

# Econ 210C Homework 3

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Due: 05/29/2023, 11:59PM PST. Submit pdf write-up and zipped code packet on Github.

## 1. Sticky Wage Model

Instead of assuming that prices are sticky for one period, we now assume that nominal wages are sticky for one period,

$$W_1 = W_0$$

The short-run equilibrium is

$$\begin{aligned} Y_1 &= A_1 N_1 \\ W_1 &= W_0 \\ \frac{W_1}{P_1} &= A_1 \\ Y_1 &= C_1 \\ \frac{M_1}{P_1} &= \zeta^{1/\nu} \left(1 - \frac{1}{Q_1}\right)^{-1/\nu} C_1^{\gamma/\nu} \\ 1 &= \beta E_1 \left\{ Q_1 \frac{P_1}{P_2} \frac{C_2^{-\gamma}}{C_1^{-\gamma}} \right\} \end{aligned}$$

The long-run equilibrium ( $t \geq 2$ ) is

$$\begin{aligned} Y_t &= A_t N_t \\ \frac{W_t}{P_t} &= A_t \\ \frac{W_t}{P_t} &= \frac{\chi N_t^\varphi}{C_t^{-\gamma}} \\ Y_t &= C_t \\ \frac{M_t}{P_t} &= \zeta^{1/\nu} \left(1 - \frac{1}{Q_t}\right)^{-1/\nu} C_t^{\gamma/\nu} \\ 1 &= \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} \end{aligned}$$

(a) Are firms on their labor curve? Explain.

- In the short-run: Firms are on their labor curve because firms have to produce  $Y_1$  whatever house-

holds demand  $C_1$ , given that  $A_1$  is exogenous and  $N_1$  is determined. The wage is determined by the marginal product of labor:

$$A_1 = \frac{W_1}{P_1} = \frac{W_0}{P_1} \quad (1)$$

which means that the price  $P_1$  will fluctuate to satisfy the equation.

- In the long run: Firms are on their labor demand curve following the same logic.

(b) Are households on their labor supply curve? Explain.

- In the short-run: the household will provide the labor supply of whatever labor is demanded at the wage  $W_1 = W_0$  set by firms. This is in order to make the labor market clearing condition hold. Therefore,  $N_1 = N_0$  is not necessarily equivalent to the household's potential optimal labor supply  $N_1^*$  (if there is no sticky wage), but we can think of  $N_1 = N_0$  as a constrained one:
  - If the wage is higher than the  $W_1 = W_0 > W_1^*$ , then  $N_1 = N_0 < N_1^*$
  - If the wage is lower than the  $W_1 = W_0 < W_1^*$ , then  $N_1 = N_0 > N_1^*$
- In the long run: there is no sticky wage and households are on their labor supply curve.

(c) How does the labor market clear?

- The labor market is clear because the price fluctuates by reflecting the change of aggregate demand  $C_t$  and  $Y_t$ . The firms demand labor to produce the output  $Y_t$  in order to meet the household aggregate demand  $C_t$ . The households supply labor to satisfy the labor that firms demand.

(d) Solve for the long-run steady state.

In the long run equilibrium, we can assume the exogenous variables to be fixed:

$$A_t = A, M_t = M \quad (2)$$

and then solve for steady state for  $C_2 = C$ ,  $P_2 = P$ , and the total resource constraints  $C = Y = AN$ :

$$Y = AN \quad (3)$$

$$\frac{W}{P} = A \quad (4)$$

$$\frac{W}{P} = \frac{\chi N^\varphi}{C^{-\gamma}} \quad (5)$$

$$Y = C \quad (6)$$

$$\frac{M}{P} = \zeta^{1/\nu} \left(1 - \frac{1}{Q}\right)^{-1/\nu} C^{\gamma/\nu} \quad (7)$$

$$1 = \beta E_t \left\{ Q \frac{P}{P} \frac{C^{-\gamma}}{C^{-\gamma}} \right\} \quad (8)$$

and then we have:

$$\frac{1}{Q} = \beta \quad (9)$$

$$N = \left( \frac{A^{1-\gamma}}{\chi} \right)^{\frac{1}{\varphi+\gamma}} \quad (10)$$

$$C = AN = A^{\frac{\varphi+1}{\varphi+\gamma}} \chi^{\frac{-1}{\varphi+\gamma}} \quad (11)$$

$$\frac{M}{P} = \zeta^{1/\nu} (1-\beta)^{-1/\nu} C^{\gamma/\nu} = \zeta^{1/\nu} (1-\beta)^{-1/\nu} A^{\frac{\varphi+1}{\varphi+\gamma} \frac{\gamma}{\nu}} \chi^{\frac{-1}{\varphi+\gamma} \frac{\gamma}{\nu}} \quad (12)$$

(e) Does the Classical Dichotomy hold in the long run? Explain.

- Yes, classical dichotomy holds in the long run. Any change in  $M$  causes a proportional change in  $P$ , leaving  $Y, C$  unchanged.

(f) Solve for output and the money market equilibrium in the short-run.

In the short-run equilibrium period 1:

$$A_1 = \frac{W_1}{P_1} = \frac{W_0}{P_1} \Rightarrow P_1 = \frac{W_0}{A_1} = \frac{W_0}{A} \quad (13)$$

$$Y_1 = A_1 N_1 = C_1 \quad (14)$$

$$1 = \beta E_1 \left\{ Q_1 \frac{P_1}{P} \frac{C^{-\gamma}}{C_1^{-\gamma}} \right\} \quad (15)$$

$$\frac{M_1}{P_1} = \zeta^{1/\nu} \left( 1 - \frac{1}{Q_1} \right)^{-1/\nu} C_1^{\gamma/\nu} \quad (16)$$

Notice that (13) implies  $P_1$  is fixed. By some algebra:

$$C_1 = Y_1 = \left\{ \frac{1}{\beta Q_1} \frac{P}{P_1} \right\}^{\frac{1}{\gamma}} C = \left\{ \frac{1}{\beta Q_1} P \frac{A}{W_0} \right\}^{\frac{1}{\gamma}} C = \left\{ \frac{1}{\beta Q_1} P \frac{A}{W_0} \right\}^{\frac{1}{\gamma}} Y \quad (17)$$

$$\frac{M_1}{P_1} = \zeta^{1/\nu} \left( 1 - \frac{1}{Q_1} \right)^{-1/\nu} \left\{ \frac{1}{\beta Q_1} P \frac{A}{W_0} \right\}^{\frac{1}{\nu}} Y^{\frac{\gamma}{\nu}} = \zeta^{1/\nu} \left( \frac{1}{Q_1 - 1} \right)^{\frac{1}{\nu}} \left\{ \frac{1}{\beta} P \frac{A}{W_0} \right\}^{\frac{1}{\nu}} Y^{\frac{\gamma}{\nu}} \quad (18)$$

$$N_1 = \frac{C_1}{A_1} = \frac{1}{A} \left\{ \frac{1}{\beta Q_1} P \frac{A}{W_0} \right\}^{\frac{1}{\gamma}} Y \quad (19)$$

(g) Does the Classical Dichotomy hold in the short run?

- The classical dichotomy does not hold in the short run. It is easy to see that in (18) the change of money  $M_1$  will influence the  $Q_1$  real interest rate and all the other endogenous variables will be influenced in the equilibrium.

(h) Explain intuitively (in words) how an increase in the money supply affects output in the short run.

- The money supply  $M_1$  increases leads to the decreases in  $Q_1$  according to equation (18). It means that the nominal rate  $Q_1$  falls to induce households to hold more money supplied.
- The real interest rate is defined as  $R = Q_1 \frac{P_1}{P}$  and it is also decreasing. The long-term inflation  $\frac{P_1}{P}$  is fixed because  $P_1$  is sticky and  $P$  is determined by the future  $M$ .

- The lower real interest rate increases the aggregate demand today, according to the equation (17):
  - the household has lower returns on savings, and the household prefers today's consumption  $C_1$  relative to future consumption  $C$ .
  - the future consumption  $C$  is determined by the aggregate supply  $Y$ .
  - there is an overall increase in consumption  $C_1$  today, and firms will hire more labor currently and  $N_1$  increases.

(i) How does productivity affect output? Explain intuitively.

- The productivity  $A$  does not influence the real output  $Y$ 
  - The productivity does not affect the real interest rate and does not affect the consumption demand
  - Because the aggregate demand  $C_1$  is fixed, the higher productivity  $A$  will only reduce the labor employment  $N$ .

(j) Derive the labor wedge. Is it procyclical or countercyclical?

- The labor wedge is defined as:

$$(1 - \tau_t^N) \equiv \frac{MRS_t}{MPL_t} \quad (20)$$

At  $t = 1$ :

$$(1 - \tau_1^N) = \frac{\chi N_1^\varphi C_1^\gamma}{\frac{Y_1}{N_1}} = \chi N_1^{\varphi+1} Y_1^{\gamma-1} = \chi N_1^{\varphi+\gamma} A_1^{\gamma-1} \quad (21)$$

- When economy is in recession, where  $N_1$  is low, the labor wedge  $\tau_1^N$  is high. Therefore, the wedge is countercyclical.

(k) What moments of the data would you use to discriminate between the predictions of the sticky price and the sticky wage model?

- The difference between the sticky price and sticky wage is which market is competitive or imperfect, goods market or labor market.
- We can try to do the survey on wage growth and price level growth in the same area. In order to distinguish between different models, we can directly choose one based on the empirical evidence, such as comparing the frequency of price changes in wages and goods.
- We can also try to compute the autocorrelation and cyclical property of the real wage (compared with the aggregate consumption).