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MA 221 Final Exam Part II

November 14, 2007

Instructions: Answer all 5 questions in the space provided. Show all necessary work, and be neat. You may use untitled Maple worksheets on your laptop computer or a calculator on this test for computational support. You need not use Maple or your calculator on every problem, but do use those resources appropriately when you wish as a tool for arithmetic, graphing, solving equations, evaluating expressions and functions, finding derivatives and integrals, checking answers, etc. When you use Maple to complete a step in a problem, describe that step and the result in usual correct mathematical language. **Do not write out any Maple commands you use, unless they are explicitly requested**. No books, notes, or networking are allowed during this test.

For Grading Use

#1	(15)	
#2	(20)	
#3	(10)	
#4	(20)	
#5	(20)	
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Total	(85)	

1. The general solution to the system of linear equations $Ax = b$ is

$$x = t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}.$$

(a) Is $Ax = b$ consistent? Explain.

(b) How many free variables are there? Explain.

(c) Is $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ in the nullspace of A? Explain.

2. A tank initially contains 100 gal of a brine in which 50 lb of salt are dissolved. A brine containing γ lb/gal of salt runs into the tank at a rate of 5 gal/min. The mixture is kept uniform by stirring and flows out at a rate of 4 gal/min.

(a) Let $x(t)$ denote the amount of salt in the tank at time t. Write an initial value problem for $x(t)$. (Your answer will be in terms of γ.)

(b) Find the amount of salt in the tank 25 minutes after the process starts. (Your answer will be in terms of γ.)

(c) Determine the value of γ so that the salt concentration in the tank is $\frac{1}{4}$ lb/gal after 1 hour.

3. Determine whether the following sets of vectors are linearly independent or linearly dependent.

(a) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \\ 5 \end{pmatrix} \right\}$

(b) $\left\{ \begin{pmatrix} 1 \\ 0 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 7 \\ 11 \end{pmatrix} \right\}$

4. A projectile of mass 1 kg is fired parallel to the ground with an initial speed of 10 m/s into a medium that offers resistance to motion of the form $F_r(t) = -v(t) - v^3(t)$ N. Assume the projectile travels in a horizontal line in the positive direction of motion and $v(t)$ satisfies the IVP $\frac{dv(t)}{dt} = -v(t) - v^3(t)$, $v(0) = 10$.

   a.) The above IVP is nonlinear but a change of variables results in a linear equation. Divide both sides by $v^3(t)$ and then use the substitution $z(t) = v^{-2}(t)$ to find a linear IVP in $z(t)$.

   b.) Solve for $z(t)$ and then for $v(t)$.

   c.) When has the speed of the projectile been reduced to 1% of its initial speed?

5. The second order initial value problem

$$\frac{1}{2}x''(t) + \mu x'(t) + 5x(t) = 6\cos(\omega t),\ x(0) = 0\,,\ x'(0) = 0$$

models a forced spring mass system with mass $m = \dfrac{1}{2}$ slug, spring constant $k = 5$ lb/ft, and damping constant μ lb-s/ft.

(a) Suppose that the system is undamped (i.e., $\mu = 0$). Determine the driving force frequency ω that permits the system to achieve resonance, solve the differential equation for $x(t)$, and plot $x(t)$ for $0 \le t \le 10$ in the space below.

(b) Now suppose that the system motion is damped, with damping constant $\mu = 2$. Determine the amplitude of the steady periodic solution as a function of ω and graph that function in the space below.

(c) Use the graph you obtained in part (b) to determine whether there is a "resonant frequency" ω. If so, calculate the resonant frequency and find the amplitude of the corresponding steady state periodic solution.