Clarifications for Lesson 2

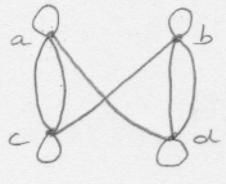
A loop in an undirected graph is counted as an edge once.

It contributes 2 to the degree of a vertex.

So in an adjacency matrix, we only enter 1 for a loop.

Handshake theorem for loops & multiple edges.

p675 # 15



$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

Enter 1 for loops because we are just counting edges here.

deg $a = 5$
deg $b = 5$
deg $c = 5$
deg $d = 5$

$\Sigma \deg = 20$
#edges $= 10$
$20 = 2 \cdot 10$.

Use 2 for loops because this is degree of the vertex here.

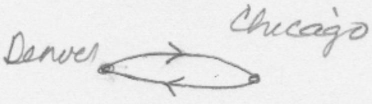
A loop in a directed graph is counted as a directed edge once.

It contributes 1 to the indegree of a vertex and contributes 1 to the outdegree of that vertex.

(See example 1 on p. 652)

An edge joining vertices a and b in an undirected graph is denoted by the set $e = \{a, b\}$. Order doesn't matter. We can write $\{b, a\}$ and it is the same edge.

An edge from vertex a to vertex b in a directed graph, is denoted by the ordered pair $e = (a, b)$. Order does matter. The directed edge starts at a and ends at b. (b, a) is not the same directed edge as (a, b). (b, a) starts at b and ends at a.



is not a multigraph because the two directed edges are not the same.

is a multigraph. The two directed edges each have multiplicity 2.

(*) Page 2-1 of the notes for Lesson 2.

$\boxed{\text{Theorem.}}$

N.B. ⇒ For a simple directed graph:

Sum of row i = $\deg v_i$ = sum of column i.

See example 5 on page 670.

This theorem does not hold for a multigraph. You need to adjust the theorem for the loops. See problem 28 on p. 676.