Operations on matrices

Matrix

Definition. A *matrix* is a rectangular array of numbers

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

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The entries of a matrix are denoted by the corresponding lowercase letter with "double subscript", such as a_{ij}.

The number a_{ij} sitting at the i-th row and the j-th column is called the *(i, j)-entry* of the matrix.

Example

To describe a 3×4 matrix, we have

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}.$$

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If B is a 3×2 matrix given by

$$B = \begin{bmatrix} 1 & 2 \\ -3 & 4 \\ 5 & -6 \end{bmatrix}$$

then $b_{12} = 2$ and $b_{21} = -3$. What is b_{32}?

Rows and columns

The horizontal lines of numbers are rows of a matrix and the vertical lines of numbers are columns of a matrix.
A matrix with m rows and n columns is called an $m \times n$ matrix (pronounced "m by n").

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The numbers m and n are called the *dimensions* of the matrix.
For instance, $[a_{11}, a_{12}, \ldots, a_{1n}]$ is the first row of the matrix.
$[a_{i1}, a_{i2}, \ldots, a_{in}]$ is the i-th row of the matrix.

$\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ and $\begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$ are the first and the j-th columns of the matrix, respectively.

Definitions

- For $n > 1$, a $1 \times n$ matrix is called a *row vector*.

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \end{bmatrix}$$

- For $m > 1$, an $m \times 1$ matrix is called a *column vector*.

$$\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$$

Definitions

• Two matrices are *equal* if they have the same dimensions and their corresponding entries are equal (i.e., they are both $m \times n$ matrices and $a_{ij} = b_{ij}$).

• The *transpose*, denoted by A^T, of an $m \times n$ matrix A is an $n \times m$ matrix whose (i, j)-entry is a_{ji}.

That is to say, let $B = A^T$ and let b_{ij} denote the (i, j)-entry of $B = A^T$. Then $b_{ij} = a_{ji}$, where a_{ji} is the (j, i)-entry of A.

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For instance, if $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & -6 & 5 \end{bmatrix}$, then $A^T = \begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 3 & -6 \\ 4 & 5 \end{bmatrix}$.

Examples

State whether matrices A and B are equal.

(a)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 1 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 1 & 5 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 5 & 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 3 \\ 5 & 1 & 2 \end{bmatrix}$$

(c)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

(d)

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

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$$A = \begin{bmatrix} 1 & 0 & 3 \\ 5 & 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 3 \\ 5 & 1 & 2 \end{bmatrix} \qquad \textit{NO}$$

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For an $n \times n$ square matrix A, the entries a_{ii}, for $i = 1, 2, \ldots, n$ are called the *diagonal* of the matrix.

$$\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}$$

A *diagonal matrix* is a square matrix such that $a_{ij} = 0$ for $i \neq j$.

For example, the following matrices are diagonal matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}.$$

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The *identity matrix* I_n is an $n \times n$ diagonal matrix such that $a_{ii} = 1$.

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

Operations

Let A and B be two $m \times n$ matrices. Let c be a scalar.

• The *sum* $C = A + B$ of matrices A and B is an $m \times n$ matrix whose (i, j)-entry c_{ij} is given by $c_{ij} = a_{ij} + b_{ij}$.

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• The *negative* $-A$ of A is an $m \times n$ matrix whose (i, j)-entry is given by $-a_{ij}$.

1. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & a & b \\ c & 2 & 1 \end{bmatrix}$, are there any values of a, b and c for which $3A = 2B^T$?

2. Given A and B as follows, find $A - 3B$ and $(2A - 3I + B^T)^T$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$$

3. Find the sum of matrices A and B, if possible, in each of the following.

(a)

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 0 & -2 & 2 \end{bmatrix} B = \begin{bmatrix} 1 & -5 & 2 \\ -2 & 4 & 6 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

(c)

$$A = \begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix} \qquad - B^T = \begin{bmatrix} 2 & 7 & 4 \end{bmatrix}$$

Next time, we will do matrix multiplication.