

reductions):

XOR p (NOT p). For all computations you perform, indicate clearly the reduction being done by underlining the abstraction used and the argument it is applied to:  $(\lambda x.M)N$ .

(b) (6pt) Does your computation indicate that XOR is behaving as the exclusive-or operator you know?



(a) Call-by-value reduction:

XOR p (NOT p)

=  $(\lambda pq.p \text{ (NOT } q) \text{ ) } q) \text{ p (NOT p)}$  [by substitution]  $\Rightarrow \lambda q. p \text{ (NOT } q) \text{ } q$

= p (NOT (NOT p)) (NOT p) [by reduction]

= p  $(\lambda qr.r \text{ p } q) \text{ (NOT p)}$  [by reduction]

=  $(\lambda pq.p) (\lambda qr.r \text{ p } q) \text{ (NOT p)}$  [by substitution]

=  $(\lambda qr.r \text{ p } q) \text{ (NOT p)}$  [by reduction]

=  $(\lambda r.\text{NOT p } r \text{ p})$  [by reduction]

Call-by-name reduction:

XOR p (NOT p)

=  $(\lambda pq.p \text{ (NOT } q) \text{ ) } q) \text{ p (NOT p)}$  [by substitution]

= p (NOT (NOT p)) (NOT p) [by reduction]

= p  $(\lambda r.\text{NOT p } r \text{ p}) \text{ (NOT p)}$  [by reduction]

=  $(\lambda p.(\lambda r.\text{NOT p } r \text{ p}) \text{ p (NOT p)})$  [by abstraction]

=  $(\lambda r.\text{NOT p } r \text{ p}) \text{ (NOT p)}$  [by reduction]

=  $(\lambda p.(\lambda r.\text{NOT p } r \text{ p}) \text{ (NOT p)})$  [by abstraction]

=  $(\lambda r.\text{NOT (NOT p) } r \text{ (NOT p)})$  [by reduction]

(b) Yes, the computation indicates that XOR behaves as the exclusive-or operator, as the result is p if the values of p and (NOT p) are different, and (NOT p) if they are the same, which is the definition of exclusive-or.