Modelling covariate effects in extremes of storm severity on the Australian North West Shelf

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Outline

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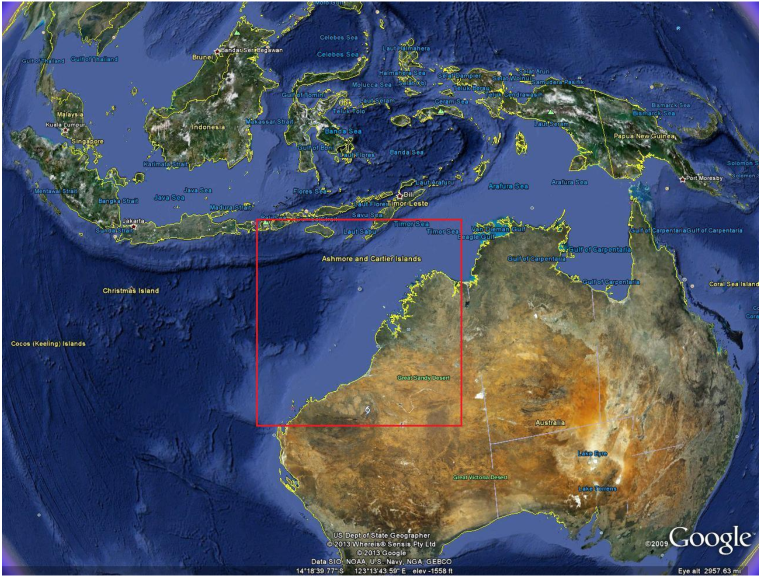
Motivation

- **Rational** design an assessment of marine structures:
  - Reducing **bias** and **uncertainty** in estimation of structural reliability.
  - Improved understanding and communication of risk.
  - Climate change.
- Other applied fields for extremes in industry:
  - Corrosion and fouling.
  - Finance.
  - Network traffic.

Australian North West Shelf

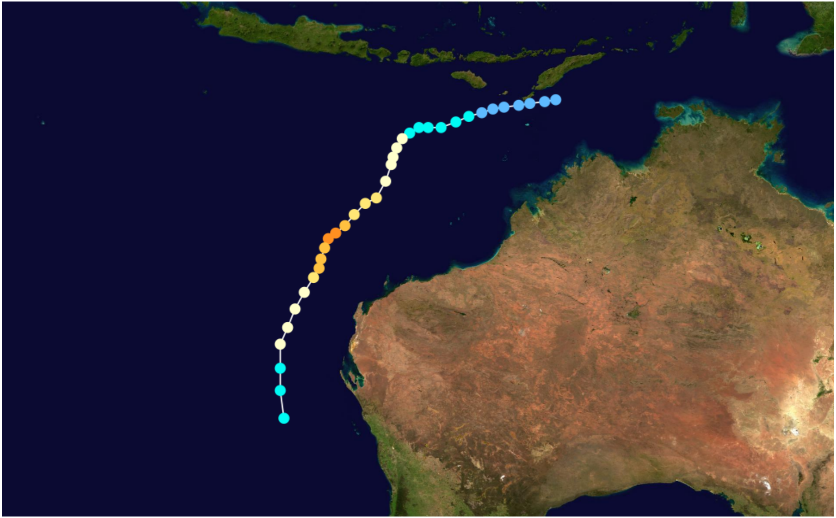


Australian North West Shelf

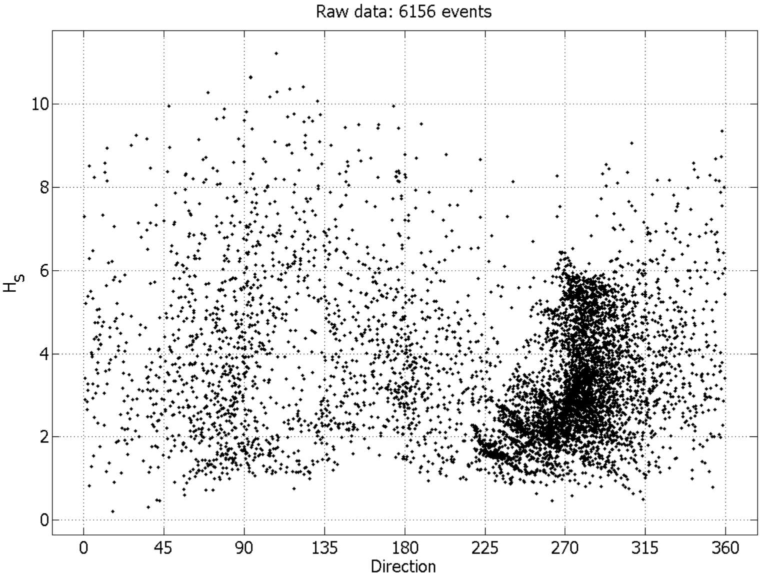
- Data consist of hindcast storms during 1970-2007.
- Model **storm peak significant wave height** H_S.
- Wave climate is dominated by westerly **monsoonal swell** and **tropical cyclones**.
- Cyclones originate from Eastern Indian Ocean and in the Timor and Arafura Sea area is also a region of **cyclogensis**.

Cyclone Narelle January 2013

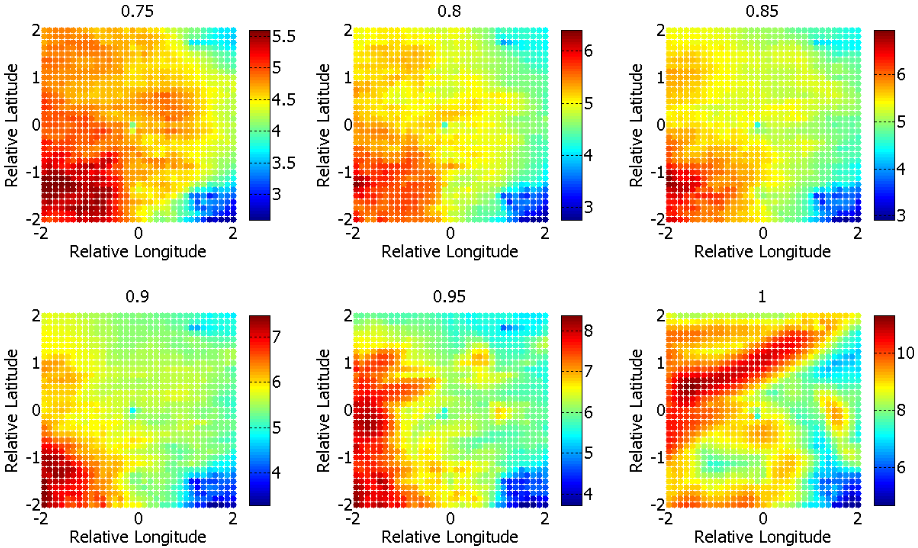
Cyclone Narelle January 2013



Storm Peak H_S by Direction



Quantiles of storm peak H_S Spatially



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Model Components

- Sample $\{\dot{z}_i\}_{i=1}^{\dot{n}}$ of $\dot{n}$ storm peak significant wave heights observed at locations $\{\dot{x}_i, \dot{y}_i\}_{i=1}^{\dot{n}}$ with storm peak directions $\{\dot{\theta}_i\}_{i=1}^{\dot{n}}$.
- Model Components
  1. **Threshold** function ϕ above which observations $\dot{z}$ are assumed to be extreme estimated using quantile regression.
  2. **Rate of occurrence** of threshold exceedances modelled using Poisson Process model with rate $\rho(\triangleq \rho(\theta, x, y))$
  3. **Size of occurrence** of threshold exceedance using a generalised Pareto (GP) model with shape and scale parameters ξ and σ.

Model Components

- Rate of occurrence and size of threshold exceedance are functionally **independent** (Chavez-Demoulin and Davison 2005).
- Equivalent to non-homogeneous Poisson point process model (Dixon et al. 1998).
- Smooth functions of covariates are estimated using P-splines (Eilers and Marx 2010)

P-Splines

- Physical considerations suggest that we should expect the model parameters ϕ, ρ, ξ and σ to vary smoothly with respect to covariates θ, x, y.
- n dimensional basis matrix B formulated using Kronecker products of marginal basis matrices

$$B = B_\theta \otimes B_x \otimes B_y$$

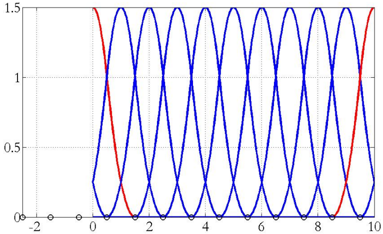
- Roughness is defined

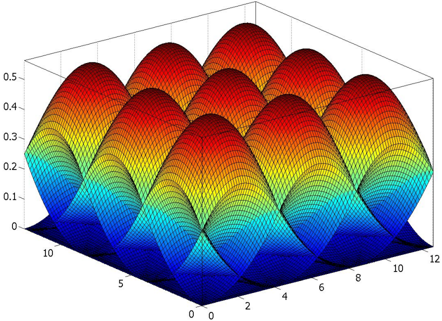
$$R = \beta' P \beta$$

where P is penalty matrix formed by taking differences of neighbouring β.

P-Splines

- Wrapped bases allows for periodic covariates such as seasonality or direction.
- High dimensional bases can easily be constructed although *number of parameters problematic*.
- Strength of roughness penalty is controlled by roughness coefficient λ: cross validation is used to choose λ optimally.





Quantile regression models threshold

- Estimate smooth quantile $\phi(\theta_i, x_i, y_i; \tau)$ for non-exceedance probability τ of storm peak H_S.

Spline basis: $$\psi(\tau, \theta) = \sum_{k=0}^{p} \Phi_{\theta k} \beta_{\tau k}$$

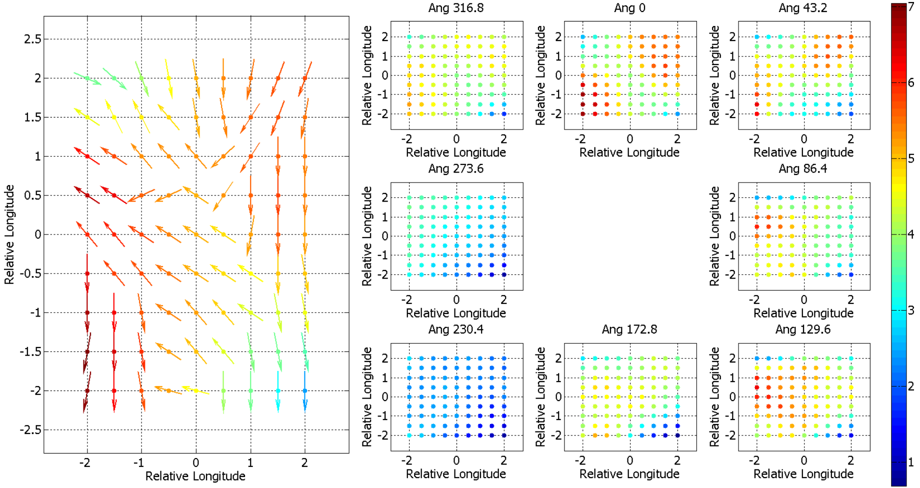
- Estimated by minimising **penalised** criterion ℓ^*_ϕ with respect to basis parameters:

$$\ell^*_\phi = \{\tau \sum_{r_i \geq 0}^{n} |r_i| + (1-\tau) \sum_{r_i < 0}^{n} |r_i|\} + \lambda_\phi R_\phi$$

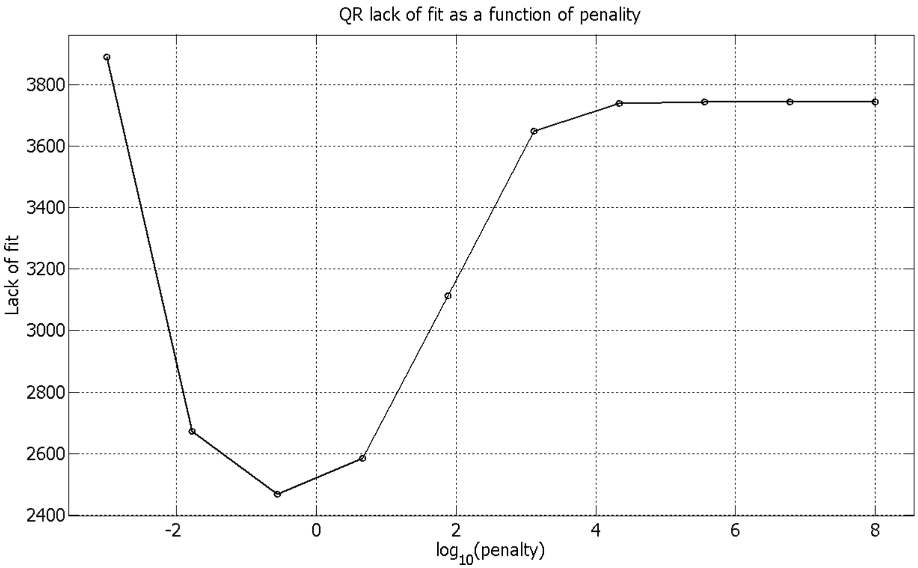
for $r_i = z_i - \phi(\theta_i, x_i, y_i; \tau)$ for $i = 1, 2, ..., n$, and **roughness** R_ϕ controlled by roughness coefficient λ_ϕ.

- Quantile regression with P-splines can be formulated and solved as a linear program (Bollaerts et al. 2006).

Spatio-Directional 50% Quantile Threshold



Cross Validation for Penalty



Poisson models rate of threshold exceedances

- Rate of occurrence of threshold exceedances is estimated by minimising the roughness penalised log likelihood

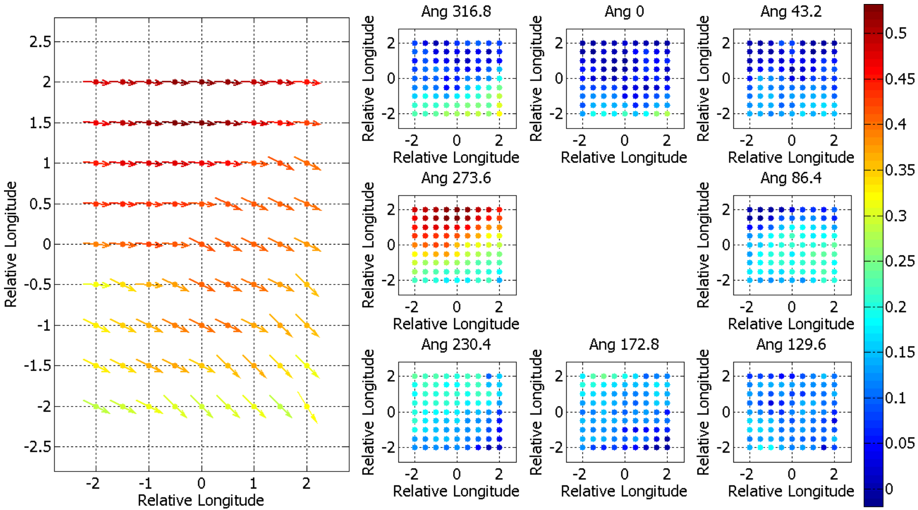
$$\ell_\rho^* = \ell_\rho + \lambda_\rho R_\rho$$

- (Negative) penalised Poisson log-likelihood for **rate of occurrence** of threshold excesses:

$$\ell_\rho = -\sum_{i=1}^{n} \log \rho(\theta_i, x_i, y_i) + \int \rho(\theta, x, y) d\theta dx dy$$

- λ_ρ is estimated using cross validation.

Spatio-Directional Rate of Threshold Exceedances



GP models size of threshold exceedances

- Generalised Pareto density (and negative conditional log-likelihood) for **sizes** of threshold excesses:

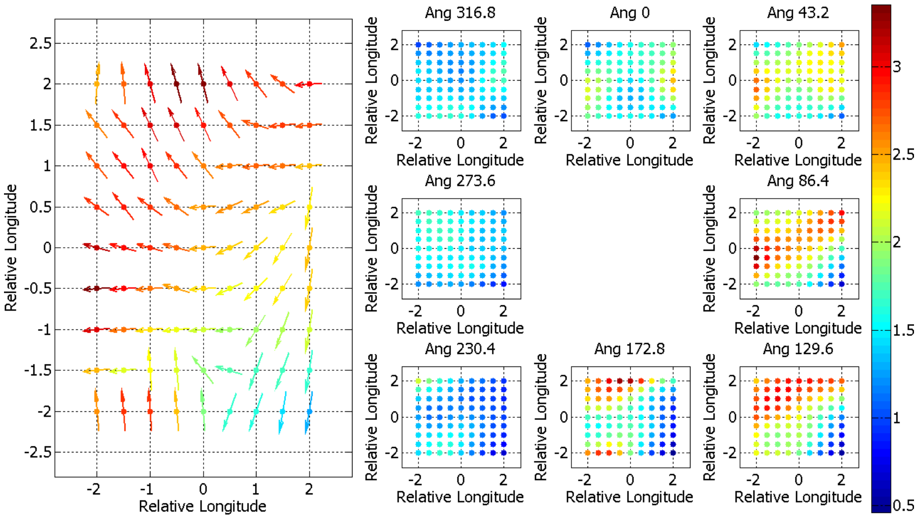
$$\ell_{\xi,\sigma} = \sum_{i=1}^{n} \log \sigma_i + \frac{1}{\xi_i} \log(1 + \frac{\xi_i}{\sigma_i}(z_i - \phi_i))$$

- Parameters: **shape** ξ, **scale** σ.
- Threshold ϕ_i set prior to estimation.
- Smoothness is imposed by minimising the roughness penalised log-likelihood.

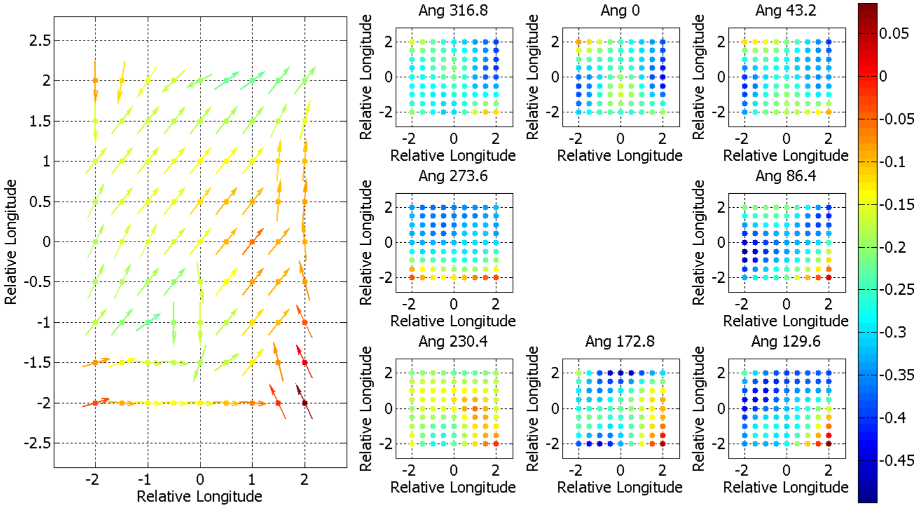
$$\ell^*_{\xi,\sigma} = \ell_{\xi,\sigma} + \lambda_\xi R_\xi + \lambda_\sigma R_\sigma$$

- λ_ξ and λ_σ are estimated using cross validation. In practice set $\lambda_\xi = \kappa\lambda_\sigma$ for fixed κ.

Spatio-Directional Scale of GP Exceedances



Spatio-Directional Shape of GP Exceedances



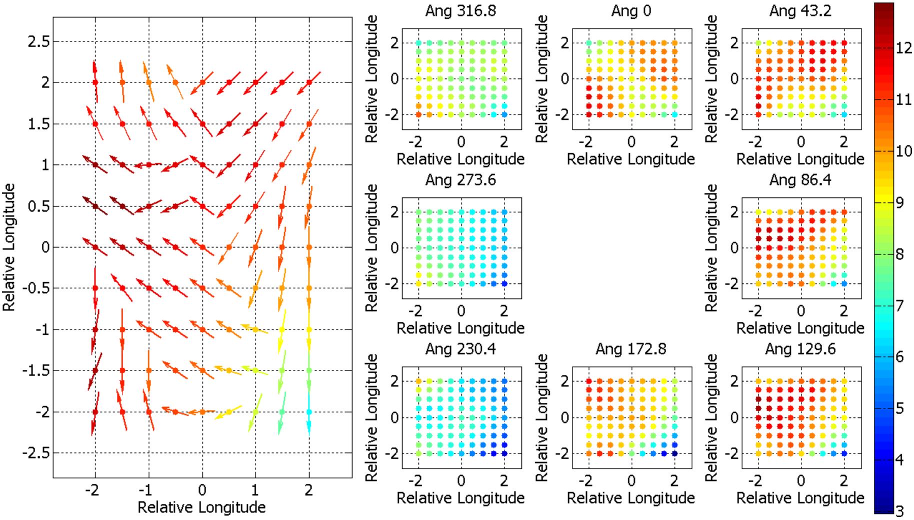
Return Values

- The return value z_T of storm peak significant wave height corresponding to some return period T , expressed in years, can be evaluated in terms of estimates for model parameters ϕ, ρ, ξ and σ

$$z_T = \phi - \frac{\sigma}{\xi}(1 + \frac{1}{\rho}(\log(1 - \frac{1}{T}))^{-\xi})$$

- z_{100} corresponds to the 100–year return value, often denoted by H_{S100}.
- Return values incorporating effects such as storm dissipation are estimated from simulation.

Spatio-Directional 100 Year Return Values



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Other Applications and Developments

- Other applications of spline extremes
  - Seasonal-directional
  - Spatio-temporal (climate change)
- Incorporation of uncertainty
  - Block bootstrapping allows quick estimates of parameter uncertainty
- Incorporation of spatial dependency
  - Composite likelihood: model (asymptotically dependent) componentwise–maxima.
  - Censored likelihood: allows extension from block-maxima to threshold exceedances.
- Multivariate Extremes
  - Conditional model for extremes with covariates; Jonathan et al. [2013].

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Thank You

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Spatio-Directional 100-year Return Value H_{S100}