H and J matrix under multivariate setting

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H matrix

$$\hat{H}_{\mu\mu} = \begin{bmatrix} -\frac{\partial^2 \ell}{\partial\left(\mu_1^{BA}\right)^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\partial^2 \ell}{\partial\left(\mu_1^{CA}\right)^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\partial^2 \ell}{\partial\left(\mu_1^{BC}\right)^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial^2 \ell}{\partial\left(\mu_2^{BA}\right)^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\partial^2 \ell}{\partial\left(\mu_2^{CA}\right)^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\partial^2 \ell}{\partial\left(\mu_2^{BC}\right)^2} \end{bmatrix}$$

where $\frac{\partial^2 \ell}{\partial\left(\mu_2^{XZ}\right)^2} = -\sum_{i \in N_{XZ}^k} \frac{1}{s_{k,i}^{XZ^2} + \tau_k^{XZ^2}}$

J matrix

Denote

$$\hat{S}_i^{XY} = \frac{\partial \left[\log\left(s_{k,i}^{XY}, \tau_k^{XY^2}\right) + \frac{\left(y_{k,i}^{XY} - \mu_k^{XY}\right)^2}{s_{k,i}^{XY} + \tau_k^{XY^2}} \right]}{\partial \eta}$$

where $\hat{S}_i^{XY}$ should be a matrix with dimension 1×6

J matrix would be

$$\begin{aligned} \hat{J}_{\mu\mu} &= \frac{1}{4} \sum_{k=1}^{2} \left\{ \sum_{\mathrm{XY} \in \{\mathrm{AB,BC,AC}\}} \sum_{i \in \mathcal{N}_{\mathrm{XY}}^k} S_i^{XY} S_i^{XY^{\mathrm{T}}} \right\} \\ &= \mathrm{Diag}\left(\left[\sum_{i \in \mathcal{N}_{\mathrm{XY}}^k} \frac{\left(y_{k,i}^{BA} - \mu_k^{BA}\right)^2}{\left(s_{k,i}^{XY} + \tau_k^{XY^2}\right)^2} \right] \right)_{6 \times 6} \end{aligned}$$

where $\eta = \left[\mu_k^{XY}\right]_{1 \times 6}$ for outcome $k = 1, 2$ and treatment $XY \in \{BA, CA, BC\}$

Covariance Matrix

$$\hat{V}(\hat{\eta}) = \hat{H}_{\mu\mu}^{-1} \hat{J}_{\mu\mu} \hat{H}_{\mu\mu}^{-1}$$

Reference

The appendix in Duan2020