Name____

MA 221 Final Exam
November 14, 2007

Part I

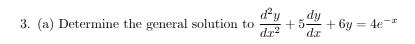
Instructions: Answer all 7 questions in the space provided. Show all necessary work, and be neat. You may use only a pen or pencil on this part of the final exam. When you finish this part, turn it in and pick up a copy of the second part of exam. You may not return to this part after turning it in.

For Grading Use

#1	(5)	
#2	(10)	
#3	(15)	
#4	(15)	
#5	(15)	
#6	(15)	
#7	(15)	
Total (Part I)	(90)	
Total (Part II)	(85)	
Grand Total	(175)	

1. Use Euler's method with h = 0.1 to approximate y(0.2) if y satisfies the IVP $\frac{dy}{dx} = y^2 + xy$, y(0) = 1.

2. Determine the solution to x'' + 25x = 5, x(0) = 1, x'(0) = 4.



(b) Determine the general solution to
$$x''(t) + 6x'(t) + 13x(t) = 0$$

(c) Determine b so that
$$x'' + bx' + 4x = 0$$
 has $x(t) = (C_1 + C_2 t)e^{-2t}$ as a general solution.

4. Determine the general solution to $t^2y' + ty = 3 + t^2$.

5. Let $M = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$.

(a) Find the eigenvalues and corresponding eigenvectors of M.

(b) Verify that the product of the eigenvalues of M equals the determinant of M.

6. Use Gaussian elimination and back substitution to solve the linear system of equations given below. Express your final answer in parametric form.

$$x_1 + 2x_2 + 5x_3 = 1$$

$$2x_1 + 4x_2 + 9x_3 = 1$$

$$x_1 + 2x_2 + 4x_3 = 0$$

- 7. An overdamped unforced spring mass system consists of an iron weight of mass m=1 kg hanging from a spring with spring constant k=2 N/m. The mass is set in motion by applying an initial displacement y(0)=1 m and an initial velocity y'(0)=-3 m/s, and it moves in a viscous medium with damping constant $\mu=3$ N-s/m.
 - (a) Write out the 2nd order initial value problem (differential equation along with initial conditions) satisfied by the displacement function y(t), and then solve the initial value problem for the displacement function y(t).

(b) In an overdamped spring mass system, the moving weight can cross the equilibrium position (y = 0) at most once. For this system, determine the time at which the weight crosses the equilibrium position or demonstrate that no such crossing occurs. Illustrate your conclusion by sketching a reasonably accurate graph of y(t).