Subtraction (three bits)

0 – 0 – 0 = 0 (borrow 0)
0 – 0 – 1 = 1 (borrow 1)
0 – 1 – 0 = 1 (borrow 1)
0 – 1 – 1 = 0 (borrow 1)
1 – 0 – 0 = 1 (borrow 0)
1 – 0 – 1 = 0 (borrow 0)
1 – 1 – 0 = 0 (borrow 0)
1 – 1 – 1 = 1 (borrow 1)

Sign and Magnitude Addition/Subtraction

- The operations are carried out similar to normal math calculations
- The resultant sign is arranged separately
  - The sign of A – B depends on the values of A and B
  - If B > A, the answer will be calculated as –(B – A), O.W., it is +(A – B)
- The location of the radix points needs to be aligned before performing the operation.
- If the provided number of bits are not enough to hold the result, it means an overflow occurred.

2's Complement Addition/Subtraction

- A subtraction operation is converted to an addition operation (after performing the *2's complement* to the operand appearing after the negative sign)

- When adding two ***positive*** numbers and finding the result is ***negative***, this means an ***overflow occurred***.
- When adding two ***negative*** numbers and finding the result is ***positive***, this means an ***overflow occurred***.

- Overflow will ***never occur*** when adding a positive number to a negative number, or vice versa.

- How about
  - subtracting a *negative* number from a *positive* number?
  - subtracting a *positive* number from a *negative* number?

2's Complement Addition/Subtraction

- *Example 1*:
  Perform $20_{10} - 10_{10}$ using 2's complement 6-bit system
- $20_{10} \rightarrow 10100_2$
- $10_{10} \rightarrow 1010_2$
- $20_{10} - 10_{10} \rightarrow 10100_2 - 1010_2$

$\rightarrow 010100_2 - 001010_2$

$\rightarrow 010100_2 + (-001010_2)$

$\rightarrow 010100_2 + 110110_2$

$\rightarrow 001010_2$

$\rightarrow +10_{10}$

Carry out to be ignored

$C_{out} == C_{in}$

```
 11 1
  010100₂
 +110110₂
 1001010₂
```

Overflow can not occur

This is the answer in 2's complement

This step is not needed. It is just for you to verify.

2's Complement Addition/Subtraction

- ***Example 2***:

  Perform $10_{10} - 20_{10}$ using 2's complement 6-bit system

- $10_{10} \rightarrow 1010_2$
- $20_{10} \rightarrow 10100_2$
- $10_{10} - 20_{10} \rightarrow 1010_2 - 10100_2$

  $\rightarrow 001010_2 - 010100_2$

  $\rightarrow 001010_2 + (-010100_2)$

  $\rightarrow 001010_2 + 101100_2$

  $\rightarrow 110110_2$

  $\rightarrow -001010_2$

  $\rightarrow -10_{10}$

This is the answer in 2's complement

This step is not needed. It is just for you to verify.

No carry out

$C_{out} == C_{in}$

$$
\begin{array}{r}
1 \\
001010_2 \\
+101100_2 \\
\hline
110110_2
\end{array}
$$

Overflow can not occur

2's Complement Addition/Subtraction

- *Example 3*:

  Perform $20_{10} + 10_{10}$ using 2's complement 6-bit system
- $20_{10} \rightarrow 10100_2$
- $10_{10} \rightarrow 1010_2$
- $20_{10} + 10_{10} \rightarrow 10100_2 + 1010_2$

  $\rightarrow 010100_2 + 001010_2$

  $\rightarrow 011110_2$

  $\rightarrow +30_{10}$

This is the answer in 2's complement

This step is not needed. It is just for you to verify.

No carry out

$C_{out} == C_{in}$

$$\begin{array}{r} 010100_2 \\ +001010_2 \\ \hline 011110_2 \end{array}$$

Overflow might occur, but it did not in this case

2's Complement Addition/Subtraction

- *Example 4*:
  Perform $-20_{10} - 10_{10}$ using 2's complement 6-bit system
- $20_{10} \rightarrow 10100_2$
- $10_{10} \rightarrow 1010_2$
- $-20_{10} - 10_{10} \rightarrow -10100_2 - 1010_2$

$\rightarrow -010100_2 - 001010_2$

$\rightarrow (-010100_2) + (-001010_2)$

$\rightarrow 101100_2 + 110110_2$

$\rightarrow 100010_2$

$\rightarrow -011110_2$

$\rightarrow -30_{10}$

This is the answer in 2's complement

This step is not needed. It is just for you to verify.

Carry out to be ignored

$C_{out} == C_{in}$

```
  1111
   101100₂
  +110110₂
  1100010₂
```

Overflow might occur, but it did not in this case

2's Complement Addition/Subtraction

- *Example 5*:
  Perform $20_{10} + 20_{10}$ using 2's complement 6-bit system
- $20_{10} \rightarrow 10100_2$

- $20_{10} + 20_{10} \rightarrow 10100_2 + 10100_2$
  $\rightarrow 010100_2 + 010100_2$

$C_{out} \neq C_{in}$

No carry out

$$\begin{array}{r} 1\;1 \\ 010100_2 \\ +010100_2 \\ \hline 101000_2 \end{array}$$

Overflow might occur, and *indeed it did in this case*

2's Complement Addition/Subtraction

- *Example 6*:
  Perform $-20_{10} - 20_{10}$ using 2's complement 6-bit system
- $20_{10} \rightarrow 10100_2$
- $-20_{10} - 20_{10} \rightarrow -10100_2 - \quad 10100_2$

  $\rightarrow \; -010100_2 - 010100_2$

  $\rightarrow (-010100_2) + (-010100_2)$

  $\rightarrow \; 101100_2 + \quad 101100_2$

Carry out to be ignored

$C_{out} \neq C_{in}$

```
 1 11
  101100₂
 +101100₂
 1011000₂
```

Overflow might occur, and *indeed it did in this case*

2's Complement Addition/Subtraction

- *Example 7*:
  Perform $20_{10} - 20_{10}$ using 2's complement 6-bit system
- $20_{10} \rightarrow 10100_2$
- $20_{10} - 20_{10} \rightarrow 10100_2 - 10100_2$

$\rightarrow 010100_2 - 010100_2$

$\rightarrow 010100_2 + (-010100_2)$

$\rightarrow 010100_2 + 101100_2$

$\rightarrow 000000_2$

$\rightarrow 0_{10}$

This is the answer in 2's complement

This step is not needed. It is just for you to verify.

Carry out to be ignored

$C_{out} == C_{in}$

```
 1111
  010100₂
 +101100₂
 1000000₂
```

Overflow can not occur

2's Complement Addition/Subtraction

- *Example 8*:
  Perform $31_{10} + 1_{10}$ using 2's complement 6-bit system
- $31_{10} \rightarrow 11111_2$
- $1_{10} \rightarrow 1_2$

- $31_{10} + 1_{10} \rightarrow 11111_2 + 1_2$
  $\rightarrow 011111_2 + 000001_2$

No carry out

$C_{out} \neq C_{in}$

$$\begin{array}{r} 11111 \\ 011111_2 \\ +000001_2 \\ \hline 100000_2 \end{array}$$

Overflow might occur, and *indeed it did in this case*

2's Complement Addition/Subtraction

- ***Example 9***:

  Perform $-31_{10} - 1_{10}$ using 2's complement 6-bit system
- $31_{10} \rightarrow 11111_2$
- $1_{10} \rightarrow 1_2$
- $-31_{10} - 1_{10} \rightarrow -11111_2 - 1_2$

  $\rightarrow (-011111_2) + (-000001_2)$

  $\rightarrow (100001_2) + (111111_2)$

  $\rightarrow 100000_2$

  $\rightarrow -100000_2$

  $\rightarrow -32_{10}$

Carry out to be ignored

$C_{out} == C_{in}$

```
 111111
  100001₂
 +111111₂
 1100000₂
```

This is the answer in 2's complement

Overflow might occur, but it did not in this case

This step is not needed. It is just for you to verify.

2's Complement Addition/Subtraction

- *Example 10*:

  Encode -3.25_{10} using 2's complement 6-bit system

- $3.25_{10} \rightarrow 11.01_2$
- $-3.25_{10} \rightarrow -0011.01_2$

  $\rightarrow 1100.11_2$

Carry out to be ignored

You can also look at it as if it is $-3_{10} - 0.25_{10}$

- $-3_{10} - 0.25_{10} \rightarrow -11_2 - 0.01_2$

  $\rightarrow (-000011_2) + (-0000.01_2)$

  $\rightarrow (111101_2) + (1111.11_2)$

  $\rightarrow 111100.11_2$

  $\rightarrow 1100.11_2$

  $\rightarrow -3.25_{10}$

$C_{out} == C_{in}$

```
1111111
 111101.00₂
+111111.11₂
1111100.11₂
```

This is the answer in 2's complement

This step is not needed. It is just for you to verify.

Overflow might occur, but it did not in this case

Binary points MUST be aligned