Q1. Assume that n is even, then there must exist an integer k that $n=2k$ for every n. So $n^2+4n+3=(2k)^2+4\times(2k)+3=4k^2+8k+3$, which could be rewritten as $2(2k^2+4k+1)+1$. Since k is an integer, $2k^2+4k+1$ is an integer and $2(2k^2+4k+1)$ is an even number, so $2(2k^2+4k+1)+1$ is odd. So n is even implies that n^2+4n+3 is odd. $\square$

Proof by contrapositive: Assume that n is not even, then there must be an integer k such that $n=2k+1$. So $n^2+4n+3=(2k+1)^2+4(2k+1)+3=4k^2+12k+8=2(2k^2+6k+4)$. Since k is an integer, $2(2k^2+6k+4)$ must be a even number. So n^2+4n+3 is odd implies that n is even. $\square$

Q2. Assume that m is odd and n is even, then there must exist integer k_1, k_2 such that $2k_1+1=m$, $2k_2=n$. So $m(n+3)=(2k_1+1)(2k_2+3)=4k_1k_2+6k_1+2k_2+3=2(2k_1k_2+3k_1+k_2+1)+1$, Since k_1, k_2 are integers, $2(2k_1k_2+3k_1+k_2+1)$ is even and $2(2k_1k_2+3k_1+k_2+1)+1$ is odd. So $m(n+3)$ is even implies m is even or n is odd $\square$.

Assume m is even or n is odd.

Case 1: Assume that m is even. Then there must exist an integer k that $m=2k$. So $m(n+3)=2k(n+3)$ Since k is an integer. So $2k(n+3)$ is even.

Case 2: Assume that n is odd. Then there must exist an integer k such that $n=2k+1$. So $m(n+3)=m(2k+1+3)$, which could be rewritten as $2m(k+2)$. Since k.

is an integer, $2m(k+2)$ is even.

These cases are exhanstive, so in either case we have prove that m is even or n is odd implies $m(n+3)$ is even $\square$

Q3. Assume an arbitrary x that $x \in (D \cup E) \setminus F$. So $x \in (D \cup E)$. and $x \notin F$. Since $x \in (D \cup E) \wedge x \notin F$, $(x \in D \vee x \in E) \wedge x \notin F$, which could be rewritten as $(x \in D \wedge x \notin F) \vee (x \in E \wedge x \notin F)$, which is equal to $x \in (D \setminus F) \vee x \in (E \setminus F)$, so we can get $x \in (D \setminus F) \cup (E \setminus F)$. Since x is arbitrary, $(D \cup E) \setminus F \subseteq (D \setminus F) \cup (E \setminus F)$ $\square$.

Assume an arbitrary x that $x \in (D \setminus F) \cup (E \setminus F)$.

Case 1: Assume that $x \in (D \setminus F)$. Since $x \in (D \setminus F)$, $x \in D$ and $x \notin F$. Since $x \in D$, $x \in (D \cup E)$, and because also $x \notin F$, we have $x \in (D \cup E) \wedge x \notin F$, so $x \in (D \cup E) \setminus F$, Since x is arbitrary, $(D \setminus F) \subseteq (D \cup E) \setminus F$.

Case 2: Assume that $x \in (E \setminus F)$. Since $x \in (E \setminus F)$, $x \in E$ and $x \notin F$, Since $x \in E$, $x \in (D \cup E)$. and because also $x \notin F$, we can have $x \in (D \cup E) \wedge x \notin F$, so $x \in (D \cup E) \setminus F$. Since x is arbitrary, $(E \setminus F) \subseteq (D \cup E) \setminus F$.

These cases are exhanstive, so in either cases we have prove that $x \in (D \setminus F) \cup (E \setminus F) \subseteq (D \cup E) \setminus F$ $\square$.

Q4. Assume we have an arbitrary x such that $x \in (A_1 \cup B_1) \cap (A_1 \cup B_2) \cap (A_2 \cup B_1) \cap (A_2 \cup B_2)$. Since $x \in (A_1 \cup B_1)$ and $x \in (A_1 \cup B_2)$, $x \in A_1 \cup (B_1 \cap B_2)$. Similarly since $x \in (A_2 \cup B_1)$ and $x \in (A_2 \cup B_2)$, $x \in A_2 \cup (B_1 \cap B_2)$. Since $x \in (B_1 \cap B_2) \cup A_1$ and $x \in (B_1 \cap B_2) \cap A_2$, we can have $x \in (A_1 \cap A_2) \cup (B_1 \cap B_2)$. Since x is arbitrary, $(A_1 \cup B_1) \cap (A_2 \cup B_1) \cap (A_1 \cup B_2) \cap (A_2 \cup B_2) \subseteq (A_1 \cap A_2) \cup (B_1 \cap B_2)$ $\square$