

Name _____ CM# _____

Section _____

MA 222

Final Exam Part II

20 Feb. 2008

Instructions: Answer all questions in the space provided. Show work appropriate for methods used, and be neat. You may not use books or notes (written or electronic). However, you may use the provided table of Laplace transforms and an untitled Maple worksheet and the Maple help files on your laptop computer or a calculator for computation and to check answers. Be sure to follow any specific instructions given in the questions.

For Grading Use

#1	10	
#2	10	
#3	10	
#4	10	

1.) Consider the RLC circuit shown, where I_1 and I_2 are the currents flowing through the inductors labeled L_1 and L_2 , respectively. The currents satisfy the system of ODEs

$$L_1 I_1' = -R_1 I_1 - R_2 I_2 + E$$

$$L_2 I_2' = -R_1 I_1 - (R_1 + R_2) I_2 + E$$

($E = E(t)$). If

$$R_1 = 5\Omega, R_2 = 10\Omega, L_1 = L_2 = 10H, E(t) = 10\sin(t) \text{ V}$$

and $I_1(0) = I_2(0) = 0A$, find and plot $I_1(t)$ for $0 \leq t \leq 10$.

2.) a.) Use the method of Laplace transforms to solve the ODE IVP

$$y'' + 2y' + 2y = f(t), \quad y(0) = 1, y'(0) = 1$$

where $f(t)$ is a Laplace transformable function.

b.) What is $y(t)$ if $f(t)$ is equal to $\sin(4t)$ for $t \in [\pi, 2\pi]$ and is equal to zero otherwise?

3.) Sketch the phase portraits of the following systems. Clearly label all equilibrium points and indicate, where possible, their type and stability.

a.) $x' = (2 - x - y)x, y' = (3 - 3x - y)y$

b.) $x' = 4x(1 - x) - xy, y' = y(3 - y) - xy$

4.) a.) Find a (real-valued) power series solution $y(x)$ of the ODE

$$y'' + xy' + 2y = 0$$

valid near $x = 0$. Clearly indicate the recurrence relation for the coefficients.

b.) If $y(0) = 1$ and $y'(0) = 0$, what is the particular solution?

c.) If $y(0) = 2$ and $y'(0) = -1$, what are $y''(0)$ and $y'''(0)$?