



Review

On environmental contours for marine and coastal design

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ABSTRACT

Environmental contours are used in structural reliability analysis of marine and coastal structures as an approximate means to locate the boundary of the distribution of environmental variables, and hence sets of environmental conditions giving rise to extreme structural loads and responses. Outline guidance concerning the application of environmental contour methods is given in recent design guidelines from many organisations. However there is lack of clarity concerning (a) the differences between approaches to environmental contour estimation reported in the literature, and (b) the relationship between the environmental contour, corresponding to some return period, and the extreme structural response for the same period. Hence there is uncertainty about precisely when environmental contours should be used, and how they should be used well. This article seeks to provide some assistance in understanding the fundamental issues regarding environmental contours and their use in structural reliability analysis. Approaches to estimating the joint distribution of environmental variables, and to estimating environmental contours based on that distribution, are described. Simple freely-available software for estimation of the joint distribution, and hence environmental contours, is illustrated. Extra assumptions required to relate the characteristics of environmental contours to structural failure are outlined. Alternative response-based methods not requiring environmental contours are summarised. The results of an informal survey of the metocean user community regarding environmental contours are presented. Finally, recommendations about when and how environmental contour methods should be used are made.

1. Introduction

1.1. Metocean design

Numerous approaches to establishing design criteria for metocean loads and responses of marine structures and coastal facilities are used by different practitioners. Some of these are included in marine industry standards and guidelines; others are internal standards of different organisations active in ocean engineering. Rigorous comparison of some methods has been reported in the literature, but there is still uncertainty in the user community regarding the relative merits of different approaches. Within the marine industry, estimation of a joint metocean description has been considered for more than thirty years. It was shown

that typically, environmental forces on marine structures may be reduced by 5%–40% by accounting for the lack of complete dependence between metocean variables (wind, wave, current, etc.) traditionally used in design (e.g. E&P Forum, 1985; Vanderschuren, 1999; Feld et al., 2019). Development of reliability methods (e.g. Madsen et al., 1986) and their implementation by some parts of the industry in the 1980s brought joint probabilities into focus: they are required for a consistent treatment of the loading in Level III reliability analysis and for assessment of the relative importance of various metocean variables during extreme load and response conditions, fatigue damage and at failure.

Until the middle of the 1990s, very few metocean data sets of sufficient quality were available, limiting development of joint probability models; this has changed during the last twenty years. Comprehensive

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hindcast data are now available for locations world-wide, including simultaneous values for wind, waves, current, sea water level, ice and snow of sufficient quality and duration. Today joint probabilities are referenced in industry standards and guidelines (e.g. IACS, 2001; DNVGL-RP-C205, 2017; NORSOK N-003, 2017; ISO 19901-1 2015). They are required for application of the Formal Safety Assessment (FSA) methodology in rule development, providing risk based goal-oriented regulations that are well balanced with respect to acceptable risk levels and economic considerations, as recommended in IMO (2001).

ISO 19901-1 (2015), DNVGL-RP-C205 (2017) and NORSOK N-003 (2017) suggest that joint probability methods should be applied if reliable simultaneous data exist. NORSOK N-003 (2017) further recommends that the duration of a data should be sufficiently long to capture the probability of occurrence for all combinations of importance regarding predictions of metocean actions and action effects. Further, in a case of wind, wave and current it recommends that at least three years of simultaneous data is required to characterise the dependence between these variables reliably in design.

Given that it is possible to establish a model for the ‘short-term’ distribution of response given sea state parameters (for example, for a 3-h sea state), NORSOK N-003 (2017) states that the designer has essentially three different risk-based approaches to estimating the ‘long-term’ distribution of response (corresponding to tens, hundreds or thousands of years): a) the so-called ‘all short-term conditions’ (or ‘all sea state’) approach, b) the ‘storm event’ approach, and c) the environmental contour method, an approximate method using only short-term analysis.

There are two distinct joint probability approaches in widespread use in coastal engineering practice in the UK (e.g. DEFRA/Environment Agency, 2005). These are (a) a simplified method that involves the use of joint probability contours (JPC, Gouldby et al., 2017) and (b) a risk-based statistical method (Hawkes et al., 2002). Both approaches are implemented within the widely-used JOIN-SEA software system (HR Wallingford 1998; Hawkes et al., 2002).

1.2. Environmental contours

The environmental contour defines a set of extreme sea state conditions. It can be used to approximate extreme values of long-term structural response extremes by considering only a few short-term metocean conditions. Environmental contours are appealing since they can be specified for a given metocean environment independently of any structure. They are also incorporated within long-established approaches to structural design, compliant with guidelines referenced in Section 1.1 above, familiar to practitioners. To establish them, joint probabilities of metocean parameters, historically in the metocean community in the form of tables, are needed. The idea behind the method is to define contours in the metocean space (for example, H_S , T_P) along which extreme responses with given return period should lay. Environmental contours are a simplified and approximate method compared with full long-term response analysis, requiring less computational effort.

All environmental contour methods have a common goal of summarising the tail of the joint distribution of environmental variables, with a view to learning about the distribution of extreme structural response within a prescribed return period. This is achieved by identifying combinations of environmental variables (sometimes referred to as ‘governing conditions’) responsible for extreme structural response. Structural responses for combinations of environmental conditions lying on the contour can be used to estimate the extreme response due to a sea state with the same return period as the contour. Importantly, only combinations of metocean parameters lying on the contour need be considered. With additional prior knowledge of the response, it is possible to limit the interval of the contour over which to evaluate structural response, substantially reducing the computational effort for calculating extreme response. An underlying assumption is that the extreme T -year response is governed by sea state conditions on the

T -year environmental contour.

Contours estimated using different methods will be different in general. Individual methods make specific assumptions in characterising the environment. Hence, the choice of environmental contour method will influence the estimation of the distribution of extreme response.

Nevertheless, the environmental contour approach may be useful for early phase concept evaluation. For example, as stated by NORSOK N-003 (2017), if the application under consideration is of a very non-linear nature, an extensive model test program may be necessary to model the short-term variability for all important metocean conditions. The environmental contour approach can help identify those conditions.

Some approaches to estimation of environmental contours (for example IFORM, Winterstein et al., 1993) make additional explicit assumptions regarding the nature of structural failure surfaces in terms of environmental variables, potentially expressed on transformed marginal scales. When these assumptions are valid, statements regarding the relative magnitude of the exceedance probability of the T -year environmental contour and the T -year structural failure probability can be made more reasonably. However it is not always clear that the additional assumptions are satisfied for a given application.

The environmental contour procedure as given by NORSOK N-003 (2017) can be summarised as: (a) establish environmental contours of the metocean characteristics (e.g. H_S , T_P) corresponding to some annual non-exceedance probability $1 - 1/T$ (for a T -year return period); (b) identify the worst metocean condition along the contour for the response under consideration; (c) for this sea state, determine the distribution function for the appropriate 3-h extreme value for the response under consideration; and (d) estimate the value of the response (corresponding to the same annual non-exceedance probability $1 - 1/T$) using the quantile of the distribution of response (from (c)) with non-exceedance probability α . A value $\alpha = 0.9$ is recommended for ULS (the ultimate limit state), and $\alpha = 0.95$ for ALS (the accidental limit state). The standard provides some guidance as to the adequacy of the approach in terms of the width of the distribution of response in (c). Although this standard discusses the environmental contour method for sea states of length 3-h, the procedure can be applied to sea states of any appropriate length (for example, 30 min); we use 3-h sea states here for consistency and ease of explanation.

DNVGL-RP-C205 (2017) recommends two environmental contour approaches: IFORM (Winterstein et al., 1993, a procedure similar to NORSOK N-003 (2017) above), and a constant probability density approach (inspired by the work of Haver, 1985). The procedure for the latter can be summarised as: (a) estimate a joint environmental model of sea state variables of interest; (b) estimate the extreme value for the governing variable for the prescribed return period, and associated values for other variables (for example, 100-year H_S and conditional median T_P); and (c) develop a contour line from the joint model or scatter diagram, as the contour of constant probability density going through the parameter combination mentioned above.

Using the environmental contour, an estimate of the extreme response is obtained by searching along the contour for the condition giving maximum characteristic extreme response. The contour method is affected by uncertainties related to metocean data and adopted joint models and has its own limitations which are pointed out by DNVGL-RP-C205 (2017) and NORSOK N-003 (2017). Naive application will tend to underestimate extreme response levels because it neglects short term variability of response between different realisations of sea states. Both standards recommend approaches based on Winterstein and Engbreten (1998) to account for this, including (a) increasing the return period corresponding to the contour, and hence ‘inflating’ the environmental contours; (b) replacing the stochastic response by a fixed fractile level higher than the median value; or (c) applying multipliers of the median extreme response estimates, to introduce more conservatism.

In the coastal engineering community, contours of joint exceedance probability of environmental variables are estimated using the JPC

method (e.g. Hames et al., 2019) (a) to find design events that form the boundary conditions for numerical and physical models for the purposes of structural design; and (b) to estimate return values of overtopping and overflow rates corresponding to some return period for use in flood mapping and risk analysis (CIRIA, 1996, 2007). A series of combinations of values of environmental variables from the contour are tested in order to find the worst case value of the response. This worst case value is then assumed to have the same return period as the return period associated with the environmental contour. Since again, without further assumptions, there is no link between environmental contour and structural response, there are obvious short-comings to this approach, which are well recognised (e.g. Gouldby et al., 2017).

The performance of different environmental contour methods has been investigated in several studies, including work by some of the current authors (including Jonathan et al., 2014a; Vanem and Bitner-Gregersen, 2015; Gouldby et al., 2017; Vanem, 2017). After consideration of the fundamental mathematical differences between different contour methods, it is unreasonable in general to expect to find any consistent trends in comparisons of contour methods across different applications. The characteristics of different environmental contour methods must be assessed on an application-by-application basis. The work of Haselsteiner et al. (2017b) provides a brief but clear review of the fundamental differences between different approaches to environmental contour estimation.

1.3. End-user survey

As part of the ECSADES research project (see Section 7), a modest survey was conducted on end-user practice in the use of environmental contours. The survey, receiving 19 respondents from industry and academia, consisted of questions aimed at establishing the following: (a) frequency of use of environmental contours in structural reliability and design; (b) variety in methods used to define the contour and popular sources of guidance and literature; (c) variety in application of contours (how is information from the contour used?); and (d) perceived advantages and disadvantages of using environmental contours.

Further details of the survey can be found in Appendix A.1. Two key insights emerged from the survey. Firstly, though respondents cited varying frequency of application of contours, they did appear to agree on contours forming an integral part of reliability assessment and design. Respondents cited contours as an ‘industry-accepted approach to approximating T -year responses quickly’ (when compared to long-term time-domain methods), especially when there is little or no knowledge about the structure being designed - the same environmental contour potentially being applicable to a range of responses. Secondly, respondents cited that there is no single standard approach to defining the contour nor to applying it in the estimation of responses. Seven key sources of guidelines were cited, part of a collection of over fifty relevant papers collected by the authors of this paper. Further, respondents expressed concern over a lack of understanding of the meaning of the contours and the risks associated with naive application of statistical methods leading to physically unreasonable contours. The level of interest in offshore applications is greater in Norway than other locations, although coastal practitioners use contours widely.

These insights highlight the need for clarity both on the modelling choices available when defining contours, and on the applicability of contours given the information available for a given structure and environment. We have attempted to address the majority of comments emerging from the survey in this paper.

1.4. Objective and outline

The objective of this article is to (a) overview the statistical ideas underpinning environmental contour methods, (b) highlight fundamental differences between methods, (c) explain the link between environmental contour and structural failure probability claimed by

some approaches, (d) provide simple software to allow a metocean practitioner to estimate a sensible model for general settings (based on extreme value analysis of a historical sample from the environment), and (e) provide basic guidance regarding when and how environmental contour methods should be used sensibly.

The layout of the article is as follows. Section 2 discusses fundamental issues regarding the meaning and definition of return values in a multivariate setting, and lack of invariance of a probability density function under transformation of variables. It also discusses the procedure typically used to estimate the distribution of so-called ‘long-term’ statistics (such as the T -year maximum response) from ‘short-term’ statistics (such as the distribution of maximum response in a 3-h sea state). Section 3 provides a description of different families of models for the joint distribution of environmental variables. Section 4 outlines the different kinds of environmental contours discussed in the literature. It outlines a rationale to relate the characteristics of an environmental contour with a structural failure surface for some response. Section 4 also describes an approach to modify environmental contours to account for the short-term variability of maximum response in a 3-h sea state. Section 5 provides a discussion of case studies used to illustrate the competing characteristics of different environmental contour methods, and the challenges of linking contour with response. Section 6 provides discussion, and a concluding protocol to aid the practitioner in deciding when and how to apply environmental contour approaches reasonably.

2. Return values, transformation of variables and long-term statistics

In this section, we start by defining a univariate return value, and considering the problem of extending this concept to the multivariate case. We illustrate the sensitivity of the probability density function for the joint distribution of environmental variables to transformation of variables. We also describe how the characteristics of a response for a sea state (‘short-term’) can be used to estimate its characteristics over an extended period of time (‘long-term’).

2.1. Univariate return values

Estimation of extremes for a single variable X is relatively straightforward and has been studied extensively (e.g. Coles, 2001). Given a representative set of independent observations of X spanning many years, extreme value analysis can be used to estimate the distribution $F_{M_{X,1}}$ of the annual maximum $M_{X,1}$. This in turn can be used to estimate the T -year return value x_T by solving the equation

$$\Pr(M_{X,1} > x_T) = 1 - F_{M_{X,1}}(x_T) = 1/T.$$

The T -year return value x_T for a single variable is therefore well-defined in terms of the tail of the cumulative distribution function $F_{M_{X,1}}$ of the annual maximum $M_{X,1}$ of X .

2.2. Multivariate return values

Unfortunately, the joint return value for two or more variables (X, Y) cannot be uniquely defined (e.g. Serinaldi, 2015). For example in the (X, Y) case, we could define the return value (x_T, y_T) in terms of the joint distribution $F_{M_{X,1}, M_{Y,1}}$ of annual maxima $M_{X,1}, M_{Y,1}$ using

$$\Pr(M_{X,1} > x_T, M_{Y,1} > y_T) = 1 - F_{M_{X,1}, M_{Y,1}}(x_T, y_T) = 1/T.$$

However, it is immediately apparent that there is no unique solution to this equation; given any pair (x_T, y_T) which satisfies the equation, we can increase x_T slightly, and reduce y_T such that the equation is still satisfied. That is, there is a continuum of solutions, which we can write as the set $\{x_T(\theta), y_T(\theta)\}$ indexed by parameter $\theta \in \mathcal{C}$, for some set \mathcal{C} . As we vary θ , the pair $(x_T(\theta), y_T(\theta))$ maps out a contour (corresponding to $\{x_T(\theta), y_T(\theta)\}$) of constant exceedance probability $1/T$ in (X, Y) -space.

We note for clarity, that a contour refers to a line segment (sometimes a closed curve) in some space ((X, Y) -space here), on which the value of some function is constant.

We can rewrite the last equation as

$$\Pr(M_{X,1} > x_T, M_{Y,1} > y_T) = \Pr(M_{Y,1} > y_T | M_{X,1} > x_T) \Pr(M_{X,1} > x_T) = 1/T \quad ,$$

motivating a definition of return value for the pair in terms of a marginal return value of $M_{X,1}$ and a conditional return value of $M_{Y,1}$ given $M_{X,1}$. Two boundary cases exist: (a) when $M_{X,1}$ and $M_{Y,1}$ are perfectly correlated, and (b) when $M_{X,1}$ and $M_{Y,1}$ are independent. In case (a), points (x_T, y_T) on the solution contour would satisfy $\Pr(M_{X,1} > x_T) = 1/T$ since $\Pr(M_{Y,1} > y_T | M_{X,1} > x_T) = 1$. In case (b), solutions (x_T, y_T) would satisfy $\Pr(M_{Y,1} > y_T) \Pr(M_{X,1} > x_T) = 1/T$ since now $\Pr(M_{Y,1} > y_T | M_{X,1} > x_T) = \Pr(M_{Y,1} > y_T)$. In general, the extent of dependence between pair $M_{X,1}$ and $M_{Y,1}$ will be somewhere between perfect dependence (a) and independence (b).

To illustrate the importance of dependence in practice, we could for example choose to use the 100-year maximum wave height, wind speed and current speed to estimate the environmental loading with a return period of 100 years. If winds, waves and currents are perfectly correlated, the probability of this combination of variables occurring would be 10^{-2} per annum as required. But if the variables were independent, the probability of this combination of variables occurring would be 10^{-6} per annum, considerably less than 10^{-2} . Some design codes and guidelines suggest taking the 100-year return period of one (dominant) variable together with the values of associated variables corresponding to shorter return periods to accommodate dependence between variables (e.g. Feld et al., 2019). The DNV-GL recommended practice for on-bottom stability of pipelines suggests the combination of the 100-year return condition for waves combined with the 10-year return condition for current or vice-versa, when detailed information about the joint probability of waves and current is not available.

In summary, the concept of return value is not uniquely defined for more than one variable. In order to specify design values for more than one variable rationally, we need to understand and exploit their joint distribution. This leads naturally to consideration of joint probabilities, of environmental contours, and of structure variables. Structure variables, such as structural response to environmental loading, capture the important joint characteristics of a multivariate environment in terms of a single ‘structure’ or response variable (see Section 4).

2.2.1. Transformation of variables

Suppose we describe the environment in terms of variables (X, Y) and a distribution with joint density $f_{X,Y}(x, y)$. Suppose further we choose also to describe the environment in terms of transformed variables $(a(X), b(Y))$. Then the corresponding density $f_{a(X),b(Y)}(a(x), b(y))$ is given by

$$\begin{aligned} f_{a(X),b(Y)}(a(x), b(y)) &= f_{X,Y}(x, y) \left| \frac{\partial(a(x), b(y))}{\partial(x, y)} \right|^{-1} \\ &\neq f_{X,Y}(x, y) \text{ in general,} \end{aligned}$$

because of the influence of the final Jacobian term on the right hand side of the first line. As a result, if the set of values $\{x(\theta), y(\theta)\}$ (for $\theta \in \mathcal{C}$, say) yields constant density in (X, Y) -space, the transformed set $\{a(x(\theta)), b(y(\theta))\}$ will not do so in $(a(X), b(Y))$ -space (unless the absolute value of the Jacobian is unity). Thus, contours of constant probability density on one scale will not be so on a different scale. However, contours defined in terms of cumulative distribution functions are invariant to monotonic transformations of variables: for example, a bivariate contour estimated in terms of X and Y would be equivalent to that estimated in terms of $a(X)$ and $b(Y)$.

2.2.2. Distribution of the T -year maximum response

In a storm-based analysis, we can formally evaluate the distribution

$F_{M_{R,T}}$ of the T -year maximum response $M_{R,T}$ (in any 3-h sea state, e.g. Jonathan and Ewans, 2007; Jonathan et al., 2008) using

$$F_{M_{R,T}}(r) = \exp[-\lambda T(1 - F_R(r))]$$

where λ is the expected number of storms per annum, and $F_R(r)$ is the distribution of maximum response R in a random storm, given by

$$F_R(r) = \int_s \left\{ \int_{x_1} \int_{x_2} \dots \int_{x_s} [F_{R|\{X_i\}}(r|\{x_i\}) f_{\{X_i\}|s}(\{x_i\}|s)] dx_1 dx_2 \dots dx_s \right\} f_S(s) ds$$

where $f_{\{X_i\}|s}(\{x_i\}|s)$ is the joint density of sea state variables for a storm of s sea states, and $f_S(s)$ is the density of the number of sea states in a storm. $F_{R|\{X_i\}}(r|\{x_i\})$ is the distribution of maximum response in a storm consisting of s sea states with variables $\{X_i\}_{i=1}^s$, which can be written as

$$F_{R|\{X_i\}}(r|\{x_i\}) = \prod_{i=1}^s F_{R|X_i}(r|x_i)$$

where $F_{R|X}(r|x)$ is the distribution of maximum response in a sea state with variables X . If we have access to all of the distributions above, we can use Monte Carlo simulation, numerical integration, importance sampling or similar to estimate $F_{M_{R,T}}$. Were this calculation to be feasible, we could directly estimate the joint distribution $F_{X|M_{R,T}}$ of the environmental variables given the occurrence of a T -year maximum response (in a 3-h sea state). These values are often called ‘associated values’ for the environmental variables given the T -year response, and could themselves be represented as environmental contours using the methods of Section 4.4-4.5. However, this calculation can be computationally complex, for example when the evaluation of $F_{R|X}$ is demanding, potentially involving time-domain simulation of environment-structure interaction, finite element analysis, etc.; other approximate approaches, including those exploiting environmental contours, are then appealing, as explained in Section 5.

The discussion here illustrates the estimation of F_R using a storm-based approach; naturally, estimation of F_R using other approaches is also possible. In this work, we consider response R to be any output of interest of the ocean environment, potentially including variables such as maximum individual wave height in a storm, structural loading (e.g. force) from an individual event or over some period, and structural responses (e.g. base shear, heave) to that loading.

In the coastal engineering literature, discussion tends to be in terms of ‘risk-based’ estimation as opposed to estimation of ‘long term response’, since key concerns are the annual probability of failure or expected annual damage, over a particular epoch (for example, ‘present day’ or ‘2050’).

3. Modelling the joint distribution of environmental variables

Given a sample $\{x_i\}_{i=1}^n$ with $x_i = \{x_{i1}, x_{i2}, \dots, x_{im}\}$ from the joint distribution of m environmental variables $X = (X_1, X_2, \dots, X_m)$, a number of different models for the joint distribution have been reported in the literature. Models can be categorised as being parametric (adopting a functional form for the density of the joint distribution) or non-parametric (typically using kernels for density estimation).

3.1. Non-parametric models

The simplest form of non-parametric density estimation is kernel density estimation. The joint probability density function $f_X(x)$ of X evaluated at x takes the form

$$f_X(x) = \sum_{i=1}^{n_k} k(x; x_i, \mathcal{P})$$

for n_k kernel functions k with common parameters \mathcal{P} centred at each of

$\{\mathbf{x}_i\}_{i=1}^{n_k}$ such that for any \mathbf{x}'

$$\int_{\mathbf{x}} k(\mathbf{x}; \mathbf{x}', \mathcal{P}) d\mathbf{x} = 1$$

A typical kernel choice might be the multivariate normal density $\varphi(\mathbf{x}; \mathbf{x}', \Sigma)$ with mean \mathbf{x}' and covariance matrix Σ . Some of the parameters \mathcal{P} can be set prior to estimation, and the remainder estimated by maximum likelihood estimation. For example, in the case of the multivariate normal density, we might set $\Sigma = h^2 I_m$ (where I_m is the $m \times m$ identity matrix) so that the model-fitting problem is reduced to estimating a single kernel width parameter h .

Kernel density models are suitable in general for the description of the body of a distribution, and the choice of kernel parameters \mathcal{P} tends not to be too critical to estimate central characteristics. Applications of kernel density methods to environmental contour estimation have been reported (e.g. Eckert-Gallup and Martin, 2016; Haselsteiner et al., 2017a). However, great care must be taken when using kernel density models to describe tails of distributions, since the tail (away from locations of observations $\{\mathbf{x}_i\}_{i=1}^n$) is strongly influenced by the choice of kernel k and kernel parameters \mathcal{P} . Extreme value analysis should be the approach of choice for describing extreme values in one or more dimensions.

3.2. Copula models

Consider a two-dimensional (H_S, T_P) environment. We might try to describe the joint density of H_S and T_P as the product of marginal densities $f_{H_S}(h)$, $f_{T_P}(t)$ for H_S and T_P , and a function $\rho_2(F_{H_S}(h), F_{T_P}(t))$ describing their dependence

$$f_{H_S, T_P}(h, t) = f_{H_S}(h) f_{T_P}(t) \rho_2(F_{H_S}(h), F_{T_P}(t))$$

where $F_{H_S}(h)$ and $F_{T_P}(t)$ are marginal cumulative distribution functions for H_S and T_P . ρ_2 is the probability density function of a two-dimensional copula, a multivariate probability distribution for which the marginal probability distribution of each variable is uniform. Copula models are useful since they focus on describing the dependence structure between random variables. Before estimating the copula model, we fit marginal distributions to H_S and T_P ; tails of marginal distributions can be estimated using extreme value models. In general for set $X = \{X_1, X_2, \dots, X_m\}$ of environmental variables, the copula density takes the form

$$f_X(\mathbf{x}) = \left[\prod_{j=1}^m f_{X_j}(x_j) \right] \rho_m(F_{X_1}(x_1), F_{X_1}(x_2), \dots, F_{X_m}(x_m))$$

where ρ_m is the density of an m -dimensional copula. There is a huge literature on copulas (e.g. Nelsen (2006); Jaworski et al., 2010), and there are many families of copulas (including the Gaussian and Archimedean), and some (so-called ‘max-stable’ or inverted max-stable copulas) more suited to the descriptions of extreme environments. Durante and Salvadori (2009) illustrate the specification and estimation of multivariate extreme value models using copulas. Gudendorf and Segers (2010) discusses the special class of extreme value (or max-stable) copulas appropriate for describing joint tails of distributions of component-wise maxima. Ribatet and Mohammed (2012) provides a review of extreme value copulas and their relationship to max-stable processes. Copula methods have received some attention in the ocean engineering literature. Fazerer-Ferradosa et al. (2018) discuss the use of copulas in metocean design. Montes-Iturrizaga and Heredia-Zavoni (2015, 2016, 2017) discuss the estimation of environmental contours using copula methods. Bender et al. (2014) and Serafin and Ruggiero (2014) propose bivariate extreme value models incorporating non-stationary marginal and dependence inference. Asymmetric copula models were found to be necessary to model H_S , T_Z by Vanem (2016).

3.3. Hierarchical conditional models

In a hierarchical model, the structure of the dependence between environmental parameters takes on a particularly advantageous form. Again consider the case of H_S and T_P ; the joint density $f_{H_S, T_P}(h, t)$ can be written in the form

$$f_{H_S, T_P}(h, t) = f_{T_P|H_S}(t|h) f_{H_S}(h).$$

It is always possible to factorise the joint density into the product of densities. For H_S and T_P , because of their physical characteristics, the densities $f_{T_P|H_S}$ and f_{H_S} assume relatively simple forms: a Weibull distribution for H_S has been used for many years, and a log-normal distribution for $T_P|H_S$ (e.g. Bitner-Gregersen and Haver (1989); Mathisen and Bitner-Gregersen (1990); Haver and Winterstein 2009). These are combined to estimate the joint density f_{H_S, T_P} . More generally, for example in the case of three environmental variables $\{X_1, X_2, X_3\}$, it is always possible to factorise the joint density as

$$f_{X_1, X_2, X_3}(x_1, x_2, x_3) = f_{X_3|X_1, X_2}(x_3|x_1, x_2) f_{X_2|X_1}(x_2|x_1) f_{X_1}(x_1)$$

with equivalent factorisations for permutations of the three variables. Estimating the joint distribution on the left hand side therefore reduces to estimating all of the distributions on the right hand side. Depending on the statistical characteristics of $\{X_1, X_2, X_3\}$, estimating all the distributions on the right hand side may be more straightforward to achieve in practice, in which case the factorisation is useful. Specifying a physically-realistic and useful conditional structure for m variables $\{X_1, X_2, \dots, X_m\}$ becomes increasingly problematic as m increases; Bitner-Gregersen and Haver (1989, 1991), Francois et al. (2007), Bitner-Gregersen (2015) provide examples with different levels of complexity of dependence structure. The conditional structure can often be usefully expressed as a graphical model (e.g. Jordan, 2004; Barber, 2012).

Once a useful conditional structure is established, we need to estimate the densities involved. In general, different functional forms are considered based on inspection of the data. For the tail of the distribution of a random variable, a standard tail distribution (for example, Weibull, Gumbel, Frechet, generalised Pareto, generalised extreme value) would seem to be a reasonable choice. Choice of a suitable generic density for conditional densities for (say) $X_2|X_1$ or $X_3|X_2, X_1$ is less obvious. Bitner-Gregersen and Haver (1989, 1991) propose a two-parameter Weibull distribution to describe the conditional distribution of wind speed given H_S . Horn et al. (2018) applies the approach to specify the joint distribution of a set of approximately ten environmental variables. Ideas from hierarchical graphical and copula models can be combined, as illustrated by Yu et al. (2014) in a metocean context.

3.4. Conditional extremes model

The conditional extremes model is motivated by the existence of an asymptotic form for the limiting conditional distribution of one or more conditioned random variables given a large value of a conditioning variable. The model is applicable to a large class of distributions (e.g. Heffernan and Tawn, 2004; Heffernan and Resnick, 2007), on particular standard marginal scales. For a set $X = \{X_1, X_2, \dots, X_m\}$ of environmental variables, it provides a flexible framework to estimate the joint distribution of $X_{-k} = \{X_1, X_2, \dots, X_{k-1}, X_{k+1}, \dots, X_m\}$ given that X_k ($k = 1, 2, \dots, m$) is extreme in its marginal distribution. The modelling procedure proceeds in four steps: (a) marginal extreme value modelling of each of $\{X_j\}$ independently, followed by (b) marginal transformation of each of $\{X_j\}$ independently to the corresponding variable in $\{\tilde{X}_j\}$ with standard Laplace marginal distribution, (c) dependence modelling of $\tilde{X}_{-k}|\tilde{X}_k > \psi$ for large ψ for each k , and (d) simulation under the fitted model to estimate return values, environmental contours, etc. The conditional extremes model with parameters $\alpha_{-k} \in [-1, 1]^{m-1}$ and

$\beta_{-k} \in (-\infty, 1]^{m-1}$ is given by

$$\tilde{\mathbf{X}}_{-k} | \{\tilde{\mathbf{X}}_k = \tilde{\mathbf{x}}_k\} = \boldsymbol{\alpha}_{-k} \tilde{\mathbf{x}}_k + \tilde{\mathbf{x}}_k^{\beta_{-k}} \mathbf{W}_{-k} \text{ for } \tilde{\mathbf{x}}_k > \psi, \text{ for each } k$$

where the equation is to be understood element-wise. Here, \mathbf{W}_{-k} is a residual process assumed to be distributed as $\mathbf{W}_{-k} \sim \text{MVN}(\boldsymbol{\mu}_{-k}, \text{diag}(\boldsymbol{\zeta}_{-k}))$, a multivariate normal distribution with mean $\boldsymbol{\mu}_{-k}$ and variance $\boldsymbol{\zeta}_{-k}$ (elements of which are positive) for model estimation only. Threshold $\psi \in \mathbb{R}$ is defined as the quantile of the standard Laplace distribution with appropriately high non-exceedance probability $\kappa \in (0, 1)$. Keef et al. (2013) provide additional constraints on the parameters of the conditional extremes model. An outline of the approach is given by Jonathan et al. (2010) in application to wave spectral characteristics.

Extensions incorporating covariates (Jonathan et al., 2014b), to conditioning on multiple locations (Papastathopoulos et al., 2017), to modelling the evolution of time-series (Winter and Tawn, 2016; Tendi et al., 2019) and the spatial distribution of extremes (Shooter et al., 2019) have recently been reported. The main advantage of the conditional extremes model compared with copula or hierarchical models is that it incorporates a full class of asymptotic extremal dependence (e.g. Coles et al. (1999)), and also allows relatively straightforward extension to higher dimensions. Of course, the conditional extremes method relies on having a sufficiently large sample to be able to estimate the marginal and conditional tails adequately.

4. Estimating contours

In this section, we outline the different types of contour typically used to describe the environment (Sections 4.3-4.5), and then discuss (Section 4.6) how the contour of environmental variables can be related to structural response. First (Section 4.2), we provide brief summaries of how environmental contours might be defined (Section 4.1), and then (Section 4.2) overview some of the literature on joint modelling of the ocean environment leading to contour estimation.

4.1. Defining an environmental contour

Any closed manifold (such as a “contour line” in two dimensions or “surface” in three dimensions) in the space of environmental variables can potentially be used as an “environmental contour”. The usefulness of such a contour relates to how it describes characteristics of the space of environmental variables (and potentially structural response) of interest. In ocean engineering, we are typically interested in describing the location and extent of the joint distribution of environmental variables; for this reason, contours are usually constructed that attempt to enclose the distribution of environmental variables in a useful fashion. There are two basic approaches to achieving this. First, we might insist that each point on the contour locally excludes values of environmental variables, in some specific sense, with a given fixed probability; we consider contours defined in this way to be examples of joint exceedance contours (see Section 4.4). Otherwise, we might insist that each point on the contour corresponds to a fixed value of the joint probability density function of environmental variables on some scale; we consider contours defined in this way to be examples of isodensity contours (see Section 4.5). Some contours (e.g. IFORM, Section 4.3, on transformed normal scale) satisfy both joint exceedance and isodensity requirements. However, we could define a contour with respect to any criterion considered to provide useful insights.

It is also important to note that, regardless of how a closed contour is defined, it is always possible to associate an exceedance probability with that contour. In ocean engineering, it is common practice to require that a contour (however defined) includes values of environmental variables with given probability, or equivalently has given exceedance probability. We can therefore talk about the “ T -year contour” for given value of T . In general, it is not possible directly to relate the value of exceedance probability (or T) to the parameters (e.g. local exceedance probability

for joint exceedance contours, or density for isodensity contours) needed for contour definition. For this reason, it is usual to use an iterative approach to find values of the contour parameters which yield the required exceedance probability. IFORM provides an example of a contouring method for which it is possible to relate exceedance probability directly to the parameters used to define the contour.

4.2. Overview of literature

Joint modelling of environmental variables, and the construction of environmental contours, has a long history. Haver (1985, 1987), Bitner-Gregersen and Haver (1989, 1991) present joint models for environmental variables from which environmental contours can be estimated. Winterstein et al. (1993) introduces the IFORM method, motivated by transformation of the joint distribution of environmental variables to standard multivariate normal using the Rosenblatt transformation.

Leira (2008) provides a comparison of stochastic process models for definition of design contours. Jonathan et al. (2014a) and Huseby et al. (2015a) present methods for estimation of joint exceedance contours based on direct Monte Carlo simulation under a model for the joint distribution of environmental variables. Haselsteiner et al. (2017b) estimate highest density contours using a numerical grid to discretise the original variable space into a finite number of grid cells; each cell's probability is then estimated and used in a numerical integration scheme. Haselsteiner et al. (2017b) also provides an illuminating discussion of the characteristics of different approaches to environmental contour estimation. The approaches of Haver (1987), Jonathan et al. (2014a) and Haselsteiner et al. (2017b) seek only to find contours which describe the distribution of environmental variables. The methods of Winterstein et al. (1993) and Huseby et al. (2015a), with extra assumptions, provide a direct link between the characteristics of the environmental contour and structural failure. Vanem and Bitner-Gregersen (2015) and Vanem (2017) provide comparisons of different approaches to contour estimation.

Other literature (e.g. Tromans and Vanderschuren, 1995; Gouldby et al., 2017; Jones et al., 2018) discusses how joint models for the environment can be combined with simple models for structural responses given environment, to estimate the characteristics of response directly. As a result, the joint distribution of environmental variables corresponding to an extreme response can be estimated. Winterstein (2015) discusses incorporating the effects of direction and other sources of non-stationarity or homogeneity in design contour estimation; Vanem (2018b) outlines a simple approach to account for seasonality. More generally, it is interesting also to consider the influence of different structural responses (and modes of failure) active for a particular structure on the desired characteristics of corresponding environmental contours. For example, if there are n_R independent structural responses, and the structure is designed so that the probability of failure with respect to each response is p_F , then the overall failure probability is $n_R p_F$; yet if the responses are perfectly correlated the overall failure probability is still only p_F . It would seem reasonable in general to design to the overall failure probability, and to adjust failure probabilities for individual responses to account for dependence; this in general would also require inflation of environmental contours. We note recent developments which seek to estimate buffered environmental contours (Dahl and Huseby, 2018) which incorporate not just structural failure, but the extent of structural failure.

Because of its prevalence in ocean engineering practice, we start our overview of methods for contour estimation using the IFORM approach. Then (in Section 4.4 and Section 4.5) we describe related approaches to estimating joint exceedance and isodensity contours. Finally, we consider direct estimation of the distribution of long-term response (in Section 4.6).

4.3. IFORM contours

The IFORM method of Winterstein et al. (1993) typically assumes a hierarchical model (Section 3.3) for the joint distribution of the environmental variables. We assume that we can describe the joint distribution of variables X sufficiently well that a transformation of variables is possible, so that the joint probability distribution of the transformed variables takes on a particularly simple form. The transformation is achieved by re-expressing the set $X = \{X_1, X_2, \dots, X_m\}$ as a set of conditional random variables $\tilde{X} = \{\tilde{X}_1, \tilde{X}_2, \dots, \tilde{X}_m\}$. For example, for appropriately ordered variables we can write $\tilde{X}_1 = X_1$, $\tilde{X}_2 = X_2|X_1$ and $\tilde{X}_3 = X_3|X_1, X_2$ and so on, with cumulative distribution functions $F_{\tilde{X}_1}, F_{\tilde{X}_2}, \dots$ such that

$$F_X(x) = \prod_{j=1}^m F_{\tilde{X}_j}(x_j) \quad .$$

That is, by design, the random variables \tilde{X} are independent of each other, and can hence be transformed independently to standard Gaussian random variables $U = \{U_1, U_2, \dots, U_m\}$ via the probability integral transform

$$F_{\tilde{X}_j}(x) = \Phi(u_j) \quad \text{for } j = 1, 2, \dots, m$$

where Φ is the cumulative distribution function of the standard Gaussian distribution. Isodensity contours in U -space, with a given non-exceedance probability, can be back-transformed to the original physical space. It should be noted (Section 2.2.1) that isodensity in U -space does not correspond to isodensity in X -space however. The non-exceedance probability corresponding to the IFORM contour can be related to the probability of structural failure, given certain assumptions, as explained in Section 4.6.

Chai and Leira (2018) extend the IFORM approach to estimate inverse second-order (ISORM) contours. Lutes and Winterstein (2014) generalise IFORM to incorporate elliptical contours in U -space, considered more appropriate for analysis of dynamic systems.

4.4. Joint exceedance contours

The equations in Section 2 illustrate the key characteristic of a T -year return value: namely that it defines a design region \mathcal{A} within boundary $\{x(\theta)\}$ for $\theta \in \mathcal{C}$ of the domain over which variables are defined associated with probability $1 - 1/T$ per annum, and a complementary set with ‘exceedance’ probability $1/T$. In a multivariate setting, for a pair of variables for simplicity, the first equation of Section 2.2 shows one way to define \mathcal{A} using the joint cumulative distribution function, leading to the so-called ‘joint exceedance’ contour; that is, the probability of jointly exceeding point $(x_1(\theta), x_2(\theta))$ (on the contour) is a constant with value $1/T$. However, \mathcal{A} could be defined quite arbitrarily, provided that it corresponds to the desired non-exceedance probability. In practice, \mathcal{A} might even correspond to the union of disjoint sets; the only requirement is that the probability p per annum associated with \mathcal{A} is $1 - 1/T$.

The IFORM procedure of Section 4.3 provides a specific approach to the estimation of region \mathcal{A} and hence of joint exceedance contours. IFORM is used typically for offshore applications. Joint exceedance contours are also widely used in coastal applications (e.g. DEFRA/Environment Agency, 2003; Gouldby et al., 2017), and their limitations in terms of naive estimation of extreme responses recognised for some time.

4.4.1. Direct sampling contours (Huseby et al., 2015a)

The IFORM-method (Section 4.3) produces a contour where the probability of any convex failure region in the transformed Gaussian U -space which do not overlap with the interior of the contour is less than or equal to a given target probability. When this contour is transformed back to the environmental X -space, however, this probabilistic

interpretation is no longer valid (as explained in Section 2.2.1). The direct sampling contour (Huseby et al., 2013, 2015a; 2015b, 2017) is constructed so that it has the same probabilistic properties in the environmental space as the IFORM contour has in transformed space. This implies that the region \mathcal{A} enclosed by the direct sampling contour is always convex; that is, for any two points in \mathcal{A} , the straight line joining them would also be in \mathcal{A} .

Estimating the direct sampling contour in two-dimensions is relatively easy, based on a simulation under a model for the joint distribution of variables X_1 and X_2 (see Section 3). For probability level α , following Huseby et al. (2015a), we first find the function $C(\theta)$, the $(1 - \alpha)$ -quantile of the distribution of the projection $X_1 \cos(\theta) + X_2 \sin(\theta)$ for each value of $\theta \in [0, 2\pi)$

$$C(\theta) = \inf\{c : \Pr([X_1 \cos(\theta) + X_2 \sin(\theta)] > c) = \alpha\}.$$

Then we estimate the contour $\mathcal{C} = \{(x_1(\theta), x_2(\theta)) : \theta \in [0, 2\pi)\}$ using

$$x_1(\theta) = C(\theta) \cos(\theta) - \frac{dC}{d\theta} \sin(\theta),$$

$$x_2(\theta) = C(\theta) \sin(\theta) + \frac{dC}{d\theta} \cos(\theta),$$

and potentially further smooth \mathcal{C} as a function of θ . Following Winterstein et al. (1993), for at T -year return period, it is recommended that the value of α be set to $1/T$. Generalisation to higher dimensions is mathematically straightforward; three-dimensional contours based on the direct sampling approach are presented in Vanem (2018a).

4.4.2. Joint exceedance contours (Jonathan et al., 2014a)

Jonathan et al. (2014a) propose joint exceedance contours for which a particular probability at any point on the contour is constant. Specifically, in two-dimensions, the closed contour $\{(x_1(\theta), x_2(\theta)) : \theta \in [0, 2\pi)\}$ is defined by

$$\Pr\left(\bigcap_{j=1}^2 (r_j(\theta; r^*) X_j > r_j(\theta; r^*) x_j(\theta))\right) = \alpha$$

for $\alpha \in (0, 1)$, where $r(\theta; r^*) = \{r_1(\theta; r^*), r_2(\theta; r^*)\}$ is defined by $r(\theta; r^*) = x(\theta) - r^*$ and r^* is a reference location for the distribution under consideration. In some cases, it is appropriate that r^* refers to some central feature (for example, mean, median or mode). In other situations, when we are interested solely in the large values of a variable X_1 (say), it might be appropriate to set $r_1^* = 0$. We estimate the contour using simulation under a model for the joint distribution of X_1, X_2 ; but we might choose to estimate the contour for any transformation of variables, in particular to independent standard normals U_1, U_2 . The probability p per annum associated with region \mathcal{A} enclosed by the contour is a (generally unknown) function of α . The value of p can be adjusted by iteration over different choices of α . Again, we can potentially further smooth the contour as a function of θ .

4.5. Isodensity contours

Another obvious approach would be to define contours using the joint probability density function f_X of the environmental variables instead of its cumulative distribution function F_X . If we assume to start that f_X is uni-modal, we might choose a value τ such that

$$f_X(x_\tau) = \tau$$

defines a closed contour $\{x_\tau(\theta)\}$ for $\theta \in \mathcal{C}$ enclosing a set \mathcal{A} in X -space such that $f_X > \tau$ within \mathcal{A} . This defines an ‘isodensity’ contour, or a contour of constant probability density (e.g. Haver, 1985; Haver, 1987, DNVGL-RP-C205, 2017). The approach can be extended to include multi-modal f_X (e.g. using kernel density estimation, Haselsteiner et al., 2017b).

Specification of τ is typically made such that the environmental

contour encloses the appropriate probability (see Section 4.1), so that the isodensity contour can be associated with a desired exceedance probability. For isodensity contours in particular, the value of τ can be set iteratively, such that the desired exceedance probability for the contour is achieved. Sometimes it is useful to consider specification of τ in terms of a decomposition of f_X into the product of marginal, conditional and/or dependence terms.

As discussed in Section 3.1, kernel density estimation is a useful tool for estimating the central features of a probability density function. It can therefore be used for estimation of isodensity contours, but this choice is problematic for estimating the tails of distributions since the relatively arbitrary choice of kernel function and its width tend to dominate the shape of the tail; we would prefer that the shape of the tail was informed more directly by the data.

As stated in Section 2.2.1, a probability density function is not invariant to non-linear transformation of variables. For this reason, isodensity contours estimated using different transformations of variables result in different environmental contours. There are examples in the literature of many different versions of isodensity contours: (a) the contour methods described by [Haver \(1985\)](#), [Haselsteiner et al. \(2017b\)](#) and [DNVGL-RP-C205 \(2017\)](#), all based on constant probability density in physical space, and (b) the contour method of [Winterstein et al. \(1993\)](#), based on constant probability density in a transformed standard normal space. In conjunction with other method differences present, including specification of τ , this leads to a number of variants of isodensity contours.

4.6. Relating environmental contours to response

If the objective of a study is to estimate environmental conditions corresponding to extreme structural responses, the obvious approach is direct estimation of the characteristics of the T -year maximum response and the environments which generate it. In contrast to Section 4.4-4.5, the purpose of this analysis is not characterisation of extreme environments, but rather of environments related to extreme responses. Response-based methods obviously require at least some information about the response function. Of course, if structural response is monotonically related to a dominant environmental driver variable, then the resulting contours may be quite similar; but this is not always the case. A number of different approaches relying on some knowledge of the response have been developed and applied over the past thirty years (e.g. [Coles and Tawn \(1990\)](#); [Tawn \(1992\)](#); [Winterstein et al. \(1993\)](#); [Tromans and Vanderschuren \(1995\)](#); [Hawkes et al. \(2002\)](#)).

4.6.1. Approximating the response

When evaluation of $R|X$ is demanding (see Section 2.2.2), an alternative approach is to adopt a simple approximation to the response function which can be easily evaluated. Sometimes, the functional form is relatively apparent from physical considerations (for example, the semi-empirical Morison equation ([Morison et al., 1950](#)) for the drag and inertial forces on a body); in this case, it is usually necessary to set the parameters of the response function to correspond with the structure of interest. Nevertheless, once set, structural loads can be estimated quickly for given environment x . This is the basis of, for example, the generic load model of [Tromans and Vanderschuren \(1995\)](#), and the response-based joint probability model in coastal applications (e.g. [Gouldby et al., 2017](#)). More generally, a statistical model (known as an ‘emulator’) can be used to estimate $R|X$. The emulator is estimated by evaluating $R\{X=x\}$ for points x drawn from a set of representative environments X (which itself can be a computationally demanding analysis), and then fitting a statistical model such as a response surface to explain response in terms of the environmental variables. Once estimated, the emulator provides rapid evaluation of $R\{X=x\}$ for any x , and hence of associated environmental values corresponding to the T -year maximum response (in Section 2.2.2). It is a natural framework for error propagation and uncertainty quantification.

4.6.2. Sampling from environmental contours

The direct estimation of the distribution $F_{M_{R,T}}$ of the T -year maximum response $M_{R,T}$, and hence of the joint distribution $F_{X|M_{R,T}}$ of environmental variables associated with it is often computationally complex. In such cases, reducing the number of evaluations of $R|X$ made is advantageous. It is intuitive therefore that we should focus on values x of environmental variables (to evaluate $R\{X=x\}$) corresponding to extreme environments to achieve this; the T -year environmental contour provides one approach to identifying those environments. We can also associate the T -year environmental contour with the probability of structural failure in the same period, as described in Section 1.2 and outlined here.

4.6.2.1. Reliability theory. Engineering codes stipulate that marine structures should be designed to exceed specific levels of reliability, usually expressed in terms of an annual probability of failure. Reliability theory provides an approach to estimating required structural strength and environmental design conditions causing failure. Structural failure is assumed to occur when structural loads R exceed structural resistance or strength S , expressed in terms of the equation

$$S - R = g_X(X) < 0,$$

where $g_X(X) = 0$ is a limit state expression for a particular failure mechanism (or ‘failure surface’ for brevity) and the vector X represents all of environmental, hydraulic loading and structural variables appropriate to a particular problem, with joint probability density function $f_X(x)$. In the context of floating structures and ships, R might correspond to a motion response of the vessel (such as roll) and S to a limiting value for roll at which structural integrity is deemed impaired. The probability p_F of structural failure can then be evaluated using

$$p_F = \Pr(g_X(X) < 0) = \int_{g_X(x) < 0} f_X(x) dx.$$

For a given environment, p_F can clearly be reduced by increasing S , since then the region of x space for which $S - R < 0$ is reduced. In this way, structural strength can be adjusted to achieve desired p_F . Solving this equation however presents multiple challenges of first specifying g_X and f_X adequately, and then performing the computation reasonably. We note that the form of g_X is in general quite arbitrary, and that estimating g adequately for a full-scale structure is likely to be problematic. If we suspect that extreme environments X produce extreme responses R , adequate characterisation of the joint tails of f_X will be necessary to estimate p_F well, requiring careful multivariate extreme value analysis of the environment. However, for a resonant frequency response of a floating structure, estimating some tail aspects may be less critical. Other responses such as fatigue are not governed by extremes of the environment in general.

4.6.2.2. Linearising the failure surface. Due to the complexity of solving the equation above, approximate approaches have been sought, including the use of environmental contours. For example, IFORM typically adopts a hierarchical model outlined in Section 4.3 to estimate f_X and hence contours of constant probability density in the transformed Gaussian U -space, with given probabilities of non-exceedance. The direct sampling method of Section 4.4.1 generates contours in the original X -space of environmental variables with given non-exceedance probability.

Solving the equation is still not easy, since failure surface g_X is unknown. To overcome this, the direct sampling and IFORM methods make the assumption that a linear approximation to the failure surface at the design point (see Section 4.7 below) is appropriate. This approximation is made on the original X -scale for direct sampling, and on the transformed U -scale for IFORM; this is the key difference between the methods. There is no a-priori physical reason for assuming that linearisation of the failure surface is appropriate, and the assumption must be

justified on engineering grounds on a case-by-case basis. In certain applications, for example of wave loads on fixed structures, extreme loads typically correspond to severe sea states; in this situation, we might assume load to be dominated by significant wave height H_S . It is then probably reasonable to assume that the set x of values (including H_S) such that $g_X(x) < 0$ (and the corresponding set u of values such that $g_U(u) < 0$) is convex. In this case, assuming g_X (or g_U) to be linear leads to a conservative overestimate of the probability of failure associated with a given contour. However, we emphasise that there is no guarantee that either g_X or g_U is convex in general. Hence there is no guarantee that linearising the failure surface is reasonable, and that the probability of failure will be smaller than that associated with the environmental contour.

4.6.2.3. Quadratic failure surface. In IFORM, the failure surface $g_U = 0$ in U -space is approximated using a tangent plane at the design point, which can be expressed as a linear combination of variables. [Chai and Leira \(2018\)](#) propose replacing this approximation with a (hyper-)sphere centred at the origin of U -space passing through the design point. This spherical failure surface corresponds to a specific simple second-order failure boundary (expressible as a combination of linear and squared terms in the variables). The region outside the sphere is then the failure region. The ISORM approach therefore provides a more conservative approximation than that of IFORM.

4.6.2.4. Finding governing conditions. In a typical analysis (e.g. using IFORM), once the environmental contour $\{u(\theta)\}$ (the surface of a hypersphere, for $\theta \in \mathcal{C}$) is estimated in U -space, we find the point $u(\theta^*) \in \{u(\theta)\}$ corresponding to the highest value of structural failure probability p_F for given structural strength S . Then $u(\theta^*)$ is transformed back to a corresponding $x(\theta^*)$ (in terms of the original variables) which is taken as the design set corresponding to the specified failure probability. Other points of interest, for example the whole contour $\{u(\theta)\}$ in the transformed space, can be similarly transformed to the original space. In a direct sampling analysis, the point $x(\theta^*)$ can be identified directly in X -space.

4.7. Adjusting contours for model mis-specification and short-term variation

We use the T -year environmental contour for the set X to provide a computationally-fast but potentially biased estimate of the T -year response discussed in Section 4.6. A typical approach is to estimate the distribution of the maximum response (in any 3-h sea state, corresponding to a specified return period) given values of environmental variables $\{x(\theta)\}$ on the contour to identify a ‘design point’ $x(\theta^*) \in \{x(\theta)\}$ yielding the largest structural response. Then a quantile q_C with non-exceedance probability p_C (for example typically the mode, median or mean) of the distribution $F_{R|X}(r|x(\theta^*))$ is used to estimate some (possibly different) quantile q_R with non-exceedance probability p_R of the distribution $F_{M,T}(r)$ of the T -year maximum response. Quantile q_C is used as an estimate for q_R , where

$$F_{R|X}(q_C|x(\theta^*)) = p_C$$

and

$$F_{M,T}(q_R) = p_R.$$

In practice, probabilities p_C and p_R are first specified, and then used to estimate quantiles q_C and q_R by solving the two equations above. There is no guarantee that q_C and q_R will coincide. Even if $p_C = p_R$, since $F_{R|X}(r|x)$ has a long right-hand tail for any environment x , it is usually possible for ‘short-term variation’ in ‘less extreme’ sea states to contribute to the distribution $F_{M,T}$, whereas this is by definition not possible for the corresponding distribution $F_{R|X}(q_C|x(\theta^*))$ from a single sea state $x(\theta^*)$. Mis-specification of the environmental model, or

violation of assumptions concerning the relationship between environment and response, may also lead to disagreement between q_C and q_R . For this reason, it is useful to define an inflation factor Δ such that

$$q_R = \Delta q_C,$$

where we might expect $\Delta > 1$ for $p_C \approx p_R$. The factor Δ can be used to inflate the whole environmental contour if desired. Standard [DNV-RP-C205 \(2010\)](#) recommends appropriate choices of q_R , q_C and the corresponding Δ . Section 5 provides illustrations of contour adjustment for simple simulation models. It is apparent that, in situations where short-term variability is relatively large compared with long-term variability, contour-based approaches should be used with great caution.

5. Case studies: contours in practice

Section 4 outlines various forms of an environmental contour. In the absence of a unified approach to defining and applying contours (see Section 2 and Section 1.3), it is informative to consider the practicalities of environmental contour estimation. Our objective in this section is to quantify how well estimates of extreme responses (in a 3-h sea state, for a particular return period) on a contour compare with estimates obtained by direct simulation of the response. In this sense we replicate the typical approach to application of contours: looking at response for a small set of environmental conditions, in the hope that this analysis approximates the characteristics of maximum response for that return period. In doing so, we discuss the key challenges in applying contours, including: choice of contour, sampling along the contour and contour inflation. We perform our analysis for four synthetic responses, whose relationship to the environment is quantified entirely in terms of H_S and T_P , and is known to us; this of course would not be the case in actual applications. We are therefore able to simulate from the known distributions to estimate the correct characteristics of response, and hence to compare simulation-based and contour-based estimates for maximum response. Then, in Section 6, we summarise our findings regarding the estimation and application of environmental contours for metocean design, with a particular focus on the appropriate use of contours, given the extent of knowledge about the response: environmental contours are clearly useful under certain conditions, but these conditions need to be carefully defined so that the user knows when environmental contours are likely to be a good option.

For simplicity in the case studies below, we define the environment in terms of a large historical sample of sea-state H_S and T_P for a typical northern North Sea environment for the period 1979–2013, from the NORA10-WAM hindcast ([Reistad et al., 2011](#)). NORA10 (Norwegian ReAnalysis 10 km grid) is a 58-year hindcast that has been developed by the Norwegian Meteorological Institute. It is a regional HIRLAM (atmosphere) and WAM Cycle-4 (wave) hindcast covering Northern European waters. The regional model uses wind and wave boundary conditions from the ERA-40 reanalysis (1958–2002) and is extended using the ERA-Interim reanalysis from 2002 onwards. NORA10 produces three-hourly wave and wind fields at 10 km resolution. We isolate storm peak events using the procedure of [Ewans and Jonathan \(2008\)](#). We then estimate structural responses using known non-linear functions of environmental variables corresponding to each storm event.

To construct an environmental contour, we require a statistical model for the environment. Here, we achieve this by means of a conditional extremes model (Section 3.4) for the historical sample, using a penalised piecewise constant (PPC) extreme value model ([Ross et al., 2018](#)) and software (outlined in [Appendix A.2](#)). We choose the conditional extremes model because of its generality and flexibility to model different forms of extremal dependence (e.g. [Jonathan et al., 2010](#)). The PPC extreme value model allows the estimation of non-stationary marginal and conditional extremes for peaks over threshold using a simple description of non-stationarity with respect to covariates in marginal and dependence models. We use this to estimate a number of the

environmental contours discussed in Section 4 and investigate their relationship to extremes of structural response. Because of its recent popularity, we consider the direct sampling contour (Section 4.4.1) in case studies 1 and 2. In case study 2, we also consider the joint exceedance contour outlined in Section 4.4.2 and the isodensity contour (Section 4.5). To estimate any of these contour methods requires a (H_S, T_p) sample simulated under the environmental model.

Isodensity contours are inspired by the work of Haver (1985, 1987), Heffernan and Tawn (2004) and DNVGL-RP-C205 (2017) recommended practice. The specific isodensity contour used in this section is based on the conditional extremes model introduced in Section 3.4, implemented in the PPC software outlined in Appendix A.2. The procedure for estimating these contours is simple and straightforward, based on simulation under a fitted model. The procedure is consistent with the recommended practice in DNVGL-RP-C205 (2017), and could therefore be used with any model from which simulation of joint extremes is possible. Specifically, for a given fitted conditional extremes model, a sufficiently large sample of realisations is simulated on the original X -scale. The X -domain is then partitioned into a regular lattice of n_B cells $\{B_i\}$ with centres $\{b_i\}$, each of volume V_B . An estimate of the probability density at each cell centre b_i is then obtained by counting as follows

$$f_X(b_i) = \frac{1}{V_B} \frac{\#\{B_i\}}{n_B}, \text{ for } i = 1, 2, \dots, n_B,$$

where $\#\{B_i\}$ is the number of realisations in bin B_i . We note the availability of free-ware such as WDRT (REF) and ViroCon (REF) for estimation of environmental contours. We note that other software (such as WDRT, Coe et al., 2016 and ViroCon, Haselsteiner et al., 2019) for estimation of environmental contours is freely available.

5.1. Case study 1

The objective of the case study is to examine the correspondence between different estimates for the distribution of the 100-year maximum response $M_{R,100}$. We compare an estimate from direct simulation of R (taken to be accurate) and one generated from combinations of H_S and T_p on the 100-year direct sampling contour of Section 4.4.1. We make this comparison for a number of different responses.

The procedure we use is intended to reflect common practice in industry. Once the contour is estimated, we identify a ‘frontier’ interval of the contour on which the maximum response is expected to occur. In the current work, we assume that the ‘frontier’ corresponds to the whole interval of the environmental contour lying close to pairs of H_S , T_p values present in the sample. Then we estimate the distribution of maximum response in two ways: (a) using a single combination of H_S and T_p corresponding to the maximum value H_S^{\max} of H_S on the contour, and (b) using all (H_S, T_p) pairs on the whole frontier interval. Then, for approach (a), we estimate the distribution of maximum 100-year response $f_{M_{R,100}\text{Point}}(r) = f_{R|X}(r|H_S^{\max})$. For approach (b), we estimate the ensemble distribution

$$f_{M_{R,100}\text{Frontier}}(r) = \frac{1}{n_L} \sum_k f_{R|X}(r|\mathbf{x}_{100}(\theta_k))$$

where $\{\mathbf{x}_{100}(\theta_k)\}_{k=1}^L$ defines a set of equally-spaced points on the 100-year contour on the frontier interval. We can then compare quantiles of the distributions from approaches (a) and (b) with quantiles estimated from direct simulation of $M_{R,100}$. Vanem (2017) notes a trade-off between the number of points on the contour used to evaluate the response, the quality of the estimate of response and the computation time required.

A total of four responses R_1, R_2, R_3, R_4 are considered. Two responses correspond to output of a numerical simulator of maximum base shear (R_1 , for a typical fixed structure) and maximum heave (R_2 , for a floating structure), as a function of H_S and T_p for a 3-h sea state. These response

simulators assume that the most probable value of maximum response in a sea state can be written as a closed form expression in terms of a number of sea state variables, including sea state H_S and T_p . The actual value of maximum response is then simulated from a Rayleigh distribution with the most probable maximum response as scale parameter.

A further two synthetic responses are defined, which are simple deterministic functions of H_S and T_p , using the following equation

$$R_i = \frac{\alpha_i H_S}{(1 + \beta_i (T_p - T_{p0,i})^2)} \text{ for } i = 3, 4 \quad (1)$$

where $T_{p0,i}$ (in seconds) is the resonant peak period for response R_i . The values of $\{\alpha_i, \beta_i, T_{p0,i}\}$ are $\{2, 0.007, 7\}$ and $\{2, 0.005, 26\}$ for $i = 3, 4$ respectively. These combinations of parameters were chosen to provide large responses at different neighbourhoods of the environmental space, and hence to correspond to different frontier intervals. The distribution of maximum response $M_{R,100}$ for synthetic responses R_3, R_4 was estimated by generating multiple environmental simulations corresponding to periods of 100 years, calculating response per sea state and storing only the maximum response observed and the values of H_S and T_p responsible for it. For responses R_1, R_2 , joint conditional extremes models for response, H_S and T_p were estimated using PPC; simulation under these models was then again used to accumulate the distribution of $M_{R,100}$.

For each response in turn, the mean value $M_{R,100}$ of the maximum 100-year response $M_{R,100}$ is plotted in Fig. 1, and coloured by the value of $M_{R,100}$. Also plotted in the figure are direct sampling contours corresponding to 20, 30, 40, 50, 70, 100 and 200 years. Note that for each response R_i , only combinations of H_S and T_p giving rise to a least one occurrence of $M_{R_i,T}$ appear in the figure.

The figure shows typical features of the different responses. Synthetic response R_3 shows resonance effects $T_p \approx 13$ s. Maximum base shear (R_1) and synthetic response R_4 increase with increasing H_S and T_p . This is true in general for maximum heave (R_2), but there are clearly large values of $M_{R_2,100}$ within even the 20-year environmental contour. That is, there are relatively benign environmental conditions, not even exceeding the 20-year contour, which sometimes generate the 100-year maximum response. For contours to be useful, we would expect to see the largest values of 100-year maximum response lying outside the 100-year contour, and smallest values of response within it. This is approximately the case for all responses, but certainly not always true for R_2 .

The extent to which the maximum response on the 100-year environmental contour agrees with the actual distribution of $M_{R,T}$ from simulation can be assessed by comparing an estimate for the distribution of the true response against that evaluated for conditions on the contour, as illustrated in Fig. 2. It shows kernel density estimates for $\{M_{R_i,100}\}$ estimated by direct simulation (in dashed blue; which can be regarded as ‘the truth’). The figure also shows corresponding kernel density estimates for $f_{M_{R,100}\text{Frontier}}$ of $M_{R,100}$ from combinations of (H_S, T_p) lying on the contour frontier (scenario (b), shown in Fig. 1), for a range of choices of T .

There is an obvious ordering of response density estimates with increasing return period, particularly for responses R_3 and R_4 as might be expected; this effect is less clear for responses R_1 and R_2 . The location of the quantile (see Section 4.7 with $p_C = p_R$) of the distribution of maximum response with non-exceedance probability $\exp(-1)$ (henceforth the ‘ $\exp(-1)$ ’ value) of the density estimate from the 100-year environmental contour is in reasonable agreement with the location of the true $\exp(-1)$ value. Following Section 4.7, defining Δ as the ratio of the $\exp(-1)$ quantile of the true 100-year maximum response to the $\exp(-1)$ quantile of the distribution from the 100-year environmental contour, we see that the environmental contour approach underestimates the $\exp(-1)$ response by between 1% and 9% for these examples. We note, of course, that choices other than $\exp(-1)$ for the quantile level of interest could have been made.

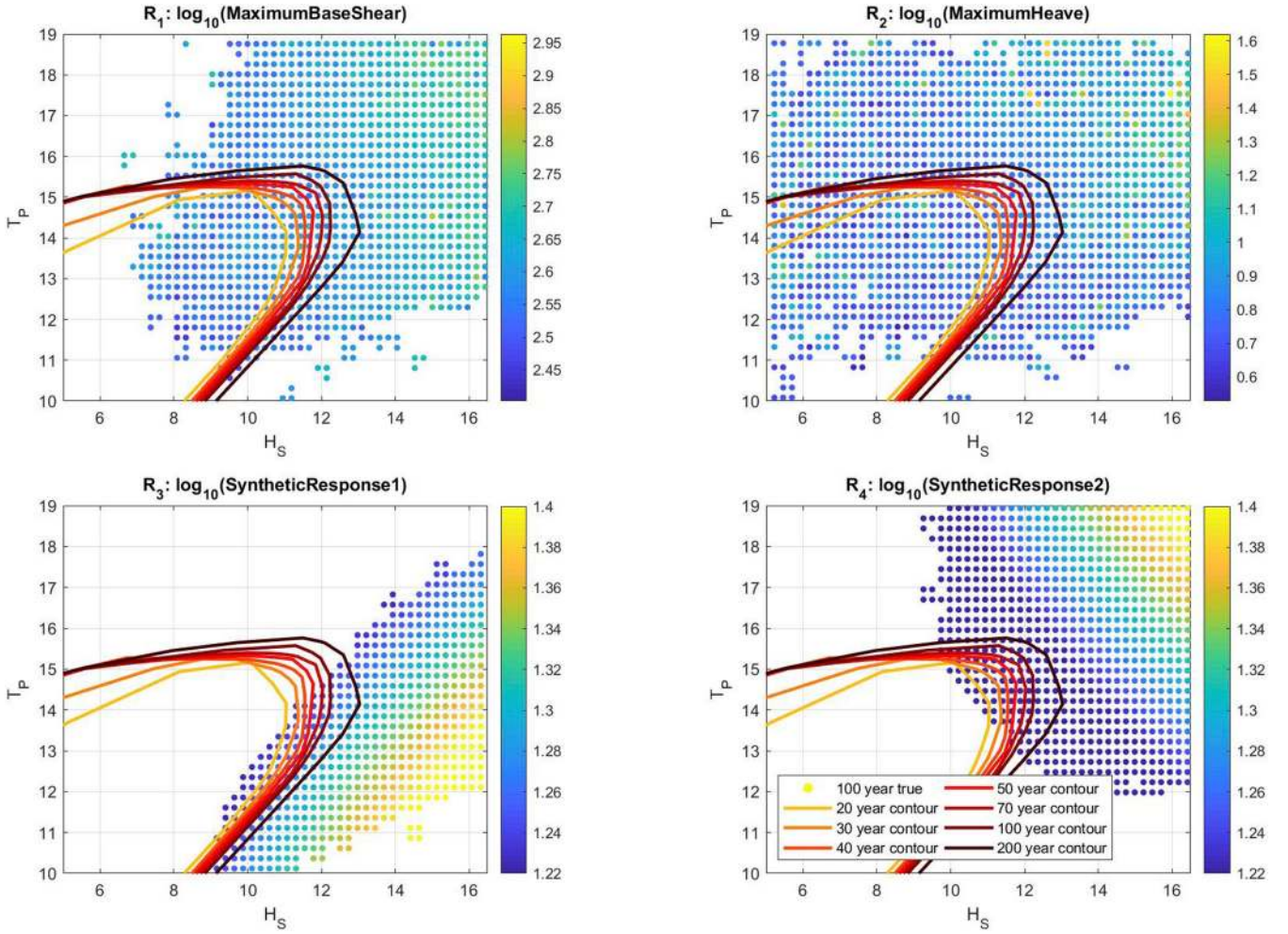


Fig. 1. Mean 100-year maximum responses $\{M_{R_i,100}\}$ as a function of H_S and T_p estimated using 1000 realisations (of length 100 years) of H_S and T_p . Points are coloured by the local mean value of maximum response estimated on a lattice of values for H_S and T_p . Also shown are T -year (H_S, T_p) direct sampling environmental contours for different values of T ; contours are coloured yellow to dark brown by return period, in order of $T = \{20, 30, 40, 50, 70, 100, 200\}$ years. Panels on top row correspond to historic responses R_1 (left) and R_2 (right); panels on bottom row correspond to synthetic responses R_3 (left) and R_4 (right). Units for variables are m (H_S , heave), s (T_p) and kN (base shear). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

We next perform a similar comparison of response distributions, this time using only (H_S, T_p) combinations near the point on the contour with maximum H_S (that is, approach (a), to estimate $f_{M_{R_i,100} \text{ Point}}$). Results, shown in Fig. 3, have similar general characteristics to those of Fig. 2. Values of Δ in the interval (0.98, 1.04) are estimated. In the current illustrations, therefore, it appears that both approaches (a) and (b) provide reasonable estimates for the $\exp(-1)$ quantile of $M_{R_i,100}$.

Case study 1 suggests that the environmental contour approach provides a reasonable estimate of the maximum response for this case. However, this estimate is subject to a number of sources of uncertainty, including for example the appropriate specification of points on the contour needed for approaches (a) and (b).

5.2. Case study 2

The end-user survey of Section 1.3 suggests that there is uncertainty about which of the many contouring methods is most appropriate for a given application. To investigate this, here we make a comparison of direct sampling contours, joint exceedance contours (Section 4.4.2), and isodensity contours (from a conditional extremes analysis in Section 3.4 and) for responses R_1 (maximum base shear) and R_2 (maximum heave). For brevity, these approaches are henceforth referred to as ‘direct sampling’, ‘joint exceedance’ and ‘isodensity’ respectively.

Fig. 4 shows minima $\{M_{R_i,100}^{\min}\}$ and maxima $\{M_{R_i,100}^{\max}\}$ values of maximum responses $\{M_{R_i,100}\}$ from the same 1000 simulations used to generate Fig. 1. The colour of each disc in the top row indicates the value of the minimum 100-year maximum response seen for that combination of H_S and T_p , using the same algorithm as for Fig. 1 to identify near neighbours. The bottom row shows corresponding values of maximum 100-year maximum response. It is clear that there is considerable variability in response for a given pair of values for H_S and T_p . 100-year environmental contours from each of the direct sampling, joint exceedance and isodensity methods are also shown in the figure. All contours have a similar frontier interval. There is good agreement between the direct sampling and joint exceedance contours in particular on the frontier interval; this is not surprising since the underlying methods have similar motivations. For the same simulation size, the isodensity contour used by us in this work is more difficult to estimate without applying considerable smoothing. Comparing the top and bottom rows of the figure, it also appears that the natural variability in the response (near the frontier interval of the contours) dominates any variability in the value of response along the contours. In this case, therefore, none of the contours is particularly preferable; any of them would give approximately the same quality of estimate for $M_{R_i,100}$.

It is interesting and intuitively appealing that the (yellow) area of largest values of maximum response (on the bottom row of the figure) is

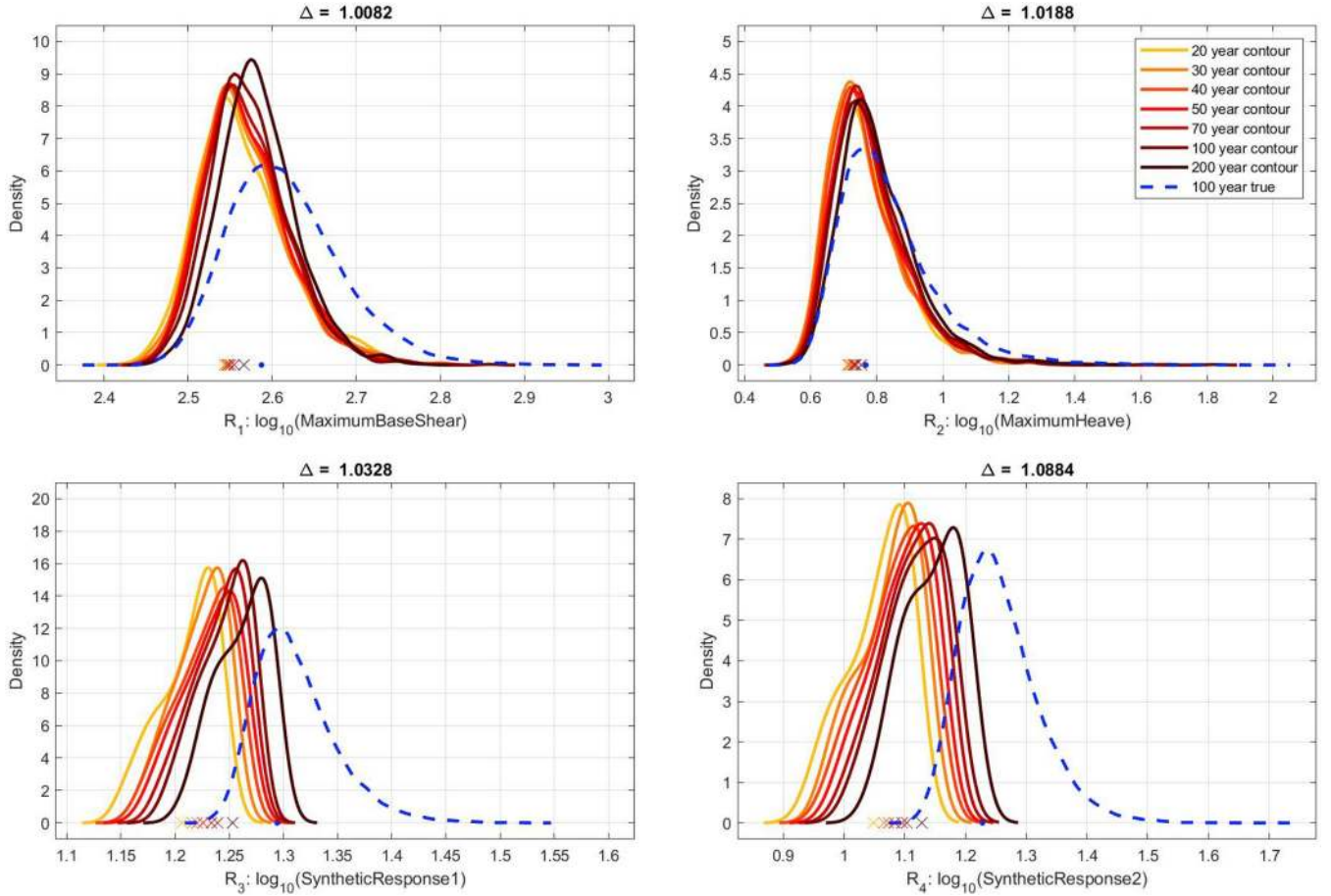


Fig. 2. Kernel density estimates for 100-year maximum responses $\{M_{R_i,100}\}$. Estimates based on direct simulation of response are shown in dashed blue. Other density estimates (solid lines) are calculated from (H_S, T_P) combinations lying near the corresponding T -year direct sampling contour shown in Fig. 1, for $T = \{20, 30, 40, 50, 70, 100, 200\}$. Coloured crosses indicate the location of the quantile of the response distribution with non-exceedance probability $\exp(-1)$ along each contour; the blue dot gives the corresponding $\exp(-1)$ ‘true’ response from the blue curve. The factor Δ by which the $\exp(-1)$ response of the 100-year environmental contour would need to be inflated to give the true $\exp(-1)$ 100-year response is given in the title to each panel. Units for variables are m (H_S , heave), s (T_P) and kN (base shear). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

centred approximately on the frontier interval of the contours. However, for synthetic response R_3 in Fig. 1, we see that the frontier interval is offset (to lower T_P) from part of the environmental contour corresponding to largest H_S : focussing on an interval on the contour corresponding to largest H_S to estimate $M_{R,100}$ would seem particularly suspect in this case, regardless of the choice of contour method.

In summary, case study 2 suggests that the key to success is ensuring that the response is quantified (using time-domain simulation or other) at a sufficient number of (H_S, T_P) combinations on or near the frontier interval of any reasonably well-defined contour.

6. Discussion and conclusions

As shown in Section 4, environmental contours provide useful characterisations of the extent of the joint distribution of environmental variables. Some contour methods assume particular parametric forms for the (conditional) distributions of environmental variables; other methods generate convex contours on particular scales; other contour approaches are only defined on part of the domain of environmental variables. There is concern in the user community that a contour should ‘look right’, closely hugging the boundaries of scatter plots of historic or simulated environmental variables.

The usual motivation for applying a contour approach in ocean engineering is to find environmental conditions efficiently (for a return period of T years say) which will generate approximately the T -year

maximum response. Environmental contours therefore provide a means of reducing the burden of running full long-term response analysis for a wide range of environmental conditions. Different types of environmental contours find favour based on their ability to estimate the T -year maximum response from the T -year environmental contour.

An environmental contour is estimated with no regard whatsoever to structural details. Since environmental contours are independent of structural specifics, they can then be used in principle to study different structures in a given environment provided that the underlying assumptions linking environment and structure are not violated.

There is no fundamental link between points on an environmental contour and structural response in general, and no reasonable expectation therefore that points on the T -year environmental contour should yield the T -year maximum response. Attempts have been made to compare different environmental contour methods using a response-based criterion, although it is mathematically obvious and generally recognised in the user community that no single approach is appropriate for all structure and response types, and that considerable ambiguity will always remain. The manner in which an environmental contour relates to extreme response depends on the specifics of the structure. However, for typical H_S -driven structures, empirical evidence suggests the responses generated from points along the environmental contour in (H_S, T_P) -space for a given return period are reasonable estimates of the actual maximum response corresponding to the same return period. In the presence of resonant response and non-extreme values of T_P , using

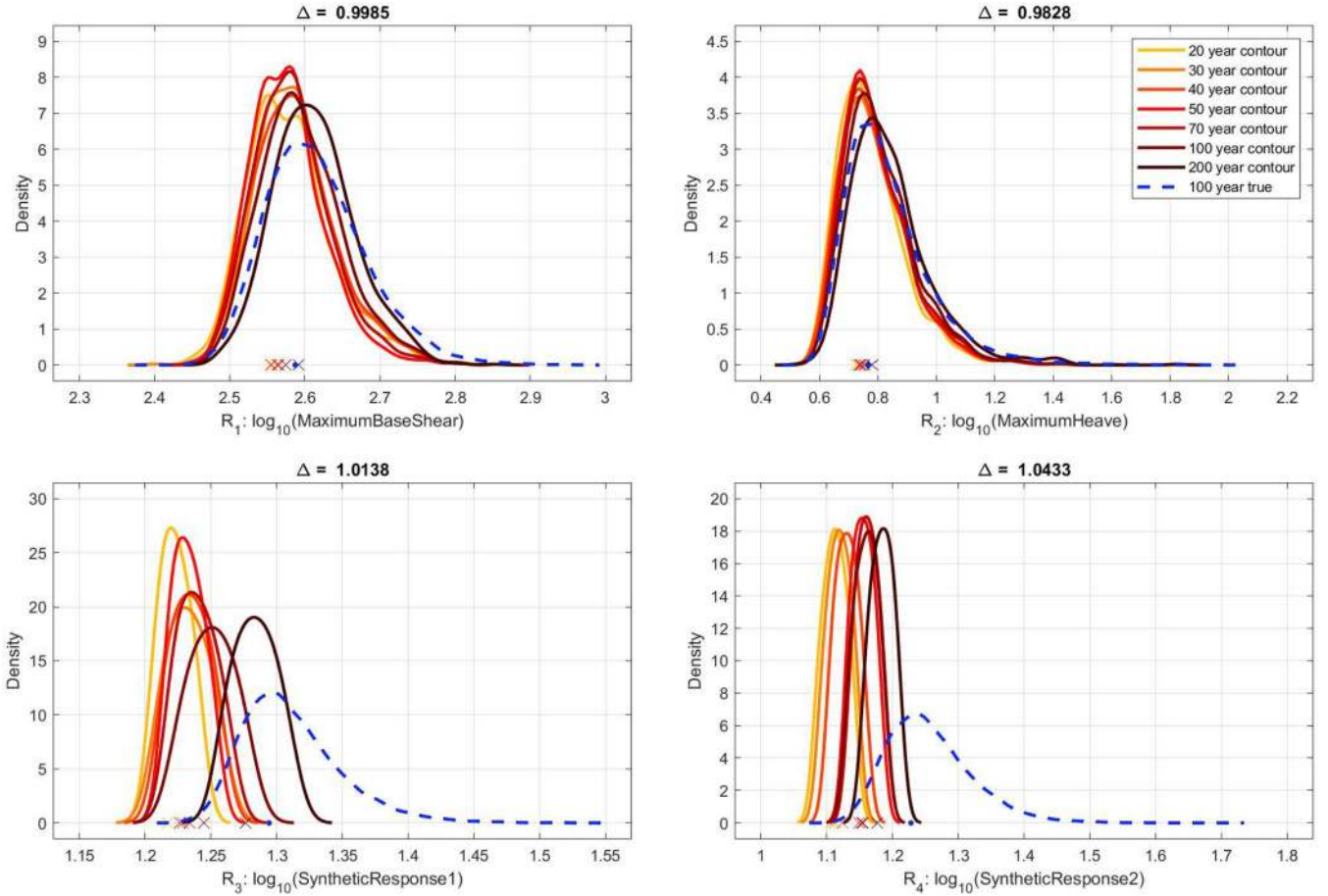


Fig. 3. Kernel density estimates for 100-year maximum responses $\{M_{R_i,100}\}$. Estimates based on direct simulation of response are shown in dashed blue. Other density estimates (solid lines) are calculated from (H_S, T_P) combinations near the point on the corresponding T -year direct sampling contour (Fig. 1) corresponding to maximum H_S , for $T = \{20, 30, 40, 50, 70, 100, 200\}$. Coloured crosses indicate the location of the quantile of the response distribution with non-exceedance probability $\exp(-1)$ along each contour; the blue dot gives the corresponding $\exp(-1)$ 'true' response from the blue curve. The factor Δ by which the $\exp(-1)$ response of the 100-year environmental contour would need to be inflated to give the true $\exp(-1)$ 100-year response is given in the title to each panel. Units for variables are m (H_S , heave), s (T_P) and kN (base shear). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

points from the contour near the maximum H_S can be misleading, since the response is not completely H_S -dominated. It is critical therefore that the dominant environmental variables are included in the estimation of environmental contour. It is apparent from physical considerations that extreme occurrences of some structural responses should not coincide with those of extreme environmental variables; the T -year environmental contour is unlikely to provide any guidance regarding the T -year maximum response for such responses. We also note methods to adjust (or inflate) contours to correct for sources of bias in estimation of extreme response, including the effects short-term variability, violation of marginal and dependence modelling assumptions, uncertainty in parameter estimates, etc.

There is some debate within the user community regarding the relative merits of using observations of the environmental variables for serially-dependent sea states, compared with near-independent storm peak characteristics. Given that the rates of occurrence of events are taken into account, both can provide useful estimates of joint models for the environment and hence environmental contours. The advantage of using sea state data is that sample size is large, potentially allowing a more detailed description of the joint distribution of environmental parameters to be estimated. However, because sea state data is serially-correlated, naive estimates of uncertainties for model parameters and inferences under the model will be too small, but can be corrected (for example, using sandwich estimators or bootstrap resampling).

Multi-modal distributions of environmental variables can be caused

by different physical processes or by covariate effects, including e.g. fetch length as a function of direction. In these cases, isodensity contours may be more reasonable summaries of the joint distribution than those based on joint exceedance, since there may be regions of low probability between modes. Further it may be useful to partition the environmental space by covariate and perform separate analyses per partition. Alternatively, it is also possible to estimate joint models incorporating covariate effects.

Basing design conditions on the T -year environmental contour alone neglects sources of bias and variability in estimation of the T -year maximum response, including short-term variability in response. Approximate methods to inflate the environmental contour, or adjust its return period, are available. It is always possible, given fully-specified environments and structural responses, to estimate inflation factors which map some quantile of the distribution of T -year maximum response onto the corresponding quantile of the distribution of maximum response given sea states on the environmental contour. However, the value of inflation factor in general will be a function of quantile level, the structure and the response. It is likely that estimating inflation factors (or adjusting contour return periods) based on comparing central characteristics (for example, mean, median or mode) of response distributions will prove more stable, since Monte Carlo simulations of a given size provide better estimates of central characteristics than those of tails.

In some applications, it may be that environmental contours will be

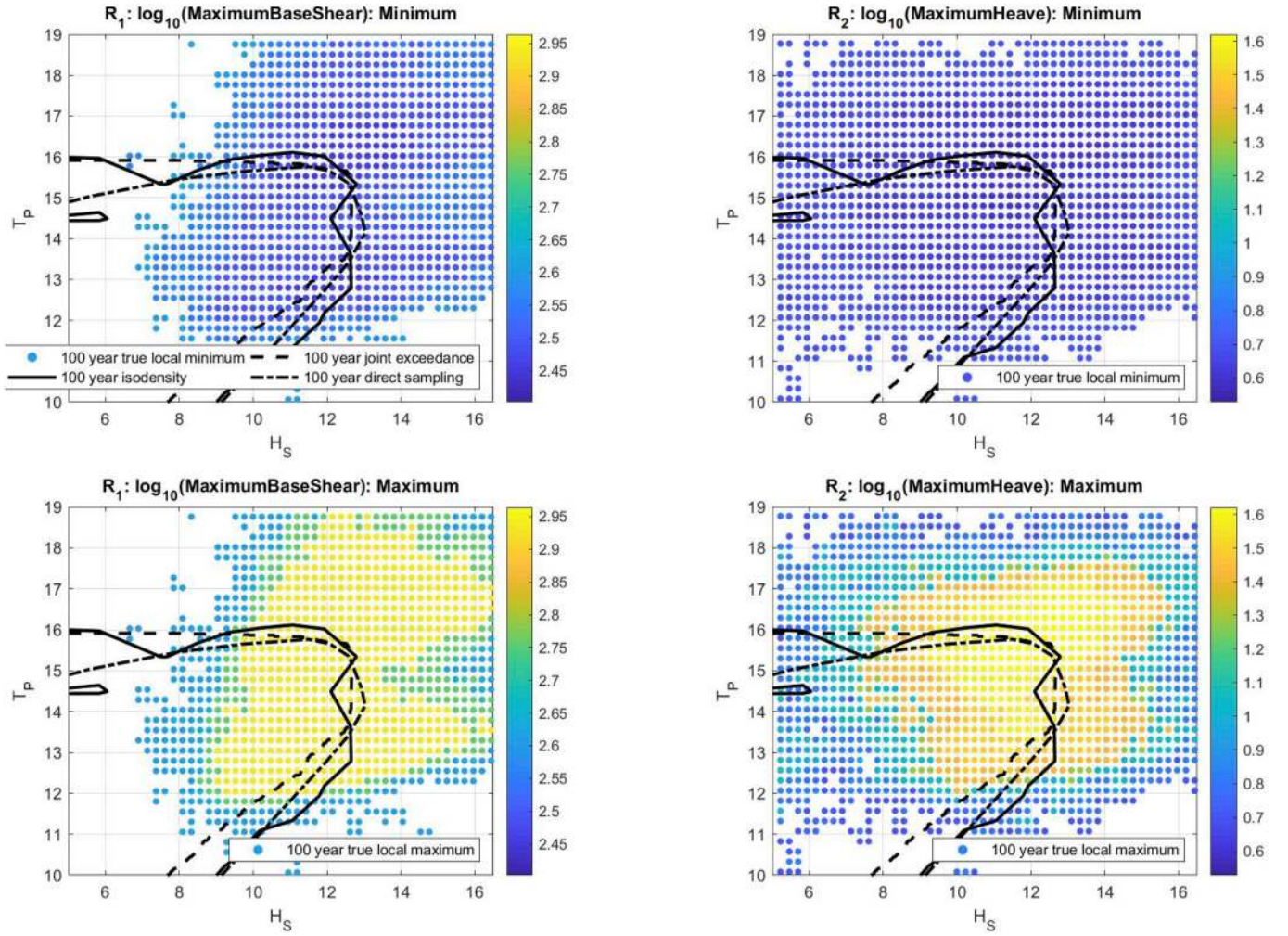


Fig. 4. Minima ($\{M_{R_i,100}^{\min}\}$, top row) and maxima ($\{M_{R_i,100}^{\max}\}$, bottom row) of the 100-year maximum response, as a function of H_S and T_P estimated using 1000 realisations (of length 100 years) of H_S and T_P for responses R_1 (left) and R_2 (right). Points are coloured by the local minimum (top) or maximum (bottom) value of maximum response estimated on a lattice of values for H_S and T_P . Each panel also shows 100-year environmental contours from each of the direct sampling (black dot-dashed), joint exceedance (black dashed) and CE density (black solid) methods. Units for variables are m (H_S , heave), s (T_P) and kN (base shear).

used to estimate multiple correlated responses. In such cases, care needs to be taken that the contour is used to estimate the responses jointly corresponding to a given return period, rather than estimating independent marginal return values.

As can be seen from the end-user survey in [Appendix A.1](#), there are valid concerns regarding a sensible definition of an environmental contour, its estimation and adjustment in relation to structural response. There are many articles in the literature comparing different contour methods, inflation factors etc. However, in reality, most sensibly-proposed contour methods (including the direct sampling, joint exceedance and isodensity examined here) locate the frontier interval of the contour within the same region of environmental space. Uncertainties due to the details of the structure, and even when the structure is defined to the natural variability of structural response given the environment, are in general issues of far greater concern. Any reasonable choice of contour, given the considerations explored in this paper, will suffice.

Methods such as IFORM and the direct sampling method are advantageous in that they impose a link between environment and structure by making assumptions about the characteristics of failure surfaces as a function of the environmental variables. Given these assumptions, it is possible to link the exceedance probability associated with a given environmental contour with structural failure probability. Although conditions from a T -year environmental contour need not result exactly

in T -year responses, IFORM and direct sampling provide at least some understanding of how an T -year environmental contour is related to the T -year maximum response. Both IFORM and direct sampling approaches assume a linearised failure boundary. The basic difference between the approaches arises from the fact that linearisation for IFORM is performed in the transformed U -space, and in direct sampling approach in the original X -space of environmental variables (e.g. [Vanem and Bitner-Gregersen, 2015](#); [Vanem, 2017](#) and references therein). For both IFORM and direct sampling contours, the relationship established is between the exceedance probability associated with the contour (on some scale) and the probability of structural failure. This does not guarantee however that searching along an IFORM or direct sampling contour for return period T will isolate the key features of the T -year maximum response; the relative performance of IFORM and direct sampling in estimating extreme responses is application-dependent.

For better specification of design conditions, a response model is necessary. To determine the frontier interval of a design contour within the domain of environmental variables, inflation factors for contours etc., some knowledge of, or working assumption regarding the response is required. As specification of the response evolves, sufficient, for example, to estimate the frontier interval and inflation factors, arguably there is already sufficient information (or supposition) to use better models to describe approximate responses and their uncertainties. From a statistical perspective, it is hard to avoid the impression that, in terms

of estimating extreme structural response, an environmental contour is just an approximation to a sample from the tail of the distribution of environmental variables. In fact, for many purposes, an appropriate sample from the tail of the distribution of environmental variables would be preferable. Furthermore, with the advent of methods such as statistical emulation (used widely in approximating complex physical systems, including metocean design e.g. Gouldby et al., 2014; Jones et al., 2018), a computationally-efficient approximate response model can be estimated, along with its uncertainty, for many if not all applications. Given this, the emulator would provide a mechanism for response-based design in all situations, avoiding the need for environmental contours completely. It is apparent that the coastal engineering community is already moving in this direction (e.g. Gouldby et al., 2017); a key assumption with statistical emulation is that a representative set of cases is available to quantify all important relationships between environment and structural loading. Nevertheless, part of the user-cited strength of the environmental contour method is its relative simplicity and computational efficiency. There is clearly a trade-off between a thorough probabilistic response-driven analysis, which may be technically more involved and less familiar to the practitioner, compared to a simple approximate approach with which the user is relatively familiar and confident.

The challenges of estimating and applying environmental contours well in practice, noted in this section, of course motivate further development of methods by academic and industrial researchers. Given the above considerations, and comments in the end-user survey, to assist the practitioner in deciding when and where to use an environmental contour approach, we present the following brief check-list. We recommend the use of environmental contours in the following circumstances.

6.1. When to use environmental contours

- **Nature of responses and environmental variables are known:** The dominant structural responses are all known. The dominant environmental variables driving each structural response are all known, and the value of response is dominated by long-term variability of the environmental variables: extreme environments produce extreme responses. The influence of short-term environmental variability is relatively small [There are some responses (e.g. fatigue) that are not dominated by extremes of the environment; environmental contour methods are therefore not appropriate.]
- **Response-based analysis is not possible:** (a) There are no adequate computationally-efficient structural response models available. [If these models are available, a response-based analysis should be performed.] (b) There are computationally-demanding structural response models available, but no time or expertise to develop approximate structural response models (for example, generic load models, statistical emulators) using these. [If these models can be estimated, a response-based analysis should be performed.]
- **At outline design stage:** The specifics of the structure may not be clear at outline design. For this reason, the environmental contour may provide a useful source of extreme sea states suitable for evaluating a range of different structures.

Once it has been decided that an environmental contour approach is suitable, the following is then recommended.

6.2. How to use environmental contours wisely

- **Reality check:** Remember that environmental contours are approximate methods that can only provide approximations to extreme responses. The use of contour approaches may need to be supported in final design by full long-term analysis.

- **Sufficient environmental data available:** There are sufficient historical data available to estimate the joint distribution of all these environmental variables adequately using, for example, a method from Section 3.
- **Estimate more than one environmental model, and consider the sensitivity of the model to arbitrary modelling choices:** The sensitivity of environmental contour estimates to arbitrary choices made when estimating a model for the joint distribution of environmental parameters should be investigated. [When different equally-plausible environmental models provide different contour estimates, the current research suggests that all contours should be considered valid and used together for choice of environmental values corresponding to extreme responses. Equally, the user might well be concerned when two different environmental models provide materially different contour estimates using a common contouring approach.]
- **If unsure which contour to use, estimate more than one type:** Each type of environmental contour is seeking to achieve different objectives. If you are not clear which contour is most suitable for your application, consider estimating contours of different types, and establish approximate consistency of inferences from different contours. [When different equally-plausible contours give materially different results, all contours should be considered valid and used together for choice of environmental values corresponding to extreme responses. We note that some contour methods (e.g. ISORM Chai and Leira, 2018, highest density contours Haselsteiner et al., 2017b) are more likely to provide conservative estimates, since they seek to exploit convex characterisations of the failure surface.]
- **Choose multiple points from the environmental contours for response evaluation:** Multiple combinations of values of environmental variables falling on or near the frontier interval of the environmental contour should be used. [When the frontier interval is not known, a wide set of combinations of values of environmental variables on or near the environmental contour should be used. If in doubt, choose more points and choose points more widely.]
- **Consider other sources of uncertainty:** (a) How influential are the effects of covariates (directionality, seasonality)? [If important, fit a non-stationary environmental model (for example, using PPC from Section 5)]; (b) Have all environmental variables influencing the response been considered in the environmental model and contours? [If not, consider estimating environmental models and contours in higher dimensions].

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Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.oceaneng.2019.106194>.

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