Image Enhancement Lecture Notes

Applied Medical Image Processing

Image Enhancement

Image enhancement is typically used to refine a given image in order to:

1. desired image features become easier for the human visual system.
2. or more likely to be detected by an automated image analysis system.

Image enhancement can be used to improve observation of details in an image, when it suffers from low SNR, low contrast, or low dynamic intensity range. To improve or enhance the image quality, we need to define measurable quantities such as contrast.

Image Contrast:

To quantify image contrast, we should typically know what is object and what is background, which are usually unknown, therefore it is desirable prior to the analysis to use object independent metrics. What follows present few quantitative metrics that can measure image contrast.

Michelson Contrast Metric:

One example of global contrast metric is called Michelson and is defined as follows:

$$C_{\text{Michelson}} = \frac{l_{\text{max}} - l_{\text{min}}}{l_{\text{max}} - l_{\text{min}}} \text{ or } \frac{\text{difference in intensity}}{\text{average intensity}}$$

Where:

l_{max}: highest intensity value in image
l_{min}: lowest intensity value in image

Assumptions are:

The number of foreground pixels is approximately equal to the number of background pixels.

Note that $C_{Michelson} \in \{0,1\}$ for 1 representing a full range of intensity

This metric is useful to determine inefficient use of available intensity. For example for an under exposed image, the majority of pixels are less than a threshold and very few with I_{max} value. This image can benefit from global contrast enhancement.

Root Mean Squared Contrast Metric:

An alternative metric, is root mean squared contrast, which takes all the pixels into account, however does not differentiate well between different intensity distributions. for $2D$ image, it is defines as:

$$\bar{l} = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} l\,(i,j)$$

$$C_{\text{rms}} \;=\; \sqrt{\frac{1}{MN-1}\sum_{i=0}^{M-11}\sum_{j=0}^{N-1}\left(l(i,j)-\bar{l}\right)^2}$$

Entropy:

This metric incorporates histogram characteristics into the contrast measure. Histograms represent frequency of intensity values in an image. For an image:

$$I(m,n) \in [0,k-1]$$

Ex. $k = 2^8 = 256$

Then $h(i) =$ Number of pixels in I with intensity value of " i " where $0 \le i < K$

or $h(i) = \mathrm{card}\{(m,n) \mid I(m,n) = i\}$
or

$$\left\{ h(i) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} \delta\,(I(m,n) - i) \right.$$

When dealing with higher bits images, such as 32 or 64 bits, it is possible to generate histograms using a range of intensities or binning.

$$h(j) = \mathrm{card}\left\{(m,n) \mid a_j \le I(m,n) < a_{j+1}\right\}$$

for $0 \le j < B$ number of bins.

bin size: $KB = K/B$ s.t. $a_j = j \cdot K/B = j \cdot K_B$

Ex.

Image : $14 - bit$

$$B = 256$$

available intensity range : $0 \to 2^{14} - 1$

K_B length $\frac{2^{14}}{256} = 64$

$$
\begin{aligned}
&a_0 = 0, \quad a_1 = 64, a_2 = 128, \ldots, a_{255} = 16{,}320 \\
&a_{256} = \alpha_B = 2^{14} = 16384 = K \\
&h(0) \quad 0 \leq I(m,n) < 64 \\
&h(1) \quad 64 \leq I(m,n) < 128 \\
&\vdots \\
&h(255) \quad 16320 \leq I(m,n) < 16384
\end{aligned}
$$

We can generate a normalized histogram as

$$h(j)_{norm} = \frac{h(j)}{\sum_k h\,(k)}$$

This provides the probability of intensity j to appear in the image. So if $h(20) = 0.05$, then there is 5% probability of a randomly selected pixel to have an intensity value of 20.

Entropy encodes average information capacity of each pixel. Information stored in a pixel with a particular intensity value is inversely proportional to the probability of its occurrence. If information is stored as a binary number, then the number of required digits can be estimated as:

stored information of intensity " j "

$$\text{stored information of intensity } j = \log_2 \frac{1}{h_{\text{norm}}\,(j)} = -\log_2 h_{\text{norm}}\,(j)$$

Then the total number of digits needed to encode whole image is equal to:

$$-\sum_{k=I_{\min}}^{I_{\max}} h_{\text{norm}}\,(k) \log_2 h_{\text{norm}}\,(k)$$

Contrast entropy can be defined as the average signal length needed.

$$C_{\text{entropy}} = -\frac{1}{MN} \sum_{k=I_{\text{min}}}^{k=I_{\text{max}}} h_{\text{norm}}\,(k) \log_2 h_{\text{nam}}\,(k)$$

Note that, entropy still does not account for intensity differences between some background and foreground objects.

Gray-level-co-occurance Matrix

Another metric is called Gray-level-co-occurance which is defines ad:

$$C_d(i,j) = \sum_{x=1}^{N}\sum_{y=1}^{m}\begin{cases}1 & \text{if } I(x,y) = i \text{ and } I(x+d,y+d) = j\\ 0 & \text{otherwise}\end{cases}$$

This calculates the rate of co-occurring intensity values in a given neighborhood. We could use a more specific definition by incorporating an angle. For example,

$$c_{\alpha_1 d}(l_1, l_2)$$

Defines the probability with which pixels with intensities I1 and I2, are d units apart at an angle alpha with x-axis. For images co-occurrence is calculated at fixed distance, for example $\mathrm{d} =$ one pixel and an arbitrary angle:

$$C_{GLCM} = \frac{1}{I_{\max}^2}\sum_{i=I_{\min}}^{I_{\max}}\sum_{j:I_{\text{mix}}}^{I_{\max}} c_d\,(i;j)\big(1+(i-j)\big)^2 - 1$$

Signal-to-noise Ratio (SNR)

Can influence the perceptibility of an object in an image and noise can have unwanted, image corrupting influence. Typically, noise is considered a random fluctuation of intensities with zero mean.

if n is noise, then $n \in N(0,\sigma^2)$ where σ^2 or σ Characterize the noise level.

We can define SNR as:

$$\mathrm{SNR}(f) = S(f)/\delta(f)$$

$S(f)$ can be: 1) $f_{\max}$ largest intensity 2) $E(f)$ average intensity

Image Enhancement Techniques

Linear Contrast Enhancement

$$g(f) = (f - f_{\min})\frac{I_{\max} - I_{\min}}{f_{\max} - f_{\min}} + I_{\min}$$

Where:

- $I_{\min}$ and $I_{\max}$

  max and min possible intensity values.

- $f_{\min}$ and $f_{\max}$

  image min & max intensity values.

- g is transformed version of f $g(f)$: Transfer function

Windowing

Windowing is an image contrast enhancement using an arbitrary intensity window range, $\mathrm{w}_{\min}$ and w_{max}. Accordingly:

$$I_{\min} < \omega_{\min} < \omega_{\max} < I_{\max}$$

Transfer function:

$$g(f) = \begin{cases} I_{\min} & \text{if } f < \omega_{\min} \\ (f - \omega_{\min})\dfrac{I_{\max} - I_{\min}}{\omega_{\max} - \omega_{\min}} + I_{\min} & if\, \omega_{min} \leq f \leq \omega_{max} \\ I_{\max} & \text{if } f > \omega_{\max} \end{cases}$$

Histogram Equalization

This is another approach to improve contrast. This approach modifies an image such that it's histogram approximates an uniform distribution. To achieve this task, we need to use "cumulative histogram" or cumulative distribution function (CDF) of intensity histogram which is defined as follows:

$$H(i) = \sum_{j=0}^{i} h\,(i) \quad 0 \leq i < K$$

or

$$H(i) = \begin{cases} h(0) & \text{for } i = 0 \\ H(i-1) + h(i) & \text{for } 0 < i < k \end{cases}$$

This is a monotonically increasing function with a max value:

$$H(k-1) = \sum_{j=0}^{K-1} h\,(j) = MN$$

Where $M \times N$ is the total number of pixels in the image. However, note that generating a true uniform distribution is not possible since we have discrete distribution. Accordingly, we get reduced individual peaks.

The goal is to make CDF approximately linear.

Given a histogram: $h(p)$ with $M \times N$ pixels such that intensity values $p \in \{p_0, p_k\}$

Then the aim would be to find a monotonic pixel brightness transformation $q = T(p)$ such that $g(q)$ (the histogram of new intensity transformed image) is uniform over intensity $q \in \{q_0, q_k\}$. Also the following relationship should hold:

$$\sum_{i=0}^{k} g\,(q_i) = \sum_{i=0}^{k} h\,(p_i)$$

due to the monotonicity of transformation.

In the continuous space, we will have:

$$\int_{q_0}^{q} g\,(s)ds = \int_{p_0}^{p} h\,(s)ds$$

$$g(s) = \frac{MN}{q_k - q_0} \text{ from the uniform distribution}$$

$$\int_{q_0}^{q} \frac{MN}{q_k - q_0} ds = \int_{p_0}^{p} h\,(s)ds$$

$$\frac{MN}{q_k - q_0} s\Big|_{q_0}^{q} = \int_{p_0}^{p} h\,(s)qs$$

$$\frac{MN(q - q_0)}{q_k - q_0} = \int_{p_0}^{p} h\,(s)ds$$

$$MN(q - q_0) = (q_k - q_0)\int_{p_0}^{p} h\,(s)ds$$

$$q - q_0 = \frac{q_k - q_0}{MN}\int_{p_0}^{p} h\,(s)ds$$

$$q = \left(\frac{q_k - q_0}{MN}\int_{p_0}^{p} h\,(s)ds\right) + q_0$$

$$q = T(p) = \frac{q_k - q_0}{MN}\sum_{i=p_0}^{p} h\,(i) + q_0$$

$$q_0 = 0$$

$$q_k = k - 1$$

$$T(p) = \lfloor\frac{K - 1}{MN} H(p)\rfloor$$

Where $H(P)$ is the CDF of our original image histogram.

Local Area Histogram Equalization

This process applies the concept of whole image histogram equalization to a small overlapping area. This is a nonlinear approach. Let’s define a normalized CDF in a 2D image as follows:

$$H_{\text{norm}}(j) = \frac{1}{MN}\sum_{i=0}^{j} h(i) \quad j = 0, \dots, K-1$$

$$T(p) = K - 1\big(H_{\text{norm}}\,(p)\big)$$

$$g(m,n) = K - 1 \cdot H_{\text{norm}}\left(\underbrace{f(m,n)}_{P \text{ at original image}}\right)$$

In this case $g(m,n)$ represents the new intensity value defined by the transfer function $T(p)$ at location m,n which was originally p. In other words $f(m,n)$ represents intensity value in the original image at location m,n and $g(m,n)$ is the transformed intensity at the same location. Now we can define local area (LA) histogram at position (m,n) as follows:

$$h_{LA}(m,n)(i) = \sum_{k=-k}^{K}\sum_{l=-L}^{L} \delta\,(f(m+l, n+k) - i)$$

Here $-k$ to k and $-L$ to L represent a small neighborhood around the pixel location at the position m,n. For example if $k = 3$ then we are looking at a 7×7 area defined by 3 pixels before and 3 pixels after the current pixel location. Accordingly we can define the local area CDF as:

$$H_{LA}(m,n)(j) = \frac{1}{(2k+1)\cdot(2L+1)}\sum_{i=0}^{i=j} h_{LA}\,(m,n)(i) \quad j = 0, \dots, P-1$$

Then, the intensity values in the new image is defined as:

$$g(m,n) = (P-1)\cdot H_{\text{LA norm}}(m,n)(fm,n)$$