CS3388B, Winter 2023

Problem Set 2

Due: January 20, 2023

Exercise 1.

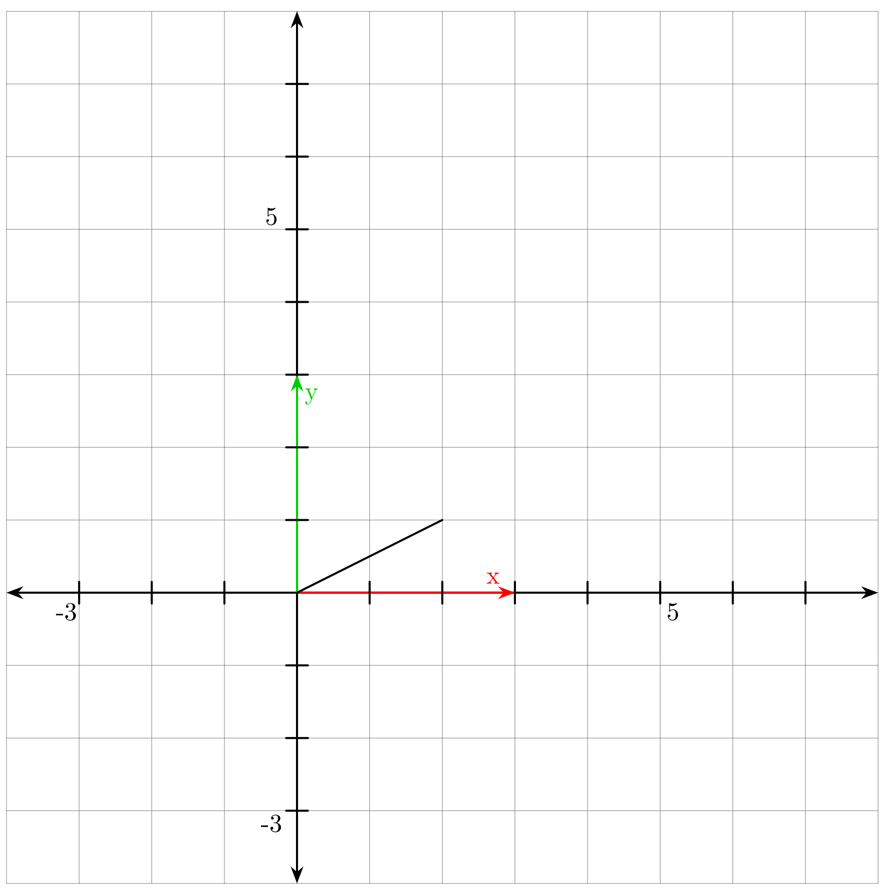
Consider a window with width 1000px and height 800px with a viewport whose opposite corners, in pixels, are $(200, 100)$ and $(800, 700)$.

Give the viewport matrix which transforms normalized device coordinates to this viewport.

Exercise 2. Consider the following affine transforms:

$$T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \cos(30) & -\sin(30) & 0 \\ \sin(30) & \cos(30) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Let $M_1 = TR$ and $M_2 = RT$ be two transformation matrices. Consider the line segment defined by $v_1 = (0, 0)$ and $v_2 = (2, 1)$. Draw the line segment when transformed by M_1 and when transformed by M_2. Describe, in words, what is the difference between the affine transforms M_1 and M_2? Why is the result different?



Exercise 3.

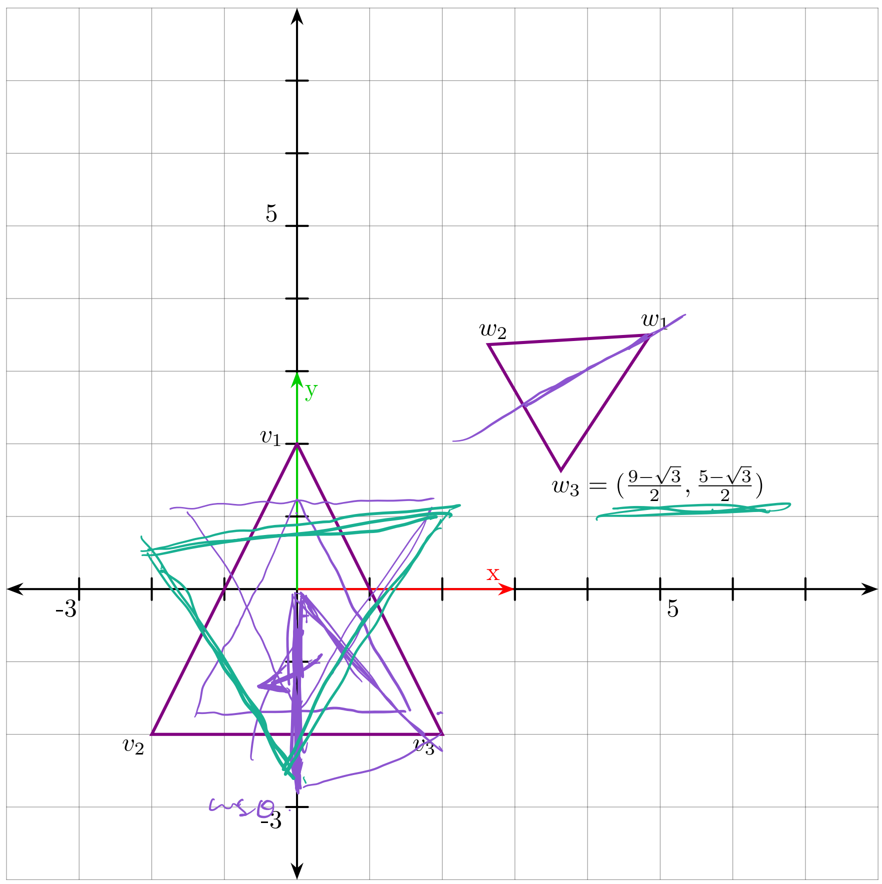
Consider the shear matrix

$$S = \begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix}.$$

Find the inverse of S in homogeneous coordinates and show that $SS^{-1} = I_3$, the 3x3 identity matrix.

Exercise 4.

The below triangle (v_1, v_2, v_3) has been affinely transformed to (w_1, w_2, w_3) by a combination of a scaling, a translation, and a rotation. Let those individual transformations be described by the matrices S, T, R, respectively.



Using homogeneous coordinates, find the matrices S, T, R. Then find (through matrix-matrix and matrix-vector multiplication) the coordinates of w_1 and w_2. What is the correct order of matrix multiplications to get the correct result?

TSRV

$$\begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -2\sqrt{2} \\ 1 \end{bmatrix}} = \begin{bmatrix} \frac{9-\sqrt{3}}{2} \\ \frac{5-\sqrt{3}}{2} \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -2\sqrt{2}s \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{9-\sqrt{3}}{2} \\ \frac{5-\sqrt{3}}{2} \\ 1 \end{bmatrix}.$$

$V_3 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ $\quad V_3' = \begin{bmatrix} 0 \\ -2\sqrt{2} \end{bmatrix}$ $\quad \theta = -\frac{\pi}{4}$

$\cos\theta = \frac{\sqrt{2}}{2} \quad \sin\theta = -\frac{\sqrt{2}}{2}$

$V_3 =$ $\quad V_3' = \begin{bmatrix} 0 \\ -2\sqrt{2} \\ 1 \end{bmatrix}$

$$R = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{matrix} 0+0 & + T_x \\ 0 & -2\sqrt{2}S + T_y \\ 0 & 0 & 1. \end{matrix}$$

$$T = \begin{bmatrix} 1 & 0 & \frac{5-\sqrt{3}}{2} \\ 0 & 1 & \frac{5-\sqrt{3}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$T_x = \frac{5-\sqrt{3}}{2}$

$-2\sqrt{2}S + T_y = \frac{5-\sqrt{3}}{2}.$

pick $S = \frac{\sqrt{2}}{2}$. $\quad T_y = \frac{9-\sqrt{3}}{2}$.

$-2\sqrt{2} \cdot \frac{\sqrt{2}}{2} = -2.$

$$S = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$R = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$V_1 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}.$$