Math 1151

Calculus I

ASSIGNMENT # 3

DUE DATE: JULY 27, 2023

BEFORE 11:00 AM

1) Calculate definite integral of f(x) at [0, 5] by Right Riemann Sums? $f(x) = \mathbf{3x^2}$

$f(x) = 3x^2 \quad [0, 5]$

$\Delta x = \frac{5-0}{n} = \frac{5}{n}$

$x_i = a + \Delta x \cdot i$

$0 + \Delta x \cdot i = 3\left(\frac{5i}{n}\right)^2 = \frac{75i^2}{n^2}$

height of each rectangle

Area of each rectangle =

$f(x_i) \cdot \Delta x = \frac{75i^2}{n^2} \cdot \frac{5}{n^1} = \frac{375 i^2}{n^3}$

$\sum_{i=1}^{n} f(x_i) \cdot \Delta x \quad \sum_{i=1}^{n} \frac{375}{n^3} i^2 = \frac{\cancel{375}}{n^{\cancel{3}2}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}$

$= \frac{125}{2} \frac{(n+1)(2n+1)}{n^2}$

$= \frac{125}{2} \frac{(2n^2 + n + 2n + 1)}{n^2} = \frac{125}{2}\left(\frac{2n^2 + 3n + 1}{n^2}\right)$

$= \frac{125}{2}\left(\frac{2\frac{n^2}{n^2} + 3\frac{n}{n^2} + \frac{1}{n^2}}{\cancel{n^2}}\right) = \boxed{\frac{125}{2}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)}$

$= \text{Area} = \sum_{n \to \infty} \frac{125}{2}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) = \frac{125}{\cancel{2}} \cdot \cancel{2} = \boxed{125}$

2) Solving simple differential equations with initial condition

Solve $y''' = e^x + 1; \quad y''(0) = 1, \quad y'(0) = 2, \quad y(0) = 3$

$y'' = \int (e^x + 1)\, dx =$

$\int e^x dx + \int 1 \cdot dx =$

$e^x + x + C_1$

$e^0 + 0 + C_1 = 1$

$1 + 0 + C_1 = 1$

$\boxed{C_1 = 0}$

$y'' = e^x + x + 0 = e^x + x$

$y' = \int (e^x + x)\, dx =$

$\int e^x dx + \int x^1 dx =$

$e^x + \frac{x^2}{2} + C_2$

$e^0 + \frac{0^2}{2} + C_2 = 2$

$1 + 0 + C_2 = 2$

$0 + 0 + C_2 = 1$

$C_2 = 1$

$y' = e^x + \frac{x^2}{2} + 1$

$y = \int \left(e^x + \frac{x^2}{2} + 1\right) dx =$

$\int e^x dx + \frac{1}{2}\int x^2 dx + \int 1 \cdot dx =$

$e^x + \frac{1}{2} \cdot \frac{x^{2+1}}{3} + x + C_3 =$

$e^x + \frac{x^3}{6} + x + C_3$

$e^0 + \frac{0^3}{6} + 0 + C_3 = 3$

$1 + 0 + 0 + C_3 = 3$

$1 + C_3 = 3$

$C_3 = 2$

$y = e^x + \frac{x^3}{6} + x + 2$

3) Suppose that the **Velocity** of a car along a track at time t is given as a function of time as: $V(t) = 1 + 2t + 3t^2 - 4t^3$ miles per second. Suppose further that: when $t = 1$, the **Position** of the car is 10 miles along the track. What is the **position** of the car along the track when $t = 1.5$?

$P = \int (1 + 2t + 3t^2 - 4t^3)\, dt$

$A + t^2 + t^3 - t^4 + C$

$1 + 1 + 1 - 1 + C = 10$

$C = 8$

$P(t) = t + t^2 + t^3 - t^4 + 8$

$P(1.5) = 1.5 + (1.5)^2 + (1.5)^3 - (1.5)^4 + 8 = 10.0625$ miles

4) Solve the equation for " y " that satisfies the initial condition.

$$2y\frac{dy}{dx} - e^{x} = 0 \quad at \quad (0,2)$$

$dx \cdot 2y \frac{dy}{dx} = -e^{x}\, dx = 0 \cdot dx$

$2y\, dy - \cancel{e^{x}\, dx} = 0$
$\quad\;\; \cancel{+e^{x}}\, dx \quad +e^{x}\, dx$

$2y\, dy = e^{x}\, dx$

$\frac{\cancel{2}\, y^{1+1}}{\cancel{2}}$

$y^2 = e^{x} + c$

Use the given Point

$2^2 = e^0 + c$

$4 = 1 + c$

$-1 \quad -1$

$3 = C$

$\sqrt{y^2} = \sqrt{e^{x} + 3}$

$y = \sqrt{e^{x} + 3}$

5) Find the Integral of the following.

a)

$$\int \frac{x}{\sqrt{25\,x^2 - 9}}\, dx$$

$U = 25x^2 - 9$

$dU = 25 \cdot 2x$

$\frac{dU}{50} = \frac{\cancel{50}\, x\, dx}{\cancel{50}}$

$\frac{}{50} dU = x \cdot dx$

$\int \frac{1}{\sqrt{U}} \frac{1}{50} dU = \frac{1}{50} \int \frac{1}{U^{\frac{1}{2}}} dU =$

$\sqrt[2]{U'} = U^{\frac{1}{2}}$

b)

$$\int \frac{4\ln(x)}{x}\, dx = \int \frac{4u}{\cancel{x}} \cancel{x} du = 2(\ln(x))^2 + C$$

$u = \ln(x)$

$du = \frac{1}{x} dx$

$dx = x\, du$

$= 4\int u$

$= \frac{4u^2}{2}$

$= 2u^2$

$= 2\ln^2 x + c$

c)

$$\int \frac{1}{\sqrt{4 - x^2}}\, dx = \arcsin\left(\frac{1}{2}x\right) + c$$

definte Integral

6) Using your graphing **calculator**, Rounded your answer to **two** decimal places and determine the area of the region

$$y = (\sin x)^2 \ on\ the\ Interval\ \left[0, \quad \frac{\pi}{2}\right].$$

7) Find the **area** of $g(x) = x^2 - 16$ and the region bounded by $x = -4, x = 4\ and\ the\ x - axis.$

$\int_0^4 (x^2 + 16)\,dx = 85.3$

8) Evaluate the Integral by using <u>appropriate technique</u> and state the **exact** answer.

$\ln = \frac{1}{x}$ use U not X $U = \frac{\ln}{x}$

<u>a)</u>

$$\int \frac{(\boldsymbol{\ln t})^{(4)}}{\boldsymbol{4\,t}} \boldsymbol{dt} = \frac{1}{4}\int (\ln t)^4 \cdot \frac{dt}{t} =$$

$U = \ln t$

$dU = \frac{1}{t}$

$dt = \frac{dt}{t}$

<u>b)</u>

$$\int_0^{\frac{\pi}{2}} \boldsymbol{4\,Cos(t)\,sin(t)dt} = \int 4\cos(t) \cdot u \cdot \frac{du}{\cos(t)}$$

Sin U dt

$u = \sin(t)$

$du = \cos(t)\,dt$

$dt = \frac{du}{\cos(t)}$

$= \int 4u\,du$

$= 4\int u\,du$

$= 4\int \frac{u^2}{2}$

$= \frac{4\sin^2(t)}{2}$

$= 2\sin^2(t) \Big|_0^{\frac{\pi}{2}}$

$= 2\sin^2(\frac{\pi}{2}) - 2\sin^2(0)$

$= 2$

9) Evaluate the following Indefinite Integrals, using the appropriate substitution:

a.

$$\int \frac{100\,x^4}{4x^5 - 6}\,dx = \int \frac{\overset{5}{\cancel{100x^4}}}{u} \cdot \frac{du}{\cancel{20x^4}} = 5 \cdot \ln(u)$$

$$= 5 \cdot \ln(4x^5 - 6) + C$$

$u = 4x^5 - 6$

$du = 20x^4\,dx$

$dx = \dfrac{du}{20x^4}$

$$= \int \frac{5}{u} \cdot du$$

$$= 5 \int \frac{1}{u}$$

b. $\displaystyle \int \frac{\sin^{-1} x}{\sqrt{1 - x^2}}\,dx = \sin^{-1}(x) \int \frac{1}{\sqrt{1 - x^2}}$ $\qquad \displaystyle \int \sin^{-1}(x)\,dx$

$$= \sin^{-1}(x) \cdot \arcsin(x)$$

10) Evaluate the following Indefinite Integrals

a.

$$\int \frac{e^{6x}}{\csc(5e^{6x} - 1)}\,dx$$

b.

$$\int \frac{\boldsymbol{sec^2(x)}}{\sqrt{\boldsymbol{25 - tan\,(x)}}}\,\boldsymbol{dx}$$

c.

$$\int \frac{secx\ tanx}{secx\ -\ 1}\,dx$$

Evaluate the following Indefinite Integrals

11) $\int x^2 \sin x^3 \, dx$

$u = 5e^{6x} - 1$

$du = 30e^{6x} \, dx$

$dx = \frac{du}{30e^{6x}}$

$\int \frac{1}{\csc u} = \int \sin u$

$u = x^3$

$du = 3x^2 dx$

$dx = \frac{du}{3x^2}$

12) $\int x \sin x^2 \cos x^2 \, dx$

$u = \sin(x^2)$

$du = \cos(x^2) \, 2x \, dx$

$dx = \frac{du}{2x\cos(x^2)}$

$= \int u \cos(x^2) \frac{du}{2x\cos(x^2)}$

$= \frac{1}{2}\int u \, du$

$= \frac{1}{2}\frac{u^2}{2} = \frac{1}{4}\sin^2(x^2) + C$

13) $\int \cos x \sin^2 x \, dx$

$\sin = u$

14) $\int \frac{dx}{x \ln x}$

u

15) $\int x e^{-3x^2} dx$

u

16) $\int 3x^2 \sqrt{2x^3 - 5} \, dx$

17) $\int x \sqrt{x - 1} \, dx$

$u = x - 1$

$x = u + 1$

$dx = du$

$du = dx$

$= \int (u+1)\sqrt{u} \, du$

$= \int (u+1) u^{\frac{1}{2}} du$

$= \int u^{\frac{3}{2}} + u^{\frac{1}{2}} \, du$

$= \frac{2}{5} u^{\frac{5}{2}} + \frac{2}{3} u^{\frac{3}{2}}$

$= \frac{2}{5}(\sqrt{x-1})^5 + \frac{2}{3}(\sqrt{x-1})^3 + C$

18) $\int \frac{\sin x}{1 + \cos x} \, dx$

u

19)

$$\int_1^e \frac{2 \ln x}{x} \, dx = 1$$