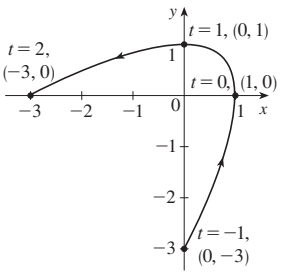
H Answers to Odd-Numbered Exercises

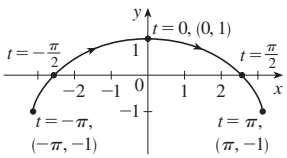
CHAPTER 10

EXERCISES 10.1 ■ PAGE 685

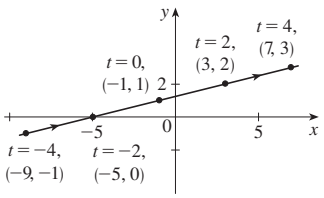
1.



3.

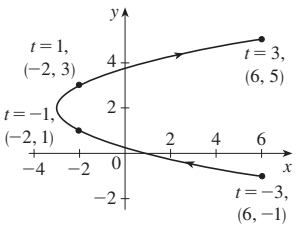


5. (a)



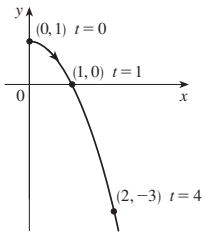
(b) $y = \frac{1}{4}x + \frac{5}{4}$

7. (a)



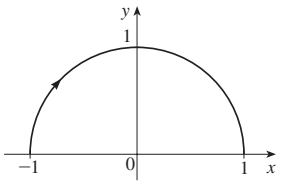
(b) $x = y^2 - 4y + 1$, $-1 \leqslant y \leqslant 5$

9. (a)

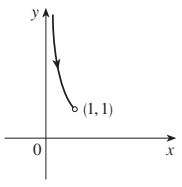


(b) $y = 1 - x^2, x \geqslant 0$

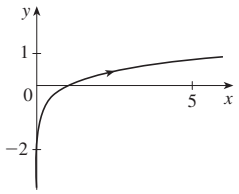
11. (a) $x^2 + y^2 = 1, y \geqslant 0$ (b)



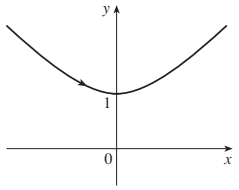
13. (a) $y = 1/x, y > 1$ (b)



15. (a) $x = e^{2y}$ (b)



17. (a) $y^2 - x^2 = 1, y \geqslant 1$ (b)

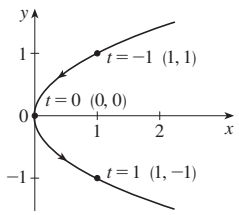


19. Moves counterclockwise along the circle $\left(\frac{x-5}{2}\right)^2 + \left(\frac{y-3}{2}\right)^2 = 1$ from (3, 3) to (7, 3)

21. Moves 3 times clockwise around the ellipse $(x^2/25) + (y^2/4) = 1$, starting and ending at $(0, -2)$

23. It is contained in the rectangle described by $1 \leqslant x \leqslant 4$ and $2 \leqslant y \leqslant 3$.

25.



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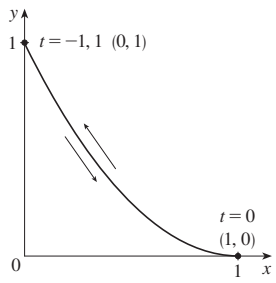

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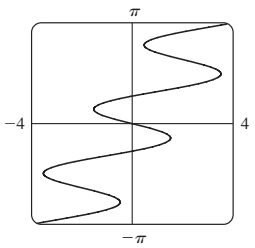


31. (b) $x = -2 + 5t, y = 7 - 8t, 0 \leqslant t \leqslant 1$

33. (a) $x = 2\cos t, y = 1 - 2\sin t, 0 \leqslant t \leqslant 2\pi$

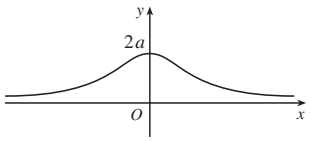
(b) $x = 2\cos t, y = 1 + 2\sin t, 0 \leqslant t \leqslant 6\pi$

(c) $x = 2\cos t, y = 1 + 2\sin t, \pi/2 \leqslant t \leqslant 3\pi/2$

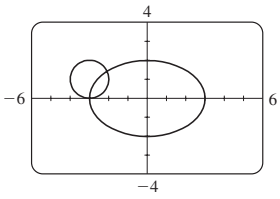
37. The curve $y = x^{2/3}$ is generated in (a). In (b), only the portion with $x \geqslant 0$ is generated, and in (c) we get only the portion with $x > 0$.

41. $x = a\cos\theta, y = b\sin\theta$; $(x^2/a^2) + (y^2/b^2) = 1$, ellipse

43.



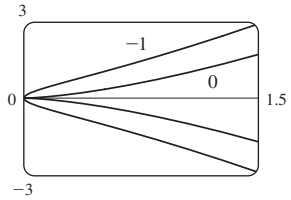
45. (a) Two points of intersection

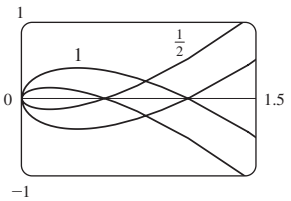


(b) One collision point at $(-3, 0)$ when $t = 3\pi/2$

(c) There are still two intersection points, but no collision point.

47. For $c = 0$, there is a cusp; for $c > 0$, there is a loop whose size increases as c increases.





49. The curves roughly follow the line $y = x$, and they start having loops when a is between 1.4 and 1.6 (more precisely, when $a > \sqrt{2}$). The loops increase in size as a increases.

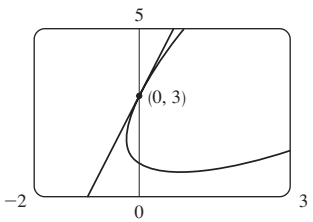
51. As n increases, the number of oscillations increases; a and b determine the width and height.

EXERCISES 10.2 ■ PAGE 695

1. $\frac{1}{2}(1 + t)^{3/2}$ **3.** $y = -x$ **5.** $y = \pi x + \pi^2$

7. $y = 2x + 1$

9. $y = 3x + 3$



11. $\dfrac{2t+1}{2t}, -\dfrac{1}{4t^3}, t < 0$ **13.** $e^{-2t}(1 - t), e^{-3t}(2t - 3), t > \frac{3}{2}$

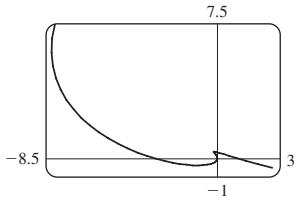
15. $\dfrac{t+1}{t-1}, \dfrac{-2t}{(t-1)^3}, 0 < t < 1$

17. Horizontal at $(0, -3)$, vertical at $(\pm 2, -2)$

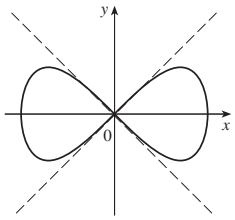
19. Horizontal at $\left(\frac{1}{2}, -1\right)$ and $\left(-\frac{1}{2}, 1\right)$, no vertical

21. $(0.6, 2); \left(5 \cdot 6^{-6/5}, e^{6^{-1/5}}\right)$

23.



25. $y = x, y = -x$



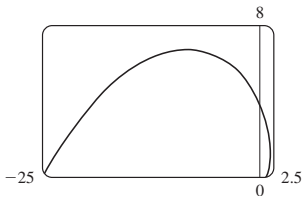
27. (a) $d \sin\theta/(r - d\cos\theta)$ **29.** $(4, 0)$ **31.** πab

33. $\frac{24}{5}$ **35.** $2\pi r^2 + \pi d^2$ **37.** $\int_0^2 \sqrt{2 + 2e^{-2t}}\, dt \approx 3.1416$

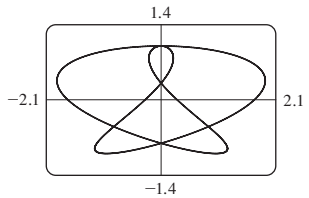
39. $\int_0^{4\pi} \sqrt{5 - 4\cos t}\, dt \approx 26.7298$ **41.** $4\sqrt{2} - 2$

43. $\frac{1}{2}\sqrt{2} + \frac{1}{2}\ln\left(1 + \sqrt{2}\right)$

45. $\sqrt{2}\,(e^{\pi} - 1)$

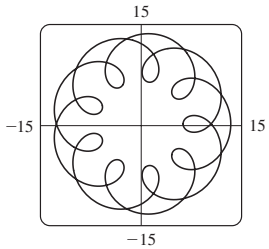


47. 16.7102



49. 612.3053 **51.** $6\sqrt{2}, \sqrt{2}$

55. (a)



$t \in [0, 4\pi]$

(b) 294

57. $\int_0^{\pi/2} 2\pi t \cos t \sqrt{t^2 + 1}\, dt \approx 4.7394$

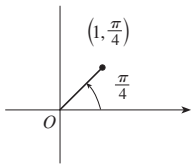
59. $\int_0^1 2\pi e^{-t}\sqrt{1 + 2e^t + e^{2t} + e^{-2t}}\, dt \approx 10.6705$

61. $\frac{2}{1215}\pi\left(247\sqrt{3} + 64\right)$ **63.** $\frac{6}{5}\pi a^2$

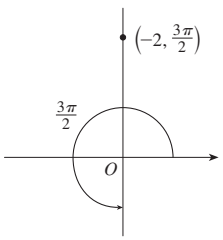
65. $\frac{24}{5}\pi\left(949\sqrt{26} + 1\right)$ **71.** $\frac{1}{4}$

EXERCISES 10.3 ■ PAGE 706

1. (a)

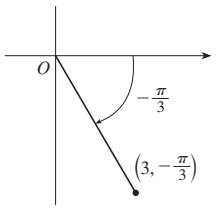


(b)



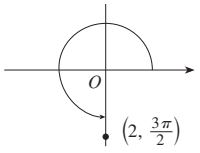
$(1, 9\pi/4), (-1, 5\pi/4)$ $(2, \pi/2), (-2, 7\pi/2)$

(c)



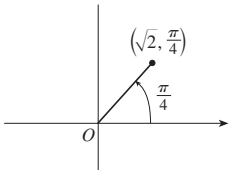
$(3, 5\pi/3), (-3, 2\pi/3)$

3. (a)



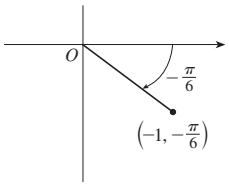
$(0, -2)$

(b)



$(1, 1)$

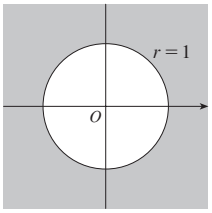
(c)



$\left(-\sqrt{3}/2, 1/2\right)$

5. (a) (i) $\left(4\sqrt{2}, 3\pi/4\right)$ (ii) $\left(-4\sqrt{2}, 7\pi/4\right)$
(b) (i) $(6, \pi/3)$ (ii) $(-6, 4\pi/3)$

7.



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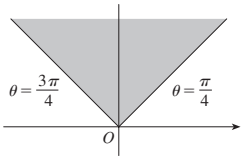

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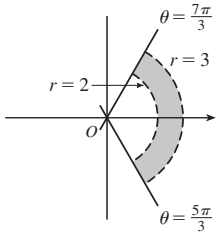
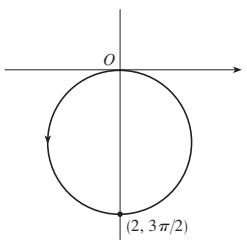


13. $2\sqrt{7}$ **15.** Circle, center O, radius $\sqrt{5}$
17. Circle, center $(5/2, 0)$, radius $5/2$
19. Hyperbola, center O, foci on x-axis
21. $r = 2 \csc\theta$ **23.** $r = 1/(\sin\theta - 3\cos\theta)$
25. $r = 2c\cos\theta$ **27.** (a) $\theta = \pi/6$ (b) $x = 3$

29.



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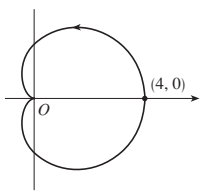

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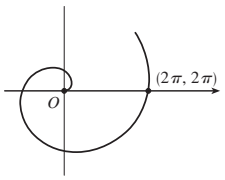

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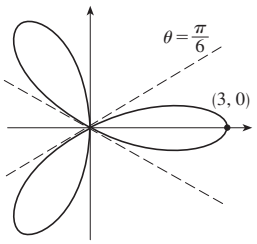

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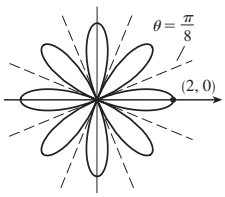

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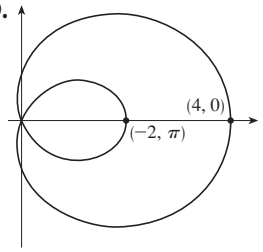

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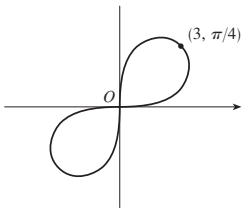

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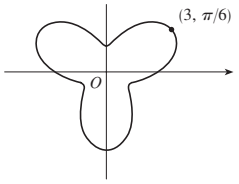

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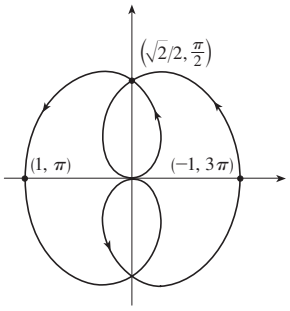

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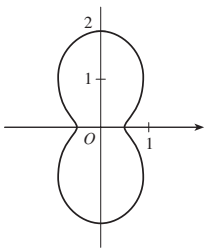

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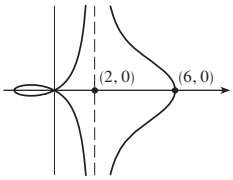

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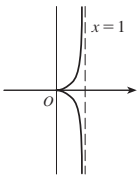
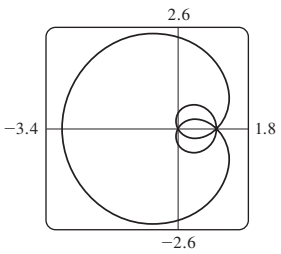


53. (a) For $c < -1$, the inner loop begins at $\theta = \sin^{-1}(-1/c)$ and ends at $\theta = \pi - \sin^{-1}(-1/c)$; for $c > 1$, it begins at $\theta = \pi + \sin^{-1}(1/c)$ and ends at $\theta = 2\pi - \sin^{-1}(1/c)$.
55. $1/\sqrt{3}$ **57.** $-\pi$ **59.** 1
61. Horizontal at $\left(3/\sqrt{2}, \pi/4\right), \left(-3/\sqrt{2}, 3\pi/4\right)$; vertical at $(3, 0), (0, \pi/2)$
63. Horizontal at $\left(\frac{3}{2}, \pi/3\right)$, $(0, \pi)$ [the pole], and $\left(\frac{3}{2}, 5\pi/3\right)$; vertical at $(2, 0), \left(\frac{1}{2}, 2\pi/3\right), \left(\frac{1}{2}, 4\pi/3\right)$
65. Center $(b/2, a/2)$, radius $\sqrt{a^2 + b^2}/2$

67.



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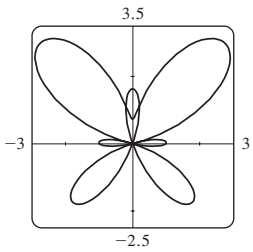

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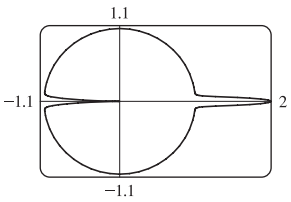


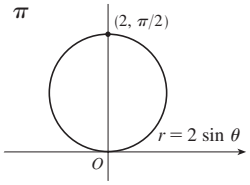
73. By counterclockwise rotation through angle $\pi/6$, $\pi/3$, or α about the origin

75. For $c = 0$, the curve is a circle. As c increases, the left side gets flatter, then has a dimple for $0.5 < c < 1$, a cusp for $c = 1$, and a loop for $c > 1$.

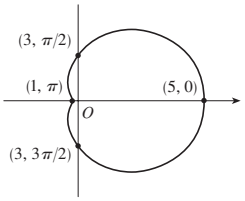
EXERCISES 10.4 ■ PAGE 712

1. $e^{-\pi/4} - e^{-\pi/2}$ **3.** $\pi/2$ **5.** $\frac{1}{2}$ **7.** $\frac{41}{4}\pi$

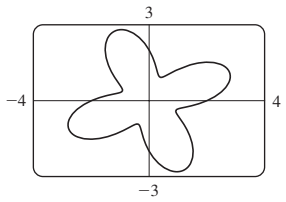
9. π



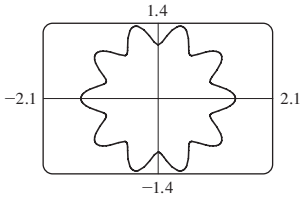
11. 11π



13. $\frac{9}{2}\pi$



15. $\frac{3}{2}\pi$



17. $\frac{4}{3}\pi$ **19.** $\frac{1}{16}\pi$ **21.** $\pi - \frac{3}{2}\sqrt{3}$ **23.** $\frac{4}{3}\pi + 2\sqrt{3}$

25. $4\sqrt{3} - \frac{4}{3}\pi$ **27.** π **29.** $\frac{9}{8}\pi - \frac{9}{4}$ **31.** $\frac{1}{2}\pi - 1$

33. $-\sqrt{3} + 2 + \frac{1}{3}\pi$ **35.** $\frac{1}{4}(\pi + 3\sqrt{3})$

37. $(\frac{1}{2}, \pi/6)$, $(\frac{1}{2}, 5\pi/6)$, and the pole

39. $(1, \theta)$ where $\theta = \pi/12, 5\pi/12, 13\pi/12, 17\pi/12$ and $(-1, \theta)$ where $\theta = 7\pi/12, 11\pi/12, 19\pi/12, 23\pi/12$

41. $(\frac{1}{2}\sqrt{3}, \pi/3)$, $(\frac{1}{2}\sqrt{3}, 2\pi/3)$, and the pole

43. Intersection at $\theta \approx 0.89, 2.25$; area ≈ 3.46

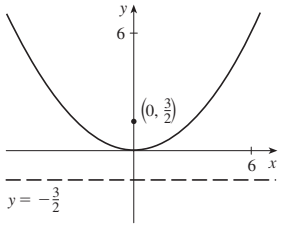
45. 2π **47.** $\frac{8}{3}[(\pi^2 + 1)^{3/2} - 1]$ **49.** $\frac{16}{3}$

51. 2.4221 **53.** 8.0091

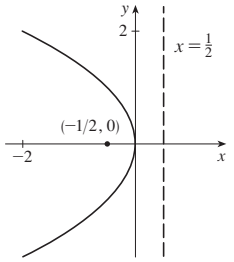
55. (b) $2\pi(2 - \sqrt{2})$

EXERCISES 10.5 ■ PAGE 720

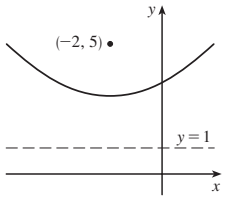
1. $(0, 0)$, $(0, \frac{3}{2})$, $y = -\frac{3}{2}$



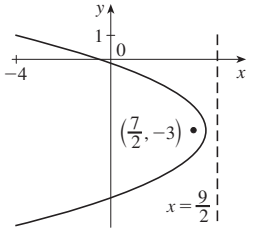
3. $(0, 0)$, $(-\frac{1}{2}, 0)$, $x = \frac{1}{2}$



5. $(-2, 3)$ $(-2, 5)$, $y = 1$

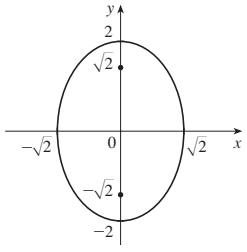


7. $(4, -3)$, $(\frac{7}{2}, -3)$, $x = \frac{9}{2}$

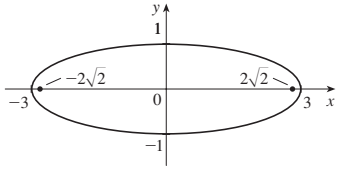


9. $x = -y^2$, focus $(\frac{1}{4}, 0)$, directrix $x = \frac{1}{4}$

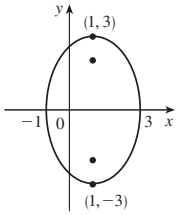
11. $(0, \pm 2)$, $(0, \pm\sqrt{2})$



13. $(\pm 3, 0)$, $(\pm 2\sqrt{2}, 0)$

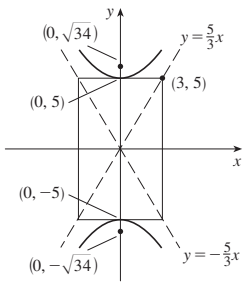


15. $(1, \pm 3)$, $(1, \pm\sqrt{5})$

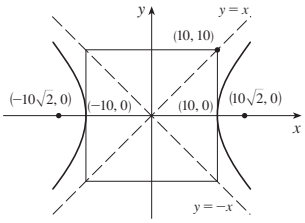


17. $\frac{x^2}{4} + \frac{y^2}{9} = 1$, foci $(0, \pm\sqrt{5})$

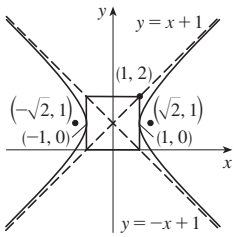
19. $(0, \pm 5)$; $(0, \pm\sqrt{34})$; $y = \pm\frac{5}{3}x$



21. $(\pm 10, 0), (\pm 10\sqrt{2}, 0), y = \pm x$



23. $(\pm 1, 1), (\pm\sqrt{2}, 1), y - 1 = \pm x$



25. Hyperbola, $(\pm 1, 0), (\pm\sqrt{5}, 0)$

27. Ellipse, $(\pm\sqrt{2}, 1), (\pm 1, 1)$

29. Parabola, $(1, -2), (1, -\frac{11}{6})$

31. $y^2 = 4x$ **33.** $y^2 = -12(x + 1)$

35. $(y + 1)^2 = -\frac{1}{2}(x - 3)$

37. $\dfrac{x^2}{25} + \dfrac{y^2}{21} = 1$ **39.** $\dfrac{x^2}{12} + \dfrac{(y-4)^2}{16} = 1$

41. $\dfrac{(x+1)^2}{12} + \dfrac{(y-4)^2}{16} = 1$ **43.** $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

45. $\dfrac{(y-1)^2}{25} - \dfrac{(x+3)^2}{39} = 1$ **47.** $\dfrac{x^2}{9} - \dfrac{y^2}{36} = 1$

49. $\dfrac{x^2}{3{,}763{,}600} + \dfrac{y^2}{3{,}753{,}196} = 1$

51. (a) $\dfrac{121x^2}{1{,}500{,}625} - \dfrac{121y^2}{3{,}339{,}375} = 1$ (b) ≈248 mi

55. (a) Ellipse (b) Hyperbola (c) No curve

59. 15.9

61. $\dfrac{b^2c}{a} + ab\ln\left(\dfrac{a}{b+c}\right)$ where $c^2 = a^2 + b^2$

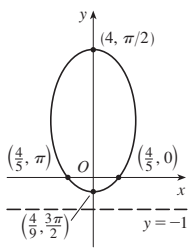
63. $(0, 4/\pi)$

EXERCISES 10.6 ■ PAGE 728

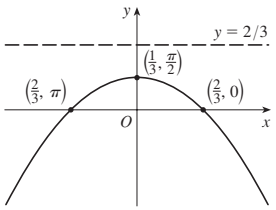
1. $r = \dfrac{4}{2 + \cos\theta}$ **3.** $r = \dfrac{6}{2 + 3\sin\theta}$

5. $r = \dfrac{10}{3 - 2\cos\theta}$ **7.** $r = \dfrac{6}{1 + \sin\theta}$

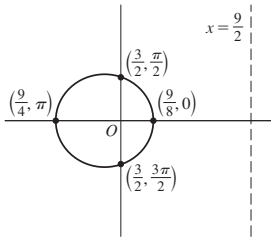
9. (a) $\frac{4}{5}$ (b) Ellipse (c) $y = -1$
(d)



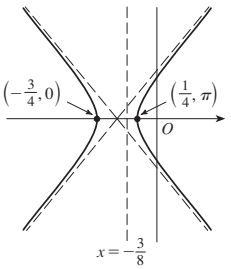
11. (a) 1 (b) Parabola (c) $y = \frac{2}{3}$
(d)



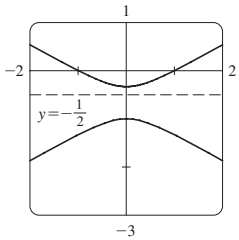
13. (a) $\frac{1}{3}$ (b) Ellipse (c) $x = \frac{9}{2}$
(d)



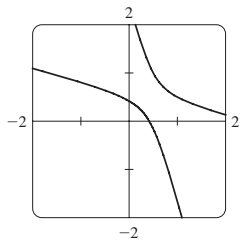
15. (a) 2 (b) Hyperbola (c) $x = -\frac{3}{8}$
(d)



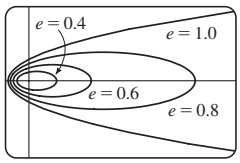
17. (a) $2, y = -\frac{1}{2}$



(b) $r = \dfrac{1}{1 - 2\sin(\theta - 3\pi/4)}$



19. The ellipse is nearly circular when e is close to 0 and becomes more elongated as $e \to 1^-$. At $e = 1$, the curve becomes a parabola.



25. $r = \dfrac{2.26 \times 10^8}{1 + 0.093\cos\theta}$ **27.** $r = \dfrac{1.07}{1 + 0.97\cos\theta}$; 35.64 AU

29. 7.0×10^7 km **31.** 3.6×10^8 km

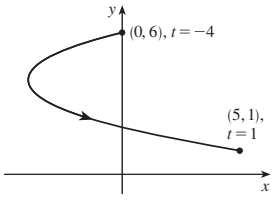
CHAPTER 10 REVIEW ■ PAGE 729

True-False Quiz

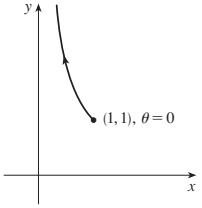
1. False **3.** False **5.** True **7.** False **9.** True

Exercises

1. $x = y^2 - 8y + 12$

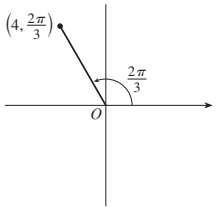


3. $y = 1/x$



5. $x = t,\ y = \sqrt{t}$; $x = t^4,\ y = t^2$; $x = \tan^2 t,\ y = \tan t,\ 0 \leq t < \pi/2$

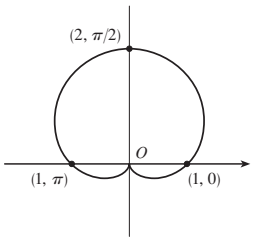
7. (a) $(4, \frac{2\pi}{3})$



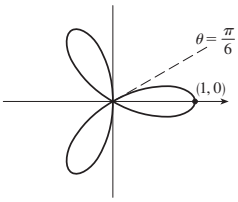
(b) $(3\sqrt{2}, 3\pi/4)$, $(-3\sqrt{2}, 7\pi/4)$

$(-2, 2\sqrt{3})$

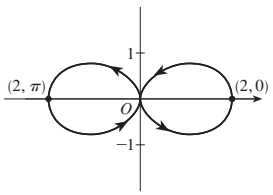
9.



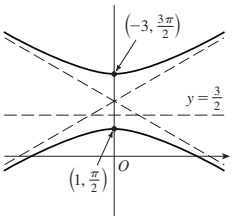
11.



13.

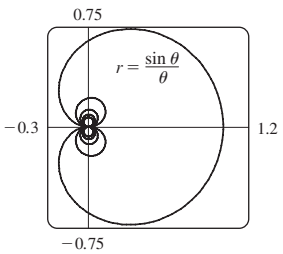


15.



17. $r = \dfrac{2}{\cos\theta + \sin\theta}$

19.

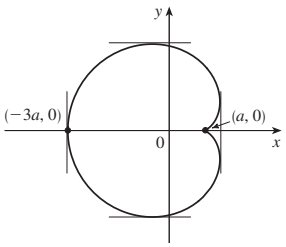


21. 2 **23.** −1

25. $\dfrac{1 + \sin t}{1 + \cos t},\ \dfrac{1 + \cos t + \sin t}{(1 + \cos t)^3}$

27. $(\frac{11}{8}, \frac{3}{4})$

29. Vertical tangent at $(\frac{3}{2}a, \pm\frac{1}{2}\sqrt{3}\,a)$, $(-3a, 0)$; horizontal tangent at $(a, 0)$, $(-\frac{1}{2}a, \pm\frac{3}{2}\sqrt{3}\,a)$



31. 18 **33.** $(2, \pm\pi/3)$ **35.** $\frac{1}{2}(\pi - 1)$

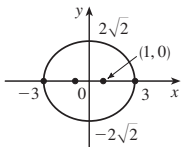
37. $2(5\sqrt{5} - 1)$

39. $\dfrac{2\sqrt{\pi^2 + 1} - \sqrt{4\pi^2 + 1}}{2\pi} + \ln\left(\dfrac{2\pi + \sqrt{4\pi^2 + 1}}{\pi + \sqrt{\pi^2 + 1}}\right)$

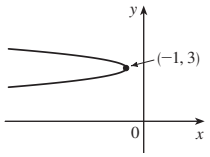
41. $471{,}295\pi/1024$

43. All curves have the vertical asymptote $x = 1$. For $c < -1$, the curve bulges to the right. At $c = -1$, the curve is the line $x = 1$. For $-1 < c < 0$, it bulges to the left. At $c = 0$ there is a cusp at $(0, 0)$. For $c > 0$, there is a loop.

45. $(\pm 1, 0)$, $(\pm 3, 0)$



47. $(-\frac{25}{24}, 3)$, $(-1, 3)$



49. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ **51.** $\dfrac{y^2}{72/5} - \dfrac{x^2}{8/5} = 1$

53. $\dfrac{x^2}{25} + \dfrac{(8y - 399)^2}{160{,}801} = 1$ **55.** $r = \dfrac{4}{3 + \cos\theta}$

57. $x = a(\cot\theta + \sin\theta\cos\theta)$, $y = a(1 + \sin^2\theta)$

PROBLEMS PLUS ■ PAGE 732

1. $\ln(\pi/2)$ **3.** $\left[-\frac{3}{4}\sqrt{3}, \frac{3}{4}\sqrt{3}\right] \times [-1, 2]$

CHAPTER 11

EXERCISES 11.1 ■ PAGE 744

Abbreviations: C, convergent; D, divergent

1. (a) A sequence is an ordered list of numbers. It can also be defined as a function whose domain is the set of positive integers.
(b) The terms a_n approach 8 as n becomes large.
(c) The terms a_n become large as n becomes large.

3. $\frac{2}{3}, \frac{4}{5}, \frac{8}{7}, \frac{16}{9}, \frac{32}{11}$ **5.** $\frac{1}{5}, -\frac{1}{25}, \frac{1}{125}, -\frac{1}{625}, \frac{1}{3125}$ **7.** $\frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \frac{1}{720}$

9. 1, 2, 7, 32, 157 **11.** $2, \frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{2}{9}$ **13.** $a_n = 1/(2n)$

15. $a_n = -3\left(-\frac{2}{3}\right)^{n-1}$ **17.** $a_n = (-1)^{n+1}\dfrac{n^2}{n+1}$

19. 0.4286, 0.4615, 0.4737, 0.4800, 0.4839, 0.4865, 0.4884, 0.4898, 0.4909, 0.4918; yes; $\frac{1}{2}$

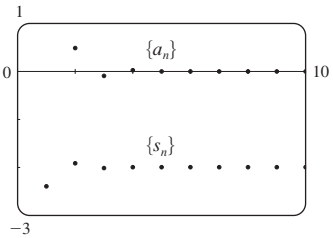
21. 0.5000, 1.2500, 0.8750, 1.0625, 0.9688, 1.0156, 0.9922, 1.0039, 0.9980, 1.0010; yes; 1

23. 5 **25.** D **27.** 0 **29.** 1 **31.** 2
33. D **35.** 0 **37.** 0 **39.** D **41.** 0 **43.** 0
45. 1 **47.** e^2 **49.** $\ln 2$ **51.** $\pi/2$ **53.** D **55.** D

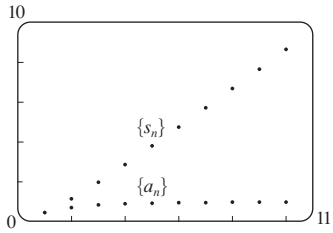
57. D **59.** $\pi/4$ **61.** D **63.** 0
65. (a) 1060, 1123.60, 1191.02, 1262.48, 1338.23 (b) D
67. (b) 5734 **69.** $-1 < r < 1$
71. Convergent by the Monotonic Sequence Theorem; $5 \leqslant L < 8$
73. Decreasing; yes **75.** Not monotonic; no
77. Increasing; yes
79. 2 **81.** $\frac{1}{2}(3 + \sqrt{5})$ **83.** (b) $\frac{1}{2}(1 + \sqrt{5})$
85. (a) 0 (b) 9, 11

EXERCISES 11.2 ■ PAGE 755

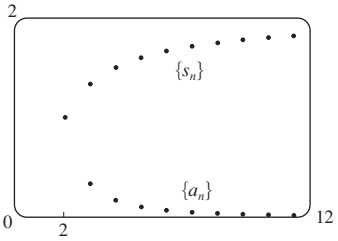
1. (a) A sequence is an ordered list of numbers whereas a series is the *sum* of a list of numbers.
(b) A series is convergent if the sequence of partial sums is a convergent sequence. A series is divergent if it is not convergent.
3. 2
5. 0.5, 0.55, 0.5611, 0.5648, 0.5663, 0.5671, 0.5675, 0.5677; C
7. 1, 1.7937, 2.4871, 3.1170, 3.7018, 4.2521, 4.7749, 5.2749; D
9. −2.40000, −1.92000, −2.01600, −1.99680, −2.00064, −1.99987, −2.00003, −1.99999, −2.00000, −2.00000; convergent, sum = −2



11. 0.44721, 1.15432, 1.98637, 2.88080, 3.80927, 4.75796, 5.71948, 6.68962, 7.66581, 8.64639; divergent



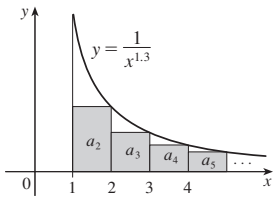
13. 1.00000, 1.33333, 1.50000, 1.60000, 1.66667, 1.71429, 1.75000, 1.77778, 1.80000, 1.81818; convergent, sum = 2



15. (a) Yes (b) No **17.** D **19.** $\frac{25}{3}$ **21.** $\frac{400}{9}$
23. $\frac{1}{7}$ **25.** D **27.** D **29.** D **31.** 9 **33.** D
35. $\dfrac{\sin 100}{1 - \sin 100}$
37. D **39.** D **41.** $e/(e - 1)$ **43.** $\frac{3}{2}$ **45.** $\frac{11}{6}$
47. $e - 1$
49. (b) 1 (c) 2 (d) All rational numbers with a terminating decimal representation, except 0
51. $\frac{8}{9}$ **53.** $\frac{838}{333}$ **55.** 45,679/37,000
57. $-\dfrac{1}{5} < x < \dfrac{1}{5}$; $\dfrac{-5x}{1 + 5x}$
59. $-1 < x < 5$; $\dfrac{3}{5 - x}$
61. $x > 2$ or $x < -2$; $\dfrac{x}{x - 2}$ **63.** $x < 0$; $\dfrac{1}{1 - e^x}$
65. 1 **67.** $a_1 = 0$, $a_n = \dfrac{2}{n(n + 1)}$ for $n > 1$, sum = 1
69. (a) 120 mg; 124 mg
(b) $Q_{n+1} = 100 + 0.20Q_n$ (c) 125 mg
71. (a) 157.875 mg; $\frac{3000}{19}(1 - 0.05^n)$ (b) 157.895 mg
73. (a) $S_n = \dfrac{D(1 - c^n)}{1 - c}$ (b) 5 **75.** $\frac{1}{2}(\sqrt{3} - 1)$
79. $\dfrac{1}{n(n + 1)}$ **81.** The series is divergent.
87. $\{s_n\}$ is bounded and increasing.
89. (a) $0, \frac{1}{9}, \frac{2}{9}, \frac{1}{3}, \frac{2}{3}, \frac{7}{9}, \frac{8}{9}, 1$
91. (a) $\frac{1}{2}, \frac{5}{6}, \frac{23}{24}, \frac{119}{120}$; $\dfrac{(n + 1)! - 1}{(n + 1)!}$ (c) 1

EXERCISES 11.3 ■ PAGE 765

1. C



3. C **5.** D **7.** D **9.** C **11.** C **13.** D
15. C **17.** C **19.** D **21.** D **23.** C **25.** C
27. f is neither positive nor decreasing.
29. $p > 1$ **31.** $p < -1$ **33.** $(1, \infty)$
35. (a) $\frac{9}{10}\pi^4$ (b) $\frac{1}{90}\pi^4 - \frac{17}{16}$
37. (a) 1.54977, error $\leqslant 0.1$ (b) 1.64522, error $\leqslant 0.005$
(c) 1.64522 compared to 1.64493 (d) $n > 1000$
39. 0.00145 **45.** $b < 1/e$

EXERCISES 11.4 ■ PAGE 771

1. (a) Nothing (b) C **3.** C **5.** D **7.** C **9.** D
11. C **13.** C **15.** D **17.** D **19.** C **21.** D
23. C **25.** D **27.** C **29.** C **31.** D
33. 0.1993, error $< 2.5 \times 10^{-5}$
35. 0.0739, error $< 6.4 \times 10^{-8}$
45. Yes

EXERCISES 11.5 ■ PAGE 776

1. (a) A series whose terms are alternately positive and negative (b) $0 < b_{n+1} \leqslant b_n$ and $\lim_{n\to\infty} b_n = 0$, where $b_n = |a_n|$ (c) $|R_n| \leqslant b_{n+1}$
3. D **5.** C **7.** D **9.** C **11.** C **13.** D
15. C **17.** C **19.** D **21.** −0.5507 **23.** 5
25. 5 **27.** −0.4597 **29.** −0.1050
31. An underestimate
33. p is not a negative integer. **35.** $\{b_n\}$ is not decreasing.

EXERCISES 11.6 ■ PAGE 782

Abbreviations: AC, absolutely convergent; CC, conditionally convergent

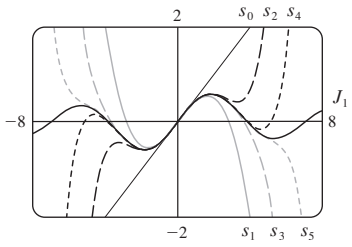
1. (a) D (b) C (c) May converge or diverge
3. CC **5.** AC **7.** AC **9.** D **11.** AC
13. AC **15.** D **17.** AC **19.** AC **21.** AC
23. D **25.** AC **27.** AC **29.** D **31.** CC
33. AC **35.** D **37.** AC **39.** D **41.** AC
43. (a) and (d)
47. (a) $\frac{661}{960} \approx 0.68854$, error < 0.00521
(b) $n \geqslant 11$, 0.693109
53. (b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$; $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

EXERCISES 11.7 ■ PAGE 786

1. D **3.** CC **5.** D **7.** D **9.** C **11.** C
13. C **15.** C **17.** C **19.** C **21.** D **23.** D
25. C **27.** C **29.** C **31.** D
33. C **35.** D **37.** C

EXERCISES 11.8 ■ PAGE 791

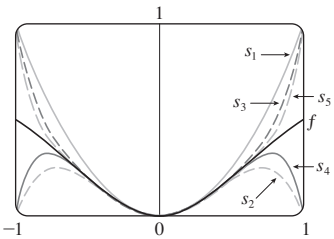
1. A series of the form $\sum_{n=0}^{\infty} c_n(x-a)^n$, where x is a variable and a and the c_n's are constants
3. $1, (-1, 1)$ **5.** $1, [-1, 1)$
7. $\infty, (-\infty, \infty)$ **9.** $4, [-4, 4]$
11. $\frac{1}{4}, \left(-\frac{1}{4}, \frac{1}{4}\right]$ **13.** $2, [-2, 2)$
15. $1, [1, 3]$ **17.** $2, [-4, 0)$
19. $\infty, (-\infty, \infty)$ **21.** $b, (a-b, a+b)$ **23.** $0, \left\{\frac{1}{2}\right\}$
25. $\frac{1}{5}, \left[\frac{3}{5}, 1\right]$ **27.** $\infty, (-\infty, \infty)$
29. (a) Yes (b) No
31. k^k **33.** No
35. (a) $(-\infty, \infty)$
(b), (c)



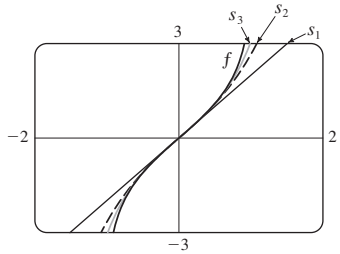
37. $(-1, 1)$, $f(x) = (1+2x)/(1-x^2)$ **41.** 2

EXERCISES 11.9 ■ PAGE 797

1. 10 **3.** $\sum_{n=0}^{\infty} (-1)^n x^n, (-1, 1)$ **5.** $2\sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^n, (-3, 3)$
7. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{2^{4n+4}}, (-2, 2)$ **9.** $-\frac{1}{2} - \sum_{n=1}^{\infty} \frac{(-1)^n 3x^n}{2^{n+1}}, (-2, 2)$
11. $\sum_{n=0}^{\infty} \left(-1 - \frac{1}{3^{n+1}}\right) x^n, (-1, 1)$
13. (a) $\sum_{n=0}^{\infty} (-1)^n (n+1) x^n, R = 1$
(b) $\frac{1}{2}\sum_{n=0}^{\infty} (-1)^n (n+2)(n+1) x^n, R = 1$
(c) $\frac{1}{2}\sum_{n=2}^{\infty} (-1)^n n(n-1) x^n, R = 1$
15. $\ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n5^n}, R = 5$
17. $\sum_{n=0}^{\infty} (-1)^n 4^n (n+1) x^{n+1}, R = \frac{1}{4}$
19. $\sum_{n=0}^{\infty} (2n+1) x^n, R = 1$
21. $\sum_{n=0}^{\infty} (-1)^n x^{2n+2}, R = 1$



23. $\sum_{n=0}^{\infty} \frac{2x^{2n+1}}{2n+1}, R = 1$



25. $C + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}, R = 1$
27. $C + \sum_{n=1}^{\infty} (-1)^n \frac{x^{n+3}}{n(n+3)}, R = 1$
29. 0.044522 **31.** 0.000395
33. 0.19740
35. (b) 0.920 **39.** $[-1, 1], [-1, 1), (-1, 1)$

EXERCISES 11.10 ■ PAGE 811

1. $b_8 = f^{(8)}(5)/8!$ **3.** $\sum_{n=0}^{\infty} (n+1) x^n, R = 1$
5. $x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$
7. $2 + \frac{1}{12}(x-8) - \frac{1}{288}(x-8)^2 + \frac{5}{20{,}736}(x-8)^3$
9. $\frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{6}\right)^3$
11. $\sum_{n=0}^{\infty} (n+1) x^n, R = 1$ **13.** $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, R = \infty$

15. $\sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!} x^n, R = \infty$ **17.** $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}, R = \infty$

19. $50 + 105(x-2) + 92(x-2)^2 + 42(x-2)^3 + 10(x-2)^4 + (x-2)^5, R = \infty$

21. $\ln 2 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n2^n} (x-2)^n, R = 2$

23. $\sum_{n=0}^{\infty} \frac{2^n e^6}{n!} (x-3)^n, R = \infty$

25. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!} (x-\pi)^{2n+1}, R = \infty$

31. $1 - \frac{1}{4}x - \sum_{n=2}^{\infty} \frac{3 \cdot 7 \cdot \cdots \cdot (4n-5)}{4^n \cdot n!} x^n, R = 1$

33. $\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2^{n+4}} x^n, R = 2$

35. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1} x^{4n+2}, R = 1$

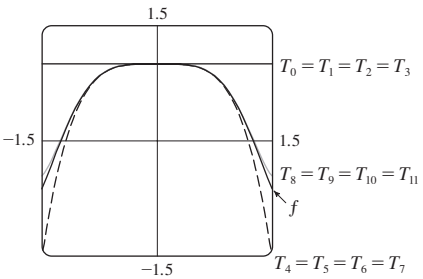
37. $\sum_{n=0}^{\infty} (-1)^n \frac{2^{2n}}{(2n)!} x^{2n+1}, R = \infty$

39. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2^{2n}(2n)!} x^{4n+1}, R = \infty$

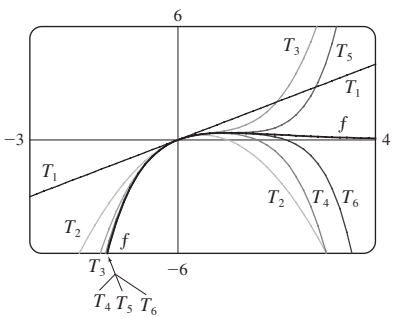
41. $\frac{1}{2}x + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{n! 2^{3n+1}} x^{2n+1}, R = 2$

43. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{2n-1}}{(2n)!} x^{2n}, R = \infty$

45. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} x^{4n}, R = \infty$



47. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n-1)!} x^n, R = \infty$



49. 0.99619

51. (a) $1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^n n!} x^{2n}$

(b) $x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{(2n+1)2^n n!} x^{2n+1}$

53. $C + \sum_{n=0}^{\infty} \binom{\frac{1}{2}}{n} \frac{x^{3n+1}}{3n+1}, R = 1$

55. $C + \sum_{n=1}^{\infty} (-1)^n \frac{1}{2n(2n)!} x^{2n}, R = \infty$

57. 0.0059 **59.** 0.40102 **61.** $\frac{1}{2}$ **63.** $\frac{1}{120}$ **65.** $\frac{3}{5}$

67. $1 - \frac{3}{2}x^2 + \frac{25}{24}x^4$ **69.** $1 + \frac{1}{6}x^2 + \frac{7}{360}x^4$

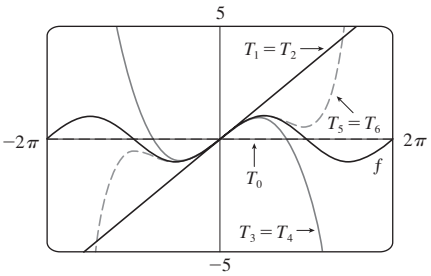
71. $x - \frac{2}{3}x^4 + \frac{23}{45}x^6$

73. e^{-x^4} **75.** $\ln \frac{8}{5}$

77. $1/\sqrt{2}$ **79.** $e^3 - 1$

EXERCISES 11.11 ■ PAGE 820

1. (a) $T_0(x) = 0, T_1(x) = T_2(x) = x, T_3(x) = T_4(x) = x - \frac{1}{6}x^3$, $T_5(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5$

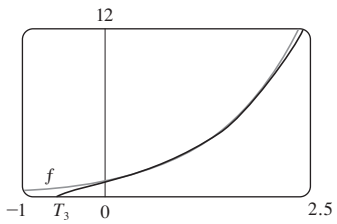


(b)

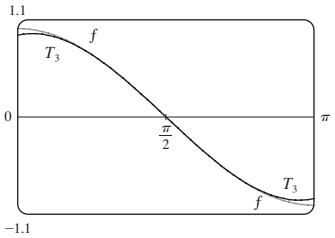
x	f	T_0	$T_1 = T_2$	$T_3 = T_4$	T_5
$\pi/4$	0.7071	0	0.7854	0.7047	0.7071
$\pi/2$	1	0	1.5708	0.9248	1.0045
π	0	0	3.1416	−2.0261	0.5240

(c) As n increases, $T_n(x)$ is a good approximation to $f(x)$ on a larger and larger interval.

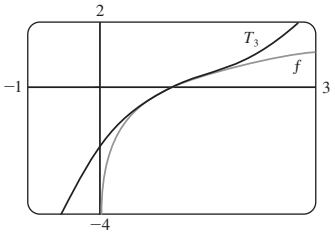
3. $e + e(x-1) + \frac{1}{2}e(x-1)^2 + \frac{1}{6}e(x-1)^3$



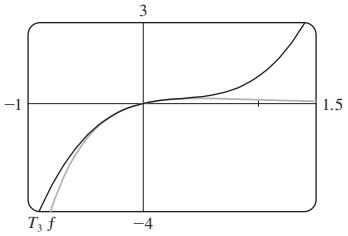
5. $-\left(x-\frac{\pi}{2}\right)+\frac{1}{6}\left(x-\frac{\pi}{2}\right)^3$



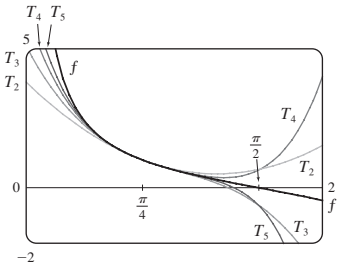
7. $(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3$



9. $x-2x^2+2x^3$



11. $T_5(x)=1-2\left(x-\frac{\pi}{4}\right)+2\left(x-\frac{\pi}{4}\right)^2-\frac{8}{3}\left(x-\frac{\pi}{4}\right)^3+\frac{10}{3}\left(x-\frac{\pi}{4}\right)^4-\frac{64}{15}\left(x-\frac{\pi}{4}\right)^5$



13. (a) $1-(x-1)+(x-1)^2$ (b) 0.006 482 7
15. (a) $1+\frac{2}{3}(x-1)-\frac{1}{9}(x-1)^2+\frac{4}{81}(x-1)^3$ (b) 0.000 097
17. (a) $1+\frac{1}{2}x^2$ (b) 0.0015
19. (a) $1+x^2$ (b) 0.000 06 **21.** (a) $x^2-\frac{1}{6}x^4$ (b) 0.042
23. 0.17365 **25.** Four **27.** $-1.037 < x < 1.037$
29. $-0.86 < x < 0.86$ **31.** 21 m, no
37. (c) They differ by about 8×10^{-9} km.

CHAPTER 11 REVIEW ■ PAGE 824

True-False Quiz

1. False **3.** True **5.** False **7.** False **9.** False
11. True **13.** True **15.** False **17.** True
19. True **21.** True

Exercises

1. $\frac{1}{2}$ **3.** D **5.** 0 **7.** e^{12} **9.** 2 **11.** C
13. C **15.** D **17.** C **19.** C **21.** C **23.** CC
25. AC **27.** $\frac{1}{11}$ **29.** $\pi/4$ **31.** e^{-e} **35.** 0.9721
37. 0.189 762 24, error $< 6.4 \times 10^{-7}$
41. 4, $[-6, 2)$ **43.** 0.5, [2.5, 3.5)

45. $\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\left[\frac{1}{(2n)!}\left(x-\frac{\pi}{6}\right)^{2n}+\frac{\sqrt{3}}{(2n+1)!}\left(x-\frac{\pi}{6}\right)^{2n+1}\right]$

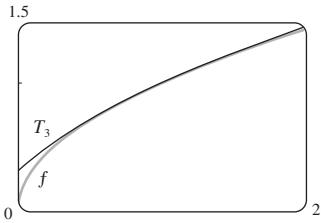
47. $\sum_{n=0}^{\infty}(-1)^n x^{n+2}$, $R=1$ **49.** $\ln 4-\sum_{n=1}^{\infty}\frac{x^n}{n4^n}$, $R=4$

51. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{8n+4}}{(2n+1)!}$, $R=\infty$

53. $\frac{1}{2}+\sum_{n=1}^{\infty}\frac{1\cdot 5\cdot 9\cdot\cdots\cdot(4n-3)}{n!\,2^{6n+1}}x^n$, $R=16$

55. $C+\ln|x|+\sum_{n=1}^{\infty}\frac{x^n}{n\cdot n!}$

57. (a) $1+\frac{1}{2}(x-1)-\frac{1}{8}(x-1)^2+\frac{1}{16}(x-1)^3$
(b)



(c) 0.000 006

59. $-\frac{1}{6}$

PROBLEMS PLUS ■ PAGE 827

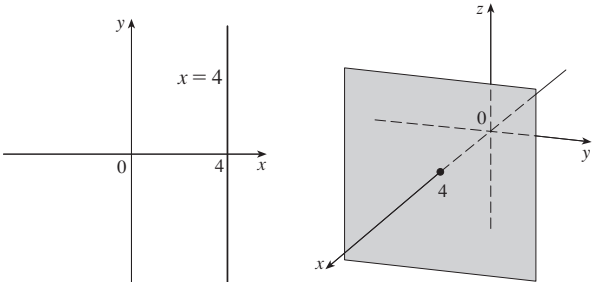
1. $15!/5! = 10{,}897{,}286{,}400$
3. (b) 0 if $x=0$, $(1/x)-\cot x$ if $x \neq k\pi$, k an integer
5. (a) $s_n=3\cdot 4^n$, $l_n=1/3^n$, $p_n=4^n/3^{n-1}$ (c) $\frac{2}{5}\sqrt{3}$
9. $\frac{3\pi}{4}$ **11.** $(-1, 1)$, $\frac{x^3+4x^2+x}{(1-x)^4}$ **13.** $\ln\frac{1}{2}$
17. (a) $\frac{250}{101}\pi(e^{-(n-1)\pi/5}-e^{-n\pi/5})$ (b) $\frac{250}{101}\pi$
19. $\frac{\pi}{2\sqrt{3}}-1$
21. $-\left(\frac{\pi}{2}-\pi k\right)^2$, where k is a positive integer

CHAPTER 12

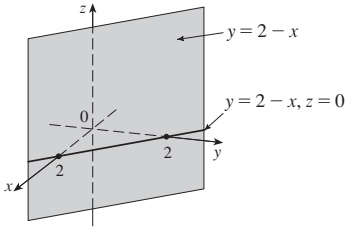
EXERCISES 12.1 ■ PAGE 836

1. $(4, 0, -3)$ **3.** C; A

5. A line parallel to the y-axis and 4 units to the right of it; a vertical plane parallel to the yz-plane and 4 units in front of it.



7. A vertical plane that intersects the xy-plane in the line $y = 2 - x$, $z = 0$



9. (a) $|PQ| = 6$, $|QR| = 2\sqrt{10}$, $|RP| = 6$; isosceles triangle

11. (a) No (b) Yes

13. $(x + 3)^2 + (y - 2)^2 + (z - 5)^2 = 16$; $(y - 2)^2 + (z - 5)^2 = 7$, $x = 0$ (a circle)

15. $(x - 3)^2 + (y - 8)^2 + (z - 1)^2 = 30$

17. $(1, 2, -4)$, 6 **19.** $(2, 0, -6)$, $9/\sqrt{2}$

21. (b) $\frac{5}{2}, \frac{1}{2}\sqrt{94}, \frac{1}{2}\sqrt{85}$

23. (a) $(x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 36$

(b) $(x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 4$

(c) $(x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 9$

25. A plane parallel to the yz-plane and 5 units in front of it

27. A half-space consisting of all points to the left of the plane $y = 8$

29. All points on or between the horizontal planes $z = 0$ and $z = 6$

31. All points on a circle with radius 2 with center on the z-axis that is contained in the plane $z = -1$

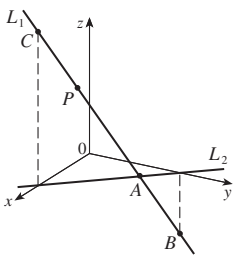
33. All point on a sphere with radius 2 and center $(0, 0, 0)$

35. All points on or between spheres with radii 1 and $\sqrt{5}$ and centers $(0, 0, 0)$

37. All points on or inside a circular cylinder of radius 3 with axis the y-axis

39. $0 < x < 5$ **41.** $r^2 < x^2 + y^2 + z^2 < R^2$

43. (a) $(2, 1, 4)$ (b)



45. $14x - 6y - 10z = 9$, a plane perpendicular to AB

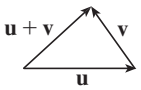
47. $2\sqrt{3} - 3$

EXERCISES 12.2 ■ PAGE 845

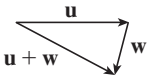
1. (a) Scalar (b) Vector (c) Vector (d) Scalar

3. $\overrightarrow{AB} = \overrightarrow{DC}$, $\overrightarrow{DA} = \overrightarrow{CB}$, $\overrightarrow{DE} = \overrightarrow{EB}$, $\overrightarrow{EA} = \overrightarrow{CE}$

5. (a)

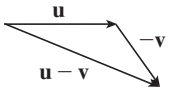


(b)

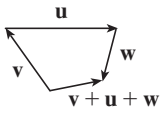


(c)

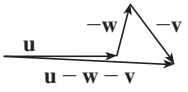
(d)



(e)

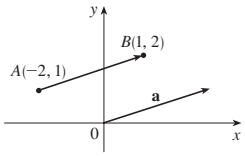


(f)

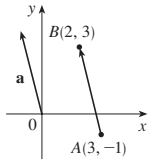


7. $\mathbf{c} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$, $\mathbf{d} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$

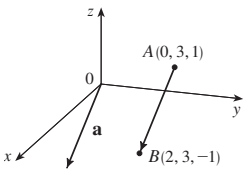
9. $\mathbf{a} = \langle 3, 1 \rangle$



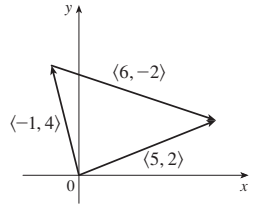
11. $\mathbf{a} = \langle -1, 4 \rangle$



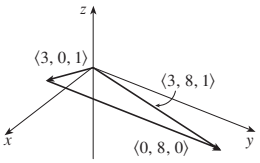
13. $\mathbf{a} = \langle 2, 0, -2 \rangle$



15. $\langle 5, 2 \rangle$



17. $\langle 3, 8, 1 \rangle$



19. $\langle 6, 3 \rangle$, $\langle 6, 14 \rangle$, 5, 13

21. $6\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$, $20\mathbf{i} - 12\mathbf{j}$, $\sqrt{29}$, 7

23. $\left\langle \frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}} \right\rangle$ **25.** $\frac{8}{9}\mathbf{i} - \frac{1}{9}\mathbf{j} + \frac{4}{9}\mathbf{k}$ **27.** 60°

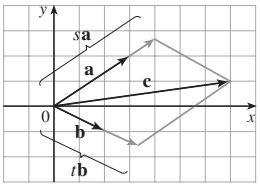
29. $\langle 2, 2\sqrt{3} \rangle$ **31.** ≈ 45.96 ft/s, ≈ 38.57 ft/s

33. $100\sqrt{7} \approx 264.6$ N, $\approx 139.1°$

35. $\sqrt{493} \approx 22.2$ mi/h, N8°W

37. $\approx -177.39\,\mathbf{i} + 211.41\,\mathbf{j}$, $\approx 177.39\,\mathbf{i} + 138.59\,\mathbf{j}$; ≈ 275.97 N, ≈ 225.11 N
39. (a) At an angle of 43.4° from the bank, toward upstream (b) 20.2 min
41. $\pm(\mathbf{i} + 4\,\mathbf{j})/\sqrt{17}$ **43.** $\mathbf{0}$

45. (a), (b)



(d) $s = \frac{9}{7}, t = \frac{11}{7}$

47. A sphere with radius 1, centered at (x_0, y_0, z_0)

EXERCISES 12.3 ■ PAGE 852

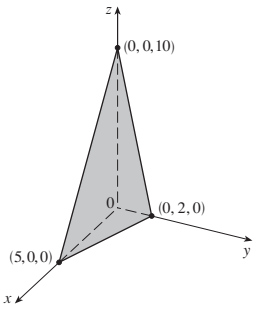
1. (b), (c), (d) are meaningful **3.** -3.6 **5.** 19 **7.** 1
9. $14\sqrt{3}$ **11.** $\mathbf{u}\cdot\mathbf{v} = \frac{1}{2}, \mathbf{u}\cdot\mathbf{w} = -\frac{1}{2}$
15. $\cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right) \approx 63°$ **17.** $\cos^{-1}\left(-\frac{5}{6}\right) \approx 146°$
19. $\cos^{-1}\left(\dfrac{7}{\sqrt{130}}\right) \approx 52°$ **21.** 48°, 75°, 57°
23. (a) Orthogonal (b) Neither (c) Parallel (d) Orthogonal
25. Yes **27.** $(\mathbf{i} - \mathbf{j} - \mathbf{k})/\sqrt{3}$ $\left[\text{or } (-\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{3}\right]$
29. 45° **31.** 0° at (0, 0), ≈8.1° at (1, 1)
33. $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$; 48°, 71°, 48°
35. $1/\sqrt{14}, -2/\sqrt{14}, -3/\sqrt{14}$; 74°, 122°, 143°
37. $1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}$; 55°, 55°, 55° **39.** $4, \left\langle -\frac{20}{13}, \frac{48}{13} \right\rangle$
41. $\frac{1}{9}, \left\langle \frac{4}{81}, \frac{7}{81}, -\frac{4}{81} \right\rangle$ **43.** $-7/\sqrt{19}, -\frac{21}{19}\,\mathbf{i} + \frac{21}{19}\,\mathbf{j} - \frac{7}{19}\,\mathbf{k}$
47. $\left\langle 0, 0, -2\sqrt{10} \right\rangle$ or any vector of the form $\left\langle s, t, 3s - 2\sqrt{10} \right\rangle, s, t \in \mathbb{R}$
49. 144 J **51.** $2400\cos(40°) \approx 1839$ ft-lb
53. $\frac{13}{5}$ **55.** $\cos^{-1}(1/\sqrt{3}) \approx 55°$

EXERCISES 12.4 ■ PAGE 861

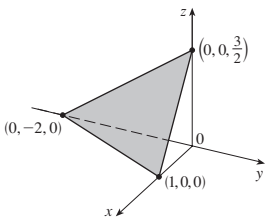
1. $15\,\mathbf{i} - 10\,\mathbf{j} - 3\,\mathbf{k}$ **3.** $14\,\mathbf{i} + 4\,\mathbf{j} + 2\,\mathbf{k}$
5. $-\frac{3}{2}\,\mathbf{i} + \frac{7}{4}\,\mathbf{j} + \frac{2}{3}\,\mathbf{k}$ **7.** $(1 - t)\,\mathbf{i} + (t^3 - t^2)\,\mathbf{k}$
9. $\mathbf{0}$ **11.** $\mathbf{i} + \mathbf{j} + \mathbf{k}$
13. (a) Scalar (b) Meaningless (c) Vector (d) Meaningless (e) Meaningless (f) Scalar
15. $96\sqrt{3}$; into the page **17.** $\langle -7, 10, 8 \rangle, \langle 7, -10, -8 \rangle$
19. $\left\langle -\dfrac{1}{3\sqrt{3}}, -\dfrac{1}{3\sqrt{3}}, \dfrac{5}{3\sqrt{3}} \right\rangle, \left\langle \dfrac{1}{3\sqrt{3}}, \dfrac{1}{3\sqrt{3}}, -\dfrac{5}{3\sqrt{3}} \right\rangle$
27. 20 **29.** (a) $\langle 0, 18, -9 \rangle$ (b) $\frac{9}{2}\sqrt{5}$
31. (a) $\langle 13, -14, 5 \rangle$ (b) $\frac{1}{2}\sqrt{390}$
33. 9 **35.** 16 **39.** $10.8 \sin 80° \approx 10.6$ N · m
41. ≈417 N **43.** 60°
45. (b) $\sqrt{97/3}$ **53.** (a) No (b) No (c) Yes

EXERCISES 12.5 ■ PAGE 871

1. (a) True (b) False (c) True (d) False (e) False (f) True (g) False (h) True (i) True (j) False (k) True
3. $\mathbf{r} = (2\,\mathbf{i} + 2.4\,\mathbf{j} + 3.5\,\mathbf{k}) + t(3\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k})$; $x = 2 + 3t, y = 2.4 + 2t, z = 3.5 - t$
5. $\mathbf{r} = (\mathbf{i} + 6\mathbf{k}) + t(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$; $x = 1 + t, y = 3t, z = 6 + t$
7. $x = 2 + 2t, y = 1 + \frac{1}{2}t, z = -3 - 4t$; $(x - 2)/2 = 2y - 2 = (z + 3)/(-4)$
9. $x = -8 + 11t, y = 1 - 3t, z = 4$; $\dfrac{x + 8}{11} = \dfrac{y - 1}{-3}, z = 4$
11. $x = -6 + 2t, y = 2 + 3t, z = 3 + t$; $(x + 6)/2 = (y - 2)/3 = z - 3$
13. Yes
15. (a) $(x - 1)/(-1) = (y + 5)/2 = (z - 6)/(-3)$ (b) $(-1, -1, 0), \left(-\frac{3}{2}, 0, -\frac{3}{2}\right), (0, -3, 3)$
17. $\mathbf{r}(t) = (6\mathbf{i} - \mathbf{j} + 9\mathbf{k}) + t(\mathbf{i} + 7\mathbf{j} - 9\mathbf{k}), 0 \leqslant t \leqslant 1$
19. Skew **21.** $(4, -1, -5)$ **23.** $x - 2y + 5z = 0$
25. $x + 4y + z = 4$ **27.** $5x - y - z = 7$
29. $6x + 6y + 6z = 11$ **31.** $x + y + z = 2$
33. $5x - 3y - 8z = -9$ **35.** $8x + y - 2z = 31$
37. $x - 2y - z = -3$ **39.** $3x - 8y - z = -38$
41.



43.

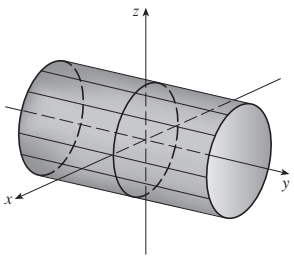


45. $(-2, 6, 3)$ **47.** $\left(\frac{2}{5}, 4, 0\right)$ **49.** $1, 0, -1$
51. Perpendicular **53.** Neither, $\cos^{-1}\left(-\dfrac{1}{\sqrt{6}}\right) \approx 114.1°$
55. Parallel
57. (a) $x = 1, y = -t, z = t$ (b) $\cos^{-1}\left(\dfrac{5}{3\sqrt{3}}\right) \approx 15.8°$
59. $x = 1, y - 2 = -z$
61. $x + 2y + z = 5$
63. $(x/a) + (y/b) + (z/c) = 1$
65. $x = 3t, y = 1 - t, z = 2 - 2t$
67. P_2 and P_3 are parallel, P_1 and P_4 are identical
69. $\sqrt{61/14}$ **71.** $\frac{18}{7}$ **73.** $5/\left(2\sqrt{14}\right)$
77. $1/\sqrt{6}$ **79.** $13/\sqrt{69}$
81. (a) $x = 325 + 440t, y = 810 - 135t, z = 561 + 38t, 0 \leqslant t \leqslant 1$ (b) No

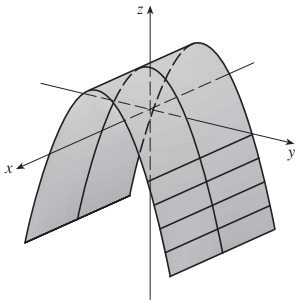
EXERCISES 12.6 ■ PAGE 879

1. (a) Parabola
(b) Parabolic cylinder with rulings parallel to the z-axis
(c) Parabolic cylinder with rulings parallel to the x-axis

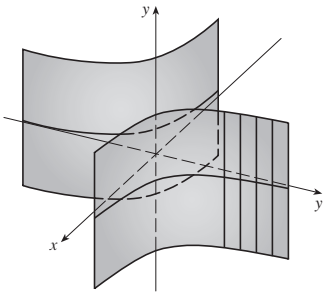
3. Circular cylinder



5. Parabolic cylinder

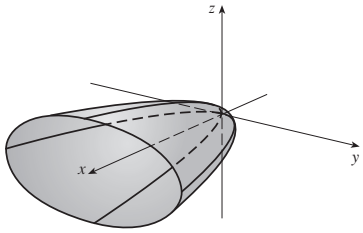


7. Hyperbolic cylinder

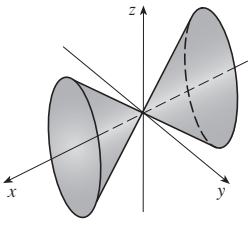


9. (a) $x = k$, $y^2 - z^2 = 1 - k^2$, hyperbola ($k \neq \pm 1$);
$y = k$, $x^2 - z^2 = 1 - k^2$, hyperbola ($k \neq \pm 1$);
$z = k$, $x^2 + y^2 = 1 + k^2$, circle
(b) The hyperboloid is rotated so that it has axis the y-axis
(c) The hyperboloid is shifted one unit in the negative y-direction

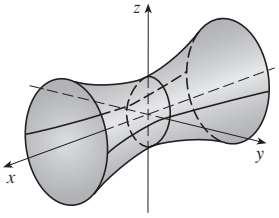
11. Elliptic paraboloid with axis the x-axis



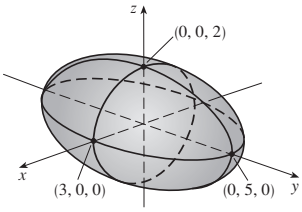
13. Elliptic cone with axis the x-axis



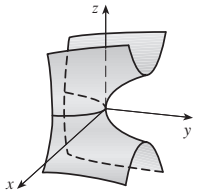
15. Hyperboloid of one sheet with axis the x-axis



17. Ellipsoid

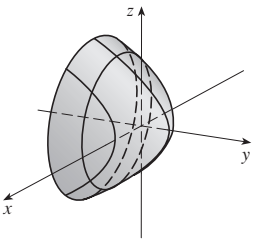


19. Hyperbolic paraboloid



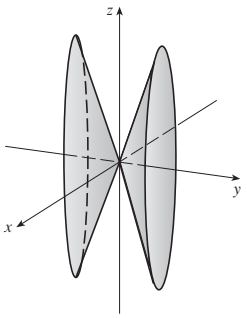
21. VII **23.** II **25.** VI **27.** VIII

29. Circular paraboloid



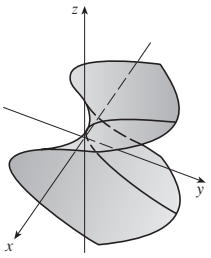
31. $y^2 = x^2 + \dfrac{z^2}{9}$

Elliptic cone with axis the y-axis

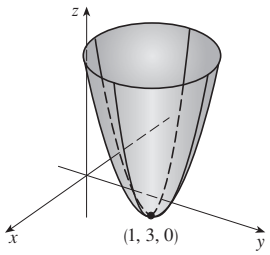


33. $y = z^2 - \dfrac{x^2}{2}$

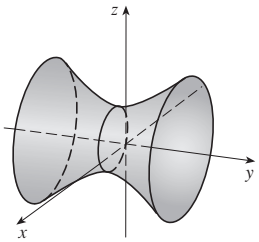
Hyperbolic paraboloid



35. $z = (x - 1)^2 + (y - 3)^2$
Circular paraboloid with vertex $(1, 3, 0)$ and axis the vertical line $x = 1, y = 3$



37. $\dfrac{(x-2)^2}{5} - \dfrac{y^2}{5} + \dfrac{(z-1)^2}{5} = 1$
Hyperboloid of one sheet with center $(2, 0, 1)$ and axis the horizontal line $x = 2, z = 1$



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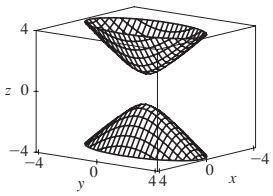

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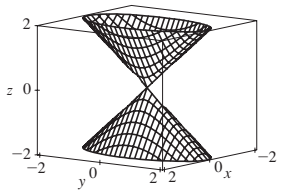

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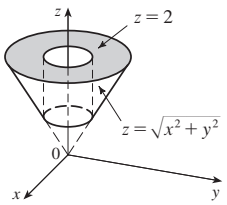
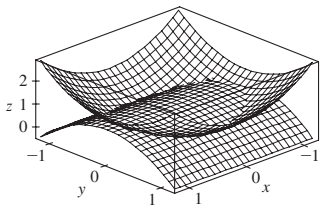


45. $x = y^2 + z^2$ **47.** $-4x = y^2 + z^2$, paraboloid

49. (a) $\dfrac{x^2}{(6378.137)^2} + \dfrac{y^2}{(6378.137)^2} + \dfrac{z^2}{(6356.523)^2} = 1$

(b) Circle (c) Ellipse

53.



CHAPTER 12 REVIEW ■ PAGE 882

True-False Quiz

1. False **3.** False **5.** True **7.** True **9.** True
11. True **13.** True **15.** False **17.** False
19. False **21.** True

Exercises

1. (a) $(x + 1)^2 + (y - 2)^2 + (z - 1)^2 = 69$
(b) $(y - 2)^2 + (z - 1)^2 = 68, x = 0$
(c) Center $(4, -1, -3)$, radius 5

3. $\mathbf{u} \cdot \mathbf{v} = 3\sqrt{2}$; $|\mathbf{u} \times \mathbf{v}| = 3\sqrt{2}$; out of the page

5. $-2, -4$ **7.** (a) 2 (b) -2 (c) -2 (d) 0

9. $\cos^{-1}\left(\frac{1}{3}\right) \approx 71°$ **11.** (a) $\langle 4, -3, 4 \rangle$ (b) $\sqrt{41}/2$

13. ≈ 166 N, ≈ 114 N

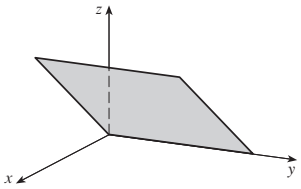
15. $x = 4 - 3t, y = -1 + 2t, z = 2 + 3t$

17. $x = -2 + 2t, y = 2 - t, z = 4 + 5t$

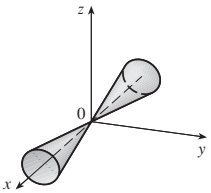
19. $-4x + 3y + z = -14$ **21.** $(1, 4, 4)$ **23.** Skew

25. $x + y + z = 4$ **27.** $22/\sqrt{26}$

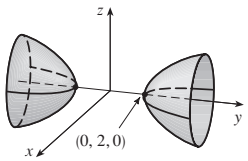
29. Plane



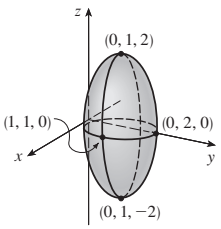
31. Cone



33. Hyperboloid of two sheets



35. Ellipsoid



37. $4x^2 + y^2 + z^2 = 16$

PROBLEMS PLUS ■ PAGE 884

1. $\left(\sqrt{3} - \frac{3}{2}\right)$ m

3. (a) $(x + 1)/(-2c) = (y - c)/(c^2 - 1) = (z - c)/(c^2 + 1)$
(b) $x^2 + y^2 = t^2 + 1, z = t$ (c) $4\pi/3$

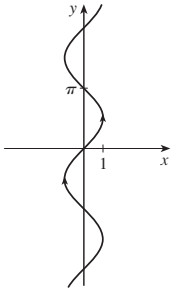
5. 20

CHAPTER 13

EXERCISES 13.1 ■ PAGE 893

1. $(-1, 3)$ **3.** $\mathbf{i} + \mathbf{j} + \mathbf{k}$ **5.** $\langle -1, \pi/2, 0 \rangle$

7.



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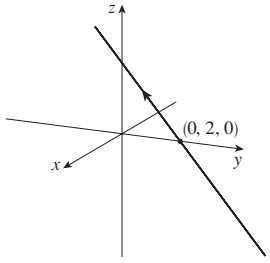

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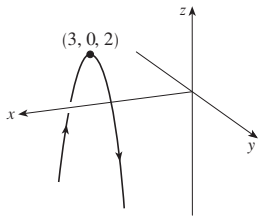

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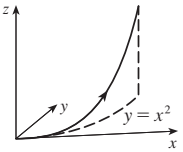

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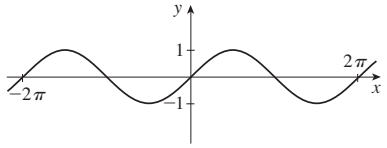
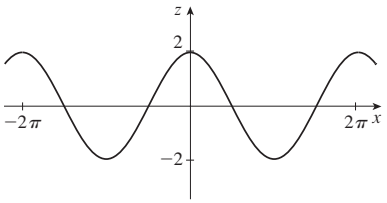
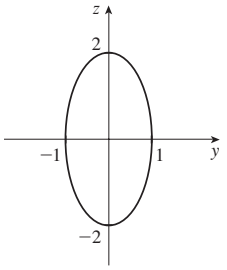
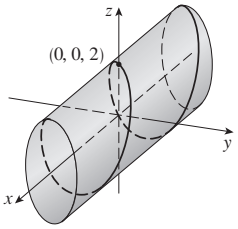






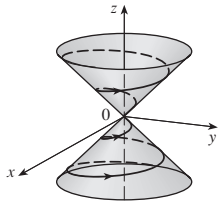


17. $\mathbf{r}(t) = \langle 2 + 4t, 2t, -2t \rangle$, $0 \leqslant t \leqslant 1$;
$x = 2 + 4t$, $y = 2t$, $z = -2t$, $0 \leqslant t \leqslant 1$

19. $\mathbf{r}(t) = \left\langle \frac{1}{2}t, -1 + \frac{4}{3}t, 1 - \frac{3}{4}t \right\rangle$, $0 \leqslant t \leqslant 1$;
$x = \frac{1}{2}t$, $y = -1 + \frac{4}{3}t$, $z = 1 - \frac{3}{4}t$, $0 \leqslant t \leqslant 1$

21. II **23.** V **25.** IV

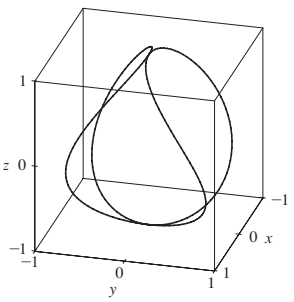
27.



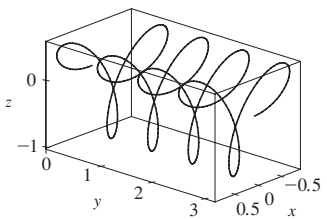
29. $y = e^{x/2}$, $z = e^{x}$, $z = y^2$

31. (0, 0, 0), (1, 0, 1)

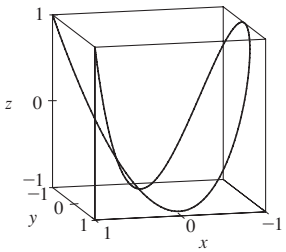
33.



35.



37.



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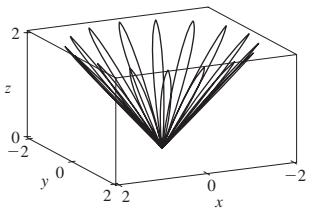
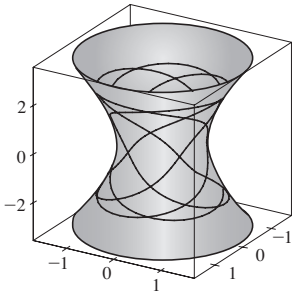


43. $\mathbf{r}(t) = t\,\mathbf{i} + \frac{1}{2}(t^2 - 1)\,\mathbf{j} + \frac{1}{2}(t^2 + 1)\,\mathbf{k}$

45. $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \cos 2t\,\mathbf{k}$, $0 \leqslant t \leqslant 2\pi$

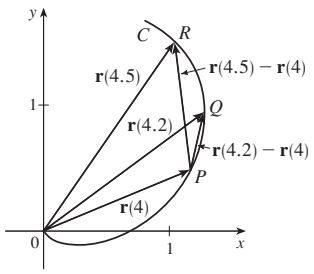
47. $x = 2\cos t$, $y = 2\sin t$, $z = 4\cos^2 t$, $0 \leqslant t \leqslant 2\pi$ **49.** Yes

51. (a)

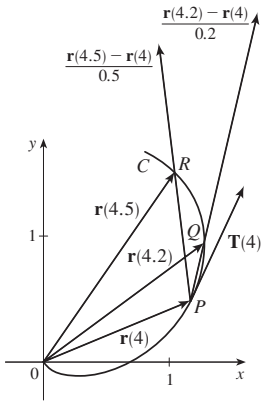


EXERCISES 13.2 ■ PAGE 900

1. (a)

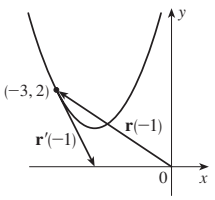


(b), (d)



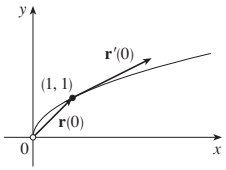
(c) $\mathbf{r}'(4) = \lim_{h \to 0} \dfrac{\mathbf{r}(4 + h) - \mathbf{r}(4)}{h}$; $\mathbf{T}(4) = \dfrac{\mathbf{r}'(4)}{|\mathbf{r}'(4)|}$

3. (a), (c)



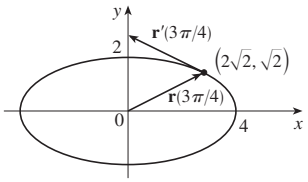
(b) $\mathbf{r}'(t) = \langle 1, 2t \rangle$

5. (a), (c)



(b) $\mathbf{r}'(t) = 2e^{2t}\,\mathbf{i} + e^{t}\,\mathbf{j}$

7. (a), (c)



(b) $\mathbf{r}'(t) = 4\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j}$

9. $\mathbf{r}'(t) = \left\langle \dfrac{1}{2\sqrt{t-2}}, 0, -\dfrac{2}{t^3} \right\rangle$

11. $\mathbf{r}'(t) = 2t\,\mathbf{i} - 2t\sin(t^2)\,\mathbf{j} + 2\sin t\cos t\,\mathbf{k}$

13. $\mathbf{r}'(t) = (t\cos t + \sin t)\,\mathbf{i} + e^{t}(\cos t - \sin t)\,\mathbf{j} + (\cos^2 t - \sin^2 t)\,\mathbf{k}$

15. $\mathbf{r}'(t) = \mathbf{b} + 2t\mathbf{c}$ **17.** $\left\langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \right\rangle$ **19.** $\frac{3}{5}\,\mathbf{j} + \frac{4}{5}\,\mathbf{k}$

21. $\langle 1, 2t, 3t^2 \rangle, \left\langle 1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14} \right\rangle, \langle 0, 2, 6t \rangle, \langle 6t^2, -6t, 2 \rangle$

23. $x = 2 + 2t, y = 4 + 2t, z = 1 + t$

25. $x = 1 - t, y = t, z = 1 - t$

27. $\mathbf{r}(t) = (3 - 4t)\,\mathbf{i} + (4 + 3t)\,\mathbf{j} + (2 - 6t)\,\mathbf{k}$

29. $x = t, y = 1 - t, z = 2t$

31. $x = -\pi - t, y = \pi + t, z = -\pi t$

33. 66° **35.** $2\,\mathbf{i} - 4\,\mathbf{j} + 32\,\mathbf{k}$

37. $(\ln 2)\,\mathbf{i} + (\pi/4)\,\mathbf{j} + \frac{1}{2}\ln 2\,\mathbf{k}$

39. $\tan t\,\mathbf{i} + \frac{1}{8}(t^2+1)^4\,\mathbf{j} + \left(\frac{1}{3}t^3\ln t - \frac{1}{9}t^3\right)\mathbf{k} + \mathbf{C}$

41. $t^2\,\mathbf{i} + t^3\,\mathbf{j} + \left(\frac{2}{3}t^{3/2} - \frac{2}{3}\right)\mathbf{k}$

47. $2t\cos t + 2\sin t - 2\cos t\sin t$ **49.** 35

EXERCISES 13.3 ■ PAGE 908

1. $10\sqrt{10}$ **3.** $e - e^{-1}$ **5.** $\frac{1}{27}(13^{3/2} - 8)$

7. 18.6833 **9.** 10.3311 **11.** 42

13. (a) $s(t) = \sqrt{26}\,(t-1)$;

$$\mathbf{r}(t(s)) = \left(4 - \frac{s}{\sqrt{26}}\right)\mathbf{i} + \left(\frac{4s}{\sqrt{26}} + 1\right)\mathbf{j} + \left(\frac{3s}{\sqrt{26}} + 3\right)\mathbf{k}$$

(b) $\left(4 - \dfrac{4}{\sqrt{26}}, \dfrac{16}{\sqrt{26}} + 1, \dfrac{12}{\sqrt{26}} + 3\right)$

15. $(3\sin 1, 4, 3\cos 1)$

17. (a) $\left\langle 1/\sqrt{10}, \left(-3/\sqrt{10}\right)\sin t, \left(3/\sqrt{10}\right)\cos t \right\rangle$, $\langle 0, -\cos t, -\sin t \rangle$ (b) $\frac{3}{10}$

19. (a) $\dfrac{1}{e^{2t}+1}\left\langle \sqrt{2}e^{t}, e^{2t}, -1 \right\rangle, \dfrac{1}{e^{2t}+1}\left\langle 1 - e^{2t}, \sqrt{2}e^{t}, \sqrt{2}e^{t} \right\rangle$

(b) $\sqrt{2}e^{2t}/(e^{2t}+1)^2$

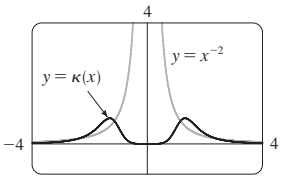
21. $6t^2/(9t^4 + 4t^2)^{3/2}$ **23.** $\dfrac{\sqrt{6}}{2(3t^2+1)^2}$

25. $\frac{1}{7}\sqrt{\frac{19}{14}}$ **27.** $12x^2/(1 + 16x^6)^{3/2}$

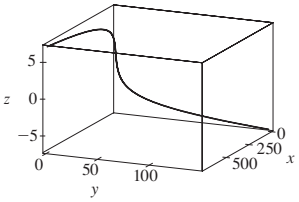
29. $e^{x}|x + 2|/[1 + (xe^{x} + e^{x})^2]^{3/2}$

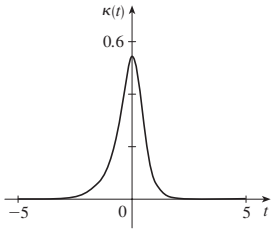
31. $\left(-\frac{1}{2}\ln 2, 1/\sqrt{2}\right)$; approaches 0 **33.** (a) P (b) 1.3, 0.7

35.



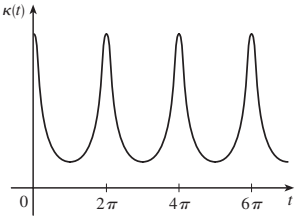
37.





39. a is $y = f(x)$, b is $y = \kappa(x)$

41. $\kappa(t) = \dfrac{6\sqrt{4\cos^2 t - 12\cos t + 13}}{(17 - 12\cos t)^{3/2}}$



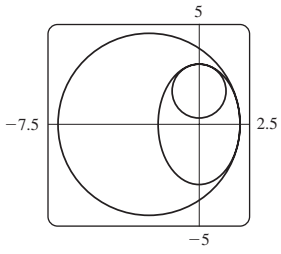
integer multiples of 2π

43. $6t^2/(4t^2 + 9t^4)^{3/2}$

45. $1/(\sqrt{2}e^{t})$ **47.** $\left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle, \left\langle -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \right\rangle, \left\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle$

49. $x - 2z = -4\pi, 2x + z = 2\pi$

51. $\left(x + \frac{5}{2}\right)^2 + y^2 = \frac{81}{4}, x^2 + \left(y - \frac{5}{3}\right)^2 = \frac{16}{9}$



53. $(-1, -3, 1)$

55. $2x + y + 4z = 7, 6x - 8y - z = -3$

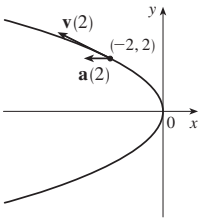
65. $2/(t^4 + 4t^2 + 1)$ **67.** 2.07×10^{10} Å ≈ 2 m

EXERCISES 13.4 ■ PAGE 918

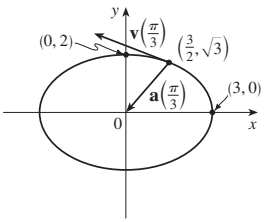
1. (a) $1.8\,\mathbf{i} - 3.8\,\mathbf{j} - 0.7\,\mathbf{k}, 2.0\,\mathbf{i} - 2.4\,\mathbf{j} - 0.6\,\mathbf{k}, 2.8\,\mathbf{i} + 1.8\,\mathbf{j} - 0.3\,\mathbf{k}, 2.8\,\mathbf{i} + 0.8\,\mathbf{j} - 0.4\,\mathbf{k}$

(b) $2.4\,\mathbf{i} - 0.8\,\mathbf{j} - 0.5\,\mathbf{k}$, 2.58

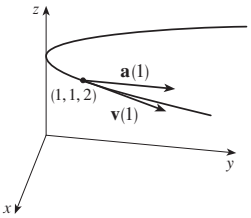
3. $\mathbf{v}(t) = \langle -t, 1 \rangle$
$\mathbf{a}(t) = \langle -1, 0 \rangle$
$|\mathbf{v}(t)| = \sqrt{t^2 + 1}$



5. $\mathbf{v}(t) = -3 \sin t\,\mathbf{i} + 2 \cos t\,\mathbf{j}$
$\mathbf{a}(t) = -3 \cos t\,\mathbf{i} - 2 \sin t\,\mathbf{j}$
$|\mathbf{v}(t)| = \sqrt{5 \sin^2 t + 4}$



7. $\mathbf{v}(t) = \mathbf{i} + 2t\,\mathbf{j}$
$\mathbf{a}(t) = 2\,\mathbf{j}$
$|\mathbf{v}(t)| = \sqrt{1 + 4t^2}$



9. $\langle 2t + 1, 2t - 1, 3t^2 \rangle$, $\langle 2, 2, 6t \rangle$, $\sqrt{9t^4 + 8t^2 + 2}$

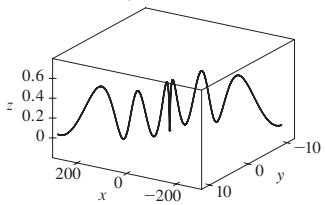
11. $\sqrt{2}\,\mathbf{i} + e^t\,\mathbf{j} - e^{-t}\,\mathbf{k}$, $e^t\,\mathbf{j} + e^{-t}\,\mathbf{k}$, $e^t + e^{-t}$

13. $e^t[(\cos t - \sin t)\mathbf{i} + (\sin t + \cos t)\mathbf{j} + (t + 1)\mathbf{k}]$,
$e^t[-2 \sin t\,\mathbf{i} + 2 \cos t\,\mathbf{j} + (t + 2)\mathbf{k}]$, $e^t\sqrt{t^2 + 2t + 3}$

15. $\mathbf{v}(t) = (2t + 3)\,\mathbf{i} - \mathbf{j} + t^2\,\mathbf{k}$,
$\mathbf{r}(t) = (t^2 + 3t)\,\mathbf{i} + (1 - t)\,\mathbf{j} + \left(\frac{1}{3}t^3 + 1\right)\mathbf{k}$

17. (a) $\mathbf{r}(t) = \left(\frac{1}{3}t^3 + t\right)\mathbf{i} + (t - \sin t + 1)\,\mathbf{j} + \left(\frac{1}{4} - \frac{1}{4}\cos 2t\right)\mathbf{k}$

(b)



19. $t = 4$

21. $\mathbf{r}(t) = t\,\mathbf{i} - t\,\mathbf{j} + \frac{5}{2}t^2\,\mathbf{k}$, $|\mathbf{v}(t)| = \sqrt{25t^2 + 2}$

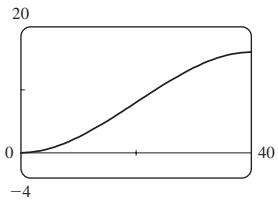
23. (a) ≈3535 m (b) ≈1531 m (c) 200 m/s

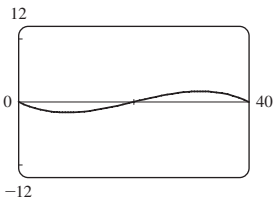
25. ≈30 m/s **27.** ≈544 ft/s

29. $13.0° < \theta < 36.0°$, $55.4° < \theta < 85.5°$

31. $(250, -50, 0)$; $10\sqrt{93} \approx 96.4$ ft/s

33. (a) 16 m (b) ≈23.6° upstream





35. The path is contained in a circle that lies in a plane perpendicular to $\mathbf{c}$ with center on a line through the origin in the direction of $\mathbf{c}$.

37. $\frac{4 + 18t^2}{\sqrt{4 + 9t^2}}$, $\frac{6t}{\sqrt{4 + 9t^2}}$ **39.** 0, 1 **41.** $\frac{7}{\sqrt{30}}$, $\sqrt{\frac{131}{30}}$

43. 4.5 cm/s², 9.0 cm/s² **45.** $t = 1$

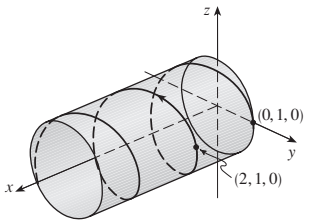
CHAPTER 13 REVIEW ■ PAGE 921

True-False Quiz

1. True **3.** False **5.** False **7.** False
9. True **11.** False **13.** True

Exercises

1. (a)



(b) $\mathbf{r}'(t) = \mathbf{i} - \pi \sin \pi t\,\mathbf{j} + \pi \cos \pi t\,\mathbf{k}$,
$\mathbf{r}''(t) = -\pi^2 \cos \pi t\,\mathbf{j} - \pi^2 \sin \pi t\,\mathbf{k}$

3. $\mathbf{r}(t) = 4 \cos t\,\mathbf{i} + 4 \sin t\,\mathbf{j} + (5 - 4 \cos t)\mathbf{k}$, $0 \leqslant t \leqslant 2\pi$

5. $\frac{1}{3}\,\mathbf{i} - (2/\pi^2)\mathbf{j} + (2/\pi)\mathbf{k}$ **7.** 86.631 **9.** 90°

11. (a) $\frac{1}{\sqrt{13}}\langle 3 \sin t, -3 \cos t, 2 \rangle$ (b) $\langle \cos t, \sin t, 0 \rangle$

(c) $\frac{1}{\sqrt{13}}\langle -2 \sin t, 2 \cos t, 3 \rangle$

(d) $\frac{3}{13 \sin t \cos t}$ or $\frac{3}{13}\sec t \csc t$

13. $12/17^{3/2}$ **15.** $x - 2y + 2\pi = 0$

17. $\mathbf{v}(t) = (1 + \ln t)\,\mathbf{i} + \mathbf{j} - e^{-t}\,\mathbf{k}$,
$|\mathbf{v}(t)| = \sqrt{2 + 2 \ln t + (\ln t)^2 + e^{-2t}}$, $\mathbf{a}(t) = (1/t)\,\mathbf{i} + e^{-t}\,\mathbf{k}$

19. $\mathbf{r}(t) = (t^3 + t)\,\mathbf{i} + (t^4 - t)\,\mathbf{j} + \left(3t - t^3\right)\mathbf{k}$

21. ≈37.3°, ≈157.4 m

23. (c) $-2e^{-t}\,\mathbf{v}_d + e^{-t}\,\mathbf{R}$

PROBLEMS PLUS ■ PAGE 924

1. (a) $\mathbf{v} = \omega R(-\sin \omega t\,\mathbf{i} + \cos \omega t\,\mathbf{j})$ (c) $\mathbf{a} = -\omega^2\mathbf{r}$

3. (a) 90°, $v_0^2/(2g)$

5. (a) ≈0.94 ft to the right of the table's edge, ≈15 ft/s
(b) ≈7.6° (c) ≈2.13 ft to the right of the table's edge

7. 56°

9. $(a_2b_3 - a_3b_2)(x - c_1) + (a_3b_1 - a_1b_3)(y - c_2)$
$+ (a_1b_2 - a_2b_1)(z - c_3) = 0$

CHAPTER 14

EXERCISES 14.1 ■ PAGE 939

1. (a) −27; a temperature of −15°C with wind blowing at 40 km/h feels equivalent to about −27°C without wind.
(b) When the temperature is −20°C, what wind speed gives a wind chill of −30°C? 20 km/h
(c) With a wind speed of 20 km/h, what temperature gives a wind chill of −49°C? −35°C

(d) A function of wind speed that gives wind-chill values when the temperature is $-5°C$
(e) A function of temperature that gives wind-chill values when the wind speed is 50 km/h

3. ≈ 94.2; the manufacturer's yearly production is valued at \$94.2 million when 120,000 labor hours are spent and \$20 million in capital is invested.

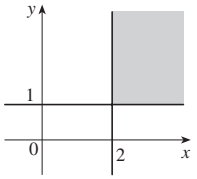
5. (a) ≈ 20.5; the surface area of a person 70 inches tall who weighs 160 pounds is approximately 20.5 square feet.

7. (a) 25; a 40-knot wind blowing in the open sea for 15 h will create waves about 25 ft high.
(b) $f(30, t)$ is a function of t giving the wave heights produced by 30-knot winds blowing for t hours.
(c) $f(v, 30)$ is a function of v giving the wave heights produced by winds of speed v blowing for 30 hours.

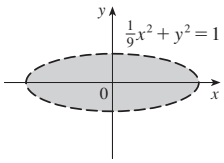
9. (a) 1 (b) $\mathbb{R}^2$ (c) $[-1, 1]$

11. (a) 3
(b) $\{(x, y, z) \mid x^2 + y^2 + z^2 < 4, x \geqslant 0, y \geqslant 0, z \geqslant 0\}$, interior of a sphere of radius 2, center the origin, in the first octant

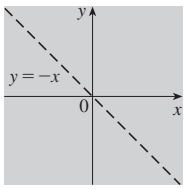
13. $\{(x, y) \mid x \geqslant 2, y \geqslant 1\}$



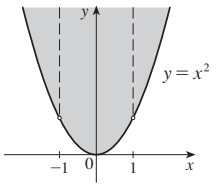
15. $\left\{(x, y) \mid \frac{1}{9}x^2 + y^2 < 1\right\}$



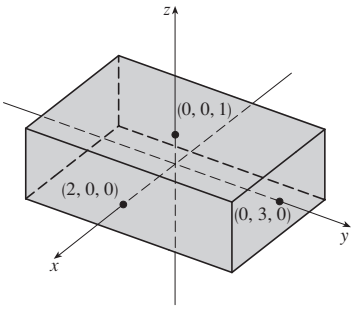
17. $\{(x, y) \mid y \neq -x\}$



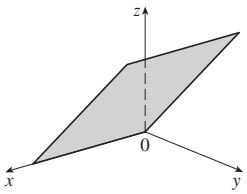
19. $\{(x, y) \mid y \geqslant x^2, x \neq \pm 1\}$



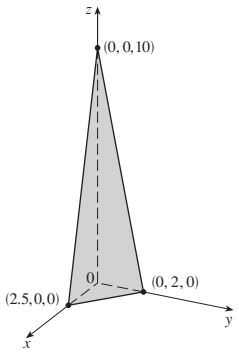
21. $\{(x, y, z) \mid -2 \leqslant x \leqslant 2, -3 \leqslant y \leqslant 3, -1 \leqslant z \leqslant 1\}$



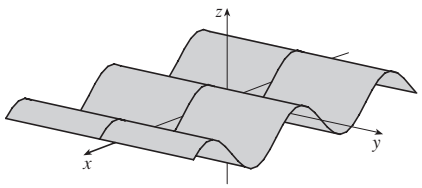
23. $z = y$, plane through the x-axis



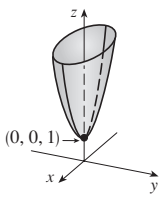
25. $4x + 5y + z = 10$, plane



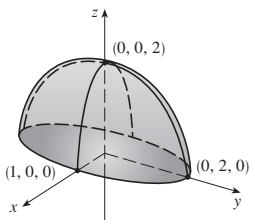
27. $z = \sin x$, cylinder



29. $z = x^2 + 4y^2 + 1$, elliptic paraboloid

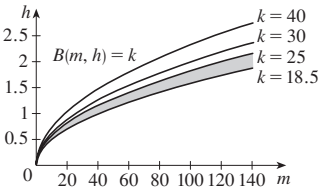


31. $z = \sqrt{4 - 4x^2 - y^2}$, top half of ellipsoid



33. ≈ 56, ≈ 35 **35.** 11°C, 19.5°C **37.** Steep; nearly flat No

39.



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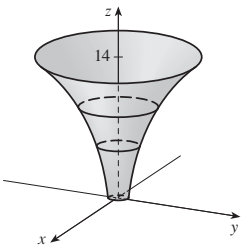

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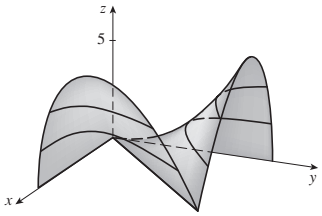
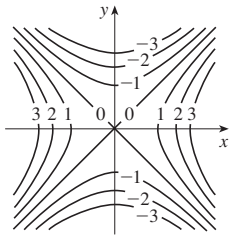
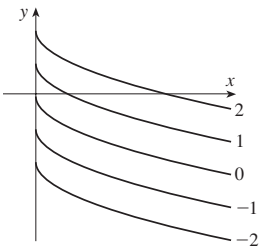


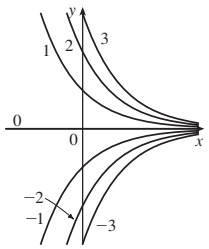
45. $x^2 - y^2 = k$



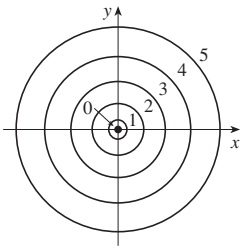
47. $y = -\sqrt{x} + k$



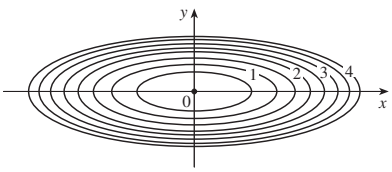
49. $y = ke^{-x}$

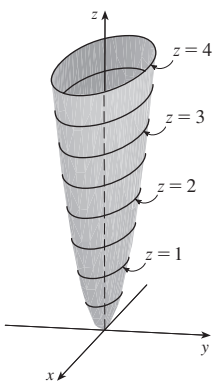


51. $x^2 + y^2 = k^3 \ (k \geqslant 0)$



53. $x^2 + 9y^2 = k$





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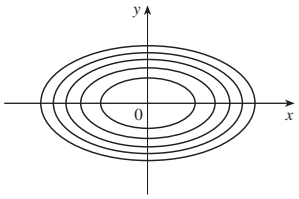

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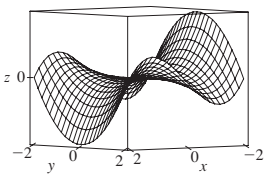

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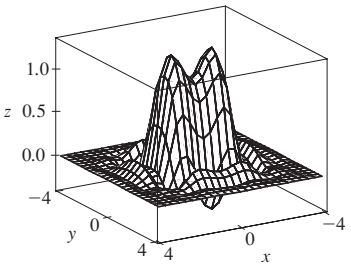
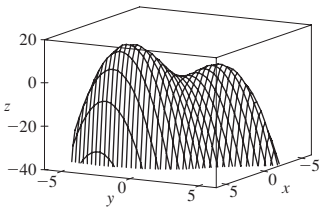


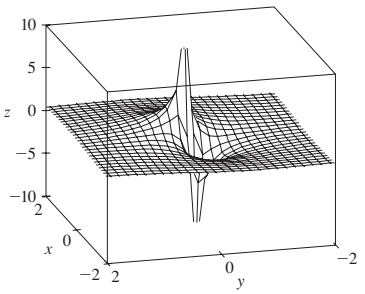
61. (a) C (b) II **63.** (a) F (b) I
65. (a) B (b) VI **67.** Family of parallel planes
69. Family of circular cylinders with axis the x-axis ($k > 0$)
71. (a) Shift the graph of f upward 2 units
(b) Stretch the graph of f vertically by a factor of 2
(c) Reflect the graph of f about the xy-plane
(d) Reflect the graph of f about the xy-plane and then shift it upward 2 units

73.



f appears to have a maximum value of about 15. There are two local maximum points but no local minimum point.

75.

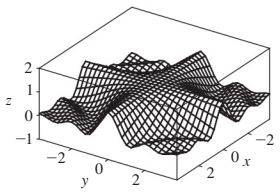


The function values approach 0 as x, y become large; as (x, y) approaches the origin, f approaches $\pm\infty$ or 0, depending on the direction of approach.

77. If $c = 0$, the graph is a cylindrical surface. For $c > 0$, the level curves are ellipses. The graph curves upward as we leave the origin, and the steepness increases as c increases. For $c < 0$, the level curves are hyperbolas. The graph curves upward in the y-direction and downward, approaching the xy-plane, in the x-direction giving a saddle-shaped appearance near $(0, 0, 1)$.
79. $c = -2, 0, 2$ **81.** (b) $y = 0.75x + 0.01$

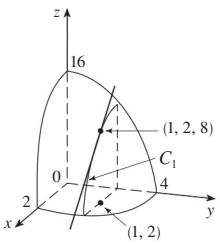
EXERCISES 14.2 ■ PAGE 950

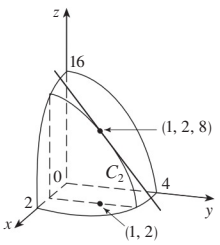
1. Nothing; if f is continuous, $f(3, 1) = 6$ **3.** $-\frac{5}{2}$
5. 56 **7.** $\pi/2$ **9.** Does not exist **11.** Does not exist
13. 0 **15.** Does not exist **17.** 2 **19.** $\sqrt{3}$
21. Does not exist
23. The graph shows that the function approaches different numbers along different lines.
25. $h(x, y) = (2x + 3y - 6)^2 + \sqrt{2x + 3y - 6}$; $\{(x, y) \mid 2x + 3y \geqslant 6\}$
27. Along the line $y = x$ **29.** $\mathbb{R}^2$
31. $\{(x, y) \mid x^2 + y^2 \neq 1\}$ **33.** $\{(x, y) \mid x^2 + y^2 \leqslant 1, x \geqslant 0\}$
35. $\{(x, y, z) \mid x^2 + y^2 + z^2 \leqslant 1\}$
37. $\{(x, y) \mid (x, y) \neq (0, 0)\}$ **39.** 0 **41.** -1
43. f is continuous on $\mathbb{R}^2$



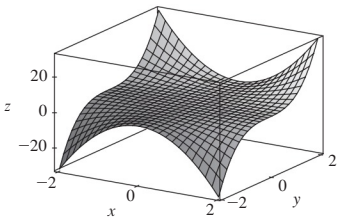
EXERCISES 14.3 ■ PAGE 963

1. (a) The rate of change of temperature as longitude varies, with latitude and time fixed; the rate of change as only latitude varies; the rate of change as only time varies
(b) Positive, negative, positive
3. (a) $f_T(-15, 30) \approx 1.3$; for a temperature of -15°C and wind speed of 30 km/h, the wind-chill index rises by 1.3°C for each degree the temperature increases. $f_v(-15, 30) \approx -0.15$; for a temperature of -15°C and wind speed of 30 km/h, the wind-chill index decreases by 0.15°C for each km/h the wind speed increases.
(b) Positive, negative (c) 0
5. (a) Positive (b) Negative
7. (a) Positive (b) Negative
9. $c = f$, $b = f_x$, $a = f_y$
11. $f_x(1, 2) = -8 =$ slope of C_1, $f_y(1, 2) = -4 =$ slope of C_2

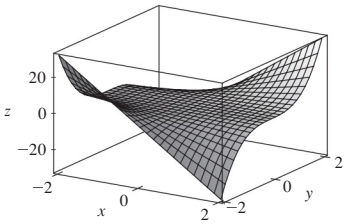




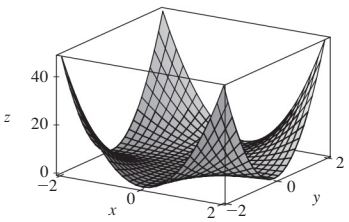
13.



$f(x, y) = x^2y^3$



$f_x(x, y) = 2xy^3$



$f_y(x, y) = 3x^2y^2$

15. $f_x(x, y) = 4x^3 + 5y^3$, $f_y(x, y) = 15xy^2$
17. $f_x(x, t) = -t^2e^{-x}$, $f_t(x, t) = 2te^{-x}$
19. $\dfrac{\partial z}{\partial x} = \dfrac{1}{x + t^2}$, $\dfrac{\partial z}{\partial t} = \dfrac{2t}{x + t^2}$
21. $f_x(x, y) = 1/y$, $f_y(x, y) = -x/y^2$
23. $f_x(x, y) = \dfrac{(ad - bc)y}{(cx + dy)^2}$, $f_y(x, y) = \dfrac{(bc - ad)x}{(cx + dy)^2}$
25. $g_u(u, v) = 10uv(u^2v - v^3)^4$, $g_v(u, v) = 5(u^2 - 3v^2)(u^2v - v^3)^4$
27. $R_p(p, q) = \dfrac{q^2}{1 + p^2q^4}$, $R_q(p, q) = \dfrac{2pq}{1 + p^2q^4}$
29. $F_x(x, y) = \cos(e^x)$, $F_y(x, y) = -\cos(e^y)$
31. $f_x = 3x^2yz^2$, $f_y = x^3z^2 + 2z$, $f_z = 2x^3yz + 2y$
33. $\partial w/\partial x = 1/(x + 2y + 3z)$, $\partial w/\partial y = 2/(x + 2y + 3z)$, $\partial w/\partial z = 3/(x + 2y + 3z)$
35. $\partial p/\partial t = 2t^3/\sqrt{t^4 + u^2\cos v}$, $\partial p/\partial u = u\cos v/\sqrt{t^4 + u^2\cos v}$, $\partial p/\partial v = -u^2\sin v/\left(2\sqrt{t^4 + u^2\cos v}\right)$
37. $h_x = 2xy\cos(z/t)$, $h_y = x^2\cos(z/t)$, $h_z = (-x^2y/t)\sin(z/t)$, $h_t = (x^2yz/t^2)\sin(z/t)$
39. $\partial u/\partial x_i = x_i/\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$
41. 1 **43.** $\frac{1}{6}$ **45.** $f_x(x, y) = y^2 - 3x^2y$, $f_y(x, y) = 2xy - x^3$
47. $\dfrac{\partial z}{\partial x} = -\dfrac{x}{3z}$, $\dfrac{\partial z}{\partial y} = -\dfrac{2y}{3z}$
49. $\dfrac{\partial z}{\partial x} = \dfrac{yz}{e^z - xy}$, $\dfrac{\partial z}{\partial y} = \dfrac{xz}{e^z - xy}$

51. (a) $f'(x), g'(y)$ (b) $f'(x+y),\ f'(x+y)$

53. $f_{xx} = 12x^2y - 12xy^2,\ f_{xy} = 4x^3 - 12x^2y = f_{yx},\ f_{yy} = -4x^3$

55. $z_{xx} = \dfrac{8y}{(2x+3y)^3},\ z_{xy} = \dfrac{6y-4x}{(2x+3y)^3} = z_{yx},$

$z_{yy} = -\dfrac{12x}{(2x+3y)^3}$

57. $v_{ss} = 2\cos(s^2-t^2) - 4s^2\sin(s^2-t^2),$
$v_{st} = 4st\sin(s^2-t^2) = v_{ts},$
$v_{tt} = -2\cos(s^2-t^2) - 4t^2\sin(s^2-t^2)$

63. $24xy^2 - 6y,\ 24x^2y - 6x$ **65.** $(2x^2y^2z^5 + 6xyz^3 + 2z)e^{xyz^2}$

67. $\frac{3}{4}v(u+v^2)^{-5/2}$ **69.** $4/(y+2z)^3, 0$ **71.** $6yz^2$

73. $\approx 12.2, \approx 16.8, \approx 23.25$ **83.** R^2/R_1^2

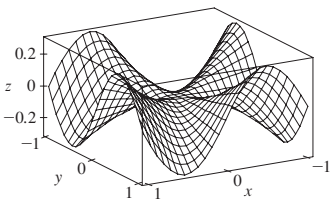
87. $\dfrac{\partial T}{\partial P} = \dfrac{V-nb}{nR},\ \dfrac{\partial P}{\partial V} = \dfrac{2n^2a}{V^3} - \dfrac{nRT}{(V-nb)^2}$

91. (a) ≈ 0.0545; for a person 70 inches tall who weighs 160 pounds, an increase in weight causes the surface area to increase at a rate of about 0.0545 square feet per pound.
(b) ≈ 0.213; for a person 70 inches tall who weighs 160 pounds, an increase in height (with no change in weight) causes the surface area to increase at a rate of about 0.213 square feet per inch of height.

93. $\partial P/\partial v = 3Av^2 - \dfrac{B(mg/x)^2}{v^2}$ is the rate of change of the power needed during flapping mode with respect to the bird's velocity when the mass and fraction of flapping time remain constant; $\partial P/\partial x = -\dfrac{2Bm^2g^2}{x^3v}$ is the rate at which the power changes when only the fraction of time spent in flapping mode varies; $\partial P/\partial m = \dfrac{2Bmg^2}{x^2v}$ is the rate of change of the power when only the mass varies.

97. No **99.** $x = 1+t, y = 2, z = 2-2t$ **103.** -2

105. (a)



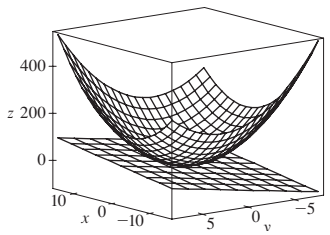
(b) $f_x(x,y) = \dfrac{x^4y + 4x^2y^3 - y^5}{(x^2+y^2)^2},\ f_y(x,y) = \dfrac{x^5 - 4x^3y^2 - xy^4}{(x^2+y^2)^2}$

(c) 0, 0 (e) No, since f_{xy} and f_{yx} are not continuous.

EXERCISES 14.4 ■ PAGE 974

1. $z = 4x - y - 6$ **3.** $z = x - y + 1$ **5.** $x + y + z = 0$

7.



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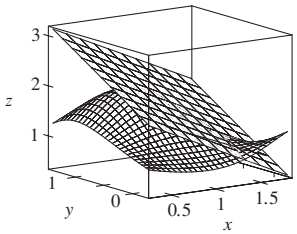


11. $6x + 4y - 23$ **13.** $2x + y - 1$

15. $2x + 2y + \pi - 4$ **19.** 6.3

21. $\frac{3}{7}x + \frac{2}{7}y + \frac{6}{7}z$; 6.9914 **23.** $4T + H - 329$; 129°F

25. $dz = -2e^{-2x}\cos 2\pi t\,dx - 2\pi e^{-2x}\sin 2\pi t\,dt$

27. $dm = 5p^4q^3\,dp + 3p^5q^2\,dq$

29. $dR = \beta^2\cos\gamma\,d\alpha + 2\alpha\beta\cos\gamma\,d\beta - \alpha\beta^2\sin\gamma\,d\gamma$

31. $\Delta z = 0.9225, dz = 0.9$ **33.** 5.4 cm^2 **35.** 16 cm^3

37. $\approx -0.0165mg$; decrease **39.** $\frac{1}{17} \approx 0.059\ \Omega$

41. (a) $0.8264m - 34.56h + 38.02$ (b) 18.801

43. $\varepsilon_1 = \Delta x,\ \varepsilon_2 = \Delta y$

EXERCISES 14.5 ■ PAGE 983

1. $2t(y^3 - 2xy + 3xy^2 - x^2)$

3. $\dfrac{1}{2\sqrt{t}}\cos x\cos y + \dfrac{1}{t^2}\sin x\sin y$

5. $e^{y/z}[2t - (x/z) - (2xy/z^2)]$

7. $\partial z/\partial s = 5(x-y)^4(2st - t^2),\ \partial z/\partial t = 5(x-y)^4(s^2 - 2st)$

9. $\dfrac{\partial z}{\partial s} = \dfrac{3\sin t - 2t\sin s}{3x+2y},\ \dfrac{\partial z}{\partial t} = \dfrac{3s\cos t + 2\cos s}{3x+2y}$

11. $\dfrac{\partial z}{\partial s} = e^r\left(t\cos\theta - \dfrac{s}{\sqrt{s^2+t^2}}\sin\theta\right),$

$\dfrac{\partial z}{\partial t} = e^r\left(s\cos\theta - \dfrac{t}{\sqrt{s^2+t^2}}\sin\theta\right)$

13. 42 **15.** 7, 2

17. $\dfrac{\partial u}{\partial r} = \dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial r} + \dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial r},\ \dfrac{\partial u}{\partial s} = \dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial s},$

$\dfrac{\partial u}{\partial t} = \dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial t}$

19. $\dfrac{\partial T}{\partial x} = \dfrac{\partial T}{\partial p}\dfrac{\partial p}{\partial x} + \dfrac{\partial T}{\partial q}\dfrac{\partial q}{\partial x} + \dfrac{\partial T}{\partial r}\dfrac{\partial r}{\partial x},$

$\dfrac{\partial T}{\partial y} = \dfrac{\partial T}{\partial p}\dfrac{\partial p}{\partial y} + \dfrac{\partial T}{\partial q}\dfrac{\partial q}{\partial y} + \dfrac{\partial T}{\partial r}\dfrac{\partial r}{\partial y},$

$\dfrac{\partial T}{\partial z} = \dfrac{\partial T}{\partial p}\dfrac{\partial p}{\partial z} + \dfrac{\partial T}{\partial q}\dfrac{\partial q}{\partial z} + \dfrac{\partial T}{\partial r}\dfrac{\partial r}{\partial z}$

21. 1582, 3164, -700 **23.** $2\pi, -2\pi$

25. $\frac{5}{144}, -\frac{5}{96}, \frac{5}{144}$ **27.** $\dfrac{2x + y\sin x}{\cos x - 2y}$

29. $\dfrac{1 + x^4y^2 + y^2 + x^4y^4 - 2xy}{x^2 - 2xy - 2x^5y^3}$

31. $-\dfrac{x}{3z}, -\dfrac{2y}{3z}$ **33.** $\dfrac{yz}{e^z - xy}, \dfrac{xz}{e^z - xy}$

35. 2°C/s **37.** ≈ -0.33 m/s per minute

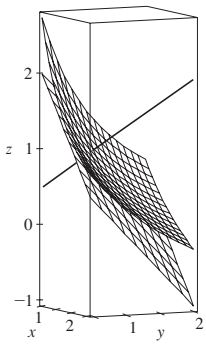
39. (a) 6 m^3/s (b) 10 m^2/s (c) 0 m/s

41. ≈ -0.27 L/s **43.** $-1/(12\sqrt{3})$ rad/s

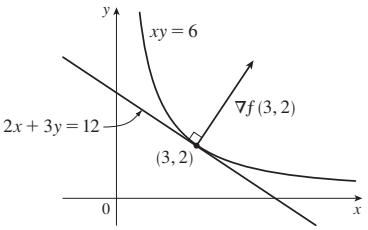
45. (a) $\partial z/\partial r = (\partial z/\partial x)\cos\theta + (\partial z/\partial y)\sin\theta$,
$\partial z/\partial\theta = -(\partial z/\partial x)\, r\sin\theta + (\partial z/\partial y)\, r\cos\theta$
51. $4rs\,\partial^2 z/\partial x^2 + (4r^2 + 4s^2)\partial^2 z/\partial x\,\partial y + 4rs\,\partial^2 z/\partial y^2 + 2\,\partial z/\partial y$

EXERCISES 14.6 ■ PAGE 996

1. ≈ -0.08 mb/km **3.** ≈ 0.778 **5.** $\sqrt{2}/2$
7. (a) $\nabla f(x, y) = (1/y)\mathbf{i} - (x/y^2)\mathbf{j}$ (b) $\mathbf{i} - 2\mathbf{j}$ (c) -1
9. (a) $\langle 2xyz - yz^3, x^2z - xz^3, x^2y - 3xyz^2\rangle$
(b) $\langle -3, 2, 2\rangle$ (c) $\frac{2}{5}$
11. $\dfrac{4 - 3\sqrt{3}}{10}$ **13.** $7/(2\sqrt{5})$ **15.** 1 **17.** $\frac{23}{42}$
19. $\frac{2}{5}$ **21.** $\sqrt{65}$, $\langle 1, 8\rangle$ **23.** 1, $\langle 0, 1\rangle$
25. $\frac{3}{4}$, $\langle 1, -2, -2\rangle$ **27.** (b) $\langle -12, 92\rangle$
29. All points on the line $y = x + 1$ **31.** (a) $-40/(3\sqrt{3})$
33. (a) $32/\sqrt{3}$ (b) $\langle 38, 6, 12\rangle$ (c) $2\sqrt{406}$
35. $\frac{327}{13}$ **39.** $\frac{774}{25}$
41. (a) $x + y + z = 11$ (b) $x - 3 = y - 3 = z - 5$
43. (a) $x + 2y + 6z = 12$ (b) $x - 2 = \dfrac{y - 2}{2} = \dfrac{z - 1}{6}$
45. (a) $x + y + z = 1$ (b) $x = y = z - 1$
47.



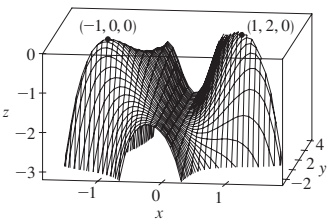
49. $\langle 2, 3\rangle$, $2x + 3y = 12$



55. No **59.** $\left(-\frac{5}{4}, -\frac{5}{4}, \frac{25}{8}\right)$
63. $x = -1 - 10t$, $y = 1 - 16t$, $z = 2 - 12t$
65. $(-1, 0, 1)$; $\approx 7.8°$
69. If $\mathbf{u} = \langle a, b\rangle$ and $\mathbf{v} = \langle c, d\rangle$, then $af_x + bf_y$ and $cf_x + df_y$ are known, so we solve linear equations for f_x and f_y.

EXERCISES 14.7 ■ PAGE 1007

1. (a) f has a local minimum at (1, 1).
(b) f has a saddle point at (1, 1).
3. Local minimum at (1, 1), saddle point at (0, 0)
5. Minimum $f\left(\frac{1}{3}, -\frac{2}{3}\right) = -\frac{1}{3}$
7. Saddle points at (1, 1), $(-1, -1)$
9. Minima $f\left(\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}\right) = f\left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right) = -\dfrac{1}{4}$,
saddle point at (0, 0)
11. Maximum $f(-1, 0) = 2$, minimum $f(1, 0) = -2$,
saddle points at $(0, \pm 1)$
13. Maximum $f(0, -1) = 2$, minima $f(\pm 1, 1) = -3$,
saddle points at (0, 1), $(\pm 1, -1)$
15. None
17. Minima $f(x, y) = 1$ at all points (x, y) on x- and y-axes
19. Minima $f(0, 1) = f(\pi, -1) = f(2\pi, 1) = -1$,
saddle points at $(\pi/2, 0)$, $(3\pi/2, 0)$
23. Minima $f(1, \pm 1) = f(-1, \pm 1) = 3$
25. Maximum $f(\pi/3, \pi/3) = 3\sqrt{3}/2$,
minimum $f(5\pi/3, 5\pi/3) = -3\sqrt{3}/2$, saddle point at (π, π)
27. Minima $f(0, -0.794) \approx -1.191$, $f(\pm 1.592, 1.267) \approx -1.310$,
saddle points $(\pm 0.720, 0.259)$,
lowest points $(\pm 1.592, 1.267, -1.310)$
29. Maximum $f(0.170, -1.215) \approx 3.197$,
minima $f(-1.301, 0.549) \approx -3.145$, $f(1.131, 0.549) \approx -0.701$,
saddle points $(-1.301, -1.215)$, $(0.170, 0.549)$, $(1.131, -1.215)$,
no highest or lowest point
31. Maximum $f(0, \pm 2) = 4$, minimum $f(1, 0) = -1$
33. Maximum $f(\pm 1, 1) = 7$, minimum $f(0, 0) = 4$
35. Maximum $f(0, 3) = f(2, 3) = 7$, minimum $f(1, 1) = -2$
37. Maximum $f(1, 0) = 2$, minimum $f(-1, 0) = -2$
39.



41. $2/\sqrt{3}$ **43.** $(2, 1, \sqrt{5}), (2, 1, -\sqrt{5})$ **45.** $\frac{100}{3}, \frac{100}{3}, \frac{100}{3}$
47. $8r^3/(3\sqrt{3})$ **49.** $\frac{4}{3}$ **51.** Cube, edge length $c/12$
53. Square base of side 40 cm, height 20 cm **55.** $L^3/(3\sqrt{3})$
57. (a) $H = -p_1 \ln p_1 - p_2 \ln p_2 - (1 - p_1 - p_2)\ln(1 - p_1 - p_2)$
(b) $\{(p_1, p_2) \mid 0 < p_1 < 1, p_2 < 1 - p_1\}$
(c) $\ln 3$; $p_1 = p_2 = p_3 = \frac{1}{3}$

EXERCISES 14.8 ■ PAGE 1017

1. $\approx 59, 30$
3. Maximum $f(\pm 1, 0) = 1$, minimum $f(0, \pm 1) = -1$
5. Maximum $f(1, 2) = f(-1, -2) = 2$,
minimum $f(1, -2) = f(-1, 2) = -2$
7. Maximum $f(2, 2, 1) = 9$, minimum $f(-2, -2, -1) = -9$
9. Maximum $f(1, \pm\sqrt{2}, 1) = f(-1, \pm\sqrt{2}, -1) = 2$,
minimum $f(1, \pm\sqrt{2}, -1) = f(-1, \pm\sqrt{2}, 1) = -2$
11. Maximum $\sqrt{3}$, minimum 1
13. Maximum $f\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) = 2$,
minimum $f\left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right) = -2$
15. Minimum $f(1, 1) = f(-1, -1) = 2$
17. Maximum $f(0, 1, \sqrt{2}) = 1 + \sqrt{2}$,
minimum $f(0, 1, -\sqrt{2}) = 1 - \sqrt{2}$
19. Maximum $\frac{3}{2}$, minimum $\frac{1}{2}$
21. Maximum $f(3/\sqrt{2}, -3/\sqrt{2}) = 9 + 12\sqrt{2}$,
minimum $f(-2, 2) = -8$
23. Maximum $f(\pm 1/\sqrt{2}, \mp 1/(2\sqrt{2})) = e^{1/4}$,
minimum $f(\pm 1/\sqrt{2}, \pm 1/(2\sqrt{2})) = e^{-1/4}$
31–43. See Exercises 41–55 in Section 14.7.
45. Nearest $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, farthest $(-1, -1, 2)$
47. Maximum ≈ 9.7938, minimum ≈ -5.3506
49. (a) c/n (b) When $x_1 = x_2 = \cdots = x_n$

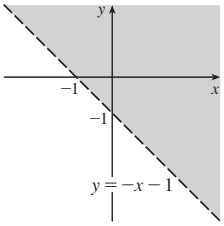
CHAPTER 14 REVIEW ■ PAGE 1022

True-False Quiz

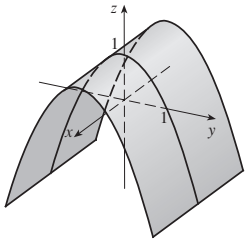
1. True **3.** False **5.** False **7.** True **9.** False
11. True

Exercises

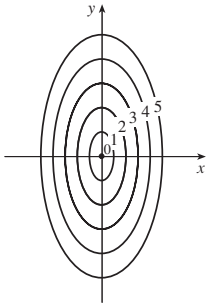
1. $\{(x, y) \mid y > -x - 1\}$



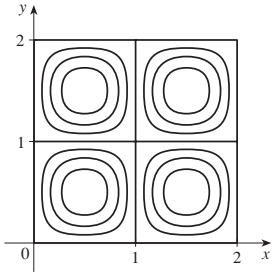
3.



5.



7.



9. $\frac{2}{3}$
11. (a) $\approx 3.5°\text{C/m}, -3.0°\text{C/m}$
(b) $\approx 0.35°\text{C/m}$ by Equation 14.6.9 (Definition 14.6.2 gives $\approx 1.1°\text{C/m}$.)
(c) -0.25
13. $f_x = 32xy(5y^3 + 2x^2y)^7$, $f_y = (16x^2 + 120y^2)(5y^3 + 2x^2y)^7$
15. $F_\alpha = \dfrac{2\alpha^3}{\alpha^2 + \beta^2} + 2\alpha \ln(\alpha^2 + \beta^2)$, $F_\beta = \dfrac{2\alpha^2\beta}{\alpha^2 + \beta^2}$
17. $S_u = \arctan\left(v\sqrt{w}\right)$, $S_v = \dfrac{u\sqrt{w}}{1 + v^2w}$, $S_w = \dfrac{uv}{2\sqrt{w}(1 + v^2w)}$
19. $f_{xx} = 24x$, $f_{xy} = -2y = f_{yx}$, $f_{yy} = -2x$
21. $f_{xx} = k(k-1)x^{k-2}y^lz^m$, $f_{xy} = klx^{k-1}y^{l-1}z^m = f_{yx}$,
$f_{xz} = kmx^{k-1}y^lz^{m-1} = f_{zx}$, $f_{yy} = l(l-1)x^ky^{l-2}z^m$,
$f_{yz} = lmx^ky^{l-1}z^{m-1} = f_{zy}$, $f_{zz} = m(m-1)x^ky^lz^{m-2}$
25. (a) $z = 8x + 4y + 1$ (b) $\dfrac{x-1}{8} = \dfrac{y+2}{4} = \dfrac{z-1}{-1}$
27. (a) $2x - 2y - 3z = 3$ (b) $\dfrac{x-2}{4} = \dfrac{y+1}{-4} = \dfrac{z-1}{-6}$
29. (a) $x + 2y + 5z = 0$
(b) $x = 2 + t, y = -1 + 2t, z = 5t$
31. $\left(2, \frac{1}{2}, -1\right), \left(-2, -\frac{1}{2}, 1\right)$
33. $60x + \frac{24}{5}y + \frac{32}{5}z - 120$; 38.656
35. $2xy^3(1 + 6p) + 3x^2y^2(pe^p + e^p) + 4z^3(p\cos p + \sin p)$
37. $-47, 108$
43. $\langle 2xe^{yz^2}, x^2z^2e^{yz^2}, 2x^2yze^{yz^2}\rangle$ **45.** $-\frac{4}{5}$
47. $\sqrt{145}/2, \left\langle 4, \frac{9}{2}\right\rangle$ **49.** $\approx \frac{5}{8}$ knots/mi
51. Minimum $f(-4, 1) = -11$
53. Maximum $f(1, 1) = 1$; saddle points $(0, 0), (0, 3), (3, 0)$
55. Maximum $f(1, 2) = 4$, minimum $f(2, 4) = -64$
57. Maximum $f(-1, 0) = 2$, minima $f(1, \pm 1) = -3$, saddle points $(-1, \pm 1), (1, 0)$
59. Maximum $f\left(\pm\sqrt{2/3}, 1/\sqrt{3}\right) = 2/\left(3\sqrt{3}\right)$, minimum $f\left(\pm\sqrt{2/3}, -1/\sqrt{3}\right) = -2/\left(3\sqrt{3}\right)$
61. Maximum 1, minimum -1
63. $\left(\pm 3^{-1/4}, 3^{-1/4}\sqrt{2}, \pm 3^{1/4}\right), \left(\pm 3^{-1/4}, -3^{-1/4}\sqrt{2}, \pm 3^{1/4}\right)$
65. $P\left(2 - \sqrt{3}\right), P(3 - \sqrt{3})/6, P\left(2\sqrt{3} - 3\right)/3$

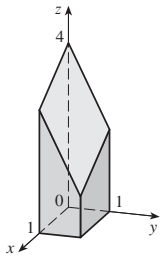
PROBLEMS PLUS ■ PAGE 1025

1. $L^2W^2, \frac{1}{4}L^2W^2$ **3.** (a) $x = w/3$, base $= w/3$ (b) Yes
7. $\sqrt{3/2}, 3/\sqrt{2}$

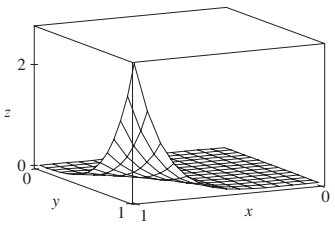
CHAPTER 15

EXERCISES 15.1 ■ PAGE 1039

1. (a) 288 (b) 144 **3.** (a) 0.990 (b) 1.151
5. $U < V < L$ **7.** (a) ≈ 248 (b) ≈ 15.5
9. $24\sqrt{2}$ **11.** 3 **13.** $2 + 8y^2, 3x + 27x^2$
15. 222 **17.** $\frac{5}{2} - e^{-1}$ **19.** 18
21. $\frac{15}{2}\ln 2 + \frac{3}{2}\ln 4$ or $\frac{21}{2}\ln 2$ **23.** 6
25. $\frac{31}{30}$ **27.** 2 **29.** $9 \ln 2$
31. $\frac{1}{2}\left(\sqrt{3} - 1\right) - \frac{1}{12}\pi$ **33.** $\frac{1}{2}e^{-6} + \frac{5}{2}$
35.



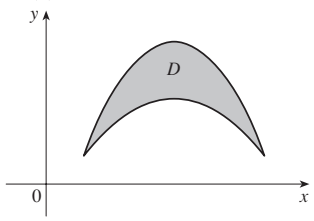
37. 51 **39.** $\frac{166}{27}$ **41.** $\frac{8}{3}$ **43.** $\frac{64}{3}$
45. $21e - 57$



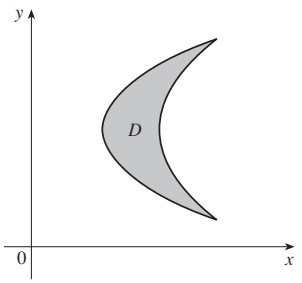
47. $\frac{5}{6}$ **49.** 0
51. Fubini's Theorem does not apply. The integrand has an infinite discontinuity at the origin.

EXERCISES 15.2 ■ PAGE 1048

1. $\frac{868}{3}$ **3.** $\frac{1}{6}(e - 1)$ **5.** $\frac{1}{3}\sin 1$
7. $\frac{1}{4}\ln 17$ **9.** $\frac{1}{2}(1 - e^{-9})$
11. (a)



(b)



13. Type I: $D = \{(x, y) \mid 0 \leqslant x \leqslant 1, 0 \leqslant y \leqslant x\}$, type II: $D = \{(x, y) \mid 0 \leqslant y \leqslant 1, y \leqslant x \leqslant 1\}$; $\frac{1}{3}$

15. $\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} y\,dy\,dx + \int_1^4 \int_{x-2}^{\sqrt{x}} y\,dy\,dx = \int_{-1}^{2} \int_{y^2}^{y+2} y\,dx\,dy = \frac{9}{4}$

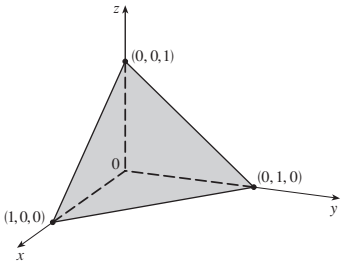
17. $\frac{1}{2}(1 - \cos 1)$ **19.** $\frac{11}{3}$ **21.** 0 **23.** $\frac{3}{4}$

25. $\frac{31}{8}$ **27.** $\frac{16}{3}$ **29.** $\frac{128}{15}$ **31.** $\frac{1}{3}$

33. 0, 1.213; 0.713 **35.** $\frac{64}{3}$

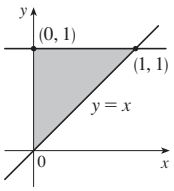
37. $\dfrac{10}{3\sqrt{2}}$ or $\dfrac{5\sqrt{2}}{3}$

39.

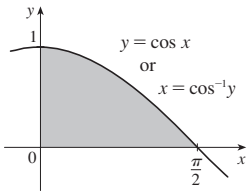


41. 13,984,735,616/14,549,535 **43.** $\pi/2$

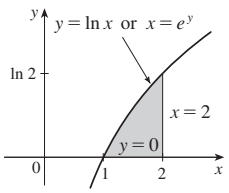
45. $\int_0^1 \int_x^1 f(x, y)\,dy\,dx$



47. $\int_0^1 \int_0^{\cos^{-1} y} f(x, y)\,dx\,dy$



49. $\int_0^{\ln 2} \int_{e^y}^{2} f(x, y)\,dx\,dy$



51. $\frac{1}{6}(e^9 - 1)$ **53.** $\frac{2}{9}\left(2\sqrt{2} - 1\right)$

55. $\frac{1}{3}\left(2\sqrt{2} - 1\right)$ **57.** 1

59. $\dfrac{\sqrt{3}}{2}\pi \leqslant \iint_S \sqrt{4 - x^2y^2}\,dA \leqslant \pi$

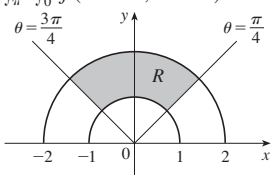
61. $\frac{3}{4}$ **65.** 9π **67.** $a^2b + \frac{3}{2}ab^2$ **69.** $\pi a^2 b$

EXERCISES 15.3 ■ PAGE 1054

1. $\int_0^{2\pi} \int_2^5 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$

3. $\int_{\pi}^{2\pi} \int_0^1 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$

5.



$3\pi/4$

7. $\frac{1250}{3}$ **9.** $(\pi/4)(\cos 1 - \cos 9)$

11. $(\pi/2)(1 - e^{-4})$ **13.** $\frac{3}{64}\pi^2$ **15.** $\pi/12$

17. $\dfrac{\pi}{3} + \dfrac{\sqrt{3}}{2}$ **19.** $\frac{625}{2}\pi$ **21.** 4π **23.** $\frac{4}{3}\pi a^3$

25. $(\pi/3)\left(2 - \sqrt{2}\right)$ **27.** $(8\pi/3)\left(64 - 24\sqrt{3}\right)$

29. $(\pi/4)(1 - e^{-4})$ **31.** $\frac{1}{120}$ **33.** 4.5951

35. 1800π ft^3 **37.** $2/(a + b)$ **39.** $\frac{15}{16}$

41. (a) $\sqrt{\pi}/4$ (b) $\sqrt{\pi}/2$

EXERCISES 15.4 ■ PAGE 1064

1. 285 C **3.** $42k, \left(2, \frac{85}{28}\right)$ **5.** $6, \left(\frac{3}{4}, \frac{3}{2}\right)$ **7.** $\frac{8}{15}k, \left(0, \frac{4}{7}\right)$

9. $\frac{1}{8}(1 - 3e^{-2}), \left(\dfrac{e^2 - 5}{e^2 - 3}, \dfrac{8(e^3 - 4)}{27(e^3 - 3e)}\right)$

11. $\left(\frac{3}{8}, 3\pi/16\right)$ **13.** $(0, 45/(14\pi))$

15. $(2a/5, 2a/5)$ if vertex is $(0, 0)$ and sides are along positive axes

17. $409.2k$, $182k$, $591.2k$

19. $7ka^6/180$, $7ka^6/180$, $7ka^6/90$ if vertex is $(0, 0)$ and sides are along positive axes

21. $\rho bh^3/3$, $\rho b^3h/3$; $b/\sqrt{3}$, $h/\sqrt{3}$

23. $\rho a^4\pi/16$, $\rho a^4\pi/16$; $a/2$, $a/2$

25. $m = 3\pi/64$, $(\bar{x}, \bar{y}) = \left(\dfrac{16384\sqrt{2}}{10395\pi}, 0\right)$,

$I_x = \dfrac{5\pi}{384} - \dfrac{4}{105}$, $I_y = \dfrac{5\pi}{384} + \dfrac{4}{105}$, $I_0 = \dfrac{5\pi}{192}$

27. (a) $\frac{1}{2}$ (b) 0.375 (c) $\frac{5}{48} \approx 0.1042$

29. (b) (i) $e^{-0.2} \approx 0.8187$

(ii) $1 + e^{-1.8} - e^{-0.8} - e^{-1} \approx 0.3481$ (c) 2, 5

31. (a) ≈ 0.500 (b) ≈ 0.632

33. (a) $\iint_D k\left[1 - \frac{1}{20}\sqrt{(x - x_0)^2 + (y - y_0)^2}\right] dA$, where D is the disk with radius 10 mi centered at the center of the city

(b) $200\pi k/3 \approx 209k$, $200\left(\pi/2 - \frac{8}{9}\right)k \approx 136k$, on the edge

EXERCISES 15.5 ■ PAGE 1068

1. $12\sqrt{35}$ **3.** $3\sqrt{14}$ **5.** $(\pi/6)\left(13\sqrt{13} - 1\right)$

7. $(\pi/6)\left(17\sqrt{17} - 5\sqrt{5}\right)$ **9.** $(2\pi/3)\left(2\sqrt{2} - 1\right)$

11. $a^2(\pi - 2)$ **13.** 3.6258

15. (a) ≈ 1.83 (b) ≈ 1.8616

17. $\frac{45}{8}\sqrt{14} + \frac{15}{16}\ln\left[\left(11\sqrt{5} + 3\sqrt{70}\right)/\left(3\sqrt{5} + \sqrt{70}\right)\right]$

19. 3.3213 **23.** $(\pi/6)\left(101\sqrt{101} - 1\right)$

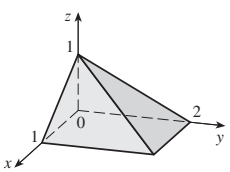
EXERCISES 15.6 ■ PAGE 1077

1. $\frac{27}{4}$ **3.** $\frac{16}{15}$ **5.** $\frac{5}{3}$ **7.** $\frac{2}{3}$ **9.** $\frac{27}{2}$ **11.** $9\pi/8$

13. $\frac{65}{28}$ **15.** $\frac{8}{15}$ **17.** $16\pi/3$ **19.** $\frac{16}{3}$ **21.** $\frac{8}{15}$

23. (a) $\int_0^1 \int_0^x \int_0^{\sqrt{1-y^2}} dz\,dy\,dx$ (b) $\frac{1}{4}\pi - \frac{1}{3}$

25. ≈ 0.985 **27.**



29. $\int_{-2}^{2}\int_{0}^{4-x^2}\int_{-\sqrt{4-x^2}-y/2}^{\sqrt{4-x^2}-y/2} f(x,y,z)\,dz\,dy\,dx$
$= \int_{0}^{4}\int_{-\sqrt{4-y}}^{\sqrt{4-y}}\int_{-\sqrt{4-x^2}-y/2}^{\sqrt{4-x^2}-y/2} f(x,y,z)\,dz\,dx\,dy$
$= \int_{-1}^{1}\int_{0}^{4-4z^2}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\,dx\,dy\,dz$
$= \int_{0}^{4}\int_{-\sqrt{4-y}/2}^{\sqrt{4-y}/2}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\,dx\,dz\,dy$
$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}/2}^{\sqrt{4-x^2}/2}\int_{0}^{4-x^2-4z^2} f(x,y,z)\,dy\,dz\,dx$
$= \int_{-1}^{1}\int_{-\sqrt{4-4z^2}}^{\sqrt{4-4z^2}}\int_{0}^{4-x^2-4z^2} f(x,y,z)\,dy\,dx\,dz$

31. $\int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{2-y/2} f(x,y,z)\,dz\,dy\,dx$
$= \int_{0}^{4}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{2-y/2} f(x,y,z)\,dz\,dx\,dy$
$= \int_{0}^{2}\int_{0}^{4-2z}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz$
$= \int_{0}^{4}\int_{0}^{2-y/2}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dz\,dy$
$= \int_{-2}^{2}\int_{0}^{2-x^2/2}\int_{x^2}^{4-2z} f(x,y,z)\,dy\,dz\,dx$
$= \int_{0}^{2}\int_{-\sqrt{4-2z}}^{\sqrt{4-2z}}\int_{x^2}^{4-2z} f(x,y,z)\,dy\,dx\,dz$

33. $\int_{0}^{1}\int_{\sqrt{x}}^{1}\int_{0}^{1-y} f(x,y,z)\,dz\,dy\,dx = \int_{0}^{1}\int_{0}^{y^2}\int_{0}^{1-y} f(x,y,z)\,dz\,dx\,dy$
$= \int_{0}^{1}\int_{0}^{1-z}\int_{0}^{y^2} f(x,y,z)\,dx\,dy\,dz = \int_{0}^{1}\int_{0}^{1-y}\int_{0}^{y^2} f(x,y,z)\,dx\,dz\,dy$
$= \int_{0}^{1}\int_{0}^{1-\sqrt{x}}\int_{\sqrt{x}}^{1-z} f(x,y,z)\,dy\,dz\,dx = \int_{0}^{1}\int_{0}^{(1-z)^2}\int_{\sqrt{x}}^{1-z} f(x,y,z)\,dy\,dx\,dz$

35. $\int_{0}^{1}\int_{y}^{1}\int_{0}^{y} f(x,y,z)\,dz\,dx\,dy = \int_{0}^{1}\int_{0}^{x}\int_{0}^{y} f(x,y,z)\,dz\,dy\,dx$
$= \int_{0}^{1}\int_{z}^{1}\int_{y}^{1} f(x,y,z)\,dx\,dy\,dz = \int_{0}^{1}\int_{0}^{y}\int_{y}^{1} f(x,y,z)\,dx\,dz\,dy$
$= \int_{0}^{1}\int_{0}^{x}\int_{z}^{x} f(x,y,z)\,dy\,dz\,dx = \int_{0}^{1}\int_{z}^{1}\int_{z}^{x} f(x,y,z)\,dy\,dx\,dz$

37. 64π **39.** $\frac{3}{2}\pi, \left(0, 0, \frac{1}{3}\right)$

41. $a^5, (7a/12, 7a/12, 7a/12)$

43. $I_x = I_y = I_z = \frac{2}{3}kL^5$ **45.** $\frac{1}{2}\pi khA^4$

47. (a) $m = \int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y}\sqrt{x^2+y^2}\,dz\,dy\,dx$
(b) $(\bar{x}, \bar{y}, \bar{z})$, where
$\bar{x} = (1/m)\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} x\sqrt{x^2+y^2}\,dz\,dy\,dx$,
$\bar{y} = (1/m)\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} y\sqrt{x^2+y^2}\,dz\,dy\,dx$,
and $\bar{z} = (1/m)\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} z\sqrt{x^2+y^2}\,dz\,dy\,dx$
(c) $\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y}(x^2+y^2)^{3/2}\,dz\,dy\,dx$

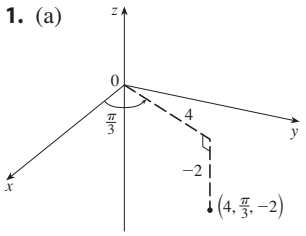
49. (a) $\frac{3}{32}\pi + \frac{11}{24}$
(b) $\left(\frac{28}{9\pi+44}, \frac{30\pi+128}{45\pi+220}, \frac{45\pi+208}{135\pi+660}\right)$
(c) $\frac{1}{240}(68+15\pi)$

51. (a) $\frac{1}{8}$ (b) $\frac{1}{64}$ (c) $\frac{1}{5760}$ **53.** $L^3/8$

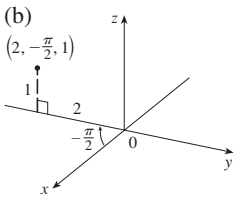
55. (a) The region bounded by the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$
(b) $4\sqrt{6}\pi/45$

EXERCISES 15.7 ■ PAGE 1083

1. (a)



(b)



$(2, 2\sqrt{3}, -2)$ $(0, -2, 1)$

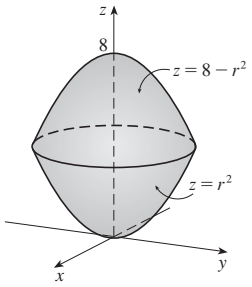
3. (a) $(\sqrt{2}, 3\pi/4, 1)$ (b) $(4, 2\pi/3, 3)$

5. Circular cylinder with radius 2 and axis the z-axis

7. Sphere, radius 2, centered at the origin

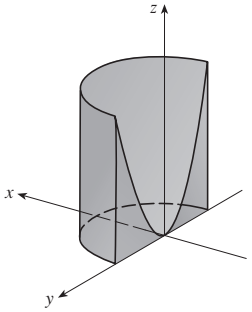
9. (a) $z^2 = 1 + r\cos\theta - r^2$ (b) $z = r^2\cos 2\theta$

11.



13. Cylindrical coordinates: $6 \le r \le 7, 0 \le \theta \le 2\pi, 0 \le z \le 20$

15. 4π



17. 384π **19.** $\frac{8}{3}\pi + \frac{128}{15}$ **21.** $2\pi/5$ **23.** $\frac{4}{3}\pi\left(\sqrt{2}-1\right)$

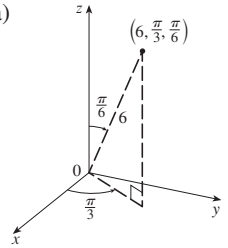
25. (a) $\frac{512}{3}\pi$ (b) $\left(0, 0, \frac{23}{2}\right)$

27. $\pi Ka^2/8, (0, 0, 2a/3)$ **29.** 0

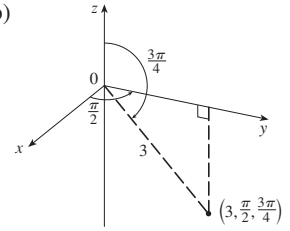
31. (a) $\iiint_C h(P)g(P)\,dV$, where C is the cone
(b) $\approx 3.1 \times 10^{19}$ ft-lb

EXERCISES 15.8 ■ PAGE 1089

1. (a)



(b)



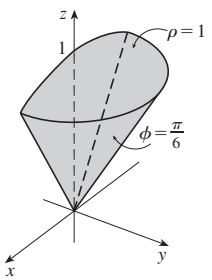
$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}, 3\sqrt{3}\right)$ $\left(0, \frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$

3. (a) $(2, 3\pi/2, \pi/2)$ (b) $(2, 3\pi/4, 3\pi/4)$

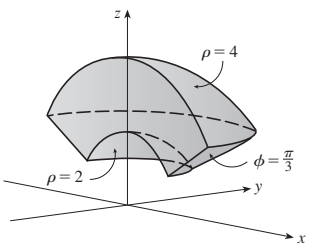
5. Half-cone **7.** Horizontal plane

9. (a) $\rho = 3$ (b) $\rho^2(\sin^2\phi\cos 2\theta - \cos^2\phi) = 1$

11.

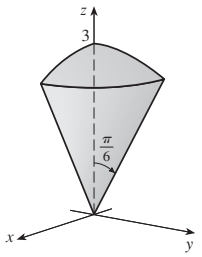


13.



15. $0 \le \phi \le \pi/4, 0 \le \rho \le \cos\phi$

17. $(9\pi/4)(2-\sqrt{3})$



19. $\int_0^{\pi/2}\int_0^3\int_0^2 f(r\cos\theta, r\sin\theta, z)\, r\, dz\, dr\, d\theta$

21. $312{,}500\pi/7$ **23.** $1688\pi/15$ **25.** $\pi/8$

27. $(\sqrt{3}-1)\pi a^3/3$ **29.** (a) 10π (b) $(0, 0, 2.1)$

31. (a) $\left(0, 0, \frac{7}{12}\right)$ (b) $11K\pi/960$

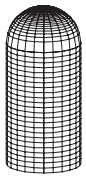
33. (a) $\left(0, 0, \frac{3}{8}a\right)$ (b) $4K\pi a^5/15$ (K is the density)

35. $\frac{1}{3}\pi(2-\sqrt{2}), \left(0, 0, 3/\left[8(2-\sqrt{2})\right]\right)$

37. (a) $\pi Ka^4h/2$ (K is the density) (b) $\pi Ka^2h(3a^2+4h^2)/12$

39. $5\pi/6$ **41.** $(4\sqrt{2}-5)/15$ **43.** $4096\pi/21$

45.



47. $136\pi/99$

EXERCISES 15.9 ■ PAGE 1100

1. -6 **3.** s **5.** $2uvw$

7. The parallelogram with vertices $(0, 0)$, $(6, 3)$, $(12, 1)$, $(6, -2)$

9. The region bounded by the line $y = 1$, the y-axis, and $y = \sqrt{x}$

11. $x = \frac{1}{3}(v-u)$, $y = \frac{1}{3}(u+2v)$ is one possible transformation, where $S = \{(u, v) \mid -1 \leq u \leq 1, 1 \leq v \leq 3\}$

13. $x = u\cos v$, $y = u\sin v$ is one possible transformation, where $S = \{(u, v) \mid 1 \leq u \leq \sqrt{2}, 0 \leq v \leq \pi/2\}$

15. -3 **17.** 6π **19.** $2\ln 3$

21. (a) $\frac{4}{3}\pi abc$ (b) $1.083 \times 10^{12}\ \text{km}^3$ (c) $\frac{4}{15}\pi(a^2+b^2)abck$

23. $\frac{8}{5}\ln 8$ **25.** $\frac{3}{2}\sin 1$ **27.** $e - e^{-1}$

CHAPTER 15 REVIEW ■ PAGE 1101

True-False Quiz

1. True **3.** True **5.** True **7.** True **9.** False

Exercises

1. ≈ 64.0 **3.** $4e^2 - 4e + 3$ **5.** $\frac{1}{2}\sin 1$ **7.** $\frac{2}{3}$

9. $\int_0^{\pi}\int_2^4 f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$

11. $(\sqrt{3}, 3, 2), (4, \pi/3, \pi/3)$

13. $(2\sqrt{2}, 2\sqrt{2}, 4\sqrt{3}), (4, \pi/4, 4\sqrt{3})$

15. (a) $r^2 + z^2 = 4$, $\rho = 2$ (b) $r = 2$, $\rho\sin\phi = 2$

17. The region inside the loop of the four-leaved rose $r = \sin 2\theta$ in the first quadrant

19. $\frac{1}{2}\sin 1$ **21.** $\frac{1}{2}e^6 - \frac{7}{2}$ **23.** $\frac{1}{4}\ln 2$ **25.** 8

27. $81\pi/5$ **29.** $\frac{81}{2}$ **31.** $\pi/96$ **33.** $\frac{64}{15}$

35. 176 **37.** $\frac{2}{3}$ **39.** $2ma^3/9$

41. (a) $\frac{1}{4}$ (b) $\left(\frac{1}{3}, \frac{8}{15}\right)$ (c) $I_x = \frac{1}{12}$, $I_y = \frac{1}{24}$; $\bar{y} = 1/\sqrt{3}$, $\bar{x} = 1/\sqrt{6}$

43. (a) $(0, 0, h/4)$ (b) $\pi a^5h/15$

45. $\ln(\sqrt{2}+\sqrt{3}) + \sqrt{2}/3$ **47.** $\frac{486}{5}$ **49.** 0.0512

51. (a) $\frac{1}{15}$ (b) $\frac{1}{3}$ (c) $\frac{1}{45}$

53. $\int_0^1\int_0^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z)\, dx\, dy\, dz$ **55.** $-\ln 2$ **57.** 0

PROBLEMS PLUS ■ PAGE 1105

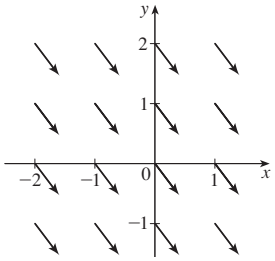
1. 30 **3.** $\frac{1}{2}\sin 1$ **7.** (b) 0.90

13. $abc\pi\left(\frac{2}{3} - \frac{8}{9\sqrt{3}}\right)$

CHAPTER 16

EXERCISES 16.1 ■ PAGE 1113

1.



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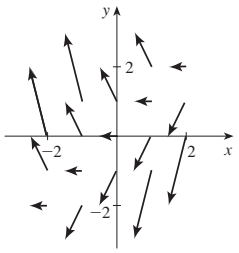

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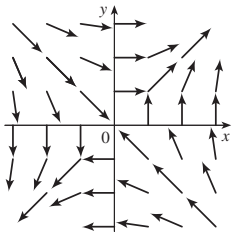

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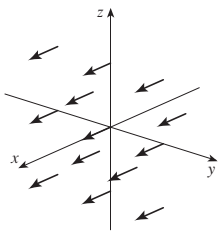
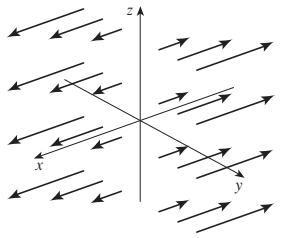
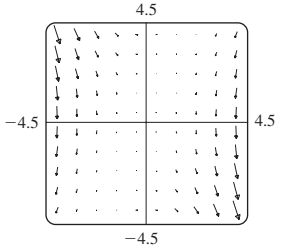


9.



11. IV **13.** I **15.** IV **17.** III

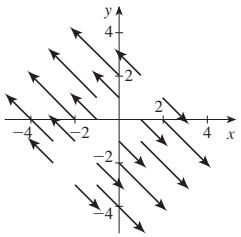
19. The line $y = 2x$



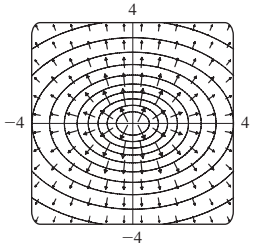
21. $\nabla f(x, y) = y^2 \cos(xy)\,\mathbf{i} + [xy\cos(xy) + \sin(xy)]\,\mathbf{j}$

23. $\nabla f(x, y, z) = \dfrac{x}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{i} + \dfrac{y}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{j} + \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{k}$

25. $\nabla f(x, y) = (x - y)\,\mathbf{i} + (y - x)\,\mathbf{j}$

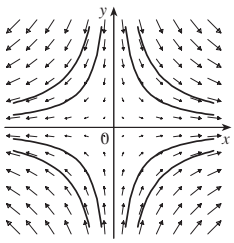


27.



29. III **31.** II **33.** (2.04, 1.03)

35. (a)



$y = C/x$ (b) $y = 1/x, x > 0$

EXERCISES 16.2 ■ PAGE 1124

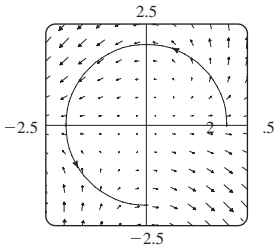
1. $\frac{4}{3}(10^{3/2} - 1)$ **3.** 1638.4 **5.** $\frac{1}{3}\pi^6 + 2\pi$ **7.** $\frac{5}{2}$

9. $\sqrt{2}/3$ **11.** $\frac{1}{12}\sqrt{14}(e^6 - 1)$ **13.** $\frac{2}{5}(e - 1)$ **15.** $\frac{35}{3}$

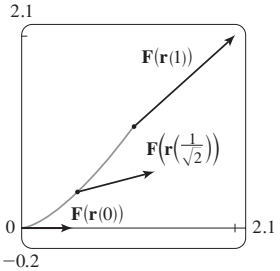
17. (a) Positive (b) Negative **19.** $\frac{1}{20}$

21. $\frac{6}{5} - \cos 1 - \sin 1$ **23.** 0.5424 **25.** 94.8231

27. $3\pi + \frac{2}{3}$



29. (a) $\frac{11}{8} - 1/e$ (b)



31. $\frac{172{,}704}{5{,}632{,}705}\sqrt{2}(1 - e^{-14\pi})$ **33.** $2\pi k$, $(4/\pi, 0)$

35. (a) $\bar{x} = (1/m)\int_C x\rho(x, y, z)\,ds$,
$\bar{y} = (1/m)\int_C y\rho(x, y, z)\,ds$,
$\bar{z} = (1/m)\int_C z\rho(x, y, z)\,ds$, where $m = \int_C \rho(x, y, z)\,ds$
(b) $(0, 0, 3\pi)$

37. $I_x = k\left(\frac{1}{2}\pi - \frac{4}{3}\right)$, $I_y = k\left(\frac{1}{2}\pi - \frac{2}{3}\right)$ **39.** $2\pi^2$ **41.** $\frac{7}{3}$

43. (a) $2ma\,\mathbf{i} + 6mbt\,\mathbf{j}$, $0 \leq t \leq 1$ (b) $2ma^2 + \frac{9}{2}mb^2$

45. $\approx 1.67 \times 10^4$ ft-lb **47.** (b) Yes **51.** ≈ 22 J

EXERCISES 16.3 ■ PAGE 1134

1. 40 **3.** Not conservative

5. $f(x, y) = ye^{xy} + K$ **7.** $f(x, y) = ye^x + x\sin y + K$

9. $f(x, y) = y^2 \sin x + x\cos y + K$

11. (b) 16 **13.** (a) $f(x, y) = \frac{1}{3}x^3y^3$ (b) -9

15. (a) $f(x, y, z) = xyz + z^2$ (b) 77

17. (a) $f(x, y, z) = ye^{xz}$ (b) 4 **19.** $4/e$

21. It doesn't matter which curve is chosen.

23. $\frac{31}{4}$ **25.** No **27.** Conservative

31. (a) Yes (b) Yes (c) Yes

33. (a) No (b) Yes (c) Yes

EXERCISES 16.4 ■ PAGE 1141

1. 120 **3.** $\frac{2}{3}$ **5.** $4(e^3 - 1)$ **7.** $\frac{1}{3}$

9. -24π **11.** $-\frac{16}{3}$ **13.** 4π

15. $\frac{1}{15}\pi^4 - \frac{4144}{1125}\pi^2 + \frac{7{,}578{,}368}{253{,}125} \approx 0.0779$

17. $-\frac{1}{12}$ **19.** 3π **21.** (c) $\frac{9}{2}$

23. $(4a/3\pi, 4a/3\pi)$ if the region is the portion of the disk $x^2 + y^2 = a^2$ in the first quadrant

27. 0

EXERCISES 16.5 ■ PAGE 1149

1. (a) $\mathbf{0}$ (b) $y^2z^2 + x^2z^2 + x^2y^2$

3. (a) $ze^x\,\mathbf{i} + (xye^z - yze^x)\,\mathbf{j} - xe^z\,\mathbf{k}$ (b) $y(e^z + e^x)$

5. (a) $-\dfrac{\sqrt{z}}{(1 + y)^2}\,\mathbf{i} - \dfrac{\sqrt{x}}{(1 + z)^2}\,\mathbf{j} - \dfrac{\sqrt{y}}{(1 + x)^2}\,\mathbf{k}$

(b) $\dfrac{1}{2\sqrt{x}(1 + z)} + \dfrac{1}{2\sqrt{y}(1 + x)} + \dfrac{1}{2\sqrt{z}(1 + y)}$

7. (a) $\langle -e^y\cos z, -e^z\cos x, -e^x\cos y\rangle$
(b) $e^x\sin y + e^y\sin z + e^z\sin x$

9. (a) Negative (b) curl $\mathbf{F} = \mathbf{0}$

11. (a) Zero (b) curl $\mathbf{F}$ points in the negative z-direction.

13. $f(x, y, z) = xy^2z^3 + K$ **15.** Not conservative

17. $f(x, y, z) = xe^{yz} + K$ **19.** No

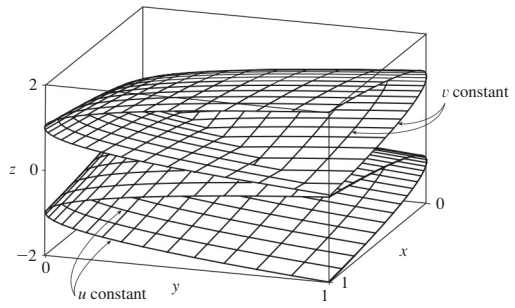
EXERCISES 16.6 ■ PAGE 1160

1. P: yes; Q: no

3. Plane through (0, 3, 1) containing vectors $\langle 1, 0, 4\rangle$, $\langle 1, -1, 5\rangle$

5. Circular cone with axis the z-axis

7.



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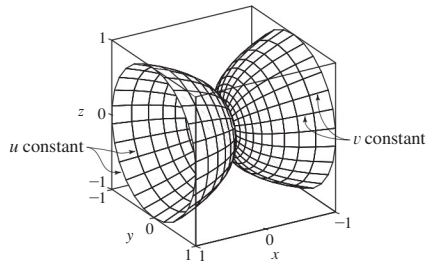

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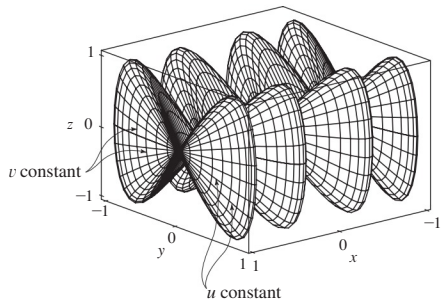


13. IV **15.** I **17.** III

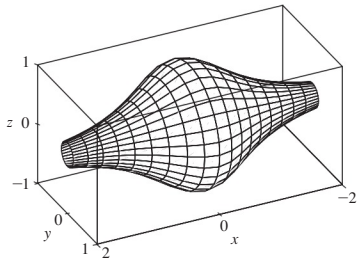
19. $x = u,\ y = v - u,\ z = -v$

21. $y = y,\ z = z,\ x = \sqrt{1 + y^2 + \frac{1}{4}z^2}$

23. $x = 2\sin\phi\cos\theta,\ y = 2\sin\phi\sin\theta,$
$z = 2\cos\phi,\ 0 \leq \phi \leq \pi/4,\ 0 \leq \theta \leq 2\pi$
$\left[\text{or } x = x,\ y = y,\ z = \sqrt{4 - x^2 - y^2},\ x^2 + y^2 \leq 2\right]$

25. $x = 6\sin\phi\cos\theta,\ y = 6\sin\phi\sin\theta,\ z = 6\cos\phi,$
$\pi/6 \leq \phi \leq \pi/2,\ 0 \leq \theta \leq 2\pi$

29. $x = x,\ y = \dfrac{1}{1 + x^2}\cos\theta,\ y = \dfrac{1}{1 + x^2}\sin\theta,$
$-2 \leq x \leq 2,\ 0 \leq \theta \leq 2\pi$



31. (a) Direction reverses (b) Number of coils doubles

33. $3x - y + 3z = 3$ **35.** $\dfrac{\sqrt{3}}{2}x - \dfrac{1}{2}y + z = \dfrac{\pi}{3}$

37. $-x + 2z = 1$ **39.** $3\sqrt{14}$ **41.** $\sqrt{14}\pi$

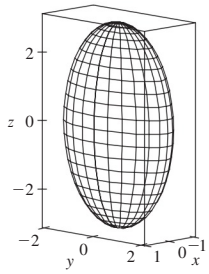
43. $\frac{4}{15}(3^{5/2} - 2^{7/2} + 1)$ **45.** $(2\pi/3)(2\sqrt{2} - 1)$

47. $(\pi/6)(65^{3/2} - 1)$ **49.** 4 **51.** $\pi R^2 \leq A(S) \leq \sqrt{3}\,\pi R^2$

53. 3.5618 **55.** (a) ≈ 24.2055 (b) 24.2476

57. $\frac{45}{8}\sqrt{14} + \frac{15}{16}\ln\left[(11\sqrt{5} + 3\sqrt{70})/(3\sqrt{5} + \sqrt{70})\right]$

59. (b)



(c) $\int_0^{2\pi}\int_0^{\pi}\sqrt{36\sin^4 u\cos^2 v + 9\sin^4 u\sin^2 v + 4\cos^2 u\sin^2 u}\,du\,dv$

61. 4π **63.** $2a^2(\pi - 2)$

EXERCISES 16.7 ■ PAGE 1172

1. ≈ -6.93 **3.** 900π **5.** $11\sqrt{14}$ **7.** $\frac{2}{3}(2\sqrt{2} - 1)$

9. $171\sqrt{14}$ **11.** $\sqrt{21}/3$ **13.** $(\pi/120)(25\sqrt{5} + 1)$

15. $\frac{7}{4}\sqrt{21} - \frac{17}{12}\sqrt{17}$ **17.** 16π **19.** 0 **21.** 4

23. $\frac{713}{180}$ **25.** $\frac{8}{3}\pi$ **27.** 0 **29.** 48 **31.** $2\pi + \frac{8}{3}$

33. 4.5822 **35.** 3.4895

37. $\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D [P(\partial h/\partial x) - Q + R(\partial h/\partial z)]\,dA,$
where D = projection of S onto xz-plane

39. $(0, 0, a/2)$

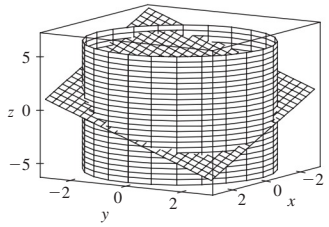
41. (a) $I_z = \iint_S (x^2 + y^2)\rho(x, y, z)\,dS$ (b) $4329\sqrt{2}\,\pi/5$

43. 0 kg/s **45.** $\frac{8}{3}\pi a^3\varepsilon_0$ **47.** 1248π

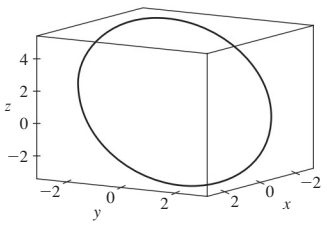
EXERCISES 16.8 ■ PAGE 1179

3. 16π **5.** 0 **7.** -1 **9.** $-\frac{17}{20}$

11. (a) $81\pi/2$ (b)



(c) $x = 3\cos t,\ y = 3\sin t,$
$z = 1 - 3(\cos t + \sin t),$
$0 \leq t \leq 2\pi$



13. -32π **15.** $-\pi$ **17.** 3

EXERCISES 16.9 ■ PAGE 1185

1. $\frac{9}{2}$ **3.** $256\pi/3$ **5.** $\frac{9}{2}$ **7.** $9\pi/2$ **9.** 0

11. π **13.** 2π **15.** $341\sqrt{2}/60 + \frac{81}{20}\arcsin(\sqrt{3}/3)$

17. $13\pi/20$ **19.** Negative at P_1, positive at P_2

21. div $\mathbf{F} > 0$ in quadrants I, II; div $\mathbf{F} < 0$ in quadrants III, IV

CHAPTER 16 REVIEW ■ PAGE 1188

True-False Quiz

1. False **3.** True **5.** False **7.** False
9. True **11.** True **13.** False

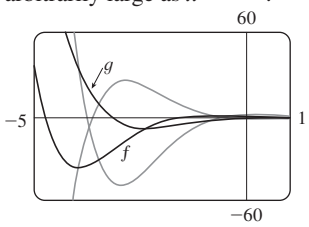
Exercises

1. (a) Negative (b) Positive **3.** $6\sqrt{10}$ **5.** $\frac{4}{15}$ **7.** $\frac{110}{3}$
9. $\frac{11}{12} - 4/e$ **11.** $f(x, y) = e^y + xe^{xy} + K$ **13.** 0
15. 0 **17.** -8π **25.** $\frac{1}{6}(27 - 5\sqrt{5})$
27. $(\pi/60)(391\sqrt{17} + 1)$ **29.** $-64\pi/3$ **31.** 0
33. $-\frac{1}{2}$ **35.** 4π **37.** -4 **39.** 21

CHAPTER 17

EXERCISES 17.1 ■ PAGE 1200

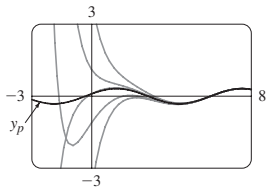
1. $y = c_1e^{3x} + c_2e^{-2x}$ **3.** $y = c_1\cos(\sqrt{2}x) + c_2\sin(\sqrt{2}x)$
5. $y = c_1e^{-x/2} + c_2xe^{-x/2}$ **7.** $y = c_1 + c_2e^{4x/3}$
9. $y = e^{2x}(c_1\cos 3x + c_2\sin 3x)$
11. $y = c_1e^{(\sqrt{3}-1)t/2} + c_2e^{-(\sqrt{3}+1)t/2}$
13. $V = e^{-2t/3}\left[c_1\cos\left(\frac{\sqrt{5}}{3}t\right) + c_2\sin\left(\frac{\sqrt{5}}{3}t\right)\right]$
15. $f(x) = e^{-x}\cos x$, $g(x) = e^{-x}\sin x$. All solution curves approach 0 as $x \to \infty$ and oscillate with amplitudes that become arbitrarily large as $x \to -\infty$.



17. $y = \cos(\sqrt{3}x) + \sqrt{3}\sin(\sqrt{3}x)$ **19.** $y = e^{-2x/3} + \frac{2}{3}xe^{-2x/3}$
21. $y = e^{3x}(2\cos x - 3\sin x)$
23. $y = \frac{1}{7}e^{4x-4} - \frac{1}{7}e^{3-3x}$ **25.** $y = -3\cos 4x + 2\sin 4x$
27. $y = 2e^{-2x} - 2xe^{-2x}$ **29.** $y = \dfrac{e-2}{e-1} + \dfrac{e^x}{e-1}$
31. No solution
33. (b) $\lambda = n^2\pi^2/L^2$, n a positive integer; $y = C\sin(n\pi x/L)$
35. (a) $b - a \neq n\pi$, n any integer
(b) $b - a = n\pi$ and $\dfrac{c}{d} \neq e^{a-b}\dfrac{\cos a}{\cos b}$ unless $\cos b = 0$, then $\dfrac{c}{d} \neq e^{a-b}\dfrac{\sin a}{\sin b}$
(c) $b - a = n\pi$ and $\dfrac{c}{d} = e^{a-b}\dfrac{\cos a}{\cos b}$ unless $\cos b = 0$, then $\dfrac{c}{d} = e^{a-b}\dfrac{\sin a}{\sin b}$

EXERCISES 17.2 ■ PAGE 1207

1. $y = c_1e^{2x} + c_2e^{-4x} + \frac{1}{4}x^2 + \frac{1}{8}x - \frac{1}{32}$
3. $y = c_1\cos(\frac{1}{3}x) + c_2\sin(\frac{1}{3}x) + \frac{1}{37}e^{2x}$
5. $y = e^{2x}(c_1\cos x + c_2\sin x) + \frac{1}{10}e^{-x}$
7. $y = e^x(\frac{9}{10}\cos 2x - \frac{1}{20}\sin 2x) + \frac{1}{10}\cos x + \frac{1}{5}\sin x$
9. $y = e^x(\frac{1}{2}x^2 - x + 2)$
11.

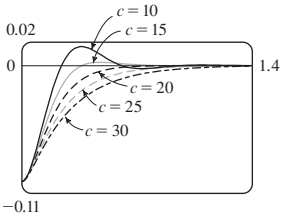


The solutions are all asymptotic to $y_p = \frac{1}{10}\cos x + \frac{3}{10}\sin x$ as $x \to \infty$. Except for y_p, all solutions approach either ∞ or $-\infty$ as $x \to -\infty$.

13. $y_p = (Ax + B)e^x\cos x + (Cx + D)e^x\sin x$
15. $y_p = Axe^x + B\cos x + C\sin x$
17. $y_p = xe^{-x}[(Ax^2 + Bx + C)\cos 3x + (Dx^2 + Ex + F)\sin 3x]$
19. $y = c_1\cos(\frac{1}{2}x) + c_2\sin(\frac{1}{2}x) - \frac{1}{3}\cos x$
21. $y = c_1e^x + c_2xe^x + e^{2x}$
23. $y = c_1\sin x + c_2\cos x + \sin x\ln(\sec x + \tan x) - 1$
25. $y = [c_1 + \ln(1 + e^{-x})]e^x + [c_2 - e^{-x} + \ln(1 + e^{-x})]e^{2x}$
27. $y = e^x\left[c_1 + c_2x - \frac{1}{2}\ln(1 + x^2) + x\tan^{-1}x\right]$

EXERCISES 17.3 ■ PAGE 1215

1. $x = 0.35\cos(2\sqrt{5}\,t)$ **3.** $x = -\frac{1}{5}e^{-6t} + \frac{6}{5}e^{-t}$ **5.** $\frac{49}{12}$ kg
7.



13. $Q(t) = (-e^{-10t}/250)(6\cos 20t + 3\sin 20t) + \frac{3}{125}$, $I(t) = \frac{3}{5}e^{-10t}\sin 20t$
15. $Q(t) = e^{-10t}\left[\frac{3}{250}\cos 20t - \frac{3}{500}\sin 20t\right] - \frac{3}{250}\cos 10t + \frac{3}{125}\sin 10t$

EXERCISES 17.4 ■ PAGE 1220

1. $c_0\sum_{n=0}^{\infty}\dfrac{x^n}{n!} = c_0e^x$ **3.** $c_0\sum_{n=0}^{\infty}\dfrac{x^{3n}}{3^n n!} = c_0e^{x^3/3}$
5. $c_0\sum_{n=0}^{\infty}\dfrac{(-1)^n}{2^n n!}x^{2n} + c_1\sum_{n=0}^{\infty}\dfrac{(-2)^n n!}{(2n+1)!}x^{2n+1}$
7. $c_0 + c_1\sum_{n=1}^{\infty}\dfrac{x^n}{n} = c_0 - c_1\ln(1 - x)$ for $|x| < 1$
9. $\sum_{n=0}^{\infty}\dfrac{x^{2n}}{2^n n!} = e^{x^2/2}$
11. $x + \sum_{n=1}^{\infty}\dfrac{(-1)^n 2^2 5^2 \cdot \cdots \cdot (3n-1)^2}{(3n+1)!}x^{3n+1}$

CHAPTER 17 REVIEW ■ PAGE 1221

True-False Quiz

1. True **3.** True

Exercises

1. $y = c_1e^{x/2} + c_2e^{-x/2}$
3. $y = c_1\cos(\sqrt{3}x) + c_2\sin(\sqrt{3}x)$
5. $y = e^{2x}(c_1\cos x + c_2\sin x + 1)$
7. $y = c_1e^x + c_2xe^x - \frac{1}{2}\cos x - \frac{1}{2}(x + 1)\sin x$

9. $y = c_1e^{3x} + c_2e^{-2x} - \frac{1}{6} - \frac{1}{5}xe^{-2x}$

11. $y = 5 - 2e^{-6(x-1)}$ **13.** $y = (e^{4x} - e^{x})/3$

15. No solution **17.** $\sum_{n=0}^{\infty} \frac{(-2)^n n!}{(2n+1)!} x^{2n+1}$

19. $Q(t) = -0.02e^{-10t}(\cos 10t + \sin 10t) + 0.03$

21. (c) $2\pi/k \approx 85$ min (d) $\approx$17,600 mi/h

APPENDIXES

EXERCISES G ■ PAGE A12

1. $8 - 4i$ **3.** $13 + 18i$ **5.** $12 - 7i$ **7.** $\frac{11}{13} + \frac{10}{13}i$

9. $\frac{1}{2} - \frac{1}{2}i$ **11.** $-i$ **13.** $5i$ **15.** $12 + 5i, 13$

17. $4i, 4$ **19.** $\pm\frac{3}{2}i$ **21.** $-1 \pm 2i$

23. $-\frac{1}{2} \pm (\sqrt{7}/2)i$ **25.** $3\sqrt{2}\,[\cos(3\pi/4) + i\sin(3\pi/4)]$

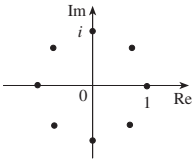
27. $5\{\cos[\tan^{-1}(\frac{4}{3})] + i\sin[\tan^{-1}(\frac{4}{3})]\}$

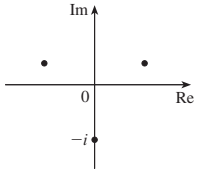
29. $4[\cos(\pi/2) + i\sin(\pi/2)]$, $\cos(-\pi/6) + i\sin(-\pi/6)$, $\frac{1}{2}[\cos(-\pi/6) + i\sin(-\pi/6)]$

31. $4\sqrt{2}\,[\cos(7\pi/12) + i\sin(7\pi/12)]$, $(2\sqrt{2})[\cos(13\pi/12) + i\sin(13\pi/12)]$, $\frac{1}{4}[\cos(\pi/6) + i\sin(\pi/6)]$

33. -1024 **35.** $-512\sqrt{3} + 512i$

37. $\pm 1, \pm i, (1/\sqrt{2})(\pm 1 \pm i)$ **39.** $\pm(\sqrt{3}/2) + \frac{1}{2}i, -i$





41. i **43.** $\frac{1}{2} + (\sqrt{3}/2)i$ **45.** $-e^2$

47. $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$, $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$