New Keynesian Model

Johannes Wieland
jfwieland@ucsd.edu

Spring 2024

Outline

New Keynesian Model: Roadmap

- Three "blocks" to the model:

1. Household: Same as in money model.
   - Optimality conditions generate "Dynamic IS" curve that gives relationship between output and real interest rate.
2. Firms: Same as imperfect competition model, with addition of persistent nominal rigidity for intermediate producers.
   - Generates a "New Keynesian Phillips Curve," a forward-looking, expectations-augmented Phillips curve.
3. Monetary authority's nominal interest rate rule closes model.

Outline

Household Problem:

$$\max_{\{C_t, N_t, B_t, M_t\}} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(\frac{C_{t+s}^{1-\gamma}}{1-\gamma} + \zeta \frac{(M_{t+s}/P_{t+s})^{1-\nu}}{1-\nu} - \chi \frac{N_{t+s}^{1+\varphi}}{1+\varphi} \right) \right\}$$

$$C_t = \frac{W_t}{P_t} N_t - \frac{B_t - Q_{t-1} B_{t-1}}{P_t} - \frac{M_t - M_{t-1}}{P_t} + TR_t + PR_t$$

- FOC

$$\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

$$\frac{M_t}{P_t} = \zeta^{1/\nu} \left(1 - \frac{1}{Q_t} \right)^{-1/\nu} C_t^{\gamma/\nu}$$

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} = E_t \{ \Lambda_{t,t+1} R_{t+1} \}$$

plus Fisher: $E_t\{R_{t+1}\} \equiv E_t\{Q_t P_t / P_{t+1}\}$

Firm Problem

- From the firm problem we derived the optimal price when able to adjust:

$$P_t^* = (1+\mu)E_t\left\{\sum_{s=0}^{\infty}\frac{\theta^s\Lambda_{t,t+s}Y_{t+s}P_{t+s}^{\varepsilon-1}}{\sum_{k=0}^{\infty}\theta^k\Lambda_{t,t+k}Y_{t+k}P_{t+k}^{\varepsilon-1}}\frac{W_{t+s}}{A_{t+s}}\right\}$$

- And the price level is a geometric average of past and new prices:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)P_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Interest Rate Rule

- The central bank sets the nominal rate following an interest rate rule

$$Q_t = \beta^{-1} \left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} e^{v_t}, \qquad \phi_\pi, \geq 0$$

where v_t is a "monetary policy shock".

  - Central bank raises nominal rate in response to inflation, $\Pi_t = (P_t/P_{t-1})$.
  - Viewed as a realistic description of central bank behavior in practice (Taylor, 1993). Also known as "Taylor rule."
  - More realistic versions also feature interest rate smoothing and response to output gap / growth.
  - Could alternatively solve for optimal monetary policy rule—we will do that soon.
  - Ignores zero lower bound constraint.

New Keynesian Model Equilibrium

A symmetric equilibrium is an allocation $\{C_{t+s}, N_{t+s}, Y_{t+s}\}_{s=0}^{\infty}$ and set of prices $\{P^*_{t+s}, P_{t+s}, W_{t+s}, Q_{t+s}\}_{s=0}^{\infty}$ along with exogenous processes $\{A_{t+s}, v_{t+s}\}_{s=0}^{\infty}$ such that:

1. Households optimize: Euler, labor-leisure, (money demand in background as central bank chooses Q_{t+s}, putting B_{t+s} in background as well).
2. Firms optimize:
   2.1 Price index follows dynamic Calvo formulation.
   2.2 Intermediate reset prices are chosen optimally.
3. Central bank follows interest rate rule with shock v_t.
4. Labor and goods (and bond) markets clear.

Equilibrium: Nonlinear Equations

$$\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

$$1 = \beta E_t\left\{Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\right\} = E_t\{\Lambda_{t,t+1} R_{t+1}\}$$

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$P_t^* = (1+\mu) E_t\left\{\sum_{s=0}^{\infty} \frac{\theta^s \Lambda_{t,t+s} Y_{t+s} P_{t+s}^{\varepsilon-1}}{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} P_{t+k}^{\varepsilon-1}} \frac{W_{t+s}}{A_{t+s}}\right\}$$

$$Y_t = C_t$$

$$Y_t = A_t N_t \left[\int_0^1 \left(\frac{N_t(i)}{N_t}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$Q_t = \beta^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} e^{v_t}$$

Outline

Log Linearization

- Log-Linearize Model Around *Zero-Inflation Steady State*.

  1. AD Block: Euler Equation $\Rightarrow$ *Dynamic IS Curve*
  2. AS Block: Pricing and MC Equations $\Rightarrow$ NK Phillips Curve
  3. Central Bank Monetary Rule

- Why log-linearize?
  - Much(!!!!) easier to analyze and understand linear system than non-linear model.

- I will skip most of the log-linearization.
  - Life is short—see Gali for details.
  - Focus on economics of NK model.

Log Linearization

- Log-linear approximation of $Y_t = f(X_t)$ around a point X .
  - First-order Taylor approx to $\log(f(X))$
- Define $x_t = \log(X_t)$ and $\hat{x}_t = x_t - x$.

$$
\begin{aligned}
y_t &= \log(f(\exp(x_t))) \\
&\approx \log(f(\exp(x))) + \frac{f'(\exp(x))\exp(x)}{f(\exp(x))}(x_t - x) \\
\hat{y}_t &\approx \frac{f'(X)X}{f(X)}\hat{x}_t
\end{aligned}
$$

- Can also derive using $\hat{x}_t \approx \frac{X_t - X}{X} \equiv \frac{dX_t}{X}$

$$
\begin{aligned}
Y + dY_t &= f(X) + f'(X)dX_t \\
\frac{dY_t}{Y}Y &= f'(X)X\frac{dX_t}{X} \\
\hat{y}_t &\approx \frac{f'(X)X}{f(X)}\hat{x}_t
\end{aligned}
$$

Log Linearization: Inflation

- Price Index:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

- The price index can be log-linearized to get

$$\hat{p}_t = \theta \hat{p}_{t-1} + (1-\theta)\hat{p}_t^*$$

  - Equivalently written in terms of inflation:

$$\hat{\pi}_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1})$$

  - Inflation is positive when new prices are higher than old prices.

Log Linearization: Reset Prices

- The reset price can be log-linearized as:

$$\hat{p}_t^* = (1-\beta\theta)E_t\left\{\sum_{s=0}^{\infty}(\beta\theta)^s\left(\hat{p}_{t+s}+\hat{mc}_{t+s}\right)\right\}$$

- We can write this recursively as:

$$\hat{p}_t^* = (1-\beta\theta)(\hat{p}_t+\hat{mc}_t)+\beta\theta E_t\{\hat{p}_{t+1}^*\}$$

Log Linearization: Phillips Curve

- Subtract $\hat{p}_{t-1}$:

$$(\hat{p}_t^* - \hat{p}_{t-1}) = (1-\beta\theta)\hat{mc}_t + \hat{\pi}_t + \beta\theta E_t\{\hat{p}_{t+1}^* - \hat{p}_t\}$$

- Plug into $\hat{\pi}_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1})$ to get an expectations-augmented Phillips curve:

$$\hat{\pi}_t = \lambda\hat{mc}_t + \beta E_t\{\hat{\pi}_{t+1}\}, \text{ where } \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

- Inflation is equal to expected future inflation plus the deviation of marginal cost from its steady state level.
    - Expected inflation: Forward looking price setters choose higher prices now if inflation is expected to be high, as nominal marginal costs will rise.

Log Linearization: Phillips Curve

- Inflation is equal to expected future inflation plus the deviation of marginal cost from its steady state level.
  - Two ways to think about marginal cost deviation:

1. Set higher prices to cover higher marginal cost.
2. When marginal costs are above desired level, markups are below desired level. Inflation as firms hike markup back to desired level. (In fact, $\hat{mc}_t = -\hat{\mu}_t$).

- Iterating forward,

$$\hat{\pi}_t = \lambda E_t \left\{ \sum_{s=0}^{\infty} \beta^s \hat{mc}_{t+s} \right\}$$

  - Inflation is the PDV of future marginal cost / markup deviations from steady state.

Log Linearization: Real Marginal Costs

$$\hat{mc}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t$$

- Combine labor-leisure, production function, and $\hat{c}_t = \hat{y}_t$:

$$\hat{w}_t - \hat{p}_t = (\gamma + \varphi)\hat{y}_t - \varphi \hat{a}_t$$

- Consequently,

$$\hat{mc}_t = (\gamma + \varphi)\hat{y}_t - (1 + \varphi)\hat{a}_t$$

- Compare to flexible price case:

$$1 = \frac{P_t(i)}{P_t} = \mu \frac{W_t}{P_t} \frac{1}{A_t}$$

so

$$\hat{mc}_t^{flex} = 0, \qquad (\gamma + \varphi)\hat{y}_t^{flex} = (1 + \varphi)\hat{a}_t$$

where $\hat{y}_t^{flex}$ is called the *natural level of output*.

Real Marginal Costs in Terms of Output Gap

- Combine:

$$\hat{mc}_t = (\gamma + \varphi)\hat{y}_t - (1+\varphi)\hat{a}_t$$
$$(\gamma + \varphi)\hat{y}_t^{flex} = (1+\varphi)\hat{a}_t$$

  to write real marginal costs in terms of output gap $\tilde{y}_t$:

$$\hat{mc}_t = (\gamma + \varphi)(\hat{y}_t - \hat{y}_t^{flex})$$

- Real marginal costs go up (and markups go down) when the output gap is high.
  - To produce more than under flex prices, markup must be lower.
  - Marginal costs high because need to hire more workers, bidding up real wage.
  - Stronger when IES and labor supply elasticity are low.
  - In Gali textbook also stronger with DRS.

The New Keynesian Phillips Curve

- Plug back into the Phillips curve $\hat{\pi}_t = \lambda \hat{mc}_t + \beta E_t\{\hat{\pi}_{t+1}\}$

$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\}, \text{ where } \kappa = \lambda(\gamma + \varphi)$$

  - This is the *New Keynesian Philips Curve*: an expectations augmented Phillips curve written in terms of the output gap.

- Solving forward,

$$\hat{\pi}_t = \kappa E_t \left\{ \sum_{s=0}^{\infty} \beta^s (\hat{y}_{t+s} - \hat{y}_{t+s}^{flex}) \right\}$$

  - Inflation is an increasing function of future output gaps.
  - Output gap high $\Rightarrow$ marginal cost high and markups low $\Rightarrow$ raise markups.

Log Linearization: The Aggregate Demand Block

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} = E_t\{\Lambda_{t,t+1} R_{t+1}\}$$

$$Y_t = C_t$$

- Log-linearize Euler around zero-inflation:

$$\hat{c}_t = -\frac{1}{\gamma}\left(\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}\right) + E_t\{\hat{c}_{t+1}\}$$

  - Steady state nominal interest rate is $i_t = \rho = -\log\beta$.
- Combine with market clearing and use $\sigma = 1/\gamma$:

$$\hat{y}_t = -\sigma\left(\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}\right) + E_t\{\hat{y}_{t+1}\}$$

- This is the *dynamic IS curve*. It relates output to future expectations of output and the real interest rate.

Dynamic IS

- Iterating forward, the current output gap depends negatively on the gap between the real interest rate and the natural rate of interest (assuming return to steady state):

$$\hat{y}_t = -\sigma E_t \left\{ \sum_{s=0}^{\infty} \left(\hat{r}_{t+s+1} \right) \right\}$$

- If you want to figure out what happens to output in the NK model, you need to figure out what happens to the path of the real interest rate.
    - Output gap determined purely by intertemporal substitution. Not old Keynesian marginal propensities to consume / invest.
    - Intuition also works well for larger NK models.

Outline

The Three Equation Model

- In sum, the log-linearized NK model boils down to three equations:

$$\hat{y}_t = -\sigma[\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}] + E_t\{\hat{y}_{t+1}\}$$
$$\hat{\pi}_t = \kappa(\hat{y}_t - \hat{y}_t^{flex}) + \beta E_t\{\hat{\pi}_{t+1}\}$$
$$\hat{i}_t = \phi_\pi \hat{\pi}_t + v_t$$

  with three unknowns: $\hat{i}_t$, $\hat{y}_t$, and $\hat{\pi}_t$ and an exogenous driving process for the output gap $\hat{y}_t^{flex}$ $(= \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t)$ and the monetary policy shock $\hat{v}_t$.
- Key new ingredient is NK Phillips curve:
  - $\beta E_t\{\hat{\pi}_{t+1}\}$: Price setters forward looking.
  - $\kappa \hat{y}_t$: Output $\uparrow \Rightarrow$ MC $\uparrow \Rightarrow$ markups $\downarrow \Rightarrow$ raise prices
- Determinacy: similar condition to lecture 2. See Gali.
- Note: Gali writes everything in terms of "gaps", $\tilde{y}_t = \hat{y}_t - \hat{y}_t^{flex}$.

Special Case: $\kappa \to \infty$

- Equivalent to flexible prices: $\theta = 0$.
- The NK Phillips Curve becomes:

$$\hat{y}_t = \hat{y}_t^{flex} = \frac{1+\varphi}{\gamma+\varphi}\hat{a}_t$$

- Output fluctuations arise only from productivity fluctuations.
- Monetary variables v_t have no real effect:
  - Drop in v_t lowers nominal rate and real interest rate.
  - All else equal increases output and marginal cost.
  - Prices today rise with marginal cost so that π_{t+1} is low and the real interest rate is constant.

$\Rightarrow$ With constant real interest rate output is unchanged.

Special Case: $\kappa = 0$

- Equivalent to perfectly rigid prices: $\theta = 1$. The NK Phillips Curve becomes $\hat{\pi}_t = 0$.
- Now output is demand determined:

$$\hat{y}_t = -\sigma \hat{i}_t + E_t\{\hat{y}_{t+1}\}$$
$$\hat{i}_t = v_t$$

- If $v_t = \rho_v v_{t-1} + \varepsilon_t$, then

$$\hat{y}_t = -\frac{\sigma}{1-\rho_v} v_t$$

- Monetary variables $\hat{v}_t$ have a real effect:
  - Drop in v_t lowers nominal rate and real interest rate.
  - Inflation is constant so output expands with the lower real interest rate.
- With rigid prices, output is independent of productivity fluctuations.

INTERMEDIATE κ

- Assume
$$v_t = \rho_v v_{t-1} + \varepsilon_t \text{ and } \hat{a}_t = 0$$

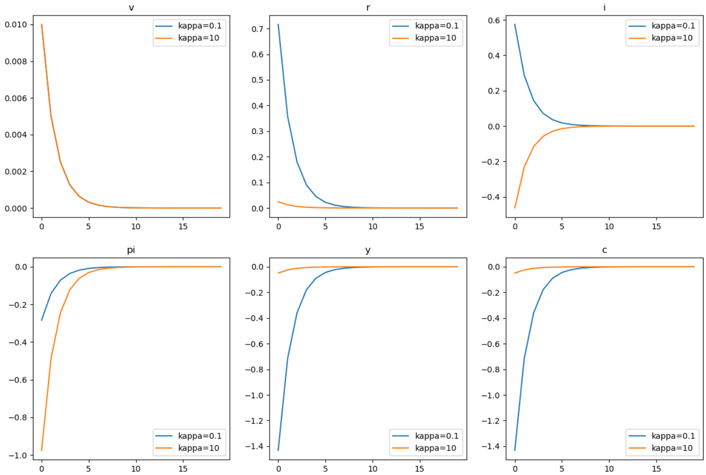
- Guess reduced form policy functions:
$$\hat{y}_t = \psi_{yv} v_t \text{ and } \hat{\pi}_t = \psi_{\pi v} v_t$$

- This gives:
$$\begin{aligned}
\psi_{\pi v} &= \kappa \psi_{yv} + \beta \rho_v \psi_{\pi v} \\
\psi_{yv} &= -\sigma(\phi_\pi \psi_{\pi v} + 1 - \rho_v \psi_{\pi v}) + \rho_v \psi_{yv}
\end{aligned}$$

- Solving by method of undetermined coeffs:
$$\psi_{yv} = -(1-\beta\rho_v)\Psi_v \text{ and } \psi_{\pi v} = -\kappa \Psi_v$$

$$\text{where } \Psi_v = \frac{1}{(1-\beta\rho_v)\gamma(1-\rho_v) + \kappa(\phi_\pi - \rho_v)} > 0$$

INTERMEDIATE κ



- Blue line: $\kappa = 0.1$. Orange line: $\kappa = 10$.

Epistemiology

- NK model a response to the Lucas critique: "fully" optimizing agents but monetary policy has real effect.
- Extensive debate on how well NK model fits the data.
  - Centers on much more complex "medium-scale" models.
  - These are the simple NK model at its core with many additional "bells and whistles" (capital, habits, indexation, rigid wages, government, etc). See references in the syllabus.
  - Everyone agrees the simple three equation model does not match the data well.
- Should view simple model as an organizing framework.
  - Communicate results: everyone knows this model and how it works.
  - How should policy respond to shocks? Why?
  - Interpret policy actions through lens of NK model.

Outline

Next Steps

- Optimal policy in NK model.

- Interpreting Federal Reserve actions / statements through the lens of the NK model.