Applied Medical Image Processing Lecture Notes

1 Image Contrast Enhancement:

1.1 Histogram Specification

Medical Image histograms typically do not follow a uniform distribution. It is more useful to find a way to match the histogram of one image to other distributions such as Gaussian.

In histogram specification, the idea is to modify an image, I_A to a get a new image (I'_A) such that P_A(Cumulative Distribution Function - CDF of I_A) matches a reference CDF (P_R) such as Gausssian CDF through a mapping. Assuming that there are K possible intensity levels, after the mapping the CDF of original image should roughly match the CDF of the reference distribution. Therefore this follows:

$P_{A'}(i) \approx P_R(i)$ for $0 \leqslant i < K$

In this process, pixel intensity with value a in the original image will be mapped to a new pixel intensity value a' using the following mapping (see Fig. 2):

$a' = P_R^{-1}\left(P_A(a)\right)$

In practice, since we are working with discrete functions, it is more convenient to use a piecewise linear function for mapping between intensity values and CDFs (see Fig. 2). We select a set of landmarks $\mathcal{L}$ that map each intensity a_k to its corresponding normalized CDF q_k. Intensity values range between 0 and $K-1$ (Ex. for 8-bit image we have intensity values within 0 - 255, K here is 256).

$$\mathcal{L} = \left[\langle a_0, q_0\rangle, \langle a_1, q_1\rangle \ldots \langle a_{N,q_N}\rangle\right]$$
$$0 \leqslant a_k < K, a_k < a_{k+1}, 0 \leqslant q_k \leq 1$$
$$\begin{cases} a_0, q_0 & \longrightarrow 0, q_0 \\ a_N, q_N & \longrightarrow K-1, 1 \end{cases}$$

For this mapping to be in invertible: $q_K < q_{k+1}$ for $0 \leqslant k < N$

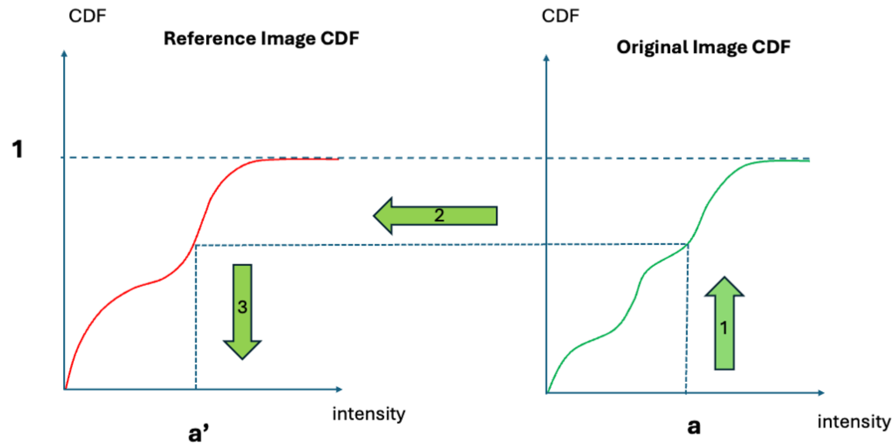

Figure 1: Mapping from the original image CDF to the reference image CDF.

First we need to map each intensity value a in the original image to its corresponding normalized CDF value. Then, we should fine the new intensity value a' in the normalized reference CDF that matched the CDF value of a. Using piecewise functions we can compute CDF of an arbitrary intensity i as:

$$P_L(i) : \begin{cases} q_m + (i - a_m) \cdot \frac{(q_{m+1} - q_m)}{(a_{m+1} - a_m)}, & \text{if } 0 \leqslant i < k - 1 \\ 1 & \text{if } i = K - 1 \end{cases}$$
$$m = \max\left\{j \in [0, N-1] \mid a_j \leqslant i\right\}$$

here:

$$\langle a_m, q_m \rangle \rightarrow \langle a_{m+1}, q_{m+1} \rangle$$

represents the index of line segment that contains arbitrary intensity i.

we also need $P_L^{-1}(b)$ for $b \in [0, 1]$ which maps a CDF value to an intensity value, inverse function.

$$P_L^{-1}(b) \begin{cases} 0 & 0 \leqslant b \leqslant p_L(0) \\ a_n + (b - q_n)\,\frac{(a_{n+1} - a_n)}{q_{n+1} - q_n} & p_L(0) \leqslant b < 1 \\ K - 1 & b \geqslant 1 \end{cases}$$
$$n = \max\{j \in [0, N-]] \mid q_j \leqslant b\}$$
$$< a_n, q_n > \rightarrow < a_{n+1}, q_{n+1} >$$
$$f_{\text{hs}}(a) = P_L^{-1}(P_A(a)) \qquad \text{for } 0 \leqslant a < K$$

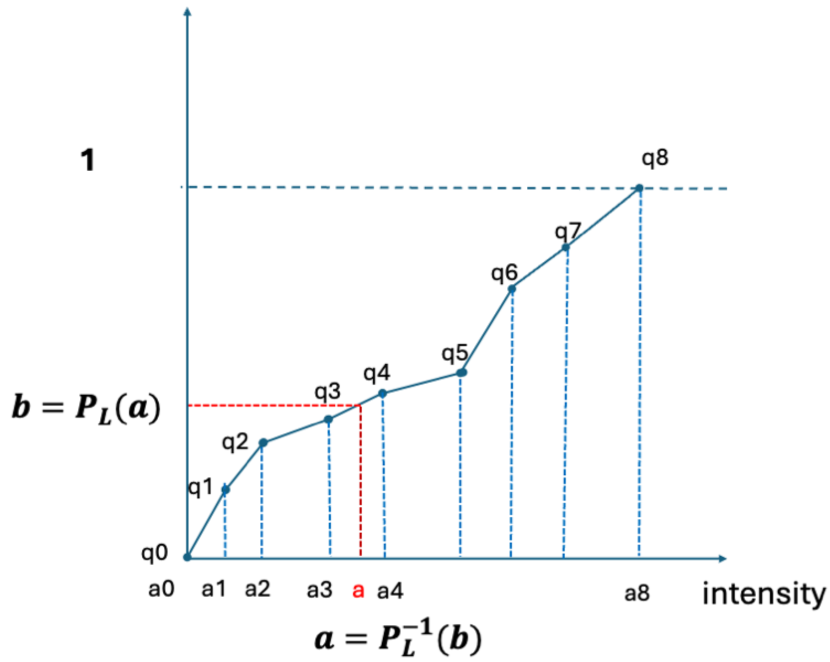

Figure 2: Piecewise linear mapping.

Putting everything together the process of histogram specification can be summarized as follows:

Algorithm 1: Pseudo code

Input : (h_A: histogram of the original image, $\mathscr{L}_R$: reference distribution function given a sequence of $N+1$ control points: $\mathcal{L}_R = [\langle a_0, q_0\rangle, \langle a_1, q_1\rangle, \ldots, \langle a_N, q_N\rangle]$;

with $0 \leq a_k < K$ and $0 \leq a_k \leq 1$;

Output: f_{hs}: Updated map of intensity values

$K \leftarrow \text{size}(h_A)$;

$P_A \longleftarrow \text{CDF}(h_A)$;

Create a table $f_{h_s}[]$ of size K;

for $a \leftarrow 0 \ldots (K-1)$ **do**

  $b \leftarrow P_A(a)$;

  if $b \leqslant a_0$ **then**

    $a' \leftarrow 0$;

  else if $b \geqslant 1$ **then**

    $a' \leftarrow K-1$;

  else

    $n \leftarrow N-1$;

    while $(n \geqslant 0) \wedge (q_n > b)$ **do**

      $a' \longleftarrow a_n + (b - q_n) \cdot \frac{(a_{n+1} - a_n)}{(q_{n+1} - q_n)}$;

      $f_{hs}[a] \longleftarrow a'$;

    end

  end

end

return f_{hs};

1.2 standardizing MR Image Intensity scale:

Histograms of medical images typically do not follow a simple Gaussian distribution; instead, they exhibit a more complex distribution. Therefore, it is more desirable to match the histogram of medical images with an arbitrary distribution to another image histogram. Many image processing algorithms perform better when the images exhibit similar intensity distributions. It is possible to generate a reference/standard histogram profile from a set of medical images that are all collected using similar structures, modalities, and image protocols. This standard histogram can then be used to generate a new set of medical images that all have a similar intensity distribution.

Let's define the following notations:

- $\mathcal{P}$: Set of MRI protocols
- $\mathcal{D}$: Set of body regions
- $V = (v, g)$: volume images
  - $v \in \mathbb{R}^3$, image coordinate
  - $g(v)$: image intensity
  - $v \in V$ such that $g(v) \geqslant 0$
- V_{PD}: all possible images that can be extracted for a particular MRI protocol and body region, such that:
  - $p \in \mathcal{P}$
  - $D \in \mathcal{D}$
- $\mathcal{H} = (G, h)$: histogram of all possible intensity values
  - $x \in G$, x is an image intensity within a range defined by G, all acceptable intensity values in the image
  - $h(x)$: histogram of voxels $v \in V$ such that $g(v) = x$

$$- m_1 = \min\{g(v) \mid v \in V, g(v) > 0\}$$
$$- m_2 = \max\{g(v) \mid v \in V, g(v) > 0\}$$

Histogram normalization/standardization constitutes the following two steps:

1. Generating standard histogram landmarks from a set of training histograms
2. Transformation from individual histograms to the standard histogram space.

For the training step, let's assume that min and max intensity on the standard scale for the range of intensity of interest (IOI) be s_1 and s_2, then for the training step, we will use algorithm 2 (see Fig. 3), followed by the transformation step (algorithm 3).

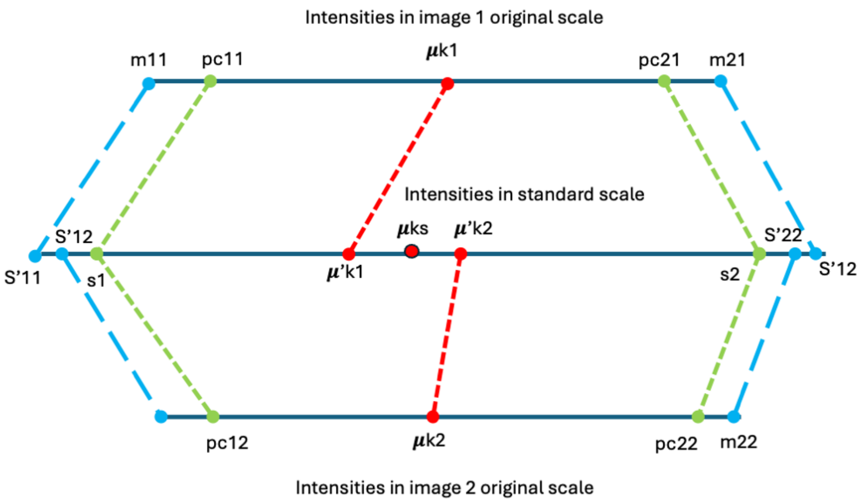

Figure 3: Image scale to standard scale mapping landmarks

Algorithm 2: Standardization

Input : A set of images V_j ($j = 1, \ldots, N$) and histogram parameters pc_1, pc_2, s_1, s_2 and landmark points $\mathcal{L} \in \{L_1, L_2, \ldots, L_M\}$

Output: $\{\mu_{\text{ks}} \mid 1 \leqslant k \leqslant n\}$

for $j = 1$ **to** N **do**

  Compute H_j of V_j;

  Determine P_{1j} and P_{2j} corresponding to pc_1 and pc_2 and find $\mu_{1j}, \mu_{2j}, \ldots, \mu_{Mj}$ in H_j;

  Map $[p_{1j}, p_{2j}]$ of H_j to $[s_1, s_2]$ (linear map);

  Find the new mapped landmarks;

  Given $\mu_j \in [P_{1j}, P_{2j}]$;

  $\mu'_j = s_1 + \frac{\mu_j - P_{1j}}{P_{2j} - P_{1j}}(s_2 - s_1)$;

end

Calculate rounded means using all subjects j mappings:

$$\mu'_{1s}, \mu'_{2s,\ldots}, \mu'_{Ms}$$

$$\mu_{ks} = \left\lfloor \frac{1}{N} \sum_{j=1}^{N} \mu'_{kj} \right\rfloor \quad \text{for } k = 1, \ldots, M$$

Algorithm 3: Transformation

Input : An Image $V_i \in V_{PD}$, $pc_1, pc_2, s_1, s_2, \mu_{1s}, \mu_{2s}, \ldots, \mu_{ms}$
Output: Transformed image V_{si} or LUT that stores intensity transformation

1 **Begin:**

1. Compute $H_i = (G_i, h_i)$ of V_i
2. Determine P_{1i}, P_{2i} corresponding to pc_1, pc_2.
3. Map sections of the scale of H_i linearly to the standard scale (see Fig. 4)
4. Map the intensity value of every voxel

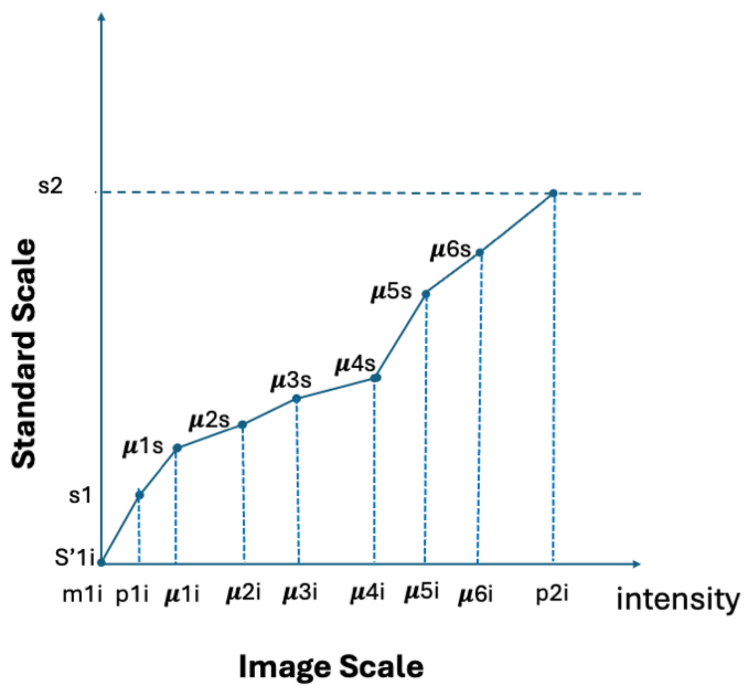

Figure 4: Image scale to standard scale mapping using landmarks

Mathematically, we will use the following equation to linearly map intensity values from the image to standard scale.

$$T_{V_i(x)} \begin{cases} \left\lceil \mu_{1s} + (x - \mu_{1i}) \frac{s_1 - \mu_{1s}}{P_{1i} - \mu_{1i}} \right\rceil & p_{1i} \leqslant x < \mu_{1i} \\ \vdots & \end{cases}$$

Bibliography

[1] Wilhelm Burger, Mark J. Burge, Digital Image Processing: An Algorithmic Introduction using Java, Chapter 5 - Point Operations, 2007

[2] Nyúl LG, Udupa JK. On standardizing the MR image intensity scale. Magnetic Resonance in Medicine. 1999;42(6):1072–1081.

[3] Nyul LG, Udupa JK, Xuan Zhang. New variants of a method of MRI scale standardization. IEEE Transactions on Medical Imaging. 2000;19(2):143–150.