ASSIGNMENT 9

* Hand in the starred problems. *

① Determine S_5 if $S_0, S_1, S_2, \cdots, S_n, \cdots$ is a sequence satisfying the given recurrence relation and initial conditions.

(a) $S_n = 3S_{n-1} - 9$ for $n \geq 1$, $S_0 = 5$

(b) $S_n = 2S_{n-1} + 3n$ for $n \geq 1$, $S_0 = 5$

→ * (c) $S_n = -S_{n-1} + 2S_{n-2} + S_{n-3} + n$ for $n \geq 3$, $S_0 = 1, S_1 = 2, S_2 = 5$

(d) $S_n = S_{n-1} - 4S_{n-2} + 3S_{n-3} + (-1)^n$ for $n \geq 3$, $S_0 = 3, S_1 = 2, S_2 = 4$

②

A consumer purchased items costing \$280 with a department store credit card that charges 1.5% interest per month compounded monthly. Write a recurrence relation and initial conditions for b_n, the balance of the consumer's account after n months if no further charges occur and the minimum monthly payment of \$25 is made.

③

Tom, a new college graduate, has just been offered a job paying \$24,000 in the first year. Each year thereafter, the salary will increase by \$1000 plus a 5% cost of living adjustment. Write a recurrence relation and initial conditions for s_n, the amount of Tom's salary after n years of employment.

④ Prove by mathematical induction that $4^n - 3^n + 1$ is a solution to the recurrence relation $S_n = 7S_{n-1} - 12S_{n-2} + 6$ for $n \geq 2$ with the initial conditions $S_0 = 1, S_1 = 2$.

⑤ Use the method of iteration to find a formula expressing S_n as a function of n for the given recurrence relation and initial conditions.

(a) $S_n = 5S_{n-1} + 3$, $S_0 = 1$

→ * (b) $S_n = S_{n-1} + 2n + 4$, $S_0 = 5$

(6) Suppose that a high school had 1000 students enrolled at the beginning of the 1990 school term. The trend in enrollment over the previous 20 years was that the number s_n of students beginning a school year was 5% less than that of the previous year.

(a) Find a recurrence relation and initial conditions representing this situation, assuming that the enrollment trend continues.

(b) Find a formula expressing s_n as a function of n.

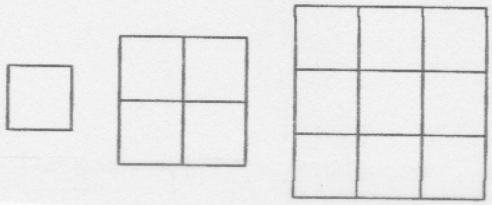
(c) If the enrollment trend continues, what number of students does the formula predict for the beginning of the school year 2000?

(7) Zebra mussels are fresh water mollusks that attack underwater structures. Suppose that the volume of mussels in a confined area grows at a rate of 0.2% per day.

(a) If there are now 10 cubic feet of mussels in a lock on the Illinois River at Peoria, Illinois, develop a recurrence relation and initial conditions that represent the volume m_n of the mussel colony n days hence.

(b) Develop a formula expressing m_n as a function of n.

(8) The figure below shows that 4 one-inch segments are needed to make a 1×1 square, 12 one-inch segments are needed to make a 2×2 square composed of four 1×1 squares, and 24 one-inch segments are needed to make a 3×3 square composed of nine 1×1 squares. How many one-inch segments are needed to make an $n \times n$ square composed of 1×1 squares?



(9) Find an explicit formula for s_n:

$s_n = s_{n-1} - 5, \quad s_0 = 100$

→ * (10) In Mayville, 90% of the existing dog licenses are reissued each year, and 1200 new licenses are issued. In 1995 there were 15,000 dog licenses issued.

(a) Write a difference equation and initial conditions describing the number of dog licenses Mayville will issue n years after 1995.

(b) How many dog licenses will Mayville issue in 2004?

(c) If the present trend continues, how many dog licenses can Mayville expect to issue after many years?

→ * STC. How does a recurrence relation illustrate the idea of creation through self-referral?