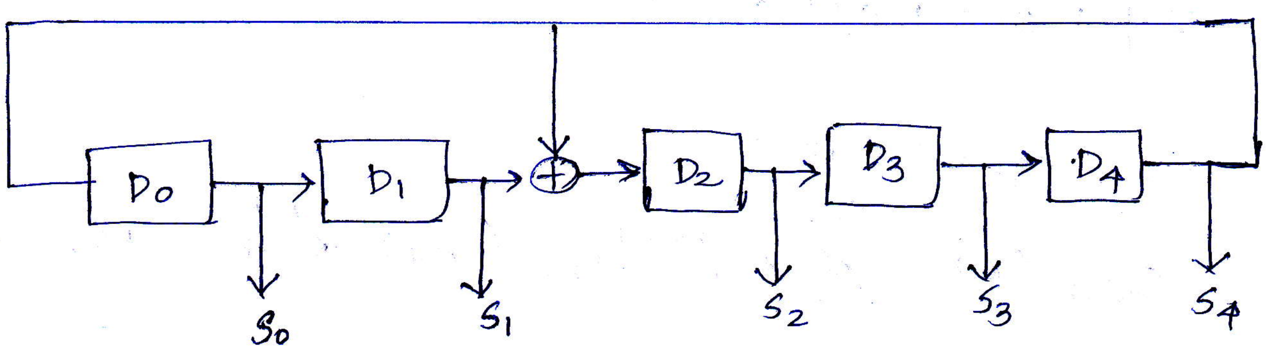
Problem 3.

12.1) $\phi(x) = x^5 + x^2 + 1$

Internal - XOR LFSR



Transition matrix / relation $S(t+1) = A * S(t)$

$$\begin{bmatrix} S_0(t+1) \\ S_1(t+1) \\ S_2(t+1) \\ S_3(t+1) \\ S_4(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} S_0(t) \\ S_1(t) \\ S_2(t) \\ S_3(t) \\ S_4(t) \end{bmatrix}$$

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} = \left[\begin{matrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{matrix}\right. \cdots$$

Initial seed

$$\cdots \left.\begin{matrix} 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{matrix}\right]$$

Cyclic repetition

sequence generated at the output of the last stage of the LFSR. $= s_4 =$

$= [1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 1]$.

using equation 12.4

$$G_{(n-1)}(x) = \frac{\sum_{i=0}^{(n-1)} \phi_i x^{n-i} \left[s_{n-1}(-n+i) x^{-n+i} + \cdots + s_{n-1}(-1) x^{-1} \right]}{1 + \sum_{i=0}^{n-1} \phi_i x^{n-i}}$$

can be rewritten as $G_{n-1}(x) = \dfrac{1}{1 + \sum_{i=0}^{(n-1)} \phi_i x^{n-i}}$

$$\Rightarrow G_{n-1}(x) = \frac{1}{x^n \phi(1/x)} =$$

$$\phi(1/x) = (1/x)^5 + (1/x)^2 + 1$$

$$\Rightarrow G_{n-1}(x) = \frac{1}{x^5 \left[(1/x)^5 + (1/x)^2 + 1 \right]}$$

$$= \frac{1}{1 + x^3 + x^5}$$

$$= 1 + x^3 + x^5 + x^6 + x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{17} + x^{18} + x^{20} + x^{21} + x^{22} + x^{24} + x^{26} + x^{31} \ldots$$

using eqn, 12.1

$$G_{n-1}(x) = \sum_{j=0}^{\infty} s_{n-1}(j) x^j$$

Comparing $G_4(x) = \sum_{j=0}^{\infty} S_4(j)x^j$

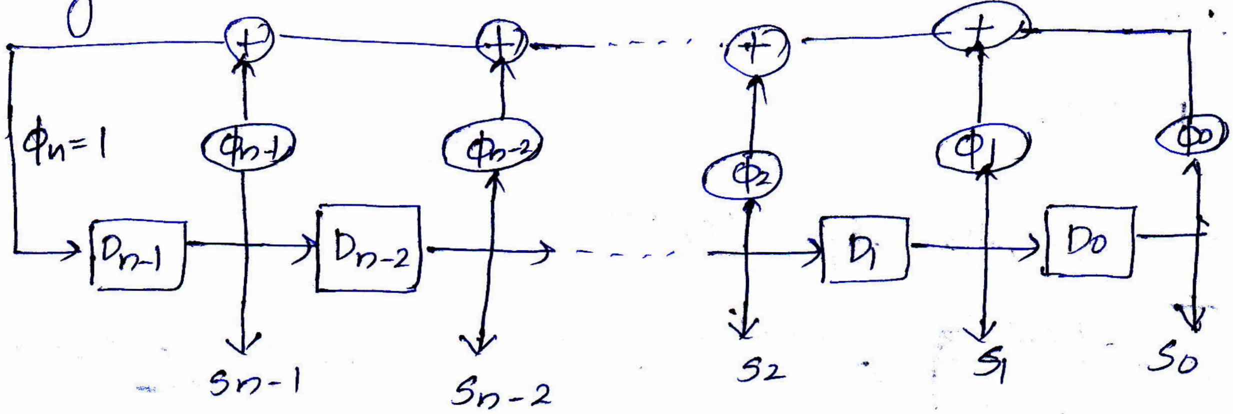
The coefficient of x^j in $G_4(x)$ are $S_4(j)$.

$\therefore S_A = [1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 1]$

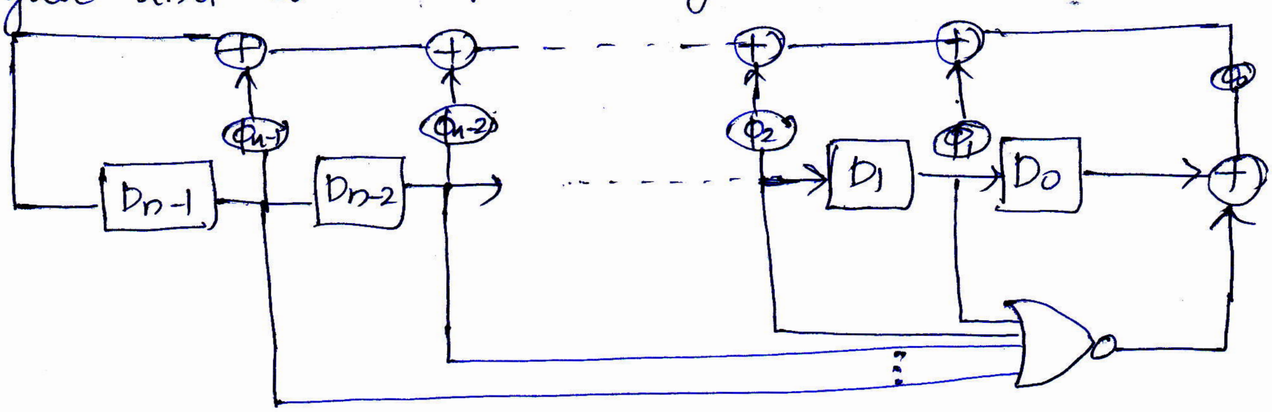
Here the two sequences are identical.

12.8)

n-stage external LFSR.



To generate a complete set of n-bit PTVs, the LFSR can be modified by the addition of an (n-1) input NOR gate and a two input XOR gate.



④

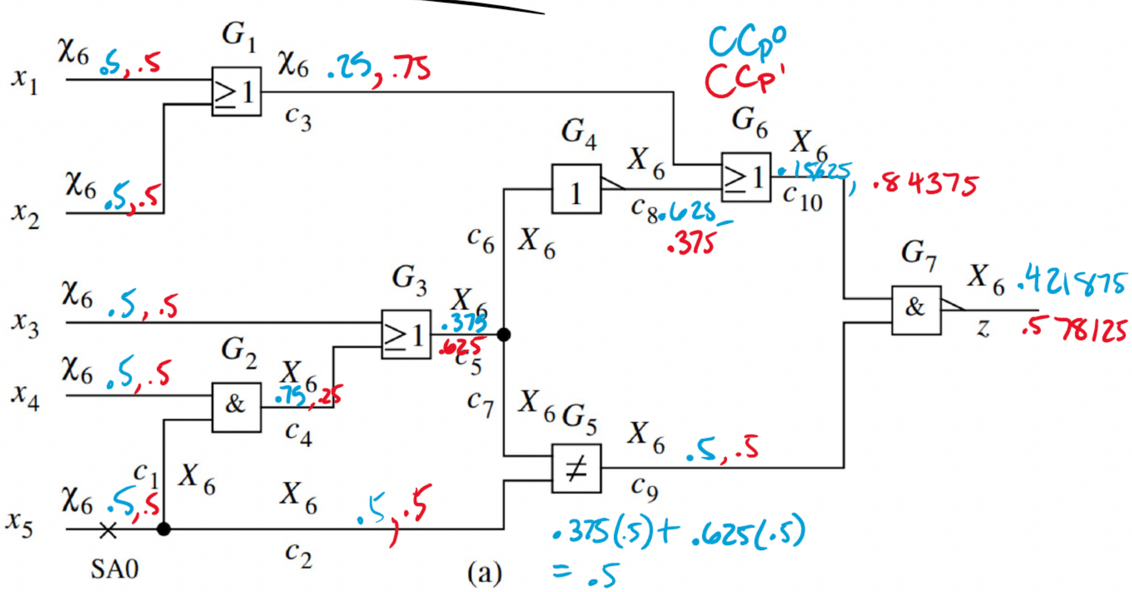
⇒ when LFSR has 0..001, the output of NOR is 1 and the last XOR injects a '0'. so, the next state will be all zero 0...000.

⇒ when LFSR has 0..000, the output of NOR is 1 again but the last XOR injects a 1, so the next state will be out of all zero state.

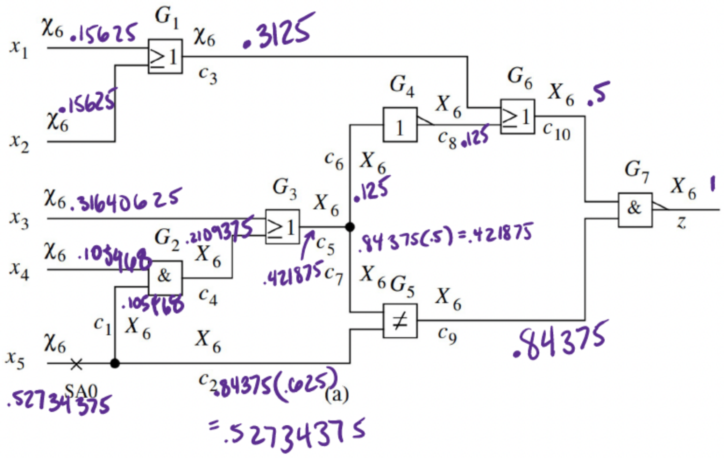
∴ This way we can generate a complete set of n-bit PTVs.

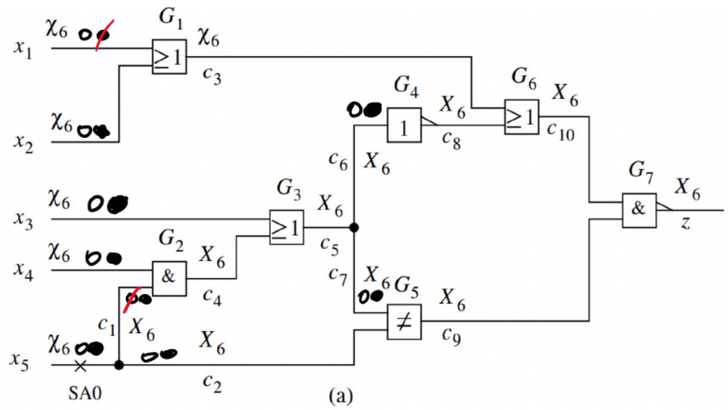
12.13 Estimate probabilistic controllabilities and observability for each line in the circuit shown in Figure 4.24(a) using the circuit traversal approach. Use the above values to obtain an estimate of the detectability profile for this circuit.

Probabilistic Controllabilities



Observabilities





-$D_p(f)=CC_p^1(c_i)O_p(c_i)$, if f is an SA0 fault at c_i

-$D_p(f)=CC_p^0(c_i)O_p(c_i)$, if f is an SA1 fault at c_i

$\kappa_f = 2^n \cdot D_p(f).$

X1 sa0
- Dp = .5*.15625 = 0.07813
- Kf = 2^5*.07813 = 2.50016

X2 sa0
- Dp = .5*.15625 = 0.07813
- Kf = 2^5*.07813 = 2.50016

X2 sa1
- Dp = .5*.15625 = 0.07813
- Kf = 2^5*.07813 = 2.50016

X3 sa0
- Dp = .5*.31640625 = 0.15820313
- Kf = 2^5*.15820313 = 5.06250016

X3 sa1
- Dp = .5*.31640625 = 0.15820313
- Kf = 2^5*.15820313 = 5.06250016

X4 sa0
- Dp = .5*.107187 = 0.053594
- Kf = 2^5*.053594 = 1.715008

X4 sa1
- Dp = .5*.107187 = 0.053594
- Kf = 2^5*.053594 = 1.715008

X5 sa0
- Dp = .5*.52734375 = 0.26367188
- Kf = 2^5*.26367188 = 8.43750016

X5 sa1
- Dp = .5*.52734375 = 0.26367188
- Kf = 2^5*.26367188 = 8.43750016

C1 sa1
- Dp = .5*.107187 = 0.053594
- Kf = 2^5*.053594 = 1.715008

c2 sa0
- Dp = .5*.52734375 = 0.26367188
- Kf = 2^5*.26367188 = 8.43750016

c2 sa1
- Dp = .5*.52734375 = 0.26367188
- Kf = 2^5*.26367188 = 8.43750016

C6 sa0
- Dp = .625*.125 = 0.0781
- Kf = 2^5*.0781 = 2.4992

C6 sa1
- Dp = .375*.125 = 0.0469
- Kf = 2^5*.0469 = 1.5008

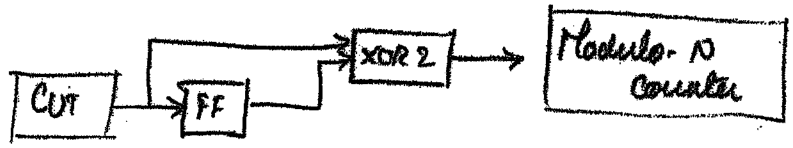
C7 sa0
- Dp = .625*.421875 = 0.263672
- Kf = 2^5*.263672 = 8.437504

C7 sa1
- Dp = .375*.421875 = 0.158203
- Kf = 2^5*.158203 = 5.062496

H = {h2.5 = 4, h5 = 3, h1.7 = 3, h8.43 = 5, h1.5 = 1}

HW#6 - PROBLEM : 1

(12.22)

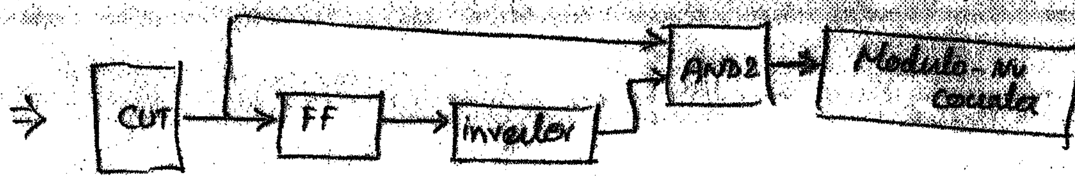


r_i	r_{i+1}	$r_i \oplus r_{i+1}$
0	0	0
0	1	1
1	0	1
1	1	0

for $0 \to 1$ transition

r_i	r_{i+1}	count
0	0	0
0	1	1
1	0	0
1	1	0

$\Rightarrow$



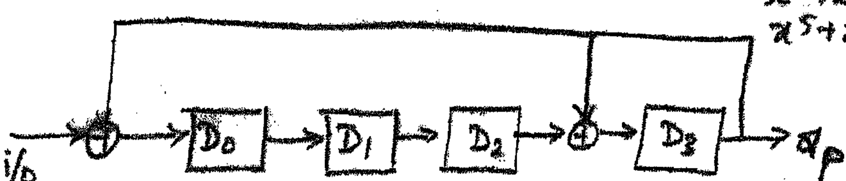
The delay through the inverter should be less compared to the D-ff clk so as to meet the timing requirements & the correct value is sampled.

(12.24) $\phi(x) = x^4 + x^3 + 1$

res : 1 1 1 0 1 0 1

$res(x) = x^6 + x^4 + x^2 + x + 1$

$$
\begin{array}{r|l}
x^4+x^3+1 & x^6+x^4+x^2+x+1 \quad\quad x^2+x \\
 & \underline{x^6+x^5+x^2} \\
 & x^5+x^4+x+1 \\
 & \underline{x^5+x^4+x} \\
 & 1 \leftarrow \text{Signature}
\end{array}
$$



Time	i/p	LFSR state	o/p
—	—	0 0 0 0	—
0	1	1 0 0 0	0
1	0	0 1 0 0	0
2	1	1 0 1 0	0
3	0	0 1 0 1	0
4	1	0 0 1 1	1
5	1	0 0 0 0	1
6	1	1 0 0 0	0

1) $Res_1(x) = x^5 + x^4 + x + 1$

$$
\begin{array}{r|l}
x^4+x^3+1 & x^5+x^4+x+1 \quad\quad x \\
 & \underline{x^5+x^4+x} \\
 & 1 \quad \text{Signature}
\end{array}
$$

2) $Res_2(x) = x^6 + x^5 + x^2 + 1$

$$
\begin{array}{r|l}
x^4+x^3+1 & x^6+x^5+x^2+1 \quad\quad x^2 \\
 & \underline{x^6+x^5+x^2} \\
 & 1 \quad \text{Signature}
\end{array}
$$

12.25 The MISR sequence can be obtained by column-wise XOR operations as follows:

$$
\begin{array}{l}
e_{11}\ \ e_{12}\ \ \cdots\ \ e_{1m} \\
\quad e_{21}\ \ e_{22}\ \ \cdots\ \ e_{2m-1}\ \ e_{2m} \\
\qquad \vdots \\
\qquad e_{N_v 1}\ \ \cdots\cdots\ \ e_{N_v m} \\
\hline
\text{MISR sequence of length } N_v + m - 1
\end{array}
$$

$N_v \rightarrow$ no of vectors

$m \rightarrow$ no of bits for each MISR output

To obtain an all-zero MISR sequence, the output of each XOR operation (each column) must be equal to 0.

(1) Now, for a column k with a_k inputs, the XOR of a_k inputs will result in an output of 0 ~~for~~ on $\left(\frac{1}{2} \times 2^{a_k}\right)$ occasions.

(2) Out of $N_v + m - 1$ columns (the length of MISR sequence), there are $(m - N_v + 1)$ columns with N_v inputs to the XOR operation

$\therefore$ The no. of combinations of the $(m - N_v + 1)$ columns to have a zero output for their respective XOR operation is:

$$C_1 = \left(\frac{1}{2} \times 2^{N_v}\right)^{m - N_v + 1} = 2^{(N_v - 1)(m - N_v + 1)}$$

For the remaining columns, the no. of possible combinations to obtain an all-zero XOR output is :

$$\text{Rem columns} = (N_V + m - 1) - (m - N_V + 1)$$
$$= 2(N_V - 1)$$

i.e $(N_V - 1)$ on either side of $(m - N_V + 1)$ columns

$\therefore$ No of possibilities

$$C_2 = \left[\left(\frac{1}{2} \times 2^1\right) \times \left(\frac{1}{2} \times 2^2\right) \left(\frac{1}{2} \times 2^3\right) \cdots \left(\frac{1}{2} \times 2^{(N_V - 1)}\right)\right]^2$$

$$= 2^{\frac{(N_V-1)N_V}{2}} \cdot \left(\frac{1}{2}\right)^{N_V - 1} \cdot 2^{\frac{(N_V-1)(N_V)}{2}} \cdot \frac{1}{2}^{(N_V - 1)}$$

$$= 2^{(N_V-1)N_V} \cdot \left(\frac{1}{2}\right)^{2(N_V - 1)}$$

$$= 2^{(N_V-1)(N_V-2)}$$

$\therefore$ Total number of possible combinations to obtain an all-zero MISR sequence :

$$n = C_1 \cdot C_2$$
$$= 2^{(N_V - 1)(m - N_V + 1)} \cdot 2^{(N_V - 1)(N_V - 2)}$$

$$= 2^{(N_V - 1)(m - N_V + 1 + N_V - 2)}$$

$$= \underline{\underline{2^{(N_V - 1)(m - 1)}}}$$

A better justification,

example:

$\begin{cases} m=3 & \text{(MISR length)} \\ N_v=5 & \text{(\# of patterns - bits fed to MISR)} \end{cases}$

$$
\begin{array}{cc|ccc|cc}
e_1 & e_2 & e_3 & & & & \\
 & e_1 & e_2 & e_3 & & & \\
 & & e_1 & e_2 & e_3 & & \\
 & & & e_1 & e_2 & e_3 & \\
 & & & & e_1 & e_2 & e_3 \\
\hline
\multicolumn{2}{c|}{m-1 \text{ columns}} & \multicolumn{3}{c|}{N_v-m+1 \text{ columns}} & \multicolumn{2}{c}{m-1 \text{ columns}}
\end{array}
$$

\# of elements in that column: $1\ 2\ 3 \ldots m-1 \mid m\ m \ldots m \mid m-1 \ldots 3\ 2\ 1$

A column with a_k elements, if we XOR them, will generate 0 for half combinations (even # of 1s in it)

This means a column with a_k elements will ~~produce~~ have $\frac{1}{2}2^{a_k}$ combinations that if we XOR will produce 0.

So, for 3 groups of columns above:

$$\left[(\tfrac{1}{2}2^1)(\tfrac{1}{2}2^2)\cdots(\tfrac{1}{2}2^{m-1})\right]\cdot\left[(\tfrac{1}{2}2^m)\cdots(\tfrac{1}{2}2^m)\right]\left[(\tfrac{1}{2}2^{m-1})\cdots(\tfrac{1}{2}2^2)(\tfrac{1}{2}2^1)\right]$$

$$=\left[\frac{1}{2^{m-1}}\cdot 2^{\frac{(m-1)m}{2}}\right]\left[\frac{1}{2^{N_v-m+1}}\cdot 2^{m(N_v-m+1)}\right]\left[\frac{1}{2^{m-1}}\cdot 2^{\frac{(m-1)m}{2}}\right]$$

$$=\frac{1}{2^{m-1+N_v\cancel{-m}\cancel{+1}\cancel{+m-1}}}\cdot 2^{\cancel{m^2}\cancel{-m}+mN_v\cancel{-m^2}\cancel{+m}}$$

$$=2^{-m+1-N_v+mN_v}=2^{m(N_v-1)-(N_v-1)}=2^{(N_v-1)(m-1)}$$