# [DOCUMENT TITLE]

[Document subtitle]

[School]
[Course title]

## Problem Statement

The Fuzzy C-Means (FCM) algorithm is an unsupervised clustering algorithm and is considered one of the most effective and widely used algorithms for medical image segmentation. It is like the traditional K-means algorithm but with "soft" memberships. Instead of binary memberships (0 or 1) that indicate whether a data point belongs to a cluster, each point in FCM has a weighted membership value, a number between 0 and 1. This value represents the degree of membership or probability of the point belonging to each cluster (Pseudocode 1).

**Step 1**: Determine the number of clusters c and $\varepsilon$

**Step 2**: Initialize the center of the cluster $v_i^{(0)}$ and $u_{ij}^{(0)}$

**Step 3**: k=1

**Step 4**: While $\left\|v_i^{(k)} - v_i^{(k-1)}\right\| > \epsilon$

Calculation of $u_{ij}^{(k)}$ and $v_i^{(k)}$ using equations (3-4)

$$u_{ij}^{(k)} = \frac{\left(1/\left\|x_j - v_i^{(k-1)}\right\|^2\right)^{\frac{1}{m-1}}}{\Sigma_{j=1}^{c}\left(1/\left\|x_j - v_i^{(k-1)}\right\|^2\right)^{1/m}}, \forall i = 1, 2, \ldots, c, j = 1, 2, \ldots, n \quad (3)$$

$$v_i^{(k)} = \frac{\Sigma_{j=1}^{n}\left(u_{ij}^{(k)}\right)^m x_j}{\Sigma_{j=1}^{N}\left(u_{ij}^{(k)}\right)^m}, \forall i = 1, 2, \ldots, c$$

k=k+1

$$(4)$$

**Step 5**: return cluster centers $v_i$ and membership function $u_{ij}$

**Pseudocode 1**: FCM Algorithm

Although the FCM algorithm is simple to implement, it has a few shortcomings such as sensitivity to the cluster center initializations, getting stuck in the local minima and low convergence rate.

Boulanouar et al. propose enhancing the quality of segmentation and the speed of convergence by using the Bat Algorithm for determining the initial cluster centers and defining a fitness function This fitness function combines intra-cluster distance with fuzzy cluster validity indices. They refer to the combined algorithm as **MFBA**.

## BAT Algorithm

The Bat Algorithm (BA) is a metaheuristic optimization technique inspired by natural processes. Specifically, it is inspired from the echolocation behavior of bats, which they use to sense distances. Bats hunting at night emit brief, intense sound pulses and analyze the returning echoes to detect obstacles or prey. Their unique auditory system enables them to determine both the size and location of objects with precision.
In the BA, the location of a bat $x_i$ , $x_i$ represents a potential solution to an optimization problem, evaluated by a fitness function that measures how close the bat is to the optimal solution (or "prey"). The goal is to optimize this fitness value, guiding the bat toward the optimal solution.
Bats fly randomly with velocity $v_i$ at position $x_i$ and emit sounds with loudness A or at varying frequency (fmin, fmax) to search for a prey.

| Parameter | Description |
|---|---|
| nBats | Number of bats |
| IterMax | Maximum number of iterations |
| $f_{min}$, $f_{max}$ | Minimum and maximum frequency |
| Loudness Coefficient | Constant parameter in range [0, 1] used to update the loudness of each bat |
| Gamma | Constant parameter in range [0, 1] used to decay pulse rate |

The steps of the algorithm could be summarized as follows:
**Step 1** - Initialize the BAT algorithm parameters:
Initialize randomly bat positions, set initial velocities to 0. Compute fitness values for each bat with initial position and data. Set up initial best position using initial bat positions.
**Step 2** – Update using best position X*, pulse frequency, the velocity, and position of the $i^{th}$ bat as follows as:

$$\boxed{\begin{aligned} f_i &= f_{\min} + (f_{\max} - f_{\min})\,\beta, \quad \beta \in [0,1], \\ V_i^{t+1} &= V_i^t + (X_i^t + X^*)\,f_i, \\ X_i^{t+1} &= X_i^t + V_i^t, \end{aligned}}$$

Where $V_i^t$, and $X_i^t$ are the velocity and position at time t (iteration t), $V_i^{t+1}$ and $X_i^{t+1}$ are the velocity and position at time t+1 (iteration t+1).

**Step 3** – If the random number is greater than $r_i$, a new solution for the bat is generated by the following equation:

$$X_{\text{new}} = X_{\text{old}} + \epsilon A^t,$$

where $\epsilon$ is a random number, $\epsilon \in [-1, 1]$, and $A^t$ represents the average loudness of all bats at time $t$.

**Step 4** - If the random number is lower than $A_i$ and $f(X_i) < f(X^*)$, the new solution is accepted. Next, update $A_i$ and $r_i$, respectively, as follows:

$$A_i^{t+1} = \alpha A_i^t,$$
$$r_i^t = r_i^0 \left[1 - e^{-\gamma t}\right],$$

where $A_i^{t+1}$ and $A_i^t$ denote the loudness at times $t$ and $t+1$, respectively; $r_i^0$ and $r_i^t$ are the initial pulse rate and pulse rate at time $t$, respectively, $\alpha$ is a constant parameter in range $[0, 1]$, $\gamma$ is a constant parameter, and $\gamma > 0$. As $t \to \infty$, $A_i^t \to 0$ and $r_i^t \to r_i^0$.

**Step 5.** Sort the bats based on their fitness and find the current optimal solution $X^*$.
**Step 6.** Return to Step 2 until the maximum number of iterations is reached; output the globally optimal solution.

## MFBAFCM

When the BAT algorithm and FCM algorithm are combined, the process involves two main steps. In the first step, the BAT algorithm selects the optimal initial clusters for the FCM algorithm. In the second step, the FCM algorithm refines these clusters to find the optimal cluster assignments and their centers. Both algorithms operate using the same cost function (Pseudocode 2).
Boulanouar et al. define as fitness function, which is minimized when the value of PC is high and the value of (Intra_cluster + SC) is low.

Fitness = (Intra_Cluster + SC) / PC

- **Intra Cluster Distance**: this metrics measures the compactness of the clusters. The goal is to minimize the distance between data points and their assigned cluster centers.
- **Partition Coefficient (PC)**: This metric quantifies the overlap between clusters, with higher values indicating better-defined clusters.
- **Classification Entropy (CE)**: Like PC, CE measures the fuzziness in cluster assignments. Lower values are preferred as they suggest more distinct cluster boundaries.

- **Partition Index (SC)**: This index measures cluster validity based on individual cluster characteristics, normalized by the fuzzy cardinality of each cluster. A higher SC value indicates better separation between clusters.
  To have more control and flexibility, we weight each term of the fitness function with scaling coefficient alpha, beta and zeta.

$$\boxed{\text{fitness} = \alpha \cdot \text{intraCluster} + \beta \cdot SC + \zeta \cdot \left(\frac{1}{PC} + CE\right)}$$

## Run test

We load an MRI image and focus on a specific slice of the brain to compare the clustering performance between segmenting the slice using the FCM algorithm alone and the combined BAT+FCM algorithm (MFBAFCM).
We apply median filtering to the image to smooth it and reduce the noise, followed by scaling the intensity values between 0 and 1 (Figure 1). Using the processed image, we create various feature vectors with a moving window approach, incorporating padding and filtering. To capture the intensity gradient landscape and preserve edges for tumor segmentation, we experimented with different filters, including Prewitt and Laplacian operators. Ultimately, we found that a simple 3x3 sliding kernel filter provided the best clustering performance. However, more in-depth exploration of window designs will be necessary in future work.
We tried different parameters for FCM and MFBA and use in final:

| Parameter | Value |
|---|---|
| nClusters | 15 |
| FCM fuzzy exponent | 2 |
| FCM max. iterations | 10 |
| FCM distance metric | Euclidean |
| FCM min. improvement | 10^(-5) |
| Fitness Alpha | 1 |
| Fitness Beta | 0.5 |
| Fitness Zeta | 1.5 |
| Bat number | 50 |
| Bat max. iterations | 10 |
| Bat $f_{min}$, $f_{max}$ | 0 and 2 |
| Bat loudness coefficient | 0.9 |
| Bat Gamma | 0.95 |

**Figure 1**: original and denoised slice.

- MFBAFCM clustering metrics are better than FCM: PC is larger, and CE, SC and S values of MFBAFCM are smallest than FCM. The proposed method MFBAFCM provides a better separation of the clusters.

| **Algorithm** | **PC** | **CE** | **SC** | **S** |
|---|---|---|---|---|
| **FCM** | 0.654 | 0.892 | 0.001 | 1771.868 |
| **BAT + FCM** | 0.667 | 0.865 | 0.001 | 1.759 |

Notice how the metric SSS is smaller with the BAT+FCM algorithm compared to FCM alone, indicating that MFBAFCM achieves better clustering performance.

**Fuzzy Separation Index (S)**: This metric assesses the separation quality of clusters. Thus, minimizing S ensures that the clusters are both compact and well-separated, which is the desired goal in clustering.

S = (Intra-cluster-compactness) / [N . minimum inter-cluster-distance]

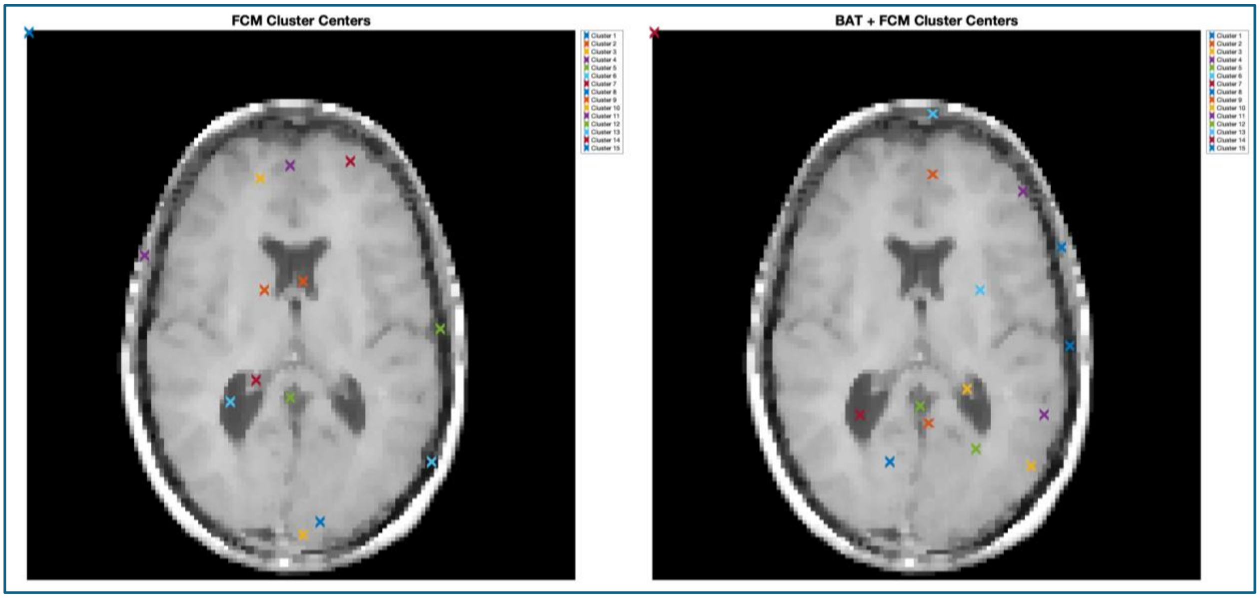


**Figure 2**: cluster centers on FCM segmented image (left) and BAT+FCM (right).

When looking at figure 2, we observe tha:

- **FCM (left image):** The cluster centers appear to be more dispersed, with some potentially located in less relevant regions of the brain slice. The cluster centers appear less compact and more spread out, possibly leading to higher intra-cluster distances and reduced clustering quality. This dispersion may indicate that FCM alone is more sensitive to initial conditions and lacks the optimization power to fine-tune cluster center placement effectively.
- **BAT+FCM (right image)**: The cluster centers are more concentrated in relevant regions, with fewer centers placed in areas without significant image features. The cluster centers are more compact and strategically positioned, which likely reduces intra-cluster distances and improves separation between clusters. This suggests that BAT+FCM provides a more optimized initialization, allowing the algorithm to focus on meaningful regions of the image.

Looking at side by side the segmented slice using FCM alone or BAT+FCM; confirms further that with BAT+FCM, the segmentation is more refined, with sharper and more distinct boundaries between clusters, indicating better clustering performance (Figure 3). The results suggest that the BAT+FCM algorithm (MFBAFCM) leads to better-defined clusters and is more suitable for tasks requiring high-quality segmentation, such as medical image analysis.

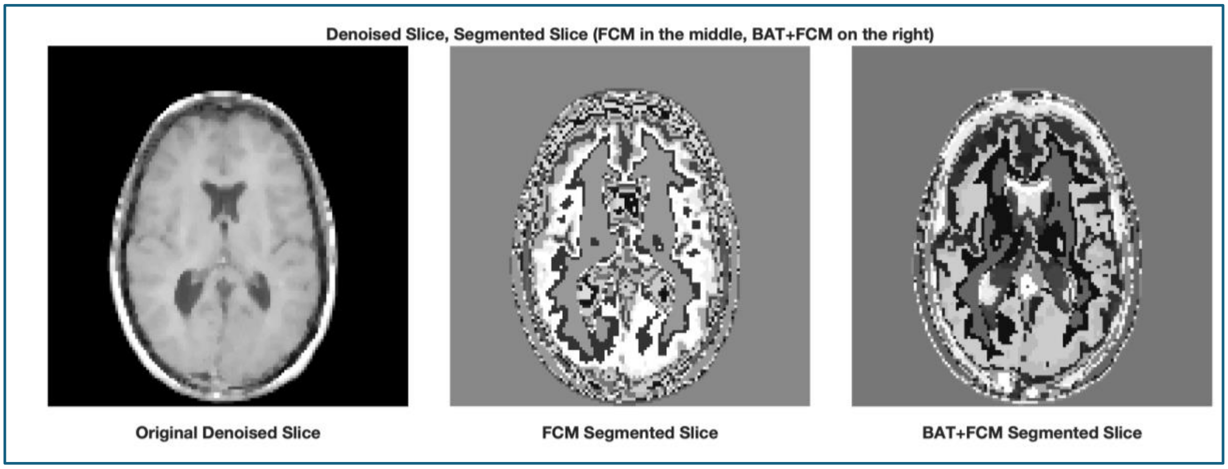
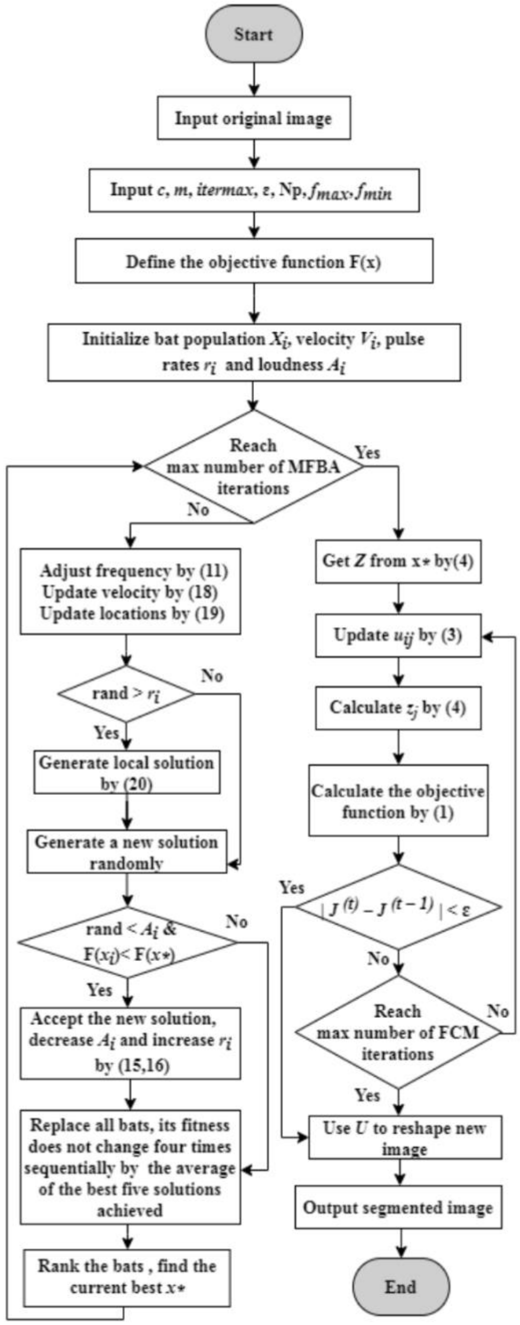



**Figure 3**: denoised slice (left), denoised segmented slice with FCM (middle), and MFBAFCM (right)




**Pseudocode 2:** MFBAFCM flow chart

[1] Gandomi A. H. and Yang X.-S., Chaotic bat algorithm, *Journal of Computational Science*. (2014) **5**, no. 2, 224–232, https://doi.org/10.1016/j.jocs.2013.10.002, 2-s2.0-84897588368.