Name_____

MA 221 Final Exam Part I

November 14, 2007

Instructions: Answer all 7 questions in the space provided. Show all necessary work, and be neat. You may use only a pen or pencil on this part of the final exam. When you finish this part, turn it in and pick up a copy of the second part of exam. You may not return to this part after turning it in.

For Grading Use

#1	(5)	
#2	(10)	
#3	(15)	
#4	(15)	
#5	(15)	
#6	(15)	
#7	(15)	
Total (Part I)	(90)	
Total (Part II)	(85)	
Grand Total	(175)	

1. Use Euler's method with $h = 0.1$ to approximate $y(0.2)$ if y satisfies the IVP $\dfrac{dy}{dx} = y^2 + xy$, $y(0) = 1$.

2. Determine the solution to $x'' + 25x = 5$, $x(0) = 1$, $x'(0) = 4$.

3. (a) Determine the general solution to $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = 4e^{-x}$

(b) Determine the general solution to $x''(t) + 6x'(t) + 13x(t) = 0$

(c) Determine b so that $x'' + bx' + 4x = 0$ has $x(t) = (C_1 + C_2 t)\mathrm{e}^{-2t}$ as a general solution.

4. Determine the general solution to $t^2y' + ty = 3 + t^2$.

5. Let $M = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$.

(a) Find the eigenvalues and corresponding eigenvectors of M.

(b) Verify that the product of the eigenvalues of M equals the determinant of M.

6. Use Gaussian elimination and back substitution to solve the linear system of equations given below. Express your final answer in parametric form.

$$\begin{aligned} x_1 + 2x_2 + 5x_3 &= 1 \\ 2x_1 + 4x_2 + 9x_3 &= 1 \\ x_1 + 2x_2 + 4x_3 &= 0 \end{aligned}$$

7. An overdamped unforced spring mass system consists of an iron weight of mass $m = 1$ kg hanging from a spring with spring constant $k = 2$ N/m. The mass is set in motion by applying an initial displacement $y(0) = 1$ m and an initial velocity $y'(0) = -3$ m/s, and it moves in a viscous medium with damping constant $\mu = 3$ N-s/m.

(a) Write out the 2nd order initial value problem (differential equation along with initial conditions) satisfied by the displacement function $y(t)$, and then solve the initial value problem for the displacement function $y(t)$.

(b) In an overdamped spring mass system, the moving weight can cross the equilibrium position ($y = 0$) at most once. For this system, determine the time at which the weight crosses the equilibrium position or demonstrate that no such crossing occurs. Illustrate your conclusion by sketching a reasonably accurate graph of $y(t)$.