Corpus Structure Discovery

Unstructured Data

CS 4417B/9117/9647

The University of Western Ontario

Latent Semantic Analysis

- SVD
  - Interpretation as compression
- Decomposition
  - Documents as sums of term-collections
  - Terms as sums of document-collections
- Representations
  - Term-term similarity
  - Document-document similarity
  - Term-document similarity
- Demo

Term-Document Matrix

```
         Docs
Terms      11224 15871 15875 17396 17953 2521 5214 5985 6657 7589
  bank         2     0     0    10     3    1    0    2    0   10
  billion      0     0     0     0     2    2    0    4    1    6
  compani      0     0     0     0     0    0    1    8    0    0
  dlrs         1     0    10     0     0    2    0    1    1    3
  mln          1     0    29     0     0    4   12    1    8    0
  pct          3     0     0     8     0    1    2   13    0   10
  reuter       6     1     1     1     1    2    2    0    2    1
  said        13     0     0    28    18   22   20   24   13    5
  will         4     1     0     2     2    8    1   14    2    3
  year         1     0     3     1     1   10   19    5   12    5
```

Each column vector represents a document, bag-of-words.

Term-Document Matrix

```
          Docs
Terms       11224 12701 14635 15871 15875 17474 17497 17953  5214  6657
  bank       5.41  0.00  0.00  0.00  0.00  8.11  0.00  8.11  0.00  0.00
  billion    0.00  0.00  2.73  0.00  0.00  2.73 10.92  5.46  0.00  2.73
  cts        0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
  dlrs       1.50  5.98  1.50  0.00 14.95  0.00  5.98  0.00  0.00  1.50
  mln        1.38  1.38  0.00  0.00 39.96  4.13 11.02  0.00 16.53 11.02
  pct        5.56  3.71  1.85  0.00  0.00 11.12  1.85  0.00  3.71  0.00
  said       6.39  4.43  1.97  0.00  0.00  5.41  3.44  8.85  9.84  6.39
  share      7.27  4.85  0.00  0.00  0.00 16.97  0.00  2.42  0.00  0.00
  will       6.37  6.37  4.78  1.59  0.00  6.37  4.78  3.19  1.59  3.19
  year       1.67  3.34  8.35  0.00  5.01 20.04  8.35  1.67 31.73 20.04
```

Each column vector represents a document, TF-IDF.

Will call this matrix M throughout these slides.

Vector representations

- Previously
  - Documents represented as vectors
  - Can represent similarity with dot prod or cosine
- Idea in This lesson
  - Compressing the TDM
    - Saves space
    - Reveals patterns
    - Improves retrieval
  - Bonus idea:
    - Each row represents a term! 🤔

Factoring the TDM

	D1	D2	D3	D4	D5	D6
cat	1	0	1	1	1	0
dog	1	0	1	1	1	0
horse	1	0	1	1	1	0
apple	0	1	0	0	1	1
orange	0	1	0	0	1	1

V^T	D1	D2	D3	D4	D5	D6
W1	1	0	1	1	1	0
W2	0	1	0	0	1	1

U	T1	T2
cat	1	0
dog	1	0
horse	1	0
apple	0	1
orange	0	1

1	0	1	1	1	0
1	0	1	1	1	0
1	0	1	1	1	0
0	1	0	0	1	1
0	1	0	0	1	1

V^T	D1	D2	D3	D4	D5	D6
W1	1	0	1	1	1	0
W2	0	1	0	0	1	1

U	T1	T2
cat	1	0
dog	1	0
horse	1	0
apple	0	1
orange	0	1

1	0	1	1	1	0
1	0	1	1	1	0
1	0	1	1	1	0
0	1	0	0	1	1
0	1	0	0	1	1

V^T	D1	D2	D3	D4	D5	D6
W1	1	0	1	1	1	0
W2	0	1	0	0	1	1

U	T1	T2
cat	1	0
dog	1	0
horse	1	0
apple	0	1
orange	0	1

1	0	1	1	1	0
1	0	1	1	1	0
1	0	1	1	1	0
0	1	0	0	1	1
0	1	0	0	1	1

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dog	1	0
horse	1	0
apple	0	1
orange	0	1

1	0	1	1	1	0
1	0	1	1	1	0
1	0	1	1	1	0
0	1	0	0	1	1
0	1	0	0	1	1

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horse	1	0
apple	0	1
orange	0	1

1	0	1	1	1	0
1	0	1	1	1	0
1	0	1	1	1	0
0	1	0	0	1	1
0	1	0	0	1	1

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apple	0	1
orange	0	1

1	0	1	1	1	0
1	0	1	1	1	0
1	0	1	1	1	0
0	1	0	0	1	1
0	1	0	0	1	1

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dog	1	0
horse	1	0
apple	0	1
orange	0	1

1	0	1	1	1	0
1	0	1	1	1	0
1	0	1	1	1	0
0	1	0	0	1	1
0	1	0	0	1	1

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cat	1	0
dog	1	0
horse	1	0
apple	0	1
orange	0	1

1	0	1	1	1	0
1	0	1	1	1	0
1	0	1	1	1	0
0	1	0	0	1	1
0	1	0	0	1	1

Singular Value Decomposition

Singular Value Decomposition

$M_{m \times n} \approx \dot{M}_{m \times n} = U_{m \times p}\, \Sigma_{p \times p}\, V^{T}{}_{p \times n}$

Such that squared reconstruction error

$(\|M_{m \times n} - \dot{M}_{m \times n}\|_F)^2$

is as small as possible.

Elements of Σ are called *singular values*

$V^{T}{}_{p \times n}\, V_{n \times p} = I_{p \times p}$
$U^{T}{}_{p \times m}\, U_{m \times p} = I_{p \times p}$

Singular Value Decomposition

$\mathrm{M}_{m\times n} \approx \dot{\mathrm{M}}_{m\times n} = \mathrm{U}_{m\times p}\, \Sigma_{p\times p}\, \mathrm{V}^{\mathrm{T}}{}_{p\times n}$

Such that squared reconstruction error

$(\|\mathrm{M}_{m\times n} - \dot{\mathrm{M}}_{m\times n}\|_{\mathrm{F}})^2$

is as small as possible.

Elements of Σ are called *singular values*
$\mathrm{V}^{\mathrm{T}}{}_{p\times n}\, \mathrm{V}_{n\times p} = \mathrm{I}_{p\times p}$
$\mathrm{U}^{\mathrm{T}}{}_{p\times m}\, \mathrm{U}_{m\times p} = \mathrm{I}_{p\times p}$

Singular Value Decomposition

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Such that squared reconstruction error

$(\;\|M_{m\times n} - \dot{M}_{m\times n}\|_{F}\;)^{2}$

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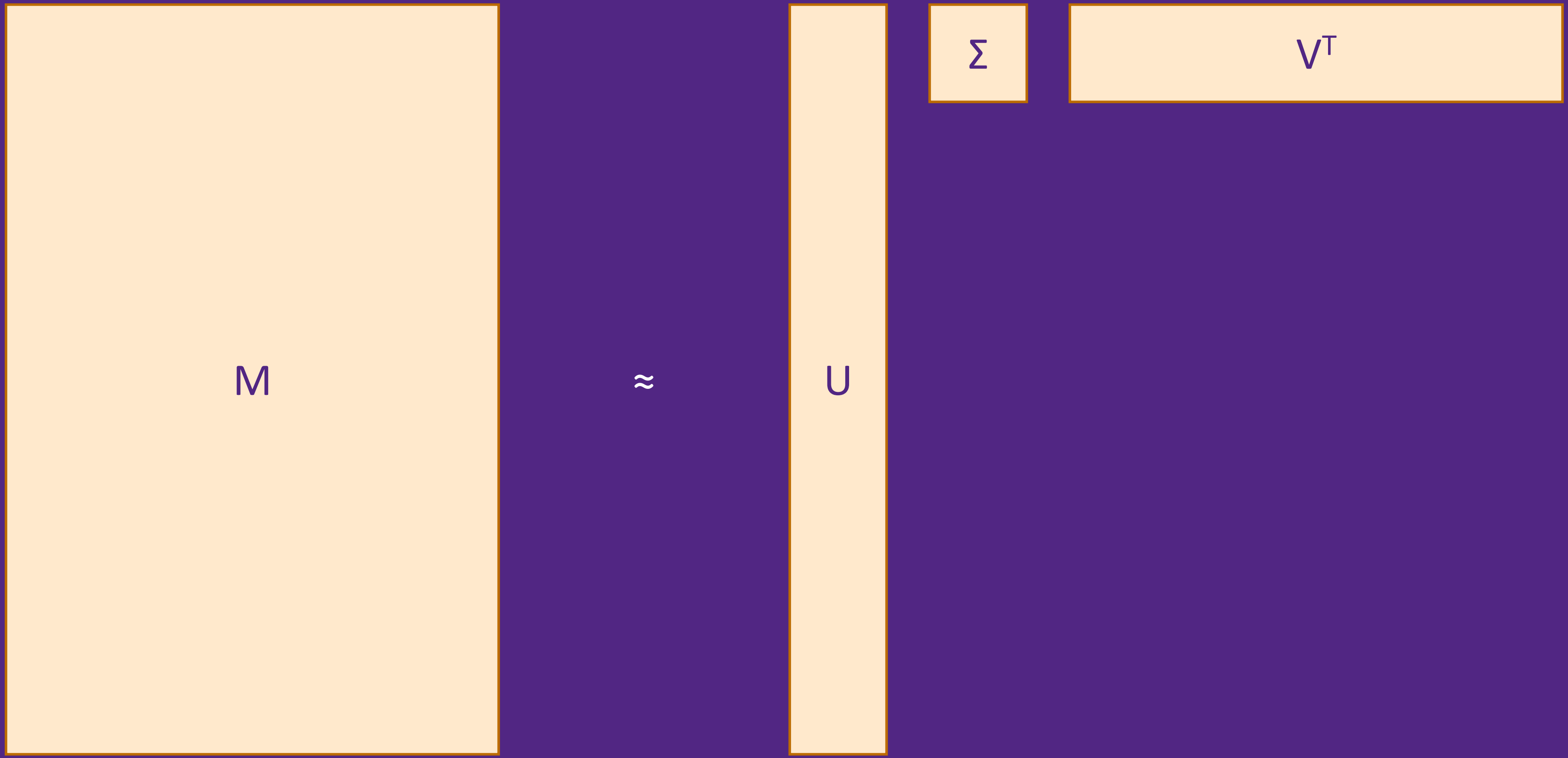
Elements of Σ are called *singular values*
$V^{T}{}_{p\times n}\,V_{n\times p} = I_{p\times p}$
$U^{T}{}_{p\times m}\,U_{m\times p} = I_{p\times p}$

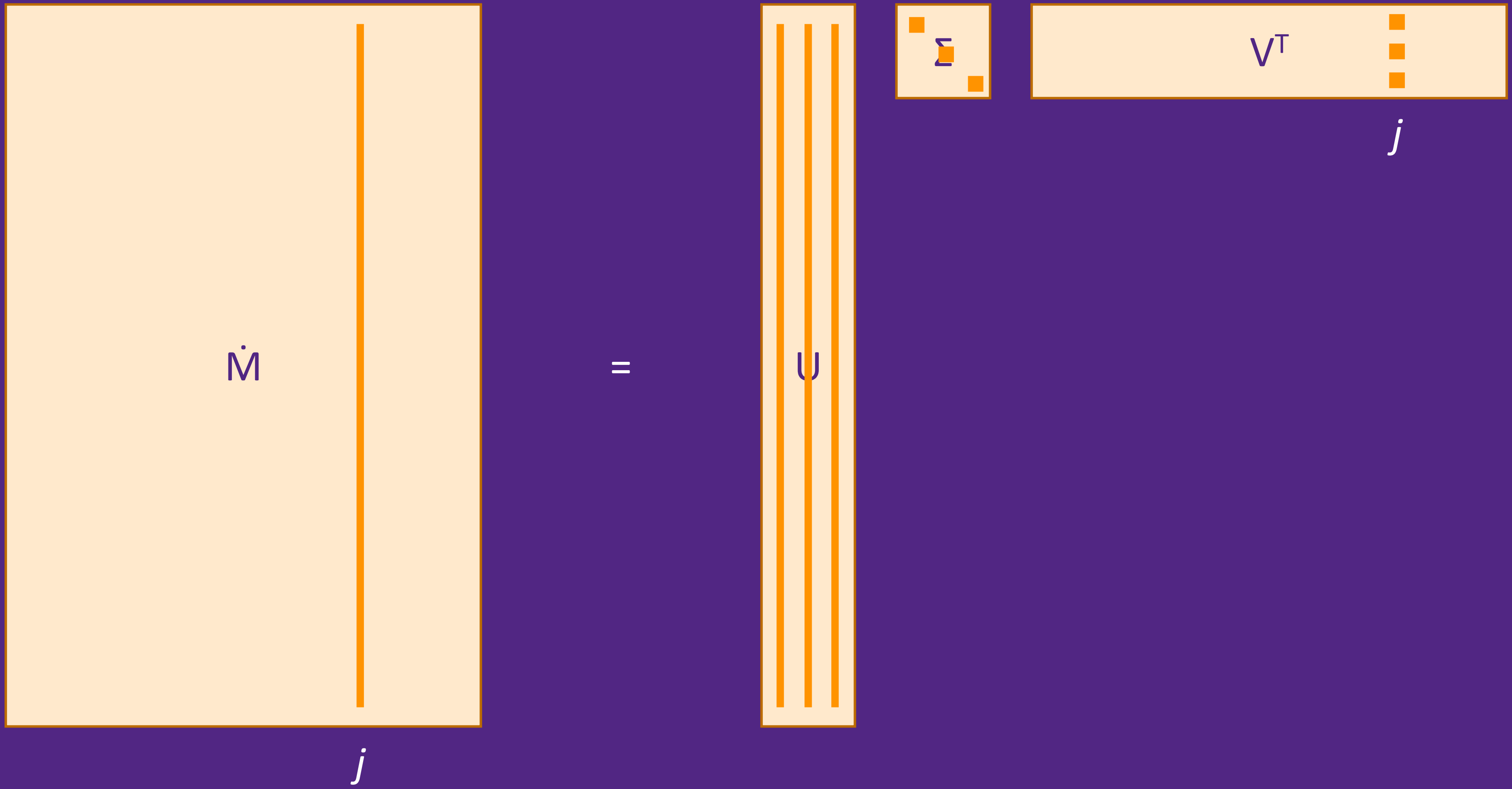
Singular Value Decomposition

$$M_{m\times n} \approx \dot{M}_{m\times n} = U_{m\times p}\ \Sigma_{p\times p}\ V^{T}{}_{p\times n}$$



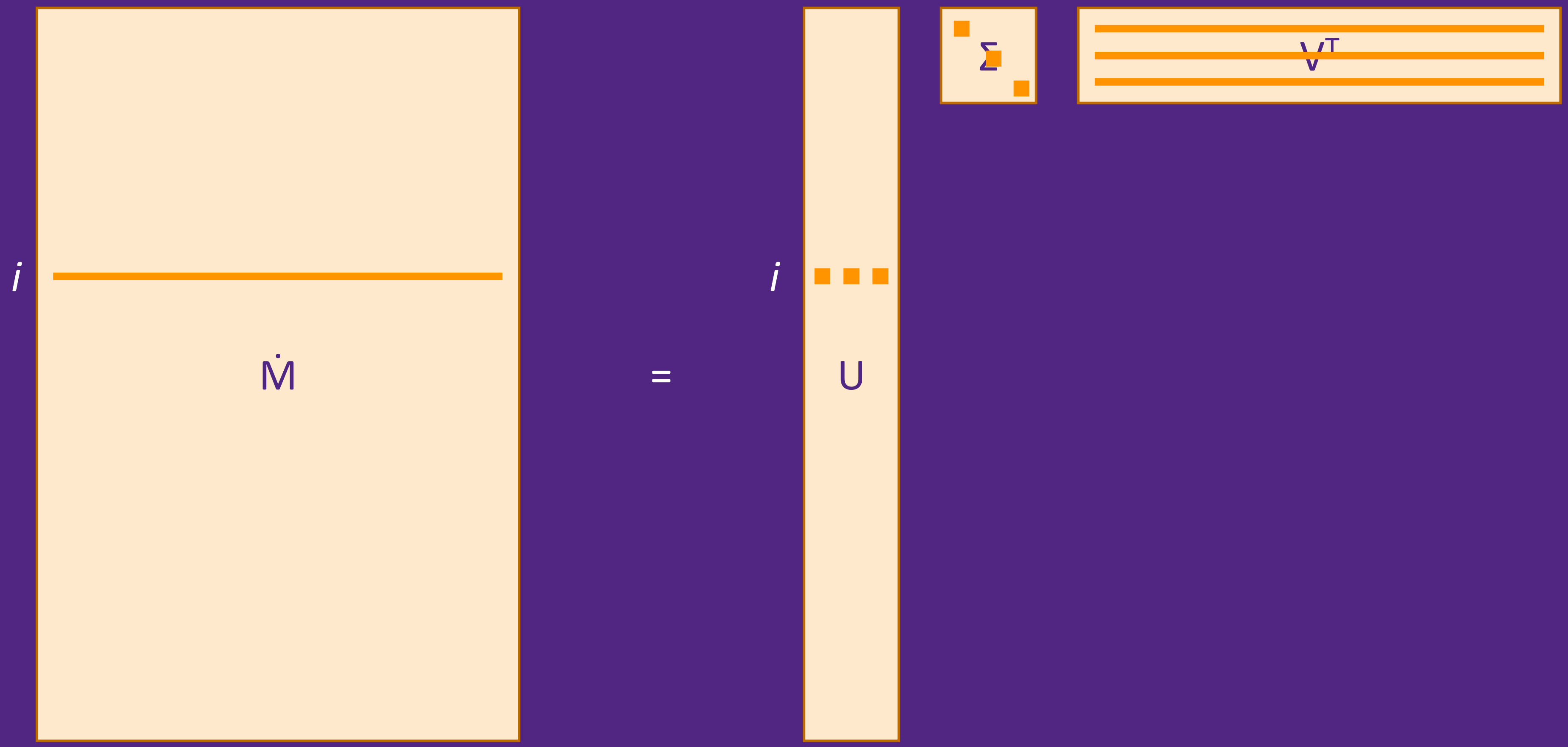
Approximating a column of M

- $\dot{M}_{:,j} = U\ \Sigma\ V^T_{j,:}$
- *A column of* Ṁ *is a weighted sum of the columns of* U



Approximating a row of M

- $\dot{M}_{i,\cdot} = U_{i,\cdot} \Sigma\, V^T$
- *A row of* $\dot{M}$ *is a weighted sum of the rows of* V^T



Latent Semantic Analysis

Latent Semantic Analysis

- Take the term-document matrix M
- Approximate it using SVD, giving

$$M_{m \times n} \approx \dot{M}_{m \times n} = U_{m \times p}\, \Sigma_{p \times p}\, V^{T}{}_{p \times n}$$

- New representations:
  - Columns of V^T represent documents using *p* dimensions
  - Rows of U represent terms using *p* dimensions
- New structure
  - Columns of U: terms that co-occur in docs (dimension *m*)
  - Rows of V^T: docs with similar words (dimension *n*)

V^T	D1	D2	D3	D4	D5	D6
W1	1	0	1	1	1	0
W2	0	1	0	0	1	1

Columns of V^T are new vector representations for each document

U	T1	T2
cat	1	0
dog	1	0
horse	1	0
apple	0	1
orange	0	1

Rows of U are new vector representations for each term

1	0	1	1	1	0
1	0	1	1	1	0
1	0	1	1	1	0
0	1	0	0	1	1
0	1	0	0	1	1

V^T	D1	D2	D3	D4	D5	D6
W1	1	0	1	1	1	0
W2	0	1	0	0	1	1

Rows of V^T identify documents that contain the same words

U	T1	T2
cat	1	0
dog	1	0
horse	1	0
apple	0	1
orange	0	1

1	0	1	1	1	0
1	0	1	1	1	0
1	0	1	1	1	0
0	1	0	0	1	1
0	1	0	0	1	1

Columns of U identify terms that co-occur in the same document

Compression

- $\underline{\dot{M}_{m\times n} = U_{m\times p}\, \Sigma_{p\times p}\, V^{T}{}_{p\times n}}$

- Consider 1M documents, 1M terms.
  Original matrix M is a terabyte if 1 byte/entry if dense. (It isn’t dense, but consider.)

- If $p = 5$, then we have 5M + 5 + 5M ≈ 10M

- Smaller by a factor of 100,000

A new basis for documents

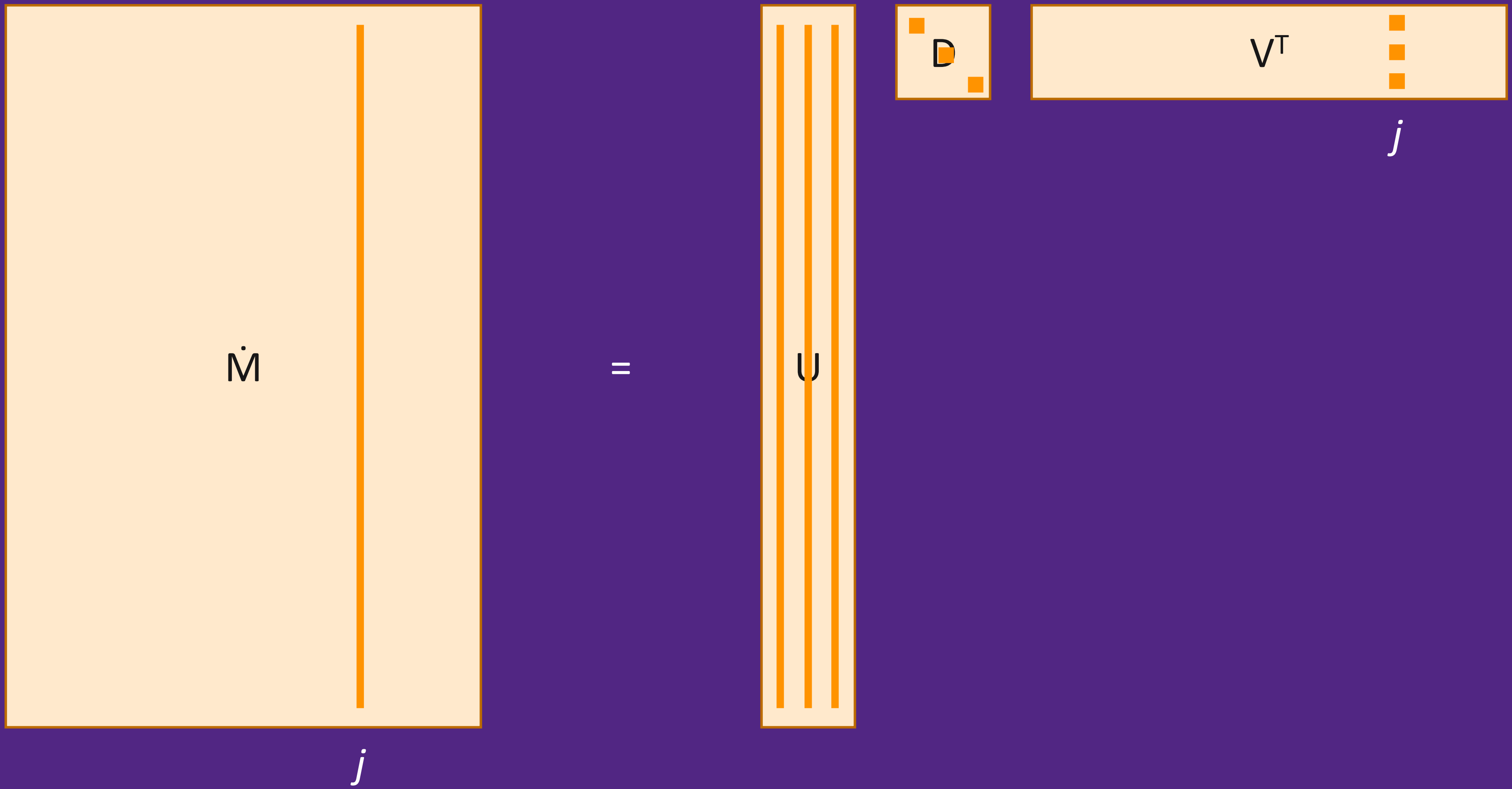
- $\dot{M}_{m\times n} = U_{m\times p}\, \Sigma_{p\times p}\, V^T{}_{p\times n}$

- All approximate document vectors must be constructed from same *p* columns of U.

- Terms that have large weights, same sign, in the same column of U, must enter the document together.

Approximating a column of M

- $\dot{M}_{:,j} = U \, \Sigma \, V^T_{:,j}$
- *A column of* $\dot{M}$ *is a weighted sum of the columns of* U



Topics from columns of U

```
share  common   stock   inc      offer   compani corp     cts
0.294   0.181   0.181   0.171    0.129   0.112   0.108    0.108
```

(This is a row of U^T; the software outputs in this format.)

Topics from U (sort of)

```
  share   common    stock    inc      offer    compani corp     cts
  0.294    0.181    0.181    0.171    0.129    0.112    0.108    0.108

     pct      bank      tonn       stg   januari      rose  februari   billion

 -0.0962   -0.0971   -0.0993   -0.1021   -0.1022   -0.1060   -0.1114   -0.5970
```

(This is a row of U^T; the software outputs in this format.)

We sort of get 2 topics per vector; the + elements and the – elements.
Words at either end of a topic vector are “anticorrelated” in documents.

A new basis for term-distributions

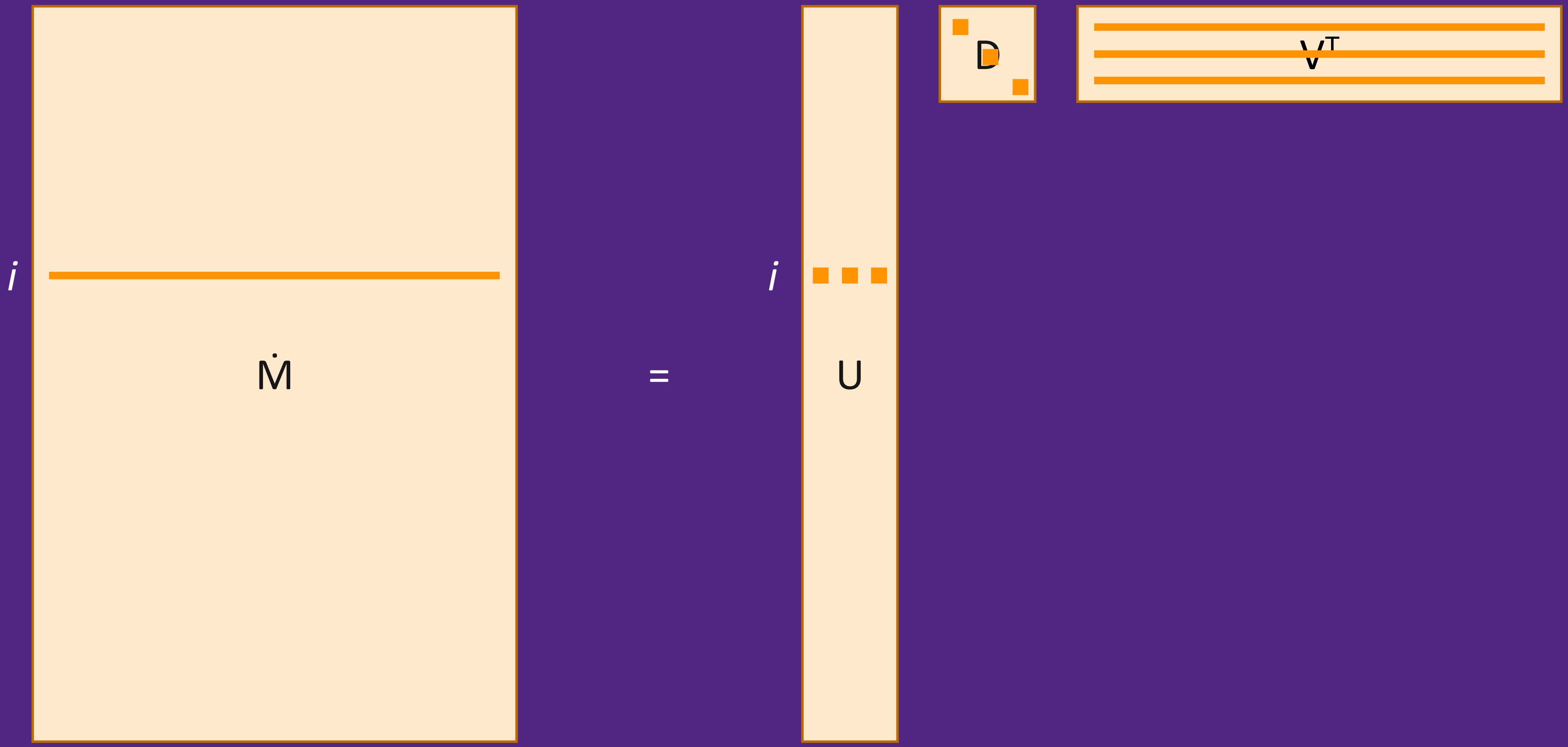
- $\dot{M}_{m \times n} = U_{m \times p}\ \Sigma_{p \times p}\ V^{T}{}_{p \times n}$

- If $p < \min(m,n)$, then we can't adjust the elements of a row of a $\dot{M}$ independently – we must take the "document amounts" from an entire row of V at once.

- Documents that have large weights, same sign, in the same row of V^{T}, must contribute together to a term's row.

Approximating a row of M

- $\dot{M}_{i,\cdot} = U_{i,\cdot}\Sigma\, V^T$
- *A row of* $\dot{M}$ *is a weighted sum of the rows of* V^T



Document clusters from V^T

```
  7200   1836  11504   2375  19897 ...
0.0762 0.0732 0.0702 0.0650 0.0605 ...
```

Structure from the corpus

…squared reconstruction error

$$\|M_{m\times n} - U_{m\times p}\,\Sigma_{p\times p}\,V^{T}{}_{p\times n}\|_F{}^2$$

is as small as possible.

- The SVD algorithm *has* to find structure in the corpus to represent it using a small number of columns of U and rows of V^T.
  - Has to find words that tend to co-occur
  - Has to find documents that tend to have similar words

- This procedure is called **Latent Semantic Analysis**
  - “Latent” because structure is hidden but discovered through the analysis. “Semantic” because the structure that is found often corresponds to groups of terms that describe similar concepts.