37. S(3, −1, 4), P(4, 3, −5) and $\mathbf{v} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \Rightarrow \overrightarrow{PS} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -4 & 9 \\ -1 & 2 & 3 \end{vmatrix} = -30\mathbf{i} - 6\mathbf{j} - 6\mathbf{k}$

$\Rightarrow d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|} = \frac{\sqrt{900+36+36}}{\sqrt{1+4+9}} = \frac{\sqrt{972}}{\sqrt{14}} = \frac{\sqrt{486}}{\sqrt{7}} = \frac{\sqrt{81 \cdot 6}}{\sqrt{7}} = \frac{9\sqrt{42}}{7}$ is the distance from S to the line

38. S(−1, 4, 3), P(10, −3, 0) and $\mathbf{v} = 4\mathbf{i} + 4\mathbf{k} \Rightarrow \overrightarrow{PS} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -11 & 7 & 3 \\ 4 & 0 & 4 \end{vmatrix} = 28\mathbf{i} + 56\mathbf{j} - 28\mathbf{k} = 28(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$

$\Rightarrow d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|} = \frac{28\sqrt{1+4+1}}{4\sqrt{1+1}} = 7\sqrt{3}$ is the distance from S to the line

39. S(2, −3, 4), x + 2y + 2z = 13 and P(13, 0, 0) is on the plane $\Rightarrow \overrightarrow{PS} = -11\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{n} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

$\Rightarrow d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = \left| \frac{-11-6+8}{\sqrt{1+4+4}} \right| = \left| \frac{-9}{\sqrt{9}} \right| = 3$

40. S(0, 0, 0), 3x + 2y + 6z = 6 and P(2, 0, 0) is on the plane $\Rightarrow \overrightarrow{PS} = -2\mathbf{i}$ and $\mathbf{n} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

$\Rightarrow d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = \left| \frac{-6}{\sqrt{9+4+36}} \right| = \frac{6}{\sqrt{49}} = \frac{6}{7}$

41. S(0, 1, 1), 4y + 3z = −12 and P(0, −3, 0) is on the plane $\Rightarrow \overrightarrow{PS} = 4\mathbf{j} + \mathbf{k}$ and $\mathbf{n} = 4\mathbf{j} + 3\mathbf{k}$

$\Rightarrow d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = \left| \frac{16+3}{\sqrt{16+9}} \right| = \frac{19}{5}$

42. S(2, 2, 3), 2x + y + 2z = 4 and P(2, 0, 0) is on the plane $\Rightarrow \overrightarrow{PS} = 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{n} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

$\Rightarrow d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = \left| \frac{2+6}{\sqrt{4+1+4}} \right| = \frac{8}{3}$

43. S(0, −1, 0), 2x + y + 2z = 4 and P(2, 0, 0) is on the plane $\Rightarrow \overrightarrow{PS} = -2\mathbf{i} - \mathbf{j}$ and $\mathbf{n} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

$\Rightarrow d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = \left| \frac{-4-1+0}{\sqrt{4+1+4}} \right| = \frac{5}{3}$

44. S(1, 0, −1), −4x + y + z = 4 and P(−1, 0, 0) is on the plane $\Rightarrow \overrightarrow{PS} = 2\mathbf{i} - \mathbf{k}$ and $\mathbf{n} = -4\mathbf{i} + \mathbf{j} + \mathbf{k}$

$\Rightarrow d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = \left| \frac{-8-1}{\sqrt{16+1+1}} \right| = \frac{9}{\sqrt{18}} = \frac{3\sqrt{2}}{2}$

45. The point P(1, 0, 0) is on the first plane and S(10, 0, 0) is a point on the second plane $\Rightarrow \overrightarrow{PS} = 9\mathbf{i}$, and $\mathbf{n} = \mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ is normal to the first plane $\Rightarrow$ the distance from S to the first plane is $d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$ $= \left| \frac{9}{\sqrt{1+4+36}} \right| = \frac{9}{\sqrt{41}}$, which is also the distance between the planes.

46. The line is parallel to the plane since $\mathbf{v} \cdot \mathbf{n} = \left(\mathbf{i} + \mathbf{j} - \frac{1}{2}\mathbf{k}\right) \cdot (\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}) = 1 + 2 - 3 = 0$. Also the point S(1, 0, 0) when t = −1 lies on the line, and the point P(10, 0, 0) lies on the plane $\Rightarrow \overrightarrow{PS} = -9\mathbf{i}$. The distance from S to the plane is $d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = \left| \frac{-9}{\sqrt{1+4+36}} \right| = \frac{9}{\sqrt{41}}$, which is also the distance from the line to the plane.

47. $\mathbf{n}_1 = \mathbf{i} + \mathbf{j}$ and $\mathbf{n}_2 = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k} \Rightarrow \theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|}\right) = \cos^{-1}\left(\frac{2+1}{\sqrt{2}\sqrt{9}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

48. $\mathbf{n}_1 = 5\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{n}_2 = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \Rightarrow \theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|}\right) = \cos^{-1}\left(\frac{5-2-3}{\sqrt{27}\,\sqrt{14}}\right) = \cos^{-1}(0) = \frac{\pi}{2}$

49. $\mathbf{n}_1 = 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{n}_2 = 2\mathbf{i} - 2\mathbf{j} - \mathbf{k} \Rightarrow \theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|}\right) = \cos^{-1}\left(\frac{4-4-2}{\sqrt{12}\,\sqrt{9}}\right) = \cos^{-1}\left(\frac{-1}{3\sqrt{3}}\right) \approx 1.76$ rad

50. $\mathbf{n}_1 = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{n}_2 = \mathbf{k} \Rightarrow \theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}\,\sqrt{1}}\right) \approx 0.96$ rad

51. $\mathbf{n}_1 = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{n}_2 = \mathbf{i} + 2\mathbf{j} + \mathbf{k} \Rightarrow \theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|}\right) = \cos^{-1}\left(\frac{2+4-1}{\sqrt{9}\,\sqrt{6}}\right) = \cos^{-1}\left(\frac{5}{3\sqrt{6}}\right) \approx 0.82$ rad

52. $\mathbf{n}_1 = 4\mathbf{j} + 3\mathbf{k}$ and $\mathbf{n}_2 = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k} \Rightarrow \theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|}\right) = \cos^{-1}\left(\frac{8+18}{\sqrt{25}\,\sqrt{49}}\right) = \cos^{-1}\left(\frac{26}{35}\right) \approx 0.73$ rad

53. $2x - y + 3z = 6 \Rightarrow 2(1-t) - (3t) + 3(1+t) = 6 \Rightarrow -2t + 5 = 6 \Rightarrow t = -\frac{1}{2} \Rightarrow x = \frac{3}{2}, y = -\frac{3}{2}$ and $z = \frac{1}{2}$
$\Rightarrow \left(\frac{3}{2}, -\frac{3}{2}, \frac{1}{2}\right)$ is the point

54. $6x + 3y - 4z = -12 \Rightarrow 6(2) + 3(3+2t) - 4(-2-2t) = -12 \Rightarrow 14t + 29 = -12 \Rightarrow t = -\frac{41}{14} \Rightarrow x = 2, y = 3 - \frac{41}{7}$, and $z = -2 + \frac{41}{7} \Rightarrow \left(2, -\frac{20}{7}, \frac{27}{7}\right)$ is the point

55. $x + y + z = 2 \Rightarrow (1+2t) + (1+5t) + (3t) = 2 \Rightarrow 10t + 2 = 2 \Rightarrow t = 0 \Rightarrow x = 1, y = 1$ and $z = 0$
$\Rightarrow (1, 1, 0)$ is the point

56. $2x - 3z = 7 \Rightarrow 2(-1+3t) - 3(5t) = 7 \Rightarrow -9t - 2 = 7 \Rightarrow t = -1 \Rightarrow x = -1 - 3, y = -2$ and $z = -5$
$\Rightarrow (-4, -2, -5)$ is the point

57. $\mathbf{n}_1 = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{n}_2 = \mathbf{i} + \mathbf{j} \Rightarrow \mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\mathbf{i} + \mathbf{j}$, the direction of the desired line; $(1, 1, -1)$ is on both planes $\Rightarrow$ the desired line is $x = 1 - t, y = 1 + t, z = -1$

58. $\mathbf{n}_1 = 3\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$ and $\mathbf{n}_2 = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k} \Rightarrow \mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} = 14\mathbf{i} + 2\mathbf{j} + 15\mathbf{k}$, the direction of the desired line; $(1, 0, 0)$ is on both planes $\Rightarrow$ the desired line is $x = 1 + 14t, y = 2t, z = 15t$

59. $\mathbf{n}_1 = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{n}_2 = \mathbf{i} + \mathbf{j} - 2\mathbf{k} \Rightarrow \mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 4 \\ 1 & 1 & -2 \end{vmatrix} = 6\mathbf{j} + 3\mathbf{k}$, the direction of the desired line; $(4, 3, 1)$ is on both planes $\Rightarrow$ the desired line is $x = 4, y = 3 + 6t, z = 1 + 3t$

60. $\mathbf{n}_1 = 5\mathbf{i} - 2\mathbf{j}$ and $\mathbf{n}_2 = 4\mathbf{j} - 5\mathbf{k} \Rightarrow \mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -2 & 0 \\ 0 & 4 & -5 \end{vmatrix} = 10\mathbf{i} + 25\mathbf{j} + 20\mathbf{k}$, the direction of the desired line; $(1, -3, 1)$ is on both planes $\Rightarrow$ the desired line is $x = 1 + 10t, y = -3 + 25t, z = 1 + 20t$

61. <u>L1 & L2</u>: $x = 3 + 2t = 1 + 4s$ and $y = -1 + 4t = 1 + 2s \Rightarrow \begin{cases} 2t - 4s = -2 \\ 4t - 2s = 2 \end{cases} \Rightarrow \begin{cases} 2t - 4s = -2 \\ 2t - s = 1 \end{cases}$
$\Rightarrow -3s = -3 \Rightarrow s = 1$ and $t = 1 \Rightarrow$ on L1, $z = 1$ and on L2, $z = 1 \Rightarrow$ L1 and L2 intersect at $(5, 3, 1)$.

L2 & L3: The direction of L2 is $\frac{1}{6}(4\mathbf{i}+2\mathbf{j}+4\mathbf{k})=\frac{1}{3}(2\mathbf{i}+\mathbf{j}+2\mathbf{k})$ which is the same as the direction $\frac{1}{3}(2\mathbf{i}+\mathbf{j}+2\mathbf{k})$ of L3; hence L2 and L3 are parallel.

L1 & L3: $x=3+2t=3+2r$ and $y=-1+4t=2+r \Rightarrow \begin{cases} 2t-2r=0 \\ 4t-\ r=3 \end{cases} \Rightarrow \begin{cases} t-r=0 \\ 4t-\ r=3 \end{cases} \Rightarrow 3t=3$ $\Rightarrow t=1$ and $r=1 \Rightarrow$ on L1, $z=2$ while on L3, $z=0 \Rightarrow$ L1 and L2 do not intersect. The direction of L1 is $\frac{1}{\sqrt{21}}(2\mathbf{i}+4\mathbf{j}-\mathbf{k})$ while the direction of L3 is $\frac{1}{3}(2\mathbf{i}+\mathbf{j}+2\mathbf{k})$ and neither is a multiple of the other; hence L1 and L3 are skew.

62. L1 & L2: $x=1+2t=2-s$ and $y=-1-t=3s \Rightarrow \begin{cases} 2t+\ s=1 \\ -t-3s=1 \end{cases} \Rightarrow -5s=3 \Rightarrow s=-\frac{3}{5}$ and $t=\frac{4}{5} \Rightarrow$ on L1, $z=\frac{12}{5}$ while on L2, $z=1-\frac{3}{5}=\frac{2}{5} \Rightarrow$ L1 and L2 do not intersect. The direction of L1 is $\frac{1}{\sqrt{14}}(2\mathbf{i}-\mathbf{j}+3\mathbf{k})$ while the direction of L2 is $\frac{1}{\sqrt{11}}(-\mathbf{i}+3\mathbf{j}+\mathbf{k})$ and neither is a multiple of the other; hence, L1 and L2 are skew.

L2 & L3: $x=2-s=5+2r$ and $y=3s=1-r \Rightarrow \begin{cases} -s-2r=3 \\ 3s+\ r=1 \end{cases} \Rightarrow 5s=5 \Rightarrow s=1$ and $r=-2 \Rightarrow$ on L2, $z=2$ and on L3, $z=2 \Rightarrow$ L2 and L3 intersect at $(1,3,2)$.

L1 & L3: L1 and L3 have the same direction $\frac{1}{\sqrt{14}}(2\mathbf{i}-\mathbf{j}+3\mathbf{k})$; hence L1 and L3 are parallel.

63. $x=2+2t,\ y=-4-t,\ z=7+3t;\ x=-2-t,\ y=-2+\frac{1}{2}t,\ z=1-\frac{3}{2}t$

64. $1(x-4)-2(y-1)+1(z-5)=0 \Rightarrow x-4-2y+2+z-5=0 \Rightarrow x-2y+z=7;$
$-\sqrt{2}(x-3)+2\sqrt{2}(y+2)-\sqrt{2}(z-0)=0 \Rightarrow -\sqrt{2}x+2\sqrt{2}y-\sqrt{2}z=-7\sqrt{2}$

65. $x=0 \Rightarrow t=-\frac{1}{2},\ y=-\frac{1}{2},\ z=-\frac{3}{2} \Rightarrow \left(0,-\frac{1}{2},-\frac{3}{2}\right);\ y=0 \Rightarrow t=-1,\ x=-1,\ z=-3 \Rightarrow (-1,0,-3);\ z=0$ $\Rightarrow t=0,\ x=1,\ y=-1 \Rightarrow (1,-1,0)$

66. The line contains $(0,0,3)$ and $\left(\sqrt{3},1,3\right)$ because the projection of the line onto the xy-plane contains the origin and intersects the positive x-axis at a 30° angle. The direction of the line is $\sqrt{3}\mathbf{i}+\mathbf{j}+0\mathbf{k} \Rightarrow$ the line in question is $x=\sqrt{3}t,\ y=t,\ z=3$.

67. With substitution of the line into the plane we have $2(1-2t)+(2+5t)-(-3t)=8 \Rightarrow 2-4t+2+5t+3t=8$ $\Rightarrow 4t+4=8 \Rightarrow t=1 \Rightarrow$ the point $(-1,7,-3)$ is contained in both the line and plane, so they are not parallel.

68. The planes are parallel when either vector $A_1\mathbf{i}+B_1\mathbf{j}+C_1\mathbf{k}$ or $A_2\mathbf{i}+B_2\mathbf{j}+C_2\mathbf{k}$ is a multiple of the other or when $|(A_1\mathbf{i}+B_1\mathbf{j}+C_1\mathbf{k})\times(A_2\mathbf{i}+B_2\mathbf{j}+C_2\mathbf{k}|=0$. The planes are perpendicular when their normals are perpendicular, or$(A_1\mathbf{i}+B_1\mathbf{j}+C_1\mathbf{k})\cdot(A_2\mathbf{i}+B_2\mathbf{j}+C_2\mathbf{k})=0$.

69. There are many possible answers. One is found as follows: eliminate t to get $t=x-1=2-y=\frac{z-3}{2}$ $\Rightarrow x-1=2-y$ and $2-y=\frac{z-3}{2} \Rightarrow x+y=3$ and $2y+z=7$ are two such planes.

70. Since the plane passes through the origin, its general equation is of the form $Ax+By+Cz=0$. Since it meets the plane M at a right angle, their normal vectors are perpendicular $\Rightarrow 2A+3B+C=0$. One choice satisfying this equation is $A=1,\ B=-1$ and $C=1 \Rightarrow x-y+z=0$. Any plane $Ax+By+Cz=0$ with $2A+3B+C=0$ will pass through the origin and be perpendicular to M.

71. The points $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ are the x, y, and z intercepts of the plane. Since a, b, and c are all nonzero, the plane must intersect all three coordinate axes and cannot pass through the origin. Thus, $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ describes all planes <u>except</u> those through the origin or parallel to a coordinate axis.

72. Yes. If $\mathbf{v}_1$ and $\mathbf{v}_2$ are nonzero vectors parallel to the lines, then $\mathbf{v}_1 \times \mathbf{v}_2 \neq \mathbf{0}$ is perpendicular to the lines.

73. (a) $\overrightarrow{EP} = c\overrightarrow{EP_1} \Rightarrow -x_0\mathbf{i} + y\mathbf{j} + z\mathbf{k} = c\left[(x_1 - x_0)\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}\right] \Rightarrow -x_0 = c(x_1 - x_0)$, $y = cy_1$ and $z = cz_1$, where c is a positive real number

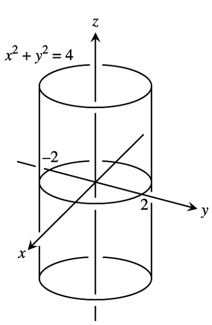
(b) At $x_1 = 0 \Rightarrow c = 1 \Rightarrow y = y_1$ and $z = z_1$; at $x_1 = x_0 \Rightarrow x_0 = 0, y = 0, z = 0$; $\lim\limits_{x_0 \to \infty} c = \lim\limits_{x_0 \to \infty} \frac{-x_0}{x_1 - x_0} = \lim\limits_{x_0 \to \infty} \frac{-1}{-1} = 1 \Rightarrow c \to 1$ so that $y \to y_1$ and $z \to z_1$

74. The plane which contains the triangular plane is $x + y + z = 2$. The line containing the endpoints of the line segment is $x = 1 - t$, $y = 2t$, $z = 2t$. The plane and the line intersect at $\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$. The visible section of the line segment is $\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = 1$ unit in length. The length of the line segment is $\sqrt{1^2 + 2^2 + 2^2} = 3 \Rightarrow \frac{2}{3}$ of the line segment is hidden from view.

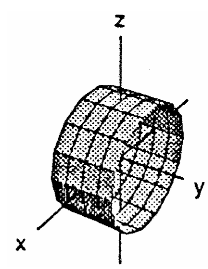
12.6 CYLINDERS AND QUADRIC SURFACES

1. d, ellipsoid
2. i, hyperboloid
3. a, cylinder
4. g, cone
5. l, hyperbolic paraboloid
6. e, paraboloid
7. b, cylinder
8. j, hyperboloid
9. k, hyperbolic paraboloid
10. f, paraboloid
11. h, cone
12. c, ellipsoid

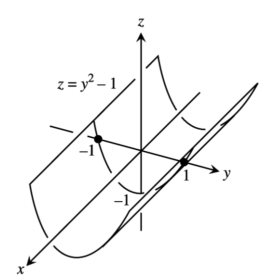
13. $x^2 + y^2 = 4$



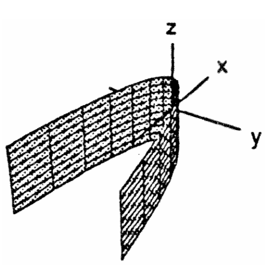
14. $x^2 + z^2 = 4$



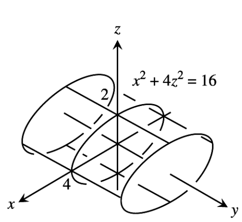
15. $z = y^2 - 1$



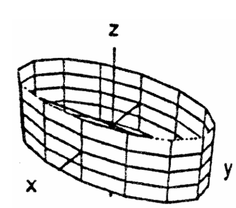
16. $x = y^2$



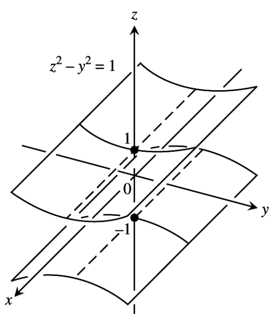
17. $x^2 + 4z^2 = 16$



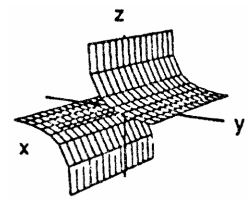
18. $4x^2 + y^2 = 36$



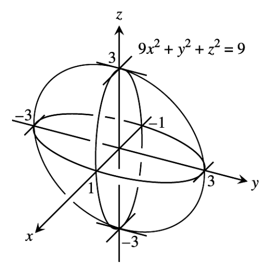
19. $z^2 - y^2 = 1$



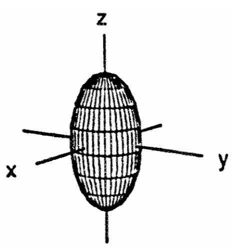
20. $yz = 1$



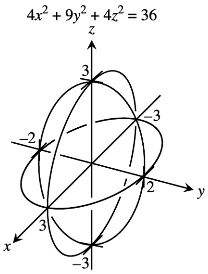
21. $9x^2 + y^2 + z^2 = 9$



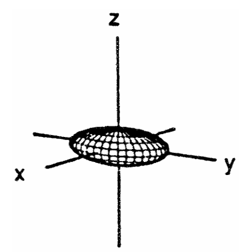
22. $4x^2 + 4y^2 + z^2 = 16$



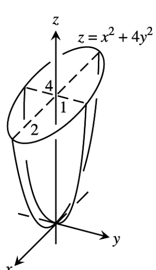
23. $4x^2 + 9y^2 + 4z^2 = 36$



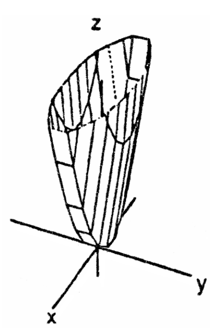
24. $9x^2 + 4y^2 + 36z^2 = 36$



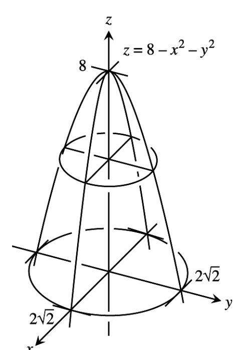
25. $x^2 + 4y^2 = z$



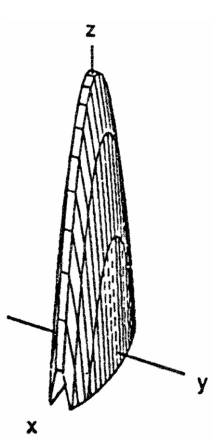
26. $z = x^2 + 9y^2$



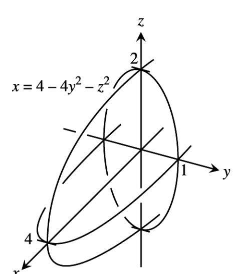
27. $z = 8 - x^2 - y^2$



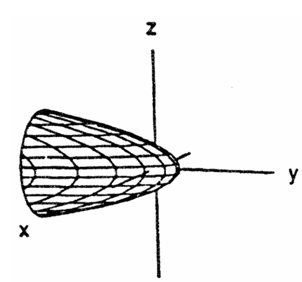
28. $z = 18 - x^2 - 9y^2$



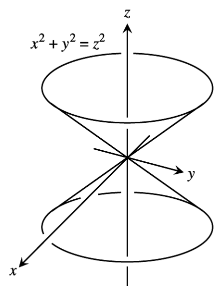
29. $x = 4 - 4y^2 - z^2$



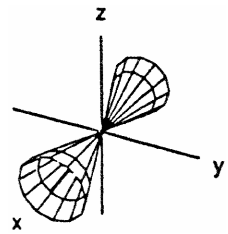
30. $y = 1 - x^2 - z^2$



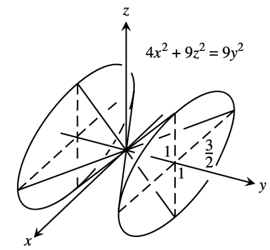
31. $x^2 + y^2 = z^2$



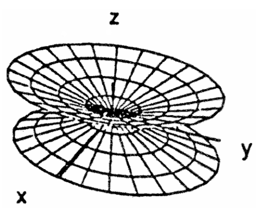
32. $y^2 + z^2 = x^2$



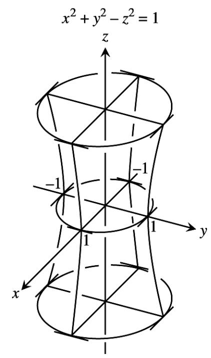
33. $4x^2 + 9z^2 = 9y^2$



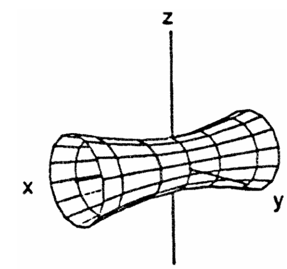
34. $9x^2 + 4y^2 = 36z^2$



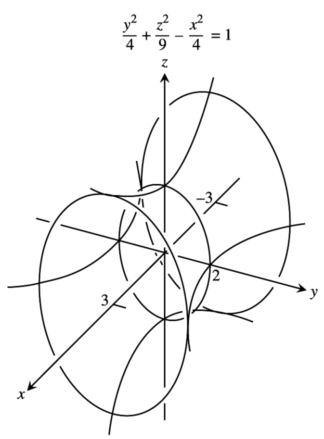
35. $x^2 + y^2 - z^2 = 1$



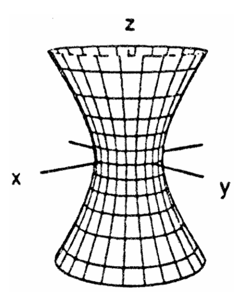
36. $y^2 + z^2 - x^2 = 1$



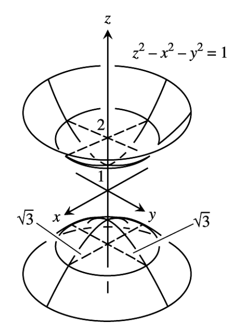
37. $\frac{y^2}{4} + \frac{z^2}{9} - \frac{x^2}{4} = 1$



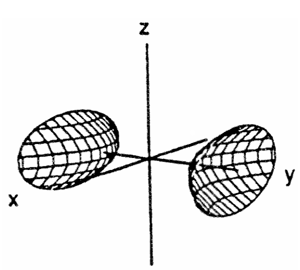
38. $\frac{x^2}{4} + \frac{y^2}{4} - \frac{z^2}{9} = 1$



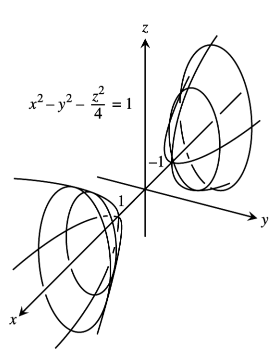
39. $z^2 - x^2 - y^2 = 1$



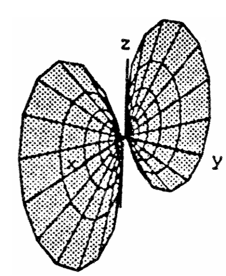
40. $\frac{y^2}{4} - \frac{x^2}{4} - z^2 = 1$



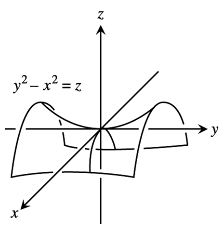
41. $x^2 - y^2 - \frac{z^2}{4} = 1$



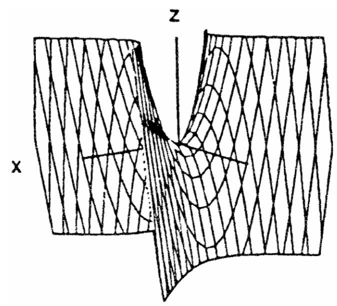
42. $\frac{x^2}{4} - y^2 - \frac{z^2}{4} = 1$



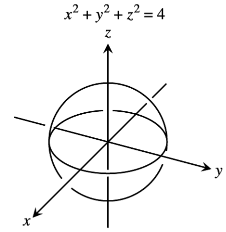
43. $y^2 - x^2 = z$



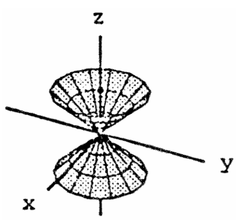
44. $x^2 = y^2 = z$



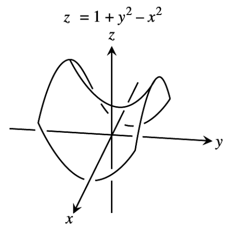
45. $x^2 + y^2 + z^2 = 4$



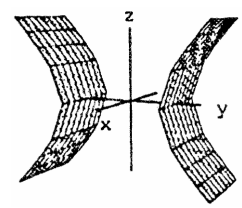
46. $4x^2 + 4y^2 = z^2$



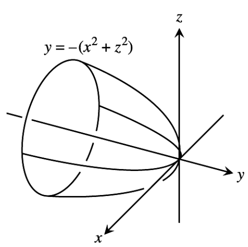
47. $z = 1 + y^2 - x^2$



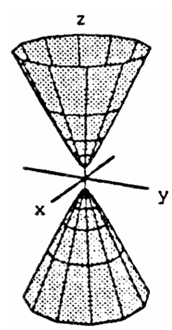
48. $y^2 - z^2 = 4$



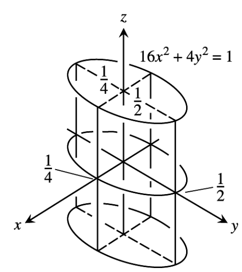
49. $y = -(x^2 + z^2)$



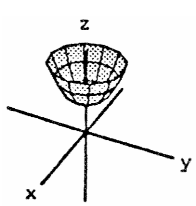
50. $z^2 - 4x^2 - 4y^2 = 4$



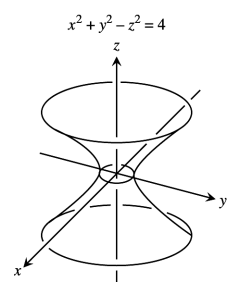
51. $16x^2 + 4y^2 = 1$



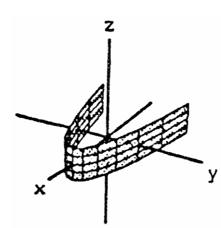
52. $z = x^2 + y^2 + 1$



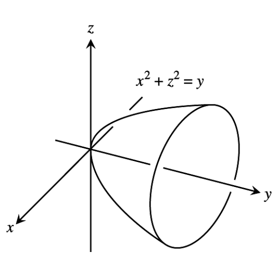
53. $x^2 + y^2 - z^2 = 4$



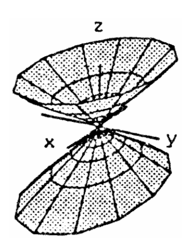
54. $x = 4 - y^2$



55. $x^2 + z^2 = y$



56. $z^2 - \frac{x^2}{4} - y^2 = 1$



57. $x^2 + z^2 = 1$