Welcome to CS2209A

- ZOOM – please ask questions! Weekly Schedule – one file
- In-person: Midterm 25%, Oct 24, 3:00 to 5:00 PM; Final 30%
- 5 assignments: 5%, 10%, 10%, 10%, 10% (9/24, 10/8, 10/22, 11/19, 12/3)
- LogiCola and other software
- We will take a (more) applied/practical approach
- Logic and its use in CS and life
- Logic: representation and proof
  - Symbolic reasoning (true or false), human intelligence
  - Proof: sound and convincing deduction
  - Started from ancient Greek philosophers
- Propositional logic, first-order logic, Prolog

A Logical Puzzle with Three Boys

HAHAHA

…

HAHAHA

…

(After a while)

Oh no… I have mud on my face!

HAHAHA

…

A

We assume that (premises):

-The description of the story …

Background (common sense) knowledge:
- boys cannot feel mud, if any, on their face
- boys have no mirrors…
- boy cannot tell others are laughing at him
- boys try to figure out if they have mud on face
- boys will be unhappy if they know to have mud on face
- … …

Prove: boy A deduced he has mud on his own face

How: A assumes that I had no mud on my face

….

Contradiction!

Therefore, I (A) must have mud on my face!

A Proof

Given (accepting) premises, draw valid conclusions

Premise 1

Premise 2

…

Prove Z

Assume NOT Z

(so, therefore, thus, hence, I prove/deduce, it must be, …)

Conclusion 1 (based on Premise i, Assumption, using Rule_x)

Conclusion 2 (based on Conclusion 1, Premise k, usnig Rule_y)

…

Convincing, justified, valid, make sense,

Symbolic reasoning (true or false), human intelligence

Rational thinking and reasoning; tool for AI ?

Real-life Cases

- Science: physics, geometry, biology, …, history?

Newton's three Laws of Motion

1. A body remains in rest or uniform motion unless an external force
2. F = *m a*
3. The forces of action and reaction are equal and opposite

Law of Gravitation: $F = G\ M\ m\ /d^2$ ($G=6.674 \times 10^{-11}$)

How to convince people? Rhetoric

- Sci-fi movies; detective movies; …
- Court
- Constitutions, laws, rules, policies
- Religions
- ……

Logic reasoning we will learn:

- If you overslept, you'll be late. You overslept, <u>Therefore</u>: You will be late
  - Rule: if A then B, A, therefore B
- If you overslept, you'll be late. You aren't late. <u>Therefore</u>: You didn't' oversleep.
  - Rule: If A then B. Not B. Therefore, Not A

- If you overslept, you'll be late. You didn't oversleep. <u>Therefore</u>: You are not late.

- If you overslept, you'll be late. You are very late. <u>Therefore</u>: You overslept.

If it rains and your tent leaks, then your sleeping bag gets wet.
Your sleeping bag does not get wet.
Your tent leaks.
Conclusion: it does not rain.

If (A and B) then C.
Not C.
B.
Conclusion: Not A

If you get up early, you'll be late. You get up early.

Conclusion: You will be late

Why a bit strange? I don't like the premise!

- Limitation of logic
  - “The economy is poor; this proves the PM is bad”
  - Probabilistic reasoning – AI – very hot these years
  - Hard to convert knowledge to logic
    - Japan’s Fifth Generation Project

Just for fun…

- Prove the sum of even numbers is even
- Prove sqrt of 2 is not rational

Chapter 3:
Basic Propositional Logic

Based on Harry Gensler's book

For CS2209A/B

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Propositional Logic

Any (formal) language has:

- Syntax: “legal” ways of writing, WFF
  - Just like syntax in Python
- Semantics: “meaning” of the WFF
  - Just like how Python codes are run

3.1 Syntax

- Need a (formal representation) language to…
  - deal with simple statements: any strings in English
  - Use capital letters: P, Q, …
  - They are called propositions
  - Use “logic connectives” to deal with “if-then”, “and”, “or”, “not”, etc. in English

Well-Formed Formula (wff)

The legal statements in logic is defined as:

1. Any capital letter is a wff.
2. The result of prefixing any wff with “~” is a wff.
3. The result of joining any two wffs by “·” or “∨” or “⊃” or “≡” and enclosing the result in ( ) is a wff.

Some examples of wff:

$$P$$
$$\sim Q$$
$$(P \cdot \sim Q)$$
$$(N \supset (P \cdot \sim Q))$$

Examples of Valid/Invalid wff

- $(P \cdot P)$, $\sim \sim \sim \sim P$
- p $(p \cdot q)$
- $(P \,\&\, Q)$, $P \cdot Q$
- $(\sim P)$, (Q), $((R \cdot Q))$, $P \cdot (Q \cdot R)$
- $P \cdot Q$, $P \cdot Q \cdot R$, $(P \cdot Q \equiv R)$,
- If P then Q

<u>Logic (including wff) is very precise</u>

The usual meaning in English

$\sim P$	=	Not-P
$(P \cdot Q)$	=	Both P and Q
$(P \vee Q)$	=	Either P or Q
$(P \supset Q)$	=	If P then Q
$(P \equiv Q)$	=	P if and only if Q

Propositional translations

- Semantics: precise definitions later
- Can help to guide translating English to wff and vise versa
- Real-life cases can be translated and proved with wff.
- “All students are rich”, “John is a student”, thus “John is rich” must be represented by three different propositions; cannot prove with this logic.

Quite subtle…

$(\sim P \cdot Q)$	=	Not-P (pause) and (pause) Q
$\sim(P \cdot Q)$	=	Not (pause) P and Q

These two also differ:

$(P \cdot (Q \supset R))$	=	P, and if Q then R
$((P \cdot Q) \supset R)$	=	If P-and-Q, then R

https://www.harryhiker.com/lc/index.htm

https://www.harryhiker.com/lc/index.htm

Home | Photos | Bike | Software | Courses | Exercises | Golden Rule | Times

Download LogiCola

LogiCola is a program to help students learn logic. It was last modified on **7 December 2020**. If you ever have problems updating the program within LogiCola, just come back to this page to re-install it.

Click here to install LogiCola on Macintosh (I suggest you first print out "Downloading LogiCola"); or click here to install LogiCola on Linux.

To install LogiCola in Windows:

CLICK HERE

After you click, you'll be asked: **Do you want to RUN or SAVE this file?** Click **RUN**; and keep insisting, if asked, that *yes* you do want to run this file. You may have to click your way around warnings; for example, if you see "Windows Defender prevented this unrecognized app from starting," then click "more info" and then "run anyway." Soon this dialog box will appear:

LogiCola Setup - 26 December 2018 version

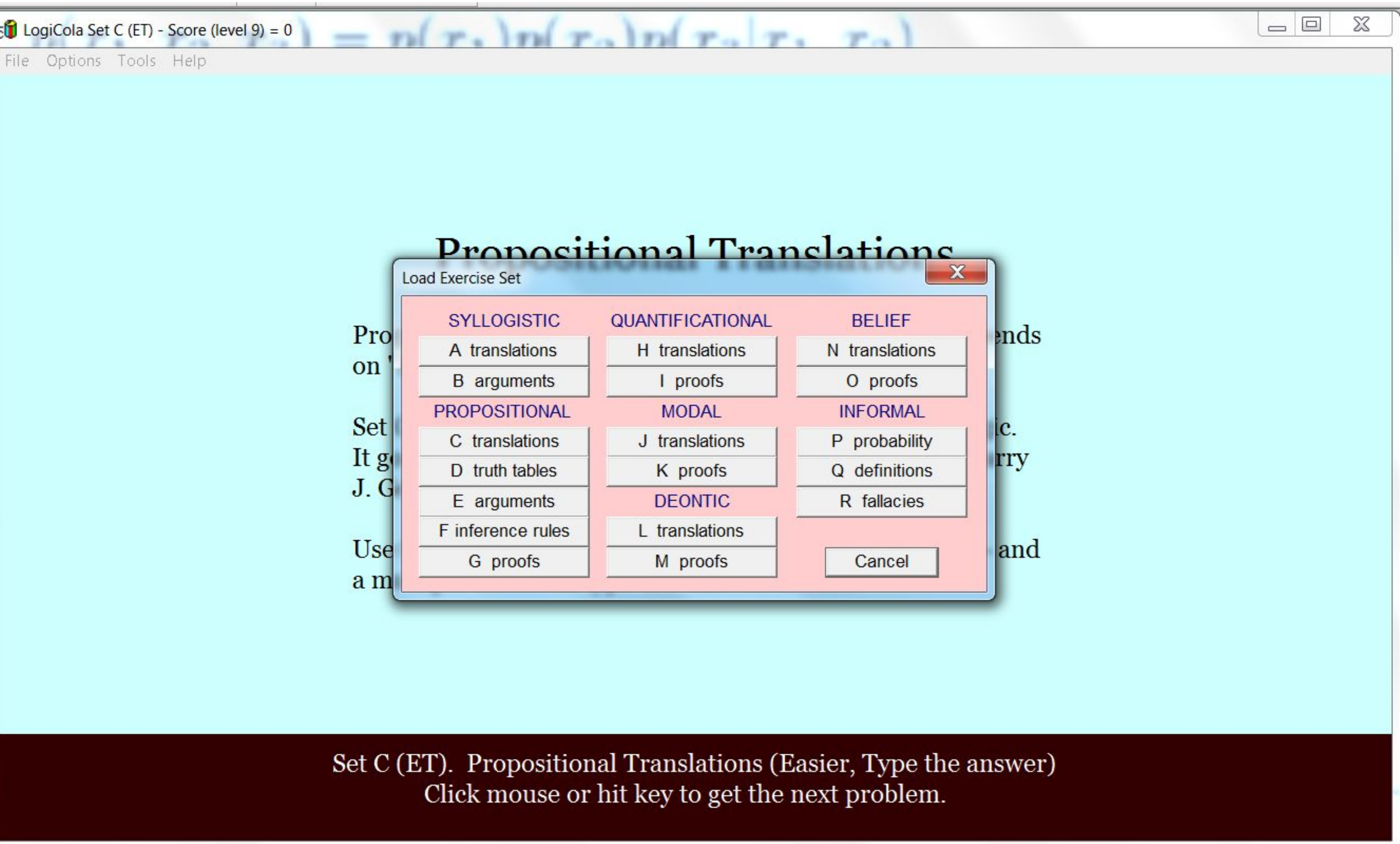
LogiCola (© 2008+ Harry J. Gensler), a program to help students learn logic, goes with Gensler's Introduction to Logic (Routledge). Where do you want it installed?

Install on my USB flash drive | Install on my computer

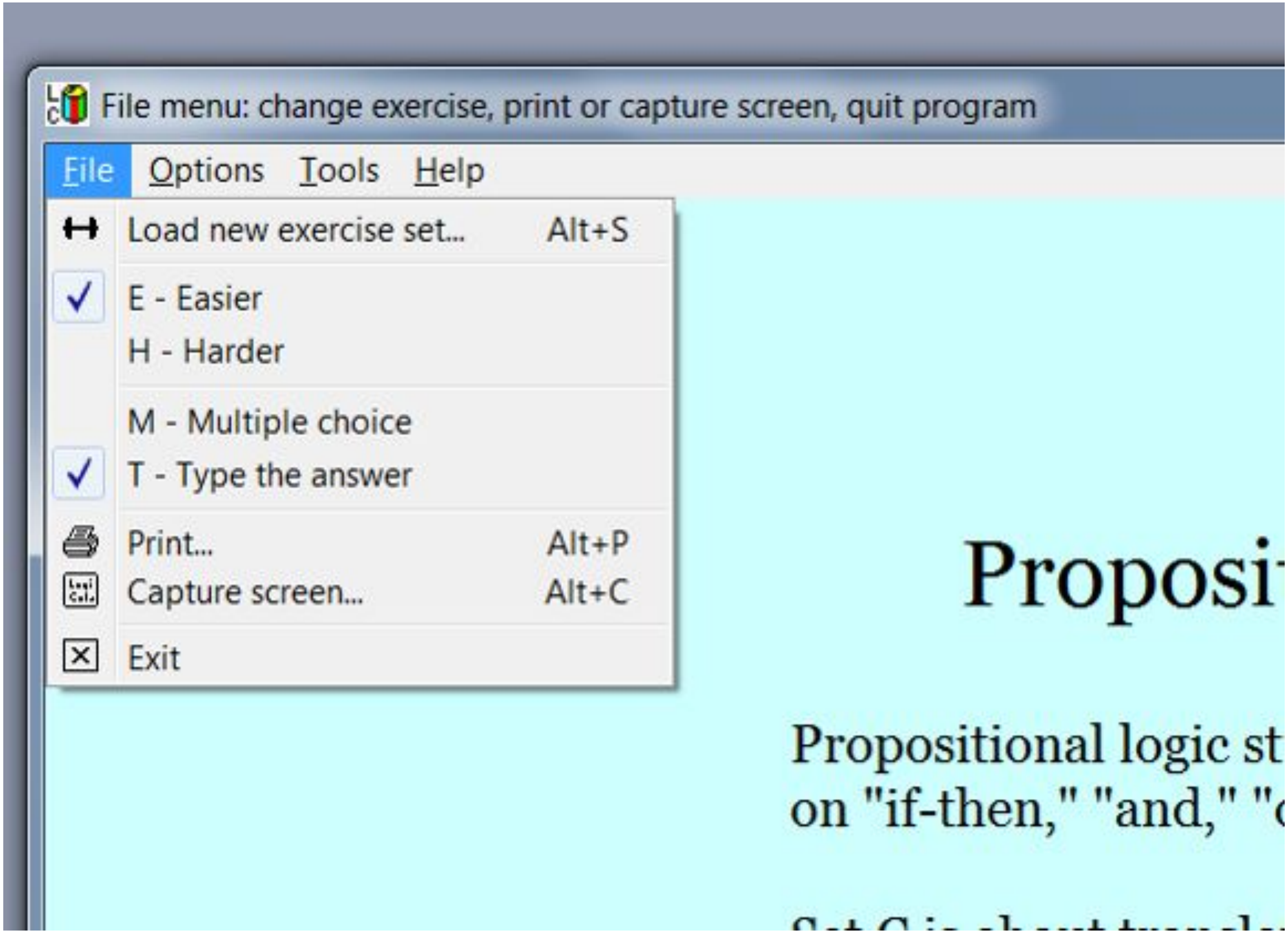
Pick this if you want | Pick this if you want

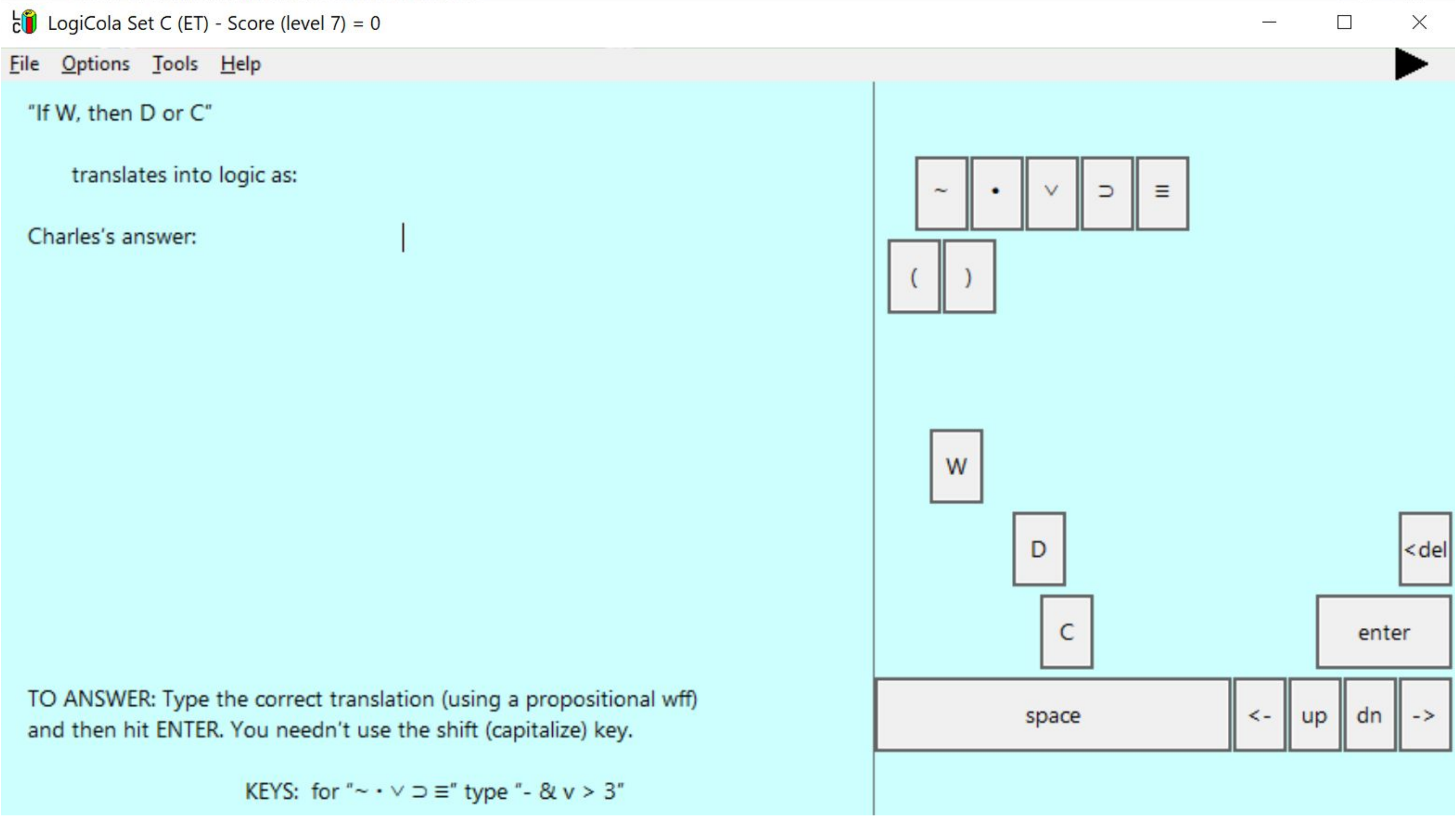
lcsetup (1).exe | lcsetup.exe | Toolkit---Accomm....pdf | Insurance contract.pdf | Show all

Exercise LogiCola C (EM & ET)



LogiCola C-ET




LogiCola Set C (ET) - Score (level 7) = 0
File Options Tools Help
"If W, then D or C"
translates into logic as:
Charles's answer:
~ • ∨ ⊃ ≡
( )
W
D
C
<del
enter
space
<- up dn ->
TO ANSWER: Type the correct translation (using a propositional wff) and then hit ENTER. You needn't use the shift (capitalize) key.
KEYS: for "~ • ∨ ⊃ ≡" type "- & v > 3"

Some useful rules in translation

- Rule: have your capital letters stand for whole statements

Here, "and" is not logic "and"

Wrong:	Bob and Lauren got married to each other	=	(B · L)
Right:	Bob and Lauren got married to each other	=	M

- Rule: put “(” wherever you see “both,” “either,” or “if.”

Either not A or B	=	$(\sim A \vee B)$
Not either A or B	=	$\sim(A \vee B)$
If both A and B, then C	=	$((A \cdot B) \supset C)$
Not both not A and B	=	$\sim(\sim A \cdot B)$

- Rule: Group together parts on either side of a comma.

If A, then B and C	=	$(A \supset (B \cdot C))$
If A then B, and C	=	$((A \supset B) \cdot C)$

If it snows then I'll go outside and I'll ski	=	If it snows, then I'll go outside and I'll ski	=	$(S \supset (G \cdot K))$

Exercise (LogiCola Exercise Set C)
Try E/H (easy/hard), M/T (mulC/type)

- 1. Not both A and B.
- 2. Both A and either B or C.
- 3. Either both A and B or C.
- 4. If A, then B or C.
- 5. If A then B, or C.
- 6. If not A, then not either B or C.
- 7. If not A, then either not B or C.
- 8. Either A or B, and C.
- 9. Either A, or B and C.
- 10. If A then not both not B and not C.
- 11. If you get an error message, then the disk is bad or it's a Macintosh disk.
- 12. If I bring my digital camera, then if my batteries don't die then I'll take pictures of my backpack trip and put the pictures on my Web site.

3.2 The meaning/semantics of wff Simple truth tables

- **<u>Define</u>** the meaning of connectives first…
  - What does P v Q mean?
- Use a **truth table** to define: It lists all possible truth-value combinations for the letters and says whether the wff is true or false, in each case.

Semantics of “~” in logic and in NL (Natural Language)
(Use 1 for True, 0 for False)

P	~P
0	1
1	0

$\sim 0 = 1$
$\sim 1 = 0$

“I *didn’t* go to Paris.”

“~P” has the *opposite* value of “P.”
“~P” is a *negation*.

Semantics of "·" in logic and in NL

"I went to Paris *and* I went to Quebec."

P	Q	(P • Q)
0	0	0
0	1	0
1	0	0
1	1	1

(0 • 0) = 0
(0 • 1) = 0
(1 • 0) = 0
(1 • 1) = 1

"(P • Q)" claims that *both* parts are true.
"(P • Q)" is a *conjunction*; P and Q are its *conjuncts*.

Semantics of “v” in logic and in NL

“I went to Paris *or* I went to Quebec.”

P	Q	$(P \vee Q)$
0	0	0
0	1	1
1	0	1
1	1	1

$(0 \vee 0) = 0$
$(0 \vee 1) = 1$
$(1 \vee 0) = 1$
$(1 \vee 1) = 1$

“$(P \vee Q)$” claims that *at least one* part is true.
“$(P \vee Q)$” is a *disjunction*; P and Q are its *disjuncts*.

Real-life "or" may be different from logic

"You can have all-you-can-eat soup or salad and bread".

- Exclusive "or": A or B but not both = $((A \vee B) \cdot \sim(A \cdot B))$
- Most logic books treat "either A or B" as exclusive OR… but this textbook treats "either A or B" as $(A \vee B)$
- "You better do this, or you will be in trouble"

Semantics of "⊃" in logic and in NL

"*If* I went to Paris, *then* I went to Quebec."

P	Q	(P ⊃ Q)
0	0	1
0	1	1
1	0	0
1	1	1

$(0 \supset 0) = 1$
$(0 \supset 1) = 1$
$(1 \supset 0) = 0$
$(1 \supset 1) = 1$

"(P ⊃ Q)" says we *don't* have the first part true and the second false.
"(P ⊃ Q)" is a *conditional*; P is the *antecedent* and Q the *consequent*.

Falsity implies anything.	$(0 \supset \ \ ) = 1$
Anything implies truth.	$(\ \ \supset 1) = 1$
Truth doesn't imply falsity.	$(1 \supset 0) = 0$

Some interesting examples of “if”

If I opened the door, … (counterfactual)

“If A then B” does NOT imply A… but is often taken otherwise.

Semantics of “≡” in logic and in NL

P	Q	(P ≡ Q)
0	0	1
0	1	0
1	0	0
1	1	1

$(0 \equiv 0) = 1$
$(0 \equiv 1) = 0$
$(1 \equiv 0) = 0$
$(1 \equiv 1) = 1$

“I went to Paris, *if and only if* I went to Quebec.”

“(P ≡ Q)” claims that both parts have the *same* truth value.
“(P ≡ Q)” is a *biconditional*.

Summarv

Basic Truth Equivalences

AND	OR	IF-THEN	IFF	NOT
$(0 \cdot 0) = 0$	$(0 \vee 0) = 0$	$(0 \supset 0) = 1$	$(0 \equiv 0) = 1$	
$(0 \cdot 1) = 0$	$(0 \vee 1) = 1$	$(0 \supset 1) = 1$	$(0 \equiv 1) = 0$	$\sim 0 = 1$
$(1 \cdot 0) = 0$	$(1 \vee 0) = 1$	$(1 \supset 0) = 0$	$(1 \equiv 0) = 0$	$\sim 1 = 0$
$(1 \cdot 1) = 1$	$(1 \vee 1) = 1$	$(1 \supset 1) = 1$	$(1 \equiv 1) = 1$	
both parts are true	*at least one* part is true	we don't have *first true & second false*	both parts have *same* truth value	*reverse* the truth value

Falsity implies anything.
Anything implies truth.
Truth doesn't imply falsity.

3.3 Truth evaluations

- We can calculate the truth value of a wff if we know the truth value of (all of) its letters.

LogiCola D (TM & TH)

If A=1 and B=1,
then "$(A \cdot \sim B)$" =

If A=0 and B=0,
then "$\sim(\sim A \supset B)$" =

If A=0, B=0, and C=0,
then "$\sim((A \vee B) \cdot \sim C)$" =

If A=0, B=1, and C=1,
then "$(\sim A \supset (\sim B \vee C))$" =

3.4 Unknown evaluations

- We can *<u>sometimes</u>* figure out a formula's truth value even if we don't know the truth value of some letters.
- **Exercise—LogiCola D (UE, UM, & UH)**
- All propositions: either true, false, or unknown

T=1 (T is true), F=0 (F is false), and U=?

$\boxed{(\sim T \cdot U)}$ $\boxed{(\sim 1 \cdot ?) = (0 \cdot ?) = 0}$

1. $(U \cdot F)$	4. $(\sim F \cdot U)$	7. $(U \supset \sim T)$	10. $(U \supset \sim F)$
2. $(U \supset \sim T)$	5. $(F \supset U)$	8. $(\sim F \vee U)$	11. $(U \cdot \sim T)$
3. $(U \vee \sim F)$	6. $(\sim T \vee U)$	9. $(T \cdot U)$	12. $(U \vee F)$

3.5 Truth tables of wff

- A wff with n distinct letters has 2^n possible truth-value combinations:

$((P \vee Q) \supset R)$

P	Q	R	$((P \vee Q) \supset R)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$((P \vee Q) \supset R)$

P	Q	R	$((P \vee Q) \supset R)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

1. $(P \equiv {\sim}Q)$
2. $({\sim}P \cdot Q)$
3. $(P \vee (Q \cdot {\sim}R))$
4. $((P \cdot {\sim}Q) \supset R)$
5. $((P \equiv Q) \supset Q)$
6. $((P \vee {\sim}Q) \supset R)$
7. $({\sim}Q \supset {\sim}P)$
8. $(P \equiv (P \cdot P))$
9. ${\sim}(P \cdot (Q \vee {\sim}R))$

Can be used to verify if two wff's are equivalent or not: iff the two truth tables are the same.

Also from truth table to wff

P	Q	R	$((P \vee Q) \supset R)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

- The truth table for “(P ∨ ~P)” is true in all cases, which makes the wff a **tautology**
  - “Good or bad”; “Tomorrow’s just another day”; “everyone is special”
- The truth table for “(P·~P)” is false in all cases: which makes the wff a **self-contradiction**
  - “This is the best time; this is the worst time”
- Otherwise (when the truth table has some 1 and some 0), the wff is a **contingent**
  - **SAT problem: satisfiable or not: NP**
  - Given a wff, how to tell if it is …

3.5a Exercise—LogiCola D (FM & FH)

Logical Paradox…

- Everything I say is a lie. Is this a lie?

- Barber paradox: An adult male barber shaves all and only men who do not shave themselves. Does he shave himself?

- One thing is certain in this world: nothing is certain.

Logical Paradox…

“Logical proof” that God does not exist. ☺

- God is supposed to be omnipotent (all-powerful).
- If He is omnipotent, then He can create a rock so big that He can't pick it up.
- If He cannot make a rock like this, then He is not omnipotent.
- If He can make a rock so big that He can't pick it up, then He isn't omnipotent either.
- Either way we demonstrated that God cannot do something.
- Therefore God is not omnipotent.
- Therefore God does not exist.

3.6 The truth-table test

- Our first “proof system”: given premises, to prove a conclusion, using truth tables (not a derivation)
- Construct a truth table showing the truth value of the premises and conclusion for all possible cases.
- The argument is **valid** (the conclusion is proved) if and only if for all rows (cases) that the premises are all true, the conclusion is also true.
- Otherwise (exists a case where premises are true but conclusion is false), the argument is **invalid**
- **Logically entail**

• If you’re a dog, then you’re an animal. You’re not an animal ∴ You’re not a dog

D A	$(D \supset A)$,	~A	~D
0 0	1	1	1
0 1	1	0	1
1 0	0	1	0
1 1	1	0	0

So, the conclusion is valid (can conclude that “you’re not an animal”

- If you're a dog, then you're an animal. You're not a dog. ∴ You're not an animal

D A	$(D \supset A)$,	$\sim D$	∴	$\sim A$	
0 0	1	1		1	
0 1	1	1		0	← **Invalid**
1 0	0	0		1	
1 1	1	0		0	

So, cannot conclude (or invalid to deduce) "you're not an animal
Cannot conclude that "you are an animal"

Short-cut table: do it faster

Letter comb	P1	P2	P3	…	C
	If any Pi is 0, no need to evaluate other P's and C (ignore this row and continue the table)				?
	If C is 1, no need to evaluate any Pi (ignore this row and continue the table)				1
	If all Pi are 1 and C is 0, stop the table. The argument is invalid.				0
	If the above case does not happen when you complete the table, the argument is valid.				

3.6a Exercise—LogiCola D (AE, AM, & AH)

3.6a Exercise (selected)

3. If television is always right, then Anacin is better than Bayer. If television is always right, then Anacin isn't better than Bayer. ∴ Television isn't always right. [Use T and B.]
4. If it rains and your tent leaks, then your down sleeping bag will get wet. Your tent won't leak. ∴ Your down sleeping bag won't get wet. [Use R, L, and W.]
7. If ethics depends on God's will, then something is good because God desires it. Something isn't good because God desires it. (Instead, God desires something because it's already good.) ∴ Ethics doesn't depend on God's will. [Use D and B; this argument is from Plato's *Euthyphro*.]
9. I'll go to Paris during spring break if and only if I'll win the lottery. I won't win the lottery. ∴ I won't go to Paris during spring break. [Use P and W.]

- If an argument “passes the truth-table test”, it means that the premises “entails” the conclusion (in semantics). $\models$
- The truth-table test can get tedious for long arguments. Arguments with 6 letters may need 64 lines, and ones with 10 letters need 1024 lines. Can have infinite # of letters
- Can we do it based only on “syntax”? Yes, see 3.10- , Chapter 4, …

3.7 The truth-assignment test

- Take a propositional argument. Set each premise to 1 and the conclusion to 0. The argument is **VALID** if and only if no consistent way of assigning 1 and 0 to the letters will make this work—so we can't make the premises all true and conclusion false.
- You **REFUTE** the argument if you can find such an assignment
  - Finding a "counter example"

$$
\begin{array}{lcl}
\begin{array}{l}
(L \vee R) = 1 \\
\sim L = 1 \\
\therefore\ R = 0
\end{array}
& \rightarrow &
\begin{array}{l}
(L^0 \vee R^0) = 1 \\
\sim L^0 = 1 \\
\therefore\ R^0 = 0
\end{array}
\end{array}
\rightarrow
\begin{array}{l}
\overset{0}{\overline{(L^0 \vee R^0)}} \neq 1 \quad \text{Valid} \\
\sim L^0 = 1 \\
\therefore\ R^0 = 0
\end{array}
$$

Exercise—LogiCola E (S); E (E)

Translate and then prove:

If you're rich, then you aren't both dishonest and not fanatical.
You aren't dishonest.
You aren't fanatical.
∴ You aren't rich.

If you're rough or a foreigner, then you're greedy and confused.
You're rough.
∴ You're greedy.

Harder Translations

- Read Chapter 3.8 by yourselves
- Read Chapter 3.9 by yourselves

- **3.8a Exercise—LogiCola C (HM & HT)**
- **3.9a Exercise—LogiCola E (F I)**

3.10/3.11 S-rules, I-rules

- **Formal proof system: “step by step”, by syntax only**
- **Inference rules,** which state that certain formulas can be derived with validity from certain other formulas, ***mechanically*** – **algorithm**
- “Deduce”, “formally deducible”: $\vdash$
- Also check mechanically if a proof is valid
- They reflect sound reasoning
- What we hope to have (see later)…
  - Everything that is deduced is indeed true. **Sound: A $\vdash$ B then A $\models$ B**
  - Everything that is true can be deduced. **Complete: A $\models$ B then A $\vdash$ B**
- Here is one proof system: using (only) S/I rules
  - You need to do it “by hand” like a machine

Note: P, Q can be any wff. E.g., $(A \cdot (B \cdot C))$

S-Rules

This and that. ∴ This. ∴ That.	$\dfrac{(P \cdot Q)}{P, Q}$	AND statement, so both parts are true.
Not either this or that. ∴ Not this. ∴ Not that.	$\dfrac{\sim(P \vee Q)}{\sim P, \sim Q}$	NOT-EITHER is true, so both are false.
False if-then. ∴ First part true. ∴ Second part false.	$\dfrac{\sim(P \supset Q)}{P, \sim Q}$	FALSE IF-THEN, so first part true, second part false.

3.10a Exercise—LogiCola F (SE & SH)

All S-rules

S-rules

$$(P \cdot Q) \rightarrow P, Q$$
$$\sim(P \vee Q) \rightarrow \sim P, \sim Q$$
$$\sim(P \supset Q) \rightarrow P, \sim Q$$
$$\sim\sim P \rightarrow P$$
$$(P \equiv Q) \rightarrow (P \supset Q), (Q \supset P)$$
$$\sim(P \equiv Q) \rightarrow (P \vee Q), \sim(P \cdot Q)$$

Also: $P \rightarrow \sim\sim P$

I-Rules

Not both are true.	$\sim(P \cdot Q)$	$\sim(P \cdot Q)$	*Deny* AND.
This one is true.	P	Q	*Affirm* one part.
∴ The other isn't.	$\sim Q$	$\sim P$	∴ *Deny* other part.

At least one is true.	$(P \vee Q)$	$(P \vee Q)$	*Affirm* OR.
This one isn't.	$\sim P$	$\sim Q$	*Deny* one part.
∴ The other is.	Q	P	∴ *Affirm* other part.

IF-THEN.	$(P \supset Q)$
Affirm first.	P
∴ Affirm second.	Q

Modus Ponens

IF-THEN.	$(P \supset Q)$
Deny second.	$\sim Q$
∴ Deny first.	$\sim P$

Modus Tolens

3.11a Exercise—LogiCola F (IE & IH)

P and Q can be any wff

- Exercise

$\sim(\sim A \vee (B \cdot C))$

$\sim(\sim A \vee (B \cdot C))$
$A, \sim(B \cdot C)$

1. $\sim((A \cdot B) \supset \sim C)$

2. $((A \cdot B) \supset \sim C)$
   $\sim(A \cdot B)$

3. $\sim((G \vee H) \cdot (I \vee J))$

4. $\sim((G \vee H) \cdot (I \vee J))$
   $(G \vee H)$

5. $((A \cdot B) \vee (C \supset D))$

6. $((A \cdot B) \vee (C \supset D))$
   C

7. $\sim((A \supset B) \vee C)$

8. $((A \supset B) \supset C)$
   $(A \supset B)$

9. $((G \equiv H) \supset \sim(I \cdot J))$
   $\sim(I \cdot J)$

Rules you can use

S-rules

$$(P \cdot Q) \rightarrow P, Q$$
$$\sim(P \vee Q) \rightarrow \sim P, \sim Q$$
$$\sim(P \supset Q) \rightarrow P, \sim Q$$
$$\sim\sim P \rightarrow P$$
$$(P \equiv Q) \rightarrow (P \supset Q), (Q \supset P)$$
$$\sim(P \equiv Q) \rightarrow (P \vee Q), \sim(P \cdot Q)$$

I-rules

$$\sim(P \cdot Q), P \rightarrow \sim Q$$
$$\sim(P \cdot Q), Q \rightarrow \sim P$$
$$(P \vee Q), \sim P \rightarrow Q$$
$$(P \vee Q), \sim Q \rightarrow P$$
$$(P \supset Q), P \rightarrow Q$$
$$(P \supset Q), \sim Q \rightarrow \sim P$$

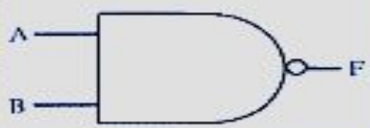
Sound but Incomplete…

- These rules are certainly sound
- But incomplete…
  - Cannot prove this

$$(A \vee B)$$
$$\therefore (\sim A \supset B)$$

Next Chapter 4: sound and complete proof system

3.14 Logic and computers

<table>
<tr><th>Name</th><th>Graphic Symbol</th><th>Algebraic Function</th><th>Truth Table</th></tr>
<tr><td>AND</td><td></td><td>$F = A \cdot B$
or
$F = AB$</td><td>A B | F
0 0 | 0
0 1 | 0
1 0 | 0
1 1 | 1</td></tr>
<tr><td>OR</td><td></td><td>$F = A + B$</td><td>A B | F
0 0 | 0
0 1 | 1
1 0 | 1
1 1 | 1</td></tr>
<tr><td>NOT</td><td></td><td>$F = \overline{A}$
or
$F = A'$</td><td>A | F
0 | 1
1 | 0</td></tr>
<tr><td>NAND</td><td></td><td>$F = (\overline{AB})$</td><td>A B | F
0 0 | 1
0 1 | 1
1 0 | 1
1 1 | 0</td></tr>
<tr><td>NOR</td><td></td><td>$F = (\overline{A + B})$</td><td>A B | F
0 0 | 1
0 1 | 0
1 0 | 0
1 1 | 0</td></tr>
</table>