Lecture 13

2 3 5 7 11 13 17 19 23 29 31

1 2 2 4 2 4 2 4 6 2

1 0 2 2 2 2 2 2 4

1 2 0 0 0 0 0 2

1 2 0 0 0 0 2

1 2 0 0 0 2

1 2 0 0 2

$\vdots$

§ 3.3

Strategy: To prove a goal of a form $\exists x\, P(x)$.

Ex: Let A be a set then $P(A) \neq \phi$

Given	Goal
A	$P(A) \neq \phi \Rightarrow \exists x \; x \in P(A)$.
	$\Rightarrow \exists B \;\; B \subseteq A$. Definition of power set.
	$\Rightarrow B = \phi$ in example.

Proof: To show $P(A) \neq \phi$, we give an element of $P(A)$

Since $\phi \subseteq A$, $\phi \in P(A)$ $\square$

Template: Let x be __

Proof of $P(x)$.

Thus, $\exists x\, P(x)$.

Strategy: To use a given form $\exists x\, P(x)$.

- Choose one variable named x
- Change the given to $P(x)$.

Template: Let x be such that $P(x)$.

Proof.

e.g. For $a, b \in \mathbb{Z}$, $a | b$ implies that $\exists k \in \mathbb{Z}$, $b = ka$.

For all integers a, b and c. if $a | b$ and $a | c$, then $a | b+c$.

Given	Goal.
	For all $a, b, c \in \mathbb{Z}$, $a \mid b \wedge a \mid c \to a \mid b+c$
$a \in \mathbb{Z}, b \in \mathbb{Z}, c \in \mathbb{Z}$.	$a \mid b+c$
$a \mid b$, $a \mid c$	
$\Rightarrow b = k_1 a$, $c = k_2 a$.	
$b + c = a(k_1 + k_2)$	

Proof: For all $a \in \mathbb{Z}$, $b \in \mathbb{Z}$, $c \in \mathbb{Z}$, and $a \mid b \wedge a \mid c$, let $k_1 \in \mathbb{Z}$ such that $b = k_1 a$, let k_2 such that $c = k_2 a$.

Then $b + c = (k_1 + k_2) a$, Since $k_1, k_2 \in \mathbb{Z}$, $k_1 + k_2 \in \mathbb{Z}$,

So $a \mid b+c$ $\square$.

Strategy: To use a given of form $\forall x\ P(x)$.

- For any given of "a" you choose, you can add add $P(a)$ as a given.

e.g.: Suppose $\mathcal{F}$ and $\mathcal{G}$ are families of set and $\mathcal{F} \cap \mathcal{G} \neq \phi$

Proove that $\cap \mathcal{F} \subseteq \cup \mathcal{G}$.

Given	Goal.
$\mathcal{F} \cap \mathcal{G} \neq \phi$	$\cap \mathcal{F} \subseteq \cup \mathcal{G}$.
$\Rightarrow \exists B\ B \in \mathcal{F} \wedge B \in \mathcal{G}$.	$\Rightarrow \forall x\, (x \in \cap \mathcal{F} \to x \in \cup \mathcal{G})$.
$x \in \cap \mathcal{F}$	$x \in \cup \mathcal{G}$.
$\Rightarrow \forall A \in \mathcal{F}\ x \in A$.	$\Rightarrow \exists A \in \mathcal{G}\ x \in A$.
$x \in B$.	$x \in B$.

Proof: Suppose $x \in \cap \mathcal{F}$ Since $\mathcal{F} \cap \mathcal{G} \neq \phi$, let B be an

element of $F \cap G$. Then $B \in F$ and $B \in G$,
Since $x \in \cap F$, we have $x \in B$. But $B \in G$, so
$x \in \cup G$ $\square$.

e.g. If $F \subseteq P(B)$, then $\cup F \subseteq B$.

Proof: Suppose $F \subseteq P(B)$, Let $x \in \cup F$.
Then $x \in A$ for some $A \in F$. Since $F \subseteq P(B)$,
$A \in P(B)$, i.e. $A \subseteq B$ Since $x \in A$, $x \in B$.
So $\cup F \subseteq B$ $\square$.