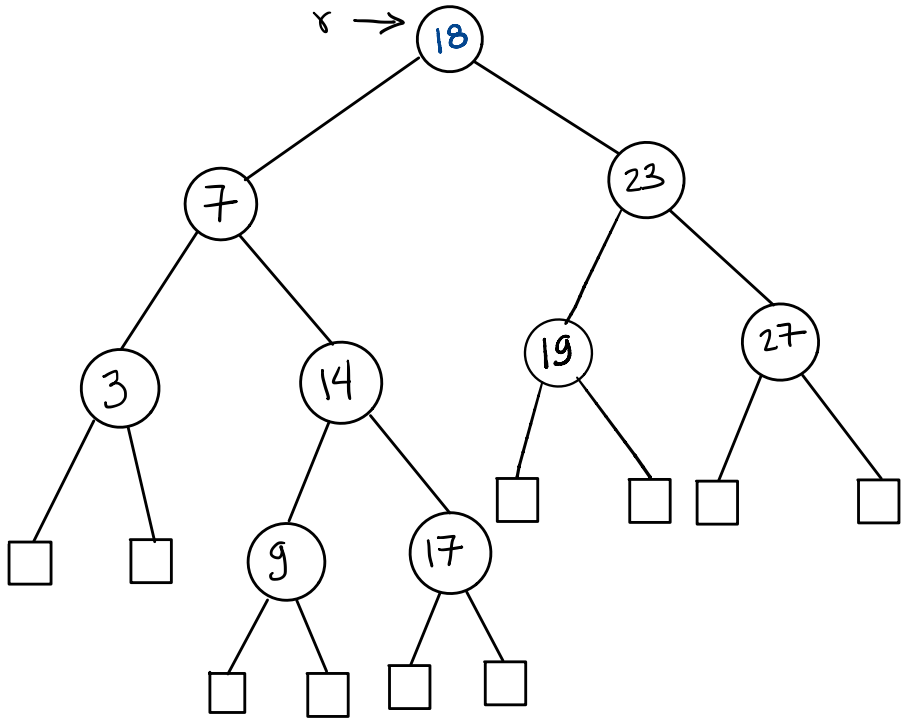
Ordered Dictionary Implemented with Binary Search Trees

Operations

get(k)
smallest()
largest()
put(k,d)
remove(k)
successor(k)
predecessor(k)

O(height of tree) time complexity



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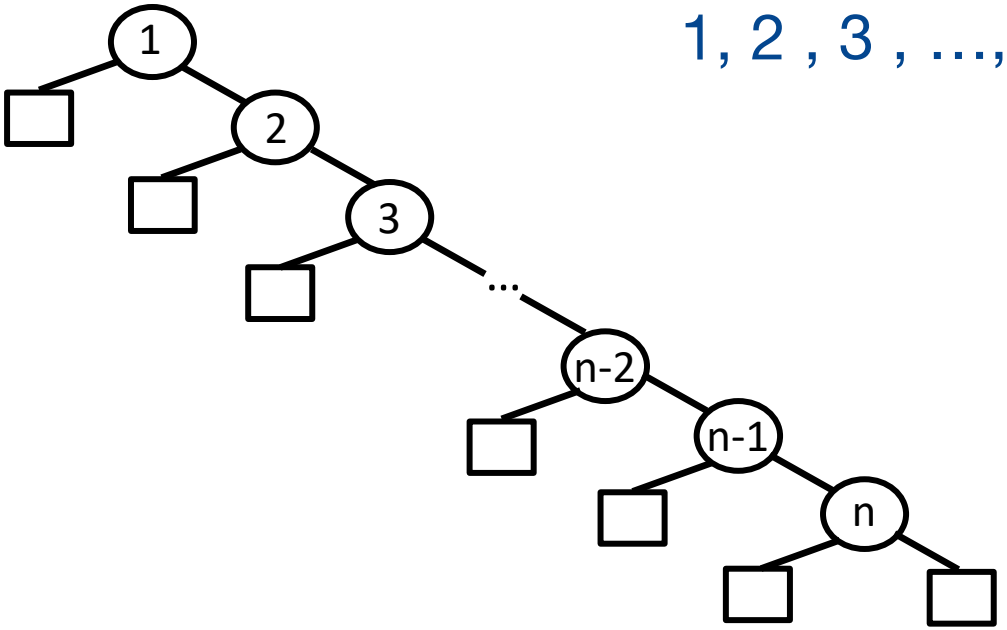
O(height of tree) time complexity

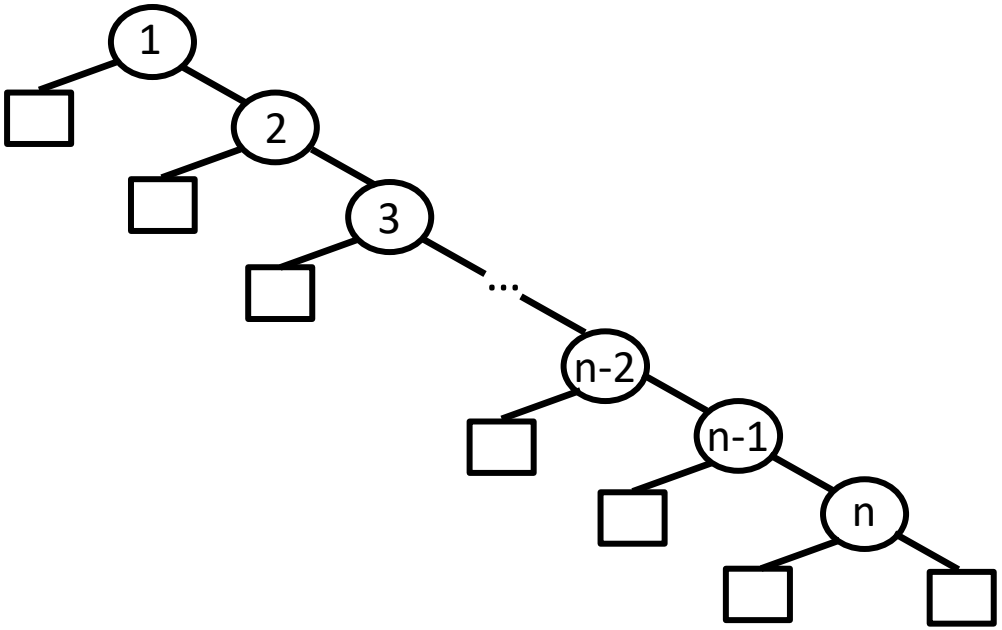
What is the maximum height of a binary search tree?

Balanced and Unbalanced Trees

Store in an initially empty tree the keys
1, 2 , 3 , …, n

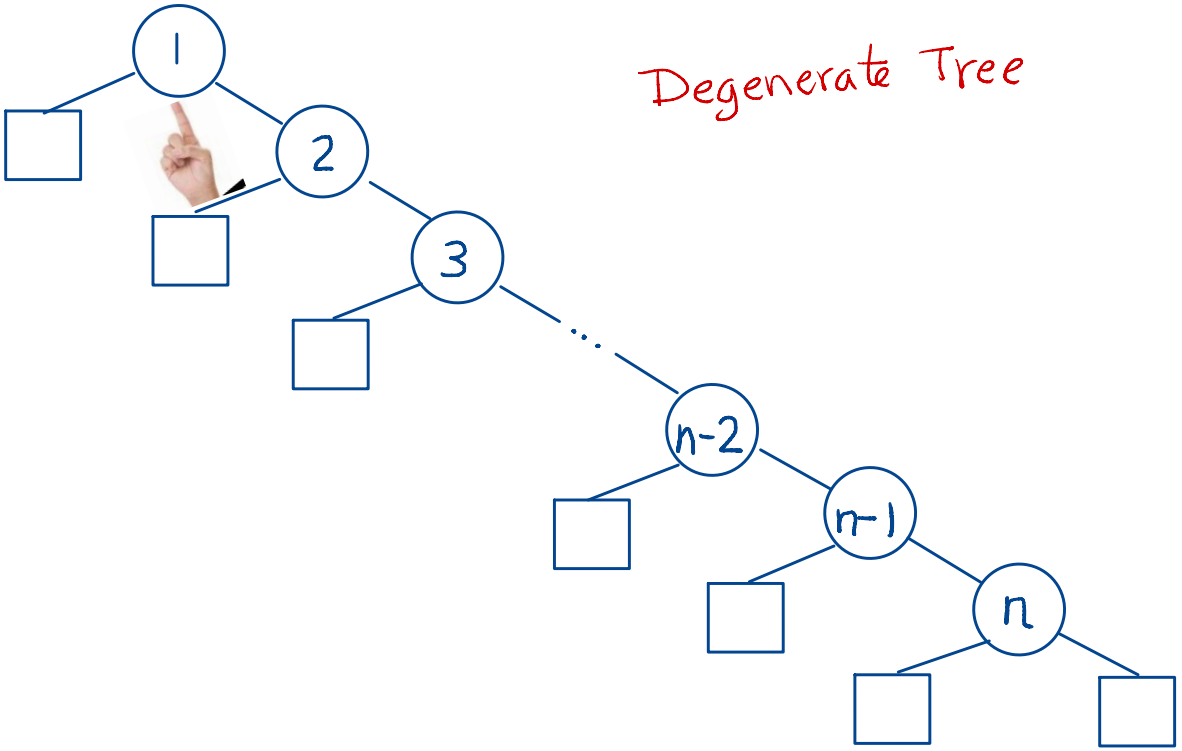


Balanced and Unbalanced Trees



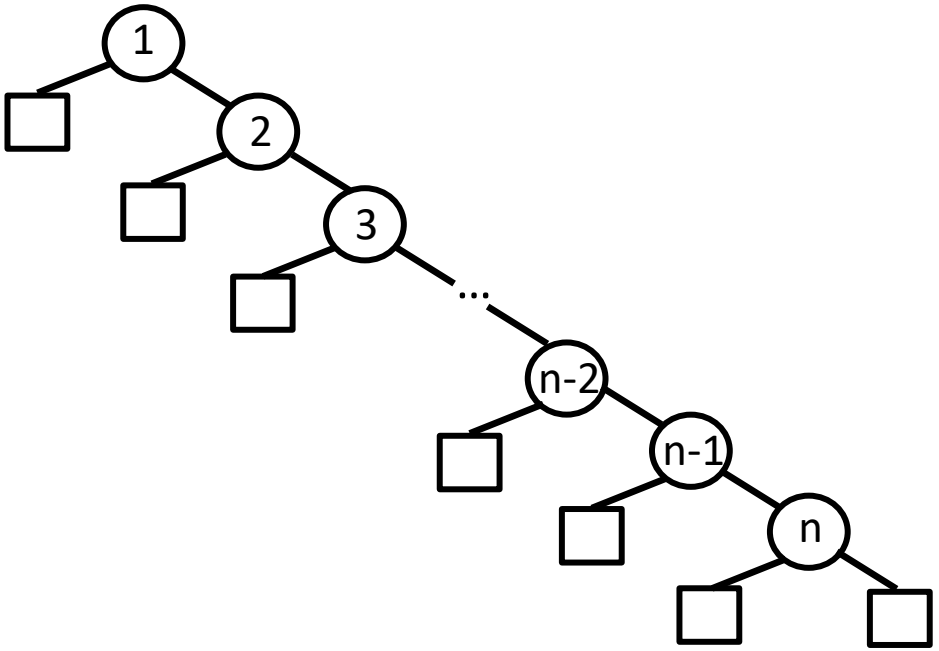
Unbalanced tree

Balanced and Unbalanced Trees

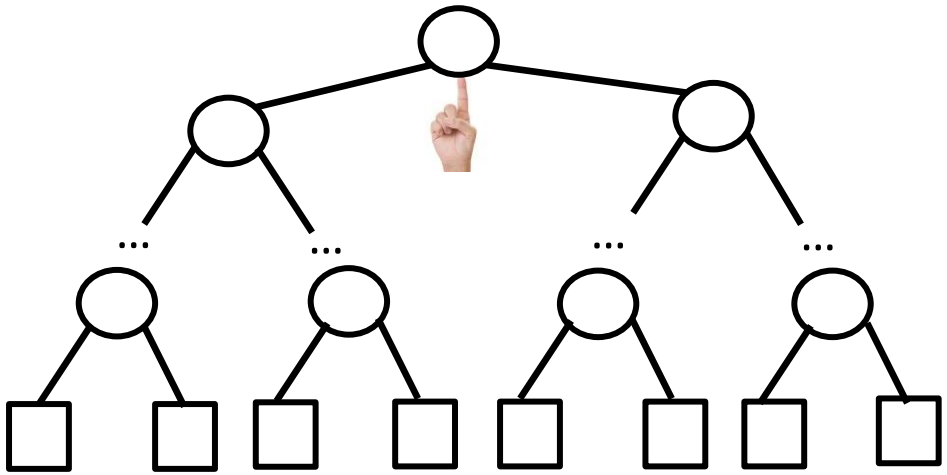


Unbalanced tree

Balanced and Unbalanced Trees



Unbalanced tree

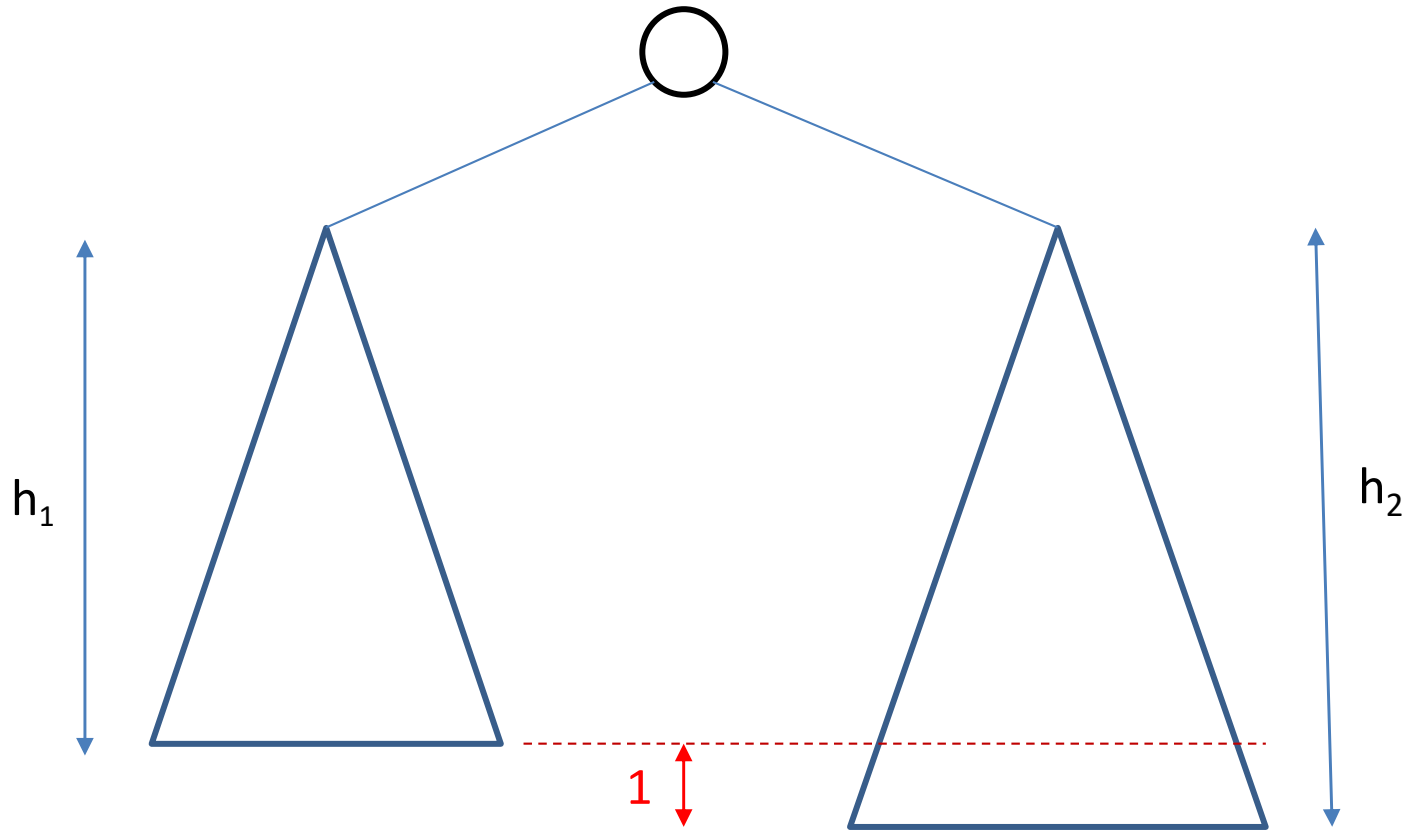


Balanced tree

AVL Trees

An AVL tree is a binary search tree in which for every internal node the heights of its two subtrees differ by at most 1.

$|h_i - h_j| \leq 1$



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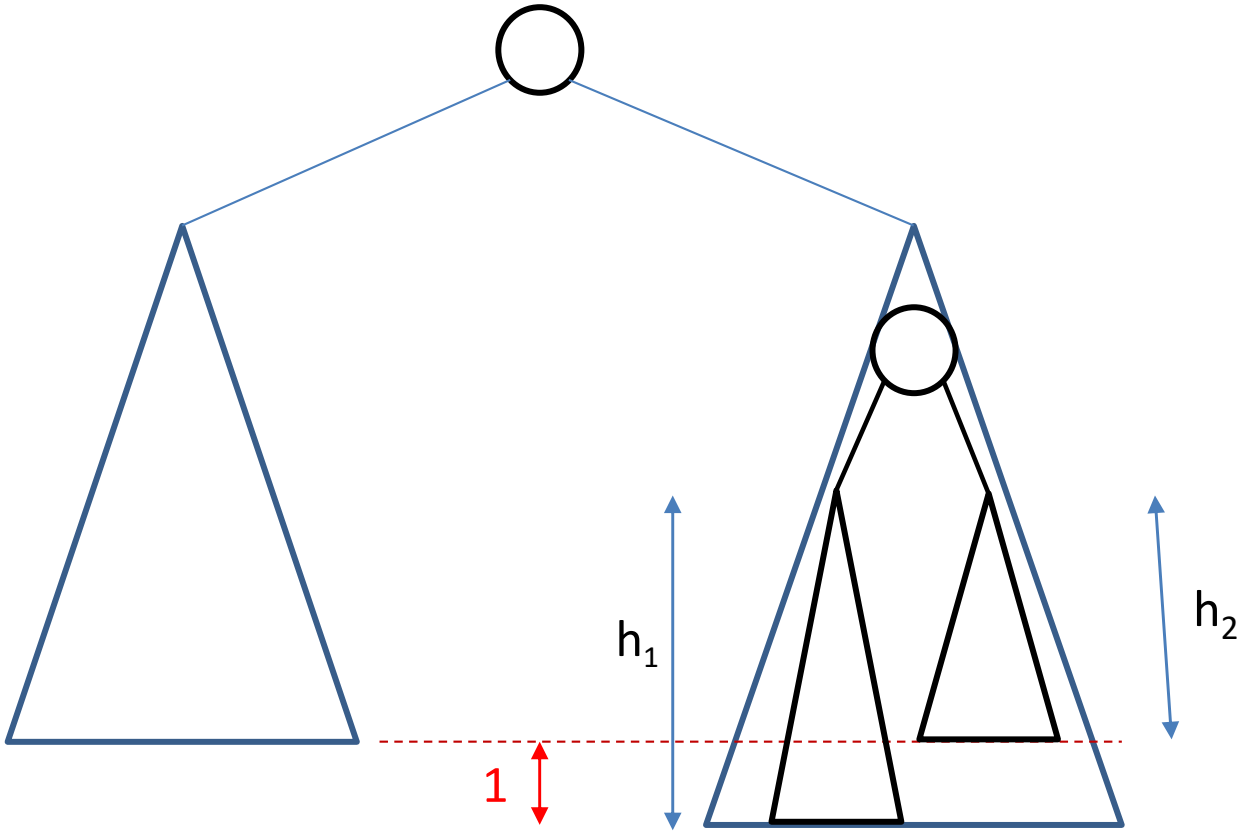
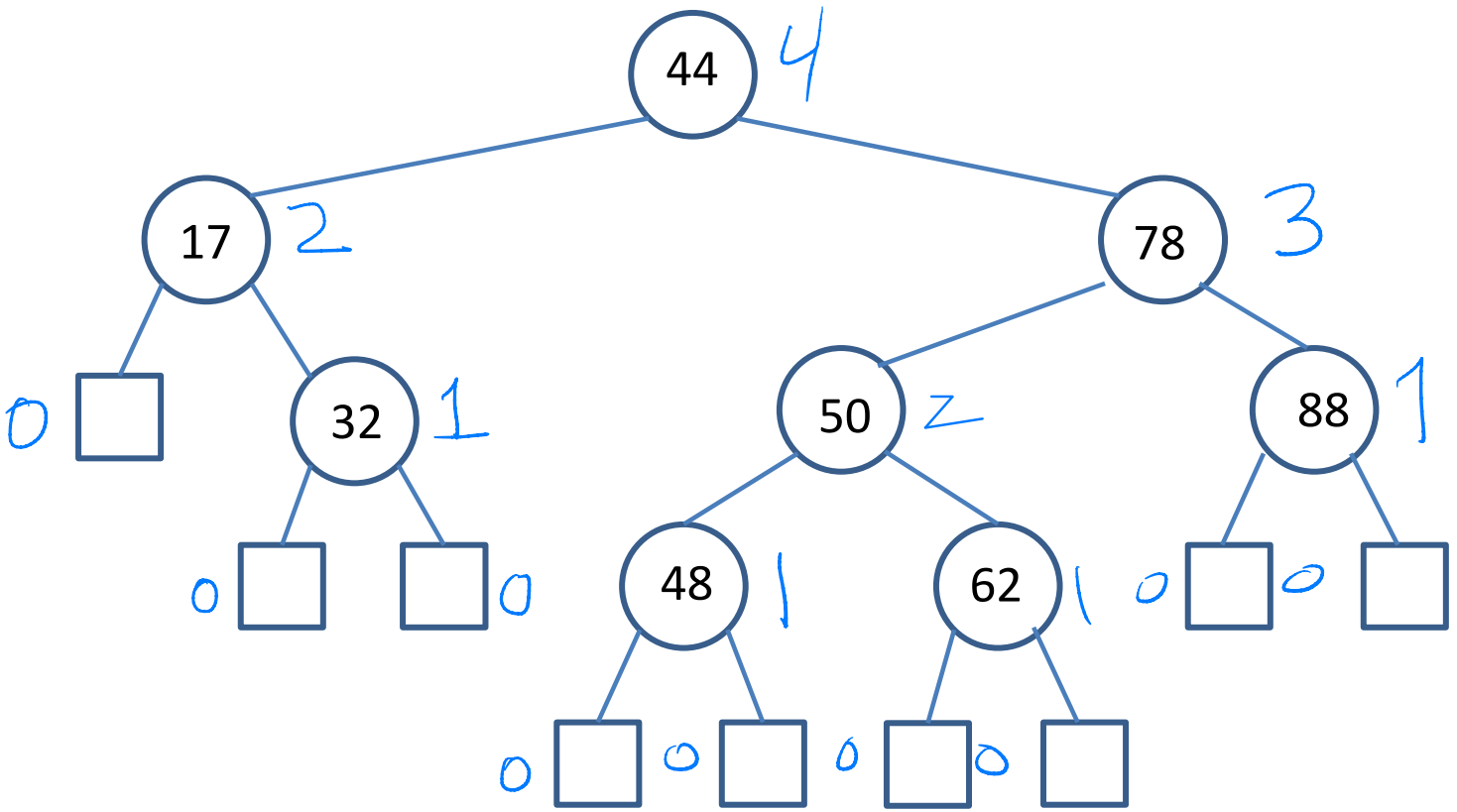
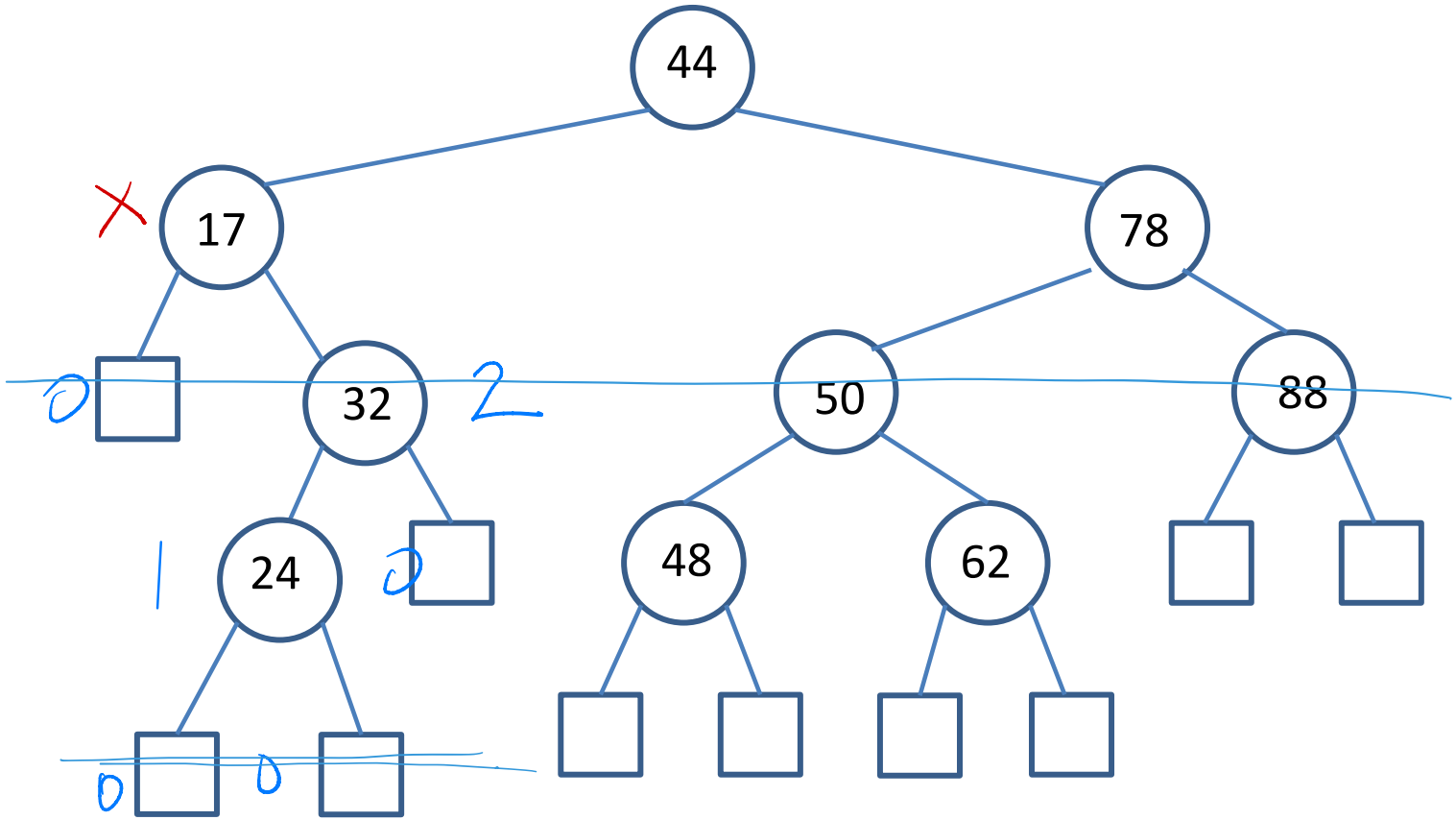


AVL Tree? Yes

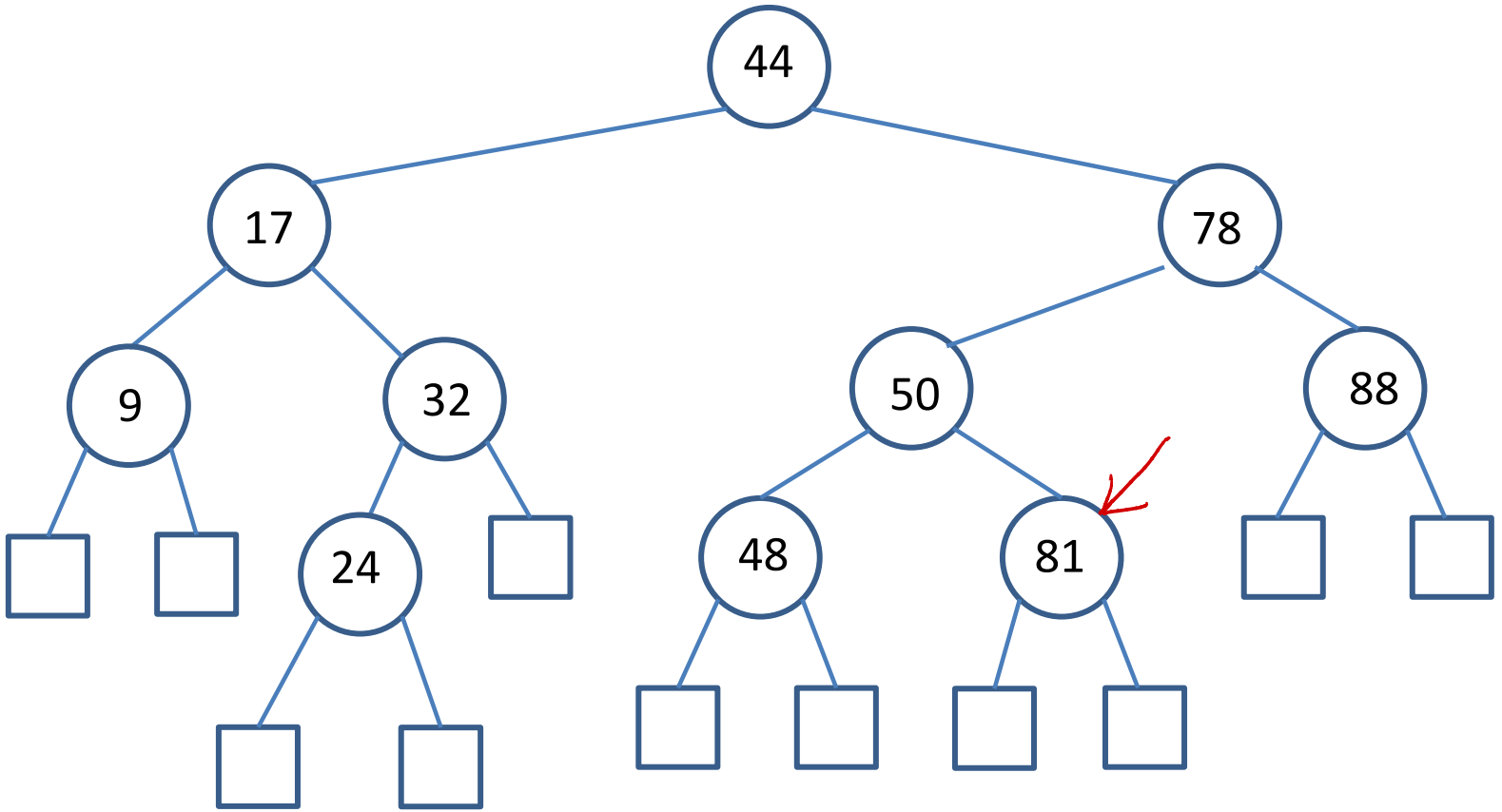


AVL Tree? No



AVL Tree? No

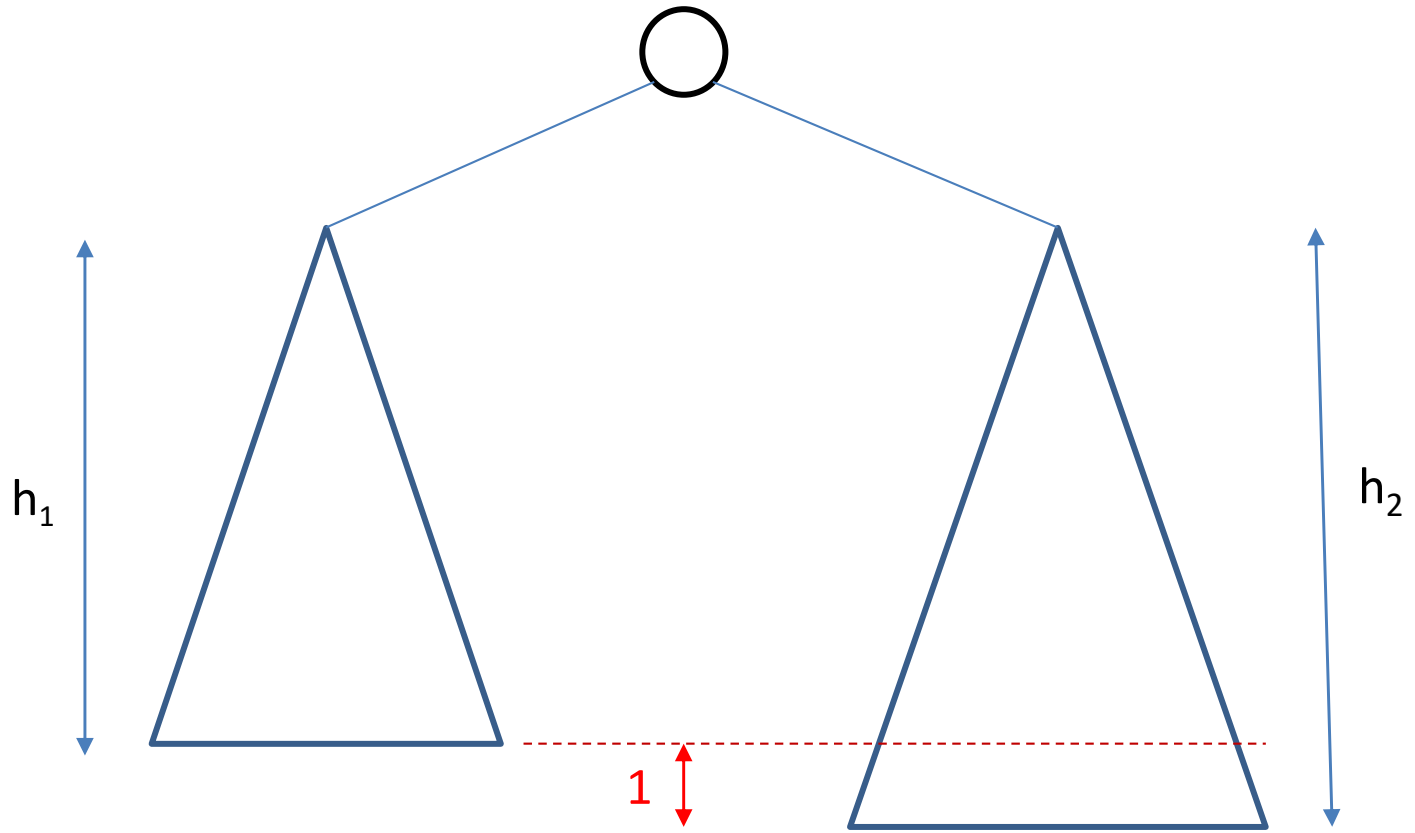
AVL Tree is a type of BST.



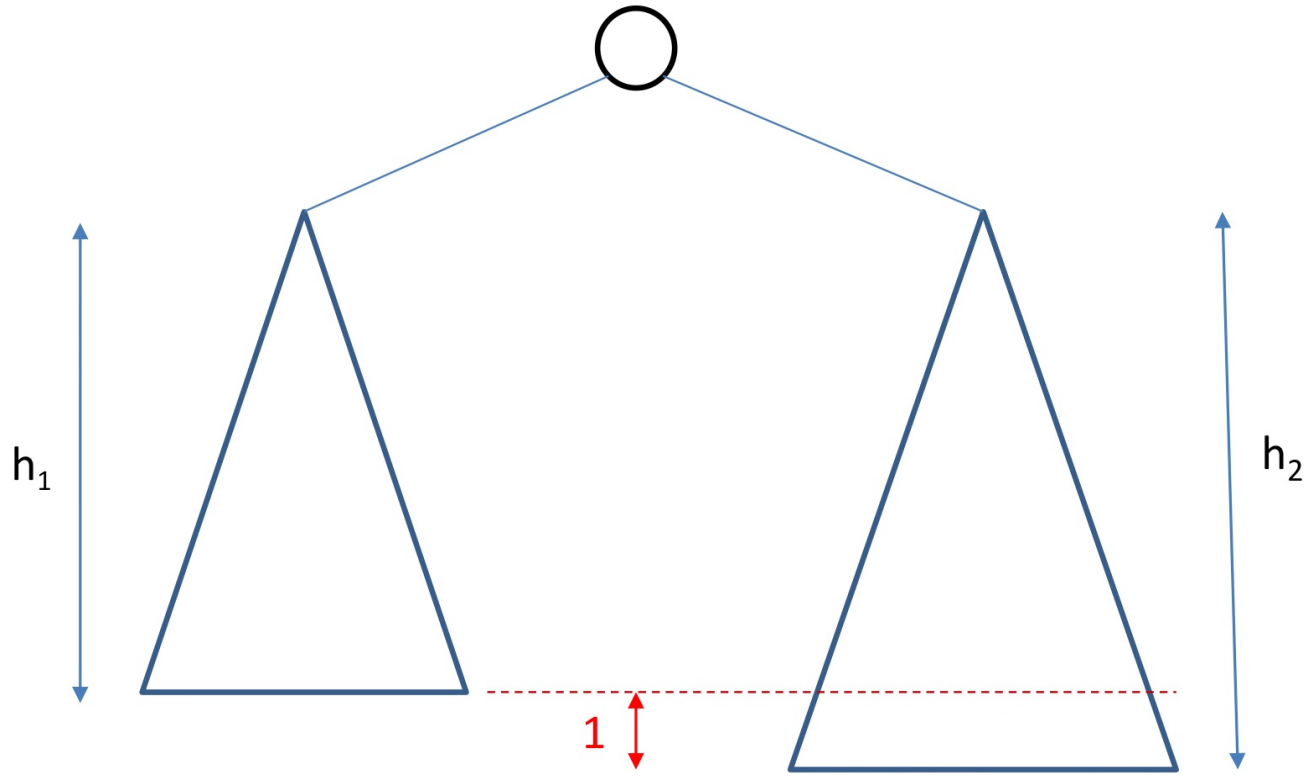
AVL Trees

Adelson-Velkskii and Landis

An AVL tree is a binary search tree in which for every internal node the heights of its two subtrees differ by at most 1.



What is the Maximum Height of an AVL Tree?



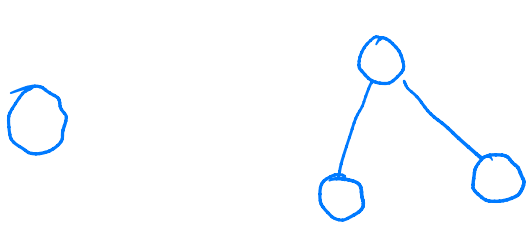
What is the Maximum Height of an AVL Tree?

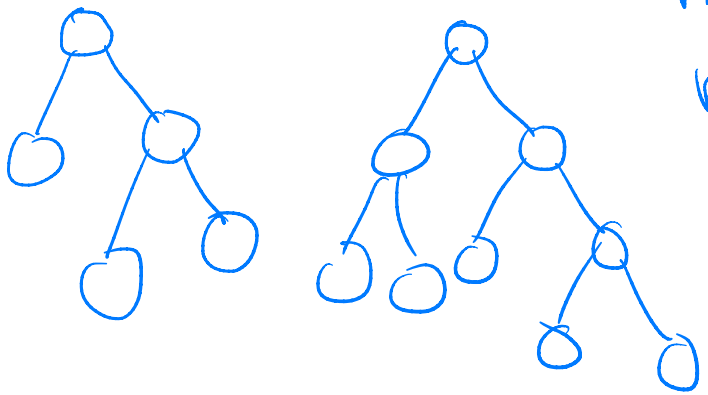
Let n(h) = minimum number of nodes in an AVL tree of height h.

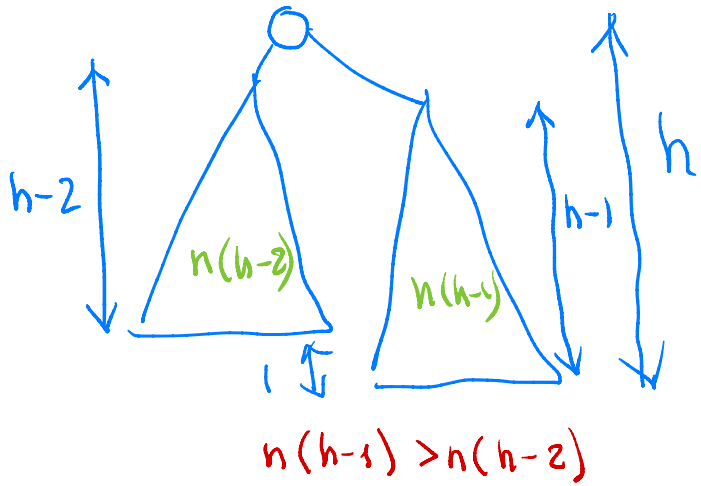
$n(0) = 1$ $n(1) = 3$ $n(2) = 5$ $n(3) = 9$ $n(4) = 15$

$n(5) = 25$

$n(6) = 41$







$n(h-1) > n(h-2)$

$n(h) = 1 + n(h-2) + n(h-1)$

$n(h) > 1 + 2n(h-2)$

$n(0) = 1$

$n(1) = 3$