Monte Carlo Simulation for Asian Option with Volatility Shifts

Goal: Develop a flexible Monte-Carlo based robustness calculation and evaluation tool to asses the pricing accuracy of path-depedent derivatives by exploring the impact of shifting volatility on the simulation over the continous timeline and mathematical limitation of real world probabilities in model-pricing

0.1 Baseline

The Monte Carlo simulation generates random paths for the option price. To simulate the path for the derivative, we use a Geometric Brownian Motion (GBM) with time-varying volatility, which follows the Stochastic Differential Equation (SDE):

$$S_{t+\Delta t} = S_t \cdot \exp\left(\left(\mu - \frac{1}{2}\sigma_{t+\Delta t}^2\right)\Delta t + \sigma_{t+\Delta t}\sqrt{\Delta t} \cdot f(Z_t)\right)$$

Where $f(Z_t)$ is defined as:

$$f(Z_t) = \begin{cases} z_{\max} & \text{if } Z_t > z_{\max} \\ Z_t & \text{if } -z_{\max} \leq Z_t \leq z_{\max} \\ -z_{\max} & \text{if } Z_t < -z_{\max} \end{cases}$$

At each time step, the volatility is updated using the equation:

$$\sigma_{t+\Delta t} = \max\left(\sigma_{\min}, \sigma_t + v \cdot \sigma_t \cdot \sqrt{\Delta t} \cdot g(Z_2)\right)$$

Where the function $g(Z_2)$ is clipped to the range $[-z_{\max}, z_{\max}]$:

$$g(Z_2) = \begin{cases} z_{\max} & \text{if } Z_2 > z_{\max} \\ Z_2 & \text{if } -z_{\max} \leq Z_2 \leq z_{\max} \\ -z_{\max} & \text{if } Z_2 < -z_{\max} \end{cases}$$

Logarithmic Update of Asset Price
The logarithmic update of the asset price is given by:

$$\log(S_{t+\Delta t}) = \log(S_t) + \left(\left(\mu - \frac{1}{2}\sigma_{t+\Delta t}^2\right)\Delta t + \sigma_{t+\Delta t}\sqrt{\Delta t} \cdot f(Z_t)\right)$$

The updated asset price is then calculated as:

$$S_{t+\Delta t} = \exp\left(\log(S_{t+\Delta t})\right)$$

To prevent the overflow of input parameters applied logarithmic
Payoff for Asian Option

The payoff for an Asian option is computed by averaging the asset prices over the simulation period:

$$\bar{S} = \frac{1}{\text{num\_steps}} \sum_{t=1}^{\text{num\_steps}} S_t$$

For an Asian call option, the payoff is:

$$\text{payoff} = \max(\bar{S} - K, 0)$$

For an Asian put option, the payoff is:

$$\text{payoff} = \max(K - \bar{S}, 0)$$

Discounted Payoff
The discounted payoff is calculated as:

$$\text{Discounted Payoff} = \exp(-rT) \cdot \text{Payoff}$$

Estimated Price of Asian Option
The estimated price of the Asian option is the average discounted payoff over all paths:

$$\text{Estimated Price} = \frac{1}{\text{num\_paths}} \sum_{i=1}^{\text{num\_paths}} \text{Discounted Payoff}_i$$

Standard Deviation of Payoffs
The standard deviation of the discounted payoffs is calculated as:

$$s = \sqrt{\frac{1}{\text{num\_paths} - 1} \sum_{i=1}^{\text{num\_paths}} (\text{Discounted Payoff}_i - \text{Estimated Price})^2}$$

Confidence Interval Calculation
The confidence interval for the estimated price is given by:

$$\text{CI} = \left[\text{Estimated Price} - z \cdot \frac{s}{\sqrt{\text{num\_paths}}}, \text{Estimated Price} + z \cdot \frac{s}{\sqrt{\text{num\_paths}}}\right]$$

Where z is the z-score corresponding to the desired confidence level (e.g., 1.96 for 95

Mean Asset Price
The mean asset price across all paths is calculated as:

$$\text{Mean Asset Price} = \frac{1}{\text{num\_paths}} \sum_{i=1}^{\text{num\_paths}} \bar{S}_i$$

Validation and benchmarking:

benchmarking- under running analytical solution for geometric average asian option price compared to the baseline montecarlo simulation, the discrepancy between the analytical solution and monte carlo simulation printed out the result as to be geometric asian option price: 5.1685 monte carlo geometric asian option price: 6.2274