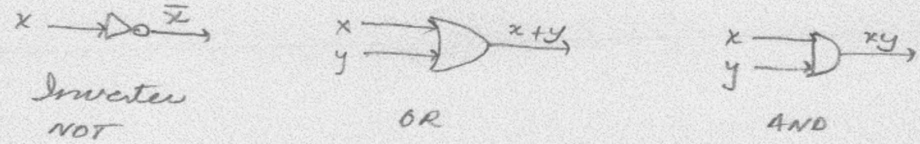
LESSON 16

Combinatorial Circuits

Parts of § 12.3 & 12.4.
p 822   p. 828

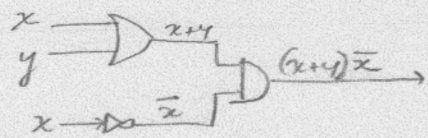
Gates from before.



multiple inputs allowed for OR and AND gates.

Examples p 823.

ⓐ $(x+y)\overline{x}$



ⓑ $\overline{x}\,\overline{(y+\overline{z})}$

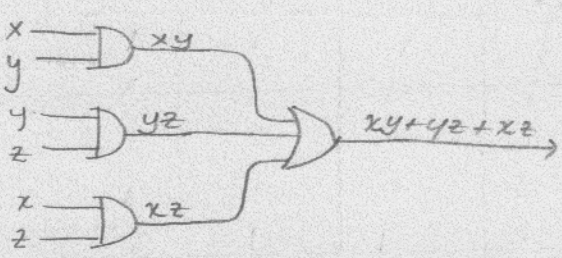
ⓒ $(x+y+z)\overline{x}\,\overline{y}\,\overline{z}$

} Try yourself before turning the page.

Useful circuits

⊛ Committee of three votes. A proposal passes if 2 out of 3 vote yes. Design a circuit that tells whether a proposal passes.

1 is a yes vote   0 is a no vote.   Refer back to Problem 12 in section 12.1. p 818



$F(x,y,z) = xy + yz + xz$
has value 1 iff at least 2 of x, y, & z have value 1.

(*) Lights controlled by more than one switch.

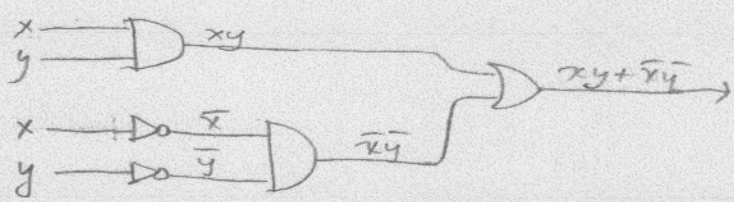
$\boxed{\text{2 switches}}$ x, y. Closed $= 1$

Light $F(x,y)$. Open $= 0$

Arbitrarily say the light is on when $F(1,1) = 1$.

x	y	$F(x,y)$	
1	1	1	switches 1 and 2 are closed - light is on
1	0	0	switch 2 is closed - light goes off
0	1	0	instead switch 1 is closed - light goes off
0	0	1	after the above, the remaining switch is closed. light turns back on.

Sum of minterms $F(x,y) = xy + \bar{x}\bar{y}$.



$\boxed{\text{3 switches}}$ As above and $F(1,1,1) = 1$ say light is on.

x	y	z	$F(x,y,z)$	
1	1	1	1	
1	1	0	0	switch z is opened, light goes off.
1	0	1	0	or switch y is opened, light goes off
1	0	0	1	switch z is closed, light goes on again
0	1	1	0	or switch x is opened, light goes off
0	1	0	1	switch z opens again, light goes on again
0	0	1	1	or switch y opens again, light goes on again
0	0	0	0	light goes off again.

Sum of minterms $F(x,y,z) = xyz + x\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z}$.

circuit shown page 826.

Omit the adders.

Simplifying circuits

The idea:

a circuit with output 1 iff $(x=y=z=1)$ or $(x=z=1$ and $y=0)$.
So the sum of minterms is

$$xyz + x\bar{y}z$$

. The circuit is on page 829.
and involves 3 gates and an inverter.

BUT. $xyz+x\bar{y}z = xz(y+\bar{y}) = xz$. which is much simpler. and has the same output value.

We aim to minimize the Boolean function:-

How? $\boxed{\text{Karnaugh maps}}$ are used to see steps of simplification
k-map
which works well up to $n=6$.

$\boxed{\text{Basic idea}}$ is that if two minterms differ by exactly one literal, then we can factor and get rid of that literal.

As above $xyz + x\bar{y}z = xz\underbrace{(y+\bar{y})}_{=1} = xz$. y has disappeared.

$\boxed{\text{Two variables}}$

Make a table

	y	$\bar{y}$
x	xy	$x\bar{y}$
$\bar{x}$	$\bar{x}y$	$\bar{x}\bar{y}$

Notice that all adjacent cells differ by exactly one literal.

Given a Boolean function $F(x,y,z) = xy + \bar{x}y$ for example, put a 1 in each part of the table if that minterm is present in $F(x,y,z)$.

	y	$\bar{y}$
x	1	0
$\bar{x}$	1	0

Circle blocks of adjacent cells with 1's in them.
This tells us visually that the xy and the $\bar{x}y$ can be simplified

Namely $xy + \bar{x}y = (x+\bar{x})y = y$.

Try $x\bar{y}+\bar{x}y$ and $x\bar{y}+\bar{x}y+\bar{x}\bar{y}$.

If all 4 minterms are present, and all 4 have a 1, then you can combine terms until you get 1, which does not involve any variables.

Goal is to identify the largest possible blocks, and to cover all the 1s with the fewest blocks using the larger blocks first.

$\boxed{\text{Three variables}}$

The K-map is

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x	xyz	$xy\bar{z}$	$x\bar{y}\bar{z}$	$x\bar{y}z$
$\bar{x}$	$\bar{x}yz$	$\bar{x}y\bar{z}$	$\bar{x}\bar{y}\bar{z}$	$\bar{x}\bar{y}z$

← carefully arranged so that, in the body, adjacent cells will differ by at most one literal.

wrap around. Cells on the far left and far right are adjacent as well.

Then 2×2 blocks and 4×1 blocks represent minterms that can be combined into a single literal.

eg. $xyz + xy\bar{z} + \bar{x}yz + \bar{x}y\bar{z}$ 4 blocks on the left.

$= xy(z+\bar{z}) + \bar{x}y(z+\bar{z})$

$= xy + \bar{x}y$

$= (x+\bar{x})y$

$= y$

In the K-map, again put 1's in blocks representing present minterms. Circle groups of blocks adjacent to each other.

e.g. $xy\bar{z} + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z}$.

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x		1	1	
$\bar{x}$	1		1	

Take one of the largest blocks first, say $xy\bar{z} + x\bar{y}\bar{z}$.

$= x\bar{z}(y+\bar{y}) = x\bar{z}$.

$f = x\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z}$

$= x\bar{z} + \bar{x}\bar{y}\bar{z} + \bar{x}yz = \underbrace{(x+\bar{x}\bar{y})}\bar{z} + \bar{x}yz = (x+\bar{y})\bar{z} + \bar{x}yz$

$= x\bar{z} + \bar{y}\bar{z} + \bar{x}yz$ ✓

$(x+\bar{x})(x+\bar{y})\bar{z} + \bar{x}y$

You try Example 3 p. 832.

$\boxed{\text{Four variables}}$ Table is 4x4

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
wx				
$w\bar{x}$				
$\bar{w}\bar{x}$				
$\bar{w}x$				

Cells on the left and right are adjacent. Also cells on the top and bottom are adjacent. Mark off blocks and continue in the same way.

Omit Quine-McCluskey method.