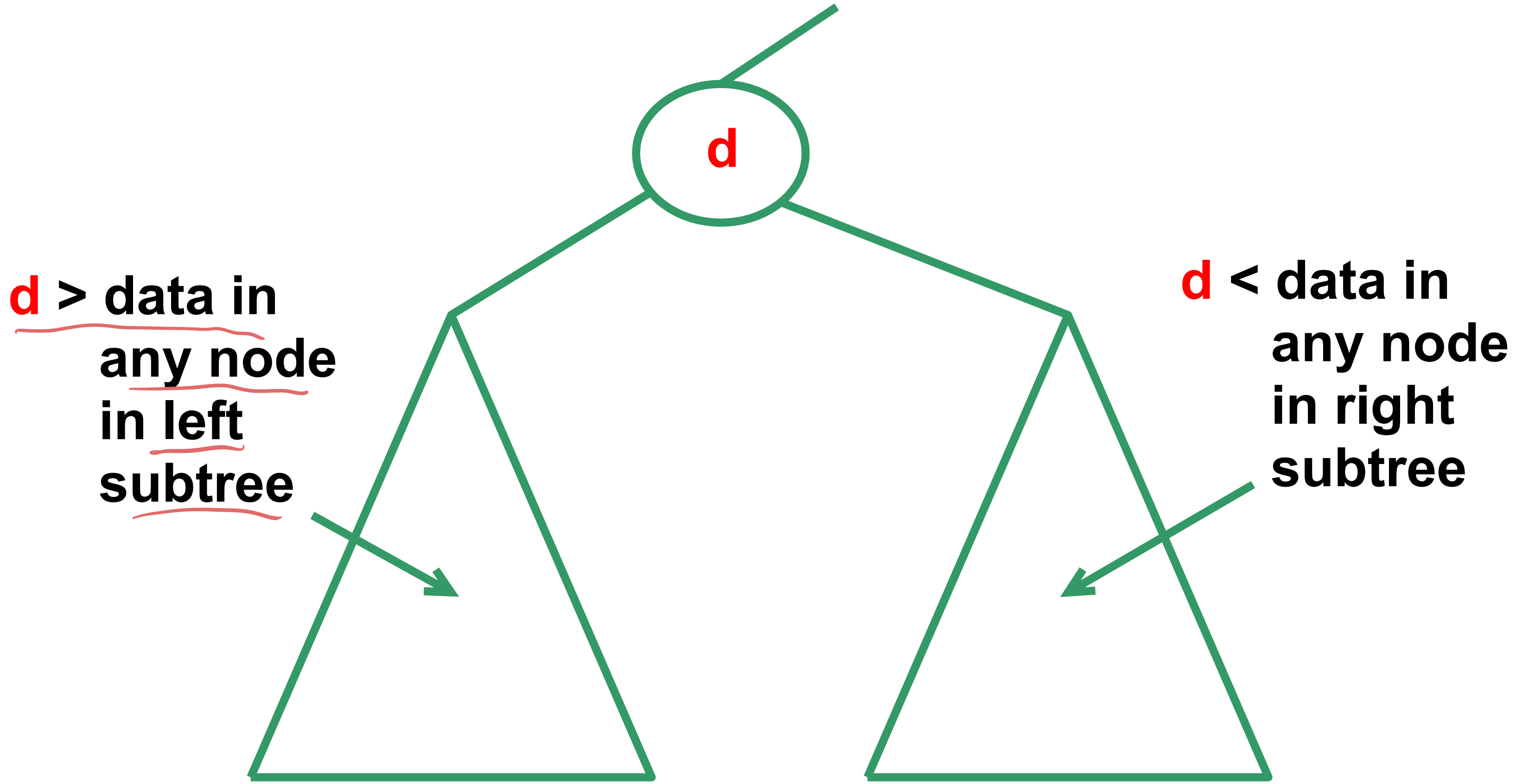
The Binary Search Tree ADT

Binary Search Tree

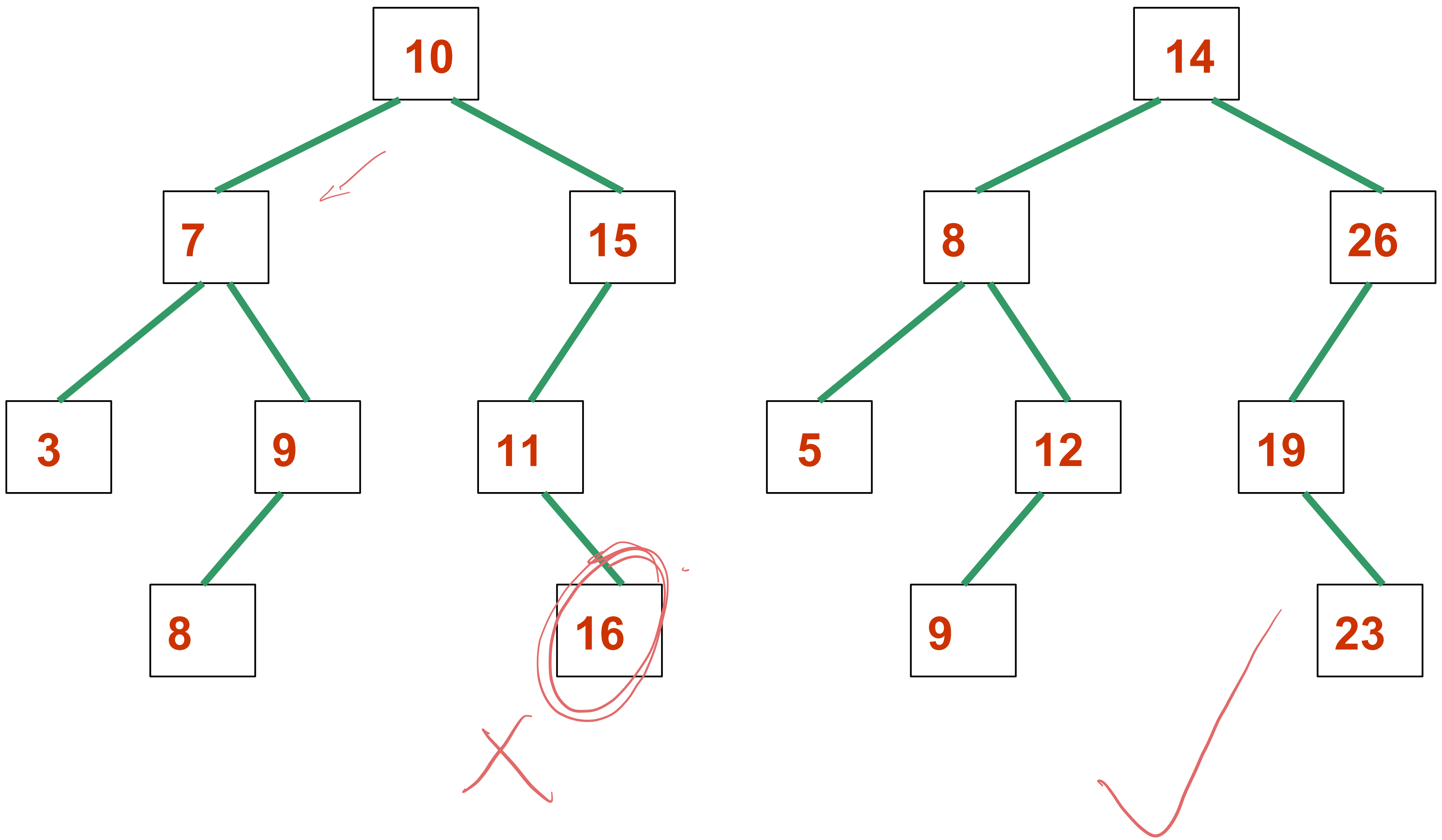
- A ***binary search tree (BST)*** **is a binary tree with an** ***ordering*** **property of its elements, such that the data in any internal node is**
  - ***Greater than*** **the data in any node in its left subtree**
  - ***Less than*** **the data in any node in its right subtree**
- ***Note*****: this definition does not allow duplicates; some definitions do, in which case we could say "*****less than or equal to*****"**

Binary Search Tree

A *binary search tree (BST)* is a binary tree with the following *ordering* property on all its internal nodes:



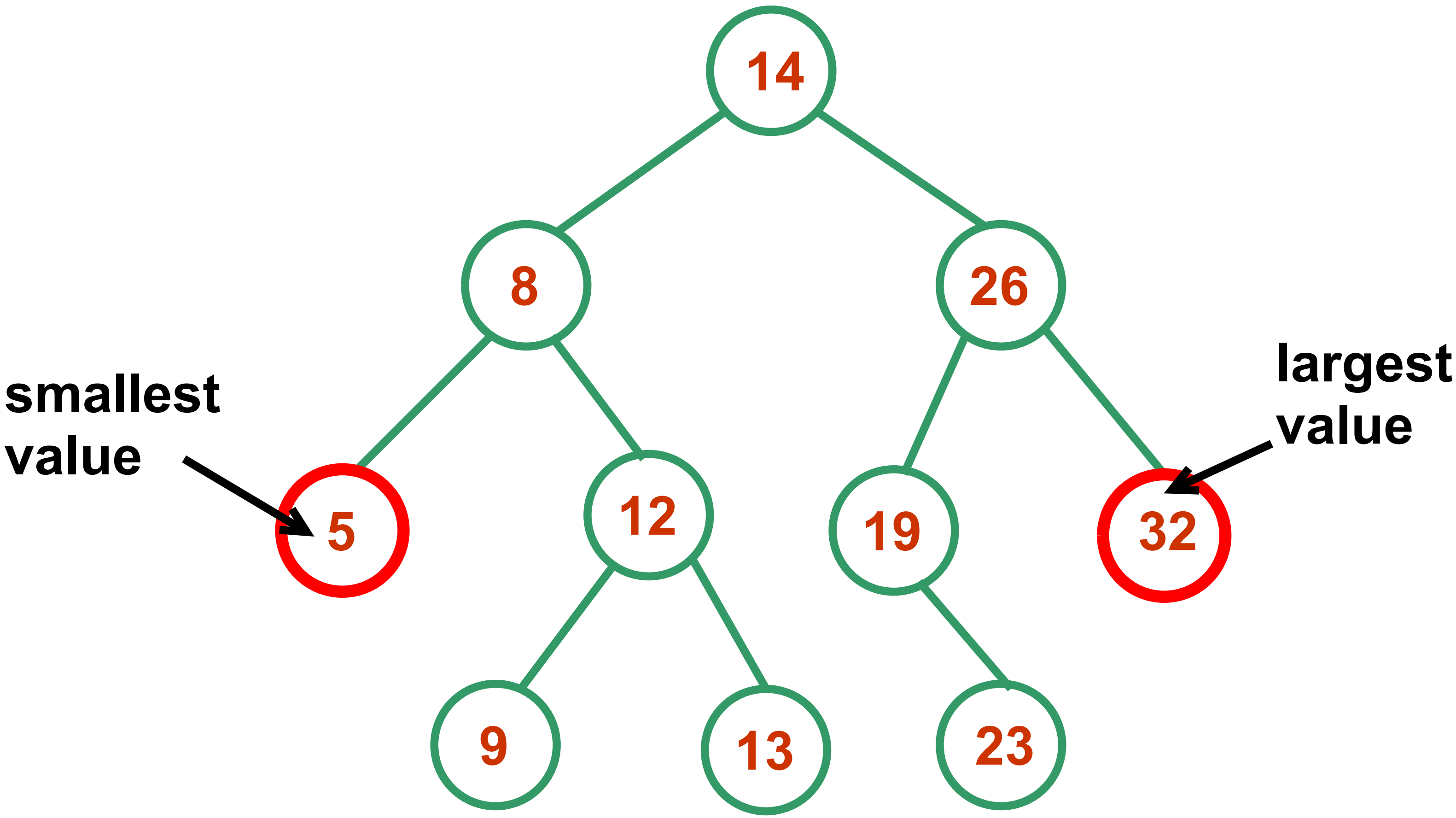
Examples: are these Binary Search Trees?



Discussion

- **Observations:**
  - **What is in the leftmost node?**
  - **What is in the rightmost node?**

Properties of Binary Search Trees



BST Operations

- A binary search tree is a special case of a binary tree
  - So, it has all the operations of a binary tree
- It also has *operations specific to a BST*:
  - *add* an element (requires that the BST property be maintained)
  - *remove* an element (requires that the BST property be maintained)
  - *remove the maximum* element
  - *remove the minimum* element

Searching in a BST

- **Why is it called a binary *search* tree?**
  - **Data is stored in such a way, that it can be more *efficiently* found than in an ordinary binary tree**

Searching in a BST

- **Algorithm to *search* for an item in a BST**
  - **Compare data item to the root of the (sub)tree**
  - **If data item = data at root, found**
  - **If data item < data at root, go to the left; if there is no left child, data item is not in tree**
  - **If data item > data at root, go to the right; if there is no right child, data item is not in tree**

```
private BinaryTreeNode<T> find (T element, BinaryTreeNode<T> r) {
    if (r == null) return null;
    else {
        Comparable<T> comparableElement = (Comparable<T>)element;
        if (comparableElement.compareTo(r.element) == 0)
            return r;
        else if (comparableElement.compareTo(r.element) > 0)
                return find(element,r.right);
            else return find(element,r.left);
    }
}
```

← Base Case.

element - r.element.

=0 => equal

>0 smaller than target

< else.

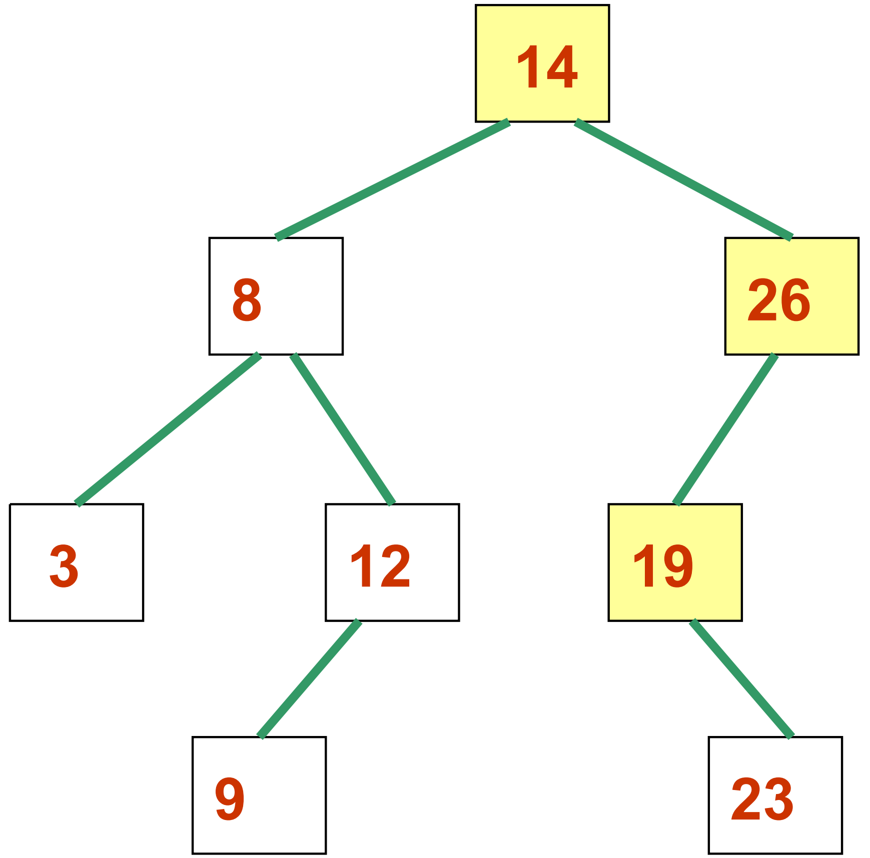
Base Case: Found => return r

not found => return null

Recursive Case: return Find.getRight() :? <

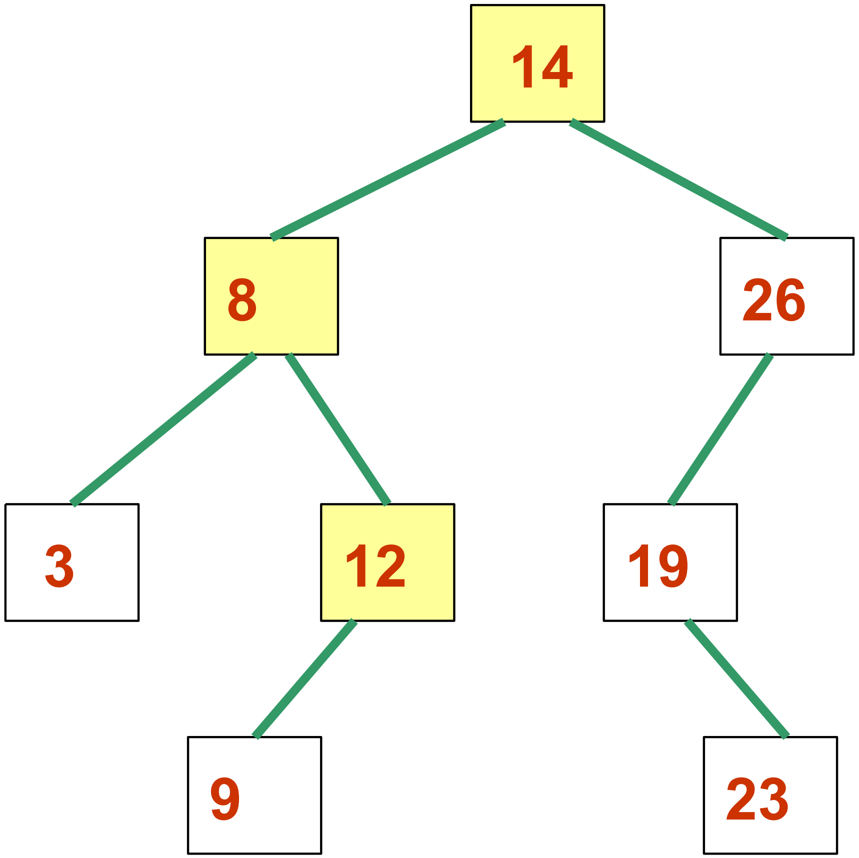
else return find.getLeft.

Search Operation



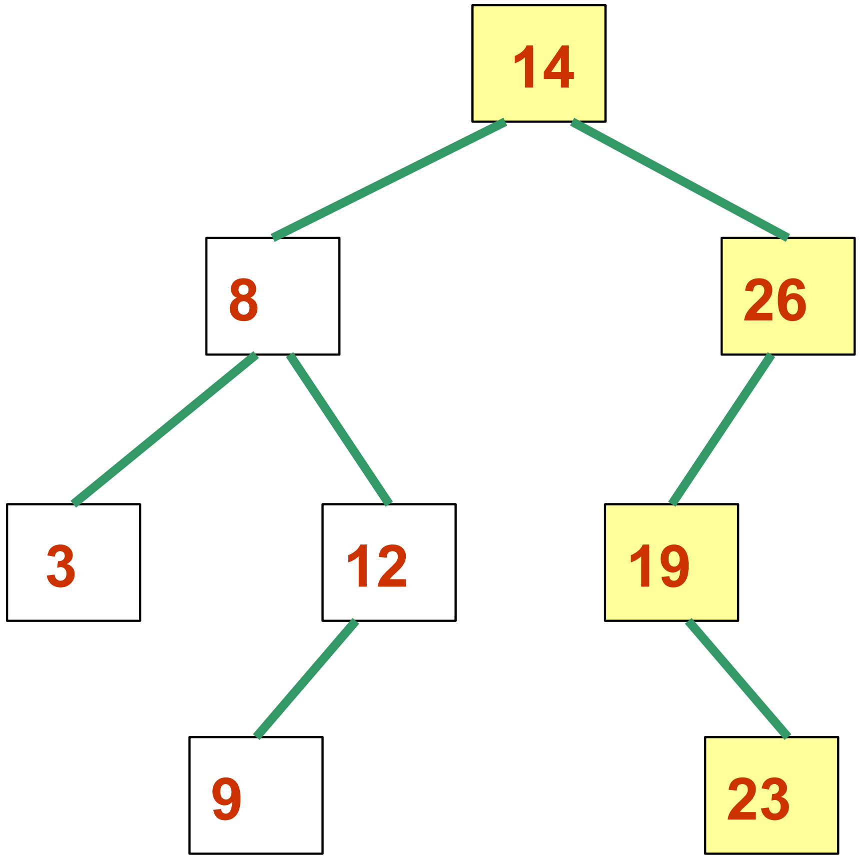
Search for 19: **visited nodes are coloured yellow; 19 > 14 so look into right child of 14, which is 26. 19 < 26 so look into left child of 26. This child is 19 and we stop because we found the target 19.**

Search Operation



Search for 13: **visited nodes are coloured yellow; return false when node containing 12 has no right child**

Search Operation



Search for 22: **return false when node containing 23 has no left child**

BST Operations: add

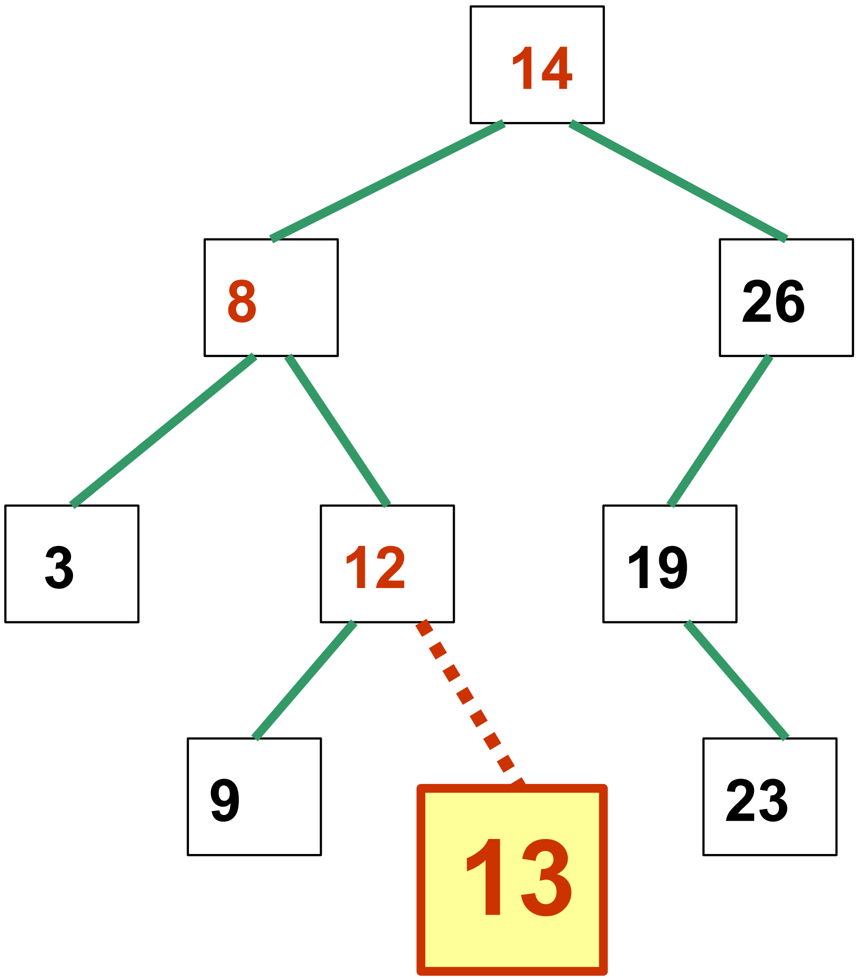
- To ***add*** an item to a BST:
  - Follow the algorithm for searching, until there is no child
  - Insert at that point

- So, new node will be added as a leaf
- (We are assuming no duplicates allowed)

```
void
public add( T node ){
      if((root > node)&&(root.getRight==null)){
            root.setRight(node);
Base case :
      else if((root < node)&&(root.getLeft==null)){
            root.setLeft(node);
```

Add Operation

Recursive Part.
else {
if (root > node) { root.getRight().add(node) }
else { root.getLeft().add(node) }.
}



To insert 13:

Same nodes are visited as when *searching* for 13.

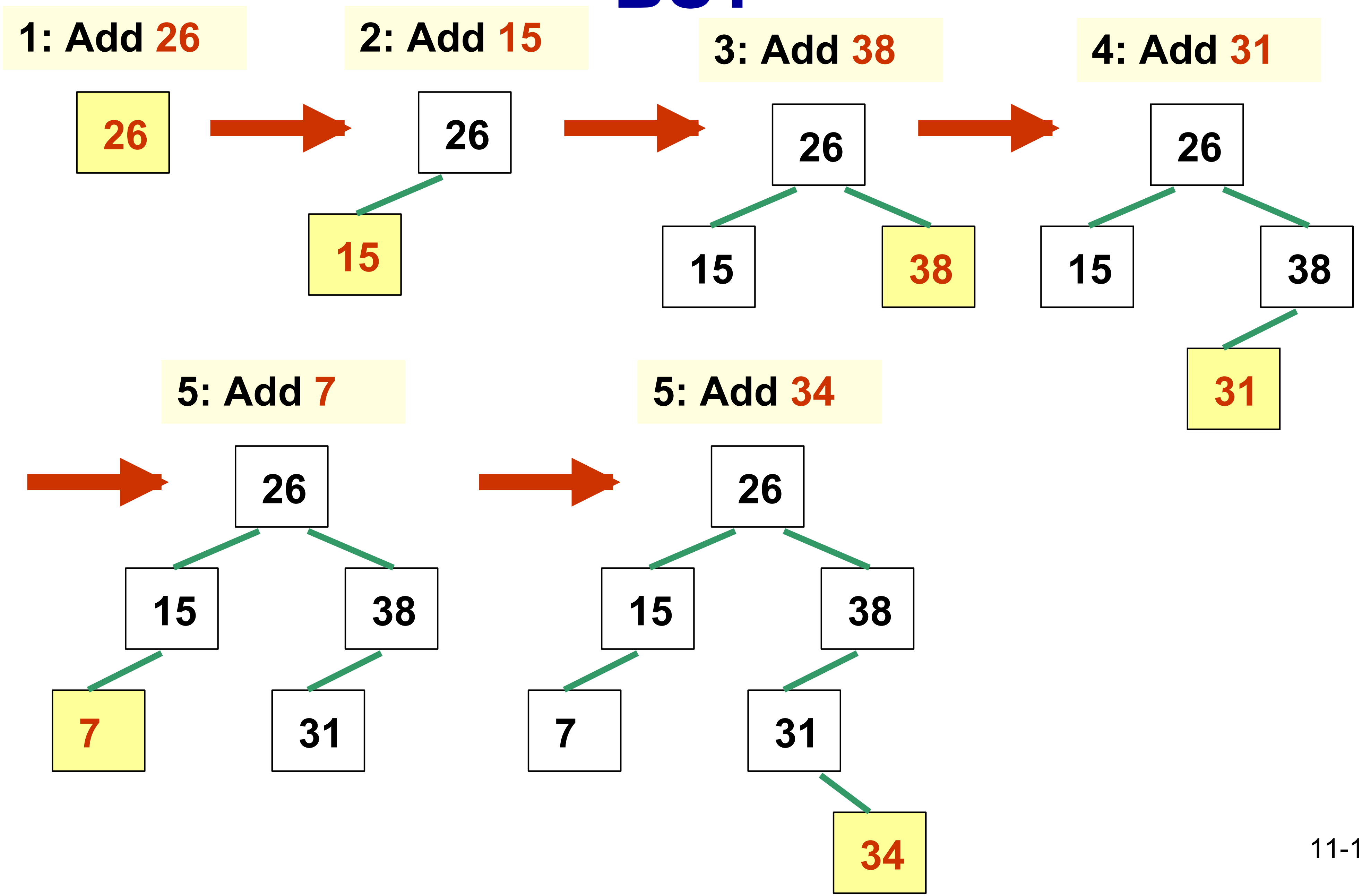
Instead of returning *false* when the node containing 12 has no right child, build the new node, attach it as the right child of the node containing 12, and return *true*.

Algorithm insert(k, r)
Input: value k, node r of a binary search tree
Output: true if k was successfully added and false if not

if tree is empty **then** {
    set new node storing k as the root of the tree
    return true
}
if k is equal to the value at r **then return** false // no duplicates allowed
else if k < value at r **then**
    if r has no left child **then** {
        set new node storing k as left child of r
        return true
    }
    else return insert (k, left child of r)
else // k > value at r
    if r has no right child **then** {
        set new node storing k as right child of r
        return true
    }
    else return insert (k, right child of r)

① Boolean
② Empty Tree Case: return false.
③. Equal Case: return false.
④ Else: return True.

Example: Adding Elements to a BST

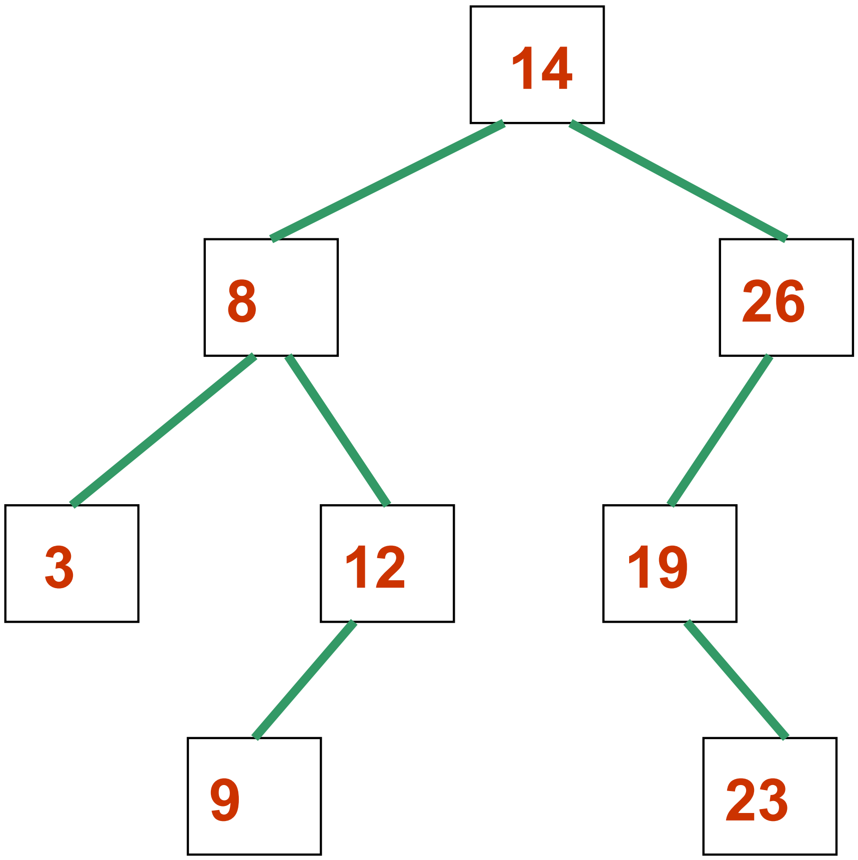


Binary Search Tree Traversals

- **Consider the traversals of a binary search tree: preorder, inorder, postorder, level-order**
- **Try the traversals on the example on the next page**
  - **Is there anything special about the *order of the data* in the BST, for each traversal?**

- ***Question*: what if we wanted to visit the nodes in *descending* order?**

Binary Search Tree Traversals



Try these traversals:

- preorder
- inorder
- postorder
- level-order

Binary Search Tree ADT

- **A binary search tree is just a binary tree with the ordering property imposed on all nodes in the tree**
- **So, we can define the BinarySearchTreeADT interface as an *extension* of the BinaryTreeADT interface**

```
public interface BinarySearchTreeADT<T> extends
                    BinaryTreeADT<T>  {
   public void addElement (T element);

   public T removeElement (T targetElement);

   public void removeAllOccurrences (T targetElement);

   public T removeMin( );

   public T removeMax( );

   public T findMin( );

   public T findMax( );
 }
```

Implementing BSTs using Links

- The special thing about a Binary Search Tree is that finding a specific element is efficient!
  - So, LinkedBinarySearchTree has a find method that *overrides* the find method of the parent class LinkedBinaryTree
    - It only has to search the appropriate side of the tree
    - It uses a recursive helper method findAgain
  - Note that it does not have a contains method that overrides the contains of LinkedBinaryTree – why not?
    - It doesn't need to, because contains just calls find

Using Binary Search Trees: Implementing Ordered Lists

- A BST can be used to provide *efficient* implementations of other collections!
- We will examine an implementation of an *Ordered List ADT* as a *binary search tree*
- Our implementation is called *BinarySearchTreeList.java* (naming convention same as before: this is a BST implementation of a List)

Using BST to Implement Ordered List

- **BinarySearchTreeList *implements* OrderedListADT**
  - **Which extends ListADT**
  - **So it also implements ListADT**
  - **So, what operations do we need to *implement*?**
    - **add**
    - **removeFirst, removeLast, remove, first, last, contains, isEmpty,size, iterator, toString**
  - **But, for which operations do we actually need to write code? …**

Using BST to Implement Ordered List

- BinarySearchTreeList *extends* our binary search tree class LinkedBinarySearchTree
  - Which extends LinkedBinaryTree
  - So, what operations have we *inherited* ?
    - addElement, removeElement, removeMin, removeMax, findMin, findMax, find
    - getRoot, isEmpty, size, contains, find, toString, iteratorInOrder, iteratorPreOrder, iteratorPostOrder, iteratorLevelOrder

Discussion

- First, let us consider some of the methods of the List ADT that we do *not* need to write code for:
  - contains method: we can just *use* the one from the LinkedBinaryTree class
  - What about the methods
    - isEmpty
    - size
    - toString

Discussion

- To implement the following methods of the OrderedListADT, we can *call* the appropriate methods of the LinkedBinarySearchTree class

  (fill in the missing ones)

  - add call addElement
  - removeFirst removeMin
  - removeLast
  - remove
  - first
  - last
  - iterator