PROVE THAT $\forall n \in \mathbb{N}$

$$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$$

- BASE CASE: FOR $\boxed{n=0}$, $\sum_{i=0}^{0} x^i = x^0 = 1$ ✓

RHS: $\frac{x^{0+1}-1}{x-1} = \frac{x^1-1}{x-1} = 1$

- INDUCTION STEP: ASSUME THAT, FOR $k \in \mathbb{N}$,

$$\boxed{\sum_{i=0}^{k} x^i} = \frac{x^{k+1}-1}{x-1} \quad (I.H)$$

WANT: $\sum_{i=0}^{k+1} x^i = \frac{x^{k+1+1}-1}{x-1} = \frac{x^{k+2}-1}{x-1}$

$$\sum_{i=0}^{k+1} x^i = \boxed{\sum_{i=0}^{k} x^i} + x^{k+1} \overset{IH}{=} \boxed{\frac{x^{k+1}-1}{x-1}} + x^{k+1} =$$

$$= \frac{x^{k+1}-1+(x-1)\cdot x^{k+1}}{x-1} = \frac{\cancel{x^{k+1}}-1+x^{k+2}\cancel{-x^{k+1}}}{x-1}$$ ✓

- CONCLUSION: BY INDUCTION, $\forall n \in \mathbb{N}$

$$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$$

LET $F(0)=0$, $F(1)=1$, $F(n+2)=F(n)+F(n+1)\quad \forall n\in\mathbb{N}$

PROVE: $\forall n\in\mathbb{N}\setminus\{0\}\ F(n-1)\cdot F(n+1)-F(n)^2=(-1)^n$

- BASE CASE: $\boxed{n=1}$: $F(0)F(2)-F(1)^2=$
$= 0\cdot(0+1)-1^2 = 0-1 = (-1)^1$ ✓

- IND. STEP: ASSUME, FOR A $k\in\mathbb{N}\setminus\{0\}$, $[k\geq 1]$

$\rightarrow \boxed{F(k-1)F(k+1)-F(k)^2}=(-1)^k \quad$ (I.H.)

WANT: $F(k+1-1)F(k+1+1)-F(k+1)^2=(-1)^{k+1}$

$F(k)\cdot F(k+2)-F(k+1)^2=(-1)^{k+1}$

$F(k)\big(F(k)+F(k+1)\big)-F(k+1)^2=$

$F(k)^2+F(k)F(k+1)-F(k+1)^2=$

$F(k)^2+F(k+1)\big(\underbrace{F(k)-F(k+1)}_{-F(k-1)}\big)$

$F(k+1)=F(k)+F(k-1)$

$-F(k-1)=F(k)-F(k+1)$

$F(k)^2-F(k+1)F(k-1)= -\boxed{\big(F(k+1)F(k-1)-F(k)^2\big)}$

$\overset{\text{I.H.}}{=} -\boxed{(-1)^k} = (-1)^{k+1}$ ✓

- CONCLUSION: BY INDUCTION, $\underbrace{\forall n\in\mathbb{N}\setminus\{0\}}_{\text{EQUIV.}\ \forall n\in\mathbb{N},\ n\geq 1}$

$F(n-1)F(n+1)-F(n)^2=(-1)^n$