Suspensions

A relatively recent constitutive model for dense suspensions (Gillissen, Ness, Petersen, Wilson & Cates, 2019 & 2020) is given by the following set of equations.

Mass conservation:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

Navier-Stokes (zero inertia version, these things tend to be thick and small-scale):

$$\nabla \cdot \mathbf{S} = 0 \qquad \mathbf{S} = -p\mathbf{I} + 2\mu\mathbf{E} + \mathbf{\Sigma} \tag{2}$$

Constitutive relation:

$$\mathbf{\Sigma} = \mu \left[\frac{\alpha_0 \mathbf{E}}{(1 - \phi/\phi^J)^2} + \frac{\chi_0 \mathbf{E}_c}{(1 - \xi/\xi^J)^2} \right] : \mathbf{b} \tag{3}$$

in which $\mathbf{b}$ is a fourth-rank tensor, related to the second-rank tensor $\mathbf{a}$ via the expression:

$$\begin{aligned} b_{ijkl} = &-\frac{1}{35} a_{mm} (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{ij}\delta_{jk}) \\ &+ \frac{1}{7} (\delta_{ij} a_{kl} + \delta_{ik} a_{jl} + \delta_{il} a_{jk} + \delta_{jk} a_{il} + \delta_{jl} a_{ik} + \delta_{kl} a_{ik}) \end{aligned} \tag{4}$$

and the second-rank tensor $\mathbf{a}$ evolves according to:

$$\frac{D}{Dt}\mathbf{a} = \mathbf{L} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{L}^T - 2\mathbf{L} : \mathbf{b} - \beta \left[\mathbf{E}_e : \mathbf{b} + \frac{\phi}{15} (2\mathbf{E}_c + \mathrm{Tr}(\mathbf{E}_c)\mathbf{I}) \right] \tag{5}$$

Here $\mathbf{u}$ is the fluid velocity, $\mathbf{L}$ is its gradient, and $\mathbf{E} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T)$ is the rate of strain tensor. $\mathbf{E}_c$ and $\mathbf{E}_e$ are its extensional and compressive parts (defined in terms of their eigenvalues; the matrix $\mathbf{E}$ is symmetric so its eigenvalues are real and can be separated into positive and negative). The scalars α_0, χ_0, β and ϕ are parameters of the model.

The other quantities can evolve with the flow, so we have more equations:

$$\xi^J = \xi_1^J (1 - f) + \xi_2^J f \qquad \phi^J = \phi_1^J (1 - f) + \phi_2^J f \qquad f = \exp\left(-\Pi^*/\Pi\right) \tag{6}$$

where ξ_1^J and ξ_2^J, ϕ_1^J and ϕ_2^J are more parameters, as is Π^*, and Π is the trace of the particle stress tensor:

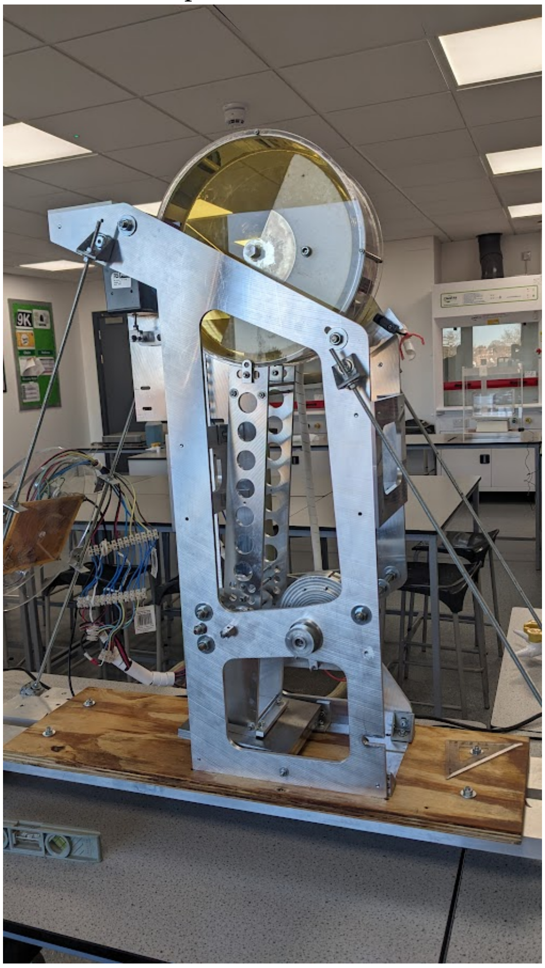
$$\Pi = \mathrm{Tr}(\mathbf{\Sigma}). \tag{7}$$

In this project you will start from these equations in some simple homogeneous flows (uniaxial strain, biaxial strain, planar strain, and simple shear) and track the evolution of the various components of $\mathbf{a}$ and of the particle stress $\mathbf{\Sigma}$. If time allows you can move on to more complex flows, such as steady shear with an imposed oscillation added, or oscillating strain flow.

Perpetual Motion

A machine has been built by Felix Isaac which, he believes, is generating more energy than is being put into it.

The mechanism consists of two or three connected parts, each of which is relatively simple to model (see photo below).



In this project you will idealise the machine as far as possible, and fully work out the potential and kinetic energy when various parts are moving at given speeds. From there we should be able to address the flow of energy within the system and thereby explain wht is going on (and why this can't be a perpetual motion machine!).