$8^{\underline{a}}$ Lista de Exercícios

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3.2.25)

Seja $u \in \mathbb{R}^n$ com $\|u\|_2 = 1$ e, definindo $P = uu^T$ com $P \in \mathbb{R}^{n\times n}$, então:

a) $Pu = u(u^T u) = u\|u\|_2^2 = u \cdot 1 = u$

b) $Pv = u(u^T v) = u(\langle u, v\rangle)^2$
Assim, caso $\langle u, v\rangle = 0$, temos que, $Pv = u \cdot 0 = 0$

c) $P^2 = P \cdot P = uu^T uu^T = u\|u\|_2^2 u^T = uu^T = P$

d) $P^T = (uu^T)^T = (u^T)^T u^T = uu^T = P$

3.2.27)

Seja $u \in \mathbb{R}^n$ com $\|u\|_2 = 1$ e, definindo $Q = I - 2uu^T$ com $Q \in \mathbb{R}^{n\times n}$, então:

a) $Qu = Iu - 2uu^T u = u - 2u\|u\|_2^2 = u - 2u \cdot 1 = -u$

b) $Qv = Iv - 2u(u^T v) = v - 2u(\langle u, v\rangle)^2$

Assim, caso $\langle u, v\rangle = 0$, temos que, $Qv = v - 2u \cdot 0 = v$

c) $Q = I - 2uu^T = I - 2(u^T)^T u^T = I - (2uu^T)^T = (I - 2uu^T)^T = Q^T$

d) Seja $Q \cdot Q^{-1} = I$, temos:

$Q \cdot Q^T = (I - 2uu^T)(I - 2uu^T)^T = (I - 2uu^T)(I - 2uu^T) = I - 2uu^T - 2uu^T + 4uu^T uu^T$

$Q \cdot Q^T = I - 4uu^T + 4u\|u\|_2^2 u^T = I - 4uu^T + 4uu^T = I$

$Q \cdot Q^T = Q \cdot Q^{-1} \Rightarrow Q^T = Q^{-1}$

e) Pelos itens *(c)* e *(d)*, podemos obter trivialmente que $Q^{-1} = Q$