Hessian Matrix and jacobian matrix

Given the treatment XZ and the outcome k, the hessian matrix (H) is

$$H_k=\begin{bmatrix} \frac{\partial^2\ell}{\partial\left(\mu_k^{xz}\right)^2} & \frac{\partial^2\ell}{\partial\mu_k^{xz}\partial\tau_k^{xz^2}} \\ \frac{\partial^2 l}{\partial\mu_k^{xz}\tau_k^{xz^2}} & \frac{\partial^2\ell}{\partial\left(\tau_k^{xz}\right)^2} \end{bmatrix}$$

The J matrix is

$$J_R=\begin{bmatrix} \operatorname{var}\left(\frac{\partial\ell}{\partial\mu_k^{xz}}\right) & \operatorname{cov}\left(\frac{\partial\ell}{\partial\mu_k^{xz}},\frac{\partial\ell}{\partial\tau_k^{xz^2}}\right) \\ \operatorname{cov}\left(\frac{\partial\ell}{\partial\mu_k^{xz}},\frac{\partial\ell}{\partial\tau_k^{xz^2}}\right) & \operatorname{var}\left(\frac{\partial\ell}{\partial\tau_k^{xz^2}}\right) \end{bmatrix}$$

where the var $(\cdot)$ and cov $(\cdot)$ are calculated based on the generated sample and the first and second derivatives are

$$\ell\left(\tau_k^{XZ^2},\mu_k^{XZ}\right)=-\frac{1}{2}\sum_{i\in N_{XZ}^k}\left[\log\left(s_{k,i}^{XZ^2}+\tau_k^{XZ^2}\right)+\frac{\left(y_{k,i}^{XZ}-\mu_k^{XZ}\right)^2}{s_{k,i}^{XZ^2}+\tau_k^{XZ^2}}\right]$$

$$\frac{\partial\ell}{\partial\tau_k^{xz^2}}=-\frac{1}{2}\sum_{i\in N_{xz}^k}\left[\frac{1}{\tau_k^{xz^2}+s_{k,i}^{xz^2}}-\frac{\left(y_{k,i}^{xz}-\mu_k^{xz}\right)^2}{\left(s_{k,i}^{xz^2}+\tau_k^{xz^2}\right)^2}\right]$$

$$\frac{\partial\ell}{\partial u_k^{xz}}=-\frac{1}{2}\sum_{i\in N_{xz}^k}\left[\frac{2\left(y_{k,i}^{xz}-\mu_k^{xz}\right)}{s_{k,i}^2+\tau_k^{xz^2}}(-1)\right]=\frac{1}{2}\sum_{i\in N_{xz}^k}\left[\frac{2\left(y_{k,i}^{xz}-\mu_k^{xz}\right)}{s_{k,i}^{xz^2}+\tau_k^{xz^2}}\right]$$

$$\frac{\partial^2\ell}{\partial\left(\tau_k^{xz^2}\right)^2}=-\frac{1}{2}\sum_{i\in N_{xz}^k}\left[-\frac{1}{\left(\tau_R^{xz^2}+s_{k_i}^{xz^2}\right)^2}+\frac{2\left(y_{k,i}^{xz}-\mu_k^{xz}\right)^2}{\left(s_{k,i}^{xz^2}+\tau_k^{xz^2}\right)^3}\right]$$

$$\frac{\partial^2\ell}{\partial\left(\mu_k^{xz}\right)^2}=-\frac{1}{2}\sum_{i\in N_{xz}^k}\left[\frac{(-2)}{s_{k,i}^{xz^2}+\tau_R^{xz^2}}\right]=\sum_{i\in N_{xz}^k}\left[\frac{1}{s_{R,i}^{xz^2}+\tau_R^{xz^2}}\right]$$

$$\frac{\partial^2\ell}{\partial\mu_k^{xz}\partial\tau_k^{xz^2}}=\sum_{i\in N_{xz}^k}\left[-\frac{\left(y_{k,i}^{xz}-\mu_k^{xz}\right)}{\left(s_{k,i}^{xz^2}+\tau_k^{xz^2}\right)^2}\right]=-\sum_{i\in N_{xz}^R}\left[\frac{\left(y_{k_{1i}}^{xz}-\mu_k^{xz}\right)}{\left(s_{k,i}^{xz^2}+\tau_k^{xz^2}\right)^2}\right]$$