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# Resolution

Translation

Connectives

Why some smaller/different sets are adequate.

Different ways of proving.

Proof argument valid { truth table.  
s-rule i-rule  
converting things into clauses.  
resolution proof.

Concepts { Disjunctive normal form  
Tautology

# What Happens With Resolution?

- So resolution is nice, but what is happening when we do it?
- What does a set of clauses mean?
  - 1  $\{A, B\}$
  - 2  $\{\sim A, \sim B, C\}$
  - 3  $\{\sim C, D\}$
- This set of clauses equivalent to the single WFF (sorry if I get  
( ) wrong sometimes)  
 $((A \vee B) * (\sim A \vee (\sim B \vee C))) * (\sim C \vee D)$

# What Happens With Resolution?

- So what happens when we do resolution?
  - 1  $\{A, B\}$
  - 2  $\{\sim A, \sim B, C\}$
  - 3  $\{\sim C, D\}$
  - 4  $\{\sim A, \sim B, D\}$  from 2,3
- This set of clauses equivalent to the single WFF  
 $((((A \vee B) * (\sim A \vee (\sim B \vee C))) * (\sim C \vee D)) * (\sim A \vee (\sim B \vee D)))$
- So we have just added the new clause to the end of the WFF

# What Happens With Resolution?

- Resolution is built on the rule (not an S/I rules but S/I can prove this)  
 $((P \vee Q) * (\sim P \vee R)) \Rightarrow (Q \vee R)$
- It is extended to more than one \*

# What Happens With Resolution?

- 1 {A, B}
- 2 { $\sim$ A,  $\sim$ B, C}
- 3 { $\sim$ C, D}

To

- 1 {A, B}
- 2 { $\sim$ A,  $\sim$ B, C}
- 3 { $\sim$ C, D}
- 4 { $\sim$ A,  $\sim$ B, D} from 2,3

# What Happens With Resolution?

- Is the same as saying

$$\begin{aligned} & (((A \vee B) * (\sim A \vee \sim B \vee C)) * (\sim C \vee D)) \Rightarrow \\ & (((A \vee B) * (\sim A \vee (\sim B \vee C))) * (\sim C \vee D)) * (\sim A \vee (\sim B \vee D))) \end{aligned}$$

- So every step of resolution we add another  $\Rightarrow$  and another term to the end
- Can't generate new terms: invalid
- Get the empty clause: valid (negation goes backwards and says we can't assume the conclusion is false so it must be true)



# What Happens With Resolution?

- Why not do 1,2?
  - 1 {A, B}
  - 2 { $\sim$ A,  $\sim$ B, C}
  - 3 { $\sim$ C, D}
  - 4 { $\sim$ A,  $\sim$ B, D} from 2,3
  - 5 {A,  $\sim$ A, C} from 1,2
- This set of clauses equivalent to the single WFF  
$$((((A \vee B) * (\sim A \vee (\sim B \vee C))) * (\sim C \vee D)) * (\sim A \vee (\sim B \vee D))) * ((A \vee \sim A) \vee C))$$

# What Happens With Resolution?

- But  $P \vee \sim P = T$   $T \vee P = T$   $T * P = P$

so

$(((((A \vee B) * (\sim A \vee (\sim B \vee C))) * (\sim C \vee D)) * (\sim A \vee (\sim B \vee D)))) * ((A \vee \sim A) \vee C))$

becomes

$(((((A \vee B) * (\sim A \vee (\sim B \vee C))) * (\sim C \vee D)) * (\sim A \vee (\sim B \vee D)))) * (T \vee C))$

$(((((A \vee B) * (\sim A \vee (\sim B \vee C))) * (\sim C \vee D)) * (\sim A \vee (\sim B \vee D)))) * T$

$(((((A \vee B) * (\sim A \vee (\sim B \vee C))) * (\sim C \vee D)) * (\sim A \vee (\sim B \vee D))))$

Which is what we started with, so no information is gained

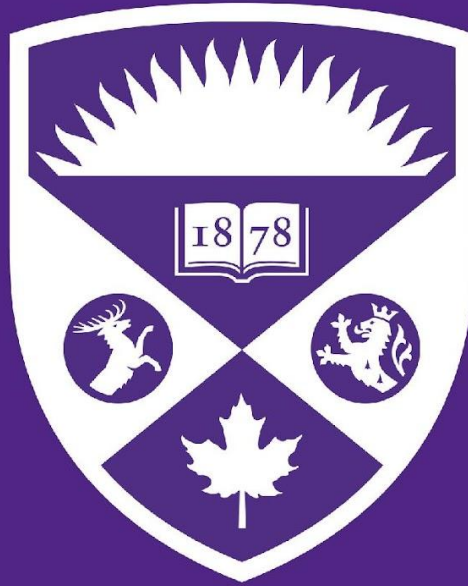
# What Happens With Resolution?

- Similarly because  $P * P = P$  we don't want to generate the same clause more than once (say we did 2,3 again)

$(((((A \vee B) * (\sim A \vee (\sim B \vee C))) * (\sim C \vee D)) * (\sim A \vee (\sim B \vee D))) * (\sim A \vee (\sim B \vee D)))$

Is equivalent to what we started with

$(((((A \vee B) * (\sim A \vee (\sim B \vee C))) * (\sim C \vee D)) * (\sim A \vee (\sim B \vee D))))$



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