The practicalities of estimating the effect of climate change on extreme values of ocean storm severity

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... with thanks to colleagues at Lancaster, Melbourne, Shell and elsewhere

Context

- Want to predict extreme quantiles and related quantities for environmental random variables
  - "Extrapolation beyond the data"
- Typical samples are
  - Small (relative to quantile level) or "non-existent" (e.g. for year 2100)
  - Uncertain (difficult to measure; theoretical models unreliable)
  - Conditional on spatial, temporal and other covariates
- Rely on asymptotic results for tails of distributions of "max stable" random variables
  - Typically a reasonable assumption
  - Rarely clear at what level the asymptotics kick in
  - Usually considerable uncertainty regarding threshold selection, or block size selection, or measurement scale.

- How do we arrive at a good description of the tail for *in-anger* applications?

Context

- Safe design of coastal and marine structures subject to fluid loading
- Design criteria conventionally specified in terms of “return values” for e.g. significant wave height, H_S
- Climate change: climate models suggest considerable uncertainty, and regional variation in extent of temporal non-stationarity of H_S
  - Different models $\Rightarrow$ different modelling assumptions (not always clear!)
  - Some models are “related” (e.g. using common components)
  - Wind fields are difficult to model in general, and winds cause waves $\Rightarrow$ are climate models any good at extreme wave prediction?

- Need to “future-proof” infrastructure
- How useful is the output of CMIP5 and CMIP6 climate models to estimate future return values of H_S?

Articles

- Climate effects East of Madagascar, South of Australia: Ewans and Jonathan [2023a]
- Climate effects Tasman Sea: Ewans and Jonathan [2023b]

- Uncertainties in extreme quantiles: Jonathan et al. [2021], Towe et al. [2023]
- Satellite-based inference: Shooter et al. [2022]

Optimal design of marine structure

Set-up

- A marine system with "strength" specifications $\boldsymbol{\mathcal{S}}$
- An ocean environment $\boldsymbol{X}$ dependent on covariates $\boldsymbol{\Theta}$
- A structural "loading" $\boldsymbol{Y}$ as a result of environment X and covariates $\boldsymbol{\Theta}$
- System utility (or risk) $U(Y|\boldsymbol{\mathcal{S}})$ for loading $\boldsymbol{Y}$ and specification $\boldsymbol{\mathcal{S}}$
- Desired U typically specified in terms of annual probability of failure
- $\boldsymbol{Y}|\boldsymbol{X}, \boldsymbol{\Theta}$ and $X|\boldsymbol{\Theta}$ (and U?) subject to uncertainty $\boldsymbol{Z}$
- $\boldsymbol{Z}, \boldsymbol{\Theta}, \boldsymbol{X}, \boldsymbol{Y}$ are multidimensional random variables

Optimal design

- A model $f_{X|\boldsymbol{\Theta},\boldsymbol{Z}}$ for the environment
- A model $f_{Y|X,\boldsymbol{\Theta},\boldsymbol{Z}}$ for environment-structure interaction
- A model $f_{\boldsymbol{\Theta}|\boldsymbol{Z}}$ for the covariates

$$\mathbb{E}[U|\boldsymbol{\mathcal{S}}] = \int_\zeta \int_y \int_x \int_\theta U(\boldsymbol{y}|\boldsymbol{\mathcal{S}}, \zeta) f_{Y|X,\Theta,Z}(\boldsymbol{y}|\boldsymbol{x}, \theta, \zeta) f_{X|\Theta,Z}(\boldsymbol{x}|\theta, \zeta) f_{\Theta|Z}(\theta|\zeta) f_Z(\zeta)\, d\theta\, d\boldsymbol{x}\, d\boldsymbol{y}\, d\zeta$$

$\Rightarrow$ solve for $\boldsymbol{\mathcal{S}}$ to achieve required (safety) utility

A model for *temporally non-stationary* peaks over threshold

- A univariate response (significant wave height, H_S)
- Neglecting non-stationarity with respect to covariates Θ

- A model for threshold ψ_t in time (quantile regression)
- A model for size X_t of threshold exceedance in time (GP, with parameters ξ_t, σ_t)
- A model for rate ρ_t of threshold exceedance in time (Poisson)

- All model parameters vary linearly in time
- Inference using adaptive MC (Roberts and Rosenthal 2009)

Example:

$$F_{\mathrm{GP}}(x|X_t > \psi_t, \psi_t, \sigma_t, \xi_t) = \begin{cases} 1 - \left[1 + (\xi_t/\sigma_t)(x - \psi_t)\right]^{-1/\xi_t} & \xi_t \neq 0 \\ 1 - \exp(-(x - \psi_t)/\sigma_t) & \xi_t = 0. \end{cases}$$

where

$$\eta_t = \eta(t) = \eta^S + \frac{t}{P}(\eta^E - \eta^S), \text{ for } t \in (0, P]$$

for any parameter η_t over period P of data, with starting and end values η_S and η_E to be estimated

Estimators of return value

- Uncertain estimates $\boldsymbol{Z}$ of model parameters (from fit to sample, say)
- Estimate distribution $F_{A|\boldsymbol{Z}}$ of annual maximum event using $\boldsymbol{Z}$ at any time t
- Estimate N-year return value by finding the $1-1/N$ quantile of $F_{A|\boldsymbol{Z}}$

- Various options available, including

$$
\begin{aligned}
q_1 &= F^{-1}_{A|\boldsymbol{Z}}(1-1/N \mid \mathbb{E}_{\boldsymbol{Z}}[\boldsymbol{Z}]) = F^{-1}_{A|\boldsymbol{Z}}(1-1/N \mid \int_{\zeta} \zeta f_{\boldsymbol{Z}}(\zeta) d\zeta) \\
q_2 &= \mathbb{E}_{\boldsymbol{Z}}[F^{-1}_{A|\boldsymbol{Z}}(1-1/N \mid \boldsymbol{Z})] = \int_{\zeta} F^{-1}_{A|\boldsymbol{Z}}(1-1/N \mid \zeta) f_{\boldsymbol{Z}}(\zeta) d\zeta \\
q_3 &= \tilde{F}^{-1}_{A}(1-1/N) \text{ where } \tilde{F}_A(x) = \int_{\zeta} F_{A|\boldsymbol{Z}}(x \mid \zeta) f_{\boldsymbol{Z}}(\zeta) d\zeta \\
q_4 &= \tilde{F}^{-1}_{A_N}(\exp(-1)) \text{ where } \tilde{F}_{A_N}(x) = \tilde{F}^{N}_{A}(x) \\
q_5 &= \text{med}_{\boldsymbol{Z}}[F^{-1}_{A|\boldsymbol{Z}}(1-1/N \mid \boldsymbol{Z})]
\end{aligned}
$$

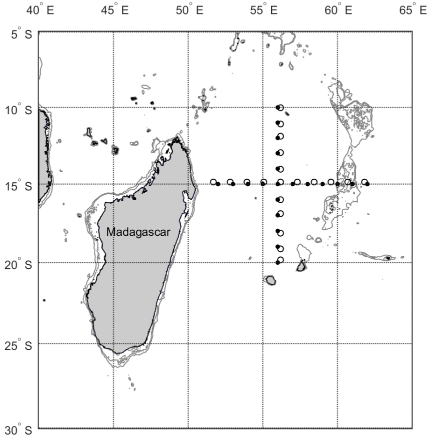
- For small samples, these have very different properties

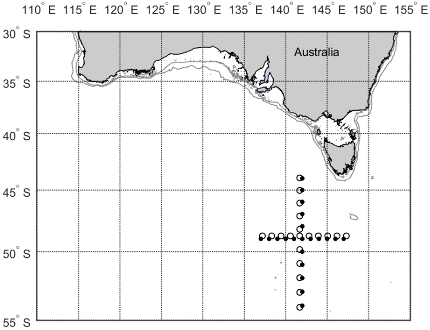
Overview: what did we do?

Estimate the change in the 100-year return value over a period of 86 years

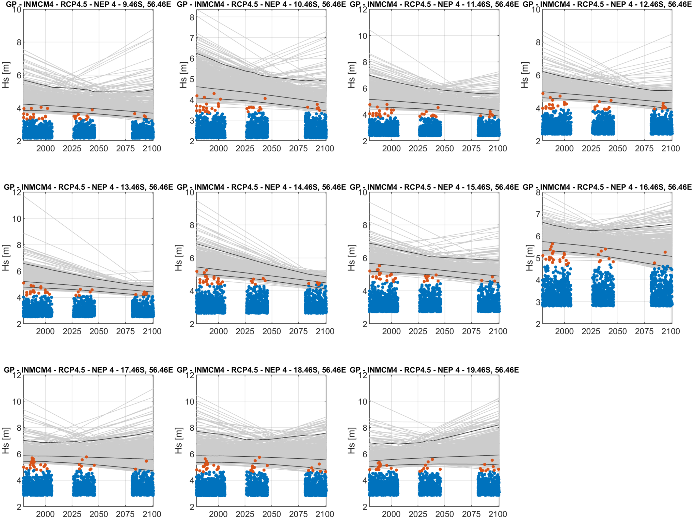
- Global coupled model-derived significant wave height, H_S
  - 7 CMIP5 models with WAVEWATCH-III (1° spatial resolution, see Meucci et al. 2020)
  - 1 CMIP6 (FIO-ESM v2.0, $[0.27, 0.54]^\circ$ meridional and 1.25° zonal resolution, see Bao et al. 2020, Song et al. 2020)
- Regions
  - East of Madagascar, EoM: predicted reduction in H_S
  - South of Australia, SoA: predicted increase in H_S
  - See Meucci et al. [2020]
- Cross-hairs of zonal and meridional transects per region
- Climate scenarios
  - CMIP5 1979-2005 (Hst), 2026-2045 (Mid) and 2081-2100 (End)
  - CMIP5 RCP4.5, RCP8.5
  - CMIP6 piControl "steady state" 301-1000 and historical 1850-2014
  - CMIP6 SSP245 ("RCP4.5"), SSP585 ("RCP8.5") for 2015-2100
- Estimation using temporally non-stationary generalised Pareto, GP
  - Threshold choice
- Other
  - Sample size for CMIP6 piControl
  - c.f. block maxima / generalised extreme value, GEV

Regions and transects



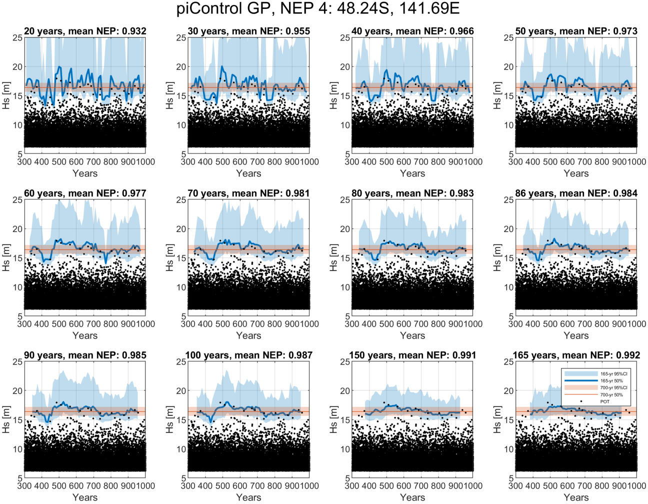


Locations of EoM (left) and SoA (right) grid points used. Dots indicate CMIP5, and circles CMIP6. Grey lines are bathymetric contours at 100m and 1,000m water depth.



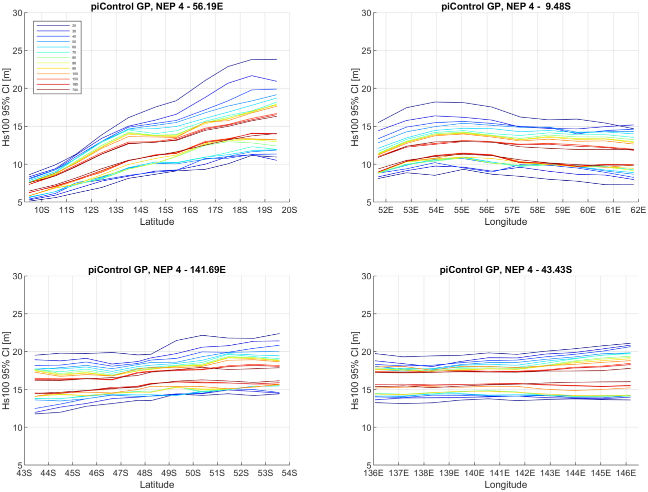
Typical illustration of the estimated variation of 100-year POT H_S with time for CMIP5 sample (available for “Historical”, “Middle” and “End” time periods) using non-stationary extreme value (EV) analysis. Each grey line corresponds to the trend of return value in time based on one sample from the MCMC chain generated. Thicker grey lines correspond to the estimated 2.5%, median and 95% levels at each time point. Underlying POT data in blue, and EV threshold exceedances in red. The data illustrated correspond to INMCM4 GCM, scenario RCP4.5, meridonal transect EoM for EV threshold level NEP4.

Effect of sample size (CMIP6 piControl, centre location SoA, NEP4)



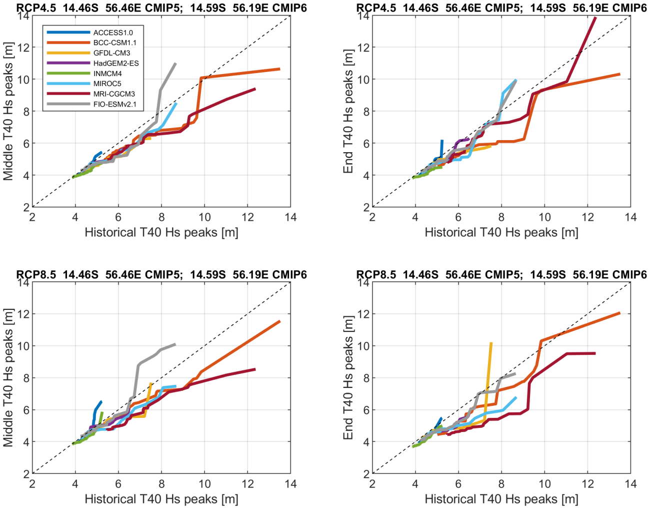
Estimates of 100-year H_S in time for the CMIP6 piControl data, using EV threshold NEP4. Each panel gives return value estimates from stationary EV analysis using an interval of data of "segment length" specified in panel title, starting at the year indicated on the x-axis, at the centre location SoA. Within each panel, the posterior median estimate is given as a blue line, and its 95% credible interval (CI) as a blue band. Also shown (in orange, and common to all panels) are the corresponding posterior median and 95% CI using the full 700 years of data for EV analysis. In each panel, the sample data is represented as black dots. The panel title also gives the value of non-exceedance probability τ corresponding to threshold level NEP4.

Summary: effect of sample size, EoM&SoA (CMIP6 piControl, NEP4)



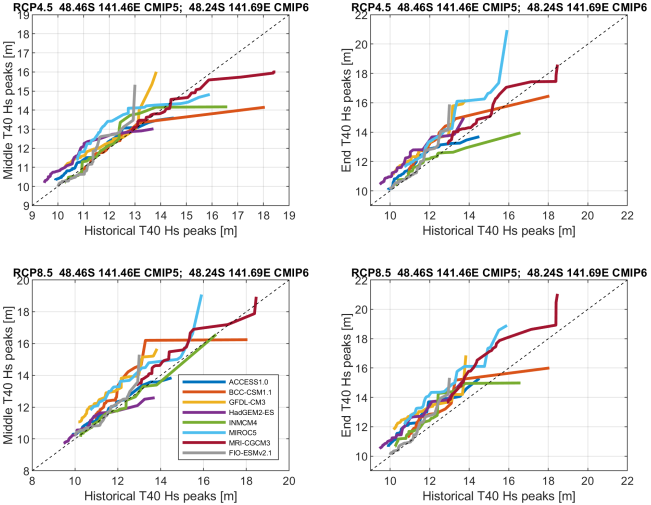
Estimates of meridional (left) and zonal (right) variation of 100-year POT H_S for EoM (top) and SoA (bottom) for the CMIP6 piControl data, using EV threshold NEP4. 13 pairs of ordered coloured lines in each panel give 95% credible intervals for the return value from stationary EV analysis using intervals of data of specific length (in years). The calculation procedure is outlined in the article text.

Scatter of sorted peaks, EoM



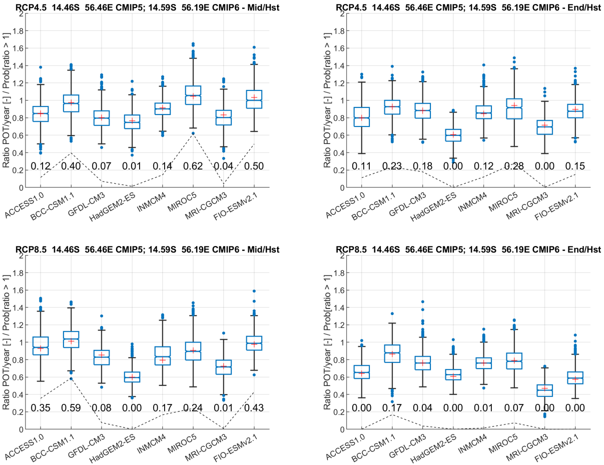
Comparison of the ordered 40 largest values (T40) of POT H_S for the "Historical" (1979–2005), "Middle" (2026-2045) and "End" (2081-2100) time periods, for all CMIP5 GCMs, and the FIO-ESM CMIP6 GCM, for forcing scenario RCP4.5 (and SSP245, top row) and RCP8.5 (and SSP585, bottom row) at the centre location EoM. Left-hand panels give scatter plots for the Middle period on the Historical period; right-hand panels give the corresponding plots for the End period on the Historical period. The line $y = x$ is added for guidance.

Scatter of sorted peaks, SoA



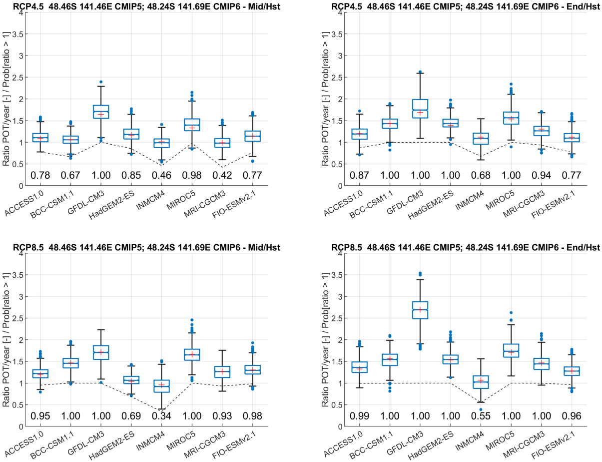
Comparison of the ordered 40 largest values of POT H_S for the Historical, Middle and End time periods, for all CMIP5 GCMs, and the FIO-ESM CMIP6 GCM, for forcing scenario RCP4.5 (top row) and RCP8.5 (bottom row) at the centre location SoA. Left-hand panels give scatter plots for the Middle period on the Historical period; right-hand panels give the corresponding plots for the End period on the Historical period.The line $y = x$ is added for guidance.

Ratio of rate of threshold exceedance, EoM



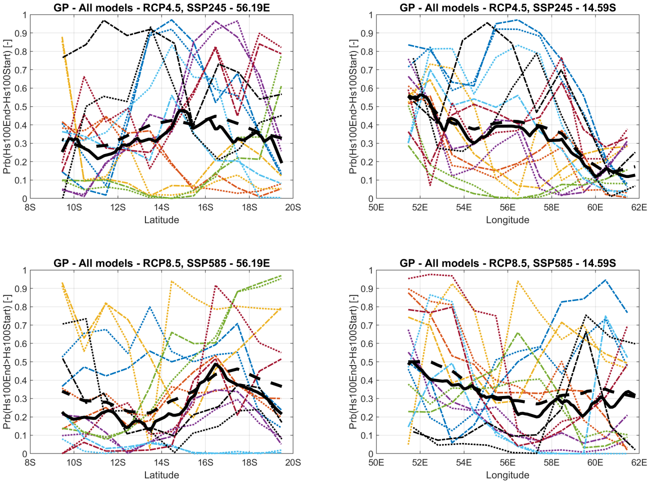
Box-whisker plots for ratio of annual rates of POT H_S at centre location EoM, for all CMIP5 and CMIP6 GCMs. Panels show ratios for the the Middle relative to Historical periods (left) and the End relative to Historical periods (right), for RCP4.5 (and SSP245, top) and RCP8.5 (and SSP585, bottom) forcing scenarios, using a POT threshold corresponding to the 80%ile of the data for the historical period. Each box-whisker indicates the mean (red crosses), median, 25%, 75% quartiles (blue lines), the smallest and largest values not exceeding 1.25 $\times$ inter-quartile range from the median (black), and outliers (blue dots). Also shown (dashed black line, and values per GCM) is the estimated probability that the ratio of annual rate exceeds unity.

Ratio of rate of threshold exceedance, SoA



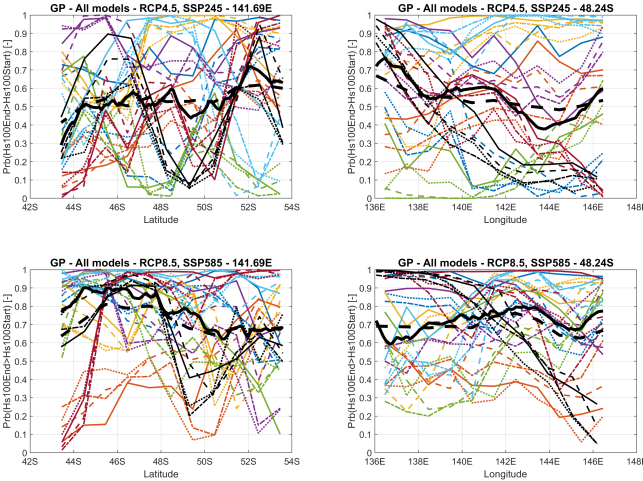
Box-whisker plots for ratio of annual rates of POT H_S at centre location SoA, for all CMIP5 and CMIP6 GCMs. Panels show ratios for the the Middle relative to Historical periods (left) and the End relative to Historical periods (right), for RCP4.5 (top) and RCP8.5 (bottom) forcing scenarios, using a POT threshold corresponding to the 80%ile of the data for the historical period. Each box-whisker indicates the mean (red crosses), the median, 25%, 75% quartiles (blue lines), the smallest and largest values not exceeding 1.25 × inter-quartile range from the median (black), and outliers (blue dots). Also shown (dashed black line, and values per GCM) is the estimated probability that the ratio of annual rate exceeds unity.

Summary: probability of increasing return value, EoM



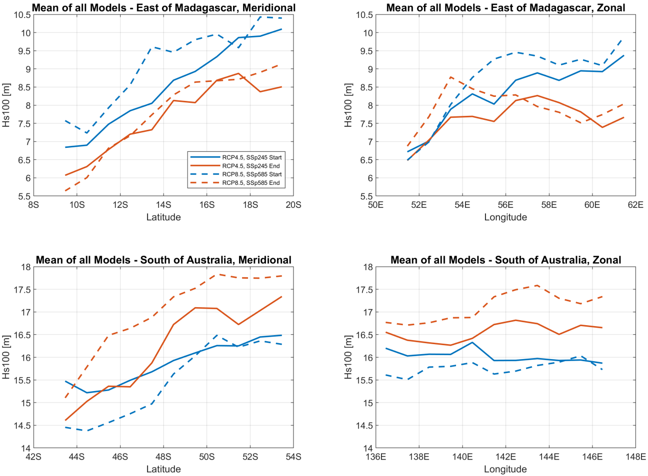
Summary of RCP trends for 100-year POT H_S and all GCMs EoM. Panels show the meridional (left) and zonal trends observed for RCP4.5 (and SSP245, top) and RCP8.5 (and SSP5-85, bottom) output. Thin coloured lines represent return value using EV threshold NEP3 (dash-dot) and NEP4 (dot) for individual CMIP5 and CMIP6 GCMs, and the thick black lines provide the global mean (solid) and the global median (dashed) over all GCMs and NEPs. The colour scheme for CMIP5 GCMs is as previously; in addition, thin black lines are used to represent the CMIP6 values.

Summary: probability of increasing return value, SoA



Summary of RCP trends for 100-year POT H_S and all GCMs SoA. Panels show the meridional (left) and zonal trends observed for RCP4.5 (top) and RCP8.5 (bottom) output. Thin coloured lines represent return value using EV threshold NEP1 (solid), NEP2 (dashed), NEP3 (dash-dot) and NEP4 (dot) for individual CMIP5 and CMIP6 GCMs, and the thick black lines provide the global mean (solid) and the global median (dashed) over all GCMs and NEPs. The colour scheme for CMIP5 GCMs is as previously; in addition, thin black lines are used to represent the CMIP6 values.

Summary: overall findings on 100-year return value, EoM&SoA



Summary of inferences for 100-year POT H_S return value from all CMIP5 and CMIP6 GCMs. Panels show the mean (over all GCMs) of the posterior median return value at the start time (blue) and the end time (orange), for RCP4.5 (and SSP245, solid) and RCP8.5 (and SSP585, dashed) for EoM (top) and SoA (bottom) meridionally (left) and zonally (right).

Conclusions

- Estimating quantiles of distributions, extreme relative to the sample size, is problematic
- Using estimates of extreme quantiles to summarise tails of distributions for design purposes is probably ill-advised; prefer full probabilistic failure analysis
- Different estimators of extreme quantiles have very different small sample performance

In the current work, estimated changes in return values 2015-2100 suggest

- Large variations between different GCMs
- Some consistency in local spatial trends
- Large variations due to threshold selection
    - High threshold needed
    - For block maxima, block length $\gg 1$ year needed EoM because of influence of rare tropical cyclone events

Analysis of the steady state CMIP6 piControl output suggests that

- Changes in return value estimates as large as $\pm$3m over 86 years are typical, using sample sizes corresponding to periods $P \in [86, 165]$ years
- There is a probability of ≈ 0.7 that a 100-year return value estimate from $P = 20$ years of data lies outside the 95% credible interval for the return value estimated using $P = 700$ years of data

Recommendations

- Work hard to get the longest relevant period P of data for analysis
- Use peaks over threshold where possible
- Incorporate the effect of threshold uncertainty in inferences
- Incorporate estimation uncertainty in inferences
- Do not rely on a single GCM. Look for a representative set of models
  - ... in terms of “climate sensitivity”
  - Some models have common components
- Avoid model mis-specification: capture the effects of non-stationarity
- Simple “one model, one vote” average over estimates of return value from plausible candidate models
  - Bayesian model average

Thank-you / Diolch!

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