

# Homework 1

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## Question 1

Problem / MIN  $\left\{ \frac{1}{T} \sum_{t=1}^T \left| \sum_{i=1}^n (R_i(t) - r_i) x_i \right| \right\}$

subject to  $\sum_{i=1}^n x_i \leq 1$

$x_1, x_2, \dots, x_n \geq 0$

$r_1 x_1 + r_2 x_2 + \dots + r_n x_n = \bar{R}$

let  $z_t = \left| \sum_{i=1}^n (R_i(t) - r_i) x_i \right| \quad t = 1, 2, \dots, T$

$z_t \geq \sum_{i=1}^n (R_i(t) - r_i) x_i \quad \text{and} \quad z_t \geq - \sum_{i=1}^n (R_i(t) - r_i) x_i$

so the linear could be

$\text{Max} \left( - \frac{1}{T} \sum_{t=1}^T z_t \right)$

subject to  $z_t \leq \sum_{i=1}^n (R_i(t) - r_i) x_i, \quad t = 1, 2, \dots, T$

$z_t \leq - \sum_{i=1}^n (R_i(t) - r_i) x_i, \quad t = 1, \dots, T$

$\sum_{i=1}^n x_i \leq 1$

$x_1 r_1 + x_2 r_2 + \dots + r_n x_n = \bar{R}$

$x_1, x_2, \dots, x_n \geq 0$

$z_1, \dots, z_T \geq 0$

## Question 2.1

Problem 2 1)  $\max \{z_6\}$   
 subject to  $x_1 + y_1 + y_2 - z_1 = 200$

$$1.005 z_1 + x_2 + y_3 \quad -1.01 y_1 - z_2 = -100$$

$$1.005 z_2 + x_3 + y_4 \quad -z_3 = 150$$

$$1.005 z_3 + y_5 + x_4 \quad -1.05 y_2 - z_4 = -50$$

$$1.005 z_4 + x_5 \quad -1.04 y_3 - z_5 = 300$$

$$1.005 z_5 - 1.02 x_5 - 1.05 y_4 - 1.03 y_5 - z_6 - (1.02)^5 x_1 \\ - (1.02)^4 x_2 - (1.02)^3 x_3 - (1.02)^2 x_4 = -600$$

$$x_t < 100, \quad x_t \geq 0 \quad y_i \geq 0 \quad z_t \geq 0 \quad i=1,2,3,4,5 \\ t=1, \dots, 6$$

and due to the formula, I got the ample code like following.

```

# decision variables
var x{1..5}>=0,<=100;
var y{1..6}>=0;
var z{1..6}>=0;

## objective function

maximize value:z[6];

## constraints
subject to cons1:x[1]+y[1]+y[2]-z[1]=200;
subject to cons2:1.005*z[1]+x[2]+y[3]-1.01*y[1]-z[2]=-100;
subject to cons3:1.005*z[2]+x[3]+y[4]-z[3]=150;
subject to cons4:1.005*z[3]+y[5]+x[4]-1.05*y[2]-z[4]=-50;
subject to cons5:1.005*z[4]+x[5]-1.04*y[3]-z[5]=300;
subject to cons6:1.005*z[5]-1.02**5*x[1]-1.02**4*x[2]-1.02*x[5]-1.02**3*x[3]-1.05*y[4]-1.02**2*x[4]-1.03*y[5]-z[6]=-600;

options solver gurobi;

solve;

display x,y,z,value;

```

## question 2.2

the code's output is below.

```

Console
AMPL
ampl: reset;
ampl: model homework2_a.mod;
Gurobi 10.0.1: Gurobi 10.0.1: optimal solution; objective 78.30606965
8 simplex iterations
:   x       y       z       :=
1   0     200        0
2   0       0        0
3   0    102        0
4   0    150     304.557
5  100   254.557        0
6   .       0     78.3061
;

value = 78.3061

ampl:

```

So we can get

1. the amount of each commercial paper is 200;0;102;150;254.557
2. the company should draw 100 from credit line at month5
3. the resluting of each month is  
month1:0;month2:0;month3:0;month4:304.557;month5:0;month6:78.3061;

## question 2.3.1

In the question we just need to increase once.

```
# decision variables
var x{1..5}>=0,<=100;
var y{1..6}>=0;
var z{1..6}>=0;

## objective function
maximize value:z[6];

## constraints
subject to cons1:x[1]+y[1]+y[2]-z[1]=200;
subject to cons2:1.005*z[1]+x[2]+y[3]-1.01*y[1]-z[2]=-100;
subject to cons3:1.005*z[2]+x[3]+y[4]-z[3]=150;
subject to cons4:1.005*z[3]+y[5]+x[4]-1.05*y[2]-z[4]=-50;
subject to cons5:1.005*z[4]+x[5]-1.04*y[3]-z[5]=300;
subject to cons6:1.005*z[5]-1.03**5*x[1]-1.03**4*x[2]-1.03*x[5]-1.03**3*x[3]-1.05*y[4]-1.03**2*x[4]-1.03*y[5]-z[6]=-600;

options solver gurobi;

solve;

display x,y,z,value;
```

```
AMPL
ampl: reset;
ampl: model homework2_b.mod;
Gurobi 10.0.1: Gurobi 10.0.1: optimal solution; objective 77.81850746
6 simplex iterations
:   x       y       z       :=
1   0      200        0
2   0        0        0
3   0     102        0
4   0     150     404.06
5   0    354.06        0
6   .        0     77.8185
;

value = 77.8185

ampl:
```

the minimum value of the interest is 3% under this amount the company never uses the credit line and borrows only by issuing commercial papers

## question 2.3.2

```
# decision variables
var x{1..5}>=0,<=100;
var y{1..6}>=0;
var z{1..6}>=0;

## objective function
maximize value:z[6];

## constraints
subject to cons1:x[1]+y[1]+y[2]-z[1]=200;
subject to cons2:1.005*z[1]+x[2]+y[3]-1.01*y[1]-z[2]=-100;
subject to cons3:1.005*z[2]+x[3]+y[4]-z[3]=150;
subject to cons4:1.005*z[3]+y[5]+x[4]-1.05*y[2]-z[4]=-50;
subject to cons5:1.005*z[4]+x[5]-1.04*y[3]-z[5]=300;
subject to cons6:1.005*z[5]-1.02**5*x[1]-1.02**4*x[2]-1.02*x[5]-1.02**3*x[3]-1.05*y[4]-1.02**2*x[4]-1.03*y[5]-z[6]=-600;
subject to cons7:y[3]=0;

options solver gurobi;

solve;

display x,y,z,value;
```

```
ampl: reset;
ampl: model homework2_c.mod;
Gurobi 10.0.1: Gurobi 10.0.1: optimal solution; objective 77.80408354
6 simplex iterations
:      x      y      z      :=
1      0    99.0099      0
2      0   100.99      0
3      0      0      0
4      0    150    199.005
5     100   255.045      0
6      .      0    77.8041
;

value = 77.8041

ampl: |
```

1. the amount of each commercial paper is 99.0099;100.99;0;150;255.045
2. the company should draw 100 from credit line at month5
3. the resluting of each month is  
month1:0;month2:0;month3:0;month4:199.005;month5:0;month6:77.8041;