Dr. Ardekani
EN.585.703.81.FA24 Applied Medical Image Processing
Module 2 Assignment
Johns Hopkins University
Student: Yves Greatti

Question 1

A convenient form of 2D Radon transform is to use the following equation:

$$\{\mathcal{R}\rho\}(t,\theta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho(x,y)\delta(x\cos\theta + y\sin\theta - t)dxdy \quad (1)$$

using this definition:

1. calculate $\{\mathcal{R}\rho\}(t,0)$ for $\rho(x,y) = \Pi\left(\frac{x}{a}\right)\Pi\left(\frac{y}{b}\right)$ (10 points), Where:

$$\Pi(x) = \begin{cases} 1, & \text{if } |x| < 1/2 \\ 0, & \text{otherwise} \end{cases}$$

For $\theta = 0$:

$$\{\mathcal{R}\rho\}(t,0) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho(x,y)\delta(x - t)dxdy \quad (2)$$

Since $\int_{-\infty}^{+\infty} \rho(x,y)\delta(x-t)dx = \rho(t\ ,y)$; then

$$\begin{aligned}
\{\mathcal{R}\rho\}(t,0) &= \int_{-\infty}^{+\infty} \Pi(\frac{t}{a})\Pi(\frac{y}{b})\ dy \\
&= \Pi(\frac{t}{a})\int_{-\infty}^{+\infty} \Pi(\frac{y}{b})\ dy \\
&= \Pi(\frac{t}{a})\int_{-\frac{b}{2}}^{\frac{b}{2}}\ dy \\
&= \Pi(\frac{t}{a})[y]_{-\frac{b}{2}}^{\frac{b}{2}} \\
&= b\ \cdot \Pi(\frac{t}{a})
\end{aligned}$$

2. Calculate and compare the Fourier transform of $\{\mathcal{R}\rho\}(t, 45°)$ for a square object defined by:

$$\rho(x,y) = \Pi\left(\frac{x}{2}\right)\Pi\left(\frac{y}{2}\right)$$

using direct approach and projection-slice theorem (20 points).

In both [a] and [b] show your work.

According to the projection-slice theorem:

$$\mathcal{F}\left(\{\mathcal{R}\rho\}\right)(t, 45°) = \mathcal{F}\left(\mathcal{R}\rho\right)(t, 45°)$$

We make the change of variables from x and y to u and v. With

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$\Rightarrow$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

With $\theta = 45°$

$$x = \frac{u - v}{\sqrt{2}}$$
$$y = \frac{u + v}{\sqrt{2}}$$

And

$$\text{Jacobian} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} = \cos\theta{\cdot}\cos\theta - (-\sin\theta){\cdot}\sin\theta = \cos^2\theta + \sin^2\theta = 1$$

$$\begin{aligned} \mathcal{R}f(t, 45°) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho(x,y)\delta(x\cos(45°) + y\sin(45°) - t)\,dx\,dy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Pi\left(\frac{x}{2}\right)\Pi\left(\frac{y}{2}\right)\delta\left(x\cos(45°) + y\sin(45°) - t\right)\,dx\,dy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Pi\left(\frac{u-v}{2\sqrt{2}}\right)\Pi\left(\frac{u+v}{2\sqrt{2}}\right)\delta\left(t - u\right)\,du\,dv \end{aligned}$$

$$
\begin{aligned}
\mathcal{R}f(t, 45^\circ) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Pi\left(\frac{u-v}{2\sqrt{2}}\right)\Pi\left(\frac{u+v}{2\sqrt{2}}\right)\delta(t-u)\,du\,dv \\
&= \int_{-\infty}^{\infty} \Pi\left(\frac{t-v}{2\sqrt{2}}\right)\Pi\left(\frac{t+v}{2\sqrt{2}}\right)\,dv \\
&= \text{[Further steps to evaluate the integral based on the properties of the } \Pi \text{ function]}
\end{aligned}
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