Question:	1	2	3	4	5	6	7	Total
Marks:	6	2	5	2	7	6	5	33
Score:								

Name (print): ______________________________

Signature: ______________________________

UWO ID number: ______________________________

Circle the numbers of your section and lab section in the tables below:

001	MWF 12:30	Matthias Franz
002	MWF 11:30	Derek Krepski

003	Thu 1:30	A. Ghorbanpour	007	Wed 10:30	J. Rastegari Koopaei
004	Thu 12:30	A. Ghorbanpour	008	Wed 9:30	Y. Yan
005	Thu 2:30	J. Haradyn	009	Wed 10:30	G. Wang
006	Thu 10:30	J. Rastegari Koopaei			

THE UNIVERSITY OF WESTERN ONTARIO
DEPARTMENT OF MATHEMATICS

MATHEMATICS 1600B MIDTERM EXAMINATION
31 January 2013 7:00–8:30 PM

INSTRUCTIONS:

1. This exam is 7 pages long. It is printed double-sided. There are 7 questions.
2. All questions must be answered in the space provided. Indicate your answer clearly. Should you need extra space, a blank page is provided at the end of the booklet.
3. Show all your of your work and explain your answers fully. Unjustified, irrelevant or illegible answers will receive little or no credit.
4. Do not unstaple the exam booklet.
5. **No aids are permitted. In particular, calculators, cell phones, ipods etc. are not allowed and may be confiscated.**
6. If not stated otherwise, all vectors and equations involve real numbers.
7. In your final answers you must give all numbers in $\mathbb{Z}_n$ as a number between 0 and $n-1$.

1. For each of the following statements, circle **T** if the statement is always true and **F** if it can be false. Give a one-sentence justification for your answer.

   (a) (2 marks) If two non-zero vectors are parallel, then the angle between them is 0 degrees.

   ~~**T**~~ **F**

   (b) (2 marks) For any two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$, $\|\mathbf{u} \times \mathbf{v}\| \leq \|\mathbf{u}\|\|\mathbf{v}\|$.

   T ~~**F**~~

   (c) (2 marks) Any vector in the plane is a linear combination of the standard unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$.

   T **F**

2. (2 marks) Let $\mathbf{u} = [3, 2, 1]$ and $\mathbf{v} = [1, 3, 2]$ be in $(\mathbb{Z}_5)^3$. Find all scalars b in $\mathbb{Z}_5$ such that $(\mathbf{u} + b\mathbf{v}) \cdot (b\mathbf{u} + \mathbf{v}) = 1$. (Read again the last instruction on the front page!.)

$(u+bv) = (3+b, 2+3b, 1+2b).$

$(bu+v) = (3b+1, 2b+3, b+2).$

$(u+bv)\cdot(bu+v) = 9b+3+3b^2+b+4b+6+6b^2+9b+b+2+2b^2+4b$

$= 11b^2+28b+11$

$\because$ in $\mathbb{Z}_5$ $\therefore$ $= b^2+3b+1.$

$b=0$ $=1$ ✓

$b=1$ $=0$

$b=2$ $=1$ ✓

$b=3$ $=4.$

$b=4$ $=4.$

$\therefore b=0$ or $b=2.$

3. Let $\mathbf{v} = [\sqrt{3}, 0, 1]$ and $\mathbf{w} = [-1, \sqrt{2}, 1]$.

(a) (2 marks) Compute the length of $\mathbf{v} + \mathbf{w}$.

$v+w = [\sqrt{3}-1, \sqrt{2}, 2]$

$\|v+w\| = \sqrt{3-2\sqrt{3}+1+2+4}$

$= \sqrt{10-2\sqrt{3}}.$

(b) (1 mark) Find a unit vector pointing in the same direction as $\mathbf{w}$.

assume that unit vector

$\vec{n} = [-a, \sqrt{2}a, a]$

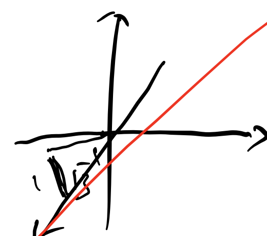
$a = \frac{1}{2}.$

(c) (2 marks) Find all unit vectors $\mathbf{x}$ in the xy-plane that make an angle of 30 degrees with $\mathbf{v}$.

assume vector $x = [x_1, y_1, 0]$.

$\cos 30^\circ = \frac{x \cdot v}{\|x\| \cdot \|v\|}$.

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}x_1}{\sqrt{x_1^2+y_1^2} \cdot 2}$.



$x_1 = 1$

$y_1 = 0.$ $\therefore x = [1, 0, 0]$

4. (2 marks) Give an example of a valid UPC code $\mathbf{x}$ with the following two properties:

- If one reads the UPC code backwards, one gets again a valid UPC code.
- The 'backwards' code is not the same as the original code. (That is, codes of the form $\mathbf{x} = [0\,1\,2\,3\,4\,5\,5\,4\,3\,2\,1\,0]$ are not permitted.)

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(Recall that the check vector for UPC is $\mathbf{c} = [3\,1\,3\,1\,3\,1\,3\,1\,3\,1\,3\,1]$.)

~~$x = [1\,1\,4\,5\,1\,4\,2\,8\,1\,8\,8\,4]$.~~

Let x be $[x_1\, x_2 \cdots x_{12}]$

$3x_1 + x_2 + \cdots + x_{12} = 0.$

$x_1 + 3x_2 + \cdots + 3x_{12} = 0.$

$\begin{cases} x_1 + x_3 + \cdots + x_{11} = 0. \\ x_2 + x_4 + \cdots + x_{12} = 0. \end{cases}$

Choose $x_2 = x_4 = \cdots = x_{12} = 0.$

Choose $x_1 = 1$ $x_3 = 9$ $\cdots = 0.$

one possible solution is

$x = [1\,0\,9\,0\,0\,0\,0\,0\,0\,0\,0\,0]$.

vector form : $\begin{bmatrix}2\\2\\4\end{bmatrix} + s\begin{bmatrix}3\\2\\-1\end{bmatrix} + t\begin{bmatrix}1\\2\\-1\end{bmatrix}$.

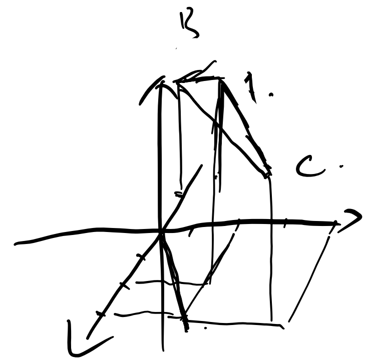
5. Let $A(2, 2, 4)$, $B(-1, 0, 5)$, $C(3, 4, 3)$ be three points in $\mathbb{R}^3$.

(a) (2 marks) Write a vector form of the equation of the plane containing the triangle ΔABC.

$$\boxed{\begin{aligned}\vec{BA} &= (3, 2, -1).\\ \vec{AC} &= (1, 2, -1).\end{aligned}}$$

$L_1 : (2, 2, 4) + a(3, 2, -1).$

$L_2 : (2, 2, 4) + b(1, 2, -1).$



(b) (2 marks) Give the normal form of the equation of the plane containing the triangle ΔABC.

$\vec{n} = u \times v = \begin{bmatrix}0\\-2\\-4\end{bmatrix}.$

normal form of the equation

$= n \cdot A = -20$

(c) (2 marks) What is the area of the triangle ΔABC?

area $= \frac{1}{2} \|u \times v\|$

$= \sqrt{5}.$

(d) (1 mark) Find a point in the plane from part (a) that lies *inside* (not including the edges) of the triangle ΔABC.

$x \in (-1, 2)$

$y \in (0, 2)$

$z \in (3, 4).$

$(3, 4, 3) + x(-2, 3, 1).$

point $P : (2, \frac{5}{2}, \frac{5}{2}).$

$Q = \boxed{\frac{1}{3}(A + B + C)}$

思考一下这是什么?

6. Consider the system of linear equations

$$\begin{aligned} x + 3y + 2z \quad &= 1 \\ -z + 2w &= 0 \\ 3x + 5y - z \quad &= 5 \\ 2x + 2y - 2z - 2w &= 4 \end{aligned}$$

(a) (1 mark) Write down the augmented matrix of this linear system.

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 1 \\ 0 & 0 & -1 & 2 & 0 \\ 3 & 5 & -1 & 0 & 5 \\ 2 & 2 & -2 & -2 & 4 \end{array}\right].$$

(b) (3 marks) Compute the reduced row-echelon form of the augmented matrix above. Indicate all elementary row operations that you are performing.

$$\left.\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 1 \\ 1 & 1 & -1 & -1 & 2 \\ 0 & 0 & 1 & -2 & 0 \\ 3 & 5 & -1 & 0 & 5 \end{array}\right.$$

$1 \quad 0 \quad -\frac{5}{2}$

$\downarrow$

$$\left.\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 1 \\ 0 & -2 & -3 & -1 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ -0 & -4 & -7 & 0 & 2. \end{array}\right.$$

$$\left.\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 1 \\ 0 & 1 & \frac{3}{2} & \frac{1}{2} & -\frac{1}{2}. \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & \frac{7}{4} & 0 & -\frac{1}{2}. \end{array}\right.$$

(c) (2 marks) Use the result of the previous part to find all solutions of the linear system.

7. Let $\mathcal{P}$ be the plane through the point $A(2,-1,-1)$ with normal vector $\mathbf{n} = [2, 2, -1]$.

(a) (2 marks) Compute parametric equations of the plane $\mathcal{P}$.

参数

$P: (2,-1,-1) + t(x,y,z).$

$2x+2y-z=0.$

$$\boxed{\begin{aligned} u &= [1,0,2] \\ v &= [0,1,2] \end{aligned}}$$

$x = A + su + tv.$

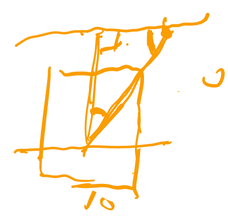
$x = 2+s$

$y = -1+t$

$z = -1+2s+2t$

(b) (2 marks) Compute the distance from $\mathcal{P}$ to the origin.

$\mathrm{proj}_n \vec{AO} = \frac{\vec{AO}\cdot n}{n^2} \cdot |n|.$



$\mathrm{Proj}_u V = V \cdot \cos\langle u, v\rangle$

$= V \cdot \frac{u\cdot v}{|u|\cdot|v|}.$

$\cos\theta = \frac{\vec{AO}\cdot\vec{n}}{|\vec{AO}|\cdot|\vec{n}|}.$

$\vec{AO} =$

$\mathrm{Proj}_u V = \frac{u\cdot v}{u\cdot u} \cdot u.$

(c) (1 mark) Find a plane $\mathcal{P}'$ that is parallel to $\mathcal{P}$ and has distance 2 from $\mathcal{P}$.

Use this page if you need extra space for your work.

Did you write your name and student ID on the first page?
Did you give full explanations and show all of your work?