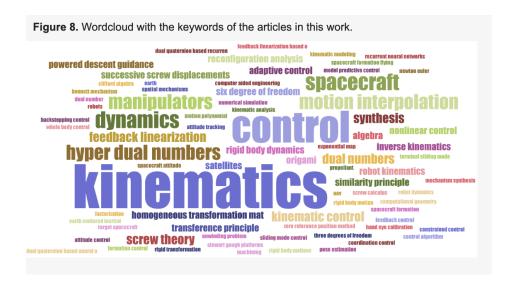Discuss the advantages of using quaternions for rotations in medical imaging compared to other methods. Provide examples of specific applications where quaternions offer significant benefits. Additionally, explain how quaternions are implemented in common medical imaging software.

Advantages of quaternions compared to other methods:

- **Prevent the loss of degree of freedom of orientation**: In the Euler system, different sequences of Euler angles can lead to the same final orientation; the Gimbal lock; which happens when two of the rotational axes align, causing the loss of one degree of freedom. This could lead to the loss of control of orientation in one direction. On the other hand, a rotation is associated with two distinct quaternions.
- **Compact Representation**: the representation of a rotation using a quaternion is more compact since it uses only four components while rotation matrices use nine components (3 x 3).
- **Numerical stability:** Floating-point machine precision errors and numerical drifts are easier to handle with quaternions by assuring that the quaternion used for rotation is unitary, rotation matrices require more advanced mathematical concepts like orthonormalization or condition number.
- **Simplicity of Representation of an Inverse Rotation:** q^{-1} represents a new rotation with same rotation axis but with negated angle.Raising a unit quaternion to an integer power means applying the same rotation multiple times. Negating all 4 of a quaternion component, corresponds to flipping its orientation in 4D space (but keeping its direction and length).
- **Spherical Linear Interpolation (Slerp) or Normalized Linear Interpolation (Nlerp):** are geodesic interpolations along an arc or a line; they are powerful type of linear interpolation used for 3D animations, easily implemented using quaternions.

For the different advantages listed above, quaternions are useful in medical applications where there is a need of exact localization like body kinetics [2]; rigid-body movement predictions, surgery navigation systems, and many other scientific domains as illustrated by a Wordcloud of a paper reviewing applications of quaternions:

Figure 8. Wordcloud with the keywords of the articles in this work.

In its most simple form, quaternions are implemented in popular software packages as the combination of a scalar and vector [4] [5] [6],and the arithmetic operations are implemented on top of this representation. In Nifti, if (p,q,r) is a unit 3-vector, the rotation of angle h about that direction is represented by the quaternion:

[a,b,c,d] = [cos(h/2), p*sin(h/2), q*sin(h/2), r*sin(h/2)].

[1]: https://splines.readthedocs.io/en/latest/index.html
[2]: Challis, John H. "Quaternions as a solution to determining the angular kinematics of human movement." *BMC biomedical engineering* vol. 2 5. 23 Mar. 2020, doi:10.1186/s42490-020-00039-z
[3]: Farias JG, De Pieri E, Martins D. A Review on the Applications of Dual Quaternions. *Machines*. 2024; 12(6):402. https://doi.org/10.3390/machines12060402
[4] Splines in Euclidean Space and Beyond
https://github.com/AudioSceneDescriptionFormat/splines/tree/58e77e1bf3473136b83d181a786e1d9eb6ebb8d2
[5]: Johnson, H. J., McCormick, M. M., & Ibanez, L. (2015). The ITK Software Guide: Design and Functionality of ITK, 4th Edition. Kitware:
The itk::QuaternionRigidTransform class implements a rigid transformation in 3D space. The rotational part of the transform is represented using a quaternion while the translation is represented with a vector
[6] Quaternion representation in Nifti: https://nifti.nimh.nih.gov/nifti-1/documentation/nifti1fields/nifti1fields_pages/quatern.html