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Extension of HITS(Cont’d)

Define, where , %FontSize=12
%TeXFontSize=12
\documentclass{article}
\pagestyle{empty}
\begin{document}
$B \in R^{n \times k_2}$, \end{document}and . For any , and :



Consider two weights vectors %FontSize=12
%TeXFontSize=12
\documentclass{article}
\pagestyle{empty}
\begin{document}
$(x^{\top}_{\alpha} \vdots x^{\top}_{\beta}) = x \in (\mathbbm{R}^{+_{}})^k$\end{document}, %FontSize=12
%TeXFontSize=12
\documentclass{article}
\pagestyle{empty}
\begin{document}
$y \in R^n$\end{document}, i.e.

%FontSize=12
%TeXFontSize=12
\documentclass{article}
\pagestyle{empty}
\begin{document}

$$\left\{ \begin{array}{l}     x_{\alpha} = \mbox{ImapactFactor} (\ {\mbox{items}}) \in     (\mathbbm{R}^{+_{}})^{k_1}\\     x_{\beta} = \mbox{Coeff} (\ {\mbox{features}}) \in     (\mathbbm{R}^{+_{}})^{k_2}\\     y = \mbox{Importance} (\ {\mbox{user}}) \in (\mathbbm{R}^{+_{}})^n   \end{array} \right.$$

\end{document}

**Algorithm** 1*EHITS*

%FontSize=12
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\documentclass{article}
\usepackage[english]{babel}
\usepackage{amsmath,bbm,latexsym,xcolor}
\catcode`\<=\active \def<{
\fontencoding{T1}\selectfont\symbol{60}\fontencoding{\encodingdefault}}
\newcommand{\nocomma}{}
\newcommand{\noplus}{}
\newcommand{\tmtextbf}[1]{{\bfseries{#1}}}
\newcommand{\tmtextit}[1]{{\itshape{#1}}}
\newcommand{\tmtextup}[1]{{\upshape{#1}}}
\newenvironment{proof}{\noindent\textbf{Proof\ }}{\hspace*{\fill}$\Box$\medskip}
\newenvironment{tmindent}{\begin{tmparmod}{1.5em}{0pt}{0pt} }{\end{tmparmod}}
\newenvironment{tmparmod}[3]{\begin{list}{}{\setlength{\topsep}{0pt}\setlength{\leftmargin}{#1}\setlength{\rightmargin}{#2}\setlength{\parindent}{#3}\setlength{\listparindent}{\parindent}\setlength{\itemindent}{\parindent}\setlength{\parsep}{\parskip}} \item[]}{\end{list}}
\newenvironment{tmparsep}[1]{\begingroup\setlength{\parskip}{#1}}{\endgroup}
\newtheorem{theorem}{Theorem}

\begin{document}
\tmtextup{Extending HITS algorithm to non square case. }
    
    \tmtextbf{Initialization Step}
    
    $x = \frac{1}{\sqrt{k}} 1_k$, $y = \frac{1}{\sqrt{n}} 1_n$,
    $i_{\mbox{iter}} = 0$, MAX\_ITER, and tolerance $\epsilon > 0$.
    
    \tmtextbf{Iteration Step}
    
    \tmtextbf{\tmtextit{while}} \ ($i_{\mbox{iter}}$<MAX\_ITER)
    
    $\quad {\color[HTML]{800000}\forall i = 1, \ldots, n} y_i^{(\mbox{new})} =
    \sum_{j = 1}^{k_1} A_{i j} x_{\alpha \nocomma, j} + \sum_{j = 1}^{k_2}
    B_{i j} x_{\beta \nocomma, j}$
    
    $\quad {\color[HTML]{800000}\forall i = 1, \ldots, n} y_i^{(\mbox{new})} =
    y_i^{(\mbox{new})} /\|y_i^{(\mbox{new})} \|_{\ell_2}$ \
    {\color[HTML]{008000} \ \tmtextit{// $\ell_2$-normalize}}
    
    ${\color[HTML]{800000}\quad \forall i = 1, \ldots, k_1 } x_{\alpha
    \nocomma, i}^{(\mbox{new})} = \sum_j y^{(\mbox{new})}_j A_{j i}$
    
    $\quad {\color[HTML]{800000}\forall i = 1, \ldots, k_2 } x_{\beta
    \nocomma, i}^{(\mbox{new})} = \sum_j y^{(\mbox{new})}_j B_{j i}$
    
    ${\color[HTML]{800000}\quad \forall i = 1, \ldots, k_1 } x^{(\mbox{new})}
    = (x^{\top}_{\alpha} \vdots x^{\top}_{\beta})^{(\mbox{new})}$ \ \ \ \
    {\color[HTML]{008000}\tmtextit{ \ // $\ell_2$-normalize}}
    
    $\quad {\color[HTML]{800000}\forall i = 1, \ldots, k_2 }
    x_i^{(\mbox{new})} = x_i^{(\mbox{new})} /\|x_i^{(\mbox{new})} \|_{\ell_2}$
    \ {\color[HTML]{008000}\tmtextit{// $\ell_2$-normalize}}
    
    \quad\tmtextbf{\tmtextit{if}} ($\|x - x^{(\mbox{new})} \|+\|y -
    y^{(\mbox{new})} \| \leq \epsilon$)
    
    \tmtextbf{\tmtextit{{\hspace{3em}}break};}
    
    \quad\tmtextit{\tmtextbf{endif}}
    
    \quad$i_{\mbox{iter}}$++ \
    
    \tmtextit{\tmtextbf{end of while}}
    
    \tmtextbf{\tmtextit{if}} ( $i_{\mbox{iter}}$=MAX\_ITER)
    
    \quad\tmtextbf{}\tmtextbf{error}(``NOT CONVERGE'')
    
    \tmtextbf{\tmtextit{endif}}
    
    \tmtextbf{DONE}.
    
    {\noindent}\begin{tabular}{l}
      \hline
      \quad
    \end{tabular}
  \end{tmparsep}
\end{tmindent}{\hspace*{\fill}}{\medskip}
\end{document}

**Theorem** 1

The EHITS algorithm converge to:

(1)

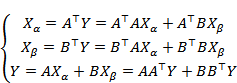
Here  is the eigenvector of square matrix  corresponding to the largest eigenvalue.

*Proof*:

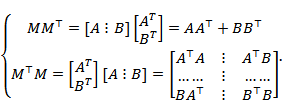
The matrix form of EHITS is indeed

%FontSize=12
%TeXFontSize=12
\documentclass{article}
\pagestyle{empty}
\begin{document}
 \[ \left\{ 
    \begin{array}{l@{=}l}
         X_{\alpha} & A^{\top} Y\\
         X_{\beta} & B^{\top} Y\\
         Y & A X_{\alpha} + B X_{\beta}
       \end{array} \right.
       \]
       \end{document}





Notice  hence



Combined with ([eq:EHITS]), the following is equivalent to the **iteration step** in Algorithm 1.



As we performed normalization in each iteration step, the algorithm converges to (1).

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