Coordinate System Lecture Notes

Applied Medical Image Processing

Coordinate Systems

Examples of Anatomical Coordinate System

LPS (left, posterior, superior): In this coordinate system positive x, y, and z axes are defined as follows (Fig. 1):

$$\begin{array}{ll} +x & Right \rightarrow Left \\ +y & Anterior \rightarrow Posterior \\ +z & Inferior \rightarrow Superior \end{array}$$

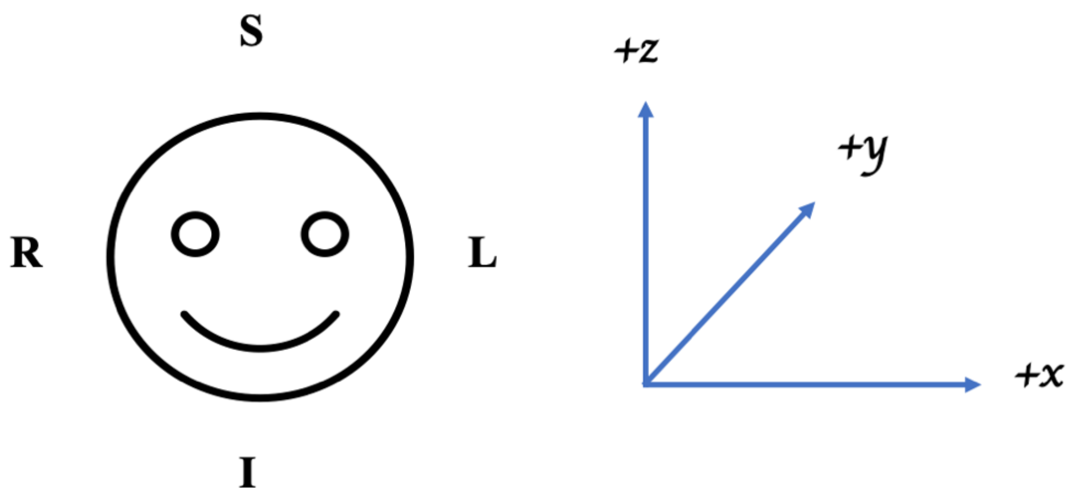

Fig. 1: LPS (right hand coordinate system)

RAS (Right, Anterior, Superior): This is coordinate system is used as neurological convention in which positive x, y, and z axes are defined as follows (Fig. 2):

$$\begin{array}{ll} +x & Left \rightarrow Right \\ +y & Posterior \rightarrow Anterior \\ +z & Inferior \rightarrow Superior \end{array}$$

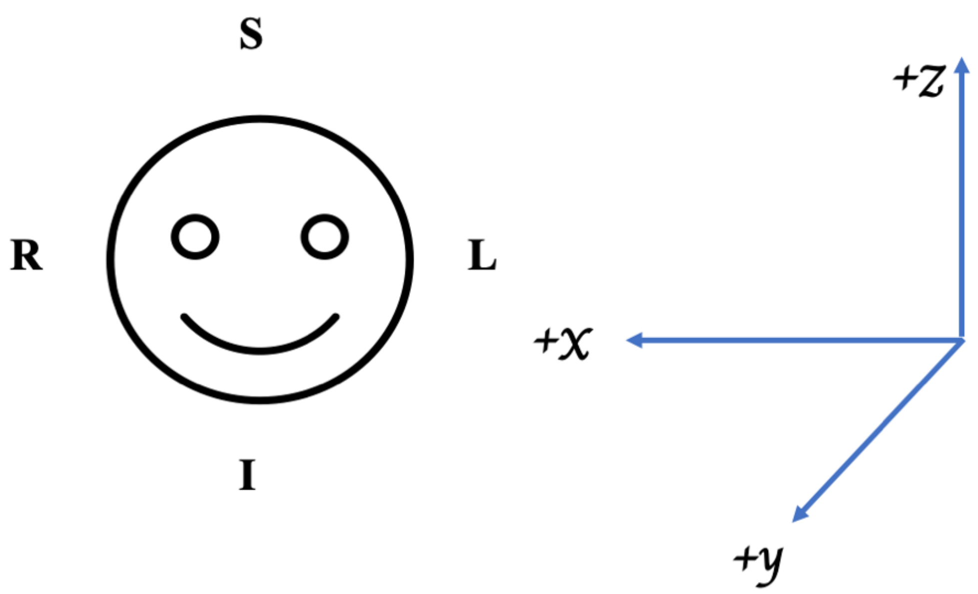

Fig. 2: RAS (right hand coordinate system)

LAS (Left, Anterior, Superior): This is coordinate system is used as radiological convention in which positive x, y, and z axes are defined as follows (Fig. 3):

$$\begin{array}{ll} +x & Right \rightarrow Left \\ +y & Posterior \rightarrow Anterior \\ +z & Inferior \rightarrow Superior \end{array}$$

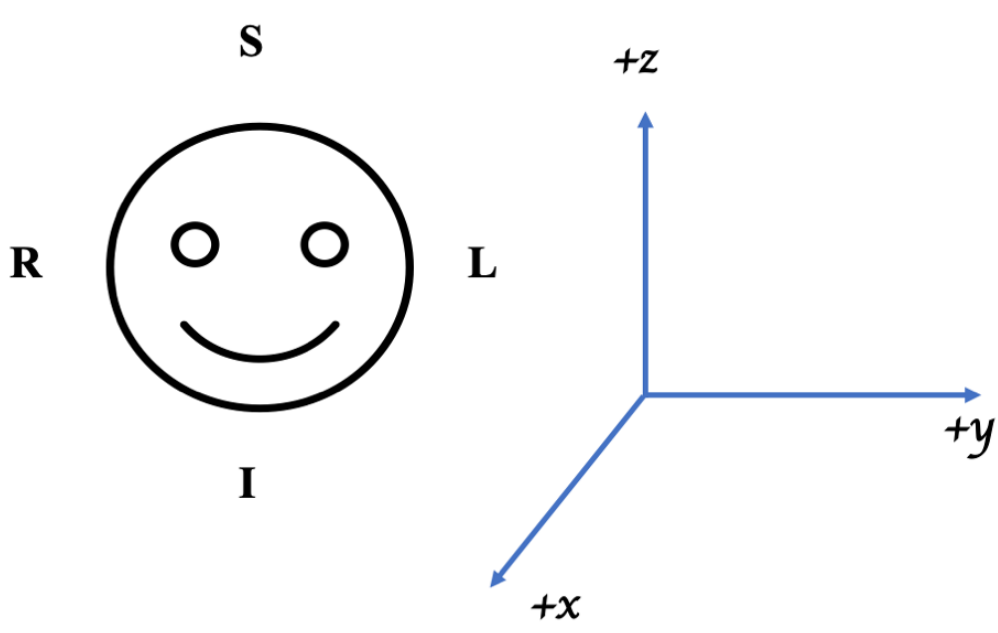

Fig. 3: LAS (Left hand coordinate system)

In general, voxels in a sequence travel in a row, one raw after another, and one slice after another slice. Directions of rows and slices depend on many factors. For example, in an axial slice using LAS system, voxels are ordered from R-L, to store a raw, rows are ordered from P-A to store a slice. Series are stored from I-S.

DICOM Coordinate System

In DICOM images, tag 0020,0032 defines the x, y, and z coordinates of the top left pixel in the scanner system and tag 0020,0037 defines the direction of cosine vectors along the the first row and column.

For example, an arbitrary vector $\vec{v}$ in a 3D coordinate system can be defined as:

$$\vec{v} = v_x\overrightarrow{e_x} + v_y\overrightarrow{e_y} + v_z\overrightarrow{e_z}$$

here $\overrightarrow{e_{xyz}}$ are unit vectors defining the basis of coordinate system.

Then, direction cosines are defined as:

$$\alpha = \frac{\vec{v} \cdot \overrightarrow{e_x}}{|\vec{v}|}$$

$$= \frac{v_x}{\sqrt{v_x^2 + v_y^2 + v_z^2}}$$

$$\beta = \frac{\vec{v} \cdot \overrightarrow{e_y}}{|\vec{v}|}$$

$$= \frac{v_y}{\sqrt{v_x^2 + v_y^2 + v_z^2}}$$

$$\gamma = \frac{\vec{v} \cdot \overrightarrow{e_z}}{|\vec{v}|}$$

$$= \frac{v_z}{\sqrt{v_x^2 + v_y^2 + v_z^2}}$$

if $|\vec{v}| = 1$ then:

$$\alpha = v_x$$
$$\beta = v_y$$
$$\gamma = v_z$$

To convert between patient and scanner coordinate systems using the DICOM coordinate system, we could use the following convention. Let's assume that the data in a DICOM coordinate system is stored in memory in such a way that the top left corner of a 2D array is the first data and i index starting from 0 and incremented by 1 along columns and j index similarly starts from 0 and incremented from one row to another. In this setup, positive row axis is from left to right and positive column axis is defined from top to bottom. If we define the patient coordinate system by a vector $\vec{p}$ as follows:

$$\vec{P} = \left[P_x P_y P_z\right]^T$$

Then patient and image coordinate systems are correlated to each other as follows:

$$\begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = \begin{bmatrix} x_x \Delta i & y_x \Delta j & 0 & s_x \\ x_y \Delta i & y_y \Delta j & 0 & s_y \\ x_z \Delta i & y_z \Delta_j & 0 & s_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ j \\ 0 \\ 1 \end{bmatrix}$$

Here:

S_{xyz}: The 3 values of Image position attribute stored in DICOM tag code $(0020{,}0032)$ which is the location in mm from the origin of the scanner and represents the first pixel at top left corner in the image.

X_{xyz} : First row direction cosine which is stored at DICOM tag attribute $(0020{,}0.37)$
Y_{xyz} : First column direction cosine which is stored at DICOM tag attribute $(0020{,}0.37)$
i: index of column
j: index of row
Δi: Pixel resolution along the columns. This information is stored in DICOM tag attribute $(0028{,}0030)$ which is referred to as pixel spacing.
Δj: Pixel resolution along the rows. This information is stored in DICOM tag attribute $(0028{,}0030)$ which is referred to as pixel spacing.

NIFTI file Coordinate System

The main information regarding how the coordinate system is defined in NIFTI files are stored in a header variable called dimension (dim). This is an array with the following format:

$dim[0]$: Stores the number of dimensions in the image a value between 1 and 7
$dim[1-3]$: reserved for x, y, and z coordinates.
$dim[4]$: reserved for time coordinates.
$dim[5-7]$: reserved for anything else.
Voxel indexing is based on i, j, k variables, where i takes values from 0 to $dim[1]-1$, j takes values from 0 to $dim[2]-1$, and k takes values from $dim[3]-1$. $pixdim[i]$ is another important variable stored in the NIFTI file header that stores voxel width size along dimension i. Units are specified in "xyz t-units" variable. As an example, consider the following functional MRI data stored as NIFTI:

$$\begin{aligned} &\mathrm{dim}[0] = 4 \\ &\mathrm{dim}[1] = 64, \text{pixdim}[1] = 3.75, \mathrm{NIFT\_UNITS\_mm} \\ &\mathrm{dim}[2] = 64, \text{pixdim}[2] = 3.75, \mathrm{NIFT\_UNITS\_mm} \\ &\mathrm{dim}[3] = 20, \text{pixdim}[3] = 5.0, \mathrm{NIFT\_UNITS\_mm} \\ &\mathrm{dim}[4] = 120, \text{pixdim}[4] = 2.0, \mathrm{NIFT\_UNITS\_SEC} \end{aligned}$$

In NIFTI, there are 3 different methods to define coordinate systems. The type of method is used to define the coordinate system is stored in header file using a variable named *qform*. For Method 1, this variable is zero. Method 1 is not a recommended method since it uses the Analyze 7.5 format coordinate system. In this method converting between i, j, k indices in the image and the real-world coordinate system is expressed by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} pixdim[1] & 0 & 0 \\ 0 & pixdim[2] & 0 \\ 0 & 0 & pixdim[3] \end{bmatrix} \begin{bmatrix} i \\ j \\ k \end{bmatrix}$$

This method only incorporates pixel resolution information and does not include orientation information. Please note that the way this data is stored in memory allows the index i varies most rapidly followed by j and k indices. Accordingly, voxel i, j, k information is stored at location $(i + j \cdot dim[1] + k \cdot dim[1] \cdot dim[2]) \cdot bitpixel/8$

For method 2 *qform* code is greater than 0 and the transformation matrix between image coordinate i, j, k and the world coordinate is defined by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} \text{pixdim } [1] \cdot i \\ \text{pixdim } [2] \cdot j \\ \text{qfac} \cdot \text{ pixdim } [3] \cdot k \end{bmatrix} + \begin{bmatrix} \text{qoffset } x \\ \text{qoffset } y \\ \text{qoffset } z \end{bmatrix}$$

Here *qfac* takes two values $-1{,}1$ and is included to ensure coordinate system are not flipped. For example, left and right sides of anatomy in image is not incorrectly flipped. As an example, if the data is stored based on the RAS coordinate system in the anatomical coordinate system, then

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the determinant of R matrix is equal to -1 resulting in flipping of coordinate system. If we use qfac $= -1$, then

$$R\colon \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\colon \quad |R| = 1$$

Resources

Slicer Community. (n.d.). *Coordinate systems*. 3D Slicer. https://slicer.readthedocs.io/en/latest/user_guide/coordinate_systems.html

National Institutes of Health. (n.d.). *Neuroimaging informatics technology initiative*. https://nifti.nimh.nih.gov/