Econ 210C Final Exam

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1 Model Simulation

Consider the new Keynesian model

$$
\begin{aligned}
\hat{y}_t &= E_t\hat{y}_{t+1} - E_t\left(\hat{i}_t - \hat{\pi}_{t+1} - \hat{r}_t^n\right) \\
\hat{\pi}_t &= \beta E_t\hat{\pi}_{t+1} + \kappa\hat{y}_t \\
\hat{i}_t &= \phi_\pi\hat{\pi}_t + \bar{i}_t, \quad \phi_\pi > 1 \\
\hat{r}_t^n &= \epsilon_t^n, \quad \epsilon_t^n \sim N\left(0, \sigma_n^2\right) \\
\bar{i}_t &= \epsilon_t^i, \quad \epsilon_t^i \sim N\left(0, \sigma_i^2\right)
\end{aligned}
$$

where $\bar{i}_t$ is an exogenous monetary policy shock and $\hat{r}_t^n$ is a shock to the natural rate of interest.
(a) Using either the sequence space methods or the method of undetermined coefficients, plot IRFs for the following variables to a ϵ_t^n shock and to a ϵ_t^i shock:

- $\bar{i}_t$, $\hat{y}_t$, $\hat{\pi}_t$, $\hat{i}_t$, $\hat{r}_t = \hat{i}_t - E_t\hat{\pi}_{t+1}$, $\hat{r}_t^n$

Use the following parameters:

$$\beta = 0.99, \kappa = 0.01, \phi_\pi = 2.5, \sigma_n = 0.01, \sigma_i = 0.001.$$

The suggested horizon is 5 quarters. Suppose:

$$
\begin{aligned}
\hat{y}_t &= \eta_{yr}\hat{r}_t^n + \eta_{yi}\bar{i}_t \\
\hat{\pi}_t &= \eta_{\pi r}\hat{r}_t^n + \eta_{\pi i}\bar{i}_t \\
\hat{i}_t &= \eta_{ir}\hat{r}_t^n + \eta_{ii}\bar{i}_t
\end{aligned}
$$

For $\hat{y}_t$:

$$
\begin{aligned}
\hat{y}_t &= E_t[\eta_{yr}\hat{r}_{t+1}^n + \eta_{yi}\bar{i}_{t+1}] - E_t\left(\eta_{ir}\hat{r}_t^n + \eta_{ii}\bar{i}_t - (\eta_{\pi r}\hat{r}_{t+1}^n + \eta_{\pi i}\bar{i}_{t+1}) - \hat{r}_t^n\right) \\
\eta_{yr}\hat{r}_t^n + \eta_{yi}\bar{i}_t &= (1 - \eta_{ir})\hat{r}_t^n - \eta_{ii}\bar{i}_t
\end{aligned}
$$

For $\hat{\pi}_t$:

$$\begin{aligned}
\hat{\pi}_t &= \beta E_t[\eta_{\pi r}\hat{r}^n_{t+1} + \eta_{\pi i}\bar{i}_{t+1}] + \kappa(\eta_{yr}\hat{r}^n_t + \eta_{yi}\bar{i}_t) \\
\eta_{\pi r}\hat{r}^n_t + \eta_{\pi i}\bar{i}_t &= \kappa\eta_{yr}\hat{r}^n_t + \kappa\eta_{yi}\bar{i}_t
\end{aligned}$$

For $\hat{i}_t$:

$$\begin{aligned}
\hat{i}_t &= \phi_\pi\hat{\pi}_t + \bar{i}_t \\
\eta_{ir}\hat{r}^n_t + \eta_{ii}\bar{i}_t &= \phi_\pi\eta_{\pi r}\hat{r}^n_t + \phi_\pi\eta_{\pi i}\bar{i}_t + \bar{i}_t
\end{aligned}$$

Therefore

$$\eta_{yr} = 1 - \eta_{ir} \tag{1}$$
$$\eta_{yi} = -\eta_{ii} \tag{2}$$
$$\eta_{\pi r} = \kappa\eta_{yr} \tag{3}$$
$$\eta_{\pi i} = \kappa\eta_{yi} \tag{4}$$
$$\eta_{ir} = \phi_\pi\eta_{\pi r} \tag{5}$$
$$\eta_{ii} = \phi_\pi\eta_{\pi i} + 1 \tag{6}$$

We have:

$$\eta_{yr} = \frac{1}{1+\phi_\pi\kappa} \tag{7}$$
$$\eta_{yi} = -\frac{1}{1+\phi_\pi\kappa} \tag{8}$$
$$\eta_{\pi r} = \frac{\kappa}{1+\phi_\pi\kappa} \tag{9}$$
$$\eta_{\pi i} = -\frac{\kappa}{1+\phi_\pi\kappa} \tag{10}$$
$$\eta_{ir} = \frac{\phi_\pi\kappa}{1+\phi_\pi\kappa} \tag{11}$$
$$\eta_{ii} = \frac{1}{1+\phi_\pi\kappa} \tag{12}$$

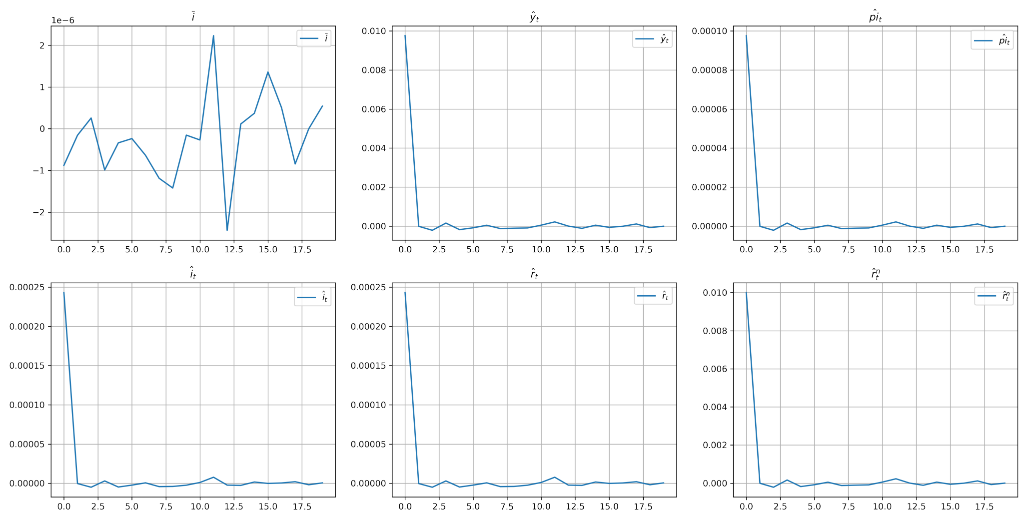
Therefore,

$$\hat{y}_t = \frac{1}{1+\phi_\pi\kappa}\hat{r}^n_t - \frac{1}{1+\phi_\pi\kappa}\bar{i}_t = \frac{1}{1+\phi_\pi\kappa}\epsilon^n_t - \frac{1}{1+\phi_\pi\kappa}\epsilon^i_t \tag{13}$$
$$\hat{\pi}_t = \frac{\kappa}{1+\phi_\pi\kappa}\hat{r}^n_t - \frac{\kappa}{1+\phi_\pi\kappa}\bar{i}_t = \frac{\kappa}{1+\phi_\pi\kappa}\epsilon^n_t - \frac{\kappa}{1+\phi_\pi\kappa}\epsilon^i_t \tag{14}$$
$$\hat{i}_t = \frac{\phi_\pi\kappa}{1+\phi_\pi\kappa}\hat{r}^n_t + \frac{1}{1+\phi_\pi\kappa}\bar{i}_t = \frac{\phi_\pi\kappa}{1+\phi_\pi\kappa}\epsilon^n_t + \frac{1}{1+\phi_\pi\kappa}\epsilon^i_t \tag{15}$$

and

$$\begin{aligned}
\hat{r}_t &= \hat{i}_t - E_t \hat{\pi}_{t+1} \\
&= \frac{\phi_\pi \kappa}{1+\phi_\pi \kappa}\epsilon_t^n + \frac{1}{1+\phi_\pi \kappa}\epsilon_t^i - E_t\left[\frac{\kappa}{1+\phi_\pi \kappa}\epsilon_t^n - \frac{\kappa}{1+\phi_\pi \kappa}\epsilon_t^i\right] \\
&= \frac{\phi_\pi \kappa}{1+\phi_\pi \kappa}\epsilon_t^n + \frac{1}{1+\phi_\pi \kappa}\epsilon_t^i \qquad (16)
\end{aligned}$$

Figure 1: ϵ_t^n Shock



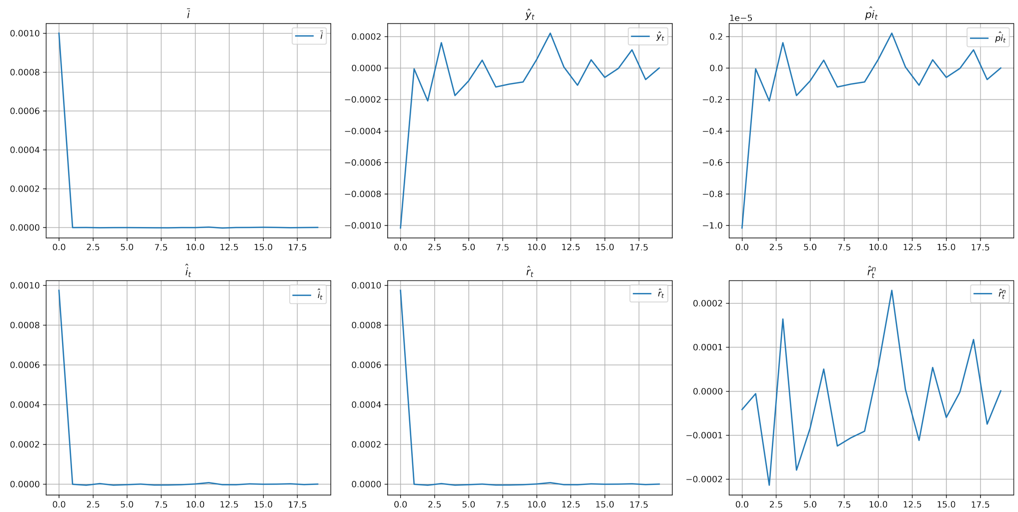
(b) Explain intuitively how the monetary policy shock affects output, inflation, the nominal interest rate, and the real interest rate in the model.

- Notice that in the model the real interest rate is exogenous and not influenced by the monetary policy shock.
- The positive (contractionary) monetary policy shock will lead to the increase of nominal interest rate i_t which enters the first equation and leads to the output decrease.
- The decreased output will enter the second equation and lead the inflation to decrease. However, the magnitude is determined by the κ.
- Then expected inflation and expected output will decrease in the next period, but the magnitude is governed by ϕ_π and β.

(c) Explain intuitively how the natural rate of interest shock affects output, inflation, the nominal interest rate, and the real interest rate in the model.

- The positive (contractionary) natural rate of interest shock will lead to an increase in the output because it directly enters the first equation.

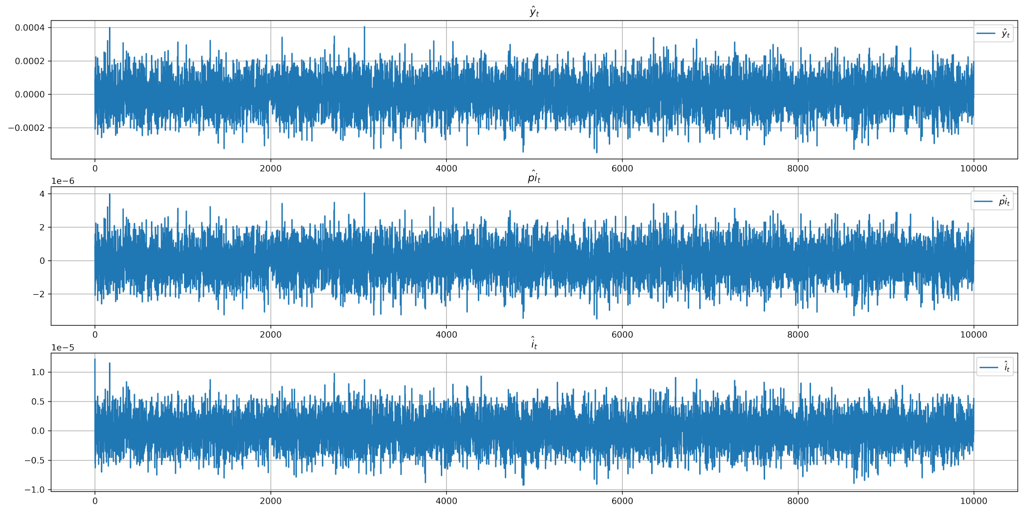
Figure 2: ϵ_t^i Shock



- The increased output will enter the second equation and lead inflation to increase. However, the magnitude is determined by the κ.
- The nominal interest rate will also increase in response to the increase of inflation, and the magnitude is governed by ϕ_π
- Then expected inflation and expected output will increase in the next period, but the magnitude is governed by ϕ_π and β.

(d) Simulate a time series of length 10000 for $\hat{y}_t, \hat{\pi}_t, \hat{i}_t$. Plot each of the three-time series. Note that the values for σ_i and σ_r are different!

Figure 3: Simulated Series



(e) Using your simulated time series, estimate the following regression by OLS and report your estimate for γ :

$$\hat{y}_t = \gamma \hat{i}_t + \eta_t$$

- The estimate γ is 34.32 if we take σ_i and σ_n as standard deviation in the simluation
- The estimate γ is 1.34 if we take σ_i and σ_n as variance in the simluation

(f) Explain why your estimate for γ does not recover the causal effect of monetary policy on output in the model.

The regression is the output against the nominal interest rate. However, the nominal interest rate itself is influenced by inflation and therefore influenced by the output as well. Therefore, there are omitted variables and therefore suffering endogeneity. The estimator γ can not be interpreted as a causal effect.

(g) Suppose you had data on $\bar{i}_t$. How would you use it to identify the causal effect of monetary policy on output in the model? Describe in words, do not implement yet.

If I have the exogenous monetary policy shock $\bar{i}_t$, there are multiple ways to do this

- I would apply the event study design with high-frequency identification: take each shock as an event and therefore each period can be taken as a cross-sectional observation. Then I will take the difference in the output before and after the shock and run regression with the monetary shock. Essentially I can estimate the local projection estimators.
- I can also add the monetary policy shock into the VAR system, including the output data, inflation data, and monetary policy rate data, and specify the monetary policy shock as an exogenous variable.

(h) Suppose you had data on $\hat{r}^n_t$. How would you use it to identify the causal effect of monetary policy on output in the model? Describe in words, do not implement yet.

If I have the exogenous natural rate shock $\hat{r}^n_t$,

- I would construct a VAR system, including the output data, inflation data, and nominal interest rate, and specify the natural rate shock as an exogenous variable. Then plot the IRF to analyze the monetary policy shock's effect.

(i) How would you use a Cholesky decomposition to identify the causal effect of monetary policy on output in the model? Describe in words, do not implement yet.

- It is possible to construct a Structural VAR, which includes the contemporaneous effects between different variables. Still, we can specify the exogenous monetary policy shock in the structural VAR.
- The Cholesky decomposition helps us to get the lower triangle matrix of the coefficient and therefore the error terms in the structural VAR can be used to identify the coefficient.

(j) Explain which of these approaches do you expect to be the most reliable in the data.

- To me, the most reliable method is the event study design, with controlling other variables, because it uses the variation of the shock itself and directly regress the change on change. However, the output data is not high-frequent enough usually.
- Considering that data limitation, the simulated data is constructed by exogenous shock, and the output, inflation, and nominal interest rate are linear combinations of these shocks. It may suffer from collinearity when estimating VAR. Therefore, I prefer IV estimation taking the $\bar{i}_t$ as an instrument.

(k) Implement your preferred approach in the model and show that it recovers the causal effect of monetary policy on output from the model. Taking the $\bar{i}$ as an instrument, we get the estimation as -1.443.

	(1) y
i	-1.443
	(1.040)
_cons	-0.000*
	(0.000)
N	10000
r2	-0.073

2 Data Analysis

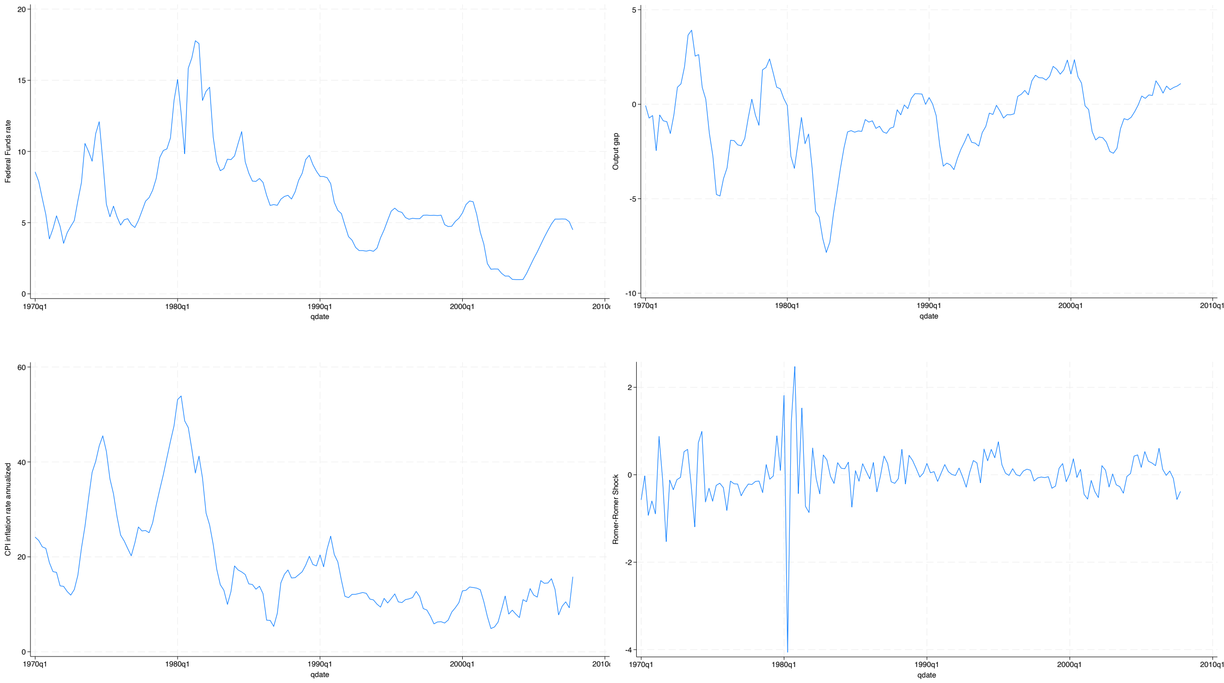
(a), (b), (c), (d), (e) in the Stata code

(f) Plot each time series from 1970Q1 to 2007Q4. You should have no missing observations!

1. Federal Funds rate (i_t)
2. Output gap (gap_t)
3. CPI inflation rate annualized (π_t)
4. Romer-Romer shock (RR_t).

Make sure all graphs are appropriately labelled.

Figure 4: Time Series Plot



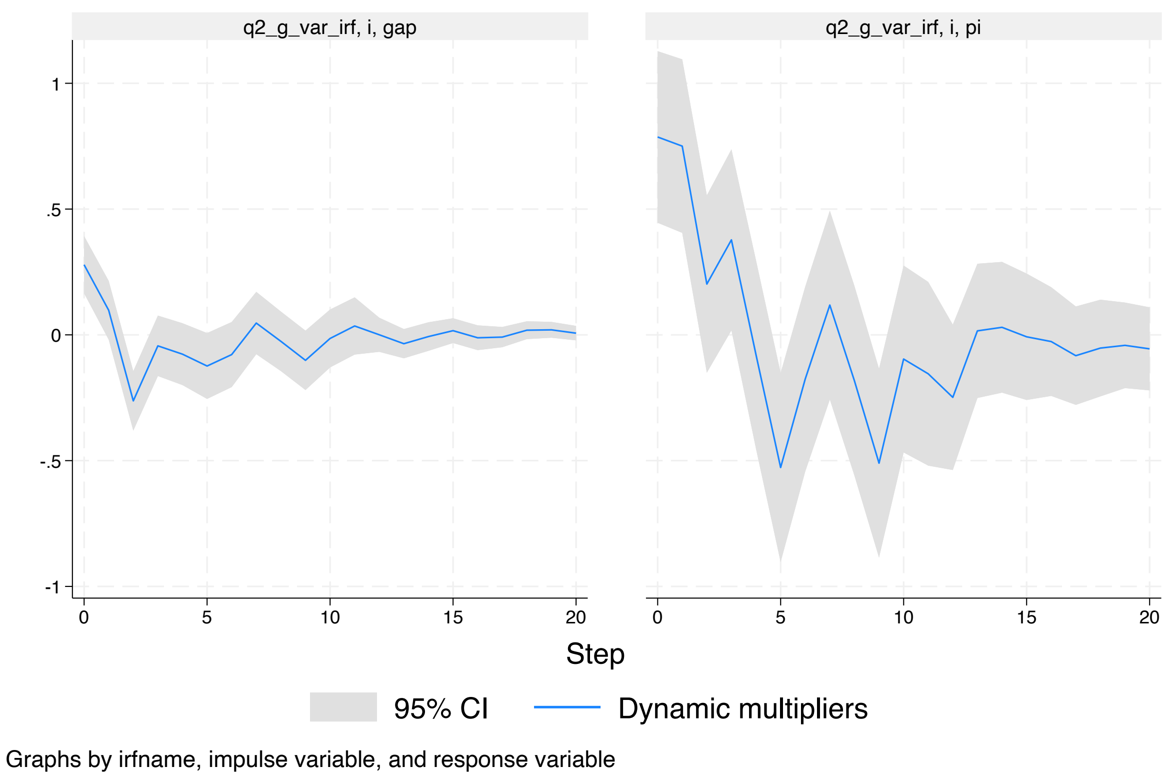
(g) Construct the IRFs from the estimation equation

$$y_t = \alpha + \sum_{s=1}^{8} \beta_s y_{t-s} + \sum_{s=0}^{12} \gamma_s i_{t-s} + \epsilon_t$$

where $y_t \in [\pi_t, gap_t]$ are the outcome variables and i_t is the Federal Funds Rate. This means you plot the response $\{y_s\}_{s=0}^{T}$ for an initial shock $i_0 = 1$ and $i_s = 0$ for all $s \geqslant 1$. A good value to start is $T = 16$.

$$\begin{bmatrix} \pi_t \\ gap_t \end{bmatrix} = \begin{bmatrix} \alpha \\ \alpha \end{bmatrix} + \beta \times \begin{bmatrix} \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ \pi_{t-4} \\ \pi_{t-5} \\ \pi_{t-6} \\ \pi_{t-7} \\ \pi_{t-8} \\ gap_{t-1} \\ gap_{t-2} \\ gap_{t-3} \\ gap_{t-4} \\ gap_{t-5} \\ gap_{t-6} \\ gap_{t-7} \\ gap_{t-8} \end{bmatrix} + \gamma \times \begin{bmatrix} i_t \\ i_{t-1} \\ i_{t-2} \\ i_{t-3} \\ i_{t-4} \\ i_{t-5} \\ i_{t-6} \\ i_{t-7} \\ i_{t-8} \\ i_{t-9} \\ i_{t-10} \\ i_{t-11} \\ i_{t-12} \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix} \tag{17}$$

Figure 5: IRF: i



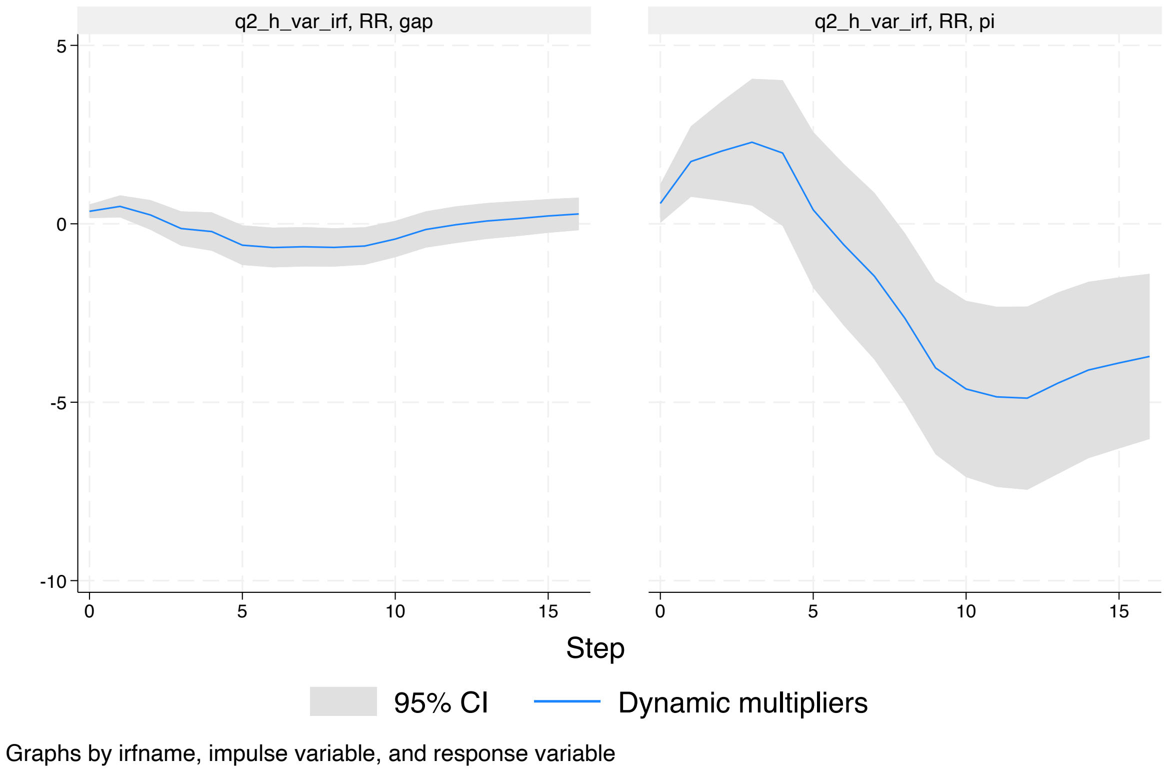
(h) Now plot the IRFs from the estimation equation

$$y_t = \alpha + \sum_{s=1}^{8} \beta_s y_{t-s} + \sum_{s=0}^{12} \gamma_s RR_{t-s} + \epsilon_t$$

where $y_t \in [\pi_t, gap_t]$ are the outcome variables and RR_t are the Romer-Romer shocks.

$$\begin{bmatrix} \pi_t \\ gap_t \end{bmatrix} = \begin{bmatrix} \alpha \\ \alpha \end{bmatrix} + \beta \times \begin{bmatrix} \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ \pi_{t-4} \\ \pi_{t-5} \\ \pi_{t-6} \\ \pi_{t-7} \\ \pi_{t-8} \\ gap_{t-1} \\ gap_{t-2} \\ gap_{t-3} \\ gap_{t-4} \\ gap_{t-5} \\ gap_{t-6} \\ gap_{t-7} \\ gap_{t-8} \end{bmatrix} + \gamma \times \begin{bmatrix} RR_t \\ RR_{t-1} \\ RR_{t-2} \\ RR_{t-3} \\ RR_{t-4} \\ RR_{t-5} \\ RR_{t-6} \\ RR_{t-7} \\ RR_{t-8} \\ RR_{t-9} \\ RR_{t-10} \\ RR_{t-11} \\ RR_{t-12} \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix} \quad (18)$$

Figure 6: IRF: RR



(i) Explain why these impulse response functions look different.

- The obvious reason is that the exogenous variables in these two VAR systems are different. The Federal Fund rate is the effective interest rate influenced by the Fed. The Romer-Romer shock is the unanticipated component in the Fed's decision.
- In the first VAR, the inflation does respond too much in the long run after the initial increase, which is counter-intuitive. Although in the first VAR system, we set the Fed fund rates i to be exogenous, in the data-generating process, the Fed will take the current and future economic condition into consideration when making the monetary policy decision, so-called foresight endogeneity.
- In the second VAR system, the inflation goes up first for a short time and then goes down after the shock. The Romer and Romer shock RR is constructed from changes in the Fed's target rate and is conditional on the Greenbook forecast, it relieves the concerns of foresight endogeneity a little bit.

(j) The output gap in our data measures the distance of output from full employment. How is this different from the output gap in our New Keynesian model?

- The output gap in the New Keynesian model is defined as the distance of the actual output between the natural level of output.
  - The natural level of output means that when the firms can flexibly adjust their price, to reflect the productivity/demand/policy shocks, what the output will be, denoted $\hat{y}^{\text{flex}}$

  - The actual output means that when the firms face the sticky price constraint in the short-term, what the output will be $\hat{y}$

- The output gap in data measures means the distance between the actual output and the potential output if there is no unemployment.
  - The (GDPPOT) Real potential GDP is the CBO's estimate of the output the economy would produce with a high rate of use of its capital and labor resources. The data is adjusted to remove the effects of inflation.
  - If there is no unemployment, then the output will assumably reach the maximum level of output, given everything else is equal.

- In the New Keynesian model, the natural level of output is not necessarily the max level of output because it just reflects the output level if the price can be freely adjusted.

- In the New Keynesian we learned in class there is no employment friction, however, in the real data, there will be employment frictions such as searching and matching. The data are not necessarily matched with the model we specified