Research and discuss alternative edge-preserving smoothing algorithms for medical imaging, such as the Total Variation (TV) denoising and the Guided Filter.

- How do these algorithms compare to methods like Geometry Preserving Diffusion Filters and unsharp masking in terms of edge preservation, noise reduction, and computational efficiency?
- Evaluate the potential clinical benefits and challenges of implementing these new algorithms in medical imaging, considering factors such as image quality, diagnostic accuracy, and integration into existing imaging systems.
- What are the future directions for research in edge-preserving smoothing techniques, and how might these innovations improve patient outcomes in medical diagnostics?

Total variation

Since edges or noise are defined as a sharp variation in intensity, total variation (TV) algorithm, approximates the input noisy image Y with an image X but which has limited variation:

$$\min_{X} \| X - Y \|^2 + \lambda \, TV(X)$$

By minimizing the total variation in an image, the algorithm tends to ignore minor, localized fluctuations (noise), without affecting sharp transitions like edges.

- The first part of the algorithm, $\| X- Y\|^2$ aims to approximate the input image Y with X
- The second part of the filter includes the regularization parameter, λ, which moderates the effect of the regularization: TV(X); TV(X) controls the characteristics of the estimated image X.

The total variation of a color image $\mathbf{X}$ can be expressed as

$$TV(\mathbf{X}) = \sum_{i,j \in \mathcal{N}} \|\mathbf{x}_i - \mathbf{x}_j\|_p^q$$

where $\mathcal{N}$ defines the pixel neighborhood (usually the horizontal and vertical cent pixels) and $\|\cdot\|_p^q$ is the ℓ_p norm to the power of q.

- Classical ℓ_1 TV computed independently on each color component.

$$\|\mathbf{x}\|_1 = \sum_k \|\mathbf{x}_k\|_1 \quad (p = 1, q = 1)$$

- ℓ_2 TV computes the Euclidean norm of the vector.

$$\|\mathbf{x}\|_2 = \left(\sum_k x_k^2\right)^{\frac{1}{2}} \quad (p = 2, q = 1)$$

- Squared ℓ_2 TV computes the squared Euclidean norm of the vector.

$$\|\mathbf{x}\|_2 = \sum_k x_k^2 \quad (p = 2, q = 2)$$

- The l_1 or l_2 TV keep both sharp edges. Mathematically for no-smooth functions, TV could be unstable, which could lead to artifacts around sharp edges. l_1 could also introduce artifacts as indicated in Fig. 1.
- TV could be computationally intensive and there have been a variety of approaches to improve its efficiency (Bregman method) or using TV with fast-gradient algorithms (FISTA [1])

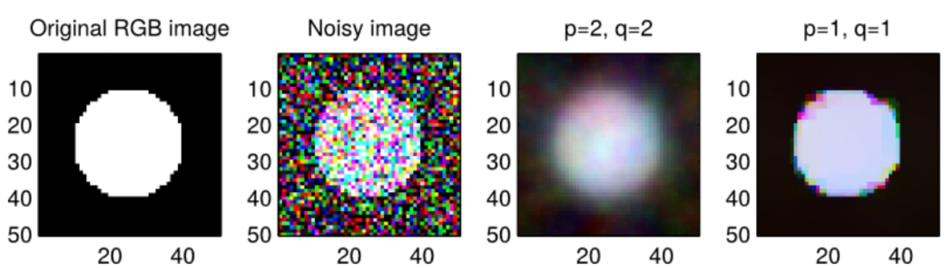

Fig. 1: this figure shows that l_2 TV can lead to a smooth image at the cost of losing sharp edges and details (3rd plot), while l_1TV can lead to color artifacts around the edges (4th plot).

Guided Filter

- For a given pixel, it finds, in a guidance image **G**, the corresponding pixel and its corresponding spatial neighborhood:
  - If the guidance image **G**, is the same as the input image to filter, the output image remains the same as the input image.
  - Otherwise, the output image is, locally, a linear transformation of the guidance image G.
- The linear optimization aims in minimizing the error (least squares method) between the output image and the input image.
- Parameter free, effective and efficient.
- Can introduce blurry effects and artifacts.

TV denoising and guided filters by removing noise in medical images, maintaining the edges, improving low-contrast regions; enhance the quality of images; which lead to better visualization of subtle anatomical features, and pathologies. Higher-quality medical images, in turn, contribute to:

- Increase diagnostic accuracy
- Reduced radiation exposure
- Improve planning of treatments and surgery

TV denoising minimizes the total variation of the image and guided filters use a template image (the guiding image); in term of edge handling, they might be less sensitive to edge structure compared to gradient filter:

- Geometry Preserving Diffusion Filters calcute for each pixel (u,v) of the image eigenvalues corresponding to largest variation in image gradient and its perpendicular direction
- Gradient filters will use gradient to calculate the edge amplitude and its direction or perform edge sharpening using the second derivative, or Laplacian to enhance pixels around edge boundaries. One fundamental problem of these algorithms is that they detect sharp edges and have difficulties to handle edges with smaller intensity transition.

TV denoising and guided filters, require relatively a few parameters to tune. This simplicity makes them easier to integrate into existing systems. However, the limited number of parameters can also restrict the filters flexibility and generalizability, as they may not adapt well to a variety of noise patterns or edge characteristics.

In contrast, deep learning models trained on large datasets can capture diverse noise patterns and learn more robust edge-preserving behaviors. These models benefit from exposure to big datasets, allowing them to generalize better across different types of images.

Future approaches may involve hybrid models, where deep learning methods learn from simpler models like TV denoising. This combination could enable the network to apply first principles while leveraging its adaptive capabilities, potentially leading to more effective denoising across diverse imaging data.

[1] A. Beck and M. Teboulle, "Fast Gradient-Based Algorithms for Constrained Total Variation Image Denoising and Deblurring Problems," in IEEE Transactions on Image Processing, vol. 18, no. 11, pp. 2419-2434, Nov. 2009, doi: 10.1109/TIP.2009.2028250. keywords: {Image denoising;Iterative algorithms;TV;Deburring;Acceleration;Noise reduction;Iterative methods;Convergence;Anisotropic magnetoresistance;Image restoration;Convex optimization;fast gradient-based methods;image deblurring;image denoising;total variation},