The Effect Of Directional Extremes On Ocean Design Criteria

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1.1 Motivation

- Extremal behaviour varies with direction, season, space and (longer-term) with time
  - ⇒ Considerable scope to improve current ocean design practice
  - ⇒ Use extreme value model with smoothly directionally-varying parameters
- (Dependent) data from neighbouring locations routinely combined
  - ⇒ Bootstrapping for realiable estimation of model parameter uncertainty
- No industry agreement on specification of directional design criteria
  - ⇒ Statistical basis for consistent design specification

1.2 References

- P. Jonathan, K. Ewans (2007) The effect of directionality on extreme wave design, *Ocean Engineering doi:10.1016/j.oceaneng.2007.03.003*, In Press.
- K. Ewans, P. Jonathan (2007) The effect of directionality on Northern North Sea extreme wave design criteria, *26th International Conference on Offshore Mechanics and Arctic Engineering, June 10-15, 2007, San Diego, USA*, OMAE2007-29657
- Coles S and Simiu E (2003) "Estimating uncertainty in the extreme value analysis of data generated by a hurricane simulation model" *J Engineering Mechanics* 129 1288-1294
- Coles SG and Tawn JA (1994) Statistical methods for multivariate extremes: an application to structural design" *Applied Statistics* 43 1-48.

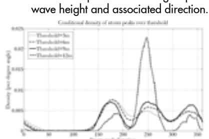
1.3 Background: Estimating directional extremes

- Environmental design criteria inherently uncertain
  Natural variability; Sampling uncertainty; Hindcast estimates
- Northern North Sea specifics
  Extra-tropical storms (on relatively large spatial scale)
  Considerable fetch variability
- Extremal behaviour dependent on storm direction
  e.g. due to hurricane track (e.g. in Gulf of Mexico) or fetch variability
  Usually ignored in practice
- Combining data from dependent locations ("site averaging")
  In principle: increases sample size for modelling; accounts for effects of random storm track
  However: data from even quite largely separated locations are highly dependent; difficult to determine the reliability (or equivalently the degree of uncertainty) of models and estimates.

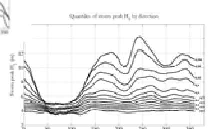
2.1 The Northern North Sea (NNS) data

Values from NEXTRA hindcast study (Oceanweather, 2002), for October 1964 to September 1998 inclusive. 180 minute sampling interval, 100 grid points covering a region of 5° longitude, 3° latitude – centred at 2° E, 61° N.

For each storm period for each grid point, we isolated a storm peak significant wave height and associated direction.

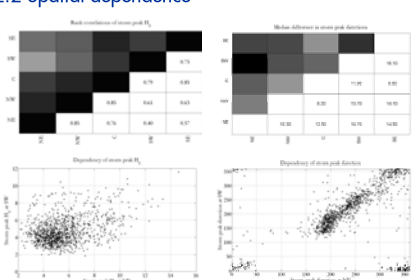


Land shadows limit fetch for some directions



Tail characteristics depend on direction

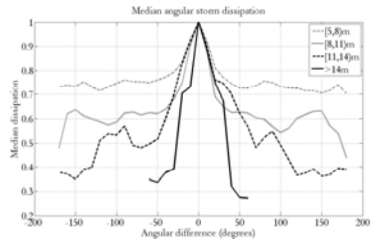
2.2 Spatial dependence



Considerable spatial dependence

2.3 Directional storm dissipation

The directional dissipation of a storm is the minimum reduction in H_S (expressed as a fraction of H_S at storm peak) as a function of angular difference from the storm peak direction. It quantifies the effect of storms on extremes in directions other than the storm peak direction



3.1 The directional GDP model

Storm peaks over threshold are modelled using the generalised Pareto distribution, with directionally-varying parameters and threshold.

$$F_{X_i|\theta_i,u}(x) = P\left(X_i \le x \mid \theta_i, u\right) = 1 - \left(1 + \frac{\gamma(\theta_i)}{\sigma(\theta_i)}(x-u)\right)_+^{-\frac{1}{\gamma(\theta_i)}}$$

$\gamma(\theta_i)$ shape parameter $\quad \sigma(\theta_i)$ scale parameter $\quad u(\theta_i)$ threshold

Parameters vary smoothly with direction (e.g. Robinson & Tawn, 1997)

$$\gamma(\theta) = \sum_{k=0}^{p}\left[A_{11k}\left(\cos(k\theta)\right) + A_{12k}\left(\sin(k\theta)\right)\right] \qquad p = 10$$

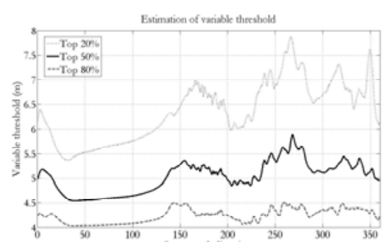
$$\sigma(\theta) = \sum_{k=0}^{p}\left[A_{21k}\left(\cos(k\theta)\right) + A_{22k}\left(\sin(k\theta)\right)\right]$$

Parameters estimated by roughness-penalised maximum likelihood

$$l^*(\{A_{abk}\};\{X_i\}_{i=1}^{n}) = \left(\sum_{i=1}^{n} l_i\right) - \lambda(R_\gamma + \tfrac{1}{w}R_\sigma) \qquad R_\gamma = \int_0^{2\pi}\left(\tfrac{\partial^2\gamma}{\partial\theta^2}\right)^2 d\theta = \sum_{k=1}^{p}\pi k^4\left(\sum_{b=1}^{2}A_{1bk}^2\right) \quad \text{etc.}$$

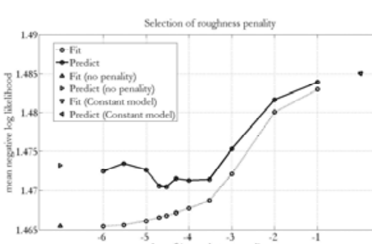
3.2 Variable directional threshold

The threshold u for EV modelling is estimated smoothly as a function of storm peak direction.



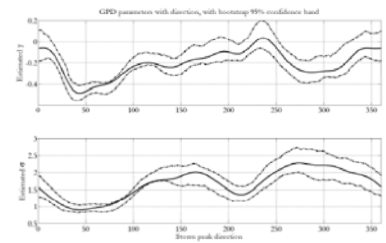
3.3 Model fitting – choice of roughness

The value of the roughness parameter λ is selected using cross-validation to maximise model predictive performance at "unseen" locations.



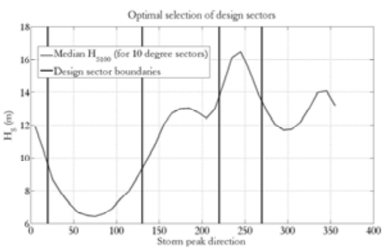
3.4 Estimating parameter uncertainty

Estimates of parameter uncertainty from asymptotics (or profile likelihood) are inappropriate for samples obtained by aggregating dependent data. Bootstrapping (stormwise) gives resistant confidence intervals.



4.1 Choice of directional design sectors

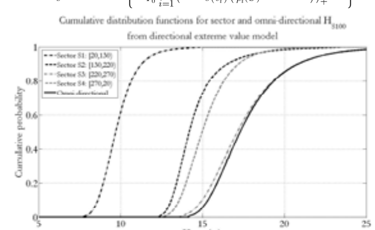
The selection of directional design sectors should reflect engineering requirements and constraints, and prevailing oceanographic conditions at the location. For the current application, we specify directional sectors which are as homogeneous as possible in terms of the 10° sector median H_{S100} using at iterative procedure



4.2 Estimation of cumulants for H_{S100}

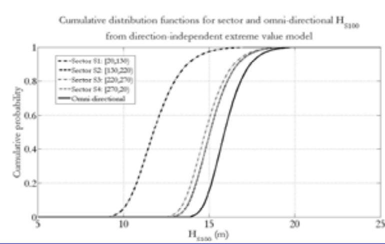
For period *P* and directional sector *S*, we estimate the sector maximum cdf as:

$$F_{X\max_S}(x) = \exp\left\{-\frac{P}{P_0}\sum_{i=1}^{n_0}\left(1 + \frac{\gamma(\theta_i)}{\sigma(\theta_i)}\left(\frac{x}{\rho_i(S)} - u(\theta_i)\right)\right)_+^{-\frac{1}{\gamma(\theta_i)}}\right\}$$



4.3 The effect of ignoring directionality

Standard engineering practice is to ignore directional effects. Setting EV scale and shape parameters as constant with respect to direction underestimates extremes.



5.1 Specifying design criteria

The 100-yr design H_s can be calculated for a particular non-exceedance probability $q_{100Omni}$ from $q_{100Omni} = P\left(X_{\max 100Omni} \le x_{100Omni}\right)$

Specification of $q_{100Omni}$ does not allow unique values of the sector 100-yr design H_s but can calculate:

$$q_{100OmniS_i} = P\left(X_{\max 100S_i} \le x_{100Omni}\right) \quad \text{and} \quad \tilde{q}_{100Omni} = \prod_{i=1}^{4} q_{100OmniS_i}$$

Case I: "Omni-directional Hs100"

Design to median omni-directional storm peak $H_{S100}, x_{100Omni}$

Case II: "Equal sector non-exceedance"

Design to same non-exceedance probability $q_{100S_i} = \left(\tilde{q}_{100Omni}\right)^{1/4}$

Case III: "Risk-cost optimal design"

Design to minimise cost criterion $RC = \frac{1}{100}\sum_{i=1}^{4}\left(x_{100S_i} - 14\right)^2$ for $\tilde{q}_{100Omni} = \prod_{i=1}^{4} q_{100S_i}$

5.2 Outline design values

Design values based on the directional model are different to their counterparts obtained by analysis ignoring the directional dependence of storms.

The risk-cost optimal design reduces the range of non-exceedence probabilities for design to omni-directional, and reduces the range of storm peak H_S values for design to equal sector non-exceedence.

Sector	Angle	Risk-Cost Optimal			Omni-directional			Equal Non-exceedence probability		
Directional model		RiskCost	Value (m)	Quantile	RiskCost	Value (m)	Quantile	RiskCost	Value (m)	Quantile
S1	[20,130)	5.44	14.00	0.99	11.97	17.30	1.00	8.94	10.75	0.82
S2	[130,220)		15.50	0.81		17.30	0.94		15.58	0.82
S3	[220,270)		18.31	0.69		17.30	0.53		19.53	0.82
S4	[270,20)		16.40	0.81		17.30	0.91		16.44	0.82
Constant model		RiskCost	Value (m)	Quantile	RiskCost	Value (m)	Quantile	RiskCost	Value (m)	Quantile
S1	[20,130)	7.66	14.00	0.92	10.11	15.90	0.99	7.80	13.16	0.83
S2	[130,220)		16.08	0.80		15.90	0.76		16.20	0.83
S3	[220,270)		15.80	0.79		15.90	0.81		15.97	0.83
S4	[270,20)		16.06	0.80		15.90	0.76		16.21	0.83

6.1 Conclusions

It is essential to capture directionality of extreme sea states when developing design criteria.

Omni-directional extreme values derived from a directional model can be materially different from a direction-independent derivation ignoring directional effects. For the current application, a directional model characterises the hindcast data significantly better than a conventional (direction-independent) model.

An extremal threshold which varies with direction is used to characterise the changing extremal properties of storm peak significant wave height with direction.

A high-order Fourier form ensures the directional extreme value model is sufficiently flexible to characterise variation in extremal behaviour with storm direction.

A roughness penalty ensures that extreme value estimates are as smooth as possible consistent with the data within a maximum likelihood framework.

Cross-validation is used to estimate the appropriate roughness penalty.

Care must be taken in aggregating dependent data for extreme value analysis

Asymptotics and profile likelihood, applied naively, give under-estimates of parameter uncertainties. Bootstrapping (storm-wise) provides more realistic estimates.

Extremal properties of storm peak significant wave height are modelled as a smooth function of storm peak direction.

Storm events are taken to be independent statistically for a given location. In estimating design criteria for a directional sector, we accommodate the effects of all sea states of all storms whose wave directions fall within that sector, regardless of the wave direction at storm peak, by quantifying the directional dissipation of every storm at every location and incorporating its influence on all directional sectors.

In the current application, the rate of occurrence of storms peaks is dependent on storm peak direction. In general, distributions of storm peaks will be directionally-dependent, even when extremal characteristics (e.g. GPD shape and scale) are independent of storm peak direction.

Directional design criteria provide more precise estimates of extreme offshore conditions enabling risk to be minimised given available resources.

Yet directional design criteria are not uniquely defined given only an omni-directional design criterion. We suggest a risk-cost criterion, which minimises design cost for a given omni-directional design specification, as an objective basis for estimation of directional criteria.

Comprehensive metocean data allows the effect of the storm direction, season and location to be accommodated in estimation of design criteria.

This work is currently being extended to incorporate both directional and spatial variation in extremal behaviour and design specification.

We are also assessing the value of stratifying extremes sea states into wind-sea and swell components for directional analysis.