Estimate the Attenuation Effect

A Toy Model

A representative household solves:

$$\max u(c, l) = \max \ln c - l,$$

subject to

$$c \leq wl + rk, \quad c \geq 0, \quad 0 \leq l.$$

- The household supplies its capital, k, inelastically to the market. Each household owns the same amount, k, of capital.

A representative competitive firm operates the following production function:

$$Y = K^{\alpha} L^{1-\alpha}, \quad 0 < \alpha < 1,$$

and maximizes profits by choosing L, K to maximize

$$Y - wL - rK$$

subject to its production function.

Solution

A competitive equilibrium satisfies:

- Households maximize utility and firms maximize profit.
- Labor market clears: $l = L$, capital market clears: $k = K$, goods market clears: $c = Y$

The household's Lagrangian is:

$$\mathcal{L} = u + \lambda(wl + rk - c)$$

with first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial c} = u_c - \lambda = 0, \quad \frac{\partial \mathcal{L}}{\partial l} = u_l + \lambda w = 0$$

Using the FOCs:

$$-\frac{u_l}{u_c} = c = wL + rK = K^\alpha L^{1-\alpha} = w = (1-\alpha)\left(\frac{K}{L}\right)^\alpha$$

The solution is:

$$L = 1 - \alpha, \quad c = Y = K^\alpha L^{1-\alpha}, \quad r = \alpha\left(\frac{K}{L}\right)^{1-\alpha}$$

Assumption

A competitive equilibrium satisfies:

- Households maximize utility and firms maximize profit.
- Labor market clears: $l = L$, capital market clears: $k = K$, goods market clears: $c = Y$

Households know:

- Each household has k units of capital and will supply all of it to the market.
- Because households make their decisions simultaneously and **without coordination or communication**, **they choose l and c based on a belief about r and w.**
- Because each household is atomistic, its actions have no impact on r, w.

The solution should be revised conditioning on labor L:

$$w(L) = (1-\alpha)\left(\frac{K}{L}\right)^\alpha \tag{1}$$

$$r(L) = \alpha\left(\frac{K}{L}\right)^{1-\alpha} \tag{2}$$

Estimate The Best Response Function

With belief on $w(L)$ and $r(L)$, the household decides $l(L)$ and $c(L)$ based on the constraint:

$$c = w(L)l + r(L)k$$

The household payoff function conditioning on strategy l is:

$$U(l, L) = \ln(w(L)l + r(L)k) - l$$

Its best response to L is:

$$\arg\max_{l \geq 0} U(l, L)$$

The FOC is:

$$l = \max\left\{0, 1 - \frac{r(L)K}{w(L)}\right\} = \max\left\{0, 1 - \frac{r(L)K}{w(L)L}L\right\}$$

Therefore, the best response l to L is piecewise-linear:

$$l = f(L) = \begin{cases} 1 - \frac{\alpha}{1-\alpha}L & \text{if } 0 \leq L \leq \frac{1-\alpha}{\alpha} \\ 0 & \text{if } L > \frac{1-\alpha}{\alpha} \end{cases}$$

Empirical Plan

Given the above theoretical intuition in mind, the regression equation I am going to explore is the following:

$$\Delta y_{i,t} = \gamma_t + \alpha \Delta E_t^i [Y_t] + \beta X_{it} + \epsilon_{it} \tag{3}$$

The $y_{i,t}$ is the individual's choice variable, which could be the labor input, firm capital investment, or bank loan origination. $E_t^i [Y_t]$ is the individual's expectation on the aggregated variable. α is the estimation of interest. X_{it} are control variables.