Lecture 9.

HW 3. Fri 11:59 p.m.

e.g. $x \in P(A)$.
iff $x \subseteq A$.
iff $\forall y \in x \to y \in A$.

$x \in P(A \cup B)$
iff $x \subseteq A \cup B$
iff $\forall y \in x \to y \in A \cup B$
$\to y \in A \cup y \in B$.

$x \in [P(A) \cup P(B)]$.
iff $x \in P(A) \cup x \in P(B)$. Definition of the union.
iff $x \subseteq A \cup x \subseteq B$.
iff $[\forall y \in x \to y \in A] \cup [\forall y \in x \to y \in B]$.
not equals to $y \in A \cup y \in B$.
e.g. $A = \{1,2,3\}$ $B = \{4,5,6\}$.
$\{3,4\} \in P(A \cup B) = P(\{1,2,3,4,5,6\})$.
$\notin P(A)$
$\notin P(B)$
$\notin P(A) \cup P(B)$.

$P(A) \cup P(B) \underset{\neq}{\subseteq} P(A \cup B)$

§2.3 Indexed sets.

Ways to define a set:

1. List all elements: e.g. $\{7, 8, 9\}$.
2. truth sets : $\{x \mid P(x).\}$. ⇐ variables on the left
3. indexed sets: $\{n^2 \mid n \in \mathbb{N}\}$ ⇐ variables on the right

Let P_i denotes the i^{th} prime. $P_1 = 2$ $P_2 = 3$, $P_3 = 5$ ....

$\{P_i \mid i \in [1,100]\}$. $= \{1^{st}$ hundered primes$\}$.

$= \{P_i \mid i \in I\}$. $I = \{k \in N, k \in [1,100]\}$.

$\{[\frac{1}{n}, 1] \mid n \in N^*\}$.

In general: $\{f(i) \mid i \in I\}$.

$S = \{$student in school$\}$.

For $x \in S$, let $C_x = \{$courses x takes$\}$.

$\{C_x \mid x \in S\}$ => all courses that x takes.

$C = \{$all courses$\}$

$\{C_x \mid x \in S\} \subseteq P(C)$.

Intersection & Union Family.

If $\mathcal{F}$ is a family of sets. (i.e. a set of sets).

$\bigcap \mathcal{F} = \{x \mid \forall A \in \mathcal{F}, x \in A\}$.

$\bigcup \mathcal{F} = \{x \mid \exists A \in \mathcal{F}, x \in A\}$.

e.g. $\mathcal{F} = \{\{1,2,3\}, \{2\}, \{2,4,6\}\}$.

$\bigcap \mathcal{F} = \{2\}$ all sets have this element.

$\bigcup \mathcal{F} = \{1,2,3,4,6\}$. all elements in these sets.

e.g. $\mathcal{F} = \{[\frac{1}{n}, 1] \mid n \in N^*\}$.

$\bigcap \mathcal{F} = [\frac{1}{1}, 1] \wedge [\frac{1}{2}, 1] \wedge [\frac{1}{3}, 1] \cdots = \{1\}$.

$\bigcup \mathcal{F} = \quad \vee \quad \vee \quad \cdots = (0, 1]$

$\bigcup \{C_x \mid x \in S\}$ = all classes that students taken.

$\bigcap \{C_x \mid x \in S\}$ = courses that every student taken.

For $n \in N$, let $D(n) = \{d \in \mathbb{Z} \mid d \mid n \quad n \in N\}$.

dividers of the number.

$$\bigcap_{n \in N} D(n) = \bigcap \{D(n) \mid n \in N\} = \{1\}.$$

=> divider of all nature number.

$$\bigcup_{n \in N} D(n) = Z^+$$

$$D(0) = Z^+$$

Puzzle:

- 3 kids.
- product : 72.
- sum = house number.
- oldest child like strawberries.

age ? house number ?