Exercise 03 – October 07-09, 2025

1. You have 8 people, and you need to select and arrange 4 of them in a row for a photo. How many different ways can you arrange them?

   Solution:
   Here, n = 8 (total people) and r = 4 (people to arrange).
   We apply the permutation formula:

   $$P(8,4) = \frac{8!}{(8-4)!} = \frac{8!}{4!}$$

   First, calculate 8! and 4!:

   8! = 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 40,320
   4! = 4 × 3 × 2 × 1 = 24

   Now, calculate P(8, 4):

   $$P(8,4) = \frac{40,320}{24} = 1,680$$

   So, there are 1,680 ways to arrange 4 people out of 8 in a row.

2. You have 7 books, and you want to choose 4 to take on a trip. How many different ways can you select the books?

   Solution:
   This is a combination problem where n = 7 and r = 4:

   $$C(7,4) = \frac{7!}{4!(7-4)!} = \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1} = \frac{840}{24} = 35$$

   So, there are 35 ways to choose 4 books from a set of 7.

3. A bag contains 10 red balls and 15 blue balls. If you randomly select 5 balls without replacement, what is the probability that exactly 3 of the selected balls are red?

   Solution:

   N = 25 (total balls/ population),
   k = 10 (total red balls),
   n = 5 (balls selected/ sample),
   x = 3 (we want to find the probability of selecting 3 red balls).

   Using the hypergeometric formula:

   $$p = \frac{{}_kC_x \ {}_{(N-k)}C_{(n-x)}}{{}_NC_n}$$

   Remember Combination formula:

   $${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

$$p = \frac{{}_{10}C_3 \quad {}_{(25-10)}C_{(5-3)}}{{}_{25}C_5}$$

$${}_{10}C_3 = \frac{10!}{7!\,3!} = \frac{10 \text{ x } 9 \text{ x } 8}{3 \text{ x } 2 \text{ x } 1} = 120$$

$${}_{15}C_2 = \frac{15!}{13!\,2!} = \frac{15 \text{ x } 14}{2 \text{ x } 1} = 105$$

$${}_{25}C_5 = \frac{25!}{20!\,5!} = \frac{25 \text{ x } 24 \text{ x } 23 \text{ x } 22 \text{ x } 21}{5 \text{ x } 4 \text{ x } 3 \text{ x } 2 \text{ x } 1} = 53{,}130$$

Now, calculate the probability:

$$p = \frac{120\ x\ 105}{53{,}130} = \frac{12{,}600}{53{,}130} \approx 0.2372$$

So, the probability of drawing exactly 3 red balls is approximately 0.2372 or 23.72%.

4. A card is drawn from a standard deck of 52 cards, and then a coin is flipped. What is the probability of drawing a "King" from the deck and flipping a "Tail"?

   Solution:
   Probability of drawing a "King" from a deck of cards:
   There are 4 Kings in a deck of 52 cards, so:

   P(King) = $\frac{4}{52} = \frac{1}{13}$

   Probability of flipping a "Tail":

   P(Tail) = $\frac{1}{2}$

   Since the two events are independent, the probability of both events happening together is:

   P(King and Tail) = P(King) × P(Tail) = $\frac{1}{13} \times \frac{1}{2} = \frac{1}{26}$

5. Calculate the probability of getting exactly 3 heads when flipping a fair coin 5 times (where getting heads is considered a success)

   Solution:
   N = 5 (number of trials)
   x = 3 (number of successes)
   π = 0.5 (probability of success, i.e., getting heads)

   Using the formula:

   $P(x = 3) = \frac{N!}{x!(N-x)!}\pi^{x}(1-\pi)^{N-x}$

   $P(x = 3) = \frac{5!}{3!\ 2!}\,0.5^{3}\,0.5^{2}$

   $P(x = 3) = \frac{5\ x\ 4}{2\ x\ 1}\,0.5^{5} = 10 \text{ x } \frac{1}{32} = \frac{10}{32} = \frac{5}{16}$

Thus, the probability of getting exactly 3 heads in 5 flips of a fair coin is $\frac{5}{16}$ or approximately 0.3125

6. In a basketball game, a player has a free throw success rate of 80%. If the player takes 15 free throws, what is the probability that they make at least 12 successful free throws?

   Solution:
   To find the probability of making at least 12 successful free throws, we need to calculate
   $$P(x \geq 12) = P(x = 12) + P(x = 13) + P(x = 14) + P(x = 15)$$

   N = 15 (number of trials)
   $\pi = 0.8$ (probability of success)

   For x = 12
   $$P(x = 12) = \frac{15!}{12!\ 3!} 0.8^{12}\, 0.2^{3} \approx 0.227$$

   For x = 13
   $$P(x = 13) = \frac{15!}{13!\ 2!} 0.8^{13}\, 0.2^{2} \approx 0.236$$

   For x = 14
   $$P(x = 14) = \frac{15!}{14!\ 1!} 0.8^{14}\, 0.2^{1} \approx 0.137$$

   For x = 15
   $$P(x = 15) = \frac{15!}{15!} 0.8^{15}\, 0.2^{0} \approx 0.035$$

   So, the probability that the player will make at least 12 successful free throws is approximately 0.227 + 0.236 + 0.137 + 0.035 = 0.635.

7. Suppose the average number of lions seen on a 1-day safari is 5. What is the probability that tourists will see **fewer** than four lions on the next 1-day safari?

   Solution:
   This is a Poisson experiment in which we know the following:

   - $\mu = 5$; since 5 lions are seen per safari, on average.
   - x = 0, 1, 2, or 3; since we want to find the likelihood that tourists will see fewer than 4 lions; that is, we want the probability that they will see 0, 1, 2, or 3 lions.
   - e = 2.71828; since *e* is a constant equal to approximately 2.71828.

   To solve this problem, we need to find the probability that tourists will see 0, 1, 2, or 3 lions. Thus, we need to calculate the sum of four probabilities: P(0; 5) + P(1; 5) + P(2; 5) + P(3; 5). To compute this sum, we use the Poisson formula:

$P(x \leq 3, 5) = P(0; 5) + P(1; 5) + P(2; 5) + P(3; 5)$

$P(x \leq 3, 5) = [(e^{-5})(5^0) / 0!] + [(e^{-5})(5^1) / 1!] + [(e^{-5})(5^2) / 2!] + [(e^{-5})(5^3) / 3!]$

$P(x \leq 3, 5) = [(0.006738)(1) / 1] + [(0.006738)(5) / 1] + [(0.006738)(25) / 2] + [(0.006738)(125) / 6]$

$P(x \leq 3, 5) = [0.0067] + [0.03369] + [0.084224] + [0.140375]$

$P(x \leq 3, 5) = 0.2650$

Thus, the probability of seeing fewer than 4 lions is 0.2650.