Tutorial 06: Combinational Circuits

Computer Science Department
CS2208: Introduction to Computer Organization and Architecture
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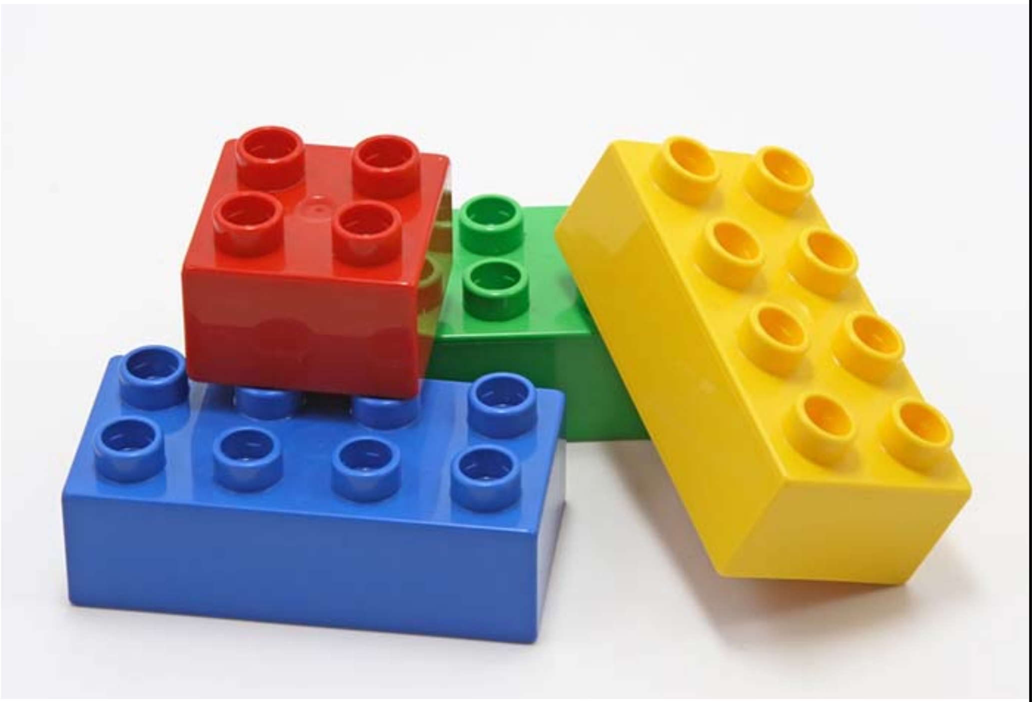
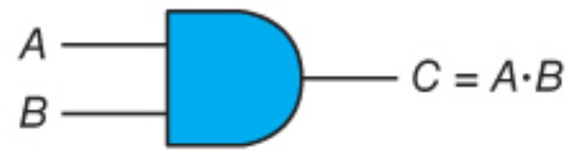
Gates

- Gates
  - Are our building blocks when considering hardware realization
  - Can be used to build combinational circuits, as well as latches, flip-flops, and sequential circuits
  - Come in many flavors, including
    - AND
    - OR
    - NOT
    - NAND (Not AND)
    - NOR (Not OR)
    - XOR

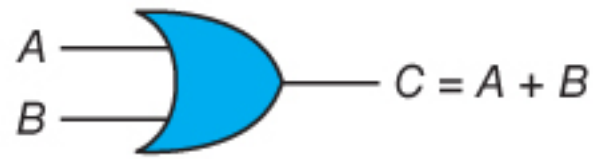


Gates

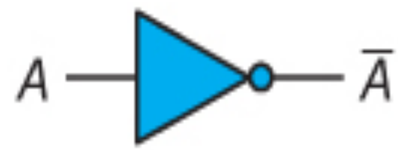
- AND:
  - True only when all input are true



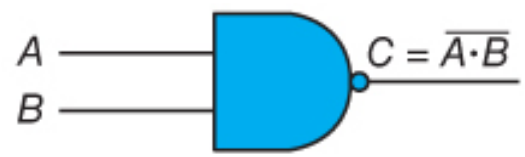
- OR:
  - True only when at least one input is true



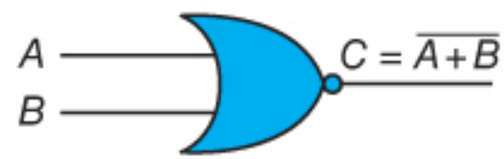
- NOT:
  - Complementing the input Boolean value



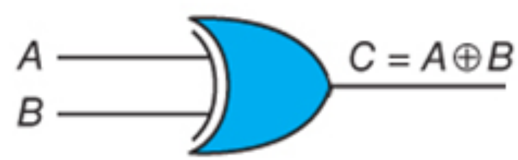
- NAND:
  - The complement of an AND result



- NOR:
  - The complement of an OR result



- XOR:
  - True only when an odd number of inputs are true

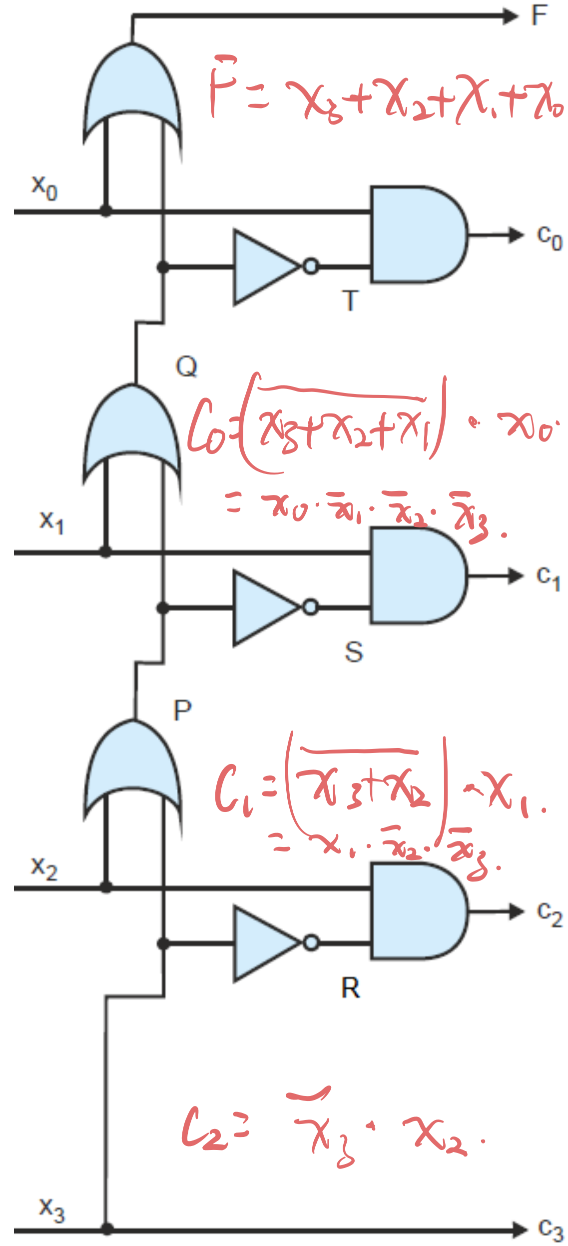


Example 1

X_0	X_1	X_2	X_3	P	S	Q	T	R	C_0	C_1	C_2	C_3	F
0	0	0	0	0	1	0	1	1					
0	0	0	1	1	0	1	0	0					
0	0	1	0	1	0	1	0	1					
0	0	1	1	1	0	1	0	0					
0	1	0	0	0	1	1	0	1					
0	1	0	1	1	0	1	0	0					
0	1	1	0	1	0	1	0	1					
0	1	1	1	1	0	1	0	0					
1	0	0	0	0	1	0	1	1					
1	0	0	1	1	0	1	0	0					
1	0	1	0	1	0	1	0	1					
1	0	1	1	1	0	1	0	0					
1	1	0	0	0	1	1	0	1					
1	1	0	1	1	0	1	0	0					
1	1	1	0	1	0	1	0	1					
1	1	1	1	1	0	1	0	0					

B	A	$C = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

A	$\bar{A}$
1	0
0	1



$F = x_3 + x_2 + x_1 + x_0$

$C_0 = \left(\overline{x_3 + x_2 + x_1}\right) \cdot x_0 = x_0 \cdot \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3$

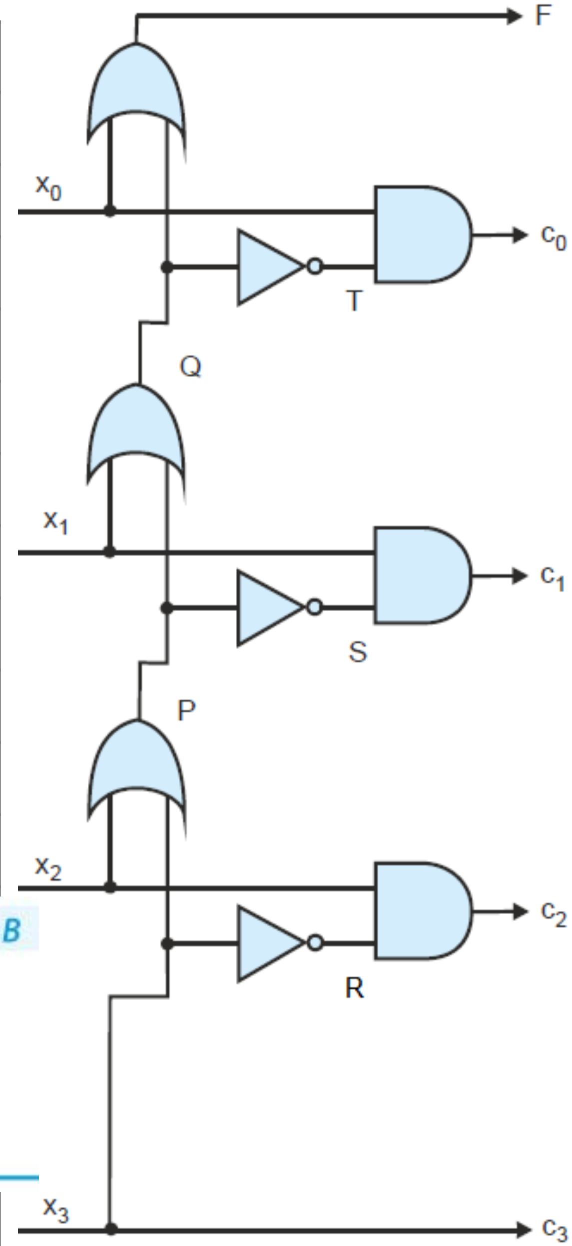
$C_1 = \left(\overline{x_3 + x_2}\right) \cdot x_1 = x_1 \cdot \bar{x}_2 \cdot \bar{x}_3$

$C_2 = \bar{x}_3 \cdot x_2$

Example 1

X_0	X_1	X_2	X_3	P	S	Q	T	R	C_0	C_1	C_2	C_3	F
0	0	0	0	0	1	0	1	1	0	0			
0	0	0	1	1	0	1	0	0	0	0			
0	0	1	0	1	0	1	0	1	0	0			
0	0	1	1	1	0	1	0	0	0	0			
0	1	0	0	0	1	1	0	1	0	1			
0	1	0	1	1	0	1	0	0	0	0			
0	1	1	0	1	0	1	0	1	0	0			
0	1	1	1	1	0	1	0	0	0	0			
1	0	0	0	0	1	0	1	1	1	0			
1	0	0	1	1	0	1	0	0	0	0			
1	0	1	0	1	0	1	0	1	0	0			
1	0	1	1	1	0	1	0	0	0	0			
1	1	0	0	0	1	1	0	1	0	1			
1	1	0	1	1	0	1	0	0	0	0			
1	1	1	0	1	0	1	0	1	0	0			
1	1	1	1	1	0	1	0	0	0	0			

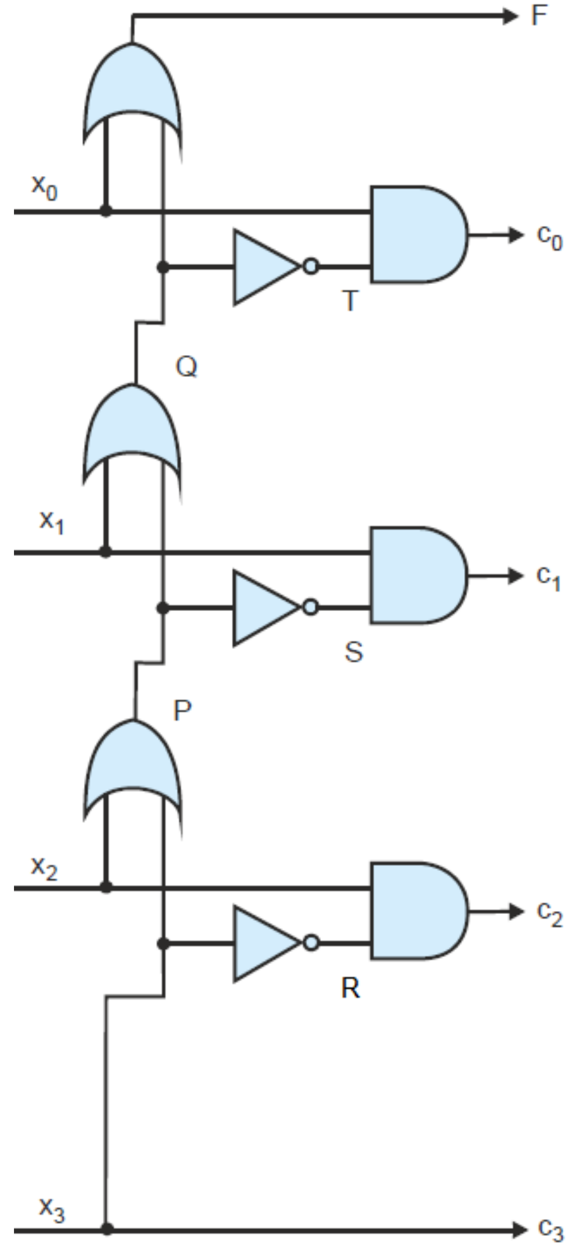
B	A	$C = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1



Example 1

X_0	X_1	X_2	X_3	P	S	Q	T	R	C_0	C_1	C_2	C_3	F
0	0	0	0	0	1	0	1	1	0	0	0	0	
0	0	0	1	1	0	1	0	0	0	0	0	1	
0	0	1	0	1	0	1	0	1	0	0	1	0	
0	0	1	1	1	0	1	0	0	0	0	0	1	
0	1	0	0	[illegible]	[illegible]	[illegible]	0	1	0	1	0	0	
0	1	0	1	[illegible]	[illegible]	[illegible]	0	0	0	0	0	1	
0	1	1	0	[illegible]	[illegible]	[illegible]	0	1	0	0	1	0	
0	1	1	1	[illegible]	[illegible]	[illegible]	0	0	0	0	0	1	
1	0	0	0	0	1	0	1	1	1	0	0	0	
1	0	0	1	1	0	1	0	0	0	0	0	1	
1	0	1	0	1	0	1	0	1	0	0	1	0	
1	0	1	1	1	0	1	0	0	0	0	0	1	
1	1	0	0	0	1	1	0	1	0	1	0	0	
1	1	0	1	1	0	1	0	0	0	0	0	1	
1	1	1	0	1	0	1	0	1	0	0	1	0	
1	1	1	1	1	0	1	0	0	0	0	0	1	

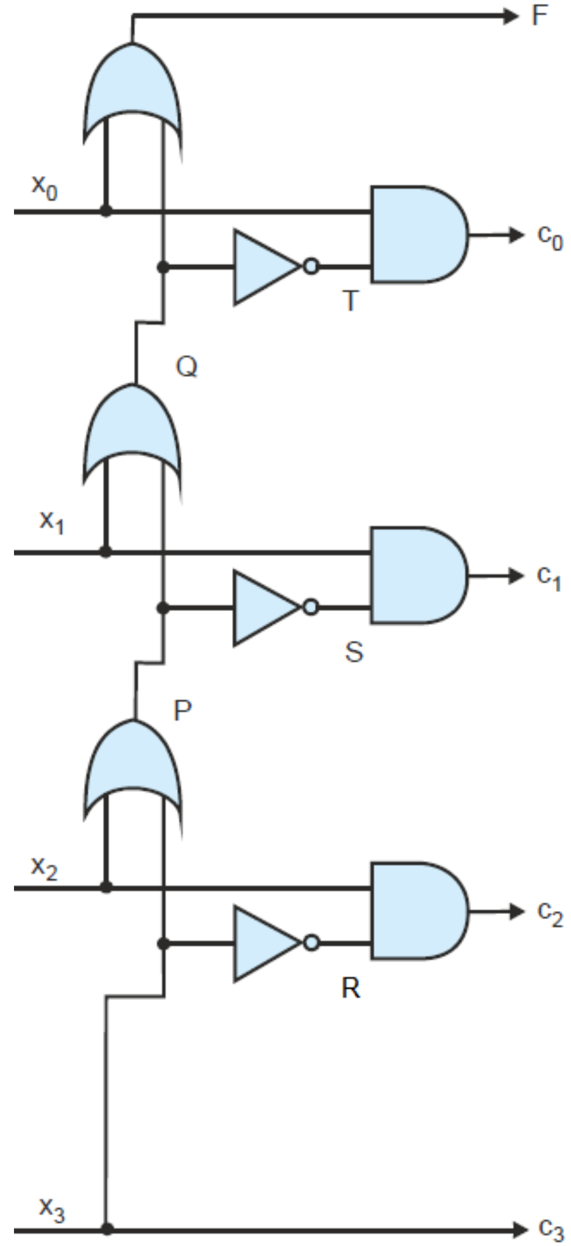
B	A	$C = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1



Example 1

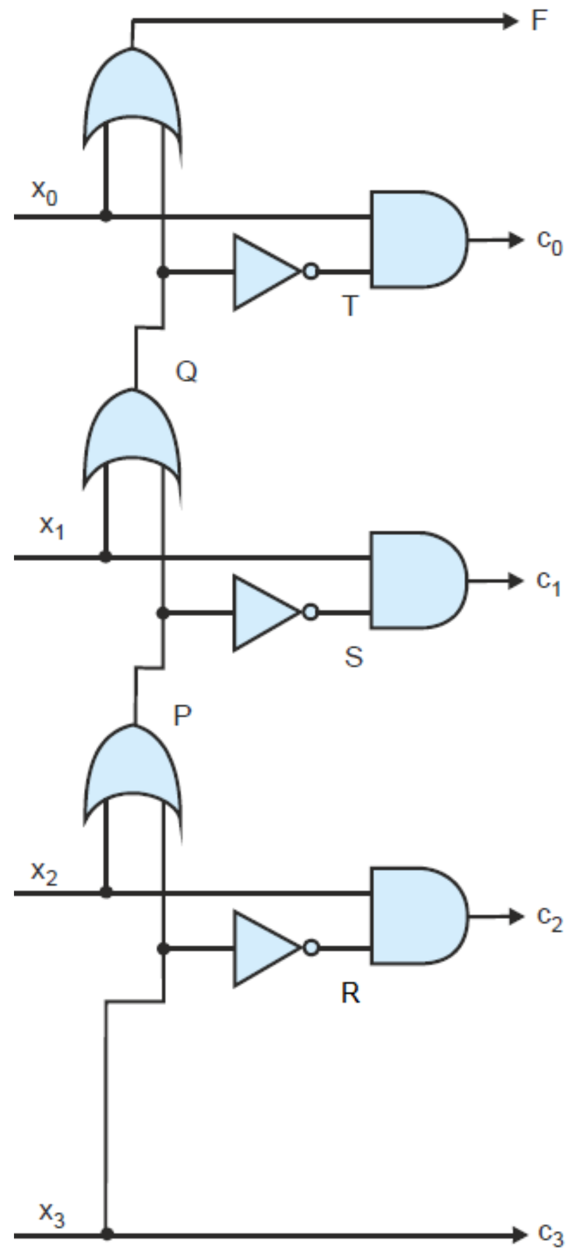
X_0	X_1	X_2	X_3	P	S	Q	T	R	C_0	C_1	C_2	C_3	F
0	0	0	0	0	1	0	1	1	0	0	0	0	0
0	0	0	1	1	0	1	0	0	0	0	0	1	1
0						1	0	1	0	0	1	0	1
0						1	0	0	0	0	0	1	1
0						1	0	1	0	1	0	0	1
0						1	0	0	0	0	0	1	1
0	1	1	0	1	0	1	0	1	0	0	1	0	1
0	1	1	1	1	0	1	0	0	0	0	0	1	1
1	0	0	0	0	1	0	1	1	1	0	0	0	1
1	0	0	1	1	0	1	0	0	0	0	0	1	1
1	0	1	0	1	0	1	0	1	0	0	1	0	1
1	0	1	1	1	0	1	0	0	0	0	0	1	1
1	1	0	0	0	1	1	0	1	0	1	0	0	1
1	1	0	1	1	0	1	0	0	0	0	0	1	1
1	1	1	0	1	0	1	0	1	0	0	1	0	1
1	1	1	1	1	0	1	0	0	0	0	0	1	1

B	A	$C = A + B$
0	0	0
0	1	1
1	0	1
1	1	1



Example 1

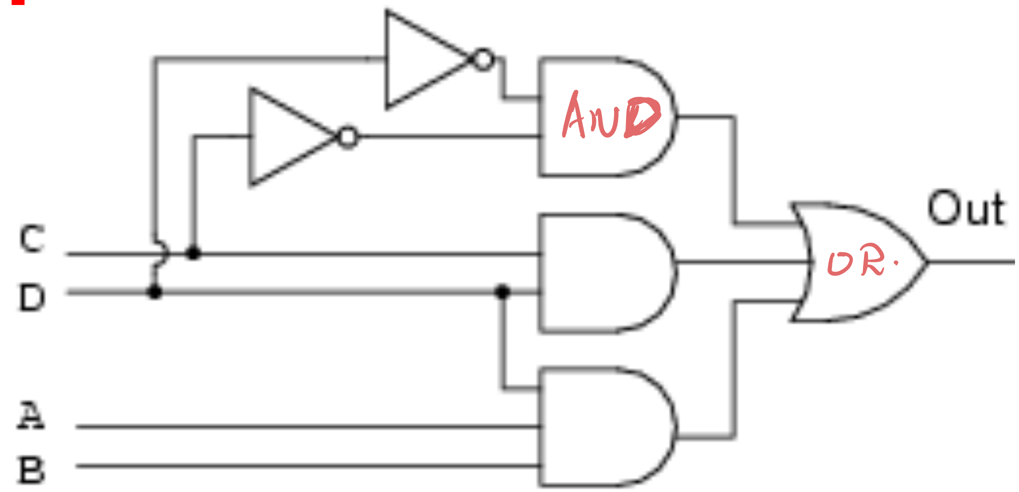
X_0	X_1	X_2	X_3	P	S	Q	T	R	C_0	C_1	C_2	C_3	F
0	0	0	0	0	1	0	1	1	0	0	0	0	0
0	0	0	1	1	0	1	0	0	0	0	0	1	1
0	0	1	0	1	0	1	0	1	0	0	1	0	1
0	0	1	1	1	0	1	0	0	0	0	0	1	1
0	1	0	0	0	1	1	0	1	0	1	0	0	1
0	1	0	1	1	0	1	0	0	0	0	0	1	1
0	1	1	0	1	0	1	0	1	0	0	1	0	1
0	1	1	1	1	0	1	0	0	0	0	0	1	1
1	0	0	0	0	1	0	1	1	1	0	0	0	1
1	0	0	1	1	0	1	0	0	0	0	0	1	1
1	0	1	0	1	0	1	0	1	0	0	1	0	1
1	0	1	1	1	0	1	0	0	0	0	0	1	1
1	1	0	0	0	1	1	0	1	0	1	0	0	1
1	1	0	1	1	0	1	0	0	0	0	0	1	1
1	1	1	0	1	0	1	0	1	0	0	1	0	1
1	1	1	1	1	0	1	0	0	0	0	0	1	1



- $P = X_2 + X_3$
- $S = \overline{P} = \overline{(X_2 + X_3)} = \overline{X_2} \,.\, \overline{X_3}$ (De Morgan law)
- $Q = X_1 + P = X_1 + X_2 + X_3$
- $T = \overline{Q} = \overline{X_1 + X_2 + X_3} = \overline{X_1} \,.\, \overline{X_2} \,.\, \overline{X_3}$ (De Morgan law)
- $R = \overline{X_3}$
- $C_0 = X_0 \,.\, T = X_0 \,.\, \overline{X_1} \,.\, \overline{X_2} \,.\, \overline{X_3}$
- $C_1 = X_1 \,.\, S = X_1 \,.\, \overline{X_2} \,.\, \overline{X_3}$
- $C_2 = X_2 \,.\, R = X_2 .\, \overline{X_3}$
- $C_3 = X_3$
- $F = X_0 + Q = X_0 + X_1 + X_2 + X_3$

Example 2

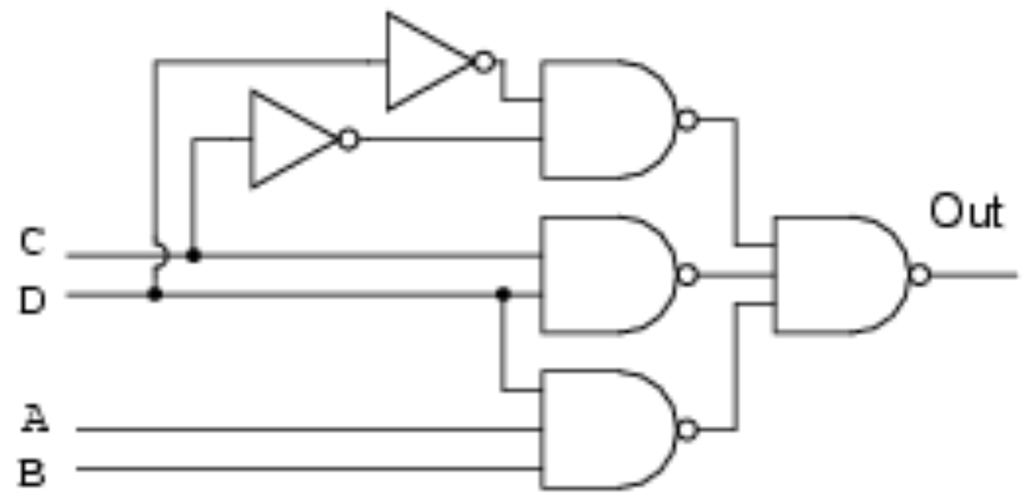
- $out = \bar{C}\,\bar{D} + CD + ABD$
- $out = \overline{\overline{(\bar{C}\,\bar{D} + CD + ABD)}}$
- $out = \overline{(\overline{\bar{C}\,\bar{D}}\;.\;\overline{CD}\;.\;\overline{ABD})}$



NAND.

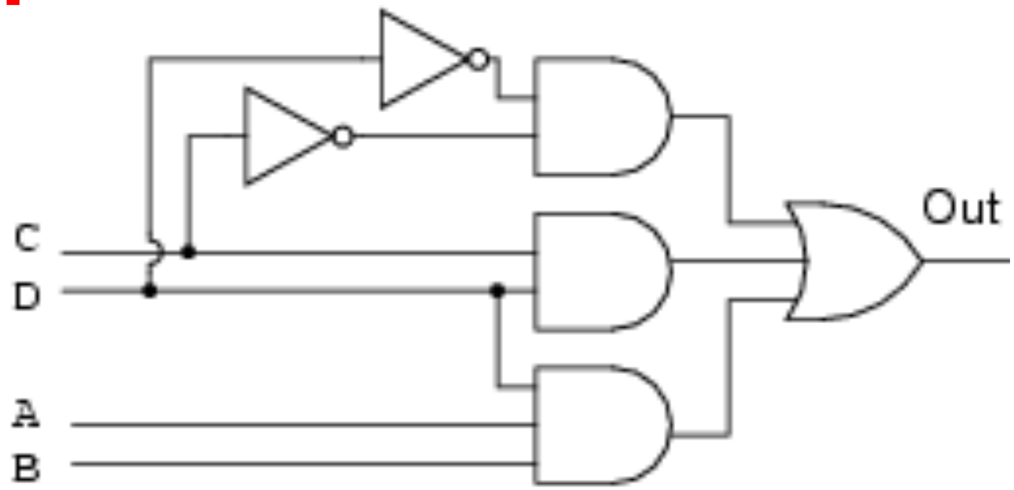
- An inverter can be also replaced by a NAND gate, as $\overline{X\,.\,X} = \bar{X}$

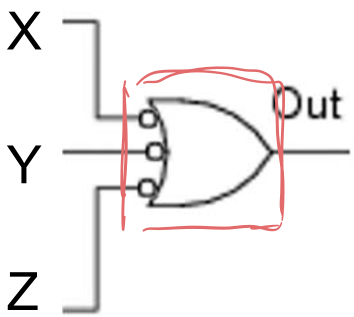
- The sum-of-products can be implemented by NAND gates only.



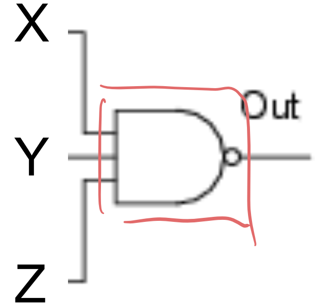
Example 2

- $out = \overline{C}\,\overline{D} + CD + ABD$
- $out = \overline{\overline{(\overline{C}\,\overline{D} + CD + ABD)}}$
- $out = \overline{(\overline{\overline{C}\,\overline{D}}\,.\,\overline{CD}\,.\,\overline{ABD})}$



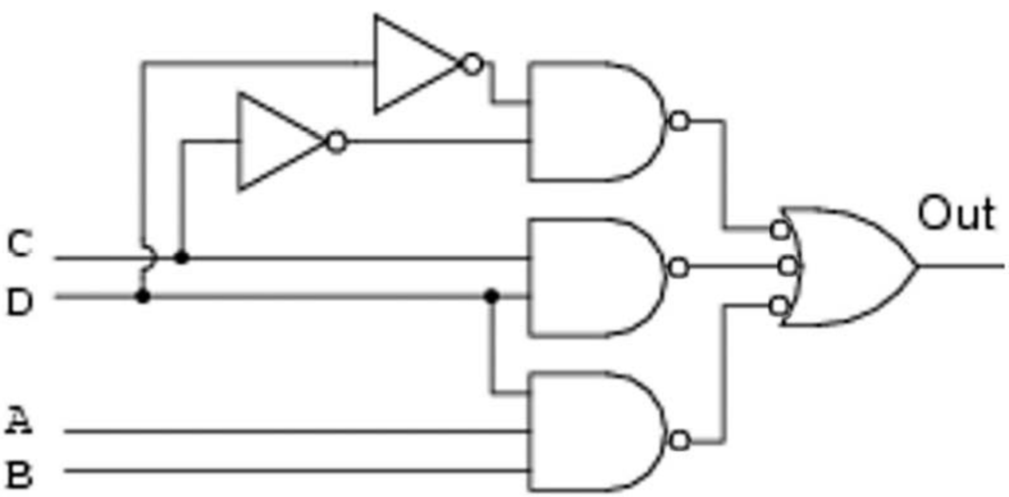


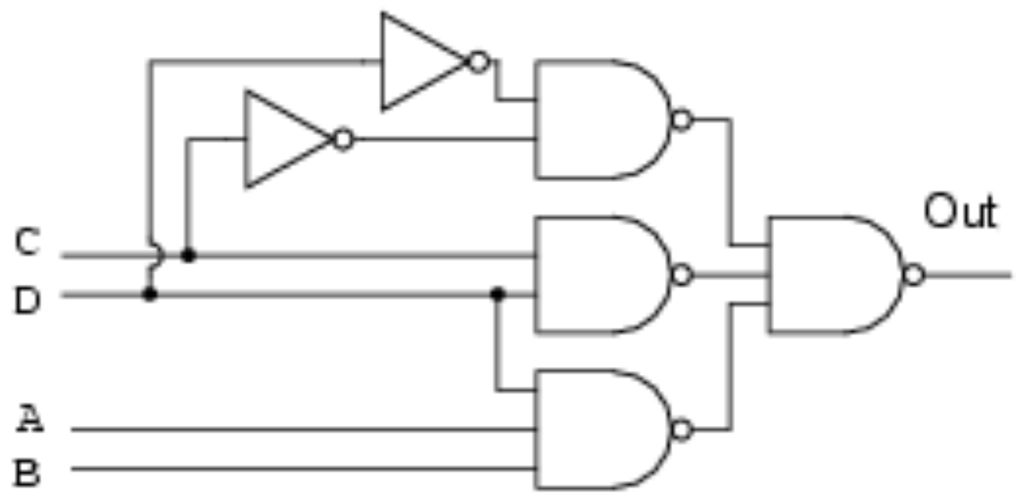
$\overline{X} + \overline{Y} + \overline{Z}$



$\overline{X} + \overline{Y} + \overline{Z}$

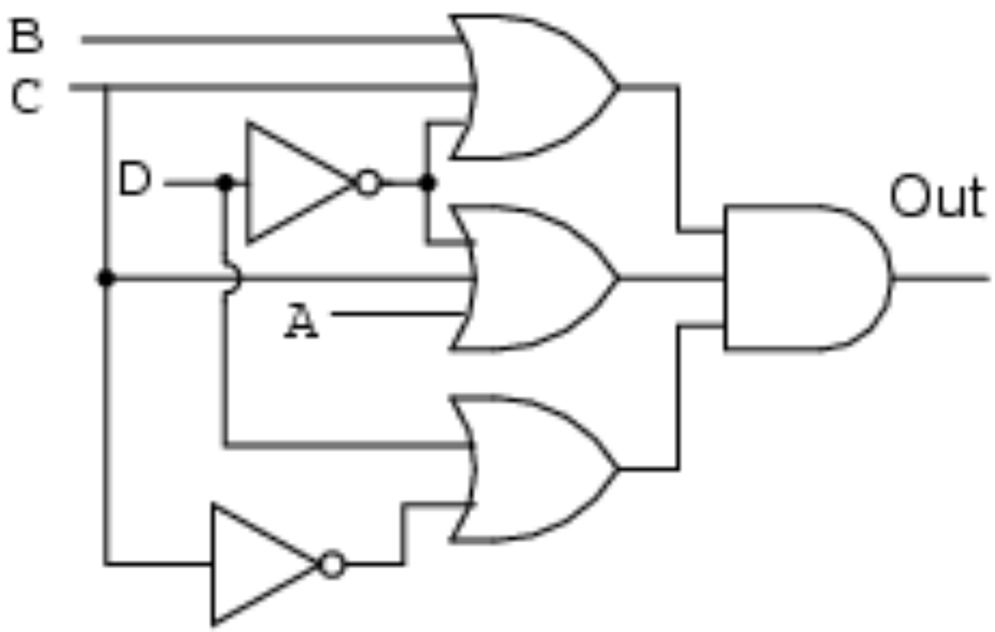
$\overline{x \cdot y \cdot z \cdot}$

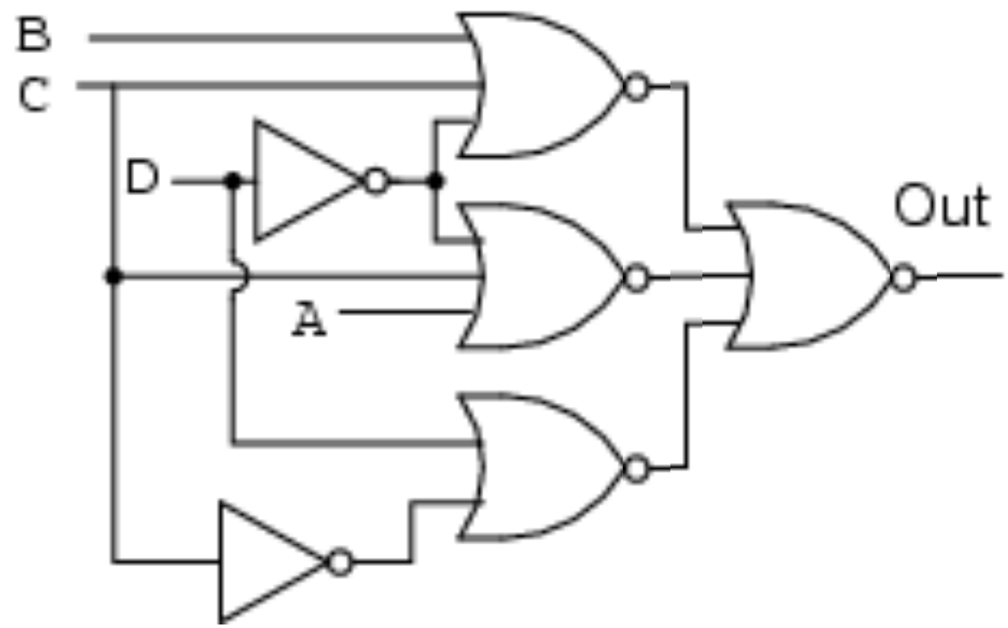




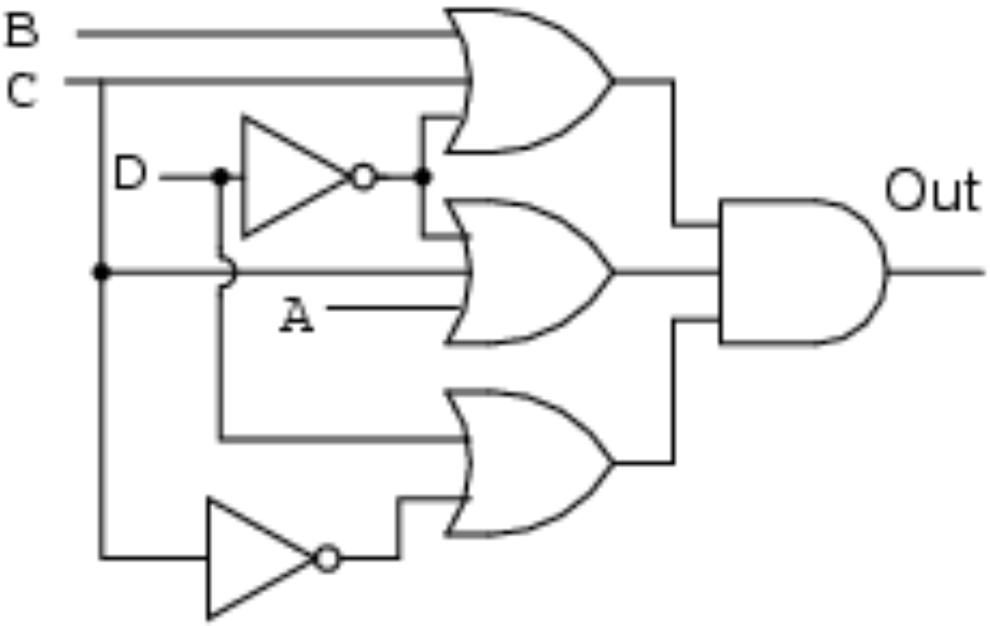
Example 3

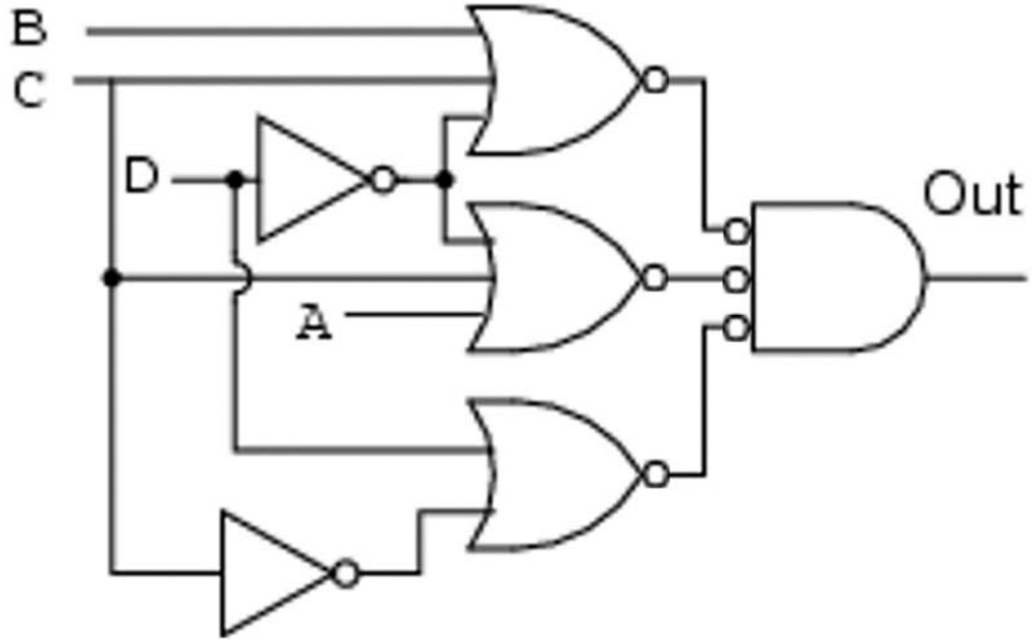
- $out = (B + C + \overline{D}).(A + C + \overline{D}).(\overline{C} + D)$
- $out = \overline{\overline{(B + C + \overline{D}).(A + C + \overline{D}).(\overline{C} + D)}}$
- $out = \overline{(\overline{(B + C + \overline{D})} + \overline{(A + C + \overline{D})} + \overline{(\overline{C} + D)})}$
- An inverter can be also replaced by a NOR gate, as $\overline{X + X} = \overline{X}$
- The product-of-sums can be implemented by NOR gates only.

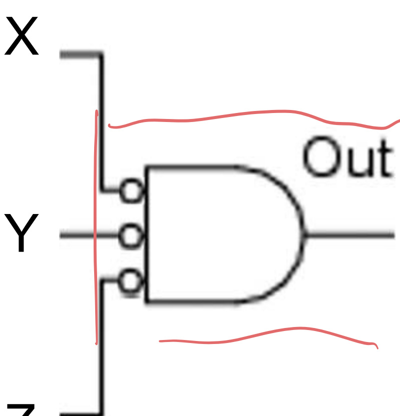




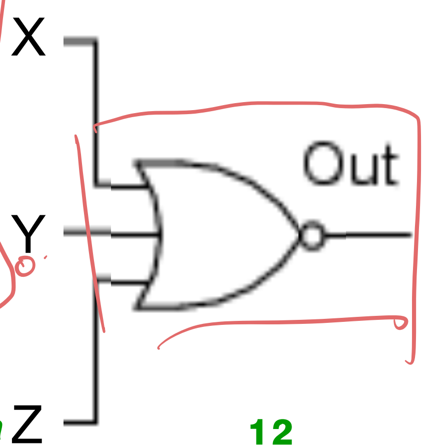
Example 3





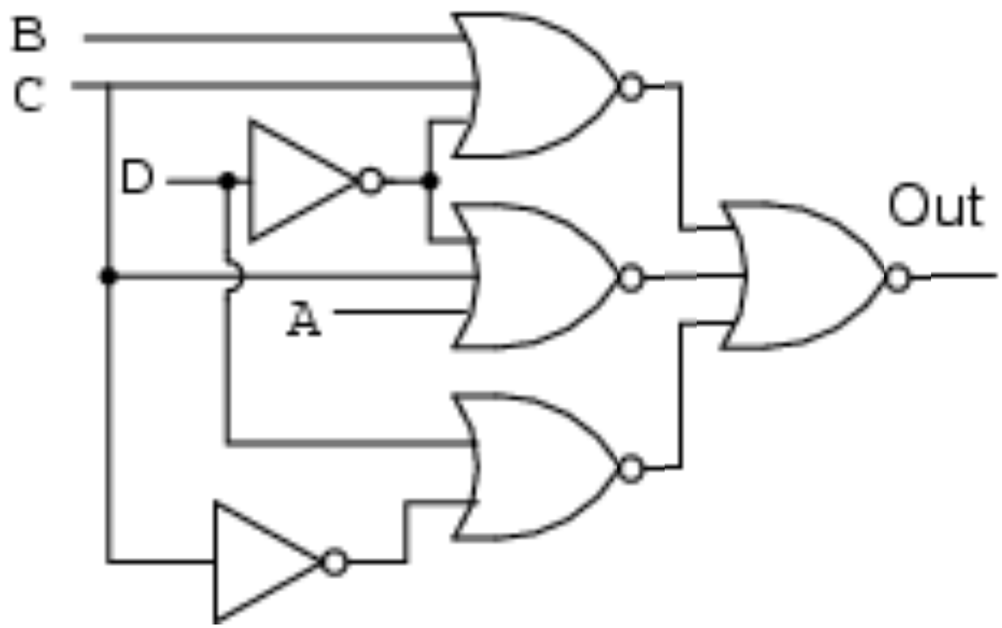


$\overline{X} . \overline{Y} . \overline{Z}$



$\overline{X} . \overline{Y} . \overline{Z}$

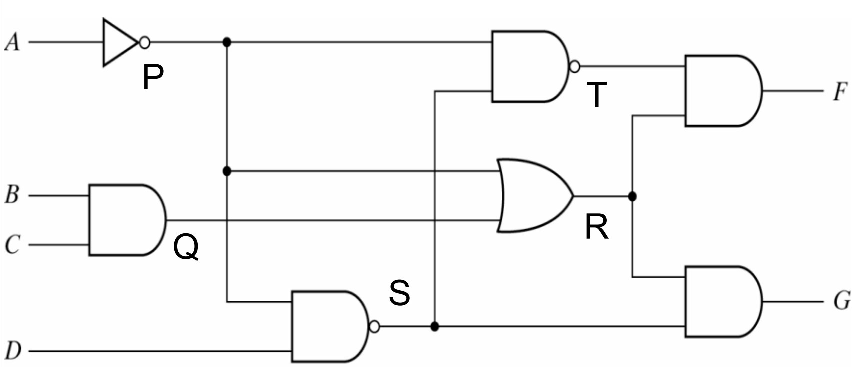
Similar as and



Example 4

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1						
0	0	0	1	1						
0	0	1	0	1						
0	0	1	1	1						
0	1	0	0	1						
0	1	0	1	1						
0	1	1	0	1						
0	1	1	1	1						
1	0	0	0	0						
1	0	0	1	0						
1	0	1	0	0						
1	0	1	1	0						
1	1	0	0	0						
1	1	0	1	0						
1	1	1	0	0						
1	1	1	1	0						

$$P = \overline{A}$$



$P = \overline{A}$

$Q = B \cdot C$

$S = \overline{\overline{A} D}$

$= A + \overline{D}.$

$T = \overline{P \cdot S} = \overline{\overline{A} \cdot (A + \overline{D})}. = \overline{\overline{A} A + \overline{A}\overline{D}} = A + D.$

$R = P + Q = \overline{A} + B \cdot C$

$F = T \cdot R = (A + D) \cdot (\overline{A} + B \cdot C).$

$= ABC + \overline{A} D + DBC.$

$G = R \cdot S = (\overline{A} + B \cdot C) \cdot (A + \overline{D}).$

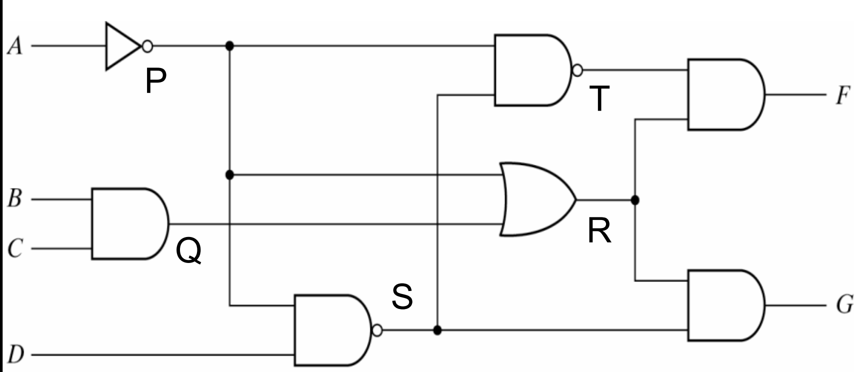
$= \ldots$

Example 4

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0					
0	0	0	1	1	0					
0	0	1	0	1	0					
0	0	1	1	1	0					
0	1	0	0	1	0					
0	1	0	1	1	0					
0	1	1	0	1	1					
0	1	1	1	1	1					
1	0	0	0	0	0					
1	0	0	1	0	0					
1	0	1	0	0	0					
1	0	1	1	0	0					
1	1	0	0	0	0					
1	1	0	1	0	0					
1	1	1	0	0	1					
1	1	1	1	0	1					

$$P = \overline{A}$$

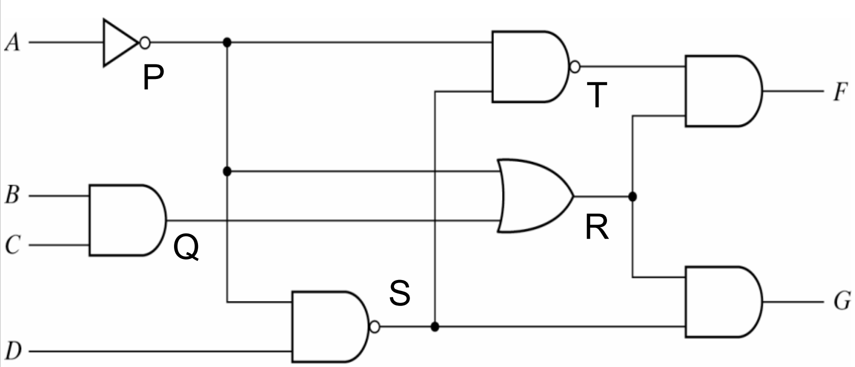
$$Q = B \,.\, C$$



Example 4

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0	1				
0	0	0	1	1	0	1				
0	0	1	0	1	0	1				
0	0	1	1	1	0	1				
0	1	0	0	1	0	1				
0	1	0	1	1	0	1				
0	1	1	0	1	1	1				
0	1	1	1	1	1	1				
1	0	0	0	0	0	0				
1	0	0	1	0	0	0				
1	0	1	0	0	0	0				
1	0	1	1	0	0	0				
1	1	0	0	0	0	0				
1	1	0	1	0	0	0				
1	1	1	0	0	1	1				
1	1	1	1	0	1	1				

$$P = \overline{A}$$
$$Q = B \, . \, C$$
$$R = P + Q = \overline{A} + B \, . \, C$$



Example 4

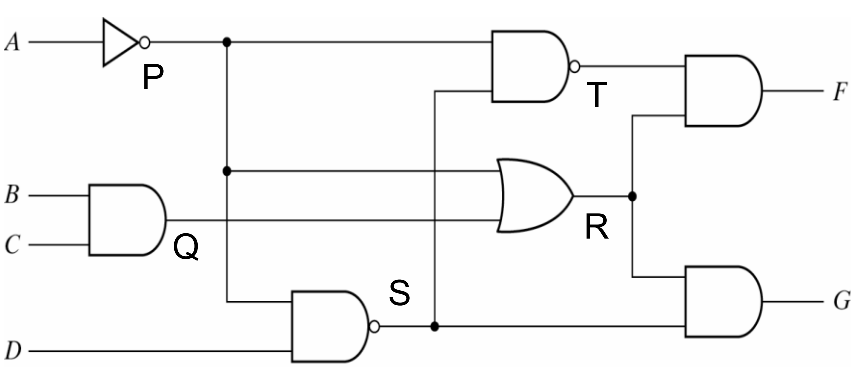
A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0	1	1			
0	0	0	1	1	0	1	0			
0	0	1	0	1	0	1	1			
0	0	1	1	1	0	1	0			
0	1	0	0	1	0	1	1			
0	1	0	1	1	0	1	0			
0	1	1	0	1	1	1	1			
0	1	1	1	1	1	1	0			
1	0	0	0	0	0	0	1			
1	0	0	1	0	0	0	1			
1	0	1	0	0	0	0	1			
1	0	1	1	0	0	0	1			
1	1	0	0	0	0	0	1			
1	1	0	1	0	0	0	1			
1	1	1	0	0	1	1	1			
1	1	1	1	0	1	1	1			

$$P = \overline{A}$$

$$Q = B\,.\,C$$

$$R = P + Q = \overline{A} + B\,.\,C$$

$$S = \overline{P\,.\,D} = \overline{\overline{A}\,.\,D} = \overline{\overline{A}} + \overline{D} = A + \overline{D}$$



Example 4

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0	1	1	0		
0	0	0	1	1	0	1	0	1		
0	0	1	0	1	0	1	1	0		
0	0	1	1	1	0	1	0	1		
0	1	0	0	1	0	1	1	0		
0	1	0	1	1	0	1	0	1		
0	1	1	0	1	1	1	1	0		
0	1	1	1	1	1	1	0	1		
1	0	0	0	0	0	0	1	1		
1	0	0	1	0	0	0	1	1		
1	0	1	0	0	0	0	1	1		
1	0	1	1	0	0	0	1	1		
1	1	0	0	0	0	0	1	1		
1	1	0	1	0	0	0	1	1		
1	1	1	0	0	1	1	1	1		
1	1	1	1	0	1	1	1	1		

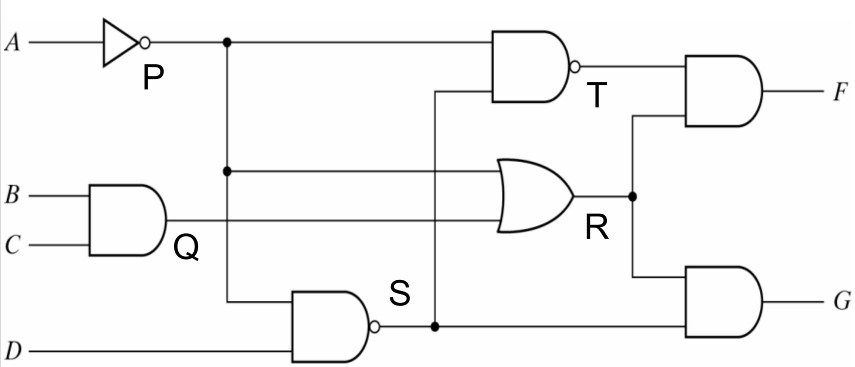
$$P = \overline{A}$$

$$Q = B \, . \, C$$

$$R = P + Q = \overline{A} + B \, . \, C$$

$$S = \overline{P \, . \, D} = \overline{\overline{A} \, . \, D} = \overline{\overline{A}} + \overline{D} = A + \overline{D}$$

$$T = \overline{P \, . \, S} = \overline{\overline{A} \, . \, (A + \overline{D})} = \overline{\overline{A} . A + \overline{A} . \overline{D}} = \overline{\overline{A} . \overline{D}} = A + D$$



Example 4

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0	1	1	0	0	
0	0	0	1	1	0	1	0	1	1	
0	0	1	0	1	0	1	1	0	0	
0	0	1	1	1	0	1	0	1	1	
0	1	0	0	1	0	1	1	0	0	
0	1	0	1	1	0	1	0	1	1	
0	1	1	0	1	1	1	1	0	0	
0	1	1	1	1	1	1	0	1	1	
1	0	0	0	0	0	0	1	1	0	
1	0	0	1	0	0	0	1	1	0	
1	0	1	0	0	0	0	1	1	0	
1	0	1	1	0	0	0	1	1	0	
1	1	0	0	0	0	0	1	1	0	
1	1	0	1	0	0	0	1	1	0	
1	1	1	0	0	1	1	1	1	1	
1	1	1	1	0	1	1	1	1	1	

$$P = \overline{A}$$

$$Q = B\,.\,C$$

$$R = P + Q = \overline{A} + B\,.\,C$$

$$S = \overline{P\,.\,D} = \overline{\overline{A}\,.\,D} = \overline{\overline{A}} + \overline{D} = A + \overline{D}$$

$$T = \overline{P\,.\,S} = \overline{\overline{A}\,.\,(A + \overline{D})} = \overline{\overline{A}.A + \overline{A}.\overline{D}} = \overline{\overline{A}.\overline{D}} = A + D$$

$$F = T\,.\,R = (A + D)\,.\,(\overline{A} + B\,.\,C)$$

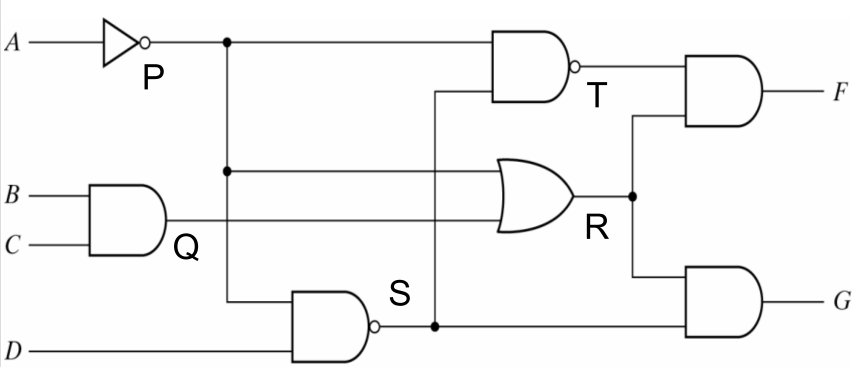
$$= A.\overline{A} + A.B.C + \overline{A}.D + B.C.D$$

$$= 0 + A.B.C + \overline{A}.D + (A.B.C.D + \overline{A}.B.C.D)$$

$$= (A.B.C + A.B.C.D) + (\overline{A}.D + \overline{A}.D.B.C)$$

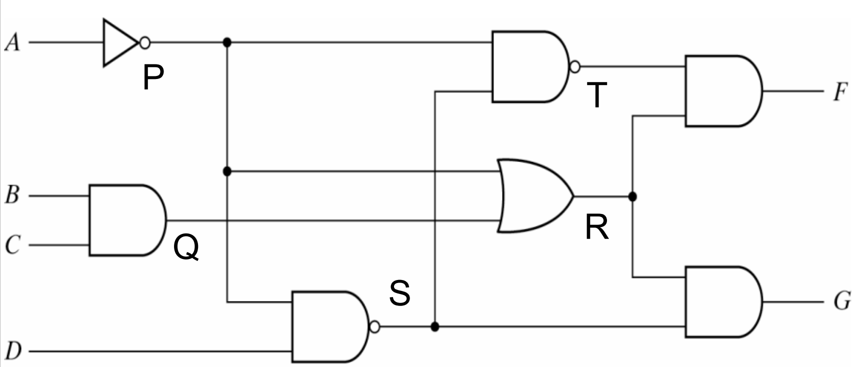
$$= A.B.C.(1 + D) + \overline{A}.D.(1 + B.C)$$

$$= A.B.C + \overline{A}.D$$



Example 4

A	B	C	D	P	Q	R	S	T	F	G
0	0	0	0	1	0	1	1	0	0	1
0	0	0	1	1	0	1	0	1	1	0
0	0	1	0	1	0	1	1	0	0	1
0	0	1	1	1	0	1	0	1	1	0
0	1	0	0	1	0	1	1	0	0	1
0	1	0	1	1	0	1	0	1	1	0
0	1	1	0	1	1	1	1	0	0	1
0	1	1	1	1	1	1	0	1	1	0
1	0	0	0	0	0	0	1	1	0	0
1	0	0	1	0	0	0	1	1	0	0
1	0	1	0	0	0	0	1	1	0	0
1	0	1	1	0	0	0	1	1	0	0
1	1	0	0	0	0	0	1	1	0	0
1	1	0	1	0	0	0	1	1	0	0
1	1	1	0	0	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1



$$P = \overline{A}$$

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$$T = \overline{P\,.\,S} = \overline{\overline{A}\,.\,(A + \overline{D})} = \overline{\overline{A}.A + \overline{A}.\overline{D}} = \overline{\overline{A}.\overline{D}} = A + D$$

$$\begin{aligned}
F = T\,.\,R &= (A + D)\,.\left(\overline{A} + B\,.\,C\right) \\
&= A.\overline{A} + A.B.C + \overline{A}.D + B.C.D \\
&= 0 + A.B.C + \overline{A}.D + (A.B.C.D + \overline{A}.B.C.D) \\
&= (A.B.C + A.B.C.D) + (\overline{A}.D + \overline{A}.D.B.C) \\
&= A.B.C.(1 + D) + \overline{A}.D.(1 + B.C) \\
&= A.B.C + \overline{A}.D
\end{aligned}$$

$$\begin{aligned}
G = S\,.\,R &= \left(A + \overline{D}\right).\left(\overline{A} + B\,.\,C\right) \\
&= A.\overline{A} + A.B.C + \overline{A}.\overline{D} + B.C.\overline{D} \\
&= 0 + A.B.C + \overline{A}.\overline{D} + (A.B.C.\overline{D} + \overline{A}.B.C.\overline{D}) \\
&= (A.B.C + A.B.C.\overline{D}) + (\overline{A}.\overline{D} + \overline{A}.\overline{D}.B.C) \\
&= A.B.C.\left(1 + \overline{D}\right) + \overline{A}.\overline{D}.(1 + B.C) \\
&= A.B.C + \overline{A}.\overline{D}
\end{aligned}$$