Econ 210C Homework 1

Instructor: Johannes Wieland

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1. Complementarity of Money and Consumption

Suppose the utility function in our classical monetary model is now

$$U(X_t, L_t) = \frac{X_t^{1-\gamma} - 1}{1-\gamma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi}$$

where X_t is a composite of consumption and money,

$$X_t = \left[(1-\vartheta) C_t^{1-\nu} + \vartheta \left(\frac{M_t}{P_t} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}}$$

(a) Derive the first-order conditions for this economy.

The budget constraint is

$$P_t C_t + B_t + M_t \leqslant W_t N_t + Q_{t-1} B_{t-1} + M_{t-1} + P_t \left(TR_t + PR_t \right)$$

The Lagrangian is

$$L_t = \mathrm{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \Bigg[\left(\frac{X_{t+s}^{1-\gamma} - 1}{1-\gamma} - \chi \frac{N_{t+s}^{1+\varphi}}{1+\varphi} \right) + \lambda_{t+s} \left(W_{t+s} N_{t+s} + Q_{t+s-1} B_{t+s-1} + M_{t+s-1} + P_{t+s} \left(TR_{t+s} + PR_{t+s} \right) \right.$$
$$\left. - P_{t+s} C_{t+s} - B_{t+s} - M_{t+s} \right) \Bigg].$$

FOCs are:

$$\begin{aligned} \frac{\partial L_t}{\partial C_{t+s}} &= \frac{\partial L_t}{\partial X_{t+s}} \frac{\partial X_{t+s}}{\partial C_{t+s}} = X_{t+s}^{-\gamma} \frac{1}{1-\nu} \left[(1-\vartheta) C_{t+s}^{1-\nu} + \vartheta \left(\frac{M_{t+s}}{P_{t+s}} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}-1} (1-\vartheta)(1-\nu) C_{t+s}^{-\nu} - \lambda_{t+s} P_{t+s} \\ &= X_{t+s}^{\nu-\gamma} (1-\vartheta) C_{t+s}^{-\nu} - \lambda_{t+s} P_{t+s} = 0 \qquad (1) \end{aligned}$$

$$\begin{aligned} \frac{\partial L_t}{\partial M_{t+s}} &= \frac{\partial L_t}{\partial X_{t+s}} \frac{\partial X_{t+s}}{\partial M_{t+s}} = X_{t+s}^{-\gamma} \frac{1}{1-\nu} \left[(1-\vartheta) C_{t+s}^{1-\nu} + \vartheta \left(\frac{M_{t+s}}{P_{t+s}} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}-1} \vartheta (1-\nu) \left(\frac{M_{t+s}}{P_{t+s}} \right)^{-\nu} \frac{1}{P_{t+s}} \\ &- \lambda_{t+s} + \beta E_{t+s}[\lambda_{t+s+1}] = X_{t+s}^{\nu-\gamma} \vartheta \left(\frac{M_{t+s}}{P_{t+s}} \right)^{-\nu} \frac{1}{P_{t+s}} - \lambda_{t+s} + \beta E_{t+s}[\lambda_{t+s+1}] = 0 \qquad (2) \end{aligned}$$

$$
\begin{aligned}
\frac{\partial L_t}{\partial N_{t+s}} &= -\chi N_{t+s}^{\varphi} + \lambda_{t+s} W_{t+s} = 0 && (3) \\
\frac{\partial L_t}{\partial B_{t+s}} &= -\lambda_{t+s} + \beta Q_{t+s} E_{t+s}[\lambda_{t+s+1}] = 0 && (4)
\end{aligned}
$$

By rearranging FOCs (1) and (4), we have euler equation:

$$
\frac{X_{t+s}^{\nu-\gamma} C_{t+s}^{-\nu}}{P_{t+s}} = \beta Q_{t+s} \mathrm{E}_{t+s} \left(\frac{X_{t+s+1}^{\nu-\gamma} C_{t+s+1}^{-\nu}}{P_{t+s+1}} \right) \tag{5}
$$

By (1) and (3), we get labor and leisure:

$$
\frac{W_{t+s}}{P_{t+s}} = \frac{\chi N_{t+s}^{\varphi}}{(1-\vartheta) C_{t+s}^{-\nu} X_{t+s}^{\nu-\gamma}} \tag{6}
$$

By (1) and (2), we get money equation:

$$
\frac{X_{t+s}^{\nu-\gamma} C_{t+s}^{-\nu}}{P_{t+s}} = \beta \mathbb{E}_{t+s} \left[\frac{X_{t+s+1}^{\nu-\gamma} C_{t+s+1}^{-\nu}}{P_{t+s+1}} \right] + X_{t+s}^{\nu-\gamma} \frac{\vartheta}{1-\vartheta} \left(\frac{M_{t+s}}{P_{t+s}} \right)^{-\nu} \frac{1}{P_{t+s}} \tag{7}
$$

and combine the (5) and (7), we have

$$
\begin{aligned}
\left(1 - \frac{1}{Q_{t+s}}\right) \frac{X_{t+s}^{\nu-\gamma} C_{t+s}^{-\nu}}{P_{t+s}} &= X_{t+s}^{\nu-\gamma} \frac{\vartheta}{1-\vartheta} \left(\frac{M_{t+s}}{P_{t+s}} \right)^{-\nu} \frac{1}{P_{t+s}} \\
\left(1 - \frac{1}{Q_{t+s}}\right) &= \frac{\vartheta}{1-\vartheta} \left(\frac{M_{t+s}}{P_{t+s}} \right)^{-\nu} C_{t+s}^{\nu} && (8)
\end{aligned}
$$

Consider the firm's problem and it wants to maximize the profit:

$$
\max_{N_t} A_t N_t - \frac{W_t}{P_t} N_t \tag{9}
$$

therefore the FOC is

$$
A_t - \frac{W_t}{P_t} = 0 \tag{10}
$$

(b) Under what conditions does this economy predict that money is neutral? Explain why. Because

$$
X_t = \left[(1-\vartheta) C_t^{1-\nu} + \vartheta \left(\frac{M_t}{P_t} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}} \tag{11}
$$

the nominal money term P_t is always in the X_t. Therefore, we can see that if $\vartheta = 0$, then the economy is money neutral.

What's more, if $\nu = \gamma$, then we can eliminate the X_t term in the FOCs, therefore, in this case, the economy is money-neutral.

(c) Solve analytically for the steady state of the model (as far as you can), assuming $A = 1$.

From (10), we have

$$A = \frac{W}{P} = 1 \Rightarrow W = P \tag{12}$$

according to the production function and good market clearing:

$$Y = AN = N = C \tag{13}$$

Then from the labor and leisure FOC:

$$\frac{\chi N^{\varphi}}{(1-\vartheta)C^{-\nu}X^{\nu-\gamma}} = \frac{W}{P} = 1 \tag{14}$$

and given $C = N$, the above becomes:

$$(1-\vartheta)X^{\nu-\gamma} = \chi C^{\varphi+\nu} \tag{15}$$

In the steady state, the real interest rate is also static:

$$R = Q\frac{1}{\Pi} = Q \tag{16}$$

where $\Pi_{t+s} = \frac{P_{t+s}}{P_{t+s-1}}$. Combine it with the Euler equation:

$$X^{\nu-\gamma}C^{-\nu} = \beta Q\frac{1}{\Pi}X^{\nu-\gamma}C^{-\nu} \tag{17}$$

we have:

$$\frac{Q}{\Pi} = \frac{1}{\beta} = R \tag{18}$$

Similarly, the money equation becomes:

$$\begin{aligned}
X^{\nu-\gamma}C^{-\nu} &= \beta\frac{1}{\Pi}X^{\nu-\gamma}C^{-\nu} + X^{\nu-\gamma}\frac{\vartheta}{1-\vartheta}\left(\frac{M}{P}\right)^{-\nu} && (19)\\
1 &= \frac{\beta}{\Pi} + \frac{\vartheta}{1-\vartheta}\left(\frac{M}{P}\right)^{-\nu}C^{\nu} && (20)\\
\frac{M}{P} &= \left(1-\frac{\beta}{\Pi}\right)^{-\frac{1}{v}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{v}}C && (21)
\end{aligned}$$

Then plug this into the definition of X:

$$X = \left[(1-\theta) + \theta\left(1-\frac{\beta}{\Pi}\right)^{-\frac{1-v}{v}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1-v}{v}}\right]^{\frac{1}{1-v}}C \tag{22}$$

and with some algebra:

$$C = \left\{ \frac{1-\theta}{\chi} \left[(1-\theta) + \theta \left(1 - \frac{\beta}{\Pi}\right)^{-\frac{1-\upsilon}{\upsilon}} \left(\frac{\theta}{1-\theta}\right)^{\frac{1-\upsilon}{\upsilon}} \right]^{\frac{\upsilon-\gamma}{1-\upsilon}} \right\}^{\frac{1}{\varphi+\gamma}} \tag{23}$$

where $\Pi = 1$ because the price does change:

$$Q = \frac{1}{\beta} = R \tag{24}$$

(d) Based on your steady state equations describe an algorithm for how to solve for the steady state.

- set the parameters $\vartheta, \chi, \beta, \nu$, φ as input and we can get the level of consumption C^*, real interest rate R^*, and bond price Q^*.
- according to the steady states equations above, Y^* and N^* can be determined by C^*.
- the X^* can be determined by (14).
- the $\frac{M^*}{P^*}$ can be determined by the definition formula of X.

(e) How would you calibrate ϑ given knowledge of ν? (I.e., what moments of the data would you use and how?)

- To calibrate ϑ, we want to use the equation (23) and matching the LHS C with the long-term mean consumption.
- given the assumption that in the steady state there is no inflation, then $\Pi = 1$. Therefore, we can back out the θ from (23).

(f) Given knowledge of other parameters, how would you set M such that $P = 1$ in steady state?

To set $P = 1$, we can use the equation (20) and set $P = 1$, then we get a formula of M:

$$\begin{aligned}
\frac{M}{P} &= \left((1-\beta)\frac{1-\vartheta}{\vartheta}\right)^{-\frac{1}{\nu}} C && (25)\\
M &= \left((1-\beta)\frac{1-\vartheta}{\vartheta}\right)^{-\frac{1}{\nu}} C \\
&= \left((1-\beta)\frac{1-\vartheta}{\vartheta}\right)^{-\frac{1}{\nu}} \left\{ \frac{1-\theta}{\chi} \left[(1-\theta) + \theta \left(1 - \frac{\beta}{\Pi}\right)^{-\frac{1-\upsilon}{\upsilon}} \left(\frac{\theta}{1-\theta}\right)^{\frac{1-\upsilon}{\upsilon}} \right]^{\frac{\upsilon-\gamma}{1-\upsilon}} \right\}^{\frac{1}{\varphi+\gamma}} && (26)
\end{aligned}$$

(g) Derive the log-linearized model.

$$\begin{aligned}
\frac{W_{t+s}}{P_{t+s}} &= \frac{\chi N_{t+s}^{\varphi}}{(1-\vartheta) C_{t+s}^{-\nu} X_{t+s}^{\nu-\gamma}} \\
\ln(W_t) - \ln(P_t) &= \ln(\chi) + \varphi \ln(N_t) - \ln(1-\varphi) + \nu \ln(C_t) - (\nu - \gamma) \ln(X_t)
\end{aligned}$$

Take the Taylor expansion:

$$\begin{aligned}
\ln(W)+\frac{1}{W}(W_t-W)-\left(\ln(P)+\frac{1}{P}(P_t-P)\right) &= \ln(\chi)+\varphi\left(\ln(N)+\frac{1}{N}(N_t-N)\right)\\
&- \ln(1-\varphi)+\nu\left(\ln(C)+\frac{1}{C}(C_t-C)\right)\\
&- (\nu-\gamma)\left(\ln(X)+\frac{1}{X}(X_t-X)\right)
\end{aligned} \tag{27}$$

Notice that $\log X_t-\log X=\frac{1}{X}(X_t-X)-\frac{1}{X}(X-X)=\frac{X_t-X}{X}$, we can denote $\hat{X}_t=\log X_t-\log X=\frac{X_t-X}{X}$ as growth rate:

$$\ln(W)+\hat{W}_t-\left(\ln(P)+\hat{P}_t\right) \quad =\ln(\chi)+\varphi\left(\ln(N)+\hat{N}_t\right)-\ln(1-\varphi)+\nu\left(\ln(C)+\hat{C}_t\right)-(\nu-\gamma)\left(\ln(X)+\hat{X}_t\right)$$

In steady states, we have $W=P$, $C=N$ and $(1-\vartheta)X^{\nu-\gamma}=\chi C^{\varphi+\nu}$ and plug these in the above equations:

$$\hat{w}_t-\hat{p}_t=\varphi\hat{n}_t+\nu\hat{c}_t-(\nu-\gamma)\hat{x}_t \tag{28}$$

Similarly, and according to the formula of linearization:

$$f_1(X,Y,Z)X\hat{x}+f_2(X,Y,Z)Y\hat{y}+f_3(X,Y,Z)Z\hat{z}=g'(W)W\hat{w} \tag{29}$$

we have the Euler equation:

$$\nu(\hat{c}_{t+1}-\hat{c}_t)+(\hat{p}_{t+1}-\hat{p}_t)=\hat{q}_t+(\nu-\gamma)(\hat{x}_{t+1}-\hat{x}_t) \tag{30}$$

and money FOC (8) with the steady state conditions (18) and (20):

$$\begin{aligned}
\hat{q}_t &= v(Q-1)(\hat{c}_t+\hat{p}_t-\hat{m}_t)\\
\hat{q}_t &= v\left(\frac{1}{\beta}-1\right)(\hat{c}_t+\hat{p}_t-\hat{m}_t)
\end{aligned} \tag{31}$$

and from the firm's FOC:

$$\hat{a}_t=\hat{w}_t-\hat{p}_t \tag{32}$$

The log-linearization of X_t definition formula is:

$$\begin{aligned}
X\hat{x}_t &= (1-\theta)X^vC^{1-v}\hat{c}_t+\theta X^v\left(\frac{M}{P}\right)^{1-v}(\hat{m}_t-\hat{p}_t)\\
X^{1-v}\hat{x}_t &= (1-\theta)C^{1-v}\hat{c}_t+\theta M^{1-v}(\hat{m}_t-\hat{p}_t)
\end{aligned} \tag{33}$$

The log-linearization of the production function is:

$$\hat{y}_t=\hat{a}_t+\hat{n}_t \tag{34}$$

The log-linearization of the good market clear condition:

$$\hat{y}_t = \hat{c}_t \tag{35}$$

- The equation systems are (28), (30), (31), (32), (33), (34) and (35).

(h) Following your calibration strategy for each of $\nu \in \{0.25, 0.5, 1, 2, 4\}$, solve the model using sequence space methods using the following parameters:

$$\gamma = 1, \varphi = 1, \chi = 1, \beta = 0.99, \rho_m = 0.99$$

where $\hat{m}_t = \rho_m \hat{m}_{t-1} + \epsilon_t^m$.

Report the IRFs for consumption, prices, and the nominal interest rate. Your graph for each variable should contain all five cases, appropriately labeled.

- Given the equation system (28), (30), (31), (32), (33), (34) and (35).

Define the **H** matrix as:

$$\mathbf{H}(\mathbf{Y}, \epsilon) = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} = \begin{bmatrix} \hat{y}_t - \hat{c}_t \\ \hat{c}_t + \hat{p}_t - \hat{m}_t - \frac{1}{v(Q-1)}\hat{q}_t \end{bmatrix} = \begin{bmatrix} \hat{y}_1 - \hat{c}_1 \\ \vdots \\ \hat{y}_T - \hat{c}_T \\ \hat{c}_1 + \hat{p}_1 - \hat{m}_1 - \frac{1}{v(Q-1)}\hat{q}_1 \\ \vdots \\ \hat{c}_T + \hat{p}_T - \hat{m}_T - \frac{1}{v(Q-1)}\hat{q}_T \end{bmatrix} = 0 \tag{36}$$

For $\forall\ t = 1, 2, \cdots, T$, define

$$\begin{aligned} \mathbf{Y} &= [\hat{y}_t, \hat{w}_t, \hat{c}_t, \hat{q}_t, \hat{x}_t] && (37) \\ \mathbf{Z} &= [\hat{m}_t] && (38) \\ \mathbf{U} &= [\hat{n}_t, \hat{p}_t] && (39) \end{aligned}$$

To compute the IRFs for consumption, prices, and the nominal interest rate, we want to compute:

$$\frac{d\mathbf{Y}}{d\mathbf{Z}} \quad \text{and} \quad \frac{d\mathbf{U}}{d\mathbf{Z}} \tag{40}$$

Because

$$\begin{aligned} \mathbf{H}_\mathbf{U} &= \frac{\partial \mathbf{H}}{\partial \mathbf{Y}} \frac{\partial \mathbf{Y}}{\partial \mathbf{U}} && (41) \\ \mathbf{H}_\mathbf{Z} &= \frac{\partial \mathbf{H}}{\partial \mathbf{Y}} \frac{\partial \mathbf{Y}}{\partial \mathbf{Z}} && (42) \\ d\mathbf{U} &= \mathbf{H}_\mathbf{U}^{-1} \mathbf{H}_\mathbf{Z} d\mathbf{Z} = \left(\frac{\partial \mathbf{H}}{\partial \mathbf{Y}} \frac{\partial \mathbf{Y}}{\partial \mathbf{U}} \right)^{-1} \left(\frac{\partial \mathbf{H}}{\partial \mathbf{Y}} \frac{\partial \mathbf{Y}}{\partial \mathbf{Z}} \right) d\mathbf{Z} && (43) \\ d\mathbf{Y} &= \frac{\partial \mathbf{Y}}{\partial \mathbf{U}} d\mathbf{U} + \frac{\partial \mathbf{Y}}{\partial \mathbf{Z}} d\mathbf{Z} = \left(\frac{\partial \mathbf{Y}}{\partial \mathbf{U}} \left(\frac{\partial \mathbf{H}}{\partial \mathbf{Y}} \frac{\partial \mathbf{Y}}{\partial \mathbf{U}} \right)^{-1} \left(\frac{\partial \mathbf{H}}{\partial \mathbf{Y}} \frac{\partial \mathbf{Y}}{\partial \mathbf{Z}} \right) + \frac{\partial \mathbf{Y}}{\partial \mathbf{Z}} \right) d\mathbf{Z} && (44) \end{aligned}$$

we need to figure out the following components $\frac{\partial \mathbf{H}}{\partial \mathbf{Y}}, \frac{\partial \mathbf{Y}}{\partial \mathbf{U}}, \frac{\partial \mathbf{Y}}{\partial \mathbf{Z}}$:

$$
\begin{aligned}
\frac{\partial \mathbf{H}}{\partial \mathbf{Y}} &= \begin{bmatrix} \frac{\partial \mathbf{H}_1}{\partial \mathbf{y}} & \frac{\partial \mathbf{H}_1}{\partial \mathbf{w}} & \frac{\partial \mathbf{H}_1}{\partial \mathbf{c}} & \frac{\partial \mathbf{H}_1}{\partial \mathbf{q}} & \frac{\partial \mathbf{H}_1}{\partial \mathbf{x}} \\ \frac{\partial \mathbf{H}_2}{\partial \mathbf{y}} & \frac{\partial \mathbf{H}_2}{\partial \mathbf{w}} & \frac{\partial \mathbf{H}_2}{\partial \mathbf{c}} & \frac{\partial \mathbf{H}_2}{\partial \mathbf{q}} & \frac{\partial \mathbf{H}_2}{\partial \mathbf{x}} \end{bmatrix} \qquad (45) \\
&= \begin{bmatrix} \frac{\partial \hat{\mathbf{y}}_t - \hat{\mathbf{c}}_t}{\partial \mathbf{y}} & \frac{\partial \hat{\mathbf{y}}_t - \hat{\mathbf{c}}_t}{\partial \mathbf{w}} & \frac{\partial \hat{\mathbf{y}}_t - \hat{\mathbf{c}}_t}{\partial \mathbf{c}} & \frac{\partial \hat{\mathbf{y}}_t - \hat{\mathbf{c}}_t}{\partial \mathbf{q}} & \frac{\partial \hat{\mathbf{y}}_t - \hat{\mathbf{c}}_t}{\partial \mathbf{x}} \\ \frac{\partial \hat{\mathbf{c}}_t + \hat{\mathbf{p}}_t - \hat{\mathbf{m}}_t - \frac{1}{v(Q-1)} \hat{\mathbf{q}}_t}{\partial \mathbf{y}} & \frac{\partial \hat{\mathbf{c}}_t + \hat{\mathbf{p}}_t - \hat{\mathbf{m}}_t - \frac{1}{v(Q-1)} \hat{\mathbf{q}}_t}{\partial \mathbf{w}} & \frac{\partial \hat{\mathbf{c}}_t + \hat{\mathbf{p}}_t - \hat{\mathbf{m}}_t - \frac{1}{v(Q-1)} \hat{\mathbf{q}}_t}{\partial \mathbf{c}} & \frac{\partial \hat{\mathbf{c}}_t + \hat{\mathbf{p}}_t - \hat{\mathbf{m}}_t - \frac{1}{v(Q-1)} \hat{\mathbf{q}}_t}{\partial \mathbf{q}} & \frac{\partial \hat{\mathbf{c}}_t + \hat{\mathbf{p}}_t - \hat{\mathbf{m}}_t - \frac{1}{v(Q-1)} \hat{\mathbf{q}}_t}{\partial \mathbf{x}} \end{bmatrix}
\end{aligned}
$$

$$
\frac{\partial \mathbf{Y}}{\partial \mathbf{U}} = \begin{bmatrix} \frac{\partial \mathbf{y}}{\partial \mathbf{n}} & \frac{\partial \mathbf{y}}{\partial \mathbf{p}} \\ \frac{\partial \mathbf{w}}{\partial \mathbf{n}} & \frac{\partial \mathbf{w}}{\partial \mathbf{p}} \\ \frac{\partial \mathbf{c}}{\partial \mathbf{n}} & \frac{\partial \mathbf{c}}{\partial \mathbf{p}} \\ \frac{\partial \mathbf{q}}{\partial \mathbf{n}} & \frac{\partial \mathbf{q}}{\partial \mathbf{p}} \\ \frac{\partial \mathbf{x}}{\partial \mathbf{n}} & \frac{\partial \mathbf{x}}{\partial \mathbf{p}} \end{bmatrix} \qquad (46)
$$

$$
\frac{\partial \mathbf{Y}}{\partial \mathbf{Z}} = \begin{bmatrix} \frac{\partial \mathbf{y}}{\partial \mathbf{m}} \\ \frac{\partial \mathbf{w}}{\partial \mathbf{m}} \\ \frac{\partial \mathbf{c}}{\partial \mathbf{m}} \\ \frac{\partial \mathbf{q}}{\partial \mathbf{m}} \\ \frac{\partial \mathbf{x}}{\partial \mathbf{m}} \end{bmatrix} \qquad (47)
$$

where the equations are

$$
\begin{aligned}
\hat{c}_t &= \frac{1}{\nu} \left(\hat{w}_t - \hat{p}_t + (\nu - \gamma) \hat{x}_t - \varphi \hat{n}_t \right) \\
\hat{q}_t &= \nu (\hat{c}_{t+1} - \hat{c}_t) + (\hat{p}_{t+1} - \hat{p}_t) - (\nu - \gamma)(\hat{x}_{t+1} - \hat{x}_t) \\
\hat{w}_t &= \hat{a}_t + \hat{p}_t \\
\hat{x}_t &= (1 - \theta) \left(\frac{C}{X} \right)^{1-v} \hat{c}_t + \theta \left(\frac{M}{X} \right)^{1-v} (\hat{m}_t - \hat{p}_t) \\
\hat{y}_t &= \hat{a}_t + \hat{n}_t \\
\hat{y}_t &= \hat{c}_t \\
\hat{q}_t &= v \left(\frac{1}{\beta} - 1 \right) (\hat{c}_t + \hat{p}_t - \hat{m}_t)
\end{aligned}
$$

The details can be found in the code.

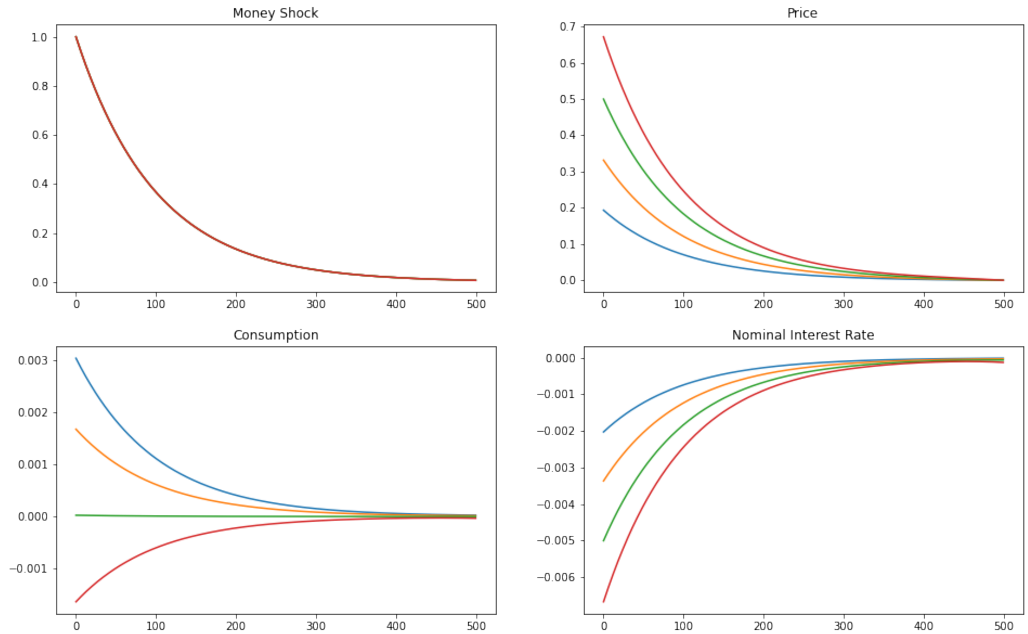

Figure 1: IRF

(i) Intuitively explain your results.

When ν increases, the response of price and consumption increases, and the response of nominal interest rates decreases. This is because ν governs the elasticity of substitution between consumption c and $\frac{m}{p}$.

- When $v < 1$, m increase will decrease the marginal utility of c. Then the labor supply n and income will decrease, and the production y and consumption c will decrease. The price p will increase because m increases. The money demand is increased, so the interest rates q decrease.
- When $v > 1$, m increase will increase the marginal utility of c. Therefore, there will be more labor supply n, and the production y and consumption c will increase. Compared to $v < 1$, agents prefer consumption now and future, therefore the money demand is increased less than in the previous one. Then the price increases more and interest rates drop more.
- When $v = 1$, m increase has no impact on the marginal utility of c.

(j) If you had evidence that an increase in the money supply increases consumption, which values for ν can you rule out? Explain why.

We know that only when $\nu > 1$, consumption increases with a positive money supply shock. Thus, we can rule out $\nu \leqslant 1$.

(k) Make sure your code packet contains a file that produces your graphs with a single click. (It does not need to save the graphs.) Upload it to Github.