

# Chapter 1. Propositional Logic

## § 1.1 Deductive reasoning & Logical connectors.

$R$ : it will rain tmr

$S$ : it will snow tmr

$\Rightarrow$  These phrases are unambiguous, they are either true or false.

Logical connectors:

$\vee$  or

$\wedge$  and

$\neg$  not (negation)

e.g.  $R \wedge S$ : It will rain and snow tmr

$R \vee S$ : It will rain or snow tmr

$(\neg R \wedge S) \vee (R \wedge \neg S)$ : It will either rain or snow tmr (exclusive or)

English to logic:  $T$ : it will be sunny tmr

e.g. Either it won't rain and it won't snow, or it would be sunny

$\Rightarrow (\neg R \wedge \neg S) \vee T$

It won't rain, and either it will snow or it would be sunny.

$\Rightarrow \neg R \wedge (S \vee T)$

Translate to logical expression

e.g.  $3 \leq \pi \Rightarrow (3 < \pi) \vee (3 = \pi)$

$3 \leq \pi \leq 4 \Rightarrow [(3 < \pi) \vee (3 = \pi)] \wedge (\pi \leq 4)$

Well-formed formula (WFF)

$P \neg \wedge Q$		$\Leftarrow$	These are bad examples!
$\wedge P Q \vee$			
$(P \wedge Q)$			

## §1.2. Truth tables.

AND	P	Q	$P \wedge Q$	OR	P	Q	$P \vee Q$
	T	T	T		T	T	T
	T	F	F		T	F	T
	F	T	F		F	T	T
	F	F	F		F	F	F

NOT	P	$\neg P$
	T	F
	F	T

e.g.	P	Q	$\neg(P \wedge Q)$	P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
	F	F	T	F	F	T	T	T
	F	T	T	F	T	T	F	T
	T	F	T	T	F	F	T	T
	T	T	F	T	T	F	F	F

$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$  These are equivalent formulas.

"De Morgan's Law"

e.g.	P	Q	$P \vee (P \wedge Q)$
	F	F	F
	F	T	F
	T	F	T
	T	T	T

$\Rightarrow P \equiv P \vee (P \wedge Q)$ .