Lab 4

September 26, 2021

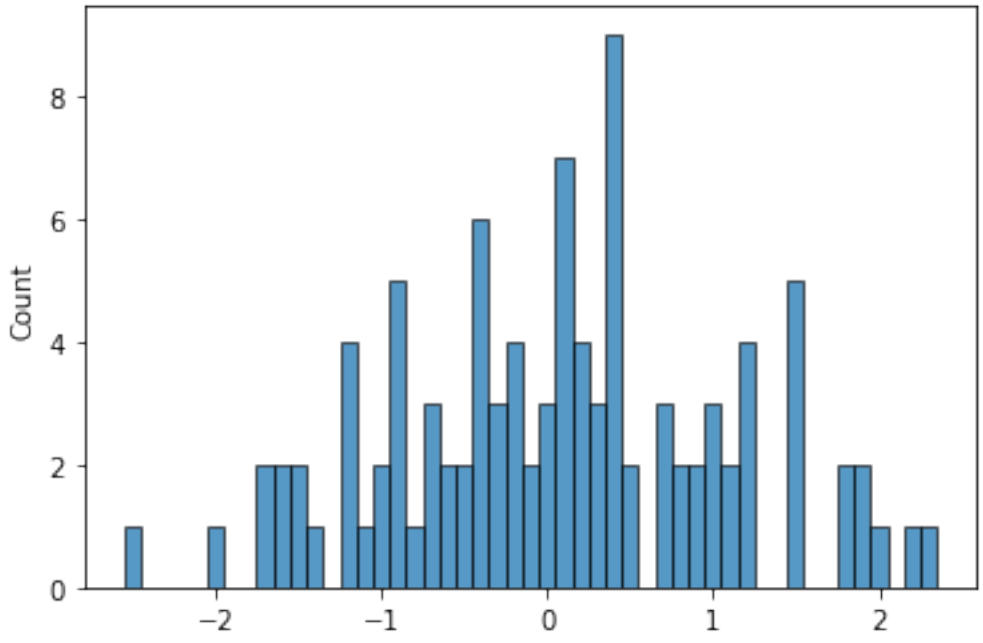
In this lab we discuss normal data and calculation of normal proportions.

1 Standard Normal Distribution (mean of zero and variance of one)

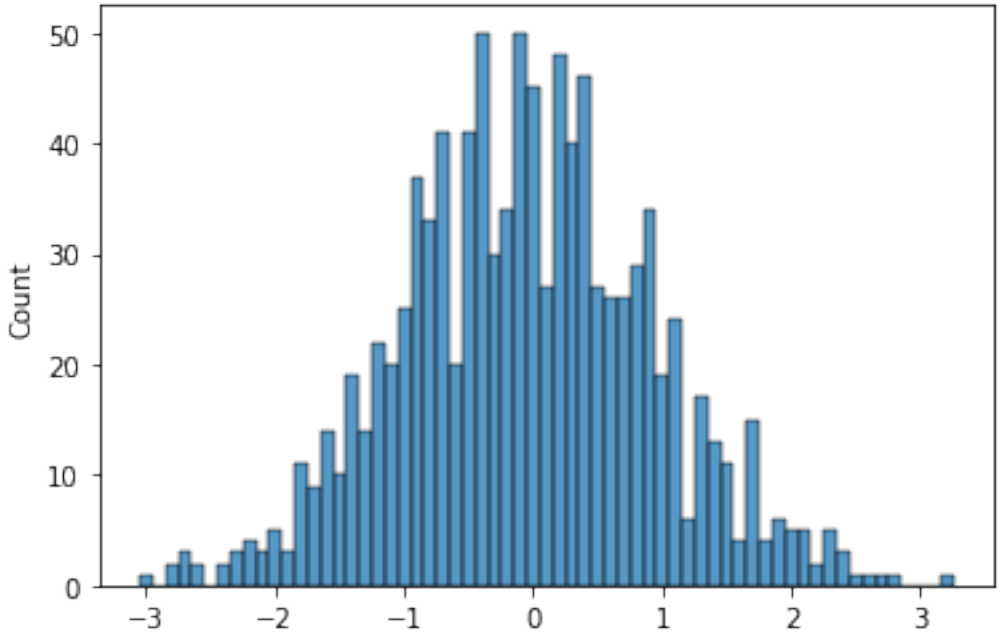
numpy.random.randn: https://numpy.org/doc/stable/reference/random/generated/numpy.random.randn.html

[1]:
```
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
from scipy.stats import norm
from numpy.random import randn
```

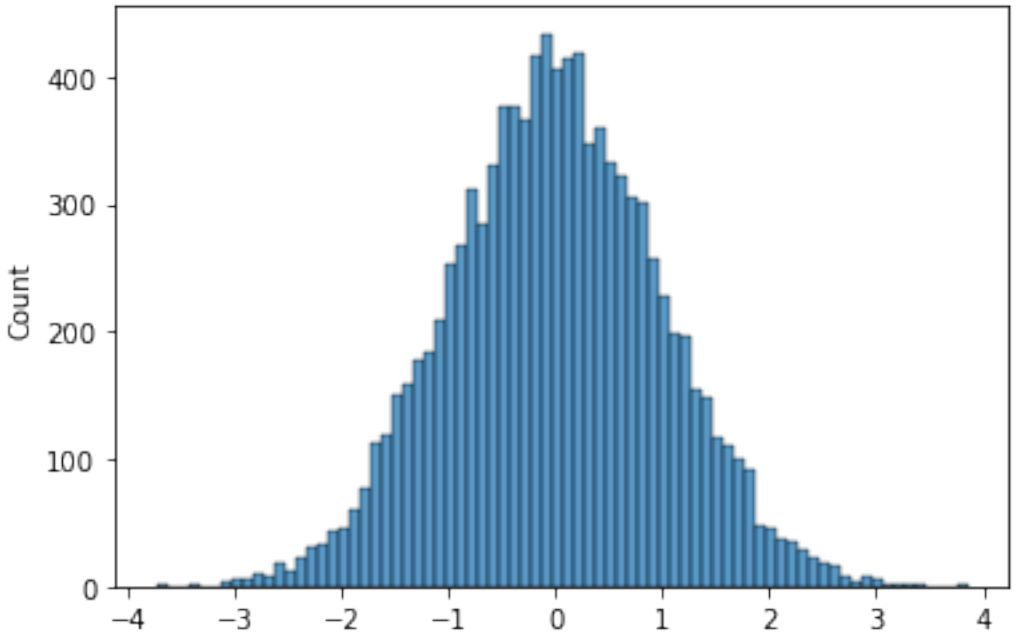
[2]:
```
# Setting the seed for the random generation
# We need to set a seed in order to be able to reproduce the results.
np.random.seed(0)

# Standard normal distribution with 100 points
# randn() returns a sample from the "standard normal" distribution.
data_100 = randn(100)
sns.histplot(data_100, binwidth = 0.1)
plt.show()
```



[3]:
```
# Standard normal distribution with 1000 points
data_1000 = randn(1000)
sns.histplot(data_1000, binwidth = 0.1)
plt.show()
```



[4]:
```
# Standard normal distribution with 10000 points
data_10000 = randn(10000)
sns.histplot(data_10000, binwidth = 0.1)
plt.show()
```

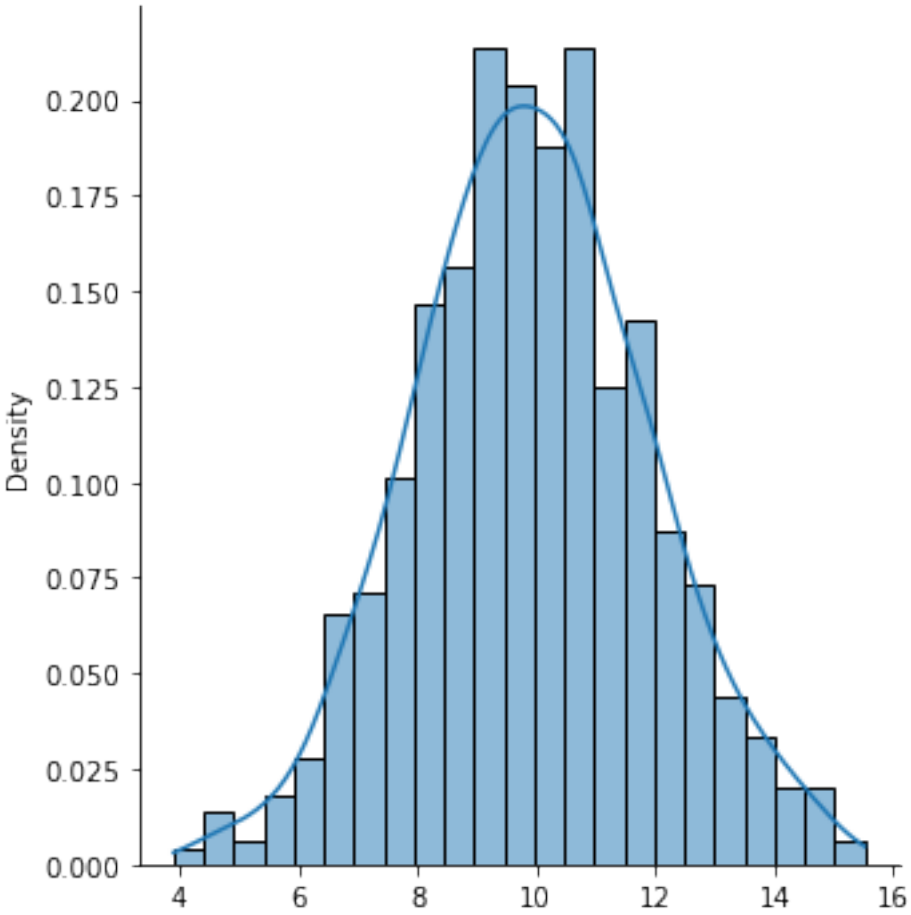


2 Normal Distribution

scipy.stats.norm: https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.norm.html

2.1 Generating 1000 data points from a normal distribution with mean of 10 and standard deviation of 2.

[5]:
```
# loc and scale denote mean and standard deviation, respectively.
normal = norm.rvs(loc = 10, scale = 2, size = 1000, random_state = 0)
sns.displot(normal, stat = 'density', kde = True)
plt.show()
```



2.2 Finding the probability that the variable has a value less than or equal to 12.

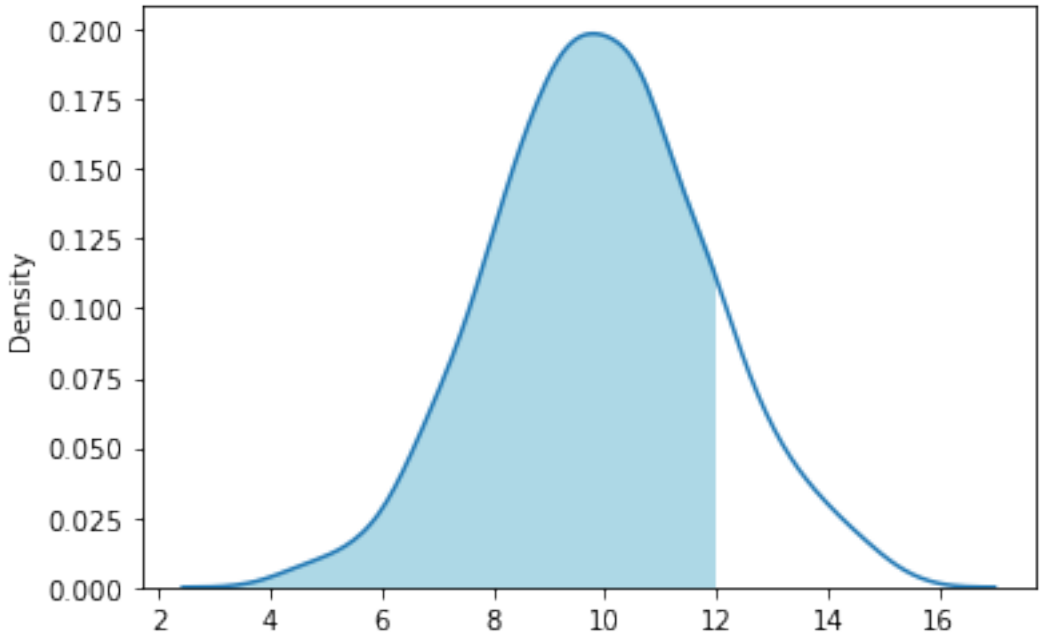
[6]:
```
norm.cdf(12, loc = 10, scale = 2)
```

[6]:
```
0.8413447460685429
```

This probability corresponds to the following area under the density curve:

[7]:
```
points = sns.kdeplot(normal).get_lines()[0].get_data()
x = points[0]
y = points[1]
plt.fill_between(x,y, where = x <=12, color='lightblue')
```

[7]:
```
<matplotlib.collections.PolyCollection at 0x12185aa00>
```



2.3 We can also calculate the probability above using the z-score and the standard normal distribution.

[8]:
```
z = (12 - 10)/2
print(z)
```

```
1.0
```

[9]:
```
norm.cdf(1) ## in this case the default for loc and scale is 0 and 1, a␣
↪standard normal distribution
```

[9]:
```
0.8413447460685429
```

2.4 Finding the probability that the variable has a value greater than or equal to 12. This probability is equal to 1 - norm.cdf(12, loc = 10, scale = 2).

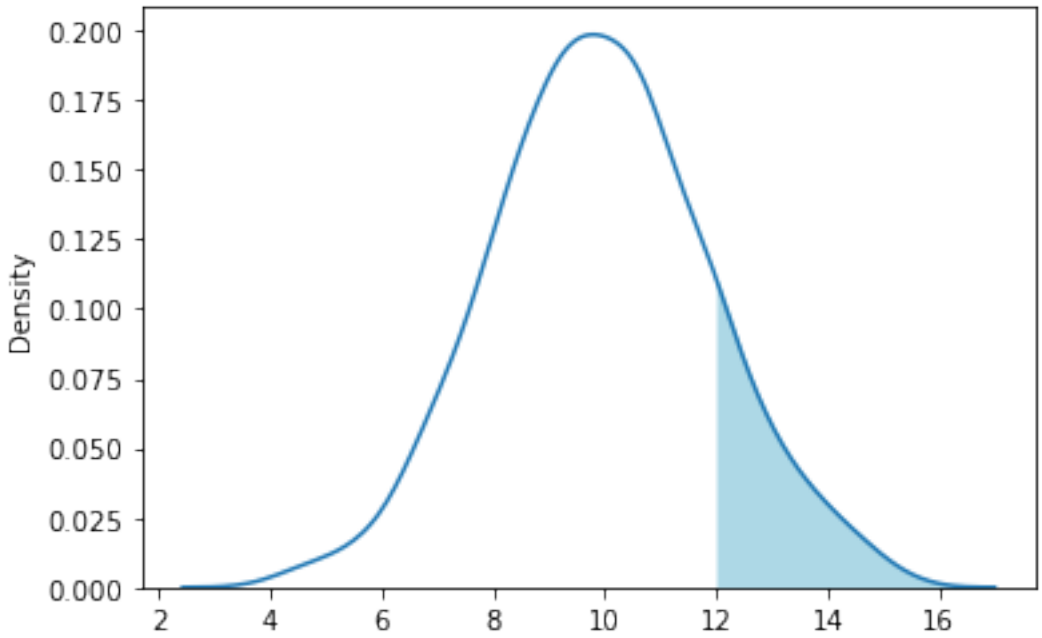
[10]:
```
1 - norm.cdf(12, loc = 10, scale = 2)
```

[10]:
```
0.15865525393145707
```

This probability corresponds to the following area under the density curve:

[11]:
```
points = sns.kdeplot(normal).get_lines()[0].get_data()
x = points[0]
y = points[1]
plt.fill_between(x,y, where = x >=12, color='lightblue')
```

[11]: <matplotlib.collections.PolyCollection at 0x1219d7a90>



2.5 Finding the variable value that gives you a certain probability.

Example 1: which variable value x gives you a probability of 0.84 for that variable to be less than or equal to x? The output is very close to 12.

[12]:
```
norm.ppf(0.84, loc = 10, scale = 2)
```

[12]: 11.988915766419506

Example 2: which variable value x gives you a probability of 0.025 for the variable to be less than or equal to x? The output is close to 6.

[13]:
```
norm.ppf(0.025, loc = 10, scale = 2)
```

[13]: 6.080072030919891

Example 3: which variable value x gives you a probability of 0.16 for the variable to be greater than or equal to x? The output is close to 12.

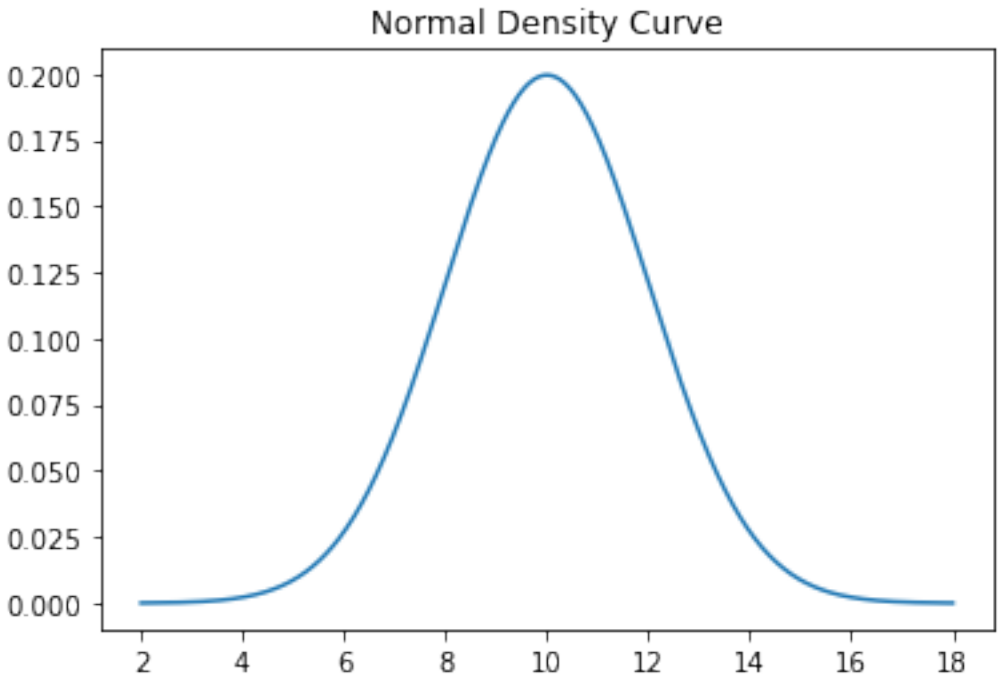
[14]:
```
norm.ppf(1-0.16, loc = 10, scale = 2) ## this value is the same as the one in␣
↪Example 1
```

[14]: 11.988915766419506

3 The 68–95–99.7 Rule

3.1 Plotting the probability density function (pdf) corresponding to a normal distribution with a mean (mu) of 10 and a standard deviation (sigma) of 2.

[15]:
```
mu = 10
sigma = 2
x = np.linspace(mu - 4 * sigma, mu + 4 * sigma, 1000) # creating a range of x␣
 ↪values
pdf = norm.pdf(x, loc = mu, scale = sigma) # calculating the density values␣
 ↪corresponding to each x
plt.plot(x, pdf) # ploting density versus x
plt.title('Normal Density Curve')
plt.show()
```



3.2 Percentage of observations between (mu - sigma) and (mu + sigma):

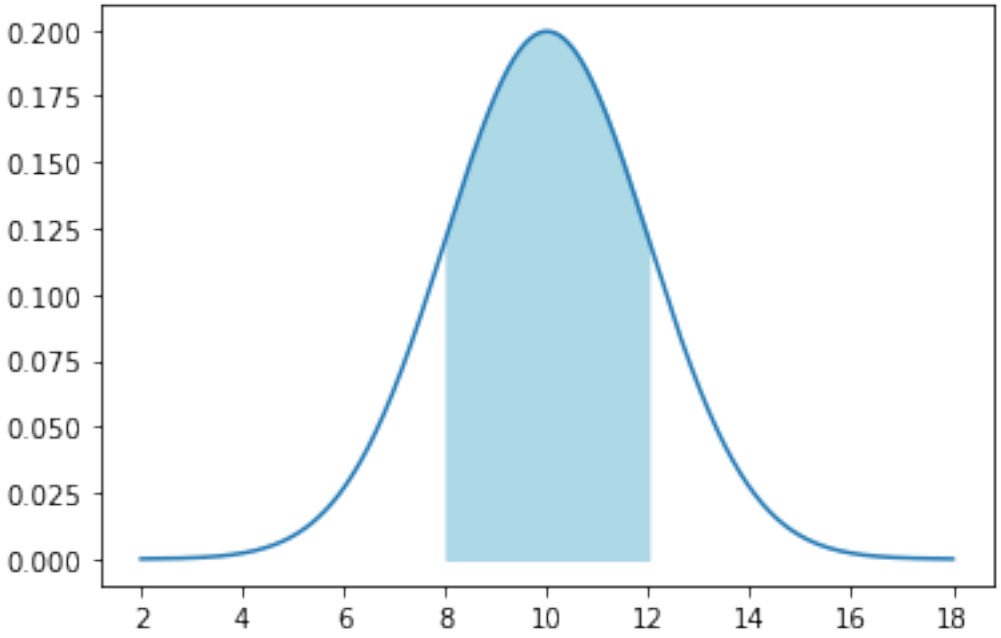
[16]:
```
cdf_upper_limit = norm.cdf(mu + sigma, loc = mu, scale = sigma)
cdf_lower_limit = norm.cdf(mu - sigma, loc = mu, scale = sigma)
percent = cdf_upper_limit - cdf_lower_limit
print(np.round(percent * 100, 2), '% of observations fall between (mu - sigma)␣
 ↪and (mu + sigma).')
```

```
68.27 % of observations fall between (mu - sigma) and (mu + sigma).
```

This percentage corresponds to the following area under the curve:

[17]:
```
plt.plot(x, pdf)
plt.fill_between(x,pdf, where = (x<=12) & (x>=8), color='lightblue')
```

[17]:
```
<matplotlib.collections.PolyCollection at 0x121adf430>
```



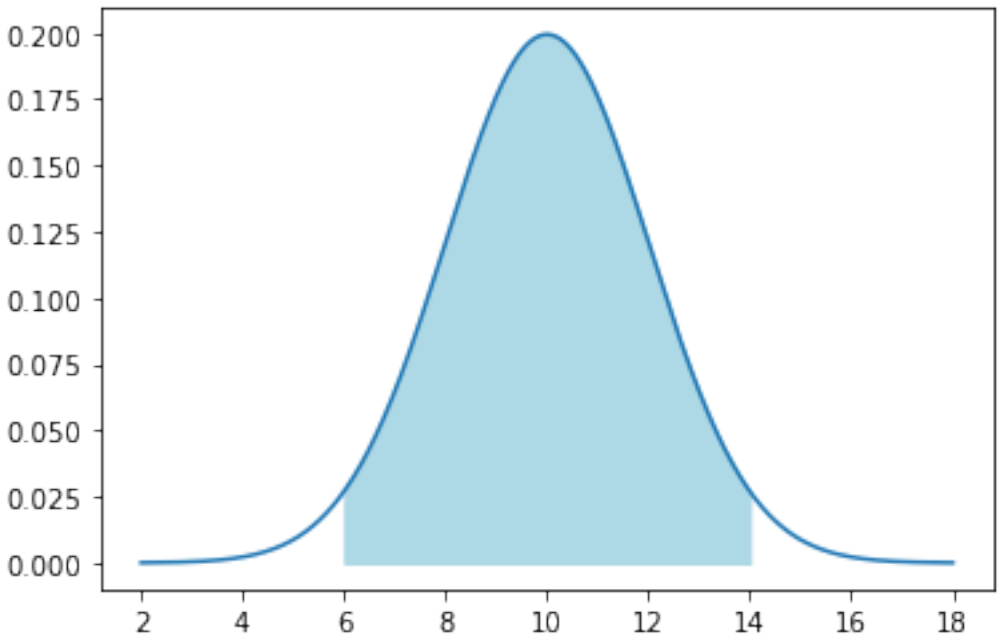
3.3 Percentage of observations between (mu - 2 * sigma) and (mu + 2 * sigma):

[18]:
```
cdf_upper_limit = norm.cdf(mu + 2 * sigma, loc = mu, scale = sigma)
cdf_lower_limit = norm.cdf(mu - 2 * sigma, loc = mu, scale = sigma)
percent = cdf_upper_limit - cdf_lower_limit
print(np.round(percent * 100, 2), '% of observations fall between (mu - 2 *␣
↪sigma) and (mu + 2 * sigma).')
```

```
95.45 % of observations fall between (mu - 2 * sigma) and (mu + 2 * sigma).
```

[19]:
```
plt.plot(x, pdf)
plt.fill_between(x,pdf, where = (x<=14) & (x>=6), color='lightblue')
```

[19]:
```
<matplotlib.collections.PolyCollection at 0x121702d90>
```



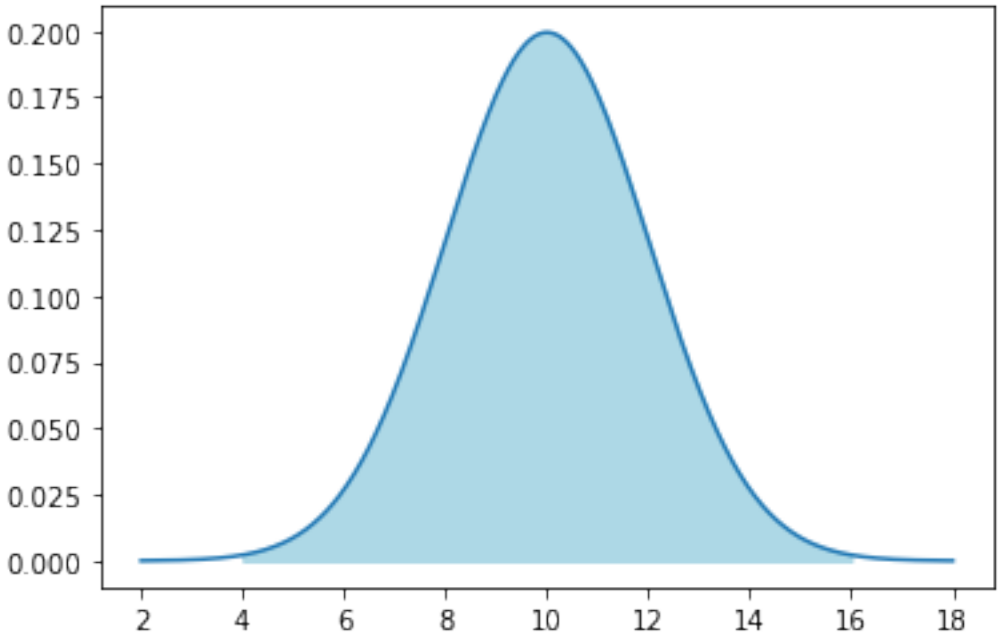
3.4 Percentage of observations between (mu - 3 * sigma) and (mu + 3 * sigma):

[20]:
```
# Percentage of observations between (mu - 3 * sigma) and (mu + 3 * sigma)
cdf_upper_limit = norm.cdf(mu + 3 * sigma, loc = mu, scale = sigma)
cdf_lower_limit = norm.cdf(mu - 3 * sigma, loc = mu, scale = sigma)
percent = cdf_upper_limit - cdf_lower_limit
print(np.round(percent * 100, 2), '% of observations fall between (mu - 3 *␣
↪sigma) and (mu + 3 * sigma).')
```

```
99.73 % of observations fall between (mu - 3 * sigma) and (mu + 3 * sigma).
```

[21]:
```
plt.plot(x, pdf)
plt.fill_between(x,pdf, where = (x<=16) & (x>=4), color='lightblue')
```

[21]:
```
<matplotlib.collections.PolyCollection at 0x1215c4520>
```


0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
2
4
6
8
10
12
14
16
18

[ ]: