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CS 2209A

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Assignment 3

Due: Thursday Oct. 22, 2020 before 6:30 PM to be uploaded in Gradescope as a single pdf file.

Please write your name and student number on your submission. Justify each step carefully using previously proven results either in the zyBook or the videos/zoom sessions. When in doubt prove the statement you are going to use. Solutions are graded for correctness as well as clarity.

Exercise 1 (10 point). Let x, y be any non-zero real numbers. Given

$$\frac{x}{y} + \frac{y}{x} = 2$$

Show that $x = y$ by “direct proof”.

Exercise 2 (15 point). Prove the following proposition by contraposition:

For every positive integer a , if a is strictly greater than 1 then a does not divide $2a^2 + a + 1$.

$$\forall a, \quad a > 1 \quad 2a^2 + a + 1 \% a \neq 0$$

Exercise 3 (15 points). Prove that $\sqrt[3]{2}$ is not a rational number by contradiction.

Hint: You may use the fact that if a^3 is an even number then a must also be even, with proper justification.

Exercise 4 (10 point). Solve the following equations for real numbers

$$|x^2 - 1| = |x - 1|$$

using case analysis.

1. $\because x \neq 0, y \neq 0$.

$$\frac{x}{y} + \frac{y}{x} = 2$$

$$x^2 + y^2 - 2xy = 0$$

$$(x - y)^2 = 0$$

$$x = y.$$

2. directly proof:

For every a greater than 1 and divided $2a^2 + a + 1$,
there's no positive integer n .

Then, find if there's a meet requirements.

$$\frac{2a^2 + a + 1}{a} = 2a + 1 + \frac{1}{a}.$$

if $a > 1$, then $\frac{1}{a}$ cannot be an integer.

Then the proposition is true.

3. assume that $\sqrt[3]{2}$ is a rational number.

then there must be integer, a, b which are relative prime.

$$2 = \frac{a^3}{b^3} \quad \therefore a \text{ is an even number.}$$

$$\text{then } a^3 = 2b^3 = b^3 + b^3.$$

however, a is an even number to meet the requirement.

also, b is an even number.

\therefore the function $2 = \frac{a^3}{b^3}$ is unreachable.

$\therefore \sqrt[3]{2}$ is an irrational number.

4. $|x^2 - 1| = |x - 1|$.

$$|(x+1)(x-1)| = |x-1|$$

$\therefore x=1$: infinity solutions.

$$\therefore x \neq 1 : x+1 = 1$$
$$x = 0.$$

$$\therefore x=1 \text{ or } x=0.$$