Chapter 9

A Two-Period Model:
The Consumption–Savings Decision and Credit Markets

Topics

- Consumer's consumption/savings decision – responses of consumer to changes in income and interest rates.
- Government budget deficits and the Ricardian Equivalence Theorem.

Intro

- Intertemporal decisions – Intertemporal decision involve economic trade-offs across periods of time
- Consumer's consumption/savings intertemporal decisions – responses of consumer to changes in income and interest rates.
- Government intertemporal decisions concerning the financing of government expenditures - budget deficits/surplus/balance and the Ricardian Equivalence Theorem.
- A **two-period model**, is the simplest framework for understanding intertemporal choice and dynamic issues.

Intro

- A **two-period model**, is the simplest framework for understanding intertemporal choice and dynamic issues.

Consumer Behavior

- There are m consumers

- Each consumer lives for two period, current and future

- Consumers do not make a work-leisure decision in either period, but receives an exogenous income

- Focus is on consumer's consumption-savings decision

Consumer Behavior – Current Period Budget Constraint

- Let y be real income in the current period and $y^/$ in the future period

- each consumer pay lump-sum taxes, t , in the current period and $t^/$ in the future period

- let s be consumer's savings in the current period

- The consumer's current-period budget constraint:

$$C + S = y - t$$

Consumer Behavior

- Let r be the real interest rate at which consumer can lend or borrow

- The consumer's future-period budget constraint:

$$C' = y' - t' + (1 + r)s$$

- Solve the future-period budget constraint for s:

$$s = \frac{c' - y' + t'}{1 + r}$$

- if *s > 0 (s < 0),* then the consumer is a lender (borrower) on the credit market.

Consumer Behavior

- *We suppose that the financial asset that is* traded in the credit market is a bond.

- In the model, bonds can be issued by consumers as well as by the government.

- Two important Assumptions:
    - All bonds are indistinguishable
    - Bonds are traded directly in the credit market

Consumer Behavior

- In our model, one bond issued in the current period is a promise to pay 1+r units of the consumption good in the future period, so that the real interest rate on each bond is r.

- Because this implies that one unit of current consumption can be exchanged in the credit market for 1+r units of the future consumption good, the relative price of future consumption in terms of current consumption is 1/(1 + r)

Consumer Behavior – Consumer's Lifetime Budget Constraint

- Substitute in the current-period budget constraint obtaining lifetime budget constraint:

$$c + \frac{c' - y' + t'}{1 + r} = y - t$$

Consumer's Lifetime Budget Constraint

- Substitute in the current-period budget constraint obtaining lifetime budget constraint:

$$c + \frac{c'}{1+r} = y - t + \frac{y' - t'}{1+r}$$

Consumer's Lifetime Wealth

- Substitute in the current-period budget constraint obtaining lifetime budget constraint:

$$we = y - t + \frac{y' - t'}{1 + r}$$

- Simplified Lifetime Budget Constraint:

$$c + \frac{c'}{1 + r} = we$$

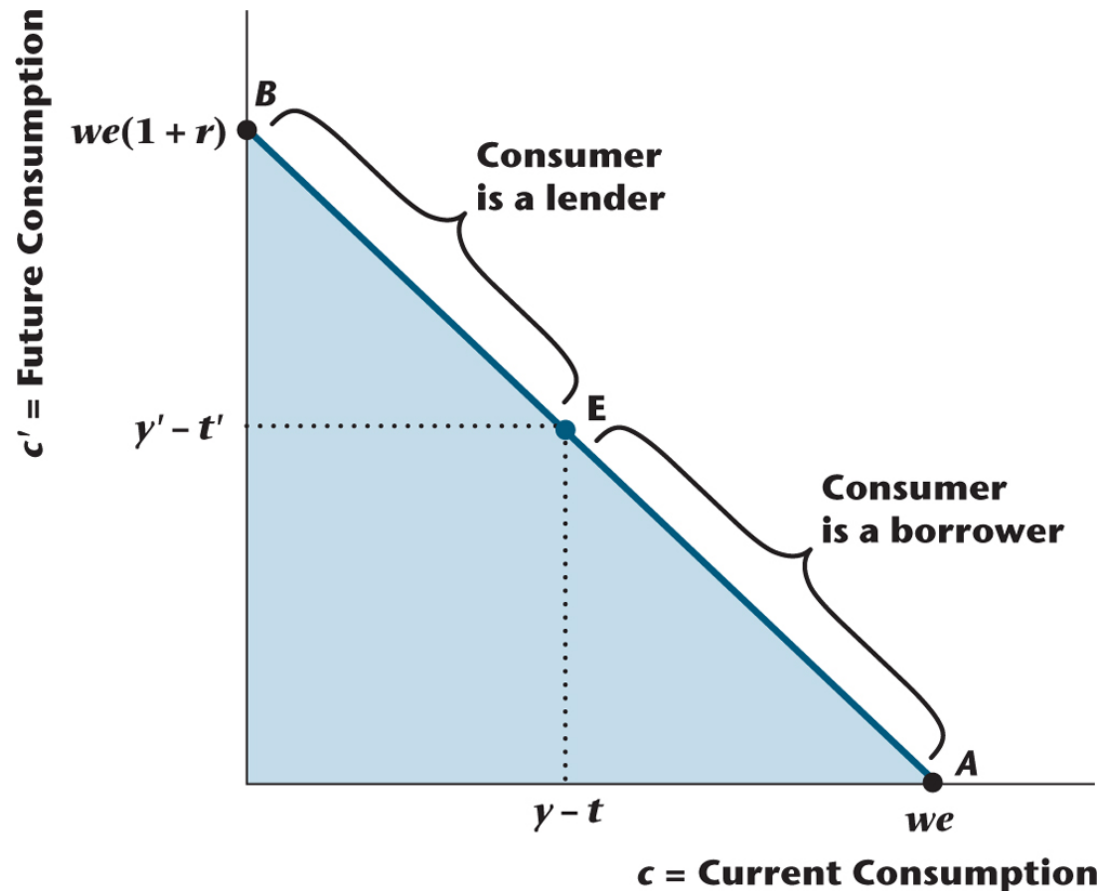# Simplified Lifetime Budget Constraint: Slope-Intercept

- Simplified Lifetime Budget Constraint in slope and intercept form:

$$c' = -(1 + r)c + we(1 + r)$$

- Vertical intercept: we(1+ r)

- Horizontal intercept: we

- Slope: - (1+ r)

Figure 9.1
Consumer's Lifetime Budget Constraint


The lifetime budget constraint defines the quantities of current and future consumption the consumer can acquire, given current and future income and taxes, through borrowing and lending on the credit market. To the northwest of the endowment point E, the consumer is a lender with positive savings; to the southeast of E, he or she is a borrower with negative savings.

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Figure 9.2
A Consumer's Indifference Curves
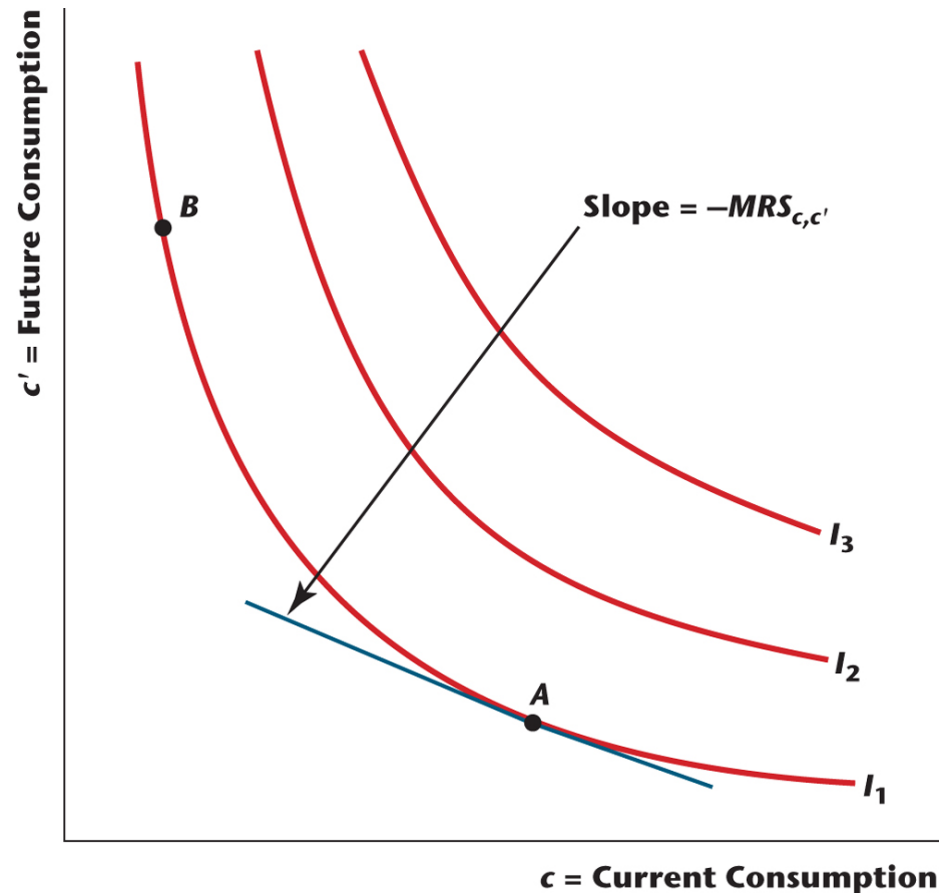
The figure shows the indifference map of a consumer. Indifference curves are convex and downward-sloping. Minus the slope of an indifference curve is the marginal rate of substitution of current consumption for future consumption.
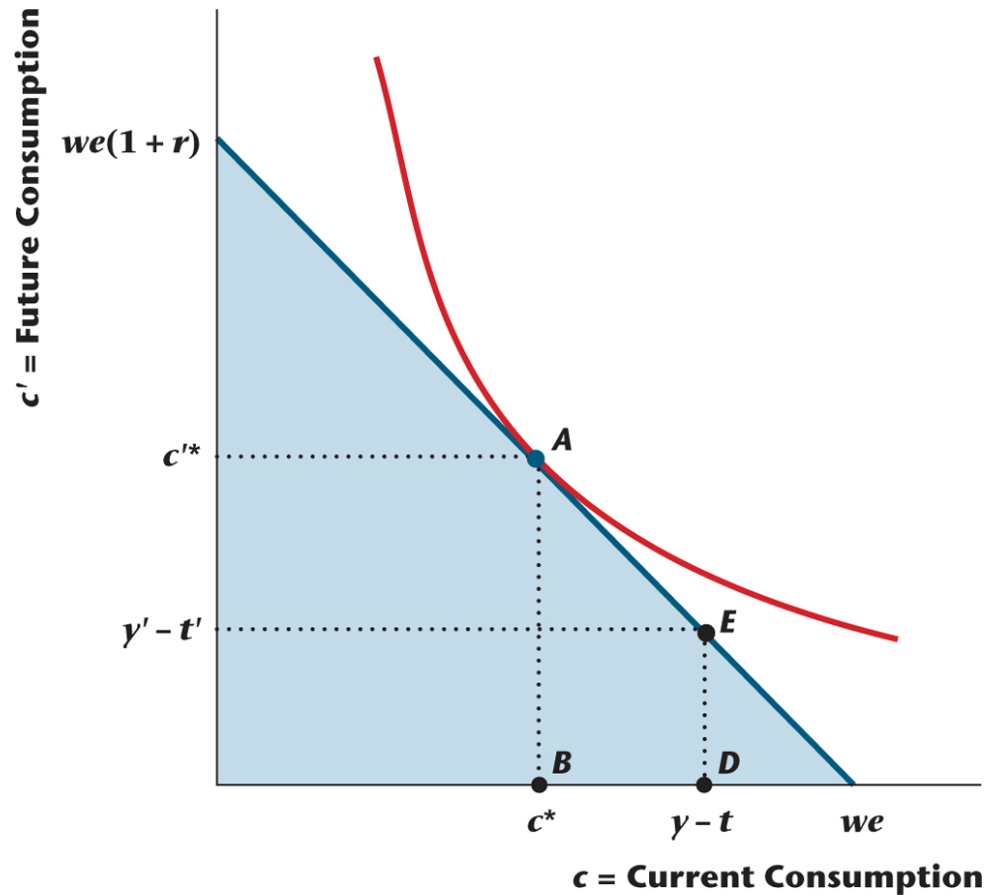
Optimization

- Marginal condition that holds when the consumer is optimizing:

$$MRS_{c,\,s'} = 1 + r$$

Figure 9.3
A Consumer Who is a Lender

The optimal consumption bundle for the consumer is at point A, where the marginal rate of substitution (minus the slope of an indifference curve) is equal to $1 + r$ (minus the slope of the lifetime budget constraint). The consumer is a lender, as the consumption bundle chosen implies positive savings, with E being the endowment point.

Figure 9.4
## A Consumer Who is a Borrower
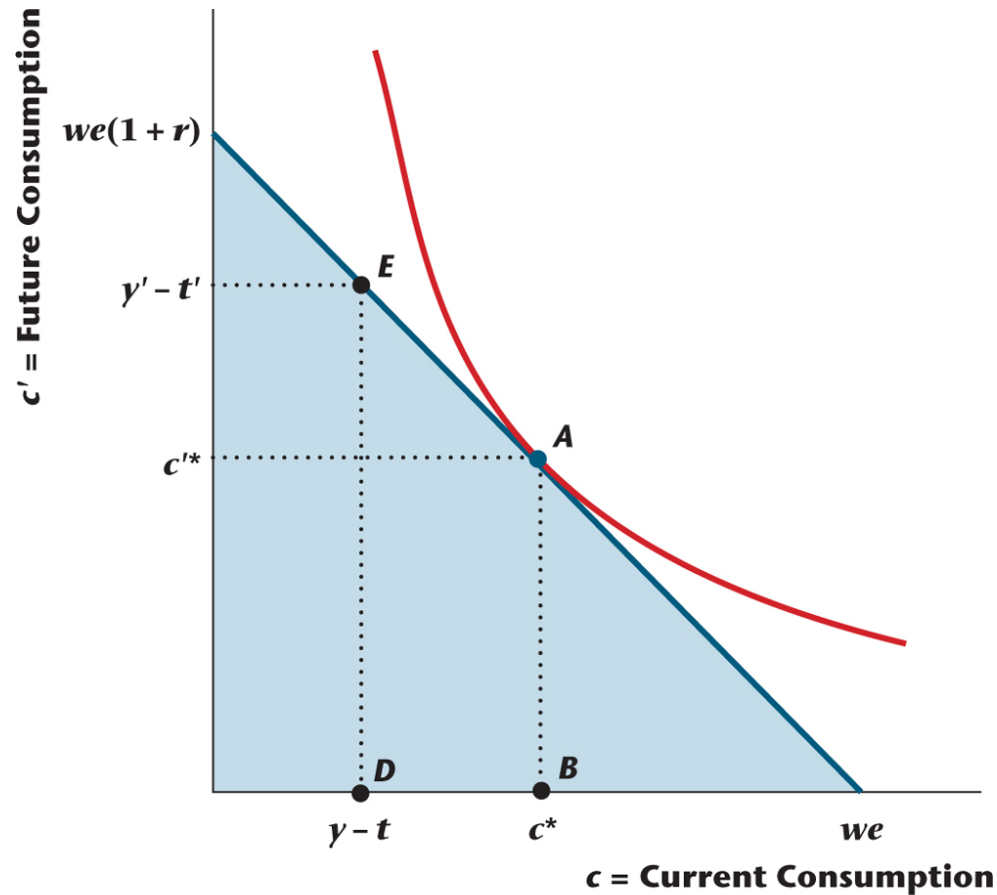
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The optimal consumption bundle is at point *A*. Since current consumption exceeds current disposable income, saving is negative, and so the consumer is a borrower.

An Increase in Current Income for the Consumer

- Current and future consumption increase.
- Saving increases.
- The consumer acts to smooth consumption over time.

Figure 9.5
## The Effects of an Increase in Current Income for a Lender
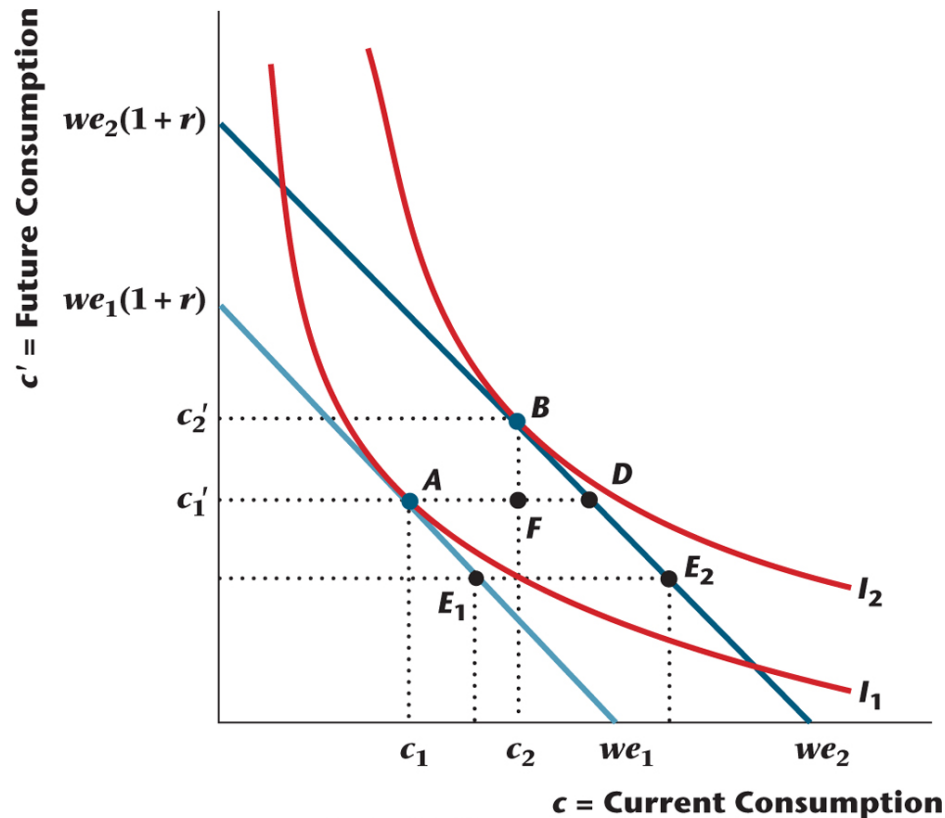
When current income increases, lifetime wealth increases from we_1 to we_2. The lifetime budget constraint shifts out, and the slope of the constraint remains unchanged, since the real interest rate does not change. Initially, the consumer chooses A, and he or she chooses B after current income increases. Current and future consumption both increase (both goods are normal), and current consumption increases by less than the increase in current income.

Observed Consumption-Smoothing Behavior

- Aggregate consumption of non-durables and services is smooth relative to aggregate income, but the consumption of durables is more volatile than income.

- This is because durables consumption is economically more like investment than consumption.

Figure 9.6
Percentage Deviations from Trend in GDP and Consumption 1961–2018.
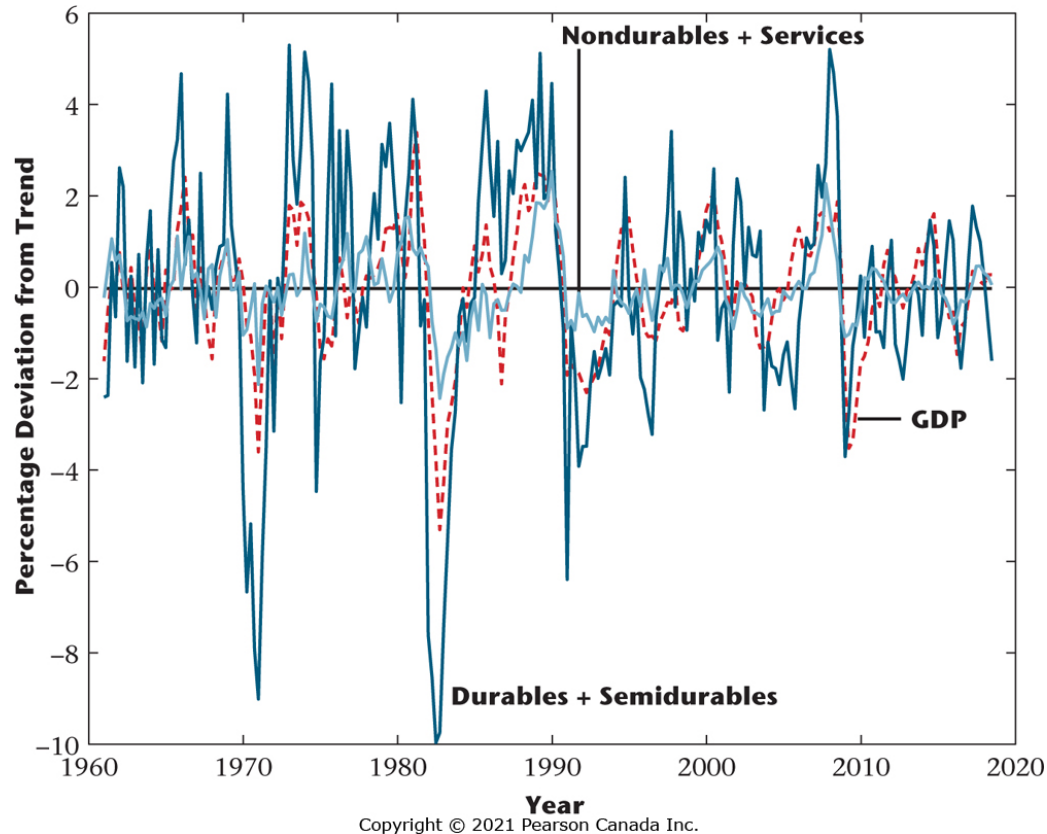

Consumption of durables and semi-durables is much more variable than GDP, which is much more variable than consumption of nondurables and services. The consumption-smoothing behaviour of consumers is clearly reflected in the behaviour of nondurables and services.

An Increase in Future Income for the Consumer

- Aggregate consumption of non-durables and services is smooth relative to aggregate income, but the consumption of durables is more volatile than income.

- This is because durables consumption is economically more like investment than consumption.

Figure 9.7
An Increase in Future Income
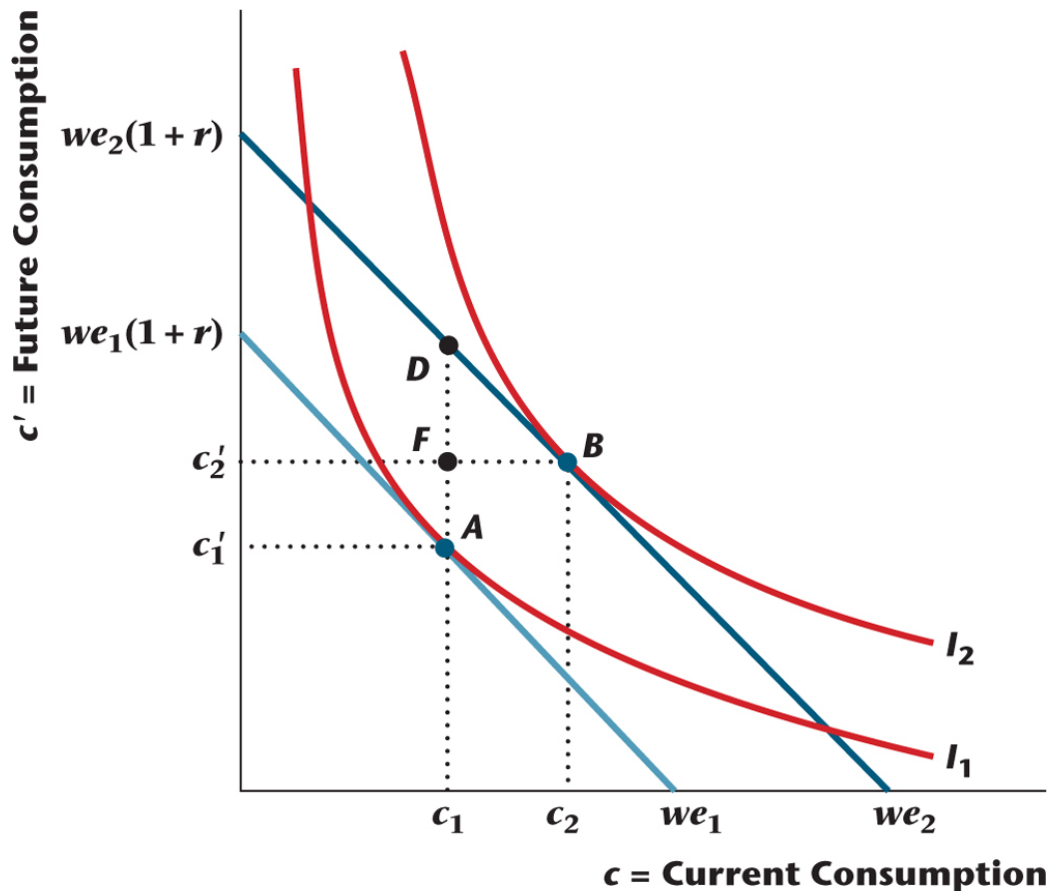
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An increase in future income increases lifetime wealth from we_1 to we_2, shifting the lifetime budget constraint up and leaving its slope unchanged. The consumer initially chooses point A, and he or she chooses B after the budget constraint shifts. Future consumption increases by less than the increase in future income, saving decreases, and current consumption increases.
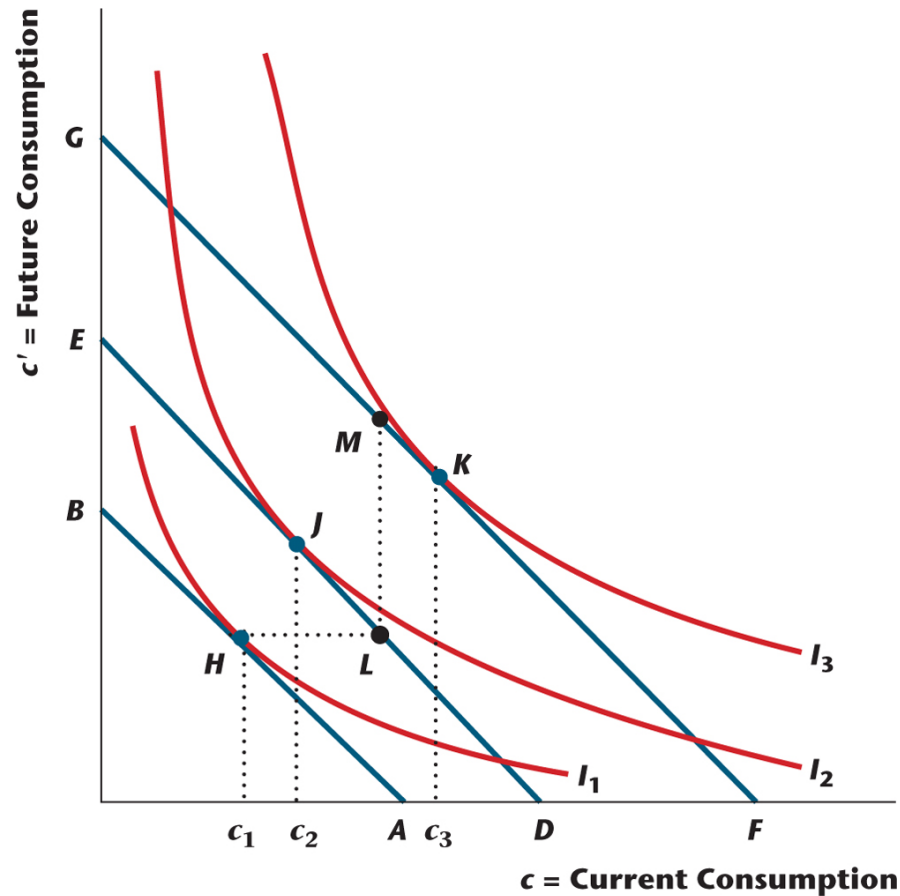
Temporary and Permanent Increases in Income

- As a permanent increase in income will have a larger effect on lifetime wealth than a temporary increase, there will be a larger effect on current consumption.

- A consumer will tend to save most of a purely temporary income increase.

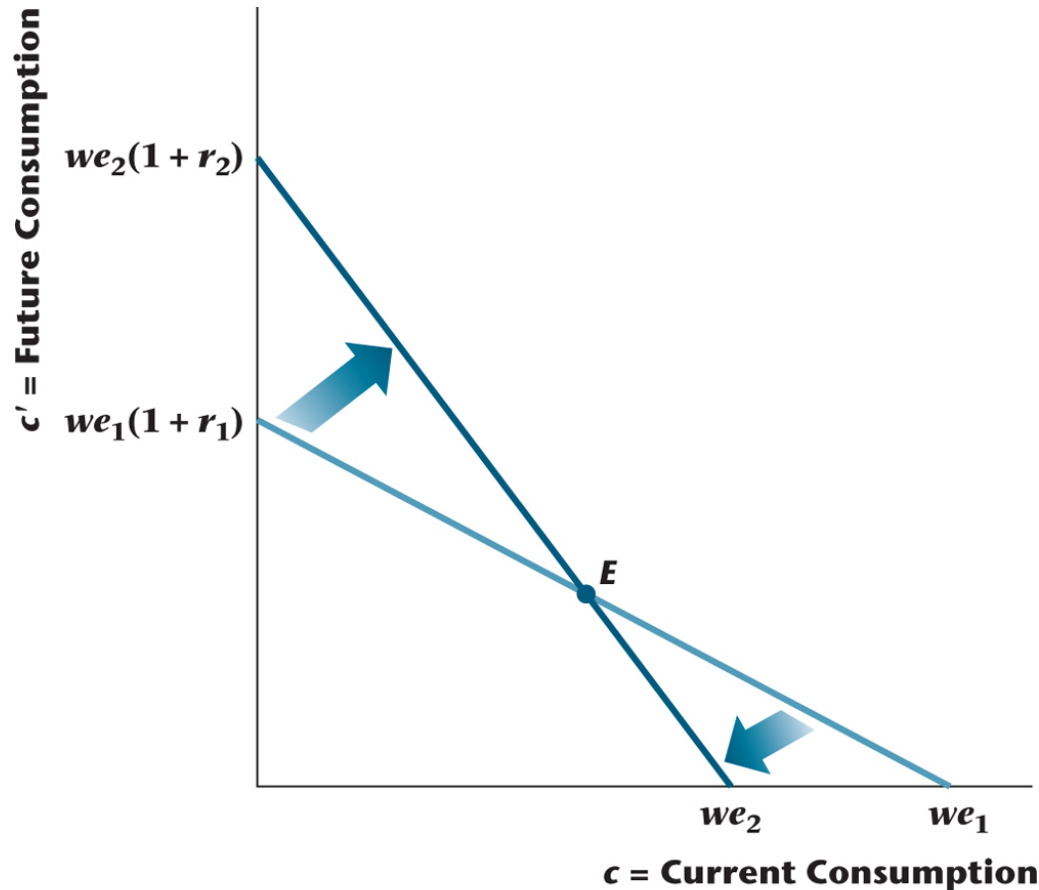# Figure 9.8
Temporary vs. Permanent Increases in Income

A temporary increase in income is an increase in current income, with the budget constraint shifting from *AB* to *DE*, and the optimal consumption bundle changing from *H* to *J*. When there is a permanent increase in income, current and future income both increase, and the budget constraint shifts from *AB* to *FG*, with the optimal consumption bundle changing from *H* to *K*.

Figure 9.11
An Increase in the Real Interest Rate

An increase in the real interest rate causes the lifetime budget constraint of the consumer to become steeper and to pivot around the endowment point *E*.

An Increase in the Market Real Interest Rate

- An increase in the market real interest rate decreases the relative price of future consumption goods in terms of current consumption goods – this has income and substitution effects for the consumer.

Figure 9.12
An Increase in the Real Interest Rate for a Lender
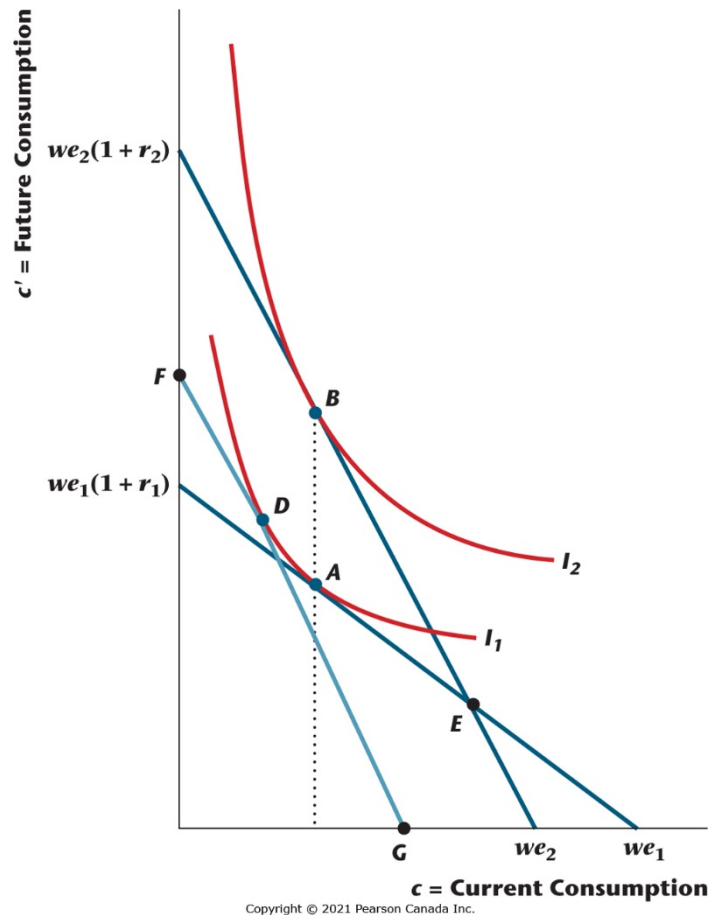
- Future consumption must increase, as both the income and substitution effects work in the same direction.
- Current-period consumption may increase or decrease
- Saving increases if the substitution effect is larger than the income effect, and saving decreases otherwise

When the real interest rate increases for a lender, the substitution effect is the movement from *A* to *D*, and the income effect is the movement from *D* to *B*. Current consumption and saving may rise or fall, while future consumption increases.

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Figure 9.13
An Increase in the Real Interest Rate for a Borrower

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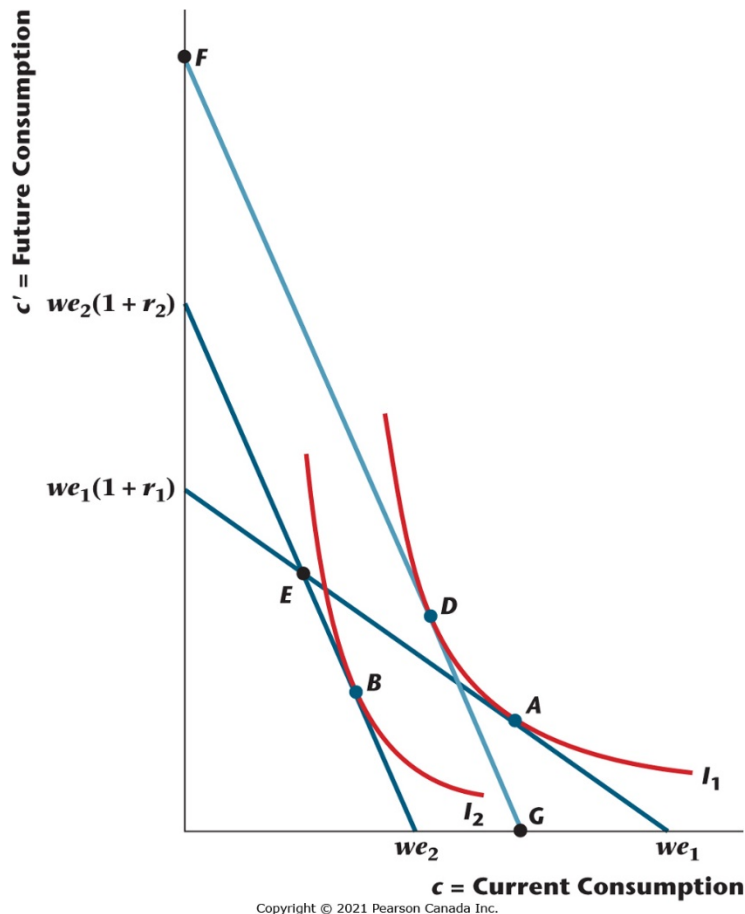
- The income effect in this case is negative for both current and future consumption.
- Current consumption falls but future consumption may rise or fall, depending on how strong the opposing substitution and income effects are.
- Savings must rise, as current consumption falls and current disposable income is held constant

When the real interest rate increases for a borrower, the substitution effect is the movement from A to D, and the income effect is the movement from D to B. Current consumption decreases, saving increases, and future consumption may rise or fall.

An Increase in the Real Interest Rate

- For both lenders and borrowers, there is an **intertemporal substitution effect** of an increase in the real interest rate.
- A higher real interest rate
    - lowers the relative price of future consumption in terms of current consumption,
    - This leads to a substitution of future consumption for current consumption and, therefore, to an increase in savings.
- Borrower & Lender:
    - Consumption decreases for borrower
    - Consumption of lenders depends on the strength of opposing income and substitution effects.
- There is no theoretical guarantee that aggregate consumption will fall when the real interest rate rises

Effects of an Increase in the Real Interest Rate for a <mark>Lender</mark>

Table 9.2 Effects of an Increase in the Real Interest Rate for a Lender

Current consumption	?
Future consumption	Increases
Current savings	?

Effects of an Increase in the Real Interest Rate for a <mark>Borrower</mark>

Table 9.3 Effects of an Increase in the Real Interest Rate for a Borrower

Current consumption	Decreases
Future consumption	?
Current savings	Increases

Perfect Complements Example

- With perfect complements, the ratio of future consumption to current consumption is constant.

$$c' = ac$$

- The consumer's budget constraint must hold.

$$c + \frac{c'}{1+r} = we$$

- With perfect complements we can solve explicitly for current and future consumption:

$$c = \frac{we(1+r)}{1+r+a}$$

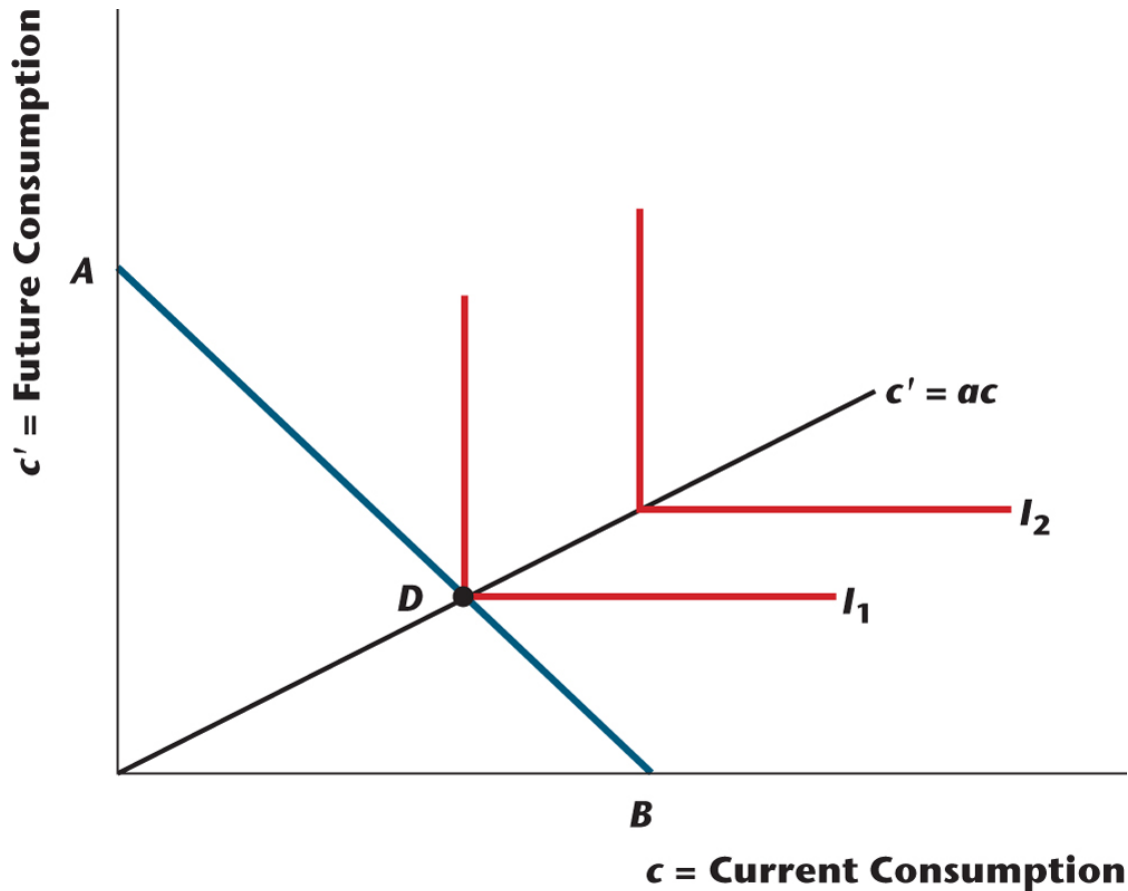$$c' = \frac{awe(1+r)}{1+r+a}$$

Perfect Complements Example

- Substituting for lifetime wealth gives:

$$c = \frac{(y - t)(1 + r) + y' - t'}{1 + r + a}$$

$$c' = a \left[\frac{(y - t)(1 + r) + y' - t'}{1 + r + a} \right]$$

Figure 9.14
Example with Perfect Complements Preferences

The consumer desires current and future consumption in fixed proportions, with $c' = ac$. With indifference curves representing perfect complementarity between current and future consumption, the optimal consumption bundle is at point D on the lifetime budget constraint AB.

Government Budget Constraints

The government's current-period budget constraint:

$$G = T + B$$

- **The** current-period government deficit, *G - T, is financed by issuing bonds.*

- The government's budget constraints allow for the possibility that B < 0.

- If B < 0 this would imply that the government was a lender to the private sector, rather than a borrower from it.

Government Budget Constraints

The government's future-period budget constraint:

$$G' = (1 + r)B = T'$$

- The government's budget constraints allow for the possibility that B < 0.

- If B < 0 this would imply that the government was a lender to the private sector, rather than a borrower from it.

Government Budget Constraints

- In practice, the government engages in direct lending to the private sector, and it issues debt to private economic agents, so that it is a lender and a borrower.

- The government's *present-value budget constraint*:

$$G + \frac{G'}{1 + r} = T + \frac{T'}{1 + r}$$

- An interpretation of the government present-value budget constraint is that the government must eventually pay off all of its debt by taxing its citizens

Credit Market Equilibrium Condition

- In a competitive equilibrium for this two-period economy, the following three conditions must hold:
  - Each consumer chooses first- and second-period consumption and savings optimally given the real interest rate *r.*
  - The government present-value budget constraint, holds.
  - The credit market clears.
- That private savings is equal to the quantity of government bonds issued in the current period.

$$S^p = B$$

Income-Expenditure Identity

- Credit market equilibrium implies that the income-expenditure identity holds.

$$Y = C + G$$

$$S^p = B$$

$$S^p = Y - C - T \quad \& $$

$$B = G - T$$

$$Y - C - T = G - T$$

$$Y = C + G$$

Ricardian Equivalence

- The Ricardian Equivalence theorem states that a change in the timing of taxes by the government is neutral.

- That is a change in current taxes, exactly offset in present-value terms by an equal and opposite change in future taxes, has no effect on the real interest rate or on the consumption of individual consumers.

Ricardian Equivalence

- Each of the N consumers shares an equal amount of the total tax burden in the current and future periods, with T = Nt and T′ = Nt′,

- substituting in the government's present-value budget constraint

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

$$G + G'/(1 + r) = Nt + Nt'/(1 + r)$$

$$t + \frac{t'}{1 + r} = \frac{1}{N}\left(G + \frac{G'}{1+r}\right)$$

- Key equation: The consumer's lifetime tax burden is equal to the consumer's share of the present value of government spending – the timing of taxation does not matter for the consumer.

Ricardian Equivalence

- Then, substitute in the consumer's budget constraint

$$c + c'/(1 + r) = [y + y'/(1 + r)] - [t + t'/(1 + r)]$$

$$t + \frac{t'}{1+r} = \frac{1}{N}\left(G + \frac{G'}{1+r}\right)$$

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \frac{1}{N}\left[G + \frac{G'}{1+r}\right]$$

- taxes do not matter in equilibrium for the consumer's lifetime wealth, just the present value of government spending.

Ricardian Equivalence

- suppose that there is a cut in current taxes, so that $\Delta t < 0.$

- the government must issue more debt today to finance the tax cut, and it will have to increase taxes in the future to pay off this higher debt.

- Consumers anticipate this, and they increase their savings by the amount of the tax cut, because this is how much extra they have to save to pay the higher taxes they will face in the future.

- In the credit market, there is an increase in savings by consumers, which just matches the increase in borrowing by the government, so there is no effect on borrowing and lending among consumers, and therefore, no effect on the market real interest rate.

Figure 9.16
Ricardian Equivalence with a Cut in Current Taxes for a Lender
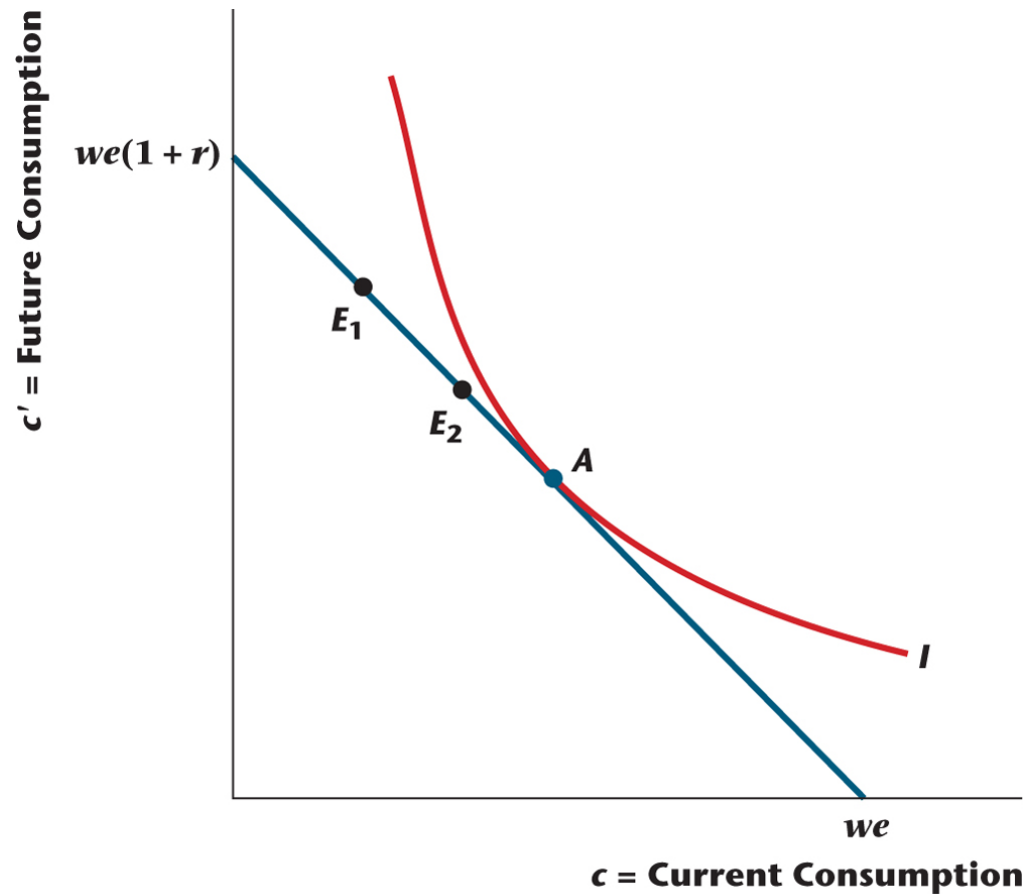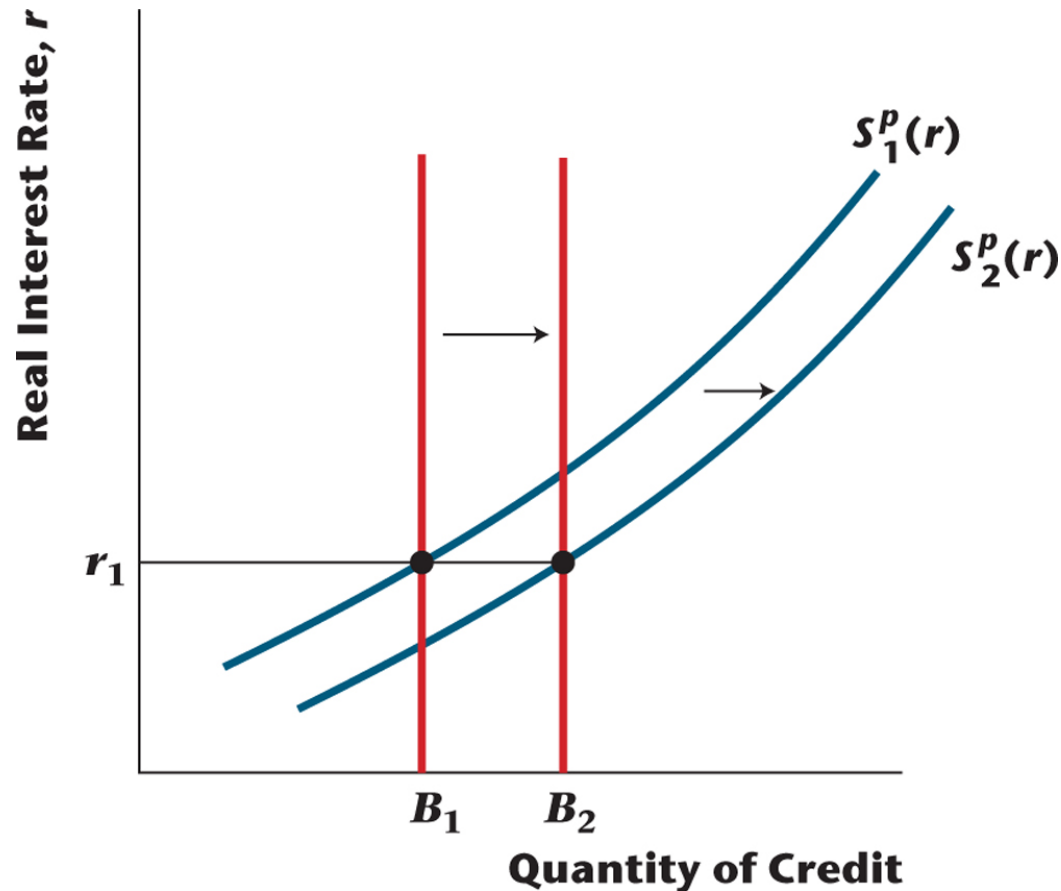

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A current tax cut with a future increase in taxes leaves the consumer's lifetime budget constraint unchanged, and so the consumer's optimal consumption bundle remains at A. The endowment point shifts from E_1 to E_2, so that there is an increase in saving by the amount of the current tax cut.

Figure 9.17
Ricardian Equivalence and Credit Market Equilibrium

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With a decrease in current taxes, government debt increases from B_1 to B_2, and the credit supply curve shifts to the right by the same amount. The equilibrium real interest rate is unchanged, and private saving increases by an amount equal to the reduction in government saving.

Why Might Ricardian Equivalence Fail in Practice?

- Redistributional effects of taxes: tax changes affect the wealth of different consumers differently.

- Intergenerational redistribution: debt issued by the government today is paid off by future generations.

- Taxes are not lump sum; they cause distortions.

- Credit market frictions: Chapter 10.