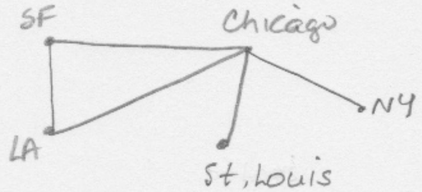
LESSON 1
DISCRETE MATH
§ 10.1, 10.2



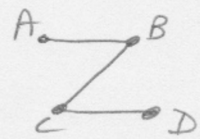
Objects and relationships between some of the objects.

Example. Cities and non-stop flights.

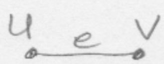
Definition A GRAPH consists of a finite, nonempty set V of objects (called vertices or nodes) and a set E of 2-element subsets of V (called edges)

Example Computers and the flow of information between them.

$V = \{A, B, C, D\}$
$E = \{\{A, B\}, \{B, C\}, \{C, D\}\}$

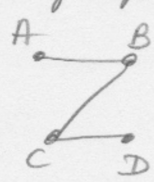


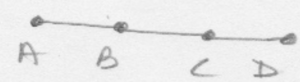
Language If $e = \{u, v\}$ is an edge, we say

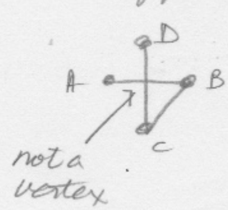


e connects u and v
e is incident with u
e is incident with v
u is incident with e
v is incident with e
u and v are adjacent.

The same graph may be drawn in different ways.

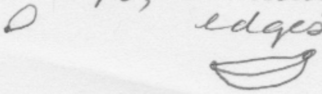






Types of graphs

* Simple graph
no loops, no multiple edges.



* Complete graph on n vertices
K_n

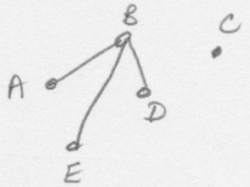
K_3

K_4

* Directed graph
Each edge has a direction associated with it.

Degree of a vertex

deg V = # edges incident to V (include a loop twice)



deg A = 1
deg B =
deg C =
deg D =
deg E =

Theorem

In any graph, sum of the degrees $= \underbrace{2 \cdot (\#\text{edges})}_{\text{an even number}}$

↑
each edge is counted twice

For directed graphs, we define

indegree of V = $\deg^- V$ = # edges with terminal vertex V.

outdegree of V = $\deg^+ V$ = # edges with initial vertex V.

The above theorem becomes

Theorem sum of the indegrees = sum of the outdegrees = # edges

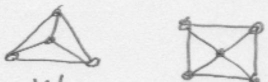
more types of (undirected) graphs.

Cycles

C_3 C_4 C_5 C_6

Wheels

Adding one vertex to a cycle and connecting it to every vertex.

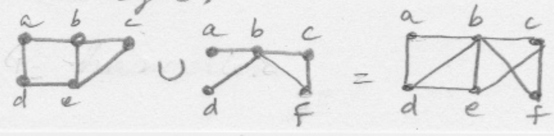


W_3 W_4 W_5 W_6

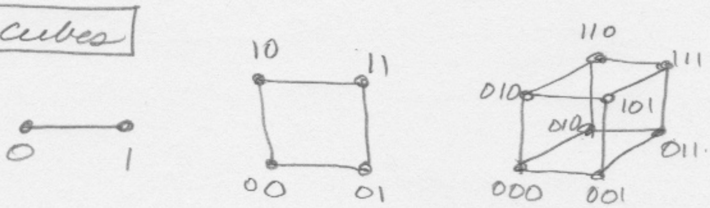
Union of two graphs

Put in all the vertices and all the edges.

If $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ then $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$.



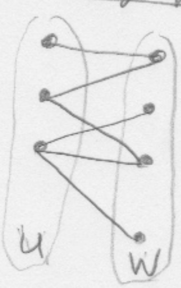
n-cubes



Two adjacent vertices differ by exactly one 0-1 digit.

Bipartite graphs

V



Only edges between the two sets of vertices are present

Examples C_6

Not K_3

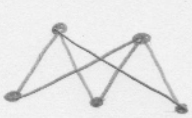
Vertices in U are not connected to each other.
Vertices in W are not connected to each other.

Theorem A simple graph is bipartite iff we can assign one of two different colors to each vertex of the graph in such a way that no two adjacent matrices have the same color.

Proof. ($\Leftarrow$) Assume the colors have been assigned, red and blue. Let U = all red vertices, W = all blue vertices. Then $U \cup W = V$ and U and V are disjoint and there are no edges connecting a vertex in U with a vertex in V.

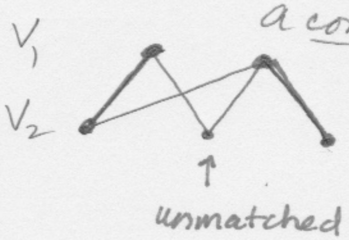
($\Rightarrow$) Let V be bipartite. By defn, $V = U \cup W$ where $U \cap W = \emptyset$ and each edge connects a vertex in U with a vertex in W. Color the vertices in U red and the vertices in W blue. No two adjacent vertices have the same color.

Complete bipartite graph



$K_{2,3}$

Matching Given a bipartite graph.



A <u>completematching</u> pairs each vertex in V_1 with exactly one vertex in V_2 and vice versa.

A <u>matching</u> pairs some of the vertices in V_1 with some of the vertices in V_2, and there are no other edges. If $\{s, t\}$ and $\{u, v\}$ are edges then s, t, u, v are all distinct.

For a complete matching, # vertices in V_1 = # vertices in V_2.

Hall's Marriage Theorem

If $G = (V, E)$ is bipartite with bipartition (V_1, V_2), then G has a complete matching from V_1 to V_2 iff for all subsets A of V_1,

vertices in $N(A)$ $\geq$ # vertices in A.

set of vertices in G that are adjacent to at least one vertex in A. (neighborhood of A)

Proof is optional (on page 658).