Conditional Expected Shortfall

Nonparametric Estimation

Young-geun Kim
ygeunkim.github.io

2019711358, Department of Statistics

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Simulation

Reviewed Paper

[1] Z. Cai and X. Wang. "Nonparametric estimation of conditional VaR and expected shortfall". In: *Journal of Econometrics* 147.1 (2008), pp. 120-130. ISSN: 0304-4076. DOI: 10.1016/j.jeconom.2008.09.005.

Simulation

AR(1)-GARCH(1, 0)

$$
\begin{cases}
X_t = Y_{t-1} \\
Y_t = 0.01 + 0.62X_t + \sigma_t \epsilon_t \\
\sigma_t^2 = -0.15 + 0.65\sigma_{t-1}^2 \\
\epsilon_t \sim N(0, 1)
\end{cases}
$$

True conditional distribution

Since $\epsilon_t \sim N(0, 1)$,

$$\sigma_t \epsilon_t \sim N(0, \sigma_t)$$

$$Y_t \mid X_t \sim N\left(0.01 + 0.62X_t, \sigma_t^2\right)$$

True CES

For each X_t, `pnorm(x, mean, sd)` gives the conditional cdf value.
Inverting $S(y \mid x) = 1 - F(y \mid x)$ gives $\nu_p(x)$.

$$\nu_p(x) = S^{-1}(p \mid x)$$

Plugging-in method gives $\mu_p(x)$.

$$\mu_p(x) = \frac{1}{p} \int_{\nu_p(x)}^{\infty} y f(y \mid x) dy$$

Goal of MC Simulation

Compute the error between the true $\mu_p(x)$ and $\hat{\mu}_p(x)$
Is the estimator of Cai and Wang (2008) good?

Random number generation

Monte Carlo Samples:

For fixed x_t
Generate GARCH(1, 0): (σ_t, ϵ_t)
$X_t = Y_{t-1}$
AR(1): $Y_t = 0.01 + 0.62Y_{t-1} + \sigma_t\epsilon_t$

References

Cai, Zongwu, and Xian Wang. 2008. "Nonparametric estimation of conditional VaR and expected shortfall." *Journal of Econometrics* 147 (1): 120–30. https://doi.org/10.1016/j.jeconom.2008.09.005.